Cyclostationary Feature Detection using Wavelet Transform

A Dissertation submitted towards the partial fulfilment of the requirement for the award of degree of

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in
Microwave and Optical Communication

Submitted by
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CERTIFICATE

This is to certify that the abstract title "**Cyclostationary Feature Detection using Wavelet Transform**" submitted by **ABHITI ABROL, Roll. No. 2K15/MOC/02**, in partial fulfilment for the award of degree of Master of Technology in "**Microwave and Optical Communication (MOC)",** run by Department of Electronics & Communication Engineering in Delhi Technological University.

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ABSTRACT

This work presents the statistic cyclostationary feature detection algorithm that uses the analytic signal via Wavelet Transform (W.T.) and compares the proposed method with the cyclostationary feature detection algorithm using analytic signal via Hilbert Transform (H.T.). The method has the advantage of providing flexible sampling rate and step size of cyclic frequency. The wavelet transform based method presents higher signal-to-noise ratio as compared to the Hilbert Transform method, as wavelet transform has time frequency localization characteristics.

DECLARATION

This work is being submitted for the degree of Master of Technology in Microwave and Optical Communication at the Delhi Technological University. The matter presented in this thesis reported has not been submitted by me in any other university/Institute for the award of Master of Technology degree.

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Chapter 1: INTRODUCTION

Objective:

This work provides a comparative study for cyclostationary Feature Detection using Hilbert Transform and Wavelet Transform. It proves that peak detection is better with wavelet transform as compared to Hilbert transform. The method has the advantage of providing flexible sampling rate and step size of cyclic frequency. The wavelet transform based method presents higher signal-to-noise ratio as compared to the Hilbert Transform method, as wavelet transform has time frequency localization characteristics.

Cyclostationary Feature Detection:

Spectrum sensing algorithms are used to detect the signal peaks and allows cognitive radio networks to use the bands that are vacant. Cyclostationary feature detection is commonly used for this purpose. Spectrum sensing is a key component in cognitive radio networks, which allows secondary users to communicate without causing harmful interference to primary users. Cyclostationary feature based spectrum sensing has proven preferable to other methods under low signal-to-noise ratio conditions. To detect the presence of primary signals, conventional cyclostationary feature based schemes tend to simply compare the values of signal features to a predefined threshold. [1] Many of the signal processing techniques used for analysing noise contaminated communications signal use probabilistic models based on stationary statistics. This limits the amount of data that can be used to derive the features in the signal and the resulting information. Most manmade signals that are typically encountered in communication are cyclostationary processes that exhibit underlying periodicities in their signal structures [2]. Cyclostationary processes are random process for which the second order statistics such as mean and autocorrelation change periodically with time. [3]

A cyclostationary process has statistical properties that vary periodically over time. Cyclostationary feature detection method deals with the inherent cyclostationary properties or features of the signal. Such features have a periodic statistics and spectral correlation that cannot be found in any interference signal or stationary noise. [4]

Cognitive Radio (CR) falls under wireless communication wherein a transceiver has the ability to detect which channels are in use and which are vacant. It can switch to the vacant channels and avoid the ones in use. This minimises the interference with the users. It is a technology that uses Software defined radios, hence it is known as hybrid technology and is applied to the spread spectrum. The, idea of cognitive radio is to utilize the available bandwidth more efficiently since spectrum occupancy measurements show that there are large temporal and spatial variations in the spectrum occupancy [5]-[6]. Cognitive radios have different functions such as spectrum sensing which employs the detection of signals presence in a desired frequency band. Different types of spectrum sensing techniques had been developed including energy detection, matched filter, and cyclostationary features extraction. [7]

In Cognitive Radio, it used by its users to sense the spectrum for adaptation and switch to the frequency that is available without disturbing other users. On the other hand, collaborative networks may provide CR networks with the method of cooperative spectrum sensing, which can boost its performance of detection. It is of great advantage as it helps to calculate features of the received signal such as frequency, power, sample rate etc. Therefore, different detection algorithm for the same are proposed.

Literature Survey

In paper [1] spectrum sensing scheme is used for energy detection. Detection performance of an energy detector used for cooperative spectrum sensing in a cognitive radio network is investigated in the paper over channels with both multipath fading and shadowing.

In paper [2] for year 2009, a survey of all the available spectrum sensing methodologies for cognitive radio is done. Various aspects of spectrum sensing problem are studied. It also introduces the multi-dimensional spectrum sensing concept. Challenges associated with spectrum sensing are described. Reviews of the spectrum sensing methods are performed. The paper also explains the cooperative spectrum sensing concept. External sensing algorithms are studied. In addition to these, statistical modelling of network traffic and utilization of these static models for the accurate prediction of the behaviour of primary user is done.

Paper [3] of 2011 examines how cognitive radio can be utilized in short range systems based on Ultra-Wideband (UWB). UWB is a technology in wireless communication used for high speed data transmission with low power utilization with applications in military, radar, sensor, tracking, data collection or even commercial application. UWB can move between very low data rate or very high data rate, also between short range and long-range distance applications. Impulse radio UWB shows International Journal of Computer Applications (0975 – 8887) Volume 47– No.21, June 2012 13 some characteristics in short-range communications with varieties of throughput options that include high data rates. The strong synergy between the aims of cognitive radio and features of IR-UWB has been shown in this paper.

Paper [4] explains that the detection performance is often compromised with multipath fading, shadowing and receiver uncertainty issues. To mitigate the impact of these issues, cooperative spectrum sensing has been shown to be an effective method to improve the detection performance by exploiting spatial diversity.

Paper [5] proposes light-weight cooperation in sensing based on hard decisions to reduce the sensitivity requirements of individual radios. Cognitive Radios have been advanced as a technology for the opportunistic usage of under-utilized spectrum since they are able to sense the spectrum and use frequency bands if and only if no primary user is detected. However, the required sensitivity is very demanding since any individual radio might face a deep fade

Paper [6] proposed a spectrum sensing method based on the correlation between primary user signal and noise. It has been shown that this method significantly outperforms conventional detector, especially in low SNR environment. Through the image denoising and machine learning, the characteristic points could be fully utilized. In addition, it was confirmed that CDM can be used for the signals with different modulation types.

In 2012 paper [7] they have proposed a technique for the classification of cyclostationary signals with compressed data length and studied the effect of reducing signals features. The results show low classification error in low SNR environments. For efficient performance of signals classifiers and features detectors in real time, limited number of features are required. In this paper we introduce a method to compress the cyclostationary features of digital signals using Discrete Wavelet Transform

In 2011 paper [8] the available spectrum is used, and cognitive radios are used to detect the vacant spectrum band reliably. They have simulated cyclostationary processing method for the detection of signal. The presence or absence of a signal are determined by comparing the value at the cyclic frequencies where the pilot carriers were detected, to a threshold value.

In paper [9] of 2017, they have implemented and demonstrated a cyclostationary feature based spectrum sensing scheme via LRS decomposition for CR networks. The cyclostationary features in the SCF of received signal are employed, and GoDec algorithm is applied to accomplish the decomposition of the SCF. In this way, noise and interference are completely excluded and the cyclostationary features of PU signal are extracted. Simulation results have proven that the proposed scheme outperforms conventional energy detector and cyclostationary detector. Furthermore, the proposed scheme has the ability for signal detection even at very low SNR, which validates its robustness.

Organisation of the Thesis

The organisation of the thesis is as follows:

Chapter 2 presents brief theory of the cyclostationary feature detection and statistic cyclostationary feature detection. Chapter 3 presents analytic signal using Hilbert Transform and Wavelet Transform. Chapter 4 presents the proposed method. Chapter 5 presents the simulation result. Chapter 6 provides the conclusions and future scope of the work.

Chapter 2: Cyclostationary Feature Detection

2.1 Cyclostationary Feature detection: Introduction

A cyclostationary process is the one in which a signal varies cyclically with time. Periodic sine waves are generated during modulation. A stationary stochastic process is followed by the data with some features to be periodic. Whereas the noise is wide sense stationary with no correlation. Hence a spectral correlation function is used to detect the noise from the modulated signal.

The received signal can be called cyclostationary if it satisfies the following conditions as stated in the paper [6]:

$$m_s(t) = E[s(t)] = m_s(t + T_0)$$

(2.1)

$$R_s(t,\tau) = E\{s\left(t + \frac{\tau}{2}\right) s^*\left(t - \frac{\tau}{2}\right)\}]$$
$$= R_s(t + T, \tau)$$

(2.2)

Where

T: period of the signal

E (.): Average of the signal

$R_s(t,\tau)$: Autocorrelation function

Taking the example of a signal s(t), where

$$s(t) = Ae^{i(2\pi f t + \theta)}$$

(2.3)

Where:

A is Amplitude

F is frequency

Θ is the phase

During the process some Additive white Gaussian noise (AWGN) is added to (2.3) the process, as a result of which the received signal is:

$$r(t) = Ae^{i(2\pi ft+\theta)} + n(t)$$

(2.4)

Where n(t) is noise.

The Fourier series of the autocorrelation function can be defined as follows as it is a periodic function:

$$R_S^{\alpha}(\tau) = \frac{1}{T_0}\int_{T_0} R_S(t,\tau)e^{-i2\pi\alpha\tau}\,d\tau$$

(2.5)

Where τ is cyclic frequency and $R_s(\tau)$ is called cyclic autocorrelation function (CAF).

On ignoring noise, cyclic autocorrelation function (CAF) in (2.5) can also be written as:

$$R_S^{\alpha}(\tau) = \frac{1}{T_0}\int_{T_0} Ae^{i\left[2\pi f\left(t+\frac{\tau}{2}\right)+\theta\right]}Ae^{-i\left[2\pi f\left(t-\frac{\tau}{2}\right)+\theta\right]}e^{-i2\pi\alpha\tau}d\tau$$

$$= \begin{Bmatrix} \frac{A^2}{T_o}, & \alpha = f \\ 0 & \end{Bmatrix}$$

(2.6)

Cyclic autocorrelation function when calculated for different cyclic frequency gives us the received frequency of the signal. Which in return helps us to calculate the received frequency of the signal. But the signal is mostly affected by noise, hence for this scenario to work out we can calculate the Fourier transform of CAF as:

$$S_S^{\alpha}(f) = \int_{-\infty}^{+\infty} R_S^{\alpha}(\tau)e^{-i2\pi f\tau}\,d\tau$$

(2.7)

This is called cyclostationary spectral density (CSD).

As for the stationary and stochastic signal like AWGN whose power will be zero. Whereas for cyclostationary signal CAF and CSD are going to be non-zero. So, signals can be distinguished from noise by the calculation of CSD and CAF of the signal.

DISADVANTAGES:

There are certain disadvantages of the above method stated. Which are as follows:

1.) If the signal that is being transmitted is not cyclostationary, the detection performance may not be satisfactory.

2.) It is a complicated process which requires a longer period to be calculated.

2.2 Statistic Cyclostationary Detection

In this algorithm instead of calculating the cyclic autocorrelation function, $e^{-2\pi\alpha t}$ is added to the received signal, which is in relation to the frequency of the received signal. Later we can calculate its average to get the desired result i.e. the received frequency.

Therefore,

$r(t) = n(t)$ in the absence of Licensed Users

$r(t) = s(t) + n(t)$ when both signal and noise are received by cognitive users

So, this factor is added to r(t) i.e. the received signal and the following expression is obtained:

$$x(\alpha, t) = r(t)\, e^{-i2\pi\alpha t} \tag{2.8}$$

On calculating the average of the above expression (2.8), we obtain:

$$M(\alpha, t_o) = \frac{1}{t_o} \left| \int_0^{t_o} x(\alpha, t) \right| = \left| \left| \int_0^{t_o} r(t)\, e^{-i2\pi\alpha t} \right| \right| \tag{2.9}$$

If we assume the received signal is complex without noise, then the result of detection can be given as:

$$\begin{aligned} M(\alpha, t_o) &= \left| \int_0^{t_o} s(t)\, e^{-i2\pi\alpha t} \right| / t_o \\ &= \left| \int_0^{t_o} A e^{i(2\pi f t + \theta)}\, e^{-i2\pi\alpha t} \right| / t_o \\ &= \left| A e^{i\theta} \int_0^{t_o} e^{i2\pi(f-\alpha)t}\, dt \right| / t_o \\ &= \left| A e^{i\theta} \int_0^{t_o} \delta(f - \alpha) \right| / t_o \end{aligned} \tag{2.10}$$

Hence $M(\alpha, t_o)$reaches its peak when the cyclic frequency is equal to the frequency of r(t). "In other words, Cognitive Users can detect the received signal for certain and proper detecting time, and set different cyclic frequencies in a band. According to the peaks of the result,
the probable spectrum of that signal can be yielded. According to paper [6]" This can be obtained irrespective of the fact how long the detecting time is.
So, on ignoring the peak value, which ideally is supposed to be infinite, then result can be simplified to variable that is related to cyclic frequency. After setting a certain and proper detecting time $t0$, detector can acquire the frequency of received signal by changing cyclic frequency to find the peak value.

After setting a certain and proper detecting time, detector can acquire.

Generally electromagnetic signal is in real form i.e. $Acos(2\pi ft + \varphi)$. Then the equations (2.10) would be as follows

$$M(\alpha, t_o) = \frac{1}{t_o}\left| \int_0^{t_o} x(\alpha, t) \right| = \left| \int_0^{t_o} r(t)\, e^{-i2\pi\alpha t} \right|$$

$$M(\alpha, t_o) = \left| \int_0^{t_o} Acos(2\pi ft + \varphi)\, e^{-i2\pi\alpha t} \right| / t_o$$

$$M(\alpha, t_o) = |A| \left| e^{-i\varphi}\, \psi_1(f, \alpha) + e^{i\varphi}\, \psi_2(f, \alpha) \right| / 2t_o$$

$$\psi_1(f, \alpha) = \int_0^{t_o} e^{-i2\pi(f+\alpha)t}$$

$$\psi_2(f, \alpha) = \int_0^{t_o} e^{-i2\pi(f-\alpha)t}$$

(2.11)

Where

Both $\psi_1(f, \alpha)$ and $\psi_2(f, \alpha)$ are frequency related functions.

CUs can detect the received signal for certain and proper detecting time and frequencies. Hence the probable spectrum of the signal can be yielded.

2.3 Implementation of Statistic Cyclostationary Algorithm

We used signal $s(t) = Ae^{i(2\pi ft+\theta)}$ which has been affected by AWGN and resulted into $r(t) = Ae^{i(2\pi ft+\theta)} + n(t)$. This signal is then digitized into r(m) and is converted into a digital signal. This undergoes a Hilbert transform which generates an imaginary counterpart. This imaginary part and the real signal the added to generate an entire signal. The average for this signal is then calculated and its peak values are identified wherein its peak corresponds to that of the frequency of the given signal.

Final result for detection for digital signals as given in the paper [10] can be written as:

$$M(\alpha) = \frac{\sum_{k=1}^{N} | \sum_{m=1}^{No} \{r(m) + jH[r(m)]\} \; \mathrm{e}^{-\mathrm{i}2\pi\alpha\mathrm{m}} |}{\mathrm{NxNo}}$$

(2.12)

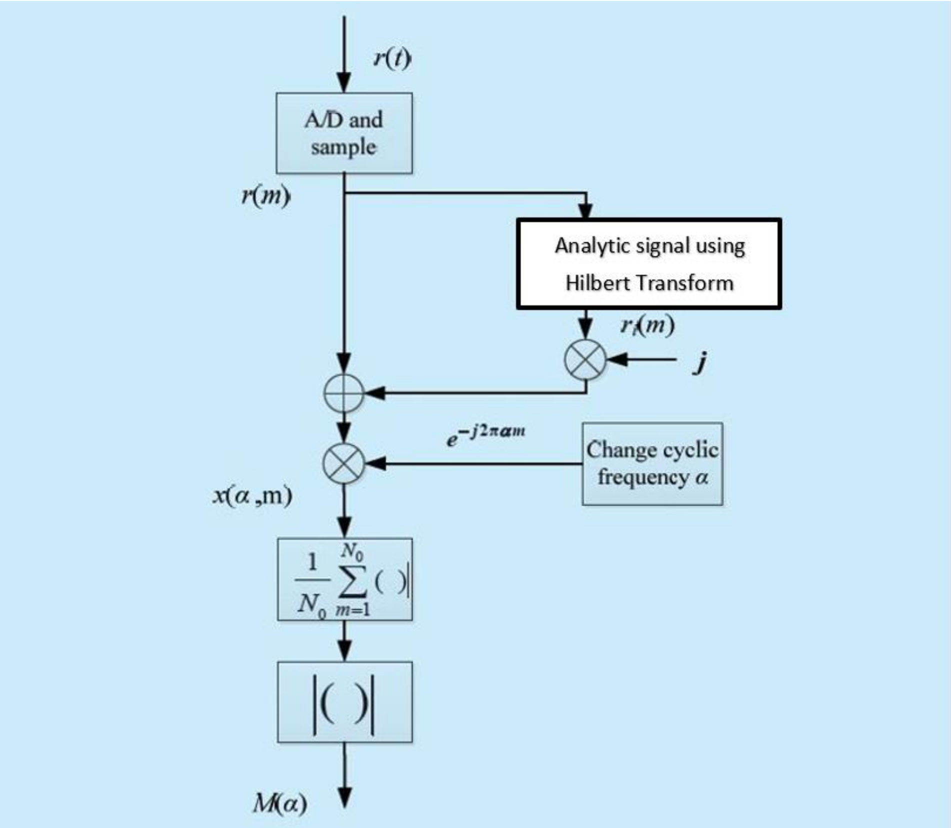
Where:

r(m); received signal

$M(\alpha)$: Detecting result

H(r(m)): Hilbert Transform of r(m)

The implementation of the above process is shown in the following flow chart:



2.4 Implementation of Statistic Cyclostationary Algorithm using Wavelet Transform

The above process can be implemented by replacing Hilbert transform with Wavelet Transform. In the equation used for detection, replacing the imaginary part by wavelet transform yields better results as compared to this approach.

In terms of Low SNR, we can yield better results with the new approach and can detect the signal from a noisy signal.

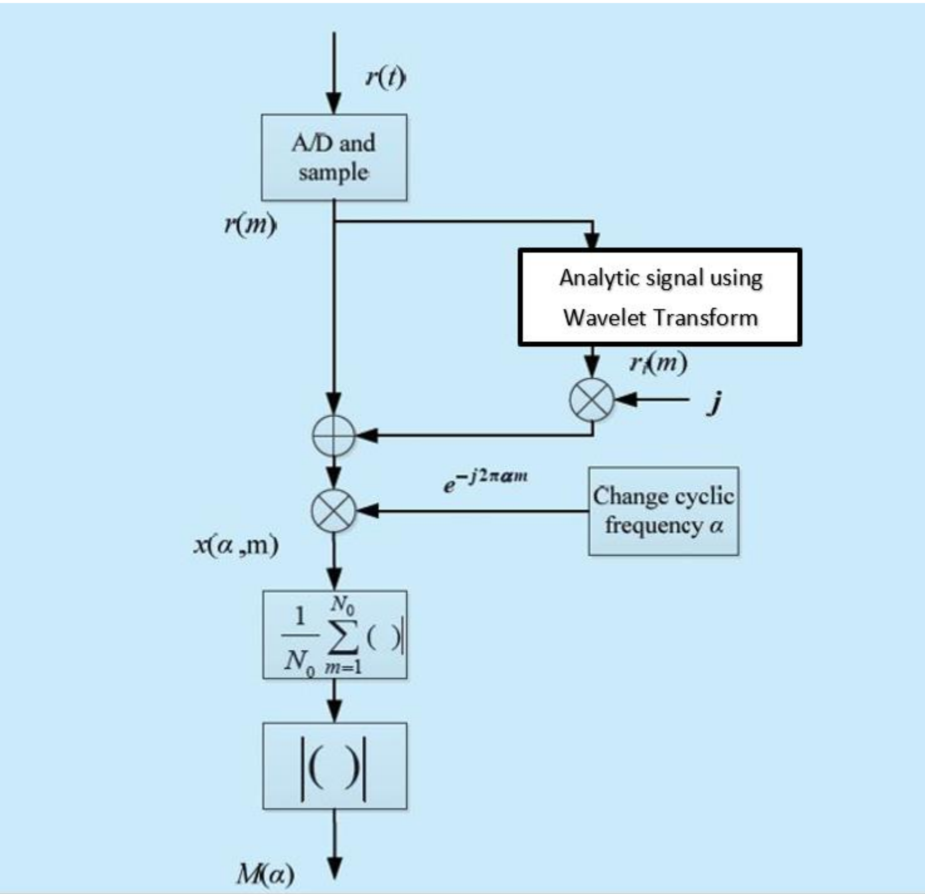
Wavelet Transform of r(m) is:

$$R(b) = \frac{1}{a}\int_{-\infty}^{+\infty} r(t)\bar{g}\left(\frac{t-b}{a}\right) dt$$

$$M(\alpha) = \frac{\sum_{k=1}^{N} | \sum_{m=1}^{No}\{\mathrm{R(b)x}\, e^{-j2\pi\alpha m}\}X\ \mathrm{e}^{-i2\pi\alpha m}|}{\mathrm{NxNo}} \quad (2.13)$$

Where:

r(m): received signal
$M(\alpha)$: Detecting result
R(b): Wavelet Transform



Chapter 3: Analytic Signal

The analytic signal can be generated using The Hilbert transform. Representation of the analytic signal is a function in the complex form. It is formed by the addition of quadrature of Hilbert transform to the original signal [11].

$$x(t) = s(t) + js^{t} = \mathrm{A(t)}e^{j\theta(t)} \tag{3.1}$$

Where

s(t) is the input signal
s^(t) is its Hilbert Transform
x(t) is the analytic signal.

x(t) is also known as pre-envelope of s(t).

In polar form the analytic signal can be expressed as:

$$x(t) = \mathrm{A(t)}e^{j\theta(t)} \tag{3.2}$$

A(t) is the envelope of x(t) and can be defined as follows:

$$\mathrm{A(t)} = \sqrt{\mathrm{s}^2\mathrm{(t)} + s^{\square 2}(t)} \tag{3.3}$$

Whereas, $\theta(t)$ is the phase of x(t)

$$\theta(t) = \arctan\left(\frac{\mathrm{s}^{(t)}}{s(t)}\right) \tag{3.4}$$

3.1 Hilbert Transform:

In 1905 while working on an analytic function that was posed by Riemann and was later known as Riemann Hilbert problem. This gave rise to the Hilbert Transform. In 1928 it was proved that Hilbert transform can be defined for u in $L^p(R)$ for $1<p<\infty$. The Euler formula was derived by a famous mathematician that is:

$$e^{j\omega t} = \cos\omega t + \mathrm{j}\sin\omega t \tag{3.5}$$

Definition:

Hilbert Transform is a signal processing tool and is used for deriving the analytic representation of a signal. It is a linear operator that takes in a periodic function and produces a real valued function.

The Hilbert Transform of a function x(t) is as follows:

$$x\hat{}(t) = H[x(t)] = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}d\tau \tag{3.6}$$

Thus, The Hilbert Transform of the signal is also a time dependant signal. Due to the singularity at, the Hilbert transform is not calculated like the integral function, rather the above integral is considered as the Cauchy principal value of the integral function [25] having the pole at τ = t.

Properties of Hilbert Transform:

Let

$$y(t) = H\big(x(t)\big)$$

$$y_1(t) = H\big(x_1(t)\big)$$

$$y_2(t) = H\big(x_2(t)\big)$$

and a, b, c be some arbitrary constants.

Then the Hilbert transform satisfies the following basic properties:

(i) Linearity: H.T satisfies the linearity property which states the Cauchy principal value [25]. It states that if any arbitrary constant is added or multiplied in the signal, it is reflected in the H.T.

$$H(ax_1(t) + bx_2(t)) = aH\big(x_1(t)\big) + bH\big(x_2(t)\big)$$

(ii) Time shift: According to this property any shift in time by any arbitrary value is reflected with the same shift in the H.T. as well

$$H\big(x(t-a)\big) = y(t-a)$$

(iii) Scaling: If the signal's time period is decreased or increased by a constant, it's H.T is also shifted by same amount.

$$H(x(a.t)) = y(a.t)$$

(iv) Time reversal: If the signal is reversed in time by some constant, it's Hilbert transform is also reversed in the axis.

$$H\big(x(-a.t)\big) = -y(-a.t), a > 0$$

(v) Derivative:

$$H(x_0(t)) = \big(y_0(t)\big)$$

3.2 Wavelet Transform:

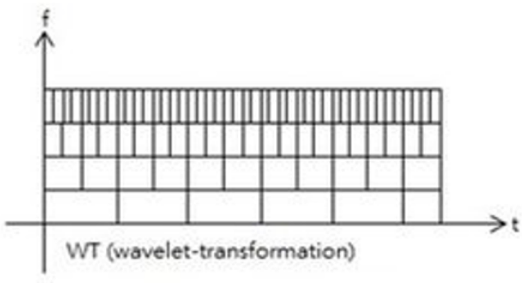
A wavelet is like an oscillation, whose amplitude begins at zero, and increases like a wave and then decreases back to zero. Wavelets are useful for signal processing with specific properties. It is basically used to extract information from the signal by convolving the wavelets with the signal. If the unknown signal contains information of similar frequency, the wavelet correlates with it. This is the core of the application. The meaning of the term wavelet is small wave [13]. Their energy is concentrated around a point in time. Over the past century, wavelets have been developed and worked upon in many different fields, independently. The first system was developed by Haar in 1910, which was an alternative to the Fourier transform (developed by Fourier, in 1807) [13]. The next such system was created by Ricker [13], which models the seismic waves as they travel through the Earth.

Scaling function: Mother wavelet and scaling function in the time domain are the ones that define wavelets. Effectively the wavelet function i.e. the mother wavelet is a band-pass filter and the scaling function does the work of scaling it for each level which halves its bandwidth. It ensures that the complete spectrum covered. A window function of a small duration is applied to the signal and its Fourier transform is taken. This process was repeated at various different parts of the signal. One of the limitations of Fourier transform method is that the length of the window is constant. The wavelets help in recovering weak signals from noise, which proves helpful in signal processing.

PRINCIPLE

The principle of changes only in the time extension but not shape is the fundamental idea behind the wavelet transforms, a suitable basis function is chosen for this purpose. According to the uncertainty principle in the signal processing, where 't' represents time and 'w' represents the angular frequency with w= 2πf, and f is the Tenable frequency. Hence the higher is the time resolution, frequency resolution has to be lower.

The signal received after transformation provides information about the time and frequency. Wavelet transformation has similarity to the short-time-fourier-transform, but with a few special properties[25]. The difference for both the transforms is depicted in the below figure which was taken from the Wikipedia page as:



From this figure it can be concluded that wavelet transformation is better in time resolution of high frequencies, whereas the frequency resolution is remarkable for slowly varying functions.

This transform gives the time and frequency representation of the signal simultaneously, giving time- frequency representation of the signal. Wavelet transform is similar to Fourier Transform or windowed Fourier Transform having a different function [25]. They differ on the basis of decomposition, Fourier Transform decomposes the signal into sine and cosines, basically in Fourier space, whereas the wavelet transform uses both the real and Fourier space.

For ψ to be wavelet function or mother wavelet, following are the necessary conditions.

(i) The first condition being the wavelet function must have zero mean.
It should be oscillatory, hence the word 'wavelet'. The integral over time of this function must be zero. Mathematically [28],

$$\int \psi(t)dt = 0$$

It ensures that wavelet has zero dc components, or in other words, any excursions the wavelet function makes above zero must be cancelled out by the excursions below zero.

(ii) The second condition is that the wavelet should be of finite energy. The function should lead towards zero with time.

$$\int_{-\infty}^{+\infty} \psi^2(t) < \infty$$

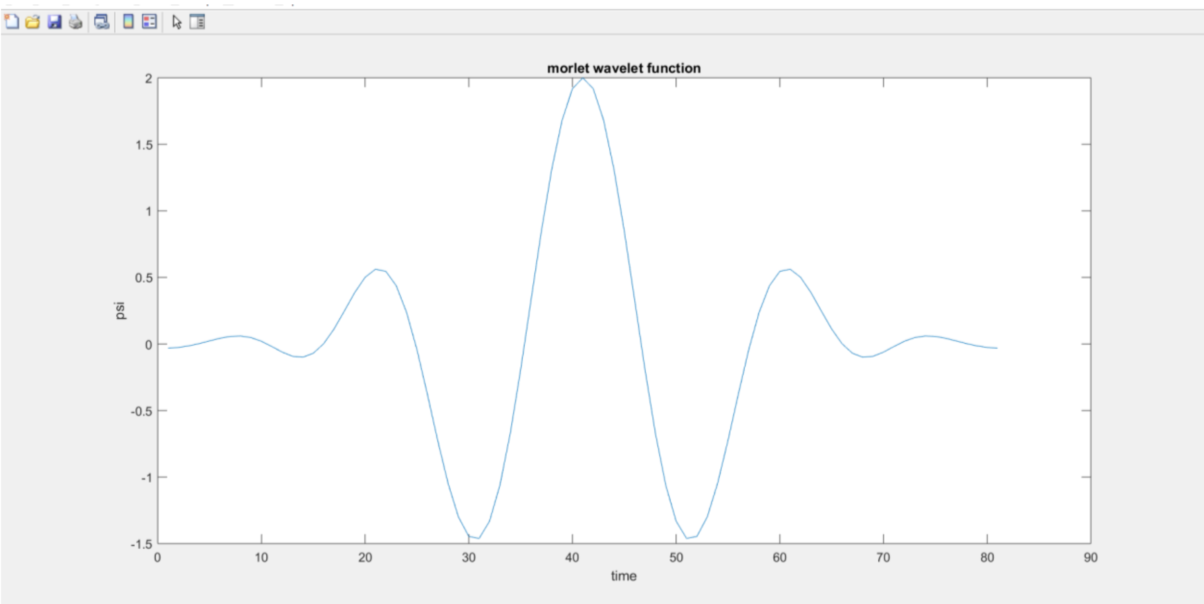
The energy of the wavelet function is usually equal to one. Sinusoidal functions such as cosine and sine do not have finite energy, hence they cannot be used as wavelet functions since they violate the second condition. This is an implicit requirement that, while the wavelet function has finite energy, it must have some energy, so the integral must be greater than zero.

(iii) The third requirement is known as the admissibility condition, which states that the Fourier transform of the wavelet function cannot have zero frequency component. One of the wavelets satisfying above conditions is the Morlet wavelet. It is a real valued wavelet function that has a small but greater than zero value for the zero-frequency value of its Fourier transform

(iv) Another condition is that the wavelet function should have effective support. While the wavelet functions for the continuous wavelet transform are usually mathematical functions that extend to infinity, effective support means that the wavelet functions are effectively zero outside a certain range.

3.2.1 Morlet Wavelet

Morlet or Gabor Wavelet is composed of a carrier which is a complex exponential and a Gaussian window.



The morlet wavelet used for the analysis is as follows:

$$g(t) = e^{i\omega t}\, e^{-0.5(t)^2} \tag{3.7}$$

The modified Morlet wavelet (Morlet wavelet modified for seismic data processing) is

$$g(t) = e^{i\omega t}\, e^{-0.5[(\sqrt{2}\sigma m/2\pi\tau)t]^2} \tag{3.8}$$

$$g(t) = \cos(mt)\, e^{-0.5\left[\left(\frac{\sqrt{2}\sigma m}{2\pi\tau}\right)t\right]^2} + j\sin(mt)\, e^{-0.5\left[\left(\frac{\sqrt{2}\sigma m}{2\pi\tau}\right)t\right]^2} \tag{3.9}$$

Let
m is the angular frequency;
τ the number of cycles of the carrier wave in an envelope;
And σ is a real number related to precision.

ADVANTAGES

With every method there are a few pros and cons which come with it, similarly advantages of using the wavelet transform are as follows:

- Windows of Wavelet transform vary which make this method customisable according to the use.

- Basis function requirement vary for different signals, to remove discontinuities one may require some very short basis functions, whereas to obtain details of the frequency analysis, long basis functions are needed. Wavelet Transform solves this purpose easily.

- Wavelet Transform provides immediate access to information which otherwise can be infected by other methods such as time-frequency Fourier analysis.

DISADVANTAGES

The main drawbacks of Wavelet Transform are:

- It is computationally complex for fine analysis.
- The discrete wavelet transform is less efficient and discrete.

It takes a lot of precision to choose the correct wavelet

3.3 Analytic Signal using Wavelet Transform

In a real valued signal, (implying finite energy) the analytic signal can be calculated using the wavelet transform based method. It has been stated in [27]-[28] that with the help of continuous wavelet transform, the Gibbs phenomenon never exceeds the Gibbs phenomenon in Fourier transform; hence there are no jump discontinuities. Thus [29] proposed a method of computation of the analytic signal using wavelet transform. Because of this, the discontinuity that occurs, when the analytic signal is formed by the Hilbert transform, is reduced greatly and there was no Gibbs effect. The analytic signal generated using wavelet transform has better performance in terms of noise reduction as compared to the analytic signal using Hilbert transform.

If g(t) refers to the analytic wavelet function

and $\hat{g}(\omega)$ is the Fourier transform of g(t)

$$g(t) \epsilon\, L^1(R, dt) \cap\, L^2(R, dt)$$

$$\hat{g}(\omega)\, \epsilon\, L^1(R\{0\}, d\omega/|\omega|) \cap\, L^2(R\{0\}, d\omega/|\omega|)$$

(3.10)

Wavelet Transform of s(t) (the given signal) with respect to the wavelet function g(t)

$$S(b.a) = \frac{1}{a}\int_{-\infty}^{+\infty} s(t)\bar{g}\left(\frac{t-b}{a}\right) dt$$

(4.2)

In the above expression, t and b are the part of the real-number set. It is assumed that a>0,

$\bar{g}$(t) is the complex conjugate of wavelet function.

According to the two theorems in [29]:

Theorem 1: If the wavelet function g(t) is analytic, $\hat{g}(\omega)$ is its Fourier transform and s(t) is taken as a signal with finite value energy, S (b, a) is the wavelet transform of s(t). Then with respect to the scale (a>0) and the shift b, S(b, a) will be a complex function. For a constant value of a, the Hilbert transform of the original signal will be the complex function's imaginary part [34].

Theorem 2: If the wavelet function g(t) is analytic and it has even real part g_R(t) and

$$C_g = \int_0^{+\infty} (\, \hat{g}(\omega)\, /\omega\; d\omega) \qquad (4.3)$$

with 0<Cg<∞ .

Then for signal s(t) ε L^2(R, dt)

$$\frac{1}{Cg}\int_0^\infty \frac{S(t,a)da}{a} = s(t) + jH[s(t)]$$

(4.4)

Where, S (t, a) is taken from equation (3.22) and H[s(t)] is the Hilbert transform of s(t)

Proof:

$$S(t,a) = \frac{1}{a}\int_{-\infty}^{+\infty} s(b)\bar{g}\left(\frac{b-t}{a}\right)db$$

$$S(t,a) = \frac{1}{a}\int_{-\infty}^{+\infty} s(b)[gr\left(\frac{b-t}{a}\right) - jgI\left(\frac{b-t}{a}\right)]db$$

$$= \frac{1}{a}\int_{-\infty}^{+\infty} s(b)[gr\left(\frac{b-t}{a}\right)db \; - j\,\frac{1}{a}\int_{-\infty}^{+\infty} s(b)\;gI\left(\frac{b-t}{a}\right)db$$

$$= \; S_R(t,a) + jS_I(t,a)$$

Where

$$g_R(t) = Re(g(t))$$

$$g_I(t) = Im\big(g(t)\big)$$

$$S_R(t,a) = \frac{1}{a}\int_{-\infty}^{+\infty} s(b)g_R\left(\frac{b-t}{a}\right)db$$

$$S_I(t,a) = \frac{-1}{a}\int_{-\infty}^{+\infty} s(b)g_I\left(\frac{b-t}{a}\right)db$$

As per Theorem 1;

$$S(t,a) = S_R(t,a) + j\,S_I(t,a)$$

$$= S_R(t,a) + j\,H[S_R(t,a)]$$

$$= \frac{1}{a}\int_{-\infty}^{+\infty} s(b)g_R\left(\frac{b-t}{a}\right)db + jH[\frac{1}{a}\int_{-\infty}^{+\infty} s(b)g_R\left(\frac{b-t}{a}\right)db]$$

If we multiply both sides of equation (3.25) by a factor of $\frac{1}{C_g a}$ and then integrating it with

respect to a, we get:

$$\frac{1}{Cg}\int_0^\infty \frac{S_R(t,a)da}{a}$$

$$= \frac{1}{Cg}\int_0^{\infty}\frac{1}{a^2}\int_{-\infty}^{+\infty}s(b)g_R\left(\frac{b-t}{a}\right)db\,da$$

$$= s(t)$$

Morlet Wavelet:

Morlet wavelet function:

$$g(t) = e^{i\omega t}\, e^{-0.5(t)^2}$$

(4.5)

Modified Morlet wavelet function [31] :

$$g(t) = e^{i\omega t}\, e^{-0.5[(\sqrt{2}\sigma m/2\pi\tau)t]^2}$$

(4.6)

Let C=$\left(\sqrt{2}\sigma m/2\pi\tau\right)$.

In the above equation ‘m’ represents angular frequency, the number of carrier wave cycles is represented by the symbol ‘ʊ’, ‘σ’ signifies the real number related to precision. From the point of view of numerical calculation, in the case where m2 = (4C2) is large enough then the wavelet defined in the equation (3.30) satisfies Theorem 2. Equation (3.30) wavelet function is taken as the reference for all the examples in this thesis.

Compact support is not provided to the wavelets in (3.29) and (3.30) but the decay rate of their amplitude is and when it is away from its center, hence the wavelet transform can provide better and accurate results compared to the Hilbert transform. The decay rate of the filter factor of Hilbert transform is 1/t.

Instantaneous parameters are defined as:

$$e(t) = \sqrt{s^2(t) + H^2[s(t)]}$$

$$\theta(t) = \arctan\left(\frac{H[s(t)]}{s(t)}\right)$$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\arctan\left(\frac{H[s(t)]}{s(t)}\right)\right]$$

(4.7)

Where e(t) is the instantaneous amplitude, $\theta(t)$ is the instantaneous phase angle, f(t) is the instantaneous frequency of the signal s(t) . H denotes the Hilbert transform.

3.3 Wavelet Transform vs Hilbert Transform

Dissimilarities Between Hilbert And Wavelet Transforms

The dissimilarities between these two kinds of transforms are as followed:

- The individual wavelet functions are localized in space whereas, sine and cosine functions are not localized.
- The property of localization of wavelets makes them "sparse" when transformed into the wavelet domain. This can be used in certain applications such as data compression, removing noise etc.
- In Hilbert Transform, the negative frequencies are not computed, hence the Hilbert Transform of the windowed signals are not taken. Only a single peak is seen.
- In wavelet transform, the width of the window can be changed foe every single spectral component unlike Hilbert transform.

3.3.1 Applications

Wavelet Transform Applications in Signal Processing Noise Reduction Detection of trends and discontinuities in higher derivatives Compression (JPEG2000, FBI fingerprints database) Image edge detection Watermarking Features extraction for image segmentation

Applications of Wavelet Transform are:

- Computer and human vision: In 1980s, David Marr, an expert on the human visual system started his work on artificial vision for robots with his goal to know as to why the first attempt to construct a robot capable of understanding was unsuccessful.
- Federal Bureau of Investigation(FBI) Fingerprint Compression: Between 1924 and today, the US Federal Bureau of Investigation has collected about 30 million sets of fingerprints. A number of jurisdictions are experimenting with digital storage of the prints, incompatibilities between data formats have recently become a problem.
- Denoising noisy Data: In diverse fields from planetary science faced with the problem of recovering a true signal from incomplete, indirect or noisy data.

Chapter 4: Proposed Method

We used the WT based analytic signal method for cyclostationary feature detection as explained in section 2.4 and compared the proposed method with the HT based cyclostationary feature detection.

So, the analytic signal is generated using wavelet transform with gives better result as compared to the Hilbert Transform due to the time frequency localization property. This is also explained using a few applications of the wavelet transform wherein it replaced the Hilbert Transform.

The received signal is made by the combination of two sine waves with different frequencies, that are 150kHz and 400 kHz, with different frequencies each.
The conventional approach is used [3] and the new approach using morlet wavelet are used and the results are compared.

For the conventional approach the formula used was as followed:

$$M(\alpha) = \frac{\sum_{k=1}^{N} | \sum_{m=1}^{No} \{r(m) + jH[r(m)]\} X \ \mathrm{e}^{-\mathrm{i}2\pi\alpha\mathrm{m}} |}{\mathrm{NxNo}}$$

Where:

R(m); received signal

Whereas the Hilbert Transform is replaced the wavelet transform in the above formula and observed for better results are obtained. The peak to average ratio is also calculated.

PRINCIPLE

Following formula is used to calculate using wavelet transform:

Wavelet Transform of r(m) is:

$$R(b) = \frac{1}{a} \int_{-\infty}^{+\infty} r(t) \bar{g}\left(\frac{t-b}{a}\right) dt$$

$$M(\alpha) = \frac{\sum_{k=1}^{N} | \sum_{m=1}^{No} \{\mathrm{R(b)x}\, e^{-j2\pi\alpha\mathrm{m}}\} X \ \mathrm{e}^{-\mathrm{i}2\pi\alpha\mathrm{m}} |}{\mathrm{NxNo}}$$

The signal that is received is:

$$s(t) = Ae^{i(2\pi ft + \theta)}$$

After adding noise to the signal, we receive:

$r(m) = \sin(2\pi f_1 t) + \sin\,(2\pi f_2 t)$

Where

f1= 150kHz

f2=400kHz

The wavelet used for the analysis of the signal is given in (2)

$$g(t) = \cos(mt)\, e^{-0.5\left[\left(\frac{\sqrt{2}\sigma m}{2\pi\tau}\right)t\right]^2} + j\sin(mt)\, e^{-0.5\left[\left(\frac{\sqrt{2}\sigma m}{2\pi\tau}\right)t\right]^2}$$

Wavelet Transform is given as:

$$S(b.a) = \frac{1}{a}\int_{-\infty}^{+\infty} s(t)\bar{g}\left(\frac{t-b}{a}\right) dt$$

Wavelet Transform of r(m) is:

$$R(b) = \frac{1}{a}\int_{-\infty}^{+\infty} r(t)\bar{g}\left(\frac{t-b}{a}\right) dt$$

Using the following formula, we obtain results as shown in the following chapter.

$$M(\alpha) \;=\; \frac{\sum_{k=1}^{N} \left| \sum_{m=1}^{No} \mathrm{R(b)x}\, e^{-j2\pi\alpha m} \} X\ \mathrm{e}^{-i2\pi\alpha m} \right|}{\mathrm{NxNo}}$$

Calculating Peak to Average Ratio is as follows:

Peak-to-Average Ratio = $\frac{|x_{peak}^2|}{x_{rms}^2}$

CHAPTER 5: SIMULATION RESULTS

The result of simulation with spectrum resolution ratio is compared with Hilbert Transform and Wavelet Transform

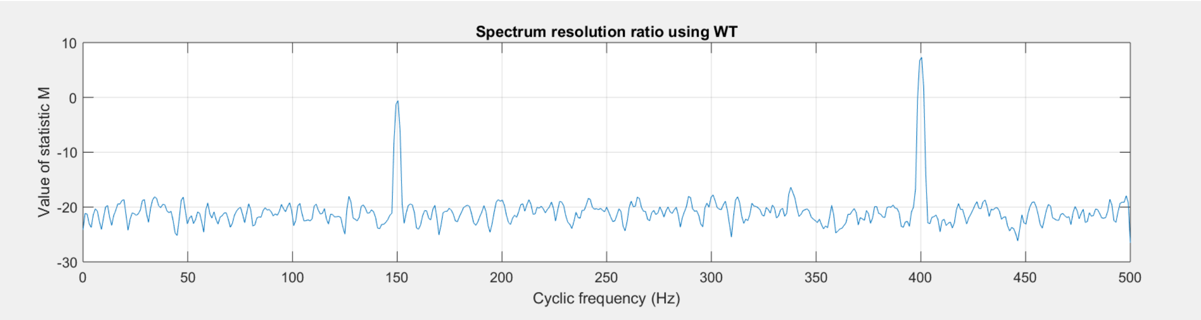


Fig (1a): Value of Value of statistic M (α) using Hilbert Transform based method.

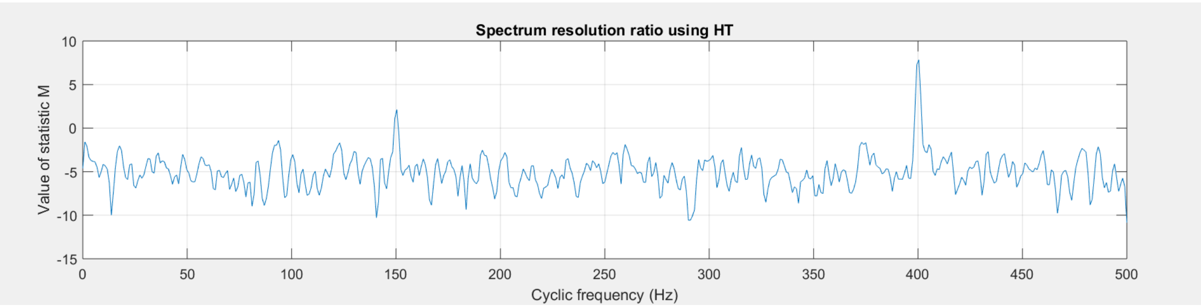


Fig (1b): Value of Value of statistic M (α) using Wavelet Transform based method.

Peak to average ratio

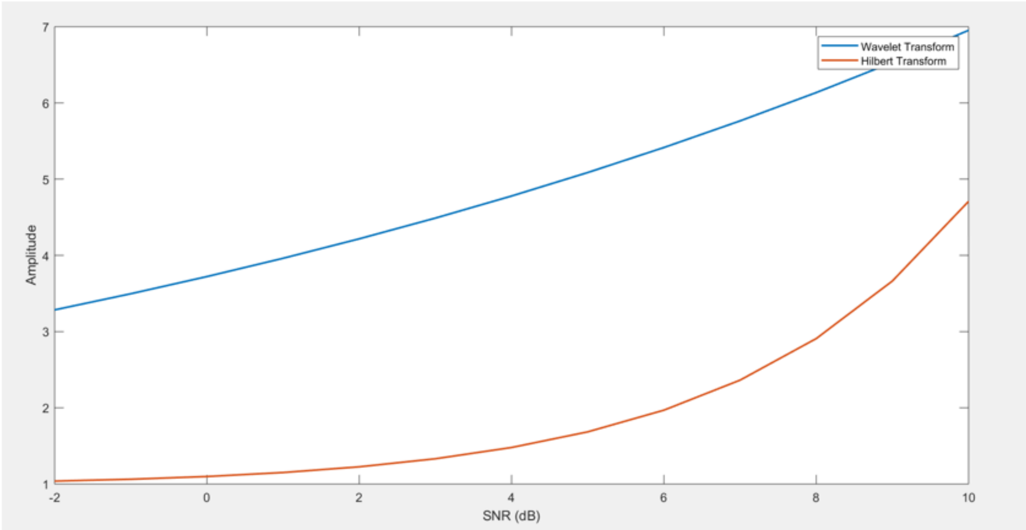


Fig (2): Peak to average ratio using WT based method and HT based method.

	Peak-to-Average Ratio
Hilbert Transform	0.0958
Wavelet Transform	0.1281

Table 1: Peak-to-Average Ratio

We implemented the proposed method in MATLAB software. Fig (1) shows the plot of M (α) using WT based method and HT based method. Table1, shows the peak to average ratio for WT and HT based method. Larger peak to average ratio is obtained due to time frequency localization property of the WT.

Chapter 6

Conclusions and Future Scope

Conclusion:

We have described a method that detects statistic cyclostationary feature detection via Wavelet Transform (WT) and compares the proposed method with the cyclostationary feature detection algorithm using analytic signal via Hilbert Transform (HT).
It provides flexible sampling rate and step size of cyclic frequency. The wavelet transform based method presents higher signal-to-noise ratio as compared to the Hilbert Transform method, as wavelet transform has time frequency localization characteristics.

Future Scope:

Noise Free signals can be extracted using this method wherein wavelet transform is used, and their features can be studied easily.

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