CONTROL OF TWO DEGREE OF FREEDOM BALL PLATE BALANCER

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE OF

MASTER OF TECHNOLOGY

IN

CONTROL AND INSTRUMENTATION

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DECLARATION

I, AVANTIKA, Roll No.2K16/C&I/05 of M.Tech. (Control and Instrumentation), hereby declare that the project dissertation entitled "Control of Two Degree of Freedom Ball Plate Balancer" is submitted to the Department of Electrical Engineering, Delhi Technological University, Delhi, India in partial fulfilment of the requirement for the award of the degree of Master of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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CERTIFICATE

I hereby certify that the Project Dissertation titled "control of two degree of freedom ball and plate balancer'' by avantika, Roll No. 2K16/C&I/05, Department of electrical engineering, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the student under my supervision. To the best of my knowledge this work has not been submitted or full for any Degree or Diploma to this University or elsewhere.

APPROVAL FOR SUBMISSION

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ABSTRACT

Ball and plate balancer is one of the nonlinear and unstable electromechanical system. Balancing of nonlinear system is a challenge to the control engineers and researcher. Ball plate control system is one of the benchmark problem in control engineering. There are several controllers such as fuzzy controller, PID controller robust LQR controller and which have been documented in the literature and have been applied to stabilize the ball and plate system. Controllers design are one of the important steps during the modelling of the systems. To achieve a good controller, it is needed to define suitable gain values for the controller coefficients. This thesis studied the performance of few of the control strategies that consist of conventional controller, modern controller and intelligent controller for ball and plate system with a comparison among these controllers. LQR being a modern controller is a full state feedback controller. The purpose of using LQR algorithm is to reduce the calculation burden of the system. This thesis describes the mathematical modelling and transfer function of the proposed system. The model of the system is also linearized in order to be used with the linear controllers. The works followed by the implementation of the controllers in MATLAB/SIMULINK. Fuzzy logic controller is one of new and intelligent controller which evolves with learning mechanism combination of Fuzzy and PID has also been simulated for our control system in MATLAB/SIMULINK. Each controller performance is analysed and compared which is based on common input criteria of step response.

ACKNOWLEDGEMENT

I would like to express my gratitude towards all the people who have contributed their precious time and effort to help me without whom it would not have been possible for me to understand and complete the project. I would like to thank Dr. S.K. VULLERU, Department of Electrical Engineering, my Project guide, support, motivation and encouragement throughout the period this work was carried out. His readiness for consultation at all times, his educative comments, his concern and assistance even with practical things have been invaluable.

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CHAPTER:1 INTRODUCTION

Balancing systems are one of the most challenging problems in the area of control system. Among those, many control systems like cart-pole system, ball-beam system, double and multiple inverted pendulum are reported in the literature. In the real world, it is observed that most of the physical systems display nonlinear dynamic characteristics which require a complex nonlinear mathematical analysis to investigate their performances. The ball balancing system in a 2-degree of freedom is a unique platform to test and identify different aspects of control, as the non-linearities increase with the increase in degree of freedom. There are two degrees of freedom for the ball movement on the plate, one is on x axis and another is on y axis. The ball-on-plate system is a promoted version of the traditional ball-on-beam control problem. It can be modelled as decoupled ball and beam systems where we assume x axis servo motor which only affects the ball's position in x direction. Similarly, y axis servo motor affects ball's position in y direction only. The ball on plate balancing system is a nonlinear and unstable system. It consists of a plate for which its deviation can be manipulated in two perpendicular directions x-axis and y-axis. The initial objective of the project is to maintain a static ball position on the plate, rejecting position disturbances. If this is extended further, trajectories such as a circle can be followed given judicious choices for ball position requests. To control the ball on plate system, a servo control system is required. The job of servo motor here is to maintain a specific angle. This thesis is organized in seven sections as follows: chapter II contains the literary review of the work done in field of control related to ball and plate system and other nonlinear system chapter III briefly describes the mathematical model for the ball and plate system and transfer function deduction. It is difficult and time-consuming to obtain a model of complex system. Simplified model is obtained using linearization which is required for linear controllers. Chapter IV deals with the design implementation of different controller's viz. combination of PD and P controller combination of fuzzy and PID controller, robust LQR ball plate balancing system followed by the result and discussions in chapter V. Finally, chapter VI conclusion and chapter VII are the references

OBJECTIVE

The objective of thesis is a comprehensive mathematical modelling of ball and plate balancing system and design of different controllers such as PD, robust LQR and fuzzy-PID for positioning control of ball on the plate. A linearized model of ball and plate balancing system is derived to implement linear controllers. Controller and dynamic performance of ball position is determined through using MATLAB/Simulink. Stability of open loop ball and plate system is discussed using simulation response of open loop system without controller and bode plot, root locus and Nyquist plot are plotted to discuss the open loop behaviour of ball plate system. We assume the angle of the x-axis servo only affects the ball movement in the x direction. Similarly, for the y ball motion. Therefore, in the thesis we derive mathematical modelling of two degree of freedom Ball plate balancer de-coupled, into one dimension. The design of control is implemented for one-dimension ball plate balancer system.

Fig :1 General structure of ball and plate balancer

The two DOF Ball plate Balancer, i.e. 2DBB, pictured in Figure 1.1 consists of a plate on which a ball can be placed and is free to move. By mounting the plate on a two degree of freedom (two DOF) gimbal, the plate is allowed swivel about any direction. The overhead USB camera is used with a vision system to measure the position of the ball. The two servos that are underneath the plate are rotary Servo Base Unit devices. Each of them is connected to a side of the plate, also using two DOF gimbals. By controlling the position of the servo load gears, the tilt angle of the plate can be adjusted to balance the ball to desired position.

CHAPTER:2 LITERARY REVIEW

As a product, the ball-on-plate system presents little in the way of a commercially viable product, except perhaps as an exercise in control systems education. In that pursuit, however, there are still several prior projects that tread the same path as this system. While investigating other projects, and additional search was made for designs or technology related to this system. This, however, yielded no systems related or similar in operation to this project in the listings of protected research and designs.

Research work published by David Debono and Marvin Begena in year 2007, [1] This paper investigates two control regimes linear full state feedback controller and sliding mode control to the ball and plate control problem. The sliding mode control strategy was selected for its robust and order reduction properties. The nonlinear properties of the ball and plate control system are first presented and experimental setup designed and built. The paper then implements and evaluates using experimental results. The sliding controller manages to obtain a faster and more accurate operation for continuously changing reference inputs. The robustness of the proposed control scheme is also verified, since the system's performance is shown to be insensitive to parameter variations. The non-linearities become more dominant with faster responses and larger ranges of operation. Faster specifications always result in smaller ranges of operation and stability. Hence the performance requirement is limited to ensure that the ball reached the desired trajectory even if its initial conditions are not on the desired trajectory. [1]

Research work published by Huan-Wen Tzeng, Sheng-Kai Hung in year 2009, [2] This paper proposes an experience-learning model based on architecture of neural network theory. Neural fuzzy system theory is based on learning procedure from an experienced operator control actions. These actions are difficult to describe in a series linguistic rules, as the learning procedure includes knowledge acquisition from an experienced controller. he controls application using a Neural-Fuzzy System algorithm to control the Ball-Beam balance system, through learning, simulation, and implementation. First, the fuzzy control rules automatically generated using control measurement input to control system and then the neural-fuzzy system undergoes a series of learning processes to achieve the best simulation results. The difficulty lies in this method is several fuzzy rule groups, with great degree of complex calculations, which require a great deal of time to determine.[2]

Research work published by Xicheng Dong, Zhang, Jiagui Tao in year 2009, This paper proposes and tests the genetic algorithm Fuzzy Neural Network Control (GA-FNNC) scheme is applied for the stabilization problem at the designed point for the ball and pate system. The simulation result is fine, especially near the stabilization point such as high accuracy, no oscillation and no overshoot. This control scheme when compared with fuzzy control scheme, the result shows that our scheme has better control performance. Optimizing then genetic algorithm in Fuzzy neural network control system of ball and plate system is helpful to improve the dynamic properties and stability. Based on the fuzzy logic, overcome the shortcomings in information processing and control, the FNNC is more quickly, accurately and efficiently in control of the angle in ball and plate system. Design and simulation with MATLAB, shows GA-FNNC is suitable to process large amounts of information fuzzy control in real time system and is effective in control of the angle in ball and plate system. Especially, when the system work situation changes or disturbing signal is input, the control performance of GA-FNNC is better than conventional fuzzy controller. The parameters and rules optimized by genetic algorithm are excellent to adapt to varying system, which makes controlling performance reach optimization or near optimization. [3]

R**esearch work published by Mohd Fuaad Rahmat, Herman Wahid and Norhaliza Abdul Wahab in year 2010**. This paper investigates the performance and compares the different control strategies that consist of conventional controller, modern controller and intelligent controller for a ball and beam system. Each controller performance will be analysed and compared against the step input response finally the paper shows the comparison of the entire controllers w.r.t set point and the output responses. the research work founds that the designed PID controller has an overall better performance than P, LQR and neural network controller, though the PID gives fastest response time with the reasonable percentage of overshoot and steady state error. [4]

Research work published by Zheng Fei, Qian Xialong, Li Xiaoli, Wang shanguin in year 2011. In the field of mathematical modelling, a radial basis function network is an artificial neural network that radial basis function as activation functions. The output of the network is a linear combination of radial basis functions of the inputs and neuron parameters. Radial basis function networks have many uses. Based on this simplified linearized model, a PID control strategy-based RBF neural network tuning scheme simulation technique is proposed for position control of the ball and plate system. The simulation results show that the control scheme output of the system can track the set point value approximately. The study in this paper found that in comparison to the conventional PID controller and the fuzzy controller, the PID controller-based RBF neural network turning has the merit of easy realization and learning ability. Therefore, derived that the PID controller-based RBF neural network is a best tuning method for the control of the ball and plate system.[5]

Research published by M. keshmiri in year 2012. This paper investigates the PID controller and combination of PID and fuzzy logic controller to control ball and beam balancer. Jacobian linearization method is followed for the linear controller. The research paper founds that the combination PID-fuzzy more efficient than PID controller. The desired point of experimental is 10 cm far from the right-hand side of the beam middle point. The steady state error, settling time and overshoot of LQR-PID is lower than PID controller. thus, model-based controller is more efficient than the non-model-based controller, based on the derived result, the response of system to be simulated with high accuracy LQR must be optimized using genetic algorithm. Also required voltage for optimized PQR is lower than PID controller. [6]

Research published by Umar Farooq, Jason Gu, Muhammad Usman Asad in year2013. This paper investigates a simple interval type 2 fuzzy proportional derivative controller for ball and beam system. The proposed controller is simulated using MATLAB/SIMULINK environment. the paper also compared it with simple interval type 1 fuzzy logic controller. Study shows that the proposed type 2 fuzzy controller is robust to beam angle and ball position disturbances, as well as to measurement noise and errors. The soft computing techniques like fuzzy logic and neural networks can have better performance than classical controllers like PID in case of uncertainties and noises present in the system as they do not need complete mathematical description of the system. The proposed Interval fuzzy controller type 2 has performed better the interval type 1 controllers in terms of reduced settling time, percentage overshoot and improved disturbance rejection ability. [7]

Research work published Byehsan Alc and Warang Asphiratasakun in year 2015. This paper implements PID controller to control the ball and beam system.to increase the system the system response time accuracy, the multiple controllers are piped through a serial protocol to boost the processing fewer and overall performance. The paper uses the processing-based approach i: e feedback mechanism is used to calculate the response. The paper found that in control systems acceptable real time performance can be achieved by decentralizing the processing unit into several PSU [8]

Research work published by PV malini mani, G. Prabhakar and S. selvapermal in year 2016. The paper investigates two control methods are designed and implemented using Proportional Derivative Integral as non-model based control method, Proportional Derivative and Proportional integral combination of model based and non-model-based control methods. Open loop and closed loop system are designed using transfer function and steady space model obtained from mathematical modelling followed by linearization of the nonlinear equation obtained. The system is designed by using two Degrees-of Freedom. Lagrange method is used to find stability .it is based on energy balance principle. The nonlinear characteristics is regulated using PID controller. The parameters are tuned using PID tuning algorithm.th paper found the system is improved using PID. [9]

Research published by S K. Valluru, Madhusudan Singh and Supriya Singh in year 2017. This paper addresses the modelling and control of ball and beam system. Stepper motor is used instead of a servo motor which is relatively more economical. The various control strategies were proposed in this paper using PID, state space, lag-lead, robust LQR, observer based LQG controllers, and performance characteristics of the system is presented. Lyapunov direct method is used describe the unstable behaviour of ball and beam system. Unlike. Other systems. The study found that robust LQR controller approaches the desirable performance when compared to other controllers. [10]

CHAPTER:3 MATHEMATICAL MODELLING

In order to design a control system that will accurately control the position of the ball, a highly accurate model of the entire system's dynamics must be developed. As the accuracy of the model increases, the uncertainty to be dealt with by the control effort will decrease. The model for our plate dynamics comes from the general equation of motion for a multibody system. Since the 2 DOF Ball Balancer uses two Rotary Servo Base Unit (SRV02) devices and the table is symmetrical, it is assumed that the dynamics of each axis is the same. The 2 DOF Ball Balancer is therefore modelled as two de-coupled "ball and beam" systems where we assume the angle of the x-axis servo only affects the ball movement in the x direction. Similarly, for the y ball motion. The equation of motion representing the ball's motion along the x-axis relative to the plate angle is developed. The servo angle is introduced into the model and is then represented as a transfer function. [11]

3.1) Nonlinear Equation of Motion

Modelling the plate in one direction on one dimension. The free body diagram of the Ball and plate system is illustrated in Figure 1. Using this diagram, the equation of motion, relating the motion of the ball, x, to the angle of the beam, α, can be found. Based on Newton's First Law of Motion, the sum of forces acting on the ball along the beam equals,

$$m_b \ddot{x}(t) = \sum F = F_{x,t} - F_{x,r} \tag{3.1}$$

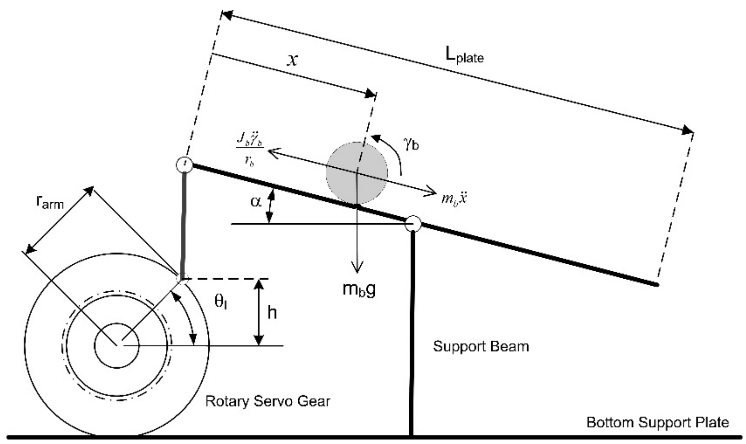


Fig:2 Modelling ball on plate in one dimension.

Modelling conventions:

- Applying a positive voltage causes the servo load gear to move in the positive, counter clockwise (CCW) direction. This moves the beam upwards and causes the ball to roll in the positive direction (i.e., away from the servo towards the left). Thus, $V_m(X) > 0$, $\Theta > 0$, $x > 0$.
- Ball position is zero, $x = 0$, when located in the centre of the plate,
- Servo angle is zero, $\Theta = 0$, when the plate is parallel to the ground

For the ball to be stationary at a certain moment, i:e, be in equilibrium, the force from the ball's momentum must be equal to the force produced by gravity. From the schematic diagram illustrated in fig 2 the force $F_{x,t}$ in the direction of x axis that is caused by gravity can be found as:

$$F_{X,}t = m_b \sin\alpha(t)$$

The force caused by rotation of the ball is,

$$F_{x,\,r} = \frac{\tau_b}{r_b}$$

where, r_b is the radius of the ball and τ_b is the torque which equals,

$$\tau_b = J_b \ddot{\Upsilon}_{b(t)}$$

where, $\Upsilon_{b(t)}$ is the ball angle. Using the sector formula, $x(t)= \Upsilon_{b(t)}\, r_b$, we can convert between linear and angular displacement. Then, the force acting on the ball in the x direction from its momentum becomes:

$$F_{x,\,r} = \frac{J_b \ddot{x}(t)}{r_b^2}$$

Now, by substituting the rotational and translational forces into equation (3.1), we can get the nonlinear equation of motion for the ball and beam as;

$$m_b \ddot{x}(t) = m_b g \sin\alpha(t) - \frac{J_b \ddot{x}\,(t)}{{r_b}^2}$$

solving, for the linear acceleration gives:

$$\ddot{x}(t) = \frac{m_b g sin\alpha(t) r_b{}^2}{mbrb^2 + J_b} \tag{3.2}$$

The equation of motion representing the position of the ball relative to the angle of the servo load gear is derived. The obtained equation will be nonlinear as it includes the trigonometric term. Therefore, it will have to be linearized to use in a control design.

3.2) Linearization of the system

With this linearized system, we can design a linear controller that will control the plate's angular position based on voltage input. In addition to the plate dynamics, the ball's response to plate position must be considered. using the schematic diagram in fig 2. or our purposes, it should suffice to treat the ball and plate system as two decoupled ball on beam systems. In addition, we will ignore any rolling friction that may occur between the ball and plate. This greatly simplifies the model and makes for easier control design. Using the schematic diagram given in Figure 3.1, consider the beam and servo angles required to change the height of the beam by h. taking the sine of the beam angle give the expression

$$\sin\alpha(t) = \frac{2h}{L_{plate}}$$

Where, taking the sine of servo load shaft angle results in the equation,

$$\sin\theta_l(t) = \frac{h}{r_{arm}}$$

thus, we obtain the relationship between the beam and servo angle as,

$$\text{Sin}\alpha(t) = \frac{2r_{arm}\sin\theta_l(t)}{L_{plate}} \tag{3.3}$$

To find the equation of motion represents the ball motion with respect to the servo angle $\theta_l(t)$ we need to linearize the equation of motion about the servo angle, $\theta_l(t) = 0$. insert the servo and plate angle relationship into nonlinear equation (3.3) into equation (3.2) as,

$$\ddot{x}(t) = \frac{2 m_b g r_{arm} r_b^2}{L_{plate}} \sin\theta_l(t) \tag{3.4}$$

About angle zero, the sine function can be approximated by $\sin\theta_l(t) \approx \theta_l(t)$. Applying this to the nonlinear equation of motion gives the linear equation of the motion of the ball,

$$\ddot{x}(t) = \frac{2 m_b g r_{arm} r_b^2}{L_{plate}(L_{plate} r_b^2 + J_b)} \theta_l(t) \tag{3.5}$$

3.3) Obtaining the transfer function

The complete open loop system of the two degrees for freedom ball on plate balancing system by the block diagram shown in fig 3.1. The servo motor transfer function $P_s(s)$, represents the dynamics between angle of the servo input motor voltage and resulting load angle. The dynamics between the angle of the servo load gear and the position of the ball is described by transfer function $P_{bb}(s)$. This is the decoupled model i: e it is assumed the x axis servo does not affect the y axis.

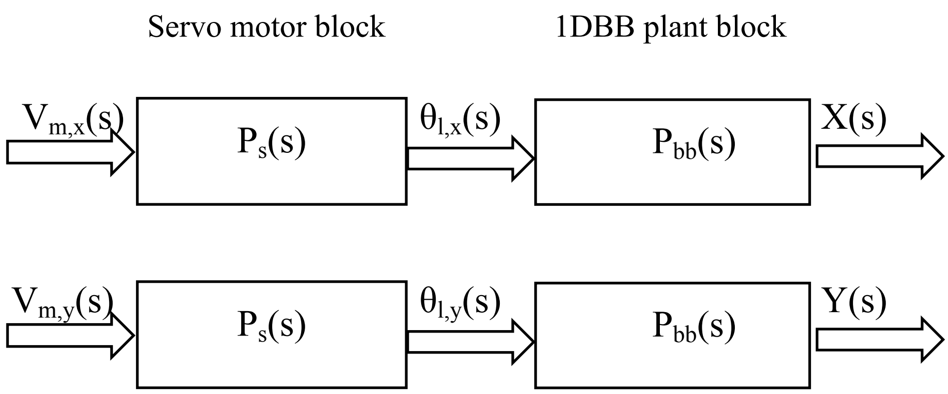


Fig: 3.1 2DOF Ball Balancer open loop diagram

The block diagram of the single axis of the 2 DOF ball and plate balancer denoted as 1DBB is shown in fig.

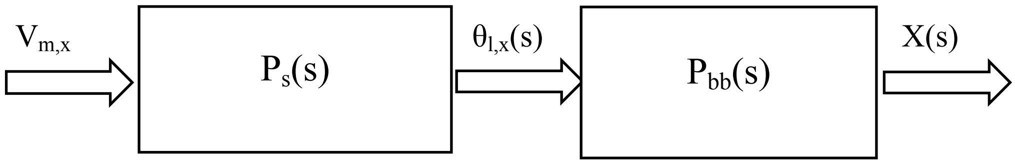


Fig: 3.2 1-D open loop block diagram of 2-D Ball plate Balancer

The overall transfer function to obtain 1DBB ball on plate balancer

$$P(s) = P_{bb}(s)P_s(s) \quad (3.6)$$

The servo angle to ball position transfer function, P_{bb} (s), can be found by taking the Laplace transform of the linear equation of motion in equation 1.5 as

$$P_{bb}(s) = \frac{X(s)}{\theta_l(s)} = \frac{K_{bb}}{s^2} \quad (3.7)$$

And, we obtain the transfer function of servomotor as,

$$\frac{\theta(s)}{V_m(s)} = \frac{K}{S(\tau S + 1)} \quad (3.8)$$

However, by inserting the plate position transfer function, (1.7) and the voltage servo transfer function, (1.8) into equation (1.6), we can derive the complete process transfer function P(s) as:

$$P(s) = \frac{X(s)}{V_m(s)} = \frac{K_{bb}K}{(s^3\tau s + 1)} \quad (3.9)$$

This is the servo voltage to ball displacement transfer function.

The servo motor plant model from equation (1.8), it can also be represented as,

$$\ddot{\theta}_l = -\frac{1}{\tau}\dot{\theta}_l + \frac{K}{\tau}V_m \quad (3.10)$$

and from equation (3.5) ball's motion can also be written as,

$$\ddot{x} = K_{bb}\theta_l \tag{3.11}$$

The linearized system of equation of motion of ball can also be represented in full state-space form. The state space model is used for LQR control design. For the state space model as the ball's position (x) and velocity ($\dot{x}$) from equation as the two of state variables. Besides, other two state variables the motor gear angle (θ_l) and motor angular velocity ($\dot{\theta}_l$) from equation (1.10) as another state variables, and the motor input voltage, V_m, as the input. The state-space representation is shown in equation (3.12), whereas equation (3.13) shows the output equation for this system.

3.4) State space model

The mathematical model in state space form is used to design the linear quadratic regulation (LQR) controller in the next section. In differential equation, servomotor model can be written as,

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & K_{bb} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{\tau} \end{bmatrix} V_m \tag{3.12}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{3.13}$$

Figure 2 illustrates the angles and dimensions for one axis of the 2 DOF Ball Balancer system. The x-direction of the 2D Balance Table is illustrated in Figure 2. It includes various dimensions and shows the variables α, θ and x that are associated with the system (for the x-axis). Some of the parameters listed in Table 1 are used in the mathematical model

Table 1: 1D ball and plate parameter

r_b	Radius of the ball	14.6	cm
m_b	Mass of the ball	0.03	Kg
L_{plate}	Table length	27.5	cm
r_{arm}	Distance between servo output gear length and coupled joint	2.54	cm
g	accerlation due to gravity	9.8	m/s^2
τ	Torque constant	0.0285	-

The ball plate balancing system parameters is provided for calculating the transfer function and the steady state model for the control system.

The mathematical model is based on the principle of balancing of forces and torques acting on the ball and the dynamic model of servo motor. The real time behaviour of our control system can only be observed by taking into account by including an approximation of linear mechanical losses, depending on the speed of the rotational motion. Mechanical losses in case of moving ball are proportional to the square of opposition translational speed of movement of the ball.

3.5) Stability analysis of open loop 1D ball and plate system:

In this section, we analyse and studied the stability of a open loop behaviour of the ball plate balancer system for one dimension, which we choose to be x-axis. Simulink model is shown in Fig 4.1 and open loop system response without is also shown in fig 4.2.

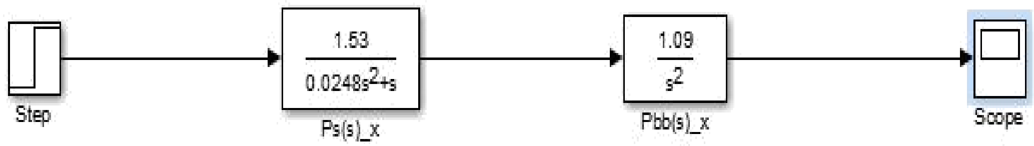


Fig 3.1) open loop system simulation model

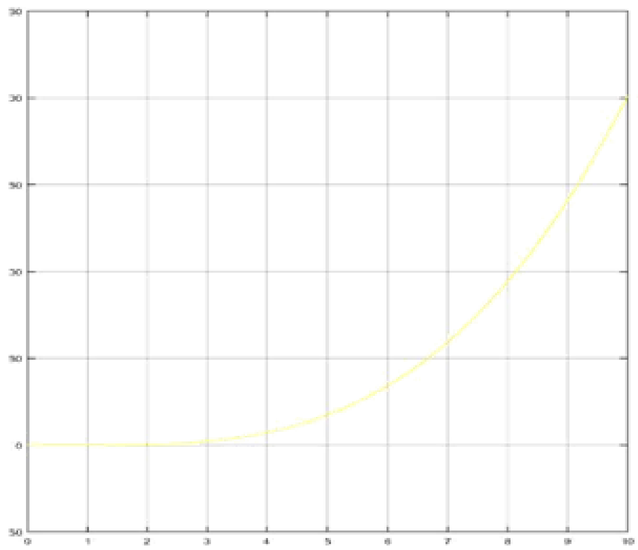


Fig 3.2) open loop response

The model dynamics and the system behaviour such as closed loop time response root locus, frequency response, bode plot, Nichols plot and Nyquist plot are shown in Fig3.3, Fig 3.4, Fig 3.5.

3.5.1) BODE PLOT

In electrical engineering and control system, a bode plot is a graph of the frequency response of a system. It is usually a combination of a Bode magnitude plot, expressing the magnitude usually in decibels of the frequency response, and a Bode phase plot, expressing the phase shift.

- The phase crossover frequency, w_{pc}, is the frequency where phase shift is equal to -180^{o}.
- The gain crossover frequency, w_{gc}, is the frequency where the amplitude ratio is 1, or when log modulus is equal to 0.

Stability criteria for bode plot:

If at the phase crossover frequency, the corresponding log modulus of $G(iw_{pc})$ is less than 0 dB, then the feedback system is stable.

The MATLAB command shown as below,

h= tf(n);

bode(h);

Thus, from the figure 3.3 we found the open loop system is unstable.

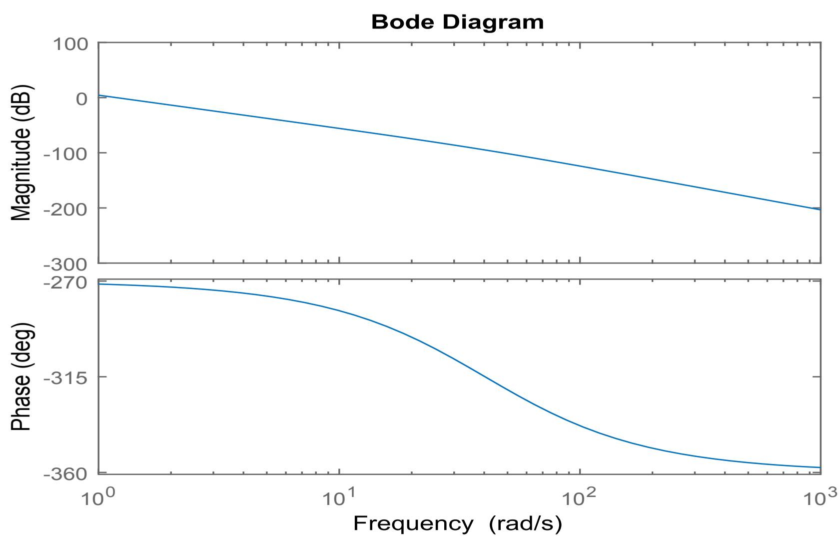


Fig:3.3) bode plot of open loop system

3.5.2) Root locus

The root locus technique in control system was first introduced in the year 1948 by Evans. In root locus technique in control system we will evaluate the position of the roots, their locus of movement and associated information. The MATLAB command will plot the root locus of the system. MATLAB code is shown below as,

```
h=tf(n);
rlocus(h);
```

This information will be used to comment upon the system performance. From the figure 4.4 it is clear to us the open loop system root locus shows the poles lies on right hand side of axis. Hence, open loop ball plate balancer is unstable system.

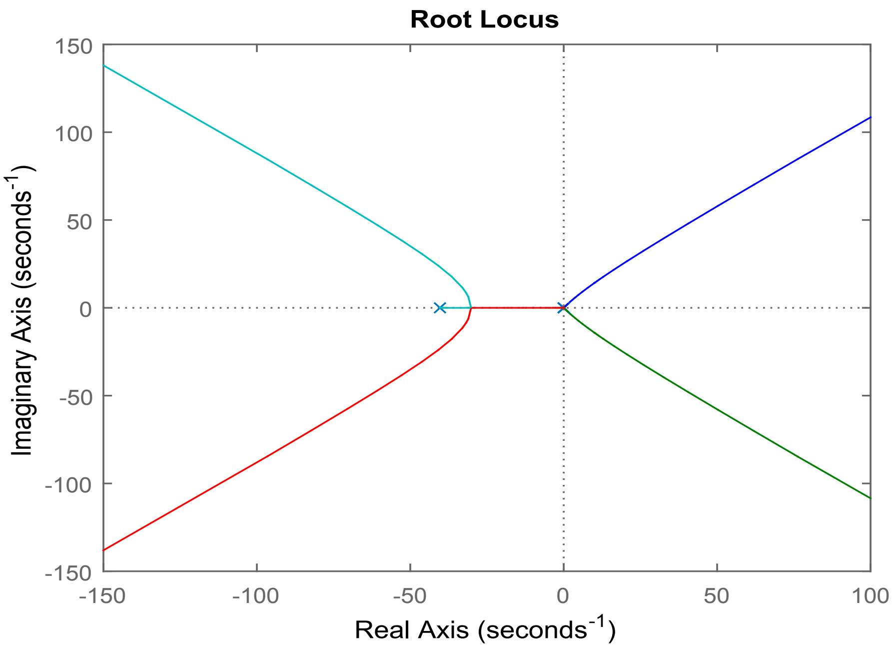


Fig 3.4 root locus of open loop system

3.5.3) Nyquist plot

In control theory and stability theory, the Nyquist stability criterion, discovered by Swedish-American electrical engineer Harry Nyquist at Bell Telephone Laboratories in 1932.It is a graphical technique for determining the stability of a system. While Nyquist is one of the most general stability tests, it is still restricted

to linear, time-invariant systems. The command for plotting in MATLAB is shown below as,

h= tf(n);

nyquist(h);

The figure 3.5 shows that the open loop ball plate balancer system is unstable.

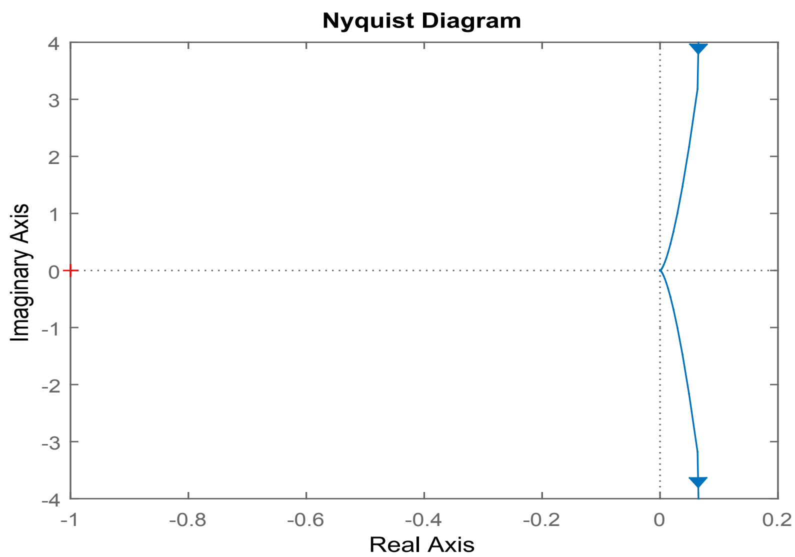


Fig 3.5

Nyquist plot for open loop system

It should be observed that 1D ball and plate control system is an unstable system and interesting control problem. The model dynamics and the system behaviour such as open loop time response, root locus, frequency response, bode plot and Nyquist plot are shown in Fig 3, which conforms the ball and plate system is unstable.

CHAPTER 4: CONTROL DESIGN AND IMPLEMENTATION

The controller realization is implemented using three control approaches, namely PD-P controller, LQR controller and fuzzy-PID controller. For decision making of controller design, a few design specifications have been desired. The time domain requirement for controlling the position of the ball for both the x and y axes on the 2 degree of freedom Ball plate balancer are given as,

Specification 1: 4% settling time, $t_s \leq 3.0$ s

Specification 2: percentage overshoot PO ≤ 10%

Specification 3: steady state error, $e_{ss} \leq 5$mm

The three second is chosen to determine the effectiveness of the designed controllers in term of fast response. Whereas, the low overshoot is required to avoid the ball run out of the beam especially at the end points of the beam.

4.2) PD-P Controller Design

We are analysing PD-P control ball plate balancer system is a fourth order system. It is quite difficult to design a controller for higher order system. Hence, to make design of controller easier and realizable, the overall control system is separated into two feedback loops as shown in figure below. The purpose of the inner loop is to control the servo gear angle position(θ_l). Controller C1 should be designed so that gear angle tracks the reference signal (ref θ). The outer loop uses the inner feedback loop to control the ball position. Therefore, the inner loop definitely must be designed before the outer loop. For the inner loop, PD controller is selected instead of PID because servomotor model is a second order system, thus PD controller will change the second order system to third order system which is quite hard to control whereby PD controller will preserve its second order. Thus, the equation for P controller for inner loop is C1(s)=Kp, where Kp is proportional gain is for servo control and PD controller C2(s) is used for outer loop i:e for ball plate plant. By using Ziegler Nichols's method, PD parameter has been tuned to be

Kp=3.44 rad/s and Kd= 2.1 rad/. This diagram represents full view of the Simulink model of closed loop system for 1D Ball and plate balancer. Ball and plate balancer is modelled by calculating mathematical modelling which is implemented by MATLAB/SIMULINK. The step input value 1 unit is given to the summer then to get the output of transfer function. Summer send to the PD controller which is provide the good steady state response. Also, it decreases the overshoot. The transfer function output and input signal are added and send to the scope.[11]

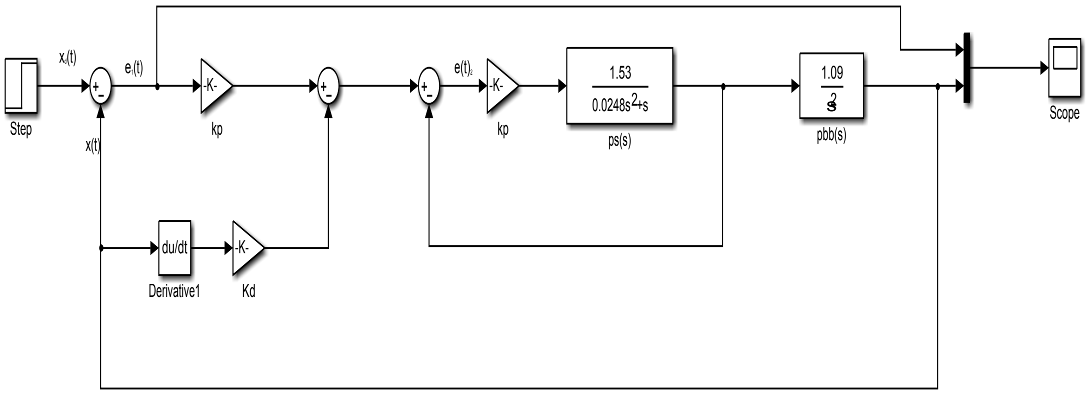


Fig :4 Simulink model of 1-D ball plate with PD control

4.2) LQR controller design

The performance index of the LQR is linear quadratic, which is easy to analysis, process and calculate. In the process of designing LQR, the main problem is how to select the weighted matrices Q and R. Generally, for obtaining the matrices hit and trial error method is used which is quite simple but inefficient for higher order systems. we often randomly select Q and R and judge them whether or not meet the requirements by simulation. The optimal feedback matrix K will be achieved if they meet the requirements. Otherwise, we continue to adjust Q and R, until they meet those requirements. The method is suit for low-order systems and the difficulty will increase for the increase in order of the system.

To design an LQR controller, firstly we check whether ball plate balancer system is controllable and observable or not. Controllability means we can influence all system states of given dynamic system through control input. In order to see what is going on inside the system under observation, the system must be observable. The observability means every change can be reflected in output. To verify this, we must ensure that the Controllability and Observability matrices are full rank:

$$C = [B\ AB\ A^2B\ A^3\ B \ldots\ldots\ldots A^nB]$$

$$0 = [C\ CA\ CA^2\ CA^3 \ldots\ldots\ldots CA^n]$$

For our linearized systems as given in Equations (3.12) and (3.13). Since the rows of the matrix are linearly independent, then, i.e. the system under consideration is observable. Another way to test the completeness of the rank of square matrices is to find their determinants. The rank of the system equals the number of controllable states in the system. Here, the rank of matrix = n = 4, thus our control system is fully controllable however unstable. they are both fully controllable and observable, so we may continue on to the control design phase.

Now we can design PQR controller for the ball and late system since the system is fully controllable and observable. For plate control a robust LQR controller is designed now. We are utilizing the state feedback equation and output equation as in equation (3.12) and equation (3.13). The characteristic equation for the closed-loop system is given by the determinant of [sI(A-BK)], where I is identity matrix, while A and B are the system matrix and input matrix respectively from state space equation in (3.12). and (3.13). For this system the A and B*K are both 4×4 matrices. Hence, there should be four poles for this system and four eigen values of the closed loop system where, K is full-state feedback gain.

Here, Q is the state penalty matrix, and R is the control penalty matrix. These are user defined matrices, whose proper selection will achieve the desired time-domain response. The optimal feedback gain matrix, K can be found using K = R^{-1}BTP where P is a positive definite solution to the Steady State algebraic Riccati Equation.

An algebraic Riccati equation is a type of nonlinear equation that arises in the context of infinite-horizon optimal control problems in continuous time or discrete equation. A typical algebraic Riccati equation is similar to one of the following is the continuous time algebraic Riccati equation:

$$\dot{S} + A^TS + SA - SBR - B^TS + Q = 0 \quad (4.1)$$

Where, S is the solution matrix, Q and R are state penalty and control penalty matrices and K is a state feedback gain matrix. A and B being a state variable matrix and input matrix respectively. The eigen values of system is e = eig (A-B*S, I) where I is the identity matrix.LQR give the optimal control for certain assumption value. The 'LQR' function (in MATLAB) allows us to choose two parameters, regulator (R) and quadratic (Q), which will balance the relative importance of the input and state in the cost function that we are trying to optimize. The simplest case is to assume R=1, and Q=C^T*C, where C is output matrix from equation (3.12) and (3.13). C^T is matrix transpose of C. The optimization of Q can be achieved by hit and trial method i: e changing the nonzero elements in the Q matrix to get a desirable response. Thus, that element will be used to weight the output response. The strategy is described in MATLAB command as shown below[9]:

R = 1;

Q = [x 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];

K = lqr (A, B, Q, R)

From above command, by increasing x, the settling time (Ts) and rise time (Tr) can be decreased. For this design, the value of x is set to 40000 and let R remain one.

```
clear all;
```

```
clc;
display('------------Linear Quadratic Regulator-----------------')
A=[0 1 0 0; 0 0 -0.437 5 0;0 0 0 1;0 0 0 -50];
B=[0;0;0;85];
C=[1 0 0 0];
D=[0];
[b,a]=ss2tf(A,B,C,D);
sys1=tf(b,a)

W=1
if W==1
    Q=[4000 0 0 0;0 0 0 0;0 0 0 0;0 0 0 0];
else
    Q=transpose(C)*C
end

R=[1];

Y=input('if want to enter value of N manually enter 1 else 2 = ')
if Y==1
    N=input('enter value of N = ')
else
    %%
    N=0
end
[K,S,e]=lqr(A,B,Q,R,N)
sys=ss(A,B,C,D)

n=length(K);
AA=A - B * K
for i=1:n
    BB(:,i)=B * K(i);
end
display(BB)
CC=C
DD=D
for i=1:n
     sys(:,i)=ss(AA,BB(:,i),CC,DD);
end
subplot(111)
step(sys(:,1))
```

4.2) Combination of Fuzzy and PID controller

i) Design of Fuzzy Controller

The fuzzy controller consists of four modules: rule base, inference engine, fuzzification, and defuzzification modules. The fuzzification module converts input measurements into fuzzy membership parameters. Every measurement, as acquired from human experience, should be examined for the design of the controller and only the measurements of successful balance control. Control cases should be used for NFS learning. there are two steps of fuzzy controller design procedures as shown below.

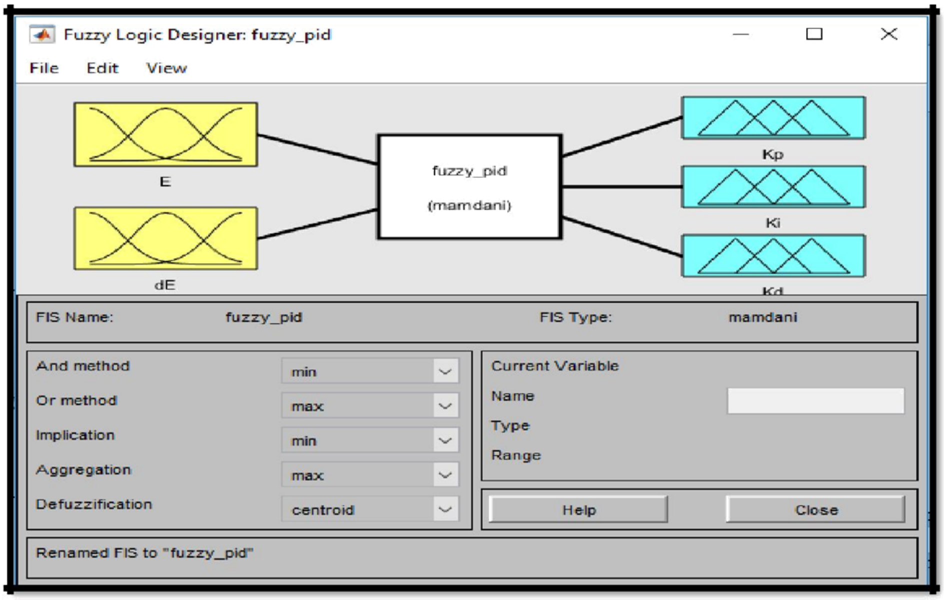


Figure 5: *Fuzzy Editor*

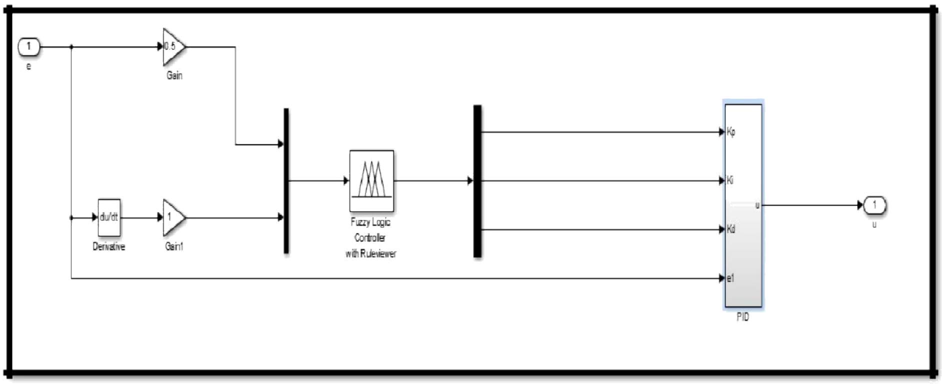


Fig:6.1 Fuzzy_pid simulation

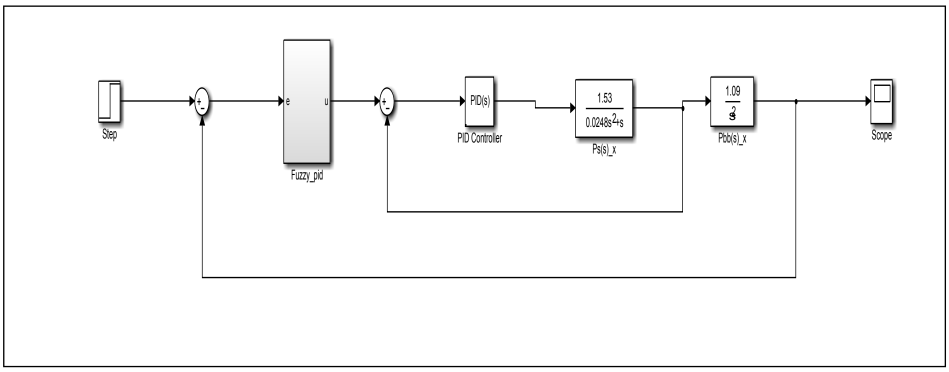


Fig :6.2 fuzzy_pid simulation

ii) Define the input and output variables*:* After identifying the input and output variables of the controller and the range of their values, some linguistic states are selected for each variable. The ball position is expressed as linguistic variable “P”, beam angle is θ, and the voltage of the servomotor is “V”. We suppose there are five states for every linguistic input variable, which are: NL-negative large, NS-negative small, ZE-zero, PS-positive small, and PL-positive large. A triangular-shaped function is selected for fuzzification. [13]

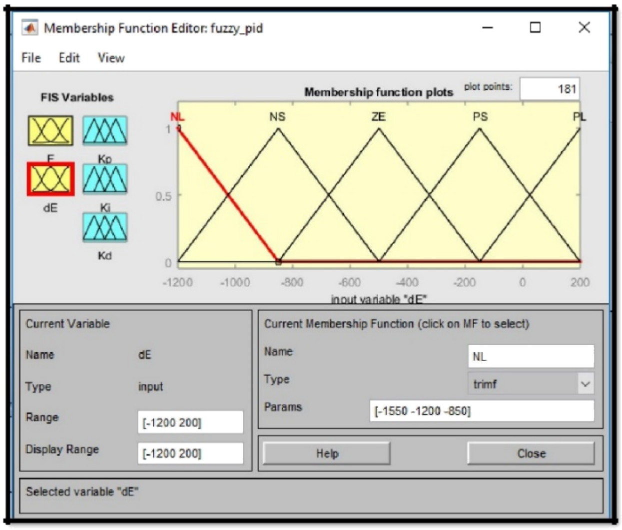


Fig: 7 fuzzy input variables

We suppose there are five states for every linguistic variable, which are: PVS-positive very small, PS-positive small, PMS-positive medium small, PM-positive medium, PML-positive medium large, PL-positive large, PVL-positive very large.

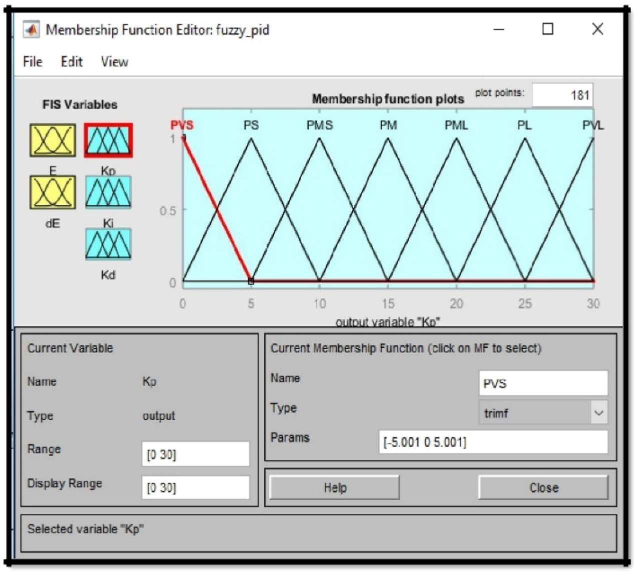


Fig:8 fuzzy output variables

iv) Establish the fuzzy rules: According to the defined format of fuzzy rules, with seven linguistic states, the system establishes all relative rules for the rule-base. Some examples of rules are shown below:

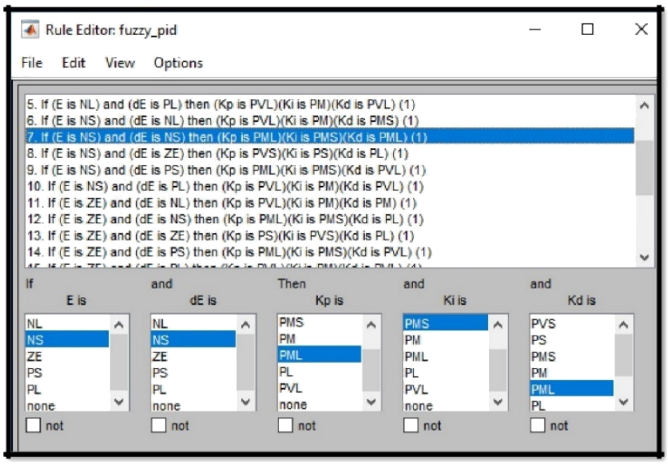


Figure:9 Fuzzy Rule Table

The first step of establishing fuzzy rules is data acquisition of control measurements, which are then divided into input and output parameters. While an experienced operator works the system, another will act as a position sensor for visual feedback, while the joystick operations returns control signals to the servomotor. Thus, allowing the human brain to generate control rules and inferences for the control system. The data acquisition procedures.

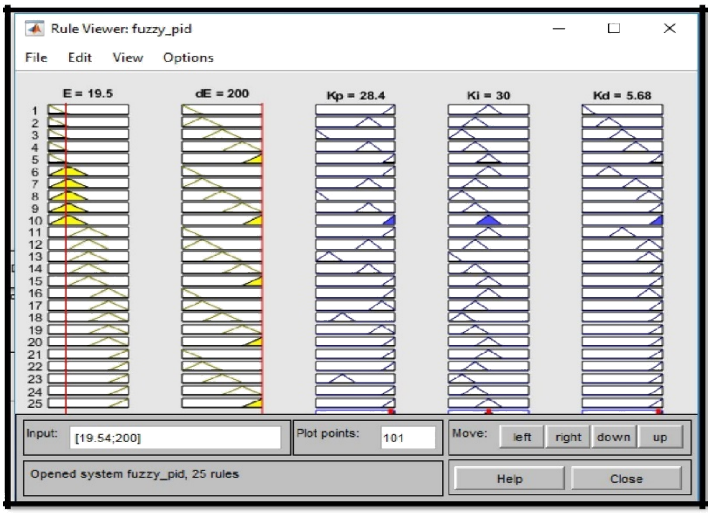


Fig :10 fuzzy rule viewer

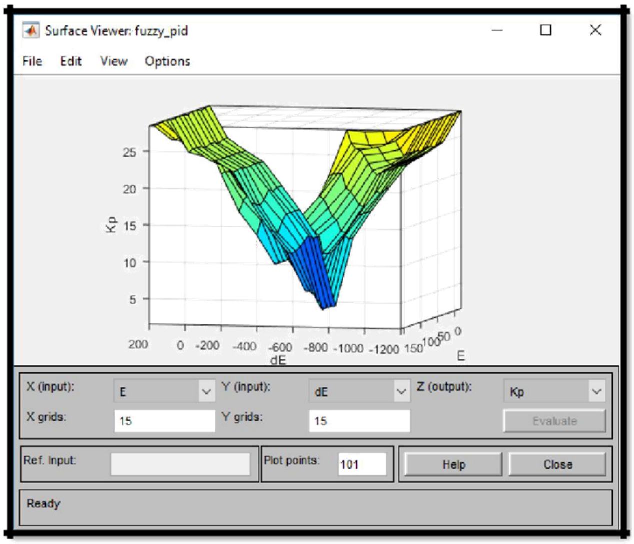


Fig:11 fuzzy surface view of K_p

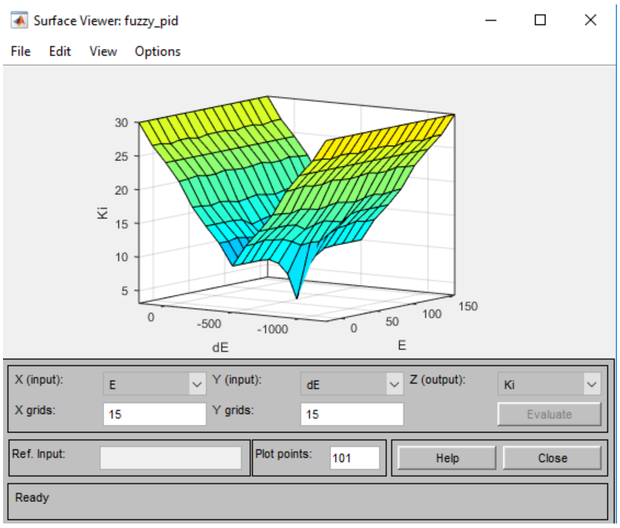


Fig:12 fuzzy surface view for K_i

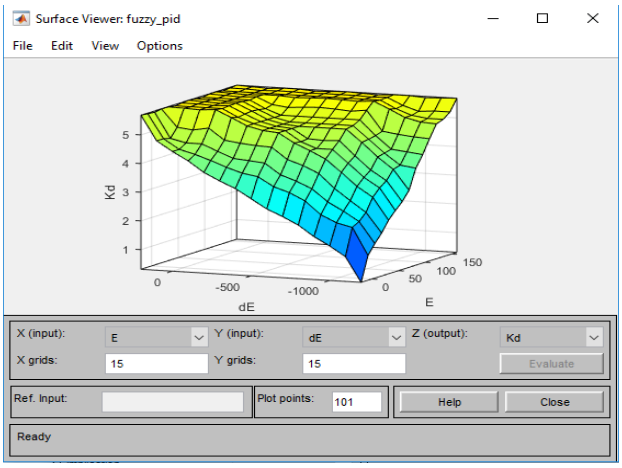
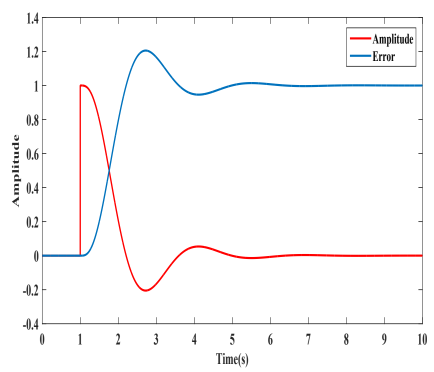


Fig:13 fuzzy surface view for K_d

CHAPTER :5 RESULTS AND DISCUSSION

5.1) Results for combination of Proportional (P) and Proportional Derivative (PD) Controller

Proportional derivative controller is the most basic strategy for the feedback control law. fortunately, this controller is capable of maintaining the output steady state value at the desired with zero steady state error as shown in Figure 14. From this figure, Kp is set to be 3.44 and K_d is set to be 2.1. While increased Kp gain, the output will response faster because it decreases the rise time (Tr). However, Kp gain is limited by the dynamics of the system, where for ball and plate system the response is limited by the length of the plate. The effect of Kp and Kd terms in PD controller tend to make the closed loop system become more stable. A proportional controller (Kp) will have the effect of reducing the rise time and will reduce but never eliminate, the steady-state error. A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Table :2 PD control specifications.



Proportional gain	3.44
Proportional gain	3.44
Derivative gain	2.1
Settling time	4.2s
Rise time	3.2s
Percentage overshoot	20%
Steady state error	Zero % of final value
% undershoot	10%

Fig: 14 Response of1D ball plate system with PD control

The result for PD controller is shown in Figure 14. The PD controller gives a good steady state response, whereby the steady state error with zero steady state error. From the graph it is observed to be high as 20% from its final value. The ball finally settles in 6.2 seconds and almost smoothens in 10 sec.

5.2) Results for State Feedback Controller: LQR controller

The Figure 15, shows the simulation result for a ball and plate system that is utilizing the LQR controller. The controller gives a very fast response with the settling time of 2 second and rising time that less than 1.3 second. In addition, the LQR tend to produce no steady state error, however it produces a small overshoot of about 7.000%. Thus, the controller satisfied the design requirements that, LQR produces a very fast response w.r.t to conventional PD controller. The optimization process in LQR controller made it the better controller than a conventional controller .by changing the value of variable Q matrix, we can set the rise time and peak time according to our design requirement. However, the response can also be altered by tuning the value of variable R and the trade-off between the variable Q and R matrix, the best combination value of R and matrix Q will give a satisfactory response. In this design, we fix the value of R to one in order to simplify the design process. By increasing the value of first element of x in matrix Q, we should able to get a better settling time and rising time.

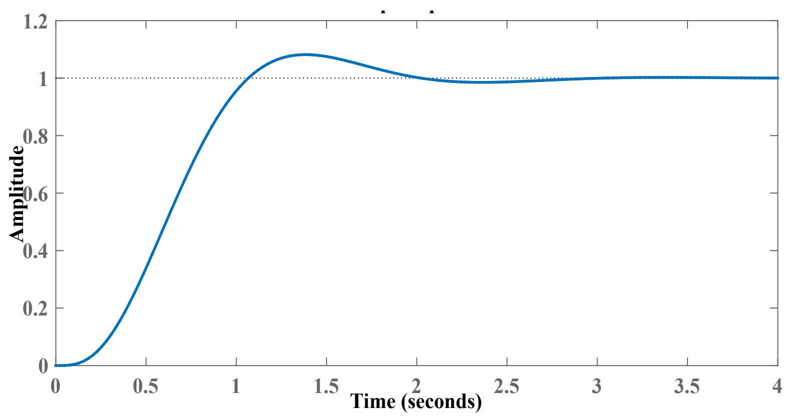


Fig: 15 1D ball plate balancer with LQR control

But the problem using LQR controller is that, it will increase the percentage of overshoot in the output response. In our case, if we decrease the x value, the overshoot specification can be met, however, it will take a longer response time.

5.3) Combination of fuzzy and PID controller

The simulation result for fuzzy-PID controller is shown in Figure 16.A small steady state error of 0.004% is generated with large existing of the overshoot. However, the response time is a little bit slower than the other LQR controller due to the time consume on the learning and training process. A delay of 1 sec can also be seen in the response which is undesirable for us. This controller gives the settling time of 4 second and rising time of 2.9 second at the reference input of step response. The figure 16 also shows the error in figure. In further, if the set-point is increased to a maximum limit, it takes a longer response time with the settling time is equal to 4 second. Hence, we can summarize that fuzzy logic network controller is able to control the ball and plate system.

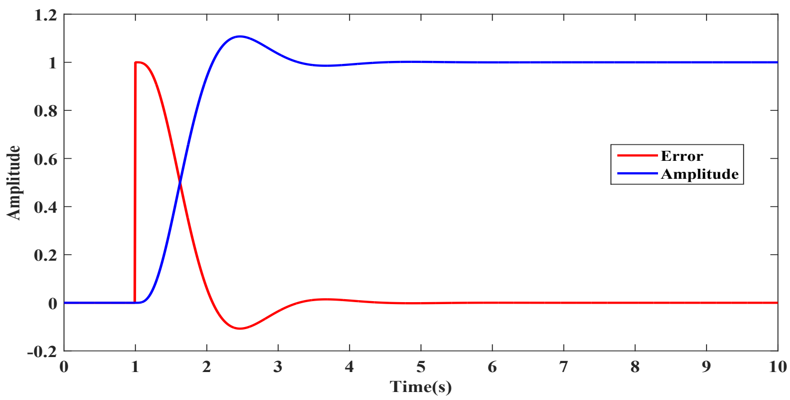


Fig:16 response of 1D ball plate balancer with Fuzzy-PID controller

5.4) Overall comparison of the controller performance

The graph in Figure 17 shows the output response for comparison of the all the controllers for a common input response of one magnitude. It can be seen that

designed LQR controller has an overall better performance than PD, fuzzy controller, though it seems that the LQR gives fastest response time with the reasonable percentage of overshoot and no steady state error. the LQR response shown no lag time and achieve steady state response very soon with lesser transient cycles than PD and fuzzy controller. Comparing the overshoot response, LQR has least overshoot followed by fuzzy and relatively largest overshoot of 20% can be seen in PD controller. LQR shows no time lag in response, while fuzzy and PD response shows a significant lag of 1 sec which is undesirable for our system.

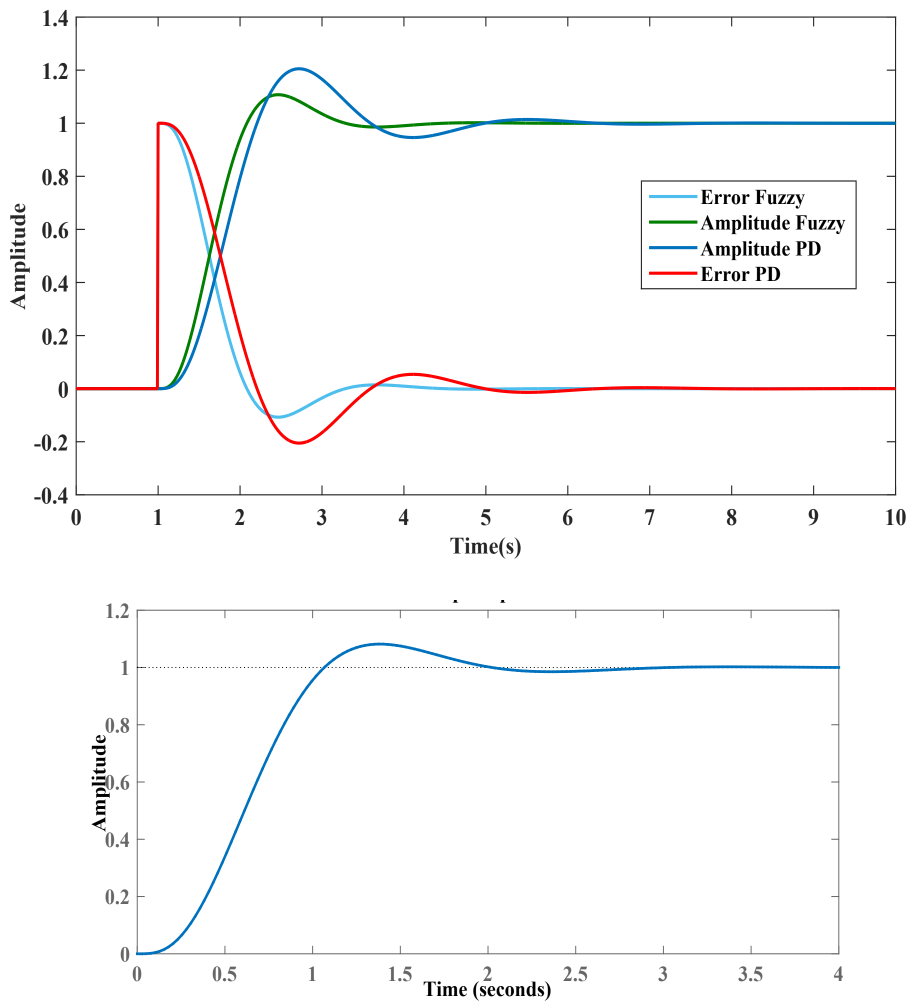


Fig:17comparison between controllers

CHAPTER:6 CONCLUSION AND FUTURE WORK

In this thesis, the mathematical model for a ball and beam system has been derived successfully. The plant is consisting of three main components which are servomotor model, angle conversion gain, and ball on the beam dynamic equation. Both servomotor and ball beam dynamic have the second order transfer function**.** the two degree of freedom ball plate balancer was taken and studied in a one dimensional assuming our system to be symmetric for both the axis. different controllers have been designed and simulated in order to meet the desired specifications. This system emulates real control problem such as horizontal stabilization of an aircraft during landing and while experiencing turbulence. Based on the transfer function and state space model, open loop system and closed loop system are designed. The MATLAB simulation results along with the graphs for the controllers have been included. From the output comparison graphs it can be seen that robust LQR controller approaches the desirable performance when compared with other controllers. With the basic configuration, seem like the intelligent controllers not giving a good transient response, but still can be an alternative to replace the conventional and modern controller.

The closed loop performance of the ball and plate system will be analysed using fuzzy logic and also the entire system will be implemented as hardware prototype. For example, we can take Quanser 2 DOF ball and plate system. Then the comparison will be made between the hardware and software results. Various other controllers can also be studied like intelligent controllers like sliding mode control, neural network and other like conventional controllers like PID controller.

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