## **Final Year Dissertation on**

# **Volatility measurement In Indian FOREX market using GARCH Model**

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#### **CERTIFICATE FROM INSTITUTE**

This is to certify that Final Year Dissertation on "Volatility measurement In Indian FOREX market by GARCH(1,1) Model" is a bona fide work carried out by Ms. Kanika Singh of MBA 2015-17 Batch and submitted to Delhi School of Management, Delhi Technological University in partial fulfillment of the requirement for the award of degree of Masters of Business Administration.

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#### **DECLARATION**

I, Kanika Singh, student of MBA 2015-17 of Delhi School of Management, Delhi Technological University, hereby declare that Final Year Dissertation on **"Volatility measurement in Indian FOREX market by GARCH Model"** submitted in partial fulfilment of Degree of Masters of Business Administration is the original work conducted by me. The information and data given in the report is authentic to the best of my knowledge.This report is not being submitted to any other University for award of any Degree, Diploma and Fellowship.

Kanika Singh

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#### **ABSTRACT**

This dissertation is a study on volatility measurement of Foreign exchange market in India using GARCH model. This study includes an overview of Indian Foreign exchange market and its position with respect to global Forex market .Regimes of Indian foreign exchange market have been studied to bring out the impact of high volatility on the foreign trade and economic growth in India .The periods of high volatility have causes a substantial decrease in foreign trade and economic activity in the country emphasizing the importance to forecast volatility so that the competent authority can take corrective measures to check high volatility. Different models that had been used to forecast volatility by researchers have been summarized in the literature review. Also in the literature review have been analyzed that GARCH model gives more accurate results in forecasting volatility than the other available models. The objectives of this study are to

- (a) Summarize different models available for forecasting volatility
- (b) Forecast volatility of Indian foreign exchange market using GARCH model

Vast literature on the subject of volatility measurement of Forex have justified that volatility can be expressed as conditional variance and time series data modeling can be used to measure volatility. Some models use standard deviation to predict volatility that gives biased results. GARCH model use conditional variance and many researchers have studied the accuracy of volatility measurement using GARCH model in other foreign countries and found that it gives satisfactory results with the use of constraints of stationarity. A number of different types of GARCH models have been developed for improving the accuracy of volatility forecast.

INR and USD currency pair data from January 2007-January 2017 is used for this study as it is the currency pair in which major part of foreign exchange trading in India is done. Analysis of volatility forecast by GARCH model shows that although the errors are not normally distributed but the estimators of volatility are consistent and GARCH model can be satisfactorily used for volatility forecast of Indian foreign exchange market.



### **TABLE OF CONTENTS**

#### **CHAPTER 1 INTRODUCTION**

Foreign Exchange Reserves or simply Forex reserves is assets held by the central bank of country that can be used to pay the liabilities when and where required. So in reference to above line one can easily draw conclusion that Forex reserves are very vital component of any country's economy. India, a growing super power and brightest spot in world economy is no exception. According to recent article published in national dailies dated 12 May 2017 India's forex reserve is all time high at \$375.71 Billion in the week ending at May 05 2017.But as it is said that hard times brings struggles, struggles brings challenges, challeneges brings opportunity and opportunity makes history. So simply having a decent forex reserve is neither sufficient nor a guarantee of a sound economy. In addition to this we should have a stable forex market. Difference in demand and supply of currencies is responsible for fluctuations in exchange rates. The more is volatility the greater is risk in investment. Therefore modern economist has always shown great interests in study of Forex Volatility and considered it as a prime area of research.

#### **1.1 Overview of Indian Foreign exchange market**

Global Forex market trading is averaged as \$5.1 trillion per day in April 2016 as reported by Bank of International settlement.US dollar is the dominant currency and accounts for 88% of all trades. The following table shows the data about the different participants in global foreign exchange market.



Currency distribution of OTC foreign exchange turnover

<sup>1</sup>Adjusted for local and cross-border inter-dealer double-counting (ie "net-net" basis). <sup>2</sup> Because two currencies are involved in each transaction, the sum of the percentage shares of individual currencies totals 200% instead of 100%. <sup>3</sup> Turnover for years prior to 2013 may be underestimated owing to incomplete reporting of offshore trading in previous surveys. Methodological changes in the 2013 survey ensured more complete coverage of activity in emerging market and other currencies. <sup>4</sup> Turnover may be underestimated owing to incomplete reporting of offshore trading

Source: Bank of International Settlement triennial report for Forex market

From the table it is clear that although the share of India in Global Forex turnover is insignificant but it is increasing over the years and it implies that more and more investors are putting their money in investments as compared to previous years and in past 2-3 years India is one of most liked destination for investors. So risk analysis of forex market in India is important and measurement can be done by measuring volatility.

#### **1.2 Volatility and its types**

Volatility means standard deviation of change in value of a financial instrument over a specified period of time. Volatility Foreign exchange market is the variation in foreign exchange rate over a given period of time. High volatility means high risk. Volatility is measured by calculating standard deviation of returns. Standard deviation tells how far the values of are dispersed from the average.

Volatility can be specified as

- Daily Volatility
- Hourly Volatility
- Weekly Volatility
- Annualized volatility



### **1.3 Volatility in India Forex Market**

Indian foreign market has seen a drastic change from the start of 90's as the government of India has adopted a liberalized approached and opened the country for investors. It has

observed episodes of heightened volatility, the latest being post May 22, 2013 volatility on fears of tapering of quantitative easing by the US Fed. Excessive volatility of forex market directly effect the foreign trade and investment. Forex market play a critical role in facilitating cross-border trade, investment, and financial transactions. It allow firms making transactions in foreign currencies to convert the currencies or deposits they have into the currencies or deposits of their choice. The importance of foreign exchange markets has grown with increased global economic activity, trade, and investment, and with technology that makes real-time exchange of information and trading possible. In a market determined exchange rate system, excessive exchange rates volatility, which is out of line with economic fundamentals, can impose real costs on the economy through its effects on international trade and investment. Moreover, at times, pressures from foreign exchange markets could complicate the conduct of monetary policy. So it is a prime concern of policy makers and researchers especially in a growing market like India.

So for our study purpose we can broadly divide the last two decades of Indian forex market volatility into following five major phases:

- i. Liberalization to Mexican Crisis-1990-96
- ii. East Asian Crisis-1997-98
- iii. Episode of Global Crisis due to terror attack-2000-01
- iv. Global Financial Crisis 2008-09 to 2011-12
- v. Euro Zone Crisis 2011-12

#### **I. Liberalization to Mexican Crisis-1990-96**

With the advent of liberalized approach in Indian market post 1990 reforms India forex market saw a major change. It marked capital inflows on the account of liberalization. As a result there was a drastic increase in FDI from 1993 at US\$341 million to US\$ 620 million in 1994.But at the same time CAD increased from 0.4 % of GDP to 1.6% of GDP resulting in increased WPI inflation from 8.4% to 12.6% for the aforesaid period. The GDP growth accelerated from 5.7% to 7.3%. The period from 1995 to 1996 witnessed intense market volatility due to Mexican Currency Crisis of 1994. In this period the rupee depreciated from 31.40 per US\$ to 36.48 per US\$ as data compiled by RBI. The graphical presentation of same is given below:



*Forex Volatility August 1995- March 1996*

Timely intervention by government and our central bank RBI has brought checks to this volatility.

### **II. East Asian Crisis-1997-98**

The period of 1997-98 again saw a huge volatility in Indian forex market due to the economic sanctions imposed on India by many industrialized nations in the wake of Pokharan Nuclear Tests. The monthly average of Rs-\$ exchange rate which was quite stable at 35.92 per US\$ shooted to 42.76 per US\$. The same is graphically represented as under:



*Forex Volatility August 1997-August 1998*

### **III. Episode of Global Crisis due to terror attack-2000-01**

Due to major terrorist attack on 11 September 2011 in America the world forx market was badly shaken and India was no exception. This created an environment of uncertainty resulting into high volatility.



*Global Crisis due to Terror Attacks 2001*

### **IV. Global Financial Crisis 2008-09 to 2011-12**

The most challenging time of Indian Forex market came in year 2008. With a stable global environment and with robust economic environment country was witnessing a GDP atover 9% and strong capital inflows. But sudden failure of financial intuitions like Lehman Brothers and a gloomy global market lead to severe forex market volatility. Due to slowdown the export level of country decreased resulting in rise in Rs-\$ exchange rates as shown below:



*Global Financial Crisis*

#### **V. Euro Zone Crisis 2011-12**

In the aftermath of the global financial crisis and the Euro zone debt crisis, emerging economies like India faced higher uncertainty and volatility. In this period the rupee saw a sharp downfall of around 20%. The same is represented here as under:



So from above division of it is very evident that in the last two decades our Forex Market is been hit by various internal and external factors which has caused market volatility. But same is successfully averted till now by effectively predicting the volatility and prompt actions and policy making by our central bank RBI. One more aspect which is shown in the case study of India is that flexibility and pragmatism have been the key of success of our economy. According to Pattanaik and Sohoo  $(2001)$ : Kohli $(2000)$  and RBI $(2005-06)$  "An important aspect of the policy response in India to the various episodes of volatility has been market intervention combined with monetary and administrative measures to meet the threats to fi3nancial stability while complementary or parallel recourse has been taken to communications through speeches and press releases. Empirical evidence in the Indian case has generally suggested that in the present day managed fl oat regime of India, intervention has served as a potent instrument in containing the magnitude of exchange rate volatility of the rupee and the intervention operations do not influence as much the level of rupee."Thus it is very clear from above that an effective tool is nece1.3ssary for predicting and acting over market volatility.

**1.4 Impact of volatility on Foreign exchange market** The foreign exchange volatility has a great impact in pricing of currency derivatives. According to Santis et al. 1998 "A major part of global foreign exchange market includes forward contracts and currency swaps .This means possessing the knowledge of currency volatility will help an individual to formulate hedging and investment strategies .Hedging strategies also depend upon the volatility of

foreign exchange rate. Hence, investing in foreign markets that are exposed to this foreign currency exchange rate risk should hedge for any source of risk that is not compensated in terms of expected returns."

**1.5 Development of models to forecast volatility** Many researchers are carried out from time to time on the characteristics of the foreign currency volatility. To name a few like Friedman and Vandersteel (1982) which defined "returns are non-linear temporal dependence and the distribution of exchange rate returns are leptokurtic."Their studies have found that large and small changes in returns are ' clustered' together over time, and that their distribution is bell-shaped, symmetric and fat-tailed. These characteristics of data are normally thought to be captured by using the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982) or the Generalised ARCH (GARCH) model developed by Bollerslev (1986), which is an extension of the ARCH model to allow for a more flexible lag structure.

Initially use of ARCH/GARCH model is very common in predicting stock market volatility in finance and economics but the use of ARCH/GARCH model in foreign exchange volatility gave very interesting results. Hseih(1989) was the pioneer who used ARCH in modeling the currency exchange volatility. In his studies on foreign exchange volatility he concluded that the data contain no linear correlation rather he was having substantial evidence which indicates the presence of nonlinearity in a multiplicative rather than additive form. He further concludes that a generalized ARCH (GARCH) model can explain a large part of the nonlinearities for all five exchange rates.

Since then the applications of ARCH and GARCH in modeling foreign exchange volatility have increased manifold and a number of studies and papers are published where each one indicating some improvement over other. To state an example Bollerslevet at. (1992) indicated that "the squared returns of not only exchange rate data, but all speculative price series, typically exhibit autocorrelation in that large and small errors tend to cluster together in contiguous time periods in what has come to be known as volatility clustering." Further French et at. 1987; Franses and Van Dijk 1996; Choo et al. 1999 proved that small lag such as GARCH(1,l) is sufficient to model the variance changing over long sample periods

**1.6 Scope and relevance of research** Although the GARCH model is very effective in removing the excess kurtosis in returns, it failed to cope with the skewness of the distribution of returns, especially the financial time series which are commonly skewed. Hence, the forecasts and forecast error variances from a GARCH model can be expected to give biased results for skewed time series. To reolve this short coming a few modifications to the GARCH model have been suggested time to time by various economists and researchers which explicitly take into account skewed distributions. One such suggestion was given by Nelson (1990) of the alternatives of non-linear models that can cope with skewness is the Exponential GARCH or EGARCH model.

For simplicity this study is limited to volatility forecast by basic GARCH model and further research can be done in this subject by using Exponential GARCH or EGARCH model.

#### **CHAPTER 2**

#### **LITERATURE REVIEW**

Many investors and generations of finance students often have an incomplete appreciation of the differences between volatility, standard deviation, and risk. It is worth elucidating some of the conceptual issues here. In finance, volatility is often used to refer to standard deviation, σ, or variance,  $\sigma^2$ , computed from a set of observations as,

$$
\sigma = \frac{1}{N-1} \sum_{T=1}^{N} (R_T - R)^2
$$
 (1)

Where R is the mean return. The sample standard deviation statistic  $\sigma$  is a distribution free parameter representing the second moment characteristic of the sample. Only when  $\sigma$  is attached to a standard distribution, such as a normal or a t distribution, can the required probability density and cumulative probability density be derived analytically. Indeed,  $\sigma$  can be calculated from any irregular shape distribution, in which case the probability density will have to be derived empirically. In the continuous time setting,  $\sigma$  is a scale parameter that multiplies or reduces the size of the fluctuations generated by the standard wiener process. Depending on the dynamic of the underlying stochastic process and whether or not the parameters are time varying, very different shapes of returns distributions may result. So it is meaningless to use s as a risk measure unless it is attached to a distribution or a pricing dynamic. When s is used to measure uncertainty, the users usually have in mind, perhaps implicitly, a normal distribution for the returns distribution.

Standard deviation, σ, is the correct dispersion measure for the normal distribution and some other distributions, but not all. Other measures that have been suggested and found useful include the mean absolute return and the inter-quantile range. However, the link between volatility and risk is tenuous; in particular, risk is more often associated with small or negative returns, whereas most measures of dispersion make no such distinction. The Sharpe ratio, for example, defined as return in excess of riskfree rate divided by standard deviation, is frequently used as an investment performance measure. It incorrectly penalizes occasional high returns. The idea of "semivariance," an early suggestion by Harry Markowitz (1991), which only uses the squares of returns below the mean, has not been widely used, largely because it is not operationally easy to apply in portfolio construction.

**2.1 Volatility Definition and Measurement** As mentioned previously, volatility is often calculated as the sample standard deviation, which is the square root of equation (1). Stephen Figlewski (1997) notes that since the statistical properties of sample mean make it a very inaccurate estimate of the true mean, especially for small samples, taking deviations around zero instead of the sample mean as in equation (1) typically increases volatility forecast accuracy. There are methods for estimating volatility that are designed to exploit or reduce the influence of extremes. While equation (1) is an unbiased estimate of  $\sigma^2$ , the square root of  $\sigma^2$  is a biased estimate of  $\sigma$  due to Jensen inequality. Zhuanxin Ding, Clive Granger, and Robert Engle (1993) suggest measuring volatility directly from absolute returns.To understand the continuous time analogue of (1), we assume for the ease of exposition that the instantaneous returns are generated by the continuous time martingale,

$$
\mathrm{dpt} = \sigma_t \, \mathrm{dW}_{p,t} \tag{2}
$$

where  $dW_{p,t}$  denotes a standard wiener process. From (2) the conditional variance for the oneperiod returns, r t+ 1= p t+ 1- p<sub>t</sub>, is  $\int_0^1 \sigma_{t+\tau}^2$  $\int_0^1 \sigma_{t+\tau}^2 d\tau$ , which is also known as the integrated volatility over the period t to  $t + 1$ . This quantity is of central importance in the pricing of derivative securities under stochastic volatility (see John Hull and Alan White 1987). While pt can be observed at time t,  $\sigma_t$  is an unobservable latent variable that scales the stochastic process  $dW_{p,t}$ continuously through time.

Time t volatility is theoretically observable from the sample path of the return process so long as the sampling process is frequent enough. The term realized volatility has been used in William Fung and David Hsieh (1991), and Torben Andersen and Tim Bollerslev (1998), to mean the sum of intraday squared returns at short intervals such as fifteen- or five-minutes. Such a volatility estimator has been shown to provide an accurate estimate of the latent process that defines volatility. Characteristics of financial market data used in these studies suggest that returns measured at an interval shorter than five minutes are plagued by spurious serial correlation caused by various market microstructure effects including nonsynchronous trading, discrete price observations, intraday periodic volatility pattern, and bid-ask bounce. Andersen and Bollerslev (1998) and George Christodoulakis and Satchell (1988) show how the inherent noise in the approximation of actual and unobservable volatility by square returns results in misleading forecast evaluation. These theoretical results turn out to have a major implication for volatility forecasting research.

**2.2 Stylized Facts about Financial Market Volatility** There are several salient features about financial time series and financial market volatility that are now well documented. These include fat tail distributions of risky asset returns, volatility clustering, asymmetry and mean reversion and comovements of volatilities across assets and financial markets. More recent research finds correlation among volatility is stronger than that among returns and both tend to increase during bear markets and financial crises. Since volatility of financial time series has complex structure, Francis Diebold et al. (1998) warn that forecast estimates will differ depending on the current level of volatility, volatility structure (e.g. the degree of persistence and mean reversion, etc.) and the forecast horizon. These will be made clearer in the discussions below. If returns are iid(independent and identically distributed, or strict white noise), then variance of returns over a long horizon can be derived as a simple multiple of single period variance. But, this is clearly not the case for many financial time series because of stylized facts listed above. While a point forecast of  $\sigma_t$  becomes very noisy as t tends to ∞, a cumulative forecast becomes more accurate because of errors cancellation and volatility mean reversion unless there is a fundamental change in the volatility level or structure. Some studies find volatility time series appear to have a unit root (Philip Perry 1982, and Adrian Pagan and G. William Schwert 1990). Other papers find some volatility measures of daily and intra-day returns have a long memory property (see Granger, Ding, and Scott Spear 2000 for examples and references). The autocorrelations of variances, and particularly those of mean absolute deviations, stay positive and significantly above zero for lags up to a thousand or more. These findings are important because they imply that a shock in the volatility process will have a long-lasting impact. Complication in relation to the choice of forecast horizon is partly due to volatility mean reversion. In general, volatility forecast accuracy improves as data sampling frequency increases relative to forecast horizon (Andersen, Bollerslev, and Steve Lange 1999). However, for volatility forecasts over a long horizon, Figlewski (1997) finds forecast error doubled in size when daily data, instead of monthly data, is used to forecast volatility over 24 months. In some cases, e.g. when the forecast horizon exceeds ten years, a volatility estimate calculated using weekly or monthly data is better because volatility mean reversion is difficult to adjust using high frequency data. In general, model based forecasts lose supremacy when the forecast horizon increases with respect to the data frequency. For forecast horizons that are longer than six months, a simple historical method using low frequency data over a period at least as long as the forecast horizon works best (Andrew Alford and James Boatsman 1995; and Figlewski 1997). As far as sampling frequency is concerned, Feike Drost and Theo Nijman (1993) prove, theoretically and for a special case (i.e. the  $GARCH(1,1)$  process), that volatility structure should be preserved through intertemporal aggregation. This means that whether one models volatility at the hourly, daily, or monthly intervals, the volatility structure should be the same. But it is well known that this is not the case in practice; volatility persistence, which is highly significant in daily data, weakens as the frequency of data decreases. This further complicates any attempt to generalize volatility patterns and forecasting results.

**2.3. Models Used in Volatility Forecasting** In this section are described various popular time series volatility models that use the historical information set to formulate volatility forecasts and a second approach that derives market estimates of future volatility from traded option prices. Nonparametric methods for volatility forecasting have been suggested. But, as nonparametric methods were reported to perform poorly (Pagan and Schwert 1990; and Kenneth West and Dongchul Cho 1995), they will not be discussed here. Also excluded from discussion here are volatility models that are based on neural networks (Michael Hu and Christ Tsoukalas 1999; genetic programming, e.g. Zumbach, Pictet, and Masutti 2001; time change and duration, e.g. Cho and Frees 1988, and Engle and Russell 1998).

**2.3.1 Times Series Volatility Forecasting Models** Stephen Brown (1990), Engle (1993), and Abdurrahman Aydemir (1998) contain lists of time series models for estimating and modelling volatility. Kroner (1996) explains how volatility forecasts can be created and used. All models described in this section capture volatility persistence or clustering. Others take into account volatility asymmetry also. It is quite easy to construct a supply and demand model for financial assets, with supply a constant and demand partly driven by an external instrument that enters nonlinearity, that will produce a model for financial returns that is heteroskedastic. Such a model is to some extent "theory based" but is not necessarily realistic. The pure time series models discussed in this section are not based on theoretical foundations but are selected to capture the main features of volatility found with actual returns. If successful in this, it is reasonable to expect that they will have some forecasting ability.

**2.3.1.1 Predictions Based on Past Standard Deviations** This group of models starts on the basis that  $\sigma_{ti}$  for all t > 0 is known or can be estimated at time t 1. The simplest historical price model is the Random Walk model, where  $\sigma_{t-1}$  is used as a forecast for  $\sigma_t$ .

Extending this idea, we have the Historical Average method, the simple Moving Average method, the Exponential Smoothing method and the Exponentially Weighted Moving Average method. The Historical Average method makes use of all historical standard deviations while the Moving Average method discards the older estimates. Similarly, the Exponential Smoothing method uses all historical estimates, and the Exponentially Weighted Moving Average (EWMA ) method uses only the more recent ones. But unlike the previous two, the two exponential methods place greater weights on the more recent volatility estimates. All together, the four methods reflect a tradeoff between increasing the number of observations and sampling nearer to time t. The Riskmetrics ™ model uses the EWMA method. The Smooth Transition Exponential Smoothing model, proposed by James Taylor (2001), is a more flexible version of exponential smoothing where the weight depends on the size, and sometimes the sign as well, of the previous return. Next we have the Simple Regression method that expresses volatility as a function of its past values and an error term. The Simple Regression method is principally autoregressive. If past volatility errors are also included, one gets the ARMA model for volatility. Introducing a differencing order I(d), we get ARIMA when  $d = 1$  and ARFIMA when  $d < 1$ . Finally, we have the Threshold Autoregressive model, where the thresholds separate volatility into states with independent simple regression models and noise processes for volatility in each state. Apart from Random Walk and Historical Average, successful applications of models described in this section normally involve searching for the optimal lag length or weighting scheme in an estimation period for out-of-sample forecasting. Such optimization generally involves minimizing insample volatility forecast errors. A more sophisticated forecasting procedure would involve constant updating of parameter estimates when new information is observed and absorbed into the estimation period.

**2.3.1.2 ARCH Class Conditional Volatility Models** A more sophisticated group of time series models is the ARCH family, which is extensively surveyed in Anil Bera and Matthew Higgins (1993), Bollerslev, Ray Chou, and Kenneth Kroner (1992), Bollerslev, Engle, and Nelson (1994), and Diebold and Jose Lopez (1995). In contrast to other models described ARCH class models do not make use of sample standard deviations, but formulate conditional variance, ht, of returns via maximum likelihood procedure. Moreover, because of the way ARCH class models are constructed,  $h_t$  is known at time  $t_1$ . So the one-step ahead forecast is readily available. Forecasts that are more than one step ahead can be formulated based on an iterative procedure. The first example of ARCH model is  $ARCH(q)$  (Engle 1982) where  $h_t$  is a function of q past squared returns. In GARCH (p, q) (Bollerslev 1986, and Taylor 1986), additional dependencies are permitted on  $p$  lags of past  $h_t$ . Empirical findings suggest that GARCH is a more parsimonious model than ARCH, and GARCH(1,1) is the most popular structure for many financial time series. It turns out that Riskmetrics ™ EWMA is a non-stationary version of GARCH(1,1) where the persistence parameters sum to 1 and there is no finite fourth moment. Such a model is often called an integrated model, which should not be confused with integrated volatility.While unconvincing theoretically as a volatility generating process, an integrated model for volatility can nevertheless be estimated and has been shown to be powerful for prediction over a short horizon, as it is not conditioned on a mean level of volatility, and as a result it adjusts to changes in unconditional volatility quickly.

The EGARCH (Exponential GARCH) model (Nelson 1991) specifies conditional variance in logarithmic form, which means that there is no need to impose estimation constraint in order to avoid negative variance. With appropriate conditioning of the parameters, this specification captures the stylized fact that a negative shock leads to a higher conditional variance in the subsequent period than a positive shock would. Other models that allow for nonsymmetrical dependencies are the TGARCH (Threshold GARCH) which is similar to the GJRGARCH (Lawrence Glosten, Ravi Jagannathan, and David Runkle 1993), QGARCH (Quadratic GARCH) and various other nonlinear GARCH reviewed in Philip Franses and Dick van Dijk (2000). Both ARCH and GARCH models have been implemented with a James Hamilton (1989) type regime switching framework, where volatility persistence can take different values depending on whether it is in high or low volatility regimes. The most generalized form of regime switching model is the RS-GARCH(1,1) model used in Stephen Gray (1996) and Franc Klaassen (2002). As mentioned before, volatility persistence is a feature that many time series models are designed to capture. A GARCH model features an exponential decay in the autocorrelation of conditional variances. However, it has been noted that squared and absolute returns of financial assets typically have serial correlations that are slow to decay, similar to those of an I( d) process. A shock in the volatility series seems to have very "long memory" and impact on future volatility over a long horizon. The Integrated GARCH (IGARCH) model of Engle and Bollerslev (1986) captures this effect but a shock in this model impacts upon future volatility over an infinite horizon, and the unconditional variance does not exist for this model. This gives rise to FIGARCH( p, d, q) in Richard Baillie, Bollerslev, and Hans Mikkelsen (1996) and FIEGARCH(p, d, q) in Bollerslev and Mikkelsen (1996) with d<sup>3</sup> 0. Provided that  $d < 0.5$ , the fractional integrated model is covariance stationary. However, as Soosung Hwang and Satchell (1998) and Granger (2001) point out, positive I( d) process has a positive drift term or a time trend in volatility level which is not observed in practice. This is a major weakness of the fractionally integrated model for it to be adopted as a theoretically sound model for volatility. It is important to note that there are many data generating processes, other than an I( d) process, that also exhibit long memory in covariances. The short-memory stationary series with occasional breaks in mean in Granger and Namwon Hyung (2000) is an example. Diebold and Atsushi Inoue (2001) show stochastic regime switching can be easily confused with long memory if only a small amount of regime switching occurs. Gilles Zumbach (2002), on the other hand, captures long memory using IGARCH(2) (i.e. the sum of two IGARCH) and an LM model which aggregates high frequency squared returns with a set of power law weights. 3.1.3 Stochastic Volatility Models In the stochastic volatility (SV) modelling framework, volatility is subject to a source of innovations that may or may not be related to those that drive returns. Modelling volatility as a stochastic variable immediately leads to fat tail distributions for returns. Autoregressive term in the volatility process introduces persistence, and correlation between the two innovative terms in the volatility process and the return process produces volatility asymmetry (Hull and White 1987, 1988). Long memory SV models have also been proposed by allowing volatility to have a fractional integrated order (see Andrew Harvey 1998). For an excellent survey of SV work see Eric Ghysels, Harvey, and Eric Renault (1996), but the subject is rapidly changing. The volatility noise term makes the SV model a lot more flexible, but as a result the SV model has no closed form, and hence cannot be estimated directly by maximum likelihood. The quasi-maximum likelihood estimation (QMLE) approach of Harvey, Esther Ruiz, and Neil Shephard (1994) is inefficient if volatility proxies are nonGaussian (Andersen and Bent Sorensen 1997). The alternatives are the generalized method of moments (GMM) approach through simulations (Durrell Duffie and Kenneth Singleton 1993), or analytical solutions (Singleton 2001), and the likelihood approach through numerical integration (Moshe Fridman and Lawrence Harris 1988) or Monte Carlo integration using either importance sampling (Jon Danielsson 1994; Michael Pitt and Shephard 1997; J. Durbin and S. J. Koopman 2000) or Markov Chain (e.g. Eric Jacquier, Nicholas Polson, and Peter Rossi 1994; Sangjoon Kim, Shephard, and Siddhartha Chib 1998).

**2.4. Comparing Forecast Errors of Different Models** In the special case where the error distribution of one forecasting model dominates that of another forecasting model, the comparison is straightforward (Granger 1999). In practice, this is rarely the case, and most comparisons are based on the average figure of some statistical measured described above. For statistical inference, West (1996), West and Cho (1995), and West and M. McCracken (1998) show how standard errors for ME, MSE, MAE, and RMSE may be derived taking into account serial correlation in the forecast errors and uncertainty inherent in model parameters estimates that were used to produce the forecasts. In general, West' s (1996) asymptotic theory works for recursive scheme only, where newly observed data is used to expand the estimation period. However, a rolling fixed estimation period method, where the oldest data is dropped whenever new data is added, might be more appropriate if there is non stationarity or time variation in model parameters estimates. Diebold and Roberto Mariano (1995) propose three tests for "equal accuracy" between two forecasting models. The tests relate prediction error to some very general loss function and analyze loss differential derived from errors produced by two competing models. The three tests include an asymptotic test that corrects for series correlation and two exact finite sample tests based on the sign test and the Wilcoxon's signedrank test. Simulation results show that the three tests are robust against non-Gaussian, nonzero mean, serially, and contemporaneously correlated forecast errors. The two sign-based tests in particular continue to work well among small samples. Instead of striving to make some statistical inference, model performance could be judged on some measures of economic significance. Examples of such an approach include portfolio improvement based on volatility forecasts (Fleming, Chris Kirby, and Ostdiek 2000, 2002). Some papers test forecast accuracy by measuring the impact on option pricing errors (G. Andrew Karolyi 1993). In this case, if there is any pricing error in the option model, the mistake in volatility forecast will be cancelled out when the option implied is reintroduced into the pricing formula. So it is not surprising that evaluation that involves comparing option pricing errors often prefers the implied volatility method to all other time series methods. What has not yet been done in the literature is to separate the forecasting period into "normal" and "exceptional" periods. It is conceivable that different forecasting methods are suited for different trading environments.

**2.4.1 Regression Based Forecast Efficiency and Orthogonality Test** The regression-based method for examining the informational content of forecastsentails regressing the "actual", Xi, on the forecasts, Xi, as shown below

$$
X_i = \alpha + \beta X_i + \nu_t \tag{3}
$$

Conditioning upon the forecast, the prediction is unbiased only if  $\alpha = 0$  and  $\beta = 1$ . The standard errors of the parameter estimates are often computed based on Hansen and Hodrick (1980) since the error term, u<sup>i</sup> , is heteroskedastic and serially correlated when overlapping forecasts are evaluated. In cases where there are more than one forecasting models, additional forecasts are added to the right-hand side of (3) to check for incremental explanatory power. Such forecast encompassing testing dates back to Henri Theil (1966). Yock Chong and David Hendry (1986), and Ray Fair and Robert Shiller (1989, 1990), provide further theoretical exposition of such method for testing forecast efficiency. The first forecast is said to subsume information contained in other forecasts if these additional forecasts do not significantly increase the adjusted regression  $\mathbb{R}^2$ . Alternatively, an orthogonality test may be conducted by regressing the residuals from (3) on other forecasts. If these forecasts are orthogonal, i.e. do not contain additional information, then the regression coefficients will not be different from zero. While it is useful to have an unbiased forecast, it is important to distinguish between biasness and predictive power. A biased forecast can have predictive power if the bias can be corrected. An unbiased forecast is useless if forecast errors are always big. For  $X_i$  to be considered as a good forecast,  $Var(u_i)$  should be small and  $R^2$  for the regression should tend to 100 percent

**2.5. Volatility Forecasting Based On Time Series Models** In this section, have been reviewed major findings in that construct volatility forecasts based on historical information only. We will make some references to implied volatility forecasts when we discuss forecasting performance of SV and long memory volatility models. Main findings regarding implied volatility forecasts will be discussed in section

**2.5.1 Pre-ARCH Era and Non-ARCH Debate** Taylor (1987) is one of the earliest to test time series volatility forecasting models before ARCH/GARCH permeated the volatility literature. Taylor (1987) studies the use of high, low, and closing prices to forecast one to twenty days DM/\$ futures volatility and finds a weighted average composite forecast to perform best. Wiggins (1992) also gives support to extreme value volatility estimators. In the pre-ARCH era, there were many other findings covering a wide range of issues. Dimson and Marsh (1990) find ex ante time-varying optimized weighting schemes do not always work well in out-of-sample forecasts. Sill (1993) finds S&P 500 volatility is higher during recession and that commercial T-Bill spread helps to predict stock market volatility. Andrew Alford and James Boatman (1995) find, from a sample of 6,879 stocks, that adjusting historical volatility towards volatility estimates of comparable firms in the same industry and size provides a better five-year ahead volatility forecast. Alford and Boatman (1995), Figlewski (1997), and Figlewski and Green (1999) all stress the importance of having a long enough estimation period to make good volatility forecasts over a long horizon.

**2.5.2 The Explosion of ARCH/GARCH Forecasting Contests** Vedat Akigray (1989) is one of the earliest to test the predictive power of GARCH and is commonly cited in many later GARCH studies, though an earlier investigation appeared in Taylor (1986). In the following decade, there were no less than twenty papers testing GARCH's predictive power against other time series methods and against option implied volatility forecasts. The majority of these forecast volatility of major stock indices and exchange rates. The ARCH/GARCH models and their variants have many supporters. Akgiray finds GARCH consistently outperforms EWMA and HIS (i.e. historical volatility derived from standard deviation of past returns over a fixed interval) in all subperiods and under all evaluation measures. Pagan and Schwert (1990) find EGARCH is best especially in contrast to nonparametric methods. Despite a low R2, Cumby, Figlewski, and Hasbrouck (1993) conclude that EGARCH is better than naïve historical methods. Figlewski (1997) finds GARCH superiority confined to stock market and for forecasting volatility over a short horizon only. Cao and Tsay (1992) find TAR provides the best forecast for large stocks and EGARCH gives the best forecast for small stocks, and they suspect that the latter might be due to a leverage effect. Bali (2000) documents the usefulness of GARCH models, the nonlinear ones in particular, in forecasting one-week ahead volatility of U.S. T-Bill yields. Other studies find no clear-cut result. These include Keun Yeong Lee (1991), West and Cho (1995), Chris Brooks (1998), and David McMillan, Alan Speight, and Dwain Gwilym (2000). Some models work best under different error statistics (e.g. MAE, MSE), different sampling schemes (e.g. rolling fixed sample estimation, or recursive expanding sample estimation), different time periods and for different assets. Timothy Brailsford and Robert Faff (1996) comment that the GJR-GARCH model has a marginal lead while Franses and Van Dijk (1996) claim the GJR forecast cannot be recommended. Many of these inconclusive studies share one or more of the following Characteristics:

- (i) they test a large number of very similar models all designed to capture volatility persistence;
- (ii) they use a large number of error statistics, each of which has a very different loss function;
- (iii) they forecast and calculate error statistics for variance and not standard deviation, which makes the difference between forecasts of different models even smaller, yet the standard error is large as the fourth moment is unstable; and
- (iv) they use squared daily, weekly, or monthly returns to proxy daily, weekly, or monthly "actual volatility," which results in extremely noisy volatility estimates; the noise makes the small differences between forecasts of similar models indistinguishable.

Unlike the ARCH class model, the "simpler" methods, including the EWMA method, do not separate volatility persistence from volatility shocks and most of them do not incorporate volatility mean reversion. The GJR model allows the volatility persistence to change relatively quickly when return switches sign from positive to negative and vice versa. If unconditional volatility of all parametric volatility models is the same, then GJR will have

the largest probability of an underforecast. The "simpler" methods tend to provide larger volatility forecasts most of the time because there is no constraint on stationarity or convergence to the unconditional variance, and may result in larger forecast errors. This possibly explains why GJR was the worst performing model in Franses and Van Dijk (1996) because they use MedSE (median squared error) as their sole evaluation criteria. In Brailsford and Faff (1996), the GJR(1,1) model outperforms the other models when MAE, RMSE, and MAPE are used.

There are some merits to using "simpler" methods, and especially models that include long distributed lags. As ARCH class models assume variance stationarity, the forecasting performance suffers when there are changes in volatility level. Parameters estimation becomes unstable when data period is short or when there is a change in volatility level. This has led to GARCH convergence problem in several studies (e.g. Tse and Tung 1992, and Walsh and Tsou 1998). Stephen Taylor (1986), Tse (1991), Tse and Tung (1992), Boudoukh, Richardson, and Whitelaw (1997), Walsh and Tsou (1998), Ederington and Guan (1999), Ferreira (1999), and James Taylor(2001) all favor some forms of exponential smoothing method to GARCH for forecasting volatility of a wide range of assets across equities, exchange rates and interest rates. In general, models that allow for volatility asymmetry came out well in the forecasting contest because of the strong negative relationship between volatility and shock. Charles Cao and Ruey Tsay (1992), Ronald Heynen and Harry Kat (1994), Lee (1991), and Adrian Pagan and G. William Schwert (1990) favor the EGARCH model for volatility of stock indices and exchange rates, whereas Brailsford and Faff (1996) and Taylor (2001) find GJR-GARCH to outperform GARCH in stock indices. Turan Bali (2000) finds a range of nonlinear models works well for interest rate volatility**.** The different types of GARCH models are summarized below-



**2.6 The Arrival of SV Forecasts** The SV model has an additional innovative term in the volatility dynamics and, hence, is more flexible than ARCH class models and was found to fit financial market returns better and have residuals closer to standard normal. It is also closer to theoretical models in finance and especially those in derivatives pricing. However, largely due to the computation difficulty, volatility forecast based on the SV model was not studied until the mid 1990's, a decade later than ARCH/GARCH development. In a PhD thesis, Heynen (1995) finds SV forecast is best for a number of stock indices across several continents. At the time of writing, there are only six other SV studies and their view about SV forecasting performance is by no means unanimous. Heynen and Kat (1994) forecast volatility for seven stock indices and five exchange rates and find SV provides the best forecast for indices but produces forecast errors that are ten times larger than EGARCH's and GARCH's for exchange rates. Jun Yu (2002) ranks SV top for forecasting New Zealand stock market volatility, but the margin is very small, partly because the evaluation is based on variance and not standard deviation. Lopez (2001) finds no difference between SV and other time series forecasts using conventional error statistics. All three papers have the 1987 crash in the in-sample period, and the impact of the 1987 crash on their result is unclear. Three other studies—Hagen Bluhm and Yu (2000); Chris Dunis, Jason Laws, and Stephane Chauvin (2000); and Eugenie Hol and Koopman (2002)—compare SV and other time series forecasts with option implied volatility forecast. Dunis et al. (2000) find combined forecast is the best for six exchange rates so long as the SV forecast is excluded. Bluhm and Yu (2000) rank SV equal to GARCH. Both Bluhm and Yu (2000) and Hol and Koopman (2002) conclude that implied is better than SV for forecasting stock index volatility. 5.4 Recent Development in Long Memory Volatility Models Volatility forecasts based on models that exploit the long memory (LM) characteristics of volatility appear rather late in the literature. These include Andersen, Bollerslev, Diebold and Labys (2002), Jon Vilasuso (2002), Zumbach (2002) and three other papers that compare LM forecasts with option implied volatility, viz. Kai Li (2002), Martens Martens and Jason Zein (2002), and ShiuYan Pong et al. (2002). It has been pointed out that other short memory models (e.g. extreme values, breaks, mixture of distribution, and regime switching) are also capable of producing long memory in second moments, and each of them entails a different data generating process. At the time of writing, there is no direct contest between these and the LM models. An earlier LM paper by Hwang and Satchell (1998) uses LM models to forecast Black-Scholes implied volatility of equity option. This paper contains some useful insights about properties of LM models, but since we are focusing on forecasting volatility of the underlying asset rather than implied volatility, the results of Hwang and Satchell (1998) will not be discussed here. Examples of LM models include the FIGARCH in Baillie, Bollerslev, and Mikkelsen (1996) and FIEGARCH in Bollerslev and Mikkelsen (1996). In Andersen, Bollerslev, Diebold, and Labys (2002) a vector autoregressive model with long distributed lags was built on realized volatility of three exchange rates, which they called the VAR-RV model. In Zumbach (2002) the weights apply to time series of realized volatility following a power law, which he called the LM-ARCH model. As noted before in section 3.1.2, all fractional integrated models of volatility have a non-zero drift in the volatility process. In practice the estimation of fractional integrated models requires an arbitrary truncation of the infinite lags and as a result, the mean will be biased. Zumbach's (2002) LM-ARCH will not have this problem because of the fixed number of lags and the way in which the weights are calculated. Hwang and Satchell's (1998) scaled-truncated logARFIMA model is mean adjusted to control for the bias that is due to this truncation and the log transformation. Among the historical price models, Vilasuso (2002) finds FIGARCH produces significantly better one- and ten-day ahead volatility forecasts for five major exchange rates. Zumbach (2002) produces only one-day ahead forecasts and finds no difference among model performance. Andersen, Bollerslev, Diebold, and Labys (2002) find the realized volatility constructed VAR model, i.e. VAR-RV, produces the best one- and tenday ahead volatility forecasts. It is difficult to attribute this superior performance to the LM model alone because the VAR structure allows a cross series linkage that is absent in all other univariate models, and we also know that the more accurate realized volatility estimates would result in improved forecasting performance, everything else equal. The other three papers that compare forecasts from LM models with implied volatility forecasts generally find implied volatility forecast produces the highest explanatory power. Martiens and Zein (2002) find log-ARFIMA forecast beats implied in S&P 500 futures but not in ¥US\$ and crude oil futures. Li (2002) finds implied produces better short-horizon forecast, whereas the ARFIMA provides better forecast for a sixmonth horizon. However, when regression coefficients are constrained to be  $\alpha = 0$  and  $\beta = 1$ , the regression  $\mathbb{R}^2$  becomes negative at long horizons. From our discussion in section 4.3, this suggests that volatility at the six-month horizon might be better forecast using the unconditional variance instead of model-based forecasts. As all LM papers in this group were written very recently and after the publication of Andersen and Bollerslev (1998), the realized volatilities are constructed from intra-day high frequency data. When comparison is made with implied volatility forecast, however, the implied volatility is usually extracted from daily closing option prices. Despite the lower data frequency, option implied volatility appears to outperform forecasts from LM models built on high-frequency data.

As given in the introduction that many researchers are carried out from time to time on the characteristics of the foreign currency volatility. To name a few like Friedman and Vandersteel (1982) which defined "returns are non-linear temporal dependence and the distribution of exchange rate returns are leptokurtic."Their studies have found that large and small changes in returns are 'clustered' together over time, and that their distribution is bellshaped, symmetric and fat-tailed. These characteristics of data are normally thought to be captured by using the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982) or the Generalised ARCH (GARCH) model developed by Bollerslev (1986), which is an extension of the ARCH model to allow for a more flexible lag structure.

Hseih(1989) was the pioneer who used ARCH in modeling the currency exchange volatility. In his studies on foreign exchange volatility he concluded that the data contain no linear correlation rather he was having substantial evidence which indicates the presence of nonlinearity in a multiplicative rather than additive form. He further concludes that a generalized ARCH (GARCH) model can explain a large part of the nonlinearities for all five exchange rates.

Bollerslevet at. (1992) indicated that "the squared returns of not only exchange rate data, but all speculative price series, typically exhibit autocorrelation in that large and small errors tend to cluster together in contiguous time periods in what has come to be known as volatility clustering.‖ Further French et at. 1987; Franses and Van Dijk 1996; Choo et al. 1999 proved that small lag such as GARCH(1,l) is sufficient to model the variance changing over long sample periods. GARCH model is very effective in removing the excess kurtosis in returns.

### **CHAPTER 3**

#### **RESEARCH METHODOLOGY**

In this study INR-US\$ pair daily data from January 2007 to January 2017 has been used. 3779 data points have been used for the purpose of volatility forecast. The period is chosen so that 2008 financial crisis can be included in this period.

This long time horizon and has been taken with a view to include significant volatility jumps in the last decade. The frequency of data is considered to be high for a time horizon of 10 years. The source of secondary data is Quandl .com

Eviews has been used for calculation of statistics. Least square method is used to find out the standard error and a histogram is plotted for residuals so that the condition of normality of errors can be validated. Volatility forecasting equation by GARCH with model is used lag period 30.

#### **CHAPTER 4**

### **ANALYSIS**

#### **4.1Validating the condition of normality of errors-**

i. Least square method for calculating standard error

EViews - [Equation: UNTITLED Workfile: CUR-INR::Untitled\]



#### ii. The plot for residuals is given as below:



#### **4.2 Result**

EViews - [Equation: UNTITLED Workfile: CUR-INR::Untitled\]



Dependent Variable: RATE Method: ML - ARCH (Marquardt) - Normal distribution Date: 05/15/17 Time: 01:11 Sample: 1/01/2007 5/07/2017 Included observations: 3778 Convergence achieved after 22 iterations Presample variance: backcast (parameter = 0.7)  $GARCH = C(3) + C(4)*RESID(-1)^n2 + C(5)*GARCH(-1)$ 



Plot for Residual<br> **S** EViews - [Equation: UNTITLED Workfile: CUR-INR::Untitled\I  $\Box$  $\times$ File Edit Object View Proc Quick Options Add-ins Window Help  $-2$ View Proc Object | Print Name | Freeze | Estimate | Forecast | Stats | Resids 70 60  $15<sup>1</sup>$ 50  $10$ 40  $5 30$  $\mathbf{0}$  $-5$ 07 08 09  $10$  $11$  $12$  $13$  $14$  $15<sup>15</sup>$  $16$  $17$ Residual Actual Fitted  $WE = CI$ ENG  $01:12$  $\bullet$  Type here to search  $\begin{picture}(20,20) \put(0,0){\line(1,0){10}} \put(15,0){\line(1,0){10}} \put(15,0){\line(1$  $\left(\fbox{)}\right]$ e 'n Ê E 一区  $\Rightarrow$  $1505201$ 

#### Forecast predicted by GARCH(1,1)



## Conditional Variance





*Conditional Standard Deviation*



*Rate Residuals*



Standard Residuals





### Correlogram of standard residuals

#### **CHAPTER 5 DISCUSSION AND RECOMMENDATIONS**

Normal probability plots and histograms of the fitted GARCH(1,1) models for INR/USD exchange rate indicate that the distribution of the errors of the GARCH(1,1) models are not normal. Closer examination of the skewness and kurtosis of each distribution indicates that each distribution is reasonably symmetric and perhaps slightly skewed left, with tails heavier than those found in the normal distribution. There is no serial correlation and no ARCH effect but the residuals are not normally distributed as can be seen from the histogram. Although residuals are not normal but the estimators of volatility are consistent. So we can use GARCH(1,1) model for measuring volatility in Forex Market.

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