STRUCTURAL RELIABILITY OF

A MULTISTORY BUILDING

A PROJECT REPORT

SUBMITTED IN PARTIAL FULFILMENT OF THE
REQUIREMNETS FOR THE AWARD OF THE DEGREE

OF

MASTER OF TECHNOLOGY

IN

STRUCTURAL ENGINEERING

Submitted by:

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<u>STUDENT DECLARATION</u>

I hereby declare that the project work entitled "**STRUCTURAL RELIABILITY OF A MULTISTORY BUILDING "** Submitted to Department of Civil Engineering, DTU is a record of an original work done by NAVIN KUAMR under the guidance of Mr.GP Awadhiya (ASSOCIATE PROFESSOR) Department of Civil Engineering, DTU, and this project work has not been performed for the award of any Degree or diploma/fellowship and similar project.

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CERTIFICATE

This is to certify that the project entitled "**STRUCTURAL RELIABILITY OF A MULTISTORY BUILDING "** Submitted by **NAVIN KUMAR,** in partial fulfillment of the requirements for award of the degree of **MASTER OF TECHNOLOGY (STRUCTURAL ENGINEERING)** to Delhi Technological University is the record of student's own work and was carried out under my supervision.

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ABSTRACT

The failure of structural systems in civil engineering is a result of decisions taken from the conditions that are not certain and failures of various natures such as design failures, temporary failures and failures resulting from natural hazards that are needed to be tackled.

The art of formulating a mathematical model over which one can get answer to the questions: "What is the probability that a structure behaves in a specified way when given that one or more of its material properties or geometric dimensions and properties are of a random or incompletely known nature, and/or that the actions on the structure in some respects have random or incompletely known properties?"

Reliability of a structure is an extension to the analysis of a structure that is deterministic in nature which leads to the formulation of a mathematical model by which one get the answer to the question: "How is a structure behaving when its material properties, geometric properties and actions all are uniquely given?"

ACKNOWLEDGEMENT

I take this opportunity to express my profound gratitude and deep regards to **Mr. GP Awadhiya**, Department of Civil Engineering, DTU for the consent encouragement, guidance and support throughout the course of this project work.

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CHAPTER 1

INTRODUCTION

1.1.General

The failure of structural systems in civil engineering is a result of decisions taken from the conditions that are not certain and failures of various natures such as design failures, temporary failures and failures resulting from natural hazards that are needed to be tackled.

The art of formulating a mathematical model over which one can get answer to the questions: “What is the probability that a structure behaves in a specified way when given that one or more of its material properties or geometric dimensions and properties are of a random or incompletely known nature, and/or that the actions on the structure in some respects have random or incompletely known properties?”

Probabilistic analysis of a structure is an extension to the analysis of a structure that is deterministic in nature which leads to the formulation of a mathematical model by which one get the answer to the question: “How is a structure behaving when its material properties, geometric properties and actions all are uniquely given?”

“The results can support in determining the reliability of a structure which under a configuration of a given load has sufficient load carrying capacity that are predicted down even to the minute detail.”

Software are available in the modern period to study the reliability of the structure, One of the software that is used in this project is named as COMREL.

Basically any deviation from the maximum load parameters value and any deviation from the load carrying capacity values of a structure when expressed through a load parameter value in the limiting situation leads to raise a question about the safety of a structure, The analysis helps in determining "how much larger than the maximal load parameter -evaluated according to the best conviction - should the ultimate load value be taken in the carrying capacity model in order that the engineer can guarantee that the structure will not fail under service or, at least, that there is an extremely small risk that a failure will occur". The difference in these two values is called the safety margin.

1.2. Objectives and basis of study

Following are the prime objectives :-

- To extravagant the alternative method of probabilistic structures analysis.
- To study the safety margin problems specifically used for the analysis of a structure.
- To determine the reliability of beams and columns as per Indian Standard codes using different probability distribution curves and methods of reliability.

CHAPTER 2

LITRATURE REVIEW

2.1 RESEARCH PAPER SUMMARY:

From quite a long time, a lot of scholars and researchers have found the concept of structural reliability analysis and design of vast interest. There have been different approaches, analysis and design methodologies that have been devised and worked upon subsequently. During the course of this project, guidance from the research papers of some of the renowned scholars in this field has been taken. The review of their papers has been explained in the subsequent section.

R.Ranganathan[1999], The aim is to introduce the probabilistic bases of structural reliability, the techniques and methods of evaluating the reliability of structural components and systems, the methodology in the development of reliability based design criteria ,and the evaluation of partial safety factors. Probabilistic concepts are used in reliability analysis, and in the design of the existing structures. It can also be used for doveloping a design criterion, that is, calibration of codes and development of partial safety factors, the use of which will result in designs with an accepted level of reliability.

A.Der Kurighian, FORM and SORM were used to present the geometry of various random vibrations and solutions. The problems of standard normal random variables which are geometricalrandom vibration problems are identified as obtained from the discretization of the input process. Linear systems when subjected to the excitation as entainled by Guassian, the curiosity problems get characterized by modest geometric forms, which arevector,half space,ellipsoid, and wedge. For responses which are non-Gaussian in nature, the problems are characterized by forms which are non-linearily geometric. The problems which are approximate in nature,solutions to such problems are obtained by use of the first-order and second-order methods of reliability(FORM and SORM).A new outlook for such problems which of random variation has been approximated for their solution.

A.Der kiureghian, and P.-L. Liu, The problem of structural reliability is often formulated in terms of a basic random variables vector X = [Xj . . . X,,]T, which representsquantities which are uncertain in nature such as loads, environmental factors, material properties,structural dimensions, and variables that are introduced to account for errors in modeling and prediction, and a performance function *g(X)* that describes the limit state of the structure in terms of X. The performance function is formulated by convention on account that *g(X)* < 0 denotes the failure structure and *g(X)* > 0 denotes the survival of the structureis known as the *limit-state surface*. The boundaries that are betweenthe failure and safe sets theories are Consistent with Ditlevsen's notion which is based on generalized reliability index, under the probability information which is insufficient in nature,Hence we do seek a formal distribution model for *X* and for transformation *T(-)* such that *Y* becomes a standard normal. As per the ground rules based on selection of the transformation and distribution,

The requirements are stipulated which are as follows-:

1. Simplicity - The strength needed to compute the index of reliability shall be appropriate with the information and quality that is accessible.

2. Consistency - The distribution model shall be able to satisfy the probability rules and it must be consistent with the information available.

3. Invariance - The reliability index β, must be invariant in respect to all conjointly consistent formulations of the transformation and the distribution model.

4. Operability - The distribution model must apply to random numbers and it should

be capable of combining any and all information available.

2.2 BRIEF REVIEW OF PROBABILISTIC PARAMETERS

General

In the conventional deterministic design method, it is assumed that all parameters are not subjected to probabilistic variations. However, it is well known that loads (live load on floors, wind load, ocean waves, earthquake, etc. coming on the structures are random variables. Similarly, the strengths of materials (strength of concrete, steel etc.)and the geometric parameters(dimension of section, effective depth, diameter of bars etc) are subjected to statistical variations. Hence, to be rational in estimation of the structural safety, the random variations of the basic parameters are to be taken into account. Since the load and strength are random variables, the safety of the structure is also a statistical variable.

In overcoming the uncertainties in the design parameters, the safety factor is ensured by taking the smallest value of the strength and the largest value of the load. This way of fixing the safety in design is very conservative and leads to an economical design.

2.2.1 Random variables

The performance of an engineering system, facility or installation is modeled in mathematical terms in conjunction with empirical relations. For a given set of model, system performance is determined on the basis of the model. The basic random variables can be defined as the parameter that carries the uncertain parameters that are considered in the model. These variables must be able to represent any type of uncertainty that are to be included in analysis. The uncertainty, that must be considered are physical uncertainty, the statistical uncertainty and the model uncertainty. The physical uncertainties are typically the uncertainties that are associated with certain kind of loading, the structural geometry, the properties of the material and qualities of the repair. The statistical uncertainty arises due to uncertainty in the statistical information as an example smaller number of material tests undertaken. Finally, the uncertainty related to model must be considered so as to take into account the uncertainty associated with the descriptions that are idealized mathematically so as to approximate any behavior of the structure identified physically.

2.2.2 Mean and Variance

The central tendency (central value) of any random variable is measured by Sample mean. This is the best statistic so as to numerically summarize a distribution and the center of gravity of data.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Where $X_i = X_1, X_2, X_3 \ldots\ldots, X_n$.

The variability or dispersion of any data set is a significant characteristic of the set of data. This dispersion may be described by the sample variance given by

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

2.2.3 Probability density function and cumulative density function

If there's a record of random function $x(t)$.

The values in between are measured and correspondingly time intervals are evaluated. The ratio is to be given by

$$P(X_1 \le X \le X_2) = \frac{\Delta t_1 + \Delta t_2 + \cdots \Delta t_n}{T}$$

Moreover,P(X) gives the probability for X having the value between X_1&X_2during the random process.

Similarly, the probability of *X(t)* smaller than value of *X* is expressed as

$$P(X) = P[X(t) < X] = \lim_{t\to\infty}\sum_i \Delta T_i$$

The delta is for the function *X(t)*which has a value smaller than that specified for X. The function *P(X) is* known as the cumulative density function in equation of the function *X(t)*. The cumulative density function when graphically plotted is a function which increases monotonically.

2.2.4 Some useful probability distributions

Probability distribution

It can be thought of as a mathematical function, that stated in simple terms provides for probability of occurrence of different possible outcomes in an experiment.

In more technical terms, the probability distribution describes a random phenomenon in terms of the probabilities of events. Examples for random phenomena includes the result from an experiment or survey. A probability distribution defined in terms of an underlying sample space, It is the set of all possible outcomes of the random phenomenon being observed.

Normal (Gaussian) Distribution

In probability theory,the most common continuous probability distribution function is normal distribution.Physical quantities expected to be the sum of many independent processes often have distributions that are very nearly normal,used as such in measurement of errors.Many results and methods are derived analytically when the variables are distributed normally usually when there is propogation of uncertainity and least square parameters are involved

The probability density of the normal distribution is:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{\frac{1}{2}(X-\bar{X})^2}{\sigma^2}}$$

$$P(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{\frac{1}{2}(X-\bar{X})^2}{\sigma^2}}$$

Where:

- $\bar{X}$ – is the mean of the distribution.
- σ - is the standard deviation of distribution
- σ^2 -is variance

The corresponding CDF is calculated from:

$$P(x) = \int_{-\infty}^{\infty} P(E)\,d\epsilon = \emptyset\left(\frac{X - \bar{X}}{\sigma}\right)$$

Where

$\emptyset(-)$denotes the standard normal distribution function

$\varphi(-)$ - denotes its probability distribution function

which are defined:

Standard Normal PDF $\varphi(X) = \frac{1}{\sqrt{2\pi}} \int e^{-X^2/2}$

Standard Normal CDF $\emptyset(X) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-U^2/2dU}$

If variables involved are multivariate in nature

then a multivariate normal PDF will be required.

A vector process is used, and the multivariate normal PDF is stated as,

$$P(x) = \left(\frac{1}{2\pi}\right)^{\frac{P}{2}} \frac{1}{\sqrt{|\rho|}} e^{-\frac{x^2}{2}}$$

Where,

$\bar{X}$ is a vector of p-dimensional random variable,

$\bar{x}$ is a vector of their realizations

χ^2 is a scalar calculated from the product

Scalar:

$$\chi^2 = (\bar{x} - \overline{m})^T \rho^{-1} (\bar{x} - \overline{m})$$

$\overline{m}$ is a vector of mean values and

ρ is the covariance matrix of $\bar{x}$

Modulus of row denotes the determinant of ρ

Lognormal Distribution

This is another commonly used distribution. If the variable X has distribution which is normal with specific mean and variance, then the random variable. $Y = e^x$ is distributed lognormally which is written as exponential function of X:

$$Y = e^X \; and \; X = \ln Y$$

Using equation the PDF of the random variable $Y = e^x$, can be obtained as written

Lognormal PDF:

$$P(X) = \frac{1}{\sigma_x \sqrt{2\pi}} \frac{1}{y} e^{-\frac{1}{2}\left(\frac{\ln y - m}{\sigma_x}\right)^2} \quad \text{For}(y>0)$$

Gamma Distribution

It represents the sum of R independently distributed exponential random variables, and those random variables which always take the positive values.

PDF and CDF function are defined as below:

Gamma Dist., PDF: $f_X(X) = \frac{\lambda}{\Gamma(R)} (\lambda X)^{R-1} e^{-\lambda X}$ if $(x \geq 0, \lambda \geq 0)$

Gamma Dist., CDF: $1 - \sum_{k=o}^{R-1} \frac{1}{k!} (\lambda X)^k (\lambda X)^k$ if $(x \geq 0, \lambda \geq 0)$

In which $\Gamma(.)$ represents gamma function as defined:

Gamma function: $\Gamma(x) = \int_0^\infty e^{-u} u^{(x-1)} du$

CHAPTER 3

INTRODUCTION TO STRUCTURAL RELIABILITY

3.1 Introduction

The failure of civil engineering structures are a consequence of decisions made under uncertain conditions and under different type of failures characterized as temporary failures, maintenance failure, design failure, failures caused due to natural hazards are to be addressed. For example, collapse of a bridge is a permanent failure, however if there is a traffic jam on the bridge, it is a temporary failure. If there is overflow in a filter or a pipe due to heavy rainfall, it is a temporary failure. *Thus failure definition is important. It is expressed as in the terms of failure probability and assessed by structure inability to perform the intended function for a specified period of time.* The converse of failure probability is called reliability which is defined in terms of success of a system, therefore reliability of a system *is the probability of a system so as to perform its required function adequately for specified period of time under the stated conditions.*

For convenience, the reliability R_0 is defined in terms of the probability of failure, P_f, which is taken as

$$R_0 = 1 - P_f$$

1. Reliability can be expressed as a probability
2. A quality performance is expected
3. It is expected over a desired period of time
4. The performance is expected under specified conditions

3.2 Levels of reliability methods

The term '**level**' can be described and is best characterised by the extent upto which the information to the problem associated can be used and provided.The methods of safety analysis suggested can be characterised under four basic “levels” (namely level IV, III, II, and I) depending upon the degree of sophistication smeared to the treatment of the several problems.

3.2.1level I methods, Deterministic methods of reliability that uses only one ‘characterstic’ value to ascertain uncertain variable.This method is analogous to the method of deterministic design.

3.2.2 level II methods,Reliability methods employ two values of specific uncertain parameter (i.e., mean and variance) which is supplemented with a measure of corelation to those parameters usually the covariance.

3.2.3 Level III methods,the joint probability function of density of random variables is extended over safety domain. Reliability as expressed in terms of suitable indices of safety, viz., reliability index, β and probabilities of failure.

3.2.4 Level IV methods the methods compare structural prospects with the prospects that are in reference as per the principles of economic ananlys of engineering which are under uncertainity.

3.3 Computation of structural reliability

There has been a need for solving complex problems that have led to the development and use of advanced quantitative methods for modeling. For example, the finite element method has been proved as a valuable concept to determine stability, deformation, earthquake response analysis of problems. There has been a rapid development of computers and computing methods that has facilitated the use for any of such methods. , The question of uncertainty of parameters and their randomness is central to design and analysis. However, it is well known fact that the information that has been derived from methods of analysis will be useful only if inputs are available and only if that data is reliable.

Decisions are made on the basis of information which is incomplete. Hence, It is desirable to use those methods and concepts in planning and design that facilitate evaluation and analysis of uncertainty. Probabilistic methods enable a logical analysis for uncertainty made and these provide a quantitative basis so as to assess the reliability of structures. Consequently, these methods are subsequently used to exercise an engineering judgment.

The basic structural reliability problem takes into account load effect (*s)* which is resisted by resistance *(r)*. they are described by a probability density function, $f_S(\ )$ and $f_R(\ )$ respectively. It is essential that rand *s to*be expressed in the same units. For ease, but without any loss of generality,safety of a structural element will be measured and, the structural element will be considered to have failed if its resistance *r*is less than the resultant stress*s* acting on it. The probability $P_f$of failure of the structural element can be stated in any of the following ways,

$$P_f = P(r \leq s)$$

$$= P(r - s \leq 0)$$

$$= P({}^{r}/_{s} \leq 1)$$

$$= P(ln r - ln \mathbf{s}) \leq 0)$$

or, in general

$$= P(g(r,s) \leq 0)$$

Where *G( )* is designated the *limit state function* and the probability of failure is similar with theprobability of limit state violation.

$$P_f = P(r - s \leq 0) = \iint_D f_{rs}(R,S)dRdS$$

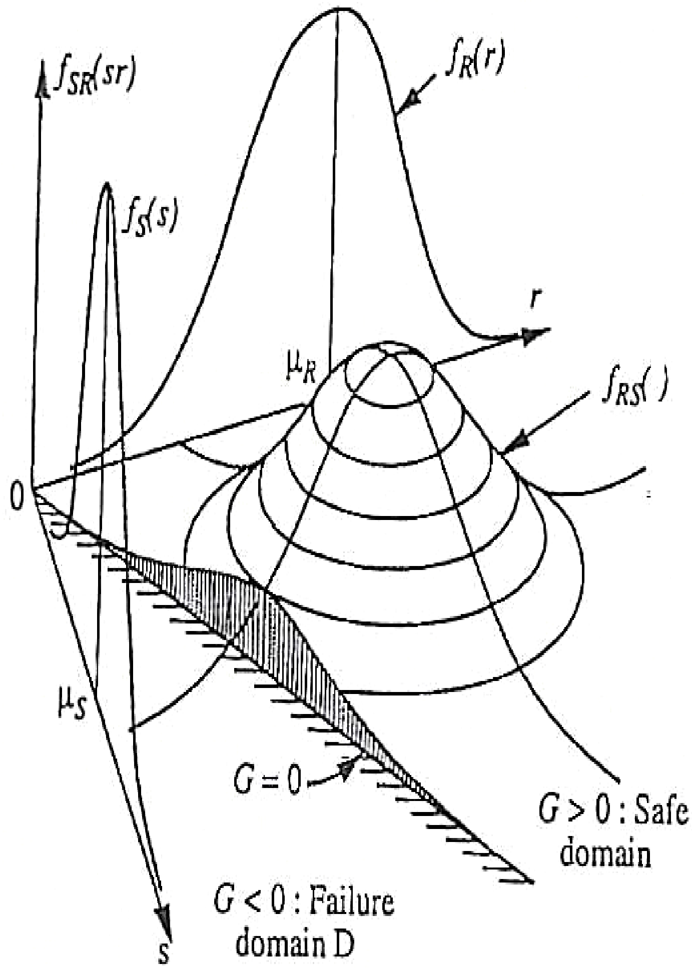


Fig. 1 -Joint density function $f_{RS}(r,s)$, marginal density functions $f_R(r)$ and $f_S(s)$ and failure domain *D*

When *R* as well as S is an independent function,

$$f_{RS}(rs) = f_R(r)f_S(s)$$

Moreover, equation for probability of failures then becomes:

$$P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{s\geq r} f_R(r)f_S(s)drds = \int_{-\infty}^{\infty} F_R(x)f_s(x)dx$$

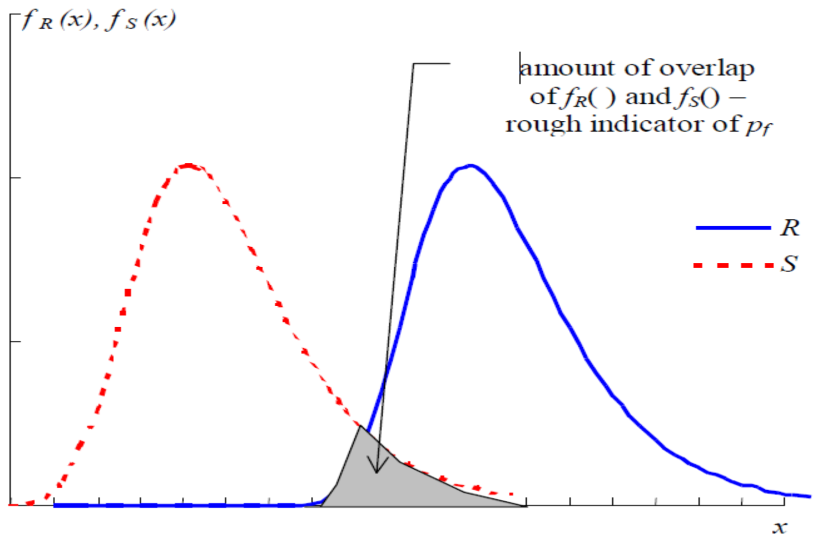


Fig 2 : Basic *R*-*S* problem: f_R( ) f_S( ) representation

Space of State Variables

For analysis, there's a need to define state variables of any problem. The *state variables or parameters* are load and resistance parameters used for formulation of the performance function. For 'n' number of state variables, the specified limit state function represents function of 'n' parameters.

If all loads (or load effects) are represented by the variable Q and total resistance (or capacity) by R, then the space of state variables is a two-dimensional space as shown in Figure 1. Within the space, there is a separation of the "safe domain" from that of "failure domain"; the intersection area between the domains describes the limit state function g(R,Q)=0.

Since both R and Q are some basic random variables, these can be defined as a joint density function f_{RQ} (r, q). A general joint density function is plotted in Figure 2. Again, the function of limit state separates the domains of safe and failure function. The integration of the joint density function over the failure domain represents the joint density function[i.e., the region for which g(R, Q) <0]. It is often very difficult to evaluate this probability, so the concept structural reliability can be quantified by a reliability index.

The standard normal distribution function (zero mean and unit variance) denoted by $\Phi(\ )$.The random variable $Z = R\text{-}S$ as shown in Figure, which is represented by the region $Z \leq 0$ as shown shaded. Using equations above, it follows that

$$P_f = \Phi\left[\frac{-(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)^{\frac{1}{2}}}\right] = \Phi(-\beta)$$

Where, $\beta = {}^{\mu_Z}/_{\sigma_Z}$ is defined as ***reliability (safety) index***.

If the standard deviations σ_R and σ_S or both are subsequently increased, the square bracket term in expression above tends to become smaller which further increases P_f. Similarly, the difference between the mean of load effect and the mean of the resistance if reduced, P_f increases. The observations as above can also be deduced from Figure 5 below, taking the overlap of $f_R(\ )$ and $f_S()$ as a rough indicator of P_f.

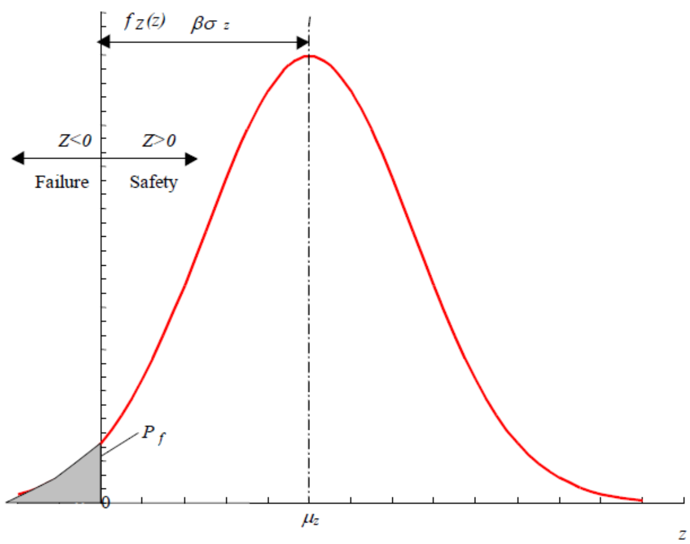


Fig. 3 - Distribution of safety margin *Z* = *R* – *S*

Reduced Variables

It is useful in particular situations to transform random variables to their "standard form" which is also a nondimensional form of variables. For variables R and Q which are basic, the standard form can be expressed as

$$Z_R = \frac{R - \mu_R}{\sigma_R}$$

$$Z_Q = \frac{R - \mu_Q}{\sigma_Q}$$

The variables as indicated in the above expression Z_R and Z_Q, are called *reduced variables.*

By reorganizing Equation, the resistance *R* and the load *Q* can best be expressed in terms ofreduced variables as follows:

$$R = \mu_R + Z_R\sigma_R$$

$$Q = \mu_Q + Z_Q\sigma_Q$$

The limit state function as represented by $g(R, Q) = R - Q$ isstated in terms of the reduced variables and the result obtained is

$$g(Z_R, Z_Q) = \mu_R + Z_R\sigma_R - \mu_Q - Z_Q\sigma_Q = (\mu_R - \mu_Q) + Z_R\sigma_R - Z_Q\sigma_Q$$

The above equation represents a straight line in the space of reduced variables Z_R and Z_Q.

The line corresponding to $g(Z_R, Z_Q) = 0$that separates the safe and failure domain in the space.

Reliability Index

Reliability index can be defined as an inverse to the coefficient of variation. The reliability index alternatively is the perpendicular or shortest distance measured from the origin of reduced variables to the failure point also called as design pointwhich is illustrated as in Fig., line $G(Z_R, Z_Q) = 0$.

The definitionwas given by Hasofer and Lind. Using the geometry one can easily determine the reliability index i.e.(the shortest distance) from the following formula

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

where β repressents the inverse of coefficient of variation of function

$g(R, Q) = R - Q$.

R represents the resistance of the structure

Q represents the action or load on the structure

then the reliability index whenrelated to probability of failure is given by:

$\beta = -\phi^{-1}(P_f)\ or\ P_f = \phi(-\beta)$

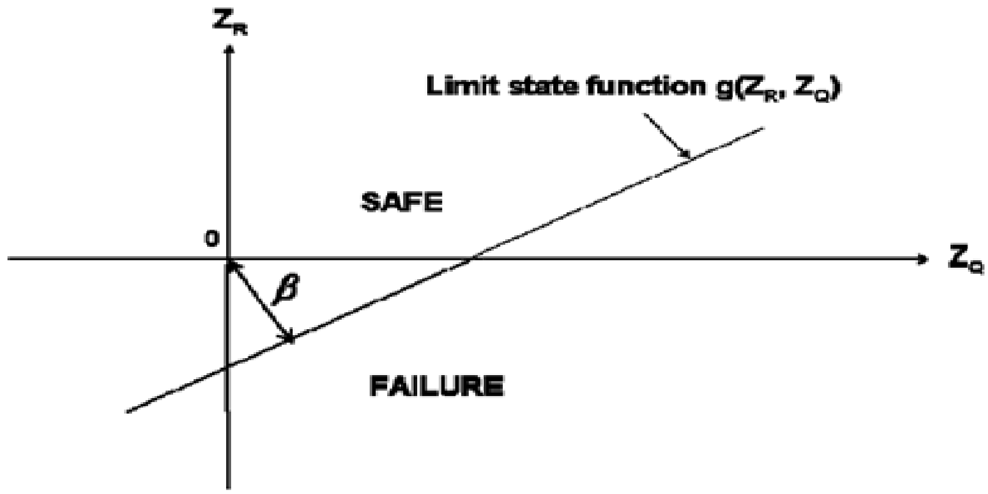


Fig. 4 Representation of Reliability index for a limit state function

CHAPTER 4

FIRST ORDER RELIABILITY METHOD

4.1 Introduction of First order reliability method (FORM)

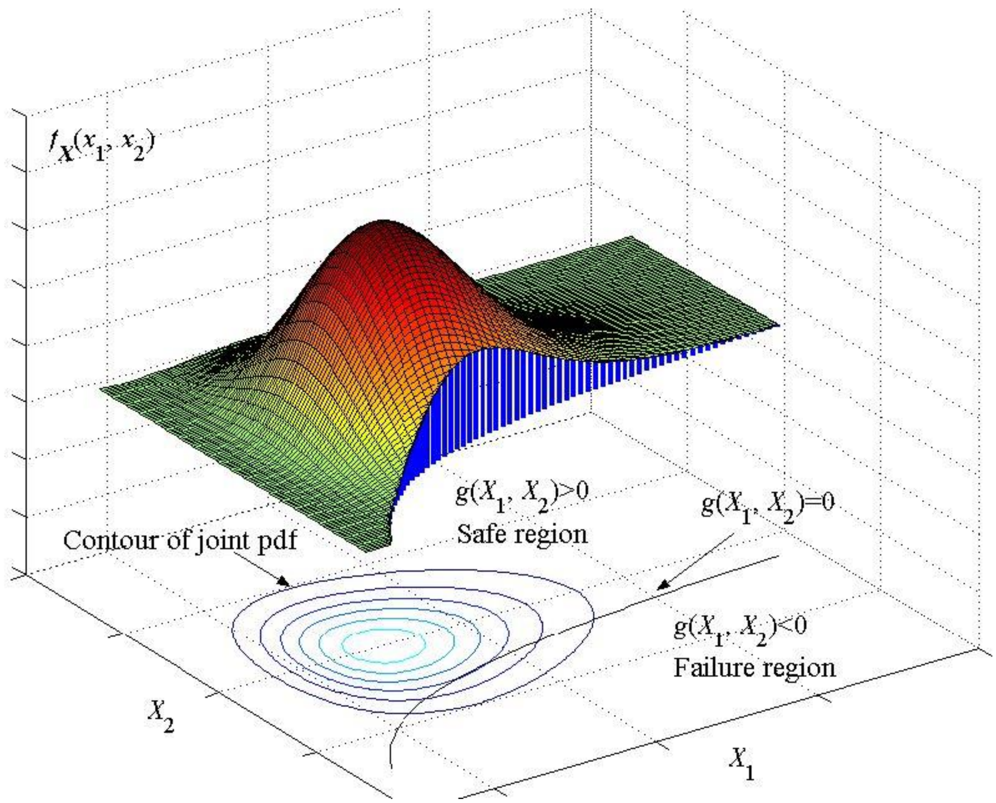


Fig 5: First order reliability method representation

THIS TECHNIQUE OF FIRST-ORDER APPROXIMATION OF TAYLOR SERIES OF THE FUNCTION IS AIMED AT LINEARIZED MEAN VALUES OF RANDOM VARIABLE KNOWN AS FIRST-ORDER SECOND MOMENT (MVFOSM)METHOD;

IT THEREFORE USES ONLYSTATISTICS (I.E.MEAN & VARIANCE) RELATED TO SECOND MOMENT OF THE RANDOM VARIABLES. INITIALLY APPROACH WAS BASED ON THE BASIC ASSUMPTION THAT THE RESULTING PROBABILITY OF *Z* DISTRIBUTION IS NORMAL, THE RELIABILITY INDEX WAS DEFINED USING THE RATIO OF THE EXPECTED VALUE OF Z OVER ITS STANDARD DEVIATION. THE RELIABILITY INDEX (B_C)AS GIVEN BY CORNELL IS AN ABSOLUTE VALUE OFANY ORDINATE OF POINT CONVERGING TO Z = 0 AS NORMAL STANDARDISED PROBABILITY FOR PLOT AS SHOWN IN FIGURE

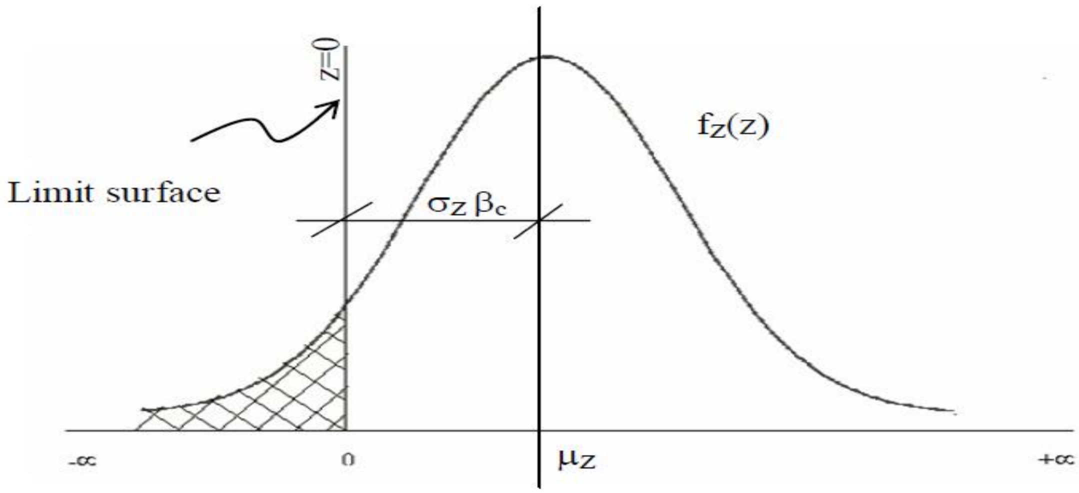


Fig. 6 – Definition of limit state and reliability index

And the equation given by:

$$\beta_c = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

An approximation can be obtained usingfirst order relaibility method(FORM) approach.

An ideal conditionis approximated for the general case .

where *X* indicates a vector for Gaussian variables with zero mean and standard deviation as unity,

where *g(X)* is a linear function.

The probability of failure P_f is then

$$P_f = P(g(X) < 0) = P\left(\sum_{i=1}^{n} \alpha_i X_i - \beta < 0\right) = \phi(-\beta)$$

Where,

α_i representsthe cosine direction of random variable

X_i, β represents the distance between an origin and its hyperplane

$g(X)=0$

n represents the number of basic random variables for X,

Φ represents thestandard normal distribution function.

The above formulations are generalized for many random variables as denoted by vector. Let performance function is given as:

$$Z= g(X) = g\,(X_1, X_2....X_n)$$

Using the Taylor series expansion, the performance function for the mean value as given by the equation

$$Z = g(\mu_X) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(X_i - \mu_{X_i}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 g}{\partial X_i\, \partial X_j}(X_i - \mu_{X_i})\left(X_j - \mu_{X_j}\right) + \cdots$$

Where derivatives are given at the mean values of random variables $(X_1, X_2... X_n)$

μ_{X_i} is the mean value of X_i.

The series when expressed in linear terms,

the mean and variance for first order of Z is obtained as:

$$\mu_Z \approx g\left(\mu_{X_1}, \mu_{X_2,..........}\mu_{X_n}\right)$$

And,

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} var\left(X_i, X_j\right)$$

Where $var(X_i, X_j)$ is covariance of X_i and X_j. If variances are uncorrelated, then the variance for z is given as

$$\sigma_Z^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 var(X_i)$$

The reliability index calculated by taking ratio of mean (μ_z) and standard deviation of Z (σ_z) as:

$$\beta = \frac{\mu_Z}{\sigma_Z}$$

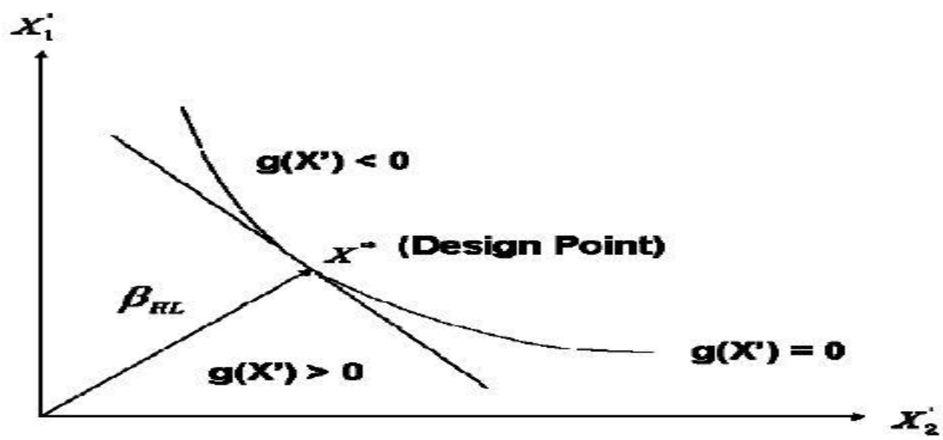


Fig 7 : Design point representation

4.1.1 Reliability Index proposed by Hasofer and Lind

A modified reliability index was proposed by Hasofer and Lind that did not exhibited the problem of invariance. The "correction" to evaluate the limit state function at a point is known as the "design point" instead of mean values. The design point as defined is a point at the failure surface g = 0. Since this point is not known previously, the technique of iteration must be used (in general) so as to solve for the problem of reliability index.

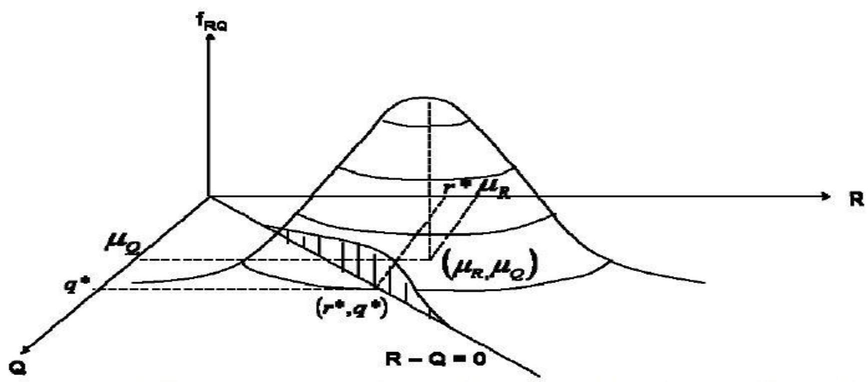


Figure 10 - Design point on the failure boundary for the linear limit state function g = R – Q

4.1.2. A FOSM Method for Normal Variables

The *Hasofer-Lind* (H-L) method as applicable for normal random variables defines the reduced variables as

$$X_i = \frac{X - \mu}{\sigma} \ (i=1,2\ldots..n)$$

Where, X_i denotes a random variable with zero mean and unity as standard deviation.

Above equation can be used to transform the original limit state g(X) = 0 to the limit state reduced to g(X`)= 0. The coordinate system of X is referred to as *original coordinate system.* The X` coordinate system can be referred to as the *transformed* or *reduced* system. Note that if X_i is normal. The safety index β can be defined as the minimum distance from the origin of the axes in the reduced coordinate system to the limiting state surface (or the failure surface).

For a failure surface non-linear in nature, the shortest distance of the origin (in normalized coordinate system)is referred to the failure surface that is not unique as in the case of linear failure surface. The computation of failure surface probability involves integration. The tangential plane to the design point is used to approximate the value of β. If the failure surface towards the origin is concave, approximation will be on the safer side, while for the convex surface it will be on the unsafe side.

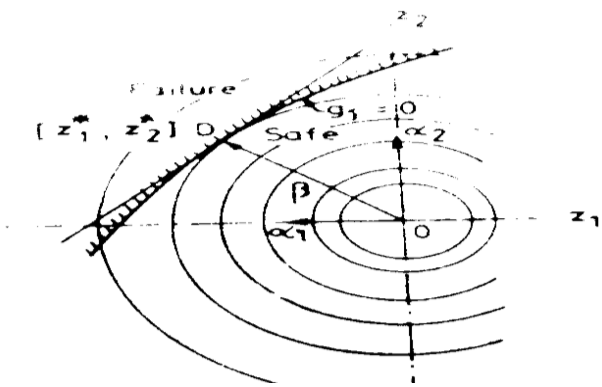


Fig. 8 – Formulation of safety analysis in normalized coordinates.

CHAPTER 5

5.1.3 Component reliability (COMREL)

COMREL is a software that performs reliability analysis that are time invaraint in nature of various individual failure modes on the basis of advanced methodologies of FORM/SORM. Various algorithms to find the most likely failure point (β-point) are to be implementedthat includes an algorithm that is gradient-free for non-differentiable criteria of failure (state functions). Alternatively other computational options include MVFO(Mean Value First Order),simulation by Monte Carlo, Sampling, Adaptive and Spherical Sampling, other Important Sampling schemes and Simulation by subset method.

Specifically Built-in functions includes all thetrigonometric, logarithmic,hyperbolic, elementary and some other special functions like Gamma functions, Gaussian distribution function and their inverse. It also includes alternatives for differentiation, numerical integration, and finding of a root is available along with testing functions and comparative operators.User-definedfunctions, auxillary as well as reference functions, can also be defined.

5.1.4 STAAD.Pro V8i

The most dynamic and popular engineering software product used in structural engineering to carry out analysis of the beams and columns of a structure.It is useful for post printing important and significant results when a structure is subjected to different types of loadings such as joint displacements,supportreactions,deflections,bending moments and shear values,not only these values are helpful in analysis but also these values are specifically used for designing.The software has additional advantages for 3-D model generation and multi-material designing.It is an integration to several modeling softwares and products of design.

5.1.5 ETABS

ETABS is a program for static, nonlinear, dynamic and linear analysis, and the design of building systems. From a systematic standpoint, multistorey buildings constitute a very distinct class of structures and therefore deserve distinct treatment. The concept of special programs for building structures was familiarized over 40 years ago and lead to the development of the TABS series of computer programs.

The innovative and ground-breaking new ETABS is the ultimate integrated software suite for the structural analysis and design of buildings. Combining 40 years of incessant research and development, this latest ETABS bids unmatched 3D object based demonstrating and visualization tools, very fast linear and nonlinear analytical power, refined and comprehensive design competencies for a wide-range of materials, and astute graphic displays, schematic drawings, and reports that allow users to decipher quickly and easily and apprehend analysis and design results.

From the beginning of design commencement through the production of schematic drawings, it integrates every aspect of engineering design process. Intuitive drawing commands allow for the swift and speedy generation of floor and elevation framing. AutoCAD drawings can be converted straight into ETABS models or used as prototypes onto which ETABS objects may be overlaid. The state-of-the-art SAPFire solver allows extremely large and multifaceted models to be rapidly analyzed and provisions nonlinear modeling techniques such as time effects (e.g., creep and shrinkage) and construction sequencing.

The numerical solution, input and output techniques of this software are specifically designed to take benefit of the unique numerical and physical characteristics related to building type structures. As a result, this analysis and design tool further execution throughput, data preparation, and output interpretation.

The need for special purpose programs has never been more apparent as Structural Engineers put non-linear dynamic analysis into run-through and use the greater computer power accessible today to create larger analytical models.

Over the past decades, it has several mega-projects to its credit and has established itself as the standard of the industry. This software is clearly recognized as the utmost practical and efficient tool for the dynamic and static analysis of shear wall buildings and multistorey frames.

ETABS is also capable of performing time variant earthquake analysis such as response spectrum analysis, time history analysis, etc.

CHAPTER 6

METHODOLOGY

It is a general tendency of a beam or a column to dovelop moments and shear when subjected to loading either in form concentrated or UDL.

To evaluate or to dovelop an equation for Moment of Resistance of the section there’s always a need to have knowledge about various physical parameters of the section.But in normalconditions these parameters are subjected to statistical variation and are probabi -listic in nature.Hence a method must be formulated so as to account for these uncertainities. One of such methods used is given by Hasofer and Lind that gives a theoritical defination of reliability index (β).The method takes into account the statistical variations of physical parameters by using mean and standard deviation values.

STAAD PRO has been used to evaluate critical bending moment values and axial forces.Further,these values are then exported to COMREL for anlysis which through first order and second order reliability methods evaluates the value of reliability index through various iterations and its inverse giving the value of probability of failure.

As an example to the explain the complete methodology a sample beam problem has been expalined further:-

ANALYSIS OF A SAMPLE BEAM PROBLEM

6.1 Reliability of the beam against the limit state of collapse in flexure

Calculate the reliability index of the beam (against the limit state of collapse in flexure) shown in fig., subjected to a self-weight Q_1 and a live load Q_2.The Flexural resistance moment capacity of the beam is R. It is given that

Q_1=wL N   L= 5 m

μ_{Q1}= 500 N μ_{Q2}=6000 N μ_R=15000N-m

σ_{Q1} = 10 N σ_{Q2}=2500 N σ_R=1100N-m

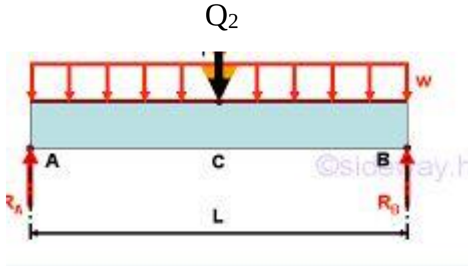


Fig 9. Simply supported beam Problem

6.1.1 Manual calculation

Solution:

Maximum bending moment due to external loads is

$$M_e = Q_1L/8 + Q_2L/4$$
$$=Q_1(5/8)+ Q_2(5/4)$$

Hence, Action=$0.625Q_1+1.25Q_2$

The failure function (R-S) is

$$G(Q_1,Q_2,R) = R - Q_1/2 - Q_2$$

This is a linear function of variables R, Q_1 and Q_2

$$M=R - .625Q_1 -1.25Q_2$$

Using Equations. Above

$$\mu_m=\mu_R - .625\mu_{Q1} -1.25\ \mu_{Q2}$$

$$\sigma_m^2=\sigma_R^2 + (.625)^2(\sigma_{Q1}^2) + (1.25)^2\sigma_{Q2}^2$$

Substituting the given data, we have

$$\mu_m=15– (0.5)(.625)– (1.25)(6)$$

$$\boldsymbol{\mu_m= 7.1875\ KN}$$

$$\sigma_m^2=1.1^2+(.625)^2(.01^2) + (2.5)^2(1.25^2)$$

$$\boldsymbol{\sigma_m = 3.31\ KN}$$

Hence the reliability index β is,

$$\boldsymbol{\beta=(7.1875/3.31) = 2.17}$$

6.1.2 Solved using COMREL

β and p_f obtained using FORM

```
*********************************************
```

Numerical Results

```
          **********************************************
          ----------- Comrel-TI (Version 9) ------------
          ---- (c) Copyright: RCP GmbH (1989-2015) -----
          **********************************************
------------------------------------------------------------------------------
   Job name ............ :                                             maj1
   Failure criterion no. :1
   Comment : reliability index of the beam
   Transformation type   : Rosenblatt
   Optimization algorithm: RFLS
 -----------------------------------------------------------------------------
-
 -----------------------------------------------------------------------------
-
```

```
 Iteration No.1; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.2825E-09; BETA =      0.0000; BETA/||U||=      0.0000
   Multipl.=   9.216    ; Step-length=    1.0000; State Func.calls:    5
--------------------------------------------------------------------------------

  Iteration No.2; CPU-seconds(cumulative):     0.016
   Scaled St.F(U) = -0.3790E-15; BETA =      2.1466; BETA/||U||=      1.0000
   Multipl.=   9.216    ; Step-length=    1.0000; State Func.calls:    9

                       FORM-beta=1.947;
                       FORM-Pf=1.21E-02;

             --------- Statistics after COMREL-TI ---------
              State Function calls              =       10
              State Funct. gradient evaluations =     2
              Total computation time (CPU-secs.)=     0.03
              The error indicator (IER) was =    0
             **********************************************

Reliability analysis is finished
```

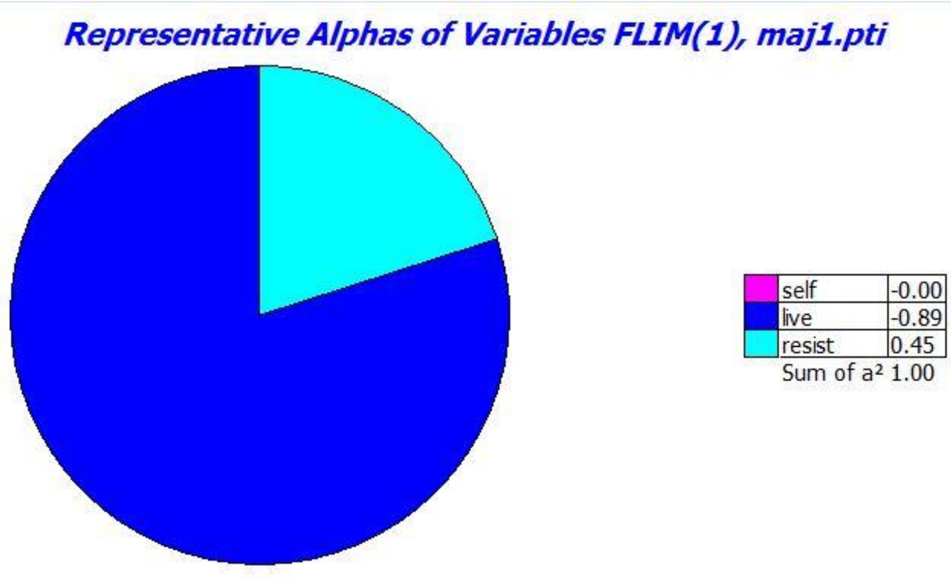
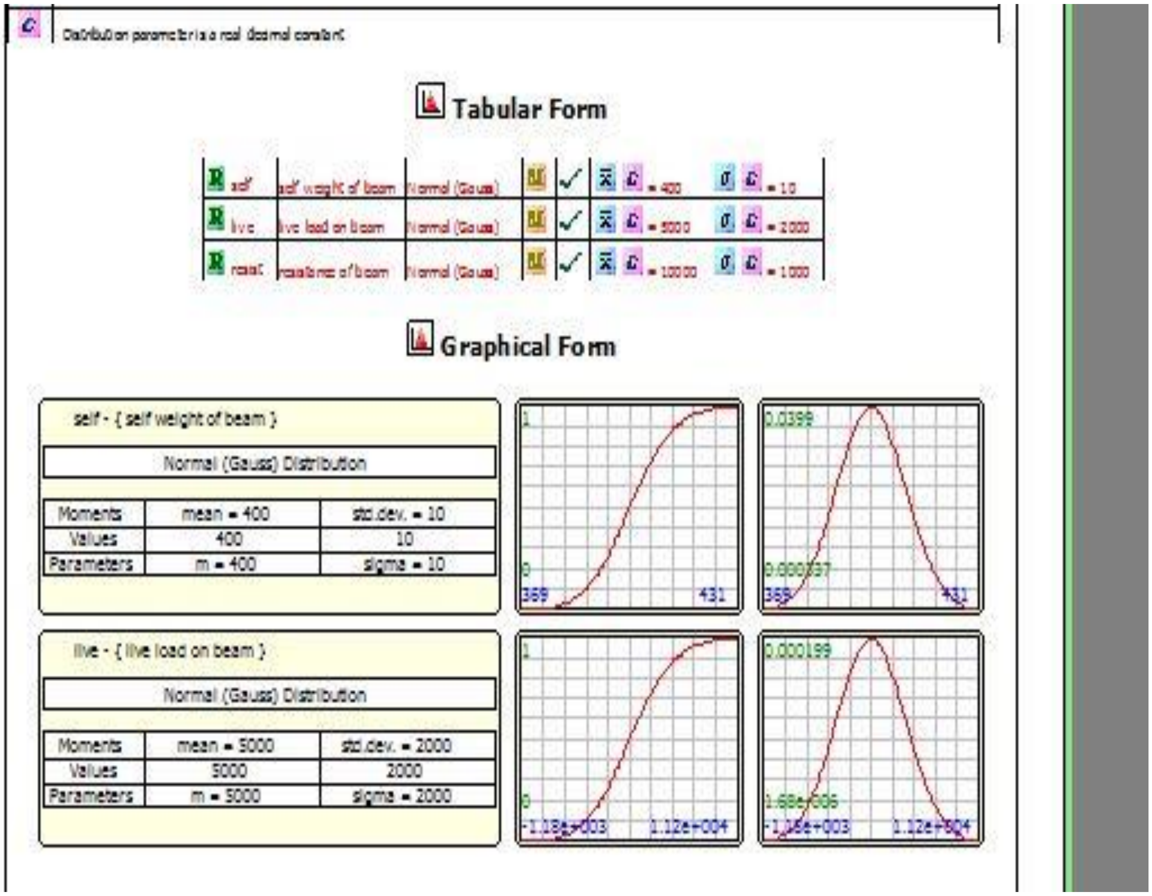
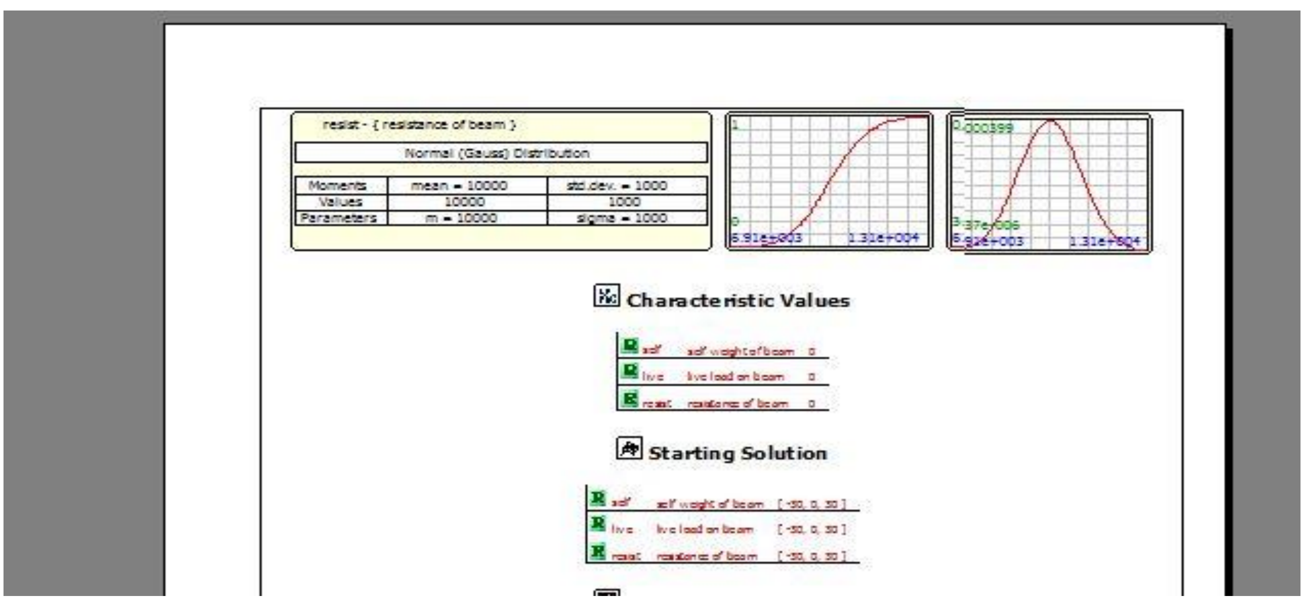


Fig.11 - α values obtained for all the three variables at design point





Limit State Functions

```
FLIM(1){reliability of beam}=
resist-self/2-live
```

Variables in FLIM(1)

Resist	**R**	R
Self	**R**	Self weight of beam
Live	**R**	live load on beam

Summary Symbolic Variables

Resist	**R**	resistance of beam
Self	**R**	self weight of beam
Live	**R**	live load on beam

Summary Numerical Constants

User	-1
User	2

CHAPTER 7

7.1 Reliability of corner and central column for a MULTI-STOREY building under seismic load definition as per IS:1893-2002/2005.

A corner and central column has been analyzed for a multi-storey G+10 steel building model when subjected to an earthquake loading as per IS 1893 2002 seismic load definition for Delhi zone i.e. (zone IV) region .The analysis were performed in STAAD PRO to get the values of most critical axial load and bending moment values acting along Y and Z direction on the column taking into account the different load combinations.

The results obtained in STAAD PRO were then transferred to MS Excel file to clearly study and note values of axial load and biaxial moments. The most critical values for different load combinations were obtained through STAAD PRO analysis that were used for the reliability analysis in COMREL. As per IS:800-2007, the buckling criteria for the column has been used for axial loading and biaxial bending in Y and Z direction which, is given as:

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} < 1$$

The final failure limiting equation is formulated using the above values and formulae which was then used for analysis in COMREL, the analysis were formulated using the different probability density functions such as normal, logarithmic, Gumbel(max.) and they are optimized for achieving the reliability of the structure. The first and second order analysis were performed for the reliability and the failure probability was evaluated.

9.3 Combined Axial Force and Bending Moment

Under combined axial force and bending moment, section strength as governed by material failure and member strength as governed by buckling failure shall be checked in accordance with **9.3.1** and **9.3.2** respectively.

9.3.1 *Section Strength*

9.3.1.1 *Plastic and compact sections*

In the design of members subjected to combined axial force (tension or compression) and bending moment, the following should be satisfied:

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \leq 1.0$$

Conservatively, the following equation may also be used under combined axial force and bending moment:

$$\frac{N}{N_d} + \frac{M_y}{M_{dy}} + \frac{M_z}{M_{dz}} \leq 1.0$$

where

M_y, M_z = factored applied moments about the minor and major axis of the cross-section, respectively;

M_{ndy}, M_{ndz} = design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone (*see* **9.3.1.2**);

N = factored applied axial force (Tension, T or Compression, P);

N_d = design strength in tension, T_d as obtained from **6** or in compression due to yielding given by $N_d = A_g f_y / \gamma_{m0}$;

M_{dy}, M_{dz} = design strength under corresponding moment acting alone (*see* **8.2**);

A_g = gross area of the cross-section;

α_1, α_2 = constants as given in Table 17; and

γ_{m0} = partial factor of safety in yielding.

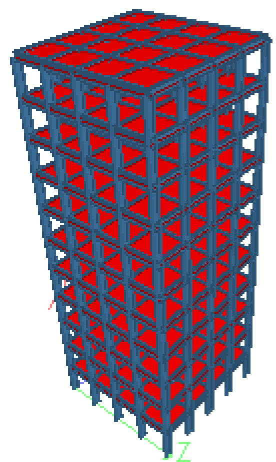
c) *For standard I or H sections*

$$\text{for } n \leq 0.2 \quad M_{ndy} = M_{dy}$$

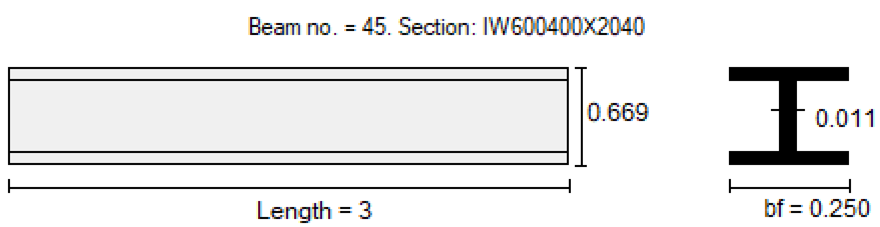
$$\text{for } n > 0.2 \quad M_{ndy} = 1.56\, M_{dy}\,(1-n)\,(n+0.6)$$

$$M_{ndz} = 1.11\, M_{dz}\,(1-n) \leq M_{dz}$$

,



<u>*Fig 12: STEEL BUILDING G+10 MODEL USED FOR STAAD ANALYSIS*</u>



Physical Properties (Unit: m)

Ax	0.0425	Ix	9.86e-006
Ay	0.007359	Iy	0.0003617
Az	0.007	Iz	0.00357691
D	0.669	W	0.25

Assign/Change Property

Material Properties

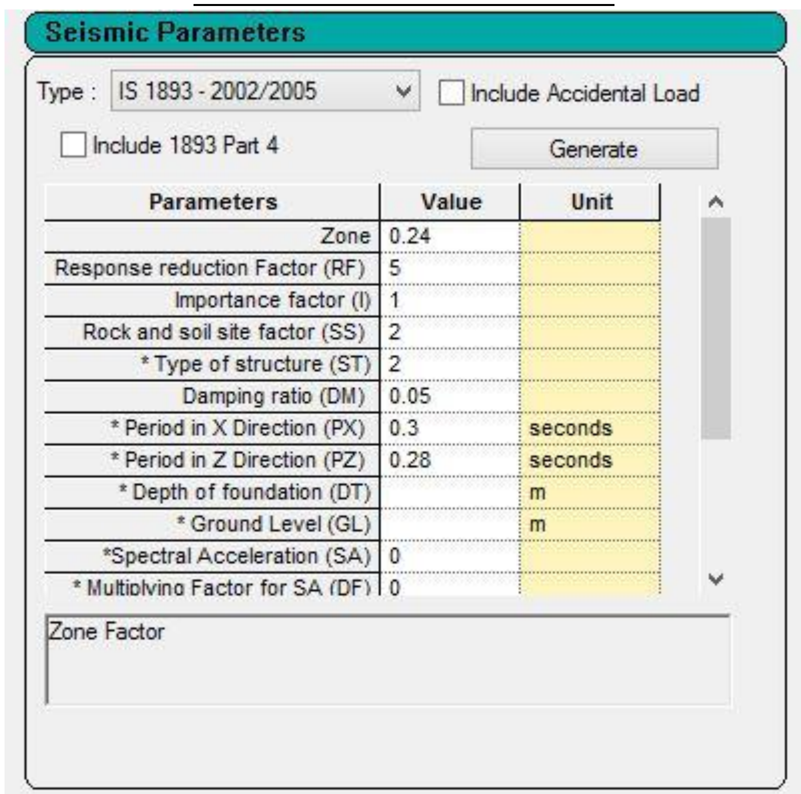
Elasticity(kN/mm2)	205	Density(kg/m3)	7833.41
Poisson	0.3	Alpha	1.2e-005

STEEL

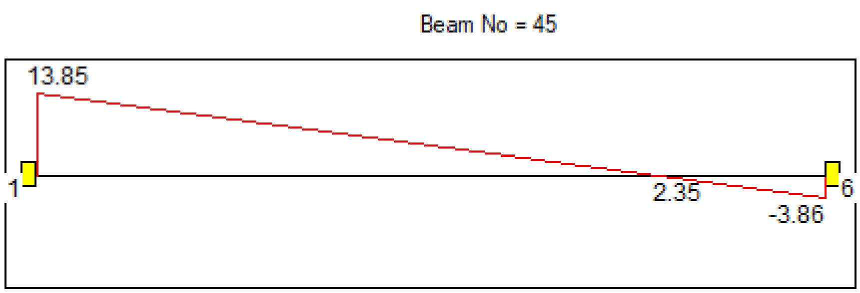
Assign Material

<u>*SECTION PROPERTIES*</u>

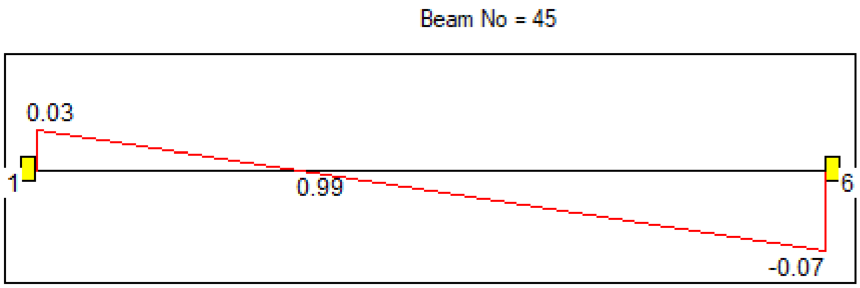
SEISMIC DEFINATION



CRITICAL VALUES FOR CORNER COLUMN MEMBER



Mz (BENDING MOMENT IN ZZ DIRECTION)



My (BENDING MOMENT IN YY DIRECTION)

7.2 COMREL ANALYSIS

Numerical Results for beam no.445

```
-----------------------------------------------------------------------------
   Job name ............ :                                          beam 445
   Failure criterion no. :      1
   Comment : LIMIT STATE ANAYSIS OF COLUMN
   Transformation type    : Rosenblatt
   Optimization algorithm: RFLS
 -----------------------------------------------------------------------------
 -----------------------------------------------------------------------------
  Iteration No.  1; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.3660     ; BETA =      0.0000; BETA/||U||=      0.0000
   Multipl.=   3.488     ; Step-length=    1.0000; State Func.calls:    6
 -----------------------------------------------------------------------------
  Iteration No.  2; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.2165     ; BETA =      1.3206; BETA/||U||=      0.6581
   Multipl.=   18.81     ; Step-length=    0.5243; State Func.calls:   12
 -----------------------------------------------------------------------------
  Iteration No.  3; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.1388     ; BETA =      2.0069; BETA/||U||=      0.7757
   Multipl.=   39.97     ; Step-length=    0.4442; State Func.calls:   18
 -----------------------------------------------------------------------------
  Iteration No.  4; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.9433E-01; BETA =      2.5871; BETA/||U||=      0.8371
   Multipl.=   73.14     ; Step-length=    0.3861; State Func.calls:   24
 -----------------------------------------------------------------------------
  Iteration No.  5; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.6691E-01; BETA =      3.0906; BETA/||U||=      0.8735
   Multipl.=   121.4     ; Step-length=    0.3433; State Func.calls:   30
 -----------------------------------------------------------------------------
  Iteration No.  6; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.4902E-01; BETA =      3.5381; BETA/||U||=      0.8973
   Multipl.=   188.4     ; Step-length=    0.3111; State Func.calls:   36
 -----------------------------------------------------------------------------
  Iteration No.  7; CPU-seconds(cumulative):     0.000
   Scaled St.F(U) =  0.3682E-01; BETA =      3.9432; BETA/||U||=      0.9137
   Multipl.=   278.5     ; Step-length=    0.2861; State Func.calls:   42
 -----------------------------------------------------------------------------
  Iteration No.  8; CPU-seconds(cumulative):     0.016
   Scaled St.F(U) =  0.2821E-01; BETA =      4.3156; BETA/||U||=      0.9257
   Multipl.=   396.9     ; Step-length=    0.2663; State Func.calls:   48
 -----------------------------------------------------------------------------
```

```
 Iteration No.  9; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.2196E-01; BETA =      4.6620; BETA/||U||=      0.9348
  Multipl.=   549.5    ; Step-length=    0.2501; State Func.calls:   54
-------------------------------------------------------------------------------
 Iteration No. 10; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.1733E-01; BETA =      4.9872; BETA/||U||=      0.9419
  Multipl.=   743.5    ; Step-length=    0.2367; State Func.calls:   60
-------------------------------------------------------------------------------
 Iteration No. 11; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.1383E-01; BETA =      5.2947; BETA/||U||=      0.9476
  Multipl.=   987.3    ; Step-length=    0.2254; State Func.calls:   66
-------------------------------------------------------------------------------
 Iteration No. 12; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.1115E-01; BETA =      5.5872; BETA/||U||=      0.9523
  Multipl.=   1291.    ; Step-length=    0.2157; State Func.calls:   72
-------------------------------------------------------------------------------
 Iteration No. 13; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.9059E-02; BETA =      5.8670; BETA/||U||=      0.9562
  Multipl.=   1664.    ; Step-length=    0.2073; State Func.calls:   78
-------------------------------------------------------------------------------
 Iteration No. 14; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.7416E-02; BETA =      6.1354; BETA/||U||=      0.9596
  Multipl.=   2121.    ; Step-length=    0.1999; State Func.calls:   84
-------------------------------------------------------------------------------
 Iteration No. 15; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.6111E-02; BETA =      6.3940; BETA/||U||=      0.9624
  Multipl.=   2676.    ; Step-length=    0.1934; State Func.calls:   90
-------------------------------------------------------------------------------
 Iteration No. 16; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.5064E-02; BETA =      6.6437; BETA/||U||=      0.9649
  Multipl.=   3346.    ; Step-length=    0.1876; State Func.calls:   96
-------------------------------------------------------------------------------
 Iteration No. 17; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.4219E-02; BETA =      6.8854; BETA/||U||=      0.9670
  Multipl.=   4150.    ; Step-length=    0.1824; State Func.calls:  102
-------------------------------------------------------------------------------
 Iteration No. 18; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.3531E-02; BETA =      7.1198; BETA/||U||=      0.9690
  Multipl.=   5109.    ; Step-length=    0.1777; State Func.calls:  108
-------------------------------------------------------------------------------
 Iteration No. 19; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.2968E-02; BETA =      7.3475; BETA/||U||=      0.9707
  Multipl.=   6247.    ; Step-length=    0.1735; State Func.calls:  114
-------------------------------------------------------------------------------
 Iteration No. 20; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.2504E-02; BETA =      7.5691; BETA/||U||=      0.9722
  Multipl.=   7591.    ; Step-length=    0.1698; State Func.calls:  120
-------------------------------------------------------------------------------
 Iteration No. 21; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.2119E-02; BETA =      7.7849; BETA/||U||=      0.9735
  Multipl.=   9173.    ; Step-length=    0.1665; State Func.calls:  126
-------------------------------------------------------------------------------
 Iteration No. 22; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.1797E-02; BETA =      7.9955; BETA/||U||=      0.9747
  Multipl.=  0.1103E+05; Step-length=    0.1638; State Func.calls:  132
-------------------------------------------------------------------------------
 Iteration No. 23; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.1528E-02; BETA =      8.2013; BETA/||U||=      0.9757
  Multipl.=  0.1319E+05; Step-length=    0.1617; State Func.calls:  138
-------------------------------------------------------------------------------
 Iteration No. 24; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.1300E-02; BETA =      8.4028; BETA/||U||=      0.9766
  Multipl.=  0.1571E+05; Step-length=    0.1606; State Func.calls:  144
-------------------------------------------------------------------------------
 Iteration No. 25; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.1106E-02; BETA =      8.6006; BETA/||U||=      0.9772
  Multipl.=  0.1863E+05; Step-length=    0.1607; State Func.calls:  150
-------------------------------------------------------------------------------
 Iteration No. 26; CPU-seconds(cumulative):     0.078
  Scaled St.F(U) =  0.9381E-03; BETA =      8.7955; BETA/||U||=      0.9775
  Multipl.=  0.2204E+05; Step-length=    0.1631; State Func.calls:  156
-------------------------------------------------------------------------------
 Iteration No. 27; CPU-seconds(cumulative):     0.078
  Scaled St.F(U) =  0.7899E-03; BETA =      8.9892; BETA/||U||=      0.9773
```

```
 Multipl.=  0.2602E+05; Step-length=    0.1696; State Func.calls:  162
-------------------------------------------------------------------------------
Iteration No. 28; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.6536E-03; BETA =      9.1842; BETA/||U||=      0.9759
 Multipl.=  0.3074E+05; Step-length=    0.1854; State Func.calls:  168
-------------------------------------------------------------------------------
Iteration No. 29; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.5138E-03; BETA =      9.3860; BETA/||U||=      0.9715
 Multipl.=  0.3650E+05; Step-length=    0.2303; State Func.calls:  174
-------------------------------------------------------------------------------
Iteration No. 30; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.2678E-03; BETA =      9.6086; BETA/||U||=      0.9451
 Multipl.=  0.4419E+05; Step-length=    0.5135; State Func.calls:  180
-------------------------------------------------------------------------------
Iteration No. 31; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) = -0.9892E-03; BETA =      9.8172; BETA/||U||=      0.9302
 Multipl.=  0.5809E+05; Step-length=    1.0000; State Func.calls:  185
-------------------------------------------------------------------------------
Iteration No. 32; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) = -0.9584E-03; BETA =      6.8315; BETA/||U||=      0.6576
 Multipl.=   9249.     ; Step-length=    0.0252; State Func.calls:  191
-------------------------------------------------------------------------------
Iteration No. 33; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) = -0.9251E-03; BETA =      7.1032; BETA/||U||=      0.6949
 Multipl.=   9252.     ; Step-length=    0.0277; State Func.calls:  197
-------------------------------------------------------------------------------
Iteration No. 34; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) = -0.8811E-03; BETA =      7.3692; BETA/||U||=      0.7346
 Multipl.=   9133.     ; Step-length=    0.0359; State Func.calls:  203
-------------------------------------------------------------------------------
Iteration No. 35; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) = -0.8215E-03; BETA =      7.6628; BETA/||U||=      0.7808
 Multipl.=   8835.     ; Step-length=    0.0478; State Func.calls:  209
-------------------------------------------------------------------------------
Iteration No. 36; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) = -0.7356E-03; BETA =      7.9683; BETA/||U||=      0.8329
 Multipl.=   8272.     ; Step-length=    0.0679; State Func.calls:  215
-------------------------------------------------------------------------------
Iteration No. 37; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) = -0.5996E-03; BETA =      8.2636; BETA/||U||=      0.8900
 Multipl.=   7348.     ; Step-length=    0.1070; State Func.calls:  221
-------------------------------------------------------------------------------
Iteration No. 38; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) = -0.3643E-03; BETA =      8.5133; BETA/||U||=      0.9480
 Multipl.=   5981.     ; Step-length=    0.1954; State Func.calls:  227
-------------------------------------------------------------------------------
Iteration No. 39; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) = -0.1004E-04; BETA =      8.6627; BETA/||U||=      0.9920
 Multipl.=   4210.     ; Step-length=    0.4236; State Func.calls:  233
-------------------------------------------------------------------------------
Iteration No. 40; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.8583E-05; BETA =      8.6819; BETA/||U||=      1.0002
 Multipl.=   2442.     ; Step-length=    1.0000; State Func.calls:  238
-------------------------------------------------------------------------------
Iteration No. 41; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.4686E-07; BETA =      8.6805; BETA/||U||=      0.9999
 Multipl.=   1441.     ; Step-length=    1.0000; State Func.calls:  243
-------------------------------------------------------------------------------
Iteration No. 42; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) = -0.1248E-09; BETA =      8.6812; BETA/||U||=      1.0000
 Multipl.=   1449.     ; Step-length=    1.0000; State Func.calls:  248
-------------------------------------------------------------------------------
Iteration No. 43; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) = -0.6040E-10; BETA =      8.6812; BETA/||U||=      1.0000
 Multipl.=   1446.     ; Step-length=    0.5132; State Func.calls:  254

FORM-beta=   8.681; SORM-beta=   --   ; beta(Sampling)=   --    (IER=   0)
FORM-Pf=  1.98E-18; SORM-Pf=    --    ; Pf(Sampling)=    --

           --------- Statistics after COMREL-TI ---------
            State Function calls               =      255
            State Funct. gradient evaluations =    43
            Total computation time (CPU-secs.)=     0.17
            The error indicator (IER) was =    0
```

****

Reliability analysis is finished

FORM-beta	8.681
FORM-Pf	1.98E-018

TABLE 1: RELIABILITY ANALYSIS FOR BEAM NO.445

Limit State Functions

```
FLIM(1){LIMIT STATE EQUATION FOR CORNER COLUMN}=
1-(My/(63.89*N*fy+2409647*fy*fy-0.000635*N*N))^(0.0002*N/fy)-(Mz/(2.75*N*fy+104011
*fy*fy-0.0002*N*N))^2
```

Variables in FLIM(1)

My	R	MOME
N	R	AXIAL LOAD
fy	R	YIELD STRESS
Mz	R	MOMENT IN ZZ DIRCETION

Summary Symbolic Variables

My	R	MOMENT IN YY DIRECTION
N	R	AXIAL LOAD
fy	R	YIELD STRESS
Mz	R	MOMENT IN ZZ DIRCETION

Tabular Form

R	My	MOMENT IN Y DIR	Lognormal	M	✓	$\bar{x}$ c = 1.985e+007	σ c = 1.4e+007
R	Mz	MOMENT IN Z DIRN	Lognormal	M	✓	$\bar{x}$ c = 3.7201e+007	σ c = 2.804e+007
R	N	AXIAL FORCE IN X DIR	Lognormal	M	✓	$\bar{x}$ c = 1.596e+006	σ c = 526000
R	fy	YIELD STRESS	Normal (Gauss)	M	✓	$\bar{x}$ c = 250	σ c = 21.04

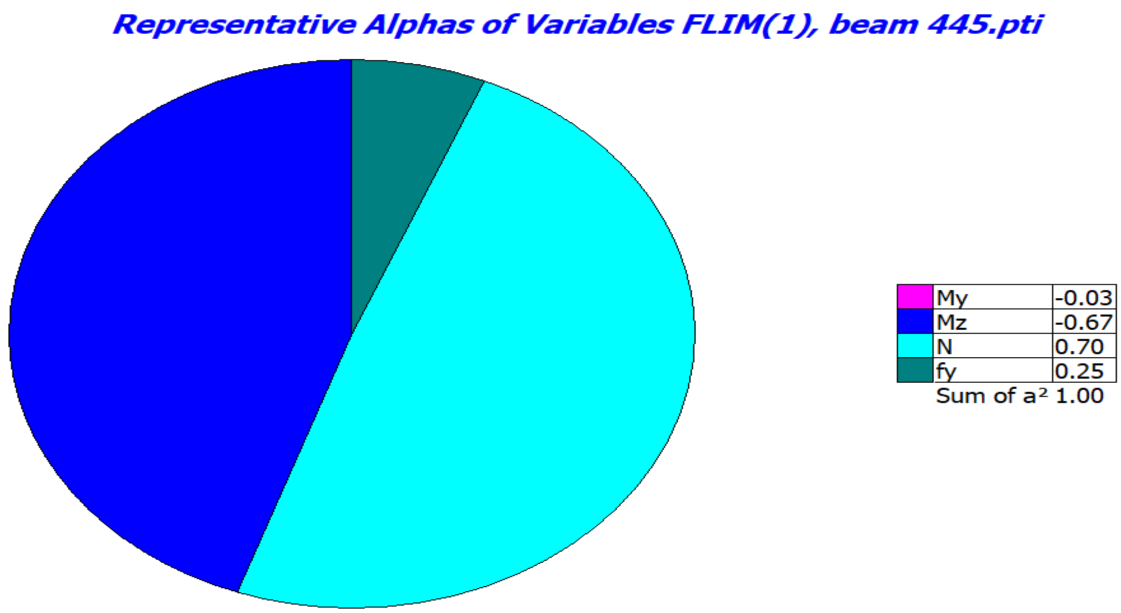


Fig.13: REPRESENTATIVE ALPHAS FOR THE VARIABLES FOR BEAM NO.445

Numerical Results for beam no.45

Comment : LIMIT STATE EQUATION FOR CENTRAL COLUMN

Transformation type : Rosenblatt

Optimization algorithm: RFLS

```
Iteration No.  1; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.3618     ; BETA =      0.0000; BETA/||U||=      0.0000
  Multipl.=   16.86     ; Step-length=    1.0000; State Func.calls:     6
-------------------------------------------------------------------------------
 Iteration No.  2; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.2141     ; BETA =      2.9035; BETA/||U||=      0.6629
  Multipl.=   89.37     ; Step-length=    0.5229; State Func.calls:    12
-------------------------------------------------------------------------------
 Iteration No.  3; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.1375     ; BETA =      4.3798; BETA/||U||=      0.7792
  Multipl.=   188.3     ; Step-length=    0.4424; State Func.calls:    18
-------------------------------------------------------------------------------
 Iteration No.  4; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.9357E-01; BETA =      5.6210; BETA/||U||=      0.8395
  Multipl.=   342.5     ; Step-length=    0.3843; State Func.calls:    24
-------------------------------------------------------------------------------
 Iteration No.  5; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.6647E-01; BETA =      6.6956; BETA/||U||=      0.8753
  Multipl.=   566.0     ; Step-length=    0.3417; State Func.calls:    30
-------------------------------------------------------------------------------
 Iteration No.  6; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.4877E-01; BETA =      7.6492; BETA/||U||=      0.8986
  Multipl.=   875.4     ; Step-length=    0.3096; State Func.calls:    36
-------------------------------------------------------------------------------
 Iteration No.  7; CPU-seconds(cumulative):     0.016
```

```
 Scaled St.F(U) =  0.3668E-01; BETA =      8.5124; BETA/||U||=      0.9148
 Multipl.=   1291.    ; Step-length=    0.2848; State Func.calls:   42
------------------------------------------------------------------------------
 Iteration No.  8; CPU-seconds(cumulative):     0.016
 Scaled St.F(U) =  0.2813E-01; BETA =      9.3057; BETA/||U||=      0.9265
 Multipl.=   1835.    ; Step-length=    0.2651; State Func.calls:   48
------------------------------------------------------------------------------
 Iteration No.  9; CPU-seconds(cumulative):     0.016
 Scaled St.F(U) =  0.2193E-01; BETA =     10.0436; BETA/||U||=      0.9355
 Multipl.=   2537.    ; Step-length=    0.2491; State Func.calls:   54
------------------------------------------------------------------------------
 Iteration No. 10; CPU-seconds(cumulative):     0.016
 Scaled St.F(U) =  0.1732E-01; BETA =     10.7365; BETA/||U||=      0.9425
 Multipl.=   3428.    ; Step-length=    0.2357; State Func.calls:   60
------------------------------------------------------------------------------
 Iteration No. 11; CPU-seconds(cumulative):     0.016
 Scaled St.F(U) =  0.1384E-01; BETA =     11.3921; BETA/||U||=      0.9481
 Multipl.=   4548.    ; Step-length=    0.2245; State Func.calls:   66
------------------------------------------------------------------------------
 Iteration No. 12; CPU-seconds(cumulative):     0.016
 Scaled St.F(U) =  0.1116E-01; BETA =     12.0161; BETA/||U||=      0.9527
 Multipl.=   5940.    ; Step-length=    0.2149; State Func.calls:   72
------------------------------------------------------------------------------
 Iteration No. 13; CPU-seconds(cumulative):     0.031
 Scaled St.F(U) =  0.9080E-02; BETA =     12.6131; BETA/||U||=      0.9565
 Multipl.=   7658.    ; Step-length=    0.2065; State Func.calls:   78
------------------------------------------------------------------------------
 Iteration No. 14; CPU-seconds(cumulative):     0.031
 Scaled St.F(U) =  0.7440E-02; BETA =     13.1867; BETA/||U||=      0.9598
 Multipl.=   9761.    ; Step-length=    0.1992; State Func.calls:   84
------------------------------------------------------------------------------
 Iteration No. 15; CPU-seconds(cumulative):     0.031
 Scaled St.F(U) =  0.6136E-02; BETA =     13.7395; BETA/||U||=      0.9626
 Multipl.=  0.1232E+05; Step-length=    0.1926; State Func.calls:   90
------------------------------------------------------------------------------
 Iteration No. 16; CPU-seconds(cumulative):     0.031
 Scaled St.F(U) =  0.5090E-02; BETA =     14.2740; BETA/||U||=      0.9650
 Multipl.=  0.1541E+05; Step-length=    0.1868; State Func.calls:   96
------------------------------------------------------------------------------
 Iteration No. 17; CPU-seconds(cumulative):     0.047
 Scaled St.F(U) =  0.4245E-02; BETA =     14.7919; BETA/||U||=      0.9671
 Multipl.=  0.1913E+05; Step-length=    0.1815; State Func.calls:  102
------------------------------------------------------------------------------
 Iteration No. 18; CPU-seconds(cumulative):     0.047
 Scaled St.F(U) =  0.3557E-02; BETA =     15.2949; BETA/||U||=      0.9690
 Multipl.=  0.2357E+05; Step-length=    0.1768; State Func.calls:  108
------------------------------------------------------------------------------
 Iteration No. 19; CPU-seconds(cumulative):     0.047
 Scaled St.F(U) =  0.2993E-02; BETA =     15.7844; BETA/||U||=      0.9707
 Multipl.=  0.2887E+05; Step-length=    0.1724; State Func.calls:  114
------------------------------------------------------------------------------
 Iteration No. 20; CPU-seconds(cumulative):     0.062
 Scaled St.F(U) =  0.2529E-02; BETA =     16.2613; BETA/||U||=      0.9722
 Multipl.=  0.3515E+05; Step-length=    0.1684; State Func.calls:  120
------------------------------------------------------------------------------
 Iteration No. 21; CPU-seconds(cumulative):     0.062
 Scaled St.F(U) =  0.2145E-02; BETA =     16.7268; BETA/||U||=      0.9735
 Multipl.=  0.4256E+05; Step-length=    0.1648; State Func.calls:  126
------------------------------------------------------------------------------
 Iteration No. 22; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.1825E-02; BETA =     17.1816; BETA/||U||=      0.9747
 Multipl.=  0.5129E+05; Step-length=    0.1614; State Func.calls:  132
------------------------------------------------------------------------------
 Iteration No. 23; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.1557E-02; BETA =     17.6265; BETA/||U||=      0.9758
 Multipl.=  0.6154E+05; Step-length=    0.1584; State Func.calls:  138
------------------------------------------------------------------------------
 Iteration No. 24; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.1333E-02; BETA =     18.0623; BETA/||U||=      0.9768
 Multipl.=  0.7351E+05; Step-length=    0.1556; State Func.calls:  144
------------------------------------------------------------------------------
 Iteration No. 25; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.1143E-02; BETA =     18.4895; BETA/||U||=      0.9777
 Multipl.=  0.8747E+05; Step-length=    0.1532; State Func.calls:  150
```

```
-----------------------------------------------------------------------------
 Iteration No. 26; CPU-seconds(cumulative):     0.094
  Scaled St.F(U) =  0.9825E-03; BETA =     18.9089; BETA/||U||=      0.9785
  Multipl.=  0.1037E+06; Step-length=    0.1511; State Func.calls:  156
-----------------------------------------------------------------------------
 Iteration No. 27; CPU-seconds(cumulative):     0.109
  Scaled St.F(U) =  0.8457E-03; BETA =     19.3211; BETA/||U||=      0.9792
  Multipl.=  0.1226E+06; Step-length=    0.1496; State Func.calls:  162
-----------------------------------------------------------------------------
 Iteration No. 28; CPU-seconds(cumulative):     0.109
  Scaled St.F(U) =  0.7286E-03; BETA =     19.7272; BETA/||U||=      0.9797
  Multipl.=  0.1445E+06; Step-length=    0.1487; State Func.calls:  168
-----------------------------------------------------------------------------
 Iteration No. 29; CPU-seconds(cumulative):     0.109
  Scaled St.F(U) =  0.6275E-03; BETA =     20.1281; BETA/||U||=      0.9801
  Multipl.=  0.1698E+06; Step-length=    0.1489; State Func.calls:  174
-----------------------------------------------------------------------------
 Iteration No. 30; CPU-seconds(cumulative):     0.125
  Scaled St.F(U) =  0.5390E-03; BETA =     20.5257; BETA/||U||=      0.9802
  Multipl.=  0.1994E+06; Step-length=    0.1511; State Func.calls:  180
-----------------------------------------------------------------------------
 Iteration No. 31; CPU-seconds(cumulative):     0.125
  Scaled St.F(U) =  0.4599E-03; BETA =     20.9230; BETA/||U||=      0.9797
  Multipl.=  0.2340E+06; Step-length=    0.1575; State Func.calls:  186
-----------------------------------------------------------------------------
 Iteration No. 32; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) =  0.3850E-03; BETA =     21.3258; BETA/||U||=      0.9779
  Multipl.=  0.2755E+06; Step-length=    0.1750; State Func.calls:  192
-----------------------------------------------------------------------------
 Iteration No. 33; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) =  0.2983E-03; BETA =     21.7476; BETA/||U||=      0.9703
  Multipl.=  0.3276E+06; Step-length=    0.2439; State Func.calls:  198
-----------------------------------------------------------------------------
 Iteration No. 34; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) = -0.1197E-03; BETA =     22.2195; BETA/||U||=      0.9005
  Multipl.=  0.4064E+06; Step-length=    1.0000; State Func.calls:  203
-----------------------------------------------------------------------------
 Iteration No. 35; CPU-seconds(cumulative):     0.156
  Scaled St.F(U) = -0.1132E-03; BETA =     10.6002; BETA/||U||=      0.4407
  Multipl.=  0.1037E+06; Step-length=    0.0307; State Func.calls:  209
-----------------------------------------------------------------------------
 Iteration No. 36; CPU-seconds(cumulative):     0.156
  Scaled St.F(U) = -0.1070E-03; BETA =     11.3654; BETA/||U||=      0.4842
  Multipl.=  0.9808E+05; Step-length=    0.0307; State Func.calls:  215
-----------------------------------------------------------------------------
 Iteration No. 37; CPU-seconds(cumulative):     0.172
  Scaled St.F(U) = -0.1012E-03; BETA =     11.9847; BETA/||U||=      0.5225
  Multipl.=  0.9003E+05; Step-length=    0.0307; State Func.calls:  221
-----------------------------------------------------------------------------
 Iteration No. 38; CPU-seconds(cumulative):     0.172
  Scaled St.F(U) = -0.9575E-04; BETA =     12.4728; BETA/||U||=      0.5559
  Multipl.=  0.8091E+05; Step-length=    0.0307; State Func.calls:  227
-----------------------------------------------------------------------------
 Iteration No. 39; CPU-seconds(cumulative):     0.172
  Scaled St.F(U) = -0.9073E-04; BETA =     12.8504; BETA/||U||=      0.5848
  Multipl.=  0.7169E+05; Step-length=    0.0307; State Func.calls:  233
-----------------------------------------------------------------------------
 Iteration No. 40; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) = -0.8368E-04; BETA =     13.1403; BETA/||U||=      0.6140
  Multipl.=  0.6295E+05; Step-length=    0.0404; State Func.calls:  239
-----------------------------------------------------------------------------
 Iteration No. 41; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) = -0.7136E-04; BETA =     13.4303; BETA/||U||=      0.6518
  Multipl.=  0.5267E+05; Step-length=    0.0615; State Func.calls:  245
-----------------------------------------------------------------------------
 Iteration No. 42; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) = -0.3973E-04; BETA =     13.7299; BETA/||U||=      0.7117
  Multipl.=  0.3990E+05; Step-length=    0.1171; State Func.calls:  251
-----------------------------------------------------------------------------
 Iteration No. 43; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.2384E-03; BETA =     12.0252; BETA/||U||=      0.8714
  Multipl.=  0.2355E+05; Step-length=    0.4021; State Func.calls:  257
-----------------------------------------------------------------------------
 Iteration No. 44; CPU-seconds(cumulative):     0.188
```

```
  Scaled St.F(U) = -0.7629E-02; BETA =     11.1846; BETA/||U||=      0.9974
  Multipl.=   4373.    ; Step-length=    1.0000; State Func.calls:  262
 -----------------------------------------------------------------------------
 Iteration No. 45; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) = -0.2941E-02; BETA =     10.2069; BETA/||U||=      1.0035
  Multipl.=   351.2    ; Step-length=    0.5517; State Func.calls:  268
 -----------------------------------------------------------------------------
 Iteration No. 46; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) = -0.1200E-04; BETA =     11.1574; BETA/||U||=      1.0024
  Multipl.=   322.2    ; Step-length=    1.0000; State Func.calls:  273
 -----------------------------------------------------------------------------
 Iteration No. 47; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) = -0.1265E-05; BETA =     10.1230; BETA/||U||=      1.0000
  Multipl.=   325.7    ; Step-length=    1.0000; State Func.calls:  278
 -----------------------------------------------------------------------------
 Iteration No. 48; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) = -0.1116E-08; BETA =     10.1229; BETA/||U||=      1.0000
  Multipl.=   342.1    ; Step-length=    1.0000; State Func.calls:  283
 -----------------------------------------------------------------------------
 Iteration No. 49; CPU-seconds(cumulative):     0.219
  Scaled St.F(U) =  0.4009E-12; BETA =     10.1229; BETA/||U||=      1.0000
  Multipl.=   342.3    ; Step-length=    1.0000; State Func.calls:  288

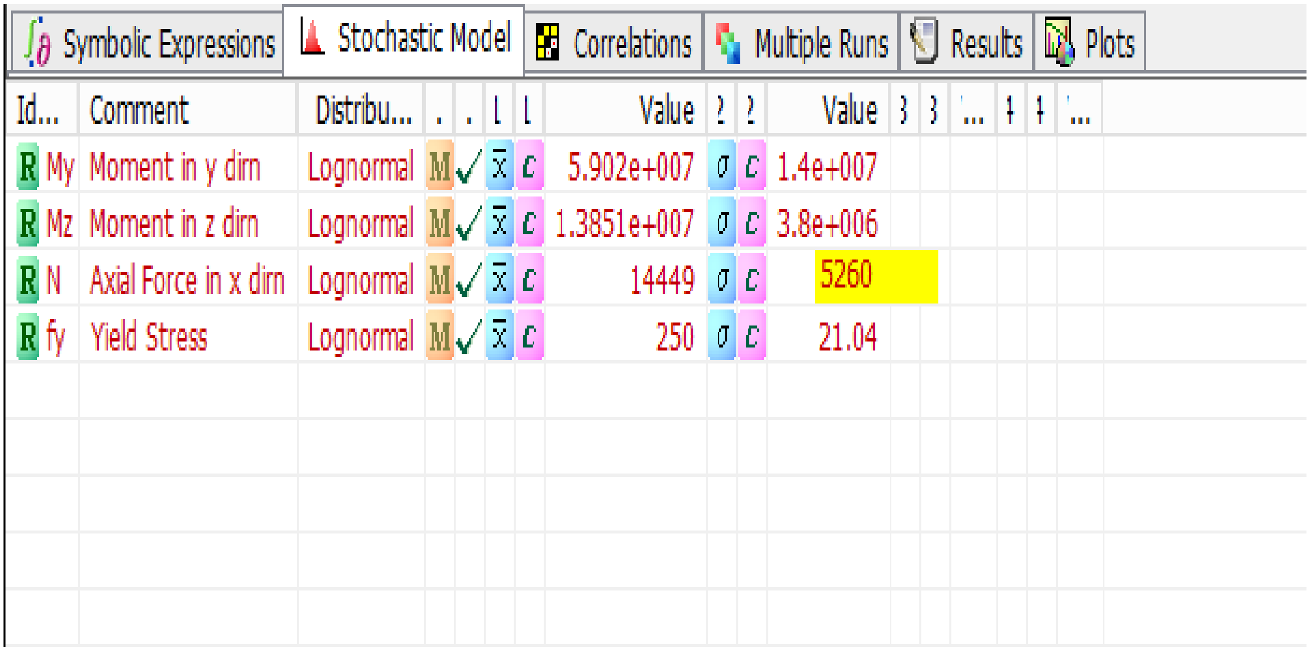
 FORM-beta=  8.123; SORM-beta=   --   ; beta(Sampling)=   --    (IER=   0)
 FORM-Pf=  1.43E-06; SORM-Pf=    --    ; Pf(Sampling)=    --

            --------- Statistics after COMREL-TI ---------
             State Function calls               =      289
             State Funct. gradient evaluations =     49
             Total computation time (CPU-secs.)=     0.23
             The error indicator (IER) was =     0
            **********************************************

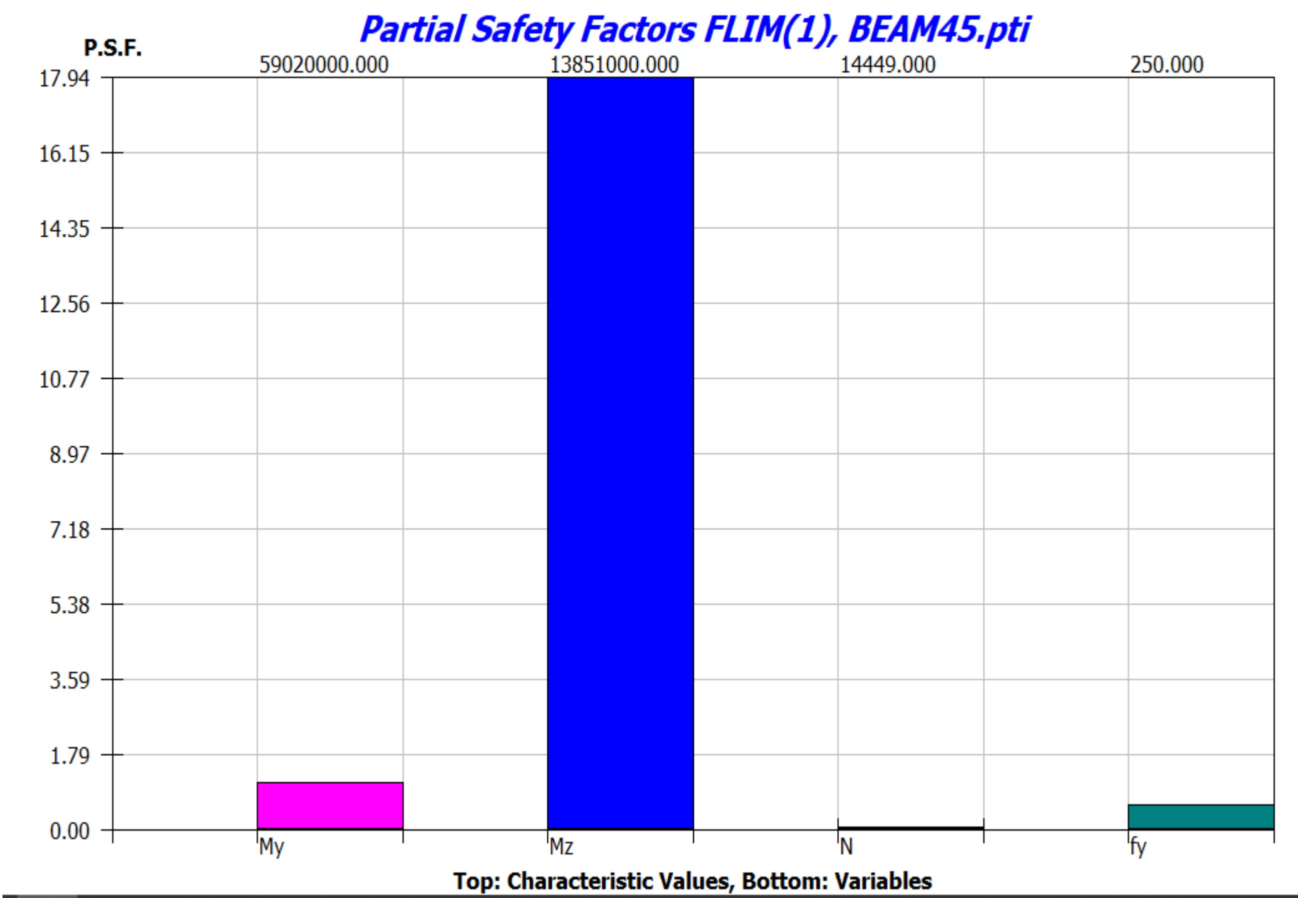
Reliability analysis is finished
```

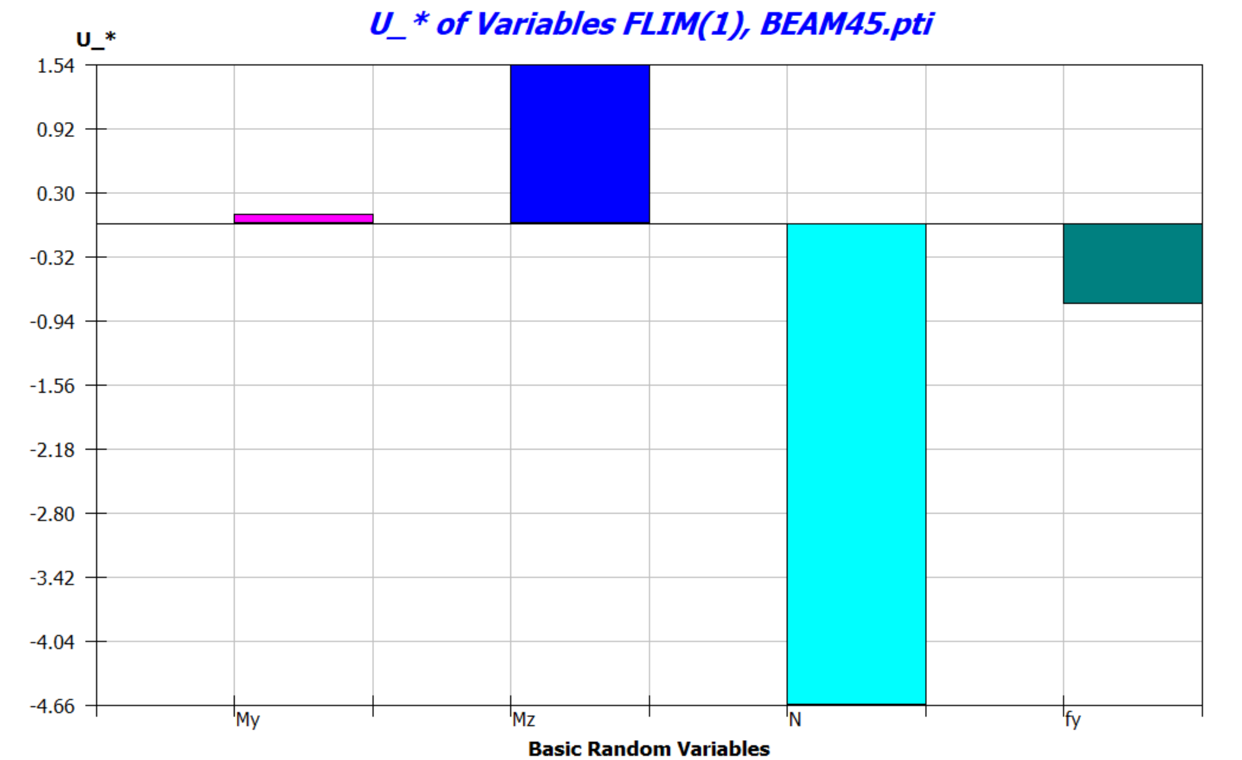
FORM-beta	8.123
FORM-Pf	1.43E-06

TABLE 2: RELIABILITY ANALYSIS FOR BEAM NO.45



Id...	Comment	Distribu...	Value	Value
My	Moment in y dirn	Lognormal	5.902e+007	1.4e+007
Mz	Moment in z dirn	Lognormal	1.3851e+007	3.8e+006
N	Axial Force in x dirn	Lognormal	14449	5260
fy	Yield Stress	Lognormal	250	21.04




U_* of Variables FLIM(1), BEAM45.pti
U_*
1.54
0.92
0.30
-0.32
-0.94
-1.56
-2.18
-2.80
-3.42
-4.04
-4.66
My
Mz
N
fy
Basic Random Variables

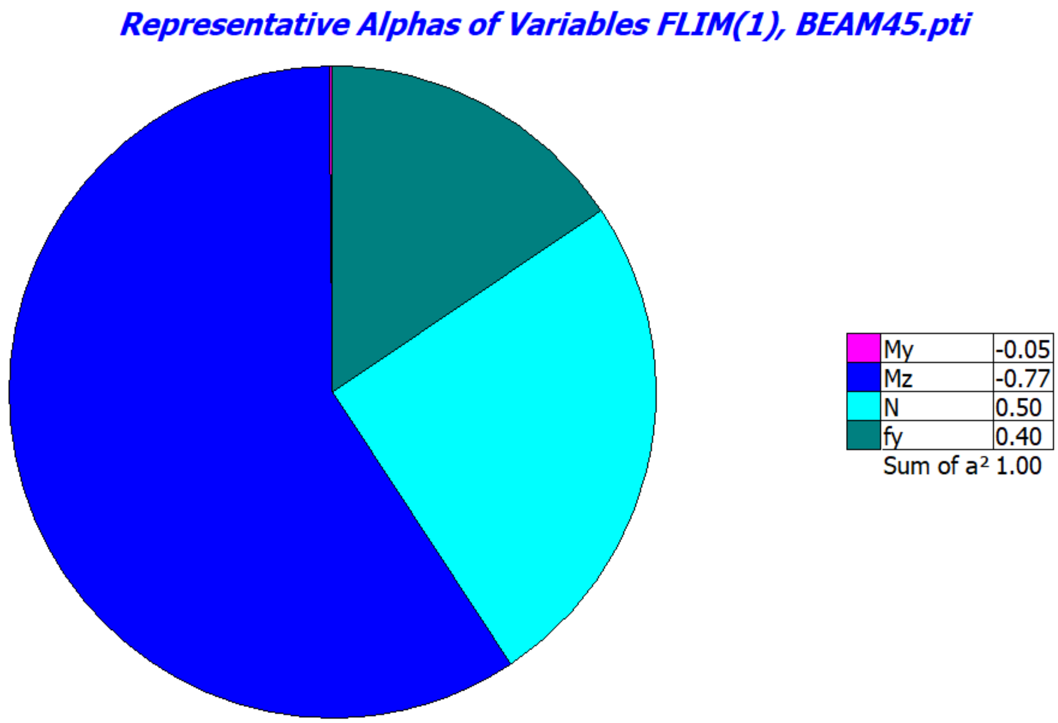
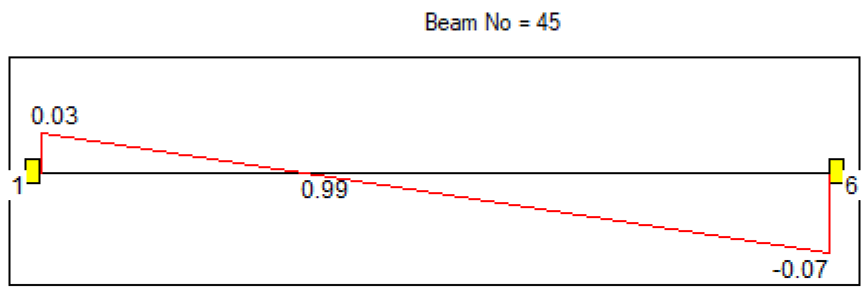
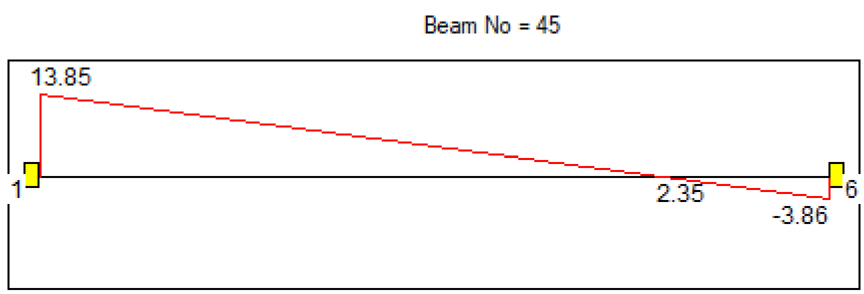


Fig 14: REPRESENTATIVE ALPHAS FOR THE VARIABLES FOR BEAM NO.45





Numerical Results for Beam no. 245

Comment : LIMIT STATE EQUATION FOR CORNER CENTRAL COLUMN

Transformation type : Rosenblatt

Optimization algorithm: RFLS

```
 Iteration No.  1; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.9956    ; BETA =      0.0000; BETA/||U||=      0.0000
  Multipl.=  0.3234E+74; Step-length=    0.0432; State Func.calls:    7
--------------------------------------------------------------------------------
 Iteration No.  2; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.9345    ; BETA =      2.1627; BETA/||U||=      0.3697
  Multipl.=  0.2993E+06; Step-length=    0.0432; State Func.calls:   13
--------------------------------------------------------------------------------
 Iteration No.  3; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.1440E-01; BETA =      4.1950; BETA/||U||=      0.2124
  Multipl.=   657.7    ; Step-length=    1.0000; State Func.calls:   18
--------------------------------------------------------------------------------
 Iteration No.  4; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.5202E-02; BETA =     19.7326; BETA/||U||=      0.8692
  Multipl.=   9367.    ; Step-length=    1.0000; State Func.calls:   23
--------------------------------------------------------------------------------
 Iteration No.  5; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.1838E-02; BETA =     22.6675; BETA/||U||=      0.8863
  Multipl.=  0.2859E+05; Step-length=    1.0000; State Func.calls:   28
--------------------------------------------------------------------------------
 Iteration No.  6; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.4988E-03; BETA =     25.4282; BETA/||U||=      0.9015
  Multipl.=  0.8521E+05; Step-length=    1.0000; State Func.calls:   33
--------------------------------------------------------------------------------
 Iteration No.  7; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.4714E-03; BETA =     25.6605; BETA/||U||=      0.9139
  Multipl.=  0.2171E+06; Step-length=    0.0432; State Func.calls:   39
--------------------------------------------------------------------------------
 Iteration No.  8; CPU-seconds(cumulative):     0.016
  Scaled St.F(U) =  0.4428E-03; BETA =     24.7421; BETA/||U||=      0.8875
  Multipl.=  0.1978E+06; Step-length=    0.0432; State Func.calls:   45
--------------------------------------------------------------------------------
 Iteration No.  9; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.4124E-03; BETA =     23.5320; BETA/||U||=      0.8528
  Multipl.=  0.1716E+06; Step-length=    0.0432; State Func.calls:   51
--------------------------------------------------------------------------------
 Iteration No. 10; CPU-seconds(cumulative):     0.031
  Scaled St.F(U) =  0.3801E-03; BETA =     22.0560; BETA/||U||=      0.8105
  Multipl.=  0.1406E+06; Step-length=    0.0432; State Func.calls:   57
--------------------------------------------------------------------------------
 Iteration No. 11; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.3464E-03; BETA =     20.4411; BETA/||U||=      0.7645
  Multipl.=  0.1087E+06; Step-length=    0.0432; State Func.calls:   63
--------------------------------------------------------------------------------
 Iteration No. 12; CPU-seconds(cumulative):     0.047
  Scaled St.F(U) =  0.3126E-03; BETA =     18.8855; BETA/||U||=      0.7211
  Multipl.=  0.8029E+05; Step-length=    0.0432; State Func.calls:   69
--------------------------------------------------------------------------------
 Iteration No. 13; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.2800E-03; BETA =     17.5578; BETA/||U||=      0.6862
  Multipl.=  0.5791E+05; Step-length=    0.0432; State Func.calls:   75
--------------------------------------------------------------------------------
 Iteration No. 14; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.2495E-03; BETA =     16.5240; BETA/||U||=      0.6620
  Multipl.=  0.4157E+05; Step-length=    0.0432; State Func.calls:   81
--------------------------------------------------------------------------------
 Iteration No. 15; CPU-seconds(cumulative):     0.062
  Scaled St.F(U) =  0.2215E-03; BETA =     15.7628; BETA/||U||=      0.6480
  Multipl.=  0.3006E+05; Step-length=    0.0432; State Func.calls:   87
--------------------------------------------------------------------------------
 Iteration No. 16; CPU-seconds(cumulative):     0.062
```

```
 Scaled St.F(U) =  0.1959E-03; BETA =     15.2171; BETA/||U||=      0.6422
 Multipl.=  0.2200E+05; Step-length=    0.0432; State Func.calls:   93
------------------------------------------------------------------------------
 Iteration No. 17; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.1727E-03; BETA =     14.8297; BETA/||U||=      0.6425
 Multipl.=  0.1633E+05; Step-length=    0.0432; State Func.calls:   99
------------------------------------------------------------------------------
 Iteration No. 18; CPU-seconds(cumulative):     0.078
 Scaled St.F(U) =  0.1518E-03; BETA =     14.5550; BETA/||U||=      0.6472
 Multipl.=  0.1229E+05; Step-length=    0.0432; State Func.calls:  105
------------------------------------------------------------------------------
 Iteration No. 19; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.1329E-03; BETA =     14.3596; BETA/||U||=      0.6551
 Multipl.=   9377.    ; Step-length=    0.0432; State Func.calls:  111
------------------------------------------------------------------------------
 Iteration No. 20; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.1159E-03; BETA =     14.2198; BETA/||U||=      0.6652
 Multipl.=   7241.    ; Step-length=    0.0432; State Func.calls:  117
------------------------------------------------------------------------------
 Iteration No. 21; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.1006E-03; BETA =     14.1192; BETA/||U||=      0.6768
 Multipl.=   5658.    ; Step-length=    0.0432; State Func.calls:  123
------------------------------------------------------------------------------
 Iteration No. 22; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.8692E-04; BETA =     14.0462; BETA/||U||=      0.6894
 Multipl.=   4469.    ; Step-length=    0.0432; State Func.calls:  129
------------------------------------------------------------------------------
 Iteration No. 23; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.7461E-04; BETA =     13.9926; BETA/||U||=      0.7026
 Multipl.=   3567.    ; Step-length=    0.0432; State Func.calls:  135
------------------------------------------------------------------------------
 Iteration No. 24; CPU-seconds(cumulative):     0.094
 Scaled St.F(U) =  0.6354E-04; BETA =     13.9529; BETA/||U||=      0.7161
 Multipl.=   2874.    ; Step-length=    0.0432; State Func.calls:  141
------------------------------------------------------------------------------
 Iteration No. 25; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) =  0.5359E-04; BETA =     13.9231; BETA/||U||=      0.7299
 Multipl.=   2338.    ; Step-length=    0.0432; State Func.calls:  147
------------------------------------------------------------------------------
 Iteration No. 26; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) =  0.4464E-04; BETA =     13.9006; BETA/||U||=      0.7435
 Multipl.=   1918.    ; Step-length=    0.0432; State Func.calls:  153
------------------------------------------------------------------------------
 Iteration No. 27; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) =  0.3661E-04; BETA =     13.8833; BETA/||U||=      0.7571
 Multipl.=   1588.    ; Step-length=    0.0432; State Func.calls:  159
------------------------------------------------------------------------------
 Iteration No. 28; CPU-seconds(cumulative):     0.109
 Scaled St.F(U) =  0.2946E-04; BETA =     13.8701; BETA/||U||=      0.7704
 Multipl.=   1324.    ; Step-length=    0.0432; State Func.calls:  165
------------------------------------------------------------------------------
 Iteration No. 29; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.2314E-04; BETA =     13.8598; BETA/||U||=      0.7835
 Multipl.=   1113.    ; Step-length=    0.0432; State Func.calls:  171
------------------------------------------------------------------------------
 Iteration No. 30; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.1764E-04; BETA =     13.8520; BETA/||U||=      0.7961
 Multipl.=   942.4    ; Step-length=    0.0432; State Func.calls:  177
------------------------------------------------------------------------------
 Iteration No. 31; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.1298E-04; BETA =     13.8460; BETA/||U||=      0.8084
 Multipl.=   803.6    ; Step-length=    0.0432; State Func.calls:  183
------------------------------------------------------------------------------
 Iteration No. 32; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.9185E-05; BETA =     13.8416; BETA/||U||=      0.8203
 Multipl.=   689.8    ; Step-length=    0.0432; State Func.calls:  189
------------------------------------------------------------------------------
 Iteration No. 33; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.6297E-05; BETA =     13.8385; BETA/||U||=      0.8317
 Multipl.=   596.0    ; Step-length=    0.0432; State Func.calls:  195
------------------------------------------------------------------------------
 Iteration No. 34; CPU-seconds(cumulative):     0.125
 Scaled St.F(U) =  0.4363E-05; BETA =     13.8364; BETA/||U||=      0.8426
 Multipl.=   518.1    ; Step-length=    0.0432; State Func.calls:  201
```

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--------------------------------------------------------------------------------
 Iteration No. 35; CPU-seconds(cumulative):     0.125
  Scaled St.F(U) =  0.3439E-05; BETA =     13.8352; BETA/||U||=      0.8530
  Multipl.=   453.1    ; Step-length=    0.0432; State Func.calls:  207
--------------------------------------------------------------------------------
 Iteration No. 36; CPU-seconds(cumulative):     0.125
  Scaled St.F(U) =  0.3575E-05; BETA =     13.8348; BETA/||U||=      0.8630
  Multipl.=   398.5    ; Step-length=    0.0432; State Func.calls:  213
--------------------------------------------------------------------------------
 Iteration No. 37; CPU-seconds(cumulative):     0.125
  Scaled St.F(U) =  0.4820E-05; BETA =     13.8351; BETA/||U||=      0.8725
  Multipl.=   352.4    ; Step-length=    0.0432; State Func.calls:  219
--------------------------------------------------------------------------------
 Iteration No. 38; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) =  0.7210E-05; BETA =     13.8359; BETA/||U||=      0.8815
  Multipl.=   313.3    ; Step-length=    0.0432; State Func.calls:  225
--------------------------------------------------------------------------------
 Iteration No. 39; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) =  0.1077E-04; BETA =     13.8372; BETA/||U||=      0.8900
  Multipl.=   279.9    ; Step-length=    0.0432; State Func.calls:  231
--------------------------------------------------------------------------------
 Iteration No. 40; CPU-seconds(cumulative):     0.141
  Scaled St.F(U) =  0.1549E-04; BETA =     13.8390; BETA/||U||=      0.8980
  Multipl.=   251.3    ; Step-length=    0.0432; State Func.calls:  237
--------------------------------------------------------------------------------
 Iteration No. 41; CPU-seconds(cumulative):     0.156
  Scaled St.F(U) =  0.2138E-04; BETA =     13.8410; BETA/||U||=      0.9055
  Multipl.=   226.6    ; Step-length=    0.0432; State Func.calls:  243
--------------------------------------------------------------------------------
 Iteration No. 42; CPU-seconds(cumulative):     0.156
  Scaled St.F(U) =  0.2838E-04; BETA =     13.8434; BETA/||U||=      0.9127
  Multipl.=   205.3    ; Step-length=    0.0432; State Func.calls:  249
--------------------------------------------------------------------------------
 Iteration No. 43; CPU-seconds(cumulative):     0.172
  Scaled St.F(U) =  0.3643E-04; BETA =     13.8459; BETA/||U||=      0.9193
  Multipl.=   186.8    ; Step-length=    0.0432; State Func.calls:  255
--------------------------------------------------------------------------------
 Iteration No. 44; CPU-seconds(cumulative):     0.172
  Scaled St.F(U) =  0.4546E-04; BETA =     13.8486; BETA/||U||=      0.9256
  Multipl.=   170.6    ; Step-length=    0.0432; State Func.calls:  261
--------------------------------------------------------------------------------
 Iteration No. 45; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.5536E-04; BETA =     13.8515; BETA/||U||=      0.9314
  Multipl.=   156.5    ; Step-length=    0.0432; State Func.calls:  267
--------------------------------------------------------------------------------
 Iteration No. 46; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.6600E-04; BETA =     13.8544; BETA/||U||=      0.9368
  Multipl.=   144.1    ; Step-length=    0.0432; State Func.calls:  273
--------------------------------------------------------------------------------
 Iteration No. 47; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.7725E-04; BETA =     13.8573; BETA/||U||=      0.9419
  Multipl.=   133.1    ; Step-length=    0.0432; State Func.calls:  279
--------------------------------------------------------------------------------
 Iteration No. 48; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.8894E-04; BETA =     13.8603; BETA/||U||=      0.9466
  Multipl.=   123.4    ; Step-length=    0.0432; State Func.calls:  285
--------------------------------------------------------------------------------
 Iteration No. 49; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.1009E-03; BETA =     13.8632; BETA/||U||=      0.9510
  Multipl.=   114.8    ; Step-length=    0.0432; State Func.calls:  291
--------------------------------------------------------------------------------
 Iteration No. 50; CPU-seconds(cumulative):     0.188
  Scaled St.F(U) =  0.1130E-03; BETA =     13.8661; BETA/||U||=      0.9550
  Multipl.=   107.1    ; Step-length=    0.0432; State Func.calls:  297
--------------------------------------------------------------------------------
 Iteration No.  1; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) =  0.6864E-01; BETA =     13.8689; BETA/||U||=      1.0000
  Multipl.=   100.3    ; Step-length=    1.0000; State Func.calls:  303
--------------------------------------------------------------------------------
 Iteration No.  2; CPU-seconds(cumulative):     0.203
  Scaled St.F(U) =  0.1221E-01; BETA =     13.8280; BETA/||U||=      0.9957
  Multipl.=   24.44    ; Step-length=    1.0000; State Func.calls:  308
--------------------------------------------------------------------------------
 Iteration No.  3; CPU-seconds(cumulative):     0.203
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```
 Scaled St.F(U) = -0.5816E-02; BETA =     13.8768; BETA/||U||=      0.9991
 Multipl.=   29.39    ; Step-length=    1.0000; State Func.calls:  313
------------------------------------------------------------------------------
 Iteration No.  4; CPU-seconds(cumulative):     0.219
 Scaled St.F(U) = -0.2552E-02; BETA =     13.8865; BETA/||U||=      1.0004
 Multipl.=   23.83    ; Step-length=    1.0000; State Func.calls:  318
------------------------------------------------------------------------------
 Iteration No.  5; CPU-seconds(cumulative):     0.219
 Scaled St.F(U) = -0.1421E-02; BETA =     13.8799; BETA/||U||=      1.0002
 Multipl.=   22.93    ; Step-length=    1.0000; State Func.calls:  323
------------------------------------------------------------------------------
 Iteration No.  6; CPU-seconds(cumulative):     0.234
 Scaled St.F(U) = -0.8589E-03; BETA =     13.8769; BETA/||U||=      1.0001
 Multipl.=   22.33    ; Step-length=    1.0000; State Func.calls:  328
------------------------------------------------------------------------------
 Iteration No.  7; CPU-seconds(cumulative):     0.234
 Scaled St.F(U) = -0.5574E-03; BETA =     13.8752; BETA/||U||=      1.0000
 Multipl.=   21.81    ; Step-length=    1.0000; State Func.calls:  333
------------------------------------------------------------------------------
 Iteration No.  8; CPU-seconds(cumulative):     0.234
 Scaled St.F(U) = -0.3805E-03; BETA =     13.8741; BETA/||U||=      1.0000
 Multipl.=   21.40    ; Step-length=    1.0000; State Func.calls:  338
------------------------------------------------------------------------------
 Iteration No.  9; CPU-seconds(cumulative):     0.234
 Scaled St.F(U) = -0.2707E-03; BETA =     13.8735; BETA/||U||=      1.0000
 Multipl.=   21.06    ; Step-length=    1.0000; State Func.calls:  343
------------------------------------------------------------------------------
 Iteration No. 10; CPU-seconds(cumulative):     0.250
 Scaled St.F(U) = -0.1993E-03; BETA =     13.8730; BETA/||U||=      1.0000
 Multipl.=   20.78    ; Step-length=    1.0000; State Func.calls:  348
------------------------------------------------------------------------------
 Iteration No. 11; CPU-seconds(cumulative):     0.250
 Scaled St.F(U) = -0.1510E-03; BETA =     13.8727; BETA/||U||=      1.0000
 Multipl.=   20.53    ; Step-length=    1.0000; State Func.calls:  353
------------------------------------------------------------------------------
 Iteration No. 12; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.1172E-03; BETA =     13.8724; BETA/||U||=      1.0000
 Multipl.=   20.32    ; Step-length=    1.0000; State Func.calls:  358
------------------------------------------------------------------------------
 Iteration No. 13; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.9296E-04; BETA =     13.8722; BETA/||U||=      1.0000
 Multipl.=   20.14    ; Step-length=    1.0000; State Func.calls:  363
------------------------------------------------------------------------------
 Iteration No. 14; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.7515E-04; BETA =     13.8721; BETA/||U||=      1.0000
 Multipl.=   19.98    ; Step-length=    1.0000; State Func.calls:  368
------------------------------------------------------------------------------
 Iteration No. 15; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.6179E-04; BETA =     13.8720; BETA/||U||=      1.0000
 Multipl.=   19.83    ; Step-length=    1.0000; State Func.calls:  373
------------------------------------------------------------------------------
 Iteration No. 16; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.5158E-04; BETA =     13.8719; BETA/||U||=      1.0000
 Multipl.=   19.70    ; Step-length=    1.0000; State Func.calls:  378
------------------------------------------------------------------------------
 Iteration No. 17; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.4367E-04; BETA =     13.8718; BETA/||U||=      1.0000
 Multipl.=   19.58    ; Step-length=    1.0000; State Func.calls:  383
------------------------------------------------------------------------------
 Iteration No. 18; CPU-seconds(cumulative):     0.266
 Scaled St.F(U) = -0.3744E-04; BETA =     13.8717; BETA/||U||=      1.0000
 Multipl.=   19.47    ; Step-length=    1.0000; State Func.calls:  388
------------------------------------------------------------------------------
 Iteration No. 19; CPU-seconds(cumulative):     0.281
 Scaled St.F(U) = -0.3248E-04; BETA =     13.8717; BETA/||U||=      1.0000
 Multipl.=   19.37    ; Step-length=    1.0000; State Func.calls:  393
------------------------------------------------------------------------------
 Iteration No. 20; CPU-seconds(cumulative):     0.281
 Scaled St.F(U) = -0.2848E-04; BETA =     13.8716; BETA/||U||=      1.0000
 Multipl.=   19.28    ; Step-length=    1.0000; State Func.calls:  398
------------------------------------------------------------------------------
 Iteration No. 21; CPU-seconds(cumulative):     0.297
 Scaled St.F(U) = -0.2523E-04; BETA =     13.8716; BETA/||U||=      1.0000
 Multipl.=   19.19    ; Step-length=    1.0000; State Func.calls:  403
```

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-----------------------------------------------------------------------------
 Iteration No. 22; CPU-seconds(cumulative):     0.297
  Scaled St.F(U) = -0.2256E-04; BETA =     13.8716; BETA/||U||=      1.0000
  Multipl.=   19.11    ; Step-length=    1.0000; State Func.calls:  408
-----------------------------------------------------------------------------
 Iteration No. 23; CPU-seconds(cumulative):     0.312
  Scaled St.F(U) = -0.2036E-04; BETA =     13.8715; BETA/||U||=      1.0000
  Multipl.=   19.03    ; Step-length=    1.0000; State Func.calls:  413
-----------------------------------------------------------------------------
 Iteration No. 24; CPU-seconds(cumulative):     0.312
  Scaled St.F(U) = -0.1853E-04; BETA =     13.8715; BETA/||U||=      1.0000
  Multipl.=   18.96    ; Step-length=    1.0000; State Func.calls:  418
-----------------------------------------------------------------------------
 Iteration No. 25; CPU-seconds(cumulative):     0.312
  Scaled St.F(U) = -0.1700E-04; BETA =     13.8715; BETA/||U||=      1.0000
  Multipl.=   18.89    ; Step-length=    1.0000; State Func.calls:  423
-----------------------------------------------------------------------------
 Iteration No. 26; CPU-seconds(cumulative):     0.312
  Scaled St.F(U) = -0.1572E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.82    ; Step-length=    1.0000; State Func.calls:  428
-----------------------------------------------------------------------------
 Iteration No. 27; CPU-seconds(cumulative):     0.312
  Scaled St.F(U) = -0.1464E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.76    ; Step-length=    1.0000; State Func.calls:  433
-----------------------------------------------------------------------------
 Iteration No. 28; CPU-seconds(cumulative):     0.328
  Scaled St.F(U) = -0.1374E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.70    ; Step-length=    1.0000; State Func.calls:  438
-----------------------------------------------------------------------------
 Iteration No. 29; CPU-seconds(cumulative):     0.328
  Scaled St.F(U) = -0.1298E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.64    ; Step-length=    1.0000; State Func.calls:  443
-----------------------------------------------------------------------------
 Iteration No. 30; CPU-seconds(cumulative):     0.328
  Scaled St.F(U) = -0.1234E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.58    ; Step-length=    1.0000; State Func.calls:  448
-----------------------------------------------------------------------------
 Iteration No. 31; CPU-seconds(cumulative):     0.328
  Scaled St.F(U) = -0.1181E-04; BETA =     13.8714; BETA/||U||=      1.0000
  Multipl.=   18.53    ; Step-length=    1.0000; State Func.calls:  453
-----------------------------------------------------------------------------
 Iteration No. 32; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1138E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.48    ; Step-length=    1.0000; State Func.calls:  458
-----------------------------------------------------------------------------
 Iteration No. 33; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1103E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.42    ; Step-length=    1.0000; State Func.calls:  463
-----------------------------------------------------------------------------
 Iteration No. 34; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1076E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.37    ; Step-length=    1.0000; State Func.calls:  468
-----------------------------------------------------------------------------
 Iteration No. 35; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1056E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.32    ; Step-length=    1.0000; State Func.calls:  473
-----------------------------------------------------------------------------
 Iteration No. 36; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1042E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.27    ; Step-length=    1.0000; State Func.calls:  478
-----------------------------------------------------------------------------
 Iteration No. 37; CPU-seconds(cumulative):     0.344
  Scaled St.F(U) = -0.1035E-04; BETA =     13.8713; BETA/||U||=      1.0000
  Multipl.=   18.22    ; Step-length=    1.0000; State Func.calls:  483
-----------------------------------------------------------------------------
 Iteration No. 38; CPU-seconds(cumulative):     0.359
  Scaled St.F(U) = -0.1034E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   18.17    ; Step-length=    1.0000; State Func.calls:  488
-----------------------------------------------------------------------------
 Iteration No. 39; CPU-seconds(cumulative):     0.359
  Scaled St.F(U) = -0.1039E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   18.12    ; Step-length=    1.0000; State Func.calls:  493
-----------------------------------------------------------------------------
 Iteration No. 40; CPU-seconds(cumulative):     0.359
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 Scaled St.F(U) = -0.1050E-04; BETA =     13.8712; BETA/||U||=      1.0000
 Multipl.=   18.07     ; Step-length=    1.0000; State Func.calls:  498
-------------------------------------------------------------------------------
 Iteration No. 41; CPU-seconds(cumulative):     0.375
  Scaled St.F(U) = -0.1068E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   18.02     ; Step-length=    1.0000; State Func.calls:  503
-------------------------------------------------------------------------------
 Iteration No. 42; CPU-seconds(cumulative):     0.375
  Scaled St.F(U) = -0.1092E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   17.97     ; Step-length=    1.0000; State Func.calls:  508
-------------------------------------------------------------------------------
 Iteration No. 43; CPU-seconds(cumulative):     0.375
  Scaled St.F(U) = -0.1124E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   17.92     ; Step-length=    1.0000; State Func.calls:  513
-------------------------------------------------------------------------------
 Iteration No. 44; CPU-seconds(cumulative):     0.375
  Scaled St.F(U) = -0.1164E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   17.87     ; Step-length=    1.0000; State Func.calls:  518
-------------------------------------------------------------------------------
 Iteration No. 45; CPU-seconds(cumulative):     0.391
  Scaled St.F(U) = -0.1213E-04; BETA =     13.8712; BETA/||U||=      1.0000
  Multipl.=   17.81     ; Step-length=    1.0000; State Func.calls:  523
-------------------------------------------------------------------------------
 Iteration No. 46; CPU-seconds(cumulative):     0.391
  Scaled St.F(U) = -0.1271E-04; BETA =     13.8711; BETA/||U||=      1.0000
  Multipl.=   17.76     ; Step-length=    1.0000; State Func.calls:  528
-------------------------------------------------------------------------------
 Iteration No. 47; CPU-seconds(cumulative):     0.391
  Scaled St.F(U) = -0.1342E-04; BETA =     13.8711; BETA/||U||=      1.0000
  Multipl.=   17.71     ; Step-length=    1.0000; State Func.calls:  533
-------------------------------------------------------------------------------
 Iteration No. 48; CPU-seconds(cumulative):     0.406
  Scaled St.F(U) = -0.1425E-04; BETA =     13.8711; BETA/||U||=      1.0000
  Multipl.=   17.65     ; Step-length=    1.0000; State Func.calls:  538
-------------------------------------------------------------------------------
 Iteration No. 49; CPU-seconds(cumulative):     0.406
  Scaled St.F(U) = -0.1524E-04; BETA =     13.8711; BETA/||U||=      1.0000
  Multipl.=   17.60     ; Step-length=    1.0000; State Func.calls:  543
-------------------------------------------------------------------------------
 Iteration No. 50; CPU-seconds(cumulative):     0.406
  Scaled St.F(U) = -0.1642E-04; BETA =     13.8711; BETA/||U||=      1.0000
  Multipl.=   17.54     ; Step-length=    1.0000; State Func.calls:  548

 FORM-beta=  13.871; SORM-beta=   --   ; beta(Sampling)=   --    (IER=   1)
 FORM-Pf=   3.27E-007    ; SORM-Pf=    --     ; Pf(Sampling)=    --

            --------- Statistics after COMREL-TI ---------
             State Function calls              =      548
             State Funct. gradient evaluations =   100
             Total computation time (CPU-secs.)=     0.45
             The error indicator (IER) was =    1
            **********************************************

The reliability analysis returned a nonzero error state: 1
See results or monitoring file
```

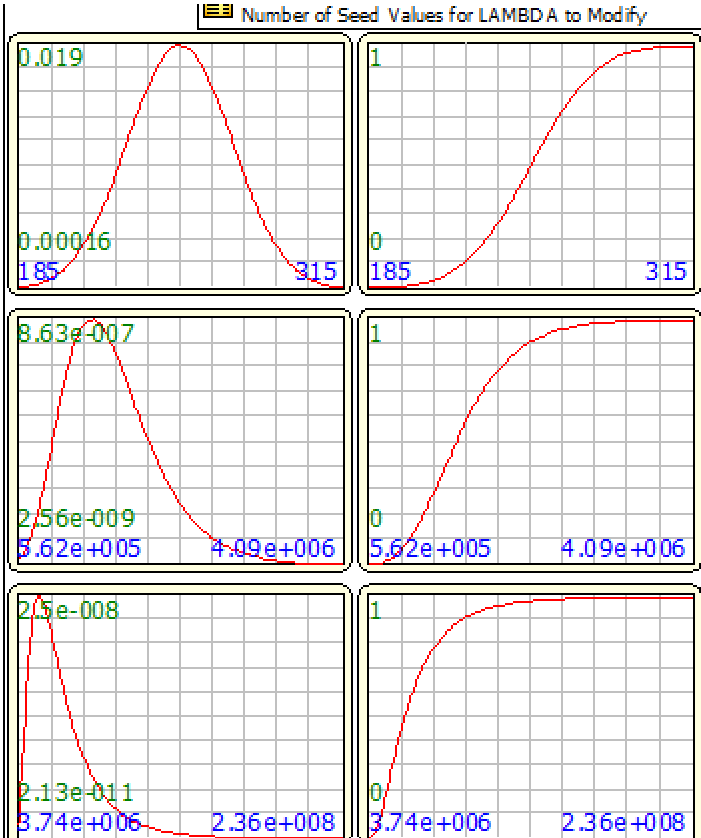
TABLE 3: RELIABILITY ANALYSIS FOR BEAM NO.245

FORM-beta	13.871
FORM-Pf	3.27E-7

Tabular Form

R My	MOMENT IN Y DIR	Lognormal	M	✓	x̄ C = 1.985e+007	σ C = 1.4e+007
R Mz	MOMENT IN Z DIRN	Lognormal	M	✓	x̄ C = 3.7201e+007	σ C = 2.804e+007
R N	AXIAL FORCE IN X DIR	Lognormal	M	✓	x̄ C = 1.596e+006	σ C = 526000
R fy	YIELD STRESS	Normal (Gauss)	M	✓	x̄ C = 250	σ C = 21.04

Number of Seed Values for LAMBDA to Modify | ADPT_NCS | 15



fy - { YIELD STRESS }

Normal (Gauss) Distribution

Moments	mean = 250	std.dev. = 21.04
Values	250	21.04
Parameters	m = 250	sigma = 21.04

N - { AXIAL FORCE IN X DIR }

Lognormal Distribution

Moments	mean = 1.596e+006	std.dev. = 526000
Values	1.596e+006	526000
Parameters	xi = 1.5158e+006	delta = 0.321115

Mz - { MOMENT IN Z DIRN }

Lognormal Distribution

Moments	mean = 3.7201e+007	std.dev. = 2.804e+007
Values	3.7201e+007	2.804e+007
Parameters	xi = 2.97073e+007	delta = 0.670733

7.2 STAAD PRO ANALYSIS

TABLE 4: STAAD ANALYSIS FOR BEAM NO.445

AXIAL FORCE-KN, SHEAR-KN ,MOMENT-KNm

MEMBER	LOAD	JT	AXIAL	SHAER Y	SHEAR Z	TORSION	MOMENT -	MOMENT-Z
445	1	245	14.491	5.902	-0.034	0	0.034	13.851
		250	-14.491	-5.902	0.034	0	0.069	3.856
	2	245	17.407	0.09	-6.214	0	10.102	0.038
		250	-17.407	-0.09	6.214	0	8.541	0.231
	3	245	449.765	0.84	-0.49	0	0.452	0.228
		250	-439.97	-0.84	0.49	0	1.018	2.293
	4	245	188.987	0.353	-0.191	0	0.176	0.085
		250	-181.487	-0.353	0.191	0	0.398	0.974
	5	245	1085.878	2.029	-1.158	0	1.066	0.532
		250	-1056.48	-2.029	1.158	0	2.408	5.554
	6	245	789.234	11.462	-0.891	0	0.826	23.934
		250	-772.583	-11.462	0.891	0	1.848	10.452
	7	245	794.192	1.581	-11.397	0	17.94	0.453
		250	-777.541	-1.581	11.397	0	16.25	4.291
	8	245	739.966	-8.606	-0.774	0	0.71	-23.16
		250	-723.316	8.606	0.774	0	1.614	-2.657
	9	245	735.009	1.275	9.731	0	-16.405	0.322
		250	-718.358	-1.275	-9.731	0	-12.788	3.504
	10	245	849.215	9.224	-0.93	0	0.859	18.413
		250	-826.732	-9.224	0.93	0	1.931	9.26
	11	245	853.006	1.668	-8.964	0	13.947	0.457
		250	-830.523	-1.668	8.964	0	12.944	4.548
	12	245	811.539	-6.122	-0.841	0	0.771	-17.6
		250	-789.056	6.122	0.841	0	1.752	-0.765

SAAD PRO ANALYSIS OF COLUMN NO-45

member	load	JOINT	Axial force KN	SHEAR -Y KN	SHAER-X KN	TORSION KNm	MOMENT-Y KNm	MOMENT-Z KNm
45	1	1	-14.491	5.902	-0.034	0	0.034	13.851
		6	14.491	-5.902	0.034	0	0.069	3.856
	2	1	-17.407	0.09	-6.214	0	10.102	0.038
		6	17.407	-0.09	6.214	0	8.541	0.231
	3	1	449.765	-0.84	0.49	0	-0.452	-0.228
		6	-439.97	0.84	-0.49	0	-1.018	-2.293
	4	1	188.987	-0.353	0.191	0	-0.176	-0.085
		6	-181.487	0.353	-0.191	0	-0.398	-0.974
CRITICAL	5	1	1085.878	-2.029	1.158	0	-1.066	-0.532
		6	-1056.477	2.029	-1.158	0	-2.408	-5.554
	6	1	739.966	8.606	0.774	0	-0.71	23.16
		6	-723.316	-8.606	-0.774	0	-1.614	2.657
	7	1	735.009	-1.275	-9.731	0	16.405	-0.322
		6	-718.358	1.275	9.731	0	12.788	-3.504
	8	1	789.234	-11.462	0.891	0	-0.826	-23.934
		6	-772.584	11.462	-0.891	0	-1.848	-10.452
	9	1	794.192	-1.581	11.397	0	-17.94	-0.453
		6	-777.541	1.581	-11.397	0	-16.25	-4.291
	10	1	811.539	6.122	0.841	0	-0.771	17.6
		6	-789.057	-6.122	-0.841	0	-1.752	0.765
	11	1	807.748	-1.434	-7.193	0	12.317	-0.356
		6	-785.265	1.434	7.193	0	9.262	-3.947
	12	1	849.215	-9.224	0.93	0	-0.859	-18.413
		6	-826.732	9.224	-0.93	0	-1.931	-9.26
	13	1	853.006	-1.668	8.964	0	-13.947	-0.457
		6	-830.523	1.668	-8.964	0	-12.944	-4.548
	14	1	764.6	-1.428	0.833	0	-0.768	-0.387
		6	-747.95	1.428	-0.833	0	-1.731	-3.898
	15	1	830.377	-1.551	0.885	0	-0.815	-0.406
		6	-807.894	1.551	-0.885	0	-1.841	-4.247

STAAD PRO ANALYSIS OF BEAM NO 245

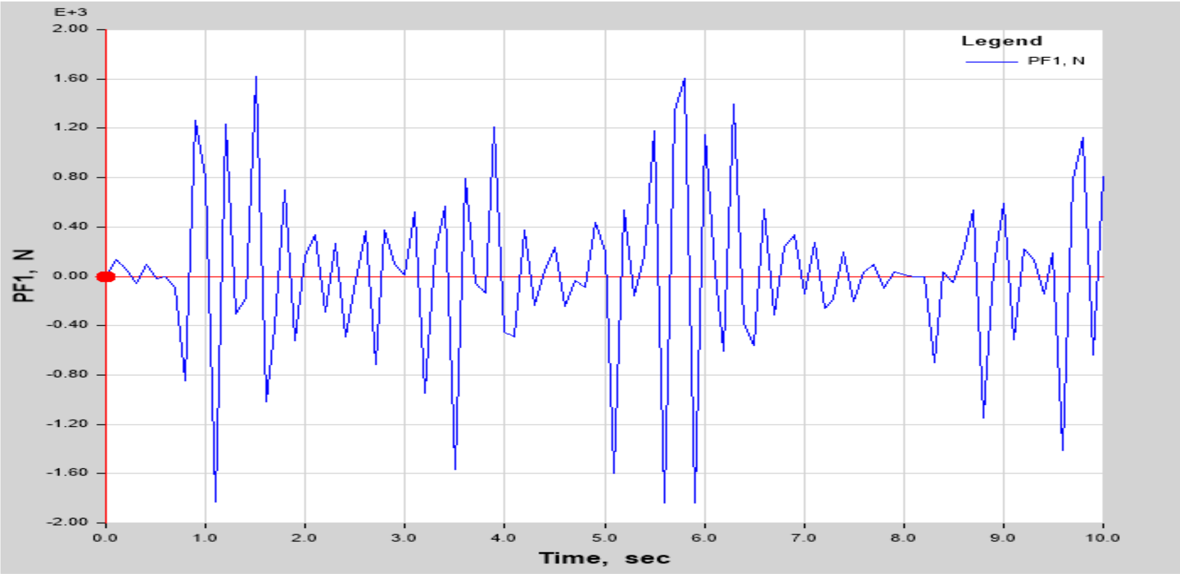
MEMBER	LOAD	JT	AXIAL	SHAER Y	SHEAR Z	TORSION	MOMENT -	MOMENT-Z
			KN	KN	KN	KNm	KNm	KNm
245	1	123	0	7.172	0	0	0	14.138
		128	0	-7.172	0	0	0	7.377
	2	123	0	0	-6.919	0	10.75	0
		128	0	0	6.919	0	10.006	0
	3	123	701.827	0	0	0	0	0
		128	-692.032	0	0	0	0	0
	4	123	237.322	0	0	0	0	0
		128	-229.822	0	0	0	0	0
Critical	5	123	1596.553	0	0	0	0	0
		128	-1567.15	0	0	0	0	0
	6	123	1193.106	12.192	0	0	0	24.034
		128	-1176.46	-12.192	0	0	0	12.541
	7	123	1193.106	0	-11.762	0	18.275	0
		128	-1176.46	0	11.762	0	17.011	0
	8	123	1193.106	-12.192	0	0	0	-24.034
		128	-1176.46	12.192	0	0	0	-12.541
	9	123	1193.106	0	11.762	0	-18.275	0
		128	-1176.46	0	-11.762	0	-17.011	0
	10	123	1220.894	9.323	0	0	0	18.379
		128	-1198.41	-9.323	0	0	0	9.59
	11	123	1220.894	0	-8.994	0	13.975	0
		128	-1198.41	0	8.994	0	13.008	0
	12	123	1220.894	-9.323	0	0	0	-18.379
		128	-1198.41	9.323	0	0	0	-9.59
	13	123	1220.894	0	8.994	0	-13.975	0
		128	-1198.41	0	-8.994	0	-13.008	0
	14	123	1193.106	0	0	0	0	0
		128	-1176.46	0	0	0	0	0
	15	123	1220.894	0	0	0	0	0
		128	-1198.41	0	0	0	0	0

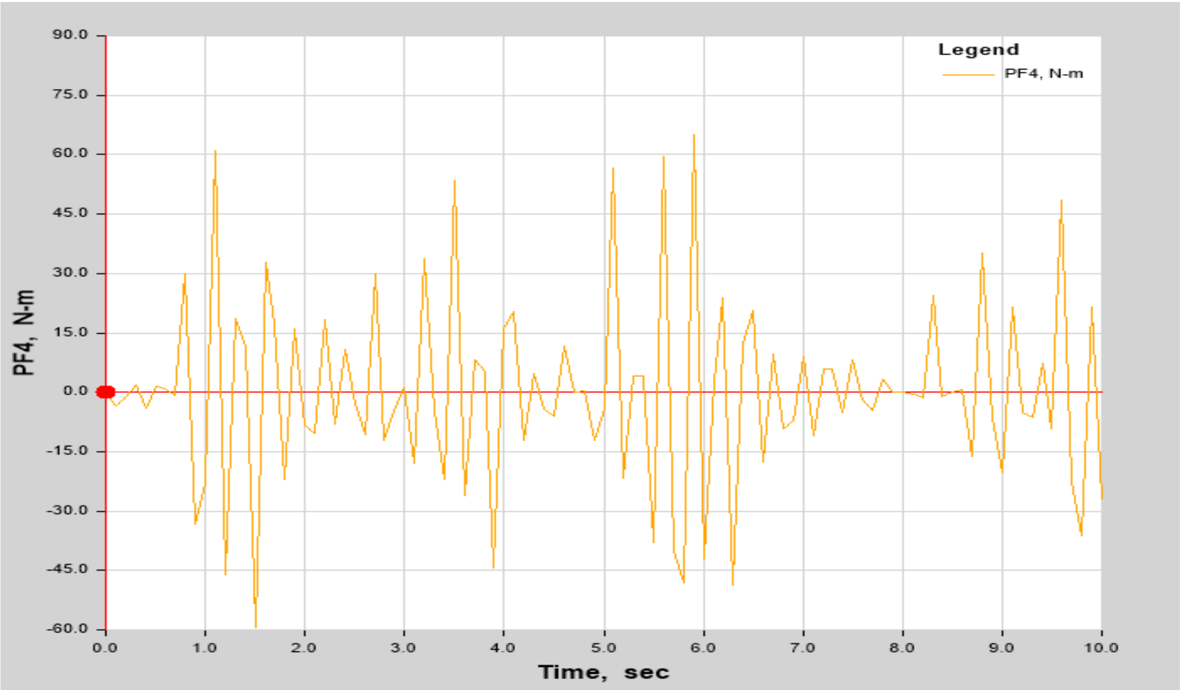
TIME HISTORY ANALYSIS USING ETABS

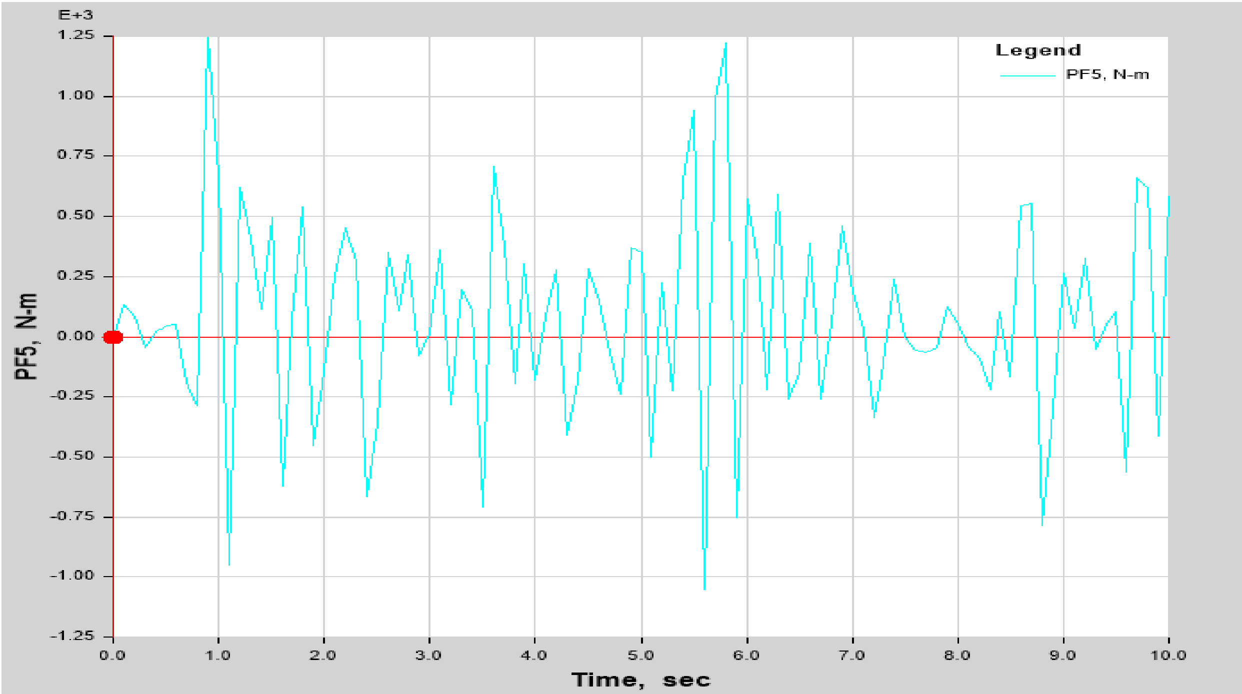
Input Data

Name	THPlot1
Load Case	Time-Accl data -x

Plots:







Utilized Plot Functions

Base FX	Type=Base Force; Component=Base shear X
PF1	Type=Column Force; Component=Axial Force; Column=C45; Rel. Dist.=0
PF2	Type=Column Force; Component=Shear 2-2; Column=C45; Rel. Dist.=0
PF3	Type=Column Force; Component=Shear 3-3; Column=C45; Rel. Dist.=0
PF4	Type=Column Force; Component=Moment 2-2 ; Column=C45; Rel. Dist.=0
PF5	Type=Column Force; Component=Moment 3-3; Column=C45; Rel. Dist.=0
PF6	Type=Column Force; Component=Torsion; Column=C45; Rel. Dist.=0

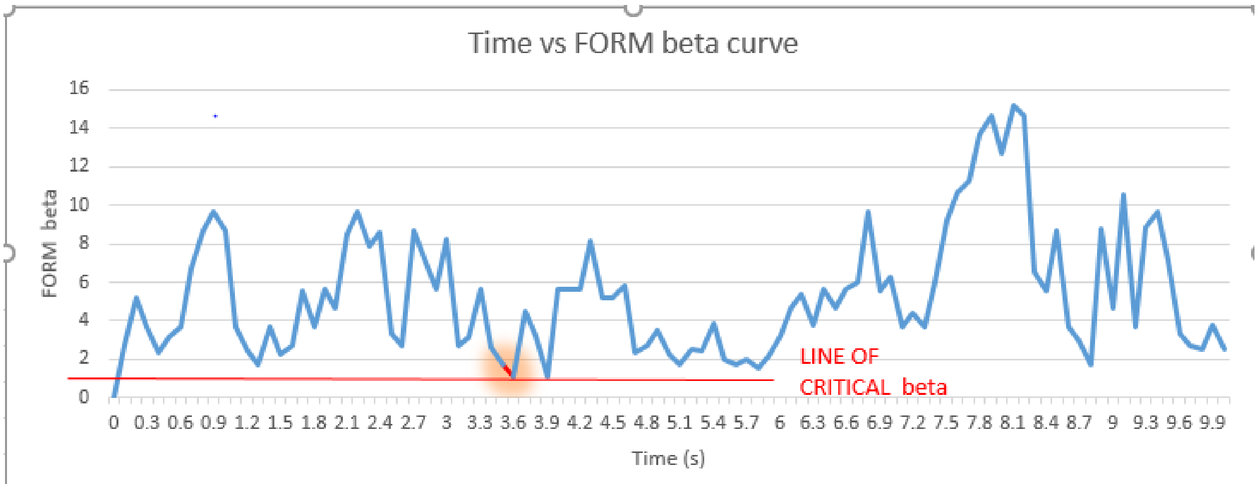
Tabulated Plot Coordinates

Time History Response Values for Column no-45

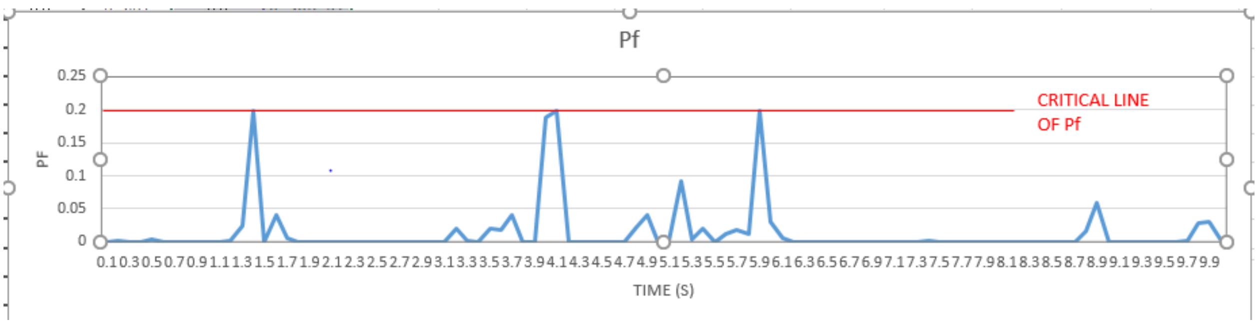
Time	PF1	PF4	PF5	FORM-beta	Pf
sec	N	N-m	N-m		
0	0	0	0	0	0
0.1	133.2162	-3.4953	132.3189	2.683	1.98E-03
0.2	60.0891	-1.2111	86.8118	5.235	2.63E-06
0.3	-61.8708	1.9352	-45.4967	3.681	4.54E-04
0.4	99.0866	-3.9401	23.1194	2.356	3.42E-03
0.5	-19.0937	1.4925	48.1004	3.123	1.88E-04
0.6	-5.5742	1.0417	49.951	3.672	5.98E-05
0.7	-88.0956	-0.924	-190.4443	6.681	1.69E-09
0.8	-848.0088	29.8076	-290.0873	8.681	2.58E-10
0.9	1263.8082	-33.4512	1247.0492	9.656	3.68E-11
1	793.3463	-22.8008	671.4694	8.681	5.78E-15
1.1	-1832.2461	60.976	-951.6442	3.653	8.28E-04
1.2	1237.5222	-46.1421	622.1403	2.442	2.38E-02

Time	PF1	PF4	PF5	FORM-beta	Pf
sec	N	N-m	N-m		
1.3	-303.0341	18.6252	410.9287	1.681	1.98E-01
1.4	-177.701	11.3084	112.5052	3.723	2.58E-04
1.5	1620.5987	-59.3834	497.8138	2.253	3.96E-02
1.6	-1017.6924	32.9279	-620.6482	2.683	6.68E-03
1.7	-415.2349	17.0597	66.6291	5.521	1.25E-07
1.8	703.3086	-22.0818	539.0125	3.681	1.28E-04
1.9	-523.3265	15.9204	-450.9366	5.662	1.38E-05
2	168.9107	-8.444	-141.4298	4.682	1.48E-04
2.1	335.4365	-10.452	253.2528	8.536	1.98E-10
2.2	-293.9663	18.2515	450.2268	9.681	2.68E-10
2.3	266.4698	-8.007	311.2061	7.856	1.22E-11
2.4	-489.8941	10.7382	-665.311	8.635	1.26E-10
2.5	-33.6636	-2.9832	-357.5888	3.362	2.59E-10
2.6	368.3555	-10.629	348.7293	2.683	1.78E-08
2.7	-714.5784	29.8024	110.5974	8.681	1.98E-10
2.8	376.1285	-12.1066	342.8046	7.258	1.23E-11
2.9	104.6881	-4.5677	-78.3731	5.653	1.98E-07
3	13.1791	1.0863	12.0371	15.23	1.52E-15
3.1	519.8536	-17.8223	359.7798	2.681	1.98E-02
3.2	-946.6283	33.7326	-283.4455	3.123	1.25E-03
3.3	186.8345	-4.8467	194.6707	5.681	1.85E-08
3.4	569.2632	-21.9031	108.1422	2.636	1.98E-02
3.5	-1569.3287	53.363	-709.8129	1.681	1.86E-02
3.6	794.9118	-26.0413	706.9392	1.128	3.98E-02
3.7	-58.9041	8.0489	376.4178	4.523	1.56E-07
3.8	-138.6234	5.2342	-195.0829	3.251	1.98E-04
3.9	1207.1152	-44.3923	302.3214	1.083	1.89E-01
4	-452.8679	15.8804	-180.0833	5.625	1.98E-01
4.1	-490.251	20.2924	85.5748	5.652	1.65E-07
4.2	374.3772	-12.161	276.5094	5.681	1.98E-08
4.3	-236.0123	4.6681	-407.6783	8.125	1.85E-06
4.4	51.7836	-4.1863	-186.2086	5.236	2.98E-07
4.5	238.239	-6.0853	285.2829	5.238	3.98E-08
4.6	-240.0312	11.5333	147.7691	5.782	1.98E-05
4.7	-37.0056	0.0112	-66.7551	2.33	2.12E-02
4.8	-90.2455	0.4583	-237.6363	2.681	3.98E-02
4.9	434.7367	-12.0154	368.1329	3.523	1.92E-04
5	199.4492	-4.0185	349.9097	2.256	1.98E-03
5.1	-1600.5723	56.5573	-501.2512	1.682	9.25E-02
5.2	540.6323	-21.7797	225.8178	2.526	3.98E-03
5.3	-155.1774	4.1178	-223.4087	2.398	1.98E-02
5.4	167.8281	4.244	644.5226	3.852	3.41E-04
5.5	1180.5974	-38.0391	940.689	1.963	1.12E-02
5.6	-1840.1739	59.536	-1049.865	1.681	1.89E-02
5.7	1343.3581	-39.8948	990.1973	1.989	1.26E-02
5.8	1601.732	-48.0969	1221.9893	1.526	1.99E-01
5.9	-1840.7305	65.0821	-752.8452	2.181	2.95E-02
6	1150.5861	-42.0909	572.8672	3.223	5.325E-03
6.1	39.5419	2.8114	301.8679	4.681	1.98E-05
6.2	-611.0845	23.8483	-220.3688	5.369	1.56E-07
6.3	1398.8753	-48.666	593.719	3.741	1.68E-04
6.4	-380.9407	12.1228	-257.4496	5.681	1.88E-07
6.5	-564.545	20.596	-152.7721	4.685	1.98E-08
6.6	546.775	-17.7219	388.626	5.691	1.48E-08
6.7	-315.1368	9.6767	-256.677	5.987	1.98E-08

Time	PF1	PF4	PF5	FORM-beta	Pf
sec	N	N-m	N-m		
6.8	241.2261	-9.3225	58.4882	9.681	1.85E-14
6.9	336.7768	-7.24	461.384	5.565	1.98E-07
7	-144.678	9.0134	187.3545	6.231	1.98E-10
7.1	273.3469	-11.0704	32.822	3.685	1.41E-04
7.2	-255.8263	5.7339	-334.5027	4.425	1.58E-05
7.3	-183.2025	5.8067	-115.5628	3.651	1.68E-03
7.4	196.7868	-5.0939	237.0256	5.963	1.28E-07
7.5	-202.4858	8.1368	-2.9181	9.235	1.68E-11
7.6	32.4911	-2.0693	-53.6334	14.681	1.48E-14
7.7	97.3703	-4.7171	-63.071	14.256	1.38E-18
7.8	-95.6144	3.3938	-43.5634	13.680	1.98E-13
7.9	38.2895	-0.0661	121.7278	14.681	1.58E-14
8	11.6308	0.1553	52.2366	12.697	1.25E-15
8.1	-1.866	-0.4127	-47.8495	15.213	1.94E-15
8.2	-0.7818	-1.481	-88.589	14.681	1.93E-15
8.3	-696.3456	24.4217	-219.8758	6.523	2.12E-10
8.4	35.0401	-1.0591	104.5034	5.562	1.85E-07
8.5	-50.518	-0.0069	-164.508	8.681	2.92E-07
8.6	190.7554	0.7498	543.6519	3.710	3.98E-04
8.7	538.6862	-16.2914	554.9296	2.985	1.57E-02
8.8	-1145.5468	35.251	-784.2123	1.688	5.98E-02
8.9	67.1699	-6.7367	-304.5871	8.795	2.35E-05
9	590.3769	-20.3339	262.4823	4.681	1.75E-06
9.1	-514.6801	21.6404	38.0794	10.564	3.58E-09
9.2	223.9796	-5.2033	327.055	3.657	1.98E-04
9.3	138.4904	-6.2903	-51.7636	8.852	1.78E-11
9.4	-146.5632	7.4487	49.0919	9.674	1.98E-12
9.5	186.482	-9.3241	104.1958	7.147	1.55E-10
9.6	-1411.3706	48.3802	-565.2899	3.295	1.98E-03
9.7	786.0085	-22.2748	657.8426	2.682	2.77E-02
9.8	1126.9354	-36.2624	623.1788	2.563	2.94E-02
9.9	-638.0649	21.3883	-415.6699	3.749	1.97E-03
10	804.952	-26.8628	582.1511	2.548	1.98E-03



Time vs FORM beta Curve



Time vs Pf curve

CONCLUSION

The objective of the project to determine the reliability of the corner column, middle corner column and central column for a G+10 building was successfully conducted. The important conclusive points are discussed below-:

1. There is an inclusion of approximate methods (first order reliability methods and second order reliability methods i.e. FORM AND SORM methods) in the project, which have an advantage of being simple in nature and their computation takes lesser time.
2. Response surface methods have been introduced that includes the basic theory and there is a simplification for complex systems through repetitive solution methods.
3. The reliability index method proposed by Hasofer and Lind accounts for the drawbacks of MVFOSM.
4. The method has the potential to greatly reduce the risk factor involved in the designing leading to the larger life expectancy of the structure.
5. The methods used are capable of producing the complete sensitive analysis that are capable of accounting any random variation in the parametric analysis.
6. The plot of FORM beta vs time, Pf vs time has been obtained and loading at critical time is identified.
7. The method proposed is one of the advanced method so as to determine the probability of failure of a structure or building components.
8. The method proposed by Hasofer and Lind accounts for the drawbacks of MVFOSM.

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