FECG EXTRACTION USING VARIOUS LMS ALGORITHMS

A DISSERTATION

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE AWARD OF THE DEGREE

OF

MASTER OF TECHNOLOGY

IN

SIGNAL PROCESSING & DIGITAL DESIGN

Submitted by:

Manoj Kumar Vimal

2K16/SPD/08

Under the supervision of

Shri Rajesh Birok

(Associate Professor)

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana road, Delhi-110042

MAY, 2018

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi 110042

CANDIDATE'S DECLARATION

I, **Manoj Kumar Vimal, Roll no. 2K16/SPD/08** of **M.Tech** (Signal Processing & Digital Design), hereby declare that the project dissertation titled "**FECG extraction using various LMS algorithms**" which is submitted by me to the department of Electronics & Communication, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is original & not copied from any source without paper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

Place: Delhi **MANOJ KUMAR VIMAL**

Date: (2K16/SPD/08)

ELECTRONICS AND COMMUNICATION ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi 110042

CERTIFICATE

I hereby certify that the Project Dissertation titled "**Fetal ECG extraction using various LMS algorithms**" which is submitted by **Manoj Kumar Vimal, 2K16/SPD/08**, Department of Electronics & Communication, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology, is a record of the project work carried out by the student under my supervision. To the best of my knowledge this work has not been submitted in part or full for any degree or diploma to this university or elsewhere.

Place: Delhi

Date:

RAJESH BIROK

(SUPERVISOR)

Associate Professor

Department of ECE

ABSTRACT

During pregnancy, it is very important to know the foetal development condition so that if there is any problem in development of foetal, it can be treated before creation of any critical condition.

Foetal development & health status can be acknowledged through various methods such as ultrasound etc., but foetal ECG plays an important role in providing important information about the health status of the baby during labor condition. Doctors always perform foetal ECG extraction during labor to know if any disease is developing in the foetal & if there is any problem, then they try to diagnose the problem accordingly.

Foetal ECG extraction is the process of separating the baby's heartbeat signal from mother's heartbeat signal. Various methods have been developed for the extraction of the foetal ECG signal such as **Principal Component Analysis** (PCA), **Blind Source Separation (Independent Component Analysis (ICA))**, **Wavelet method** etc. Adaptive filtering is one the most popular method used for the separation of foetal ECG signal.

Adaptive filtering is the method which generates an error signal corresponding to the desired output signal. Adaptive filters are based upon adaptive algorithm. Adaptive algorithms are designed in such a way that it always tries to minimize the amplitude of error signal by changing the filter coefficient values in an iterative manner. Least Mean Square is the standard adaptive algorithm which tries to minimize its cost function value. The cost function for LMS algorithm is the square of the difference

between the desired signal & the obtained output signal. In this project, we are taking pure foetal ECG signal which is our desired signal & the obtained foetal ECG signal.

In this thesis, some improvements have been implemented in standard LMS algorithm. L1 norm penalty has been applied on the LMS cost function to generate a new algorithm named as **Zero Attracting Least Mean Square (ZALMS)**. As the name suggests "Zero Attracting", this algorithm tries to make the weights of the filter equal to zero as much as possible. Higher the number of the coefficients equal to zero, higher is the sparsity of the system & this higher sparsity helps in decreasing the error signal results in increasing the performance rate. Also an adaptive algorithm named as **Normalized Least Mean Square (NLMS)** is implemented for the extraction of the foetal ECG signal. Both ZALMS & NLMS provide better results in terms of signal to noise ratio and convergence speed in comparison to standard LMS algorithm.

ACKNOWLEDGEMENT

I owe my gratitude to all the people who have helped me in this dissertation work & who have made my postgraduate college experience one of the most special periods of my life.

Firstly, I would like to express my deepest gratitude to my supervisor **Shri Rajesh Birok**, Associate Professor (ECE) for his invaluable support, guidance, motivation & encouragement throughout the period during which this work was carried out. I am deeply grateful to **Dr. S. Indu**, H.O.D. (Department. of ECE) for her support & encouragement towards me in carrying out this project successfully.

I also wish to express my heart full thanks to all faculties at Department of Electronics & Communication Engineering of Delhi Technological University for their goodwill & support that helped me a lot in successful completion of this project.

Finally, I want to thank my parents, family & friends for always believing in my abilities & showering their invaluable love & support.

MANOJ KUMAR VIMAL

(2K16/SPD/08)

Contents

List of figures

List of tables

List of abbreviations

ECG	Electrocardiogram
FECG	Fetal electrocardiogram
AECG	Abdominal electrocardiogram
MECG	Maternal electrocardiogram
FHR	Fetal heart rate
EMG	Electromyogram
ANC	Adaptive noise canceller
FIR	Finite impulse response
IIR	Infinite impulse response
MSE	Mean square error
w.r.t.	With respect to
BSS	Blind source separation
&	And
mS	millisecond
mV	millivolt
sec	Second
MATLAB	Matrix laboratory
HR	Heart rate
Fig	Figure
ICA	Independent component analysis
CT	Continuous time

BSSR	Blind source separation with the reference signal
LMS	Least mean square
NLMS	Normalized least mean square
ZALMS	Zero attracting least mean square
ANFIS	Adaptive neuro fuzzy inference system
eqn	Equation

List of some important symbols

K^+	Potassium ions
Cl^+	Chlorine ions
Na^+	Sodium ions
β	time varying step size in case of NLMS
μ	Step size in case of standard LMS
$e(n)$	Error signal
$u(n)$	Input signal in case of standard adaptive filtering
$r(n)$	Output of the linear FIR filter in adaptive filtering
$d(n)$	Desired signal
$w(n)$	Filter weights
J	Cost function
E	Expectation
$p_n(\,)$	Probability density function
$(\,)^T$	Transpose
R	Autocorrelation
P	Cross Correlation

CHAPTER 1

INTRODUCTION

1.1) OUTLINE

This chapter offers the short-term summary of a normal ECG signal, heart of the fetus, foetal ECG monitoring, polarisation plot of ECG & the literature survey.

1.2) ELECTROCARDIOGRAM (ECG)

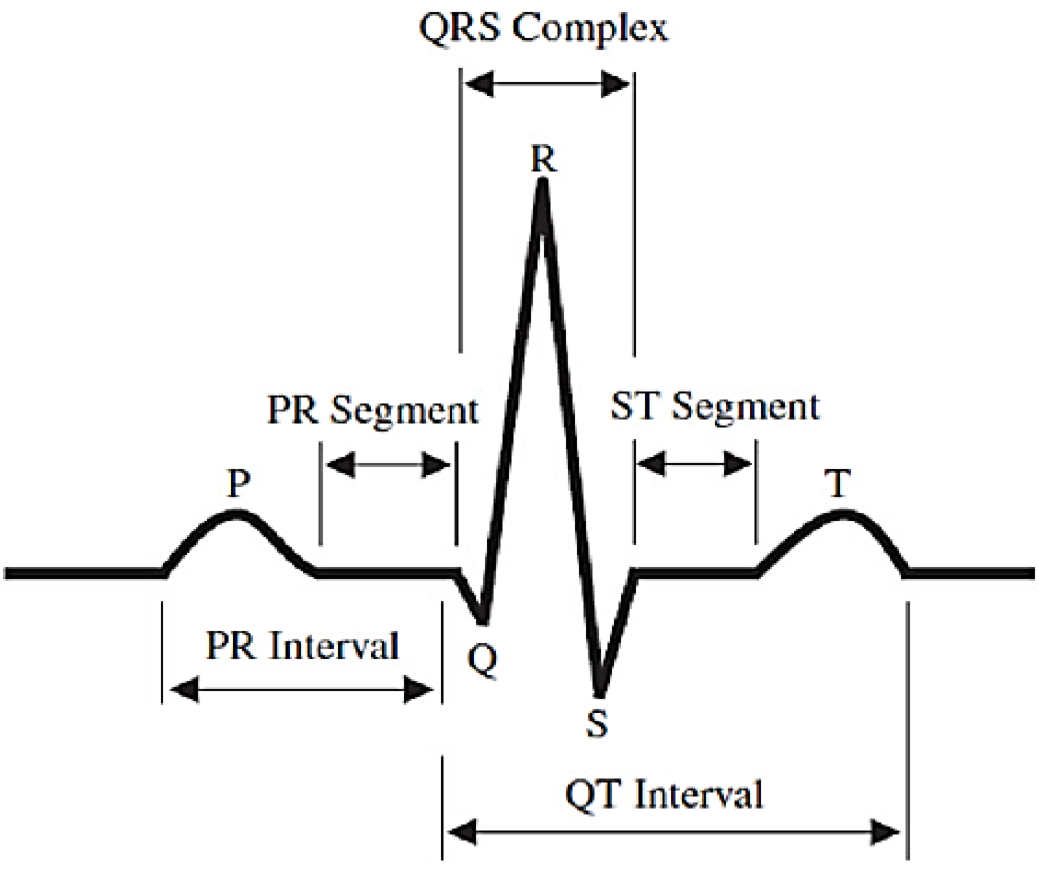

Fig 1.1 A normal ECG signal

ECG is an exploration of the electrical movement of the heart over a period of time & is noted or recorded by a device which is linked externally to the human body by using the probes (electrodes) attached to the surface of the skin of the human body. The waveform which is created by this non-invasive method is called as electrocardiogram (ECG).

An ECG helps in measuring the rate & consistency of heartbeat, the occurrence of any anomaly in the heart such as the position & size of the chambers etc. Maximum ECGs are executed for problem-solving & investigation purposes. They may also be accomplished on animals, mainly for research applications. Different waves present in an ECG signal are being described in table 1.1.

Table 1.1 ECG signal's description & duration

Features	Description	Duration
P wave	P wave signifies the atrial depolarization. Depolarization of atria ranges from sinoatrial node towards atrioventricular node, & from the right atrium to the left atrium.	Less than 80 ms
PR interval	This intermission lies between the opening of the P wave & opening of the QRS complex. This intermission discloses the interval taken by the electrical impulse to move from the sinus node through the atrioventricular node.	120 to 200 ms
	The QRS complex characterizes the right &	

QRS complex	left ventricle's quick depolarization. The QRS complex generally has larger amplitude comparing with the P wave.	80 to 100 ms
ST segment	This segment joins the T wave & the QRS complex. It characterises the time spent for the ventricles depolarization.	
T wave	The T wave signifies the ventricles repolarization.	160 ms

1.3) HEART SIGNAL POLARIZATION

Movement of positive & negative ions across the cell membrane of the heart cause change in the voltage amplitude of the ECG signal. This method is called as polarization of the heart signal.

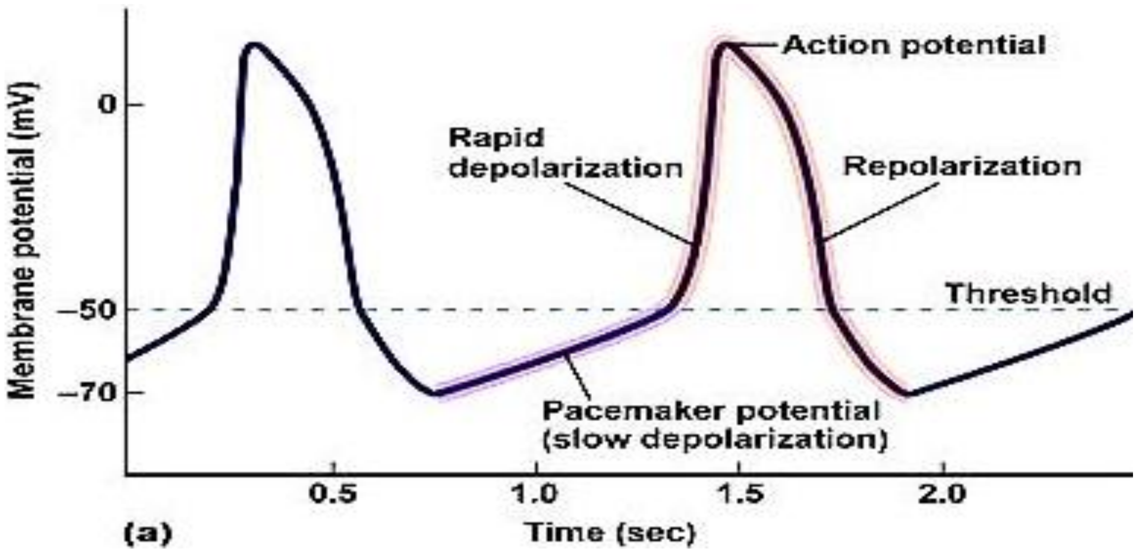

Fig 1.2 Membrane potential vs time (in seconds)

1.3.1) Action potential

Membrane potential of the cell is called as action potential. When a cell is being stimulated by an electrical current then the cell undergoes several mechanical contractions to give the action potential.

1.3.2) Resting potential

Cells have a static membrane potential which is known as resting potential. When the cells are at rest (means not stimulated anymore), the membrane freely permits ions like potassium ions (K^+) & chlorine ions (Cl^-) to move across it as it is permeable membrane. The cell is having positive charge on its external side as compared with the inner layer of the cell. Hence, for stabilizing the charge, K^+ ions move into the cell. Hence, it creates a potential difference between the inner layer & outer layer of the cell when a state of equilibrium is created means no more movements of the ions across the cell membrane. In the state of equilibrium means at rest a cell is polarised.

1.3.3) Depolarisation

In the beginning the sodium ions (Na^+) ions are not allowed to move inside the cell because the voltage-gated sodium & calcium channels closed when the resting potential is created completely. But on stimulating the cell, it initiates permitting the sodium ions to travel inside. This motion of sodium ions comprises the ionic currents. As a consequence, the obstruction of membrane diminishes for Na^+ ions movement. This movement of sodium ions in a large amount into the cell is called “Avalanche effect”. Because of a large cluster of the potassium ions in the cell, it attempts to travel towards outside of the cell but cannot travel faster than the sodium ions. As a result, the inner side of the cell gets positively charged in comparison to the outer side of the cell because of unevenness of the sodium ions. This complete process is known as depolarisation.

1.3.4) Repolarisation

The repolarization of cells relies upon the time & voltage dependent of the permeability of membrane changes for potassium ions as compared to sodium ions. During depolarization the sodium ions movement get relaxed & hence repolarization occurs because of rise in movement of the potassium ions at the termination of depolarization.

1.4) FETAL HEART

There are several body parts which develop gradually inside a fetal. Out of them one of the organs is heart of the fetal. It undergoes many variations with time through the initial phases of pregnancy. The heart starts beating after twenty two days of pregnancy. From 18^{th} - 20^{th} week of formation, the fetal heart signal can be achieved from the mother's belly. In the mother abdomen, fat & the skin have very high conductivity. Linking them form a volume conductor which supports in the spreading the ECG signals to the abdomen's surface. Figure 1.3 indicates the estimation of the fetal electrocardiogram signal.

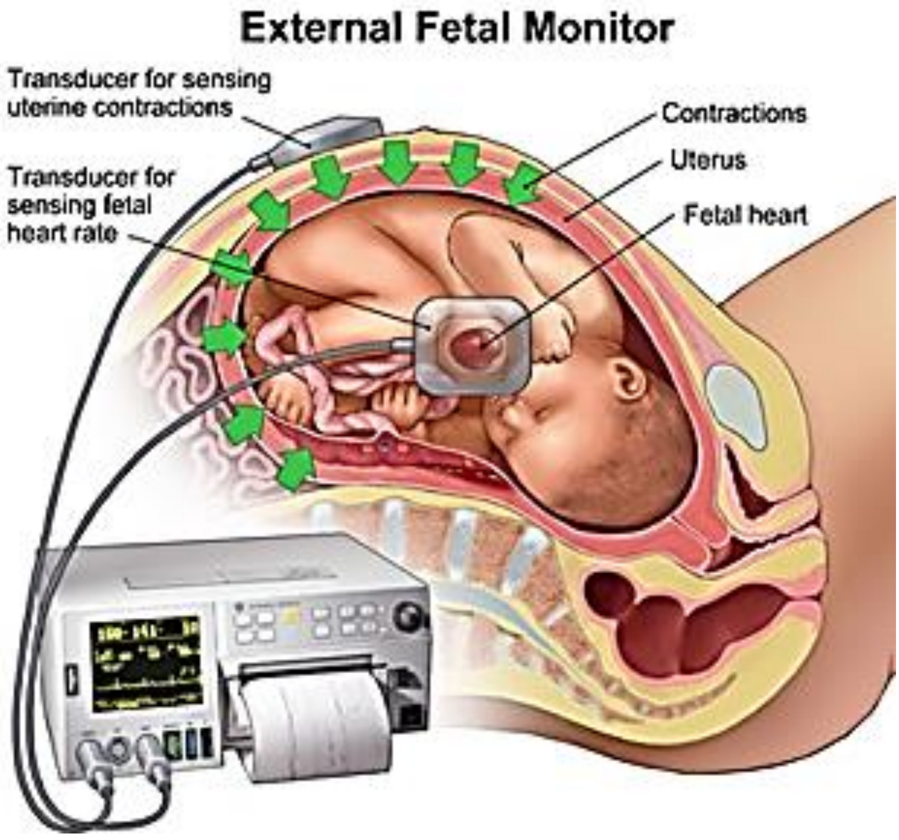

Fig 1.3 Measurement of FECG

1.5) FECG EXTRACTION

FHR monitoring is one of the promising solutions to examine health of the fetal & solve out probable deformities. During pregnancy phase, fetal monitoring allows the doctor to identify & diagnose the pathologic disorder specially asphyxia. The ECG is the simplest non-invasive problem-solving way for many heart ailments. Fetal electrocardiogram reveals the fetal heart's electrical action & delivers valuable evidence of its functional state. Non-invasive FECG has been used to attain valued medical evidence about the fetal situation during pregnancy with the help of skin electrodes positioned on the abdomen of the mother. However, all the time AECG is degraded with maternal ECG, EMG & power line interference whereas fetal ECG signal is ruined by the skin impedance, electrodes position & the gestational age. During a cardiac cycle, an ECG signal includes a P wave, QRS complex & T wave. Heart rate can be measured by detecting & counting R-peaks which are the peaks of the QRS complexes present in an ECG signal & hence physicians use this heart rate estimation method to recognize anomalies in the heart events.

Electrocardiogram signals are usually achieved the abdomen & the chest. The signal obtained from the abdomen is a mixture of fetal ECG & some other noise components & the signal obtained from the chest of the mother is mother ECG signal only. Numerous exploration methods have been suggested to extract the child ECG from the abdominal ECG such as correlation techniques, adaptive filtering, a combination of wavelet analysis & BSS method. FHR can be calculated by counting the R-peaks present in the extracted fetal ECG. However, some MECG residuals are still present in the extracted fetal ECG which makes the separation of FECG difficult.

1.6) OBJECTIVE

For separation of FECG from AECG, two signals are obtained, one is the signal obtained from chest which is the mother ECG & the other signal is obtained from the abdomen the pregnant mother & this signal is the composition of fetal ECG & other noise signals. So our main goal is to separate this FECG from abdominal ECG using proposed adaptive algorithm which we will see in this thesis later. Our goal can be described through following two points:

i) To separate the FECG signal from the AECG signal

- Restoring the waveforms of both FECG & MECG signals in software MATLAB 2017b from the data occupied using the ECG instrument.
- Use of ANC for separating the two signals FECG & MECG and to reduce the other noise components.

ii) Calculation of heart rate by counting the number of consecutive R peaks (Pan Tompkins method).

- Finding derivation of the extracted FECG signal up to 3rd order to acquire the peaks.
- Set up of a threshold value & equating the tops with this threshold to spot the top of the QRS complexes i.e. R-peaks.
- As a final point, the fetal HR can be estimated by calculating R-peaks per minute.

But we will not describe the heart rate calculation because we already have a standard method for heart rate calculation known PAN TOMPKINS ALGORITHM.

1.7) DATA ACQUISITION

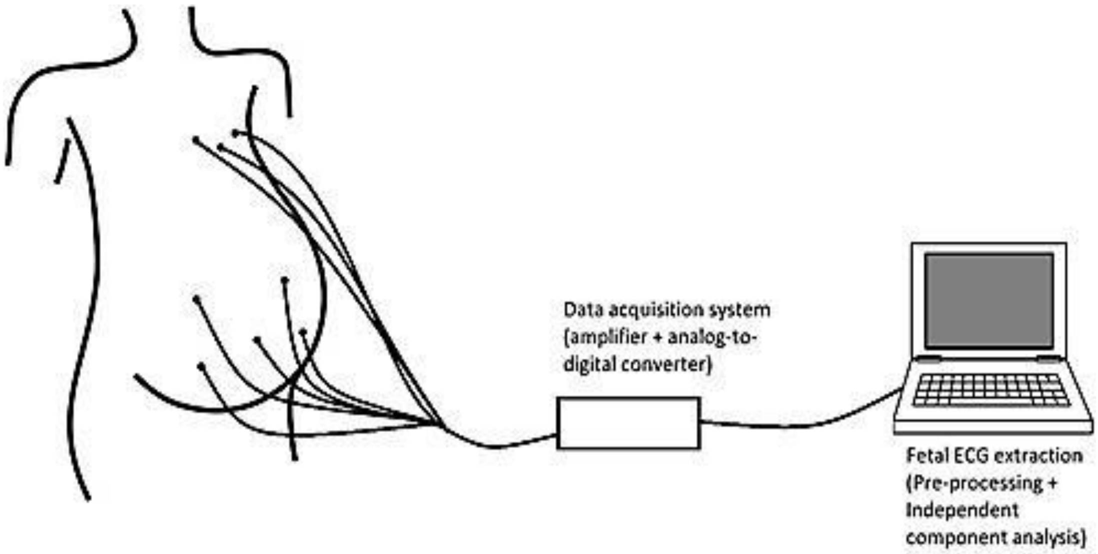

Fig 1.4 ECG data acquisition

ECG signals are usually measured from the chest & the abdomen which is clearly shown in Fig 1.4. The abdominal leads discover a compound signal, containing the involvement of both MECG & FECG while only MECG signal is acquired by chest leads.

The measured mother abdominal signal is generally ruled over by the mother heartbeat signal in comparison to fetal heartbeat signal, which spreads from the chest cavity to the abdomen.

The ECG signal which is acquired by the ECG circuit is digitalized & then transferred to the personal computer system where it is worked upon by the relevant software to get the desired parameters.

For separating fetal ECG from mother ECG signal, we will be using adaptive noise canceller. The ANC needs a reference signal to perform this task which is nothing but the MECG signal obtained from the mother's chest. The other signal needed by ANC is the abdominal signal. Just like in fetal electrocardiogram signal, some additive broadband noise is also present in the mother electrocardiogram signal.

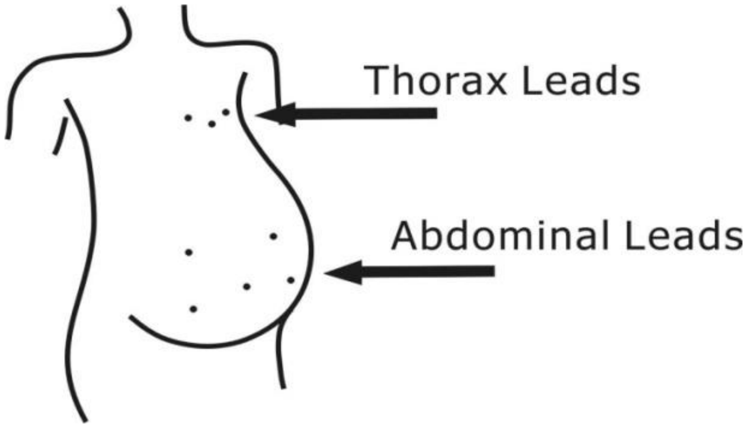

Fig 1.5 Lead locations for ECG signal measurements in a pregnant woman.

1.8) THESIS OUTLINE

Now let us have look how the proposed thesis has been outlined in the form of chapters.

Chapter 1 enlightens the framework of the FECG extraction & also provide information about literature survey on FECG extraction. Data acquisition methods are also briefly introduced.

Chapter 2 describes the Adaptive filter & standard proposed algorithms used in the adaptive filtering.

Chapter 3 describes the proposed method & the gives the detailed explanations of all the algorithms used in the proposed method.

Chapter 4 shows the comparison of the results corresponding to all the different algorithms.

Chapter 5 includes the conclusion & future scope of the proposed method.

1.9) LITERATURE REVIEW

a) An extended nonlinear Bayesian filtering framework is presented in [1] for extracting electrocardiograms (ECGs) from a solo channel as come across in the foetal ECG extraction from abdominal sensor. The recorded signals are shown off as the summation of some ECGs. Each of them is defined by a nonlinear dynamic model, formerly presented for the generation of a very realistic artificial ECG.

b) In [2], a fast & very humble procedure is defined for estimating the foetal electrocardiogram (FECG). It is established on independent component analysis (ICA). The process consists of two steps: 1) a dimensionality reduction step and 2) a computationally light post processing step used to improve the FECG signal.

c) In [3], a fresh technique is suggested for separating foetal electrocardiogram (FECG) from a thoracic ECG recording & an abdominal ECG recording of a pregnant lady. The polynomial networks technique is used to non-linearly plot the thoracic ECG signal to the abdominal ECG signal. The FECG is then taken out by deducting the mapped thoracic ECG from the abdominal ECG signal.

d) In [4], an adaptive neuro-fuzzy inference system is used for foetal ECG (FECG) withdrawal from two ECG signals recorded at the thoracic & abdominal areas of the mother's skin. The maternal part in the abdominal ECG signal is a non-linearly altered version of the MECG. An ANFIS network is used to recognize this nonlinear connection, & to align the MECG signal with the maternal component in the abdominal ECG signal. Thus, the FECG component is separated by removing the aligned form of the MECG signal from the abdominal ECG signal.

e) In [5], the author has made a comparison between two most popular techniques for separation of foetal ECG signal from mother ECG signal i.e. adaptive noise cancellation & blind source separation.

f) In [6], a novel method for separating the foetal ECG from abdominal complex signals is suggested. The method comprises of the termination of the mother's ECG signal & blind source separation with the reference signal (BSSR). The termination of the mother's ECG component was achieved by deducting the linear combination of mutually orthogonal projections of the heart vector. The BSSR is a fixed-point procedure, the Lagrange function of which contains the higher order cross-correlation among the extracted signal & the reference signal as the cost term rather than a constraint.

g) In [7], a 3 stage procedure is represented for separation of the foetal ECG signal & maternal ECG signal. In the first stage, multiscale principal component analysis & the smoothed non-linear energy operator are used for detecting maternal R-peaks & fiducial points. In second stage, again multiscale principal component analysis & the smoothed non-linear energy operator are used to detect the fetal heart beats that do not overlap with the maternal QRS complexes. In final stage, a histogram based technique is used to separate the maternal & FECG signals.

h) In [8], Bernard Widrow describes the concept of adaptive noise cancelling which is another method for approximating signals corrupted by additive interferences. The technique uses a “main” input containing the corrupted signal & a “reference” input containing noise correlated in some unknown way with the primary noise. The reference input is adaptively clarified & deducted from the main input to obtain the signal estimate.

i) In [9], ICA (Independent component analysis), which is also known as “blind source separation” is described for separating FECG signal from maternal ECG signal. ICA is used to decompose abdominal ECG signal into components which are FECG signal, maternal ECG signal & other noise signals. In the end, except FECG signal, all other signals are suppressed & the resultant is fetal heartbeat.

j) In [10], an enhancement to LMS adaptation algorithm is made by applying L1 norm to adaptive filtering coefficients to obtain better estimation result of the desired signal. The resultant algorithm is named as “Zero Attracting Least Mean Square” as it attracts most of the filter coefficient towards zero.

1.10) SUMMARY OF THE CHAPTER

In this chapter, we discussed about a normal ECG signal and its depolarization and repolarization processes. We have also defined various intervals and segments of the ECG signal with their time intervals. We have also mentioned some details of the heart of the fetus. We have also briefly discussed the general procedure of fetal ECG extraction using adaptive filters and also in the end we have mentioned about the objective of the respective thesis. We have also discussed about how to acquire the signal for the fetal ECG extraction from the mother’s body and finally we have seen a bit on the previous work done for the fetal ECG extraction for the diagnosis purpose.

In the further chapters, we will study about adaptive filtering process and the three different adaptive algorithms used. We will also see how to use this adaptive filtering for the removal of fetal ECG signal from the obtained abdominal signal.

CHAPTER 2

ADAPTIVE FILTERING

2.1) INTRODUCTION

An adaptive filter is a structure having a transfer function & the filter's behaviour is controlled by some mutable factors & these parameters or factors are changed to their optimized values with the help of an optimization algorithm called adaptive algorithm. Some famous adaptive algorithms are LMS & RLS. All the adaptive filters are digital in nature due to their complex optimization algorithm. Adaptive filters are used in various applications where parameters of the preferred processing operation are unidentified or are altered. Transfer function of the adaptive filters is being refined by diminishing the error in closed loop adaptive filters.

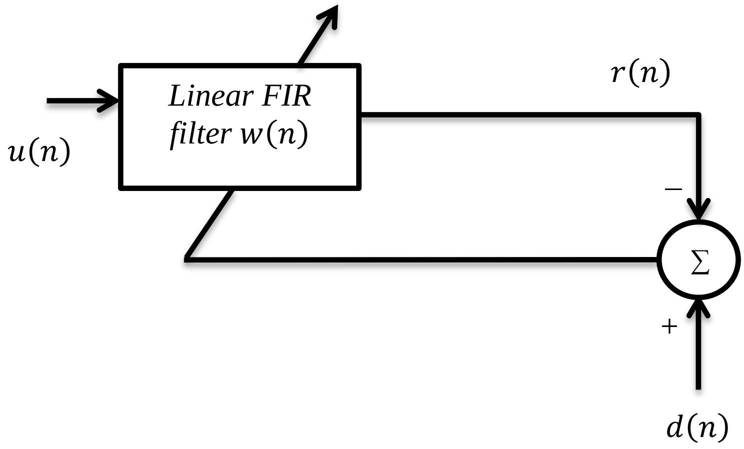

Fig 2.1 Standard adaptive filter scheme

A block diagram is shown in above figure 2.1, showing that a section from an input signal $u(n)$ is supplied to a device named as linear FIR filter, which calculates a resultant signal sample r(n) at time 'n'. Comparison is made between filter output signal r(n) & the desired signal $d(n)$ by subtracting output signal from desired signal, as given by following eqn

$$e(n) = d(n) - r(n) \tag{2.1}$$

$$r(n) = \sum_{i=0}^{N} w_i(n).u(n-i) \quad (2.2)$$

In matrix form,
$$R(n) = W^T(n).X(n) \quad (2.3)$$

Where $u(n)$ is the input signal, $d(n)$ is the desired signal, r(n) is the output obtained & $e(n)$ is called as the error signal as shown in figure 2.1.

An adaptive filter can be described by four aspects:

1. The signals which are to be handled by the filter.
2. Filter that describe how the input is transformed to out the output.
3. The structure's parameters which can be changed iteratively for altering the input-output relationship.
4. The algorithm which is made adaptive in nature describes how the parameters of the structure will change to obtain an estimate of the desired output.

Applications of the adaptive filters are:

- Adaptive noise cancellation:

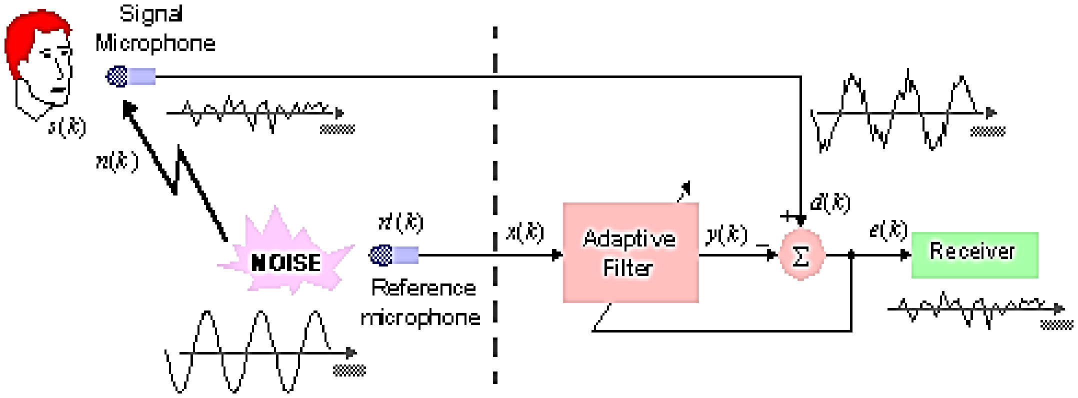

Fig 2.2 Adaptive noise cancellation

It can be seen from figure 2.2 that sending signal is corrupted with noise which is to be removed by using adaptive filter. For this the original noise signal is applied as the reference signal to one of the input terminals of filter. Adaptive filter then finds an estimate of the noise signal & remove this estimated quantity from the message signal which is corrupted with noise. The subtraction process results in an error signal which is to be minimized with the help of an optimized adaptive algorithm such as standard LMS etc.

- Echo cancellation

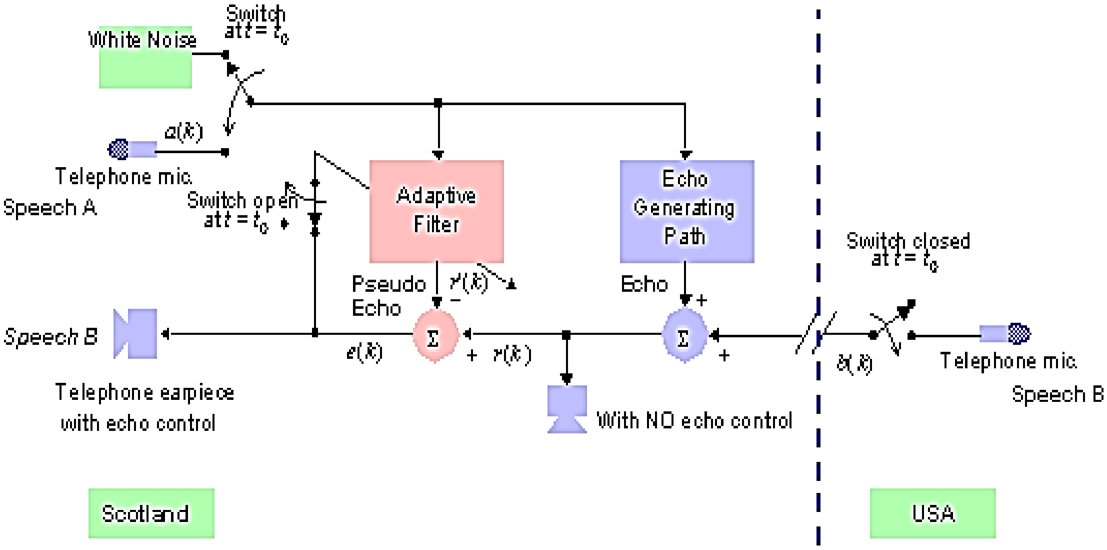

Fig 2.3 Adaptive echo cancellation

As it can be easily seen from figure 2.3 that an echo signal is moved back towards the sender end due to which sender listen the mixture receiver's reply & his own voice. To cancel out this echo signal, an adaptive filter is used to produce an error signal which is attained by removing the estimate of echo signal from the original echo signal. This error signal is minimized to achieve to zero echo at the sender end.

- Signal prediction: The purpose of signal predictor is to predict upcoming values of a signal based on its previous values.

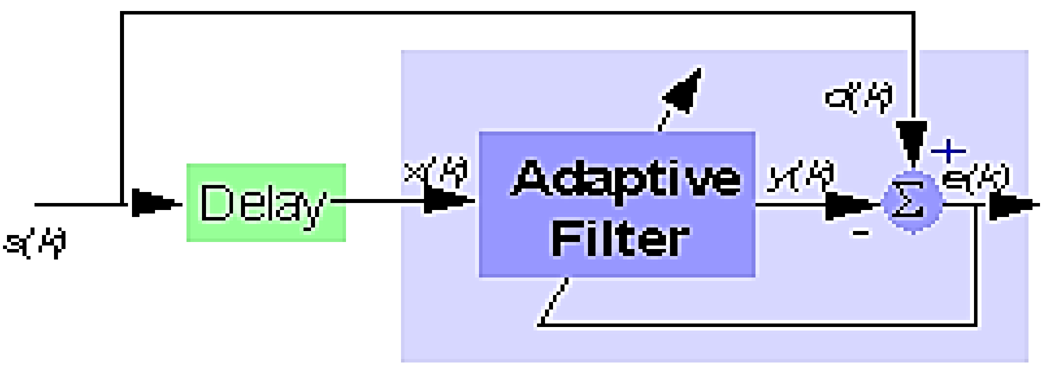

Fig 2.4 Linear prediction

The MECG signal involved in obtained AECG signal acts as a noise signal added to the FECG signal. Further we will examine how adaptive filter can be used to remove this maternal ECG signal from AECG to obtain the desired FECG signal & also how our proposed method will replace standard LMS algorithm based adaptive filtering & provides much better results.

2.2) WHY ADAPTIVE FILTERING?

One question which comes in the mind of the reader is, “If we already have the desired signal then what is the requirement of matching it using an adaptive filter?” So let us look at some reasons behind using adaptive filtering:

- The concerned quantity is not always the desired signal $d(n)$.
- Sometimes there are circumstances when the desired signal is not available.
- There are some real world conditions where the desired signal is never available.
- Also sometimes the relation between the input signal & the desired signal can differ with time. In these conditions the adaptive algorithm helps in altering the values its variables to adapt the changes in relation between $d(n)$ & $u(n)$. This process is called “tracking”.

2.3) ADAPTIVE NOISE CANCELLATION

ANC is one of the most significant adaptive filtering methods which are used to detach FECG from mother ECG. Adaptive filters required two input signals for separating operation. One signal which is the primary signal is the abdominal signal which is combination of FECG & with mother ECG signal & the other signal i.e. the reference signal, which is the noise to be cancelled (here, mother ECG signal act as noise for the fetal electrocardiogram signal). The noise present in primary signal must be well-correlated with the secondary signal.

Adaptive filters are used in various applications where the system is to be made adaptive means the system changes its parameters on its own (with the help of an adaptive algorithm) according to the changes occur in the input signal. In this thesis, we will see the medical application of adaptive filtering for extracting the fetal ECG from abdominal signal.

Adaptive filter plays an important role in fetal electrocardiography as it helps in separating a maternal heartbeat signal & a fetal heartbeat signal. Adaptive filters are based upon adaptive algorithm which helps in changing the coefficient values accordingly so that the error signal amplitude can be minimized as much as possible.

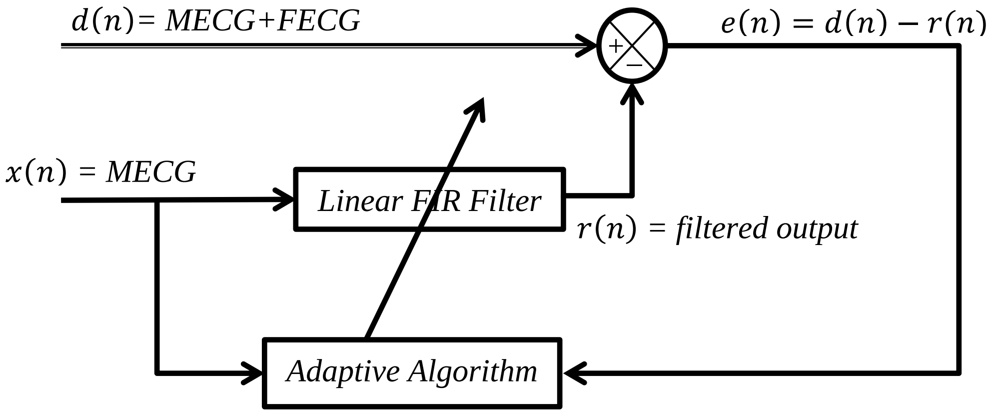

Fig 2.5 Adaptive noise cancellation (ANC) structure

In figure 2.5, $d(n)$ is the primary input signal which involve both the desired signal $s(n)$ i.e. FECG & the noise signal $v(n)$ i.e. mother ecg signal.

$$d(n) = s(n) + v(n) \tag{2.4}$$

Here $v(n)$ & s (n) are uncorrelated & the reference input $x(n)$ is similar to $v(n)$ i.e. both $v(n)$ & $x(n)$ are correlated signals.

The reference signal is used to estimate $v(n)$ & estimated signal from the adaptive filter is removed from $d(n)$ to obtain an estimate of $s(n)$.

$$s(n) = d(n) - \hat{v}_1(n) \tag{2.5}$$

Where $\hat{v}_1(n)$ is the estimate of $v(n)$.

The error signal $e(n)$ in the ANC gives us the desired signal which is further used by the adaptive filter to the filter weights up-to-date automatically. The error signal is estimated by subtracting the filter output from mixture of MECG & FECG. Subtraction will result in termination of the mother ECG signal & the remaining signal will be fetal ECG signal which is signal of interest.

2.4) ADAPTIVE ALGORITHM

An adaptive algorithm is a well-defined list of commands to perform a specific function that can familiarize in the occurrence of alterations in environment. Adaptive algorithms are capable of intelligently adjusting their actions in light of changing situations to attain the best possible output. Adaptive algorithms are designed in such a way so that the similarity between desired output & the obtained output can be maximized. Subtraction between the output signal & desired signal is known as error signal. Lesser the error signal, higher the efficiency of the adaptive algorithm.

In further sections, we will show different adaptive algorithm which are used in this project to separate FECG signal from AECG signal. In this thesis, we only think through adaptive filter structure instead of IIR filter structure because

- The FIR filter structure has a guaranteed output-input stability for all set of permanent coefficients, &
- The procedure for altering the coefficients of IIR filters is further difficult than those for altering the coefficients of FIR filters.

2.4.1) Universal form of Adaptive FIR algorithms

The adaptive FIR filtering algorithm can be generally defined by

$$W(n+1) = W(n) + \mu \,.G[e(n), u(n), \phi(n)] \tag{2.6}$$

Where $u(n)$ is the input signal vector, $e(n)$ is the error signal, μ is a *step size* parameter, $G(\,)$ is a vector-valued nonlinear function, & $\phi(n)$ is a state vector that supply relevant facts about the features of the error signal & input signal and/or the coefficients at earlier time moments. In general, $\phi(n)$ is never used, & the only evidence desirable for adjusting the coefficients at time 'n' are the step size, error signal vector & input signal vector.

The step size μ determines the modification to be occupied by the adaptive algorithm for defining a valuable coefficient vector. Success & failure of the adaptive algorithm is decided by the value of μ is chosen for the best performance of the algorithm.

2.4.2) Cost Function

The structure of function $G(\,)$ in eqn 2.6 is defined by the cost function preferred in specific adaptive filtering mission. Now a specific cost function will be studied that produces a famous adaptive algorithm & that is the Mean Squared Error (MSE) cost function defined as:

$$J_{MSE}(n) = \frac{1}{2}\int_{-\infty}^{\infty} e^2(n) p_n\big(e(n)\big).\, de(n) \tag{2.7}$$

$$J_{MSE}(n) = \frac{1}{2}\; E\,\{\, e^2(n)\,\} \tag{2.8}$$

Where $p_n(n)$ symbolizes the error signal's probability density function at time 'n' & $E\{e^2(n)\}$ is the expectation integral of squared error signal in eqn 2.7. For the adaptive filters, MSE cost function plays a convenient role because of the following reasons:

- As $J_{MSE}(n)$ is quadratic in nature, it has a distinct lowest value w.r.t. the parameters in $W(n)$
- The values of the coefficient acquired at this least point are the values that reduces the error signal's power to its least value, signifying that $r(n)$ has come close to $d(n)$, and
- $J_{MSE}(n)$ is a continuous function for all the factors in $W(n)$, such that $J_{MSE}(n)$ can be differentiated w.r.t. each of the factors in $W(n)$.

The last reason assists us to decide the finest coefficient values, assumed the statistical knowledge of desired signal $d(n)$ & the input signal $u(n)$ as well as a humble iterative way for correcting the parameters of an adaptive filter.

2.4.3) Wiener filter

Statistical knowledge of the desired signal $d(n)$ & the input signal $u(n)$ helps in determining the well-defined coefficients of $W(n)$ which minimize the cost function $J_{MSE}(n)$. "Wiener" was the first who derived the statement of this problem in case of CT signals & the resulting justification. Hence, Wiener gave an ideal coefficient vector $W_{MSE}(n)$. This vector is also known as Wiener solution in the field of adaptive filtering.

To conclude $W_{MSE}(n)$, it should be noted that $J_{MSE}(n)$ in eqn 2.8 is quadratic in nature (as the maximum power in its eqn is two) & the function is also differentiable. By its quadratic nature we can understand that it will be having only a minimum value. Thus, we can make use of optimization theory result which states that at a certain point, the derivation of a continuous quadratic cost function w.r.t. its parameters is equal to zero on the surface of cost function error curve. This specific point where the derivative is zero is called as the minimizing point where the curve attains its minimum value. Thus, the following system of eqns will help in finding the optimum wiener solution $W_{MSE}(n)$

$$\frac{\partial\, J_{MSE}(n)}{\partial\, w_i(n)} = 0, \qquad 0 \le i \le L-1 \tag{2.9}$$

Differentiating $J_{MSE}(n)$ & considering $e(n)$ & $r(n)$ as given in eqns 2.1 & 2.2 respectively, we get

$$\frac{\partial\, J_{MSE}(n)}{\partial\, w_i(n)} = E\left\{e(n)\frac{\partial\, e(n)}{\partial\, w_i(n)}\right\} \tag{2.10}$$

$$= -E\left\{e(n)\frac{\partial\, r(n)}{\partial\, w_i(n)}\right\} \tag{2.11}$$

$$= -E\{e(n)u(n-i)\} \tag{2.12}$$

$$= -\left(E\{d(n)u(n-i)\} - \sum_{j=0}^{L-1} E\{u(n-i)u(n-j)\}\, w_j(n)\right) \tag{2.13}$$

To expand the last result in 2.12, we have used the eqns of e(n) & of $r(n)$ as given in eqns 2.1 & 2.2 respectively.

Let us define vector $P_{du}(n)$ & the matrix $R_{uu}(n)$ as:

$$R_{uu}(n) = E\{U(n)U^T(n)\} \tag{2.14}$$

$$P_{du}(n) = E\{d(n)U(n)\} \tag{2.15}$$

Eqns 2.14 & 2.15 can be combined to achieve the vector form of system of eqns as

$$R_{uu}(n)W_{MSE}(n) - P_{du}(n) = O \tag{2.16}$$

where O is the null matrix. So the best wiener solution for this problem maintaining the condition that matrix $R_{uu}(n)$ should be invertible is

$$W_{MSE}(n) = R_{uu}^{-1}(n)P_{du}(n) \tag{2.17}$$

2.4.4) Method of steepest descent

This is an eminent optimization technique for diminishing the value of the cost function $J(n)$ w.r.t. a set of adaptable parameters $W(n)$. The method of steepest descent alters all parameter of the system agreeing to the following weight updating eqn

$$w_i(n+1) = w_i(n) - \mu(n)\frac{\partial\, J(n)}{\partial\, w_i(n)} \tag{2.18}$$

In other words, the i^{th} weight of the system is transformed in line with the differentiation of the cost function w.r.t. the i^{th} parameter. Writing the vector forms of these eqns, we get

$$W(n+1) = W(n) - \mu(n)\frac{\partial\, J(n)}{\partial\, W(n)} \tag{2.19}$$

Where $\frac{\partial\, J(n)}{\partial\, W(n)}$ is a one-dimensional matrix of derivatives $\frac{\partial\, J(n)}{\partial\, w_i(n)}$.

For a finite impulse response adaptive filter which minimizes the mean squared error cost function, we can make use of the outcome in (2.13) for openly providing formula for the procedure of steepest descent. Replacing these outcomes into (2.18) produces the reorganized eqn for $W(n)$ as

$$W(n+1) = W(n) + \mu(n)(\, P_{du}(n) - R_{uu}(n)W(n)) \qquad (2.20)$$

Still, this process of steepest descent is influenced by the statistical measures $E\{u(n-i)u(n-j)\}$ & $E\{d(n)u(n-i)\}$ contained in $R_{uu}(n)$ & $P_{du}(n)$ respectively. Generally, only measurement of both desired & input signal is available for the adaptation technique. While determining appropriate evaluation of the statistical quantities desirable for eqn (2.20), with the help of signals $u(n)$& $d(n)$, we, as an alternative, produce an estimated form of steepest descent procedure which is influenced by the signal values themselves. This process is recognized as the Least Mean Square algorithm.

2.4.5) Standard Least Mean Square (LMS)

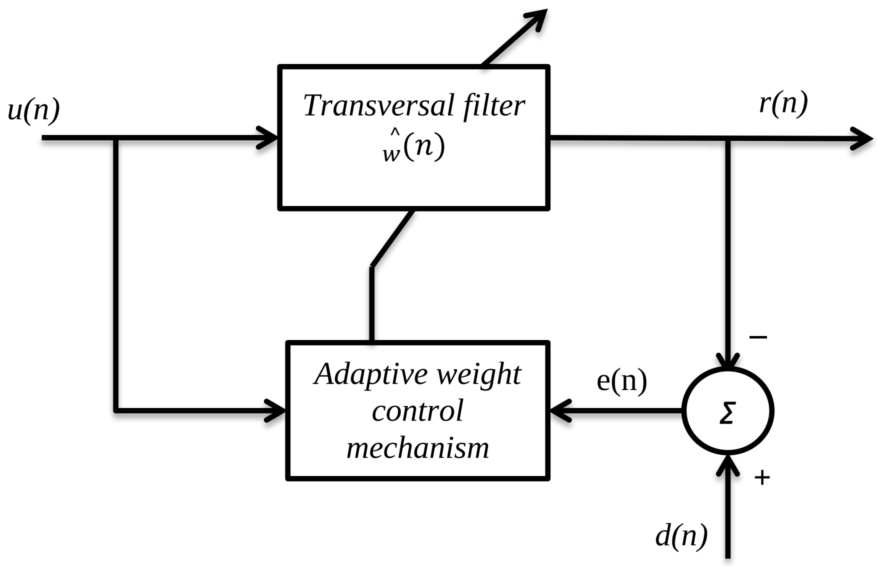

Fig 2.6 Adaptive filter block diagram

Bernard Widrow & Ted Hoff bring together a popular technique for adaptive system identification recognized as the LMS algorithm.

The LMS algorithm is a linear filtering algorithm which is adaptive in nature, and generally it involves two simple processes:

1. A filtering procedure involving two main tasks:
   a) Calculating the linear filter output in reaction of an input signal and
   b) Calculating an approximation error by subtracting this filter output with the desired signal.
2. An adaptive procedure, which includes the instinctive correction of the parameters of the filter according to the variation in estimation error.

Coefficient solution for the adaptive filtering is obtained with the help of the cost function $J(n)$ which is selected for "the steepest descent algorithm" in eqn (2.18). If the mean square error cost function (in eqn 2.8) is chosen, the resultant system is being influenced by the statistical measures of $d(n)$ & $u(n)$ for the reason that expectation operation describes this cost function. As only the measurements of $d(n)$ & $u(n)$ are accessible, we place a different cost function which is dependent upon these measurements only. Least mean square is such type of cost function given as

$$J_{LS}(n) = \sum_{k=0}^{n} \alpha(k)(d(k) - W^T(n)U(k))^2 \tag{2.21}$$

Where $\alpha(n)$ is an appropriate weighting sequence for the terms within the summation.

Although LMS error cost function is complex as it calls for various calculations to compute its value as well as its differentiation w.r.t. each $w_i(n)$, a well-organized recursive procedure for its minimization can be established. On the other hand, we recommend the simplified cost function $J_{MSE}(n)$ given by

$$J_{MSE}(n) = \frac{1}{2}\, E\{e(n)^2\} \tag{2.22}$$

This simplified version can be understood as an instantaneous estimate of the mean square error cost function, as $J_{MSE}(n) = E\{\, J_{LMS}(n)\,\}$. Even if it might not seems to be beneficial, the resultant procedure attained when $J_{LMS}(n)$ is used for $J(n)$ in (2.18) is exceptionally useful for real-world applications.

Differentiating $J_{LMS}(n)$ w.r.t. the components of $W(n)$ & replacing the result into (2.18), we attain the Least Mean Square adaptive algorithm given by

$$W(n+1) = W(n) + \mu(n)e(n)U(n) \tag{2.23}$$

The average behaviour of the Least Mean Square procedure is pretty analogous to that of the steepest descent algorithm which is explicitly influenced by the statistical estimation of the desired & input signals. In actuality, the iterative idea for the updates of the least mean square coefficient is a type of time-averaging that smooth out the miscalculation in the instant gradient measurements to get a more sensible approximation of the exact gradient.

LMS algorithm can be summarized using following steps:

1. Obtain the output of the filter

$$r(n) = w^T(n).u(n) \tag{2.24}$$

2. Get the error or difference signal $e(n)$ as given below

$$e(n) = d(n) - r(n) \tag{2.25}$$

3. Update the filter weights as given below

$$w(n+1) = w(n) + \mu.u(n).e(n) \tag{2.26}$$

Where μ is defined the filter's step size, $w(n)$ is the filter weights vector, the error signal $e(n)$ & the input signal $u(n)$ which is the mixture of fetal & mother ECG signal. The range of step size, μ is $0 < \mu < \frac{2}{\lambda_{max}}$ where λ_{max} is the greatest Eigen value of the autocorrelation matrix $R = E\{u(n)u'_{(n)}\}$ where $u'_{(n)}$ is the conjugate transpose of input signal matrix.

Applications of LMS algorithm include channel equalization, echo cancellation, interference cancellation etc.

2.4.6) Normalized Least Mean Square (NLMS)

The main downside of the standard LMS adaptive procedure is that it is influenced by scaling of its input signal $u(n)$ which makes it extremely tough to pick the step size value which makes the algorithm stable.

NLMS is a process which is an improved version of the standard LMS algorithm. This algorithm solves out the disadvantage of LMS by normalizing with the input signal power. In NLMS algorithm, filter coefficients are modified as

$$w(n+1) = w(n) + \beta . e(n) . \frac{u(n)}{||u(n)||^2} \qquad (2.27)$$

Where β is the step size which varies with time & lies between $0 < \beta < 2$ for best converging performance of the algorithm.

So NLMS algorithm is summarised using following steps:

1. Obtain the filter output as

$$r(n) = w^T(n) . u(n) \qquad (2.28)$$

2. Get the error or difference signal e(n)

$$e(n) = d(n) - r(n) \qquad (2.29)$$

3. Modify the structure weights using eqn given below

$$w(n+1) = w(n) + \beta . \frac{u(n)}{||u(n)||^2} . e(n) \qquad (2.30)$$

2.4.7) Zero Attracting Least Mean Square (ZALMS)

For obtaining Zero Attracting LMS algorithm, the cost function of the standard least mean square procedure is altered by adding a l_1 norm penalty of the coefficient vector as shown below

$$L_1\,(n) = \frac{1}{2}\; e(n)^2 + \Upsilon\; ||w(n)||_1 \tag{2.31}$$

In ZALMS, the filter weights are updated as:

$$w(n+1) = w(n) - \rho\; sgn\; w(n) + \mu.u(n)e(n) \tag{2.32}$$

Where $\rho = \mu\, \Upsilon$ & sgn ( ) is a component wise sign function defined as

$$sgn(u) = \begin{cases} \frac{u}{|u|} & u \neq 0 \\ 0 & u = 0 \end{cases} \tag{2.33}$$

On comparison between ZALMS & standard LMS, it can be easily seen that ZALMS is having a supplementary term "$-\rho\, sgn\, w(n)$". This term always attract filter coefficients to zero. In ZALMS, more the number of zero filter coefficients more will be the convergence speed of the algorithm.

So ZALMS can be summarised using following steps:

1. Obtain the filter output as

$$r(n) = w^T(n).u(n) \tag{2.34}$$

2. Obtain the error signal as given below

$$e(n) = d(n) - r(n) \tag{2.35}$$

3. Modify the coefficients of the filter as

$$w(n+1) = w(n) - \rho\; sgn\; w(n) + \mu.u(n)e(n) \tag{2.36}$$

2.5) SUMMARY OF THE CHAPTER

In this chapter, we have seen the detailed explanation of the adaptive filtering and the various adaptive algorithms. We have seen how the standard least mean square algorithm derived from the method of steepest descent. We have covered the explanation of the cost function which describes the efficiency of the adaptive algorithm. And finally we have also discussed the two new algorithms named as “Normalized least mean square” and “Zero attracting least mean square” which are the modified version of the standard LMS algorithm.

The next chapter we will be seeing the use of these two new adaptive algorithms in fetal ECG extraction and in the further chapter, we will be comparing the results of the three algorithms.

CHAPTER 3

PROPOSED METHOD

3.1) OVERVIEW

This chapter describes the proposed method which uses two different enhanced version of LMS adaptive algorithm for separation of foetal ECG signal & abdominal signal. Enhanced versions of LMS adaptive algorithm used in the proposed method are named as NLMS adaptive algorithm (Normalized Least Mean Square) & ZALMS (Zero Attracting Least Mean Square) adaptive algorithm.

Further, with the help of a flow diagram, we'll try to understand the working of the proposed model that how the proposed method extract the foetal ECG signal from abdominal ECG signal. In the next chapter, we will see that the proposed method provide better results than the standard LMS adaptive algorithm.

3.2) PROPOSED METHOD

The proposed method for foetal ECG extraction is shown below:

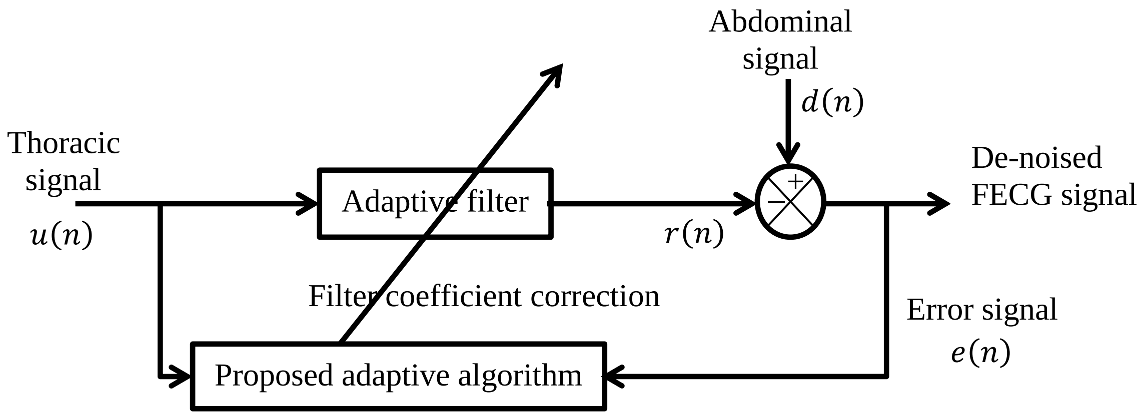

Fig 3.1 proposed method block diagram

Block diagram as shown in fig 3.1 describes how an adaptive filtering algorithm helps in reducing error signal. The proposed algorithms are named "ZERO ATTRACTING LEAST MEAN SQUARE" and "NORMALIZED MEAN SQUARE".

The primary input signal $u(n)$ for the filtering process is the abdominal signal which involves the foetal ECG signal corrupted with some noise signals (including mother ECG signal) & the other input signal $d(n)$ is the thoracic which contains noise correlated in some unknown way to the noise present in primary signal. The secondary input is applied to the adaptive filter. The output of this adaptive filter is then subtracted from the desired signal & the resultant signal is the error signal $e(n)$.

$$e(n) = d(n) - r(n) \tag{3.1}$$

This difference signal is applied as feedback to the adaptive filter via passing through adaptive algorithm block. The adaptive algorithm updates filter coefficients according to the error signal obtained & tries to reduce the cost function i.e. square of the error signal. After updating the coefficients of the filter, the adaptive filter again generates the result for the input thoracic signal & again the error signal corresponding to this "regenerated result" & the abdominal signal is generated. This process keeps on going until the error signal or the cost function obtained its minimum value.

We have applied all the three adaptive algorithms one by one for the foetal ECG extraction.

3.2.1) Standard Least Mean Square

- Obtain the filter output

$$y(n) = w^T(n).u(n)$$

- Get the difference signal $e(n)$ using:

$$e(n) = d(n) - y(n)$$

- Modify the weights of the filter using following eqn:

$$w(n+1) = w(n) + \mu.u(n).e(n)$$

Where μ is the step size.

3.2.2) Zero Attracting Least Mean Square

- Obtain the filter output

$$y(n) = w^T(n).u(n)$$

- Obtain the difference signal $e(n)$ using:

$$e(n) = d(n) - y(n)$$

- Update the weighs of the filter as

$$w(n+1) = w(n) - \rho\, sgn\, w(n) + \mu . u(n) e(n)$$

3.2.3) Normalized Least Mean Square

- Obtain the filter output

$$y(n) = w^T(n). x(n)$$

- Get the difference signal $e(n)$ using:

$$e(n) = d(n) - y(n)$$

- Update the filter weights using following eqn:

$$w(n+1) = w(n) + \beta . \frac{u(n)}{\left|\left|u(n)\right|\right|^2} . e(n)$$

3.3) SUMMARY OF THE CHAPTER

In this chapter, we discussed the simple linear filtering which is made adaptive with the help of an adaptive algorithm. An adaptive algorithm is a set of instruction which must be followed by the linear FIR filter to adjust its coefficients to attain the maximum efficiency foe a specific task.

Till now the most standard adaptive algorithm was used which is known as Least Mean Square whose cost function is the mean square error, but we have proposed two new adaptive algorithms named as “Normalized least mean square” and “Zero attracting least mean square” that can be used in place of the standard LMS algorithm.

In the respective thesis, our main task is to separate the fetal ECG signals and the maternal ECG signal which are obtained in a mixed signal form in the abdominal signal of the pregnant mother. So we have applied the adaptive filtering for the specified task of fetal ECG extraction using all the three algorithms separately and in the next chapter, we will see the results of the three different approaches of adaptive algorithm.

CHAPTER 4

RESULTS AND DISCUSSION

This chapter includes the results of standard LMS adaptive filtering & the proposed method of adaptive filtering used for the extraction of FECG from the abdominal signal.

The proposed method is implemented on MATLAB 2017b software & the obtained results show that the NLMS & ZALMS performance is better than standard LMS algorithm which we will see further with the help of plots obtained from MATLAB software. We will see the results one by one.

4.1) AVERAGE OF ABDOMINAL & THORACIC SIGNAL

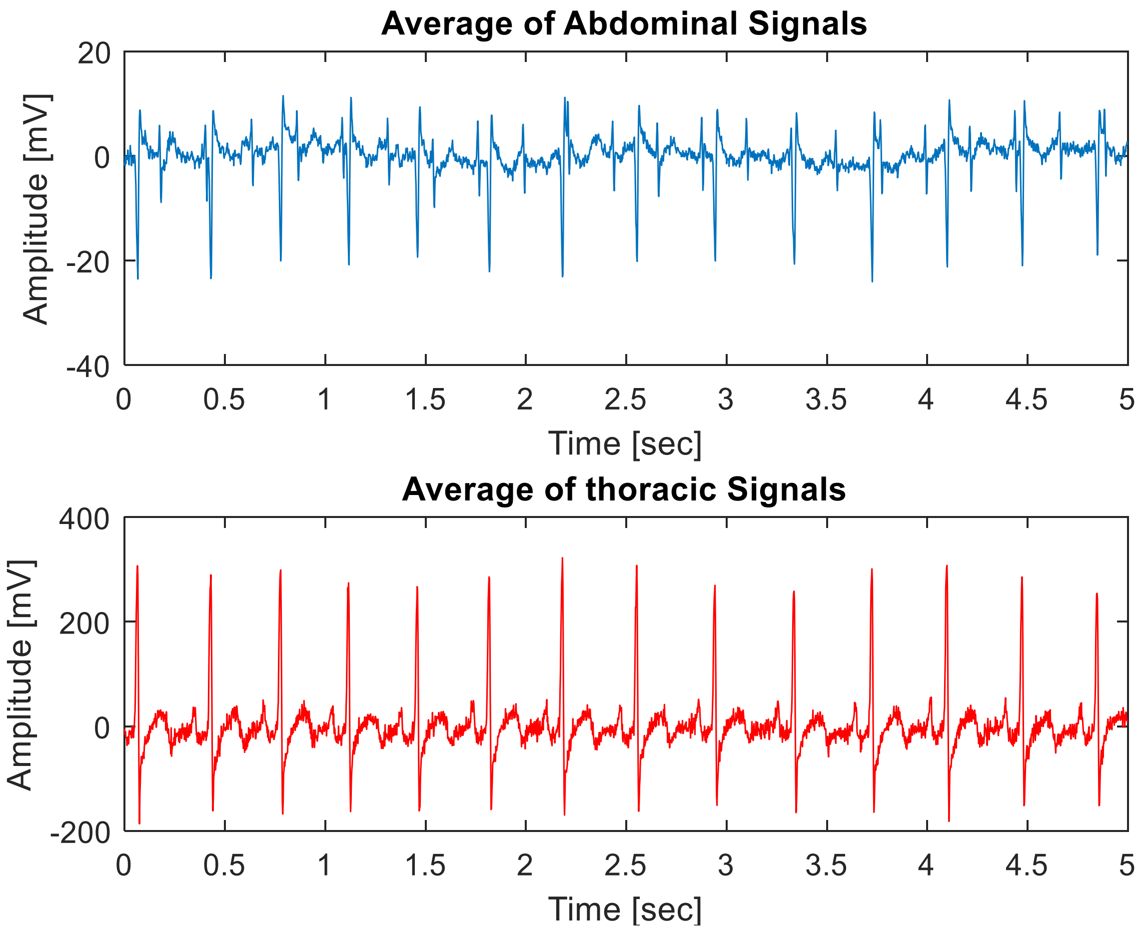

Fig 4.1 average of abdominal & thoracic signal

4.2) STANDARD LEAST MEAN SQUARE

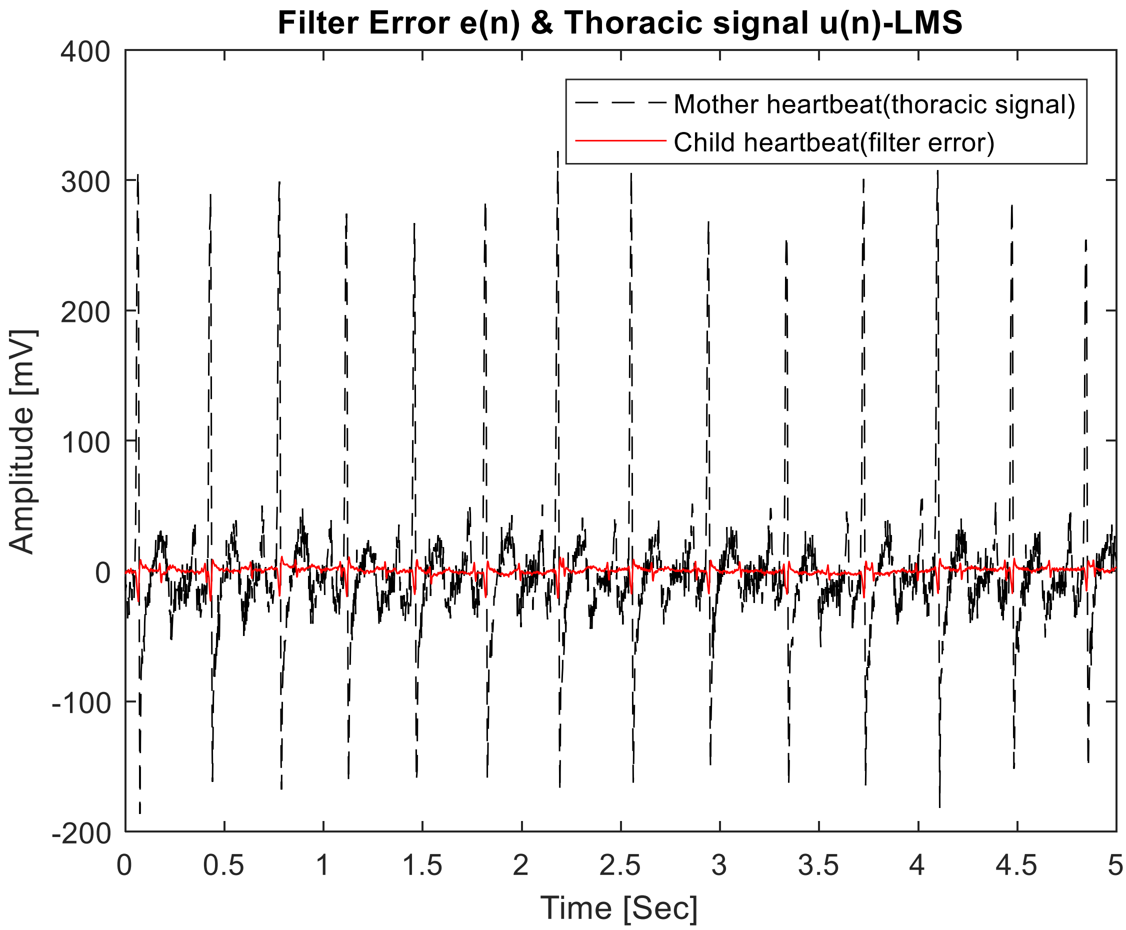

Fig 4.2 mother heartbeat (black) & child heartbeat (red) in case of LMS for order=8 and $\mu = 0.00000001$

Figure 4.2 shows the mother signal (thoracic signal) in black dotted line & the extracted FECG signal (child heartbeat) in red colour. The FECG is extracted using standard LMS algorithm.

Convergence of the algorithm depends on value of step size μ. The optimal range of μ for best convergence of the algorithm is

$$\text{is } 0 < \mu < \frac{2}{\lambda_{max}}$$

4.3) NORMALISED LEAST MEAN SQUARE

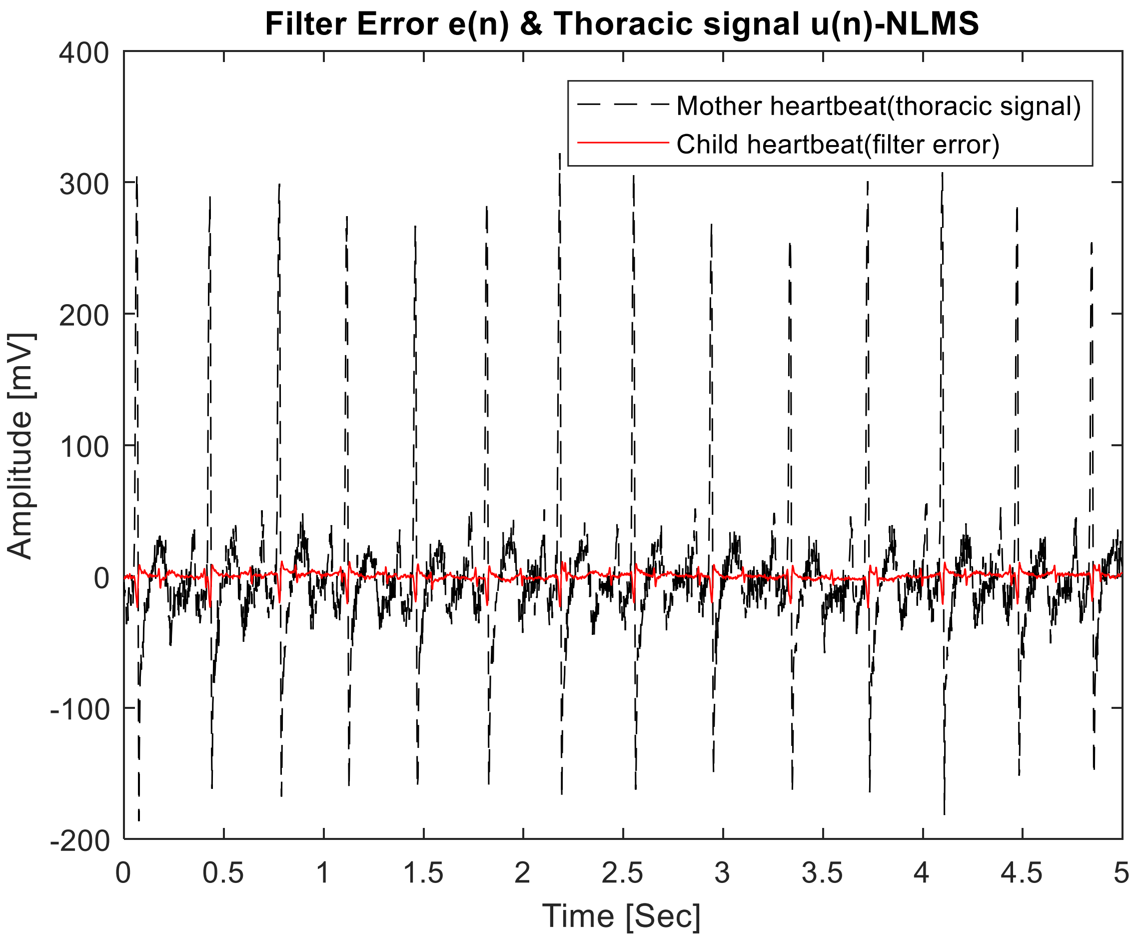

Fig 4.3 mother heartbeat (black) & child heartbeat (red) in case of NLMS for order=8 and $\mu = 0.000009$

Figure 4.3 shows mother heartbeat (thoracic signal) in black colour & the FECG signal (child heartbeat) in red colour. The ECG signal of the fetal is extracted using Normalised Least Mean Square.

It can be easily seen that the amplitude of the child heartbeat signal in case of NLMS is somewhat greater than in case of Standard LMS.

Convergence of the algorithm depends on time varying step size parameter β where

$$0 < \beta < 2$$

4.4) ZERO ATTRACTING LEAST MEAN SQUARE

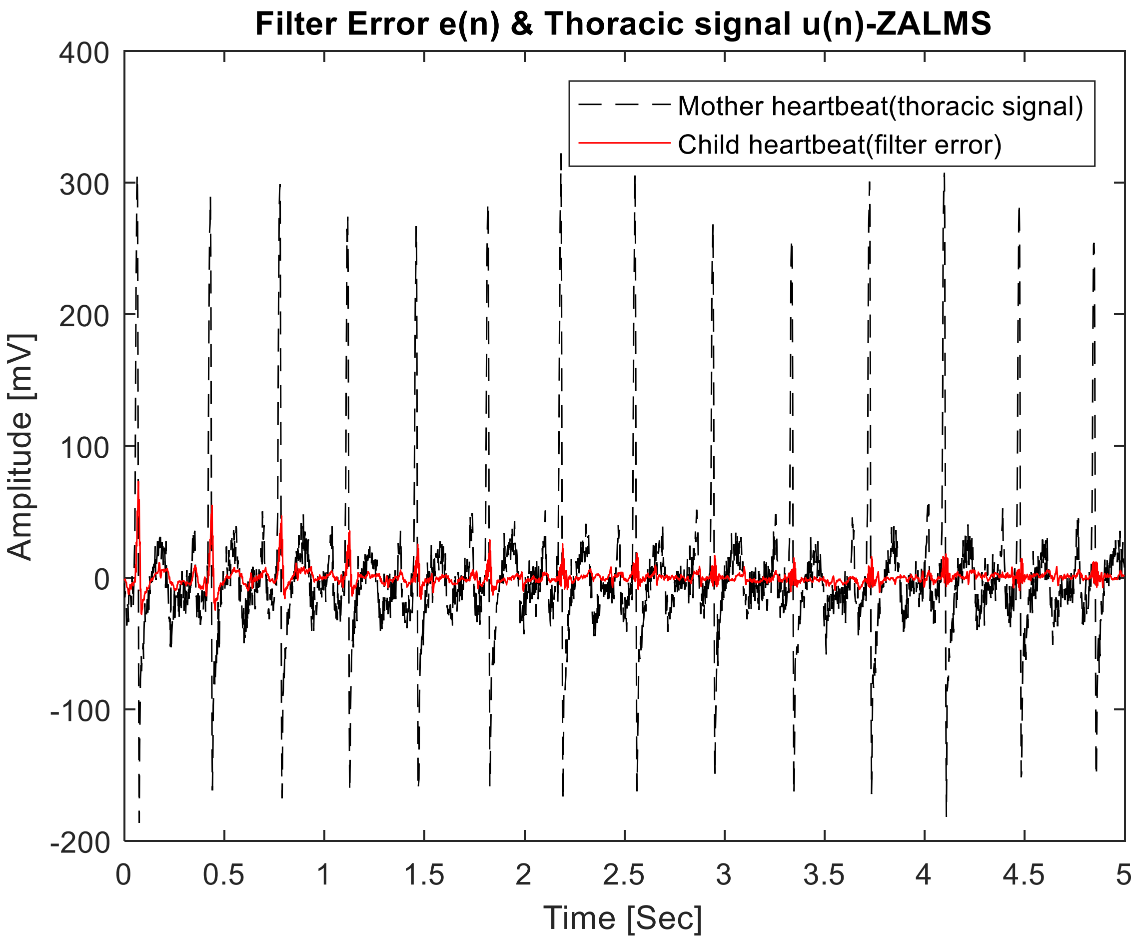

Fig 4.4 mother heartbeat (black) & child heartbeat (red) in case ZALMS for order=5 and μ = *0.0000001*

Figure 4.4 shows mother heartbeat in black colour & the extracted child heartbeat in red colour using Zero Attracting Least Mean Square.

Convergence of the algorithm depends on parameter ρ which is dependent on μ as

$$\rho = \mu \Upsilon$$

The term $-\rho\ sgn\ w(n)$ plays an important role in convergence f the algorithm

4.5) OVERLAPPED PLOT OF FECG SIGNAL USING THREE ADAPTIVE ALGORITHM

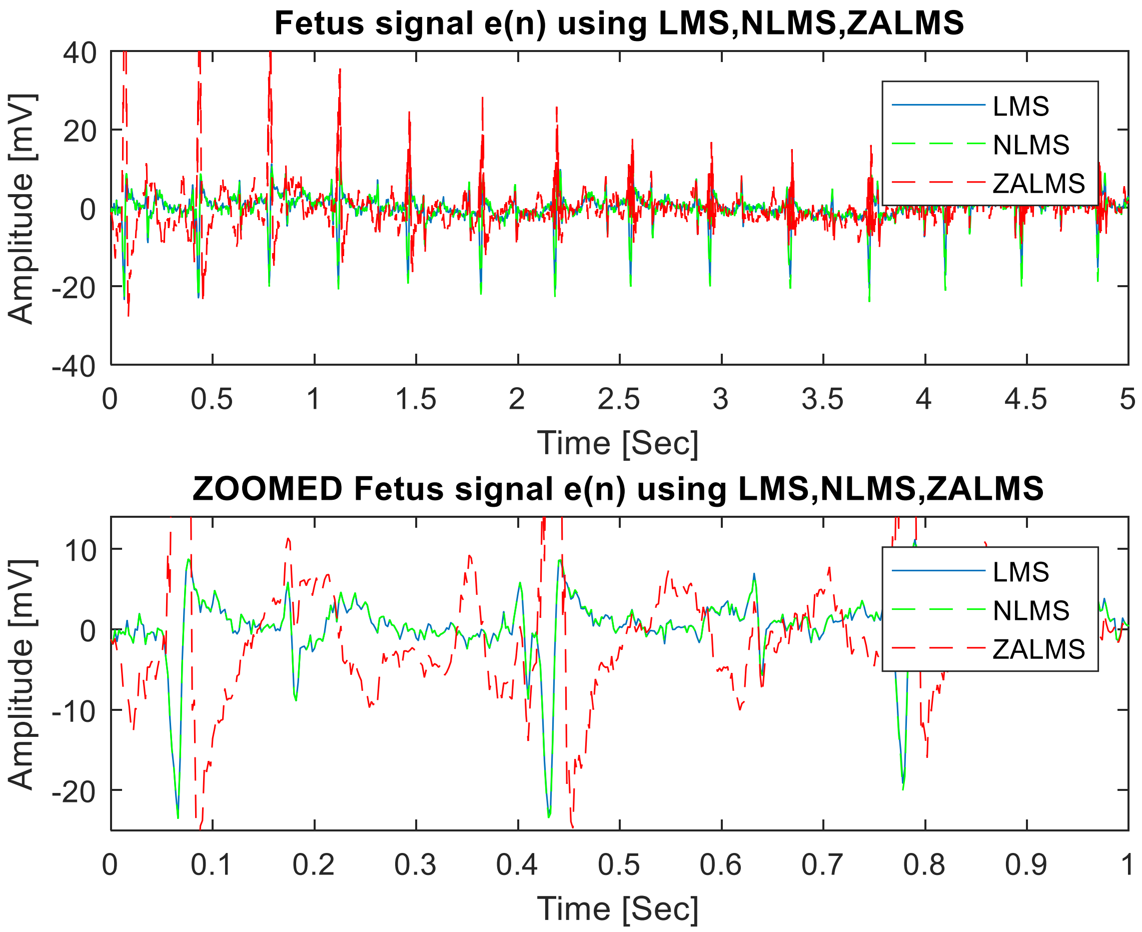

Fig 4.5 Extracted FECG signal (error signal e(n)) & its ZOOMED version using LMS (blue), NLMS (green) & ZALMS (red)

Figure 4.5 shows the overlapped plot of the error signal i.e. extracted FECG signal using all the three described adaptive algorithm & second plot shows its zoomed version.

4.6) ADAPTIVE FILTER OUTPUT $y(n)$

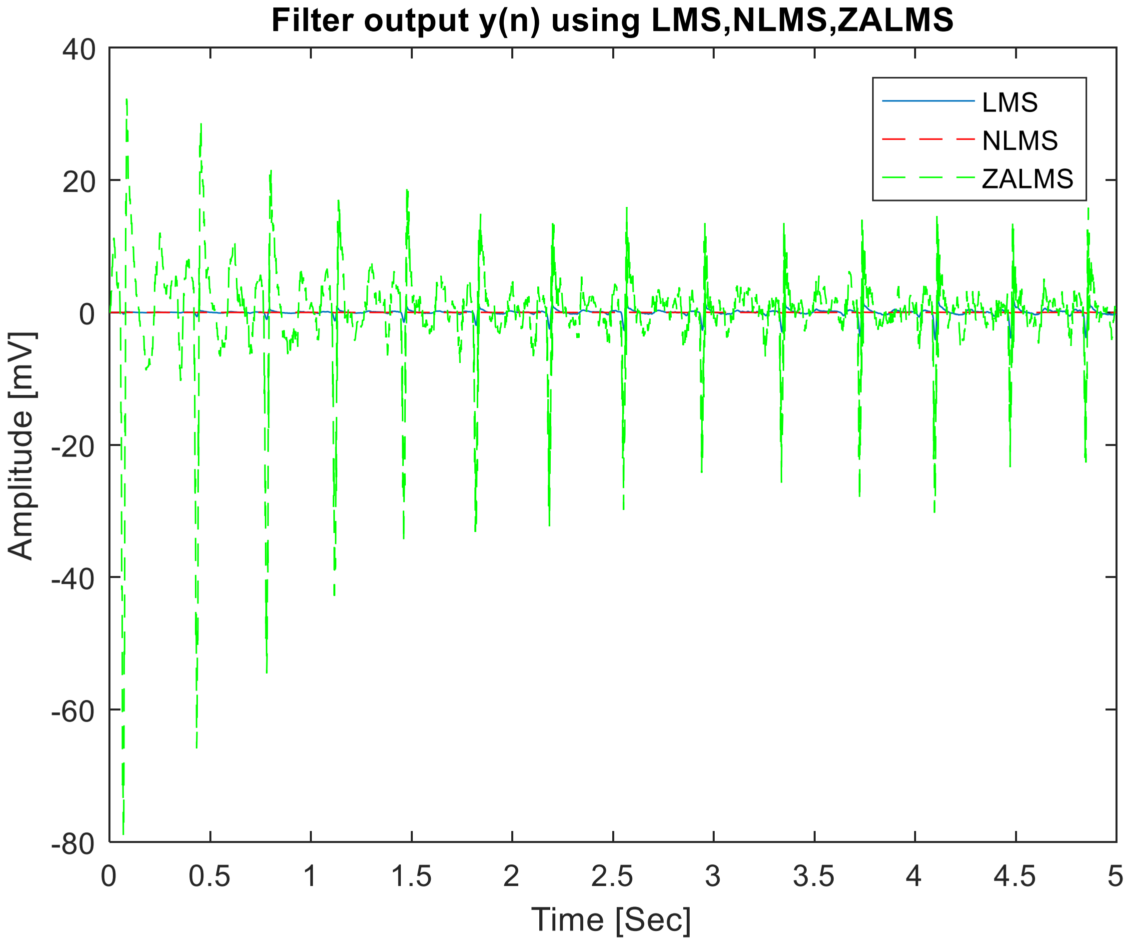

Fig 4.6 Adaptive filter output y(n) for LMS (blue), NLMS (red) & ZALMS (green)

Figure 4.6 shows the adaptive filter output $y(n)$ for the entire three adaptive algorithms i.e. LMS, NLMS, & ZALMS.

4.7) COMPARISON OF SNR VALUES

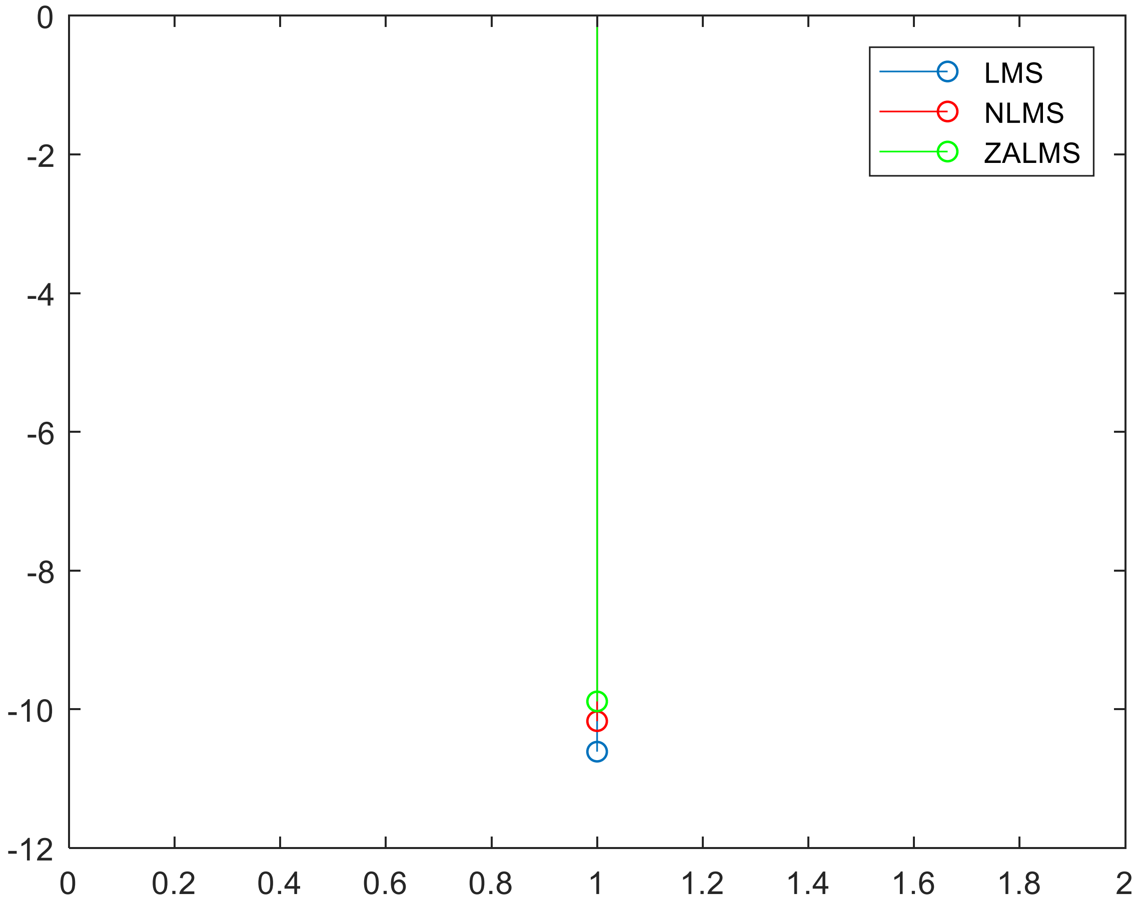

Fig 4.7 Comparison of SNR values of extracted FECG for LMS (blue), NLMS (red) & ZALMS (green)

Figure 4.7 shows the comparison between SNR values corresponding to standard least mean square (blue colour), normalised least mean square (red colour) & zero attracting least mean square (green colour).

Table 4.1 Obtained SNR values in dB for the three algorithms

Parameter	Standard Least Mean Square	Normalised Least Mean Square	Zero Attracting Least Mean Square
SNR (in dB)	-10.6117	-10.1717	-9.8870

SNR values are calculated for the extracted FECG signal in MATLAB software. More negative value shows that the more noise component is involved in the extracted FECG signal.

Hence, it can be easily seen that the noise component in ZALMS & NLMS is less than as compared to standard LMS algorithm.

4.8) SUMMARY OF THE CHAPTER

In this chapter, we discussed the results of the three algorithms and we came to know that the two new adaptive algorithms provide better results than the standard LMS algorithm. The efficiency of the two new algorithms is decided by their respective step size and the order of the linear FIR filter used in the whole process.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1) OVERVIEW

This chapter discusses about how adaptive filtering theory plays an important role in fetal electrocardiogram extraction process which is helpful to know about any medical problem in the fetus during pregnancy so that the problem can be diagnosed at the right time.

5.2) CONCLUSION

This thesis concludes that adaptive filtering plays an important role in extraction of FECG. Also we have seen that standard LMS algorithm which is the most simple & important adaptive algorithm used in adaptive filtering is being replaced by the two new algorithms proposed in this which are called as NLMS & ZALMS. The two proposed algorithms are easier to implement in MATLAB 2017b & they have shown better results than standard LMS algorithm in terms of SNR too.

5.3) FUTURE WORK

- The conclusion of the thesis made us to reach a point where we can think of implementing the proposed algorithm by replacing the standard LMS algorithm based adaptive systems.
- Zero attractor can also be implemented in Normalized Least Mean Square filter.
- Least mean square method can be replaced by another efficient algorithm named as Recursive Least Square due to higher convergence rate of RLS but RLS takes more time to update the adaptive filter in comparison to LMS.
- L1 norm penalization can be extended to other types of filters such as RLS filter and the kalman filter.
- Proposed method of adaptive filtering can be applied to noise removal area in communication field.

REFERENCES

[1]. Mohammad Niknazar, Bertrand Rivet and Christian Jutten, "Fetal ECG Extraction by Extended State Kalman Filtering Based on Single-Channel Recordings," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 60, NO. 5, MAY 2013

[2]. Ruben Mart´ın-Clemente, Jose Luis Camargo-Olivares, Susana Hornillo-Mellado, Mar Elena, and Isabel Roman, "Fast Technique for Noninvasive Fetal ECG Extraction," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 58, NO. 2, FEBRUARY 2011

[3]. Khaled Assaleh and Hasan Al-Nashash, "A Novel Technique for the Extraction of Fetal ECG Using Polynomial Networks," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 52, NO. 6, JUNE 2005

[4]. Khaled Assaleh, "Extraction of Fetal Electrocardiogram Using Adaptive Neuro-Fuzzy Inference Systems," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 54, NO. 1, JANUARY 2007

[5]. Vicente Zarzoso and Asoke K. Nandi, "Noninvasive Fetal Electrocardiogram Extraction: Blind Separation Versus Adaptive Noise Cancellation," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 48, NO. 1, JANUARY 2001

[6]. Michiyoshi Sato, Yoshitaka Kimura, Shinichi Chida, Takuya Ito, Norihiro Katayama, Kunihiro Okamura, and Mitsuyuki Nakao, "A Novel Extraction Method of Fetal Electrocardiogram From the Composite Abdominal Signal," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 54, NO. 1, JANUARY 2007

[7]. Evaggelos C. Karvounis and Dimitrios I. Fotiadis, " Maternal and fetal heart rate extraction from abdominal recordings using multi-scale principal components analysis," Proceedings of the 29th Annual International Conference of the IEEE EMBS Cité Internationale, Lyon, France August 23-26, 2007

[8]. Bernard Widrow, John R. Glover, John M. McCool, John Kaunitz, Charles s. Williams, Robert H. Hean, James R. Zeilder, Eugene Dong, and Robert C. Goodlin,

"Adaptive Noise Cancelling: Principles and Applications," PROCEEDINGS OF THE IEEE, VOL. 63, NO. 12, DECEMBER 1975

[9]. Lieven De Lathauwer, Bart De Moor and Joos Vandewalle, "Fetal Electrocardiogram Extraction by Blind Source Subspace Separation," IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL. 47, NO. 5, MAY 2000

[10]. Yilun Chen1, Yuantao Gu2 and Alfred O. Hero , "SPARSE LMS FOR SYSTEM IDENTIFICATION," Department of EECS, University of Michigan, Ann Arbor, MI 48109-2122, USA 2Department of EE, Tsinghua University, Beijing 100084, P.R. China

[11]. Radek Martinek, Radana Kahankova, Hana Skutova, Petr Koudelkar, Jan Zidek, and Jiri Koziorek, "Adaptive Signal Processing Techniques for Extracting Abdominal Fetal Electrocardiogram," 2016 10th International Symposium on Communication Systems, Networks and Digital Signal Processing (CSNDSP)

[12]. S.Ravindrakumar, K. Bommanna Raja, "Fetal ECG Extraction and Enhancement in Prenatal Monitoring - Review and Implementation Issues," Trendz in Information Sciences&Computing (TISC2010 2010

[13]. Mi Hye Song, Sung Pil Cho, Ho Dong Park, and Kyoung Joung Lee, "The Novel Method for the Fetal Electrocardiogram Extraction from the Abdominal Signal," Proceedings of the 29th Annual International Conference of the IEEE EMBS Cité Internationale, Lyon, France August 23-26, 2007

[14]. Dharmendra Gurve, Jeevan K. Pant, Sridhar Krishnan, "Real-time Fetal ECG Extraction from Multichannel Abdominal ECG Using Compressive Sensing and ICA," 2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)

[15]. Veenadevi S. V., Padmavathi C, Shanthamma B., Balaji Govindraj Abbigeri, Pavithra K. M., "Extraction of Fetal Electrocardiogram from Maternal Electrocardiogram and Classification of Normal and Abnormal Signals," 2017 IEEE 2nd International Conference on Signal and Image Processing