PARAMETRIC STABILITY

OF

RADIALLY FUNCTIONALLY GRADED CIRCULAR COLUMN

AND STUDY OF AMPLITUDE-FREQUENCY CURVE

USING

PHASE PORTRAITS

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CERTIFICATE

The undersigned have examined the dissertation entitled

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presented by Ashu Sharma
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and hereby certify that in their opinion it is worthy of acceptance

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ABSTRACT

In the proposed study parametric stability of circular columns has been investigated. In the first part of study linear system is considered while in second part non-linearity due to geometry has been taken into account.

In linear system the dynamic instability of isotropic damped and undamped circular columns subjected to periodic axial loading is studied for four different boundary conditions. The governing partial differential equation is derived for transverse vibration of column of uniform cross-section subjected to dynamic axial loading. Employing Galerkin's method, this partial differential equation is reduced into an ordinary differential equation (Mathieu type of equation) describing the column dynamic instability behaviour. Following Bolotin's method the instability regions are determined from the boundaries of instability, which represents the periodic solution of differential equation with period T and 2T to the Mathieu equation. Following the same methodology the instability regions are determined for radially functionally graded circular columns, circular column with different bending stiffness and column with four different boundary conditions.

In non-linear column, nonlinearity is introduced by the geometry of deformation which amplifies the stresses calculated based on the initial undeformed configuration of the structure. Using the Hamilton's principle and further applying Galerkin's approximation equation of motion is derived for different end conditions and the instability points on amplitude frequency curve are studied by plotting phase portraits.