IMPLEMENTATION OF PID AND FPD+I CONTROLLER ON NON-LINEAR SYSTEMS

DISSERTATION REPORT

Submitted in Partial fulfillment of the requirements for the award of the degree of

MASTER OF TECHNOLOGY

IN

CONTROL AND INSTRUMENTATION

Submitted by:

SANGEETA DEVRA

(2K11/C&I/22)

Under the esteemed guidance of

DR. BHARAT BHUSHAN

Associate Professor

Dept. of Electrical Engineering

DEPARTMENT OF ELECTRICAL ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

DELHI-110042

2011-2013

DELHI TECHNOLOGICAL UNIVERSITY

Department of Electrical Engineering

CERTIFICATE

This is to certify that the dissertation titled "**Implementation of PID and FPD+I Controller on Nonlinear Systems**" submitted in partial fulfilment of the requirement for the award of the degree of Master of Technology in Control and Instrumentation (C&I) by **Sangeeta Devra (Roll No: 2K11/C&I/22)** is a bonafide record of the candidate's own work carried out by him under my supervision and guidance.

This work has not been submitted earlier in any university or institute for the award of any degree to the best of my knowledge.

Dr. BHARAT BHUSHAN
Associate Professor
Department of Electrical Engineering
Delhi Technological University

PROF. MADHUSUDAN SINGH
HOD
Department of Electrical Engineering
Delhi Technological University

ACKNOWLEDGEMENT

I would like to thank all people who have helped and inspired me during my dissertation work.

I sincerely acknowledge the earnestness and the patronage of my guide **Dr. Bharat Bhushan**, Department of Electrical Engineering, Delhi Technological University, New Delhi, for his valuable guidance, support and motivation throughout this project work. The valuable hours of discussion and suggestion that I had with him have undoubtedly helped in supplementing my thoughts in the right direction for attaining the desired objective.

I wish to express my gratefulness to **Prof. Madhusudan Singh**, Head, Department of Electrical Engineering, Delhi Technological University, New Delhi, for providing the necessary lab facilities. And I wish to thank all faculty members whoever helped to finish my project in all aspects.

I wish express my love and gratitude to my beloved **parents, siblings** and **friends** for their understanding and endless love. Thank you for always being there for me.

I humbly extend my grateful appreciation to my friends **Sarath S. Pillai, Pramila** and **Rohit Goyal** for their time to time suggestions and cooperation without which I would not have been able to complete my work.

Above all, thanks to **Almighty** for blessing and guiding me throughout my life. Thank you God. May your name be exalted, honoured, and glorified.

Sangeeta Devra

2K11/C&I/22

ABSTRACT

Since Fuzzy controllers are nonlinear, it is more difficult to set the controller gains compared to Proportional-Integral-Derivative (PID) controller. The idea is to start with a tuned conventional PID controller, replace it with an equivalent fuzzy controller. In contrast, a PID controller is a simple, linear combination of three signals: P action proportional to error, the I-action proportional to the integral of the error and the D-action proportional to the time derivative of error. This introduces a systematic tuning procedure for fuzzy PID type controllers and study of higher order systems based on Fuzzy controller

This comprises of level control of a surge tank level system and inverted pendulum’s angular position as well as cart position using fuzzy PD+I controller with effect of different membership function on the system control as well as comparison with conventional PID Controller has been carved out. Different type of membership functions like Bell, Pi, Gaussian and Psigmoid are used in the fuzzy PD+I control of the system. The stability of the system has also been discussed upon using phase plot. The simulation results have been obtained using MATLAB.

TABLE OF CONTENTS

LIST OF FIGURES

List of Tables

CHAPTER-1

INTRODUCTION

The Fuzzy logic gives a simple way to reach definite conclusion even when the input is based on vague, noisy, imprecise, ambiguous or missing information. It provides a multi- valued truth space in [0,1]. It is capable of generating inferences even when a partial matching occurs. Here we make use of conventional PID and FPD+I controller to control the nonlinear systems. A fuzzy PID controller is a fuzzified proportional-integral-derivative (PID) controller. It acts on the same input signals, but the control strategy is formulated as fuzzy rules.

Conventional proportional integral derivative (PID) controller is widely made use of in processes due to its simplicity in structure, ease to design and robust performance in wide range of operating conditions [9]. Conventional PID controller is the most popular control tool in many industrial applications as they can improve both the transient response and steady state error of the system at the same time [18]. A PID controller is a simple, linear combination of three signals: the P-action proportional to error, the I-action proportional to the integral of the error and the D-action proportional to the time derivative of error.

The use of just a proportional control leads to an offset error and to minimize this error integral action is used. This integral part takes care of the steady state response of the system while the transient state is improved by the use of derivative action. Traditionally the parameters of Conventional PID controller, i.e. K_P, K_I & K_D, are usually fixed during operation. Such a controller is thus inefficient for controlling a system while a system is disturbed by the changes in the surrounding environment of the system or due to unknown facts [6, 17]. Integral action is necessary whenever a sustained error in steady state occurs. It will increase or decrease the control signal if there is a positive or negative error, even for small magnitudes of error. Thus, a controller with integral action will always return to the reference in steady state [12].

A fuzzy controller acts on three inputs: error, integral error and change in error. To design a rule base with three premise inputs and three linguistic terms for each would be a cumbersome task. Further, it is difficult to decide rules relating to integral action as its initial and final values depend on the load. Thus, the integral action in the crisp PID controller combined with a fuzzy PD rule base as fuzzy PD+I (FPD+I) controller is an effective way to control.

Fuzzy controllers appear in consumer product- such as washing machines, video cameras, cars- and in industry, for controlling cement kilns, underground trains, and robots. A fuzzy controller is a self-acting or self-regulating, self acting mechanism or an automatic controller that controls an object in accordance with a desired behaviour. The object can be, for instance, a robot set to follow a certain path. A fuzzy acts or regulates by means of rules in a more or less natural language, based on the distinguishing feature: fuzzy logic. The rules are invented by plant operators or design engineers, and fuzzy control is thus a branch of intelligent control.

In recent years, fuzzy logic control has been applied successfully in the area of nonlinear process control [3, 1, 4, 2]. In direct fuzzy control applications Fuzzy Logic Controller (FLC) estimates the necessary control action through rule inference by taking the linguistic description of the error behavior [5]. The literature tells that it has been successfully applied to various problems viz. temperature control of glass furnace [11], controlling the Yaw system for wind turbine generator [14], Numerical simulations using dynamic model of 3 DOF planar rigid robot manipulator [16], Fuzzy modeling of Inverted Pendulum [20]. The fuzzy PID can be built to regulate the ball's Position automatically in the experiment of Ball in Tube [10] and Motor speed control of Servo Electro Hydraulic System [18].

Here, liquid level in a surge tank system and position of angle in inverted pendulum system is controlled using both PID and Fuzzy PD+I controller. FPD+I controller has been implemented with four different membership functions viz. Bell shaped, Pi, Gaussian and Psigmoid functions. The results of each controller have been projected along with a comparative analysis. Here a plane is spanned by variables E and CE, which is a phase plane between error and its derivative. The sequence of errors and its derivative can be plotted, CE(t) against E(t), to form a phase trajectory. If the trajectory is ended in the centre (origin), then the plant stays on reference and not moving and that system is stable. For each membership function, phase plots have also been shown for stability analysis.

This dissertation comprises of five chapters. While the present ongoing introduction and literature review is included in chapter 1 and both PID and FPD+I controller is explained in the second chapter which will involve the basics of each controller with their behaviour.

Chapter 3 represents the nonlinear systems which are used for performance analysis of PID controller and FPD+I controller. One of the nonlinear systems is an inverted pendulum system where the aim is to set the pendulum rod to an upright position while the other one is a surge tank where the level of the liquid inside the tank is to be controlled.

Chapter 4 shows the simulated results of each nonlinear system on which the PID and FPD+I controller is implemented separately. The comparative analysis for PID and FPD+I controller on each nonlinear system have been done.

Chapter 5 gives the conclusion and further scope of the work and concludes the performance of both the PID and FPD+I controller individually and comparatively.

CHAPTER-2

CONTROLLER DESIGN

2.1 PID Controller

Nowadays, more than 90% of all control loops are PID, with a wide range of applications: process control, motor drives, magnetic and optical memories, automotive, flight control, instrumentation, etc [21, 22]. This is mainly due to its ability in solving a broad class of practical control problems as well as its structural simplicity, no steady-state error, allowing the operators to use it without much difficulty. But, there are also some problems, such as the parameters of the PID controller are given by the human experience, and the parameters cannot be changed once identified. For the industrial process systems with nonlinearity, large delay, parameter variability, model uncertainty and other factors, it's difficult to meet the performance requirements by using the PID controller. [21]

The combinative action of Proportional, Integral and Derivative action is an effective tool for controlling processes. It is a generic control loop feedback mechanism (controller) widely used in industrial control system. An error value is calculated by the PID Controller as the difference between a desired set point and measured process variable. By adjusting the process control inputs the error is minimized by the controller.

The PID controller is sometimes called Three-term controller as the calculation involves three separate constant parameters which are the Proportional, Integral and Derivative values denoted P, I & D. These values can be interpreted in terms of time: P depends on present error, I on the integration of past errors, and the prediction of future errors is denoted by D, based on current rate of change. The process is adjusted based on the weighted sum of these three actions via a control element such as the position of a damper, a control valve, or the power supplied to a heating element.

A chronological order of the controllers has been discussed below:

a) Proportional controller

In this, controller output is changing linearly to error input. Controller expression is

$$U \propto e$$

In discrete time, it is defined as

$$u(n) = K_p e(n) \tag{2.1}$$

In this controller, the gain is very high or infinity but the range of error it converse is very high.

$$Range\ of\ error \rightarrow Proportional\ Band$$

In the proportional controller, proportional band indicates the range of error to change controller output from 0% to 100%.

$$K_p = \frac{100}{PB}$$

PB- Proportional Band

K_p- Proportional Gain

Drawback:

Offset error: It is s permanent residual error. Complete elimination of this offset error is not possible. This is simply minimized by increasing the proportional gain.

b) Integral Controller

It integrates the previously existed errors and gives the controller output. It is also called reset controller. Controller expression of integral controller is

$$U \propto \int_0^t e\,dt$$

$$U = K_I \int_0^t e\,dt \tag{2.2}$$

K_I= Integral Gain

$$K_I = \frac{1}{T_i}$$

T_i= Integral Time or Reset time

Characteristics:

i. It is not used alone because it is having deadtime.

ii. It is added to the proportional controller (PI).

iii. Integral error adds pole to the system $\left[\frac{1}{T_i s}\right]$.

iv. It improves steady state error of the system.

v. It increases the oscillations and the stability of the system is reduced as it moves the root locus to the right half of s-plane.

vi. It is a phase lag network.

c) Derivative Controller

Controller output is proportional to rate of change of error

$$U \propto \frac{de}{dt}$$

$$U = K_d \frac{de}{dt} \qquad (2.3)$$

K_d- Derivative Gain

Characteristics:

i. Derivative controller is not preferred individually as if error is constant then controller output will be zero which is undesired and misleading.

ii. It is also called Anticipatory and Rate controller.

iii. It reduces oscillations and increases stability.

iv. It improves transient response.

v. Derivative controller adds zero to the transfer function (as s in the numerator). Root locus shifts to the left half of the S-plane. Hence Stability increases.

vi. It is also called the Lead controller or Lead network.

d) Proportional Integral Controller (PI Controller)

Controller output is combination of proportional and integral controller.

$$P = K_p \left[e + \frac{1}{T_i} \int_0^t e\, dt \right] \tag{2.4}$$

Characteristics

i. It adds pole to the system.

ii. It improves steady state response because it makes e_{ss} or offset error zero.

iii. It is a lag network or low pass filter.

iv. It increases oscillations and decreases stability.

e) Proportional Derivative Controller (PD Controller)

It is a combination of proportional and derivative controller.

$$U = K_p \left[e + T_d \frac{de}{dt} \right]$$

The discrete time PD controller is

$$u(n) = K_p \left[e(n) + T_d \frac{e(n) - e(n-1)}{T_s} \right] \tag{2.5}$$

T_s- Sampling time

Characteristics:

i. It adds a zero to the system.

ii. It improves transient response.

iii. It reduces oscillations and improves stability.

iv. It is also a lead network or high pass filter.

v. Since oscillations are decreased, it improves damping.

vi. It reduces maximum overshoot, rise time, peak time, settling time.

vii. Bandwidth is increased.

f) Proportional Integral and Derivative (PID Controller)

The controller can provide control action designed for specific process requirements by tuning the three parameters in the PID controller algorithm. The responsiveness of the controller is given by the degree to which the controller overshoots the setpoint, its response to an error and the degree of system oscillations. The block diagram for a system employing PID control with unity feedback is shown in figure 2.1.

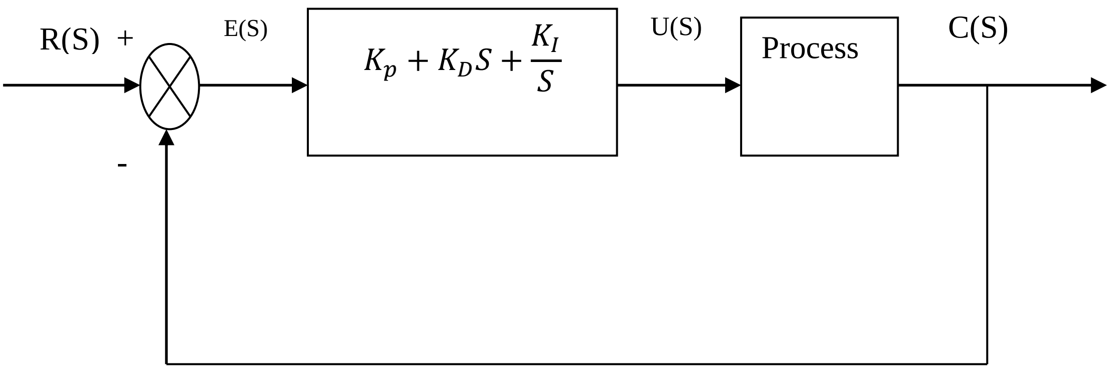


Figure 2.1. Block Diagram of PID Controller with unity feedback.

The output of the controller in continuous form is denoted by

$$u = K_p \left(e + \frac{1}{T_i} \int_0^t e * d\tau + T_d \frac{de}{dt} \right) \quad (2.6)$$

The constant K_p is the proportional gain, T_i is the integral time, T_d the derivative time, and e is the error between the reference and the process output. We are concerned with digital control, and for small sampling periods T_s, the equation may be approximated by a discrete approximation. Replacing the derivative term by a backward difference and the integral by a sum using rectangular integration, an approximation is

$$u_n = K_p \left(e_n + \frac{1}{T_i} \sum_{j=1}^{n} e_j T_s + T_d \frac{e_n - e_{n-1}}{T_s} \right) \quad (2.7)$$

Index n refers to the time instant.

Characteristics:

i. It adds both pole and zero.
ii. It improves both transient and steady state response.
iii. It decreases offset error and oscillations and increases stability.

By tuning we shall mean the activity of adjusting the parameters K_p, T_i and T_d.

2.1.1 Tuning

A PID controller has three tuning parameters. If these are adjusted in an ad hoc fashion, it may take a while for satisfactory performance to be obtained. Also, each tuning technician will end up with a different set of tuning parameters. There is plenty of motivation, then, to develop an algorithmic approach to controller tuning. The first widely used method for PID tuning was published by Ziegler and Nichols.

The Ziegler-Nichols closed-loop tuning technique was perhaps the first rigorous method to tune PID controllers. The technique is not widely used today because the closed-loop behaviour tends to be oscillatory and sensitive to uncertainty. We study the technique for historical reasons, and because it is similar to commonly used automatic tuning [24].

The closed-loop Ziegler-Nichols method consists of the following steps:

a) With P-only closed-loop control, increase the magnitude of the proportional gain until the closed-loop is in a continuous oscillation. For slightly larger values of controller gain, the closed-loop system is unstable, while for slightly lower value the system is stable.
b) The value of the controller proportional gain that causes the continuous oscillation is called the critical (or ultimate) gain K_u. The peak-to-peak period (time between successive peaks in the continuously oscillating process output) is called the critical (or ultimate) period P_u.

Depending on the controller chosen, P, PI or PID, use the values in table for the tuning parameters, based on the critical gain and period.

Table 2-1 Ziegler-Nichols Closed Loop Oscillation Method Tuning Parameters

Controller type	K_p	T_i	T_d
P-only	$0.5K_u$	-	-
PI	$0.45K_u$	$P_u/1.2$	-
PID	$0.6K_u$	$P_u/2$	$P_u/8$

The sample period may be related to the derivative gain T_d. Åström and Wittenmark (1984) suggest that the sample period should be between 1/10 and 1/2 of T_d. In connection with the Ziegler-Nichols rules, this implies that T_s should approximately equal 1-5 percent of the ultimate period T_u rule says that it should be chosen somewhat smaller than the dominating time constant in the process, for instance between 1/10 and 1/5 of that time constant.

Ziegler and Nichols also give another method called reaction curve and step response method. That method uses the open loop step response to find the gains, and this is an advantage if oscillations in the closed loop system cannot be tolerated.

Table 2-2 Rules of Thumb for tuning PID controllers

Action	**Rise time**	**Overshoot**	**Stability**
Increase K_p	Faster	Increases	Get worse
Increase T_d	Slower	Decreases	Improves
Increase $1/T_i$	Faster	Increases	Get worse

2.2 Fuzzy Controller

Traditionally, computers make rigid yes or no decisions, by means of decision rules based on two-valued logic: true-false, yes-no, or 1-0. Fuzzy logic, on the other hand, allows a graduation from true to false. The core of fuzzy controller is a collection of verbal or linguistic rules of the if-then form. Several variables may occur in each rule, both on the if-side and the then-side. Reflecting expert opinions, the rules can bring the reasoning used by computers closer to that of human beings.

If PID (proportional-integral-derivative control) is inadequate-for example, in the case of higher-order plants, systems with a long deadtime, or systems with oscillatory modes- fuzzy control is an option. But first, let us consider why one would not use a fuzzy controller:

a) The PID controller is easy to implement and well understood- both in its analog and digital forms- and it is widely used. On other hand, the fuzzy controller requires some knowledge of fuzzy logic. It also involves building arbitrary membership function.
b) The fuzzy controllers are normally nonlinear. It is not having a simple equation like the PID, and it is more difficult to analyse mathematically; approximations are required, and it follows that stability is more difficult to guarantee.
c) The fuzzy controller has more tuning parameters than the PID controller. Furthermore, it is difficult to trace the data flow during execution, which makes error correction more difficult.

On the other hand, fuzzy controllers are used in industry with success. There are several possible reasons:

a) Since the control strategy consists of if-then rules, it is easy for plant operator to read. The rules can be built from a vocabulary containing everyday words such as ‘high’, ‘low’, and ‘increasing’. Plant operators can embed their experience directly.
b) The fuzzy controller accommodates many inputs and many outputs. Variables can be combined in an if-then rule with the connectives and and or. Rules are executed in parallel, implying a recommended action from each. The recommendation may be in conflict, but the controller resolves conflicts.(Jan Jantzen)
c) One great advantageous thing of fuzzy control is that it does not require detailed mathematical model of a plant.

Fuzzy logic enables non-specialists to design control systems, and this may be the main reason for its success.

It is a technique to embody human- like thinking into a control system. A fuzzy controller can be designed to emulate human deductive thinking to infer conclusion from the past experience. Fuzzy control incorporates ambiguous human logic into computer programs. It suits the control problems that cannot be easily represented by mathematical models. Following are some of the difficulties involved in a control system design where fuzzy control can be applied:

a) Inaccurate model
b) Parameter variation problem
c) Unavailable or incomplete data
d) Very complex plants
e) Good qualitative understanding of plant or process operation

Design of such controller leads to faster development or implementation cycles. The principal design components of fuzzy logic controller are discussed below:

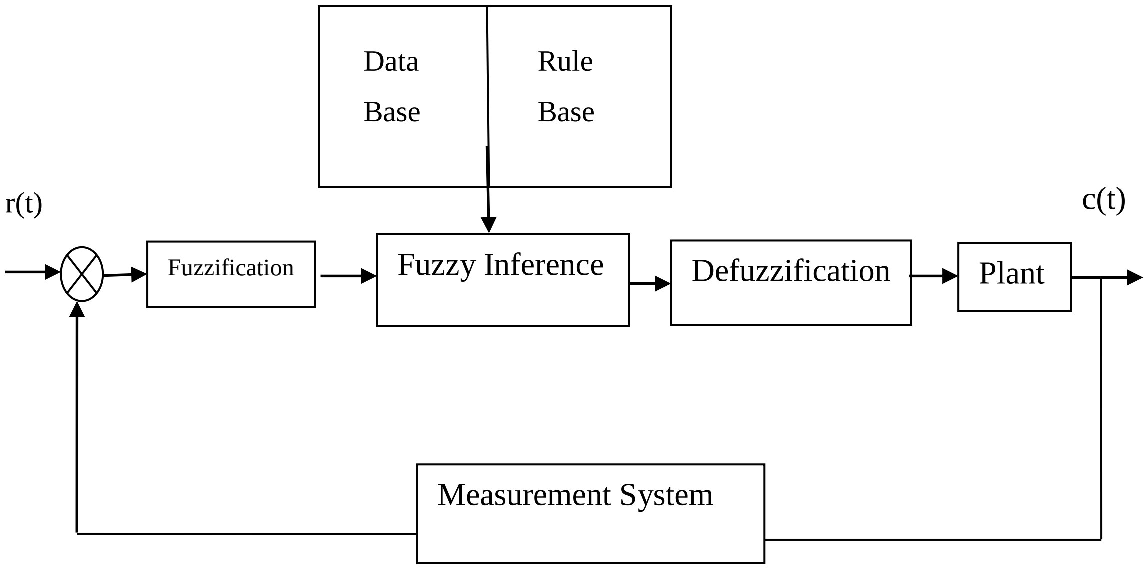


Fig 2.2. Fuzzy Logic Controller System

Fuzzifier:

This decides the fuzzification strategies and interpretation of a fuzzification operator, which basically involves discretization or normalization of a discourse, fuzzy partition of the input and output spaces, and choice of the membership function of a primary fuzzy set. In a typical fuzzy logic controller, the input variables are chosen as error and change in error, which are converted into fuzzy set using membership function. Fuzzification is the process of mapping input to the fuzzy logic controller into fuzzy sets membership values in the various input universe of discourse.

Decision need to be made regarding

a) Number of inputs
b) Size of the universe of discourse
c) Number and shapes of fuzzy sets.

The size of the universe of discourse will depend upon the expected range of the input variables.

Rule base:

The rule base is a collection of rules that describe the control strategy which depends on the choice of process state (input) variables and control (output) variables.

The rule base is constructed using a prior knowledge from either one or all of the following sources.

a) Physical laws that govern the plant dynamics.
b) Data from existing controllers.
c) Imprecise heuristic knowledge obtained from experienced experts.

Fuzzy inference machanism:

This establishes a logical connection between the input and output fuzzy sets. It is the process of mapping membership values from the input windows, through the rulebase to the output window.

Defuzzifier:

This decides the defuzzification strategy to convert the output fuzzy variable into a corresponding crisp value. Among various defuzzification methods, the most popular is the centre of gravity (COG) method.

A design strategy, which makes use of known PID design techniques, before implementing the fuzzy controller:

1. Tune a PID controller
2. Replace it with an equivalent linear fuzzy controller
3. Make the fuzzy controller nonlinear
4. Fine-tune it

It seems sensible to start the controller design with a crisp PID controller, maybe even just a P controller, and get the system stabilised. From there it is easier to go to fuzzy control. Each step will be investigated in the following.

The designer must brace himself for the next step into nonlinear control. The analytical method lack generality in the nonlinear domain, and it is difficult to predict the behaviour of a nonlinear system.

It is characteristic for a linear time-invariant system that

(1) The response has an analytical solution

(2) A sinusoidal input causes a sinusoidal output of the same frequency.

Nonlinear system does not possess similar characteristics, on the contrary, it is characteristics that the superposition principle does not hold. It is further characteristic that the output depends not only on the frequency of the input, but also on amplitude. Therefore we cannot plot anymore, or at least several plots must be drawn: one for each chosen amplitude. The overall plan for the following section is to insert a nonlinear rule base. Basic knowledge of the state-space approach is s prerequisite.

Membership Function:

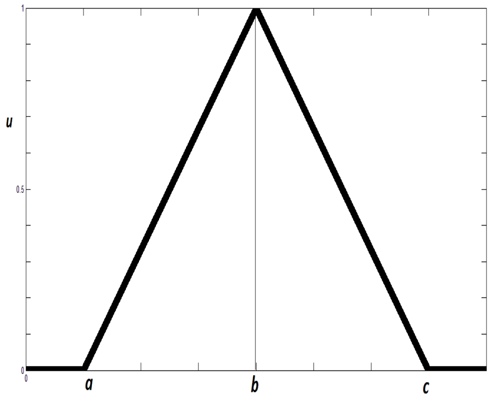
There are different types of membership function that are used for fuzzy control. There is also provision to custom-design MFs in some fuzzy control software tools [23]. Sometimes, Membership function types are hybrided for the input and output fuzzy variables. Although trapezoidal type Membership function has often been used in fuzzy control literature, triangular Membership functions are most commonly used almost intuitively for all the variables. There are two alternative ways to represent a membership function: continuous or discrete.

The general classification of Membership functions are as follows:

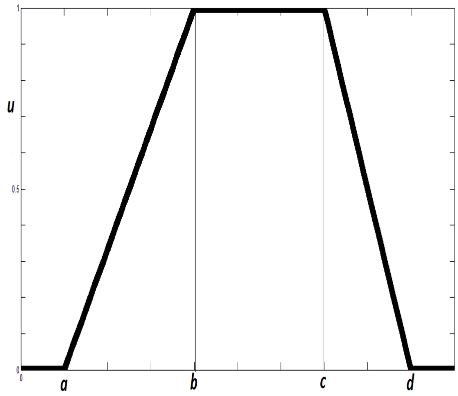
a) Triangular MF
b) Trapezoidal MF
c) Gaussian MF
d) Two side Gaussian MF
e) Bell shaped MF
f) Sigmoid- right MF
g) Sigmoid-left MF
h) Difference-sigmoid MF
i) Product-sigmoid MF
j) Polynomial-Z MF

k) Polynomial-S MF

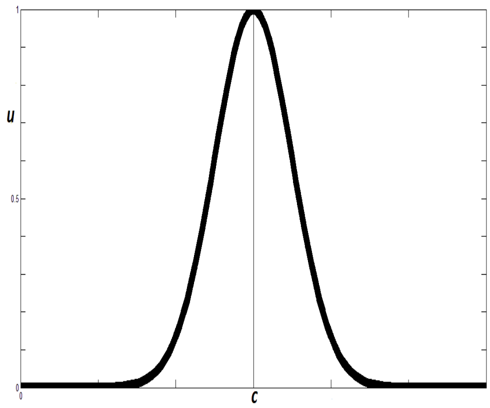
l) Polynomial-PI MF



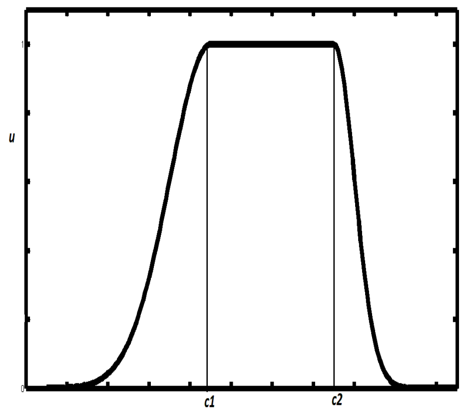
(a) Triangular MF



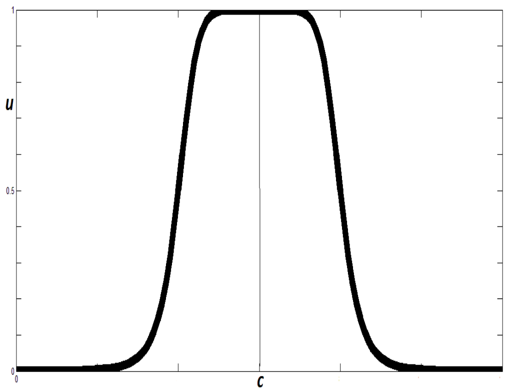
(b) Trapezoidal MF



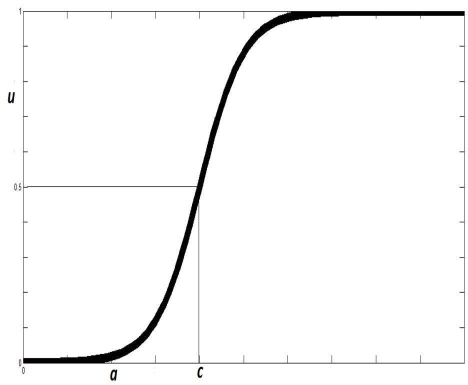
(c) Gassian MF



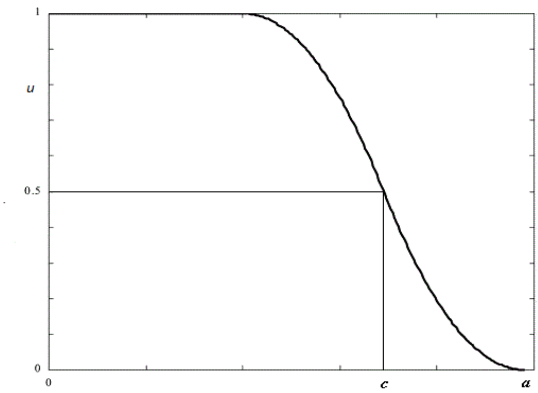
(d) Two-sided Gaussian MF



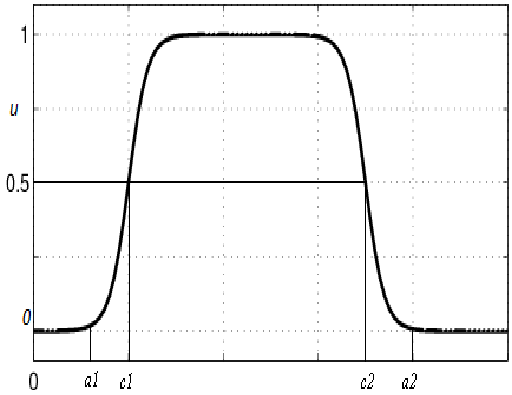
(e) Bell-shaped MF



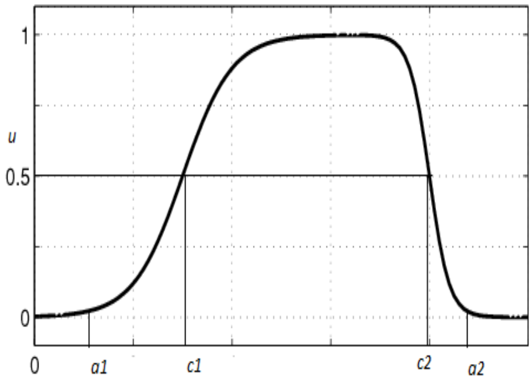
(f) Sigmoid-right MF



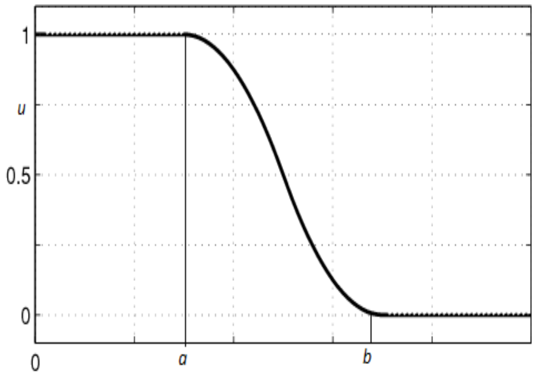
(g) Sigmoid-left MF



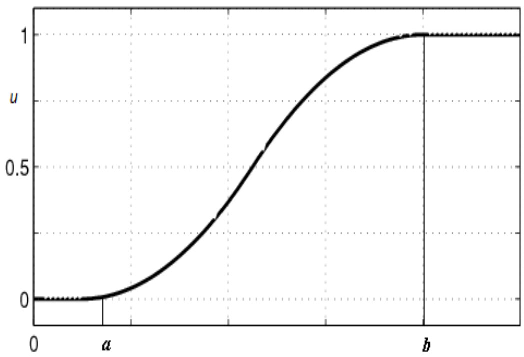
(h) Difference-sigmoid MF



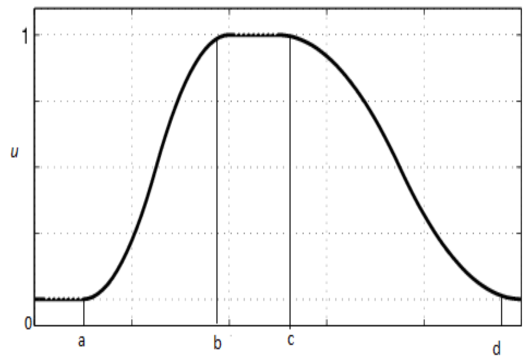
(i) Product-sigmoid MF



(j) Polynomial-Z MF



(k) Polynomial-S MF



(l) Polynomial-Pi MF

Figure 2.3: Different Type of Membership Functions

Piecewise Linear Functions:

Piecewise linear functions constitute the simplest type of Membership functions, and they may be generally either triangular or trapezoidal type. A truncated triangle gives the shape of trapezoidal membership function. These may again be asymmetrical or symmetrical in shape. Three parameters a, b and c explains a triangular membership function which gives the expression

$$f(x,a,b,c) = \max\left\{min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0\right\} \tag{2.8}$$

where the parameters a and c locate the "feet" of the triangle and the peak is located by the parameter b which is shown in figure 2.3(a).

A trapezoidal Membership function is described by four parameters which are a, b, c and d and it is given by the expression

$$f(x;a,b,c,d) = max\left\{min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right\} \tag{2.9}$$

where a and d locate the “feet” of the trapezoid and b and c locate the “shoulder”, as shown in figure 2.3(b). A trapezoidal MF can have either narrow “shoulder” or wide “shoulder”.

Gaussian Functions:

A symmetric Gaussian MF, as shown in figure 2.3(c), is given by the expression

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}} \tag{2.10}$$

Where the distance from the origin is located by parameter *c* and *σ* indicates width of the curve. In a two-sided Gaussian membership function , shown in figure 2.3(d), the first function, described by the parameters σ_1 and c_1, determines the shape of the left-side of curve, whereas the second function described by the parameters σ_2 and c_2 , determines the shape of the right-side of curve. Usually, $c_1 < c_2$ so that the function can reach the maximum value of 1. The Gaussian membership functions have the features that they are non-zero at all points and smooth.

Bell-Shaped Function:

A bell-shaped Membership function, as shown in figure 2.3(e) has symmetrical shape and is given by the expression

$$f(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \tag{2.11}$$

Where the parameter *b* is usually positive, the centre of the curve is located by parameter *c* and *a* shows width of the curve. Again, the membership function has the features of being non-zero at all points and smooth.

Sigmoidal Function:

A sigmoidal type membership function can generally be open to the right or to the left, as shown by figures 2.3(f) and 2.3(g), respectively. The former type is commonly used as activation function in neural network. The general expression of sigmoidal membership function is given as

$$f(x; a, c) = \frac{1}{1 + e^{-a(x-c)}} \tag{2.12}$$

Where distance from the origin is located by the *c* and steepness of the function is determined by *a*. If it is positive, it is open to the right, whereas if *a* is negative, the membership function is open to the left. The “very large positive” is intended by the former while the “very large negative” is intended by the later in linguistic terms. Symmetrical or asymmetrical but closed (not open to the right or left) membership functions can be constructed by using either the product or difference of the two sigmoidal membership functions described above. The membership function formed by the difference between the two sigmoidal membership functions is defined as difference-sigmoid, and the membership function formed by the product of these is defined as product-sigmoid. These are shown in figures 2.3(h) and 2.3(i), respectively. All the membership functions in this family are smooth and non-zero at all points.

Polynomial Based Functions:

Three polynomial based membership functions in this family are defined as polynomial-Z(*zmf*), polynomial-S(*smf*) and polynomial-PI(*pimf*) which are shown, respectively, in figures 2.3(j), 2.3(k) and 2.3(l). They are named according to their shapes. The former two are always asymmetric, the function *pimf* may be symmetric or asymmetric in shape. The function *zmf* is open to the left and shaped somewhat like Z and is given by

$$y = zmf(x, [ab]) \tag{2.13}$$

Where the extremes of the sloped portion of the curve indicates the parameter *a* and *b*. The function *smf* is open to the right and somewhat shaped like S is given by

$$y = smf(x, [ab]) \tag{2.14}$$

Where a and b represent also the extremes of the sloped portion of the curve. The function *Pimf* is zero at both ends but has a rise in the middle and it is given by the expression

$$y = pimf(x, [abcd]) \tag{2.15}$$

Where the parameters a and d locate the "feet" of the curve, whereas b and c locate the "shoulder". Again, all these spline-shaped function curves have the features of being smooth but not non-zero at all points.

2.1.1 Transferring gain from PID to Fuzzy:

The third step in the design procedure is to transfer the PID gains to the linear fuzzy controller.

A. Proportional controller

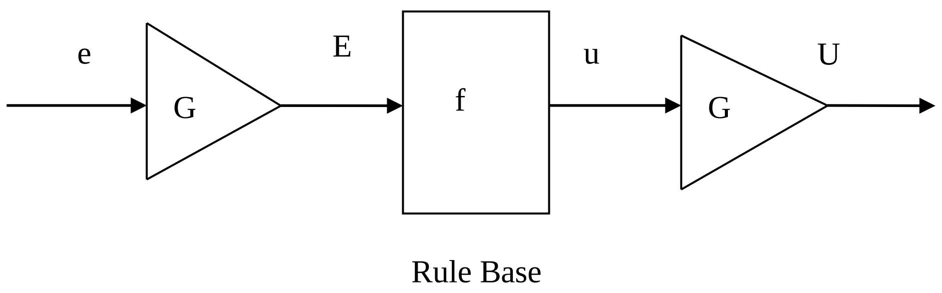


Figure 2.4: Fuzzy Proportional controller (FP)

Input to a fuzzy proportional (FP) controller is error and the output is the control signal. This is the simplest fuzzy controller. It is relevant for state- or output-feedback in a state space controller. Compared to crisp proportional control the fuzzy P controller has two gains GE and GU instead of just one. As a convention, signals are written in lower case before gains and upper case after gains, for instance E=GE*e. The gains are mainly for tuning the response, but since there are two gains, they can also be used for scaling the input signal onto the input universe to exploit it better. The controller output is the control signal U_n a nonlinear function of e_n

$$U_n = f(GE * e_n) * GU \tag{2.16}$$

The function f is the fuzzy input-output map of the fuzzy controller. Using the linear approximation $f(GE * e_n) = GE * e_n$

$$U_n = GE * e_n * GU = GE * GU * e_n \quad (2.17)$$

Compared with (2.1) the product of the gain factors is equivalent to the proportional gain, i.e;

$$GE * GU = K_p \quad (2.18)$$

The accuracy of the approximation depends mostly on the membership functions and the rules. The approximation is best, however, if we choose the same universe on both input and output side. For example [-100,100], the rule base could be given as

1. If E is Pos then u is 100
2. If E is Neg then u is -100

With Pos and Neg triangular as defined previously, is equivalent to a P-controller. Given a target Kp, for example from the Ziegler-Nichols rules helps to choose the gains. The equation has one degree of freedom, since the fuzzy P controller has one more gain factor than the crisp P controller. This is used to exploit the full range of the input universe.

Because of the process dynamics it will take some time before a change in the control signal is noticeable in the process output, and the proportional controller will be more or less late in correcting for an error.

B. Proportional and derivative control

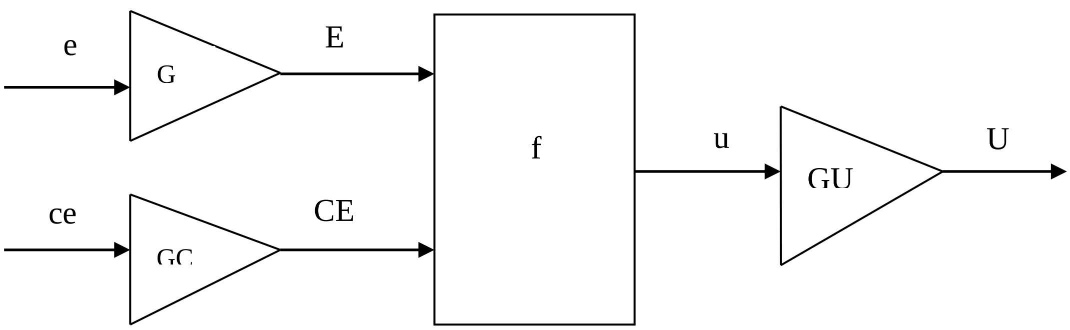


Figure 2.5: Fuzzy Proportional Derivative controller (FPD)

Derivative action helps to predict the error and the proportional-derivative controller uses the derivative action to improve closed-loop stability. The basic structure of a PD controller is

$$u_n = K_p * (e_n + T_d * (e_n - e_{n-1})/T_s) \tag{2.19}$$

The control signal is thus proportional to an estimate of the error, T_d seconds ahead, where the estimate is obtained by linear extrapolation. For T_d=0 the control is purely proportional, and when T_d is gradually increased, it will dampen oscillations. If T_d becomes too large the system becomes overdamped and it will start to oscillate again.

Input to the fuzzy proportional derivative (FPD) controller is the error and the derivative of the error figure 2.5. In fuzzy control the latter term is usually called change in error,

$$ce_n = \frac{(e_n - e_{n-1})}{T_s} \tag{2.20}$$

This is a discrete approximation to the differential quotient using a backward difference. Other approximations are possible, as in crisp PD controllers. Notice that this definition deviates from the straight difference $ce_n = (e_n - e_{n-1})$ used in the early fuzzy controllers.

The controller output is a nonlinear function of error and change in error,

$$U_n = f(GE * e_n, GCE * ce_n) * GU \tag{2.21}$$

Again the function f is the input-output map of the fuzzy controller, only this time it is a surface. Using the linear approximation $GE * e_n + GCE * ce_n$, then

$$U_n = (GE * e_n + GCE * ce_n) * GU \tag{2.22}$$

$$U_n = GE * GU \left(e_n + \frac{GCE}{GE} * ce_n\right) \tag{2.23}$$

By comparison, the gains in (2.19) and (2.23) are related as follows,

$$GE * GU = K_p \tag{2.24}$$

$$\frac{GCE}{GE} = T_d \tag{2.25}$$

The approximation corresponds to replacing the fuzzy input-output surface with a plane. The approximation is best if we choose the output universe to be the sum of the input universes. Assume, for example, that the input universes are both [-100,100] and we choose output singletons on [-200,200], then the input-output map will be the plane u=E+CE. Therefore, by that choice, we can exploit (2.24) and (2.25).

The fuzzy PD controller may be applied when proportional control is inadequate. It may be sensitive to noise as well as an abrupt change of the reference causing a derivative kick but it has an advantage that the derivative term reduces overshoot. The usual counter-measures may overcome these problems: in the former case insert a filter, and in the later use the derivative of the process output y_n instead of the error.

C. Incremental control

If there is a sustained error in steady state, integral action is necessary. The integral action will increase the control signal if there is a small positive error, no matter how small the error is; the integral action will decrease it if the error is negative. A controller with integral action will always return to zero in steady state.

It is possible to obtain a fuzzy PI controller using error and change in error as inputs to the rule base. Past experience shows that it is difficult to write rules for the integral action. Problems with the integrator windup also have to be dealt with. When the actuator has limits then windup occurs, such as maximum speed for a motor or maximum opening of a valve.

When the actuator saturates, the control action stays constant, but the error will continue to be integrated, the integrator winds up. There will be very large integral term and it will then take a long time to wind it down when the error changes sign. Large overshoots may be the consequence. For avoiding it, many methods are there.

It is often a better solution to set up the controller as an incremental controller. An incremental controller adds a change in control signal Δu to the current control signal,

$$u_n = u_{n-1} + \Delta u \quad (2.26)$$

$$\Delta u = K_p\left(e_n - e_{n-1} + \frac{e_n * T_s}{T_i}\right) \quad (2.27)$$

Using (2.7) with $T_d = 0$, it is natural to use an incremental controller when for example a stepper motor is the actuator. The controller output is an increment to the control signal, and the integration is performed by motor itself. It is an advantage that the controller output is driven directly from an integrator, and then it is easy to deal with windup and noise. A disadvantage is that it cannot include D-action well.

The fuzzy incremental (FInc) controller in figure (2.5) is almost the same configuration as the FPD controller except for the integrator on the output. The output from the rule base is therefore called change in output (cu_n), and the gain on the output has changed name accordingly to GCU. The control signal U_n is the sum of all previous increments,

$$U_n = \sum cu_i * GCU * T_s \tag{2.28}$$

Notice again that this definition deviates from the early fuzzy controllers, where $U_n = \sum(GCU * cu_i)$ - the difference is the sampling period T_s. The linear approximation to this controller is

$$U_n = \sum (E_i + CE_i) * GCU * T_s$$

$$= GCU * \sum \left[GE * e_i + GCE * \frac{(e_i - e_{i-1})}{T_s}\right] * T_s$$

$$= GCU * \left[GE * \sum e_i * T_s + GCE * \sum (e_i - e_{i-1})\right]$$

$$U_n = GCE * GCU * \left[\frac{GE}{GCE} * \sum e_i * T_s + e_n\right] \tag{2.29}$$

By comparing (2.7) and (2.29) it is clear that the linear controller is a crisp PI controller ($T_d = 0$), and gains are related in the following way,

$$GCE * GCU = K_p$$

$$\frac{GE}{GCE} = \frac{1}{T_i} \tag{2.30}$$

Notice that the proportional gain now depends on GCE. The gain on the integral action is determined by the ratio between GE and GCE, and it is the inverse of the derivative gain in FPD control. It is as if GE and GCE have changed roles. The usual problem of Integrator windup can be overcome by simply limiting the integrator.

D. Proportional, Integrator and Derivative control

In this section, Fuzzy PD+I has been applied to the surge tank system and inverted pendulum system. Three inputs had been used, error(e), change in error(ce) that will be fed to the fuzzy controller while integral error will be used as conventional integral action [16]. A rule base with three inputs, however, easily becomes rather complex and, as mentioned earlier, rules concerning the integral action are troublesome. So it is common to separate the integral action as in the fuzzy PD+I (FPD+I) controller in figure 2.6. The integral error is computed as,

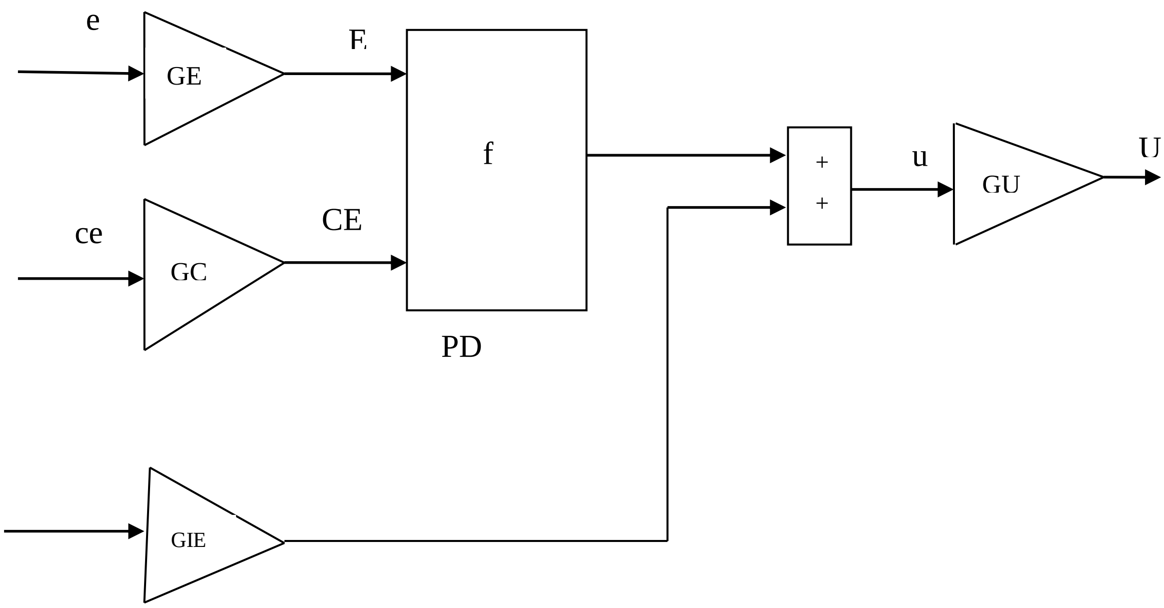


Figure 2.6: Fuzzy Proportional Derivative Integral controller (FPD+I)

$$ie_n = \sum (e_i * T_s) \qquad (2.31)$$

Thus the controller consists of a function of the three inputs

$$U_n = [f(GE * e_n, GCE * ce_n) + GIE * ie_n] * GU \qquad (2.32)$$

Its linear approximation is

$$U_n = [GE * e_n + GCE * ce_n + GIE * ie_n] * GU$$

$$U_n = GE * GU \left[e_n + \frac{GCE}{GE} * ce_n + \frac{GIE}{GE} * ie_n\right] \quad (2.33)$$

We have assumed a nonzero GE in the last line. Comparing (2.7) and (2.33) the gains are related in the following way,

$$GE * GU = K_p \quad (2.34)$$

$$\frac{GCE}{GE} = T_d \quad (2.35)$$

$$\frac{GIE}{GE} = \frac{1}{T_i} \quad (2.36)$$

2.3 Stability Analysis of Nonlinear Systems

Phase plane analysis

A phase plot is generally a plot of a variable and its time derivative plotted against each other. The fuzzy PD (FPD) controller operates with error and its derivative, hence our interest in the phase plots. In fact, the control surface is a plot of change in error against error, and can therefore as a phase plot.

Phase plane analysis is a graphical method providing valuable visual overview. It has a drawback, however: it is limited to two-dimensional plots. We can apply phase plane analysis without having to solve the differential equations analytically. Furthermore, it accommodates discontinuous (hard) nonlinearities.

Phase trajectory

Our starting point is a second-order autonomous system

Trajectory

$$\dot{x} = Ax$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_{22}$$

Where x_1 and x_2 are the state variables of the system, and A the system matrix. An autonomous system is a system without inputs, and the usual input term is missing.

Geometrically, the variable x_1 and x_2 span a two-dimensional space, called the phase plane. The solution in time domain $x(t)$ given an initial value $x(0)$ of the state vector, is a curve in the phase plane with x_2 along the y-axis and x_1 along the x-axis. The curve is called a phase trajectory.

Equilibrium point

If the system settles down to an equilibrium state, as the result of some initial state, the equilibrium is characterized by zero motion in the phase plane. Such an equilibrium point (singular point) must satisfy the equation.

$$0 = Ax$$

If the A matrix is non-singular, then the origin $x = 0$ is an equilibrium point. Otherwise there may be e collection of equilibrium points lying on a line through the origin. Nonlinear system can have several equilibrium points, one, or none at all.

Slope

$$\frac{\dot{x}_2}{\dot{x}_1} = \frac{a_{21}x_1 + a_{22}x_2}{a_{11}x_1 + a_{12}x_2}$$

Conversely, given a particular slope $S=S^*$, and solving the equation for x_2, result in an equation for a line along which the slope of all trajectories crossing it is the same. Such a curve is called an isoclines and a collection of isoclines can be used to graphically construct phase plane trajectories.

Stability

Stability requires $x^T Ax < 0$, which is equivalent to the usual criterion that all the eigen values of A must have negative real parts. There are six different types of behaviour around the equilibrium point of various A matrices:

a) **Node**

If the eigen values of the matrix A are real and negative, the equilibrium is a stable node, because $x(t)$ converges to 0 in an exponential decay. If both eigen values are positive, the equilibrium is an unstable node, because $x(t)$diverges exponentially.

b) **Focus**

If the eigen values of the A matrix are complex conjugates and having negative real parts, the equilibrium is a stable focus. If both eigen values have positive real parts, the equilibrium is an unstable focus.

c) **Centre point**

If the eigen values of the A matrix are complex conjugate, with real parts equal to zero, the equilibrium is a centre point, because $x(t)$encircles the equilibrium point, without converging or diverging. The plot of $x_1(t)$ in the time domain shows a sustained oscillation, a marginally stable system.

d) **Saddle point** If the eigen values of the A matrix are real, but one is negative and the other is positive, the equilibrium is a saddle point, Because of the unstable positive eigen value, almost all trajectories will ditto infinity.

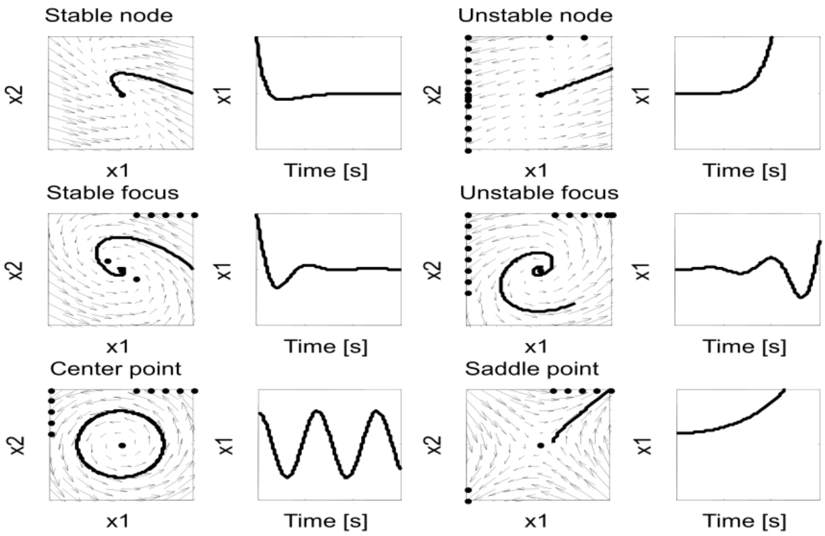


Figure 2.7. Equilibrium point

CHAPTER-3

NONLINEAR SYSTEMS

3.1 Inverted Pendulum

In recent Years, projects on the themes of robotics and mechatronics are the most attractive. Many interesting robotic benchmark systems exist in the literature in this framework. The inverted pendulum system is always considered as the most fundamental benchmark among others. This system exists in many different versions offering a variety of interesting control challenges. Figure 3.1 shows the inverted pendulum system.

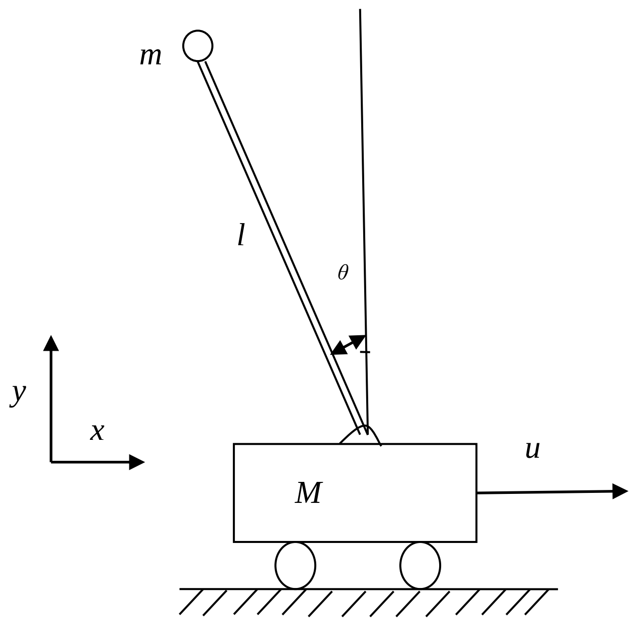


Figure 3.1: Inverted Pendulum on Cart

The system consists of an inverted pole hinged on a cart. When the cart is given a push or force in the x axis direction, it moves along the direction of force. This results in a change in the angle of the pole with the vertical. If an initial deviation in the angle is considered, the objective of the control would be to keep the angle of the pole with respect to the vertical axis zero, i.e. in the upright position. Thus, the system could be modelled by taking u as the control input variable and θ as the controlled variable. While controlling, certain constraints should be taken care of, i.e. the cart should not go beyond a specified finite value, i.e. the cart should be moving through a certain specified range. The angle of pole must lie within the range of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Model of Inverted Pendulum

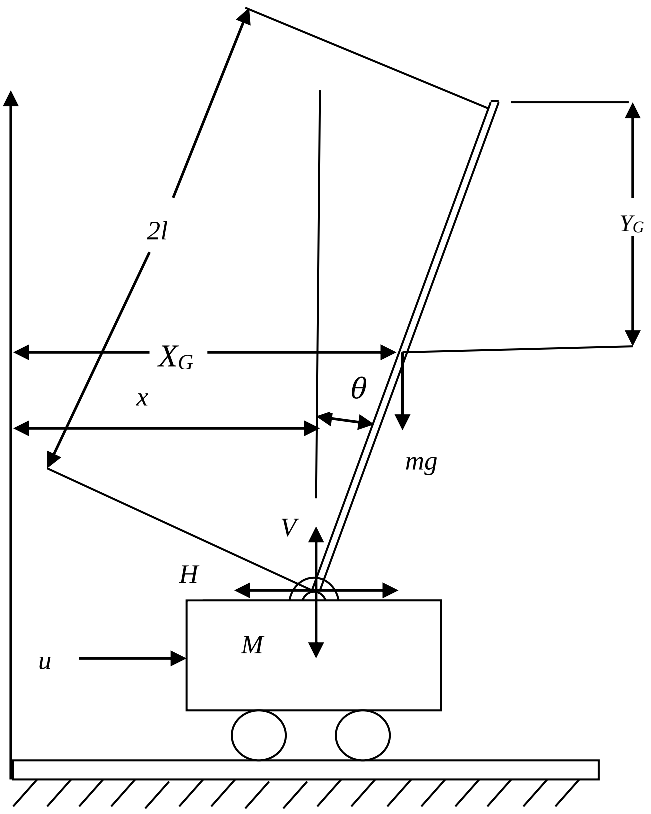


Figure 3.2: Mathematical model of Inverted pendulum on cart

M (Mass of Cart) = 3Kg

m (Mass of pendulum) = 0.2Kg

l (Length to centre of mass of pendulum) = 0.31m

u- Force applied to the cart

x- Cart position coordinate

θ- Pendulum angle from vertical

b (Frictional force of cart) = 0.1 N/ms^{-1}

The dynamical equation of the system can be derived as follows:

Force analysis and system equation

The equation describing the centre of gravity of the pendulum rod with respect to the origin on the Cartesian axis of reference can be written as

$$X_G = x + l\sin\theta$$

$$Y_G = l\cos\theta$$

The balancing the force of the rod in the vertical direction with respect to its centre of gravity is given by

$$m\frac{d^2}{dt^2} l\cos\theta = V - mg$$

$$m\left[\frac{d}{dt}\left(l\sin\theta\,\dot{\theta}\right)\right] = V - mg$$

$$m\left[-l\cos\theta\dot{\theta}^2 - l\sin\theta\ddot{\theta}\right] = V - mg$$

On completing the differentiation above, we get

$$mg - ml\ddot{\theta}\sin\theta - ml\cos\theta\,\dot{\theta}^2 = V \qquad (3.1)$$

The balance of forces of the rod with respect to its centre of gravity in the horizontal direction is given by

$$m\frac{d^2}{dt^2}(x + l\sin\theta) = H$$

$$m\left[\ddot{x} + \frac{d}{dt}\left(l\dot{\theta}\cos\theta\right)\right] = H$$

$$m\left[\ddot{x} + l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2\right] = H$$

$$m\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 = H \qquad (3.2)$$

Balancing the forces of the cart in the x direction is described by

$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} = u - H \qquad (3.3)$$

Balancing the rotational motion of the pendulum rod around its centre of gravity is given by

$$I\ddot{\theta} = Vl\sin\theta - Hl\cos\theta \qquad (3.4)$$

Put (3.2) in (3.3)

$$M\ddot{x} = u - m\ddot{x} - ml\cos\theta\,\ddot{\theta} + ml\sin\theta\,\dot{\theta}^2$$

After solving this equation, we get

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u \qquad (3.5)$$

Put (3.1) & (3.2) in (3.5)

$$I\ddot{\theta} = \left(mg - ml\ddot{\theta}\sin\theta - ml\cos\theta\,\dot{\theta}^2\right)l\sin\theta - \left(m\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2\right)l\cos\theta$$

After solving it, we get

$$(ml^2 + I)\ddot{\theta} = mgl\sin\theta - m\ddot{x}l\cos\theta \qquad (3.6)$$

From (3.6)

$$\ddot{\theta} = \frac{mgl\sin\theta - m\ddot{x}l\cos\theta}{ml^2 + I}$$

Put in (3.5)

$$(M+m)\ddot{x} + ml\left(\frac{mgl\sin\theta - m\ddot{x}l\cos\theta}{ml^2 + I}\right)\cos\theta + b\dot{x} - ml\dot{\theta}^2\sin\theta = u$$

$$(M+m)\ddot{x} + \frac{m^2l^2g\sin\theta\cos\theta}{ml^2 + I} - \frac{m^2l^2\ddot{x}cos^2\theta}{ml^2 + I} + b\dot{x} + ml\dot{\theta}^2\sin\theta = u$$

To approximate the equation, we use

$$\sin\theta = \theta$$

$$\dot{\theta}^2 = 0$$

$$\cos\theta = 1$$

$$sin^2\theta = 0$$

$$cos^2\theta = 1$$

Then we get,

$$\ddot{x} = -\frac{m^2l^2g}{(ml^2 + I)M + mI}\theta - \frac{b(ml^2 + I)}{(ml^2 + I)M + mI}\dot{x} + \frac{(ml^2 + I)}{(ml^2 + I)M + mI}u \qquad (3.7)$$

From (3.6)

$$\ddot{x} = \frac{mgl\sin\theta - (ml^2 + I)\ddot{\theta}}{ml\cos\theta}$$

Put in (3.5)

$$(M+m)\left[\frac{mgl\sin\theta - (ml^2 + I)\ddot{\theta}}{ml\cos\theta}\right] + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$

$$\frac{(M+m)g\sin\theta}{\cos\theta} - \frac{(M+m)(ml^2 + I)}{ml\cos\theta} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$

By approximating the equation, we get

$$(M+m)g\theta - \frac{(M+m)(ml^2 + I)}{ml}\ddot{\theta} + b\dot{x} + ml\ddot{\theta} = u$$

Solving the above equation, we get

$$\ddot{\theta} = \frac{(M+m)mgl}{M(ml^2+I)+mI}\theta + \frac{bml}{M(ml^2+I)+mI}\dot{x} - \frac{ml}{M(ml^2+I)+mI}u \qquad (3.8)$$

A state-space model can be devised for which further transformations have to be applied to these equations. Introducing the substitution variables for (3.7) and (3.8)

$$A = \frac{(M+m)mgl}{M(ml^2+I)+mI}$$

$$B = \frac{bml}{M(ml^2+I)+mI}$$

$$C = -\frac{m^2l^2g}{(ml^2+I)M+mI}$$

$$D = -\frac{b(m^2l+I)}{(ml^2+I)M+mI}$$

With the data, the values of the data are calculated to be

$$A = 33.75$$

$$B = 0.11$$

$$C = -0.65$$

$$D = -0.03$$

Therefore, the linear state space model is:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ A & 0 & 0 & B\\ 0 & 0 & 0 & 1\\ C & 0 & 0 & D\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ -ml/[(ml^2+I)M+mI]\\ 0\\ (m^2l+I)/[(ml^2+I)M+mI]\end{bmatrix}u$$

After inserting constant data, the state space model is:

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 33.75 & 0 & 0 & 0.11\\ 0 & 0 & 0 & 1\\ -0.65 & 0 & 0 & -0.03\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ -1.08\\ 0\\ 0.33\end{bmatrix}u$$

The state space model obtained, is converted into discrete form and we get the discrete state space representation of the above system as

$$\begin{bmatrix}\Delta\theta_{k+1}\\ \Delta\dot{\theta}_{k+1}\\ \Delta x_{k+1}\\ \Delta\dot{x}_{k+1}\end{bmatrix} = \begin{bmatrix}1 & 0.0013 & 0 & 0\\ 0.0430 & 1 & 0 & 0.0001\\ 0 & 0 & 1 & 0.0012\\ -0.0008 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\theta_{k}\\ \Delta\dot{\theta}_{k}\\ \Delta x_{k}\\ \Delta\dot{x}_{k}\end{bmatrix} + \begin{bmatrix}0\\ -1.35\times 10^{-3}\\ 0\\ 0.4125\times 10^{-3}\end{bmatrix}\Delta u_k$$

3.2 Level Control in a Nonlinear Surge Tank system

Design Problem: Consider the "surge tank" shown below with input u(t), the height of the liquid h(t) and an outlet below the tank characterized by $\tilde{d}$ which is related to the outlet pipe diameter.

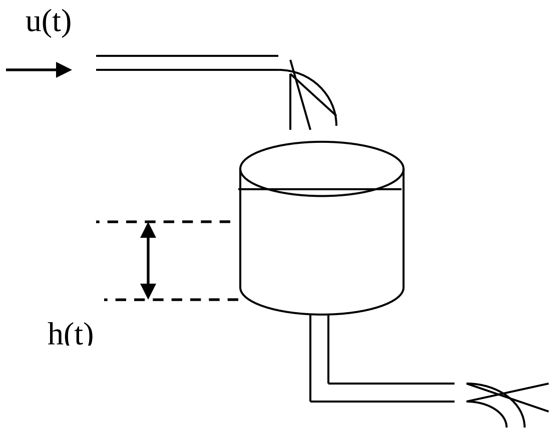


Figure 3.3: Surge Tank System

The input changes dynamically and the objective is to maintain the level of liquid in the surge tank at 'h' while u(t) is the varying input to the system. The mathematical model of the system is given as

$$h(k+1) = h(k) + T\left(\frac{-\tilde{d}\sqrt{19.6h(k)}}{\left|\tilde{a}h(k)+\tilde{b}\right|} + \frac{\tilde{c}}{\left|\tilde{a}h(k)+\tilde{b}\right|}u(k)\right) \tag{3.9}$$

The above model is derived by relating the inputs to measured outputs that needs to be regulated. The volume of liquid in the vessel varies as a function of the inlet and outlet flow rates. The end-use of the model defines the model requirements.

Assuming the density to be constant, balanced equations based on an instantaneous rate of change can be given as

$$\begin{bmatrix} \text{Rate of change of} \\ \text{total mass of fluid} \\ \text{inside the vessel} \end{bmatrix} = \begin{bmatrix} \text{mass flow rate} \\ \text{of fluid} \\ \text{into the vessel} \end{bmatrix} - \begin{bmatrix} \text{mass flow rate} \\ \text{of fluid} \\ \text{out of the vessel} \end{bmatrix}$$

Following notations are used in the modelling equation:

$u =$ inlet volumetric flow rate (volume/time);

$o =$ outlet volumetric flow rate (volume/time);

$V =$ volume of liquid in vessel;

$h =$ height of liquid in vessel;

$\rho =$ liquid density (mass/volume);

$A =$ cross-sectional area of vessel = $\left|\tilde{a}h(k) + \tilde{b}\right|$ where $\tilde{a} > 0$ and $\tilde{b} > 0$.

$\tilde{c} =$ clogging factor ϵ $(0,1)$. $\tilde{c} = 1$ shows that the filter in the actuator

$\tilde{d} =$ area of liquid discharge related to the outlet pipe diameter.

Where the total mass of fluid inside the vessel is denoted by $V\rho$, the rate of change is $\frac{dV\rho}{dt}$, and the density of the outlet stream is equal to the density of the vessel contents.

$$\frac{dV\rho}{dt} = \tilde{c}u\rho - o\rho \tag{3.10}$$

The volumetric flow rate Q can be written as $Q = Av$ where A= area and $v =$ velocity. Thus, the outlet volumetric flow rate can be expressed with $\tilde{d}$ and outlet velocity v as

$$o = \tilde{d}\, v \tag{3.11}$$

If we equate the kinetic and potential energies of liquid, we get

$$mgh = \frac{1}{2}mv^2$$

$g =$ acceleration due to gravity and $m =$ mass of liquid a. ‘m’ gets cancelled on both sides of the equation and hence we get,

$$v = \sqrt{2gh} \tag{3.12}$$

using (3.11) & (3.12) a modified (3.10), after cancelling out ρ on both sides of the equation

$$\frac{dV}{dt} = \tilde{c}u - \tilde{d}\sqrt{2gh} \tag{3.13}$$

Volume V of the tank can be expressed as $V = Ah$. Thus, the equation can be modified as,

$$\frac{dh}{dt} = \frac{\tilde{c}u - \tilde{d}\sqrt{2gh}}{A}$$

this can be modified further as

$$\frac{dh}{dt} = -\frac{\tilde{d}\sqrt{2gh}}{|\tilde{a}h(k) + \tilde{b}|} + \frac{\tilde{c}}{|\tilde{a}h(k) + \tilde{b}|}u \qquad (3.14)$$

Putting the value of g=9.8 and converting into discrete forms

$$h(k+1) = h(k) + T\left(\frac{-\tilde{d}\sqrt{19.6h(k)}}{|\tilde{a}h(k) + \tilde{b}|} + \frac{\tilde{c}}{|\tilde{a}h(k) + \tilde{b}|}u(k)\right) \qquad (3.15)$$

In the simulation model we use $\tilde{a} = 0.01, \tilde{b} = 0.2, \tilde{c} = 1, \tilde{d} = 1$ and $T = 0.1$. We assume that the plant input saturates at ± 50 so that if the controller generates an input $u(k)$, then

$$u(k) = \begin{cases} 50 & if\ u(k) > 50 \\ u(k) & if -50 < u(k) < 50 \\ -50 & if\ u(k) < -50 \end{cases}$$

Since the liquid level cannot go negative, the system model is modified as

$$h(k+1) = max\left\{0.0001, h(k) + T\left(\frac{-\tilde{d}\sqrt{19.6h(k)}}{|\tilde{a}h(k) + \tilde{b}|} + \frac{\tilde{c}}{|\tilde{a}h(k) + \tilde{b}|}u(k)\right)\right\} \qquad (3.16)$$

CHAPTER-4

SIMULATION RESULTS

Rule base is the necessary part of the Fuzzy logic controller which uses linguistic variables. Whereas an algebraic variable takes numbers as values, a linguistic variable takes words or sentences as values. Three linguistic variables for each input have been used: Neg (negative), Pos (positive), Zer (zero). There are two inputs: *e* and *ce.* The Rules are as shown in the Table below.

Table 4.1: Fuzzy Rule Base

e \ ce	**Neg**	**Zer**	**Pos**
Neg	Neg	Neg	Zer
Zer	Neg	Zer	Pos
Pos	Zer	Pos	Pos

Simulations have been performed in MATLAB 7.7 and scratches have been developed using Intel(R) Core(TM) 2 Duo2.80GHz, 4GB of RAM. Both PID and FPD+I control have been conducted with a 0.1 second sampling period.

.

4.1 Inverted Pendulum

The system was controlled with PID controller with a fixed value for K_p, K_i & K_d as well with the fuzzy PD+I controller using different membership functions. The comparative analysis of PID controller and FPD+I controller for different membership function is shown.

Bell Membership function

The controlled output for inverted pendulum's angle by using PID and FPD+I controller is as shown in figure 4.1. It can be inferred from the figure 1 that angle of pendulum is smoothly varying in case of FPD+I controller while there are lots of variations in PID controller case.

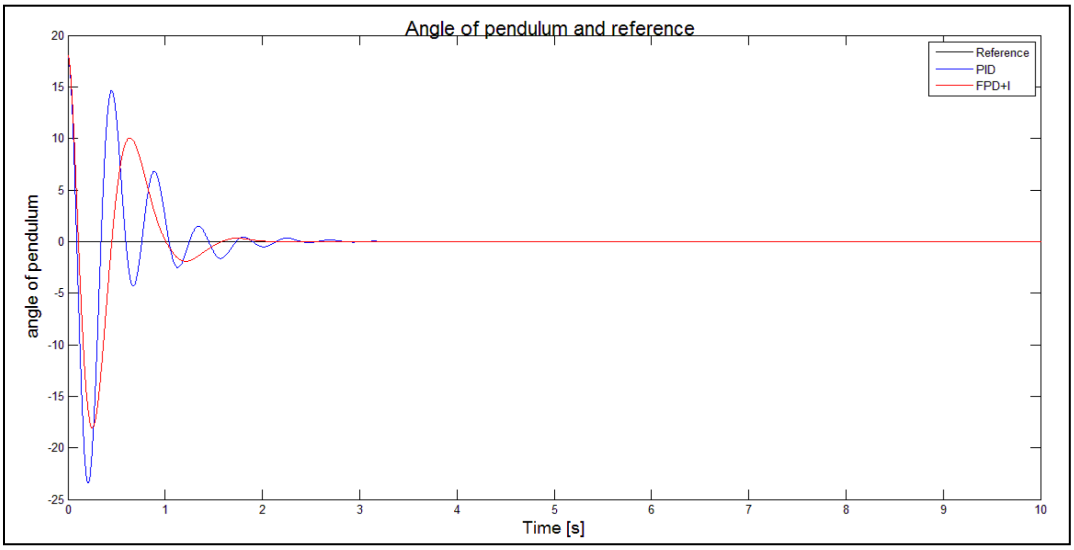


Figure 4.1. The controlled Angle of Pendulum for Bell PID and FPD+I (Bell)

Figure 4.2 shows the position of cart with respect to reference for both FPD+I and PID controller. It is evident from the figure that FPD+I settles to the desired reference much faster than the PID controller.

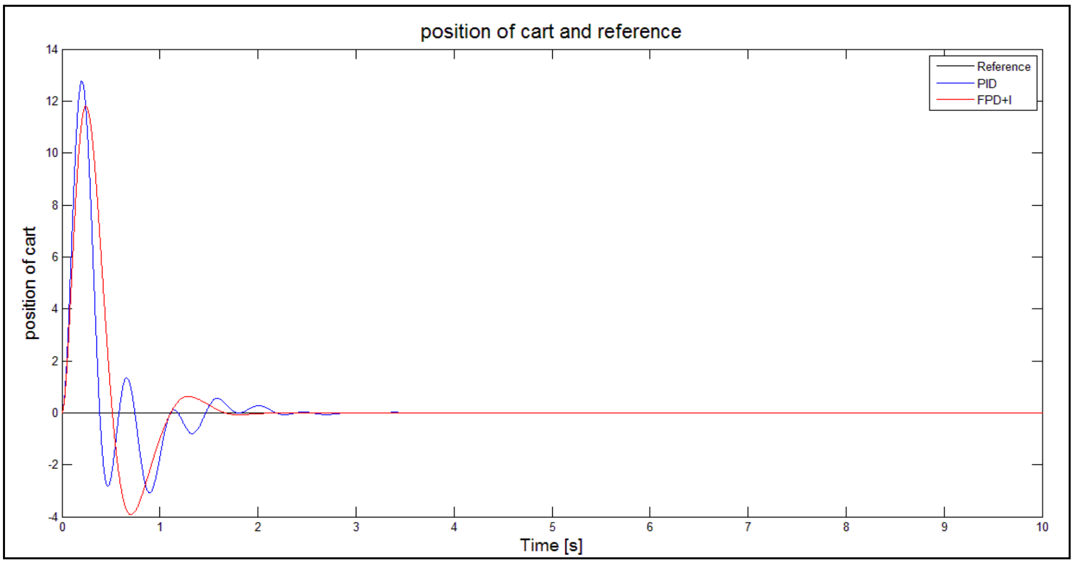


Figure 4.2. The controlled Position of Cart for PID and FPD+I (Bell)

The output of the controller for both FPD+I and PID Controllers is shown in figure 4.3. It can be noticed from the figure that the response for FPD+I controller is much smoother and having lesser overshoot than PID controller.

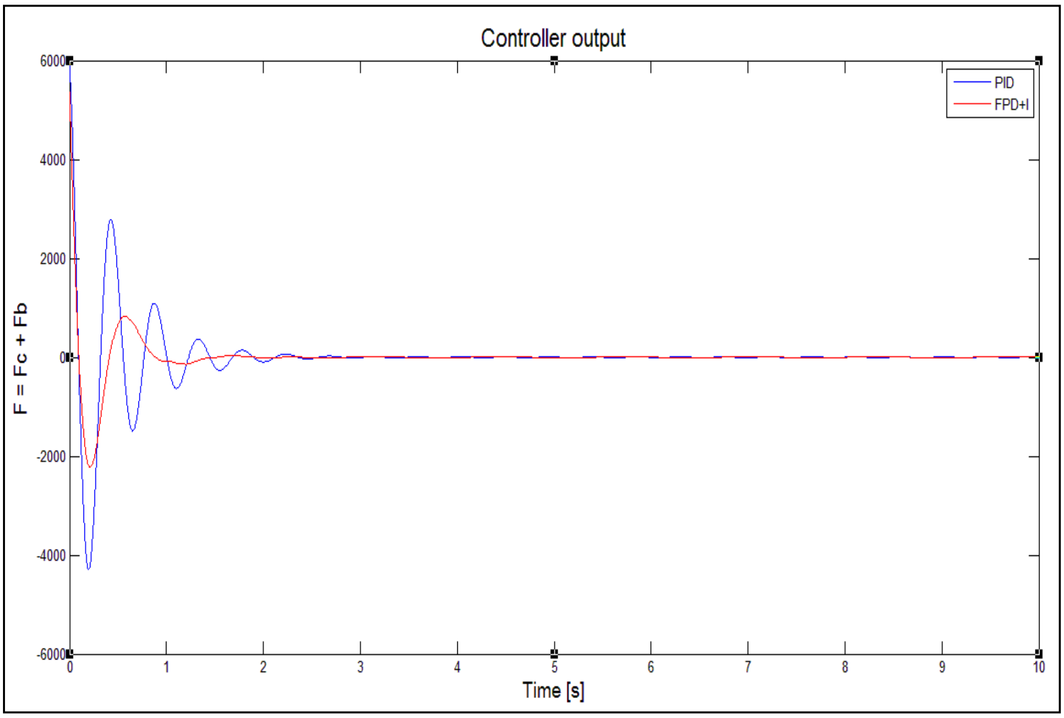


Figure 4.3. Controller output of Inverted Pendulum for PID and FPD+I (Bell)

The phase plot is shown in figure 4.4 which is obtained between CE and E which shows that the plot is terminating at zero and the system is stable.

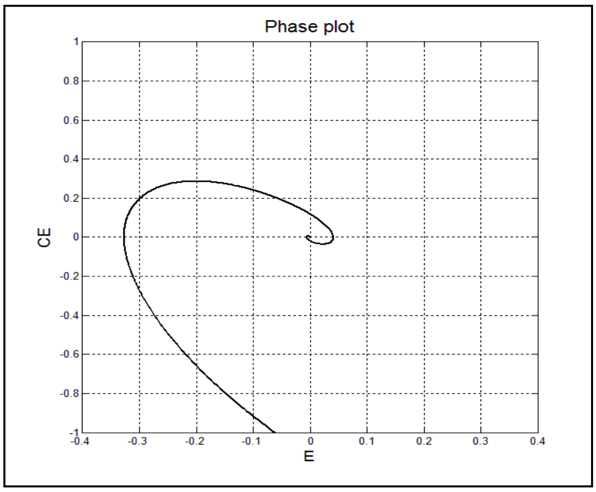


Figure 4.4. Phase Plot of Inverted Pendulum for Bell membership function

The Bell membership function is shown in figure 4.5 which was then used in the FPD+I controller to analyze the effects on the control of the system.

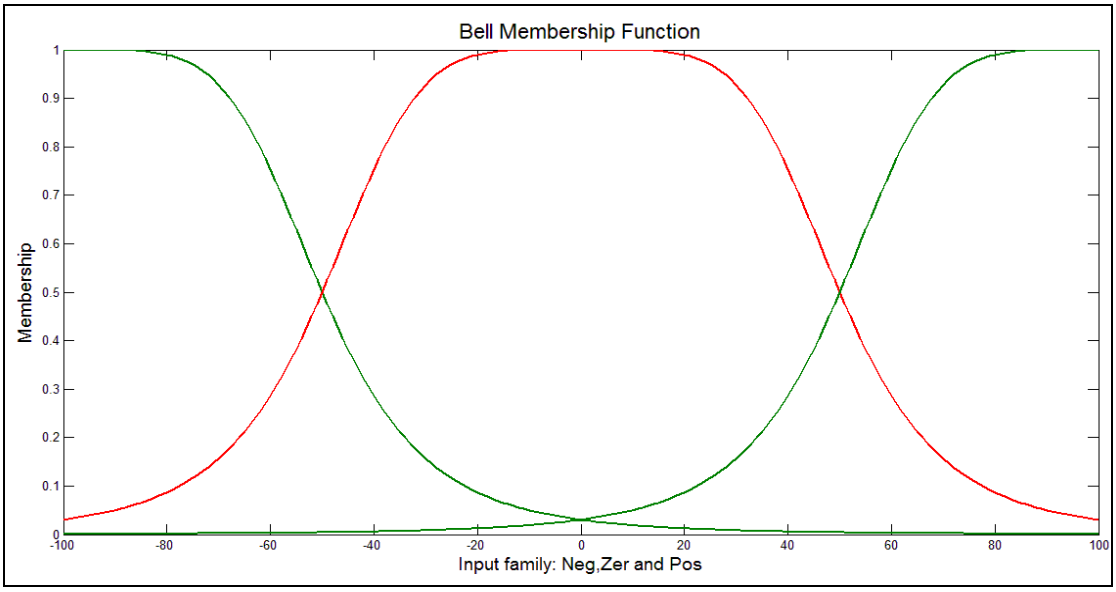


Figure 4.5. Bell Membership Function

Gaussian Membership function

The controlled output for inverted pendulum's angle by using the PID and FPD+I controller is as shown in figures 4.6. It is described by the figure that the response with the FPD+I controller is having a lesser overshoot than response with PID controller

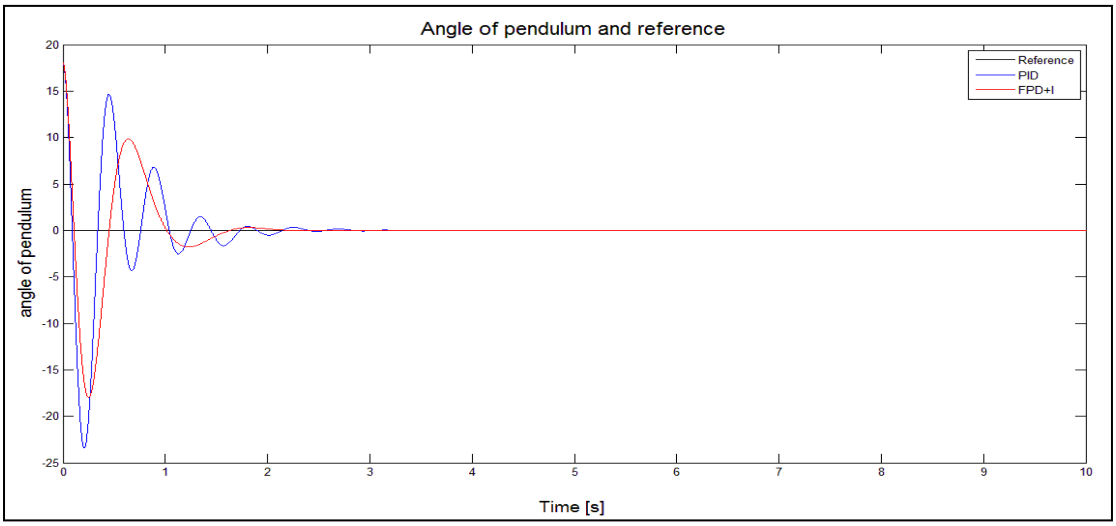


Figure 4.6. The controlled Angle of Pendulum for PID and FPD+I (Gaussian)

Figure 4.7 shows the position of cart with respect to reference for both FPD+I and PID controller. Figure shows that the response with FPD+I controller has less setting time as well as less variations than response with PID controller.

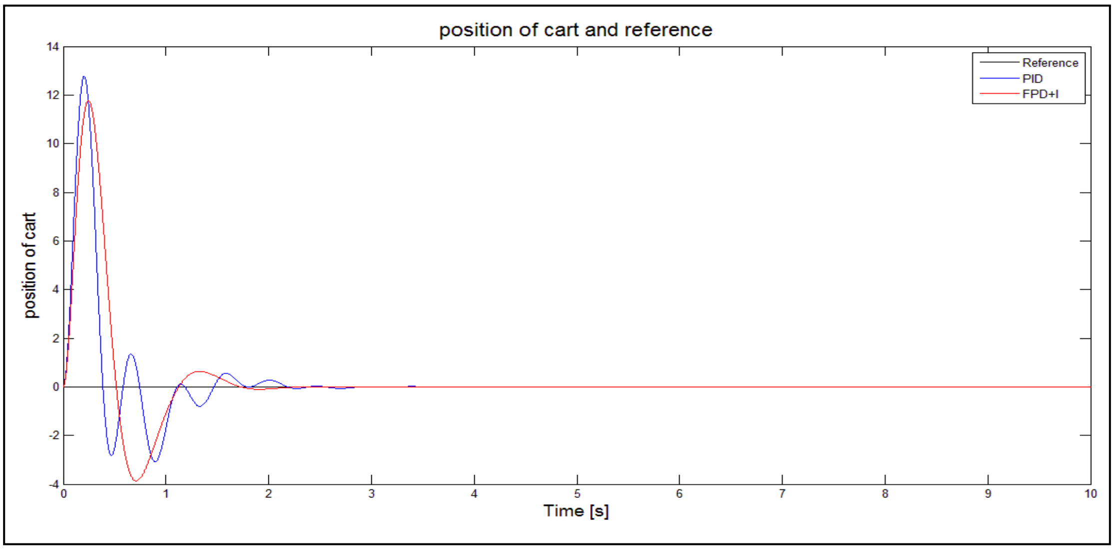


Figure 4.7. The controlled position of Cart for PID and FPD+I (Gaussian)

The output of controller is shown in figure 4.8 which is settling smoothly without much variation for FPD+I controller but in case of PID controller, it is having more oscillations and FPD+I controller is controlling better than PID controller.

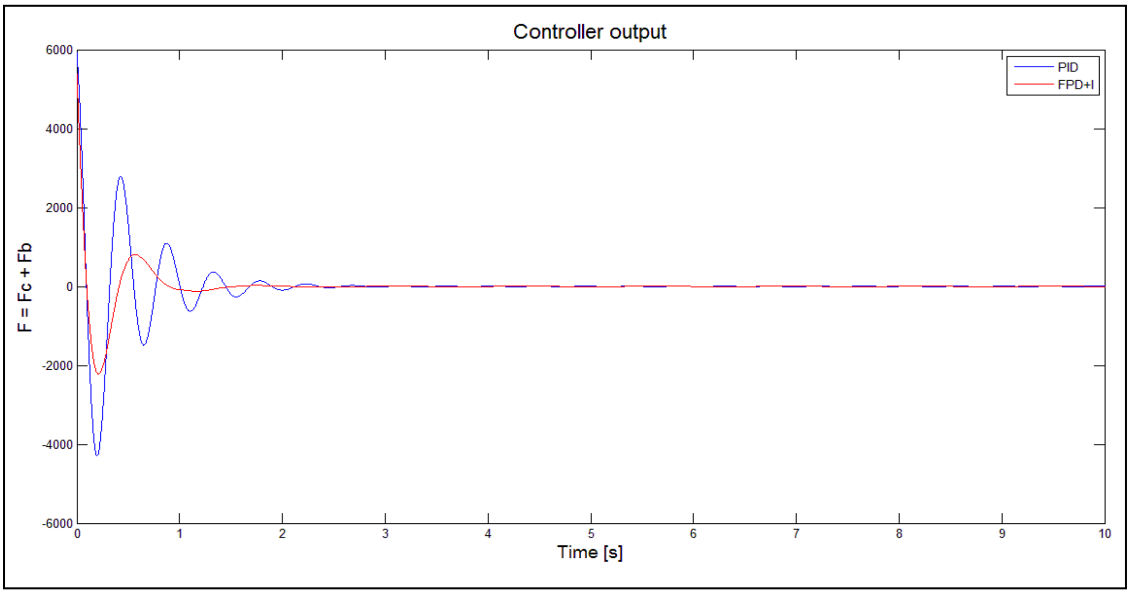


Figure 4.8. Controller output of Inverted Pendulum for PID and FPD+I (Gaussian)

The phase plot is shown in figure 4.9 which is obtained between CE and E which shows that phase plot curves towards origin indicating that the system is stable.

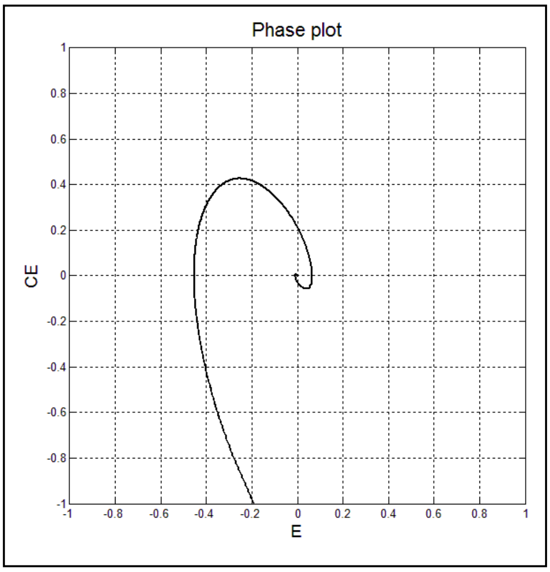


Figure 4.9. Phase Plot of Inverted Pendulum for Gaussian membership function

The Bell membership function is shown in figure 4.10 which was then used in the FPD+I controller to analyze the effects on the control of the system.

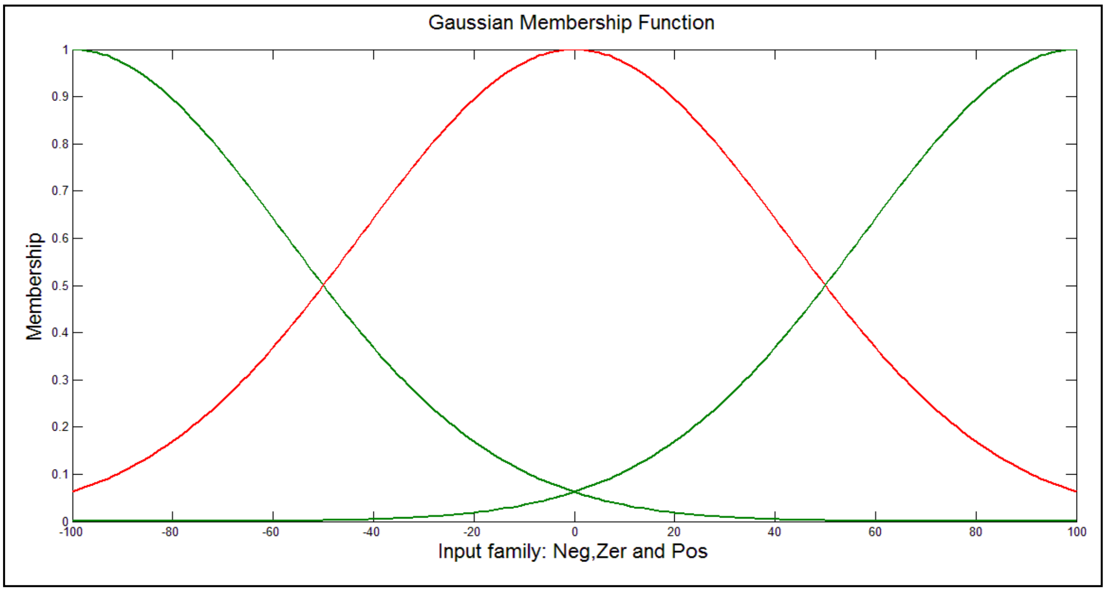


Figure 4.10. Gaussian membership function

Pi Membership function

The output of the system obtained by using PID controller and FPD+I controller with Pi membership function with respect to the reference input is shown in figure 4.11. First peak overshoot of angle of pendulum using FPD+I comes around -15° while in case of PID controller, it comes nearly -24° which shows that FPD+I controller performs better than PID controller.

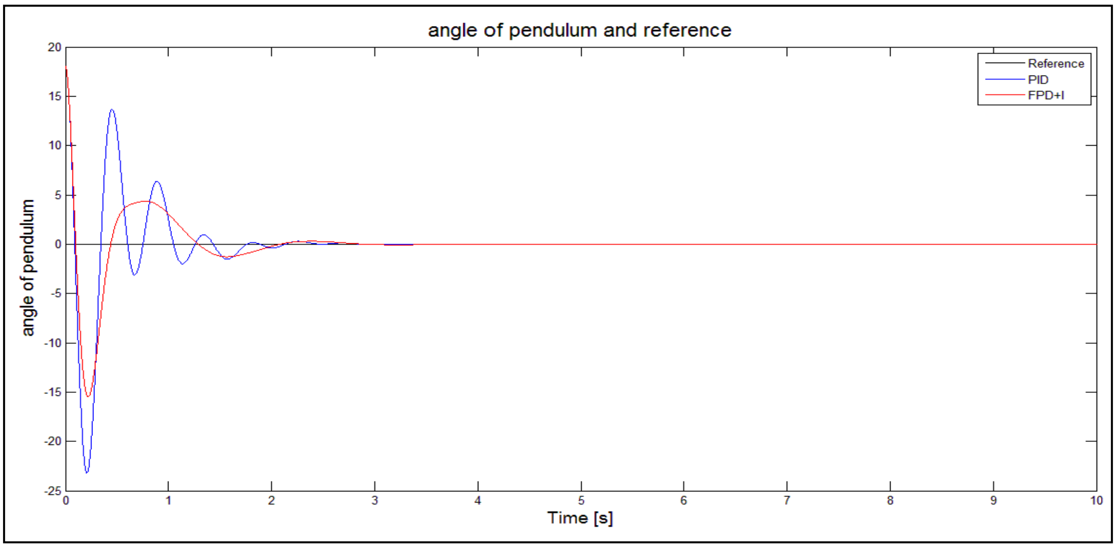


Figure 4.11. The controlled Angle of Pendulum for PID and FPD+I (Pi)

Figure 4.12 shows comparison of FPD+I and PID controller for position of cart with respect to desired reference. Number of overshoots are more in FPD+I controller than PID controller.

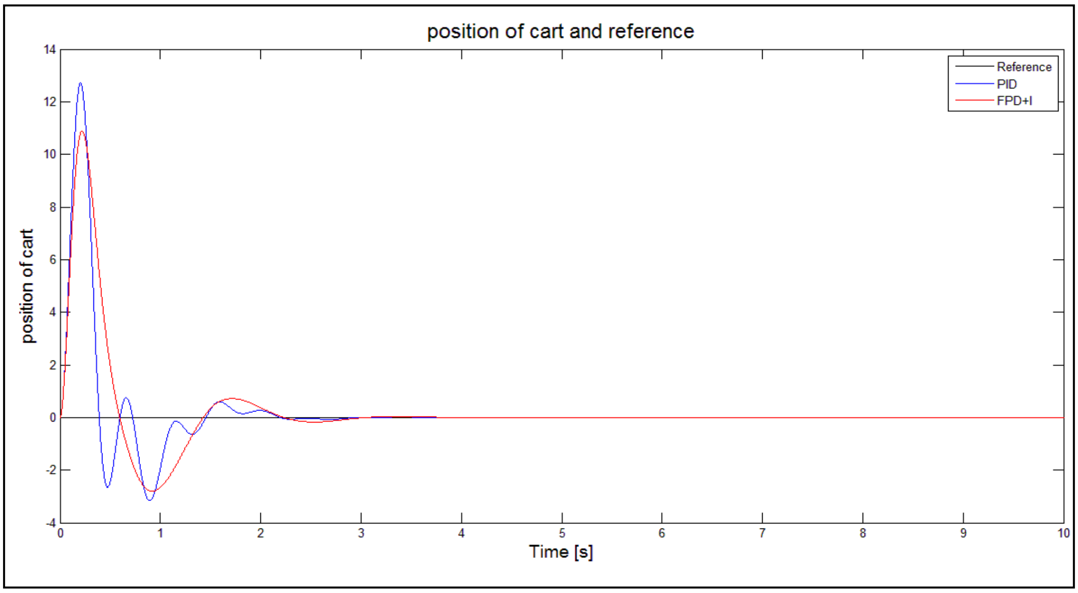


Figure 4.12. The controlled Position of Cart for PID and FPD+I (Pi)

The output of controller is shown in figure 4.13. The response for FPD+I controller shows a placid behavior with minimal disturbances when compared with its counterpart i.e. the response for the PID controller which has more heavy waves of disturbances.

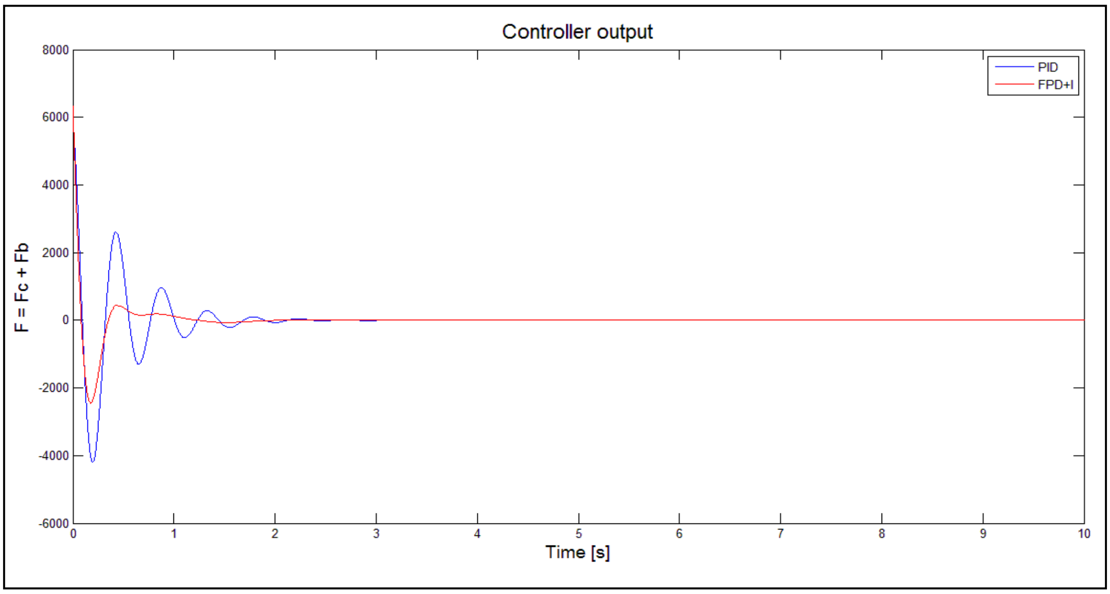


Figure 4.13. Controller output of Inverted Pendulum for PID and FPD+I (Pi)

The phase plot is shown in figure 4.14 which is obtained between CE and E which shows that the pi membership function used in the FPD+I controller successfully controls the system as the phase plot moves towards the origin and shows that system is stable.

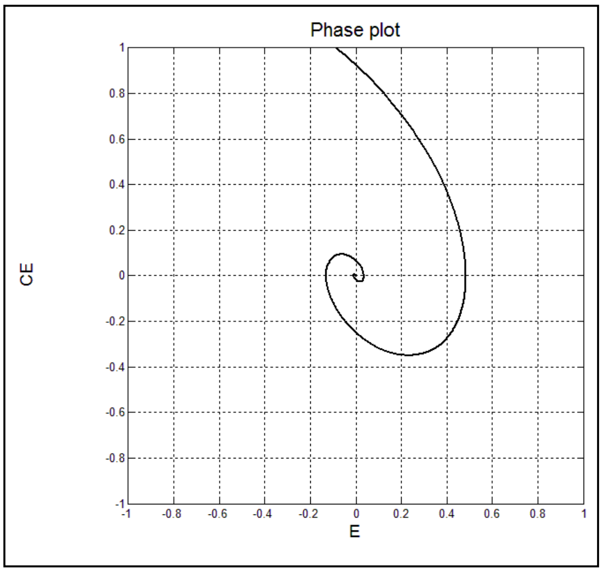


Figure 4.14. Phase Plot for Pi Membership function

The Pi membership function is shown in figure 4.15 which was then used in the FPD+I controller to analyze the effects on the control of the system.

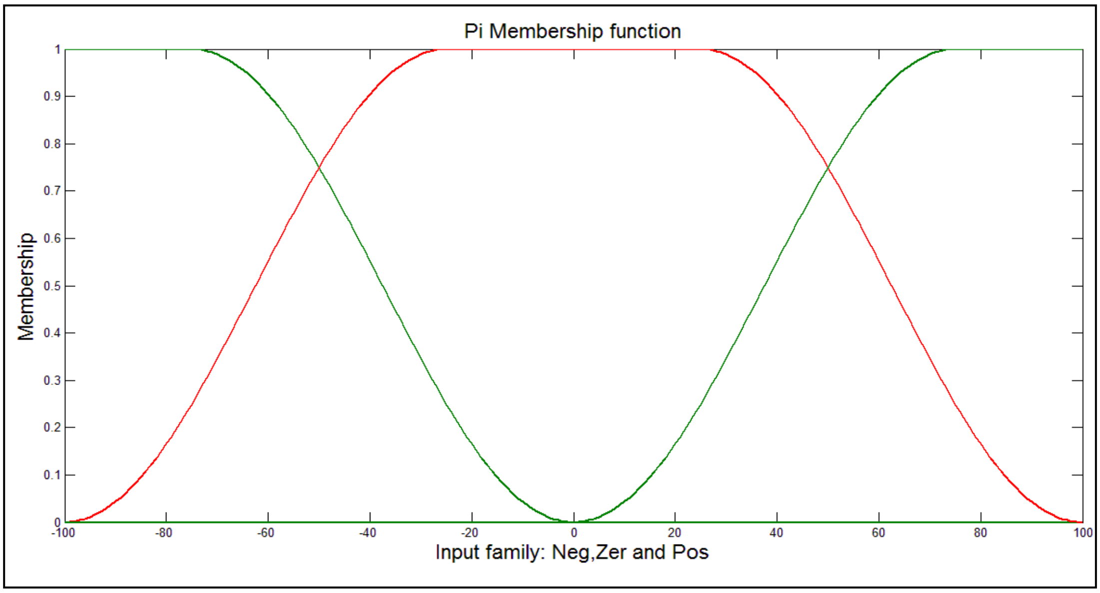


Figure 4.15. Pi Membership function

Psigmoid Membership function

The controlled output for inverted pendulum's angle by using the PID controller and FPD+I controller is as shown in figure 4.16. It can be seen from the figure that in the case of PID controller, the controller output has larger variations which indicates that it attempts to bring the output close to the desired reference trajectory.

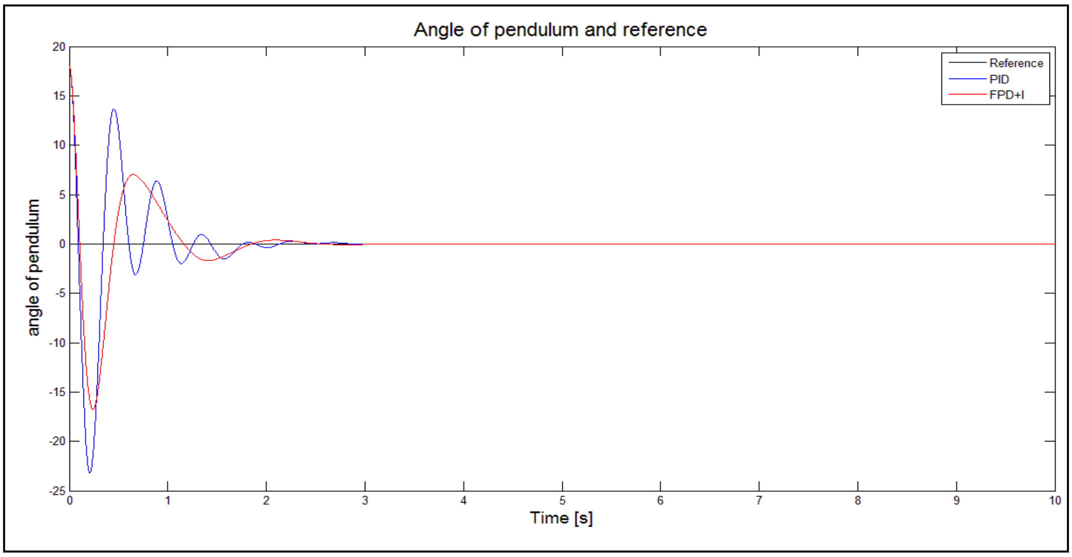


Figure 4.16. The controlled Angle of Pendulum for PID and FPD+I (Psigmoid)

Position of the cart and reference for the Psigmoid membership function is shown below in figure 4.17 for both PID and FPD+I controller. It can be seen from the figure that in case of PID controller, the position of the cart gets more displaced from the origin when compared with the displacement in the case of FPD+I controller.

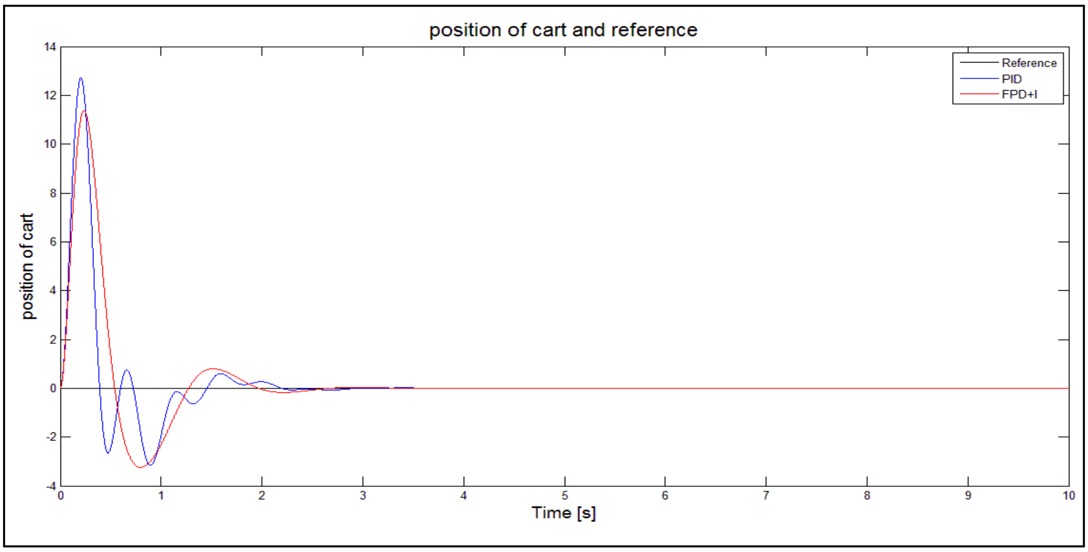


Figure 4.17. The controlled Angle of Pendulum for PID and FPD+I (Psigmoid)

The output of the controller for both PID and FPD+I with Psigmoid membership function for inverted pendulum system is shown below in figure 4.18. It shows that FPD+I controller is able to control the desired output in a very less time when compared with the PID controller.

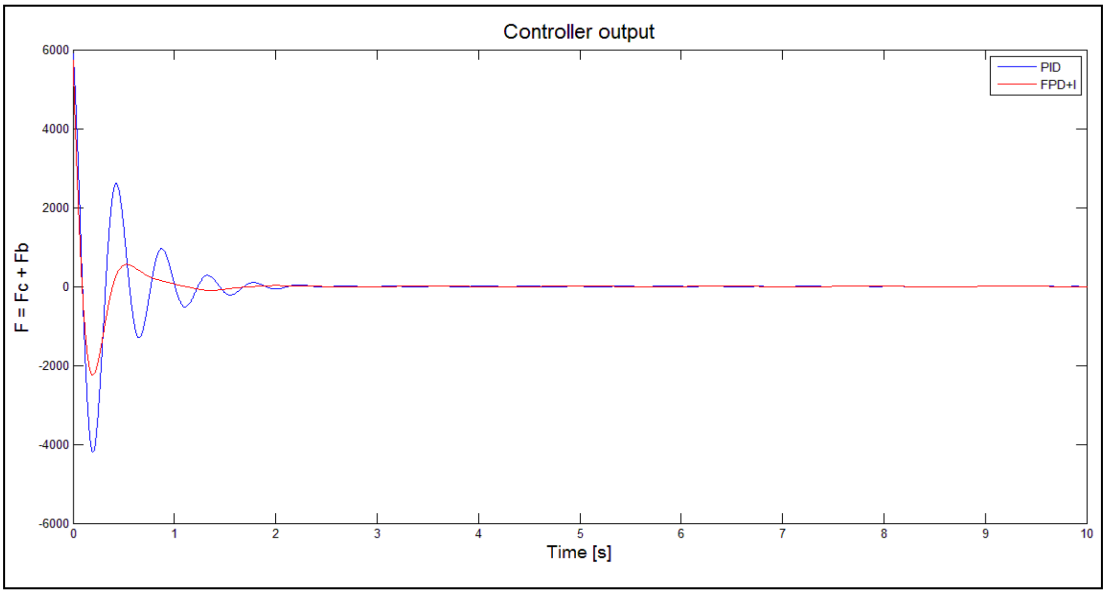


Figure 4.18. Controller output of Inverted Pendulum for PID and FPD+I (Psigmoid)

The phase plot is shown in figure 4.19 for FPD+I controller with Psigmoid membership function which is obtained between CE and E. It can be seen from the plot that the system is successfully controlled with the Psigmoid membership function and the system is stable as the plot moves to the origin smoothly.

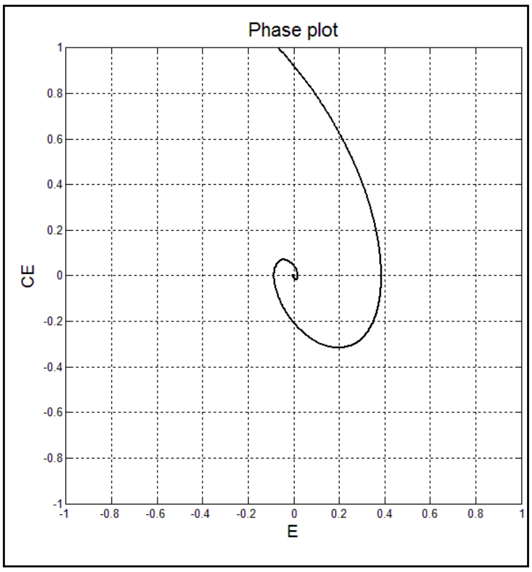


Figure 4.19. Phase Plot for Psigmoid membership function

The Psigmoid membership function is shown in figure 4.20 which was then used in the FPD+I controller to analyze the effects on the control of the system.

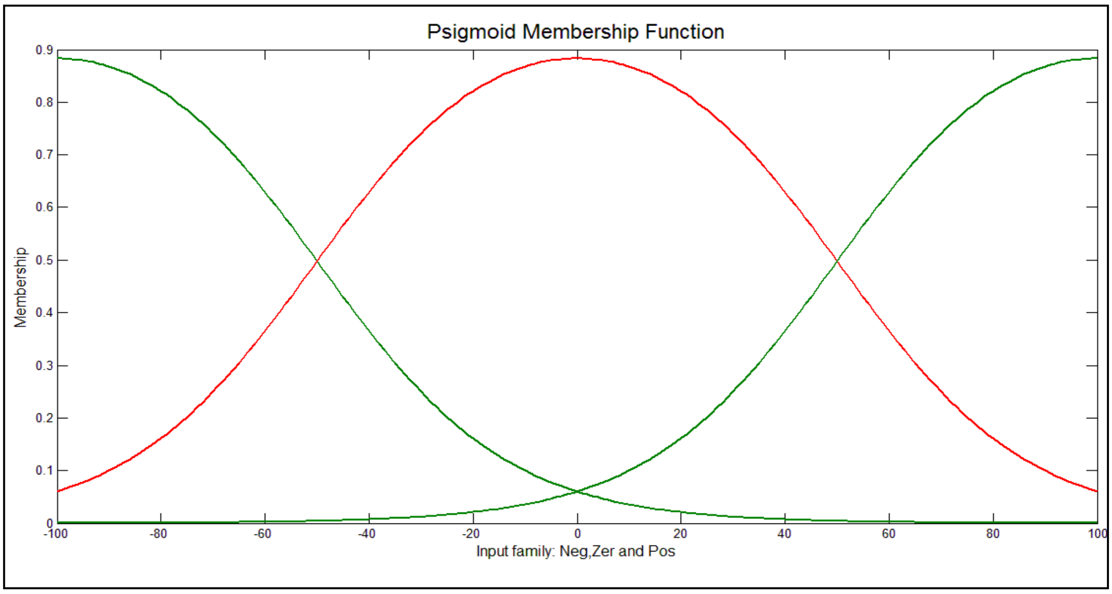


Figure 4.20. Psigmoid Membership function

4.2 Surge Tank Level

The Nonlinear Surge tank system in which the liquid level is to be controlled. The system can be modeled by the equation:

$$h(k+1) = h(k) + T\left(\frac{-\tilde{d}\sqrt{19.6h(k)}}{|\tilde{a}h(k)+\tilde{b}|} + \frac{\tilde{c}}{|\tilde{a}h(k)+\tilde{b}|}u(k)\right)$$

The liquid level is controlled using FPD+I controller with various membership function and PID controller. The comparison between PID and FPD+I is shown.

Bell Membership function

The output of the system obtained by using PID controller and FPD+I controller with Bell membership function with respect to the reference input is shown in figure 4.21. It can be seen from the figure that in the case of PID controller a large no. of subsequent overshoots can be seen which are prominent later as well while in the case FPD+I controller, the subsequent peaks die out.

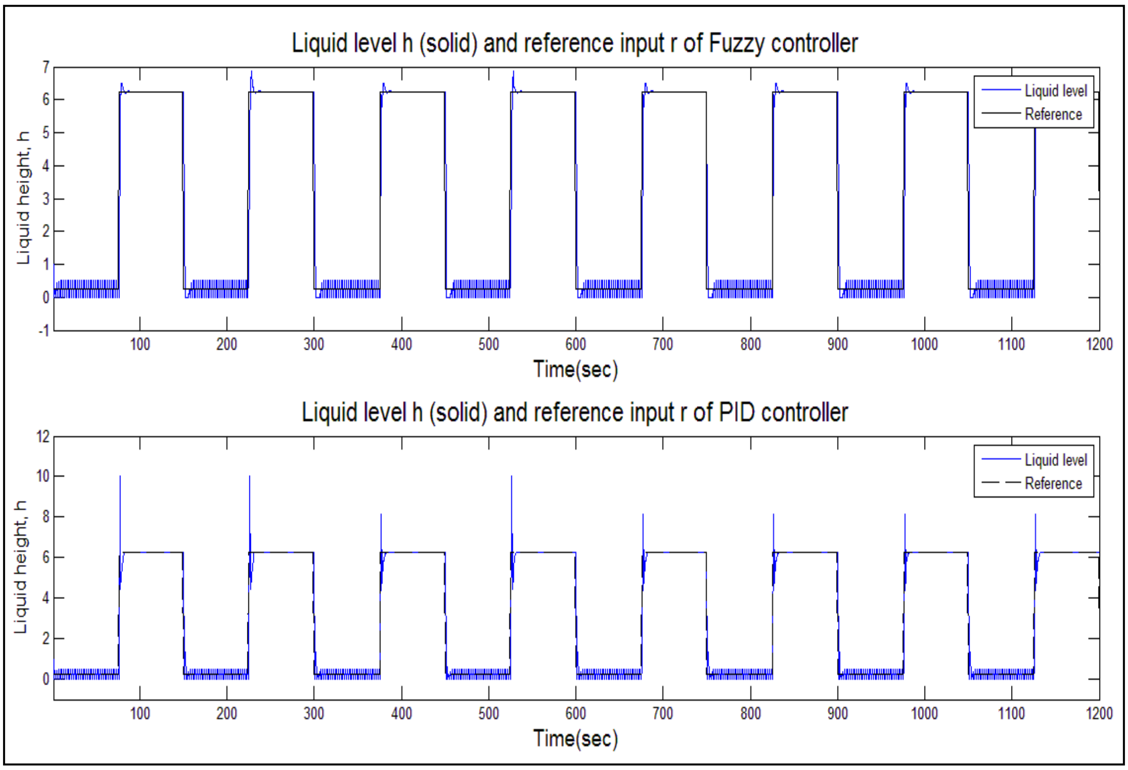


Figure 4.21. The Controlled level of the liquid using PID and FPD+I (Bell)

The output of PID as well as FPD+I controller is shown in figure 4.22. As we can see in figure, the PID controller is providing the controlling action to the height of surge tank level faster than the FPD+I controller.

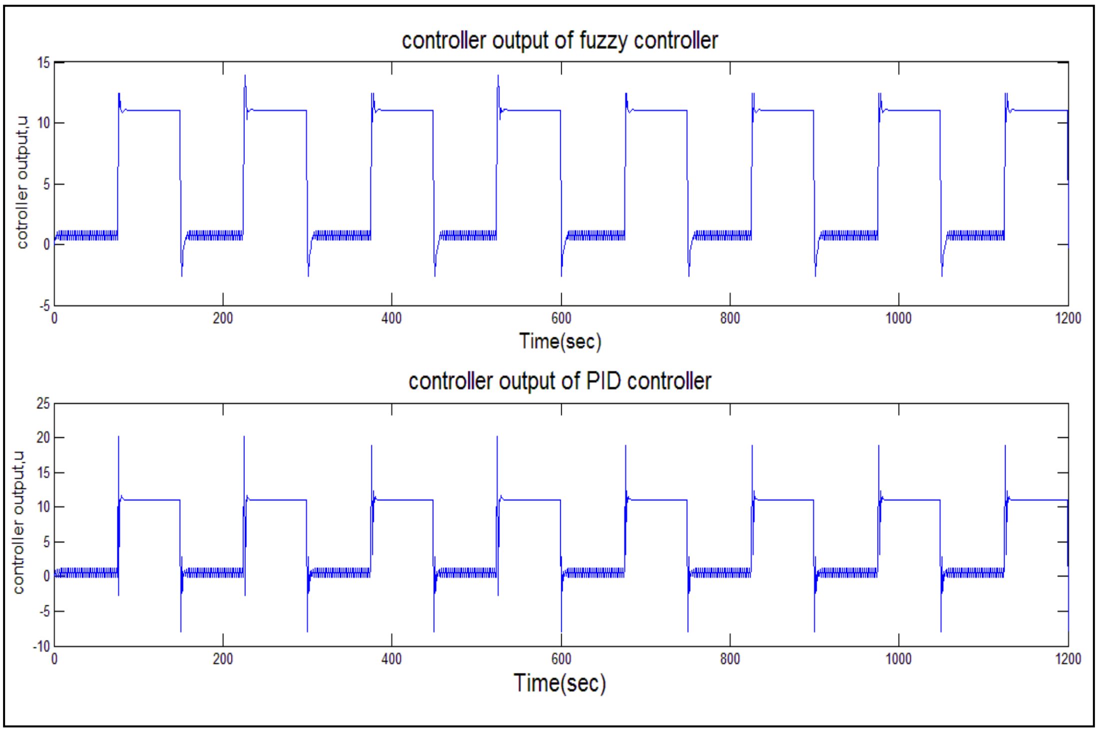


Figure 4.22. controller output (Bell)

The phase plot of FPD+I controller for bell membership function is shown in figure 4.23 which is obtained between CE and E. It is evident from the below figure that phase trajectory is converging to zero which indicates that the system is stable

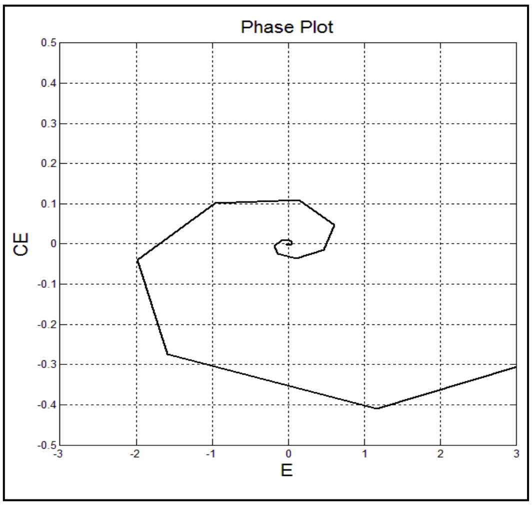


Figure 4.23. Phase Plot for Bell membership function

The Bell membership function is shown in figure 4.24 which was then used in the FPD+I controller to analyze the effects on the control of the system.

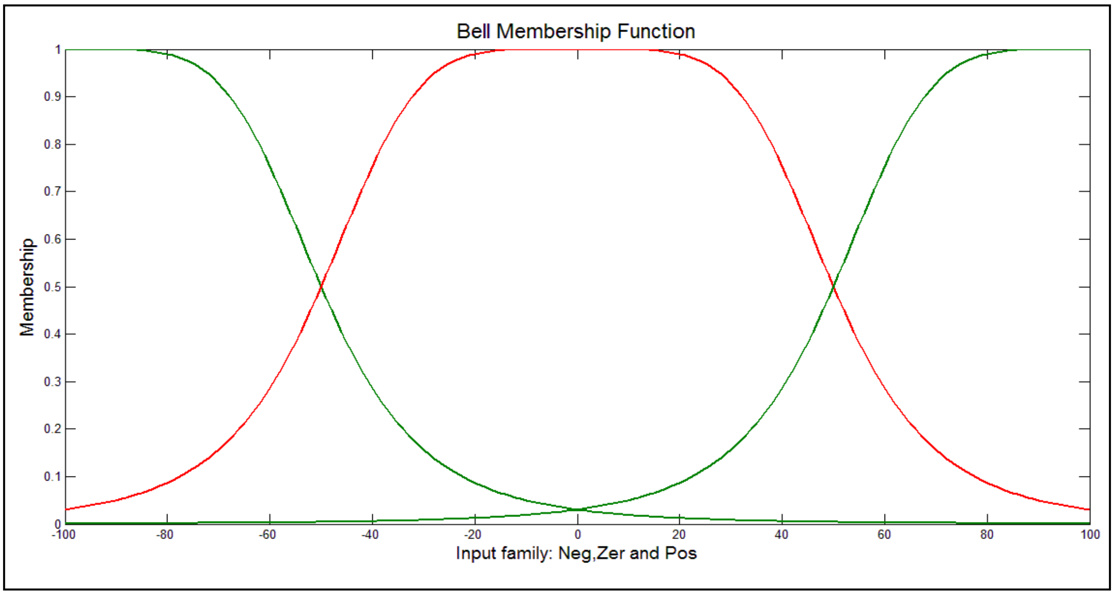


Figure 4.24. Bell Membership function

Gaussian Membership function

The output of the system obtained by using PID controller and FPD+I controller with Gaussian membership function with respect to the reference input is shown in figure 4.25. It can be observed from the figure that the peak overshoot of the liquid level using PID controller is higher than the FPD+I controller.

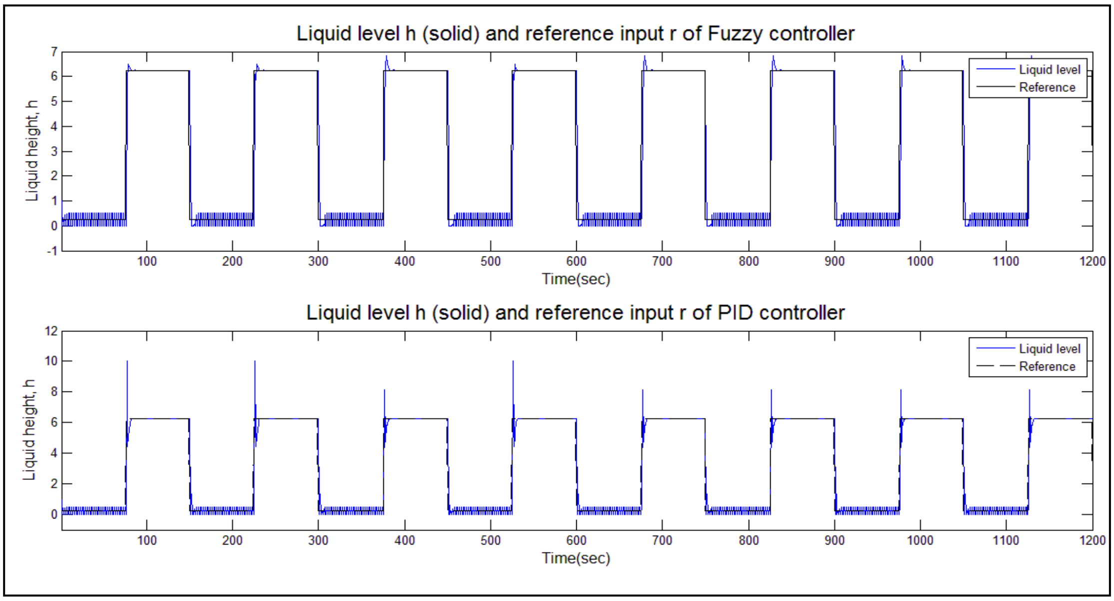


Figure 4.25. The Controlled level of the liquid using PID and FPD+I (Gaussian)

The output of FPD+I controller with Gaussian membership function and PID controller is shown in figure 4.26 which tells that the controller action provided by the FPD+I controller with Gaussian membership function is dominant over PID controller.

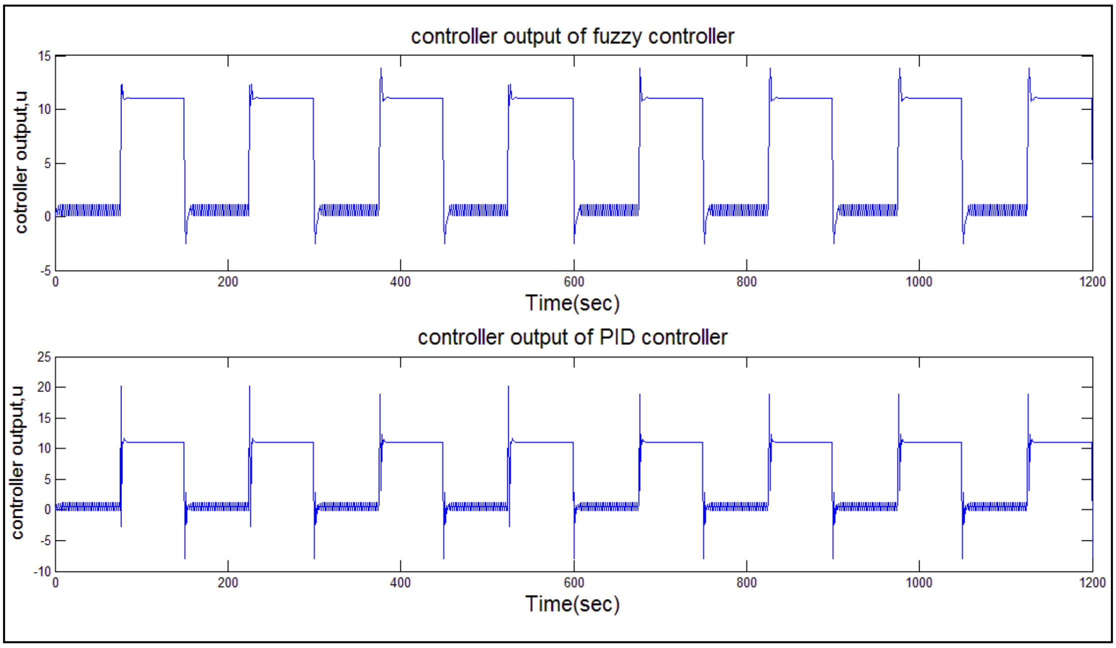


Figure 4.26. Controller output (Gaussian)

The phase plot of FPD+I controller for Gaussian membership function is shown in figure 4.27 which is obtained between CE and E. It has been seen from the figure that the phase trajectory ends in the centre (origin) which shows that output is on the reference and the system is stable.

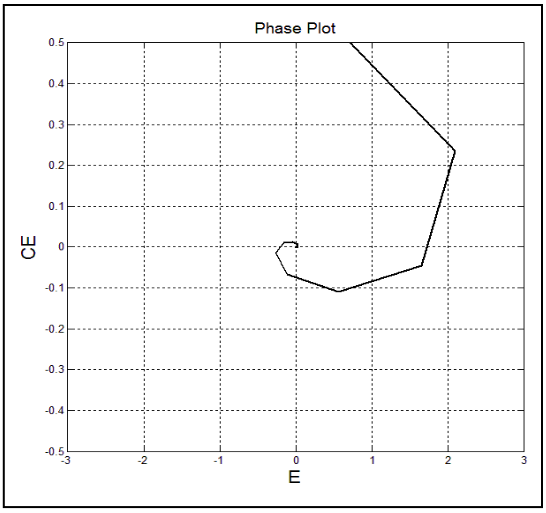


Figure 4.27. Phase plot for Gaussian Membership function

The Gaussian membership function is shown in figure 4.28 which was then used in the FPD+I controller to analyze the effects on the control of the system.

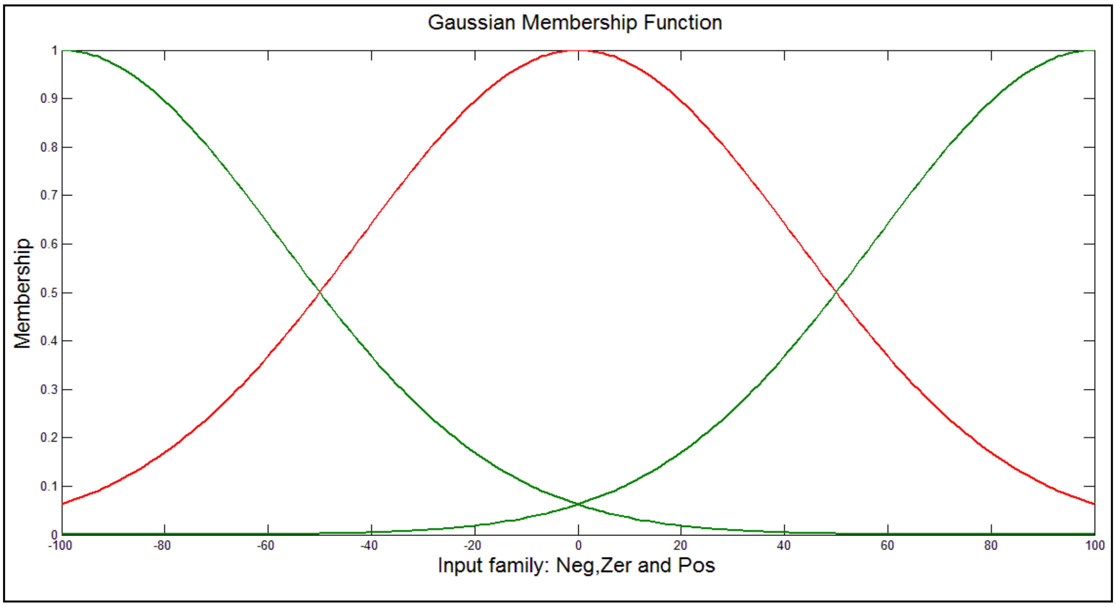


Figure 4.28. Gaussian Membership function

Pi Membership function

The output of the system obtained by using FPD+I controller with Pi membership function and PID controller with respect to the reference input is shown in figure 4.29. Liquid level using PID controller is having a large number of peak overshoots while in case of FPD+I controller, there are a negligible peaks which is more advantageous.

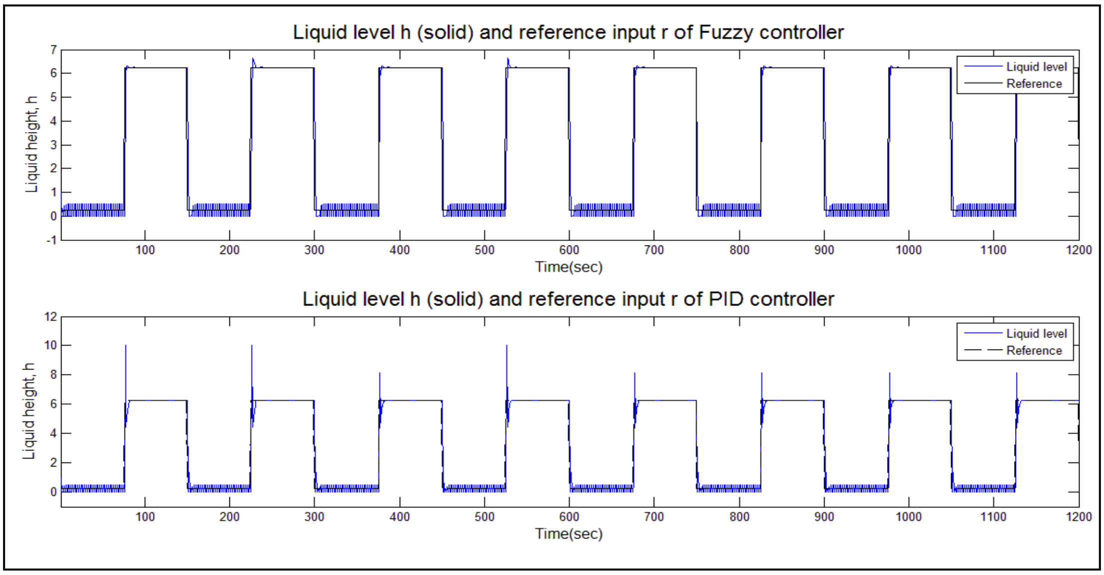


Figure 4.29: The Controlled level of the liquid using PID and FPD+I (Pi)

The output of FPD+I controller with Pi membership function and PID controller is shown in figure 4.30. In comparison to PID controller, the controlling action of FPD+I controller is faster with less peaks and this comparison can be seen in the below figure.

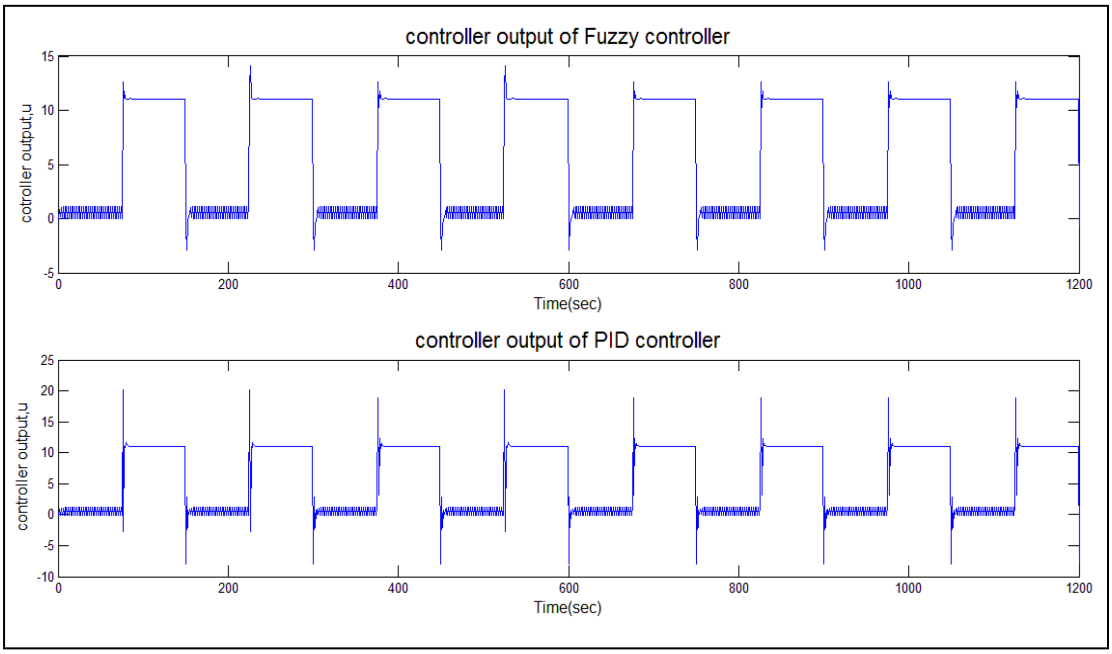


Figure 4.30 Controller output (Pi)

The phase plot is shown in figure 4.31 which is obtained between CE and E which shows that the system is stable because the phase plot is converging to the equilibrium point which is origin.

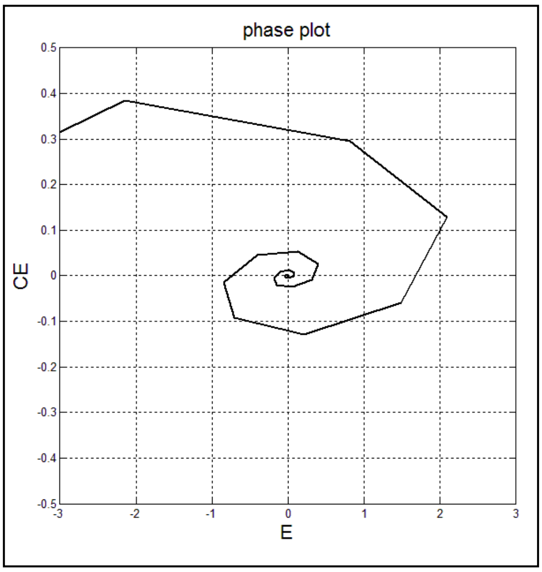


Figure 4.31. Phase Plot for Pi membership function

The Pi membership function is shown in figure 4.32 which was then used in the FPD+I controller to analyze the effects on the control of the system.

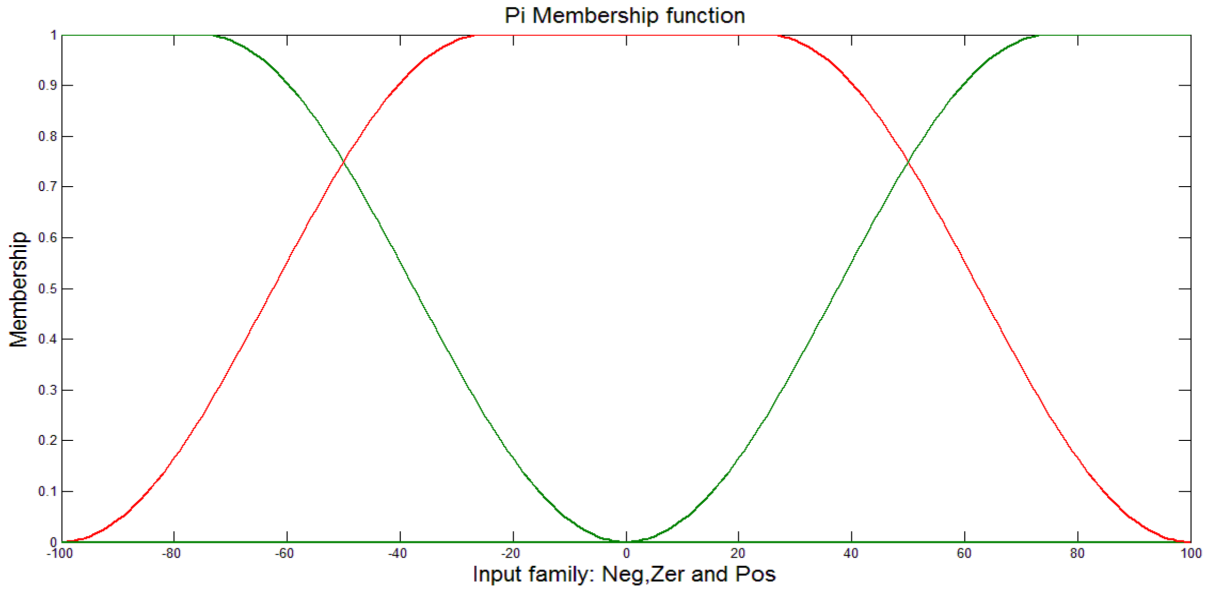


Figure 4.32. Pi Membership function

Psigmoid Membership Function

The output of the system obtained by using PID controller and FPD+I controller with Psigmoid membership function with respect to the reference input is shown in figure 4.33.

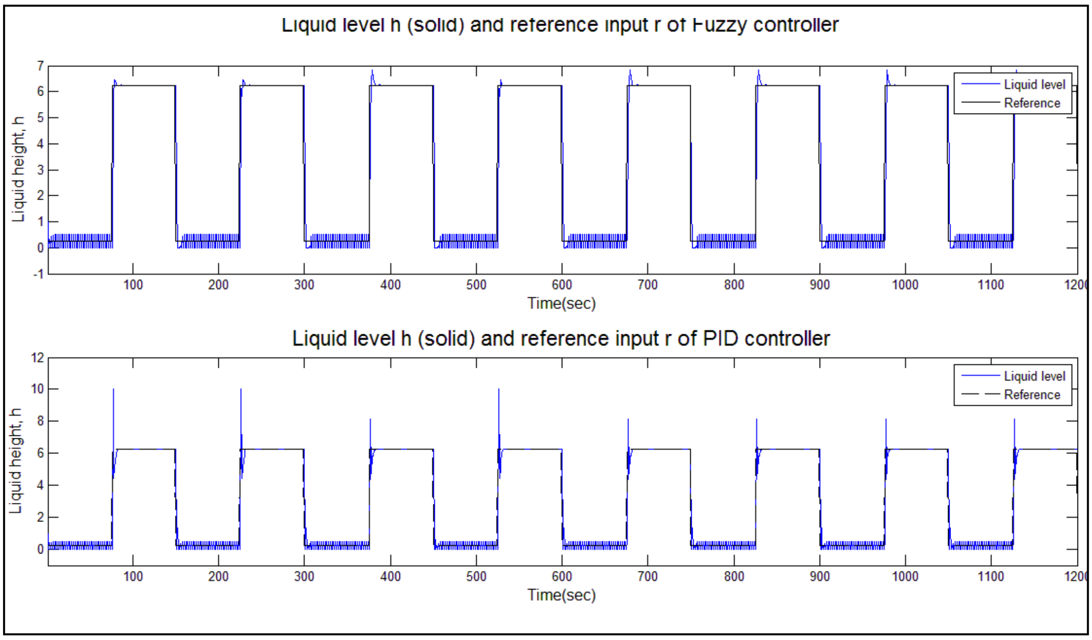


Figure 4.33. The Controlled level of the liquid using PID and FPD+I (Psigmoid)

The output of controller is shown in figure 4.34.

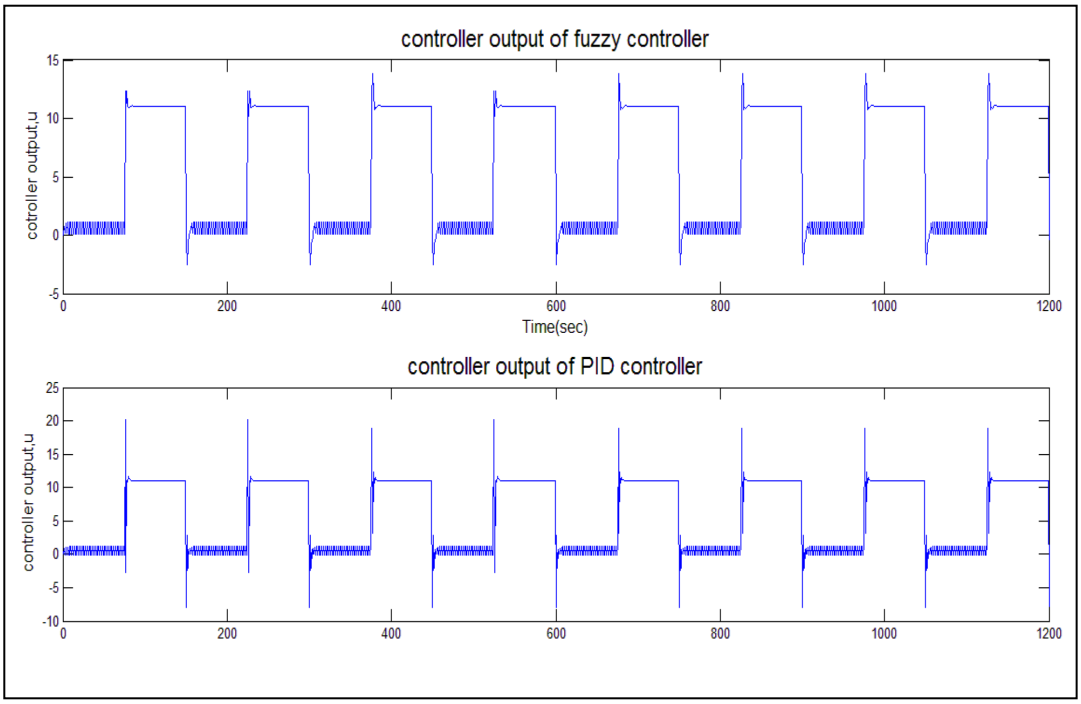


Figure 4.34. Controller output (Psigmoid)

The phase plot is shown in figure 4.35 which is obtained between CE and E which shows that the system is stable.

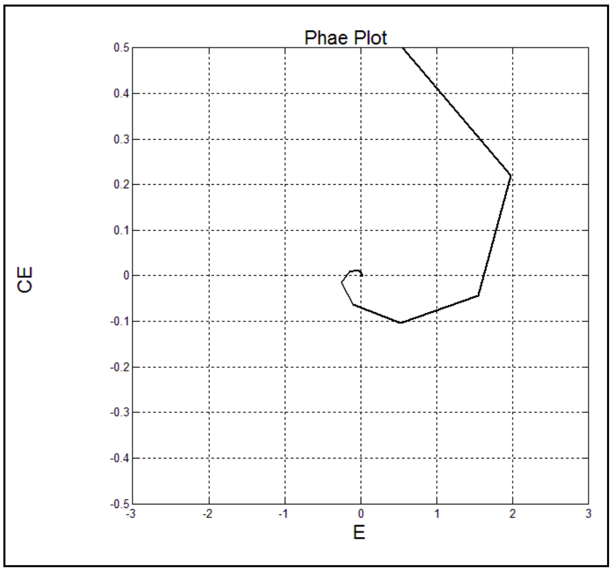


Figure 4.35. Phase Plot for Psigmoid Membeship function

The Psigmoid membership function is shown in figure 4.36 which was then used in the FPD+I controller to analyze the effects on the control of the system.

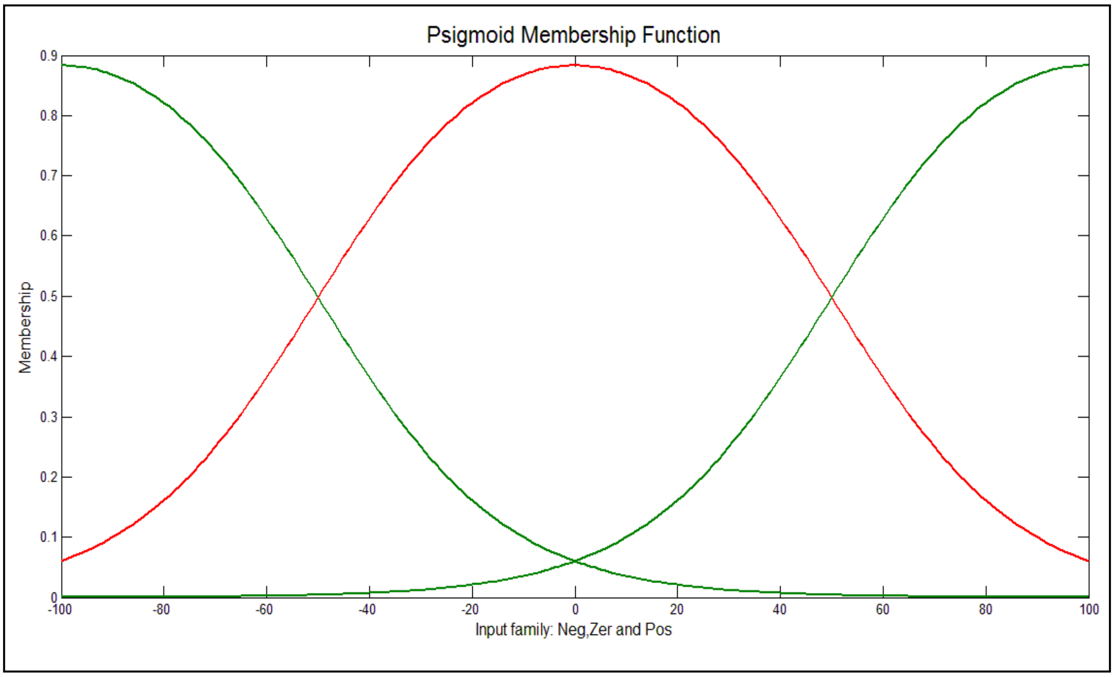


Figure 4.36. Psigmoid membership function

CHAPTER-5

CONCLUSION AND FURTHER SCOPE OF WORK

The results show that the response of Fuzzy PD+I controller with all membership functions used above gives low overshoot than the PID Controller. Four different membership functions with Fuzzy PD+I controller gives better response than conventional PID controller. The performance tells that large number of processes can be efficiently controlled by using Fuzzy Controller rather than Conventional PID Controller.

On the effect of membership functions on the system, it can be commented that performance using Gaussian and Psigmoid membership functions is almost same for surge tank level system, but Fuzzy PD+I controller using Pi and Bell membership functions gives better output than the output obtained using Gaussian and Psigmoid membership functions for the same. But in the case of inverted pendulum results using Pi and Psigmoid give better response than Gaussian and Bell membership functions. In comparison to all performances Fuzzy Controller using Pi membership function gives best output. The FPD+I helps in determining the stability of the system which is evident from the phase plots above. The plots show that the trajectory converges to the origin which shows the system is stable.

The results pertaining to both PID controller and FPD+I controller with different membership functions are shown for both the systems. It is evident from the results that FPD+I performs better than the PID controller. Effect of four different membership functions was studied and it is found that Fuzzy controller using Pi membership function gives better result. Phase plot between E & CE shows that the system is stable.

Further Scope:

A lot of research has been carried out in the region pertaining to fuzzy nonlinear control. Many different types of fuzzy membership function can be customized. Fuzzy logic is an innovative technology to design non-linear control problems and to design solutions for multi-parameter problems. Since it does not use mathematical model and instead uses man experience and experimental results. Human brains work with fuzzy patterns with ease, but computers fail to do so. Fuzzy logic tries to change that.

Different customized membership function can be used to analyze the behavior of the fuzzy control for different class of nonlinear systems. Analysis can also be done by increasing the number of rules for each customized membership function.

REFERENCES

[1] C. J. Harris, C. G. Moore and M. Brown. Intelligent Control, Aspects of Fuzzy Logic and :Veural !Vets, World Scientific: London, 1993.

[2] H. A. Malki, H. Li and G. Chen, "New Design and Stability Analysis of Fuzzy Proportional-Derivative Control Systems". IEEE Trans. on Fuzzy Systems. Vol. 2. 1994, pp. 245-254.

[3] C. W. de Silva. Intelligent Control-Fuq Logic Applications, CRC Press, New York, 1995.

[4] C-L. Chen and F-C. Kuo. "Design and Analysis of a Fuzzy Logic Controller", Int. J. Systems Sci., Vol. 26, 1995, pp. 1223-1248.

[5] G.K.I.Mann, B.-G.Hu and R.G.Gosine, "Analysis and Performance Evaluation of Linear-Like Fuzzy PI and PID Controllers," Procc. Of 6th International Conference on Fuzzy Systems, IEEE, 1997, pp. 383-390.

[6] Parnichkun, M. and C. Ngaecharoenkul., "Hybrid of fuzzy and PID in kinematics of a pneumatic system," Proceeding of the 26th Annual Conference of the IEEE Industrial Electronics Society, Japan, 2000, pp. 1485-1490.

[7] K.M. Passino,"Adaptive Control",in Biomimicry for Optimization, Control, and Automation, 1st ed.,london, Springer-verlag,2005, pp. 547-600.

[8] W. Chatrattanawuth, N. Suksariwattanagul, T. Benjanarasuth & J. Ngamwiwit, "Fuzzy I-PD Controller for Level control", SICE-ICASE International Joint Conference 2006, pp. 5649-5652.

[9] Asim Khan, N. Rapal, "Fuzzy PID Controller: Design, Tuning and Comparison with Conventional PID Controller," International Conference on Engineering of Intelligent Systems, IEEE, 2006, pp.1-6.

[10] O. Ziwei, S.Michael and W. Kexin, "The Experiment "Ball-in-tube" with Fuzzy PID Controller Based on Dspace," International Conference on Systems, Man and Cybernetics, IEEE, 2007, pp. 877-881.

[11] Qi Jianling, D. Zhenjie and Li Yezi, "Design of Fuzzy PID Controller and Application in Glass Furnace," The Eighth International Conference on Electronic Measurement and Instruments, ICEMI'2007, pp. 224-227.

[12] Jan Jantzen, "Foundations of Fuzzy Control", John Wiley & Sons, USA, 2007.

[13] L. Behara and Indrani Kar, "Intelligent Systems and Control-Principles and Applications", Oxford University Press, UK, 2009.

[14] F. CHEN and J. YANG, "Fuzzy PID Controller used in Yaw System of Wind Turbine," 3rd International Conference on Power Electronics Systems and Applications, 2009, pp. 1-4.

[15] R. Kannan, P. Ananthachristu Raj & P. Poongodi, "Design of Fuzzy Immune PID Controller for Liquid Level Control System", 2nd International Conference on Computer and Network Technology, IEEE, 2010, pp. 566-570.

[16] A.F.Amer, E.A.Sallan and W.M.Elawady, "Fuzzy Pre-compensated Fuzzy Self-tunig Fuzzy PID controller of 3 DOF Planar Robot Manipulators," International Conference on Advanced Intelligent Machatronics, IEEE/ASME, 2010, pp. 599-604.

[17] P.Pratumsuwan, S. Thongchai, and S. Tansriwong., "A Hybrid of Fuzzy and Proportional-Integral-Derivative Controller for Electro-Hydraulic Position Servo System", Energy Research Journal, vol. 1,issue 2, 2010, pp. 62–67.

[18] K. Sinthipsomboon, W. Pongaen and P. Pratumsuwan, "A Hybrid of Fuzzy and Fuzzy Self Tuning PID Controller for Servo Electro Hydraulic System," Conference on Industrial Electronics and Applications, IEEE, 2011, pp. 220-225.

[19] A.K. Yadav, P.Gaur, A.P. Mittal & M. Anzar, "Comparative Analysis of Various Control Techniques for Inverted Pendulum", India International Conference on Power Electronics (IICPE), IEEE, 2011, pp. 1-6.

[20] Dr. M P Nandakumar and R. Nalakath, "A Non-Linear PID Fuzzy Approach to Inverted Pendulum Controller with Evolutionary Parameter Optimisation," International Conference on Power, Signals, Controls and Computation, IEEE, 2012, pp. 1-6.

[21] Wen-wen cai, Li-xin Jia, Yan-bin Zhang,, Nan Ni, “Design and Simulation of Intelligent PID Controller Based on Particle Swarm Optimization,” ICEEE, IEEE, 2010, pp. 1-4.

[22] KJ Aström, T Hägglund. “The future of PID control,”Control Engineering Practice, vol. 9, November 2001, pp. 1163–1175.

[23] Jin Zhao and Bimal K. Bose, “ Evaluation Of Membership Functions For Fuzzy Logic Controlled Induction Motor Drive,” IEEE 28th Annual Conference on the Industrial Electronics Society (IECON), IEEE, 2002, pp. 229-234.

[24] B. Wayne Bequette, "Introduction", in Process Control: Modeling, Design, and Simulation,Prentice Hall Professional,NJ, USA, 2003, pp. 1-30.

[25] Anil Kumar Yadav, Prerna Gaur, A.P. Mittal and Masood Anzar, “Comparative Analysis Of Various Control Techniques for Inverted Pendulum,” India International Conference on power Electronics (IICPE), IEEE, 2011, pp.1-6.

[26] Dr. M P Nandakumar and Rehna Nalaknath, “A Non Linear Pendulum Controller With Evaluationary Parameter Optimisation,” IEEE, 2012.

[27] Stellet, Jan. "Control of an Inverted Pendulum." chicago, 2011, pp. 1-14..