IMPROVING SYSTEM AVAILABILITY THROUGH OPPORTUNISTIC MAINTENANCE

A

Project Report

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CERTIFICATE

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This is to certify that the thesis work titled "**Improving System Availability through Opportunistic Maintenance**" is an authentic work carried out by **Mr. Varun Kumar** in the requirement of partial fulfillment for the award of Degree of Master of Technology in Production Engineering at Delhi Technological University. This work was completed under my supervision and guidance. He has completed his work with utmost sincerity and diligence. The work embodied in this project has not been submitted for the award of any other degree to the best of my knowledge.

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DECLARATION

I, **Varun Kumar**, hereby declare that the report of the Post Graduate Thesis work entitled **"Improving System Availability Through Opportunistic Maintenance"**, which is being submitted to the Delhi Technological University, Delhi, in the partial fulfillment of the requirement for the award of Degree of Master of Technology in Production engineering**,** Department of Mechanical Engineering, is an authentic record of my own work carried under the supervision of **Dr. Girish Kumar** and **Dr. R K Singh**. The material contained in this report has not been submitted to any university or Institution for the award of any degree.

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ABSTRACT

Most modern systems are equipped with complex, expensive and high technology components whose maintenance costs have become an increasingly large portion of the total operating cost of these systems. Therefore, some efforts are required to reduce this cost and opportunistic maintenance is an alternate available to overcome this problem.

This project deals with opportunistic maintenance modeling for availability analysis of repairable mechanical systems using Markovian approaches. The conventional techniques such as reliability block diagram, fault tree analysis and reliability graphs are of no use when there comes repairs and other dependencies. The Markovian approaches considered for modeling as it can incorporate repair and other dependency features in the model.

In this work availability models are developed for a multi stage reciprocating air compressor system to see the gain due to opportunistic maintenance. Stochastic modelling and analysis are carried out using Markovian and Semi-Markovian approaches. The system availability is evaluated considering different types of repair actions, namely, perfect repair and imperfect repair.

Keywords: Availability, Corrective Maintenance, Opportunistic Maintenance, Perfect Repair, Imperfect Repair, Markov Process, Semi-Markov Process, Steady State Probability, Mean Sojourn time, Transition Probability, Cumulative Density Function.

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Notations

Notation	Details
$A(t)$	Point Availability
$\overline{A(t)}$	Mean Availability
$A(\infty)$	Steady-state Availability
A_I	Inherent Availability
A_A	Achieved Availability
A_o	Operational Availability
O	Operating
D	Degraded
F	Failed
λ_{ij}	Failure rate of the component where ‘i’ is the initial state and ‘j’ is the final state of component
μ_{ij}	Repair rate of the component where ‘i’ is the initial state and ‘j’ is the final state of component
y_i	The probability of the system to be in the state “i” at time t
F_{ij}	Cumulative density function
$K(t)$	Kernel Matrix
K_{ij}	Transition Probability
v_i	Steady-state probability of each state of embedded Markov chain
T_i	Mean Sojourn Time
P_i	Steady-state probability of each state of SMP model
A	Availability

β	Weibull Shape Factor
θ	Weibull Characteristic Life - hours

Abbreviations

Abbreviation	**Details**
OM	Opportunistic Maintenance
CM	Corrective Maintenance
MTBM	Mean time between maintenance action
M	Mean maintenance downtime
MTBF	Mean time between Failures
MTTR	Mean time to Repair
MTTF	Mean time to Failure
TPM	Total Productive Maintenance
RBD	Reliability Block Diagram
SMP	Semi-Markov Process
EMC	Embedded Markov chain
cdf	Cumulative density function

CHAPTER 1

INTRODUCTION

Nowadays modern technologies are used to increase the productivity of the system. To increase the production and reduce the production cost highly reliable technologies are required. These technologies should be highly available and maintainable also so that after any failure during manufacturing/service it can restore its initial stage easily. Availability of the system is increased by opportunistic maintenance. For doing this Markovian and Semi-Markovian approaches are used. In this thesis opportunistic maintenance is used to increase system availability, then Markov and Semi-Markov model is used to evaluate the availability of the system.

System maintenance is related to profitability through equipment output and equipment running cost. Maintenance work raises the level of equipment performance and availability but at the same time it adds to running cost. The objective of an industrial maintenance department should be the achievement of the optimum balance between these effects that is the balance which minimizes the department's contribution to profitability. In modern industry, equipment and machinery is an important part of the total productive efforts. The efficiency of production function solely depends on the functional reliability of the production facilities which are nothing but a combination of land, building, plants, tools, equipment and services utilized in the plant such as material handling, power plant, water supply and fire-fighting facilities, etc. All these facilities are subjected to wear and tear. Therefore, proper attention should be given to protect them from undue wear as well as tear by elapse of time or by the frequency of their use. A proper attention means lubrication, cleaning, timely inspection and systematic maintenance. The basic objective of plant maintenance is to keep all the production facilities in a constant and smooth service condition so that the intended functions are performed satisfactorily and at minimum cost.

It cannot be claimed that breakdown will not occur when there is a maintenance system in operation. Such a system however, minimizes the costly breakdowns. Machinery/equipment must be lined and leveled, wearing surfaces must be examined and replaced, and oiling schedules must be laid down at regular intervals. Thus, a machine in good operating condition subjected to regular inspection and adjustment will continue to produce quality services for a long time.

Maintenance activities include all efforts to keep production facilities and equipment in a good or acceptable operating condition. Failure or breakdown of machines and equipment in manufacturing and service industries has a direct impact on the following aspects:

- Production Capacity
- Production Cost
- Product and Service Quality
- Workers and consumer safety
- Consumer Satisfaction

Modern engineering systems, like process and energy systems, transport systems, off-shore structures, bridges, pipelines are designed to ensure successful operation throughout the anticipated service life, in compliance with given safety requirements related to the risk posed to the personnel, the public and the environment. Unfortunately, the threat of deteriorating processes is always present, so that it is necessary to install proper maintenance measures to control the development of deterioration and ensure the performance of the system throughout its service life. This requires decisions on what to inspect and maintain, how to inspect and maintain, and when to inspect and maintain. These decisions are to be taken so as to achieve the maximum benefit from the control of the degradation process while minimizing the impact on the operation of the system and other economical and safety consequences.

Engineers are always on the lookout for ways of reducing system down time and increasing availability, without compromising on required level of system reliability. The ultimate objective of any maintenance regime is to maintain the system functionality to the maximum extent possible with optimum trade-offs between the down times and cost of maintenance, avoiding any catastrophic failures. Opportunistic maintenance works out to be the perfect remedy, which utilizes the opportunity of system shutdown or module dismantle to perform any maintenance required in the immediate future and saves a substantial amount of system down-time. A system of components working in a random environment is subjected to wear and damage over time and may fail unexpectedly. The components are replaced or repaired upon failure, and such unpleasant events of failure are at the same time also considered in practice as opportunities for preventive maintenance on other components.

Opportunistic maintenance basically refers to the scheme in which preventive maintenance is carried out at opportunities, either by choice or based on the physical condition of the system. In this project, the focus is made on the situation in which the opportunities for preventive maintenance are generated by the failure epochs of individual components. At each failure epoch, the failed components are correctively repaired and other components that are still operational are also preventively serviced so that all the components are maintained and restored to certain conditions. An advantage of this opportunistic maintenance is that corrective repair combined with preventive repair can be used to save set-up costs. By combining both types of repair, one may not know in advance which repair actions should be taken, and thus sacrifices the plannable feature of preventive maintenance. However, there are many situations in which opportunistic maintenance is effective. For example, when corrective repair on some components requires dismantling of the entire system, a corrective repair on these components combined with preventive repair on other or neighbouring components might be worthwhile. Another instance is when a certain corrective repair on failed components can be delayed until the next scheduled preventive maintenance. In this work availability models are developed for a multi-stage reciprocating air compressor system to see the gain due to opportunistic maintenance. Stochastic modeling and analysis are carried out using Markovian and Semi-Markovian approaches. In the next chapter literature review is carried out.

CHAPTER 2

LITERATURE REVIEW

The chapter gives an overview of the research problem and highlights issues that are relevant to system availability. For this, the appropriate literature review is carried out and described, including identification of the research gaps and formulation of the research problem.

Xiaojun *et al.* studied opportunistic preventive maintenance scheduling for a multi-unit series system based on dynamic programming. The authors proposed an opportunistic PM scheduling algorithm for the multi-unit series system based on dynamic programming with the integration of the imperfect effect into maintenance actions. An optimal maintenance practice was determined by maximizing the short-term cumulative opportunistic maintenance cost savings for the whole system. Matlab was considered for the optimization which is based on numerical simulation. A dynamic opportunistic PM scheduling algorithm for a multi-unit series system was proposed based on short-term optimization with the integration of imperfect effect into maintenance actions. Whenever one of the units reaches its reliability threshold, a PM action will be performed on that unit.

Radouane *et al.* studied opportunistic policy for optimal preventive maintenance of a multi-component system in continuous operating units and proposed the maintenance plan which is based on opportunistic multi grouping replacement optimization for multi-component systems like chemical plants, etc. The proposed algorithm is a numerical procedure, where the life cycles are simulated and the optimal solution is numerically searched. The large number of random simulations by Monte Carlo method, which is very useful in solving nonlinear and complex optimization problems, ensures the stability of the estimates and guarantees the solution convergence to the optimal one. As all significant combinations are considered, the optimal solution cannot be missed. The most important facts revealed in the study are:

- The effectiveness of the opportunistic policy in cost saving for multi-component systems and the capability of the Monte Carlo simulations in solving these complex problems.
- The evident grouping configuration, which is sometimes adopted by maintenance managers, is not necessary cost-effective; therefore, an optimization procedure must be considered.

Tim Bedford *et al.* studied the signal model: A model for competing risks of opportunistic maintenance**.** The authors presented a competing risks reliability model for a system that releases signals each time and its condition deteriorates. The released signals are used to inform opportunistic maintenance. The proposed model can be used to support decision-making in optimizing preventive maintenance at a component level. The estimates of the underlying failure distribution can be used to identify the critical signal that would trigger maintenance of the individual component; at a multi-component system level. These estimates of the component lifetime are important when making general maintenance decisions. An important feature of the signal model is that it implicitly captures the dependence structure between failure and maintenance. Provided that full signal data is available, the model allows for the determination of the underlying system lifetime on the basis of right censored data, without having to make any un-testable assumptions.

Ding and Tian studied opportunistic maintenance for wind farms considering multi-level imperfect maintenance thresholds**.** The authors developed opportunistic maintenance approaches for wind farms to take advantage of the maintenance opportunities. Imperfect maintenance actions are considered, which addresses the practical issue that preventive maintenance does not always return components to as-good-as-new status. The proposed opportunistic maintenance policies are defined by the component's age threshold values, and different imperfect maintenance thresholds are introduced for failure turbines and working turbines. Three types of preventive maintenance actions are considered, including perfect, imperfect and two-level action. Simulation methods are developed to evaluate the costs of proposed opportunistic maintenance policies. Comparative study with the widely used corrective maintenance policy demonstrates the advantage of the proposed opportunistic maintenance methods in significantly reducing the maintenance cost.

Zhou *et al.* proposed Bottleneck-based opportunistic maintenance model for series production systems. The authors developed and presented a bottleneck-based OM optimization model with the integration of the imperfect effect as a new method to schedule maintenance activities for a series production system with buffers. While most previous researches have addressed the problem of finding an optimal maintenance policy for series production system, the current research is the first of its kind which deals with issues considering the bottleneck constraint on system capacity and diverse types of machines as a means to reduce the maintenance cost and to increase corporate profit by increasing availability and production.

Ding and Tian also studied opportunistic maintenance optimization for wind turbine systems considering imperfect maintenance actions and developed opportunistic maintenance approaches for an entire wind farm rather than individual components. The authors considered imperfect actions in the preventive maintenance tasks, which addresses the issue that preventive maintenance do not always return components to the as-good-as-new status in practice. The authors also proposed three opportunistic maintenance optimization models, where the preventive maintenance is considered as perfect, imperfect and two-level action, respectively.

These policies are defined by the age threshold value(s) at the component level. Based on failure distribution information of components, the age values of each component at each failure instant can be obtained, and the optimal policy corresponding to the minimum average cost can be decided. Simulation methods are developed to evaluate the costs of proposed opportunistic maintenance policies.

Barringer studied the reliability of critically turbo/compressor equipment and a methodology is presented to evaluate and determine the necessary level of reliability for process equipment such as centrifugal compressors and turbine in a refinery environment. Inherent availability of turbine-compressor system for different values of mean time between failures was calculated and identified the optimum replacement interval of turbine-compressor system.

Samhouri et al. proposed an intelligent opportunistic maintenance system; A genetic algorithm approach" and presents an intelligent method on how to decide whether a particular item requires opportunistic maintenance or not and if so how cost effective this opportunistic maintenance will be. Genetic algorithm was used to determine whether opportunistic maintenance is cost effective or not. This approach optimized the total cost of maintenance and gave an accurate indication about the economic of replacing a certain component under opportunistic maintenance strategy.

Tambe *et al.* introduced optimization of opportunistic maintenance of a multi component system considering the effect of failure on quality and production schedule; a case study. The authors considered a model of multi component system to take maintenance decision with a constraint on available time and the system availability requirements. The maintenance decision involves one of the three actions namely, repair, replace or do nothing to achieve the target availability with minimum maintenance cost. The model generates the maintenance decision considering maintenance cost, down-time cost, failure cost and the cost of rejections. The structure of model is complex hence an algorithm is used for optimal decision making. The approach is applied to a real life case study of a high pressure die casting machine.

Cavalcante and Rodrigo studied opportunistic maintenance policy for a system with hidden failure; a multi-criteria approach applied to an emergency diesel generator and developed a maintenance policy applicable to emergency diesel generator in a health facility. Emergency diesel generators are used when a fault in main electricity supply occurs. These systems requires cares from the moment they are put into operation and maintenance policy for such systems was successfully developed in which the failures are hidden and inspections are performed to detect the state of the system and subsystems.

Literature review on performance analysis, mainly on availability, has been carried out. Although researchers have suggested numerous approaches to model availability of mechanical systems, yet these do not yield realistic results. The realistic results imply that the obtained value of the system availability is near to the expected value. Therefore, not yielding of realistic results means the obtained value is far away from the logical or expected value. . This is due to the inappropriate assumptions, e.g. use of constant failure and repair rate, independence of components, etc. that are considered in modeling, which are far from the actual behavior. Moreover, most of the availability models are too simple to handle complexity of the systems. For large and complex systems, the modeling does become difficult and that too in a single model. For ease in modeling and simplification of the analysis, hierarchical modeling has merits. However, there is a lack of literature in this. But there have been few attempts in this using Markovian approach that restricted to constant failure and repair rates. Therefore, there is a need for carrying out and exploiting decomposition of a larger system into various hierarchical levels, which can be handled with ease and at the same time maintains its accuracy. Also, the model should be capable of handling time dependent failure and repair rates.

From the above literature review and discussion, the following research gaps are identified for taking up the proposed research work:

- Semi-Markov process model, which is capable of handling non-exponential failure and repair times, is the most appropriate for mechanical systems. Its analytical solution, however, needs to be explored.
- There is a need for developing an availability model that takes into account multi-state degradation of components.

Based on the above, research problem is formulated. Literature review and discussion in this chapter have helped in identifying the research gaps, leading to the formulation of the research problem. It is quite clear that the availability assessment of mechanical systems is difficult with the existing models and methodologies. This is mainly due to the assumptions in developing models and their inability to model complexities. The aim of the present research work was to develop models and methodologies, which can improve the applicability and accuracy of the availability assessment. The present research work focuses on the development of models with Markov and Semi-Markov approaches to see the gain due to opportunistic maintenance.

CHAPTER 3

SYSTEM AVAILABILITY & SYSTEM MAINTENANCE

Reliability, availability and maintainability are very essential factor of industrial engineering. A system is reliable only when it is available and performs its task economically over specified period of time. Reliability can be expressed in terms of availability. MTTF is the main factor of reliability evaluation, if MTTF is known then it can be easily evaluated that when the system will fail and when it require repair. But MTTF of any system cannot be evaluated exactly so range of MTTF is calculated with the help of confidence intervals. Confidence intervals give a range in which MTTF of the system varies.

This chapter concerns with the definitions of reliability, availability and its classification, maintainability, and maintenance policies, applied in a general sense, to mechanical systems.

3.1 AVAILABILITY

The degree to which a system, subsystem or equipment in a specified operable and committable state at the start of mission, when the mission is called for at an unknown, i.e. a random time. It can also be defined as the ratio of the total time a functional unit is capable of being used during a given interval to the length of interval. Availability of system is typically measured as a factor of its reliability-as reliability increases, so does availability. Availability of a system may also be increased by the strategy of focusing on diagnostics and maintenance and not only on reliability. Improving maintainability during the early design phase is generally easier than reliability. Therefore, maintainability and maintenance strategies influence the availability of the system. Availability is an important metric used to assess the performance of repairable systems, accounting for both the reliability and maintainability properties of a component or system. In the reliability study failures are measured. Failures demonstrate evidence of lack of reliability. Reliability problems are failures, and failures cost money in an economic enterprise. Therefore, the focus is on improving reliability. Improved reliability occurs at an increased capital cost but brings with it the expectation for improving availability, decreasing downtime, smaller maintenance cost and results in better chances of making money because the equipment is free from failure for longer periods of time.

3.2 CLASSIFICATION of AVAILABILITY

The classification of availability is somewhat flexible and is largely based on types of downtimes used in the computation and on the relationship with time, i.e. the span of time to which the availability refers. As a result, there are number of different classifications of availability.

3.2.1 Instantaneous or Point Availability, *A(t)*-Instantaneous (or point) availability is the probability that a system (or component) will be operational at a specific time, t. It is sometimes necessary to estimate the availability of a system at a specific time of interest, i.e. when a certain mission is to happen.

3.2.2 Average uptime Availability (Mean Availability), $\overline{A(t)}$-The mean availability is the proportion of the time during a mission or time period that the system is available for use. It represents the mean values of instantaneous availability function over the period [0 T]. For systems that have periodical maintenance, availability may be zero at regular periodic intervals. In this case, mean availability is a more meaningful measure than instantaneous availability.

3.2.3 Steady State Availability, *A (∞)*-The steady state availability of a system is the limit of availability function as time tends to infinity. Steady state availability is also called as long-run or asymptotic availability.

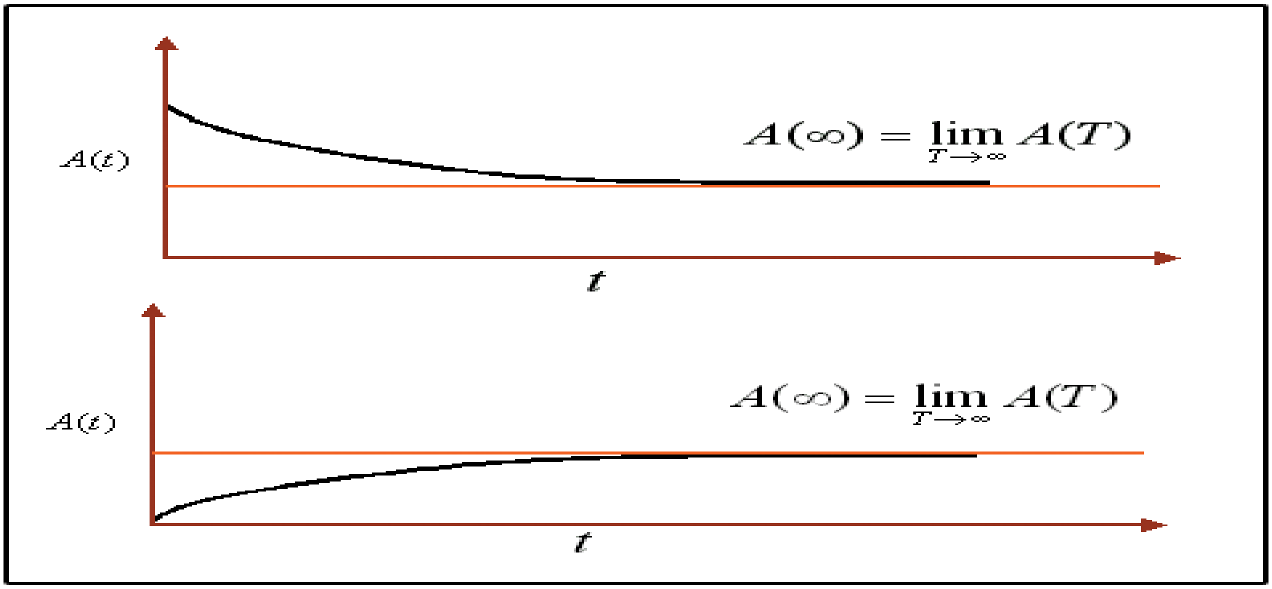

Figure 3.1: Illustration of point availability approaching steady state.

For practical considerations, the availability function will start approaching the steady state value after a time period of approximately four times the average time to failure. This varies depending upon the maintainability and the complexity of the system. Steady state availability is a stabilizing point where the system availability is roughly a constant value.

3.2.4 Inherent Availability, A_I –Inherent availability is the steady state availability when considering only the corrective maintenance (CM) downtime of a system. This classification excludes preventive maintenance downtime, logistic delays, supply delays and administrative delays. Inherent availability is expressed as:

$$A_I = {(MTTF)}\Big/{(MTTF + MTTR)}$$

3.2.5 Achieved Availability, A_A –Achieved availability is similar to inherent availability with the exception that preventive maintenance (PM) downtimes are also included. It is the steady state availability when considering corrective and preventive downtime of system. The achieved availability is sometimes referred to as the availability seen by the maintenance department which includes both corrective and preventive maintenance but does not include logistics delays, supply delays or administrative delays. It is expressed as:

$$A_A = {(MTBM)}/{(MTBM + M)}$$

where; MTBM stands for Mean time between maintenance actions and M stands for mean maintenance downtime.

$$\text{MTBM}=\frac{Uptime}{Number\ of\ system\ failures+Number\ of\ system\ downing\ PMs}$$

$$\text{M}=\frac{(CM\ downtime+PM\ downtime)}{Number\ of\ system\ failures+Number\ of\ system\ downing\ PMs}$$

3.2.6 Operational Availability, A_o - Operational availability is a measure of the real average availability over a period of time and includes all experienced sources of downtimes, such as administrative downtime, logistic downtime, etc. The operational availability is the availability that the customer actually experiences.

$$A_o = {Uptime}/{Operating\ Cycle}$$

where, the operating cycle is the overall time period of operation being investigated and uptime is the total time the system is functioning during the operating cycle.

3.3 MAINTENANCE MEASURES

In this section various measures related to availability theory are described.

3.3.1 Mean Time Between Failures (MTBF) – The MTBF is the mean time between the successive failures of a product. This definition assumes that the product in question can be repaired and placed back in operation after each failure. MTBF is used for repairable items. Refer Fig 3.2 for MTBF.

$$\text{MTBF}={Total\ test\ time}/{Number\ of\ failures\ during\ test}$$

3.3.2 Mean Time to Repair (MTTR) – It is the arithmetic mean of the time required to perform maintenance action. Refer Fig 3.2 for MTTR.

MTTR=$^{Total\ maintenance\ time}/_{Number\ of\ maintenance\ action}$

3.3.3 Maintenance action rate (μ) – It is a numerical value representing the number of maintenance action that can be carried out on particular equipment per hour and expressed as:

$$\mu = {}^{1}/_{MTTR}$$

3.3.4 Mean Time to Failure (MTTF) – It is more appropriate to assess reliability in terms of mean time to failure for non-repairable items. MTTF is the average time that an item may be expected to function before failure. MTTF is the reciprocal of constant hazard rate. Refer Fig 3.2 for MTTF and it is expressed as:

MTTF=$^{1}/_{\lambda}$

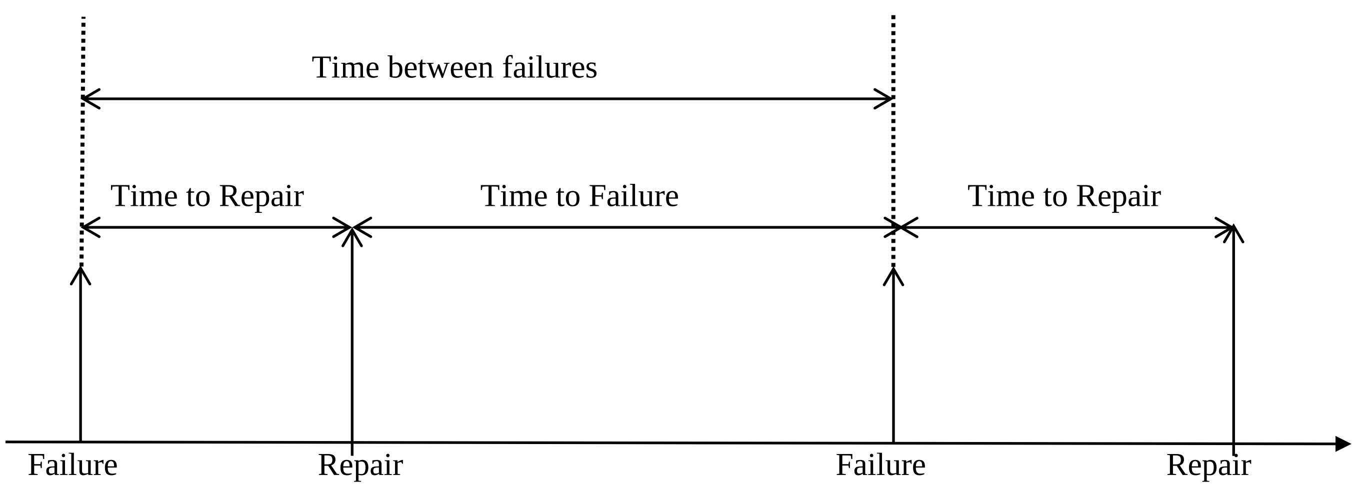

Figure 3.2: Failure and Repair Cycle.

In the next section the role of maintenance is discussed.

3.4 SYSTEM MAINTENANCE

System maintenance is related to profitability through equipment output and equipment running cost. Maintenance work raises the level of equipment performance and availability but at the same time it adds to the running cost. The objective of an industrial maintenance department should be the achievement of the optimum balance between these effects that is the balance which minimizes the department's contribution to profitability. Past and current maintenance practices in both the private and government sectors would imply that maintenance is the actions associated with equipment repair after it is broken. The dictionary defines maintenance as follows: "the work of keeping something in proper condition; upkeep." This would imply that maintenance should be actions taken to prevent a device or component from failing or to repair normal equipment degradation experienced with the operation of the device to keep it in proper working order. Unfortunately, data obtained in many studies over the past decade indicates that most private and government facilities do not expend the necessary resources to maintain equipment in proper working order. Rather, they wait for equipment failure to occur and then take whatever actions are necessary to repair or replace the equipment. Nothing lasts forever and all equipment has associated with it some predefined life expectancy or operational life. For example, equipment may be designed to operate at full design load for 5,000 hours and may be designed to go through 15,000 start and stop cycles.

The need for maintenance is predicated on actual or impending failure – ideally, maintenance is performed to keep equipment and systems running efficiently for at least design life of the component(s). As such, the practical operation of a component is time-based function. If one were to graph the failure rate a component population versus time, it is likely the graph would take the "bathtub" shape shown in Figure 3.3. In the Fig 3.3, the Y axis represents the failure rate and the X axis is time. From its shape, the curve can be divided into three distinct periods: infant mortality, useful life, and wear-out periods. The initial infant mortality period of bathtub curve is characterized by high failure rate followed by a period of decreasing failure. Many of the failures associated with this region are linked to poor design, poor installation, or misapplication. The infant mortality period is followed by a nearly constant failure rate period known as useful life. There are many theories on why components fail in this region, most acknowledge that poor operation and maintenance (O & M) often plays significant role. It is also generally agreed that exceptional maintenance practices encompassing preventive and predictive elements can extend this period. The wear-out period is characterized by a rapid increasing failure rate with time. In most cases this period encompasses the normal distribution of design life failures.

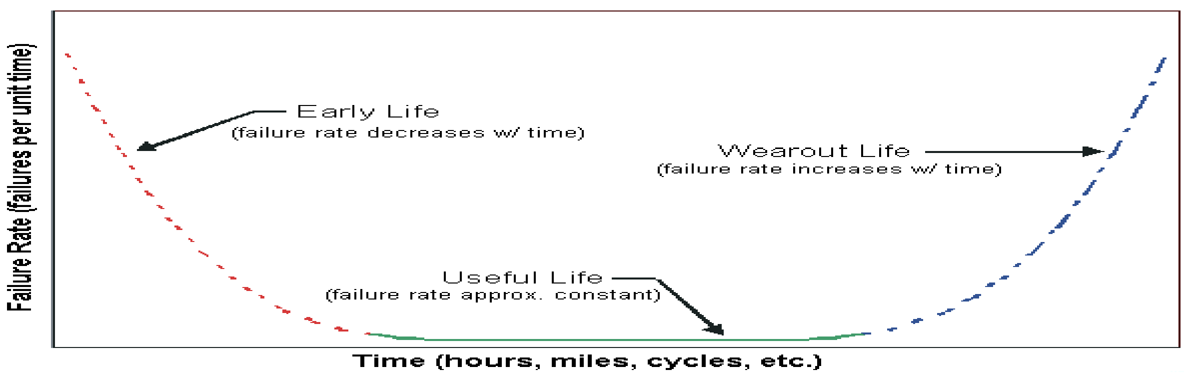

Figure 3.3: Component failure rate over time.

The design life of most equipment requires periodic maintenance. Belts need adjustment, alignment needs to be maintained, proper lubrication on rotating equipment is required, and so on. In some cases, certain components need replacement, (e.g., a wheel bearing on a motor vehicle) to ensure the main piece of equipment (in this case a car) last for its design life. When the maintenance activities intended by the equipment's designer are not performed, operating life of the equipment is shorten. But what options do we have? Over the last 30 years, different approaches to how maintenance can be performed to ensure equipment reaches or exceeds its design life have been developed. In addition, to waiting for a piece of equipment to fail (reactive maintenance), preventive maintenance, predictive maintenance, etc., can be utilized.

3.5 MAINTENANCE POLICIES

In this section various maintenance policies are discussed.

3.5.1 Reactive Maintenance:

Reactive maintenance is basically the run it till it breaks maintenance mode. No actions or efforts are taken to maintain the equipment as the designer originally intended to ensure design life is reached.

The main advantages of reactive maintenance are:

- Low cost.
- Less staff.

Disadvantages of reactive maintenance are:

- Increased cost due to unplanned downtime of equipment.
- Increased labor cost, especially if overtime is needed.
- Possible secondary equipment or process damage from equipment failure.
- Inefficient use of staff resources.

3.5.2 Preventive Maintenance:

It attempts to prevent any probable failure resulting in production stoppages. It refers to a maintenance action performed to retain a machine in a satisfactory operating condition through periodic inspections, lubrications, calibration, replacement and overhauls. It involves regular cleaning, greasing and oiling of moving parts, replacement of worn out parts before they fail to operate, periodic overhauling of the entire machine, etc. It is subdivided into running maintenance and shut down maintenance. Running maintenance means that maintenance work carried out even when machine or equipment is in service, while shut down maintenance is concerned with maintenance work carried out only when the machine or equipment is not in operation.

3.5.3 Predictive Maintenance:

Predictive maintenance can be defined as follows: Measurements that detect the onset of system degradation (lower functional state), thereby allowing causal stressors to be eliminated or controlled prior to any significant deterioration in the component physical state. Results indicate current and future functional capability. Basically predictive maintenance differs from preventive maintenance by basing maintenance need on the actual condition of machine rather then on some preset schedule.

Advantages of predictive maintenance are:

- Increased component operational life/availability.
- Allows for pre-emptive corrective actions.
- Decrease in equipment or process downtime.
- Decrease in cost of parts and labor.
- Better product quality.

Disadvantages of predictive maintenance are:

- Increased investment in diagnostic equipment.
- Increased investment in staff training.
- Saving potential not readily seen by management.

3.5.4 Unplanned Maintenance – It is an operation carried out without any prior planning. It is very urgent in nature. Such type of maintenance operations are required in case of heavy and total breakdown which may occur without any proper indication. Such breakdowns are generally harmful to the system and they may cause loss of human life also. In order to fight such unwanted situations provisions are made to provide maintenance with prior planning, preparations and scheduling, etc. Thus in most of the cases the unplanned maintenance is emergent in nature in view of the fact that here the recovery time is the most important factor in order to minimize the consequences of serious breakdowns. The typical examples are bursting of boilers or failure of pipe line carrying fluids/gases.

3.5.5 Opportunistic Maintenance – In opportunistic maintenance, when a system or module is grounded for corrective or preventive maintenance, that opportunity is utilized to do maintenance on other parts of the module, which are found to be damaged or have started to deteriorate. On one hand, this improves the safety and reliability of the system, and on the other hand it reduces the downtime by avoiding unscheduled maintenance. This in turn reduces the cost of maintenance and loss of revenue due to extra groundings.

3.5.6 Total Productive Maintenance (TPM):

In manufacturing environment of today, employing high tech, expensive machines, backed by computer control of manufacture and advanced manufacturing concepts, there is almost no place for breakdown of any type. Thus, maintenance management is now under an all time high pressure, with the only goal/aim of 'zero breakdowns'. Thus, starting with conventional repair/maintenance strategy of machines, maintenance has now reached a stage of "Total Productive Maintenance" a concept with aim/goal of 'Zero down Time'.

In the next chapter various availability assessment technique are discussed.

CHAPTER 4

AVAILABILITY MODELING TECHNIQUES

Availability is an important measure used to assess the performance of repairable systems, accounting for both the reliability and maintainability properties of a component or system. Availability is the probability that a system will work as and when required during the period of a mission. There are various methods of availability evaluation such as reliability block diagram, fault tree analysis, Monte Carlo simulation, Markov and semi-Markov state space model etc. In this thesis Markov and semi-Markov state space model is used to evaluate the availability. This is the probabilistic model in which the transition of the system from full available state to failed state is shown. The various modeling techniques for system availability are as follows:

4.1 RELIABILITY BLOCK DIAGRAM

A reliability block diagram (RBD) is a diagrammatic method of showing how component reliability contributes to the success or failure of a complex system. RBD is also known as dependence diagram (DD). RBD is drawn as a series of blocks connected in parallel or series configuration. Each block represents the component of a system with failure rate. Parallel paths are redundant, meaning that all of the parallel paths must fail for the parallel network to fail, By contrast, any failure along a series path causes entire series path to fail. Refer Fig 4.1, for typical reliability block diagram.

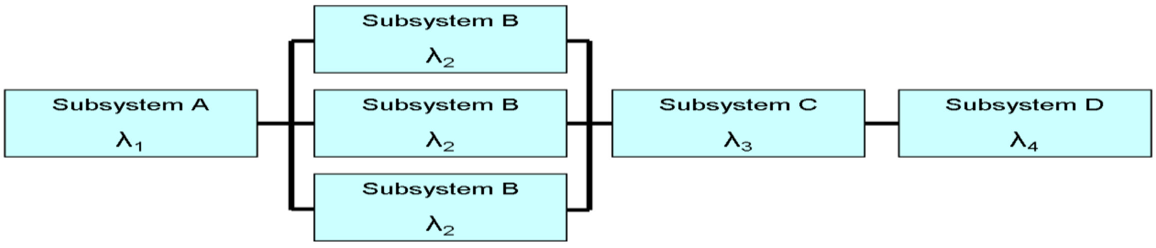

Figure 4.1: Reliability Block Diagram

An RBD may be drawn using switches in place of blocks, where a closed switch represents a working component and an open switch represents a failed component. If a path may be found through the network of switches from beginning to end, the system still works.

4.2 FAULT TREE ANALYSIS

The fault tree analysis is one of the approaches to reliability analysis of a complex system. This method is based on the causes (events) that will lead to system failure. A fault tree is a diagrammatic representation of all possible fault events, their logical combinations, and their relationship to the system failure. The events (causes) at the lowest level are known as basic events. There could be another events resulting from combination of basic events. Such events are represented as intermediate events. The failure possibilities of the basic events are combined to obtain the failure possibilities of intermediate events and finally the top events. The technique is called "fault tree" analysis due to the branching out of origin and causes . Refer, Fig 4.2, for a typical fault tree.

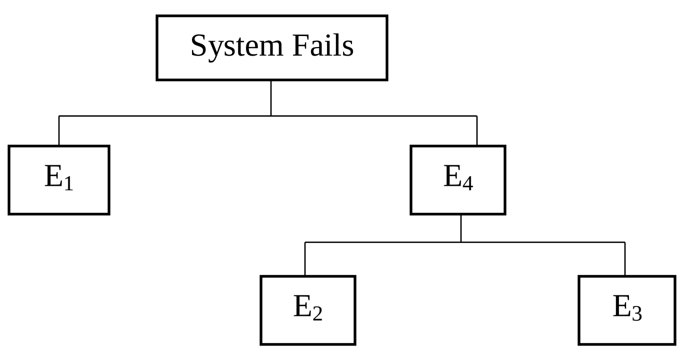

Figure 4.2: Fault Tree Diagram

In the Figure 4.2, E_1, E_2 and E_3 are the basic events. E_4 is an intermediate event which occurs when both E_2 and E_3 occur. The top event, i.e. system failure occurs when either E_1 or E_4 occurs.

4.3 MARKOV PROCESS

Markov models are frequently used where events, such as the failure or repair of a module, can occur at any point in time. The Markov model evaluates the probability of jumping from one known state into the next logical state until, depending upon the configuration of the system being considered, the system has reached the final or totally failed state or until particular mission time is achieved. The basic assumption of a Markov Process is that the behavior of a system in each state is memory less. For any given system, a Markov model consists of a list of the possible states of that system, the possible transition paths between those states, and the rate parameters of those transitions. The symbol λ denotes the rate parameter of the transition from State 1 to State 2. In addition, P_j (t), denotes the probability of the system being in State 'j' at time 't'. Refer, Fig 4.3, for a two state Markov model.

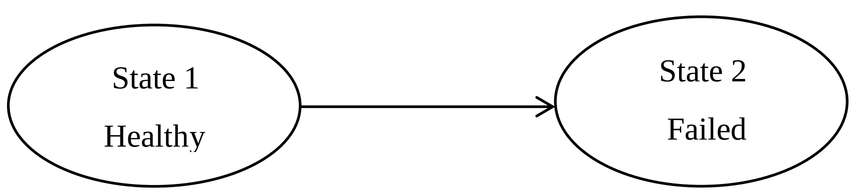

Figure 4.3: Two states i.e. healthy and failed respectively.

If the device is known to be healthy at some initial time, t = 0, the initial probabilities of the two states are, P_1 (0) = 1 and P_2 (0) = 0. Thereafter, the probability of State 1 decreases at the constant rate λ, which means that if the system, is in State 1 at any given time, the probability of making the transition to State 2 during the next increment of time dt is λdt. Therefore, the overall probability that the transition from State 1 to State 2 will occur during a specific incremental interval of time dt is given by multiplying the probability of being in State 1 at the beginning of that interval, and the probability of the transition during an interval dt given that it was in State 1 at the beginning of that increment. This represents the incremental change dP_1 in probability of State 1 at any given time, this is expressed as:

$$dP_1 = -(P_1)(\lambda dt)$$

Dividing both sides by dt it reduces to:

$$dP_1/dt = -\lambda P_1$$

This signifies that a transition path from a given state to any other state reduces the probability of the source state at a rate equal to the transition rate parameter λ multiplied by the current probability of the state. Now, since the total probability of both states must equal 1, it follows that the probability of State 2 must increase at the same rate that the probability of State 1 is decreasing. Thus, the equations for this simple model are

$$dP_1/dt = -\lambda P_1$$

$$dP_2/dt = \lambda P_2$$

$$\text{Also } P_1 + P_2 = 1$$

The solution of these equations, with the initial conditions $P_1(0) = 1$ and $P_2(0) = 0$, is

$$P_1(t) = e^{-\lambda t} \text{ and } P_2(t) = 1- e^{-\lambda t}$$

The form of this solution explains why transitions with constant rates are sometimes called 'exponential transitions', because the transition times are exponentially distributed. Also, it is clear that the total probability of all the states is conserved. Probability simply flows from one state to another.

4.4 SEMI-MARKOV PROCESS

The idea of a semi-Markov process was proposed in 1954. The semi-Markov process is similar to the Markov process in that both processes are described by a set of states whose transitions are governed by a transition probability matrix. The semi-Markov process, however, differs from the

Markov process in that the times between transitions may be random variables. Further, the amount of time spent in any state after entering it is a random variable described by a probability density that can be a function of both the state of occupancy and the states to which transitions can occur.

The statistical time behaviour of the semi-Markov process is described by a set of linear integral equations. The solution of this set of equations yields the probabilities that the process occupies the states of the system as a function of time. In many practical cases where the semi-Markov process is complex or involves many states, an analytic solution is difficult to obtain and numerical procedures must be used.

Semi-Markov Process based analytical approach is faster and more accurate than the simulation approaches used in existing software programs for system availability analysis. Semi-Markov process (SMP) which are capable of handling non-exponential distributions, are more appropriate for availability modelling and analysis of repairable mechanical systems.

CHAPTER 5

SYSTEM MODELING USING MARKOV APPROACH

In this thesis a multi stage reciprocating air compressor system is taken for availability evaluation. Availability evaluation methods are discussed in previous chapters. Main layout is focused for evaluation in which low pressure compressor, intercooler and high pressure compressor are connected in series manner. With the help of this layout state space diagram is constructed for availability evaluation. All differential and integral equations of Markov and semi-Markov model are solved with the help of Matlab.

It has already mentioned that, when a system is grounded for corrective or preventive maintenance, that opportunity is utilized to do maintenance on other parts of the system, which are found to be damaged or have started to deteriorate. In most plants, opportunistic maintenance is utilized. For example, maintaining intercooler of two-stage reciprocating air compressor system, while its compressor is being repaired. Consider a system of several subsystems, with their multistage degradation. When any one of its subsystem degrade to an unacceptable level, a corrective maintenance is undertaken, that is categorized as perfect repair. As long as the system remains under corrective maintenance, the other partially degraded subsystems have the opportunity for the maintenance. However, this maintenance should be completed before the completion of corrective maintenance. It reduces the downtime by avoiding unscheduled maintenance and improves the safety and reliability of the system.

5.1 System Description:

A two-stage reciprocating air compressor with intercooler is selected for this project. This is a series system. First of all, the fresh air is sucked from atmosphere in the low pressure (L.P.) cylinder during its suction stroke and then after compression in the L.P. cylinder, it is delivered to the intercooler. The air is cooled in the intercooler at constant pressure and moved to the high pressure (H.P.) cylinder during its suction stroke. Finally, the air, after further compression in the H.P. cylinder (i.e., second stage) is delivered by the compressor under high pressure to storage vessel. From storage vessel, it may be conveyed by pipeline to a place where the supply of compressed air is required.

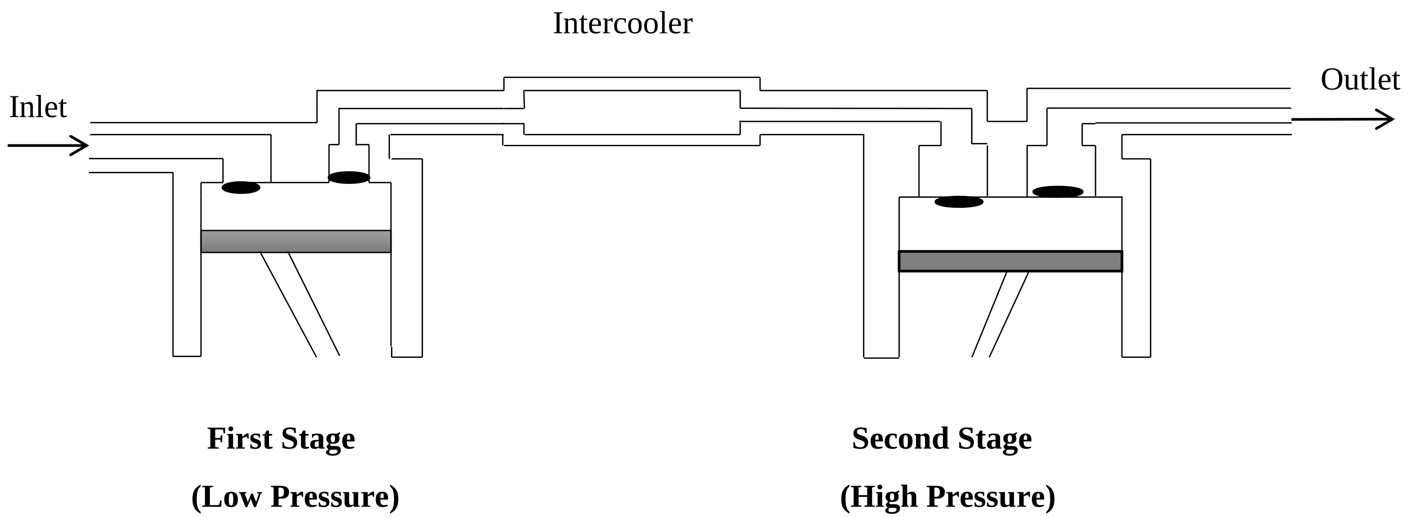

Figure 5.1: Multi-stage reciprocating air compressor system

Figure 5.2: Three states at subsystem level.

5.2 SYSTEM MODELING

In this section, system model is developed.

5.2.1 Defining State Space

Models are developed with opportunistic maintenance for system availability assessment. In the system model development, state O is considered as the original operating state, i.e. “as good as new” state, and D as degraded state and F as the failed state at the subsystem level. Refer Fig 5.2. A maintenance action, i.e. perfect repair and imperfect repair are considered, which restores the subsystem from its failed state F to the operating state O and failed state F to the operating state D respectively. The states O and D of the subsystem are considered as working states, while F is the ‘repair state’ due to performance below the unacceptable level. Let a system is considered with its three subsystems in series, with each subsystem having one original operating state, one degraded state and one failed state. It is assumed that only one subsystem is changing its state at a particular time. State of a system is dependent on the state of its subsystems. Total 20 states out of 27 possible states are feasible for the system and these are listed in the Table 5.1. All the transitions are identified for the system to develop system model.

The following assumptions are made for this model:

- In system model development, operating state, i.e., O is considered as the original operating state, i.e., “as good as new” state.
- Simultaneous failure of two or more subsystems is not considered.
- Subsystems should undergo gradual degradation i.e. from operating state to degraded state and then finally reaches the failed state.
- Time to restore a subsystem in corrective maintenance is more than restoration time of other partially degraded subsystems undergoing opportunistic maintenance.
- Failure or maintenance on any subsystem disables the system.
- Perfect corrective maintenance restores the subsystem to the original operating state, i.e., “as good as new” state.
- Imperfect corrective maintenance restores the subsystem to the state just before the failed state, i.e. degraded state.

Table 5.1: System state for three subsystem in series (perfect repair).

System State		State of subsystem 1	State of subsystem 2	State of subsystem 3	Transition to	Maintenance Possibility	
S.NO	Status					C.M	O.M
1	Working	O	O	O	2 4 9	-	-
2	Working	O	O	D	3 5 10	-	-
3	Under Repair	O	O	F	1	Yes(S_3)	No
4	Working	O	D	O	5 7 12	-	-
5	Working	O	D	D	6 8 13	-	-
6	Under Repair	O	D	F	4	Yes(S_3)	Yes(S_2)
7	Under Repair	O	F	O	1	Yes(S_2)	No
8	Under Repair	O	F	D	2	Yes(S_2)	Yes(S_3)
9	Working	D	O	O	17 12 10	-	-
10	Working	D	O	D	18 13 11	-	-
11	Under Repair	D	O	F	9	Yes(S_3)	Yes(S_1)
12	Working	D	D	O	19 15 13	-	-
13	Working	D	D	D	20 16 14	-	-
14	Under Repair	D	D	F	12	Yes(S_3)	Yes($S_{1,2}$)
15	Under Repair	D	F	O	9	Yes(S_2)	Yes(S_1)
16	Under Repair	D	F	D	10	Yes(S_2)	Yes($S_{1,3}$)
17	Under Repair	F	O	O	1	Yes(S_1)	No
18	Under Repair	F	O	D	2	Yes(S_1)	Yes(S_3)
19	Under Repair	F	D	O	4	Yes(S_1)	Yes(S_2)
20	Under Repair	F	D	D	5	Yes(S_1)	Yes($S_{2,3}$)

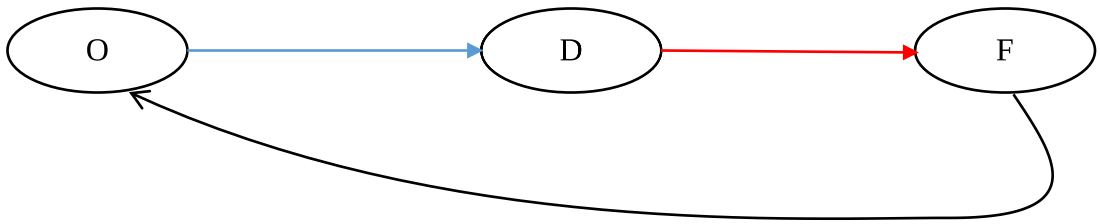

Figure 5.3: Multi-states of a subsystem and level of maintenance, i.e. perfect repair.

The logic for decision making for corrective and/or opportunistic maintenance is explained here. Table shows system states for three subsystems ‘1’, ‘2’, and ‘3’ in series, each with three states as O, D and F. The last two columns of the table give the maintenance option or possibility of ‘Corrective Maintenance’ and/or ‘Opportunistic Maintenance’ for the subsystems ‘1’, ‘2’, and ‘3’, which is decided from the system state status (Column 2); ‘Under Repair’, i.e. 3, 6, 7, 8, 11, 14 to 20. The logic applied for decision making ‘yes’ or ‘no’ is illustrated by considering the first system state status ‘Under Repair’ (Column 2) in the Table, i.e. at S.NO. 3 (Column 1). In this state, states of subsystems ‘1’, ‘2’ and ‘3’ (Column 3, 4 and 5) are O, O and F respectively. For the subsystem ‘3’ in state, F i.e. the failed state, a corrective maintenance (perfect repair) is the best choice as one needs considerable time to perform its maintenance task, including use of resources needed which are also expected to be on a higher side. In view of this, it is ‘yes’ logic for corrective maintenance (perfect repair) of subsystem ‘3’. With subsystem ‘3’ under maintenance, there is need to check if ‘Opportunistic Maintenance’ for the other subsystems, i.e. subsystems ‘1’ and ‘2’ can be undertaken. For this, one needs to check their state, which is O and O, i.e. ‘Original Operating State’, and they does not need it. Hence, the decision logic for this is ‘no’ (Column 8) for opportunistic maintenance.

In similar way, decision ‘yes’ or ‘no’ for other system states 6, 7, 8, 11, 14 to 20, which are ‘Under Repair’ are carried out.

In all, four models for the system have been developed, and out of which two models are based on corrective maintenance which is perfect repair along with and without opportunistic maintenance, while the other two models are based on corrective maintenance which is imperfect repair along with and without opportunistic maintenance. The development of model is detailed in the following subsections.

5.2.2 Models with Perfect Repair

In this section, system models are developed considering perfect repair. First model based on corrective maintenance with opportunistic maintenance is developed.

5.2.2.1 System Model – Corrective maintenance (perfect repair) with opportunistic maintenance

In this case, the opportunistic maintenance is considered along with corrective repair. In a system with three subsystems in series, out of the twenty possible system states (Table 5.1), twelve states (3, 11, 6 to 8, 14 to 20) are the 'under repair' system states, while the remaining eight states are the 'working states'. A system model is developed considering subsystem degradation, corrective repair with opportunistic maintenance and is shown in the Figure 5.4.

There are four types of transition edges in the system model. Degradation of the subsystem from O →D is represented by blue line, failure of the subsystem from D →F is represented by red line, black line for corrective repair, and continuous dotted black line for corrective repair with opportunistic maintenance. The system being in the 'under repair' states, 3, 7 and 17, only corrective repair is carried out for these as the opportunistic maintenance is not possible because the other subsystems are still in state 'O', i.e. good condition. In the system model, the corrective repair for these states is represented by lines 3 →1, 7 →1, 17 →1 for the subsystems respectively, restoring the subsystem which is under repair from its state, F to O. For the remaining nine 'under repair' states (6, 8, 11, 14, 15, 16, 18, 19, 20), the opportunistic maintenance is possible because the other subsystems are in state D, i.e. degraded state.

In the system model, the opportunistic maintenance with corrective repair is shown for arcs 6-1, 8-1, 11-1, 14-1, 15-1, 16-1, 18-1, 19-1 and 20-1. These nine arcs do represent corrective repair, restoring the under repair system from its state, F to O. Also, these lines represents the restoration of the subsystems which is under opportunistic maintenance, from state D to O. It is assumed that the time to restore a subsystem in corrective repair from state F to O is more than the restoration time of the other partially degraded states of the subsystem in opportunistic maintenance from state D to O.

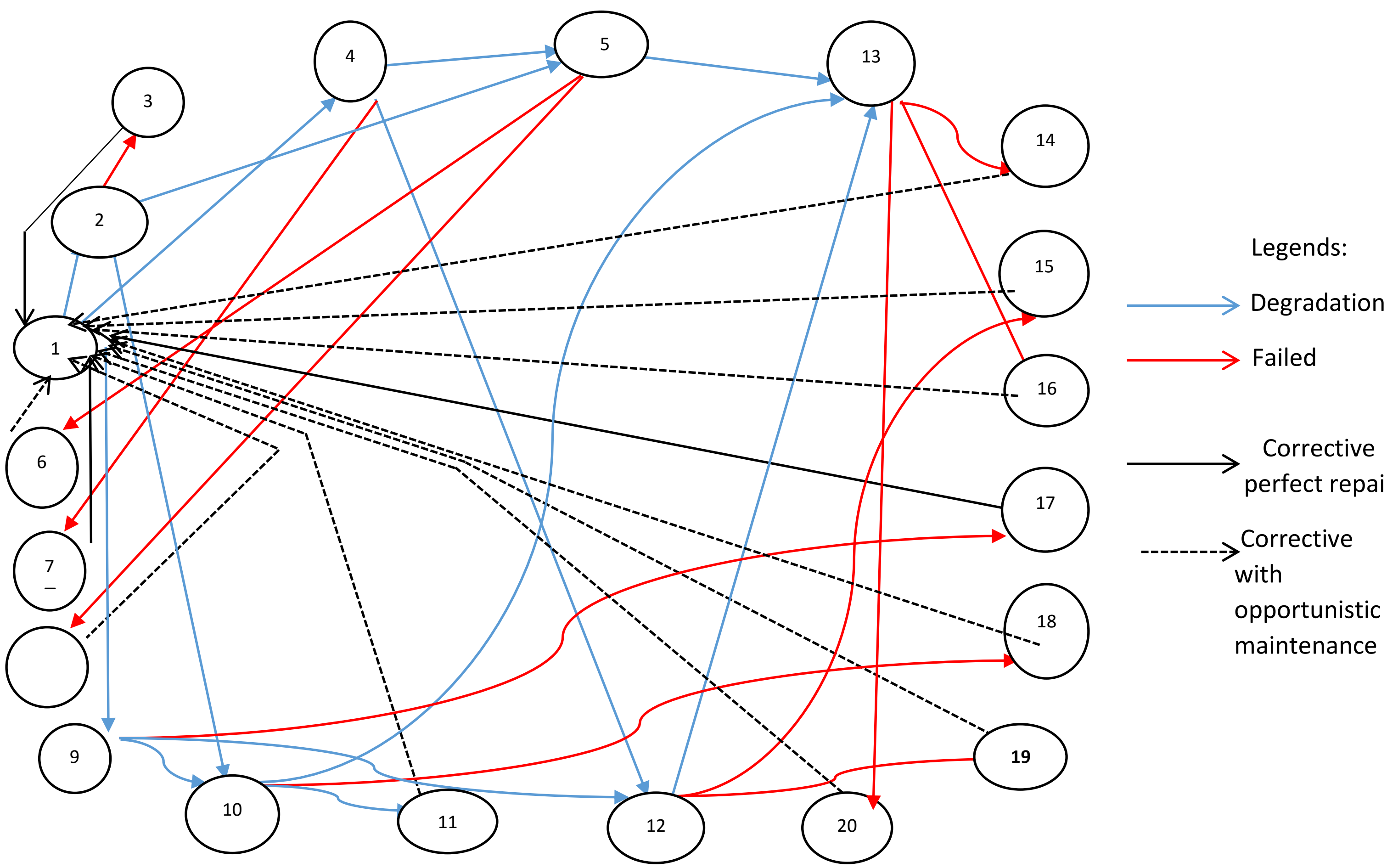

Figure 5.4: System model with corrective perfect repair and opportunistic maintenance

1) The system model developed on previous page as per Figure 5.4 is solved by Markov approach following Section 4.3.

2) For determining the mathematical equation following symbols are used:

   $P_i(t) \rightarrow$ Probability of system being in i^{th} state at time 't'.

   $dP_i/dt \rightarrow$ Rate of change of i^{th} state at time 't'.

   $\lambda_i \rightarrow$ Degradation/failure rate for i^{th} state.

   $\mu_i \rightarrow$ Repair rate for i^{th} state.

3) The equations are developed for the model (Figure 5.4).

$$dP_1/dt = -(\lambda_{12} + \lambda_{14} + \lambda_{19})\, P_1(t) + \mu_{31} P_3(t) + \mu_{61} P_6(t) + \mu_{71} P_7(t) + \mu_{81} P_8(t) + \mu_{111} P_{11}(t) + \mu_{141} P_{14}(t) + \mu_{151} P_{15}(t) + \mu_{161} P_{16}(t) + \mu_{171} P_{17}(t) + \mu_{181} P_{18}(t) + \mu_{191} P_{19}(t) + \mu_{201} P_{20}(t)$$

$$dP_2/dt = -(\lambda_{25} + \lambda_{23} + \lambda_{210})\, P_2(t) + \lambda_{12} P_1(t)$$

$$dP_3/dt = \lambda_{23} P_2(t) - \mu_{31} P_3(t)$$

$$dP_4/dt = -(\lambda_{45} + \lambda_{47} + \lambda_{412})\, P_4(t) + \lambda_{14} P_1(t)$$

$$dP_5/dt = -(\lambda_{56} + \lambda_{58} + \lambda_{513})\, P_5(t) + \lambda_{25} P_2(t) + \lambda_{45} P_4(t)$$

$$dP_6/dt = \lambda_{56} P_5(t) - \mu_{61} P_6(t)$$

$$dP_7/dt = \lambda_{47} P_4(t) - \mu_{71} P_7(t)$$

$$dP_8/dt = \lambda_{58} P_5(t) - \mu_{81} P_8(t)$$

$$dP_9/dt = -(\lambda_{910} + \lambda_{912} + \lambda_{917})\, P_9(t) + \lambda_{19} P_1(t)$$

$$dP_{10}/dt = -(\lambda_{1011} + \lambda_{1013} + \lambda_{1018})\, P_{10}(t) + \lambda_{210} P_2(t) + \lambda_{910} P_9(t)$$

$$dP_{11}/dt = \lambda_{1011} P_{10}(t) - \mu_{111} P_{11}(t)$$

$dP_{12}/dt = -(\lambda_{1213} + \lambda_{1215} + \lambda_{1219})\, P_{12}(t) + \lambda_{412}\, P_4(t) + \lambda_{912}\, P_9(t)$

$dP_{13}/dt = -(\lambda_{1314} + \lambda_{1316} + \lambda_{1320})\, P_{13}(t) + \lambda_{513}\, P_5(t) + \lambda_{1013}\, P_{10}(t) + \lambda_{1213}\, P_{12}(t)$

$dP_{14}/dt = \lambda_{1314}\, P_{13}(t) - \mu_{141}\, P_{14}(t)$

$dP_{15}/dt = \lambda_{1215}\, P_{12}(t) - \mu_{151}\, P_{15}(t)$

$dP_{16}/dt = \lambda_{1316}\, P_{13}(t) - \mu_{161}\, P_{16}(t)$

$dP_{17}/dt = \lambda_{917}\, P_9(t) - \mu_{171}\, P_{17}(t)$

$dP_{18}/dt = \lambda_{1018}\, P_{10}(t) - \mu_{181}\, P_{18}(t)$

$dP_{19}/dt = \lambda_{1219}\, P_{12}(t) - \mu_{191}\, P_{19}(t)$

$dP_{20}/dt = \lambda_{1320}\, P_{13}(t) - \mu_{201}\, P_{20}(t)$

5.2.2.2 System model – Corrective maintenance (perfect repair) without opportunistic maintenance.

In this subsection system model based on corrective maintenance without opportunistic maintenance is developed. This model is developed from the model developed in previous subsection. The model with corrective repair is obtained as shown in Figure 5.5, with removal of the lines 6-1, 8-1, 11-1, 14-1, 15-1, 16-1, 18-1, 19-1, 20-1, from the model for corrective maintenance with opportunistic maintenance, but including new lines 6-4, 8-2, 11-9, 14-12, 15-9, 16-10, 18-2, 19-4 and 20-5 representing corrective repair only.

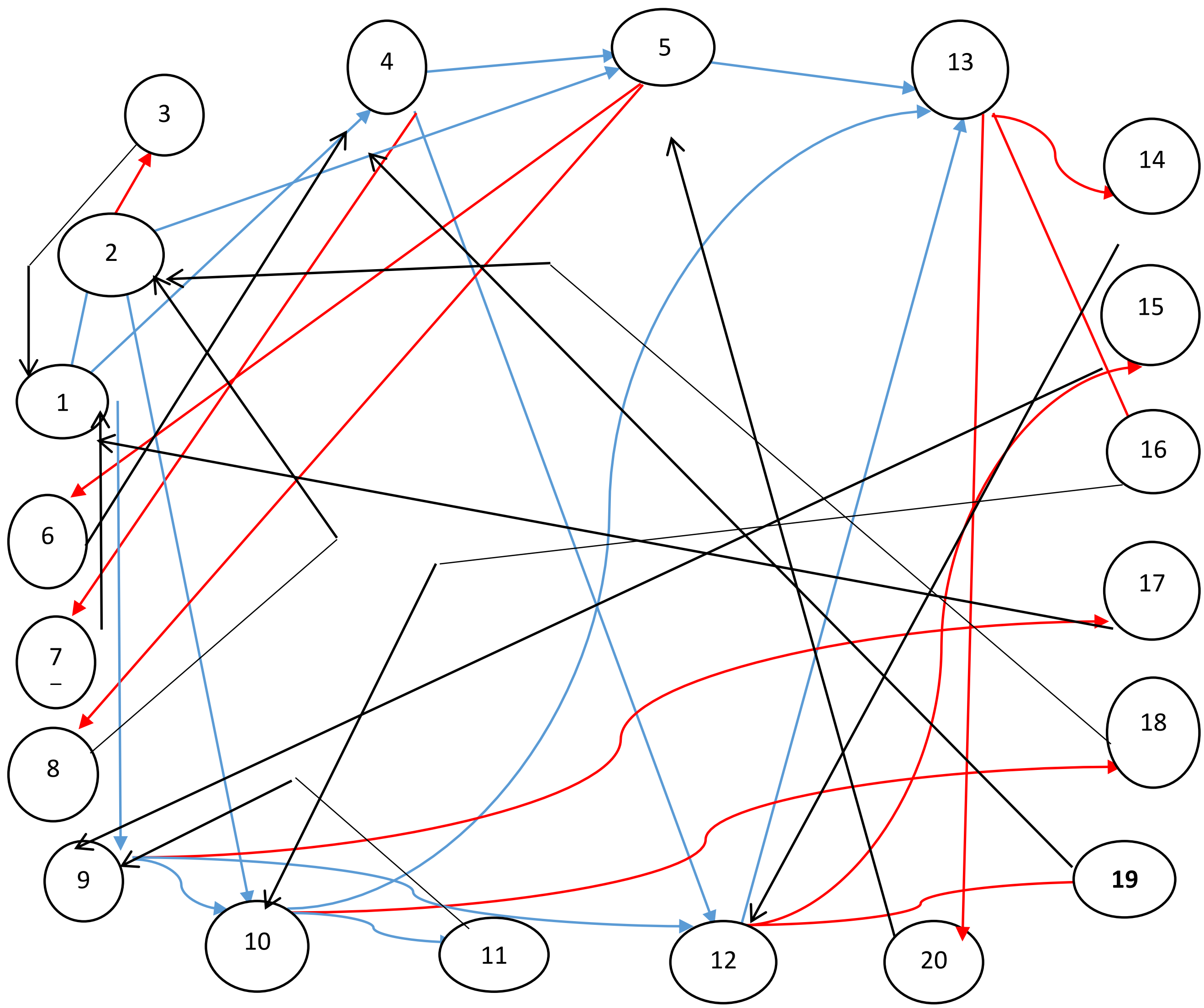

Figure 5.5: System model with corrective perfect repair

Following the methodology suggested in Section 4.3, the set of rate equations are derived as follows.

$dP_1/dt = -(\lambda_{12} + \lambda_{14} + \lambda_{19}) P_1(t) + \mu_{31} P_3(t) + \mu_{71} P_7(t) + \mu_{171} P_{17}(t)$

$dP_2/dt = -(\lambda_{25} + \lambda_{23} + \lambda_{210}) P_2(t) + \lambda_{12} P_1(t) + \mu_{82} P_8(t) + \mu_{182} P_{18}(t)$

$dP_3/dt = \lambda_{23} P_2(t) - \mu_{31} P_3(t)$

$dP_4/dt = -(\lambda_{45} + \lambda_{47} + \lambda_{412}) P_4(t) + \lambda_{14} P_1(t) + \mu_{64} P_6(t) + \mu_{194} P_{19}(t)$

$$dP_5/dt = -(\lambda_{56} + \lambda_{58} + \lambda_{513})\, P_5(t) + \lambda_{25}\, P_2(t) + \lambda_{45}\, P_4(t) + \mu_{205}\, P_{20}(t)$$

$$dP_6/dt = \lambda_{56}\, P_5(t) - \mu_{64}\, P_6(t)$$

$$dP_7/dt = \lambda_{47}\, P_4(t) - \mu_{71}\, P_7(t)$$

$$dP_8/dt = \lambda_{58}\, P_5(t) - \mu_{82}\, P_8(t)$$

$$dP_9/dt = -(\lambda_{910} + \lambda_{912} + \lambda_{917})\, P_9(t) + \lambda_{19}\, P_1(t) + \mu_{119}\, P_{11}(t) + \mu_{159}\, P_{15}(t)$$

$$dP_{10}/dt = -(\lambda_{1011} + \lambda_{1013} + \lambda_{1018})\, P_{10}(t) + \lambda_{210}\, P_2(t) + \lambda_{910}\, P_9(t) + \mu_{1610}\, P_{16}(t)$$

$$dP_{11}/dt = \lambda_{1011}\, P_{10}(t) - \mu_{119}\, P_{11}(t)$$

$$dP_{12}/dt = -(\lambda_{1213} + \lambda_{1215} + \lambda_{1219})\, P_{12}(t) + \lambda_{412}\, P_4(t) + \lambda_{912}\, P_9(t) + \mu_{1412}\, P_{14}(t)$$

$$dP_{13}/dt = -(\lambda_{1314} + \lambda_{1316} + \lambda_{1320})\, P_{13}(t) + \lambda_{513}\, P_5(t) + \lambda_{1013}\, P_{10}(t) + \lambda_{1213}\, P_{12}(t)$$

$$dP_{14}/dt = \lambda_{1314}\, P_{13}(t) - \mu_{1412}\, P_{14}(t)$$

$$dP_{15}/dt = \lambda_{1215}\, P_{12}(t) - \mu_{159}\, P_{15}(t)$$

$$dP_{16}/dt = \lambda_{1316}\, P_{13}(t) - \mu_{1610}\, P_{16}(t)$$

$$dP_{17}/dt = \lambda_{917}\, P_9(t) - \mu_{171}\, P_{17}(t)$$

$$dP_{18}/dt = \lambda_{1018}\, P_{10}(t) - \mu_{182}\, P_{18}(t)$$

$$dP_{19}/dt = \lambda_{1219}\, P_{12}(t) - \mu_{194}\, P_{19}(t)$$

$$dP_{20}/dt = \lambda_{1320}\, P_{13}(t) - \mu_{205}\, P_{20}(t)$$

5.2.3 Models with Imperfect Repair

It has already mentioned that imperfect repair restores the subsystem from its failed state F to the operating state D. In this subsystem modeling, the states O and D of the subsystems are considered as the working states, while F is the 'repair state' due to performance below unacceptable level.

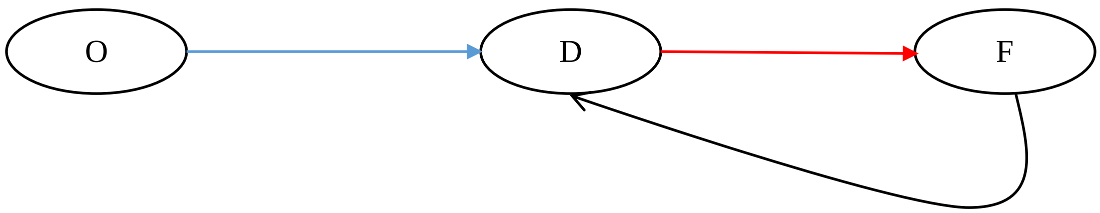

Figure 5.6: Multi-states of a subsystem with imperfect repair.

It is assumed that only one subsystem is changing its state at a particular time, i.e. from O →D, D →F and F →D. State of a system is dependent on the state of its subsystems. For the system, 20 states out of 27 possible states are feasible and these are listed in the Table 5.2. All transitions are also identified for the system.

Table 5.2: System state for three subsystem in series (imperfect repair)

System State		State of subsystem 1	State of subsystem 2	State of subsystem 3	Transition to	Maintenance Possibility	
S.NO	Status					C.M	O.M
1	Working	O	O	O	2 4 9	-	-
2	Working	O	O	D	3 5 10	-	-
3	Under Repair	O	O	F	2	Yes(S_3)	No
4	Working	O	D	O	5 7 12	-	-
5	Working	O	D	D	6 8 13	-	-
6	Under Repair	O	D	F	5	Yes(S_3)	Yes(S_2)
7	Under Repair	O	F	O	4	Yes(S_2)	No
8	Under Repair	O	F	D	5	Yes(S_2)	Yes(S_3)
9	Working	D	O	O	17 12 10	-	-
10	Working	D	O	D	18 13 11	-	-
11	Under Repair	D	O	F	10	Yes(S_3)	Yes(S_1)
12	Working	D	D	O	19 15 13	-	-
13	Working	D	D	D	20 16 14	-	-
14	Under Repair	D	D	F	13	Yes(S_3)	Yes($S_{1,2}$)
15	Under Repair	D	F	O	12	Yes(S_2)	Yes(S_1)
16	Under Repair	D	F	D	13	Yes(S_2)	Yes($S_{1,3}$)
17	Under Repair	F	O	O	9	Yes(S_1)	No
18	Under Repair	F	O	D	10	Yes(S_1)	Yes(S_3)
19	Under Repair	F	D	O	12	Yes(S_1)	Yes(S_2)
20	Under Repair	F	D	D	13	Yes(S_1)	Yes($S_{2,3}$)

5.2.3.1 System Model – Corrective maintenance (imperfect repair) with opportunistic maintenance

In this case, the opportunistic maintenance is considered along with corrective repair. In a system with three subsystems in series, out of the twenty possible system states (Table 5.2), twelve states (3, 11, 6 to 8, 14 to 20) are the 'under repair' system states, while the remaining eight states are the 'working states'. A system model is developed considering subsystem degradation, corrective repair (imperfect repair) with opportunistic maintenance and is shown in the Figure 5.7.

There are four types of transition edges in the system model. These legends are similar to model developed in case of perfect repair. The system being in the 'under repair' states, 3, 7 and 17, only corrective repair (imperfect repair) is carried out for these as the opportunistic maintenance is not possible because the other subsystems are still in state 'O', i.e. good condition. In the system model, the corrective repair (imperfect repair) for these states is represented by lines 3 →2, 7 →4, 17 →9 for the subsystems respectively, restoring the subsystem which is under repair from its state, F to D. For the remaining nine 'under repair' states (6, 8, 11, 14, 15, 16, 18, 19, 20), the opportunistic maintenance is possible because the other subsystems are in state D, i.e. degraded state.

In the system model, the opportunistic maintenance with corrective repair (imperfect repair) is shown for lines 6-2, 8-4, 11-2, 14-2, 15-4, 16-4, 18-9, 19-9 and 20-9. These nine lines do represent corrective repair (imperfect repair), restoring the under repair system from its state, F to D. Also, these lines represents the restoration of the subsystems which is under opportunistic maintenance, from state D to O.

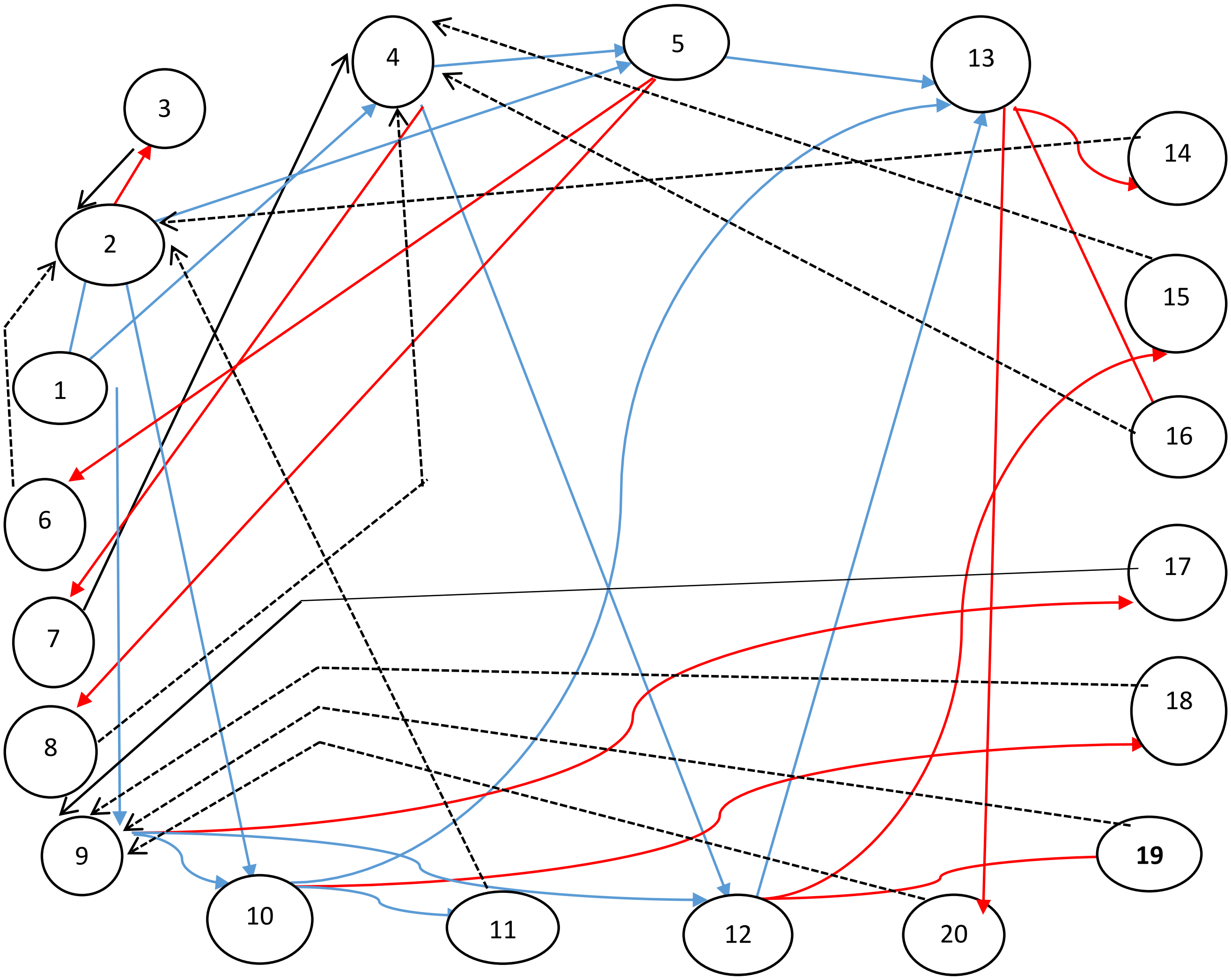

Figure 5.7: System model with corrective imperfect repair and opportunistic maintenance.

Following the methodology suggested in Section 4.3, the solution of model is obtained by Markov approach.

$dP_1/dt = -(\lambda_{12} + \lambda_{14} + \lambda_{19}) P_1(t)$

$dP_2/dt = -(\lambda_{25} + \lambda_{23} + \lambda_{210}) P_2(t) + \lambda_{12} P_1(t) + \mu_{32} P_3(t) + \mu_{62} P_6(t) + \mu_{112} P_{11}(t) + \mu_{142} P_{14}(t)$

$dP_3/dt = \lambda_{23} P_2(t) - \mu_{32} P_3(t)$

$dP_4/dt = -(\lambda_{45} + \lambda_{47} + \lambda_{412}) P_4(t) + \lambda_{14} P_1(t) + \mu_{74} P_7(t) + \mu_{84} P_8(t) + \mu_{154} P_{15}(t) + \mu_{164} P_{16}(t)$

$dP_5/dt = -(\lambda_{56} + \lambda_{58} + \lambda_{513}) P_5(t) + \lambda_{25} P_2(t) + \lambda_{45} P_4(t)$

$dP_6/dt = \lambda_{56} P_5(t) - \mu_{62} P_6(t)$

$dP_7/dt = \lambda_{47} P_4(t) - \mu_{74} P_7(t)$

$dP_8 /dt = \lambda_{58} P_5 (t) - \mu_{84} P_8 (t)$

$dP_9 /dt =-(\lambda_{910} +\lambda_{912} +\lambda_{917}) P_9 (t) + \lambda_{19} P_1 (t) +\mu_{179} P_{17} (t) + \mu_{189} P_{18} (t) + \mu_{199} P_{19} (t) + \mu_{209} P_{20} (t)$

$dP_{10} /dt =-(\lambda_{1011} +\lambda_{1013} +\lambda_{1018}) P_{10} (t) + \lambda_{210} P_2 (t) + \lambda_{910} P_9 (t)$

$dP_{11} /dt = \lambda_{1011} P_{10} (t) - \mu_{112} P_{11} (t)$

$dP_{12} /dt =-(\lambda_{1213} +\lambda_{1215} +\lambda_{1219}) P_{12} (t) + \lambda_{412} P_4 (t) + \lambda_{912} P_9 (t)$

$dP_{13} /dt =-(\lambda_{1314} +\lambda_{1316} +\lambda_{1320}) P_{13} (t) + \lambda_{513} P_5 (t) + \lambda_{1013} P_{10} (t) + \lambda_{1213} P_{12} (t)$

$dP_{14} /dt = \lambda_{1314} P_{13} (t) - \mu_{142} P_{14} (t)$

$dP_{15} /dt = \lambda_{1215} P_{12} (t) - \mu_{154} P_{15} (t)$

$dP_{16} /dt = \lambda_{1316} P_{13} (t) - \mu_{164} P_{16} (t)$

$dP_{17} /dt = \lambda_{917} P_9 (t) - \mu_{179} P_{17} (t)$

$dP_{18} /dt = \lambda_{1018} P_{10} (t) - \mu_{189} P_{18} (t)$

$dP_{19} /dt = \lambda_{1219} P_{12} (t) - \mu_{199} P_{19} (t)$

$dP_{20} /dt = \lambda_{1320} P_{13} (t) - \mu_{209} P_{20} (t)$

5.2.3.2 System model – Corrective maintenance (imperfect repair) without opportunistic maintenance

In this subsection, system model with corrective maintenance but without opportunistic maintenance is developed. This model is developed from the model developed in previous subsection. The model with corrective repair is obtained as shown in Figure 5.8, with removal of the lines 6-2, 8-4, 11-2, 14-4, 15-4, 16-9, 18-9, 19-9, 20-9, from the model for corrective maintenance (imperfect repair) with opportunistic maintenance, but including new lines 6-5, 8-5, 11-10, 14-13, 15-12, 16-13, 18-10, 19-12 and 20-13.

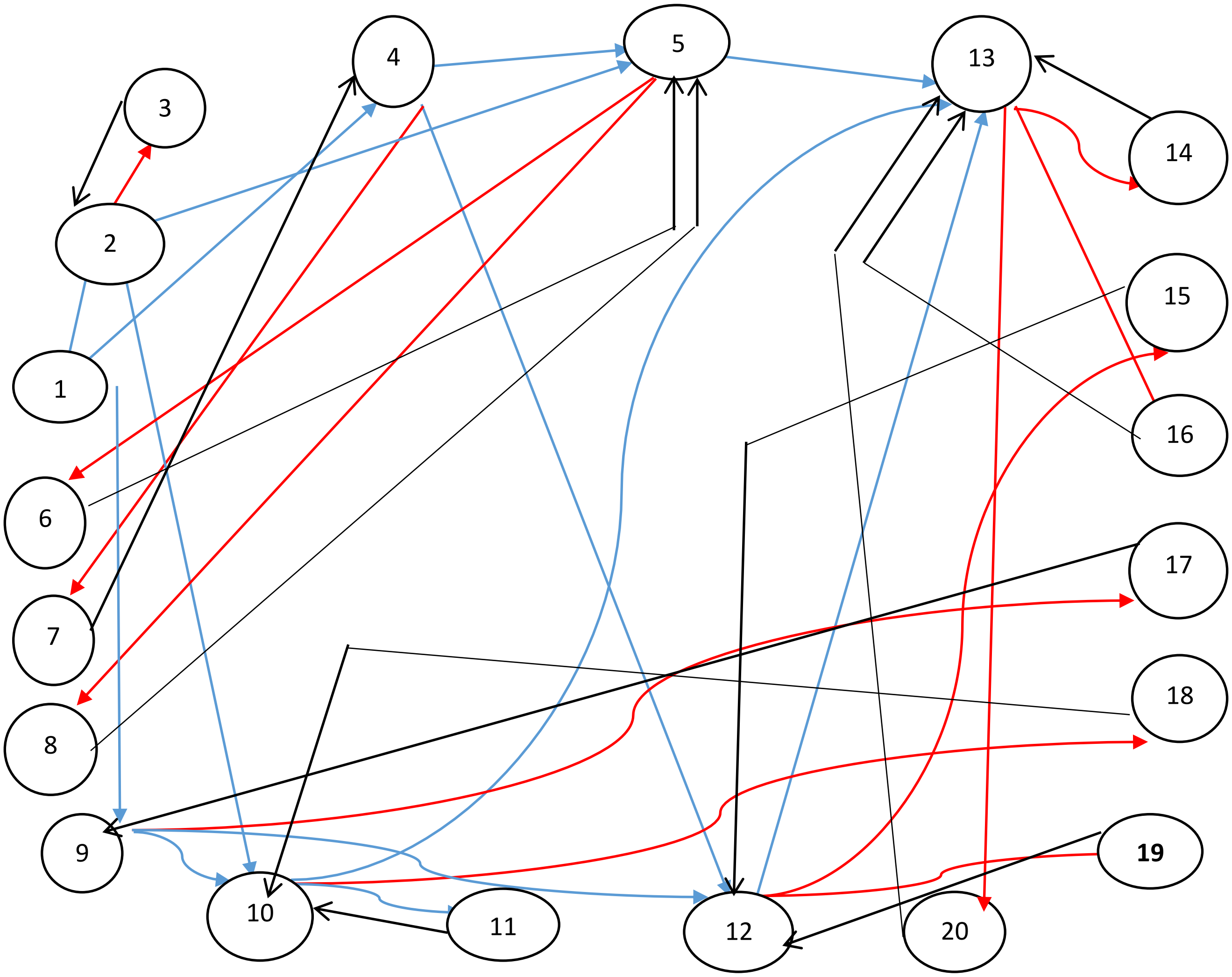

Figure 5.8: System model with corrective imperfect repair.

Following the methodology suggested in Section 4.3, the solution is obtained using Markov approach. The following rate equations are derived.

$dP_1/dt = -(\lambda_{12} + \lambda_{14} + \lambda_{19})\, P_1(t)$

$dP_2/dt = -(\lambda_{25} + \lambda_{23} + \lambda_{210})\, P_2(t) + \lambda_{12}\, P_1(t) + \mu_{32}\, P_3(t)$

$dP_3/dt = \lambda_{23}\, P_2(t) - \mu_{32}\, P_3(t)$

$dP_4/dt = -(\lambda_{45} + \lambda_{47} + \lambda_{412})\, P_4(t) + \lambda_{14}\, P_1(t) + \mu_{74}\, P_7(t)$

$dP_5/dt = -(\lambda_{56} + \lambda_{58} + \lambda_{513})\, P_5(t) + \lambda_{25}\, P_2(t) + \lambda_{45}\, P_4(t) + \mu_{65}\, P_6(t) + \mu_{85}\, P_8(t)$

$dP_6/dt = \lambda_{56}\, P_5(t) - \mu_{65}\, P_6(t)$

$dP_7 /dt = \lambda_{47} P_4 (t) - \mu_{74} P_7 (t)$

$dP_8 /dt = \lambda_{58} P_5 (t) - \mu_{85} P_8 (t)$

$dP_9 /dt =-(\lambda_{910} +\lambda_{912} +\lambda_{917}) P_9 (t) + \lambda_{19} P_1 (t) +\mu_{179} P_{17} (t)$

$dP_{10} /dt =-(\lambda_{1011} +\lambda_{1013} +\lambda_{1018}) P_{10} (t) + \lambda_{210} P_2 (t) + \lambda_{910} P_9 (t) + \mu_{1110} P_{11} (t) + \mu_{1810} P_{18} (t)$

$dP_{11} /dt = \lambda_{1011} P_{10} (t) - \mu_{1110} P_{11} (t)$

$dP_{12} /dt =-(\lambda_{1213} +\lambda_{1215} +\lambda_{1219}) P_{12} (t) + \lambda_{412} P_4 (t) + \lambda_{912} P_9 (t) + \mu_{1512} P_{15} (t) + \mu_{1912} P_{19} (t)$

$dP_{13} /dt =-(\lambda_{1314} +\lambda_{1316} +\lambda_{1320}) P_{13} (t) + \lambda_{513} P_5 (t) + \lambda_{1013} P_{10} (t) + \lambda_{1213} P_{12} (t) + \mu_{1413} P_{14} (t) + \mu_{1613} P_{16} (t) + \mu_{2013} P_{20} (t)$

$dP_{14} /dt = \lambda_{1314} P_{13} (t) - \mu_{1413} P_{14} (t)$

$dP_{15} /dt = \lambda_{1215} P_{12} (t) - \mu_{1512} P_{15} (t)$

$dP_{16} /dt = \lambda_{1316} P_{13} (t) - \mu_{1613} P_{16} (t)$

$dP_{17} /dt = \lambda_{917} P_9 (t) - \mu_{179} P_{17} (t)$

$dP_{18} /dt = \lambda_{1018} P_{10} (t) - \mu_{1810} P_{18} (t)$

$dP_{19} /dt = \lambda_{1219} P_{12} (t) - \mu_{1912} P_{19} (t)$

$dP_{20} /dt = \lambda_{1320} P_{13} (t) - \mu_{2013} P_{20} (t)$

CHAPTER 6

MATHEMATICAL MODELING USING SEMI- MARKOV APPROACH

For mechanical systems and their components, the degradation rate increases with the aging process. A weibull distribution for time to failure is appropriate for such systems. The semi-Markov approach is capable to handle non-exponential distributions. Therefore, Semi-Markov process (SMP) based availability analysis is carried out. A steady-state solution of the SMP is obtained by a two-stage analytical approach.

6.1 STEADY-STATE ANALYSIS OF SMP

In this section SMP approach is presented for steady-state solution.

6.1.1 Overview of Semi-Markov Process:

An SMP is defined as a sequence of two-dimensional random variables, $\{(X_K, T_K) : k \epsilon 1, 2, \ldots\ldots, n\}$, with the following properties:

- X_K represents its state after k transitions and is a discrete-time Markov chain taking values in a countable set, **Ω**, of possible states of system.
- The holding time, T_K-T_{K-1}, between two transitions is a random variable whose distribution depends only on the present state X_K and the state after the next transition.

In the following subsection, the steady state analytical solution of the SMP model using a two-stage method is detailed.

6.1.2 Two-Stage Analytical Solution

Let the SMP model consists of n number of states, which are numbered sequentially. The state space is denoted as **Ω** = {1, 2, 3, ……, n}. The SMP model is a graphical representation of all possible states interconnected by the transition arcs showing the respective failure and repair time cumulative density function. The two-stage method, which is easy to implement, is selected to solve the SMP model for a repairable mechanical system.

In the two-stage method for the steady–state solution, stage 1 deals with evaluation of the one-step transition probability matrix **Z** of the embedded Markov chain (EMC) of the SMP model. The one-step transition probability matrix is used to obtain steady-state probabilities, v_i, of the EMC.

In stage 2, sojourn times T_i, in each state are evaluated. The steady-state probability, P_i, of each state of the SMP model is obtained substituting the values of steady-state probabilities, v_i, of the EMC and sojourn times values of the each state in the following equation:

$$P_i = (v_i T_i) / \sum_{i=\Omega} (v_i\, T_i)\ ,\ \mathrm{i} \in \Omega$$

The system steady-state availability is evaluated as the summation of the steady-state probability, P_i, of all the working states of the SMP model.

6.2 Solution of system models using SMP

6.2.1 System SMP Model - Corrective maintenance (perfect repair) without opportunistic maintenance

In this section, the solution of the model developed in previous chapter is obtained using SMP approach. Refer Section 5.2.2.2 and Figure 5.5 for the system model.

For convenience, the twenty states of the availability model are numbered sequentially and the state space is denoted as **Ω** = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}. The SMP model is derived in terms of states and the transitions connecting these states showing the respective failure and repair time cumulative density function (cdf).

System Failure and Repair Distribution

The cdfs of the time spent in each state are given in Table 6.1. In this Table, $F_{ij,}$ denotes the cdf associated with the arc i to j, (i,j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20).

Two-Stage Analytical Approach for Steady-State Analysis

Stage I

(a) Kernel Matrix K(t).

The SMP model is completely described by its kernel matrix K(t), whose elements, $K_{ij}(t)$ are evaluated as per the procedure explained in the next section. The matrix shows zero and remaining non-zero elements, $K_{ij.}$

$$
K(t) = \begin{bmatrix}
0 & K_{12} & 0 & K_{14} & 0 & 0 & 0 & 0 & K_{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{23} & 0 & K_{25} & 0 & 0 & 0 & 0 & K_{210} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & K_{45} & 0 & K_{47} & 0 & 0 & 0 & 0 & K_{412} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & K_{56} & 0 & K_{58} & 0 & 0 & 0 & 0 & K_{513} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K_{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_{82} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{910} & 0 & K_{912} & 0 & 0 & 0 & 0 & K_{917} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{1011} & 0 & K_{1013} & 0 & 0 & 0 & 0 & K_{1018} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{119} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{1213} & 0 & K_{1215} & 0 & 0 & 0 & K_{1219} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{1314} & 0 & K_{1316} & 0 & 0 & 0 & K_{1320} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{1412} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{159} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{1610} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K_{171} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_{182} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K_{194} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & K_{205} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Table 6.1: cdf and Parameter Values for the multi-stage reciprocating air compression system (perfect repair without opportunistic maintenance)

cdf	Distribution	Parameter
F_{12}	Weibull	B_{12}=3.78 θ_{12}=10000
F_{14}	Weibull	B_{14}=1.2 θ_{12}=6000
F_{19}	Weibull	B_{19}=2.5 θ_{19}=16000
F_{23}	Weibull	B_{23}=3.78 θ_{23}=4892
F_{25}	Weibull	B_{25}= 1.2 θ_{25}=6000
F_{210}	Weibull	B_{210}=2.5 θ_{210}=16000
F_{31}	Exponential	μ_{31}=1/600
F_{45}	Weibull	B_{45}=3.78 θ_{45}=10000
F_{47}	Weibull	B_{47}= 1.2 θ_{47}=4000
F_{412}	Weibull	B_{412}= 2.5 θ_{412}=16000
F_{56}	Weibull	B_{56}= 3.78 θ_{56}=4892
F_{58}	Weibull	B_{58}= 1.2 θ_{58}=4000
F_{513}	Weibull	B_{513}= 2.5 θ_{513}=16000
F_{64}	Exponential	μ_{64}=1/600
F_{71}	Exponential	μ_{71}=1/600
F_{82}	Exponential	μ_{82}=1/600
F_{910}	Weibull	B_{910}=3.78 θ_{910}=10000
F_{912}	Weibull	B_{912}= 1.2 θ_{912}=6000
F_{917}	Weibull	B_{917}= 2.5 θ_{917}=6776
F_{1011}	Weibull	B_{1011}= 3.78 θ_{1011}=4892
F_{1013}	Weibull	B_{1013}= 1.2 θ_{1013}=6000
F_{1018}	Weibull	B_{1018}= 2.5 θ_{1018}=6776
F_{119}	Exponential	μ_{119}=1/600
F_{1213}	Weibull	B_{1213}= 3.78 θ_{1213}=10000
F_{1215}	Weibull	B_{1215}= 1.2 θ_{1215}=4000
F_{1219}	Weibull	B_{1219}= 2.5 θ_{1219}=6776
F_{1314}	Weibull	B_{1314}= 3.78 θ_{1314}=4892
F_{1316}	Weibull	B_{1316}= 1.2 θ_{1316}=4000
F_{1320}	Weibull	B_{1320}= 2.5 θ_{1320}=6776
F_{1412}	Exponential	μ_{1412}=1/600
F_{159}	Exponential	μ_{159}=1/600
F_{1610}	Exponential	μ_{1610}=1/600
F_{171}	Exponential	μ_{171}=1/600
F_{182}	Exponential	μ_{182}=1/600
F_{194}	Exponential	μ_{194}=1/600
F_{205}	Exponential	μ_{205}=1/600

Using the distribution given in the Table 6.1, the non-zero elements of kernel matrix K (t), are derived for the SMP model (Figure 5.5).

For Example, for the element K_{12}, refer to Table 6.1 for its distribution. Because there are three outgoing transition and the distribution is weibull, the expression for the element K_{12} in terms of weibull parameters is given as follows:

$$K_{12} = \int_0^t \bar{F}_{14}\bar{F}_{19}dF_{12} = \frac{\beta_{12}}{(\theta_{12})^{\beta_{12}}}\int_0^t t^{(\beta_{12}-1)}\ e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]}dt$$

For the element $K_{31,}$ refer to Table 6.1 for distributions. Because there is only one outgoing transition and the distribution is exponential, the expression for the element K_{31} in terms of exponential parameters is given as follows:

$$K_{31} = F_{31}(t) = 1 - e^{-(\mu_{31}t)}$$

Other remaining non-zero elements of the matrix K (t) are derived as explained above, and all these expressions are listed in Table 6.2.

Table 6.2: Expressions of elements of K (t)

K_{ij}	$K_{ij}(t)$	$K_{ij}(t)$ with specific distribution
K_{12}	$\int_0^t \bar{F}_{14}\bar{F}_{19}dF_{12}$	$\frac{\beta_{12}}{(\theta_{12})^{\beta_{12}}}\int_0^t t^{(\beta_{12}-1)}\ e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]}dt$
K_{14}	$\int_0^t \bar{F}_{12}\bar{F}_{19}dF_{14}$	$\frac{\beta_{14}}{(\theta_{14})^{\beta_{14}}}\int_0^t t^{(\beta_{14}-1)}\ e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]}dt$
K_{19}	$\int_0^t \bar{F}_{14}\bar{F}_{12}dF_{19}$	$\frac{\beta_{19}}{(\theta_{19})^{\beta_{19}}}\int_0^t t^{(\beta_{19}-1)}\ e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]}dt$
K_{23}	$\int_0^t \bar{F}_{25}\bar{F}_{210}dF_{23}$	$\frac{\beta_{23}}{(\theta_{23})^{\beta_{23}}}\int_0^t t^{(\beta_{23}-1)}\ e^{-\left[\left(\frac{t}{\theta_{23}}\right)^{\beta_{23}}+\left(\frac{t}{\theta_{25}}\right)^{\beta_{25}}+\left(\frac{t}{\theta_{210}}\right)^{\beta_{210}}\right]}dt$
K_{25}	$\int_0^t \bar{F}_{23}\bar{F}_{210}dF_{25}$	$\frac{\beta_{25}}{(\theta_{25})^{\beta_{25}}}\int_0^t t^{(\beta_{25}-1)}\ e^{-\left[\left(\frac{t}{\theta_{23}}\right)^{\beta_{23}}+\left(\frac{t}{\theta_{25}}\right)^{\beta_{25}}+\left(\frac{t}{\theta_{210}}\right)^{\beta_{210}}\right]}dt$

K_{210}	$\int_0^t \bar{F}_{25}\bar{F}_{23}dF_{210}$	$\frac{\beta_{210}}{(\theta_{210})^{\beta_{210}}}\int_0^t t^{(\beta_{210}-1)}\ e^{-\left[\left(\frac{t}{\theta_{23}}\right)^{\beta_{23}}+\left(\frac{t}{\theta_{25}}\right)^{\beta_{25}}+\left(\frac{t}{\theta_{210}}\right)^{\beta_{210}}\right]}dt$
K_{31}	$F_{31}(t)$	$1-e^{-(\mu_{31}t)}$
K_{45}	$\int_0^t \bar{F}_{47}\bar{F}_{412}dF_{45}$	$\frac{\beta_{45}}{(\theta_{45})^{\beta_{45}}}\int_0^t t^{(\beta_{45}-1)}\ e^{-\left[\left(\frac{t}{\theta_{45}}\right)^{\beta_{45}}+\left(\frac{t}{\theta_{47}}\right)^{\beta_{47}}+\left(\frac{t}{\theta_{412}}\right)^{\beta_{412}}\right]}dt$
K_{47}	$\int_0^t \bar{F}_{45}\bar{F}_{412}dF_{47}$	$\frac{\beta_{47}}{(\theta_{47})^{\beta_{47}}}\int_0^t t^{(\beta_{47}-1)}\ e^{-\left[\left(\frac{t}{\theta_{45}}\right)^{\beta_{45}}+\left(\frac{t}{\theta_{47}}\right)^{\beta_{47}}+\left(\frac{t}{\theta_{412}}\right)^{\beta_{412}}\right]}dt$
K_{412}	$\int_0^t \bar{F}_{47}\bar{F}_{45}dF_{412}$	$\frac{\beta_{412}}{(\theta_{412})^{\beta_{412}}}\int_0^t t^{(\beta_{412}-1)}\ e^{-\left[\left(\frac{t}{\theta_{45}}\right)^{\beta_{45}}+\left(\frac{t}{\theta_{47}}\right)^{\beta_{47}}+\left(\frac{t}{\theta_{412}}\right)^{\beta_{412}}\right]}dt$
K_{56}	$\int_0^t \bar{F}_{58}\bar{F}_{513}dF_{56}$	$\frac{\beta_{56}}{(\theta_{56})^{\beta_{56}}}\int_0^t t^{(\beta_{56}-1)}\ e^{-\left[\left(\frac{t}{\theta_{56}}\right)^{\beta_{56}}+\left(\frac{t}{\theta_{58}}\right)^{\beta_{58}}+\left(\frac{t}{\theta_{513}}\right)^{\beta_{513}}\right]}dt$
K_{58}	$\int_0^t \bar{F}_{56}\bar{F}_{513}dF_{58}$	$\frac{\beta_{58}}{(\theta_{58})^{\beta_{58}}}\int_0^t t^{(\beta_{58}-1)}\ e^{-\left[\left(\frac{t}{\theta_{56}}\right)^{\beta_{56}}+\left(\frac{t}{\theta_{58}}\right)^{\beta_{58}}+\left(\frac{t}{\theta_{513}}\right)^{\beta_{513}}\right]}dt$
K_{513}	$\int_0^t \bar{F}_{58}\bar{F}_{56}dF_{513}$	$\frac{\beta_{513}}{(\theta_{513})^{\beta_{513}}}\int_0^t t^{(\beta_{513}-1)}\ e^{-\left[\left(\frac{t}{\theta_{56}}\right)^{\beta_{56}}+\left(\frac{t}{\theta_{58}}\right)^{\beta_{58}}+\left(\frac{t}{\theta_{513}}\right)^{\beta_{513}}\right]}dt$
K_{64}	$F_{64}(t)$	$1-e^{-(\mu_{64}t)}$
K_{71}	$F_{71}(t)$	$1-e^{-(\mu_{71}t)}$
K_{82}	$F_{82}(t)$	$1-e^{-(\mu_{82}t)}$
K_{910}	$\int_0^t \bar{F}_{912}\bar{F}_{917}dF_{910}$	$\frac{\beta_{910}}{(\theta_{910})^{\beta_{910}}}\int_0^t t^{(\beta_{910}-1)}\ e^{-\left[\left(\frac{t}{\theta_{910}}\right)^{\beta_{910}}+\left(\frac{t}{\theta_{912}}\right)^{\beta_{912}}+\left(\frac{t}{\theta_{917}}\right)^{\beta_{917}}\right]}dt$
K_{912}	$\int_0^t \bar{F}_{910}\bar{F}_{917}dF_{912}$	$\frac{\beta_{912}}{(\theta_{912})^{\beta_{912}}}\int_0^t t^{(\beta_{912}-1)}\ e^{-\left[\left(\frac{t}{\theta_{910}}\right)^{\beta_{910}}+\left(\frac{t}{\theta_{912}}\right)^{\beta_{912}}+\left(\frac{t}{\theta_{917}}\right)^{\beta_{917}}\right]}dt$
K_{917}	$\int_0^t \bar{F}_{912}\bar{F}_{910}dF_{917}$	$\frac{\beta_{917}}{(\theta_{917})^{\beta_{917}}}\int_0^t t^{(\beta_{917}-1)}\ e^{-\left[\left(\frac{t}{\theta_{910}}\right)^{\beta_{910}}+\left(\frac{t}{\theta_{912}}\right)^{\beta_{912}}+\left(\frac{t}{\theta_{917}}\right)^{\beta_{917}}\right]}dt$
K_{1011}	$\int_0^t \bar{F}_{1013}\bar{F}_{1018}dF_{1011}$	$\frac{\beta_{1011}}{(\theta_{1011})^{\beta_{1011}}}\int_0^t t^{(\beta_{1011}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1011}}\right)^{\beta_{1011}}+\left(\frac{t}{\theta_{1013}}\right)^{\beta_{1013}}+\left(\frac{t}{\theta_{1018}}\right)^{\beta_{1018}}\right]}dt$
K_{1013}	$\int_0^t \bar{F}_{1011}\bar{F}_{1018}dF_{1013}$	$\frac{\beta_{1013}}{(\theta_{1013})^{\beta_{1013}}}\int_0^t t^{(\beta_{1013}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1011}}\right)^{\beta_{1011}}+\left(\frac{t}{\theta_{1013}}\right)^{\beta_{1013}}+\left(\frac{t}{\theta_{1018}}\right)^{\beta_{1018}}\right]}dt$

K_{1018}	$\int_0^t \bar{F}_{1013}\bar{F}_{1011}dF_{1018}$	$\frac{\beta_{1018}}{(\theta_{1018})^{\beta_{1018}}}\int_0^t t^{(\beta_{1018}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1011}}\right)^{\beta_{1011}}+\left(\frac{t}{\theta_{1013}}\right)^{\beta_{1013}}+\left(\frac{t}{\theta_{1018}}\right)^{\beta_{1018}}\right]}dt$
K_{119}	$F_{119}(t)$	$1-e^{-(\mu_{119}t)}$
K_{1213}	$\int_0^t \bar{F}_{1215}\bar{F}_{1219}dF_{1213}$	$\frac{\beta_{1213}}{(\theta_{1213})^{\beta_{1213}}}\int_0^t t^{(\beta_{1213}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1213}}\right)^{\beta_{1213}}+\left(\frac{t}{\theta_{1215}}\right)^{\beta_{1215}}+\left(\frac{t}{\theta_{1219}}\right)^{\beta_{1219}}\right]}dt$
K_{1215}	$\int_0^t \bar{F}_{1213}\bar{F}_{1219}dF_{1215}$	$\frac{\beta_{1215}}{(\theta_{1215})^{\beta_{1215}}}\int_0^t t^{(\beta_{1215}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1213}}\right)^{\beta_{1213}}+\left(\frac{t}{\theta_{1215}}\right)^{\beta_{1215}}+\left(\frac{t}{\theta_{1219}}\right)^{\beta_{1219}}\right]}dt$
K_{1219}	$\int_0^t \bar{F}_{1215}\bar{F}_{1213}dF_{1219}$	$\frac{\beta_{1219}}{(\theta_{1219})^{\beta_{1219}}}\int_0^t t^{(\beta_{1219}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1213}}\right)^{\beta_{1213}}+\left(\frac{t}{\theta_{1215}}\right)^{\beta_{1215}}+\left(\frac{t}{\theta_{1219}}\right)^{\beta_{1219}}\right]}dt$
K_{1314}	$\int_0^t \bar{F}_{1316}\bar{F}_{1320}dF_{1314}$	$\frac{\beta_{1314}}{(\theta_{1314})^{\beta_{1314}}}\int_0^t t^{(\beta_{1314}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1314}}\right)^{\beta_{1314}}+\left(\frac{t}{\theta_{1316}}\right)^{\beta_{1316}}+\left(\frac{t}{\theta_{1320}}\right)^{\beta_{1320}}\right]}dt$
K_{1316}	$\int_0^t \bar{F}_{1314}\bar{F}_{1320}dF_{1316}$	$\frac{\beta_{1316}}{(\theta_{1316})^{\beta_{1316}}}\int_0^t t^{(\beta_{1316}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1314}}\right)^{\beta_{1314}}+\left(\frac{t}{\theta_{1316}}\right)^{\beta_{1316}}+\left(\frac{t}{\theta_{1320}}\right)^{\beta_{1320}}\right]}dt$
K_{1320}	$\int_0^t \bar{F}_{1316}\bar{F}_{1314}dF_{1320}$	$\frac{\beta_{1320}}{(\theta_{1320})^{\beta_{1320}}}\int_0^t t^{(\beta_{1320}-1)}\ e^{-\left[\left(\frac{t}{\theta_{1314}}\right)^{\beta_{1314}}+\left(\frac{t}{\theta_{1316}}\right)^{\beta_{1316}}+\left(\frac{t}{\theta_{1320}}\right)^{\beta_{1320}}\right]}dt$
K_{1412}	$F_{1412}(t)$	$1-e^{-(\mu_{1412}t)}$
K_{159}	$F_{159}(t)$	$1-e^{-(\mu_{159}t)}$
K_{1610}	$F_{1610}(t)$	$1-e^{-(\mu_{1610}t)}$
K_{171}	$F_{171}(t)$	$1-e^{-(\mu_{171}t)}$
K_{182}	$F_{182}(t)$	$1-e^{-(\mu_{182}t)}$
K_{194}	$F_{194}(t)$	$1-e^{-(\mu_{194}t)}$
K_{205}	$F_{205}(t)$	$1-e^{-(\mu_{205}t)}$

(b) One-Step transition probability matrix Z of the EMC.

The kernel matrix K (t) is used to evaluated the one-step transition probability matrix, **Z** = K (∞), of the EMC of the SMP considering $t \rightarrow \infty$. To evaluate the elements of Z matrix, there is an additional condition that for each row the sum of elements of Z becomes 1. Because there is only single non-zero element in the 3rd, 6th, 7th, 8th, 11th, 14th, 15th, 16th, 17th, 18th, 19th, and 20th row, therefore, $K_{31}(\infty) = 1$, $K_{64}(\infty) = 1$, $K_{71}(\infty) = 1$, $K_{82}(\infty) = 1$, $K_{119}(\infty) = 1$, $K_{1412}(\infty) = 1$; $K_{159}(\infty) = 1$, $K_{1610}(\infty) = 1$; $K_{171}(\infty) = 1$, $K_{182}(\infty) = 1$, $K_{194}(\infty) = 1$ and $K_{205}(\infty) = 1$.

The expressions for $K_{ij}(t)$ with specific distribution and distribution parameter values in the Table 6.1 are used to evaluate the remaining non-zero matrix elements of matrix **Z.** Matlab software (version 7.6.0.324) is used to evaluate the integrals. The evaluated non-zero elements of the matrix **Z** are given in Table 6.3.

Table 6.3: Non-zero elements of the matrix, Z

K_{ij}	value	K_{ij}	value	K_{ij}	value	K_{ij}	value
K_{12}	0.1885	K_{47}	0.8678	K_{917}	0.3560	K_{1314}	0.2610
K_{14}	0.7367	K_{412}	0.0426	K_{1011}	0.3743	K_{1316}	0.5993
K_{19}	0.0748	K_{56}	0.3336	K_{1013}	0.4397	K_{1320}	0.1397
K_{23}	0.4915	K_{58}	0.6741	K_{1018}	0.1860		
K_{25}	0.4823	K_{513}	0.0193	K_{1213}	0.0431		
K_{210}	0.0262	K_{910}	0.0753	K_{1215}	0.7238		
K_{45}	0.0896	K_{912}	0.5687	K_{1219}	0.2331		

The complete matrix Z obtained is

$$
Z = K(\infty) = \begin{bmatrix}
0 & 0.1885 & 0 & 0.7367 & 0 & 0 & 0 & 0 & 0.0748 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.4915 & 0 & 0.4823 & 0 & 0 & 0 & 0 & 0.0262 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0896 & 0 & 0.8678 & 0 & 0 & 0 & 0 & 0.0426 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.3336 & 0 & 0.6471 & 0 & 0 & 0 & 0 & 0.0193 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0753 & 0 & 0.5687 & 0 & 0 & 0 & 0 & 0.3560 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3743 & 0 & 0.4397 & 0 & 0 & 0 & 0 & 0.1860 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0431 & 0 & 0.7238 & 0 & 0 & 0 & 0.2331 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2610 & 0 & 0.5993 & 0 & 0 & 0 & 0.1397 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(c) Steady-state probability of the states of EMC

The steady-state probabilities of the EMC are evaluated using the equation explained below for the system data given in Table 6.1. The system of equations for the model is

$[v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9\ v_{10}\ v_{11}\ v_{12}\ v_{13}\ v_{14}\ v_{15}\ v_{16}\ v_{17}\ v_{18}\ v_{19}\ v_{20}] = [v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9\ v_{10}\ v_{11}\ v_{12}\ v_{13}\ v_{14}\ v_{15}\ v_{16}\ v_{17}\ v_{18}\ v_{19}\ v_{20}] * Z$

This set of equations are to be solved using MATLAB (version 7.6.0.324) and the values of v_i, $i \in \Omega$ are listed in Table 6.4.

Table 6.4: Steady-state probabilities of the state of EMC

v_i	Probability values	v_i	Probability values	v_i	Probability values	v_i	Probability values
v_1	0.2289	v_6	0.0191	v_{11}	0.0038	v_{16}	0.0043
v_2	0.0820	v_7	0.1705	v_{12}	0.0376	v_{17}	0.0171
v_3	0.0403	v_8	0.0370	v_{13}	0.0071	v_{18}	0.0019
v_4	0.1964	v_9	0.0481	v_{14}	0.0019	v_{19}	0.0088
v_5	0.0571	v_{10}	0.0100	v_{15}	0.0272	v_{20}	0.0009

Stage 2

(a) Mean sojourn time of the states of the SMP model

The mean Sojourn time, T_i, is the time the process spends at state i. This is evaluated to obtain the steady-state probability of the state I, i.e.; p_i [8].

Sojourn times are going to be evaluated in stage 2 and these values are used for evaluation of steady-state probabilities of the system states. Sojourn time, T_i, expressions are derived using the distribution given in the Table 6.1.

Refer to Table 6.1 for the distribution of the sojourn time for state 1, T_1. Because only three states are reachable from state 1 and the distribution is Weibull, the expression of the sojourn time for state 1, T_1, is

$$T_1 = \int_0^\infty \bar{F}_{12}\bar{F}_{14}\bar{F}_{19}dt = \int_0^\infty e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]} dt$$

Refer to Table 6.2 for the distribution of the sojourn time for state 3, T_3. Because only one state is reachable from state 3 and the distribution is exponential, the expression of the sojourn time for state 3, T_3, is

$$T_3 = \int_0^\infty \bar{F}_{31}dt = \int_0^\infty e^{-(\mu_{31}t)} dt$$

Similarly, the sojourn time for the remaining states are derived and these expressions are listed in Table 6.5.

Table 6.5: Sojourn Time of System States

T_i	Sojourn time expression	Sojourn time expression with parameter value
T_1	$\int_0^\infty \bar{F}_{12}\bar{F}_{14}\bar{F}_{19}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{12}}\right)^{\beta_{12}}+\left(\frac{t}{\theta_{14}}\right)^{\beta_{14}}+\left(\frac{t}{\theta_{19}}\right)^{\beta_{19}}\right]} dt$
T_2	$\int_0^\infty \bar{F}_{23}\bar{F}_{25}\bar{F}_{210}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{23}}\right)^{\beta_{23}}+\left(\frac{t}{\theta_{25}}\right)^{\beta_{25}}+\left(\frac{t}{\theta_{210}}\right)^{\beta_{210}}\right]} dt$
T_3	$\int_0^\infty \bar{F}_{31}dt$	$\int_0^\infty e^{-(\mu_{31}t)} dt$
T_4	$\int_0^\infty \bar{F}_{45}\bar{F}_{47}\bar{F}_{412}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{45}}\right)^{\beta_{45}}+\left(\frac{t}{\theta_{47}}\right)^{\beta_{47}}+\left(\frac{t}{\theta_{412}}\right)^{\beta_{412}}\right]} dt$
T_5	$\int_0^\infty \bar{F}_{56}\bar{F}_{58}\bar{F}_{513}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{56}}\right)^{\beta_{56}}+\left(\frac{t}{\theta_{58}}\right)^{\beta_{58}}+\left(\frac{t}{\theta_{513}}\right)^{\beta_{513}}\right]} dt$
T_6	$\int_0^\infty \bar{F}_{64}dt$	$\int_0^\infty e^{-(\mu_{64}t)} dt$
T_7	$\int_0^\infty \bar{F}_{71}dt$	$\int_0^\infty e^{-(\mu_{71}t)} dt$
T_8	$\int_0^\infty \bar{F}_{82}dt$	$\int_0^\infty e^{-(\mu_{82}t)} dt$

T_9	$\int_0^\infty \bar{F}_{910}\bar{F}_{912}\bar{F}_{917}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{910}}\right)^{\beta_{910}}+\left(\frac{t}{\theta_{912}}\right)^{\beta_{912}}+\left(\frac{t}{\theta_{917}}\right)^{\beta_{917}}\right]}\ dt$
T_{10}	$\int_0^\infty \bar{F}_{1011}\bar{F}_{1013}\bar{F}_{1018}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{1011}}\right)^{\beta_{1011}}+\left(\frac{t}{\theta_{1013}}\right)^{\beta_{1013}}+\left(\frac{t}{\theta_{1018}}\right)^{\beta_{1018}}\right]}\ dt$
T_{11}	$\int_0^\infty \bar{F}_{119}dt$	$\int_0^\infty e^{-(\mu_{119}t)}\ dt$
T_{12}	$\int_0^\infty \bar{F}_{1213}\bar{F}_{1215}\bar{F}_{1219}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{1213}}\right)^{\beta_{1213}}+\left(\frac{t}{\theta_{1215}}\right)^{\beta_{1215}}+\left(\frac{t}{\theta_{1219}}\right)^{\beta_{1219}}\right]}\ dt$
T_{13}	$\int_0^\infty \bar{F}_{1314}\bar{F}_{1316}\bar{F}_{1320}dt$	$\int_0^\infty e^{-\left[\left(\frac{t}{\theta_{1314}}\right)^{\beta_{1314}}+\left(\frac{t}{\theta_{1316}}\right)^{\beta_{1316}}+\left(\frac{t}{\theta_{1320}}\right)^{\beta_{1320}}\right]}\ dt$
T_{14}	$\int_0^\infty \bar{F}_{1412}dt$	$\int_0^\infty e^{-(\mu_{1412}t)}\ dt$
T_{15}	$\int_0^\infty \bar{F}_{159}dt$	$\int_0^\infty e^{-(\mu_{159}t)}\ dt$
T_{16}	$\int_0^\infty \bar{F}_{1610}dt$	$\int_0^\infty e^{-(\mu_{1610}t)}\ dt$
T_{17}	$\int_0^\infty \bar{F}_{171}dt$	$\int_0^\infty e^{-(\mu_{171}t)}\ dt$
T_{18}	$\int_0^\infty \bar{F}_{182}dt$	$\int_0^\infty e^{-(\mu_{182}t)}\ dt$
T_{19}	$\int_0^\infty \bar{F}_{194}dt$	$\int_0^\infty e^{-(\mu_{194}t)}\ dt$
T_{20}	$\int_0^\infty \bar{F}_{205}dt$	$\int_0^\infty e^{-(\mu_{205}t)}\ dt$

Using the set of expressions explained above and the cdf and distribution parameter values given in Table 6.1, the sojourn times for all states is evaluated. Matlab software version (7.6.0.324) is used to evaluate the complex integrals occurring in sojourn time expressions. The evaluated values of sojourn time are listed in Table 6.6.

Table 6.6: Mean Sojourn times of the states of SMP model

T_i	Value (h)	T_i	Value (h)	T_i	Value (h)	T_i	Value (h)
T_1	4458.2	T_6	600	T_{11}	600	T_{16}	600
T_2	3176.8	T_7	600	T_{12}	2922.8	T_{17}	600
T_3	600	T_8	600	T_{13}	2519.4	T_{18}	600
T_4	3379	T_9	3611.9	T_{14}	600	T_{19}	600
T_5	2683.9	T_{10}	2941.2	T_{15}	600	T_{20}	600

(b) Steady-state probability of states of the SMP model

The steady-state probabilities of each state i for the SMP model are evaluated using equation described in Section 6.1.2 and the values of steady-state probabilities of EMC, v_i (Table 6.4), and the values of sojourn times, T_i (Table 6.6) as calculated above are used. The evaluated values of the steady-state probabilities of the states are listed in Table 6.7.

Table 6.7: Steady-state probabilities of the states of SMP model

P_i	Probability values	P_i	Probability values	P_i	Probability values	P_i	Probability values
P_1	0.3882	P_6	0.0044	P_{11}	0.00085842	P_{16}	0.00097674
P_2	0.0991	P_7	0.0384	P_{12}	0.0418	P_{17}	0.0039
P_3	0.0092	P_8	0.0084	P_{13}	0.0068	P_{18}	0.00042657
P_4	0.2525	P_9	0.0661	P_{14}	0.00042538	P_{19}	0.0020
P_5	0.0583	P_{10}	0.0112	P_{15}	0.0062	P_{20}	0.00022768

Availability Measure

Availability is the sum of steady-state probabilities of working states of SMP mode, i.e.

$A=P_1+ P_{2+} P_{4+} P_{5+} P_{9+} P_{10+} P_{12+} P_{13}$

The evaluated value of the steady-state availability is:

A= 0.9241

Steps explained in Section 6.2.1 is repeated for other SMP models; corrective perfect repair with opportunistic maintenance, corrective imperfect repair with opportunistic maintenance and corrective imperfect repair without opportunistic maintenance. Refer Appendix V, VI and VII for the remaining three models. The results obtained are tabulated in Table 7.1.

CHAPTER 7

RESULT & DISCUSSION

This chapter deals with the effect of opportunistic maintenance on system availability.

Markov Method

The sets of differential equations for each model is simultaneously solved with initial conditions $y_1(0)=1$, $y_2(0)=0$, $y_3(0)=0$, $y_4(0)=0$, $y_5(0)=0$, $y_6(0)=0$, $y_7(0)=0$, $y_8(0)=0$, $y_9(0)=0$, $y_{10}(0)=0$, $y_{11}(0)=0$, $y_{12}(0)=0$, $y_{13}(0)=0$, $y_{14}(0)=0$, $y_{15}(0)=0$, $y_{16}(0)=0$, $y_{17}(0)=0$, $y_{18}(0)=0$, $y_{19}(0)=0$, $y_{20}(0)=0$ and for a required mission time T. The availability (A) of the system at the end of the mission time is given by $A = y_1(T)+y_2(T)+y_4(T)+y_5(T)+y_9(T)+y_{10}(T)+y_{12}(T)+y_{13}(T)$.

Semi-Marko Method

The step by step procedure for availability analysis of system model using SMP is described in detail in previous sections. Refer to Section 6.2 for detailed analysis.

The availability values for the four models with and without opportunistic maintenance by Markov and Semi-Markov process for a mission time of 50000 hours is shown in Table 7.1.

Table 7.1: System Availability

S.No	Corrective Repair	Availability			
		Without opportunistic maintenance		With opportunistic maintenance	
		Markov	Semi-Markov	Markov	Semi-Markov
1	Perfect Repair	0.8787	0.9241	0.9083	0.9342
2	Imperfect Repair	0.7603	0.8808	0.8687	0.9062

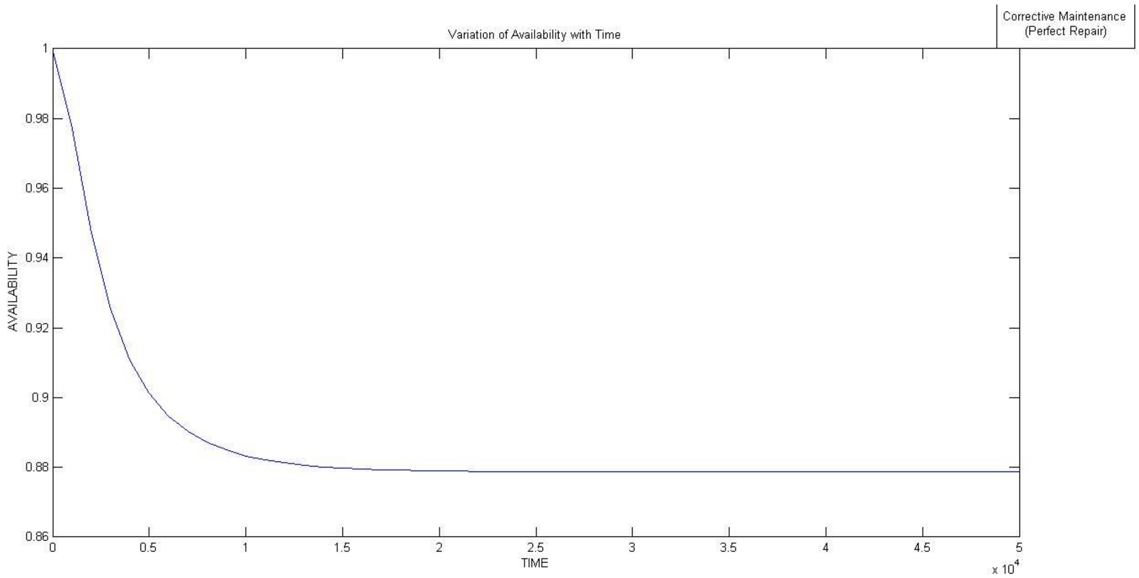

Figure 7.1: Availability- Markov model with perfect repair

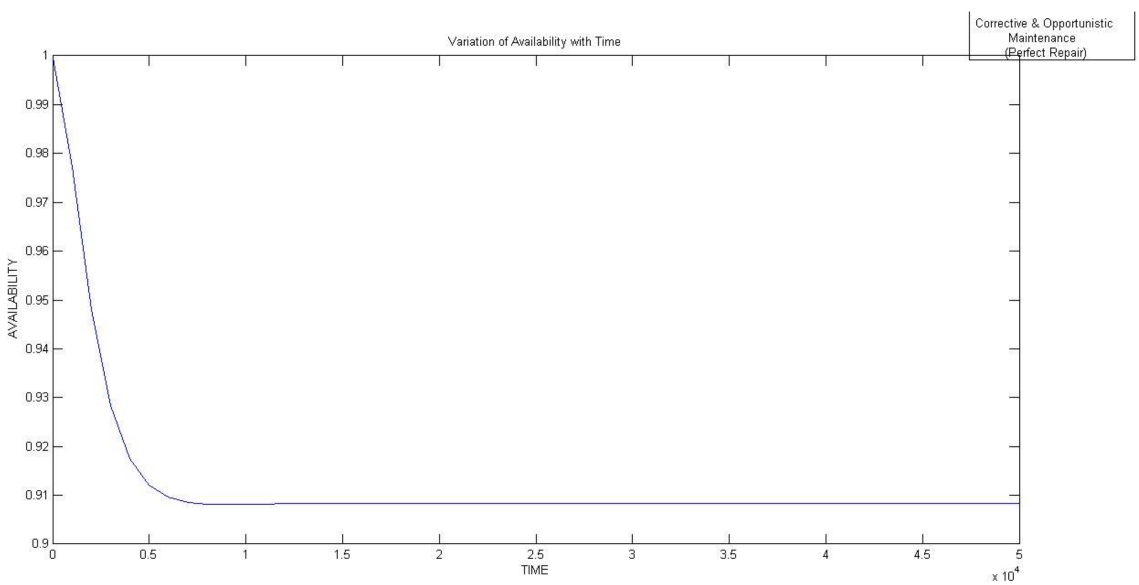

Figure 7.2: Availability- Markov model with perfect repair and opportunistic maintenance

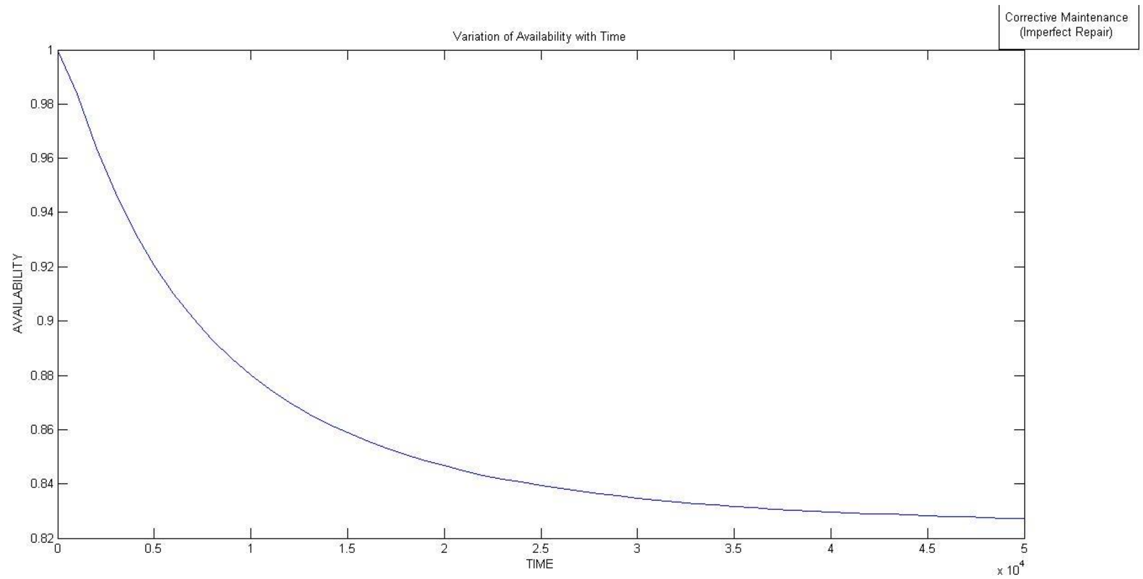

Figure 7.3: Availability- Markov model with imperfect repair

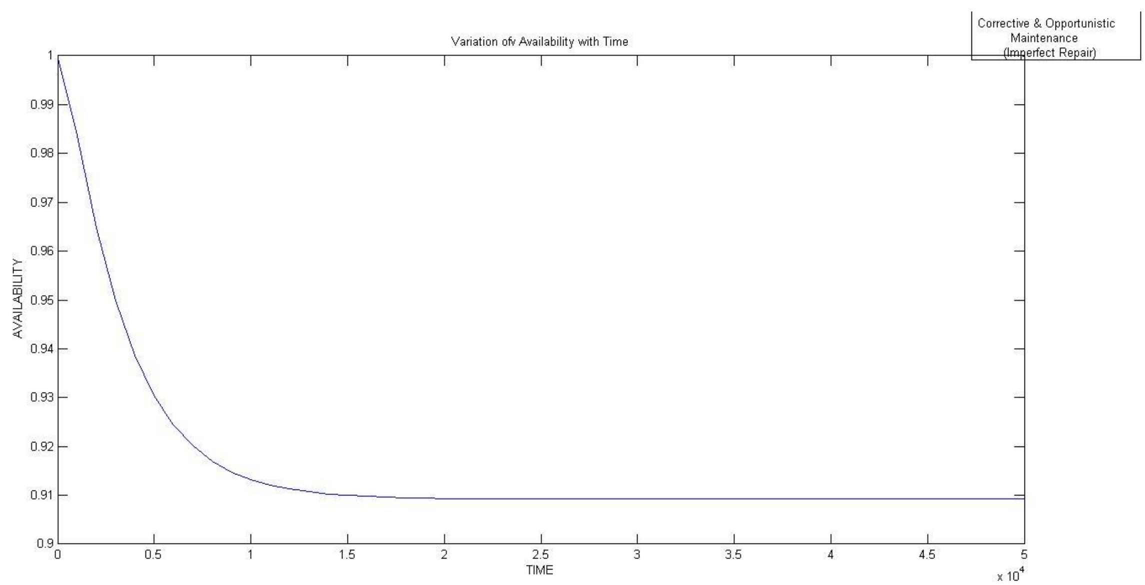

Figure 7.4: Availability- Markov model with imperfect repair and opportunistic maintenance

The results obtained provide a definite indication of the trend in the availability for different maintenance policies. These numeric results can be analyzed quantitatively to compare the relative improvement in the performance of the system in the different scenarios. For the same mission time moving from imperfect repair to perfect repair, the availability shows the increasing trend. The percentage of increase in the availability from imperfect to perfect repair in case of corrective maintenance is high in comparison to increase in availability from imperfect to perfect repair when system is under opportunistic maintenance. This clearly establishes that the imperfect repair policy is inefficient and should be seldom used unless cost of maintenance is the dictating factor.

The availability of the system increases when opportunistic maintenance is done. When the system is new, the effect of opportunistic maintenance on availability is very less and the effect increases with mission time till the steady state is reached. The system shows the slight availability increase in both types of repair. When opportunistic maintenance is considered, the system shows the same decreasing trend as it was showing in the system with corrective maintenance only.

Therefore, analysis of availability of repairable mechanical systems under different scenarios is a vital tool in creating a system/policy for a definite application to maximize its performance and the availability increases with the opportunistic maintenance.

CHAPTER-8

CONCLUSION AND SCOPE FOR FUTURE WORK

The final chapter of this project contains the conclusion of the project and the scope for improvement in this project.

8.1. CONCLUSION

The emphasis of this research was to develop models and methodologies for the availability analysis of mechanical systems with various dependency features and under select maintenance policies. This research has resulted in an alternate approach to the simulation approach for availability analysis of mechanical systems. It bridges the gaps identified from the literature and it is envisaged that it would open avenues for further research studies. The research has resulted in following major conclusions:

- An analytical framework based on Markov and Semi-Markov process for steady state availability analysis is suggested.
- SMP availability models with corrective maintenance and opportunistic maintenance under different repair scenarios are developed.
- There is gain in availability when opportunistic maintenance is combined with corrective maintenance.
- The maximum availability is observed in perfect repair when opportunistic maintenance is carried out along with corrective maintenance.
- The minimum availability is observed in imperfect repair when the system is under corrective maintenance only.

The detailed conclusions of the work are presented in the following subsections.

8.1.1 Markov process- Availability models with corrective maintenance and opportunistic maintenance

The system models are developed with opportunistic maintenance (OM) and corrective maintenance (CM) considering multi-state degradation, with degradation assumed to follow Weibull distribution. Two repair scenarios, i.e., perfect and imperfect, are considered in the model with exponential distribution.

For OM models, corrective maintenance is combined with the opportunistic maintenance. Four models are developed, out of which two models are based on corrective maintenance, i.e. perfect and imperfect repair. The rest two are developed with corrective maintenance combined with opportunistic maintenance.

8.1.2 Semi-Markov process – Analytical solution framework

Semi-Markov models, which are capable of handling non-exponential failure distributions such as, Weibull have been developed. The non-exponential distributions used in SMP models are applicable for mechanical components/subsystems. An analytical solution of the SMP model for steady state availability is suggested. A two-stage method is proposed to solve the SMP model for steady state system availability analysis. Analytical approach provides closed form solution, which is accurate and obtained in a single run under the assumptions of the model. An extended example of a multi stage reciprocating air compressor system is analyzed that showed the usefulness and portability of the approach for large scale mechanical systems.

8.2. SCOPE FOR FUTURE WORK

Unlike the studies and projects done before this, our modeling incorporates the real life factors which affect a component during its working life cycle to make it closer to reality. Also, it makes the model more practical, thereby ensuring the results obtained from its analysis are more accurate and confirmed to the actual observed values depicting a true behaviour, free from errors. Various factors considered in this project have made our model quiet close to a real one, hence, in future when such a study is taken up, a decent acceptable base model is already available in the form of this model. In the past, not much attention is paid to the above considered factors, this analysis shows how the individual parameters can contribute significantly to the enhanced availability. Hence, it can be an initiation in this regard for many firms to analyze the parameters discussed here and improve the availability of the component(s) and thereby, that of the overall system.

In this project work, only the availability analysis part is focused and discussed hence, there lies a scope for extending it for cost analysis. It will be meaningful to include it for the maintenance or repair actions. As a three component system is considered, this analysis can be extended for multi components configured in series or parallel. Further, optimization work can be carried out considering different objectives such as, cost, availability, etc. Work may be extended to optimize maintenance considering other objective functions such as: consequence of failures, performance of maintenance personnel, etc.

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APPENDIX I

Program for Markov Analysis of Perfect Repair System (without OM)

```
function dy
=cmpr(t,y,L12,L14,L19,L23,L25,L210,M31,L45,L47,L412,L56,L58,L513,M64,M71,M82,
L910,L912,L917,L1011,L1013,L1018,M119,L1213,L1215,L1219,L1314,L1316,L1320,M14
12,M159,M1610,M171,M182,M194,M205);
L12=0.000111;L14=0.000167;L19=0.0000667;L23=0.000223;L25=0.000167;L210=0.0000
667;M31=0.00166667;L45=0.000111;L47=0.0002936;L412=0.0000667;L56=0.000223;L58
=0.0002936;L513=0.0000667;M64=0.00166667;M71=0.00166667;M82=0.00166667;L910=0
.000111;L912=0.000167;L917=0.00019204;L1011=0.000223;L1013=0.000167;L1018=0.0
0019204;M119=0.00166667;L1213=0.000111;L1215=0.0002936;L1219=0.00019204;L1314
=0.000223;L1316=0.0002936;L1320=0.00019204;M1412=0.00166667;M159=0.00166667;M
1610=0.00166667;M171=0.00166667;M182=0.00166667;M194=0.00166667;M205=0.001666
67;
dy = zeros(20,1); % a column vector
y0=[1;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
dy(1) =-(L12+L14+L19)*y(1)+M31*y(3)+M71*y(7)+M171*y(17);
dy(2) =-(L25+L210+L23)*y(2)+L12*y(1)+M82*y(8)+M182*y(18);
dy(3) =-M31*y(3)+L23*y(2);
dy(4) =-(L45+L47+L412)*y(4)+L14*y(1)+M64*y(6)+M194*y(19);
dy(5) =-(L56+L58+L513)*y(5)+L25*y(2)+L45*y(4)+M205*y(20);
dy(6) = L56*y(5)-M64*y(6);
dy(7) =-M71*y(7)+L47*y(4);
dy(8) = L58*y(5)-M82*y(8);
dy(9) =-(L917+L912+L910)*y(9)+L19*y(1)+M119*y(11)+M159*y(15);
dy(10) =-(L1018+L1013+L1011)*y(10)+L210*y(2)+L910*y(9)+M1610*y(16);
dy(11) = L1011*y(10)-M119*y(11);
dy(12) =-(L1219+L1215+L1213)*y(12)+L412*y(4)+L912*y(9)+M1412*y(14);
dy(13) =-(L1320+L1316+L1314)*y(13)+L513*y(5)+L1013*y(10)+L1213*y(12);
dy(14) = L1314*y(13)-M1412*y(14);
dy(15) = L1215*y(12)-M159*y(15);
dy(16) = L1316*y(13)-M1610*y(16);
dy(17) =-M171*y(17)+L917*y(9);
dy(18) =-M182*y(18)+L1018*y(10);
dy(19) =-M194*y(19)+L1219*y(12);
dy(20) =-M205*y(20)+L1320*y(13)
```

APPENDIX II

Program for Markov Analysis of Perfect Repair System (with OM)

```
function dy =
omcmpr(t,y,L12,L14,L19,L23,L25,L210,M31,L45,L47,L412,L56,L58,L513,M61,M71,M81
,L910,L912,L917,L1011,L1013,L1018,M111,L1213,L1215,L1219,L1314,L1316,L1320,M1
41,M151,M161,M171,M181,M191,M201);
L12=0.000111;L14=0.000167;L19=0.0000667;L23=0.000223;L25=0.000167;L210=0.0000
667;M31=0.00166667;L45=0.000111;L47=0.0002936;L412=0.0000667;L56=0.000223;L58
=0.0002936;L513=0.0000667;M61=0.00166667;M71=0.00166667;M81=0.00166667;L910=0
.000111;L912=0.000167;L917=0.00019204;L1011=0.000223;L1013=0.000167;L1018=0.0
0019204;M111=0.00166667;L1213=0.000111;L1215=0.0002936;L1219=0.00019204;L1314
=0.000223;L1316=0.0002936;L1320=0.00019204;M141=0.00166667;M151=0.00166667;M1
61=0.00166667;M171=0.00166667;M181=0.00166667;M191=0.00166667;M201=0.00166667
;
dy = zeros(20,1); % a column vector
y0=[1;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
dy(1) =-
(L12+L14+L19)*y(1)+M31*y(3)+M71*y(7)+M171*y(17)+M61*y(6)+M81*y(8)+M111*y(11)+
M141*y(14)+M151*y(15)+M161*y(16)+M181*y(18)+M191*y(19)+M201*y(20);
dy(2) =-(L25+L210+L23)*y(2)+L12*y(1);
dy(3) =-M31*y(3)+L23*y(2);
dy(4) =-(L45+L47+L412)*y(4)+L14*y(1);
dy(5) =-(L56+L58+L513)*y(5)+L25*y(2)+L45*y(4);
dy(6) = L56*y(5)-M61*y(6);
dy(7) =-M71*y(7)+L47*y(4);
dy(8) = L58*y(5)-M81*y(8);
dy(9) =-(L917+L912+L910)*y(9)+L19*y(1);
dy(10) =-(L1018+L1013+L1011)*y(10)+L210*y(2)+L910*y(9);
dy(11) = L1011*y(10)-M111*y(11);
dy(12) =-(L1219+L1215+L1213)*y(12)+L412*y(4)+L912*y(9);
dy(13) =-(L1320+L1316+L1314)*y(13)+L513*y(5)+L1013*y(10)+L1213*y(12);
dy(14) = L1314*y(13)-M141*y(14);
dy(15) = L1215*y(12)-M151*y(15);
dy(16) = L1316*y(13)-M161*y(16);
dy(17) =-M171*y(17)+L917*y(9);
dy(18) =-M181*y(18)+L1018*y(10);
dy(19) =-M191*y(19)+L1219*y(12);
dy(20) =-M201*y(20)+L1320*y(13);
```

APPENDIX III

Program for Markov Analysis of Imperfect Repair System (without OM)

```
function dy =
cmir(t,y,L12,L14,L19,L23,L25,L210,M32,L45,L47,L412,L56,L58,L513,M65,M74,M85,L
910,L912,L917,L1011,L1013,L1018,M1110,L1213,L1215,L1219,L1314,L1316,L1320,M14
13,M1512,M1613,M179,M1810,M1912,M2013);
L12=0.000111;L14=0.000167;L19=0.0000667;L23=0.000223;L25=0.000167;L210=0.0000
667;M32=0.0025;L45=0.000111;L47=0.0002936;L412=0.0000667;L56=0.000223;L58=0.0
002936;L513=0.0000667;M65=0.0025;M74=0.0025;M85=0.0025;L910=0.000111;L912=0.0
00167;L917=0.00019204;L1011=0.000223;L1013=0.000167;L1018=0.00019204;M1110=0.
0025;L1213=0.000111;L1215=0.0002936;L1219=0.00019204;L1314=0.000223;L1316=0.0
002936;L1320=0.00019204;M1413=0.0025;M1512=0.0025;M1613=0.0025;M179=0.0025;M1
810=0.0025;M1912=0.0025;M2013=0.0025;
dy = zeros(20,1); % a column vector
y0=[1;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
dy(1) =-(L12+L14+L19)*y(1);
dy(2) =-(L25+L210+L23)*y(2)+L12*y(1)+M32*y(3);
dy(3) =-M32*y(3)+L23*y(2);
dy(4) =-(L45+L47+L412)*y(4)+L14*y(1)+M74*y(7);
dy(5) =-(L56+L58+L513)*y(5)+L25*y(2)+L45*y(4)+M65*y(6)+M85*y(8);
dy(6) = L56*y(5)-M65*y(6);
dy(7) =-M74*y(7)+L47*y(4);
dy(8) = L58*y(5)-M85*y(8);
dy(9) =-(L917+L912+L910)*y(9)+L19*y(1)+M179*y(17);
dy(10) =-
(L1018+L1013+L1011)*y(10)+L210*y(2)+L910*y(9)+M1110*y(11)+M1810*y(18);
dy(11) = L1011*y(10)-M1110*y(11);
dy(12) =-
(L1219+L1215+L1213)*y(12)+L412*y(4)+L912*y(9)+M1512*y(15)+M1912*y(19);
dy(13) =-
(L1320+L1316+L1314)*y(13)+L513*y(5)+L1013*y(10)+L1213*y(12)+M1413*y(14)+M1613
*y(16)+M2013*y(20);
dy(14) = L1314*y(13)-M1413*y(14);
dy(15) = L1215*y(12)-M1512*y(15);
dy(16) = L1316*y(13)-M1613*y(16);
dy(17) =-M179*y(17)+L917*y(9);
dy(18) =-M1810*y(18)+L1018*y(10);
dy(19) =-M1912*y(19)+L1219*y(12);
dy(20) =-M2013*y(20)+L1320*y(13);
```

APPENDIX IV

Program for Markov Analysis of Imperfect Repair System (with OM)

```
function dy =
omcmir(t,y,L12,L14,L19,L23,L25,L210,M32,L45,L47,L412,L56,L58,L513,M62,M74,M84
,L910,L912,L917,L1011,L1013,L1018,M112,L1213,L1215,L1219,L1314,L1316,L1320,M1
42,M154,M164,M179,M189,M199,M209);
L12=0.000111;L14=0.000167;L19=0.0000667;L23=0.000223;L25=0.000167;L210=0.0000
667;M32=0.0025;L45=0.000111;L47=0.0002936;L412=0.0000667;L56=0.000223;L58=0.0
002936;L513=0.0000667;M62=0.0025;M74=0.0025;M84=0.0025;L910=0.000111;L912=0.0
00167;L917=0.00019204;L1011=0.000223;L1013=0.000167;L1018=0.00019204;M112=0.0
025;L1213=0.000111;L1215=0.0002936;L1219=0.00019204;L1314=0.000223;L1316=0.00
02936;L1320=0.00019204;M142=0.0025;M154=0.0025;M164=0.0025;M179=0.0025;M189=0
.0025;M199=0.0025;M209=0.0025;
dy = zeros(20,1); % a column vector
y0=[1;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0];
dy(1) =-(L12+L14+L19)*y(1);
dy(2) =-(L25+L210+L23)*y(2)+L12*y(1)+M32*y(3)+M62*y(6)+M112*y(11)+M142*y(14);
dy(3) =-M32*y(3)+L23*y(2);
dy(4) =-(L45+L47+L412)*y(4)+L14*y(1)+M74*y(7)+M84*y(8)+M154*y(15)+M164*y(16);
dy(5) =-(L56+L58+L513)*y(5)+L25*y(2)+L45*y(4);
dy(6) = L56*y(5)-M62*y(6);
dy(7) =-M74*y(7)+L47*y(4);
dy(8) = L58*y(5)-M84*y(8);
dy(9) =-
(L917+L912+L910)*y(9)+L19*y(1)+M179*y(17)+M189*y(18)+M199*y(19)+M209*y(20);
dy(10) =-(L1018+L1013+L1011)*y(10)+L210*y(2)+L910*y(9);
dy(11) = L1011*y(10)-M112*y(11);
dy(12) =-(L1219+L1215+L1213)*y(12)+L412*y(4)+L912*y(9);
dy(13) =-(L1320+L1316+L1314)*y(13)+L513*y(5)+L1013*y(10)+L1213*y(12);
dy(14) = L1314*y(13)-M142*y(14);
dy(15) = L1215*y(12)-M154*y(15);
dy(16) = L1316*y(13)-M164*y(16);
dy(17) =-M179*y(17)+L917*y(9);
dy(18) =-M189*y(18)+L1018*y(10);
dy(19) =-M199*y(19)+L1219*y(12);
dy(20) =-M209*y(20)+L1320*y(13);
```

APPENDIX V

System SMP Model – Corrective maintenance (perfect repair) with opportunistic maintenance

Refer Section 5.2.2.1 and Figure 5.4 for the system model.

Table 1 - cdf and Parameter Values for the multi-stage reciprocating air compression system (perfect repair with opportunistic maintenance).

cdf	Distribution	Parameter
F_{12}	Weibull	B_{12}=3.78 θ_{12}=10000
F_{14}	Weibull	B_{14}=1.2 θ_{12}=6000
F_{19}	Weibull	B_{19}=2.5 θ_{19}=16000
F_{23}	Weibull	B_{23}=3.78 θ_{23}=4892
F_{25}	Weibull	B_{25}= 1.2 θ_{25}=6000
F_{210}	Weibull	B_{210}=2.5 θ_{210}=16000
F_{31}	Exponential	μ_{31}=1/600
F_{45}	Weibull	B_{45}=3.78 θ_{45}=10000
F_{47}	Weibull	B_{47}= 1.2 θ_{47}=4000
F_{412}	Weibull	B_{412}= 2.5 θ_{412}=16000
F_{56}	Weibull	B_{56}= 3.78 θ_{56}=4892
F_{58}	Weibull	B_{58}= 1.2 θ_{58}=4000
F_{513}	Weibull	B_{513}= 2.5 θ_{513}=16000
F_{61}	Exponential	μ_{61}=1/600
F_{71}	Exponential	μ_{71}=1/600
F_{81}	Exponential	μ_{81}=1/600
F_{910}	Weibull	B_{910}=3.78 θ_{910}=10000
F_{912}	Weibull	B_{912}= 1.2 θ_{912}=6000
F_{917}	Weibull	B_{917}= 2.5 θ_{917}=6776
F_{1011}	Weibull	B_{1011}= 3.78 θ_{1011}=4892
F_{1013}	Weibull	B_{1013}= 1.2 θ_{1013}=6000
F_{1018}	Weibull	B_{1018}= 2.5 θ_{1018}=6776
F_{111}	Exponential	μ_{111}=1/600
F_{1213}	Weibull	B_{1213}= 3.78 θ_{1213}=10000
F_{1215}	Weibull	B_{1215}= 1.2 θ_{1215}=4000
F_{1219}	Weibull	B_{1219}= 2.5 θ_{1219}=6776
F_{1314}	Weibull	B_{1314}= 3.78 θ_{1314}=4892
F_{1316}	Weibull	B_{1316}= 1.2 θ_{1316}=4000
F_{1320}	Weibull	B_{1320}= 2.5 θ_{1320}=6776
F_{1412}	Exponential	μ_{141}=1/600
F_{151}	Exponential	μ_{151}=1/600
F_{161}	Exponential	μ_{161}=1/600
F_{171}	Exponential	μ_{171}=1/600
F_{181}	Exponential	μ_{181}=1/600
F_{191}	Exponential	μ_{191}=1/600
F_{201}	Exponential	μ_{201}=1/600

Table 1: Non-zero elements of the matrix, Z

K_{ij}	Value	K_{ij}	value	K_{ij}	value	K_{ij}	value
K_{12}	0.1885	K_{47}	0.8678	K_{917}	0.3560	K_{1314}	0.2610
K_{14}	0.7367	K_{412}	0.0426	K_{1011}	0.3743	K_{1316}	0.5993
K_{19}	0.0748	K_{56}	0.3336	K_{1013}	0.4397	K_{1320}	0.1397
K_{23}	0.4915	K_{58}	0.6741	K_{1018}	0.1860		
K_{25}	0.4823	K_{513}	0.0193	K_{1213}	0.0431		
K_{210}	0.0262	K_{910}	0.0753	K_{1215}	0.7238		
K_{45}	0.0896	K_{912}	0.5687	K_{1219}	0.2331		

Table 2: Steady-state probabilities of the state of EMC

v_i	Probability values	v_i	Probability values	v_i	Probability values	v_i	Probability values
v_1	0.3075	v_6	0.0161	v_{11}	0.0012	v_{16}	0.0020
v_2	0.00580	v_7	0.1966	v_{12}	0.0227	v_{17}	0.0082
v_3	0.0285	v_8	0.0312	v_{13}	0.0033	v_{18}	0.00060457
v_4	0.2265	v_9	0.0230	v_{14}	0.00087175	v_{19}	0.0053
v_5	0.0482	v_{10}	0.0033	v_{15}	0.0165	v_{20}	0.00042368

Table 3: Mean Sojourn times of the states of SMP model

T_i	Value (h)	T_i	Value (h)	T_i	Value (h)	T_i	Value (h)
T_1	4458.2	T_6	600	T_{11}	600	T_{16}	600
T_2	3176.8	T_7	600	T_{12}	2922.8	T_{17}	600
T_3	600	T_8	600	T_{13}	2519.4	T_{18}	600
T_4	3379	T_9	3611.9	T_{14}	600	T_{19}	600
T_5	2683.9	T_{10}	2941.2	T_{15}	600	T_{20}	600

Table 4: Steady-state probabilities of the states of SMP model

P_i	Probability values	P_i	Probability values	P_i	Probability values	P_i	Probability values
P_1	0.4893	P_6	0.0034	P_{11}	0.00026054	P_{16}	0.00042866
P_2	0.0657	P_7	0.0421	P_{12}	0.0237	P_{17}	0.0018
P_3	0.0061	P_8	0.0067	P_{13}	0.0030	P_{18}	0.00012947
P_4	0.2732	P_9	0.0296	P_{14}	0.00018669	P_{19}	0.0011
P_5	0.0462	P_{10}	0.0034	P_{15}	0.0035	P_{20}	0.000099923

Availability Measure

Availability is the sum of steady-state probabilities of working states of SMP mode, i.e.

$A=P_1+ P_{2+} P_{4+} P_{5+} P_{9+} P_{10+} P_{12+} P_{13}$

The evaluated value of the steady-state availability is:

$A = 0.9342$.

APPENDIX VI

System SMP Model – Corrective maintenance (imperfect repair) without opportunistic maintenance

Refer Section 5.2.3.2 and Figure 5.8 for the system model.

Table 1 - cdf and Parameter Values for the multi-stage reciprocating air compression system (imperfect repair without opportunistic maintenance).

cdf	Distribution	Parameter
F_{12}	Weibull	B_{12}=3.78 θ_{12}=10000
F_{14}	Weibull	B_{14}=1.2 θ_{12}=6000
F_{19}	Weibull	B_{19}=2.5 θ_{19}=16000
F_{23}	Weibull	B_{23}=3.78 θ_{23}=4892
F_{25}	Weibull	B_{25}= 1.2 θ_{25}=6000
F_{210}	Weibull	B_{210}=2.5 θ_{210}=16000
F_{32}	Exponential	μ_{32}=1/400
F_{45}	Weibull	B_{45}=3.78 θ_{45}=10000
F_{47}	Weibull	B_{47}= 1.2 θ_{47}=4000
F_{412}	Weibull	B_{412}= 2.5 θ_{412}=16000
F_{56}	Weibull	B_{56}= 3.78 θ_{56}=4892
F_{58}	Weibull	B_{58}= 1.2 θ_{58}=4000
F_{513}	Weibull	B_{513}= 2.5 θ_{513}=16000
F_{65}	Exponential	μ_{65}=1/400
F_{74}	Exponential	μ_{74}=1/400
F_{85}	Exponential	μ_{85}=1/400
F_{910}	Weibull	B_{910}=3.78 θ_{910}=10000
F_{912}	Weibull	B_{912}= 1.2 θ_{912}=6000
F_{917}	Weibull	B_{917}= 2.5 θ_{917}=6776
F_{1011}	Weibull	B_{1011}= 3.78 θ_{1011}=4892
F_{1013}	Weibull	B_{1013}= 1.2 θ_{1013}=6000
F_{1018}	Weibull	B_{1018}= 2.5 θ_{1018}=6776
F_{1110}	Exponential	μ_{1110}=1/400
F_{1213}	Weibull	B_{1213}= 3.78 θ_{1213}=10000
F_{1215}	Weibull	B_{1215}= 1.2 θ_{1215}=4000
F_{1219}	Weibull	B_{1219}= 2.5 θ_{1219}=6776
F_{1314}	Weibull	B_{1314}= 3.78 θ_{1314}=4892
F_{1316}	Weibull	B_{1316}= 1.2 θ_{1316}=4000
F_{1320}	Weibull	B_{1320}= 2.5 θ_{1320}=6776
F_{1413}	Exponential	μ_{1413}=1/400
F_{1512}	Exponential	μ_{1512}=1/400
F_{1613}	Exponential	μ_{1613}=1/400
F_{179}	Exponential	μ_{179}=1/400
F_{1810}	Exponential	μ_{1810}=1/400
F_{1912}	Exponential	μ_{1912}=1/400
F_{2013}	Exponential	μ_{2013}=1/400

Table 6.1: Non-zero elements of the matrix, Z

K_{ij}	value	K_{ij}	value	K_{ij}	value	K_{ij}	value
K_{12}	0.1885	K_{47}	0.8678	K_{917}	0.3560	K_{1314}	0.2610
K_{14}	0.7367	K_{412}	0.0426	K_{1011}	0.3743	K_{1316}	0.5993
K_{19}	0.0748	K_{56}	0.3336	K_{1013}	0.4397	K_{1320}	0.1397
K_{23}	0.4915	K_{58}	0.6741	K_{1018}	0.1860		
K_{25}	0.4823	K_{513}	0.0193	K_{1213}	0.0431		
K_{210}	0.0262	K_{910}	0.0753	K_{1215}	0.7238		
K_{45}	0.0896	K_{912}	0.5687	K_{1219}	0.2331		

Table 6.2: Steady-state probabilities of the state of EMC

v_i	Probability values	v_i	Probability values	v_i	Probability values	v_i	Probability values
v_1	0.0091	v_6	0.1067	v_{11}	0.00014301	v_{16}	0.0340
v_2	0.0034	v_7	0.0440	v_{12}	0.0641	v_{17}	0.00037635
v_3	0.0017	v_8	0.2069	v_{13}	0.0652	v_{18}	0.00060457
v_4	0.0507	v_9	0.0011	v_{14}	0.0170	v_{19}	0.000071068
v_5	0.3198	v_{10}	0.00038208	v_{15}	0.0464	v_{20}	0.0091

Table 6.3: Mean Sojourn times of the states of SMP model

T_i	Value (h)	T_i	Value (h)	T_i	Value (h)	T_i	Value (h)
T_1	4458.2	T_6	400	T_{11}	400	T_{16}	400
T_2	3176.8	T_7	400	T_{12}	2922.8	T_{17}	400
T_3	400	T_8	400	T_{13}	2519.4	T_{18}	400
T_4	3379	T_9	3611.9	T_{14}	400	T_{19}	400
T_5	2683.9	T_{10}	2941.2	T_{15}	400	T_{20}	400

Table 6.4: Steady-state probabilities of the states of SMP model

P_i	Probability values	P_i	Probability values	P_i	Probability values	P_i	Probability values
P_1	0.0249	P_6	0.0261	P_{11}	0.000035055	P_{16}	0.0096
P_2	0.0066	P_7	0.0108	P_{12}	0.1148	P_{17}	0.00009225
P_3	0.00040648	P_8	0.0507	P_{13}	0.1006	P_{18}	0.00001742
P_4	0.1050	P_9	0.0023	P_{14}	0.0042	P_{19}	0.0037
P_5	0.5259	P_{10}	0.00068865	P_{15}	0.0114	P_{20}	0.0022

Availability Measure

Availability is the sum of steady-state probabilities of working states of SMP mode, i.e.

$A=P_1+P_2+P_4+P_5+P_9+P_{10}+P_{12}+P_{13}$

The evaluated value of the steady-state availability is:

$A = 0.8808.$

APPENDIX VII

System SMP Model – Corrective maintenance (imperfect repair) with opportunistic maintenance

Refer Section 5.2.3.1 and Figure 5.7 for the system model.

Table 1 - cdf and Parameter Values for the multi-stage reciprocating air compression system (imperfect repair with opportunistic maintenance).

cdf	Distribution	Parameter
F_{12}	Weibull	B_{12}=3.78 θ_{12}=10000
F_{14}	Weibull	B_{14}=1.2 θ_{12}=6000
F_{19}	Weibull	B_{19}=2.5 θ_{19}=16000
F_{23}	Weibull	B_{23}=3.78 θ_{23}=4892
F_{25}	Weibull	B_{25}= 1.2 θ_{25}=6000
F_{210}	Weibull	B_{210}=2.5 θ_{210}=16000
F_{32}	Exponential	μ_{32}=1/400
F_{45}	Weibull	B_{45}=3.78 θ_{45}=10000
F_{47}	Weibull	B_{47}= 1.2 θ_{47}=4000
F_{412}	Weibull	B_{412}= 2.5 θ_{412}=16000
F_{56}	Weibull	B_{56}= 3.78 θ_{56}=4892
F_{58}	Weibull	B_{58}= 1.2 θ_{58}=4000
F_{513}	Weibull	B_{513}= 2.5 θ_{513}=16000
F_{62}	Exponential	μ_{62}=1/400
F_{74}	Exponential	μ_{74}=1/400
F_{84}	Exponential	μ_{84}=1/400
F_{910}	Weibull	B_{910}=3.78 θ_{910}=10000
F_{912}	Weibull	B_{912}= 1.2 θ_{912}=6000
F_{917}	Weibull	B_{917}= 2.5 θ_{917}=6776
F_{1011}	Weibull	B_{1011}= 3.78 θ_{1011}=4892
F_{1013}	Weibull	B_{1013}= 1.2 θ_{1013}=6000
F_{1018}	Weibull	B_{1018}= 2.5 θ_{1018}=6776
F_{112}	Exponential	μ_{112}=1/400
F_{1213}	Weibull	B_{1213}= 3.78 θ_{1213}=10000
F_{1215}	Weibull	B_{1215}= 1.2 θ_{1215}=4000
F_{1219}	Weibull	B_{1219}= 2.5 θ_{1219}=6776
F_{1314}	Weibull	B_{1314}= 3.78 θ_{1314}=4892
F_{1316}	Weibull	B_{1316}= 1.2 θ_{1316}=4000
F_{1320}	Weibull	B_{1320}= 2.5 θ_{1320}=6776
F_{142}	Exponential	μ_{142}=1/400
F_{154}	Exponential	μ_{154}=1/400
F_{164}	Exponential	μ_{164}=1/400
F_{179}	Exponential	μ_{179}=1/400
F_{189}	Exponential	μ_{189}=1/400
F_{199}	Exponential	μ_{199}=1/400
F_{209}	Exponential	μ_{209}=1/400

Table 6.1: Non-zero elements of the matrix, Z

K_{ij}	value	K_{ij}	value	K_{ij}	value	K_{ij}	value
K_{12}	0.1885	K_{47}	0.8678	K_{917}	0.3560	K_{1314}	0.2610
K_{14}	0.7367	K_{412}	0.0426	K_{1011}	0.3743	K_{1316}	0.5993
K_{19}	0.0748	K_{56}	0.3336	K_{1013}	0.4397	K_{1320}	0.1397
K_{23}	0.4915	K_{58}	0.6741	K_{1018}	0.1860		
K_{25}	0.4823	K_{513}	0.0193	K_{1213}	0.0431		
K_{210}	0.0262	K_{910}	0.0753	K_{1215}	0.7238		
K_{45}	0.0896	K_{912}	0.5687	K_{1219}	0.2331		

Table 6.2: Steady-state probabilities of the state of EMC

v_i	Probability values	v_i	Probability values	v_i	Probability values	v_i	Probability values
v_1	0.00038862	v_6	0.0186	v_{11}	0.0006299	v_{16}	0.0017
v_2	0.0394	v_7	0.3562	v_{12}	0.0224	v_{17}	0.0031
v_3	0.0194	v_8	0.0361	v_{13}	0.0028	v_{18}	0.00031301
v_4	0.4104	v_9	0.0086	v_{14}	0.00072606	v_{19}	0.0052
v_5	0.0558	v_{10}	0.0017	v_{15}	0.0162	v_{20}	0.00038862

Table 3: Mean Sojourn times of the states of SMP model

T_i	Value (h)	T_i	Value (h)	T_i	Value (h)	T_i	Value (h)
T_1	4458.2	T_6	400	T_{11}	400	T_{16}	400
T_2	3176.8	T_7	400	T_{12}	2922.8	T_{17}	400
T_3	400	T_8	400	T_{13}	2519.4	T_{18}	400
T_4	3379	T_9	3611.9	T_{14}	400	T_{19}	400
T_5	2683.9	T_{10}	2941.2	T_{15}	400	T_{20}	400

Table 4: Steady-state probabilities of the states of SMP model

P_i	Probability values	P_i	Probability values	P_i	Probability values	P_i	Probability values
P_1	0.00088599	P_6	0.0038	P_{11}	0.00012884	P_{16}	0.00034102
P_2	0.0640	P_7	0.0729	P_{12}	0.0335	P_{17}	0.00062902
P_3	0.0040	P_8	0.0074	P_{13}	0.0036	P_{18}	0.000064027
P_4	0.7092	P_9	0.0160	P_{14}	0.00014852	P_{19}	0.0011
P_5	0.0766	P_{10}	0.0025	P_{15}	0.0033	P_{20}	0.000079493

Availability Measure

Availability is the sum of steady-state probabilities of working states of SMP mode, i.e.

$A=P_1+P_2+P_4+P_5+P_9+P_{10}+P_{12}+P_{13}$

The evaluated value of the steady-state availability is:

$A = 0.9062$.