

# **Prediction of strength of fly ash concrete**

*A project report submitted in partial fulfillment of  
the requirements for the degree of*

**MASTER OF TECHNOLOGY**

in

**STRUCTURAL ENGINEERING**

by

**VIVEK KUMAR (2K15/STE/19)**

UNDER THE GUIDANCE OF

**DR. ALOK VERMA**

(Associate Professor)



**DEPARTMENT OF CIVIL ENGINEERING  
DELHI TECHNOLOGICAL UNIVERSITY  
(FORMELY DELHI COLLEGE OF ENGINEERING)  
BAWANA ROAD, DELHI-110042**

## **DECLARATION**

I declare that the work presented in this thesis titled “Prediction of strength of flyash concrete”, submitted to Department of Civil Engineering, is an authentic record of my own work carried out under the supervision of Dr. Alok Verma, Associate Professor, Department of Civil Engineering, Delhi Technological University, Delhi.

This report does not, to the best of my knowledge, contain part of my work which has been submitted for the award of any other degree either of this university or any other university without proper citation.

Date:

Place: DTU, Delhi

Signature of candidate

# **CERTIFICATE**

**DELHI TECHNOLOGICAL UNIVERSITY**  
(Formerly DELHI COLLEGE OF ENGINEERING)

Date:-\_\_\_\_\_

This is to certify that report entitled “**Prediction of strength of flyash concrete**” by **Vivek Kumar** in the requirement of the partial fulfilment for the award of Degree of **Master of Technology (M.Tech) in Structural Engineering** at **Delhi Technological University**. This work was completed under my supervision and guidance. He has completed his work with utmost sincerity and diligence. The work embodied in this project has not been submitted for the award of any other degree to the best of my knowledge.

Dr. Alok Verma  
Associate Professor,  
Department of Civil Engineering

## **ACKNOWLEDGEMENT**

First, I would like to express my gratitude to God for giving me ideas and strengths to make my dreams true and accomplish this thesis.

To achieve success in any work, guidance plays an important role. It makes us put right amount of energy in the right direction and at right time to obtain the desired result. Express my sincere gratitude to my guide, **Dr. Alok Verma**, Associate Professor, Department of Civil Engineering for giving valuable guidance during the course of this work, for his ever encouraging and timely moral support.

I am greatly thankful to **Dr. Nirendra Dev**, Professor and Head, Department of Civil Engineering, Delhi Technological University, for his encouragement and inspiration for execution of the this work. I express my feelings of thanks to the entire faculty and staff of Department of Civil Engineering for their help, inspiration and moral support, which went a long way in the successful completion of my report work.

**VIVEK KUMAR**

**(2K15/STE/19)**

**CONTENTS**

LIST OF FIGURES	vi
LIST OF TABLES	viii
ABSTRACT	ix
Chapter - 1	
INTRODUCTION	1
Chapter - 2	
OBJECTIVE OF THE STUDY	4
Chapter - 3	
LITERATURE REVIEW	5
3.1 Paperwork	4
3.2 Mix design procedure as IS10262:1982 and IS10262:1982	9
3.3 Factors affecting compressive strength of concrete	11
3.4 Prediction of compressive strength	14
3.5 Artificial Neural Network	16
3.5.1 Artificial Neuron	17
3.5.2 Activation Functions	18
3.5.3 Back-propagation Learning Network	19
3.6 Application of Artificial Neural Network in Civil Engineering	21
3.7 Multiple Linear Regression	22
3.7.1 F-test	23
3.7.2 T-test	25
3.8 Application of Regression Analysis in Civil Engineering	26

Chapter - 4	
MATERIALS AND METHODOLOGY	28
4.1 ANN	28
4.1.1 Collection and preparation of data	29
4.1.2 Creation, training and testing of network	30
4.1.3 ANN procedure	31
4.2 Multiple linear regression analysis	36
4.2.1 MLR procedure	37
4.3 Performance Evaluation	39
4.4 Experimental Program	39
4.4.1 Calculation for mix design M40	40
4.4.2 Calculation for mix design M40	41
Chapter – 5	
RESULTS AND DISCUSSIONS	43
5.1 ANN models	43
5.2 Multiple linear regression analysis	53
5.2.1 Interpretation of result	54
Chapter - 6	
CONCLUSION	58
REFERENCE	60
Appendix – I	63

## LIST OF FIGURES

No	Caption	Page No
3.1	Compressive strength v/s w/c ratio	11
3.2	A Multi-Layer Feed Forward Neural Network	17
3.3 (a)	Log-sigmoid transfer function	19
3.3 (b)	Tan-sigmoid transfer function	19
3.4	Linear Transfer function	19
3.5	Hypothesis testing	24
3.6	Hypothesis testing for individual parameters	26
4.1	Flow chart for designing ANN model	28
4.2	Graph showing effect of selecting too small or large learning rate	31
4.3	Network Data Manager Window	32
4.4	Import of Data to Network/Data	32
4.5	Network Creation in NN toolbox	33
4.6	Created Network	34
4.7	Network Training Phase	34
4.8	Network Training Window	35
4.9	Network Training, Performance Plot	35
4.10	Network Regression Graph	36
4.11	Data Analysis Window	37
4.12	Input of data in Regression Window	38
4.13	Casting, curing and testing of cubes	42
5.1	RMSE and R- value versus different neuron number for ANN-1 model with tansig activation function	45

No	Caption	Page No
5.2	RMSE and R- value versus different neuron number for ANN-1 model with logsig activation function	45
5.3	RMSE and R- value versus different neuron number for ANN-2 model with tansig activation function	46
5.4	RMSE and R- value versus different neuron number for ANN-2 model with logsig activation function	46
5.5	RMSE and R- value versus different neuron number for ANN-3 model with tansig activation function	47
5.6	RMSE and R- value versus different neuron number for ANN-3 model with logsig activation function	47
5.7	RMSE and R-value versus different neuron number for ANN-1 model with tansig activation function and 20 number of hidden neurons	49
5.8	RMSE and R-value versus different neuron number for ANN-2 model with tansig activation function and 30 number of hidden neurons	49
5.9	RMSE and R-value versus different neuron number for ANN-3 model with tansig activation function and 20 number of hidden neurons	50
5.10	Graph between actual compressive strength vs predicted using ANN-1 model	51
5.11	Graph between actual compressive strength vs predicted using ANN-2 model	52
5.12	Graph between actual compressive strength vs predicted using ANN-3 model	52
5.13	Graph between predicted compressive strength from MLR approach versus actual compressive strength	56
5.14	Summarized performance of all models used in prediction	57



## LIST OF TABLES

No	Caption	Page No
4.1	Range of components of data sets	29
4.2	Outline of ANOVA table	38
5.1	Performance of different networks with different hidden neuron numbers and activation functions of ANN-1 model	43
5.2	Performance of different networks with different hidden neuron numbers and activation functions of ANN-2 model	44
5.3	Performance of different networks with different hidden neuron numbers and activation functions of ANN-2 model	44
5.4	Performance of networks with different learning rate for all three models	48
5.5	Mix Proportions of flyash concrete	50
5.6	Results from ANNs in prediction	51
5.7	Regression Statistics (1)	53
5.8	ANOVA table (1)	53
5.8	Regression Statistics (2)	55
5.9	ANOVA table (2)	55
5.10	Performance of MLR model	56
A.1	Data collected from previous research works	63

## **ABSTRACT**

Concrete, being widely used, is the most important building material in civil engineering. Concrete is a highly complex material, which makes modeling its behavior a very difficult task. Many attempts were taken earlier to develop suitable mathematical models for the prediction of compressive strength of different concretes, but not for flyash concrete. Those traditional methods have failed to map non-linear behavior of concrete ingredients. The present study has used artificial neural networks (ANN) to predict the compressive strength of fly ash concrete. The ANN model has been developed and validated in this research using the mix proportioning and experimental strength data of 6 different mixes. The artificial neural networks (ANN) model is constructed trained and tested (in MATLAB) using the previous researches data. A total of 149 different fly ash concrete mix design were collected from technical literature. For comparative study, 4 models were developed. Strength was modeled in ANN-1 model as a function of three input variables: w/b, cement, water. ANN-2 model was presented with 4 input parameters: w/b (water-binder ratio), cement, water and fly ash%. ANN-3 model consist of 6 input variables: w/b (water-binder ratio), cement, water, fly ash%, coarse and fine aggregates. In this study, an attempt was also made to develop a multiple regression model for predicting strength (in EXCEL) as it is being used largely by researches in prediction. Finally, these four models were compared using coefficient of determination and RMSE values, and resulted in the fact that ANNs models have performed better than MLR model in predicting compressive strength of flyash concrete. Also, ANN model presented with more description of system (with more input variables that affect strength) yield more accurate results showing better correlation with observed/ experimented/actual strength.

*Keywords:* fly ash concrete, artificial neural network, multiple linear regression

## **1 INTRODUCTION**

Concrete is the most used and common building material all over the world. Its popularity is mainly because of its ability to flow and take any shape when presented in wet condition and acquire desired strength when it hardens. This material is produced by mixing water, cement, fine and coarse aggregate in additions to admixtures. The characteristics of concrete such as durability, strength etc. depend on ingredients properties, proportions of mix and other control measures like temperature level, curing period etc.

The wide use of concrete as the basic construction material may be due to its adaptability for a wide range of strength and workability. To achieve different strength requirements, it is the “Mix-design process” that makes the difference as the basic ingredients are same all the way. Very likely to other methods of Concrete Mix Design, Guidelines recommended by Bureau of Indian Standards for concrete mix design is based on certain empirical relations established through vast number of experiments conducted upon materials used in Indian conditions. IS:10262 is the specified code to serve the purpose. This code came to being in the year 1982. So IS:10262-1982 had been evolved to guide the concreting technology being followed at that period. But at present due to demand in high strength concrete and for economic production, use of supplementary materials has become essential. With the advanced technology a number of additives have been identified and are being used extensively now-a-days. These additives are not only enhancing the quality of concreting but also make the process economic and eco-friendly too. So keeping these in view the necessary modifications were felt essential and the revised version of the code as IS: 10262-2009-“Concrete Mix Design Guidelines” has met this in time. The revised version encourages use of supplementary cementitious materials and water reducing additives. Besides, being consistent with specifications of IS: 456-2000, necessary modifications have been made. Data gathered for this present study (shown in Appendix-I) from previous researches is a collection of results obtained using

different mix design procedures (as per IS codes 10262 old as well as new, ACI guidelines etc.). This study also takes an opportunity to study and compare the comparison between IS 10262:1982 and IS 10262:2009.

Several studies independently have shown that concrete strength development is determined not only by the w/b ratio, but that it is also influenced by the content of other ingredients. The need to study compressive strength stems from its importance as it directly relates to a certain technical standard requirements. The implication exhibited by compression strength on properties like durability can also be used to the advantage of the structural designer, material engineer as well as other stakeholders in the concrete manufacturing and construction industry. This signifies the need to have a standard measure for the prediction of concrete compression strength.

Most research work on predicting strength of concrete aims in developing mathematical models which consists of rules and expressions. These rules or expressions are generally fixed equations formed from the work of previous data. These equations are constructed using statistical analysis, by which many linear and nonlinear regression equations have been constructed to model these prediction. But it has been found that if some new data with slight variation is presented to these equations then it results in large errors from actual output (which is supposed to come). Mainly, models development focused on minimizing these errors as low as possible.

In recent time, Artificial neural network (ANN) is being successfully used for modeling purpose in several fields of engineering like thermodynamics, electronics. In civil engineering, it has been applied to mainly hydrologic fields for predicting flood, rainfall-runoff correlation etc. Artificial neural network (ANN) does not require such a specific form. Instead of that, it needs sufficient input-output data.

The fundamental methodology to create a neural system based model for prediction is to train a neural network on the results of a series of experiments, thus, minimizing the absolute difference between the target (desired) outputs and the actual outputs (from model), thereby, resulting in approximate optimal solutions.

In this study, an attempt has been made to predict the 28<sup>th</sup> day compressive strength of flyash concrete using Artificial Neural Network (ANN) and also multiple linear regression model (which has been traditionally used by previous researches). For the purpose of comparative study, three ANN models, ANN-1 having 3 inputs (w/b, water, cement), ANN-2 having four inputs (including flyash %) and ANN-3 consist of six inputs variables (including flyash %, coarse aggregate and fine aggregates) were trained and tested. Here, the motive is to understand if results from these strength predicting models can be improved if there is better portrayal of the system, hence in second and third model more input variables are introduced to them. Network were trained and tested based on the 149 samples obtained from 17 previous research works. For validation of developed models, 6 concrete cubes were made in laboratory of M40 and M45 mix design with varying fly ash percentages as 0%, 20% and 30%.

## **2. Objectives of the study**

Various objectives of present project work have been to:

- Study concrete mix design procedures as per IS codes of practice (old as well as new)
- Study the effects of various factors/ parameters on compressive strength of concrete.
- Review on application of methods used for predicting compressive strength of concrete.
- Gather results from previous research works on fly ash concrete and prepare a worksheet for it.
- Develop ANN and MLR models from data collected to predict compressive strength
- Design concrete mixes for grade M40 and M45 (with varying fly ash percentages) in the laboratory for validation of those models.
- Analysis results of ANN and MLR models in prediction and compare its results with experimental/observed results obtained from laboratory.

### **3 LITERATURE REVIEW**

#### **3.1 Paperwork**

In recent years, the analytical methods using artificial intelligence (AI) such as neural network and fuzzy logic have increasingly been applied to not only to predict concrete strength only but also in other engineering fields also. Qasrawi[35] presented that ANNs has strong potential as a feasible tool for predicting the compressive strength of concrete. In this study, model has been developed on the data collected from previous researches. They were:

Nagabhushana[12] aimed in observing the variation of strength of different grades of concrete with different levels of fly ash replacement. The objective of his study was to re-establish the findings of earlier research done in the area of fly ash concrete. The grades of concrete selected for the study were M20, M35 and M50. The fly ash replacements considered for the study are 0%, 20%, 35% and 50% of cement by weight. The results of study indicated that for M20 and M35 grades of concrete, there was increase in strength with 35% cement replacement by fly ash. For M50 grade of concrete, there was decrease in strength for all replacement levels selected for the study.

Namagga and Atadero[2] investigated the use of large volumes of High Lime flyash in concrete. Varying amount of flyash was used in given mixes of concrete as partial replacement of the cement and fine aggregates. Thet test provided a general increase in the strength of concrete with addition of High Lime flyash. The test results indicated that replacing proportion of cement with High Lime flyash would provide improved strength and a most cost effective solution.

Myadaraboina et.al.,[10] aimed to produce high volume high performance fly ash concrete by reducing the drawback of low early strength of High Volume Fly Ash Concrete (HVFAC). To achieve this, few key factors were considered in mix design and investigated studying microstructure of HVFAC. The factors being the w/b ratio, fineness of fly ash added, type of mixing water and dosage of super plasticizer. The experimental results of 50% replacement of cement with ultra-fine fly ash, raw fly ash with tap water and lime water as mixing water were presented in his paper.

Kalra and Kumar[16] had replaced cement by flyash within the range of 0% (without fly ash), 25%, 40%, 50% and 60% by weight of cement for M-25 mix. Concrete mixtures were moulded, tested and compared in terms of compressive ,split and flexural strength.

Bajad et al.[9] in their study casted cubes by replacing aggregate and cement with 10%, 20%, 30%, 40% recycled coarse aggregates (RCA) and flyash (FA) and compressive strength is checked. Obtained results were then used to establish an empirical relationship between the strength of concrete by using percentage of RCA and percentage of FA. Results showed that RCA and FA up to 30% can be used for making concrete.

Naik and Ramme[8] reported the advantages of using a high quality ASTM C-618 Class C fly ash on water demand, workability and compressive strength of concrete. The research was performed at two precast/prestressed concrete plants to identify optimum mixture proportions for production of high early strength concrete with high fly ash contents. Tests were carried out on nominal 5000 psi (34 MPa) concrete utilizing fly ash produced at Wisconsin Electric Power Company's Pleasant Prairie Power Plant. Fly ash replacement improved workability, decreased water demand, and increased strength while maintaining the high early strength requirements of precast/ prestressed concrete operations.

Raju and Dharmar[5] studied the influence of combination of Fly Ash (FA) and Copper Slag (CS) on the mechanical properties of concrete. Concrete mixtures were made with 10%, 20% and 30% replacement of cement with low lime (class F) fly ash by mass and fine aggregate was replaced by CS from 0 - 100% with an increment of 20% by volume. On the hardened concrete, Destructive Test (DT) methods such as compressive strength (7, 28, 56 and 90 days), split tensile strength (28 days) and flexural strength (28 days)



were determined. Based on the experimental results, the results were favorable for concrete with industrial wastes such as Fly Ash and Copper Slag and also superior to control concrete.

Solikin[17] examined the effect of type of fly ash, kind of mixing water and the utilization of basalt fibre on the strength of concrete by using design of experiment method. It was found that the brittleness of basalt fibre in alkali environment makes it inappropriate to use as concrete fibre. Also, the combination of high volume ultra-fine fly ash with lime water as mixing water is found as the optimum mix proportion to produce high strength concrete which has similar concrete strength as ordinary portland cement concrete (OPC) starting at the concrete age of 28 days and beyond.

Han et al.[1] modeled the variation of compressive strength of fly ash concrete with aging, using apparent activation energy. After analyzing the experimental result with the model, fly ash replacement content and water–binder ratio influence on apparent activation energy was investigated. Concrete with water–binder ratio smaller than 0.40 gives nearly constant limiting relative compressive strength and initial apparent activation energy when analyzed with various water–binder ratios. However, concrete with water–binder ratio larger than 0.40 increases limiting relative compressive strength and initial apparent activation energy.

Awanti and Harwalkar[15] presented a study on development of mix proportions of high volume fly ash concrete (HFC). A series of HFC mixtures with cement replacement levels varying between 50% and 65% were prepared with water/binder ratios of 0.3 and 0.35. Compressive strength values were obtained at different ages. From the experimental results, pozzolanic efficiency ratios and mix design curves for HFC were established.

Mukherjee et al.[4] compared the compressive strength of zero slump and high slump concrete with high volume fly ash. 40% to 70% replacements of OPC (by weight) with class F fly ash had been incorporated. Superplasticizer was added at 1% of binder (cement + fly ash) to the zero slump mixture to get a slump in the range of 140 to 180mm and cubes were cast without compaction. The results showed that the apparent porosity and water absorption were higher for zero slump concrete than high slump concrete. Zero slump concrete showed better compressive strengths than superplasticized concrete with

40 to 60% fly ash addition for all curing times tested (3,7 and 28 days). Ultrasonic pulse velocity results categorized all mixes as of 'EXCELLENT' concrete quality. Based on the present experimental investigation, it can be concluded that high volume fly ash concrete is suitable for general construction applications.

Quan and Kasami[6] attempted to improve the durability of fly ash concrete. a series of experimental studies were carried out. The results show edthat by using durability improving admixture in nonair-entraining fly ash concrete, the compressive strength of fly ash concrete can be improved by 10%–20%, and the drying shrinkage is reduced by 60%.

Singh[11] presented the properties of HVFAC with 50% fly ash used on two demonstration projects in New Delhi, India. The results show that HVFAC is indeed an excellent material with later age properties superior to conventional concrete, namely - compressive strength, flexural strength, elastic modulus, abrasion resistance and permeability.

Sarika et al.[14] studied the relationship between water /binder ratio to compressive strength of high volume fly ash concrete using fly ash as an additional material in the cement concrete. The studies had indicated that the high volumes fly ash used in concrete as an additional material would lead to enhanced properties in concrete and contribute towards development of high performance and high strength concrete which is the need of the hour.

John and Ashok[7] aimed to study the mechanical strength behavior of High Volume Fly ash concrete pavement slab. In this study the mechanical properties were studied with various replacements with cement like 50%, 60%, and 70% of Fly ash. % saves the higher compressive strength. When compared with control mix the strength of HVFA concrete reduced % for 50%, 60% and 70% at 7 day and 28 day respectively.

Basha et al.[3] assess compressive strength of fly ash based cement concrete. Concrete mixes M25, M30, were designed as per the Indian standard code (IS-10262-82) by adding, 0%, 10%, 20%, 30% and 40% of fly ash. Concrete cubes of size 150mm × 150mm × 150 mm were casted and tested for compressive strength at 7 days, 14 days, 21

days and 28 days curing for all mixes and the results were compared with that of conventional concrete.

Naik and Ramme[13] aimed to identify and recommend mix designs for high fly ash content 30W and 4000 psi (21 and 28 MPa) structural grade concrete utilizing Class C fly ash. Low-cement content structural grade concrete with 70 percent Class C fly ash substitution for cement also was used for some trial construction projects. Results and recommendations obtained from limited construction experience were also presented.

Data collected from above mentioned researchers work has been shown in tabulated form in Appendix-I.

### **3.2: Mix Design Procedure as per IS10262:1982 and IS10262:2009**

The Bureau of Indian Standards (BIS) has released the final code on concrete mix proportioning in December 2009. Significant changes have been made in the new code adapting from international codes on concrete mix design. The IS 10262:2009 is an adaption of ACI method. It requires a designer to develop the w/c curve for the type of materials to be actually used to form the basis for the mix design rather than using any available curves. In the absence of such a data, the w/c ratio is to be assumed based on such available relationship as already established to start the process. Table 5 of IS 456:2000 can also be used to select the w/c ratio. It means one should be careful while selecting the initial w/c ratio, as it decides the calculated cement content.

The key modifications noticed are:

- The 1982 version considers strength as the governing criteria for durability and so also for the mix design process. But according to the revised one strength may be a factor for acceptance but may not assure durability.
- IS: 10262-1982 considers expected air content of 1% to 3% in the design process depending on the nominal maximum size of aggregates. IS: 10262-2009 eliminates consideration of air content in the mix proportion calculation as it's not of much significance.

- The old version suggested that selection of preliminary free w/c ratio may be adopted from established relationships presented in form of graph as generalized w/c ratio curves for different cement strengths. Accordingly six ready reference curves were there namely A to F for a wide range of cement strengths from 325kg/cm<sup>2</sup> to 625kg/cm<sup>2</sup>. This selected w/c ratio is to be checked against limiting w/c ratio for durability. The revised version encourages establishing the relationships for actually used material. Otherwise it suggests to consider it from the specified table (Table-5) of IS: 456 for desired exposure condition as preliminary w/c ratio that has to be further checked for limiting value ensuring durability.
- IS: 10262-1982 considers compaction factor as the measure of workability. In revised one, slump is considered as the measure of workability. Measurement of workability as slump is more convenient, widely used at sites and is better acceptable.
- The revised code provides guidelines for addition of supplementary cementitious additives. So additives like fly ash, silica fume, ground granulated blast furnace slag, rice husk ash etc. can be used in concrete mix provided the strength and durability requirement are met with. So as per the revised code the concrete is no longer a four component system (cement, sand, coarse aggregates & water) as considered in the previous version, but it is much more.
- The quantity of water to be used plays a vital role in concrete mix design. Agreeing with the old guidelines, values of water content have been specified in terms of kg per cubic meter of concrete depending upon the nominal maximum size of aggregates which can be considered as starting selection point of water content. IS: 10262-2009 allows use of water reducers/ super plasticizers and also specifies the alteration in water content accordingly. Further water adjustment was specified in terms of variation of compaction factor in the older version whereas the same has been remolded in terms of slump variation (+3% for every 25mm slump over 50mm) in the revised one.
- The 1982 publication specifies ratio of fine aggregates to all-in-aggregates from which coarse aggregates content can be derived. In revised one the volume of coarse

aggregates per unit volume of total aggregates for different zones of fine aggregates and different maximum nominal size of aggregates has been tabulated from which the fine aggregates content has to be derived. Further in the earlier guidelines necessary adjustments in sand content has been suggested depending on its grading zone, whereas the recent guidelines allow reduction in coarse aggregates content for better workability, provided other desired properties are satisfied.

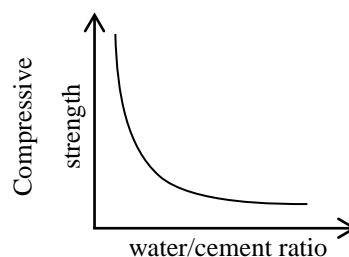
### 3.3 Factors affecting compressive strength of concrete

For producing desired properties of hardened concrete, one should know which factors affect the properties of hardened concrete. Abrams's law of compressive strength versus water/ cement ratio[21] is well known and many investigators have studied the possible effects of other factors, such as cement content and sand concentration, upon concrete strength. Erntroy and Shacklock[22] showed that reducing the aggregate/cement ratio, for a given water/cement ratio, reduced the cube crushing strength. Singh[36] confirmed this result. They plotted cube strength against water/cement ratio and obtained families of curves for different aggregate/cement ratios.

Some of the factors that affect compressive strength largely are discussed below:

#### 1) Water / Cement Ratio:

The higher the water/cement ratio, the greater the initial spacing between the cement grains and the greater the volume of residual voids not filled by hydration products. The relation between water cement ratio and strength of concrete is shown in the plot as shown below:



**Fig 3.1:** Compressive strength vs w/c ratio

There is one thing missing on the graph. For a given cement content, the workability of the concrete is reduced if the water/cement ratio is reduced. A lower water cement

ratio means less water, or more cement and lower workability. However if the workability becomes too low the concrete becomes difficult to compact and the strength reduces. For a given set of materials and environment conditions, the strength at any age depends only on the water-cement ratio, providing full compaction can be achieved.

1) Aggregate/cement

The aggregate/cement ratio, is only a secondary factor in the strength of concrete but it has been found that, for a constant water/cement ratio, a leaner mix leads to a higher strength. Some water may be absorbed by the aggregate: a larger amount of aggregate absorbs a greater quantity of water, the effective water/cement ratio being thus reduced. A higher aggregate content would lead to lower shrinkage and lower bleeding, and therefore to less damage to the bond between the aggregate and the cement paste. As a result, in a leaner mix, the voids form a smaller fraction of the total volume of concrete, and it is these voids that have an adverse effect on strength.

3) Shape and amount of aggregate

The strength of concrete mostly depends upon the strength of aggregate. The coarse aggregates act like bone in concrete. If we use low quality aggregate in concrete mix, the strength of concrete will be low. The amount of aggregate greatly affects the properties of hardened concrete. Increasing the amount of aggregate at a constant cement content reduce the concrete strength. Strength of concrete can also be affected by the type of aggregate. Such as, angular and rough surface concrete increase the concrete strength. Because, rough surface of aggregates make strong bond between cement paste and aggregates than smooth surface aggregates. In other side, round and smooth surface aggregate decrease the concrete strength. Larger size aggregate with lower water requirement can produce strong concrete. But, using larger size aggregate without decreasing water content decreases the strength of concrete. Aggregates add strength to concrete and reduce its potential for shrinkage.

4) Age of concrete

The degree of hydration is synonymous with the age of concrete provided the concrete has not been allowed to dry out or the temperature is too low. In theory, provided the

concrete is not allowed to dry out, then it will always be increasing albeit at an ever reducing rate. For convenience and for most practical applications, it is generally accepted that the majority of the strength has been achieved by 28 days.

#### 5) Temperature

The rate of hydration reaction is temperature dependent. If the temperature increases the reaction also increases. This means that the concrete kept at higher temperature will gain strength more quickly than a similar concrete kept at a lower temperature. However, the final strength of the concrete kept at the higher temperature will be lower. This is because the physical form of the hardened cement paste is less well structured and more porous when hydration proceeds at faster rate. This is an important point to remember because temperature has a similar but more pronounced detrimental effect on permeability of the concrete.

#### 6) Relative humidity:

If the concrete is allowed to dry out, the hydration reaction will stop. The hydration reaction cannot proceed without moisture.

#### 7) Curing

Detrimental effects of storage of concrete in a dry environment can be reduced if the concrete is adequately cured to prevent excessive moisture loss.

#### 8) Admixtures

There are two basic types of admixtures available: chemical & mineral. Admixtures like flyash, silicate fume, slag comes in the category of mineral admixtures. They are added to concrete to enhance the workability, improve resistance to thermal cracking and alkali–aggregate reaction and to enable reduction in cement content. Flyash is fine residue left after combustion of ground or powdered coal. They are all generally finer than cement and consist mainly of glassy–spherical particles as well as residues of hematite and magnetite, char and some crystalline phases formed during cooling. The use of flyash in concrete makes the mix economical, and improves the workability, reduces segregation, bleeding and reduced heat of hydration but also provides ecological benefits. Chemical admixtures are added to concrete in very small amounts mainly for air entrainment, reduction of water or cement content, plasticizing of fresh

concrete mixtures or to control the setting time of concrete. These admixtures can be broadly categorised as superplasticizers, accelerators, retarders, water reducers and air entraining admixtures. Superplasticizers are added to reduce the water requirement by 15 to 20% without affecting the workability leading to a high strength and dense concrete. Superplasticizers are linear polymers containing sulfonic acid groups attached to the polymer at regular intervals. The commercial formulation can be sulfonated melamine–formaldehyde condensates, sulfonated naphthalene formaldehyde condensates, and modified lignosulfonates, polycarboxylate derivatives.

Some other factors that affect compressive strength are creep, shrinkage, bleeding etc. in this study. Parameters considered in developing models are w/b ratio, quantity of cement, water, coarse and fine aggregates, and fly ash %.

### **3.4 Prediction of compressive strength**

There are several methods of compressive strength prediction methods. Some of the major methods, employed by many researchers around the world are discussed below.

#### 1) Empirical Methods

Empirical methods try to directly depict the experiment process into a certain relationship. The relationship is a measure of the constituents' role in producing the final outcome. They usually rely on rigorous experimental analysis so that the produced relationship is able to fit the outcomes in the laboratory. While predicting compressive strength, most empirical relationship try to relate the strength to the water cement ratio, as the two almost always have an inverse relationship. In other cases, the models make use of an already known compressive strength and apply empirically computed coefficients to relate the known compressive strength with the required compressive strength.

Empirical methods give a better feeling of the final outcome as interactions between constituent materials can be studied. However, they may take a considerable amount of time compared with other available methods especially if the model sought for is subject to intricacy due to a larger number of input parameters. Their application may also be limited to certain specific cases in which coefficients were set or experiments were carried out[23].



2) Computational Modelling

Finite element analysis is based on complex thermodynamic equations. Such modelling is usually computer-aided endeavor. They rely on a correct representation of the concrete microstructure. Such endeavor entails randomly casting the cement in a unit cell space, to effectively represent the hydrations (and other mechanisms) of the varying particle sizes in the cement using a computer simulation (pixel based). The mechanisms may also be encoded empirically, after calibration against experimental information.

3) Statistical Methods

Statistical methods make use of experimental data and employ mathematical equations to best describe the relationships between inputs and outputs. The most known statistical method is the multilinear regression technique. Statistical methods may be easy to understand and even easier to use, but their reliance on data can sometimes handicap a user. They also perform inferior compared to the other available methods and depend on the type of mathematical function chosen to fit the data.

4) Artificial Intelligence

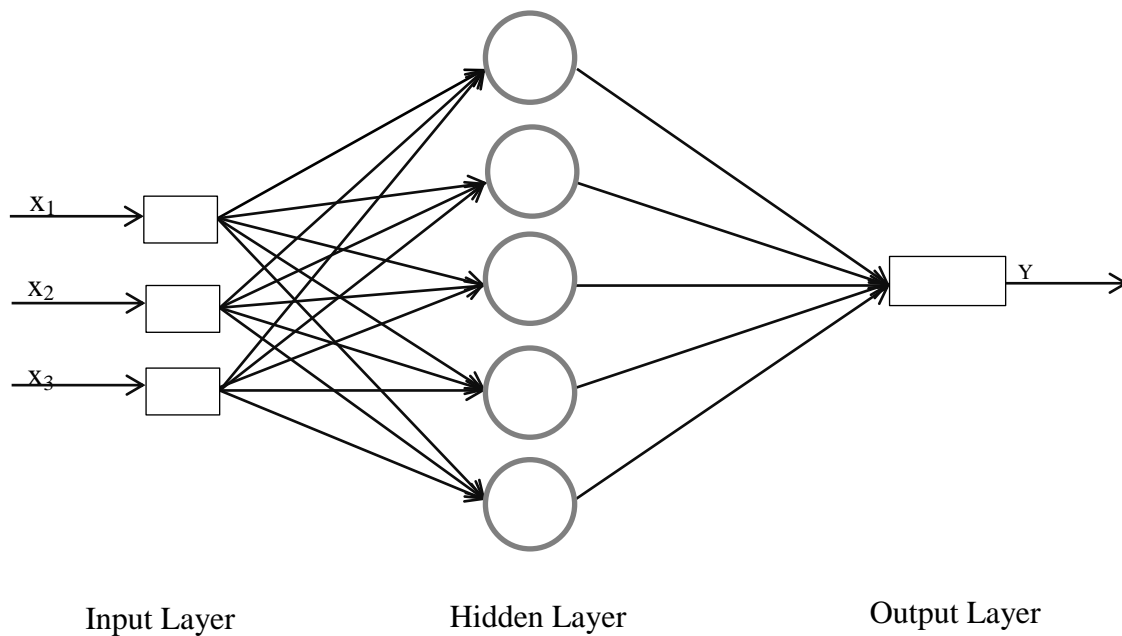
The Webster's New World College Dictionary defines artificial intelligence as the capability of computers or programs to operate in ways to mimic human thought process, such as reasoning and learning. This may bring one to inquire about the type of problems requiring computers to mimic the human brain. These include inference based on knowledge reasoning with uncertain or incomplete information, various forms of perception and learning, and applications to problems such as control, prediction, classification, and optimization [24]. In civil engineering problems, they can be used to model a phenomenon when the mechanism is not fully understood. Examples of AI methods include neural networks and genetic algorithms, which bear resemblance to microscopic biological models in structure (artificial neural networks analogous to the one in the brain) and in performance (genetic algorithms analogous to genes: the computer's program components 'mutate', produce better and better solutions as they adapt). Other relatively contemporary forms of AI include fuzzy systems and Adaptive Network based Fuzzy Inference Systems (ANFIS). ANFIS features combination of artificial neural networks and fuzzy systems. A network obtained in such manner has an excellent ability of training by means of neural networks and linguistic interpretation of variables via fuzzy logic [25].

Artificial Intelligence methods are powerful in terms of their capacity to learn and produce appropriate models even for a vastly ranging information. However, this information must be enough so that a ‘close to reality’ inference can be made.

### 3.5 Artificial Neural Network

Artificial neural networks are modeled after the organic neural networks in the brain of an organism. The fundamental unit of an organic neural network is the neuron. Neurons receive input from one or more different neurons. The strength of the effect that each input has on a neuron depends on the neuron’s proximity to the neuron from which it received the input. If the combined value of these inputs is strong enough, then the neuron receiving these signals outputs a brief pulse. When neurons combine with many other neurons (there are approximately  $10^{11}$  neurons in the human brain) to form networks, an organism can learn to think and make decisions.

Although much simpler, artificial neural networks perform much the same way as organic neural networks. Artificial neurons receive inputs from other neurons. The strength of the effect that each input has on a neuron is determined by a *weight* associated with the input. The receiving neuron then takes the sum these weighted inputs and outputs a value according to its *transfer function* (and possibly a *bias value*). Neurons can be combined into sets of neurons called *layers*. The neurons in a layer do not interconnect with each other, but interconnect with neurons in other layers. A *neural network* is made up of one or more neurons, organized into one or more layers. The layer that receives the network input is called the *input layer* and the layer that outputs the network output is called the *output layer*. Neural networks can have one or more layers between the input and output layers. These layers are called *hidden layers*. Two major components that contribute to the effectiveness of a neural network at solving a particular problem are its *architecture* and the method by which it is *trained*.



**Fig 3.2:** A Multi-Layer Feed Forward Neural Network

Different neural networks can have different architectures. In order for a neural network to learn how to correctly solve a problem, appropriate network connections and their corresponding weights must be determined through a process called training. There are many different algorithms used for training a neural network. Many different types of neural networks were designed, created, trained, tested, and evaluated in an effort to find the appropriate neural network architecture and training method for use in developing model. The creation, training, and testing of each neural network was done using the MathWorks software package MATLAB<sup>®</sup>. MATLAB<sup>®</sup> contains a “Neural Network Toolbox” that facilitates rapid creation, training, and testing of neural networks. MATLAB<sup>®</sup> was chosen to use for model development because this toolbox would save an enormous amount programming effort.

### 3.5.1 Artificial Neuron

As stated earlier, an Artificial Neuron models the behavior of the biological neuron. Each Artificial neuron receives a set of inputs. Each input is multiplied by weight analogous to synaptic strength. The sum of all weighted inputs determines the degree of firing called the activation level. Notationally, if  $I_1, \dots, I_i, \dots, I_n$ , are the input values and  $w_{1j}, \dots, w_{ij}, \dots, w_{nj}$  are synaptic weight values,  $net_j$  is the summation (over all the incoming neurons)

of the product of the incoming neuron's activation and synaptic weight of connection at the typical  $j$ th neuron expressed as  $\sum I_i w_{ij}$ . A threshold value  $\phi$  is incorporated into the output. Thus we have the resultant:

$$\text{net}_j = \sum_{i=1}^n I_i w_{ij} + \phi_i$$

where  $n$  is the number of incoming neurons.

Output from  $j^{\text{th}}$  neuron can be expressed as:

$$O_j = g(\text{net}_j)$$

where 'g' is activation function or transfer function.

There are several conventionally used choices for the activation function, which is also called as threshold function, transfer function or squashing function. The most commonly used activation functions are Linear Function, Nonlinear Function, Step Function, Sigmoid Function and Hyperbolic Tangent Function. Out of the above mentioned activation functions, Sigmoid Function is very popular. Output of  $j^{\text{th}}$  neuron using sigmoid function can be written as:

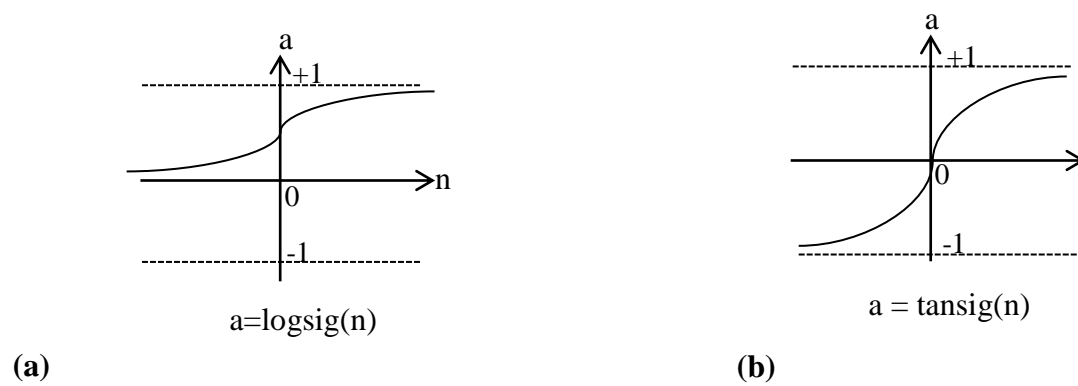
$$O_j = \frac{1}{1 + \exp(-\text{net}_j)}$$

Sigmoid Function is most commonly used activation function as it is continuous function and it has a very simple derivative that is useful for development of learning algorithms, and also it represents the processing of biological neuron.

### 3.5.2 Activation Functions

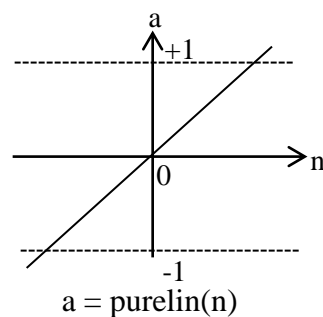
The activation function is central to the idea of neural networks. First, if there were no activation functions, the whole neural network could be reduce to a group of linear function of the network input - one linear function for each output neuron. So, without activation functions, a neural network could not learn non-linear relationships. Activation functions for the hidden units are needed to introduce non-linearity into the network. It's an function used to transform the input into an output signal. Each neuron has an activation function which species the output of a neuron to a given input.

The sigmoid function consists of two functions, *logistic* and *tangential*. The values of logistic function range from 0 and 1 and -1 to +1 for tangential function.



**Fig. 3.3:** (a) Log-sigmoid transfer function  
 (b) Tan-sigmoid transfer function

Like a linear regression, a linear activation function transforms the weighted sum inputs of the neuron to an output using a linear function.



**Fig. 3.4:** Linear Transfer function

### 3.5.3 Back-Propagation Learning Network

In ANN theory there are many paradigms developed to update synaptic weights. Out of them back-propagation is the most widely used of the neural network paradigms and has been applied successfully in application studies in a broad range of areas. Back-propagation can attack any problem that requires pattern mapping. Given an input pattern, the network produces an associated output pattern. In the present study, Backpropagation learning algorithm has been used.

A weight is associated with each connection from input to hidden units and from hidden units to output units. Each unit in the input layer is connected to every unit of hidden layer, likewise each unit in the hidden layer is connected to each unit of next hidden layer (if more than one hidden layer is present) or to each unit of output layer. Bias unit

(optional) has been employed to every layer but to output layer. This can be achieved simply by adding a constant input with an appropriate weight. This unit has a constant activation of 1. Each bias unit is connected to all units in the next higher layer, and its weights to them are adjusted like the other weights. The bias units provide a constant term in the weighted sum of the units in the next layer. This sometimes results in improvement on the convergence properties of the network.

For Back-propagation learning algorithm, an error measure known as the mean square error is used. In training phase of Back-propagation learning algorithm, the total error of the network is minimised by adjusting the weights. Gradient descent method is used for this. Each weight may be thought of as a dimension in N-dimensional error space. In error space the weights act as independent variables and the shape of the corresponding error surface is determined by error function in combination with the training set. The negative gradient of the error function with respect to the weights, thus points in the direction, which will most quickly reduce the error function. This can be expressed as

$$\Delta_p W_{ji} \propto \frac{-\partial E_p}{\partial w_{ji}}$$

where  $\Delta_p W_{ji}$  designates the change in the weight connecting a source neuron 'i' in a layer (let say L-1) and a destination neuron 'j' in next layer (L-2).

Applying chain rule to evaluate  $\frac{-\partial E_p}{\partial w_{ji}}$ :

$$\frac{-\partial E_p}{\partial w_{ji}} = \frac{-\partial E_p}{\partial \text{net}_j} \times \frac{\partial \text{net}_j}{\partial w_{ji}}$$

As  $\text{net}_j = \sum w_{ji} O_i$

where  $O_i$  is the output of all neurons in the L-1 layer.

$$\begin{aligned} \frac{\partial \text{net}_j}{\partial w_{ji}} &= \frac{\partial}{\partial w_{ji}} \left( \sum w_{ji} O_i \right) \\ &= O_i \end{aligned}$$

Hence

$$\frac{-\partial E_p}{\partial w_{ji}} = O_i \frac{-\partial E_p}{\partial \text{net}_j}$$

Defining error  $\delta_{pj}$  as  $\delta_{pj} = \frac{-\partial E_p}{\partial \text{net}_j}$

this gives  $\frac{-\partial E_p}{\partial w_{ji}} = O_i \delta_{pj}$

as discussed earlier,  $\Delta_p W_{ji} \propto \frac{-\partial E_p}{\partial w_{ji}}$

therefore  $\Delta_p W_{ji} = \eta O_i \delta_{pj}$

where  $\eta$  is learning rate parameter. The learning rate parameter determines the amounts of weight change will be used for the weight correction. To find the best results while training networks, this has to be optimized. Its value varies between 0 and 1.

### 3.6 Application of Artificial Neural Networks in Civil Engineering

ANNs have been applied to the detection of structural damage, structural system identification, modeling of material behavior, structural optimization, structural control, ground water monitoring, prediction of settlement of shallow foundation, and concrete mix proportions [26].

When looking at their application in prediction models; Aggarwal et al. [27] tried to also predict the compressive strength of self-compacting concrete using artificial neural networks to complement their work in fuzzy logic. The ANN model carried out 500 iteration with 6 hidden neurons in the hidden layer. Their results showed a similar success, with similar error-measurements.

The resulting model predicted the strength with  $R^2 = 0.9767$ , showing a better result than the one obtained using fuzzy logic. The chosen ANN architecture had a single hidden layer with 11 hidden neurons. Topcu et al. [26] found accuracy of their model increase when adding another hidden layer. The absolute percentage error found was minimum ( $=0.000515$ ) at ANN architecture of 2-hidden layers with nine and eight neurons at the first and second hidden layer, respectively.

The use of multiple layers was also applied in Subasi's ANN model[28]. His model also gave due attention to the material aspect in which chemical analysis results of fly ash, gradient and chemical compositions of sand were employed. The prediction model

showed a good result with  $R^2 = 0.9557$  for compressive strength and 0.9119 for flexural tensile strength.

In addition to assessing the effect of silica fume, Özcan et al.'s compressive strength predicting model[29] also addressed the effect of water to binder ratio on compressive strength. They found that increasing water-binder ratio resulted in a decrease in compressive strength. Their model was able to reproduce experimental results with  $R^2 = 0.9944$  and predicted the testing samples with  $R^2=0.9767$ .

Özturan et al.[30] tried to compare concrete strength prediction techniques with artificial neural network approach and shown that ANN approach gave a good prediction for concretes of low to medium strength concretes. Apart from some good performance observed in multiple linear regression model, ANN approach showed better results. Another approach, using Abrams' law showed the same performance (measured by the coefficient of determination,  $R^2$ ) in all the five system models.

Compression strength predicting models can be improved if there is a better description of the system, hence, more input variables. This has been elaborated in Özturan's paper[30]. Their ANN model showed a much better performance when early strength data (7<sup>th</sup> day compressive strength) was added to the input layer. This signifies the utilization of more clarifiers (those that contribute in the make-up of the concrete) for a better predicting model.

### **3.7 Multiple Linear Regression**

Multiple linear regression attempts to model the relationship between two or more independent variables and a dependent variable by fitting a linear equation to observed data.

A multiple linear regression model that relates a  $y$ -variable to  $k$   $x$ -variable is in the form as:

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_k x_k + \varepsilon_i.$$

where  $y_i$  is the response variable,  $x_k$  are the predictor variables,  $\beta$  are population regression coefficients.  $\varepsilon_i$  represents error ( not a constant term but random) having a normal distribution with mean 0 and constant variance  $\sigma^2$ .



Each  $\beta$  coefficient represents the change in the mean response,  $E(y)$ , per unit increase in the associated predictor variable when all the other predictors are held constant. For example,  $\beta_1$  represents the change in the mean response,  $E(y)$ , per unit increase in  $x_1$  when  $x_2, x_3, \dots, x_k$  are held constant. The intercept term,  $\beta_0$ , represents the mean response,  $E(y)$ , when all the predictors  $x_1, x_2, \dots, x_k$ , are all zero  $\beta$  are unknown parameters because of which true/perfect population regression line is also unknown. The estimates of the  $\beta$  coefficients are the values that minimize the sum of squared errors for the sample. The regression line that we obtain from a sample provides an estimate of the population regression line.

A predicted/fitted value is calculated as  $\hat{y} = b_0 + b_1x_1 + b_2x_2 + \dots + b_kx_k$ .

where  $b_k$  and  $b_0$  values are estimated values of the regression coefficients and intercepts and obtain from statistical software (in this study, EXCEL was used), and the  $x$  values are specified by us.

The difference between an actual and a predicted value of  $y$  is termed as residual

$$e_i = y_i - \hat{y}_i.$$

The least square error method aims to minimize:

$$\text{Sum of square error (SSE)} = \sum_{i=1}^n e_i^2.$$

Once a model is fitted, it has to be determine the validity of model, its goodness of fit and usefulness of each independent/explanatory variables in prediction. For these, we have to consider F-test and t-test.

### 3.7.1 F-test

In order to use the estimated regression equation we firstly have to test the significance of given estimates.

This is zero and alternative hypothesis:

$H_0: \beta_0 = \beta_1 = \beta_2 = \dots \beta_k = 0$  (null hypothesis)

$H_A$ : at least one  $\beta_i \neq 0$

According to this, we have laid, null hypothesis in that way that a linear connection between observed phenomena variations does not exist, or that  $x_1, x_2, \dots, x_k$  has not influence on  $Y$ .

If we start from the assumption that the total variability of dependent variable is conditioned by the variability of independent variables involved in the model and by the unexplained variability, we can write:

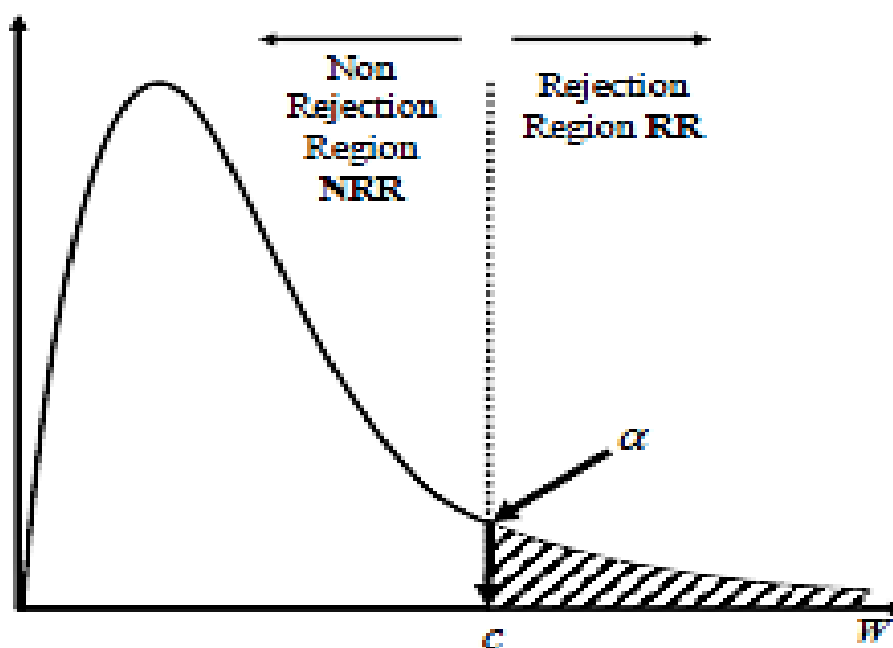
$$SSy = SSR + SSE$$

Where,  $SSy$  presents the total Sum of Squares (total variability),

$SSR$ =Regression Sum of Squares (explained variability),

$SSE$ =Error Sum of Squares (unexplained variability).

The rejection region ( $RR$ ), delimited by the critical value(s), is a set of values of the test statistic for which the null hypothesis is rejected. That is, the sample space for the test statistic is partitioned into two regions; one region (the rejection region) will lead us to reject the null hypothesis  $H_0$ , while the other will lead us not to reject the null hypothesis. Therefore, if the observed value of the test statistic  $S$  is in the critical region, we conclude by *rejecting*  $H_0$ ; if it is not in the rejection region then we conclude by *not rejecting*  $H_0$  or *failing to reject*  $H_0$ . In the process of hypothesis testing, the most subjective part is the determination of the significance level. In general, this is an arbitrary decision, though, as we have said, the 1%, 5% and 10% levels for  $\alpha$  are the most used in practice[18].



**Fig. 3.5:** Hypothesis testing

With the use of computers, hypothesis testing can be contemplated from a more rational perspective. Computer programs typically offer, together with the test statistic, a probability. This probability, which is called  $p$ -value (i.e., probability value), is also known as the critical or exact level of significance or the exact probability. Technically, the  $p$  value is defined as the lowest significance level at which a null hypothesis can be rejected. Once the  $p$ -value has been determined, we know that the null hypothesis is rejected for any  $\alpha \geq p$ -value, while the null hypothesis is not rejected when  $\alpha < p$ -value. Therefore, the  $p$ -value is an indicator of the level of admissibility of the null hypothesis: the higher the  $p$ -value, the more confidence we can have in the null hypothesis[18].

### 3.7.2 T-test

The t-test is used for checking the significance for each individual predictor. Within a multiple regression model, we may want to know whether a particular  $x$  variable is making a useful contribution to the model. That is, given the presence of the other  $x$  variables in the model, does a particular  $x$  variable help us predict or explain the  $y$  variable?

In a t-test, the null hypothesis,  $H_0$ , is that the predictor  $x_j$  is not explanatory for the dependent variable and thus the respective coefficient  $\beta_j$  is zero. The alternative hypothesis,  $H_A$  is that the coefficient explains a part of the dependent variable and thus  $\beta_j$  is not zero[19].

This may be illustrated mathematically as

$H_0: \beta_j=0$  (null hypothesis)

$H_A: \beta_j \neq 0$  (alternative hypothesis)

To carry out the test, statistical software will report  $p$  values for all coefficients in the model. Each  $p$  value will be based on a  $t$  statistic calculated as

$$t = (\text{sample coefficient} - \text{hypothesized value}) / \text{standard error of coefficient.}$$

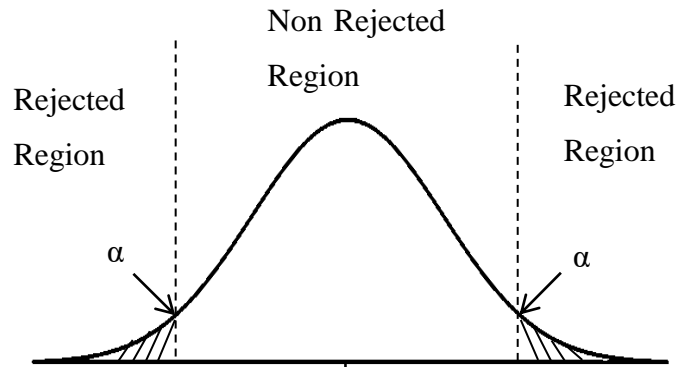
For our example above, the  $t$  statistic is:

$$t = \frac{b_j - 0}{\text{se}(b_j)} = \frac{b_j}{\text{se}(b_j)}$$

Note that the hypothesized value is usually just 0, so this portion of the formula is often omitted.

The  $t$ -value for the estimated  $\beta_j$  is then compared to the  $t$ -distribution and if the  $t$ -value falls in the region specified by the selected confidence level, the null hypothesis is regarded as supported.

Once the  $p$ -value has been determined, we know that  $H_0$  is rejected for any level of significance of  $\alpha > p$ -value, while the null hypothesis is not rejected when  $\alpha < p$ -value.



**Fig. 3.6:** Hypothesis testing for individual parameters

### 3.8 Application of Regression Analysis in Civil Engineering

Regression analysis is deemed a major aspect of statistical approach of modelling technique. Zain et al. [31] highlighted the use of statistical approach over other techniques in their relative ease of computing coefficients which can also be interpreted with regard to time saving. However, such approach is often complex and circuitous, particularly for non-linear relationships [32]. This element of their feature is temporarily shunned in this section. Statistical models are mathematically rigorous and can be used to define confidence interval for the predictions. Furthermore influence of the major ingredients on the output can be observed by carrying out correlation analysis.

Zain et al. [31] introduced slump test results and density of concrete, in addition to the mix proportions of the constituent materials for predicting the compressive strength of high performance concrete. This was done to get a better representation of the strength gaining process for the different ages of testing. Introducing those independent variables gave proved to improve the final multi-variable power equation, with a correlation

coefficient reaching 99.99%. Mahmoud [33] also forwarded his statistical compressive strength predicting model and found good correlations with experimental results. His model focused on forwarding an aid for designing concrete mixes so the inputs were generalized according to a predetermined matrix mixture.

The ease in modelling, compounded by the relatively smaller amount of time needed, has long made regression methods appealing. However, as the non-linearity between reactants and products increases (which is usually the case in concrete), the predicting power of each regression model highly subsides. This is visible in Subasi's research [28], when he tried to compare this technique with artificial neural network method and found the latter to give much better predictions.

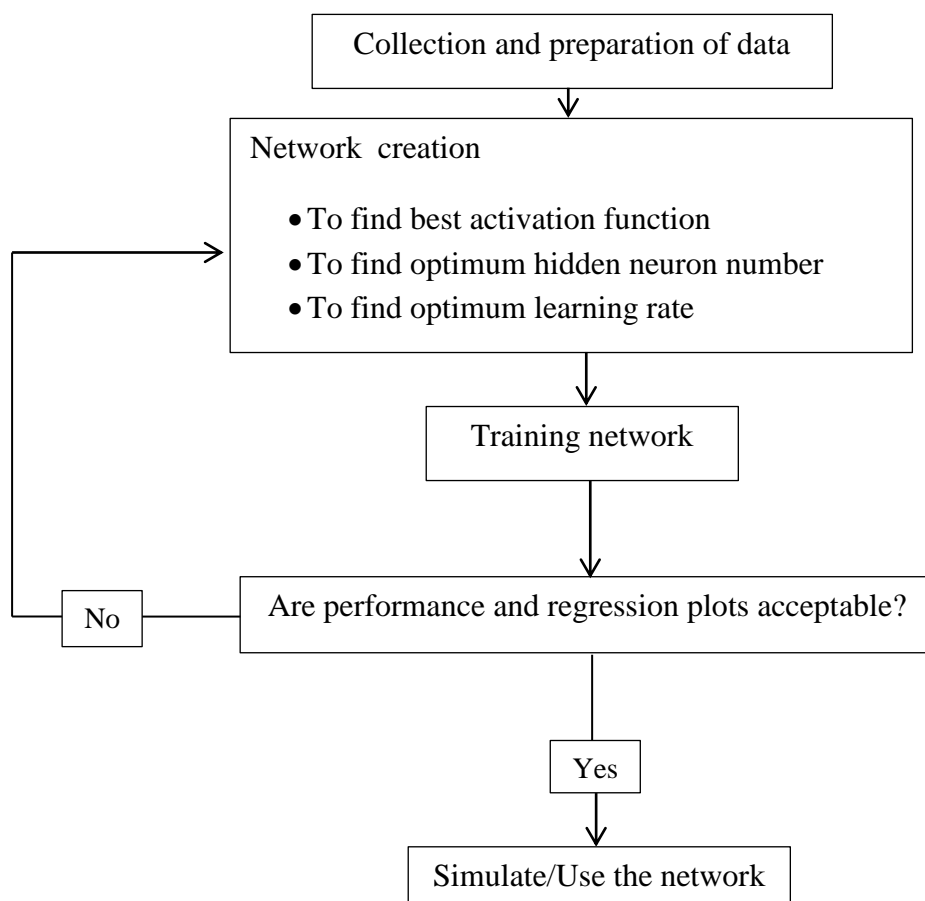
Hamid et.al. [34] also presented a shear strength prediction model for concrete beams reinforced with GFRP bars. An empirical equation was developed using multiple regression analysis from the experimental results of 16 RC beams with GFRP bars. The proposed equation involved the parameters that affected the shear strength of beams such as compressive concrete strength, shear span ratio, longitudinal reinforcement ratio and modulus elasticity of the reinforcement. The accuracy of the proposed equation was verified by predicting the available experimental data from the literature.

## 4. Materials and Methods

### 4.1 ANN

An ANN model should be designed in such a way that it produce outputs having minimum error and works efficiently overall.

This design can be easily understood with flowchart shown below:



**Fig. 4.1:** Flow chart for designing ANN model

#### 4.1.1 Collection and preparation of data

The very first step is to collect data samples to be entered into ANN model. Data sets should cover the whole range of inputs of the problem. This is due to the fact that whilst ANNs are generally capable of being trained to generalise well within the range of training data sets, they cannot correctly extrapolate beyond this range. In addition, redundant or useless data should be removed from data sets. Training intends to give the network the information as an example so that it can learn, or change its weights, to such an extent that it accurately replicates the compressive quality when new data is presented to them. This is very important step as ANN model can work as accurately as accurate data is presented to them to be trained.

As discussed earlier, data were collected for 149 concrete samples for developing. They cover a wide range of different mix proportions. Their range is shown below:

**Table 4.1:** Range of components of data sets

<b>Input Parameters</b>	<b>Range</b>
w/b	0.4-0.66
cement	55-600
Fly ash %	0-85
water	103-204
Coarse aggregate	775-1277
Fine aggregate	491-937

Of these records, 3 ANN models were developed:

- ANN-1 model was trained and tested by providing with only 3 input parameters as w/b ratio, water and cement
- ANN-3 model was trained and tested by provided with 4 input parameters as w/b ratio, water, cement and flyash %

- ANN-3 model was trained and tested by provided with 6 input parameters as w/b ratio, water, cement, flyash %, coarse aggregate and fine aggregate.

Target/Output parameter in both models was 28<sup>th</sup> day compressive strength of concrete.

All data were divided into three set:

- 70% of data was used for network learning (training) set
- 15 % for validation set
- 15% for testing set.

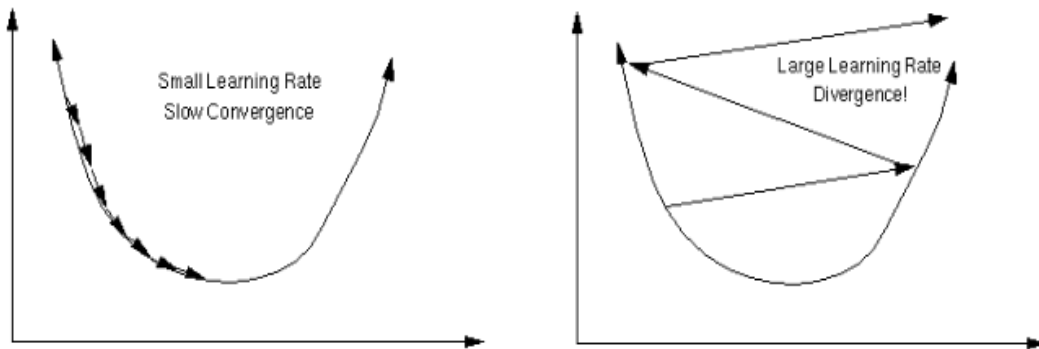
#### **4.1.2 Creation, training and testing of network**

Before simulating the network for new inputs(in this case, our lab results), we have to create few networks to find out best network having suitable activation function, optimized hidden neuron number and also learning rate. Performance of each networks were evaluated using root mean square error (RMSE) and co-efficient of correlation (R value) statistics. Best network was chosen which gave low RMSE and high R-value as it shows better estimate results. Levenberg-Marquardt backpropagation algorithm was used for training the network on the grounds that in previous researches it proved to being the fastest for network convergences.

In our study, predictive capabilities of feed-forward back-propagation were examined. The training commences with random weights, and proceeds from input layer towards output layer. After that whatever error occurs, it was propagated back to prior layers. Any difference between output values from model and actual/observed values is accounted as error. Back propagation algorithm aims to reduce this error by altering the weights assigned to each node of layers, according to error occurred. Calculation to update the weight is done from partial derivative of error function multiplied by a constant known as learning rate.

Learning rate determines how rapidly or gradually the neural network converges. The decision of learning rate largely affects the performance of the neural network. Small values for network may result in very slow convergence whereas high values for network could result in a considerable variance in the training.





**Fig. 4.2:** Graph showing effect of selecting too small or large learning rate

#### 4.1.3 ANN procedure:

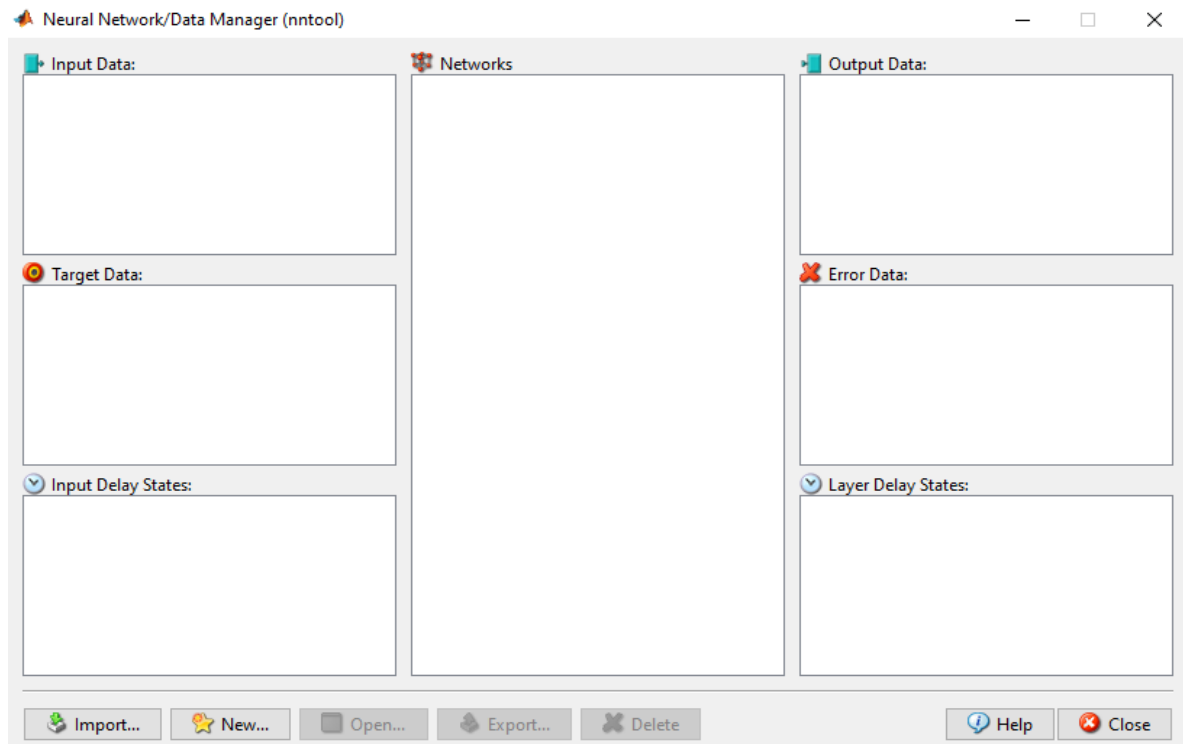
MATLAB (version R 2015a) was used to develop ANN networks, for training, validation and testing of these networks. MS-EXCEL was used for input, output (target) and samples data processing.

The following steps were followed for the developing ANN models in the Matlab Neural Network toolbox.

Step 1: Start Matlab.

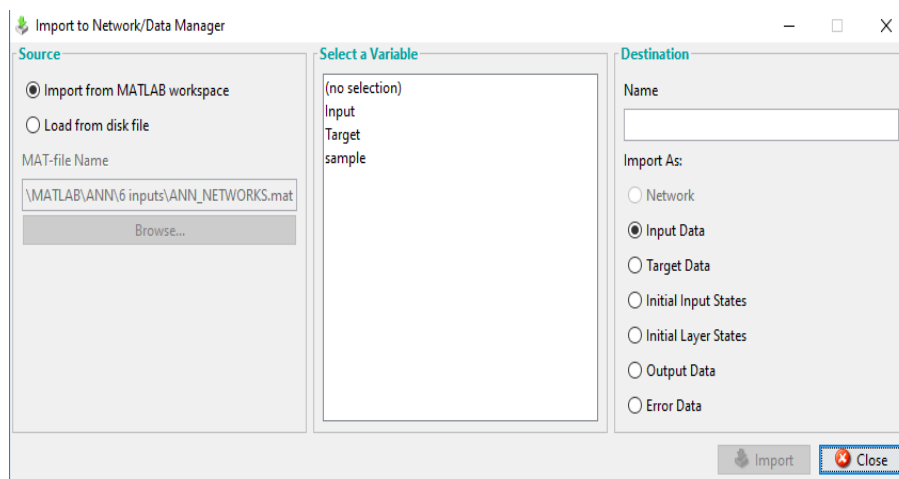
Step 2: As stated earlier, MS-EXCEL software was used to prepare the collected data into matrix form separated as input and target variables for use, and imported to MATLAB. Also there is one more set of data which was named as ‘samples’ which is new set of inputs (experimental results performed in lab) used to simulate models to predict strength.

Step 3: Type ‘nntool’ in the space provided in Command Window in Matlab. A new window named ‘Neural Network/ Data Manager (nntool)’ will pop up in the screen shown below:



**Fig. 4.3:** Network Data Manager Window

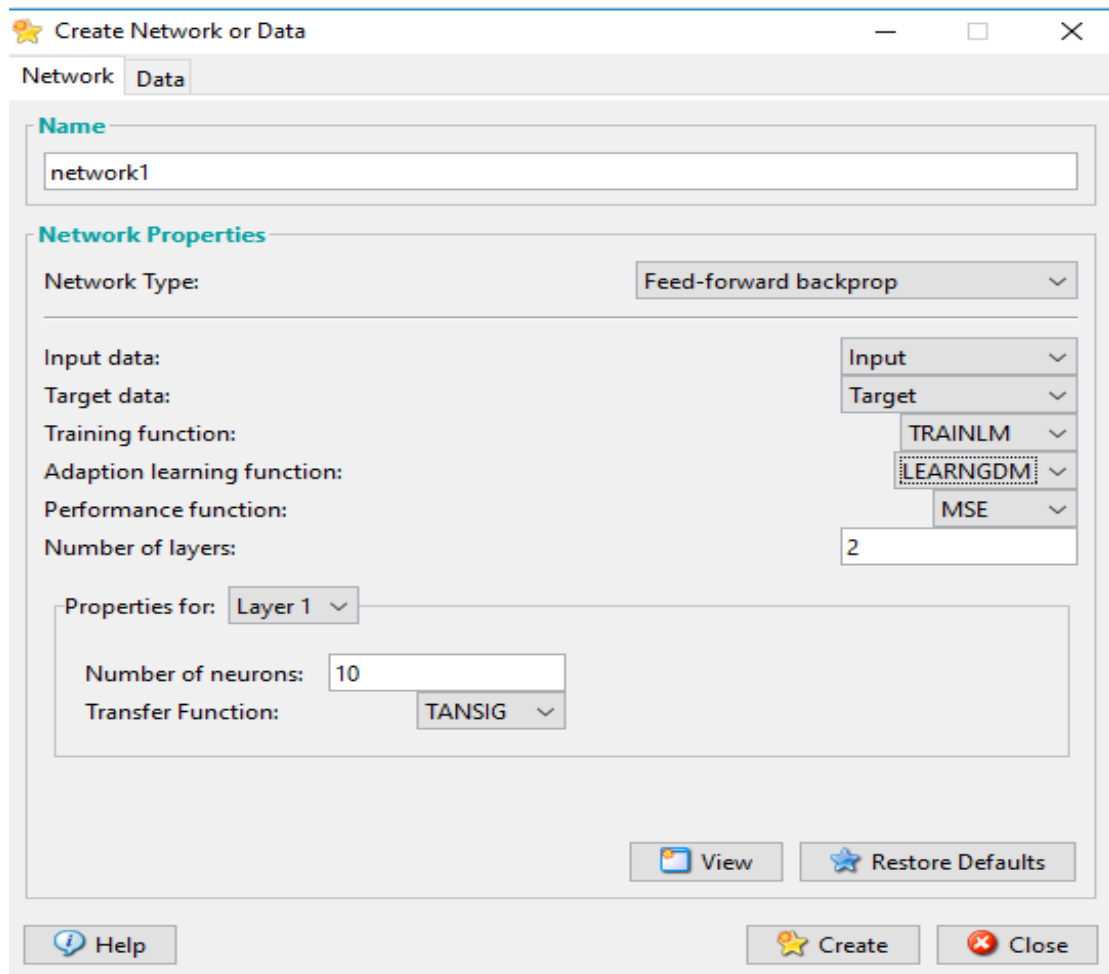
In this stage, data are imported from Workspace of Matlab to this Data Manager window, and they are split up in two groups: inputs and samples have to entered into Input Data section and Outputs (Targets) into Target Data.



**Fig. 4.4:** Import of Data to Network/Data

Step 4: Click on 'new' tab to create networks in Data Manager window. A total of 26 networks for each models (i.e.,total 78) were developed, trained and tested by

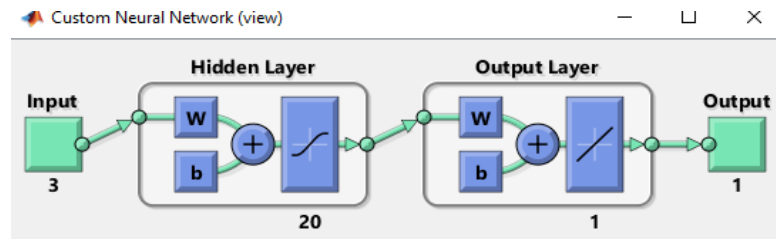
varying the transfer function, hidden neuron number and learning rate to find out best network.



**Fig. 4.5:** Network Creation in NN toolbox

Feed-forward back propagation type of network was selected among the available network types. Levenberg-Marquardt (trainlm) was used as training function to updates weight and bias value. Mean square error (MSE) was chosen to measures performance of networks.

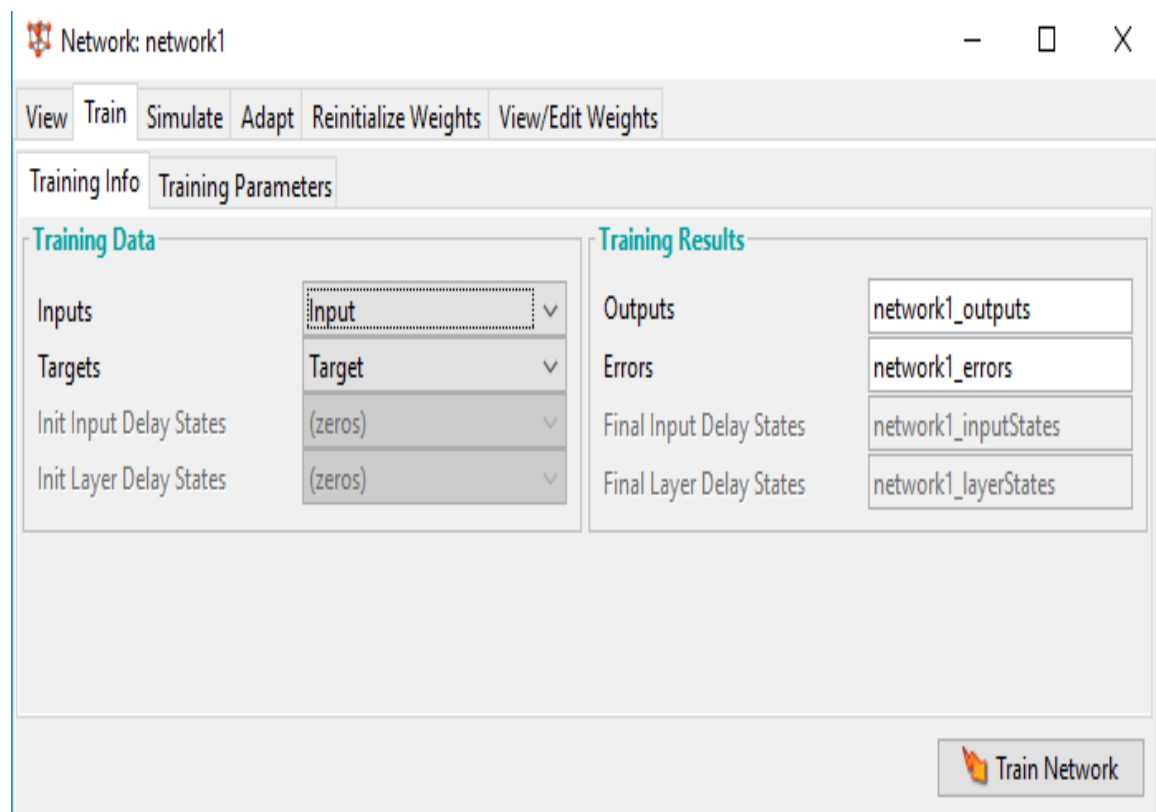
All networks made in this study had one hidden layers and number of hidden neurons numbers has to be find out. Also decision has to be made that whether to use logistic Sigmoid(LOGSIG) or tan sigmoid(TANSIG) as transfer function, whichever gives better results (having lower RMSE value and higher R-value).



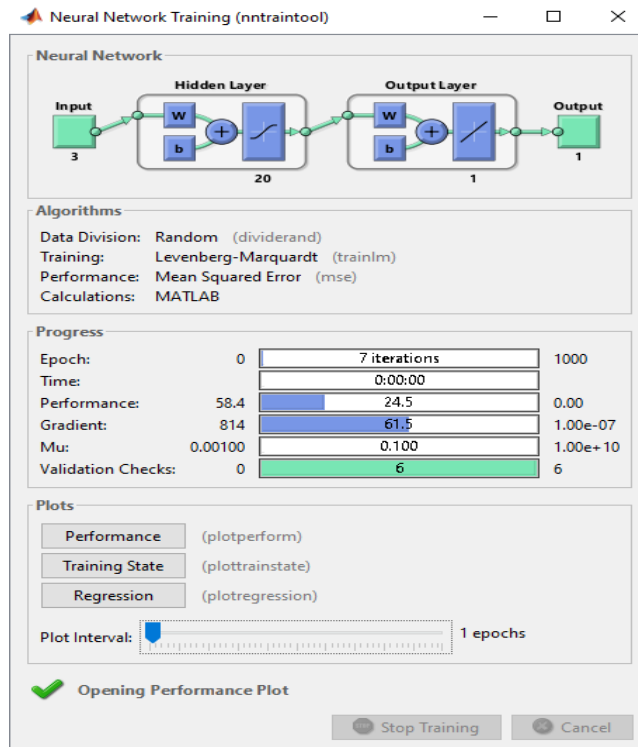
**Fig. 4.6:** Created Network

Figure shows a representation of feed forward neural network with hidden layer having tansig or logsig as transfer function and second layer, i.e. output layer has got purelin transfer function.

Step 5: Click on the Train Network under train tab as shown in figure. A pop-up windowpane as demonstrated in Figure 4.8 will appear.

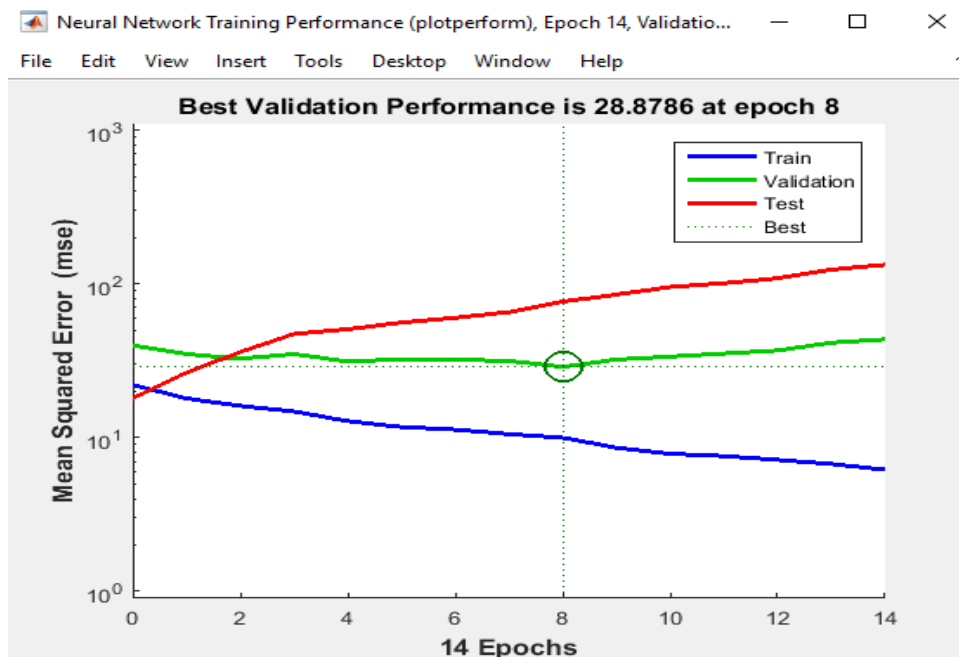


**Fig. 4.7:** Network Training Phase



**Fig. 4.8:** Network Training Window

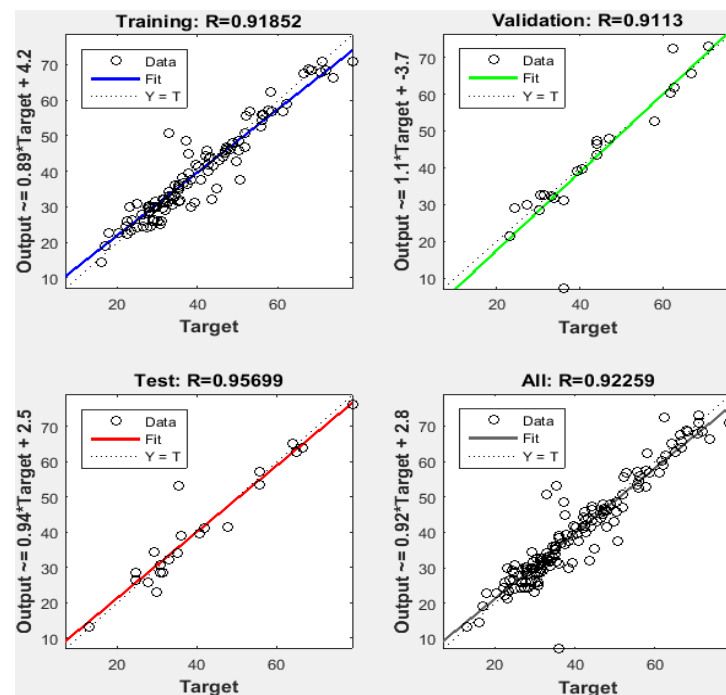
We can see here the training results like epochs, time, and performance that has been appeared through the training stage. Performance of this training phase can be analysed to locate validation errors by clicking on ‘performance’ button.



**Fig. 4.9:** Network Training, Performance Plot

In figure, performance chart is shown in which blue line shows the training error, green line shows the validation error and red line shows the testing error, while dotted line shows the finest results of the performance graph.

Regression option has to be selected in order to execute linear regression between network outputs (predicted results) and targets (observed results).



**Fig. 4.10:** Network Regression Graph

The above mentioned figure graph displays the accuracy as well as reliability of the final outcomes based on regression model, the goodness of fit is actually compared based on the coefficient of determination R values for all the three kinds of data are nearly equal to one.

Step 6: Simulate the trained network for new inputs to obtain predicted compressive strength to our experimental lab work.

#### 4.2 Multiple Linear Regression Analysis

Multiple linear regression was done on MS-EXCEL software.

The regression equation is in the form of:

$$Y = a \times (w/b) + b \times (\text{cement}) + c \times (\text{water}) + d \times (\text{flyash \%}) + e \times (\text{coarse aggregate}) + f \times (\text{fine aggregate})$$

Where  $Y$  is 28<sup>th</sup> day compressive strength of concrete in MPa, and  $a, b, c, d, e$  and  $f$  are regression co-efficients for their respective parameters.

#### 4.2.1 MLR procedure:

Step-1: Data has to be arrange in column in EXCEL spreadsheet to use regression function.

Step-2: Under the **Data** tab, select **Data Analysis** option. Within Data Analysis, chose Regression.

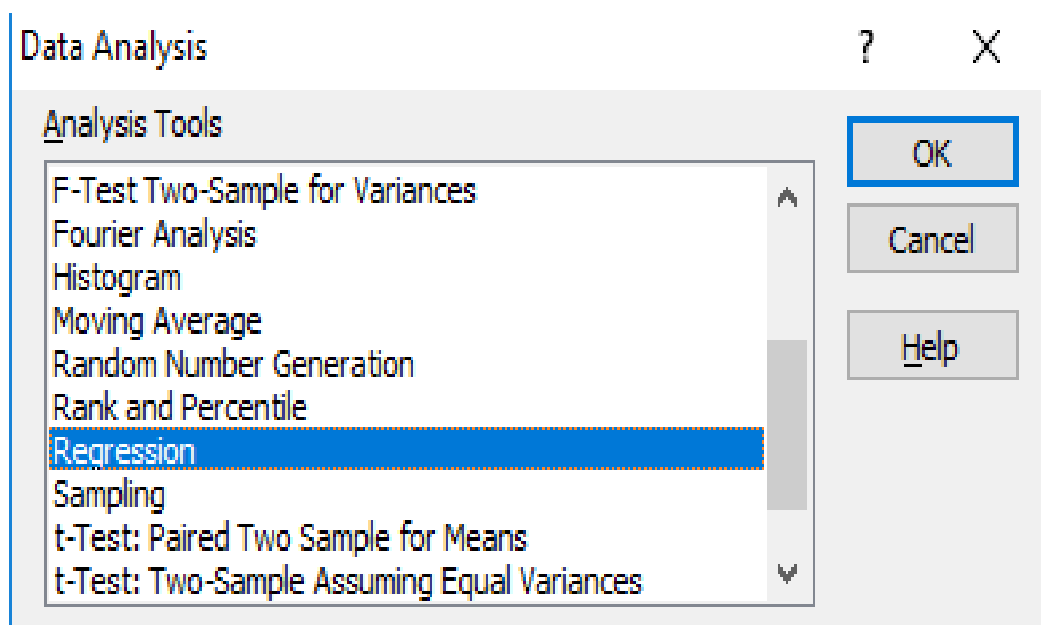
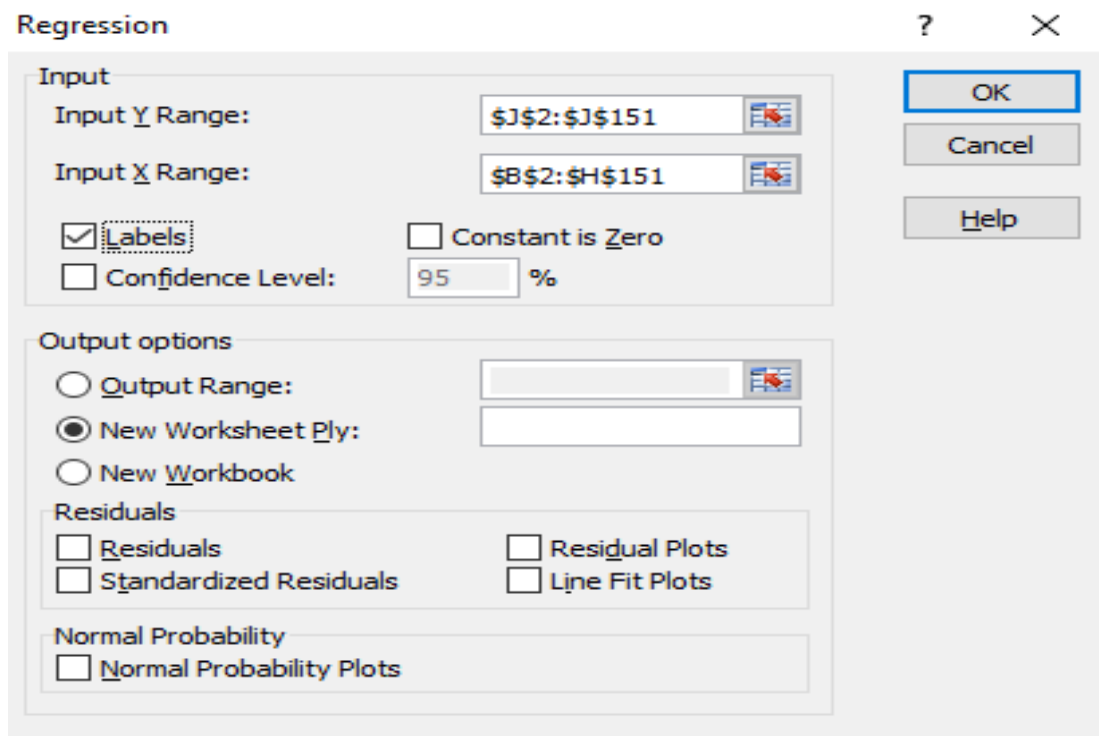


Fig. 4.11: Data Analysis Window

Step-3: Specify the Y-range (dependent variables, in our study strength) and X-range (independent variables, i.e. input parameters). If variables are also included in the column headings during arranging of data in Step-1, then check **Labels** also:



**Fig. 4.12:** Input of data in Regression Window

Step-4: Interpret the regression results that will appear after clicking on ‘OK’ button.

Apply F-test using analysis of variance (ANOVA). The table of this analysis is presented here:

**Table 4.2:** Outline of ANOVA table

Sources of variation	Degree of freedom	Sum of squares	Mean squares	F-ratio
Regression	k	SSR	$MSR = \frac{SSR}{k}$	$F = \frac{MSR}{MSE}$
Error	n-k-1	SSE	$MSE = \frac{SSE}{n-k-1}$	
Total	n-1	SSy		



## 4.2 Performance Evaluation:

In order to evaluate the prediction accuracy of above developed models, two norms were used for comparative study of performance of each model. These are Root Mean Square Error (RMSE) and coefficient of determination ( $R^2$ ).

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}}$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y}_i)^2}$$

where  $y_i$ ,  $\hat{y}_i$  and  $\bar{y}_i$  are actual, predicted and averaged actual output of the network/model respectively.

'n' is the number of data/ samples presented.

The smaller the RMSE value obtain, the better the estimates are in prediction. Higher the value of  $R^2$  (closer to 1) comes; predicted value is better correlated with actual/observed value.

## 4.4 Experimental Program

Following materials were used in preparation of concrete mixes:

- Cement (OPC of 43 grade)
- Water
- Flyash
- Coarse aggregate
- Fine aggregate
- Admixture

In the present study, total 6 mixes were prepared with varying flyash percentage. OPC was partially replaced with 0%, 20% and 30% flyash by weight. Mix design was done to prepare concrete cubes of M40 and M45 characteristic strength.

#### 4.4.1 Calculation for mix design M40:

Grade Designation = M40

Target Mean Strength =  $40 + (5 \times 1.65) = 48.25$  MPa

Assume w/c ratio = 0.45 (From Table 5 of IS456)

Assume cement content =  $350 \text{ kg/m}^3$

water content =  $0.45 \times 350 = 158$  kg (which is less than maximum water content for 20mm aggregate = 180 Kg)

Now,

$$V = \frac{[w + \frac{C}{S_c} + \frac{1}{p} (\frac{f_a}{S_{fa}})]}{1000}$$

$$c_a = \left(\frac{1-p}{p}\right) \cdot f_a \cdot \left(\frac{S_{ca}}{S_{fa}}\right)$$

$V$  = absolute volume of fresh concrete, which is equal to gross volume ( $\text{m}^3$ ) minus the volume of entrapped air,

$w$  = mass of water (kg) per  $\text{m}^3$  of concrete,

$c$  = mass of cement (kg) per  $\text{m}^3$  of concrete,

$S_c$  = specific gravity of cement (assumed 3.15),

$p$  = ratio of fine aggregate to total aggregate by absolute volume,

$f_a$ ,  $c_a$  = total mass of fine aggregate and coarse aggregate (kg) per  $\text{m}^3$  of concrete respectively,

$S_{fa}$ ,  $S_{ca}$  = specific gravities of saturated surface dry fine aggregate and coarse aggregate respectively.

For 20mm maximum size entrapped air is 2%.

Assume f.a. by % of volume of total aggregate = 36.5%

$$0.98 = \frac{[158 + (\frac{350}{3.15}) + (\frac{1}{0.365})(\frac{f_a}{2.61})]}{1000}$$

$f_a = 660$  kg

$$c_a = 1168 \text{ kg}$$

Considering 20 mm:10mm of coarse aggregates= 0.6: 0.4

Using 20 mm aggregate = 701 kg, 10 mm aggregate = 467 kg

Admixture= 0.6 % by weight of cement = 2.4 kg

#### 4.4.2 Calculation for mix design M45:

Grade Designation = M45

Target Mean Strength =  $45 + (5 \times 1.65) = 53.25 \text{ MPa}$

Assume w/c ratio= 0.4 (From Table 5 of IS456)

Assume cement content=  $400 \text{ kg/m}^3$

water content= $0.4 \times 400 = 160 \text{ kg}$  (which is less than maximum water content for 20mm aggregate = 180 Kg)

For 20mm maximum size entrapped air is 2%.

Assume f.a. by % of volume of total aggregate = 36.5%

$$0.98 = \frac{[160 + (\frac{400}{3.15}) + (\frac{1}{0.365})(\frac{f_a}{2.61})]}{1000}$$

$$f_a = 668 \text{ kg}$$

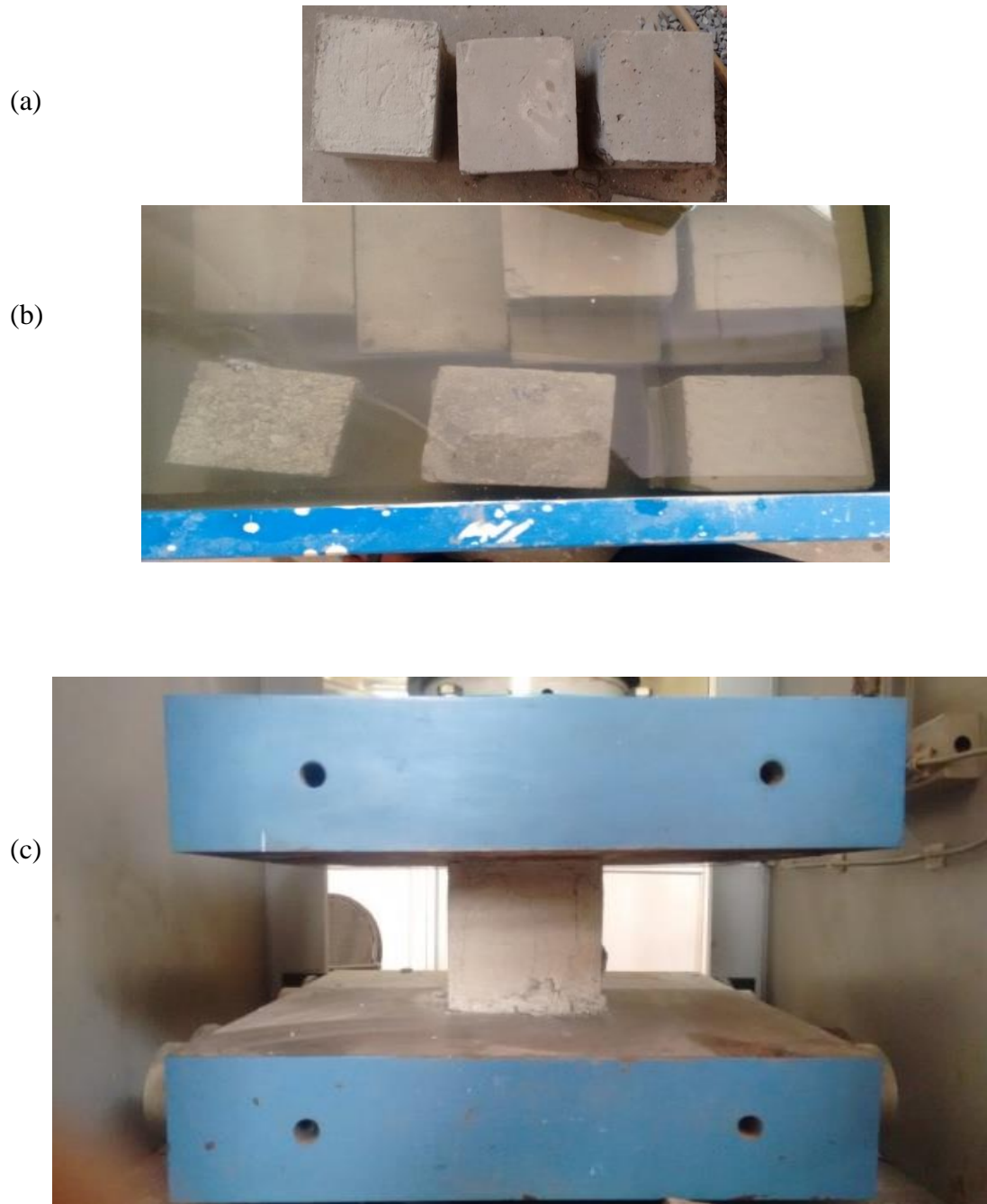
$$c_a = 1180 \text{ kg}$$

Considering 20 mm:10mm = 0.6: 0.4

Using 20 mm aggregate = 708 kg, 10 mm aggregate = 472 kg

Admixture= 0.6 % by weight of cement = 2.4 kg

Concrete cube specimens were prepared of size 150mm×150mm×150mm. The concrete was left in the mould and allowed to set for 24 hours before the specimens were demoulded and placed in curing tank. All samples were cured in curing tank for 28 days. Then 28<sup>th</sup> day compressive strength was measured from failure load obtained in compression testing machine.



**Fig. 4.13:** (a) casting , (b) curing and (c) testing of cubes

## 5. Results and Discussion

### 5.1 ANN models

As stated earlier a total of 78 networks were developed (26 for each model) to find optimum number of hidden neuron numbers, learning rate and suitable transfer function. The observed performance and correlation coefficients are summarized from Table.

**Table 5.1:** Performance of different networks with different hidden neuron numbers and activation functions of ANN-1 model

Activation function	Number of hidden neuron numbers	R values				RMSE
		Training	Validation	Testing	All	
Tansig	5	0.85	0.85	0.85	0.85	7.54
	10	0.89	0.88	0.70	0.86	7.62
	20	0.92	0.94	0.92	0.92	5.56
	30	0.93	0.98	0.94	0.91	5.89
	40	0.86	0.98	0.99	0.90	6.32
	50	0.93	0.83	0.87	0.90	6.59
	60	0.91	0.95	0.91	0.92	5.83
	70	0.90	0.94	0.89	0.90	5.83
	80	0.87	0.95	0.87	0.88	7.43
Logsig	5	0.82	0.91	0.82	0.84	7.74
	10	0.86	0.90	0.85	0.86	7.28
	20	0.88	0.94	0.92	0.90	6.38
	30	0.93	0.92	0.88	0.91	5.69
	40	0.93	0.92	0.92	0.93	6.89
	50	0.92	0.88	0.84	0.88	7.49
	60	0.93	0.84	0.80	0.87	6.96
	70	0.89	0.91	0.89	0.89	7.37
	80	0.91	0.49	0.75	0.84	9.28

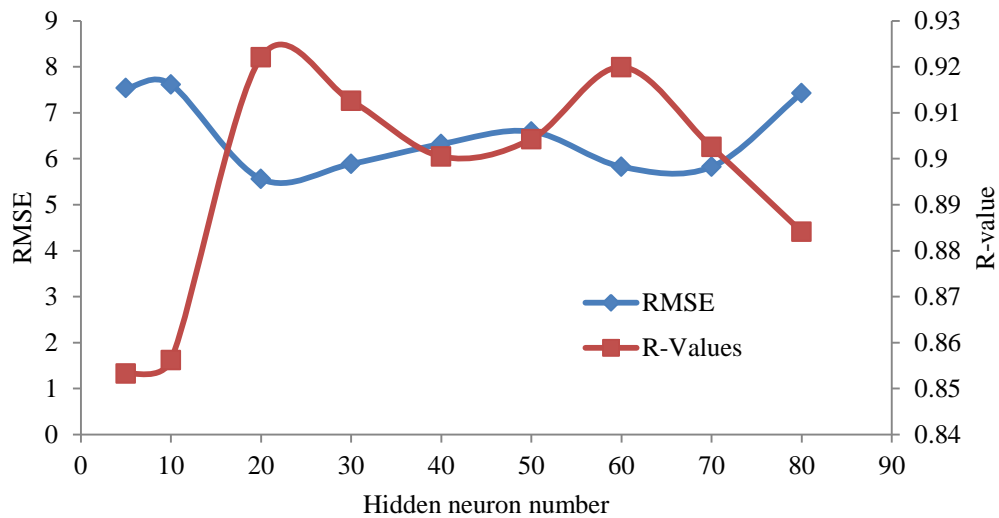
**Table 5.2:** Performance of different networks with different hidden neuron numbers and activation functions of ANN-2 model

Activation function	Number of hidden neuron numbers	R values				RMSE
		Training	Validation	Testing	All	
Tansig	5	0.86	0.84	0.86	0.85	5.14
	10	0.90	0.87	0.73	0.88	6.72
	20	0.90	0.96	0.92	0.92	5.47
	30	0.94	0.98	0.96	0.96	4.22
	40	0.86	0.98	0.91	0.90	4.80
	50	0.90	0.81	0.85	0.91	5.91
	60	0.91	0.95	0.91	0.92	6.46
	70	0.91	0.92	0.89	0.89	6.83
	80	0.86	0.91	0.86	0.82	7.53
Logsig	5	0.80	0.92	0.81	0.84	7.47
	10	0.87	0.90	0.84	0.87	7.52
	20	0.86	0.92	0.92	0.90	5.25
	30	0.91	0.90	0.91	0.90	5.69
	40	0.92	0.92	0.92	0.93	6.47
	50	0.91	0.89	0.85	0.87	7.21
	60	0.92	0.86	0.81	0.88	6.96
	70	0.89	0.90	0.88	0.88	7.32
	80	0.95	0.48	0.75	0.83	7.29

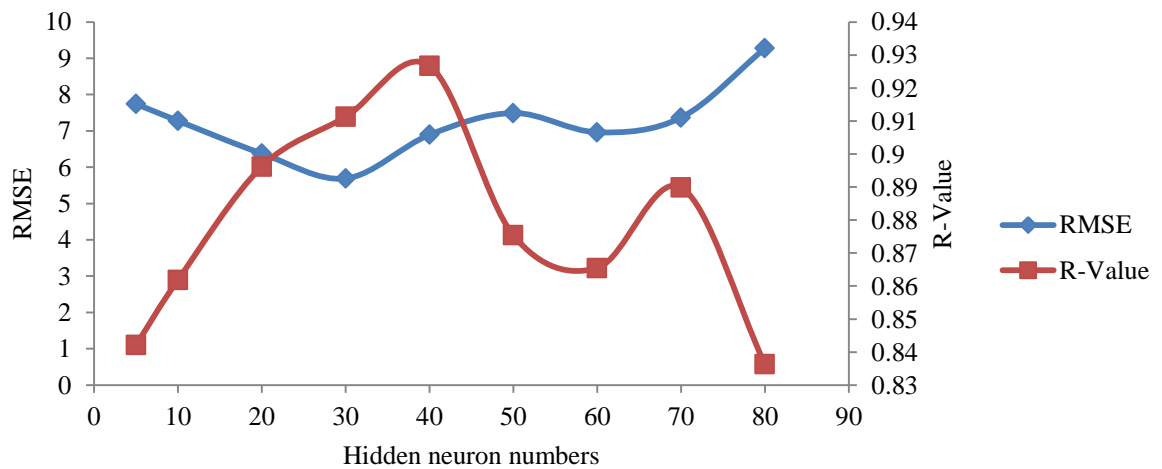
**Table 5.3:** Performance of different networks with different hidden neuron numbers and activation functions of ANN-3 model

Activation function	Number of hidden neuron numbers	R values				RMSE
		Training	Validation	Testing	All	
Tansig	5	0.91	0.90	0.87	0.90	6.21
	10	0.95	0.93	0.90	0.94	4.79
	20	0.95	0.91	0.97	0.95	4.68
	30	0.93	0.88	0.88	0.91	6.00
	40	0.94	0.85	0.67	0.91	6.54
	50	0.87	0.81	0.89	0.88	7.74
	60	0.53	0.21	0.44	0.41	7.86
	70	0.90	0.90	0.92	0.91	7.83
	80	0.86	0.91	0.81	0.83	7.83

	5	0.92	0.84	0.89	0.89	6.50
	10	0.95	0.95	0.82	0.92	5.62
	20	0.81	0.72	0.77	0.78	9.24
	30	0.97	0.82	0.80	0.93	5.24
Logsig	40	0.99	0.81	0.35	0.65	16.34
	50	0.93	0.92	0.95	0.93	5.70
	60	0.95	0.87	0.87	0.92	5.56
	70	0.99	0.83	0.64	0.88	7.75
	80	0.86	0.64	0.21	0.56	18.58

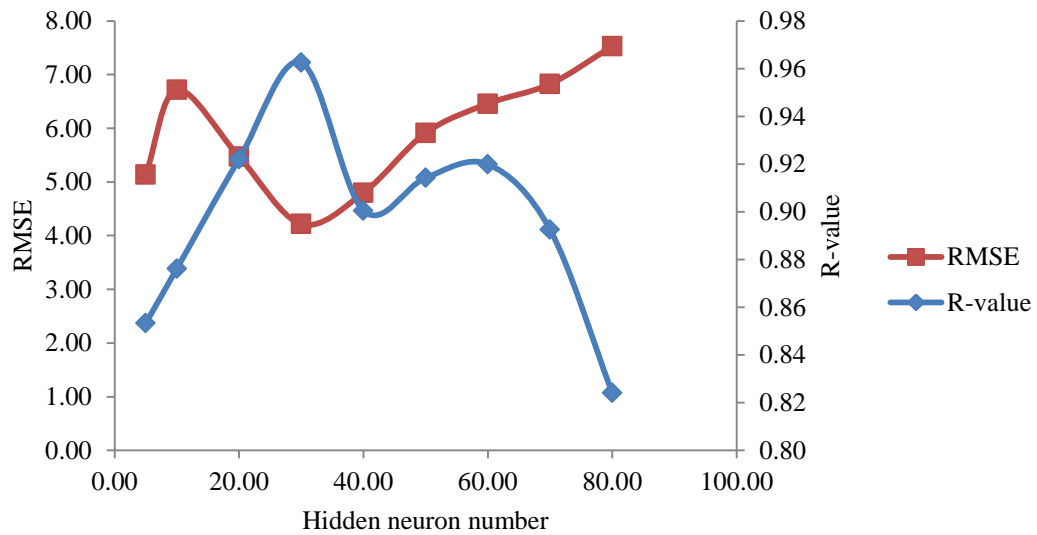


**Fig. 5.1:** RMSE and R- value versus different neuron number for ANN-1 model with tansig activation function

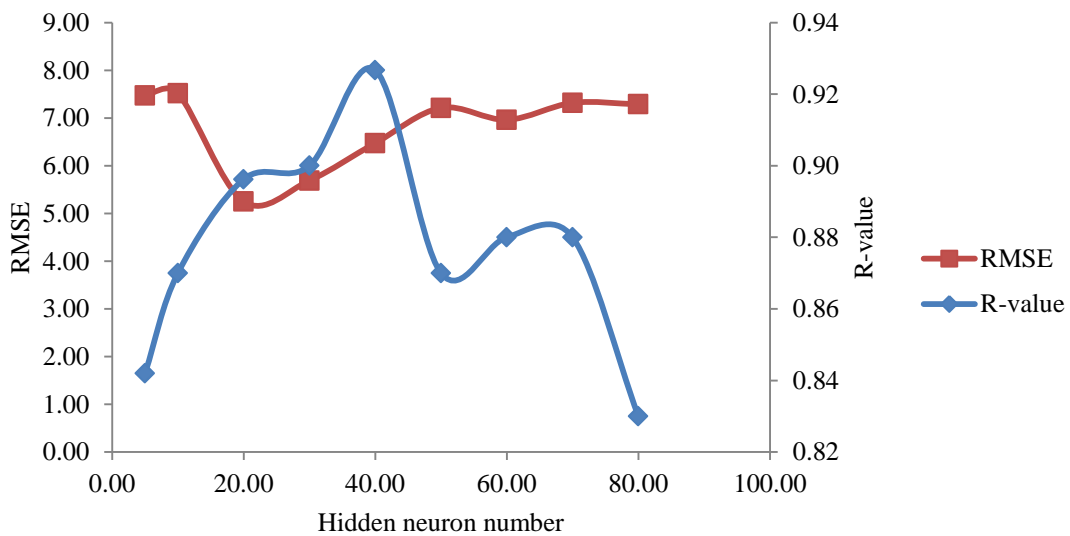


**Fig. 5.2:** RMSE and R- value versus different neuron number for ANN-1 model with logsig activation function

It can be seen from above graphs and tables that smallest RMSE (i.e. 5.6) and highest R-value (0.92) are obtained by 20 hidden neuron numbers with tansig as activation function for ANN-1 model.



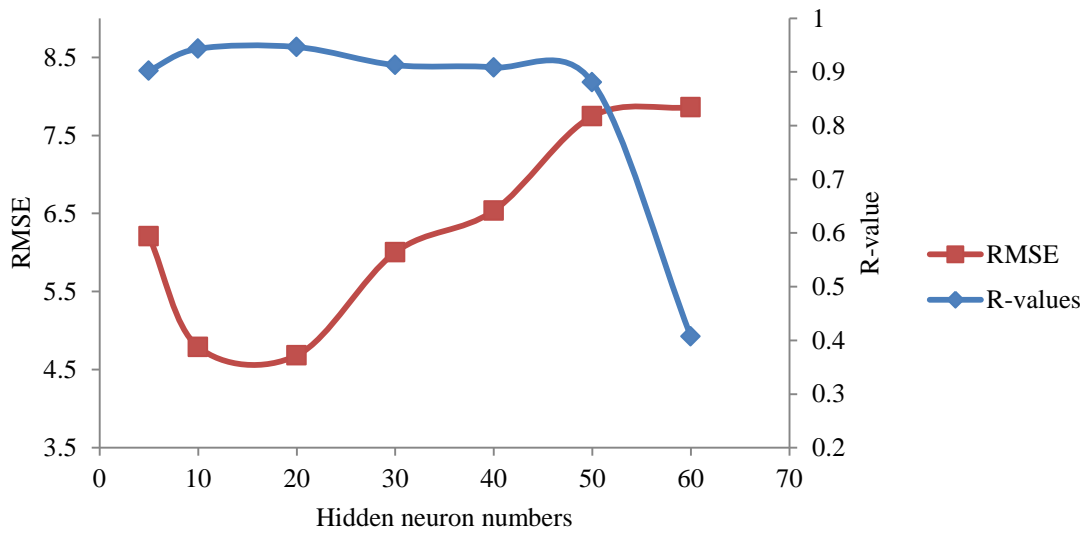
**Fig. 5.3:** RMSE and R- value versus different neuron number for ANN-2 model with tansig activation function



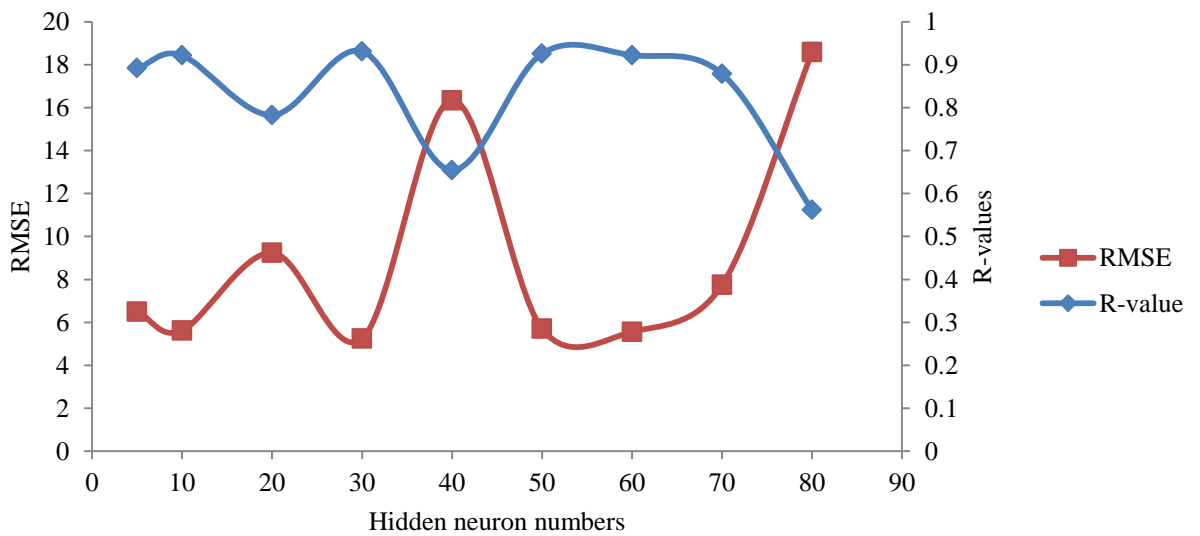
**Fig. 5.4:** RMSE and R- value versus different neuron number for ANN-2 model with logsig activation function

It can be seen from above graphs and tables that smallest RMSE (i.e. 4.22) and highest R-value (0.96) are obtained by 30 hidden neuron numbers with tansig as activation function for ANN-2 model.





**Fig. 5.5:** RMSE and R- value versus different neuron number for ANN-3 model with tansig activation function



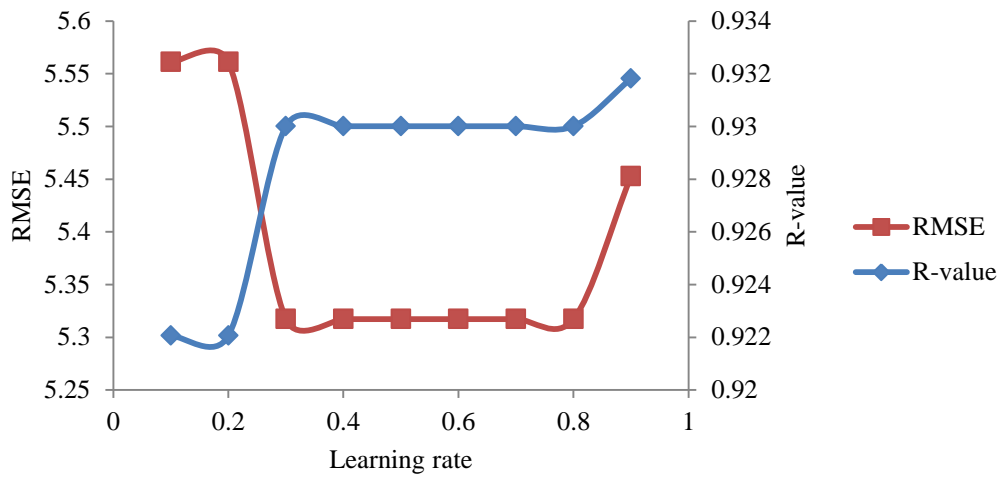
**Fig. 5.6:** RMSE and R-value versus different neuron number for ANN-3 model with logsig activation function

Again it can be seen from above graphs and tables that smallest RMSE (i.e. 4.6829) and highest R-value (0.94645) are obtained by 20 hidden neuron numbers with tansig as activation function for ANN-3 model.

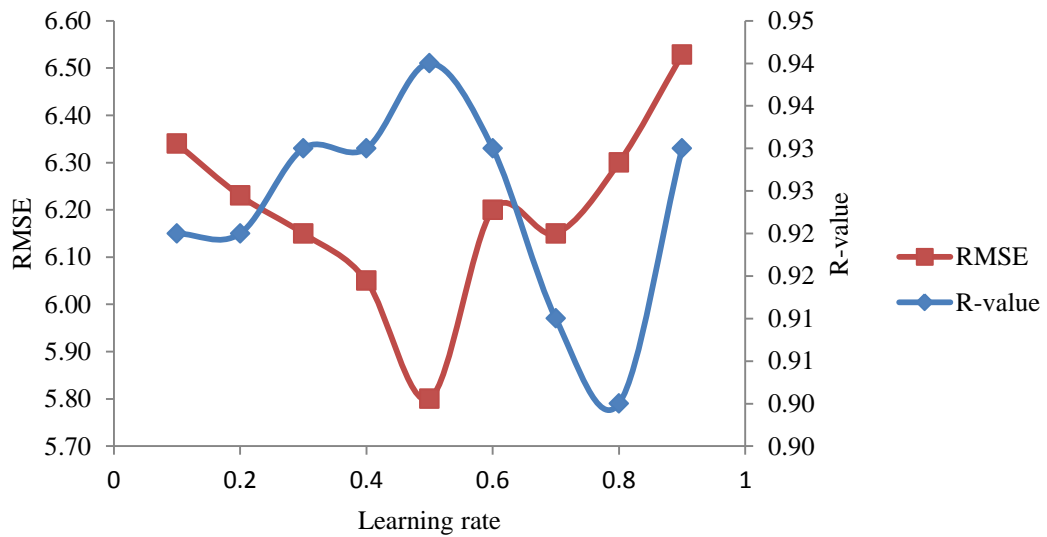
Again to find optimum learning rate, 10 networks were made with tansig as activation function and 20 hidden neuron numbers for each models. The results are summarized in table and graphs given below:

**Table 5.4:** Performance of networks with different learning rate for all three models

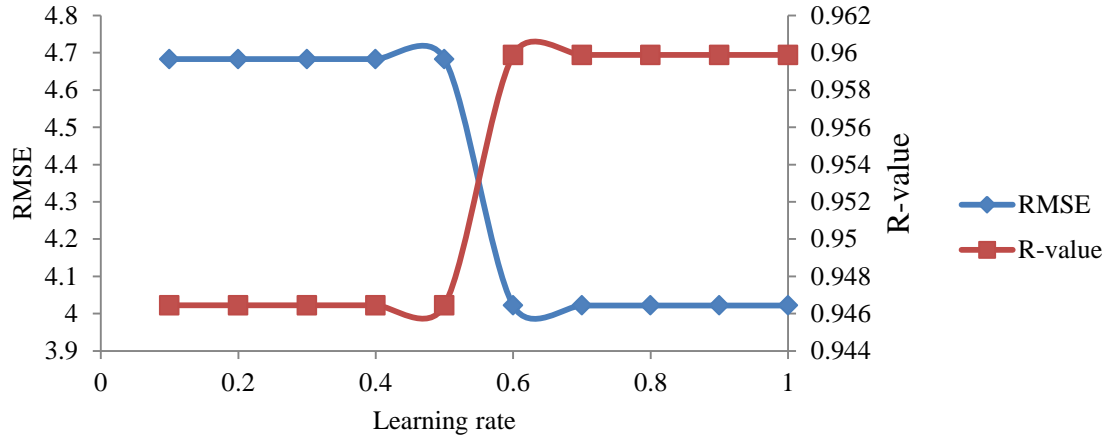
Model	Activation function	R-value				RMSE
		Training	Validation	Testing	All	
ANN-1	0.1	0.92	0.94	0.92	0.92	5.56
	0.2	0.93	0.91	0.93	0.92	5.56
	0.3	0.93	0.95	0.92	0.93	5.32
	0.4	0.93	0.95	0.93	0.93	5.32
	0.5	0.93	0.96	0.95	0.93	5.32
	0.6	0.94	0.90	0.91	0.93	5.32
	0.7	0.92	0.94	0.97	0.93	5.32
	0.8	0.92	0.93	0.94	0.93	5.32
	0.9	0.93	0.90	0.91	0.93	5.45
	1	0.92	0.94	0.92	0.92	5.56
ANN-2	0.1	0.91	0.94	0.91	0.92	6.34
	0.2	0.92	0.90	0.93	0.92	6.23
	0.3	0.93	0.90	0.92	0.93	6.15
	0.4	0.94	0.95	0.94	0.93	6.05
	0.5	0.95	0.96	0.97	0.94	5.80
	0.6	0.90	0.90	0.91	0.93	6.20
	0.7	0.92	0.94	0.97	0.91	6.15
	0.8	0.90	0.92	0.91	0.90	6.30
	0.9	0.90	0.91	0.92	0.93	6.53
	1	0.91	0.94	0.91	0.92	6.34
ANN-3	0.1	0.95	0.91	0.97	0.95	6.34
	0.2	0.94	0.98	0.97	0.95	6.23
	0.3	0.97	0.96	0.87	0.95	6.15
	0.4	0.94	0.97	0.95	0.95	6.05
	0.5	0.95	0.96	0.95	0.95	5.80
	0.6	0.96	0.98	0.96	0.96	6.20
	0.7	0.95	0.97	0.95	0.96	6.15
	0.8	0.97	0.95	0.92	0.96	6.30
	0.9	0.97	0.96	0.94	0.96	6.53
	1	0.95	0.96	0.95	0.96	6.34



**Fig. 5.7:** RMSE and R-value versus different neuron number for ANN-1 model with tansig activation function and 20 number of hidden neurons



**Fig. 5.8:** RMSE and R-value versus different neuron number for ANN-2 model with tansig activation function and 30 number of hidden neurons



**Fig. 5.9:** RMSE and R-value versus different neuron number for ANN-3 model with tansig activation function and 20 number of hidden neurons

It was found for learning rate 0.5 for ANN-1 and ANN-3 model, and 0.6 for ANN-2 model to be optimum, as it gives lowest RMSE and highest R-value among all.

Once the network is trained, experimental results from lab work with varying flyash % are presented to it to find out the accuracy of it in predicting compressive strength. The materials used in the experimental program to make concrete cube are shown in table:

**Table 5.5:** Mix Proportions of flyash concrete

	w/b	cement (kg/m <sup>3</sup> )	flyash %	water (kg/m <sup>3</sup> )	coarse aggregate (kg/m <sup>3</sup> )	fine aggregate (kg/m <sup>3</sup> )
mix 1	0.45	350	0	158	1165	660
mix 2	0.45	280	20	158	1165	660
mix 3	0.45	245	30	158	1180	660
mix 4	0.4	400	0	160	1180	668
mix 5	0.4	320	20	160	1180	668
mix 6	0.4	280	30	160	1180	668

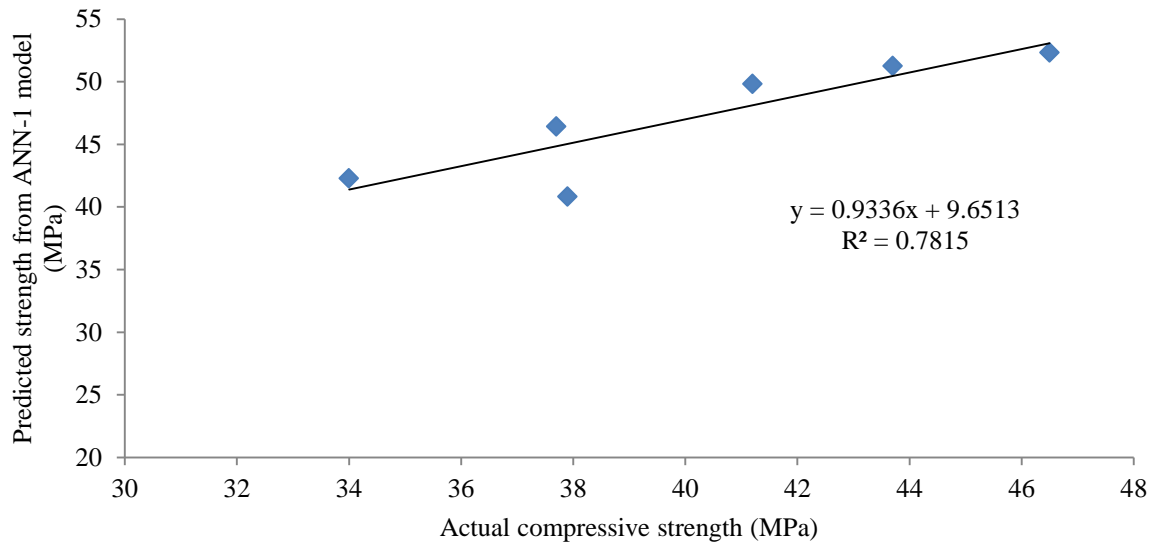
Experimental/ observed 28<sup>th</sup> compressive strength of concrete cube, predicted compressive strength from ANN-1 and ANN-2 model are presented below:

**Table 5.6:** Results from ANNs in prediction

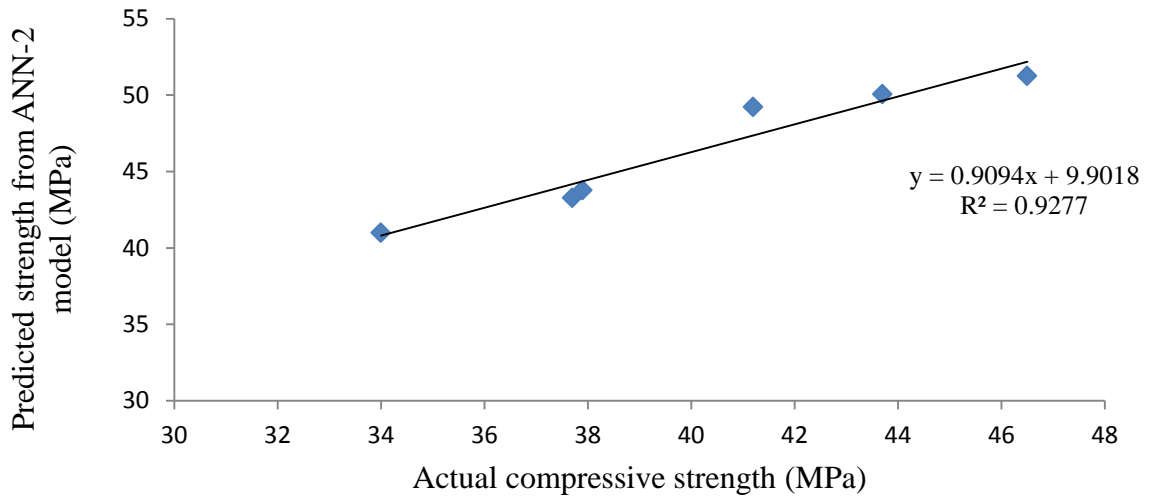
Mix no.	mix 1	mix 2	mix 3	mix 4	mix 5	mix 6
Observed/actual strength (MPa)	43.7	37.7	34	46.5	41.2	37.9
Predicted strength from ANN-1 model (MPa)	51.2	46.4	42.2	52.3	49.8	40.8
% error	17.3	23.1	24.3	12.5	20.8	7.7
RMSE	7.28					
Predicted strength from ANN-2 model (MPa)	50.1	43.2	40.9	51.2	49.2	43.7
% error	14.5	14.8	20.5	10.2	19.4	15.5
RMSE	6.33					
Predicted strength from ANN-3 model (MPa)	50.1	42.1	40.9	54.6	47.9	42.3
% error	14.5	11.7	20.5	17.4	16.4	11.8
RMSE	6.25					

From above table it can be seen the error for 3 results from ANN-1 model is found to be more than 20% while it is just for 1 result from ANN-2 model which is greater.

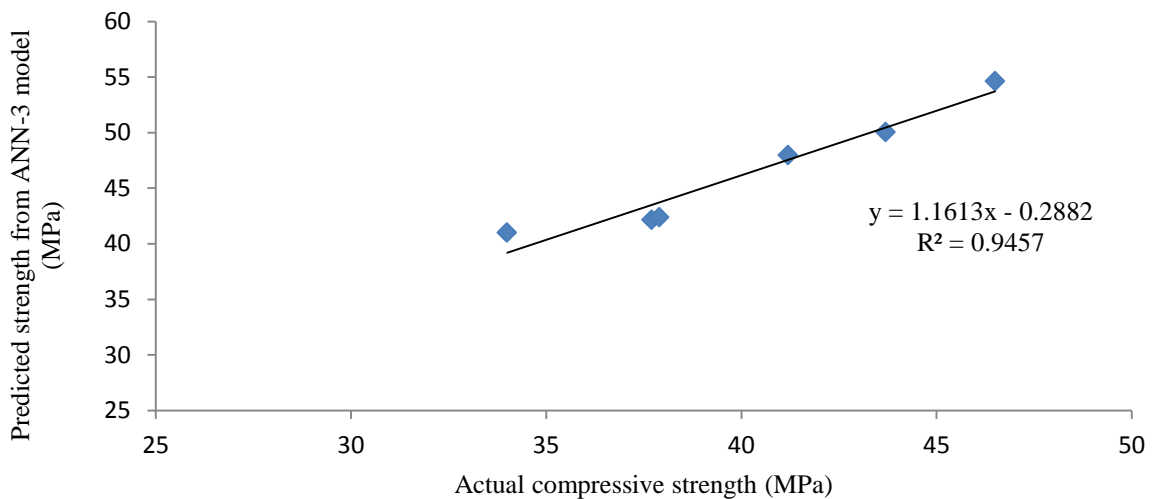
The experimented strength results are graphically plotted against predicted compressive strength from both ANN models and are shown in fig. below:



**Fig. 5.10:** Graph between actual compressive strength vs predicted using ANN-1 model



**Fig. 5.11:** Graph between actual compressive strength vs predicted using ANN-2 model



**Fig. 5.12:** Graph between actual compressive strength vs predicted using ANN-3 model

From above figures, it can be clearly stated that experimentally evaluated values of concrete compressive strength are in strong consistency with the values predicted through ANN for most of the specimens as a linear correlation can be observed and the correlation coefficient is found to be 0.7815 for ANN-1 model, 0.9277 for ANN-2 model and 0.9457 for ANN-3 model. It can also be seen from Table 5.6 that RMSE value is lower for ANN-2 model (6.33) and ANN-3 model (6.25) than that is what is obtained for ANN-1 model (7.28), and  $R^2$  value obtained is found to be higher for ANN-3 model. Thus increasing number of input parameters or it can be said that presenting more descriptive system to neural network for developing model can result in lowering error to predict strength of concrete.

### 5.2 Multiple Linear Regression Analysis

In the conventional/ traditional modeling process for prediction, regression analysis is an important tool for developing a model.

The regression equation is in the form of:

$$Y = a \times (w/b) + b \times (\text{cement}) + c \times (\text{water}) + d \times (\text{flyash}\%) + e \times (\text{coarse aggregate}) + f \times (\text{fine aggregate})$$

Where Y is 28<sup>th</sup> day compressive strength of concrete in MPa, and a,b,c,d,e and f are regression co-efficients for their respective parameters.

The result of regression analysis appear as follows:

**Table 5.7:** Regression Statistics (1)

Multiple R	0.72
R Square	0.52
Adjusted R Square	0.50
Standard Error	10.13
Observations	149

**Table 5.8:** ANOVA table (1)

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	6	16042.76	2673.793	26.04	9.61E-21
Residual	142	14576.56	102.65		
Total	148	30619.31			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	35.99	11.34	3.17	0.0018	13.56	58.40	13.57	58.42
w/b ratio	-0.94	0.72	-1.30	0.19	-2.37	0.48	-2.38	0.48
cement	0.15	0.014	9.98	4.03E-18	0.11	0.17	0.12	0.18
flyash %	0.31	0.068	4.55	1.12E-05	0.17	0.44	0.18	0.44
water	-0.30	0.039	-7.71	2E-12	-0.38	-0.23	-0.39	-0.23
coarse aggregate	-0.00056	0.001	-0.55	0.58	-0.002	0.00	0.00	0.00
fine aggregate	0.0082	0.009	0.88	0.37	-0.010	0.03	-0.01	0.03

### **5.2.1 Interpretation of result**

- Multiple R is correlation coefficient and in our case it comes out to be 0.72383865. This indicates that there exist a positive relationship between independent and dependent variables. A zero value means there exist no relationship at all. Numerically, it is square root of R-squared value.
- Coefficient of determination ( $R^2$  value) is 0.52. This means that approx. 52.3% of the variation in the dependent variable (Y-value) is explained by the independent variables (X-value). In simple words, 52.3% of the values fit the model.
- The adjusted R-square is 0.503. The Adjusted Coefficient of Determination (Adjusted R-squared) is an adjustment for the Coefficient of Determination that takes into account the number of variables in a data set. So, in multiple linear regression where we are dealing with more than one x-variable, we should consider adjusted R-square rather than R-square.
- The standard error of the regression is 10.13, which is an estimate of the variation of the observed compressive strength about the regression line.

The analysis of variance (ANOVA) table breakdown the total variation of the dependent variable in to the explained and unexplained portions. SS Regression is the variation that is explained by the regression line; SS Residual is the variation of the dependent variable that is not explained.

As discussed earlier, F-statistic (F-tests) has to be used test the overall significance for a regression model. The F-statistic is simply the ratio of the mean square regression (MS Regression) to the mean square residual (MS Residual). The p-value associated with the calculated F-statistic is a simple way to decide whether to reject null hypotheses or not.

From ANOVA table shown above, the F-value is 26.0472 and Significance F (p-value) comes out to be 9.61559E-21. The p-value is very smaller than the standard level of significance ( $\alpha=0.05$ ) indicating the validation of regression line and rejecting null hypotheses.



Looking at the p-value of the t-test for each individual predictor (x-variable) from ANOVA table-2, we can see that three predictor (w/b ratio, coarse aggregate, fine aggregate) has very high p-value ( $>0.05$ ). A predictor having a low p-value is probably going to be a meaningful addition to model because changes in the predictor will be associated with changes in the response. From table shown above, we can see predictor variables of cement, flyash% and water because of their p-values which are  $4.03E-18$ ,  $1.12E-05$  and  $2E-12$  respectively.

Hence we have to again run multiple linear regression procedure to develop but this time, we will only include cement, flyash% and water only as our predictor (independent variables).

The result of regression analysis for second time appears as follows:

**Table 5.9:** Regression Statistics (2)

Multiple R	0.71
R Square	0.51
Adjusted R Square	0.50
Standard Error	10.13
Observations	149

**Table 5.10:** ANOVA table (2)

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	15739.11	5246.37	51.12	1.33E-22
Residual	145	14880.2	102.62		
Total	148	30619.31			

	<i>Coefficients</i>	<i>Stand. Error</i>	<i>t Stat</i>	<i>p-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	44.81	7.17	4.26	4.90E-07	23.62	51.99	23.62	51.99
cement	0.15	0.013	11.2	2.19E-21	0.12	0.17	0.12	0.17
flyash %	0.30	0.062	4.89	2.56E-06	0.18	0.43	0.18	0.43
water	-0.29	0.037	-7.87	7.29E-13	-0.37	-0.22	-0.37	-0.22

- Significance F (p-value) is 1E-22 (<0.05) hence significant of model is excellent and valid, also null hypotheses is rejected.
- For t-test, p-value of all three predictor is found to be less than significance level (<0.05) indicating all three variables are statistically significant to model.

Thereby, regression equations developed from MLR approach appears in the form:

$$Y = 44.81031 + 0.150993 \times (\text{cement}) + 0.308107 \times (\text{flyash}\%) - 0.29603 \times (\text{water})$$

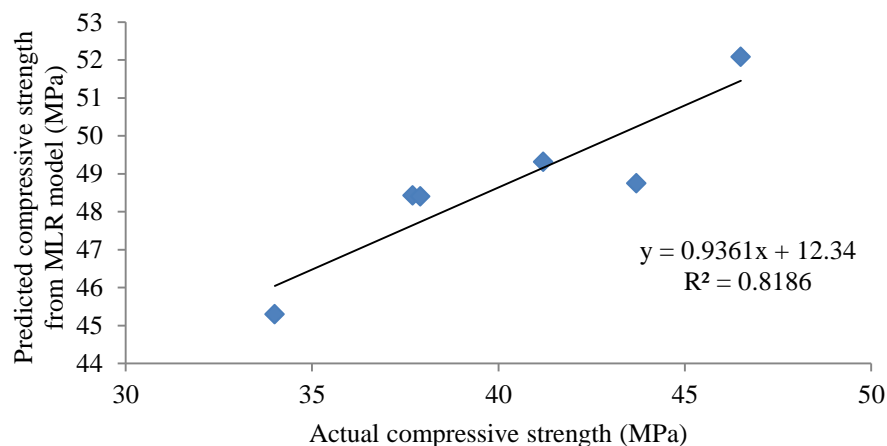
Where Y is compressive strength of concrete (in MPa).

Experimental /observed strength are compared with compressive strength predicted from MLR model shown in table:

**Table 5.11:** Performance of MLR model

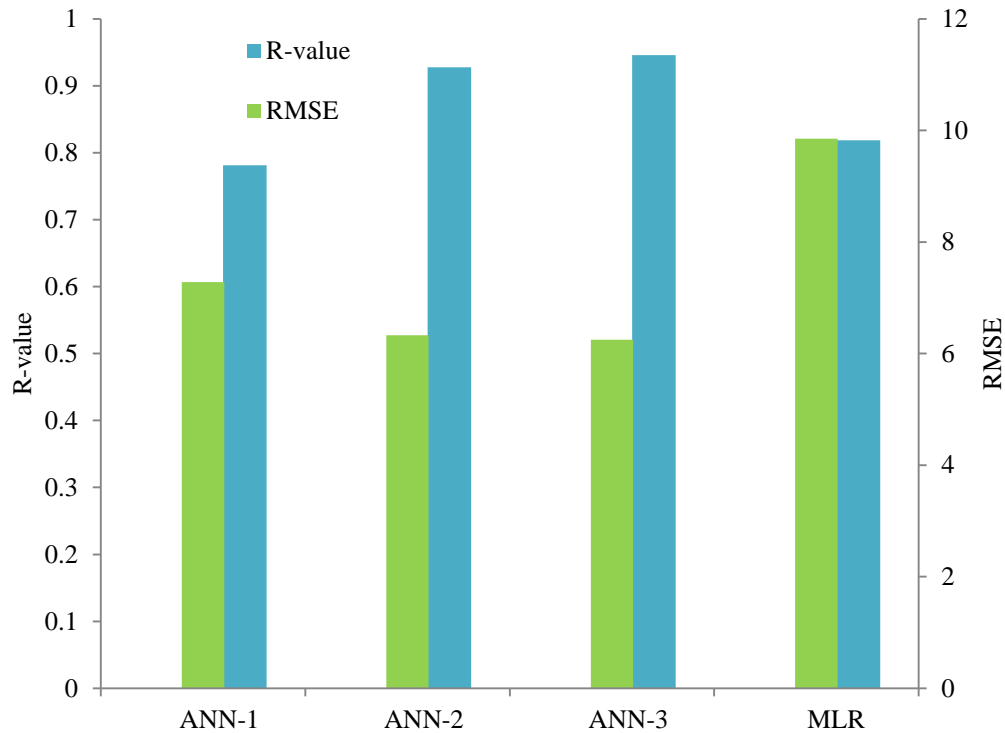
	Observed strength (MPa)	Predicted strength from MLR model (MPa)	% error	RMSE
mix 1	43.7	50.88	16.43	
mix 2	37.7	46.47	23.26	
mix 3	34	44.27	30.21	9.85
mix 4	46.5	57.84	22.75	
mix 5	41.2	51.925	26.03	
mix 6	37.9	48.96	29.18	

The experimented strength results are graphically plotted against predicted compressive strength for MLR model and are shown in fig. below:



**Fig. 5.13:** Graph between predicted compressive strength from MLR approach versus actual compressive strength

For better comparative study of results obtained from all the models developed, RMSE and  $R^2$  from all three approaches are shown below:



**Fig. 5.14:** Summarized performance of all models used in prediction

It can be observed that ANN-3 model has lowest RMSE and highest  $R^2$  value among four models developed (though there is slight variation in ANN-2 and ANN-3 model), hence indicating it provides better estimation in predicting compressive strength of concrete.

## **6. Conclusion**

This study is aimed at finding the better system model for the prediction of the 28<sup>th</sup> day compressive strength of concrete through a comparative analysis by ANN and MLR models. The performance of models is compared by calculating coefficient of determination and root mean square error of results, with models providing highest  $R^2$  and lowest RMSE values shows more confidence towards predicting results.

Based on the findings of this study it can be concluded that:

- Despite data samples collected (from previous researches) for developing models consist of works performed as per different mix procedure guidelines (ACI, BIS etc.), all four models seem to have performed satisfactorily in predicting.
- From results and discussions section, we can say that ANNs were able to outperform the conventional method in prediction (MLR models) because strength model based on the ANN was found to be more accurate than the model based on regression analysis. In the case of ANN, it is to be noted that it is not possible to repeat an experiment and get the same results, as initial weights were set randomly.
- MLR model has also performed well. But concrete being a highly complex material and interaction between ingredients of concrete is somewhat non-linear, so it may be not advisable to go for linear regression method for predicting. It is exhibited, in this study, that ANN can be used for better understanding of non-linear interactions between various factors that decide the compressive strength of concrete.

- ANN-3 model (with 6 inputs) and ANN-2 (with 4 inputs) showed better results than ANN-1 model (with 3 inputs), hence it can be inferred that results were improved when network were provided with better description of the system. Strength of concrete also depends on other various in-situ factors use of plasticizers, age of concrete, curing conditions, temperature etc. Hence to obtain higher accuracy, ANN models can be develop considering these factors as input parameters also.
- ANNs has performed satisfactorily in predicting compressive strength in this study, and it can be employed in predicting other characteristics of concrete like shrinkage, creep, its splitting tensile strength, flexural strength etc.

## References

- [1] Han, S.H., Kim, J.K. and Park, Y.D. (2003) “Prediction of compressive strength of fly ash concrete by new apparent activation energy function”, *Cement and Concrete Research*, 33(7), pp. 965-971.
- [2] Namagga, C., & Atadero, R. A. (2009, May) “Optimization of fly ash in concrete: High lime fly ash as a replacement for cement and filler material”, In *World of Coal Ash Conference (WOCA)*, Lexington, KY, USA.
- [3] Basha, S.A., Pavithra, P. and Reddy, B.S. (2014), “Compressive strength of fly ash based cement concrete”, *International Journal of Innovations in Engineering and Technology*, 4(4), pp. 141-156.
- [4] Mukherjee, S., Mandal, S., & Adhikari, U. B. (2013) “Comparative study on physical and mechanical properties of high slump and zero slump high volume fly ash concrete (HVFA)”, *Global NEST J*, 20(10), pp. 1-7.
- [5] Raju, S., & Dharmar, B. (2016) “Mechanical Properties of Concrete with Copper Slag and Fly Ash by DT and NDT”, *Periodica Polytechnica, Civil Engineering*, 60(3), pp. 313-322.
- [6] Quan, H. Z., and Kasami, H. (2014) “Experimental study on durability improvement of fly ash concrete with durability improving admixture”, *The Scientific World Journal*, 2014.
- [7] John, J. & Ashok, M. (2014) “Strength study on high volume fly ash concrete”, *International Journal of Advanced Structures and Geotechnical Engineering*, 3(2), pp. 168-171.
- [8] Naik, T.R. and Ramme, B.W. (1989) “High-strength concrete containing large quantities of fly ash”, *Materials Journal*, 86(2), pp.111-116.
- [9] Bajad, D. M., Mutha, N., Husain, H., & Kshirsagar, N. (2015) “Effect of Recycled Aggregate and Fly Ash in Concrete”, *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)* e-ISSN: 2278-1684, p-ISSN: 2320-334X, 12(2).
- [10] Myadaraboina, H., Solikin, M., Patnaikuni, I., & Setunge, S. (2014) “Development of high volume fly ash concrete using ultra-fine fly ash”.
- [11] Singh, T. P. (2007) “Field performance of high volume fly ash concrete-The indian experience”, paper presented in *ACI conference*.

- [12] Nagabhushana (2015) “Study on properties of concrete with different levels of replacement of cement by fly ash”, *International Journal of Research in Engineering and Technology*, 4(8), pp. 158-161.
- [13] Naik, T. R., & Ramme, B. W. (1990) “High early strength fly ash concrete for precast/prestressed products”, *PCI Journal*, 35(6), pp. 72-78.
- [14] Sarika, P. S., Rao, S., Sekhar, S. T., & Apparao, G. (2013) “Studies on Relationship between Water/Binder Ratio And Compressive Strength Of High Volume Fly Ash Concrete”, *American Journal of Engineering Reseach*, 2(8), pp. 115-122.
- [15] Awanti, S. S., & Harwalkar, A. B. (2016) “Mix Design Curves for High Volume Fly Ash Concrete”, *World Academy of Science, Engineering and Technology, International Journal of Civil, Environmental, Structural, Construction and Architectural Engineering*, 10(10), pp. 1304-1309.
- [16] Kalra, T., & Kumar, R. (2016) “Comparison of Normal and High Volume Flyash Concrete”, *International Journal of Research in Electronics and Communication Technology*, 3(1), pp. 11-13.
- [17] Solikin, M. (2012) “High performance concrete with high volume ultra-fine fly ash reinforced with basalt fibre”.
- [18] Uriel, E. (2013) “Hypothesis testing in the multiple regression model”, *Universidad de Valencia: Department of economics*.
- [19] Gustafsson, A., & Wogenius, S. (2014) “Modelling Apartment Prices with the Multiple Linear Regression Model”.
- [20] Hughes, B.P. and Bahramian, B. (1967) “Some factors affecting the compressive strength”, *Magazine of Concrete Research*, 19(60), pp. 165-172.
- [21] Abrams, D. A. (1927) “Water-cement ratio as a basis of concrete quality”, *Proceedings of the American Concrete Institute*, 23, pp. 452-458.
- [22] Erntroy, H. C. and Shack lock, B. W. (1955) “Design of high strength concrete mixes”, *Proceedings of a Symposium on mix design and quality control of concrete, London*, Cement and Concrete Association, pp. 55-73.
- [23] Ministry of Works, and Urban Development, "Ethiopian Building Code Standard, Structural Use of Concrete, (EBCS 2-1995)", Addis Ababa, 1995.
- [24] Munakata T., *Fundamentals of the New Artificial Intelligence*, USA: Springer-Verlag London Ltd, 2008.
- [25] Silarbi S., Abderrahmane B. and Benyettou A. (2014) “Adaptive Network Based Fuzzy Inference System for Speech Recognition Through Subtractive Clustering”, *International Journal of Artificial Intelligence & Applications (IJAIA)*, 5(6).

- [26] Topçu, İ. B., & Sarıdemir, M. (2008) “Prediction of mechanical properties of recycled aggregate concretes containing silica fume using artificial neural networks and fuzzy logic”, *Computational Materials Science*, 42(1), pp. 74-82.
- [27] Aggarwal, P., & Aggarwal, Y. (2011) “Prediction of compressive strength of self-compacting concrete with fuzzy logic”, *World Academy of Science, Engineering and Technology*, 77, pp. 847-854.
- [28] Subaşı, S. (2009) “Prediction of mechanical properties of cement containing class C fly ash by using artificial neural network and regression technique”, *Scientific Research and Essays*, 4(4), pp. 289-297.
- [29] Özcan, F., Atiş, C. D., Karahan, O., Uncuoğlu, E., & Tanyildizi, H. (2009) “Comparison of artificial neural network and fuzzy logic models for prediction of long-term compressive strength of silica fume concrete”, *Advances in Engineering Software*, 40(9), pp. 856-863.
- [30] Ozturan, M., Kutlu, B. I. R. G. Ü. L., & Ozturan, T. (2008) “Comparison of concrete strength prediction techniques with artificial neural network approach”, *Building Research Journal*, 56(1), pp. 23-36.
- [31] Zain, M. F. M., & Abd, S. M. (2009) “Multiple regression model for compressive strength prediction of high performance concrete”, *Journal of applied sciences*, 9(1), pp. 155-160.
- [32] Nagendra, S. (1998) “Practical Aspects of Using Neural Networks: Necessary Preliminary Specifications”, *Technical Paper, GE Research and Development Center*.
- [33] Ahmed, M. S. S. (2012) “Statistical modelling and prediction of compressive strength of concrete”, *Concrete Research Letters*, 3(2).
- [34] Hamid, N. A. A., Thamrin, R., Ibrahim, A., Hamid, H. A., Salleh, N., Jamellodin, Z., & Khalid, N. H. A. (2017) “Shear Strength Prediction for Concrete Beams Reinforced with GFRP Bars”, In *MATEC Web of Conferences* (Vol. 103, p. 02013). EDP Sciences.
- [35] Qasrawi, H. Y. (2000) “Concrete strength by combined nondestructive methods – Simply and reliably predicted”, *Cement and Concrete Research*, Vol. 30, pp. 739-746.
- [36] Singh, B. G. (1958) “Specific surface of aggregates related to compressive and flexural strength of concrete”, *Journal of the American Concrete Institute*, 54, pp. 897-907.



## Appendix I

Appendix I contains the tables having the data that were used to train the ANN network and also to develop MLR model.

**Table A.1:** Data collected from previous research works

Researchers	w/b ratio	Quantities in kg/m <sup>3</sup>					compressive strength( in Mpa)
		cement	flyash %	water	coarse aggregate	fine aggregate	
Nagabhushana	0.40	465.00	0.00	186.0	1069.00	707.00	26.00
	0.40	163.00	55.10	132.0	1277.00	653.00	18.00
	0.40	127.00	65.01	132.0	1270.00	649.00	17.00
	0.40	91.00	74.93	132.0	1259.00	644.00	16.00
	0.40	55.00	84.85	132.0	1252.00	640.00	13.00
Namagga & Atadero	0.50	249.48	14.99	147.0	774.28	573.79	39.30
	0.50	234.96	20.06	147.0	774.28	571.98	44.82
	0.50	220.45	25.00	147.0	774.28	569.71	42.06
	0.50	205.48	29.98	146.5	774.28	567.90	46.88
	0.50	190.96	35.03	146.5	774.28	566.08	44.13
	0.50	176.45	39.97	146.5	774.28	563.81	42.06
	0.50	161.48	44.98	146.5	774.28	562.00	40.68
	0.50	146.96	50.00	146.5	774.28	559.73	35.16
Myadaraboina et.al.	0.30	450.00	0.00	137.0	994.00	912.00	79.00
	0.30	225.00	50.00	141.0	994.00	835.00	71.00
	0.30	225.00	50.00	141.0	994.00	835.00	78.50
	0.30	225.00	50.00	139.0	994.00	811.00	66.50
Kalra & Kumar	0.46	398.00	0.00	183.0	1266.00	599.00	36.00
	0.46	298.50	25.00	183.0	1266.00	599.00	35.50
	0.46	238.80	40.00	183.0	1266.00	599.00	33.00
	0.46	199.00	50.00	183.0	1266.00	599.00	26.00
	0.46	159.20	59.97	183.0	1266.00	599.00	23.00
Bajad et.al.	0.50	371.33	0.00	185.7	1142.15	761.43	28.20
	0.50	367.38	9.93	203.9	1131.29	723.41	30.92
	0.50	326.40	19.97	203.9	1122.99	716.99	31.88
	0.50	261.22	31.92	191.8	1113.50	711.56	31.48
	0.50	371.33	0.00	185.7	1141.65	761.43	26.40
	0.50	371.33	0.00	185.7	1142.64	761.43	27.30
	0.50	371.33	0.00	185.7	799.46	761.43	24.90
	0.50	371.33	0.00	185.7	1141.66	761.43	22.10
	0.50	367.37	9.93	203.9	1131.29	723.41	30.10
	0.50	367.37	9.93	203.9	1130.83	723.41	28.80
	0.50	367.37	9.93	203.9	1130.84	723.41	26.70
	0.50	367.37	9.93	203.9	1130.83	723.41	22.60
	0.50	328.39	19.88	204.9	1122.44	717.03	29.61

	0.50	328.39	19.88	204.9	1122.89	717.03	28.88
	0.50	328.39	19.88	204.9	1122.40	717.03	27.21
	0.50	328.39	19.88	204.9	1122.40	717.03	23.10
	0.50	285.91	29.99	204.2	1130.03	711.56	29.34
	0.50	285.91	29.99	204.2	1113.51	711.56	30.21
	0.50	285.91	29.99	204.2	1113.01	711.56	27.85
	0.50	285.91	29.99	204.2	1113.51	711.56	24.61
	0.45	284.86	0.00	128.4	819.64	579.69	32.92
	0.41	259.45	11.86	119.3	830.07	586.95	37.20
Naik & Ramme	0.38	251.29	17.68	114.8	851.39	602.37	47.09
	0.36	239.50	23.26	112.5	861.37	609.17	55.71
	0.34	222.71	28.74	107.5	854.57	604.18	58.16
	0.33	208.20	34.15	103.0	855.93	621.42	57.67
	0.40	380.00	0.00	152.0	1293.00	596.00	43.20
	0.40	380.00	0.00	152.0	1293.00	520.00	44.23
	0.40	380.00	0.00	152.0	1293.00	390.00	46.34
	0.40	380.00	0.00	152.0	1293.00	260.00	44.81
	0.40	380.00	0.00	152.0	1293.00	131.00	45.42
	0.40	380.00	0.00	152.0	1293.00	0.00	45.71
	0.40	342.00	10.00	152.0	1293.00	596.00	41.34
	0.40	342.00	10.00	152.0	1293.00	520.00	44.74
	0.40	342.00	10.00	152.0	1293.00	390.00	48.90
	0.40	342.00	10.00	152.0	1293.00	260.00	44.45
	0.40	342.00	10.00	152.0	1293.00	131.00	41.90
	0.40	342.00	10.00	152.0	1293.00	0.00	48.90
Raju & Dharmar	0.40	304.00	20.00	152.0	1293.00	596.00	38.51
	0.40	304.00	20.00	152.0	1293.00	520.00	37.93
	0.40	304.00	20.00	152.0	1293.00	390.00	39.11
	0.40	304.00	20.00	152.0	1293.00	260.00	42.96
	0.40	304.00	20.00	152.0	1293.00	131.00	52.01
	0.40	304.00	20.00	152.0	1293.00	0.00	45.48
	0.40	266.00	30.00	152.0	1293.00	596.00	36.53
	0.40	266.00	30.00	152.0	1293.00	520.00	38.22
	0.40	266.00	30.00	152.0	1293.00	390.00	43.26
	0.40	266.00	30.00	152.0	1293.00	260.00	33.78
	0.40	266.00	30.00	152.0	1293.00	131.00	46.50
	0.40	266.00	30.00	152.0	1293.00	0.00	47.70
	0.30	225.00	50.00	141.0	994.00	835.00	70.90
	0.30	225.00	50.00	141.0	994.00	809.00	73.78
Solikin	0.30	225.00	50.00	139.0	994.00	785.00	52.97
	0.30	225.00	50.00	139.0	994.00	811.00	66.69
	0.60	330.00	0.00	198.0	963.00	788.00	32.90
	0.60	297.00	10.00	198.0	955.00	781.00	33.80
Han et.al.	0.60	264.00	20.00	198.0	947.00	775.00	28.00
	0.60	231.00	30.00	198.0	938.00	768.00	25.00

	0.55	350.00	0.00	193.0	962.00	787.00	36.10
	0.55	315.00	10.00	193.0	953.00	780.00	38.30
	0.55	280.00	20.00	193.0	945.00	773.00	33.80
	0.55	254.00	28.25	193.0	936.00	766.00	30.90
	0.40	420.00	0.00	168.0	1054.00	703.00	49.70
	0.40	378.00	10.00	168.0	1044.00	696.00	50.30
	0.40	336.00	20.00	168.0	1032.00	688.00	48.00
	0.40	294.00	30.00	168.0	1020.00	680.00	42.80
	0.35	480.00	0.00	168.0	1025.00	683.00	56.30
	0.35	432.00	10.00	168.0	1012.00	675.00	55.70
	0.35	384.00	20.00	168.0	999.00	666.00	56.00
	0.35	336.00	30.00	168.0	986.00	657.00	51.80
	0.32	520.00	0.00	166.0	1042.00	638.00	62.10
	0.32	468.00	10.00	166.0	1027.00	629.00	61.70
	0.32	416.00	20.00	166.0	1013.00	621.00	58.60
	0.32	364.00	30.00	166.0	998.00	612.00	44.20
	0.27	600.00	0.00	162.0	1056.00	569.00	71.80
	0.27	540.00	10.00	162.0	1039.00	560.00	67.70
	0.27	480.00	20.00	162.0	1022.00	550.00	65.00
	0.27	420.00	30.00	162.0	1005.00	541.00	51.90
	0.35	440.00	0.00	154.0	1059.00	871.00	56.10
	0.35	220.00	50.00	154.0	1059.00	807.00	42.44
	0.35	198.00	55.00	154.0	1059.00	800.00	40.62
	0.35	176.00	60.00	154.0	1059.00	794.00	35.17
Awanti & Harwalkar	0.35	155.00	64.77	154.0	1059.00	787.00	24.42
	0.30	440.00	0.00	132.0	1059.00	937.60	62.28
	0.30	220.00	50.00	132.0	1059.00	871.00	52.10
	0.30	198.00	55.00	132.0	1059.00	864.80	47.31
	0.30	176.00	60.00	132.0	1059.00	858.20	40.84
	0.30	155.00	64.77	132.0	1059.00	851.80	27.69
	0.35	400.00	0.00	140.0	1158.00	637.00	50.35
	0.35	240.00	40.00	140.0	1158.00	637.00	43.65
	0.35	200.00	50.00	140.0	1158.00	637.00	35.31
	0.35	160.00	60.00	140.0	1158.00	637.00	31.40
Mukherjee et.al.	0.35	120.00	70.00	140.0	1158.00	637.00	23.67
	0.35	240.00	40.00	140.0	1158.00	637.00	31.72
	0.35	200.00	50.00	140.0	1158.00	637.00	29.42
	0.35	160.00	60.00	140.0	1158.00	637.00	27.99
	0.35	120.00	70.00	140.0	1158.00	637.00	24.72
	0.60	303.00	0.00	182.0	1014.00	856.00	35.50
	0.60	283.00	0.00	170.0	1014.00	813.00	36.10
Quan and Kasami	0.60	235.00	15.16	166.0	1014.00	880.00	29.20
	0.60	235.00	15.16	166.0	1014.00	815.00	27.70
	0.60	205.00	24.91	164.0	1014.00	879.00	20.40
	0.60	209.00	25.09	167.0	1014.00	801.00	22.60

	0.50	364.00	0.00	182.0	1014.00	806.00	47.50
	0.50	368.00	0.00	174.0	1014.00	814.00	46.40
	0.50	340.00	0.00	170.0	1014.00	766.00	45.50
	0.50	296.00	14.94	174.0	1014.00	823.00	43.20
	0.50	282.00	15.06	166.0	1014.00	831.00	38.30
	0.50	282.00	15.06	166.0	1014.00	766.00	36.50
	0.50	258.00	25.00	172.0	1014.00	820.00	34.50
	0.50	246.00	25.00	164.0	1014.00	829.00	33.80
	0.50	251.00	25.07	167.0	1014.00	751.00	31.30
	0.43	440.00	0.00	189.0	1014.00	726.00	57.80
	0.43	414.00	0.00	178.0	1014.00	684.00	50.70
	0.43	342.00	14.93	173.0	1014.00	752.00	47.90
	0.43	348.00	14.91	176.0	1014.00	673.00	43.90
	0.43	298.00	24.94	171.0	1014.00	748.00	42.60
	0.43	309.00	25.00	177.0	1014.00	655.00	37.60
Singh	0.47	360.00	0.00	170.0	1140.00	700.00	33.30
	0.31	220.00	50.00	138.0	1112.00	681.00	30.00
	0.36	190.00	50.00	136.0	1150.00	705.00	32.70
Sarika et.al.	0.55	213.00	33.44	178.0	1134.00	718.00	35.15
	0.53	223.00	33.43	178.0	1138.00	706.00	37.38
	0.50	237.00	33.43	178.0	1140.00	693.00	39.39
	0.48	246.00	33.51	178.0	1154.00	672.00	42.55
	0.46	258.00	33.33	178.0	1165.00	651.00	44.13
	0.44	270.00	33.17	178.0	1178.00	630.00	46.91
	0.42	283.00	33.41	178.0	1190.00	609.00	48.87
	0.40	296.00	33.48	178.0	1200.00	588.00	55.80
	0.38	313.00	33.40	178.0	1209.00	566.00	61.17
	0.36	330.00	33.33	178.0	1207.00	553.00	62.66
	0.34	350.00	33.21	178.0	1205.00	539.00	64.18
	0.32	370.00	33.45	178.0	1202.00	525.00	66.40
	0.30	396.00	33.33	178.0	1196.00	511.00	68.19
0.27	440.00	33.33	178.0	1178.00	491.00	70.66	
John & Ashok	0.38	416.00	0.00	158.0	1242.00	668.00	36.13
	0.34	229.00	50.00	158.0	1184.00	610.00	30.71
	0.34	183.00	60.04	158.0	1170.00	602.00	31.00
	0.34	137.00	70.09	158.0	1158.00	596.00	29.30
Basha et.al.	0.46	219.08	99.79	771.1	605.09	605.09	44.82
	0.49	75.30	83.91	831.9	665.42	665.41	23.10
	0.35	113.40	89.81	831.9	566.99	566.99	37.92
	0.28	136.08	83.91	839.1	574.25	574.24	57.23
Naik & Ramme	0.66	192.78	0.00	127.5	821.00	730.28	27.96
	0.62	154.67	22.68	123.8	821.00	730.28	30.61
	0.60	136.08	33.33	123.4	821.00	730.28	33.02
	0.57	115.67	44.92	118.8	821.00	730.28	34.50
	0.55	95.25	55.32	117.0	821.00	730.28	31.41

*Appendix*

0.52	77.56	64.45	112.9	821.00	730.28	23.41
0.57	234.51	0.00	134.7	821.00	694.00	31.80
0.53	187.79	23.19	128.8	821.00	694.00	35.18
0.49	165.11	34.30	123.8	821.00	694.00	37.72
0.49	140.61	44.74	124.3	821.00	694.00	40.27
0.48	117.48	54.48	123.4	821.00	694.00	39.64
0.41	94.80	64.21	109.8	821.00	694.00	33.49

---