INTELLIGENT CONTROL OF ACTIVE SUSPENSION FOR 2 DOF AND 3 DOF QUARTER CAR MODEL

DISSERTATION
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Submitted by:

Naman Garg

Roll No. 2K15/C&I/12

Under the supervision of

DR. BHARAT BHUSHAN

DEPARTMENT OF ELECTRICAL ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi-110042

2017

DEPARTMENT OF ELECTRICAL ENGINEERING

DELHI TECHNOLOGICAL UNIVERSITY

(Formerly Delhi College of Engineering)

Bawana Road, Delhi-110042

CERTIFICATE

I, Naman Garg, Roll No. 2K15/C&I/12 student of M.Tech. Control and Instrumentation (C&I), hereby declare that the dissertation/project titled **"Intelligent control of Active Suspension for 2 DOF and 3 DOF Quarter Car Model"** under the supervision of Dr. Bharat Bhushan of Electrical Engineering Department, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology has not been submitted elsewhere for the award of any Degree.

Place: Delhi **(NAMAN GARG)**

Date: 26/07/2017

DR. BHARAT BHUSHAN

(SUPERVISOR)

Professor

Department of Electrical Engineering

Delhi Technological University

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NAMAN GARG

2K15/C&I/12

M. Tech. (Control and Instrumentation)

Delhi Technological University

ABSTRACT

Suspension system plays most significant role in supporting the vehicle weight and isolating vehicle body from road shocks and vibrations. The aim of suspension system is to suppress uncomforting situations from road unevenness to driver and passengers and to provide road handling and stability of vehicles. The aim of this research work is to design different types of active controller for active suspension system. By using mathematical modelling 2 DOF and 3 DOF quarter car model are designed. The objective is to determine a control strategy to achieve better road quality and ride comfort by delivering better performance with respect to seat acceleration, seat displacement, suspension deflection, sprung mass displacement, tire deflection in terms of peak overshoot, settling time etc.

Four different active controllers are designed using 2 DOF quarter car model as well as 3 DOF quarter car model to compare the performance of Four controllers with the passive suspension system. The Four controllers designed are PID, Fuzzy logic controller ,Fuzzy PID controller and Linear Quadratic Controller (LQR). Three different road profiles such as Step, Sine and Band limited white noise are taken as input disturbances. In this work, MATLAB/SIMULINK software is used for simulation purpose and simulation result demonstrate that active suspension system shows better results in comparison to passive suspension system with reference to body acceleration, suspension deflection, tire deflection. Also the result of comparison shows that Fuzzy PID controller based active suspension system gives better result and stability as compared to other active controllers and passive model.

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LIST OF ABBREVIATIONS

DOF	Degree of Freedom
PID	Proportional Integral Derivative
LQR	Linear Quadratic Regulator
FLC	Fuzzy Logic Controller
MF	Membership Function
ARE	Algebraic Riccatti Equation

LIST OF SYMBOLS

M_p	Mass of passenger
M_s	Mass of car body or sprung mass
M_{us}	Mass of wheel or un-sprung mass
K_p	Stiffness of the seat
C_p	Damping coefficient of the seat
K_s	Stiffness of car suspension
C_s	Damping coefficient of car suspension
K_t	Stiffness coefficient of the tire
r	Input road disturbance
x_p	Seat displacement
x_s	Sprung mass displacement
x_{us}	Un-Sprung mass displacement
$\dot{x}_p$	Seat velocity
$\dot{x}_s$	Sprung mass velocity
$\dot{x}_{us}$	Un-sprung mass velocity
$\ddot{x}_p$	Seat acceleration
$\ddot{x}_s$	Sprung mass acceleration
$\ddot{x}_{us}$	Unsprung mass acceleration
$(x_s - x_{us})$	Suspension travel
$(x_{us} - r)$	Tire deflection
F_a	Actuator Force
J	Performance index
u_w	Disturbance feed-forward control
u_x	State-feedback control
$\xi(t)$	Riccatti vector
K_x	Optimal state feedback controller gain
K_w	Optimal disturbance feed forward controller gain

$Q(t)$	Matrix for weighting the states
$R(t)$	Matrix for penalizing the control effort
$\lambda(t)$	Dynamic Lagrange Multipliers
$N(t)$	Matrix showing relation between states and control inputs

CHAPTER 1

INTRODUCTION

1.1 GENERAL

Modern automobiles have numerous features which were not present some few decades back as they are now transformed into complex electromechanical systems. Few examples are suspension, cruise control, antilock brakes, hybrid powertrains, differential brakes etc. The advantage of electromechanical system over purely mechanical system is that the system can be controlled by means of electrical signal. They are more flexible and accurate. Major subsystems of an automobile are chassis, engine, suspension, Drive train, steering, brakes and instrumentation.

To design an Electromechanical system for any system, detailed knowledge of system is required. There are many systems in day to day life which are or form of electromechanical system. To minimise cost and to increase efficiency there is an increasing demand for automation of a mechanical system. The automation requires sensors, controllers, actuators for measuring, computing and controlling the system Suspension system is one of the major functional subsystem of an automobile.

1.2 SUSPENSION SYSTEM

Suspension system is a combination of mechanical devices like springs, dampers, tires, linkages that supports the vehicle body from above and seclude the passengers from vibrations and road disturbances. It is an electromechanical system as it combines both electrical and mechanical processes in a single system. Figure 1.1 shows an suspension system. Springs are the elastic elements so they are flexible. When they are compressed or stretched energy is stored in to them. As they are flexible they can easily bend without breaking and can absorb energy. As springs attach the wheels with vehicle body, the rise in wheel position compresses the spring with very less increase in vehicle body. Similarly when the wheel goes down, the spring gets stretched and does not let the vehicle body to fall down rapidly. This movement of Springs due to upward and downward shifting of wheels makes springs to absorb the irregularities of road. If the vehicle system consists of only spring mechanism, then the vehicle would keep on bouncing up and down the road until the energy absorbed by spring get dissipated. The energy would keep on adding if more bumps are present . This would create oscillations in vertical movement of vehicle.

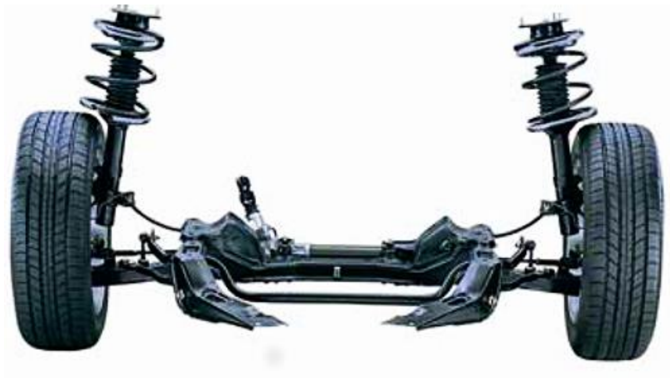

Figure 1.1- Suspension system of car

When a bump comes in a way of wheel, due to this upward movement of wheel spring compresses and energy is absorbed for a very short time. This energy is then released by getting into its original position by stretching back. Sometimes the spring gets stretched more than its original position that it again tries to compress and this cycle of expanding and compressing goes on. As all the energy is released out these oscillation stops and that energy is dissipated in the form of heat. Some part of the energy is also dissipated as frictional loss through damper. The figure 1.2 is showing the heat transfer. Dampers are the fluid filled cylinders with a moveable piston attached to them. A Damper help to lessen the effect of oscillations of spring contraction and expansion.

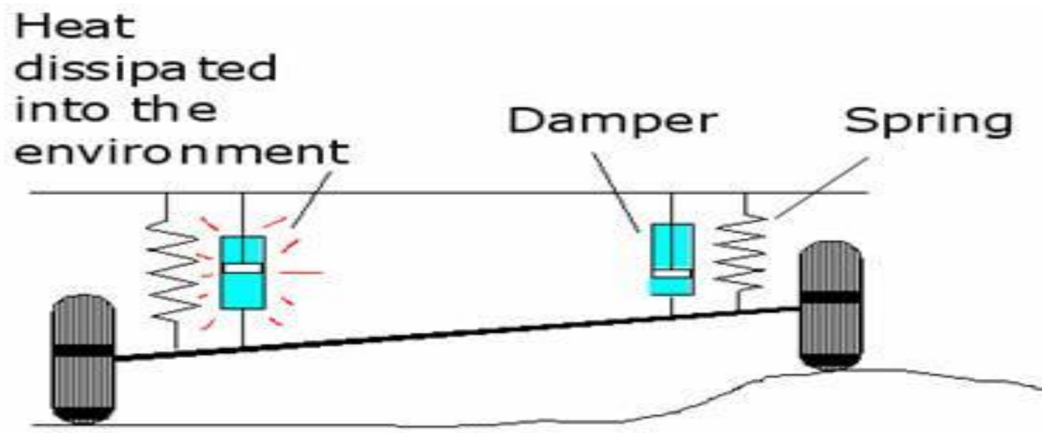

Figure 1.2 – Energy transfer in spring and damper

The main objectives of suspension system are:

- It provides isolation of chassis,
- It ensures that the tire does not go away from the road i.e. less tire deflection should be there,
- It allow very less fluctuation of tire load, i.e. all tires have uniformly distributed load,
- It keeps the alignment of wheels at proper place in every kind of road.

Classification of Suspension system is shown in figure1.3.

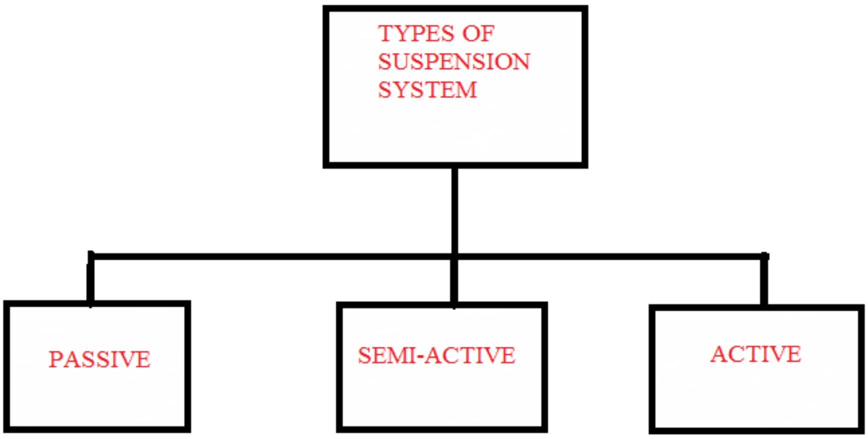

Figure 1.3 – Classification of Suspension System[52]

The basic mechanical suspension system which consists of system of dampers and spring is known as Passive suspension system . Figure 1.4 (a) is showing such system. The dampers and springs of such system have fixed value of coefficients. The energy is stored through spring while dissipated as heat by damper. Due to fix parameter value they cannot give same performance in every terrain. They are generally used in middle and low end cars as they are cheap, reliable and simple structured. It is not possible to achieve good performance in all types of road profiles.

When this suspension system of spring and damper is varied or controlled externally then it is known as active or semi active suspension system [51]. If the damping can be varied according to the road conditions then it is semi active suspension system as shown in figure 1.4 (b). The Adjustment of spring stiffness is not easy that's why only variable damping is used. They does not have any specific dynamic control component ,ECU acts an controller and gives control signal to actuator which is damping adjustable shock absorber in this system. They are cheaper than active suspension system and takes less space in comparison to latter.

If the suspension system is able to store, dissipate and can introduce energy to the system is known as Active suspension system. It consists of sensors, mini computers and system of electro-hydraulic actuators. The parameters of its components are adjustable and changes with respect to road surface and load on the vehicle. To achieve desirable performance a control strategy is need by an Active suspension therefore, the performance can be in influenced by the designing of control strategy for controllable suspension .For a comfortable ride soft damper is needed while stiffer damper gives a stable drive. Therefore, a compromise has to be made between comfort and stability. There is a separate actuator in active suspension system which exerts force if needed on the suspension system as shown in figure 1.4 (c). In addition, active suspension system also counters the inertial force generated due to deceleration or turning of the vehicle. This helps in minimizing the changes of position of car body.

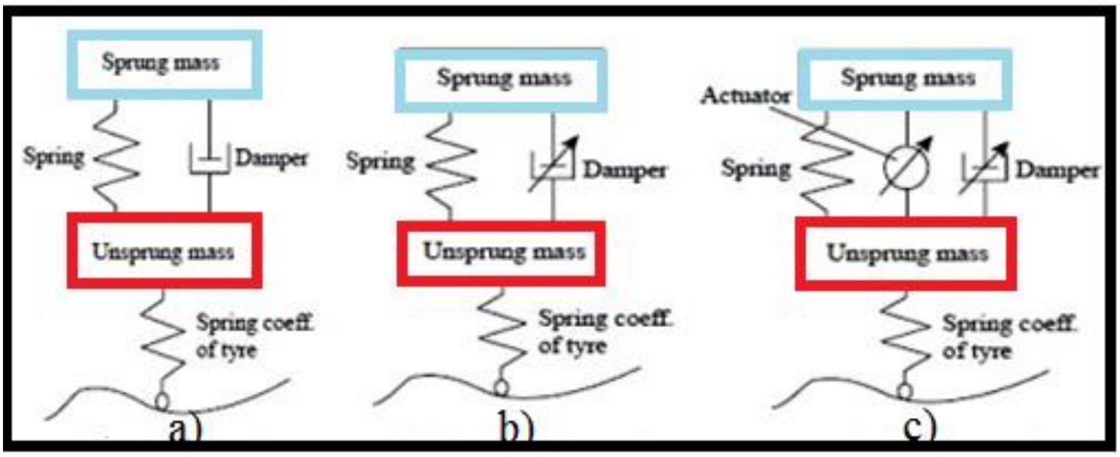

Figure 1.4- Types of Suspension System

The designing of suspension system must care about the important parameters like road handling, ride comfort and suspension travel.

- Road handling or road holding is the parameter which reflects the stability of the vehicle at high speed. For racing cars this is an important parameter .Road holding is directly related to tire road contact which is represented by tire deflection .
- Ride quality or Ride comfort is the ability of vehicle to provide comfortable feeling to the passengers and driver of vehicle on uneven road. It is also related to acceleration/ braking. Ride quality is represented as an acceleration of top most

mass. Ambulances needed top quality of ride comfort so that ill/injured patient would not get further damage.

- It is the displacement between bottom and top of the suspension strokes. It is basically wheel movement of vehicle and compression ability of suspension.

All these parameters cannot be optimised as desired at a same time but controller designing can help in getting better results by making a trade-off among these parameters. There is variety of models available on which research is going on from many years like quarter car model, half car model, and full car model etc.

1.3 MOTIVATION

Passive suspension system does not provides any controlling of vibrations and deflections caused due to change in road or added disturbances as it is an open loop system. Passive suspension system has fixed parameters and cannot be adjusted mechanically or any other way. As Passive suspension is a system of spring and damper, if heavily damped it will transfer road input to the passenger (hard suspension) but it will have better road handling, or if suspension is soft there will be loose handling and less traction control. It can achieve better suspension for certain conditions only. Hence the performance of the passive suspension depends on the type of damping (light or heavy) of damper and stiffness (low or high) spring. It also depends upon type of road profile vehicle is travelling through.

Active suspension system which is a closed loop system can give better performance by having an actuator and sensor to measure displacements. Force actuator is a mechanical element which is included inside the system that is controlled by a controller. Sensors will sense the information about road input in the form of displacements and controller will calculate the required energy needed to either add or dissipate to the system. Therefore, active suspension system design is essential in order to improve importance of the suspension system.

1.3 OBJECTIVES OF THE THESIS

- To give basic idea about suspension system in automobiles, designing and controlling of suspension system.

- To understand mathematical modelling and state space representation of 2 DOF and 3 DOF quarter car model.
- To use three different kind of road profiles such as step, sine and band limited white noise as input disturbance for analysis of suspension system.
- To design four different controlling techniques for active suspension system.
    - To model PID controller using MATLAB/Simulink's auto tuner tool.
    - To model active suspension system for quarter car model using Fuzzy logic controller in MATLAB/Simulink environment using fuzzy logic toolbox. Three different membership functions i.e. Gaussian, triangular and trapezoidal are used for analysis and designing of fuzzy logic controller.
    - To design Fuzzy based PID controller i.e. auto tuning of PID controller using fuzzy logic rule method in MATLAB/Simulink environment. Three different membership functions i.e. Gaussian, triangular and trapezoidal are used for analysis and designing of fuzzy PID controller.
    - To model active suspension system for quarter car model using LQR controller in MATLAB/Simulink environment.
- Simulation of the controllers for two and three DOF quarter car Models.
- To compare the performance characteristics of the active controllers with the passive system and analyse results of all controllers on three road profiles.

1.5 LITERATURE REVIEW

Active suspension system control is the favourite topic for research in the field of automotive control design. This section discusses some previous research work and development of controllers for active suspension system. Extensive research has been done on suspension system through both analysis and experiments. Automobile is a complex and hybrid system as it consists of many complex systems like Traction control system , cruise control system, four-wheel steering, anti-lock braking system, suspension system etc. . [1] combines physics and designing of all types of system of an automobile system. Development of several types of suspension system has been studied[4],semi active suspension system[5], Active suspension system[6][7]. The first preview in the control of an active system for a 1-DOF model was done [2], in which the disturbance used was the white noise terrain. An optimal pair of damping coefficient and spring stiffness was developed to provide a wide range of vibration isolation. Active suspension system design can allow the performance to be user selectable. For example, if a softer or a firmer ride characteristics is needed, the weights in the performance index used in the controller design can be changed, leading to different controller gains and different performance characteristics by [8]. [3] concludes that models with increasing complexity plays an important role in improving the overall performance of the system and accuracy. The 1 DOF model have potential to improve performance, 2 DOF introduced an additional handling related constraint and a more precise quantification is done in 3 DOF. A linear model is used for two different road profiles for quarter car active suspension control in a paper [9]. A PID controller is implemented and tuned for improving the automobile stability and riding comfort by [10]. [11] proposed a fuzzy PI and PD controller to control the semi-active suspension system and showed effectiveness of fuzzy-PID controller to control suspension system in comparison with PID controller. In the paper [12], performance of PID controller for active suspension system is investigated and the parameters of PID were tuned using heuristic tuning, Ziegler-Nichols (ZN) tuning, and iterative learning algorithm (ILA). Other studies on PID controller are summarised as follows: [13], [14], each one has used different design method for PID controller. For designing fuzzy logic based controller implemented fuzzy logic on active suspension system[15]. In addition, a paper implemented fuzzy using different membership functions [16]. Travel comfort of passenger is analysed by designing Fuzzy Logic controller, Sliding mode controller for 8 DOF quarter car model and 4 types of road disturbances are used[17].

In paper [18] was shown that for the optimal two DOF systems, both ride and handling can be improved by reducing the unsprung mass. The connections between LQG-optimal one DOF and two DOF models is explored and the maximum possible ride and handling improvements for two DOF systems are obtained in the limiting case of singular control with zero penalty on unsprung actuator force[19]. The paper [20], described the application of LQR control strategies in quarter car model for better ride quality. The performance of active suspension system was found to be satisfactory compared to passive suspension system. With increasing degree of freedom the characteristics of system changes, a 3 DOF system is used and a fuzzy logic controller for the suspension system is designed and analysed to compare fuzzy logic controller with that of passive suspension system[21]. In paper [22],LQR controller for a 3 DOF quarter car model was designed.

1.6 ORGANIZATION OF THE THESIS

The thesis is organised as follows:

Chapter 1, gives an introduction to the concept of suspension system and the need for the design of an active suspension and provides a review of background literature on suspension system.

Chapter 2, shows the mathematical modelling of two and three DOF quarter car model based passive and active suspension system. Designing of input disturbances is also discussed here.

Chapter 3, gives the relevant literature regarding the three proposed controller. The chapter gives step by step formulation and design of the control methods for two and three DOF quarter car model based active suspension system. The chapter describes Linear Quadratic Regulator control design for linear system with measurable disturbances[29]. It also describes Fuzzy Logic control, and Fuzzy PID control methods for active suspension system.

Chapter 4, provides the comparison between different membership functions used in Fuzzy logic and Fuzzy tuned PID controller .The graphs and results between passive and active suspension system of different controllers with 3 input road profiles is shown.

Chapter 5, gives a brief description of the project work and draws conclusions by analysing the results obtained in the thesis. Some conclusions regarding the improvement of the outcomes of the project are discussed in this chapter. Also the scope of future work is discussed in some detail.

CHAPTER 2

MATHEMATICAL MODELLING

The first and foremost task in designing and analysis of any control system is the mathematical modelling. Suspension system is an example of vibration control system [58]and modelling can be represented as state space or as transfer function[39]. In this chapter mathematical model of a quarter car model is developed. Quarter car model consists of one-fourth of the body mass, suspension components and one wheel. This model represents a basic and simple form of the full car model which represents almost all the features of full car model. Firstly, this chapter discusses how to obtain mathematical model of passive and active suspension for quarter car model. In this project the suspension system is modelled as a linear suspension system in state space. The force needed to be applied between vehicle body and the wheels.

Ideally, the vehicle-ride dynamics include roll and pitch motion as well as vertical (heave) motion of the vehicle. For both pitch and vertical motions we need a half car model which leads to four DOF model. From many previous studies it is found that for passenger car Quarter car model is enough to consider rather than half car as the coupling between vertical motion and Pitch and roll motion is irrelevant.

2.1 MATHEMATICAL MODELLING OF TWO DOF PASSIVE SUSPENSION SYSTEM

In One DOF quarter car model there is no unsprung mass to be considered in modelling while in a two DOF quarter car model both sprung mass and unsprung mass are to be modelled. A two-DOF quarter car model is simple but sufficiently detailed to capture many of the key suspension performance trade-offs, such as ride quality, handling etc. It represents a good compromise between model simplicity and accuracy. The optimal design of a 2-DOF quarter car model of passive suspension system is shown in figure 2.1. Sprung mass comprises of the total mass supported above the suspension system while Un-sprung mass is the mass of wheels and other components like wheel axles, tires, brakes etc. Larger sprung mass to un-sprung mass ratio helps in minimising effect of bumps and road disturbance to the occupants but it reduces the vehicle control.

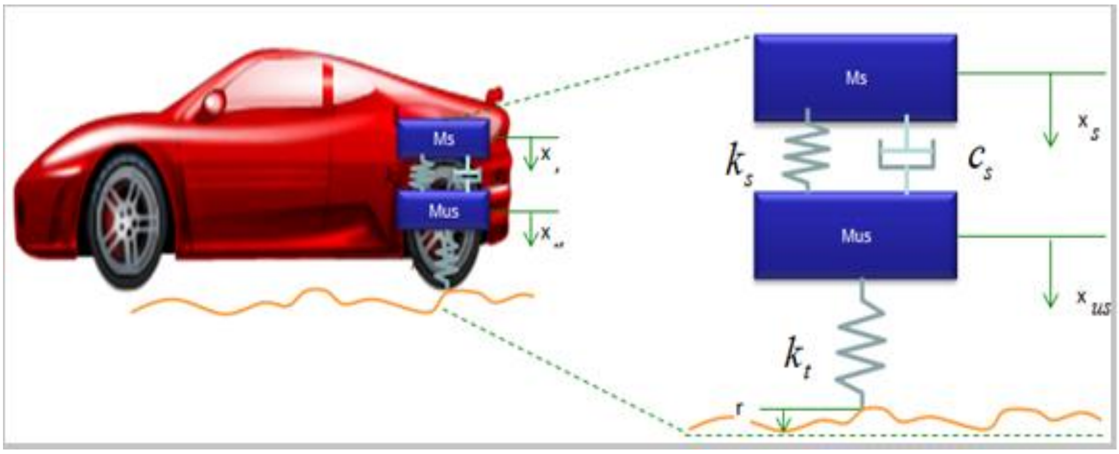

Figure 2.1- A Two DOF Quarter car passive suspension system

Selection of Un-sprung mass is done according to the requirement as lighter un-sprung mass will have better road grip but it will transfer vibrations to the occupants. Sprung mass and Un-sprung mass are connected through spring and damper. Tire is assured to have continuous contact with road and is represented as linear spring. A road disturbance is provided as input to the system. From this model, we can analyze the vehicle suspension system dynamics and establish two degrees of freedom motion differential equations. From Newton's second law of motion the differential equations[27] are found as follows

Equation of motion for sprung mass is given as follows

$$\boldsymbol{M_s \ddot{x}_s + K_s(x_s - x_{us}) + C_s(\dot{x}_s - \dot{x}_{us}) = 0} \tag{2.1}$$

Equation of motion for un-sprung mass is given as follows

$$\boldsymbol{M_{us}\ddot{x}_{us} - K_s(x_{us} - x_s) - C_s(\dot{x}_{us} - \dot{x}_s) + K_t(x_{us} - r) = 0} \tag{2.2}$$

where,

M_s=Mass of car body or sprung mass

M_{us}=Mass of wheel or un-sprung mass

K_s=Stiffness of car suspension

C_s=Damping coefficient of car suspension

K_t=Damping coefficient of the tire

r =Input road disturbance

The state variables to be used in this model are

$$x_1 = x_s \tag{2.3}$$

$$x_2 = x_{us} \tag{2.4}$$

$$x_3 = \dot{x}_s \tag{2.5}$$

$$x_4 = \dot{x}_{us} \tag{2.6}$$

Where,

x_1 = Sprung mass displacement

x_2 = Un-Sprung mass displacement

x_3 = Sprung mass velocity

x_4 = Un-sprung mass velocity

In terms of state variable the system equations can be written as follows:

$$\dot{X} = AX + GW \tag{2.7}$$

Where,

X= variable input state matrix

W=Road input matrix

Rewriting the equations in terms of states

$$\dot{x_1} = x_3 = \dot{x}_s \tag{2.8}$$

$$\dot{x_2} = x_4 = \dot{x_{us}} \tag{2.9}$$

$$\dot{x_3} = \ddot{x_s} = \frac{-K_s(x_1 - x_2) - C_s(x_3 - x_4)}{M_s} \tag{2.10}$$

$$\dot{x_4} = \ddot{x}_{us} = \frac{K_s(x_1 - x_2) - C_s(x_3 - x_4) - K_t(x_2 - r)}{M_{us}} \tag{2.11}$$

Representing the equations 2.8-2.11 in terms of matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_S}{M_s} & \frac{K_S}{M_s} & -\frac{C_s}{M_s} & \frac{C_s}{M_s} \\ \frac{K_S}{M_{us}} & \frac{-(K_t+K_S)}{M_{us}} & \frac{-C_s}{M_{us}} & \frac{C_s}{M_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_t}{M_{us}} \end{bmatrix} W \tag{2.12}$$

The output vector is defined as

$$Y = [x_s - x_{us} \quad x_s \quad x_{us} - r \quad \ddot{x}_s]^T \tag{2.13}$$

$$y_1 = (x_s - x_{us}) = \text{ suspension travel} \tag{2.14}$$

$$y_2 = x_s = \text{ Sprung mass displacement} \tag{2.15}$$

$$y_3 = (x_{us} - r) = \text{ tire deflection} \tag{2.16}$$

$$y_4 = \ddot{x}_s = \text{Sprung mass acceleration} \tag{2.17}$$

Rewriting the equations in terms of state variables as follows:

$$Y = CX + D_W W \tag{2.18}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-K_S}{M_s} & \frac{K_S}{M_s} & \frac{-C_s}{M_s} & \frac{C_s}{M_s} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} W \tag{2.19}$$

2.2 MATHEMATICAL MODELLING OF TWO DOF ACTIVE SUSPENSION SYSTEM

Here an actuator is added between Sprung mass and Un-sprung mass[41] in addition to spring and damper as shown in figure 2.2. Whenever there is any disturbance in road, it experiences vibrations and these vibrations must be dissipated in short period of time. The function of actuator is to produce the required force between the sprung mass and Un-sprung mass.

Equation of motion for sprung mass with actuator force is given as follows

$$\boldsymbol{M_s \ddot{x}_s + K_s(x_s - x_{us}) + C_s(\dot{x}_s - \dot{x}_{us}) = F_a} \tag{2.20}$$

Equation of motion for uns-prung mass with actuator force is given as follows

$$\boldsymbol{M_{us}\ddot{x}_{us} - K_s(x_{us} - x_s) - C_s(\dot{x}_{us} - \dot{x}_s) + K_t(x_{us} - r) = -F_a} \tag{2.21}$$

where,

F_a=Actuator force

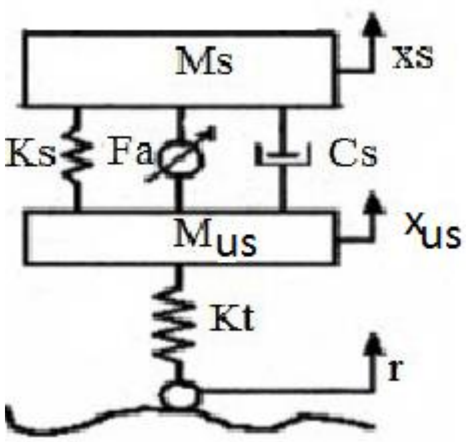

Figure 2.2- A Two DOF Quarter car Active suspension system

In terms of state variable the system equations can be written as follows:

$$\dot{X} = AX + BU + GW \tag{2.22}$$

Where,

X=variable input state matrix

U=input controlvariable matrix

W=Road input matrix

Rewriting the equations in terms of state variables as follows:

$$\dot{x_1} = x_3 = \dot{x}_s \tag{2.23}$$

$$\dot{x_2} = x_4 = \dot{x_{us}} \tag{2.24}$$

$$\dot{x_3} = \ddot{x_s} = \frac{-K_s(x_1 - x_2) - C_s(x_3 - x_4) + F_a}{M_s} \tag{2.25}$$

$$\dot{x_4} = \ddot{x}_{us} = \frac{K_s(x_1 - x_2) - C_s(x_3 - x_4) - K_t(x_2 - r) - F_a}{M_{us}} \tag{2.26}$$

Representing the equations 2.23-2.26 in terms of matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_S}{M_s} & \frac{K_S}{M_s} & -\frac{C_S}{M_s} & \frac{C_s}{M_s} \\ \frac{K_S}{M_{us}} & \frac{-(K_t+K_S)}{M_{us}} & \frac{-C_S}{M_{us}} & \frac{C_s}{M_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_s} \\ -\frac{1}{M_{us}} \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_t}{M_{us}} \end{bmatrix} W \tag{2.27}$$

The output vector is defined as

$$Y = [x_s - x_{us} \quad x_s \quad x_{us} - r \quad \ddot{x}_s]^T \tag{2.28}$$

y_1=$(x_s - x_{us})$= suspension travel (2.29)

y_2=x_s= Sprung mass displacement (2.30)

y_3=$(x_{us} - r)$= tire deflection (2.31)

y_4=$\ddot{x}_s$=Sprung mass acceleration (2.32)

Rewriting the equations in terms of state variables as follows:

$$Y = CX + DU + D_W W \tag{2.33}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-K_S}{M_S} & \frac{K_S}{M_S} & \frac{-C_S}{M_S} & \frac{C_S}{M_S} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_S} \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} W \qquad (2.34)$$

2.3 MATHEMATICAL MODELLING OF THREE DOF PASSIVE SUSPENSION SYSTEM

The base for modelling a higher DOF suspension system is provided by 2 DOF system. A three DOF quarter car model is designed [45]as shown in figure 2.3. In addition to sprung mass and un-sprung mass it consists of passenger mass which comprises of seat cushion and passenger mass. The passenger mass is connected to car body with the help of damper and spring.

The mass of vehicle suspended through spring and damper is known as Sprung mass. The mass of wheel system is said to be as Un-sprung mass . To represent Tire a spring is used.

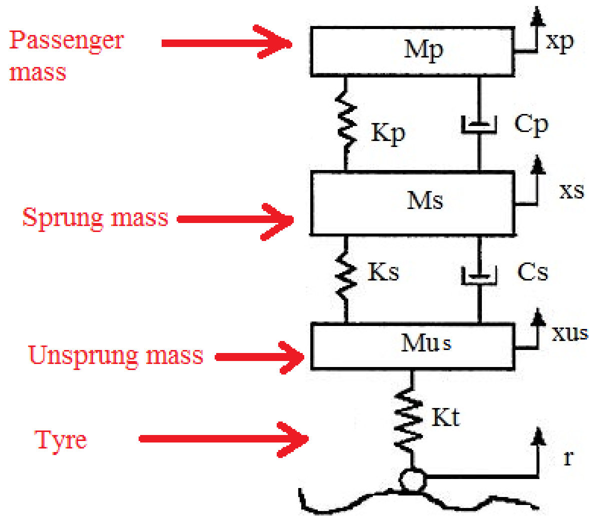

Figure 2.3- A Three DOF Quarter car Passive suspension system

Applying Newton's second law of motion

Equation of motion for seat is gives as follows

$$\boldsymbol{M_p \ddot{x}_p + K_p(x_p - x_s) + C_p(\dot{x}_p - \dot{x}_s) = 0} \qquad (2.35)$$

Equation of motion for sprung mass is gives as follows

$$\boldsymbol{M_s \ddot{x}_s - K_p(x_p - x_s) - C_p(\dot{x}_p - \dot{x}_s) + K_s(x_s - x_{us}) + C_s(\dot{x}_s - \dot{x}_{us}) = 0} \quad (2.36)$$

Equation of motion for unsprung mass is gives as follows

$$\boldsymbol{M_{us}\ddot{x}_{us} - K_s(x_s - x_{us}) - C_s(\dot{x}_s - \dot{x}_{us}) + K_t(x_{us} - r) = 0} \quad (2.37)$$

where,

M_p=Mass of passenger

M_s=Mass of car body or sprung mass

M_{us}=Mass of wheel or un-sprung mass

K_p=stiffness of the seat

C_p=Damping coefficient of the seat

K_s=Stiffness of car suspension

C_s=Damping coefficient of car suspension

K_t=Damping coefficient of the tire

r =Input road disturbance

The state variables to be used in this model are

$$x_1 = x_p \quad (2.38)$$

$$x_2 = x_s \quad (2.39)$$

$$x_3 = x_{us} \quad (2.40)$$

$$x_4 = \dot{x}_p \quad (2.41)$$

$$x_5 = \dot{x}_s \quad (2.42)$$

$$x_6 = \dot{x}_{us} \quad (2.43)$$

Where,

x_1 = Seat displacement

x_2 = Sprung mass displacement

x_3 = Un-sprung mass displacement

x_4 = Seat velocity

x_5 = Sprung mass velocity

x_6 =Unsprung mass velocity

In terms of state variable the system equations can be written as follows:

$$\dot{X} = AX + GW \quad (2.44)$$

Where,

X=State input variable matrix

W=Road input matrix

Rewriting the equations in terms of states

$$\dot{x_1} = x_4 = \dot{x}_p \tag{2.45}$$

$$\dot{x_2} = x_5 = \dot{x_s} \tag{2.46}$$

$$\dot{x_3} = x_6 = \dot{x_u} \tag{2.47}$$

$$\dot{x_4} = \ddot{x}_p = \frac{-K_p(x_1 - x_2) - C_p(x_4 - x_5)}{M_p} \tag{2.48}$$

$$\dot{x_5} = \ddot{x_s} \tag{2.49}$$

$$\dot{x_6} = \frac{K_s(x_2 - x_3) - C_s(x_5 - x_6) - K_t(x_3 - r)}{M_{us}} \tag{2.50}$$

Representing the equations 2.45-2.50 in terms of matrix form

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-K_p}{M_p} & \frac{K_p}{M_p} & 0 & \frac{-C_p}{M_p} & \frac{C_p}{M_p} & 0 \\ \frac{-K_p}{M_s} & \frac{-(K_p+K_S)}{M_s} & \frac{K_S}{M_s} & \frac{C_p}{M_s} & \frac{-(C_p+C_s)}{M_s} & \frac{C_s}{M_s} \\ 0 & \frac{K_S}{M_{us}} & \frac{-(K_t+K_S)}{M_{us}} & 0 & \frac{C_s}{M_{us}} & \frac{-C_s}{M_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{K_t}{M_{us}} \end{bmatrix} W \tag{3.51}$$

The output vector is defined as

$$Y = [x_s - x_{us} \quad x_p \quad x_s \quad x_{us} - r \quad \ddot{x}_p]^T \tag{2.52}$$

$$y_1 = (x_s - x_{us}) = \text{suspension travel} \tag{2.53}$$

$$y_2 = x_p = \text{seat displacement} \tag{2.54}$$

$$y_3 = x_s = \text{sprung mass displacement} \tag{2.55}$$

$$y_4 = \ddot{x}_p = \text{seat acceleration} \tag{2.56}$$

$$y_5 = (x_{us} - r) = \text{Tire deflection} \tag{2.57}$$

Rewriting the equations in terms of state variables as follows:

$$Y = CX + D_W W \tag{2.58}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-K_p}{M_p} & \frac{K_p}{M_p} & 0 & \frac{-C_p}{M_p} & \frac{C_p}{M_p} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} W \tag{2.59}$$

$$D=0 \tag{2.60}$$

2.4 MATHEMATICAL MODELLING OF THREE DOF ACTIVE SUSPENSION SYSTEM

Here an actuator is added between Sprung mass and Un-sprung mass in addition to spring and damper as shown in figure 2.4. The function of actuator is to produce the required force between the sprung mass and Un-sprung mass.

Equation of motion for seat is gives as follows

$$\boldsymbol{M_p \ddot{x}_p + K_p(x_p - x_s) + C_p(\dot{x}_p - \dot{x}_s) = 0} \tag{2.61}$$

Equation of motion for sprung mass is gives as follows

$$\boldsymbol{M_s \ddot{x}_s - K_p(x_p - x_s) - C_p(\dot{x}_p - \dot{x}_s) + K_s(x_s - x_{us}) + C_s(\dot{x}_s - \dot{x}_{us}) = F_a} \tag{2.62}$$

Equation of motion for unsprung mass is gives as follows

$$\boldsymbol{M_{us} \ddot{x}_{us} - K_s(x_s - x_{us}) - C_s(\dot{x}_s - \dot{x}_{us}) + K_t(x_{us} - r) = -F_a} \tag{2.63}$$

where,

F_a=Actuator Force (2.64)

And all others parameters are same as in passive system.

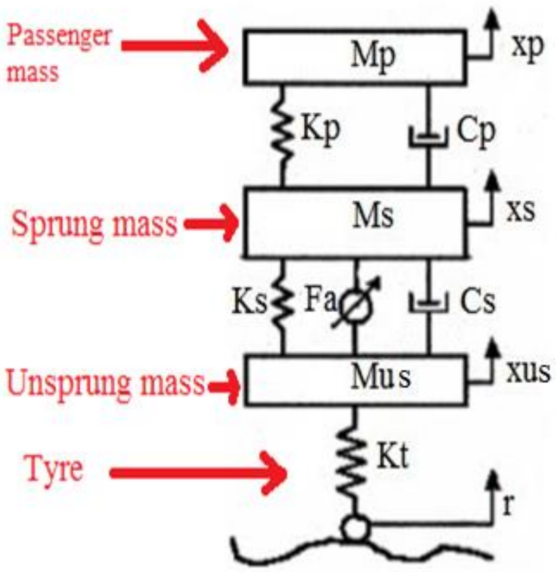

Figure 2.4- A Three DOF Quarter car active suspension system

The state variables to be used in this model are same as used in passive system.S

In terms of state variable the system equations can be written as follows:

$$\dot{X} = AX + DU + GW \tag{2.65}$$

Where,

X=State input variable matrix

U=Control input variable matrix

W=Road input matrix

Rewriting the equations in terms of state variables as follows:

$$\dot{x_1} = x_4 = \dot{x}_p \tag{2.66}$$

$$\dot{x_2} = x_5 = \dot{x}_s \tag{2.67}$$

$$\dot{x_3} = x_6 = \dot{x}_u \tag{2.68}$$

$$\dot{x_4} = \ddot{x}_p = \frac{-K_p(x_1 - x_2) - C_p(x_4 - x_5)}{M_p} \tag{2.69}$$

$$\dot{x_5} = \ddot{x_s} = \frac{K_p(x_1 - x_2) + C_p(x_4 - x_5) - K_s(x_2 - x_3) - C_s(x_5 - x_6) + F_a}{M_s} \tag{2.70}$$

$$\dot{x_6} = \ddot{x}_{us} = \frac{K_s(x_2 - x_3) - C_s(x_5 - x_6) - K_t(x_3 - r) - F_a}{M_{us}} \tag{2.71}$$

Representing the equations 2.66-2.71 in terms of matrix form

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-K_p}{M_p} & \frac{K_p}{M_p} & 0 & \frac{-C_p}{M_p} & \frac{C_p}{M_p} & 0 \\ \frac{-K_p}{M_s} & \frac{-(K_p+K_S)}{M_s} & \frac{K_S}{M_s} & \frac{C_p}{M_s} & \frac{-(C_p+C_s)}{M_s} & \frac{C_s}{M_s} \\ 0 & \frac{K_S}{M_{us}} & \frac{-(K_t+K_S)}{M_{us}} & 0 & \frac{C_s}{M_{us}} & \frac{-C_s}{M_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{M_s} \\ \frac{-1}{M_{us}} \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{K_t}{M_{us}} \end{bmatrix} W \tag{2.72}$$

The output vector is defined as

$$Y = [x_s - x_{us} \quad x_p \quad x_s \quad x_{us} - r \quad \ddot{x}_p]^T \tag{2.73}$$

$$y_1 = (x_s - x_{us}) = \text{suspension travel} \tag{2.74}$$

$$y_2 = x_p = \text{seat displacement} \tag{2.75}$$

$$y_3 = x_s = \text{sprung mass displacement} \tag{2.76}$$

$$y_4 = \ddot{x}_p = \text{seat acceleration} \tag{2.78}$$

$$y_5 = (x_{us} - r) = \text{Tire deflection} \tag{2.79}$$

Rewriting the equations in terms of state variables as follows

$$Y = CX + DU + D_W W \tag{2.80}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-K_p}{M_p} & \frac{K_p}{M_p} & 0 & \frac{-C_p}{M_p} & \frac{C_p}{M_p} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} W \tag{2.81}$$

$$D=0 \tag{2.82}$$

Table 2.1 shows value of all the system parameters used in the model[26].

Table 2.1: Parameter values of the quarter car model

System Parameters			
Symbol	**Description**	**Value in 10^3**	**Unit**
$\boldsymbol{M_p}$	Passenger mass	0.1	Kg
$\boldsymbol{M_s}$	Sprung mass	2.05	Kg
$\boldsymbol{M_{us}}$	Un-sprung mass	0.1	Kg
$\boldsymbol{K_p}$	Stiffness of seat	100	N/m
$\boldsymbol{C_p}$	Damping coefficient of seat	6	Ns/m
$\boldsymbol{K_s}$	Stiffness of suspension system	400	N/m
$\boldsymbol{C_s}$	Damping coefficient of suspension system	5	Ns/m
$\boldsymbol{K_t}$	Stiffness of tire	2000	N/m

2.5 CONTROLLABILITY AND OBSERVABILITY ANALYSIS OF THE SYSTEM

The concepts of controllability and Observability were given by Kalman. A system is said to be controllable if it is possible to transfer the system from any initial state to any other state in a finite interval of time. A system is said to be observable if it is possible to determine the state from the observation of the output at any instant over a finite time interval. They play an important role in the design of control systems in state space.

2.5.1 Controllability Analysis

In order to check controllability of a system MATLAB command ctrb(A,B) is used where A and B are the system matrix given by state space representation of the system.

An nth-order plant whose state equation is

$\dot{x}$= Ax + Bu

$$C_M=[B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] \tag{2.83}$$

is completely controllable if the rank of controllability matrix C_M is equal to n. For 2 DOF system n is 4 and the rank of C_M is found to be 4 and for 3 DOF system n is 6 and the rank of C_M is 6 as shown in Appendix. Hence the system is completely controllable.

2.5.2 Observability Analysis

In order to check observability of a system MATLAB command obsv(A,C) is used where A and C are the system matrix given by state space representation [38] of the system. An nth-order plant whose state and output equations are respectively

$\dot{x}$= Ax + Bu

and y = Cx

$$O_v=[C \quad CA \quad CA^2 \quad \dots \quad CA^{n-1}]^T \tag{2.84}$$

is completely observable if the rank of Observability matrix O_v is equal to n. . For 2 DOF system n is 4 and the rank of O_v is found to be 4 and for 3 DOF system n is 6 and the rank of O_v is 6 as shown in Appendix I. Hence the system is completely observable.

2.6 INPUT ROAD DISTURBANCES

In this thesis, to simulate the feature of road surface three types of road input signal are used. They are step input signal, sine input signal and white noise road input signal. These inputs are required to simulate the vehicle suspension system, and reflect the real road condition.

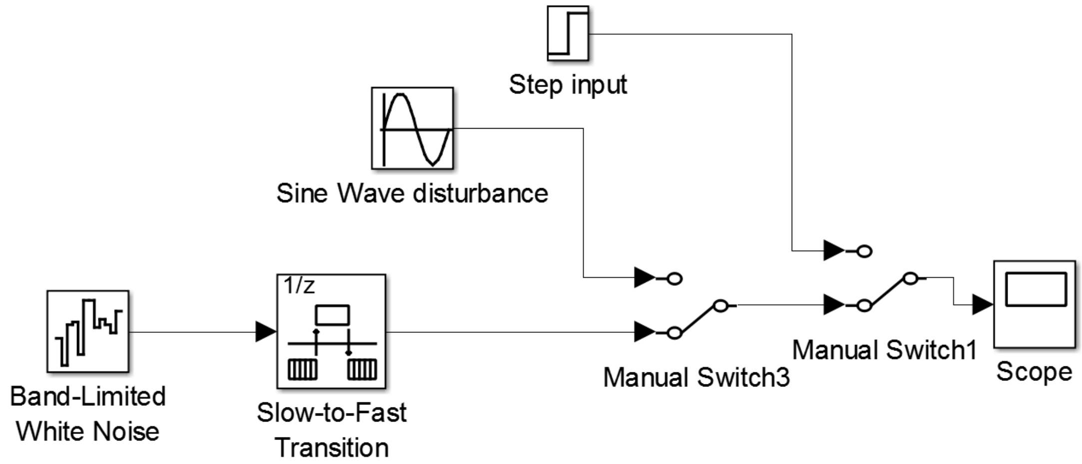

Figure 2.5 Road input Disturbance Simulink model

2.6.1 Step Road Profile

The basic input signal for simulating suspension system is step input. Suppose a vehicle goes through a sudden change of height in a very short time period. The figure 2.6 shows a change of 0.1m height after 1 sec.

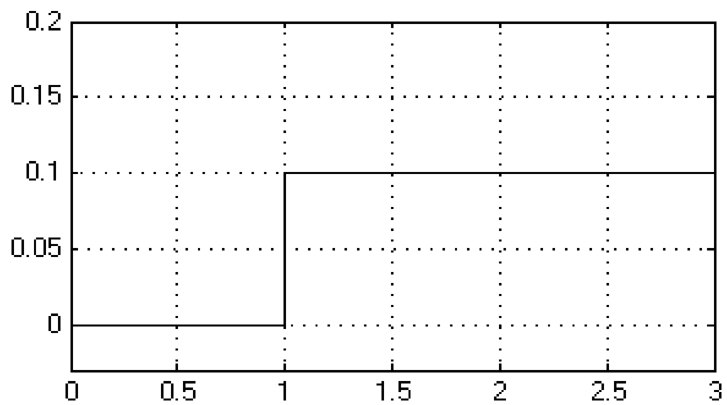

Figure 2.6 Step input road profile

2.6.2 Sine Road Profile

To simulate continuous bumps or periodic fluctuations, Sine wave input signal can be used. When the car experiences a periodic wave pavement the toughness of spring is tested. Every automotive industry make this test before a new vehicle drives on road. The sine used here has frequency 10 rad/s and amplitude 0.1m as shown in figure 2.7.

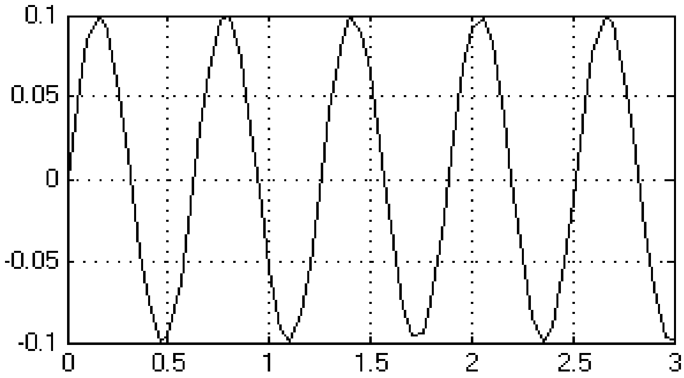

Figure 2.7 Sine input road profile

2.6.3 White noise Road Profile

Many researches show that when the speed of an automobile is constant, the road roughness is an stochastic process which is subjected to Gauss distribution, and it cannot be described accurately by means of mathematical relations. The transformation of white noise road input signal can perfectly simulate the actual pavement condition. The sampling time of roughness used in signal is 0.1 sec and noise power is 0.00001 as shown in figure 2.8.

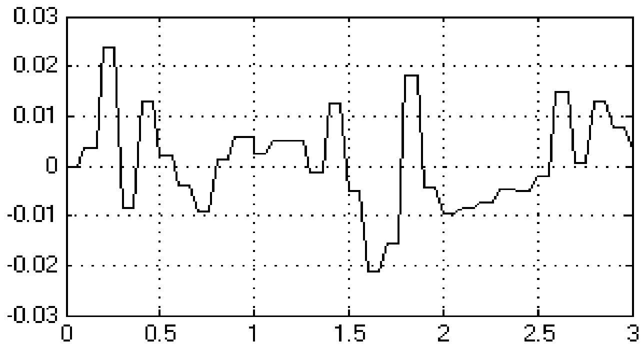

Figure 2.8 White noise input road profile

CHAPTER 3

CONTROLLER DESIGN

This chapter includes the related theory and methodology required for designing controller for the active suspension system. The control force is generated by the controller to control output parameters such as seat acceleration, tire deflection, suspension travel or suspension defection, seat displacement, sprung mass acceleration. For some given measure performance an optimal controller can provide better performance. Four controllers has been designed i.e. PID controller design using MATLAB/SIMULINK's PID tuner, LQR, fuzzy and Fuzzy PID controllers. All the three controllers are designed separately for 2 DOF and 3 DOF quarter car model for all three input road conditions. For fuzzy logic and fuzzy PID system a research is done to investigate the system response with different membership functions. The control objective of the controller design is to achieve better good handling and ride comfort and to reduce overshoot with a faster output response.

3.1 SIMULINK MODEL OF QUARTER CAR SUSPENSION SYSTEM

From modelling equations we get the Passive suspension system model for 2 DOF and 3 DOF as shown in figure 3.1 and 3.2 and Active suspension system model for 2 DOF and 3 DOF are shown in figure 3.3 and 3.4.

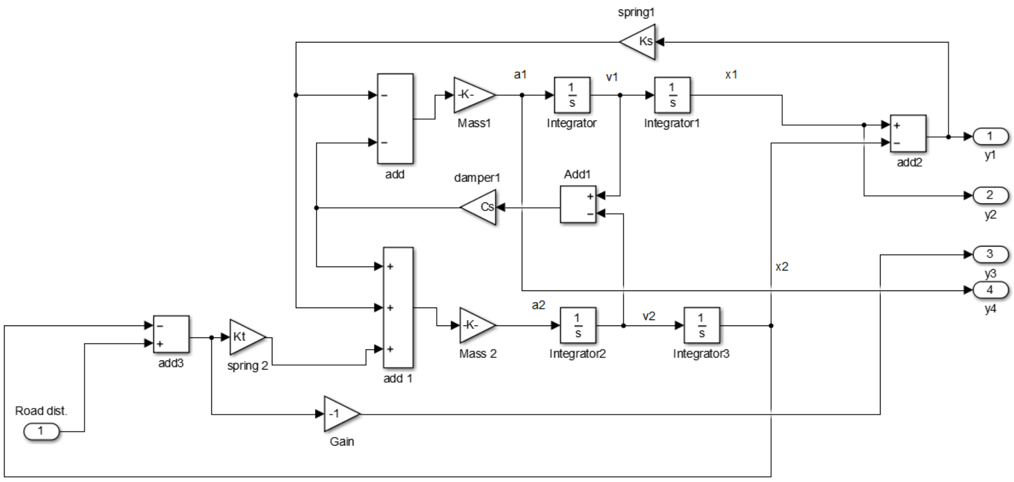

Figure 3.1 A 2 DOF Passive suspension system Simulink model

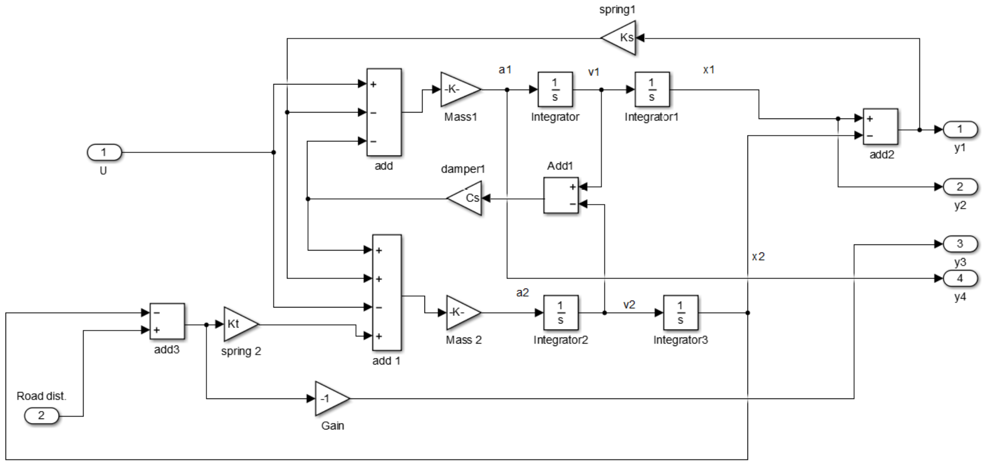

Figure 3.2 A 2 DOF Active suspension system Simulink model

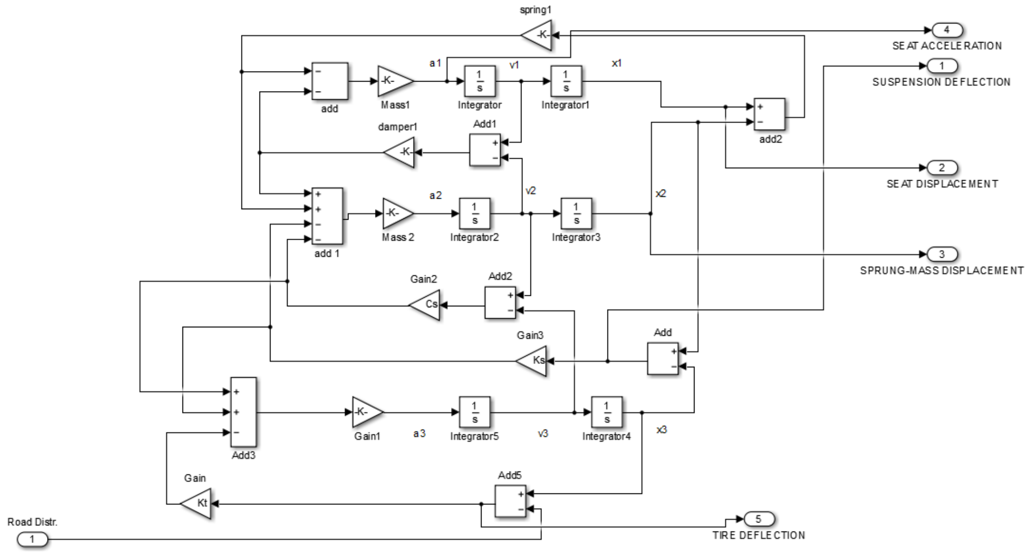

Figure 3.3 A 3 DOF Passive suspension system Simulink model

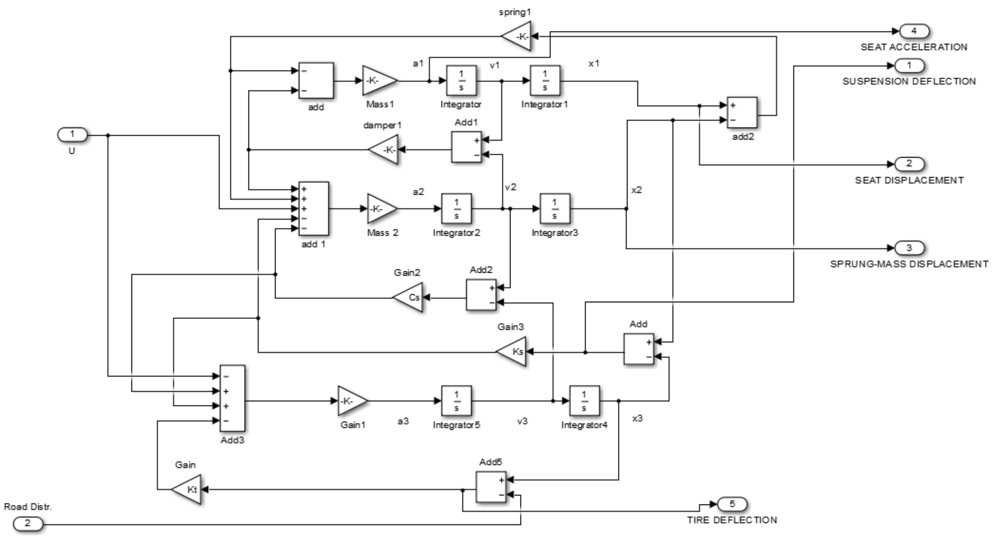

Figure 3.4 A 3 DOF Active suspension system Simulink model

From above given figures, Active system has a control input U which is nothing but Actuator force and is calculated and applied by the controller.

3.2 PID CONTROL SYSTEM

More than half of the industries uses PID controller or modified PID controller. It is the most common and practical way in control methods as it can be adjusted on site. PID controller is applicable to almost every control system. For the systems whose mathematical model is not known, PID controller find its purpose.

Sometimes response is observed according to reference input or sometimes it is set according to disturbance input. These responses from two inputs does not always get tuned at a same time in a single degree of freedom that is why most of the designing is done in Two DOF.

Block diagram of Conventional PID control is shown in Figure 3.5, the system consists of the PID controller, Process/Plant , sensor , reference input or set point, and disturbance input.

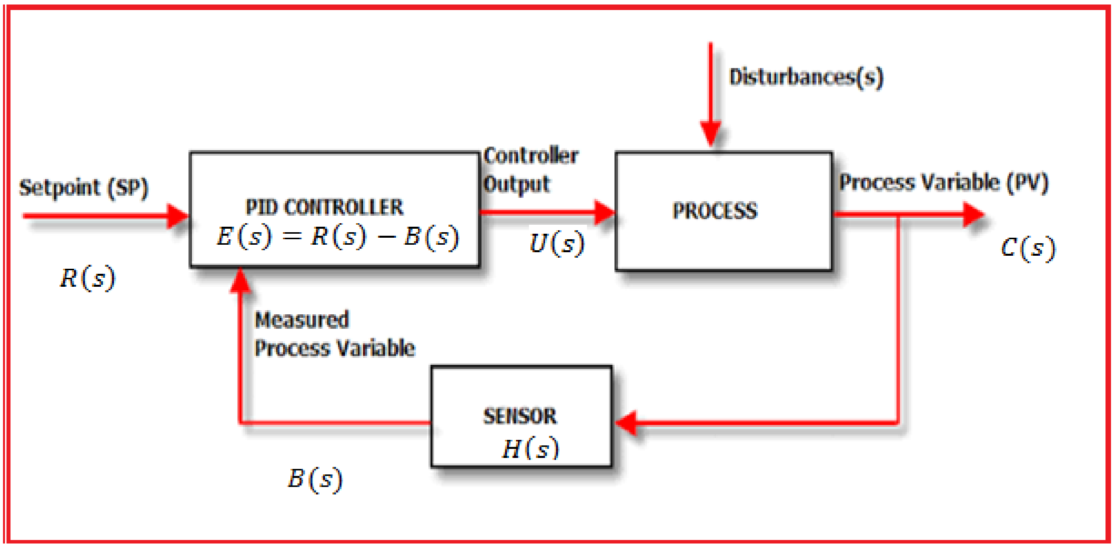

Figure 3.5 PID controller block diagram

In Laplace domain , Error is given as

$$E(s) = R(s) - B(s) \tag{3.1}$$

$B(s) = H(s)C(s)$

If $H(s) = 1$, $B(s) = C(s)$

The PID controller in Laplace is given as

$$U(s) = \left(K_p + \frac{K_i}{s} + K_d s\right) E(s) \tag{3.2}$$

Where,

K_p is the proportional gain,

K_i is the integral gain and

K_d is derivative gain,

$U(s)$ is control signal

3.2.1 Design Steps of PID controller

There are various tuning methods for tuning a PID controller. In this project PID controller used is designed using Simulink's PID tuner. PID tuner yields a fast and widely applicable PID tuning approach for tuning Simulink PID controller. In this work PID tuner is first of all applied to the 2 Degree Of Freedom (DOF) quarter car model for all the three road input conditions i.e. step, sine and band limited white noise. Same methodology is applied to 3 Degree Of Freedom (DOF) quarter car model for all the input road disturbances. The goal is to reduce the oscillation to zero in case of suspension travel, tire deflection, seat acceleration, sprung mass acceleration and to have a faster system response. Figure 3.6 & 3.7 shows the PID controller Simulink model for 2 DOF and 3DOF active suspension system compared with the passive system.

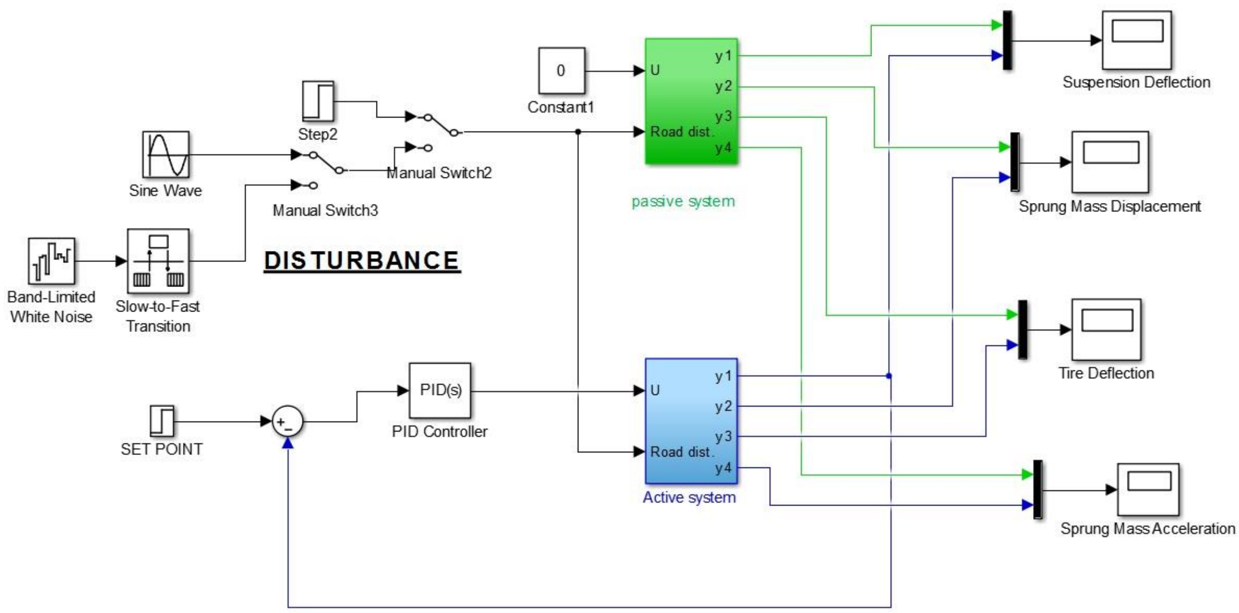

Figure 3.6 PID controller for 2 DOF model in Simulink

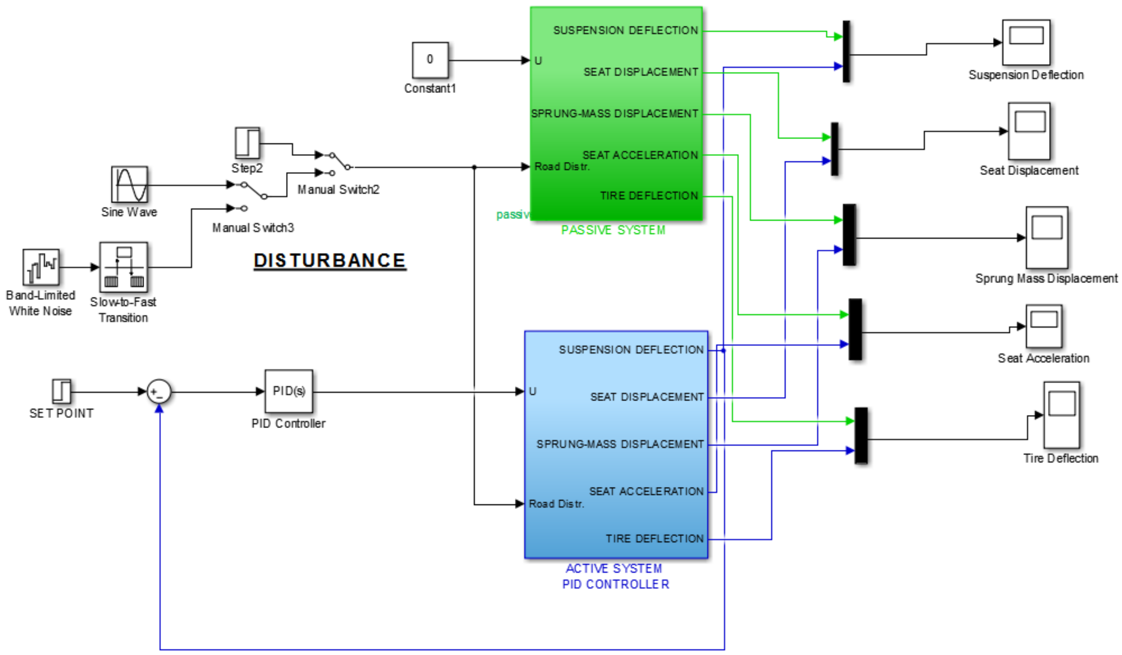

Figure 3.7 PID controller for 3 DOF model in Simulink

Design steps with the PID tuner are as follows:

- First step is to launch the PID tuner and after launching the simulink automatically generates a linear plant model and an initial controller is designed. The tuner automatically detects plant model input and output using current operating point for the linearization.
- According to design requirements of the system, response time and transient behavior slides are adjusted to finely tune the controller. In order to reduce the settling time controller response is made faster and in order to reduce overshoot transient behavior robustness is increased.
- After the parameters of the designed controller are properly tuned it is exported back to PID controller block and it is verified to compare the performance in Simulink model. In this work, suspension travel is used as the feedback signal from output to input.

- PID parameters for 2 DOF quarter car model
  - for step road input are K_p=100, K_i=51139, K_d=3000.
  - for sine road input are K_p=50, K_i=5000, K_d= 7555.855.
  - for band limited white noise input are K_p=0, K_i=5000, K_d=7500.

- PID parameters for 3 DOF quarter car model
    - for step road input are K_p=100, K_i=51139, K_d=2900.
    - for sine road input are K_p=100, K_i=51138, K_d=3500.
    - for band limited white noise road input are K_p=100, K_i=51138, K_d=3000.

3.3 LQR CONTROL SYSTEM

The quadratic optimal control provides a methodical way of computing the state feedback control. System with non-zero reference input value is known as regulator problem. Linear Quadratic Regulator control is used for optimal control of the linear systems whose states can be measured. LQR controlling can be done on linear systems without the disturbances or with the disturbances. In case of disturbed systems LQR approach requires measurable states and measurable disturbances. Suspension system consists of external disturbances so the LQR control approach used here is with measurable disturbance.

A linear time-varying system consists of disturbances can be represented as follows:

$$\dot{x} = Ax + Bu + B_w \tag{3.3}$$

$x(0) = x_0$

$$y = Cx + Dy + D_w \tag{3.4}$$

Where,

$B_w \in R^{n\times w}$

$D_w \in R^{m\times w}$.

Our aim is to find a control input vector u

$$u = -kx \tag{3.5}$$

To minimize the performance index J[56]

$$J = \int_0^{\infty} [(y)^T\, Q_y(y) + u^T R_u u]dt \tag{3.6}$$

$$J = \int_0^{\infty}[(Cx + Du)^T\, Q_y\big(Cx + Du\big) + 2(Cx + Du)^T Q_y D_w w + w^T {D_w}^T Q_y D_w w\big) + u^T R_u u]dt \quad (3.7)$$

$$=\int_0^{\infty}[x^T\, \big(C^T Q_y C\big)x + 2x^T\big(C^T Q_y D\big)u + u^T(D^T Q_y D + R_u)u]dt + \int_0^{\infty}\{2x^T C^T\, Q_y D_w w + 2w^T {D_w}^T Q_y D u + w^T {D_w}^T Q_y D_w w\}dt \quad (3.8)$$

$$= \int_0^{\infty}[(x^T\, Qx + 2x^T N u + u^T R u)\}dt + (2x^T N_{xw} w + 2w^T N_{uw} u + w^T R_w w)]dt \quad (3.9)$$

$$J = \int_0^{\infty}[F(u, x, w, t)dt \quad (3.10)$$

The control law has to be a function of both x(t) and w(t).

3.3.1Pontryagin's maximum principle

Control problem can be defined as

$$F = (u, x, w, t) + \lambda^T(Ax + Bu + B_w w) \quad (3.11)$$

$$=(x^T Qx + 2x^T N u + u^T R u) + (2x^T N_{xw} w + 2w^T N_{uw} u + w^T R_w w) + \lambda^T(Ax + Bu + B_w w) \quad (3.12)$$

The necessary condition for a local minimum is given by the Euler-Lagrange equation

$$\frac{\partial H}{\partial x}=\dot{\lambda} = -(2Qx + 2Nu + 2N_{xw} w + A^T\lambda) \quad (3.13)$$

And optimal control equation is given as

$$u = \frac{\partial H}{\partial u}=2N^T x + 2Ru + 2{N_{uw}}^T w + B^T\lambda) \quad (3.14)$$

To eliminate λ ,Riccatti transformation is assumed as

$$\lambda = 2(Px - \xi) \quad (3.15)$$

Where P^T=P is Riccatti matrix and $\xi(t)$ is Riccatti vector

Now optimal control is

$$u = -R^{-1}(N^T x + {N_{uw}}^T w + B^T P x - B^T \xi) \quad (3.16)$$

$$u = [-R^{-1}(N^T + B^T P)x] + [-R^{-1}({N_{uw}}^T w - B^T \xi)] \quad (3.17)$$

For a linear system with disturbance u is the combination of state-feedback control u_x and disturbance feed-forward control u_w

$$u = u_x + u_w \quad (3.18)$$

$$u = -K_x x - K_w w \quad (3.19)$$

K_x is Optimal state feedback controller gain and is given by

$$K_x = -R^{-1}(N^T + B^T P) \quad (3.20)$$

K_w is Optimal disturbance feed forward controller gain and is given by

$$K_w = -R^{-1}({N_{uw}}^T w - B^T \xi) \quad (3.21)$$

3.3.2 Riccatti Equation

Derivative of Riccatti transformation is

$$\dot{\lambda} = 2(\dot{P}x + P\dot{x} - \xi) \quad (3.22)$$

Using value of λ in Euler Lagrange equation (4.13) gives

$$\dot{\lambda} = -\left(2Qx + 2Nu + 2N_{xw}w + A^T 2(Px - \xi)\right) \quad (3.23)$$

$$= -2[Qx + Nu + N_{xw}w + A^T(Px - \xi)] \quad (3.24)$$

From equations 3.22 & 3.24

$$(\dot{P}x + P\dot{x} - \xi) = -[Qx + Nu + N_{xw}w + A^T(Px - \xi)] \quad (3.25)$$

From equations 3.3 & 3.17 an rearranging

$$\left(\dot{P}x + PAx + A^T Px + Qx\right) + (N + PB)u - [\xi + A^T \xi - (N_{xw} + PB)w] = 0$$

$$\left(\dot{P}x + PAx + A^TPx + Qx\right) + (N + PB)[-R^{-1}(N^T + B^TP)x] +$$

$$(N + PB)\left[-R^{-1}\left({N_{uw}}^T w - B^T\xi\right)\right] - [\xi + A^T\xi - (N_{xw} + PB)w] = 0 \quad (3.26)$$

The reduced form of the Riccati equation must be satisfied by matrix P

$$\dot{P} + PA + A^TP - (PB + N)R^{-1}(N^T + B^TP) + Q = 0 \quad (3.27)$$

And so called differential Riccati vector equation

$$\dot{\xi} + [A^T + (N + PB)R^{-1}B^T]\xi + [(N + PB)R^{-1}{N_{uw}}^T - (N_{xw} + PB_w)]w = 0 (3.28)$$

From equation 4.20,differential riccati vector is simplified as

$$\dot{\xi} + [A^T + (K_x)^TB^T]\xi + \left[(K_x)^T{N_{uw}}^T - (N_{xw} + PB_w)\right]=0 \quad (3.29)$$

For infinite time problem $\dot{P}$ (t)=0 and $\dot{\xi}$=0, from equations 3.27 and 3.29, riccati vector can be computed as

$$\dot{\xi} = -[A^T + (K_x)^TB^T]^{-1} + \left[(K_x)^T{N_{uw}}^T - (N_{xw} + PB_w)\right]w \quad (3.30)$$

By equating derivative of Riccati vector $\dot{\xi}$ =0 and putting the value of ξ, u_w can be written [29]as

$$u_w = -R^{-1}\left\{{N_{uw}}^T + B^T[A^T + (K_x)^TB^T]^{-1}\left[(K_x)^T{N_{uw}}^T - (N_{xw} + PB_w)\right]\right\}w$$

$$K_w = -R^{-1}\left\{{N_{uw}}^T + B^T[A^T + (K_x)^TB^T]^{-1}\left[(K_x)^T{N_{uw}}^T - (N_{xw} + PB_w)\right]\right\} \quad (3.31)$$

For the case of LTI systems, P(t)=P is a constant matrix which is the solution of ARE resulting in constant gain matrices K_x & K_w.

The LQR problem for the LTI systems can be solved for the objective functions defined by (3.3) and (3.4) using the command 'lqr' and 'lqry', respectively. Once the weighting matrices Q, N and R the commands 'lqr' and 'lqry' will check the controllability and Observability automatically and gives the Riccati matrix P and the optimal state-feedback gain matrix K .

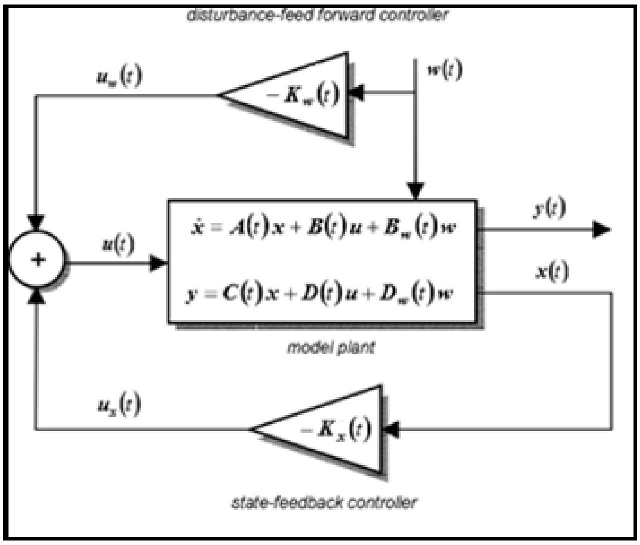

Fig.3.8 Block diagram of LQR control System With Measurable Disturbances

The optimal control u can then be derived easily by (3.17). The weighting matrices, therefore, can be considered as the design parameters of the LQR problem. Figure 3.8 shows the basic block diagram of LQR controller.

3.3.3 Design Steps of LQR controller

In this work first of all LQR controller is designed for 2 DOF quarter car model and then for 3 DOF quarter car. For each model, again LQR controller is designed and investigated with step, sine and band limited white noise road input condition. Matlab coding is used to design LQR controller .Designing steps of the LQR controller are as follows:

- The first step in designing a LQR controller is to derive the state space representation of the system under consideration.
- After the state space representation is attained, next step is to verify whether that the system is controllable. For the system to be completely state controllable, the controllable matrix must have rank n where rank of a matrix is the number of independent rows (or columns). Rank n represents the number of state variables of the system. Ranks of controllability matrix are 4 and 6 for 2 DOF and 3 DOF quarter car systems respectively which is equal to rank of both the systems. So it has 0 uncontrollable states. Hence both the models are completely controllable.

- MATLAB command `ctrb` is used to generate controllability matrix and MATLAB command `controllability` is used afterward to test the rank of the system.
- After checking the controllability criteria next step is to choose two parameters Q and R. In LQR controller design, Q can be chosen as a diagonal 6x6 matrix and R is a scalar because of existence of single control input. First step in selecting Q and R is to assume $Q = C^T C$ and $R = 1$ and output response is observed. According to the requirement the non-zero parameters of Q matrix are changed and R matrix is simultaneously changed to get the better response with the outputs.

MATLAB command `[K,S,e]=lqr(A,B,Q,R)` is used to calculate to calculate the LQR optimal gain K_x (or K) and disturbance gain K_w is calculated using LQR matrix control equations. . Figure 3.9 & 3.10 shows the LQR controller Simulink model for 2 DOF and 3DOF active suspension system compared with the passive system.

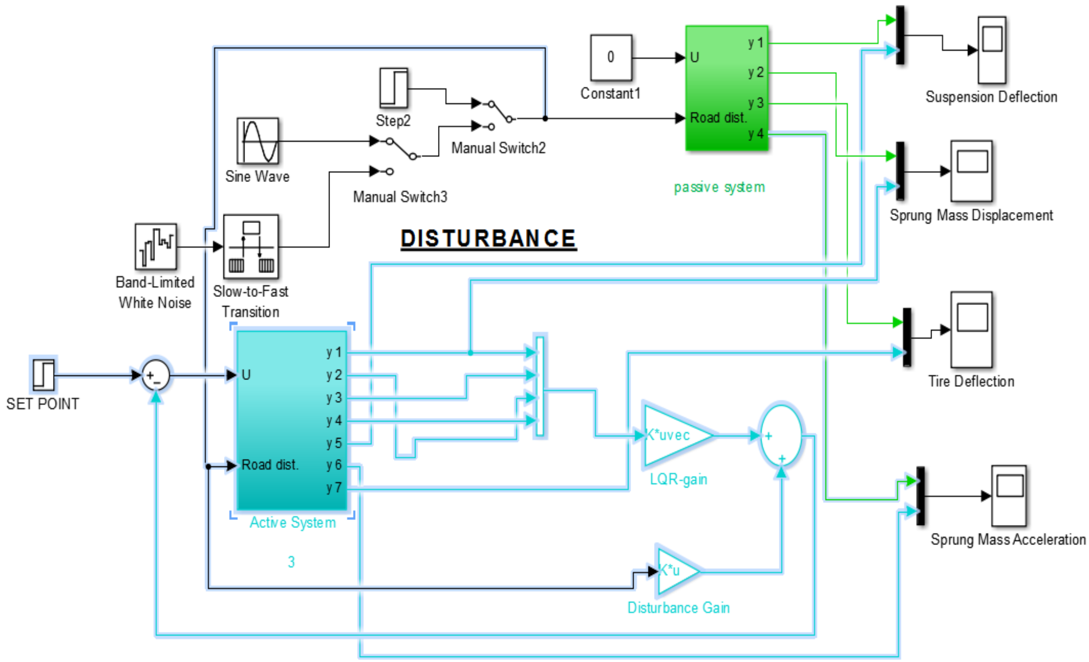

Figure 3.9 LQR controller for 2 DOF model in Simulink

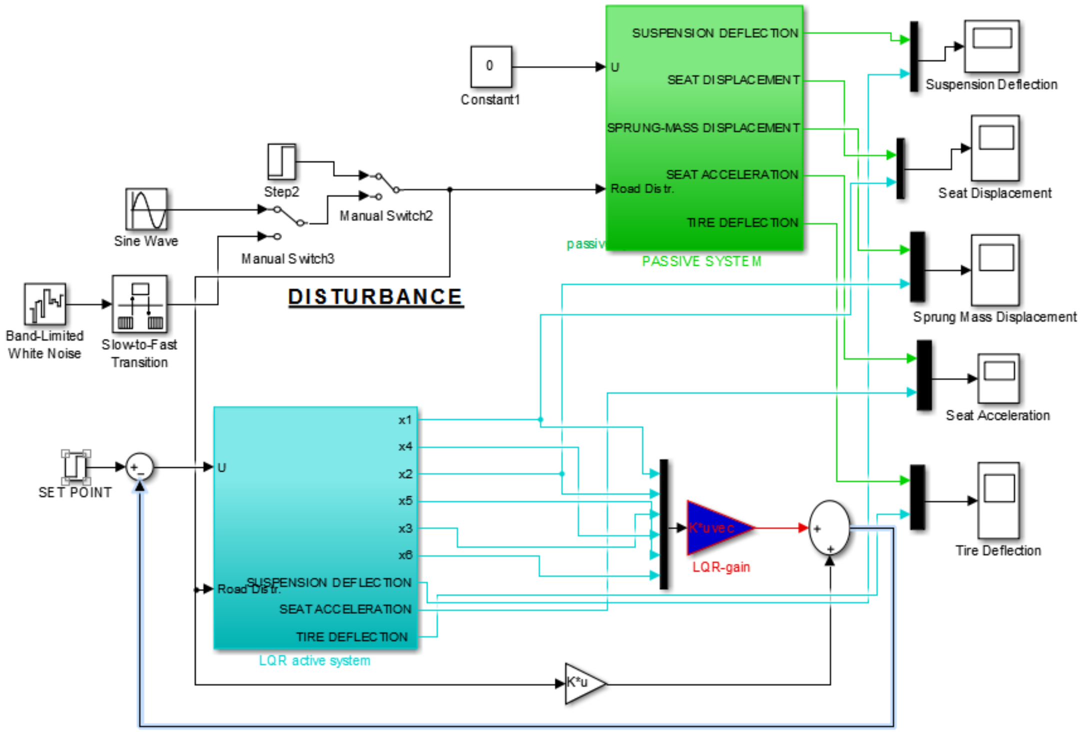

Figure 3.10 LQR controller for 3 DOF model in Simulink

3.3.4 Designed parameters of LQR controller for two DOF model

- For step input road disturbance $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 30000 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $R = 0.000399$. The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+04 *[0.0000 -1.4936 0.6194 -0.9874] and K_w=1.4936e+04.

- For sine input road disturbance $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $R = 0.00000116$. The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+05 *[0.0000 -1.4866 0.2505 -0.0990] and K_w=1.4866e+05.

- For band limited white noise input road disturbance $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 60000 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $R = 0.0000999$ The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+05 *[0.0000 -3.1041 0.4759 -0.0982] and K_w= 3.1041e+05.

3.3.5 Designed parameters of LQR controller for three DOF model

- For step input road disturbance $Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 85500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6500 \end{bmatrix}$ and

  $R = 0.00099333$. The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+04 * [-1.7091 1.7091 -1.6034 0.0313 0.6457 -0.0558] and K_w=1.6034e+04.

- For sine input road disturbance $Q == \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ and

  $R = 0.0000999$. The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+04 *[-3.7902 3.7902 -2.2374 0.0563 1.1013 0.0092]

  and K_w=2.2374e+04.

- For band limited white noise input road disturbance

  $Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15000 \end{bmatrix}$

- and $R = 0.00000999$. The values for K_x and K_w used are calculated using Matlab coding

  K_x = 1.0e+05 *[-0.8496 0.8496 -4.5894 0.0087 0.2712 -0.3370] and K_w= 2.4424e+06.

3.4 FUZZY LOGIC CONTROL SYSTEM

The knowledge of mathematical model, exact differential equations, and numerical parameters of a system is required for any classical control theory. But Fuzzy logic based control approach is a powerful problem solving concept that resembles decision making ability of human to work with subjective concepts and approximate data. It is a rule based system. Fuzzy logic gives an alternative way of design methodology for modelling of complex linear and non-linear systems. It allows modelling of complex systems using a higher level of abstraction originating from ones knowledge and experience of the system. Fuzzy logic allows expressing this knowledge with subjective concepts as fuzzy sets. Hence it does not require a mathematical model. Thus fuzzy logic controller has an advantage over classical controllers when they are applied to complex systems. The proposed fuzzy logic controller must convert objective of control contained in the fuzzy rules into a controller capable of managing all the nonlinearities and uncertainties of the active suspension system. The objective of this fuzzy logic based control system is to maintain all the output parameter as close to zero as possible in presence of road disturbances.

3.4.1 Conventional and Fuzzy Logic Design Methodologies

In conventional design approach first step is to understand the physical system and the requirement of the system. Second step is to develop a mathematical model of the system under consideration which consists of the plant, sensors and actuators. This step may require designing a state space representation of the system. The third step is to develop conventional control theory in order to design a simplified controller. The fourth step is to develop an algorithm for proper designing of the controller or for proper tuning of parameters of the control method. The last step is the simulation of the designed control theory including the effects of non-linearity, noise, and parameter variations. If the performance is found to be unsatisfactory, system should be again modelled along with modelling of controller. The first step in fuzzy Logic design is proper understanding of the system behaviour by using heuristic knowledge and operator's experience. The second step is to develop the control strategies using fuzzy rules in terms of linguistic variable using if-else statement. In this step, the fuzzy sets for the inputs and outputs, the rule set are developed and the defuzzification method is designed. The last step is to simulate the designed controller for the system. If the performance is not satisfactory further modification can be done in fuzzy rules for required performance or result. Thus fuzzy logic based methodology gives a simplified design method. . Figure 3.11 & 3.12 shows the Fuzzy Logic controller

Simulink model for 2 DOF and 3 DOF active suspension system compared with the passive system.

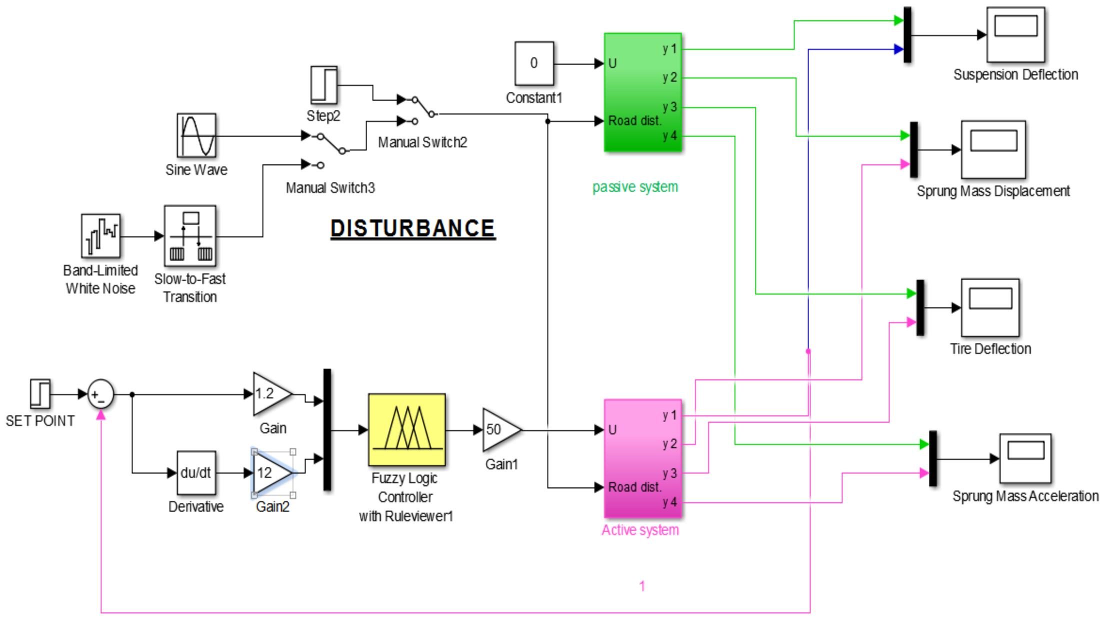

Figure 3.11 Fuzzy Logic Controller for 2 DOF model in Simulink

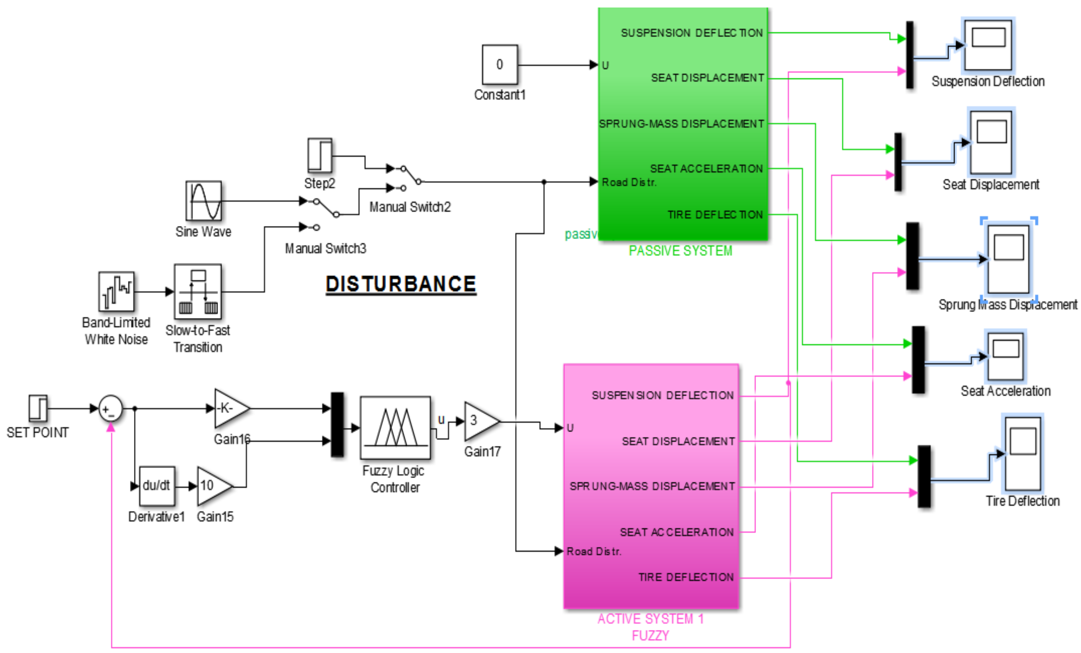

Figure 3.12 Fuzzy Logic Controller for 3 DOF model in Simulink

3.4.2 Design Steps of FLC for Two DOF System

Fuzzy controller consists of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs to the appropriate membership functions and fuzzy values. This step is known as Fuzzification [34]. The processing stage invokes each appropriate rule in the rule base and generates a result for each, the inference engine then combines the results of the rules by computing the contribution of each rule to the overall FLC output and finally the output stage converts the combined result back into a specific crisp output value. This is referred to as defuzzification. Commonly used membership functions are triangular, trapezoidal and sinusoidal. In this work, three membership functions are used and the closed loop control system output response and behaviour is analysed and observed to determine which membership function will give best response. Gaussian ,Triangular and trapezoidal MFs are used for input and output parameters of Fuzzy logic controller.

The steps in designing fuzzy logic controller are as follows:

- In this work, linguistic variables assigned to these fuzzy sets are NB, NM, NS, ZE, PS, PM and PB which are negative big, negative medium, negative small, zero, positive small, positive medium and positive big respectively. So the total rules come out to be 49.
- Method used is Mamdani type inference method and for defuzzification centroid method is used. Actuator force is the fuzzy controller output and is calculated with the help of 2 inputs i.e. the suspension travel error E and the corresponding rate of change of the suspension Travel error EC. Rules are shown in table 3.1. Fuzzy control rules for 2 DOF quarter car model are same in case of three road input conditions. For each case of road input a comparison is performed with three types of membership functions i.e. for step input with gaussian, step input with triangular and step input with trapezoidal membership function and is shown in chapter 4. "And" method is taken as "min", "Or" method as "max", "implication" method as "min" and "Aggregation" as "max"[35]. Figure 3.13 shows the fuzzy surface for the designed controller.

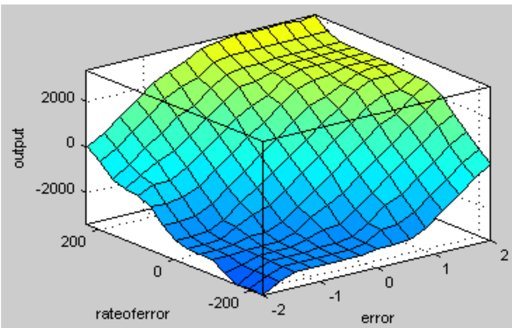

Figure 3.13 Fuzzy surface of the designed controller with 49 rules

Table 3.1: Fuzzy control rules of the designed controller for 2 DOF car model

		E						
		PB	**PM**	**PS**	**ZE**	**NS**	**NM**	**NB**
EC	**PB**	PB	PB	PB	PB	PM	PS	ZE
	PM	PM	PM	PM	PM	PS	ZE	NS
	PS	PM	PM	PM	PS	ZE	NS	NS
	ZE	PM	PM	PS	ZE	NS	NM	NM
	NS	PM	PS	ZE	NS	NM	NM	NM
	NM	PS	ZE	NS	NM	NM	NM	NB
	NB	ZE	NS	NM	NM	NM	NM	NB

3.4.3 Designed Parameters of FLC for Two DOF Model

Justification factors and Defuzzification factors are very important in designing and proper tuning of Fuzzy based controllers. Following are the justification factors and Defuzzification factors in case of step , sine and band limited white noise input condition.

- Justification factor used for step input fuzzy logic control are $K_e = 1.2$ and $K_{ec} = 12$. The Defuzzification factor K_0 is 50 .
- Justification factor used for sine input fuzzy logic control are $K_e = 5$ and $K_{ec} = 5$. The Defuzzification factor K_0 is 50.
- Justification factor used for band limited white noise input fuzzy logic control are $K_e = 0.55$ and $K_{ec} = 16$. The Defuzzification factor K_0 is 30.

3.4.4 Design Steps of FLC for Three DOF System

In this work, three membership functions are used and behaviour of the system is analysed to observe the system response to determine best membership function for 3 degree of freedom system. Gaussian, Triangular and trapezoidal MFs are used for input and output parameters of Fuzzy logic controller.

The steps in designing fuzzy logic controller are as follows:

- In this work, linguistic variables assigned to these fuzzy sets are NB, NM, NS, ZE, PS, PM and PB which are negative big, negative medium, negative small, zero, positive small, positive medium and positive big respectively. So total rules came out are 49.

- Method used is Mamdani type inference method and for defuzzification centroid method is used. Actuator force is the fuzzy controller output and is calculated with the help of 2 inputs i.e. the suspension travel error E and the corresponding rate of change of the suspension Travel error EC. Rules are shown in table 3.2. Fuzzy control rules for 3 DOF quarter car model are same in case of three road input conditions. For each case of road input a comparison is performed with three types of membership functions i.e. for step input with gaussian, step input with triangular and step input with trapezoidal membership function and is shown in chapter 4. "And" method is taken as "min", "Or" method as "max", "implication" method as "min" and "Aggregation" as "max".

Table 3.2: Fuzzy control rules of the designed controller for 3 DOF quarter car model

		E						
		PB	**PM**	**PS**	**ZE**	**NS**	**NM**	**NB**
EC	**PB**	PM	PM	PM	PB	PM	PS	ZE
	PM	PM	PM	PM	PM	PS	ZE	NS
	PS	PM	PM	PM	PS	ZE	NS	NM
	ZE	PM	PM	PS	ZE	NS	NM	NM
	NS	PM	PS	ZE	NS	NM	NM	NM
	NM	PS	ZE	NS	NM	NM	NM	NM
	NB	ZE	NS	NM	NM	NM	NM	NM

3.4.5 Designed Parameters of Fuzzy Controller for Three DOF Model

Following are the justification factors and Defuzzification factors in case of step, sine and band limited white noise input condition.

- Justification factor used for step input fuzzy logic control are $K_e = 0.09$ and $K_{ec} = 10$. Defuzzification factor K_0 is 4.5.

- Justification factor used for sine input fuzzy logic control are $K_e = 1.856$ and $K_{ec} = 12$. Defuzzification factor K_0 is 1.9.
- Justification factor used for band limited white noise input fuzzy logic control are $K_e = 0.09$ and $K_{ec} = 10$. Defuzzification factor K_0 is 3.

3.5 FUZZY TUNED PID CONTROL SYSTEM

PID tuning can be performed by many different ways. Author have already developed PID controller using PID tuner. In this section PID controller parameters are tuned using fuzzy logic and fuzzy logic tuned PID control scheme is found to give better performance than the PID tuner and this comparison is shown in chapter 4. It is an combination of fuzzy logic and PID controller. Fuzzy logic is used to calculate interval change of PID parameters i.e. K_p, K_i and K_d. In this PID controller takes relative displacement as error(E) and relative rate of change of error(EC) as inputs and outputs are adjustable PID parameters(K_p, K_i and K_d) according to the requirement using fuzzy rules. Fuzzy PID controller is designed initially for 2 DOF quarter car model and then for 3 DOF car model with three different road conditions (step, sine and band limited white noise) with three different memberships function i.e. Gaussian, triangular and trapezoidal. A comparison is made between the responses of all three membership function case which is shown in chapter 4. Figure 3.14 & 3.15 shows the Fuzzy Logic controller Simulink model for 2 DOF and 3DOF active suspension system compared with the passive system.

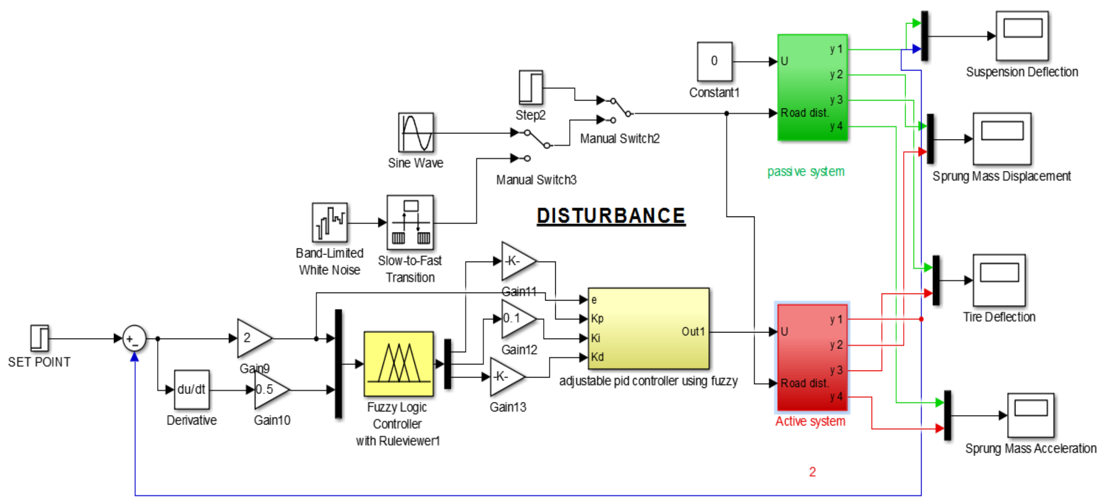

Figure 3.14 Fuzzy Logic based PID controller for 2 DOF model in Simulink

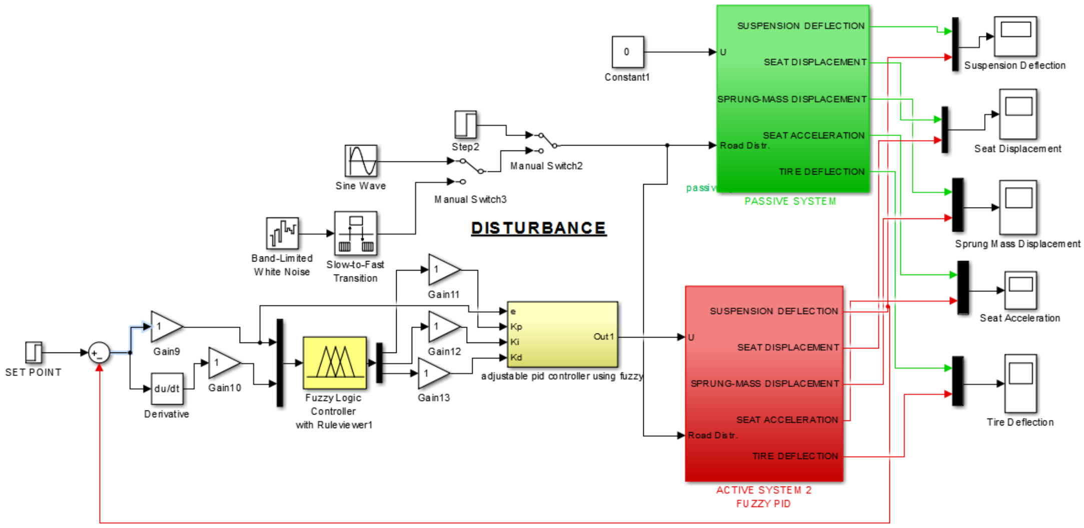

Figure 3.15 Fuzzy Logic based PID controller for 3 DOF model in Simulink

3.5.1 Design steps of Fuzzy Logic tuned PID controller for Two DOF and Three DOF Model

The design steps of the controller are as follows:

- The linguistic variables used for fuzzy sets are, N, ZE and P which are namely negative, zero and positive respectively.
- Method used is Mamdani type inference method and for defuzzification centroid method is used. Tables (3.3), (3.4) and (3.6) shows Fuzzy control rules used in controlling. Fuzzy control rules are same for both the two and three DOF quarter model but Fuzzification and defuzzification factors are different for three road input with different model conditions.

Table 3.3: Fuzzy control rules for tuning of K_p

Kp		**E**		
		Low	**Medium**	**High**
EC	**Low**	P	P	ZE
	Medium	ZE	ZE	ZE
	High	N	N	ZE

Table 3.4: Fuzzy control rules for tuning of K_i

Ki		E		
		Low	Medium	High
EC	Low	P	P	ZE
	Medium	ZE	ZE	ZE
	High	N	N	ZE

Table 3.5: Fuzzy control rules for tuning of K_d

Kd		E		
		Low	Medium	High
EC	Low	N	N	ZE
	Medium	N	ZE	P
	High	ZE	P	P

3.5.2 Designed Parameters of Fuzzy Tuned PID Controller for Two DOF Model

Following are the justification factors and Defuzzification factors in case of step, sine and white noise input condition which are used in designing of Fuzzy PID controller for two DOF quarter car model.

- Justification factor used for step input fuzzy logic control are $K_e = 1$ and $K_{ec} = 1$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=0.01, K_{02}=3 and K_{03}=0.09.
- Justification factor used for sine input fuzzy logic control are $K_e = 0.2$ and $K_{ec} = 0.5$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=0.2, K_{02}=2 and K_{03}=5.

- Justification factor used for band limited white noise input fuzzy logic control are $K_e = 0.09$ and $K_{ec} = 0.5$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=0.2, K_{02}=0.5 and K_{03}=2.

3.5.3 Designed parameters of Fuzzy tuned PID controller for Three DOF model

Following are the justification factors and Defuzzification factors in case of step, sine and band limited white noise input condition which are used in designing of Fuzzy PID controller for three DOF quarter car model.

- For step input fuzzy logic control justification factor used are $K_e = 0.65$ and $K_{ec} = 0.64$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=1, K_{02}=0.1 and K_{03}=3.
- For sine input fuzzy logic control justification factor used are $K_e = 2$ and $K_{ec} = 0.5$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=1, K_{02}=1 and K_{03}=2.
- For band limited white noise input fuzzy logic control justification factor used are $K_e = 0.9$ and $K_{ec} = 0.2$. Defuzzification factors forK_p, K_i and K_d used are K_{01}=0.5, K_{02}=0.08 and K_{03}=1.05895.

CHAPTER 4

SIMULATION AND RESULTS

The simulation of quarter car based active suspension model for different control methods and their comparison with passive suspension is discussed in this chapter. This chapter shows the simulation of passive suspension system under different road conditions. For suspension system design two different models naming 2 DOF and 3 DOF quarter car model is taken as mentioned earlier in this thesis. Again three different road input conditions step, sine and band limited white noise are chosen to check the closed loop system performance under four different designed controllers and compared with passive system. A comparison is carried out by taking different membership function for both the quarter car model with different road condition. All the project work is carried out in MATLAB environment.

4.1 COMPARISON AND RESULT FOR TWO DOF QUARTER CAR MODEL

In passive suspension system, the first model is 2 DOF quarter car model. Output parameters selected in two Degree Of Freedom (DOF) system are sprung mass displacement, suspension deflection, sprung mass acceleration and tire deflection. In the next section we will provide a comparison between different membership function based Fuzzy and Fuzzy tuned PID controller for 2DOF model with three input conditions. In the next part a comparison between the performances of four controllers with the passive i.e. uncontrolled system is presented.

4.1.1 Comparison of Membership Functions and Results of FLC for Two DOF Quarter Car Model

Figure 4.1-4.4 shows the comparison of fuzzy logic controller based active suspension system with three membership function i.e. gaussian, trapezoidal and triangular for step road input disturbance and again it is compared with passive suspension system.

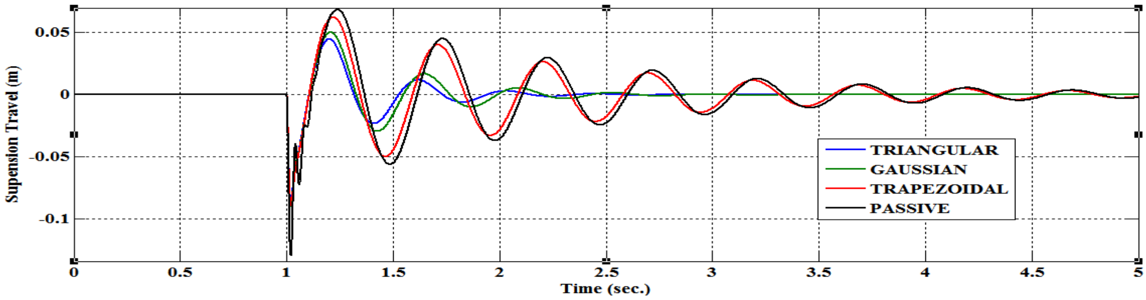

Figure 4.1- Comparison of time response of Suspension travel of fuzzy logic controller with three membership functions with step input road excitation

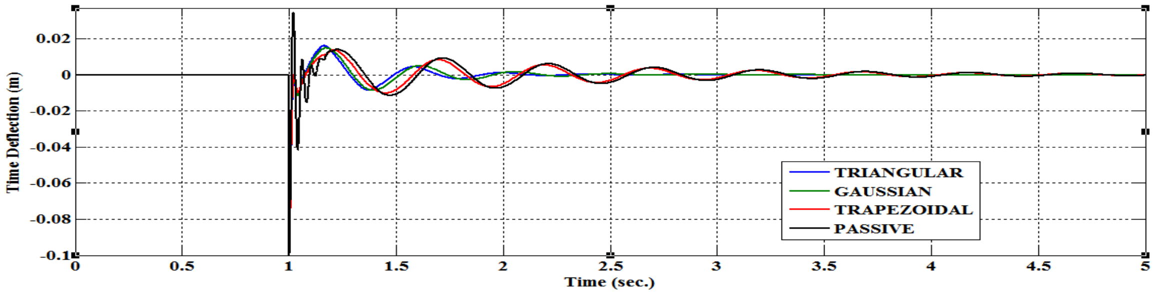

Figure 4.2- Time deflection time response of fuzzy logic controller with three membership functions with step input road excitation

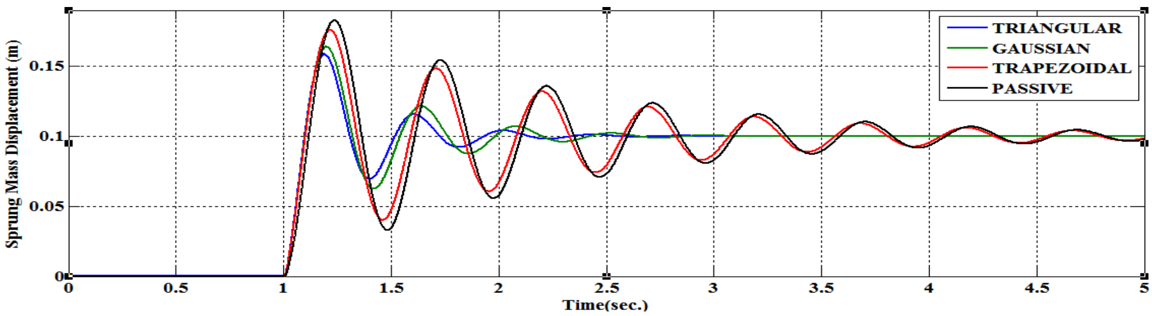

Figure 4.3- Comparison of time response of sprung mass displacement of fuzzy logic controller with three membership functions with step input road excitation

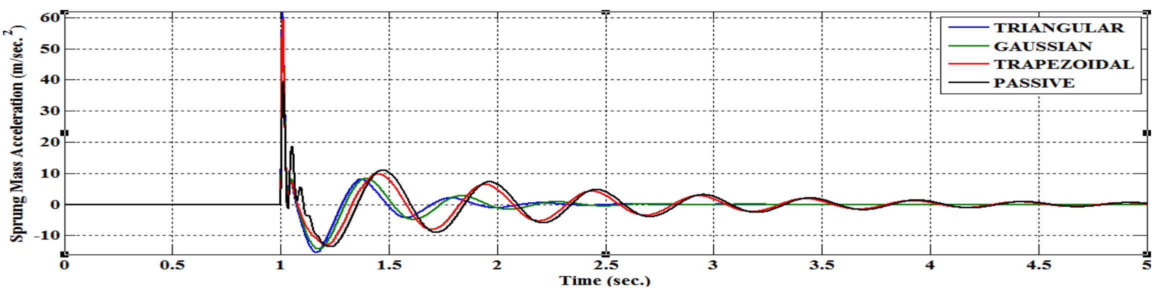

Figure 4.4- Comparison of time response of sprung mass acceleration of fuzzy logic controller with three membership functions with step input road excitation

From above figures4.1 – 4.4, it is observed that the Triangular membership function has given better results for Suspension travel and sprung mass Displacement and for Tire deflection and sprung mass acceleration Gaussian membership function is best as compared to others.

Figure 4.5-4.8 shows the comparison of fuzzy logic controller based active suspension system with three membership function i.e. gaussian, trapezoidal and triangular for sine road input disturbance and again it is compared with passive suspension system.

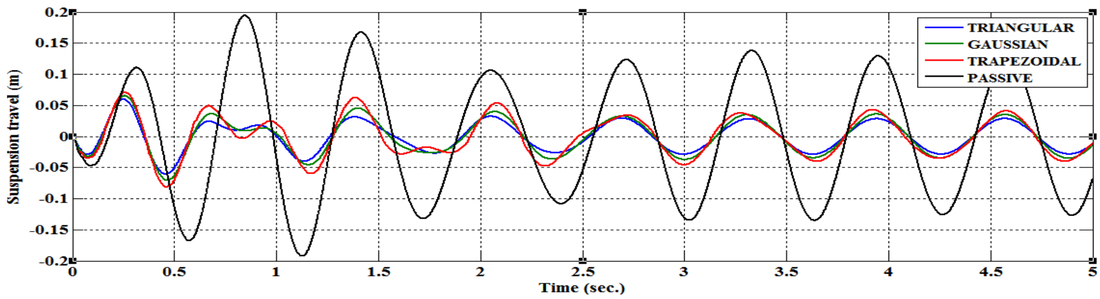

Figure 4.5- Comparison of time response of Suspension travel of fuzzy logic controller with three membership functions with sine road input

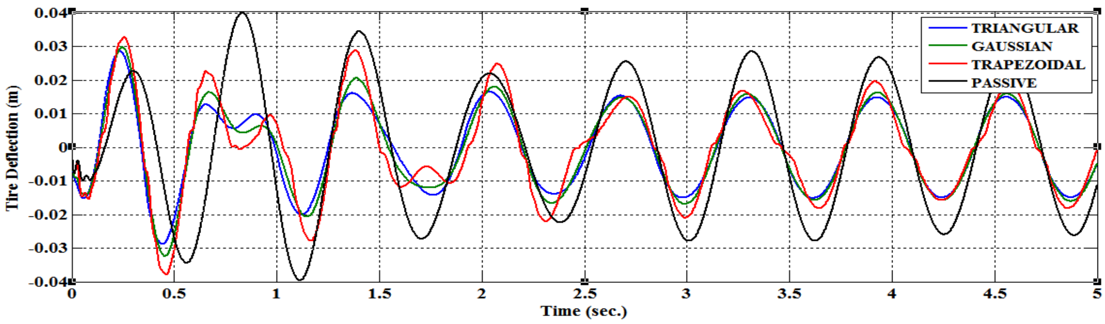

Figure 4.6- Comparison of time response of tire deflection of fuzzy logic controller with three membership functions with sine road input

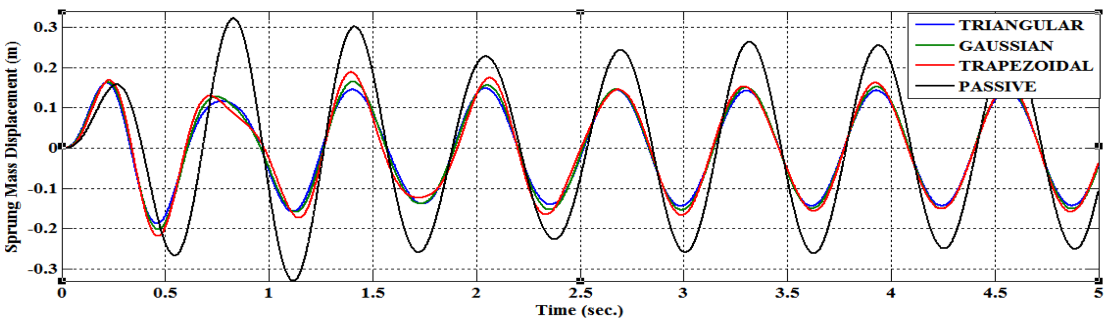

Figure 4.7- Comparison of time response of sprung mass displacement of fuzzy logic controller with three membership functions with sine road input

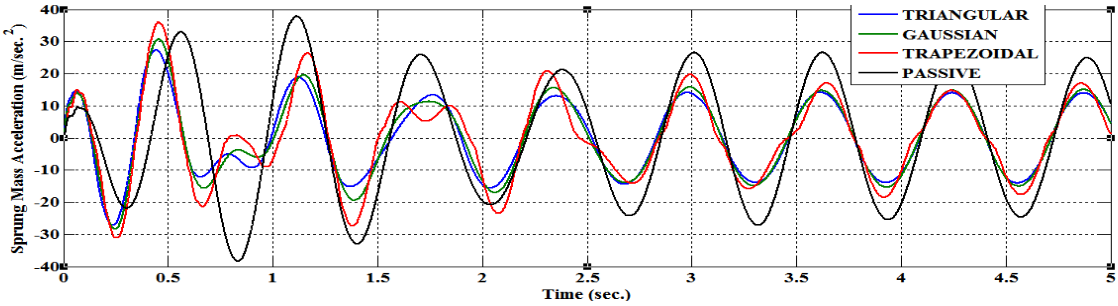

Figure 4.8- Comparison of time response of sprung mass acceleration of fuzzy logic controller with three membership functions with sine road input

Above figures show that for sine input Triangular membership function is best as compared to others in all four parameters.

Figure 4.9-4.12 shows the comparison of fuzzy logic controller based active suspension system with three membership function i.e. gaussian, trapezoidal and triangular for band limited white noise road input disturbance and again it is compared with passive suspension system.

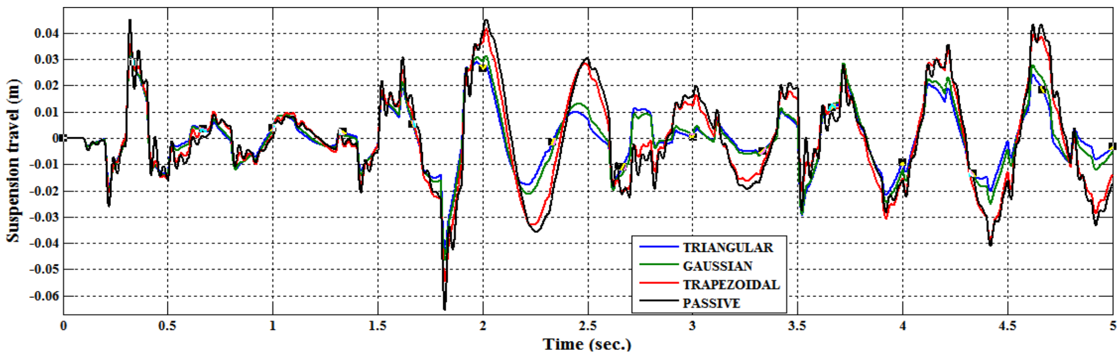

Figure 4.9- Comparison of time response of Suspension travel of fuzzy logic controller with three membership functions with band limited white noise road input

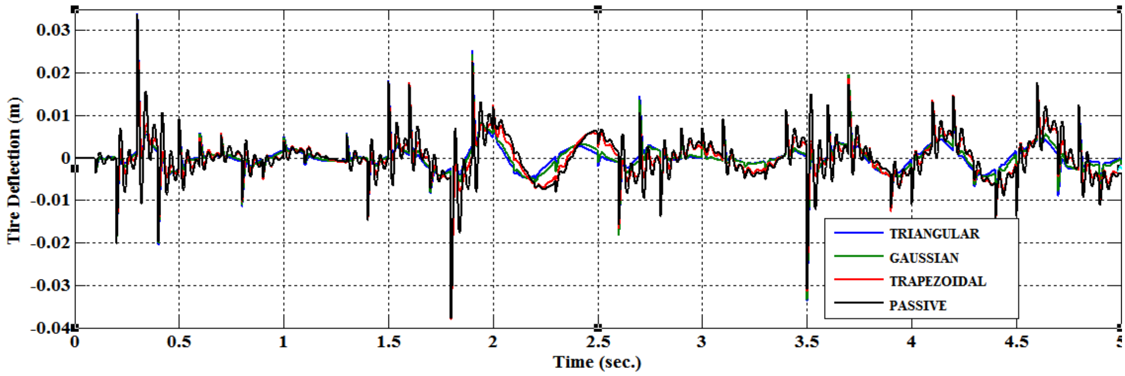

Figure 4.10- Comparison of time response of tire deflection of fuzzy logic controller with three membership functions with band limited white noise input

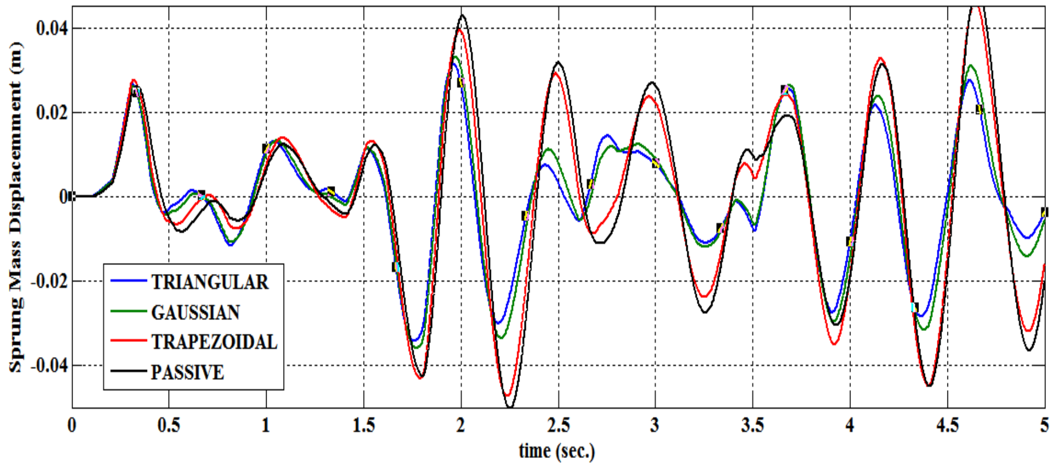

Figure 4.11- Comparison of time response of sprung mass displacement of fuzzy logic controller with three membership functions with band limited white noise input

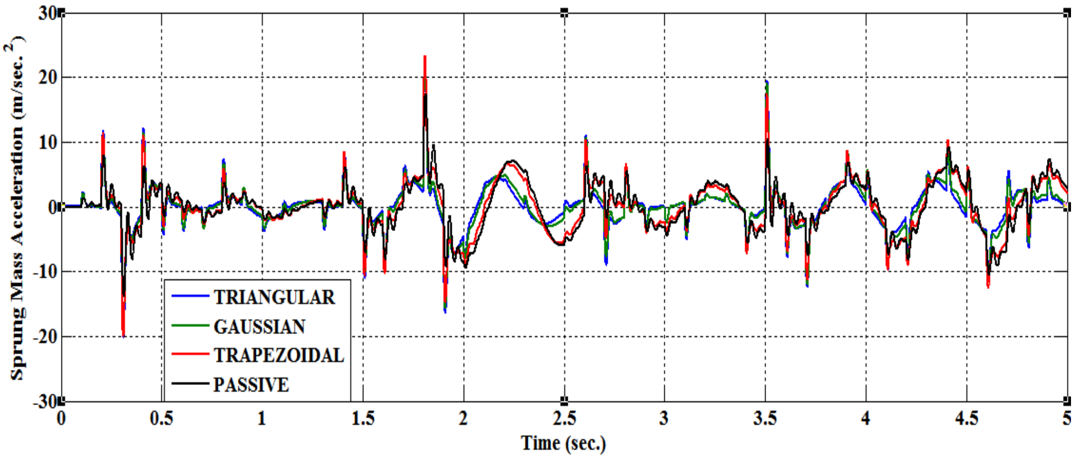

Figure 4.12- Comparison of time response of Sprung mass acceleration of fuzzy logic controller with three membership functions with band limited white noise input

For White noise type input Triangular membership function has shown better results as seen in figures 4.9- 4.12.

So from the above figures(4.1-4.12) it is evident that fuzzy with triangular membership function gives better response compared to other two membership function for sine and random signal but in case of step input, for 2 parameters Gaussian is showing better output and for other 2 parameters triangular is showing better response. So for future comparison purpose with other controllers, fuzzy controller with triangular membership function has been considered in this thesis.

4.1.2 Comparison of Membership Function and Results of Fuzzy Tuned PID Controller for two DOF Quarter Car model

Figure 4.13-4.16 shows the comparison of fuzzy tuned PID controller with three membership function i.e. gaussian, trapezoidal and triangular for step road input disturbance and again it is compared with passive suspension system.

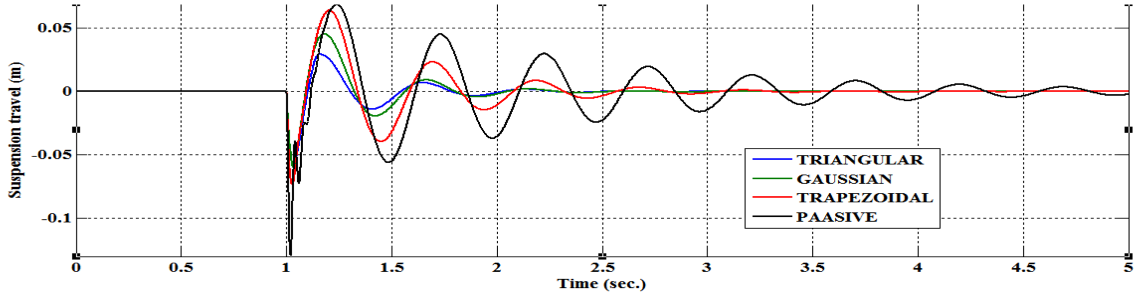

Figure 4.13- Comparison of time response of Suspension travel of fuzzy PID controller with three membership functions with step input

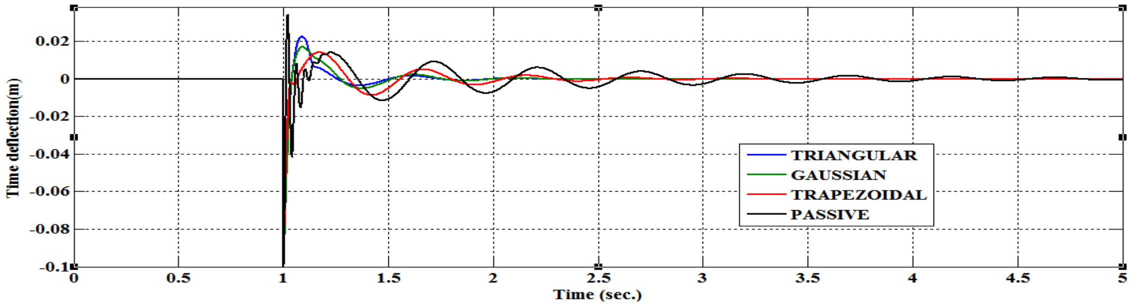

Figure 4.14- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions with step input

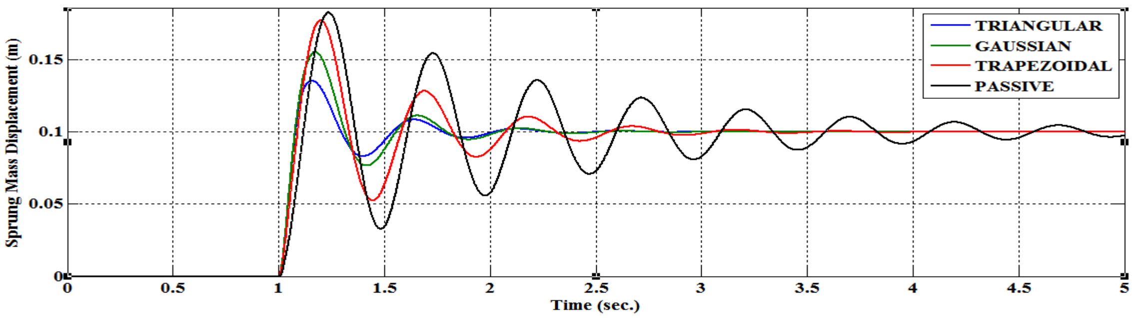

Figure 4.15- Comparison of time response of Sprung mass displacement of fuzzy PID controller with three membership functions with step input

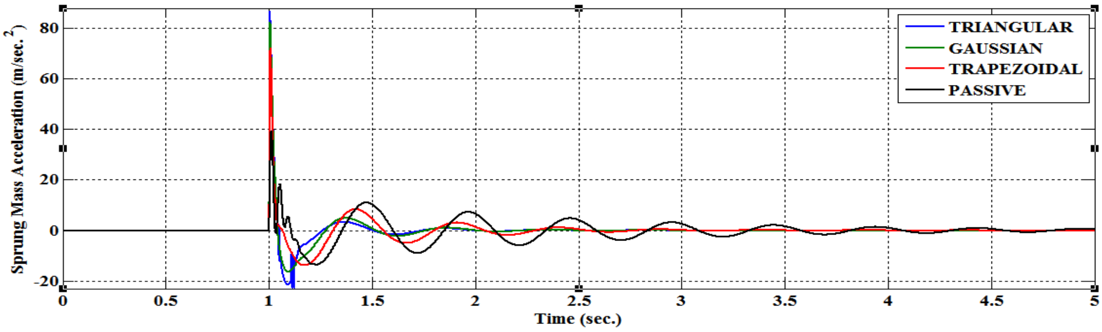

Figure 4.16- Comparison of time response of sprung mass acceleration of fuzzy PID controller with three membership functions with step input

From Figures 4.13-4.16, For step input Gaussian membership function has shown better response as compared to others.

Figure 4.17-4.20 shows the comparison of fuzzy tuned PID controller with three membership function i.e. gaussian, trapezoidal and triangular for sine road input disturbance and again it is compared with passive suspension system.

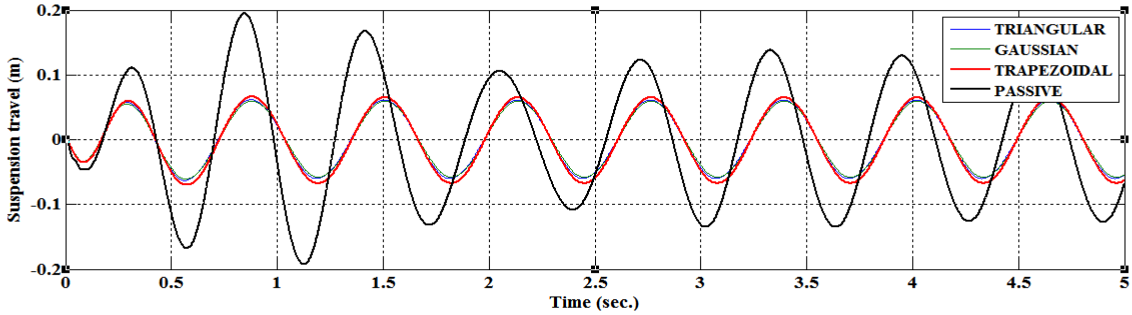

Figure 4.17- Comparison of time response of Suspension travel of fuzzy PID controller with three membership functions with sine input

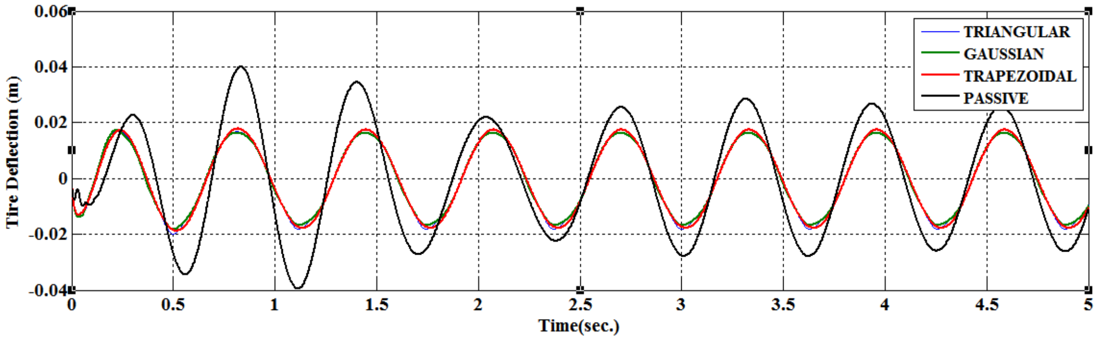

Figure 4.18- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions with sine input

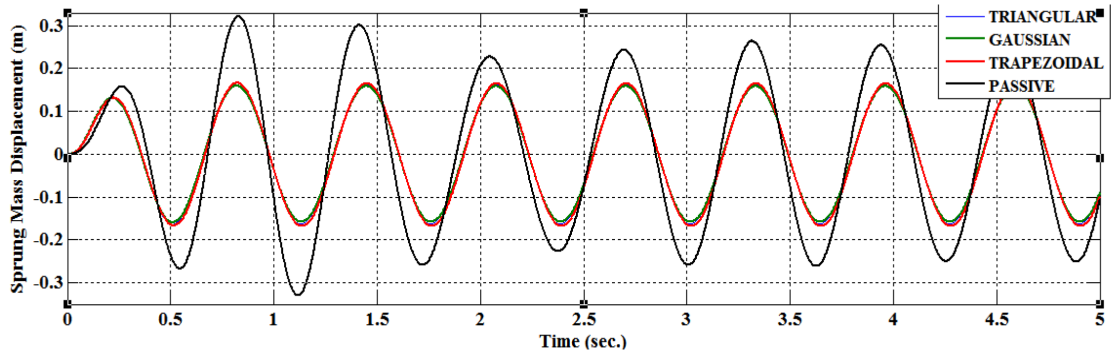

Figure 4.19- Comparison of time response of sprung mass displacement of fuzzy PID controller with three membership functions with sine input

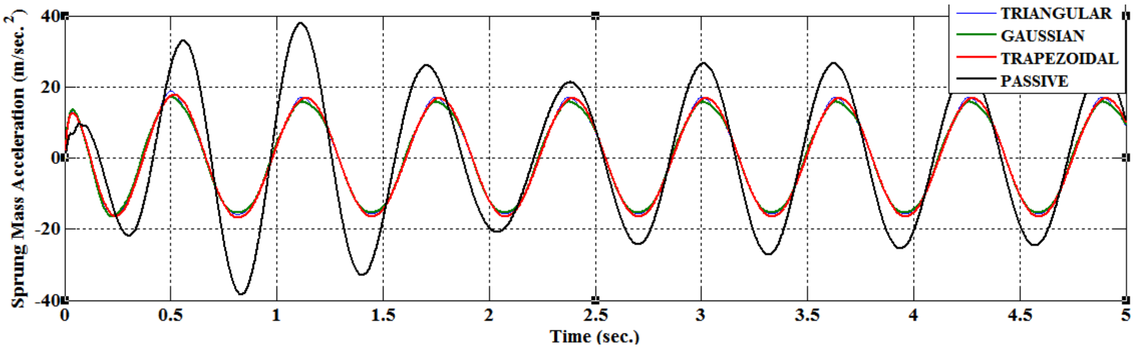

Figure 4.20- Comparison of time response of Sprung mass acceleration of fuzzy PID controller with three membership functions with sine input

As seen from figures 4.16-4.20, the result obtained for sine input is similar for all the membership functions and it is better than the Passive system.

Figure 4.21-4.24 shows the comparison of fuzzy tuned PID controller with three membership function i.e. gaussian, trapezoidal and triangular for band limited white noise input disturbance and again it is compared with passive suspension system.

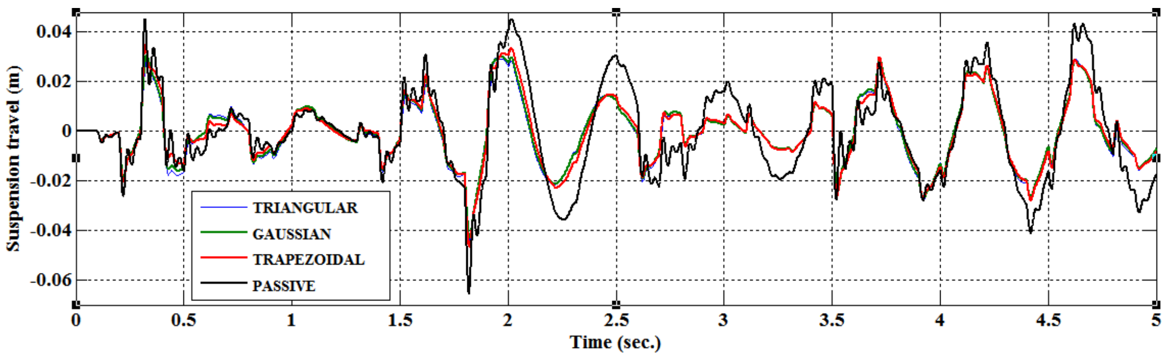

Figure 4.21- Comparison of time response of suspension travel of fuzzy PID controller with three membership functions for band limited white input[28]

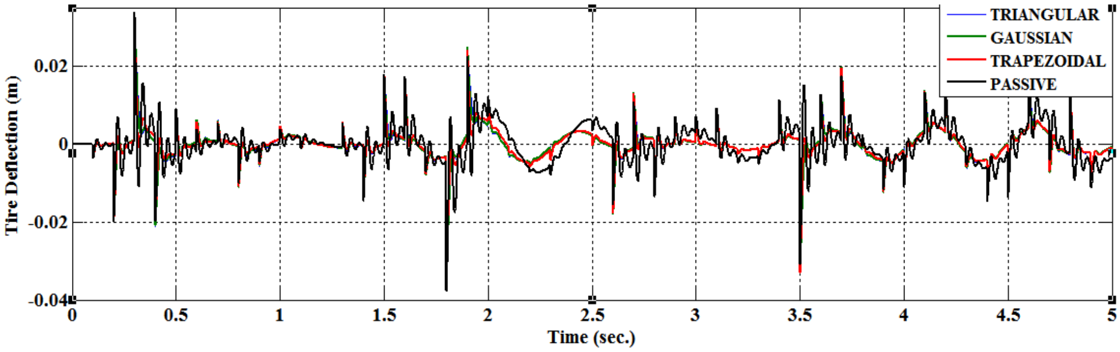

Figure 4.22- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions for band limited white input

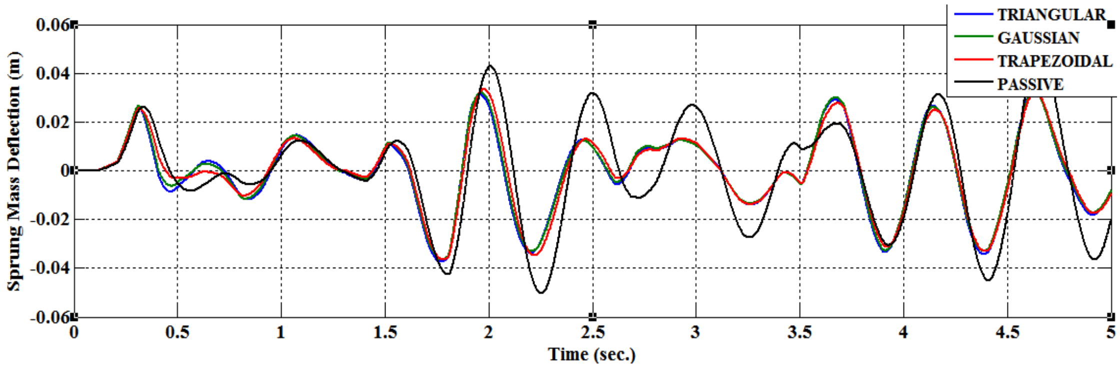

Figure 4.23- Comparison of time response of sprung mass displacement of fuzzy PID controller with three membership functions for band limited white input

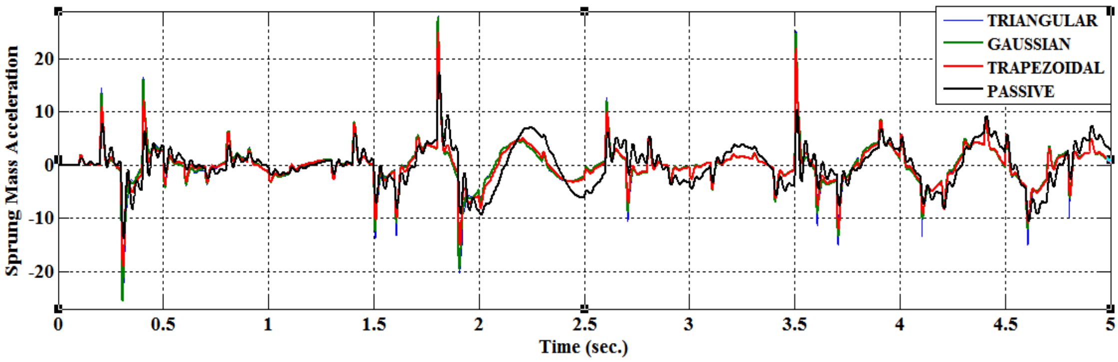

Figure 4.24- Comparison of time response of sprung mass acceleration of fuzzy PID controller with three membership functions for band limited white input

The results obtained for Gaussian membership function are better from other the results obtained for other.

So from the above comparison it is evident that fuzzy tuned PID control based active suspension system gives better response than passive suspension system but fuzzy PID with gaussian membership function gives better response compared to other two membership function for all the three types of road disturbance conditions.

So for future comparison purpose with other controllers, fuzzy PID controller with gaussian membership function has been considered in this thesis

4.1.3 Comparison of all Designed Controllers for Two DOF Quarter Car Model

A) STEP INPUT

1) Suspension Travel

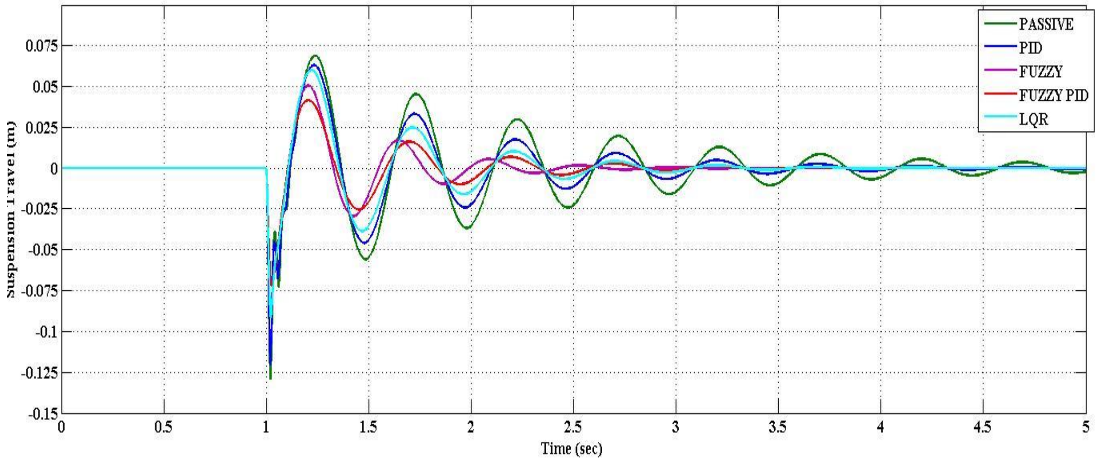

Figure 4.25.Comparison of time response of Suspension Travel for all the controllers on 2 DOF quarter car model with step road input

2) Tire Deflection

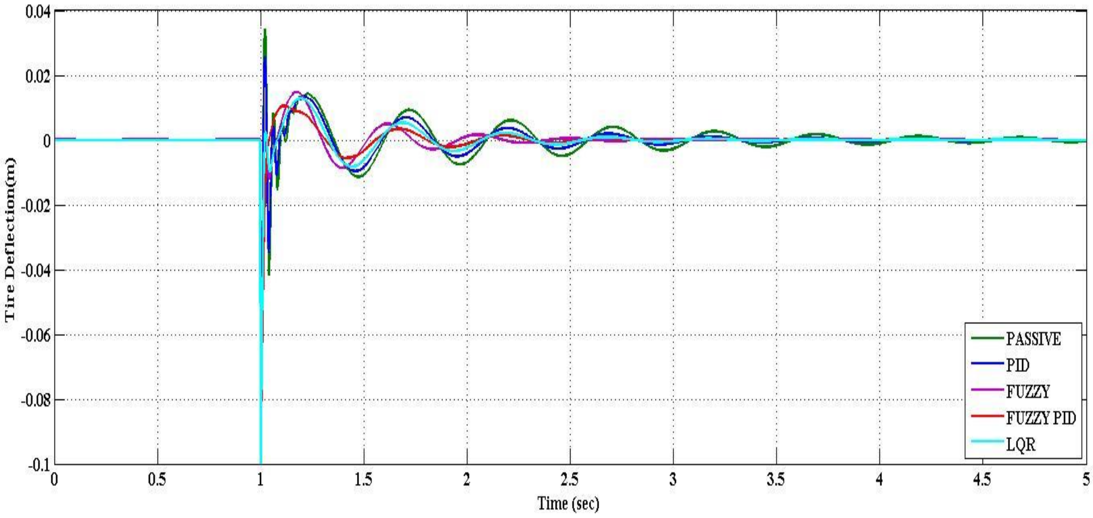

Figure 4.26.Comparison of time response of Tire Deflection for all the controllers on 2 DOF quarter car model with step road input

3) <u>Sprung Mass Displacement</u>

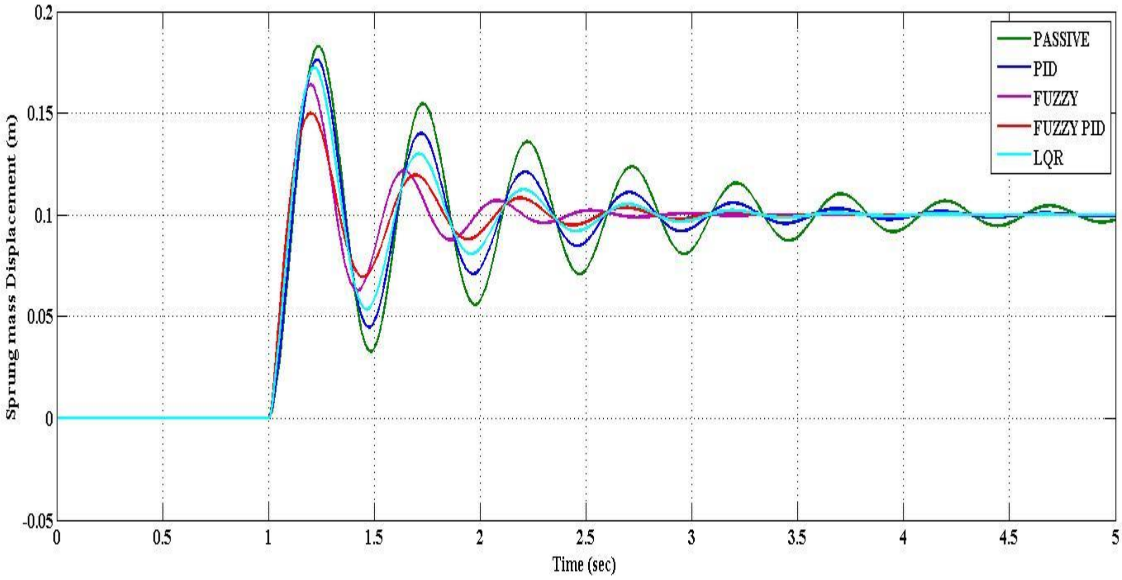

Figure 4.27.Comparison of time response of Sprung Mass Displacement for all the controllers on 2 DOF quarter car model with step road input

4) <u>Sprung Mass Acceleration</u>

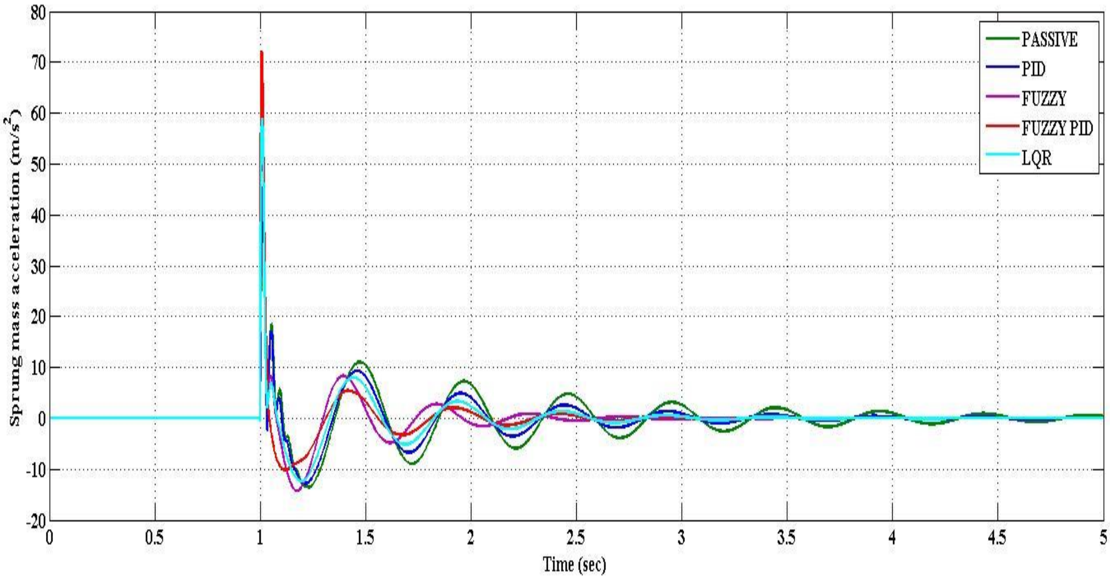

Figure 4.28.Comparison of time response of Sprung Mass Acceleration for all the controllers on 2 DOF quarter car model with step road input

Table 4.1 Calculation of time domain parameters for suspension travel on 2 DOF quarter car model with step road input

Suspension Travel					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	80	71	60	40	100
Settling time, t_s(sec)	5	3.5	3	3.2	10

Table 4.2 Calculation of time domain parameters for Tire deflection on 2 DOF quarter car model with step road input

Tire Deflection					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	71	45	40	25	100
Settling time, t_s(sec)	4	3.5	3	2.5	10

Table 4.3 Calculation of time domain parameters for Sprung Mass Displacement on 2 DOF quarter car model with step road input

Sprung Mass Displacement					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	80	65	55	40	100
Settling time, t_s(sec)	4.5	3.5	3	3.2	10

Table 4.4 Calculation of time domain parameters for Sprung Mass Acceleration on 2 DOF quarter car model with step road input

Sprung Mass Acceleration					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Settling time, t_s(sec)	5	3.5	3	3	10
Peak Overshoot (M_p)(%)	80	71	60	60	100

The peak overshoot reduction for suspension travel is minimum in case of Fuzzy tuned PID which is 60 % whereas in case of Fuzzy, LQR and PID controller the values are 40%, 29 % and 20 % respectively. Peak overshoot reduction in case of tire deflection is 75%, 60%, 55% and 29% respectively with fuzzy PID, fuzzy, LQR and PID controller. In case of sprung mass displacement the % improvement in peak overshoot are 60%, 45%.35% and 20% respectively. In case of sprung mass acceleration % improvement in peak overshoot are 40%, 40%, 29% and 20% respectively. Settling time is also considerable reduced. In case of 2 DOF model for step input all the output parameters showing better result in case of settling time reduction and peak overshoot reduction. Fuzzy PID gives the best result followed by fuzzy controller in case step input for 2 DOF model compared to LQR and auto tuned PID controller.

B) SINE INPUT

1) Suspension travel

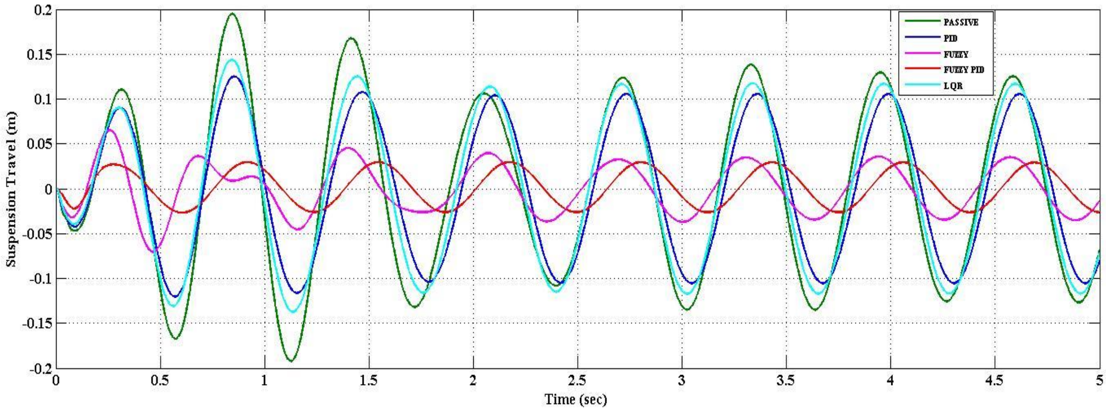

Figure 4.29.Comparison of time response of Suspension Travel for all the controllers on 2 DOF quarter car model with sine road input

2) Tire Deflection

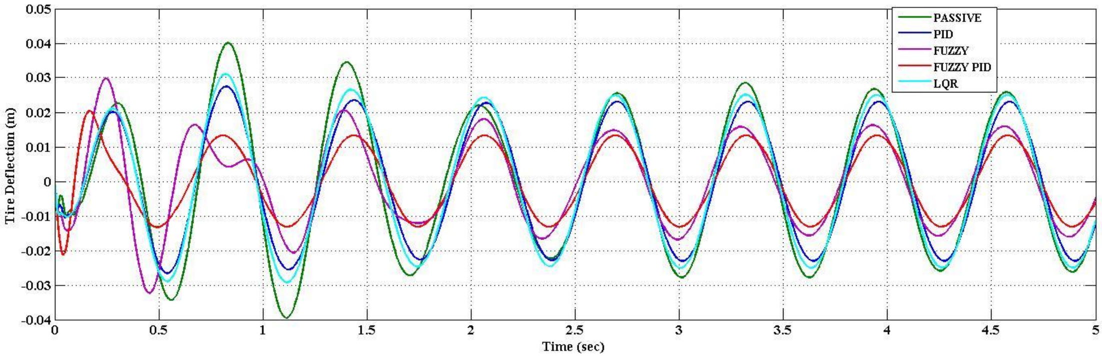

Figure 4.30.Comparison of time response of Tire Deflection for all the controllers on 2 DOF quarter car model with sine road input

3) **Sprung Mass Displacement**

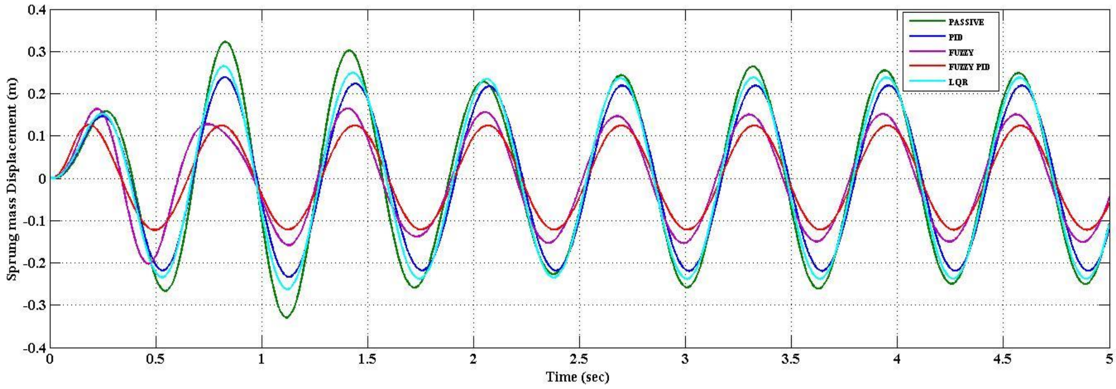

Figure 4.31.Comparison of time response of Sprung Mass Displacement for all the controllers on 2 DOF quarter car model with sine road input

4) **Sprung Mass Acceleration**

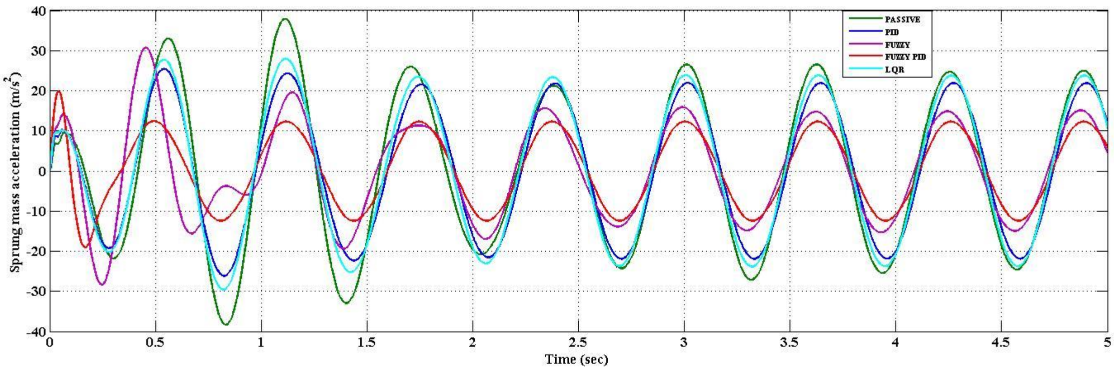

Figure 4.32.Comparison of time response of Sprung Mass Acceleration for all the controllers on 2 DOF quarter car model with sine road input

Table 4.5. Calculation of time domain parameters for suspension travel on 2 DOF quarter car model with sine road input

Suspension Travel					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.125	0.144	0.065	0.03	0.195
Max. Negative Amplitude(m)	-0.121	-0.137	-0.072	-0.026	-0.193
Min. Positive Amplitude(m)	0.12	0.1	0.04	0.027	0.105
Min. Negative Amplitude(m)	-0.12	-0.1	-0.035	-0.026	-0.105

Table 4.6 Calculation of time domain parameters for Tire deflection on 2 DOF quarter car model with sine road input

Tire Deflection					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.028	0.032	0.03	0.02	0.04
Max. Negative Amplitude(m)	-0.028	-0.032	-0.03	-0.02	-0.04
Min. Positive Amplitude(m)	0.024	0.025	0.015	0.014	0.022
Min. Negative Amplitude(m)	-0.024	-0.025	-0.015	-0.014	-0.022

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Table 4.7 Calculation of time domain parameters for Sprung Mass Displacement on 2 DOF quarter car model with sine road input

Sprung Mass Displacement					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.26	0.27	0.17	0.12	0.33
Max. Negative Amplitude(m)	-0.26	-0.25	-0.2	-0.12	-0.33
Min. Positive Amplitude(m)	0.22	0.24	0.25	0.12	0.25
Min. Negative Amplitude(m)	-0.22	-0.24	-0.25	-0.12	-0.25

Table 4.8 Calculation of time domain parameters for Sprung Mass Acceleration on 2 DOF quarter car model with sine road input

Sprung Mass Acceleration					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m/sec.^2)	25	27	30	19	37
Max. Negative Amplitude(m/sec.^2)	-30	30	-28	-18	-37
Min. Positive Amplitude(m/sec.^2)	22	24	15	12	22
Min. Negative Amplitude(m)	-22	-24	-17	-12	-22

In case of sine road input for 2 DOF system, all the controllers have been successfully developed. Amplitude of oscillations has decreased providing a good control response but Fuzzy PID gives the best result. Maximum and minimum values of amplitudes are minimum in case of all the parameters and corresponding oscillations have minimum value in case of Fuzzy PID and Fuzzy.

C) BAND LIMITED WHITE NOISE

1) Suspension Travel

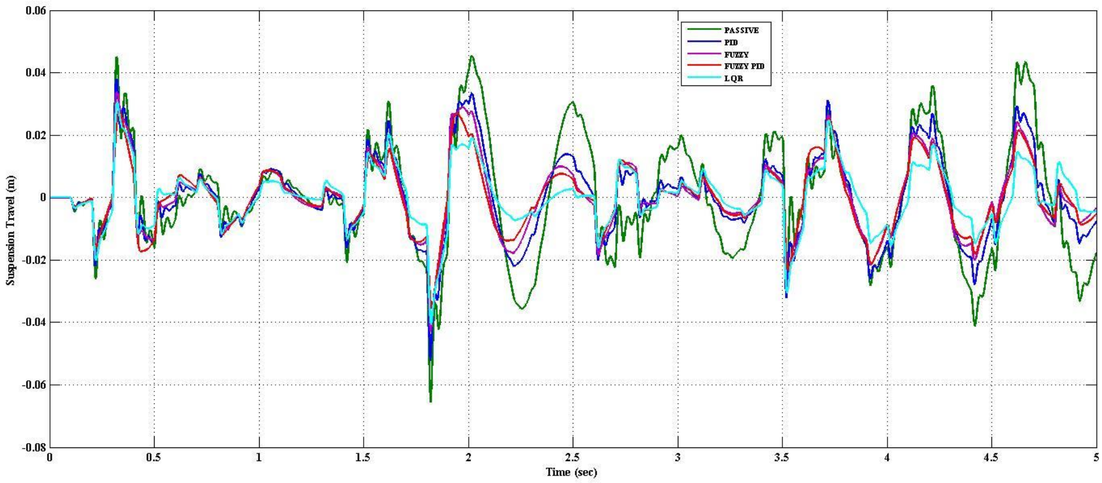

Figure 4.33.Comparison of time response of Suspension Travel for all the controllers on 2 DOF quarter car model with white noise road input

2) Tire Deflection

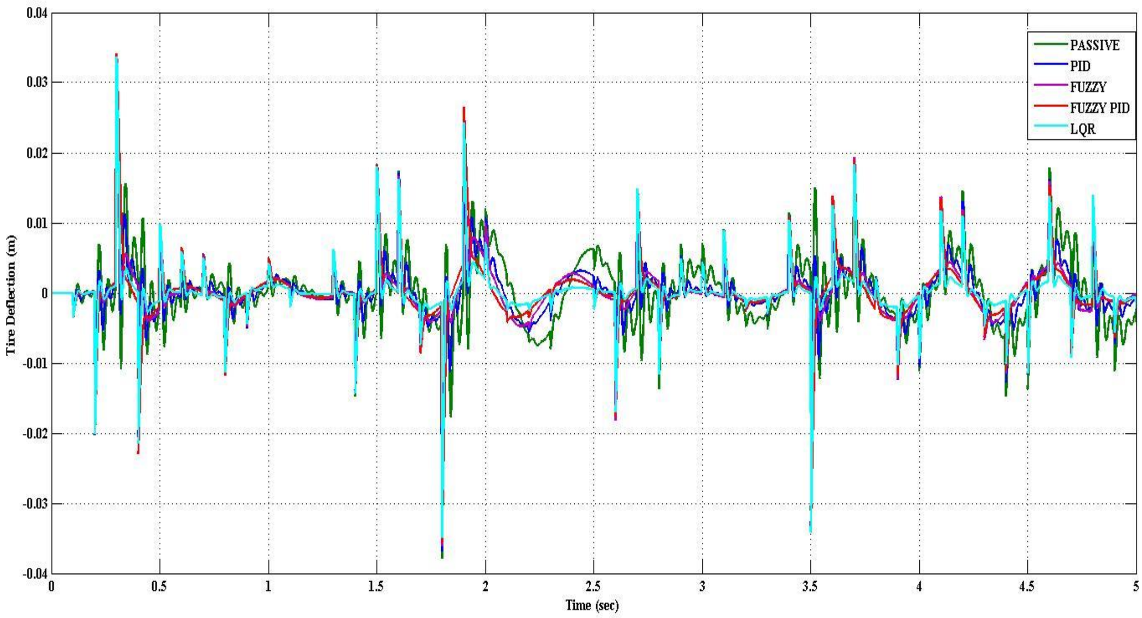

Figure 4.34.Comparison of time response of Tire Deflection for all the controllers on 2 DOF quarter car model with white noise road input

3) **<u>Sprung Mass Displacement</u>**

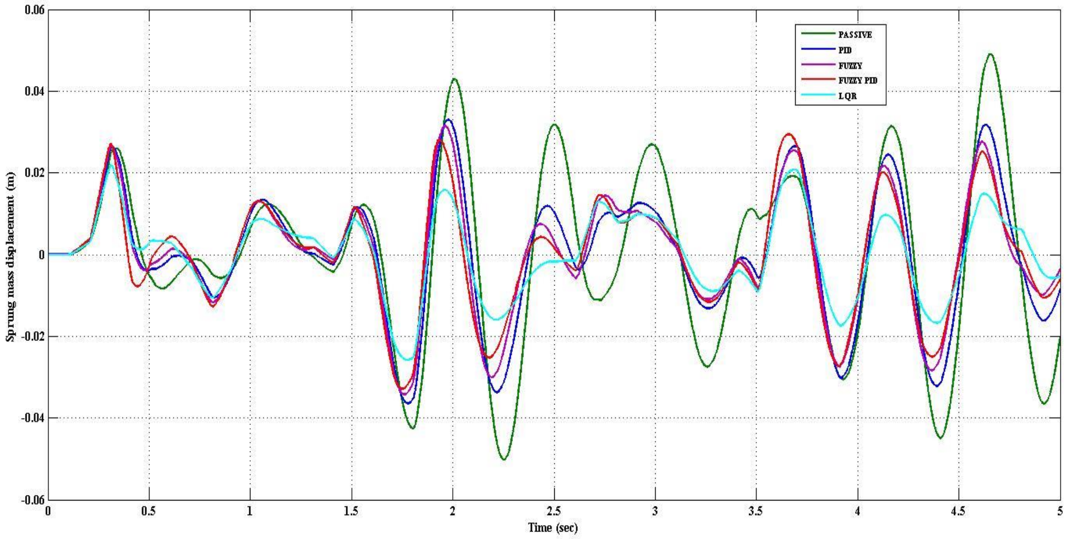

Figure 4.35.Comparison of time response of Sprung mass displacement for all the controllers on 2 DOF quarter car model with white noise road input

4) **<u>Sprung Mass Acceleration</u>**

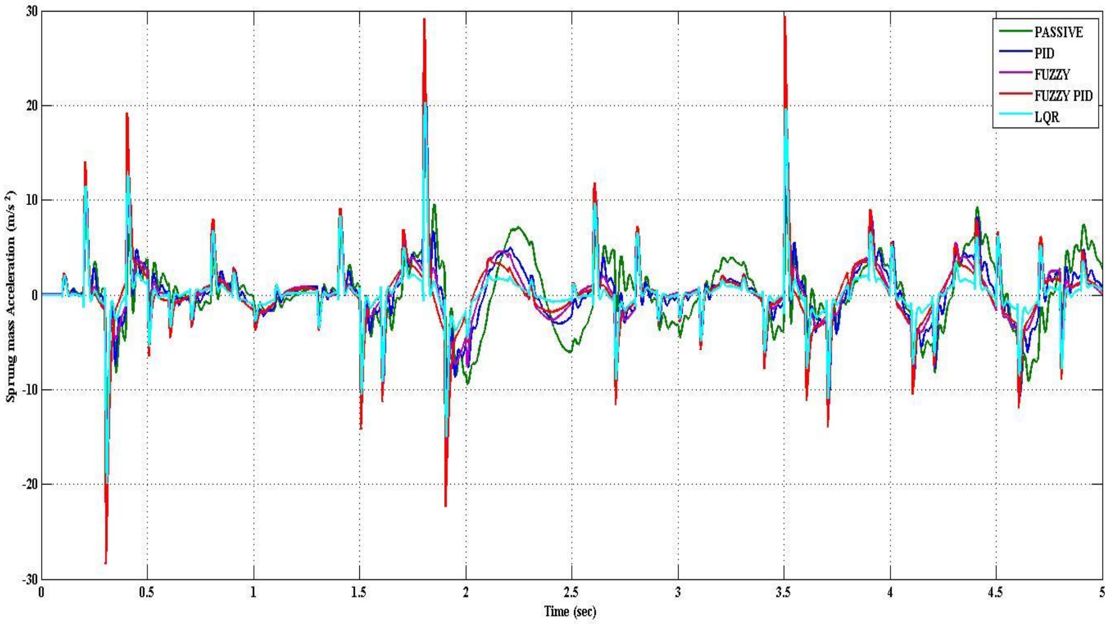

Figure 4.36.Comparison of time response of Sprung mass acceleration for all the controllers on 2 DOF quarter car model with white noise road input

Table 4.9 Calculation of time domain parameters for suspension travel on 2 DOF quarter car model with white noise road input

Suspension Travel					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.0.43	0..03	0.035	0.03	0.048
Max. Negative Amplitude(m)	-0.061	-0.035	-0.035	-0.03	-0.068

Table 4.10 Calculation of time domain parameters for Tire deflection on 2 DOF quarter car model with white road input

Tire Deflection					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.033	0.03	0.033	0.029	0.035
Max. Negative Amplitude(m)	-0.033	-0.032	-0.032	-0.029	-0.038

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Table 4.11 Calculation of time domain parameters for Sprung Mass Displacement on 2 DOF quarter car model with white road input

Sprung Mass Displacement					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.044	0.02	0.025	0.022	0.05
Max. Negative Amplitude(m)	-0.041	-0.02	-0.025	-0.024	-0.048

Table 4.12 Calculation of time domain parameters for Sprung Mass Acceleration on 2 DOF quarter car model with white road input

Sprung Mass Acceleration					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m/sec.^2)	16.1	11.8	15	15	18
Max. Negative Amplitude(m/sec.^2)	-13.2	-12.5	-13	-13	-15

For band limited white noise input, for 2 parameters LQR gives better result but the overall performance is best for Fuzzy and fuzzy PID.

4.2 COMPARISON AND RESULT FOR THREE DEGREE OF FREEDOM QUARTER CAR MODEL

Output parameters selected in three Degree Of Freedom (DOF) system are seat displacement, sprung mass displacement, suspension deflection, sprung mass acceleration and tire deflection. In the next section a comparison with different membership function based Fuzzy and Fuzzy tuned PID controller for 3 DOF model with three input conditions is provided. In the next part a comparison between the performances of four controllers with the passive i.e. uncontrolled system is presented.

4.2.1 Comparison of Membership Functions and Results of Fuzzy Logic Controller for Three DOF Car Model

Parameters selected for comparison purposes in this section are suspension travel, seat acceleration and tire deflection. Figure 4.37-4.39 shows a comparison study of fuzzy logic controller based active suspension system with three membership functions with passive suspension system for step input road excitation.

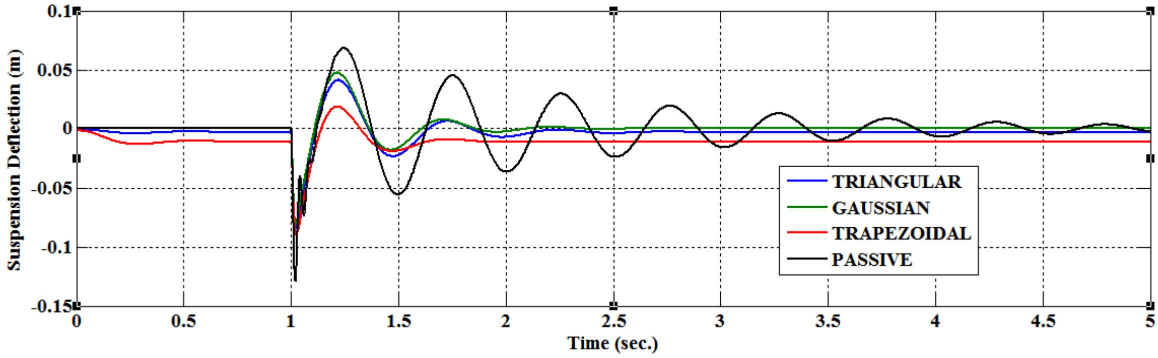

Figure 4.37- Comparison of time response of suspension travel of fuzzy logic controller with three membership functions for step input

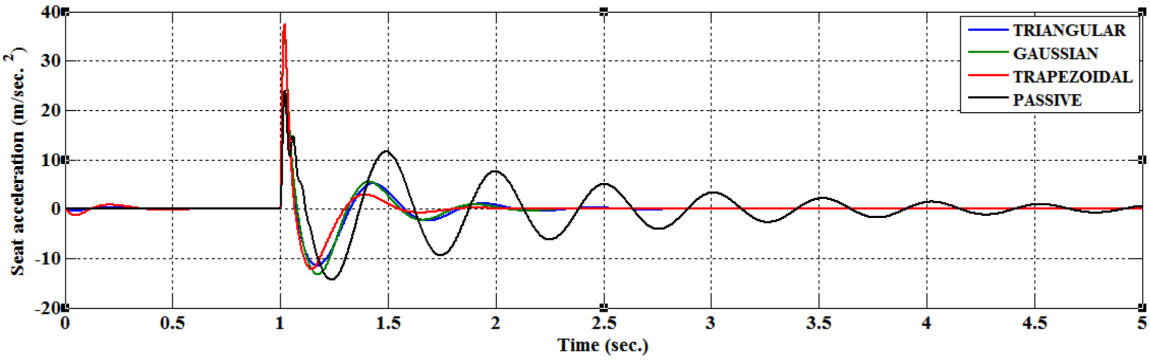

Figure 4.38- Comparison of time response of seat acceleration of fuzzy logic controller with three membership functions for step input

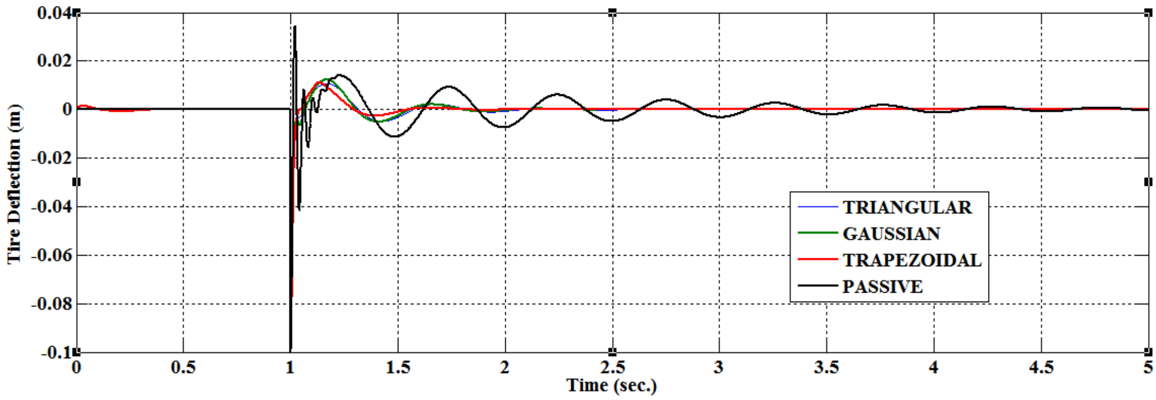

Figure 4.39- Comparison of time response of tire deflection of fuzzy logic controller with three membership functions for step input

From above figures 4.37 – 4.39, it is observed that the Trapezoidal membership function has given better results for all the three parameters namely Suspension travel, seat acceleration and Tire deflection .

Figure 4.40-4.42 shows a comparison study of fuzzy logic controller based active suspension system with three membership functions with passive suspension system for sine input road excitation.

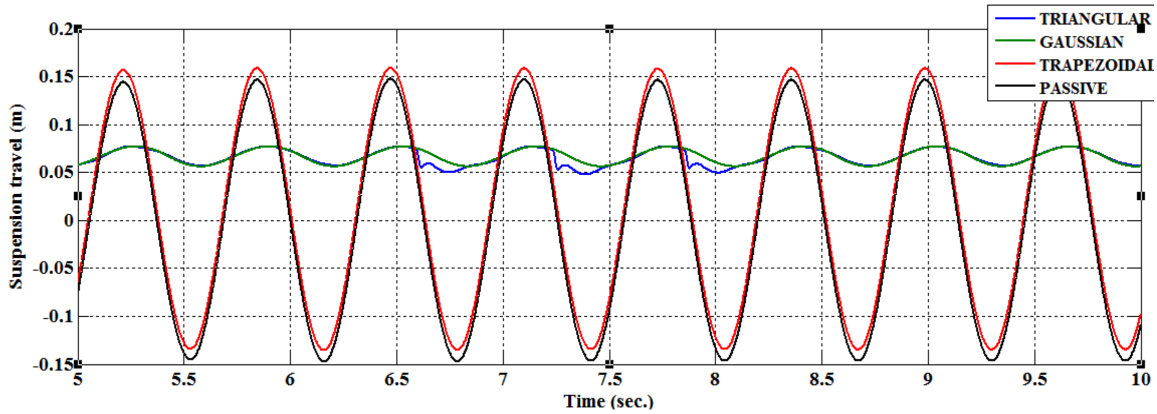

Figure 4.40- Comparison of time response of suspension travel of fuzzy logic controller with three membership functions with sine input excitation

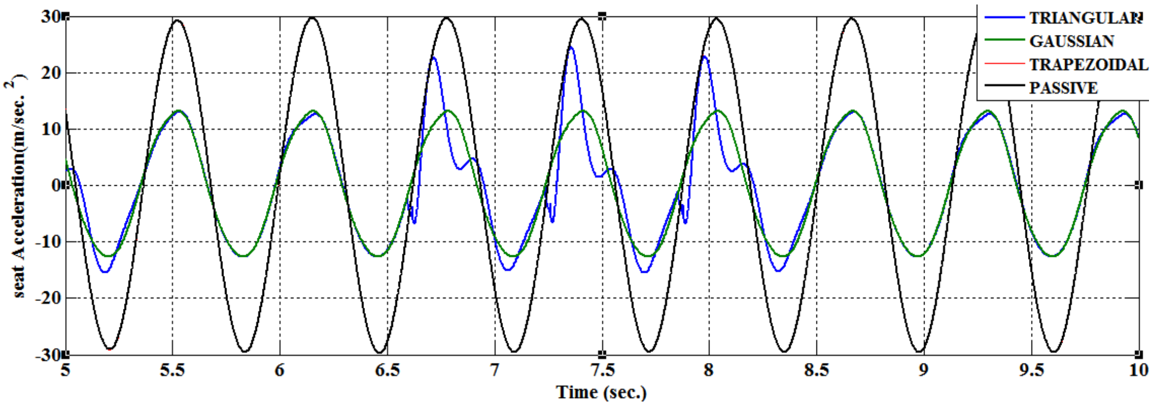

Figure 4.41- Comparison of time response of seat acceleration of fuzzy logic controller with three membership functions with sine input excitation

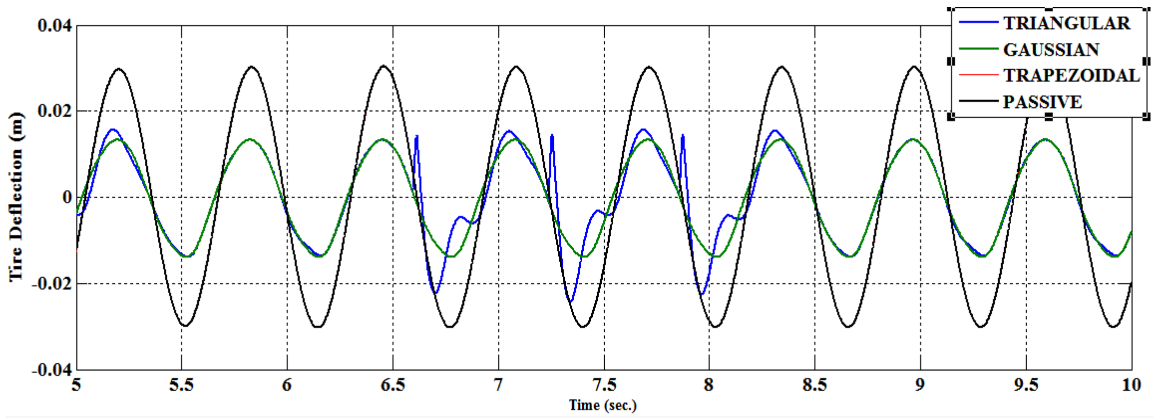

Figure 4.42- Comparison of time response of tire deflection of fuzzy logic controller with three membership functions with sine input excitation

From above figures 4.39 – 4.42, it is observed that the Gaussian membership function has given better results for all the three parameters namely Suspension travel, seat acceleration and Tire deflection .

Figure 4.43-4.45 shows a comparison study of fuzzy logic controller based active suspension system with three membership functions with passive suspension system for band limited white noise input road excitation.

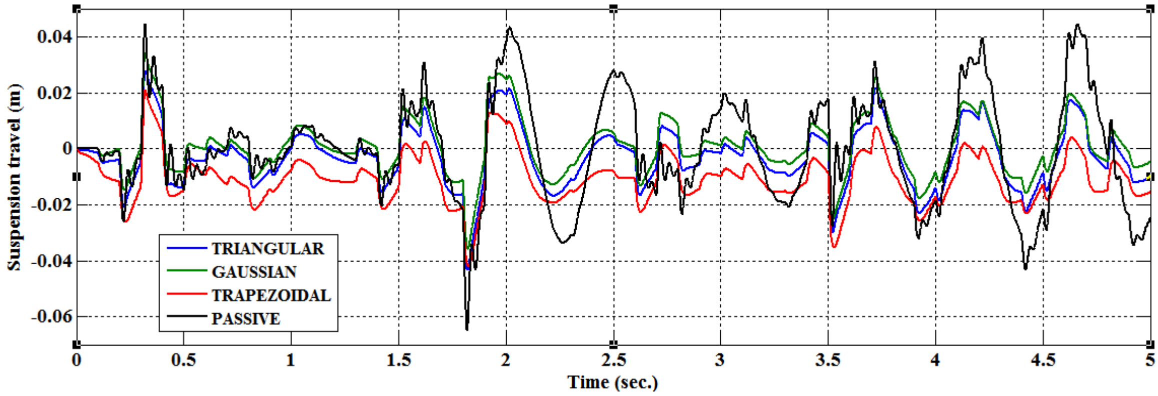

Figure 4.43- Comparison of time response of suspension travel of fuzzy logic controller with three membership functions with band limited white noise input excitation

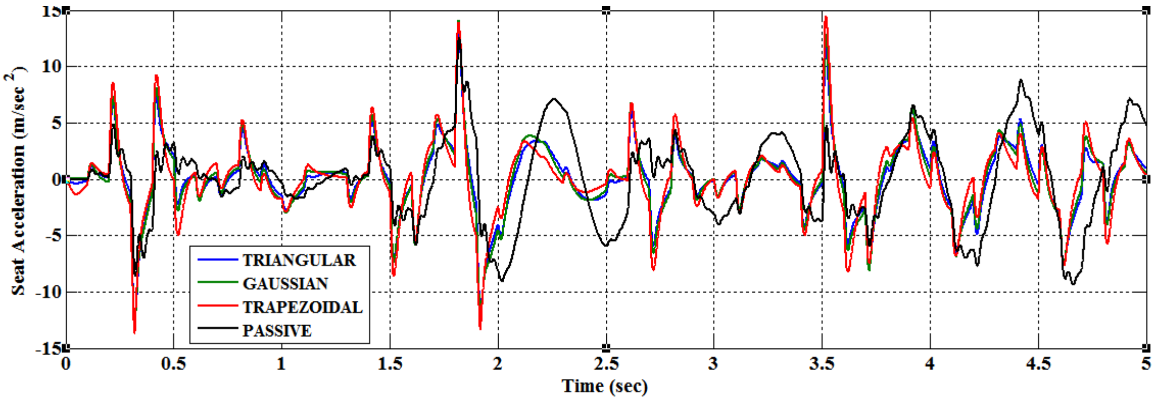

Figure 4.44- Comparison of time response of seat acceleration of fuzzy logic controller with three membership functions with band limited white noise input excitation

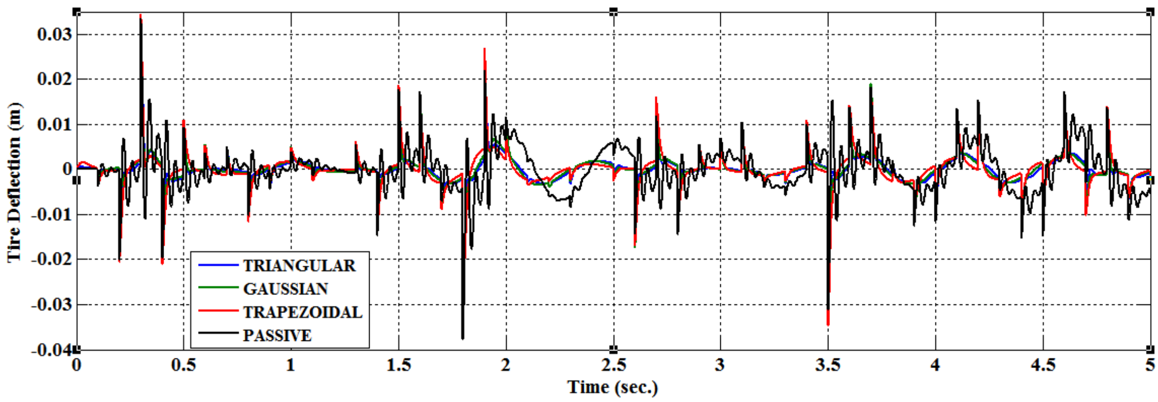

Figure 4.45- Comparison of time response of tire deflection of fuzzy logic controller with three membership functions with band limited white noise input excitation

From the above figures it is evident that for random signal Gaussian is showing better output.

4.2.2 Comparison of Membership Functions and Results of Fuzzy Tuned PID Controller for Three DOF Quarter car model

Figure 4.46-4.48 shows a comparison study of fuzzy tuned PID controller based active suspension system with three membership functions with passive suspension system for step input road excitation.

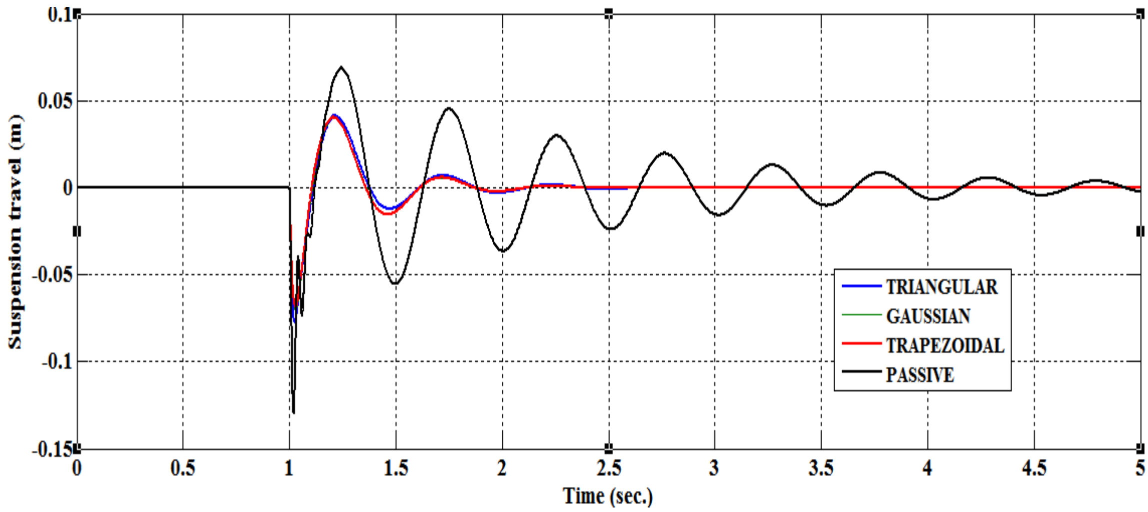

Figure 4.46- Comparison of time response of suspension travel of fuzzy PID controller with three membership functions with step input excitation

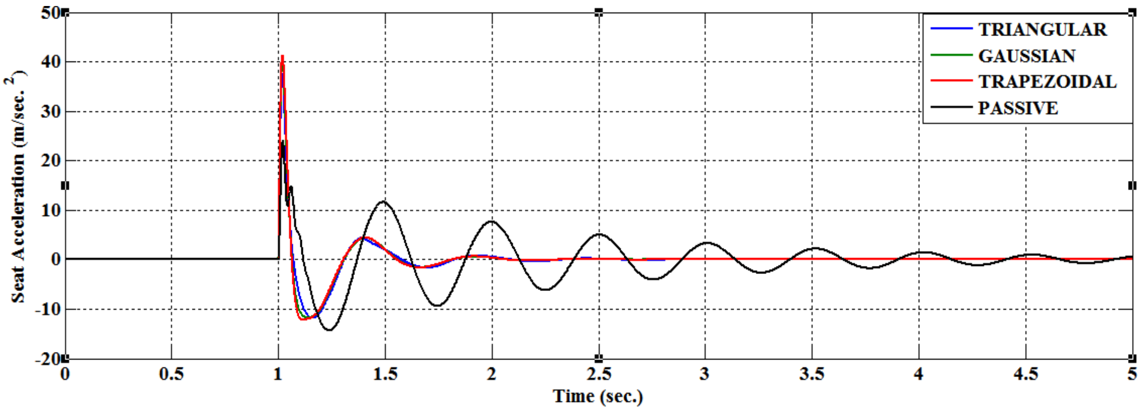

Figure 4.47- Comparison of time response of seat acceleration of fuzzy PID controller with three membership functions with step input excitation

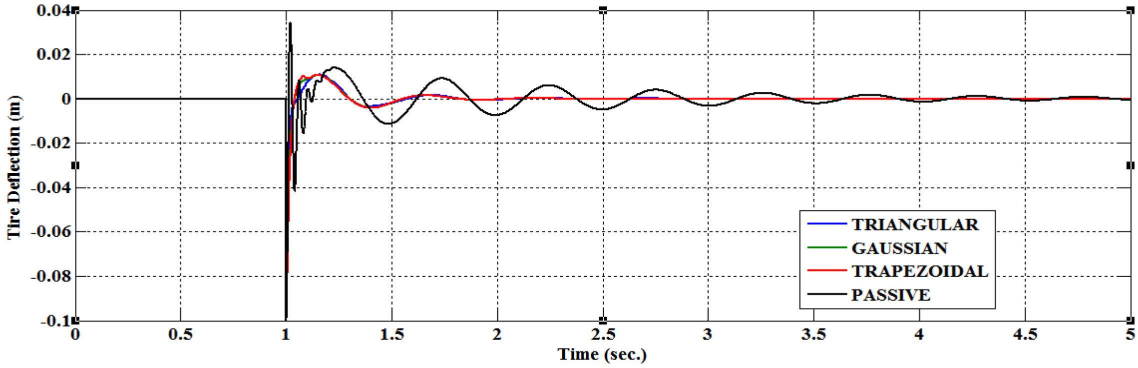

Figure 4.48- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions with step input excitation

From above figures 4.45 – 4.48, it is observed that the Trapezoidal as well as triangular membership function has given better results for all the three parameters namely Suspension travel, seat acceleration and Tire deflection .

Figure 4.49-4.51 shows a comparison study of fuzzy tuned PID controller based active suspension system with three membership functions with passive suspension system for sine input road excitation.

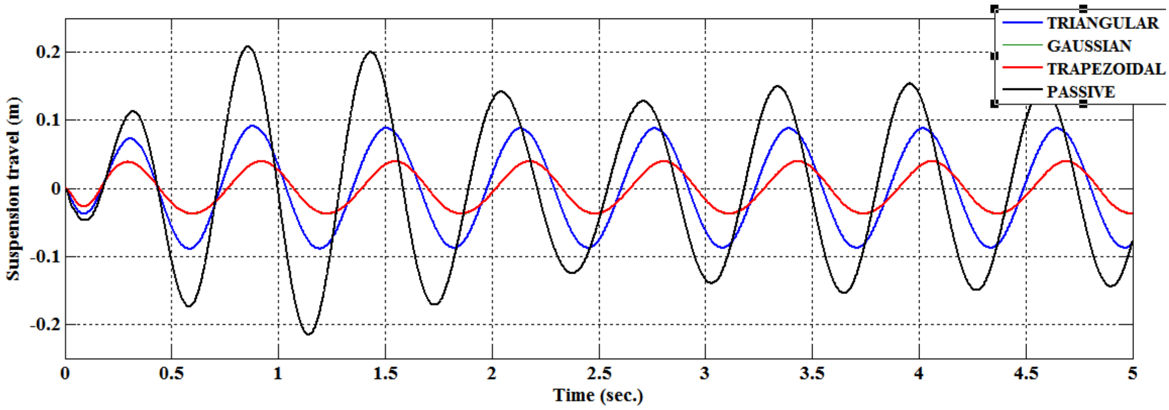

Figure 4.49- Comparison of time response of suspension travel of fuzzy PID controller with three membership functions with sine input excitation

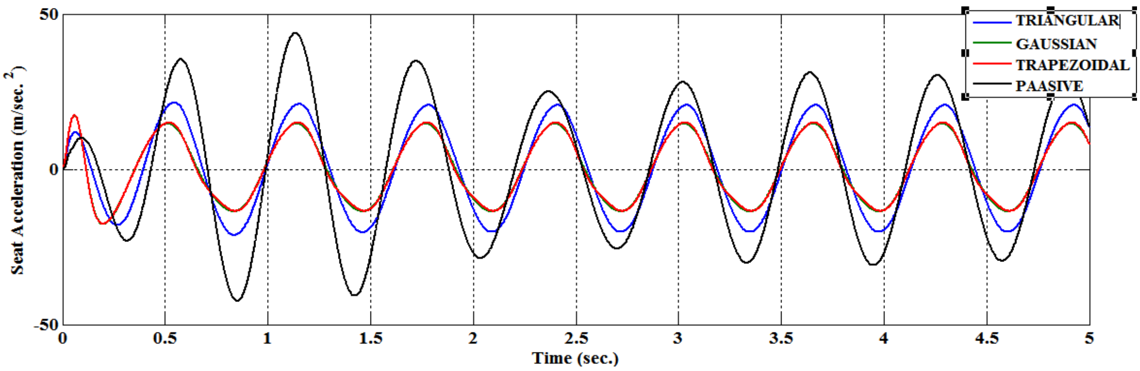

Figure4.50- Comparison of time response of seat acceleration of fuzzy PID controller with three membership functions with sine input excitation

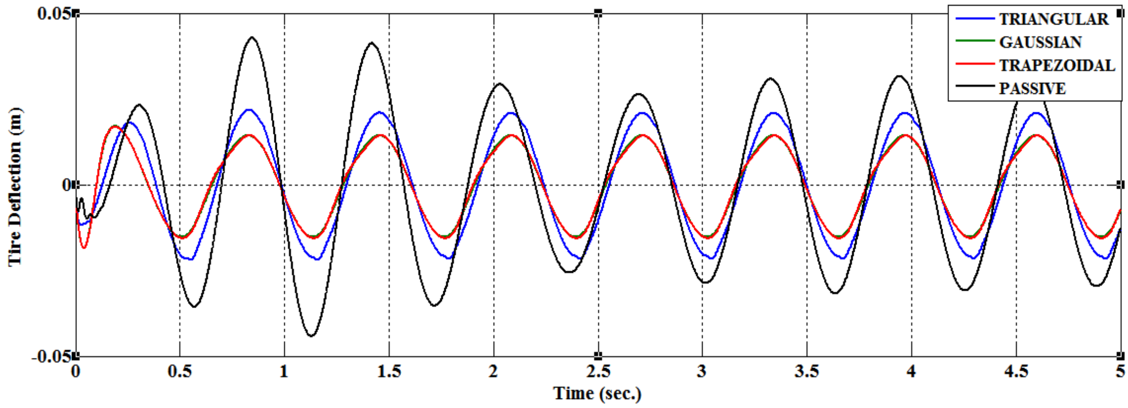

Figure 4.51- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions under sine input excitation

From above figures 4.49 – 4.51, it is observed that the Gaussian as well asTrapezoidal membership function has given better results for all the three parameters namely Suspension travel, seat acceleration and Tire deflection .But Gaussian is tending more towards zero as compared to trapezoidal MF.

Figure 4.52-4.54 shows a comparison study of fuzzy tuned PID controller based active suspension system with three membership functions with passive suspension system for band limited white noise input road excitation.

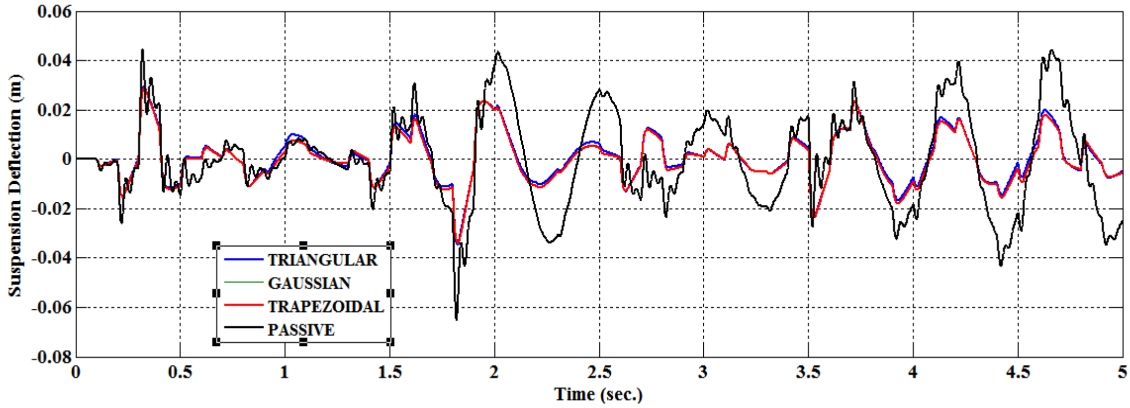

Figure 4.52- Comparison of time response of suspension travel or suspension deflection of fuzzy PID controller with three membership functions with band limited white noise

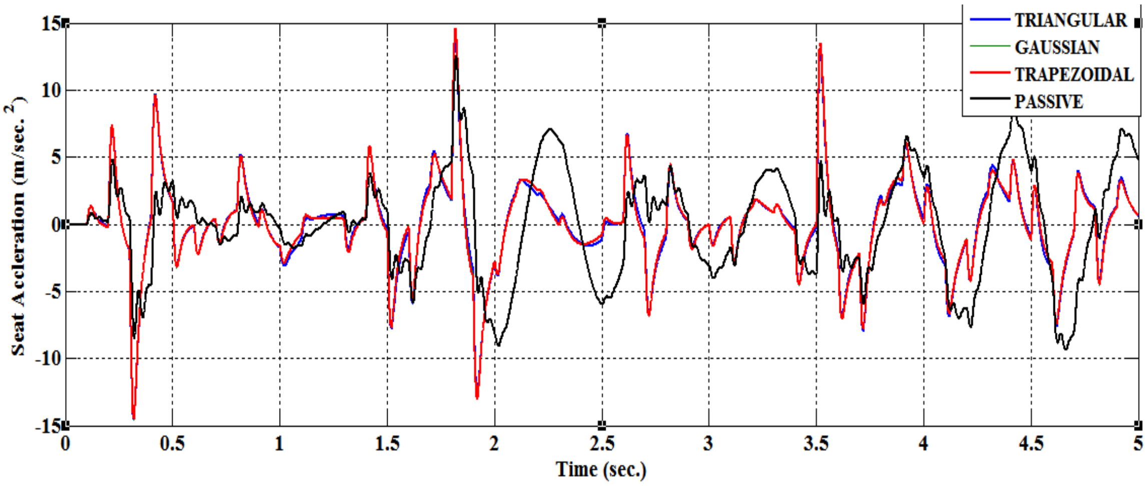

Figure 4.53- Comparison of time response of seat acceleration of fuzzy PID controller with three membership functions with band limited white noise

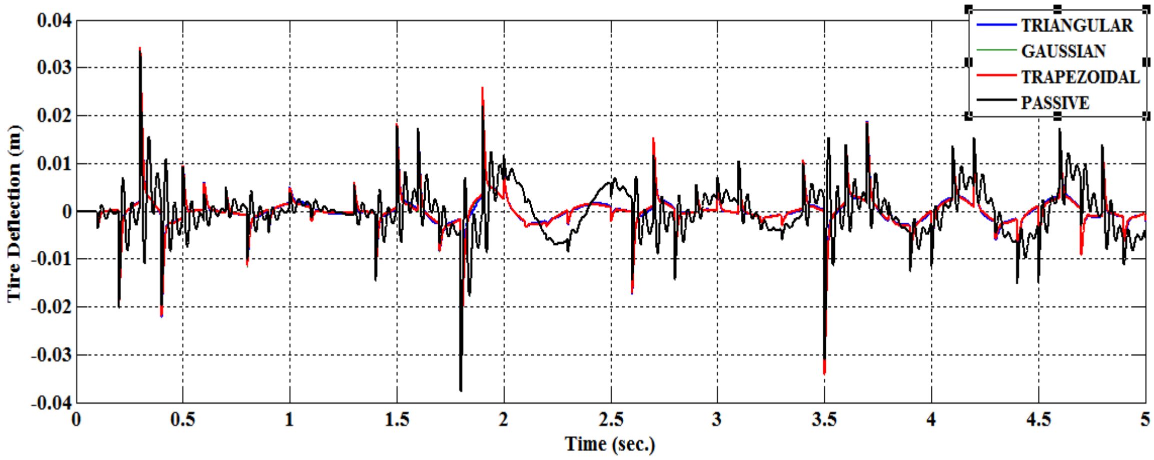

Figure 4.54- Comparison of time response of tire deflection of fuzzy PID controller with three membership functions with band limited white noise

From the above figures it is evident for step input, fuzzy with gaussian membership function gives better response compared to other two membership function and for sine and random signal, Gaussian and trapezoidal is showing better output. So for further comparison and study with other controllers, gaussian membership function has been selected in this work for design of Fuzzy tuned PID controller with all the three types of input.

4.2.3 Comparison of all the designed Controllers for 3 DOF Quarter Model

A) <u>STEP INPUT</u>

1) <u>Suspension Travel</u>

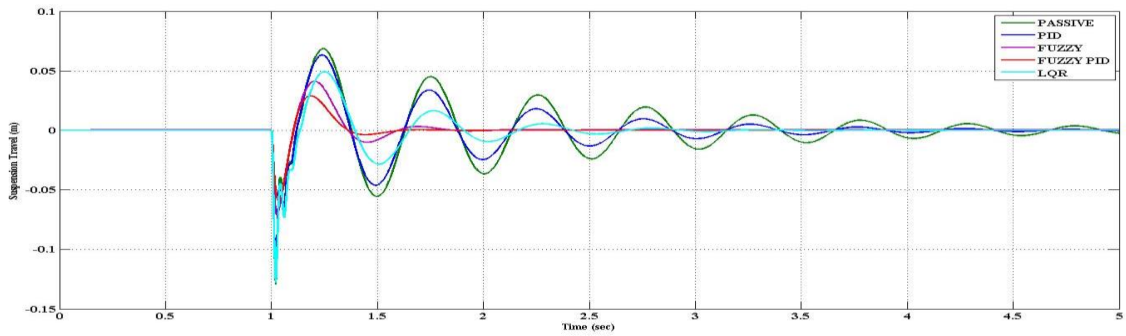

Figure 4.55 Comparison of time response of Suspension Travel for all the controllers on 3 DOF quarter car model with step road input

2) <u>Tire Deflection</u>

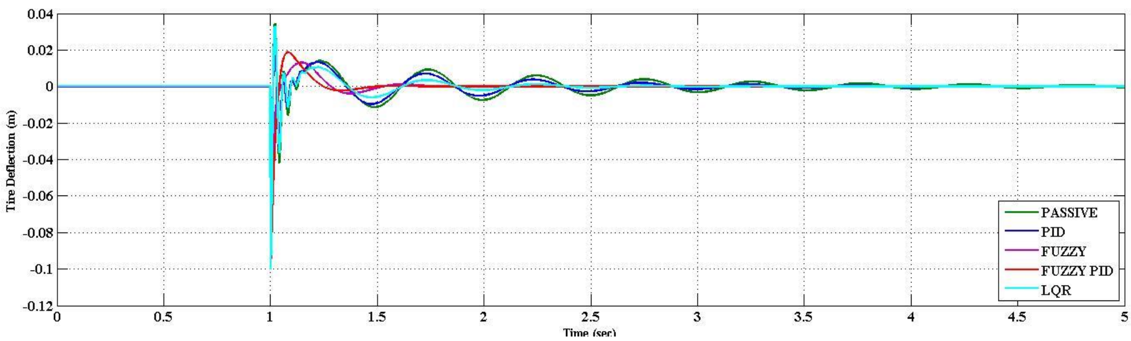

Figure 4.56 Comparison of time response of Tire Deflection for all the controllers on 3 DOF quarter car model with step road input

3) <u>Sprung Mass Displacement</u>

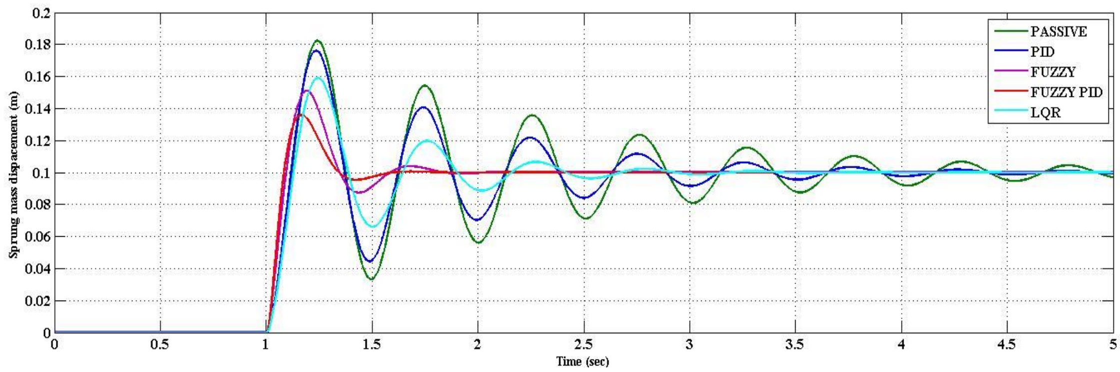

Figure 4.57 Comparison of time response of Sprung mass Displacement for all the controllers on 3 DOF quarter car model with step road input

4) Seat Displacement

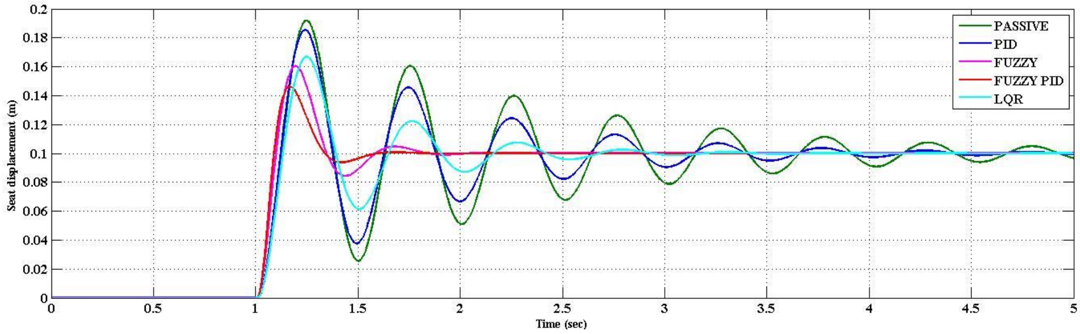

Figure 4.58 Comparison of time response of Seat Displacement for all the controllers on 3 DOF quarter car model with step road input

5) Seat Acceleration

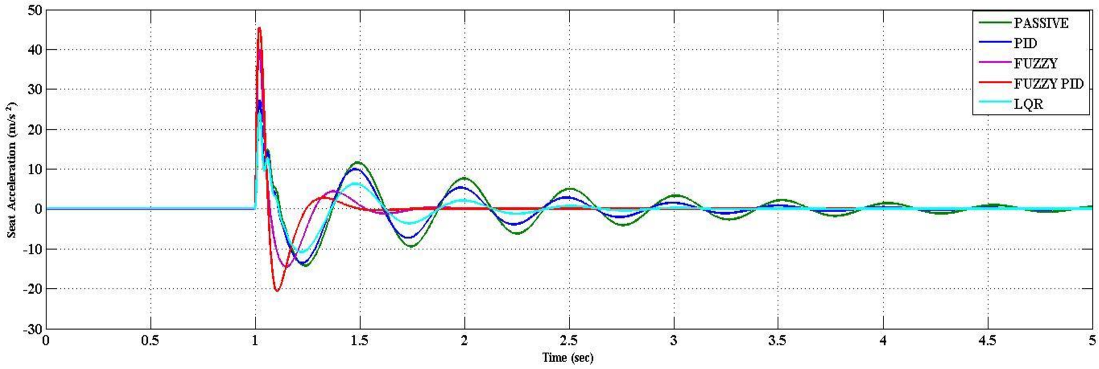

Figure 4.59 Comparison of time response of Seat Acceleration for all the controllers on 3 DOF quarter car model with step road input

Table 4.13 Calculation of time domain parameters for suspension travel on 3 DOF quarter car model with step road input

Suspension Travel					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	90	70	55	35	100
Settling time, t_s(sec)	4	2.8	1.8	1.5	10

Table 4.14 Calculation of time domain parameters for Tire deflection on 3 DOF quarter car model with step road input

Tire Deflection					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	90	70	38	50	100
Settling time, t_s(sec)	4	3	2	1.5	10

Table 4.15 Calculation of time domain parameters for Sprung Mass Displacement on 3 DOF quarter car model with step road input

Sprung Mass Displacement					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	70	60	50	40	80
Settling time, t_s(sec)	5	3.5	2	1.6	10

Table 4.16 Calculation of time domain parameters for Seat Acceleration 3 DOF quarter car model with step road input

Seat Acceleration					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Settling time, t_s(sec)	5	3.2	2.2	1.5	10
Peak Overshoot (M_p)(%)	80	80	80	90	100

Table 4.17 Calculation of time domain parameters for Seat Displacement 3 DOF quarter car model with step road input

Seat Displacement					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Peak Overshoot (M_p)(%)	72	58	41	41	80
Settling time, t_s(sec)	5	3.5	2.2	1.7	10

For step input all the controllers has provided best controlling action except in case of reduction of peak value seat acceleration i.e. in improving ride comfort. The peak overshoot reduction for suspension travel is minimum in case of Fuzzy tuned PID which is 65% whereas in case of Fuzzy, LQR and PID controller the values are 45%, 30% and 10% respectively. Peak overshoot reduction in case of tire deflection is 50%, 62%, 30% and 10% respectively with fuzzy PID, fuzzy, LQR and PID controller. In case of sprung mass displacement the % improvement in peak overshoot are 60%, 50%.40% and 30% respectively. In case of seat acceleration % improvement in peak overshoot are 10%, 20%, 20% and 20% respectively. The % improvements in seat displacement are 59%, 59%, 42% and 28%. Settling time is also considerable reduced. In case of 3 DOF model for step input all the output parameters showing better result in case of settling time reduction and peak overshoot reduction. Fuzzy PID gives the best result followed by fuzzy controller in case step input for 2 DOF model compared to LQR and auto tuned PID controller

B) SINE INPUT

1) Suspension Travel

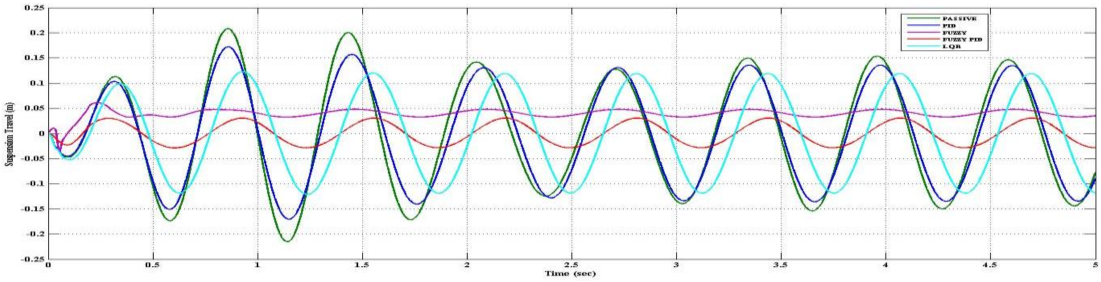

Figure 4.60 Comparison of time response of Suspension Travel for all the controllers on 3 DOF quarter car model with sine road input

2) Tire Deflection

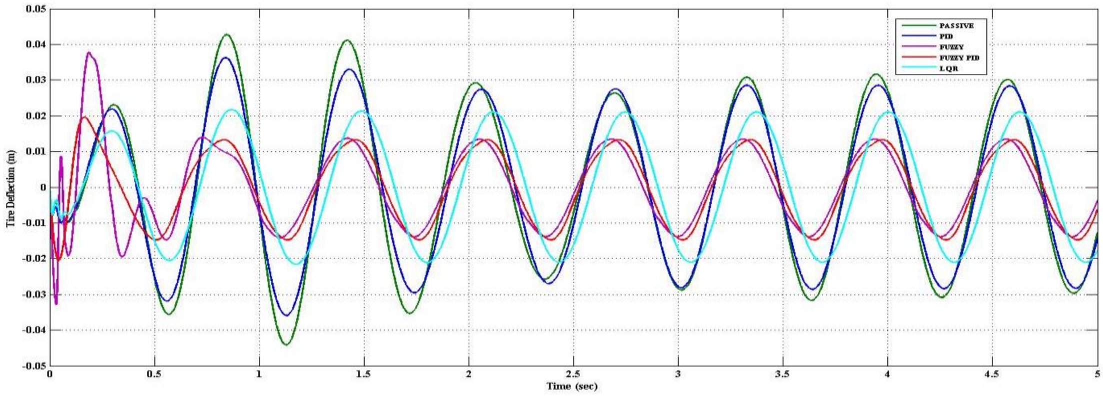

Figure 4.61 Comparison of time response of Tire Deflection for all the controllers on 3 DOF quarter car model with sine road input

3) Sprung Mass Displacement

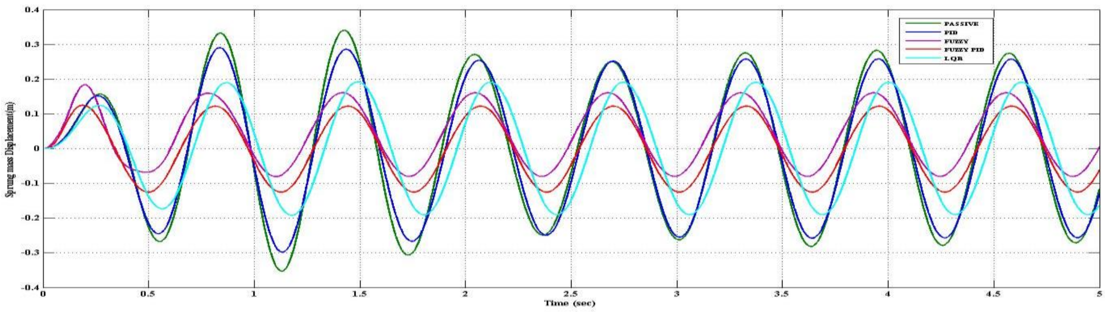

Figure 4.62 Comparison of time response of Sprung Mass Displacement for all the controllers on 3 DOF quarter car model with sine road input

4) Seat Displacement

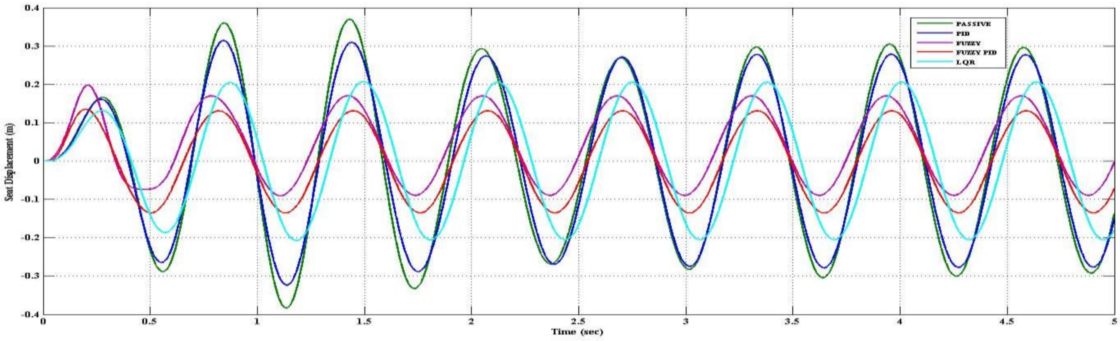

Figure 4.63 Comparison of time response of Seat Displacement for all the controllers on 3 DOF quarter car model with sine road input

5) Seat Acceleration

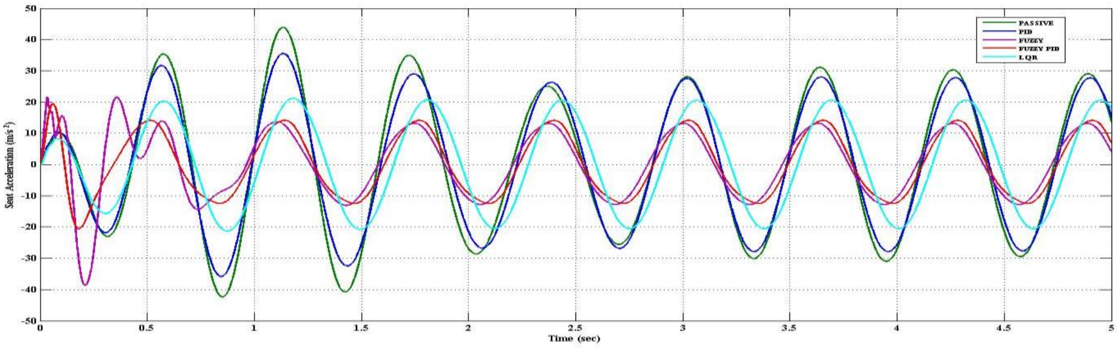

Figure 4.64 Comparison of time response of Seat Acceleration for all the controllers on 3 DOF quarter car model with sine road input

Table 4.18 Calculation of time domain parameters for suspension travel on 3 DOF quarter car model with sine road input

Suspension Travel					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Max. Positive Amplitude(m)	0.17	0.14	0.06	0.02	0.2
Max. Negative Amplitude(m)	-0.18	-0.13	-0.03	-0.02	-0.23
Min. Positive Amplitude(m)	0.12	0.1	0.04	0.018	0.15
Min. Negative Amplitude(m)	-0.12	-0.12	0.02	-0.018	-0.15

Table 4.19 Calculation of time domain parameters for Tire deflection on 3 DOF quarter car model with sine road input

Tire Deflection					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.036	0.032	0.038	0.02	0.042
Max. Negative Amplitude(m)	-0.036	-0.032	-0.032	-0.02	-0.043
Min. Positive Amplitude(m)	0.024	0.023	0.014	0.013	0.025
Min. Negative Amplitude(m)	-0.024	-0.022	-0.014	-0.013	-0.025

Table 4.20 Calculation of time domain parameters for Sprung Mass Displacement on 3 DOF quarter car model with sine road input

Sprung Mass Displacement					
Parameters \ Controllers	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.27	0.18	0.18	0.12	0.34
Max. Negative Amplitude(m)	-0.3	-0.19	-0.08	-0.12	-0.35
Min. Positive Amplitude(m)	0.24	0.13	0.18	0.12	0.25
Min. Negative Amplitude(m)	-0.24	-0.16	-0.08	-0.12	-0.25

Table 4.21 Calculation of time domain parameters for Seat Acceleration 3 DOF quarter car model with sine road input

Seat Acceleration					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m/sec.^2)	35	20	20	19	43
Max. Negative Amplitude(m/sec.^2)	-35	20	-37	-20	-42
Min. Positive Amplitude(m/sec.^2)	24	10	12	12	22
Min. Negative Amplitude(m)	-25	-15	-12	-12	-22

Table 4.22 Calculation of time domain parameters for Seat Displacement 3 DOF quarter car model with sine road input

Seat Displacement					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.31	0.2	0.2	0.13	0.37
Max. Negative Amplitude(m)	-0.32	-0.2	-0.08	-0.13	-0.39
Min. Positive Amplitude(m)	0.25	0.13	0.16	0.13	0.26
Min. Negative Amplitude(m)	-0.25	-0.17	-0.08	-0.13	-0.26

In case of sine road input for 3 DOF system, all the controllers have been successfully developed. Amplitude of oscillations has decreased providing a good control response but Fuzzy PID gives the best result. Maximum and minimum values of amplitudes are minimum in case of all the parameters and corresponding oscillations have minimum value in case of Fuzzy PID and Fuzzy compared to LQR and PID controller.

C) BAND LIMITED WHITE NOISE

1) Suspension Travel

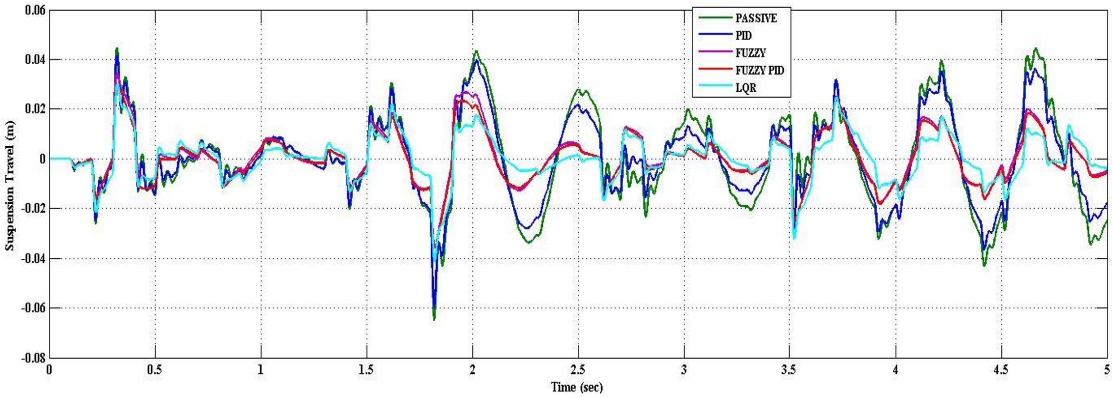

Figure 4.65 Comparison of time response of Suspension Travel for all the controllers on 3 DOF quarter car model with white noise road input

2) Tire Deflection

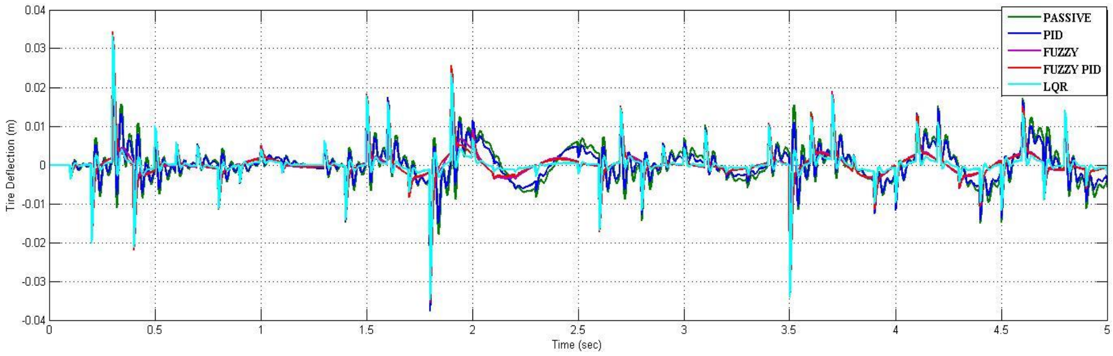

Figure 4.66 Comparison of time response of Tire Deflection for all the controllers on 3 DOF quarter car model with white noise road input

3) Sprung Mass Displacement

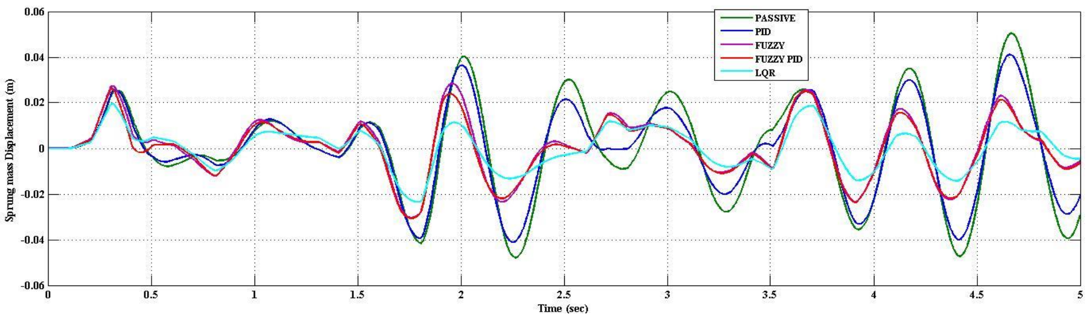

Figure 4.67 Comparison of time response of Spring Mass Displacement for all the controllers on 3 DOF quarter car model with white noise road input

4) <u>Seat Displacement</u>

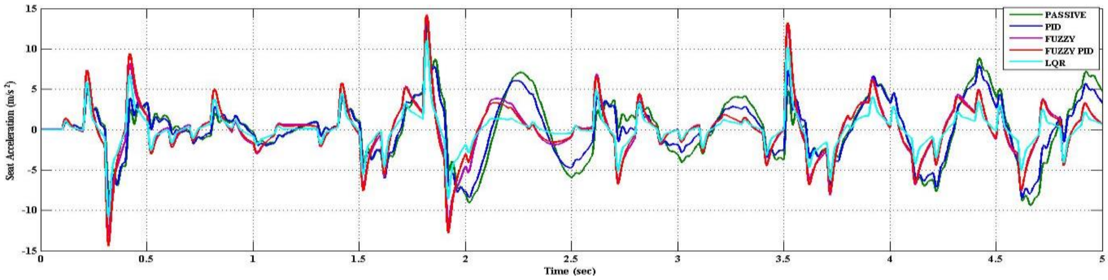

Figure 4.68 Comparison of time response of Seat Displacement for all the controllers on 3 DOF quarter car model with white noise road input

4) <u>Seat Acceleration</u>

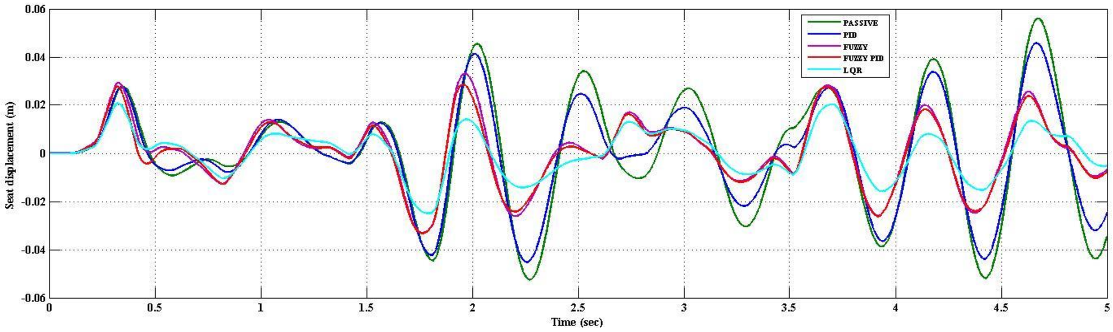

Figure 4.69 Comparison of time response of Seat Acceleration for all the controllers on 3 DOF quarter car model with white noise road input

Table 4.23 Calculation of time domain parameters for suspension travel on 3 DOF quarter car model with white noise road input

Suspension Travel					
Controllers / Parameters	**PID**	**LQR**	**FUZZY**	**FUZZY PID**	**PASSIVE**
Max. Positive Amplitude(m)	0.0.42	0..03	0.038	0.03	0.048
Max. Negative Amplitude(m)	-0.06	-0.04	-0.038	-0.03	-0.068

Table 4.24 Calculation of time domain parameters for Tire deflection on 3 DOF quarter car model with white noise road input

Tire Deflection					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.033	0.032	0.031	0.03	0.034
Max. Negative Amplitude(m)	-0.033	-0.032	-0.032	-0.03	-0.038

Table 4.25 Calculation of time domain parameters for Sprung Mass Displacement on 3 DOF quarter car model with white noise road input

Sprung Mass Displacement					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.045	0.02	0.025	0.023	0.05
Max. Negative Amplitude(m)	-0.04	-0.02	-0.025	-0.025	-0.048

Table 4.26 Calculation of time domain parameters for Seat Acceleration 3 DOF quarter car model with white noise road input

Seat Acceleration					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m/sec.^2)	16	12	15	16	17
Max. Negative Amplitude(m/sec.^2)	-13	-12	-13	-13	-14

Table 4.27 Calculation of time domain parameters for Seat Displacement
3 DOF quarter car model with white noise road input

Seat Displacement					
Controllers / Parameters	PID	LQR	FUZZY	FUZZY PID	PASSIVE
Max. Positive Amplitude(m)	0.03	0.02	0.224	0.022	0.057
Max. Negative Amplitude(m)	-0.025	-0.022	-0.028	-0.03	-0.05

For white noise input, maximum and minimum values have been reduced to some extent as shown in tables 4.23-4.27 and comparison graphs. LQR and Fuzzy tuned PID give the best response.

4.3 DISCUSSION

In case of 2 DOF model for step input all the output parameters are showing better result in case of settling time reduction and peak overshoot reduction. Fuzzy PID gives the best result followed by fuzzy controller in case of step input for 2 DOF model compared to LQR and auto tuned PID controller. In case of sine road input for 2 DOF system, all the controllers have been successfully developed. Amplitude of oscillations has decreased providing a good control response but Fuzzy PID gives the best result. Maximum values of amplitude are minimum in case of all the parameters and corresponding oscillations has been minimum in case of Fuzzy PID and Fuzzy. For band limited white noise input, for 2 parameters LQR has given better result but the overall performance is best for Fuzzy and fuzzy PID.

In case of 3 DOF model, all the controllers are properly designed and system has been controlled and road holding and ride comfort criterion have been satisfied to some extent. For step input all the controllers have provided best controlling action except in case of reduction of peak value of seat acceleration i.e. in improving ride comfort. Fuzzy PID has given the best response in terms of settling time reduction and overshoot reduction and it has been already validated in graphs and comparison tables. The results of Fuzzy and LQR are better than SIMULINK’s auto tuned PID controller. Again for sine input Fuzzy tuned PID, Fuzzy and LQR gives the better control while fuzzy control and fuzzy tuned PID giving the best response. For White noise input LQR is showing better result for 2 parameters while Fuzzy and Fuzzy PID is good overall .

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

The objective of the study is to maximize the performance of ride comfort and road holding simultaneously , to reduce settling time and amplitude of displacement of body of the passenger and car body and to reduce suspension travel. Ride comfort is represented by acceleration and Road holding is represented by tire deflection. Ride comfort and road handling depict the quality of a good suspension system.

First of all, mathematical model of two and three DOF car model has been designed in the form of differential equations and state space representation. Three inputs i.e. Step, sine and band limited white noise have been successfully tested to the designed controllers. Fuzzy tuned PID and fuzzy logic controllers have been designed using different membership functions in order to study the difference in the output coming due to different membership functions. Three membership functions have been compared with each other in designing of Fuzzy PID and fuzzy control scheme and the best membership function for each model has been estimated corresponding to different types of road input conditions. Auto tuned PID, LQR, Fuzzy and Fuzzy tuned PID controller are developed for designing active suspension system for two and three degree of freedom quarter car model. In the previous chapter a set of graph and comparison tables comparing different output variables of two and three Degree of Freedom (DOF) with three input road conditions has been provided. The objectives of this project have been achieved. All the four controllers have satisfied the design requirements and have performed better than passive system. Fuzzy PID has given the best response in terms of settling time reduction and overshoot reduction and it has been already validated in graphs and comparison tables. Fuzzy and LQR also gave better result than SIMULINK's auto tuned PID controller.

5.2 FUTURE WORK

In this work, the dynamics of the force actuator is not considered. Design of force actuator gives a more realistic study of active suspension system that can generate the required forces in order to attain desired objectives. Parameter can also affect the output performance for a suspension systems. A study can be carried out using different type of parameter in order to give different output performance. Finally, some other robust control scheme can also be used. In future study, a four and 8 Degree Of Freedom(DOF) model can be utilised which gives more realistic view of the car model. Another approach that may be used is some latest optimization techniques such as Bat, Cuckoo etc. that can be used for tuning of controller parameters.

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APPENDIX –I

MATLAB CODE/SIMULINK MODELS

```
>> Ms=2050;
Mu=100;
Ks=400000;
Cs=5000;
Kt=2000000;
A=[0 0  1 0 ;0 0 0  1 ;-Ks/Ms Ks/Ms  -Cs/Ms Cs/Ms ;Ks/Mu -(Kt+Ks)/Mu  -Cs/Ms Cs/Mu];
B=[0;0;1/Ms;-1/Mu];
Co=ctrb(A,B);
n=rank(Co);
C=[1 -1 0 0 ;1 0 0 0 ;0 1 0 0 ;-Ks/Ms Ks/Ms -Cs/Ms Cs/Ms];
Ov=obsv(A,C);
rank(Ov)

ans =

     4

fx >>
```

Fig A.1-Code to check controllability for 2 DOF Suspension System

```
>> Ms=2050;
Mu=100;
Ks=400000;
Cs=5000;
Kt=2000000;
A=[0 0  1 0 ;0 0 0  1 ;-Ks/Ms Ks/Ms  -Cs/Ms Cs/Ms ;Ks/Mu -(Kt+Ks)/Mu  -Cs/Ms Cs/Mu];
B=[0;0;1/Ms;-1/Mu];
Co=ctrb(A,B);
n=rank(Co);
C=[1 -1 0 0 ;1 0 0 0 ;0 1 0 0 ;-Ks/Ms Ks/Ms -Cs/Ms Cs/Ms];
Om=ctrb(A,B);
rank(Om)

ans =

     4

fx >>
```

Fig A.2-Code to check Observability for 2 DOF Suspension system

```
>> A=[0 0 1 0;0 0 0 1;-Ks/Ms Ks/Ms -Cs/Ms Cs/Ms;Ks/Mu -(Ks+Kt)/Mu -Cs/Mu Cs/Mu];
B=[0;0;1/Ms;-1/Mu];
G=[0;0;0;Kt/Mu];
Q=[500 0 0 0;0 1 0 0;0 0 2000 0;0 0 0 1];
R=0.000000016*diag(1);
[K,S,e]=lqr(A,B,Q,R);
x=inv(R);
y=transpose(B);
z=transpose(A);
t=transpose(K);
k=z+t*y;
g=inv(k);
Kw=-(x*(y*g)*(S*G));
```

Fig A.3-LQR Coding used for 2 DOF Suspension system

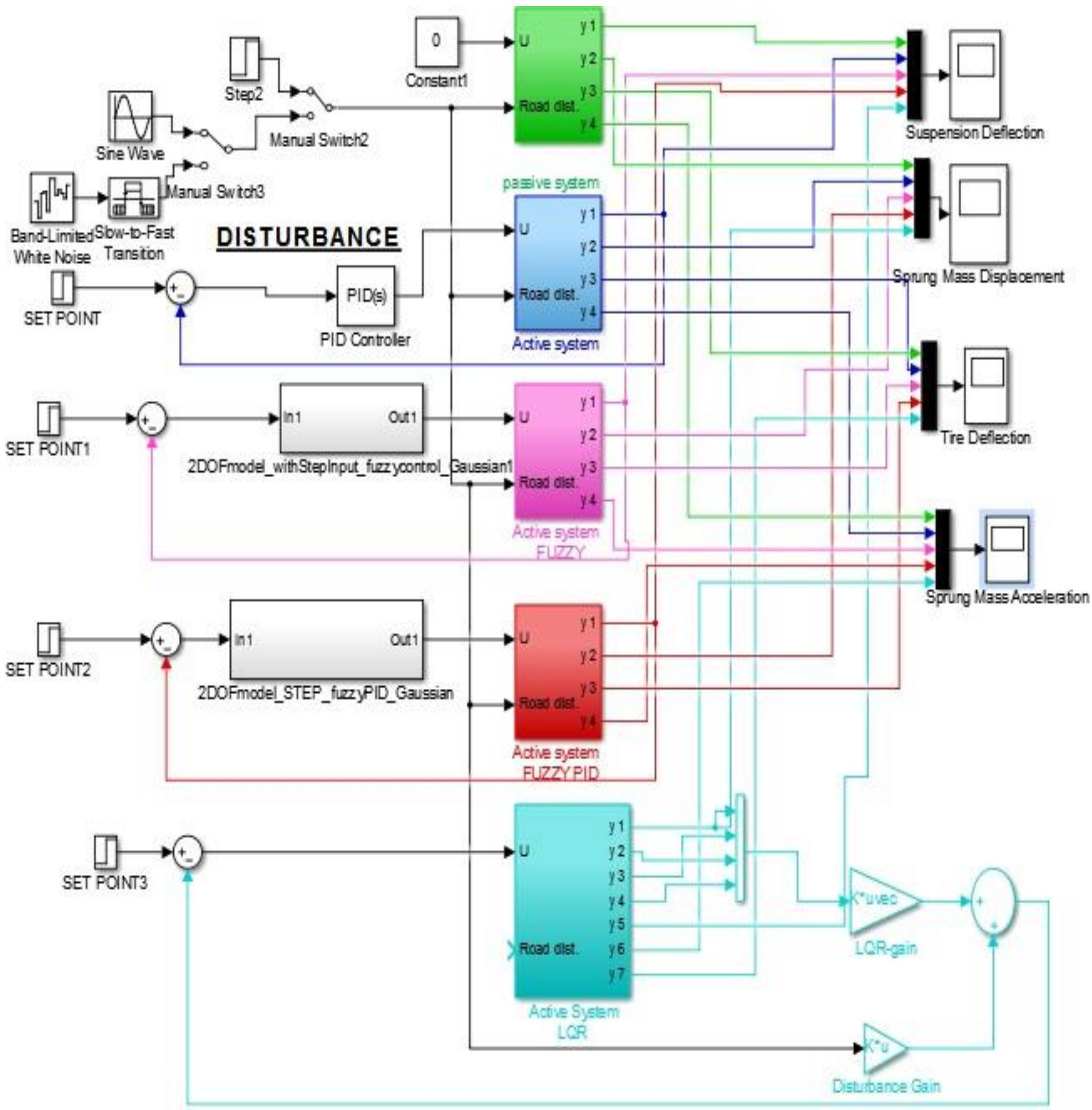

Fig.A.4 Overall setup for 2 DOF model in Simulink

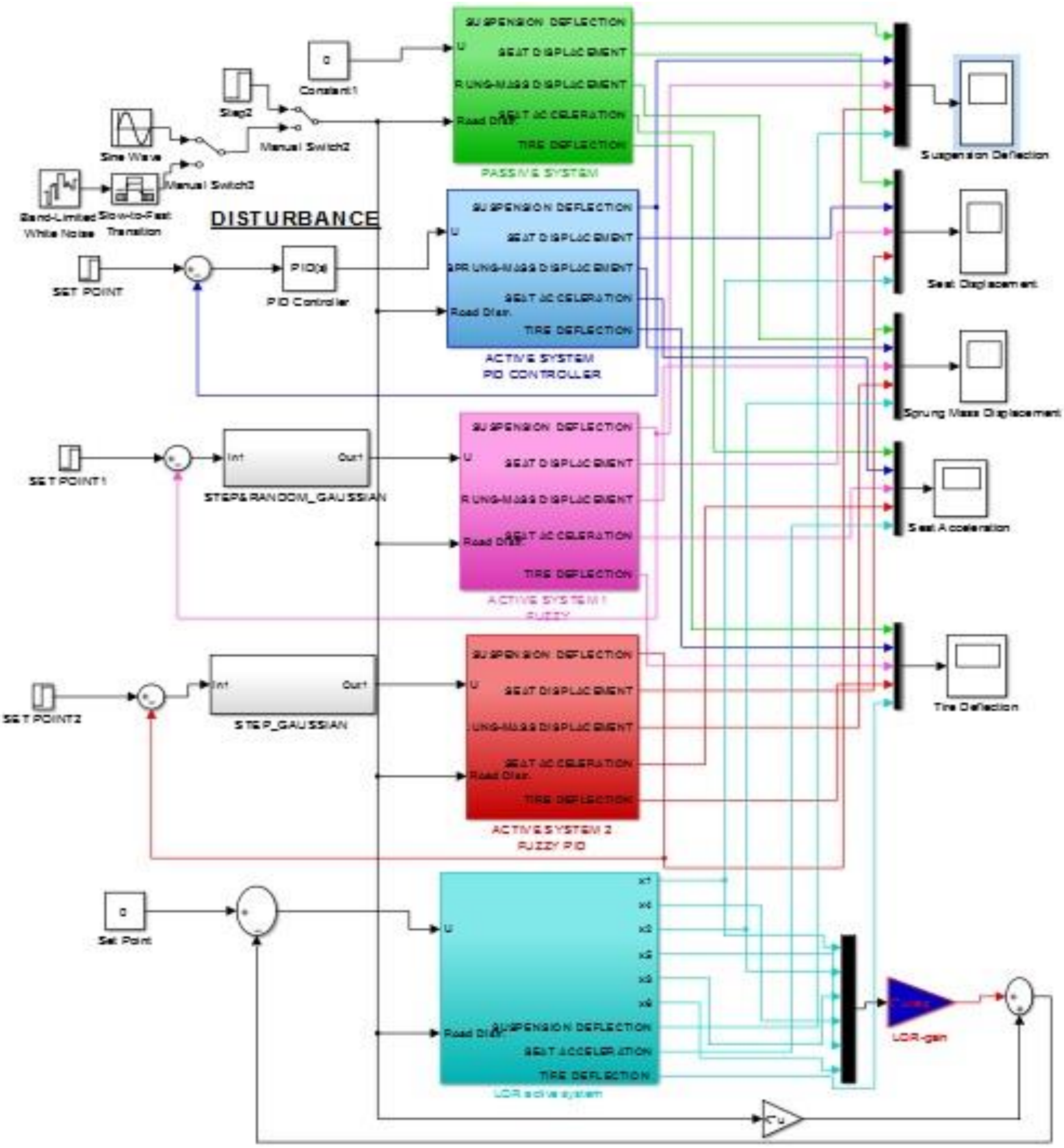

Fig.A.5 Overall setup for 3 DOF model in Simulink

APPENDIX –II

LIST OF PUBLICATION

"A 3 degree of freedom quarter car active suspension system model design approach using PID, fuzzy logic, fuzzy tuned PID". International Journal of Engineering Technology, Management and Applied Sciences (IJETMAS) ,2017.