PERFORMANCE ANALYSIS OF TWO DOF OF HELICOPTER MODEL USING PID AND FUZZY CONTROL

DISSERTATION
SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE
OF

MASTER OF TECHNOLOGY
IN
CONTROL AND INSTRUMENTATION

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CERTIFICATE

I, PANKAJ KUMARI MEENA, Roll No. 2K15/C&I/14 student of M. Tech. (CONTROL AND INSTRUMENTATION), hereby declare that the dissertation/project titled **"Performance Analysis of Two DOF of Helicopter Model using PID and Fuzzy control"** under the supervision of Prof. BHARAT BHUSHAN, Electrical Engineering Department, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Master of Technology has not been submitted elsewhere for the award of any Degree.

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Date: 26.07.2017

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ACKNOWLEDGEMENT

I am highly grateful to the Department of Electrical Engineering, Delhi Technological University (DTU) for providing this opportunity to carry out the project work.

The constant guidance and encouragement received from my supervisor Prof. BHARAT BHUSHAN of Department of Electrical Engineering, DTU, has been of great help in carrying my present work and is acknowledged with reverential thanks.

I would like to express a deep sense of gratitude and thanks to Prof. MADHUSHUDAN SINGH for providing the laboratory and other facilities to carry out the project work. Again, the help rendered by Prof. BHARAT BHUSHAN, for the literature, and for experimentation is greatly acknowledged.

Finally, I would like to expresses gratitude to other faculty members of Electrical Engineering Department, DTU for their intellectual support throughout the course of this work.

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ABSTRACT

The main objective of this thesis is the modelling and controlling of two DOF helicopter system. A Two Degree of Freedom Helicopter model has the two degree of freedoms namely pitch angle and yaw angle. The pitch angle is the vertical angle and this movement helps the helicopter to move in forward or backward direction on the other hand yaw angle is the horizontal angle and the movement in this direction helps in turning the direction of the helicopter. The controllers studied and implemented on the two DOF setup of helicopter are Linear Quadratic Regulator (LQR), Proportional–Integral-Differential (PID) and Fuzzy Logic Controller (FLC). The main motive of the controlling is to stabilize the position of the controller at the set points which are given as input to the system, all the controllers are implemented and the tuning of the parameters is done to obtain the suitable results. For FLC control the rule base is created and implemented. The designing and implementation of the controllers are done on the simulink MATLAB platform, which is connected to the real time system by interfacing devices. The results are obtained for both the non linear model of the system and the real time system and the controlling algorithms are tested on software and hardware. A comparative study is carried out to find out the performance of the controllers on the non linear model as well as on the real time system of two DOF helicopter system and finally the conclusion is drawn based on the analysis of the results obtained for all the controllers with two output parameters namely, pitch angle and yaw angle.

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LIST OF SYMBOLS

Symbol	Description
K_{pp}	Thrust force constant of yaw motor/propeller
K_{yy}	Thrust torque constant of yaw axis from yaw motor/propeller
K_{py}	Thrust torque constant acting on pitch axis from yaw motor/propeller
K_{yp}	Thrust torque constant acting on yaw axis from pitch motor/propeller
$B_{eq,p}$	Equivalent viscous damping about pitch axis
$B_{eq,y}$	Equivalent viscous damping about yaw axis
M_{heli}	Total moving mass of the helicopter (body, two propeller assemblies, etc.)
L_{cm}	Centre of mass length along helicopter body from pitch axis
$J_{eq,p}$	Total moment of inertia about pitch axis
$J_{eq,y}$	Total moment of inertia about yaw axis.
$R_{m,p}$	Armature resistance of pitch motor
$R_{m,y}$	Armature resistance of yaw motor
$K_{t,p}$	Current-torque constant of pitch motor

$K_{t,y}$	Current-torque constant of yaw motor
$J_{m,p}$	Rotor moment of inertia of pitch motor
$J_{m,y}$	Rotor moment of inertia of yaw motor
$K_{f,p}$	Pitch propeller force-thrust constant
$K_{f,y}$	Yaw propeller force-thrust constant
$m_{m,p}$	Mass of pitch motor
$m_{m,y}$	Mass of yaw motor
$K_{EC,LN,Y}$	Yaw encoder resolution (in quadrature mode)
$K_{EC,LN,P}$	Pitch encoder resolution (in quadrature mode)
$K_{EC,Y}$	Yaw encoder calibration gain
$K_{EC,P}$	Pitch encoder calibration gain
M_{shield}	Mass of propeller shield
M_{props}	Mass of pitch and yaw propellers, shields and motors
$m_{body,p}$	Mass moving about pitch axis
$m_{body,y}$	Mass moving about yaw axis
m_{shaft}	Mass of metal shaft rotating about yaw axis

L_{body}	Total length of helicopter body
L_{shaft}	Length of metal shaft rotating about yaw axis
$J_{body,p}$	Moment of inertia of helicopter body about pitch axis
$J_{body,y}$	Moment of inertia of helicopter body about yaw axis
J_{shaft}	Moment of inertia of metal shaft about yaw axis end Point
J_p	Moment of inertia of front motor/shield assembly about pitch pivot
J_y	Moment of inertia of back motor/shield assembly about yaw pivot

ABBREVIATIONS

DOF	Degree Of Freedom
LQR	Linear Quadratic Regulator
PID	Proportional Integral Derivative
FF	Feed Forward
PC	Proportional Controller
IC	Integral Controller
DC	Derivative Controller
PB	Proportional Band
FLC	Fuzzy Logic Controller
DEG	Degree

CHAPTER 1

INTRODUCTION

The design and development of helicopters started during the first half-century of 20^{th} century, when Focke-Wulf Fw 61 became the first helicopter to be operated in 1936. The earlier designs generally used to have more than one main rotor, which is the single main rotor along with configuration of anti-torque tail rotor which has become the most common configuration of helicopter.

Helicopter is a framework of rotorcraft where lift and thrust are to be supplied through rotors which permits the vertical movements i.e. takeoff and landing, and also permits the helicopter to hover, and fly in lateral, forward and backward direction. These features make it suitable to be used in isolated or congested areas where other aircrafts having fixed-wings cannot be operated.

Owing to the function in a specified manner by the helicopter its ability to vertically take off and land, and its hovering ability for long periods of time, along with the aircraft's operation in low airspeed conditions it is chosen to perform tasks that were earlier not feasible using any other aircraft, or were work or time-intensive to complete on the ground. In today’s world, helicopter is being used for transportation of cargo and people, construction, military uses, tourism, firefighting, search and rescue, law enforcement, medical transport, news and media, aerial observation and agriculture.

The flight controls are used to maintain and achieve controlled aerodynamic flight. Changes in the control system of aircraft flight are transmitted mechanically to the rotor, which in turn produces aerodynamic effects upon the blades of rotor that enables the helicopter to move in the desired way. For tilting forward and backward (pitch) or in the sideways (roll), it is required that the controls change the angle of attack of main rotor blades periodically during rotation, thus creating varying amounts of lift (force) at varied points in the period. For increasing or decreasing overall lift it is required that the controls vary the angle of attack of all the blades altogether by same amount at the same time, which will result in descent, ascent, deceleration and acceleration.

1.1 NON LINEAR HELICOPTER MODEL

The design of helicopters is a challenging task in nonlinear feedback design, because of the nonlinearity in the dynamics and the effect of strong coupling between the torques and forces developed by the actuators of vehicle. In general a helicopter can be called an under actuated mechanical system, which means a system having more degrees of freedom than self-governing control inputs.

Partial feedback linearization methods are not good for the control of this kind of a system, as the resultant zero-dynamics are only critically stable. Also, the model can be affected by un-modeled dynamics and large number of uncertainties, and this also does not offer any design method based on exact cancellation of nonlinear terms which are poorly suited.

1.2 TWO DEGREE OF FREEDOM OF HELICOPTER SYSTEM

The Systems which require two independent coordinates for describing their motion are known as two degree of freedom systems.

Physical model as well as simplified laboratory model of helicopter which consists body of a helicopter is driven by two motors which drive the both main and tail propeller. The body is connected to a base to enable the two degrees of freedom (2 DOF):

• Rotation around the vertical axis-Pitch angle (θ)

• Rotation around the horizontal axis-Yaw angle (Ψ)

1.3 FEED FORWARD (FF)

Feed-forward, generally written feed forward could be a term describing a component or pathway among an impact system that passes a dominant signal from a supply in its external atmosphere, typically a command signal from an associate external operator, to a load elsewhere in its external atmosphere. An impact system that has solely feed-forward behavior responds to its control signal during a pre-defined approach while not responding to however the load reacts; it's in distinction with a system that conjointly has feedback, that adjusts the output to require account of however it affects the load, and the way the load itself might vary unpredictably; the load is taken into account to belong to the external atmosphere of the system.

In a feed-forward system, the management variable adjustment isn't error-based. Instead it's supported data regarding the method within the kind of a mathematical model of the method and data regarding or measurements of the method disturbances.

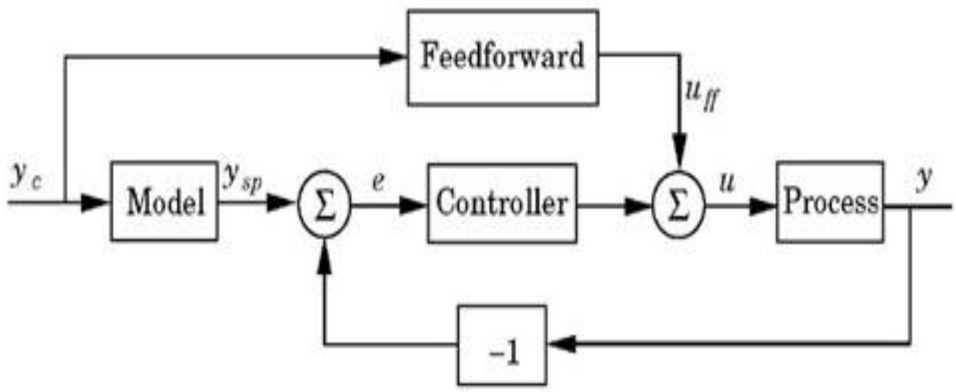

Fig 1.1: Block Diagram of Feed forward Controller

1.4 LINEAR QUADRATIC REGULATOR (LQR)

The settings of a (regulating) controller governing either a machine or method (like an airplane plane or chemical reactor) are found by employing a mathematical algorithmic program that minimizes a value operated with coefficient factors provided by an operator. The value to be operated is usually outlined as a total of the deviations of key measurements, desired altitude or method temperature, from their desired values. The algorithmic program so finds those controller settings that minimize unwanted deviations. The magnitude of the management action itself can also be enclosed within the cost function.

The LQR algorithmic program reduces the number of labor done by the management systems or engineer to optimize the controller. However, the engineer still must specify the value operate parameters, and compare the results with the desired style goals. Typically this implies that controller construction are going to be a repetitive method within which the engineer judges the "optimal" controllers made through simulation to adjust the parameters to provide a controller additional in keeping with desired goals.

The LQR algorithmic program is basically an automatic method of finding an applicable state-feedback controller. As such, it's not uncommon for management engineers to like different strategies, like full state feedback, conjointly referred to as pole placement, within which there's a clearer relationship between controller parameters and controller behavior. The issue to find the correct coefficient factors limits the applying of the LQR based mostly controller synthesis.

1.5 PID CONTROL

Proportional-Integral-Derivative (PID) control is the most commonly used control algorithm in industry. It has been accepted universally in industrial control processes. The PID controllers are robust in performance providing a wide range of operational conditions along with functional simplicity. The algorithm of PID contains three basic coefficients; proportional, integral and derivative which are altered to obtain optimal response.

1.6 FUZZY CONTROL

It is a control system which is based on fuzzy logic. In this, system takes analog input values as logical variables which accepts continuous values between 0 and 1, contrary to digital logic or classical logic, which performs its operation on either 1 or 0 (true or false, respectively). This logic is widely used in machine control. The "fuzzy" refers to the fact that the logic can also have conditions which are "partially true". It has an advantage that the solution to the problem can be stated in the terms which are familiar to human operators, and thus their experience can be used in designing the controller. It helps to mechanize tasks which are already successfully performed by humans.

1.7 LITERATURE REVIEW

The implementation of sliding mode control along with the modeling of the 2 DOF helicopter is done in [1]. A high order neural network based controlling of 2 DOF is carried out in [2], neural back-stepping and neural sliding mode block control techniques are applied in this paper to control yaw and pitch angle of helicopter. A non linear model for 3 DOF helicopter is considered in this paper [3] and a feed forward controller is designed on the mathematical model of the system to compensate the uncertainties and the disturbances on the system. This paper deals with the tracking control problem for a 2 DOF laboratory helicopter using optimal linear quadratic regulator (LQR) [4], the APSO and LQR controllers are implemented on the 2 DOF helicopter model which is a highly non linear model. A survey based a neural network is cloned from various controllers and implemented on 2 DOF model in [6]. An uncertainty modelling of 2 DOF model is done in this paper. A Fuzzy Logic Controller is implemented and compared with the LQR controlled system for 2 DOF model of helicopter [9]. The proposed gain selection and other techniques reported in the literature for LQR controllers are used to compare the response [11]. Different types of controllers are proposed to control different processes [20]. The modeling of the system, the non linear model of the system with state space representation is done, the specifications of the system and the parameters of the lab apparatus is taken from the Quanser lab manual. The controller designing and the implementation is done [22,23,24,25,26,27]. A multi variable PID algorithm is implemented in this paper [28]. Ziegler Nicholas method for tuning the parameters of Proportional Integral Differential controller is proposed in this paper [31]. A Model Predictive Control based control algorithm is developed in this paper to control the pitch and yaw angle for the non linear 2 DOF model of helicopter. A fuzzy logic based controller design is implemented for twin rotor system, with multiple inputs and multiple output system [32]. Dynamic modelling of a twin motor multiple inputs and multiple output system is done and an optimal controller designing is done in this paper [33]. Modelling and designing of a 2 DOF helicopter system is done in this paper along with the mathematical modelling of the system [34]. The modern control theory which includes the Fuzzy Logic Control and Linear Quadratic Regulator based control algorithms and concepts implemented on non linear systems [37].

CHAPTER 2

TWO DOF HELICOPTER MODEL

In this chapter, the mechanical system design and the electronic system design are presented. The electronic system design is explained in three sub-sections as main board, encoder reader and motor controller circuit.

2.1 TWO DOF HELICOPTER OVERALL COMPONENTS

The Quanser Two-Degree of Freedom (DOF) Helicopter, in Figure 2.1, a model of helicopter placed on a fixed base is having two propellers which are driven by DC motors. The elevation of helicopter nose of the pitch axis is controlled by front propeller and the back propeller enables control of the side to side motion along the yaw axis. The angles of yaw and pitch are measured by encoders of high-resolution. The slipring is used to transmit the motor signals and pitch encoder. It removes the chances of entanglement of wires on the yaw axis and also permits the yaw angle to rotate 360 degrees [5].

The 2 DOF Helicopter consists of a body of model helicopter and a metallic base. The two propellers in the helicopter are perpendicularly to each other and both are actuated using DC motors. This reproduces the function of configuration of common helicopter having an anti-torque tail rotor and main rotor. The pitch axis is controlled by front propeller which rotates the center of the body along the horizontal (i.e. front propeller moves up and down). The yaw axis is controlled by the back propeller which is the angle along the vertical base. The high-resolution encoders are used to measure both the axes. The slip ring on the vertical axis enables the body to rotate freely along the yaw angle, by elimination of the requirement of any wires being connected to the encoders and motors to the base. The intrinsic torque effect by the front propeller enables the body to rotate and should be counteracted by the tail rotor, similar to the full-sized helicopters, which can provide interesting control and modeling challenges.

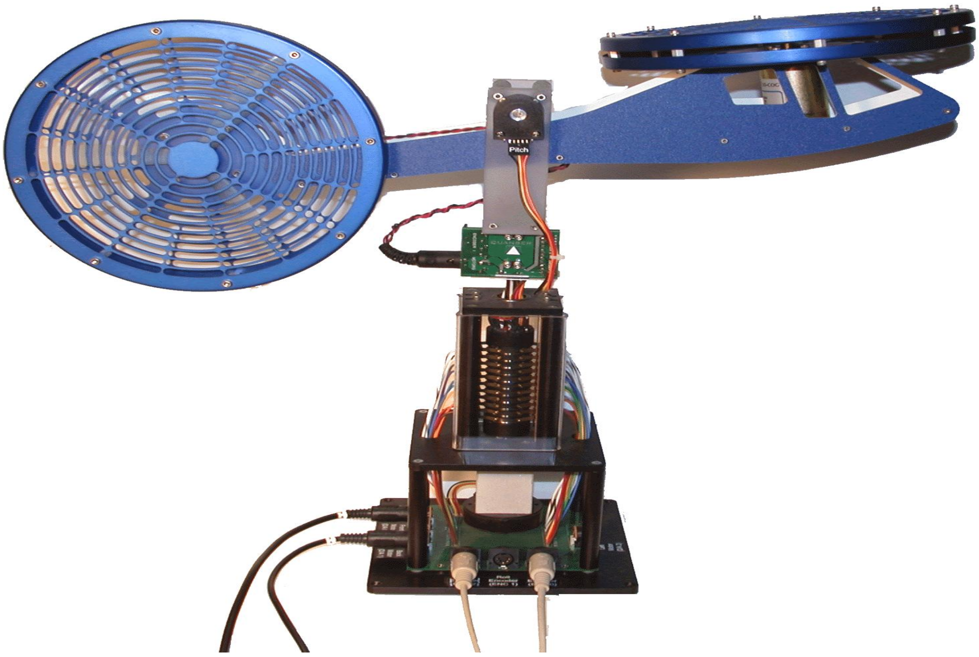

Fig 2.1: Quanser 2 DOF Helicopter

The overall components of two DOF helicopter system are described below:

1. Back Propeller
2. Back Propeller Shield
3. Yaw/Back Motor
4. Pitch Encoder
5. Yoke
6. Helicopter Body Front
7. Pitch Propeller
8. Pitch/Front Motor
9. Shield Encoder/Motor
10. Circuit

11. Motor connector on circuit
12. Metal shaft rotates about yaw axis
13. Slip ring
14. Yaw Encoder
15. Base Platform
16. Front Motor Connector
17. Back Motor Connector
18. Yaw Encoder Connector
19. Pitch Encoder Connector

2.1.1. <u>Mechanical Components</u>

The mechanical components of the two DOF helicopter system contains a shaft with two propellers on both ends. This shaft enables rotation in the horizontal as well as vertical plane. The shaft can be described by the measurement of vertical and horizontal angles by encoder.

<u>DC Motors:</u> The 2-DOF Helicopter consists of two DC motors: the yaw motor i.e. component #3, which mimics the propeller at the back and the pitch motor, component #8, which rotates the propeller in the front.

The motor for yaw is 2842 Model 006C of Faulhaber Series motor which provides terminal resistance equals to 1.6Ω and also provides constant of current-torque equals to 0.0109 N.m/A. Table 2.1 provides the full specifications about this motor. The motor for pitch which is larger in size is a Pittman Model 9234. It provides an electrical resistance equals to 0.83 Ω and also provides current-torque constant equal to 0.0182 N.m/A. The motor has rated voltage equal to 12 V but the peak voltage may be raised up to 22 V without causing any damage.

Propellers: The assemblies for yaw and pitch propeller contains the actual propeller, which is mounted to the motor shaft, along with the aluminum propeller shield. The Graupner 20/15 cm or 8/6" propellers are being used for both the yaw and pitch motors. The established thrust-force constant of value 0.104 N/V is provided by pitch motor/propeller and the thrust-force constant of 0.43 N/V is provided by yaw propeller/motor.

2.1.2. Electronic Components

The mechanical equipment of the system has to be reliable measuring instrument as compared to traditional mechanical meters.

Encoders: The system is having two encoders: the encoder for measurement of angle of pitch i.e. component #04, and encoder for measuring the angle of yaw i.e. component #15. When using quadrature mode, pitch encoder gives a resolution equal to 4096 counts in a single revolution and yaw encoder gives resolution equal to 8192 counter in a single revolution. Therefore it provides effective position resolution equal to 0.0879 degrees along pitch axis and 0.0439 degrees along the yaw axis [7].

2.1.3. Interfacing Devices

- **Q2-USB data acquisition device**: The Q2-USB gives an easy and rapid preliminary design and Hardware-In-The-Loop (HIL) environment. It provides wide range of inputs and outputs, which can be easily connected and can be used to control the variety of devices with analog as well as digital sensors, along with encoders – on a single board. It has following features :
    a. Optimized for real-time management performance with Quanser QUARC and RCP Toolkit management software
    b. High-speed interface with USB 2.0
    c. Windows 7 compatible

d. Various Q2-USB units can be used at same time

e. Robust device case

Fig 2.2: Q-2 USB Data Acquisition Device

Fig 2.3: VoltPAQ-X2Voltage amplifier

- **<u>VoltPAQ-X2 dual-channel linear voltage amplifier:</u>** The VoltPAQ-X2 is linear voltage-controlled electronic equipment ideal for all complicated controls configurations associated with academic or analysis desires. It is designed to realize high performance with Hardware-In-The-Loop (HIL) implementations. Pairing the VoltPAQ-X2 with Quanser information acquisition board and management style code, you'll build a dependable period of time platform.

VoltPAQ-X2 with 2 output channels will drive Quanser two degree-of-freedom experiments, like two DOF Inverted setup or two DOF whirlybird, or alternative motors or actuators through easy-connect terminal boards and cables.

2.2 SYSTEM SPECIFICATIONS

Table 2.1: Parameters of Two DOF Helicopter model [22,23]

Symbol	Description	Value	Unit
K_{pp}	Thrust force constant of yaw motor/propeller	0.204	N.m/V
K_{yy}	Thrust torque constant of yaw axis from yaw motor/propeller	0.072	N.m/V
K_{py}	Thrust torque constant acting on pitch axis from yaw motor/propeller	0.0068	N.m/V
K_{yp}	Thrust torque constant acting on yaw axis from pitch motor/propeller	0.0219	N.m/V
$B_{eq,p}$	Equivalent viscous damping about pitch axis	0.800	N/V
$B_{eq,y}$	Equivalent viscous damping about yaw axis	0.318	N/V
M_{heli}	Total moving mass of the helicopter (body, two propeller assemblies, etc.)	1.3872	Kg
L_m	Centre of mass length along helicopter body from pitch axis	0.186	M
$J_{eq,p}$	Total moment of inertia about pitch axis	0.0384	$kg.m^2$
$J_{eq,y}$	Total moment of inertia about yaw axis.	0.0432	$kg.m^2$

Table 2.2: Specifications of motor and encoder of Two DOF Helicopter model [24]

Symbol	Description	Value	Unit
$R_{m,p}$	Armature resistance of pitch motor	0.83	Ω
$R_{m,y}$	Armature resistance of yaw motor	1.60	Ω
$K_{t,p}$	Current-torque constant of pitch motor	0.0182	N.m/A
$K_{t,y}$	Current-torque constant of yaw motor	0.0109	N.m/A
$J_{m,p}$	Rotor moment of inertia of pitch motor	1.91×10^{-6}	$kg.m^2$
$J_{m,y}$	Rotor moment of inertia of yaw motor	1.37×10^{-4}	$kg.m^2$
$K_{f,p}$	Pitch propeller force-thrust constant (found experimentally)	0.1037	N/V
$K_{f,y}$	Yaw propeller force-thrust constant (found experimentally)	0.428	N/V
$m_{m,p}$	Mass of pitch motor	0.292	Kg
$m_{m,y}$	Mass of yaw motor	0.128	Kg
$K_{EC,LN,Y}$	Yaw encoder resolution (in quadrature mode)	8192	counts /rev
$K_{EC,LN,P}$	Pitch encoder resolution (in quadrature mode)	4096	counts /rev
$K_{EC,Y}$	Yaw encoder calibration gain	7.67×10^{-4}	rad/counts
$K_{EC,P}$	Pitch encoder calibration gain	1.50×10^{-3}	rad/counts

Table 2.3: Various mass, inertia, and length parameters Two DOF Helicopter [25,26]

Symbol	Description	Value	Unit
M_{shield}	Mass of propeller shield	0.167	Kg
M_{props}	Mass of pitch and yaw propellers, shields and motors	0.754	Kg
$m_{body,p}$	Mass moving about pitch axis	0.633	Kg
$m_{body,y}$	Mass moving about yaw axis	0.667	Kg
m_{shaft}	Mass of metal shaft rotating about yaw	0.151	Kg

L_{body}	Total length of helicopter body	0.483	M
L_{shaft}	Length of metal shaft rotating about yaw	0.280	M
$J_{body,p}$	Moment of inertia of helicopter body about	0.0123	kg.m^2
$J_{body,y}$	Moment of inertia of helicopter body about .	0.0129	kg.m^2
J_{shaft}	Moment of inertia of metal shaft about yaw axis end Point	0.0039	kg.m^2
J_p	Moment of inertia of front motor/shield assembly about pitch pivot	0.0178	kg.m^2
J_y	Moment of inertia of back motor/shield assembly about yaw pivot	0.0084	kg.m^2

2.3 COMPUTER SOFTWARE DESIGN

2.3.1 MATLAB

The MATLAB platform is optimized for determination of engineering and scientific issues. The matrix-based MATLAB language is that the world's most natural categorical machine arithmetic. Its inherent graphics create it simple to see and gain insights from knowledge. A colossal library of prebuilt toolboxes allows to start promptly with algorithms essential to domain. These MATLAB tools and capabilities are all strictly tested and designed to figure along [29].

MATLAB helps to take up ideas on the far side the desktop by enabling to run analyses on larger knowledge sets and rescale to clusters and clouds. MATLAB codes are often integrated with alternative languages, facultative to deploy algorithms and applications at intervals internet, enterprise, and production systems.

MATLAB is used in automobile active safety systems, heavenly body satellite, health observance devices, power grids, and LTE cellular networks. It is also used for machine learning, signal process, image process, pc vision, communications, machine finance management, robotics, and so on.

2.3.2 Simulink

Simulink is a block diagram based environment for multipurpose simulation and Model-Based Design. It supports various features like simulation, self-code generation, and continuous testing along with verification of embedded systems. It provides an editor in graphical format, customizable block libraries, and processors of functions for modeling and simulation of dynamic systems. It is combined with MATLAB, enabling to include MATLAB algorithms in the models and transfer simulation results to MATLAB for extended analysis. It is used everywhere by engineers to get their ideas raised from the ground, like reducing fuel emissions, development of autopilot software, and design of wireless LTE systems.

2.3.3 QUARC real time control software for MATLAB/Simulink

QUARC software by Quanser enhances capabilities of MATLAB as well as Simulink by adding powerful tools and thus making the design, development and deployment of critical real-time control applications easier. QUARC can generate real-time code from controllers designed by Simulink and compiles it in real-time on the Windows target – without use of digital signal processing or writing a single line of code.

The main features of QUARC include:

- Flexible and simple hardware interfacing
- Protocol independent communication framework
- Support for multi-threaded, multi-rate and asynchronous models
- Enhanced third-party devices support

2.4 CONTROLLER THEORY

2.4.1 PID

A PID is widely used in feedback control of industrial processes on the market in 1939 and has remained the most widely used controller in process control until today. Thus, the PID controller is the controller which takes the present, past, and future values of the error into consideration. A PID controller is composing of three controllers [30]:

1. Proportional controller (PC)
2. Integral controller (IC)
3. Derivative controller (DC)

a) Role of a Proportional Controller (PC)

The role played by a proportional controller is dependable on the present error, I on the summation of past error and D on futuristic prediction of error. The weighted function of three actions helps in performing control. In this controller, the steady state error depends inversely upon the proportional gain. The proportional response is adjusted by having product of the error and constant Kp, which is called the proportional gain. The proportional term (P) is given by:

$$P = K_p . \mathrm{error}(t) \tag{2.1}$$

A high value proportional gain gives large variation in the output for a specified change in the error. The very high value of proportional gain can make the system to be unstable. Contrary to that, the small gain produces a small output response on application of large input error. When proportional gain is of low value, the controlling action can be too small when reflecting system disturbances. Consequently, Kp will show the effect by reduction in the rise time and will reduce, but it will never eliminate, the steady-state error. In actual practice the proportional band (PB) is expressed as a percentage:

$$\mathrm{PB} = \frac{100}{\mathrm{K_p}} \tag{2.2}$$

Thus, PB of 10% ⇔K_p =10

b) <u>Role of an Integral Controller (IC)</u>

An Integral controller (IC) corresponds to both the duration of the error and the magnitude of the error. The integral in a PID controller is the summation of the instantaneous error over an interval of time and produces the added offset that must have been corrected previously. Thus, an integral control (Ki) will eliminate the steady-state error, but it negatively affects the transient response. The integral term (I) is given by:

$$\mathrm{I} = \int_0^t \mathrm{error(t)dt} \tag{2.3}$$

c) <u>Role of a Derivative Controller (DC)</u>

The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain Kd. The derivative term slows the rate of change of the controller output. A derivative control (Kd) increases the stability of the system, reduces the overshoot, and improves the transient response. The derivative term (D) is given by:

$$\mathrm{D} = \mathrm{K_d}.\frac{\mathrm{derror(t)}}{\mathrm{dt}} \tag{2.4}$$

In proportional-integral-derivative control, the proportional term, the integral term, and the derivative term are added together. PID controller is defined as:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + \mathrm{K_d}.\frac{derror(t)}{dt} \quad (2.5)$$

Where K_p is the constant diagonal proportional gain matrix, K_i is the constant diagonal integral gain matrix and Kd is the constant diagonal derivative gain matrix

PID control has control actions based on the past, the present and the future, which are due to integral, proportional and derivative terms, respectively. Pure proportional control will have a steady-state error. The steady-state error can be reduced with a higher gain but this will result in increasing the oscillation. Adding an integral term helps to eliminate the steady-state error but it will also increase the tendency for oscillation and will slow down the response of the system. Thus, a damping effect is required and an increase in system response is needed. This is provided by adding a derivative term. All three terms are dependent each other.

Effects of each of controllers K_p, K_d, and K_i on a closed-loop system is shown below in the table 2.4

Table 2.4: Effect of K_p, K_d, and K_i on a closed-loop system

Parameter	Rise time	Overshoot	Settling Time	Steady State Error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Decrease significantly
K_d	Minor decrease	Minor decrease	Minor decrease	No effect in theory

Figure 2.4 shows a structure of a PID control system. The error signal e(t) is used to generate the proportional, integral, and derivative actions, with the resulting signals weighted and summed to form the control signal u(t) applied to the plant model.

A standard PID controller structure is also known as the "three-term" controller.

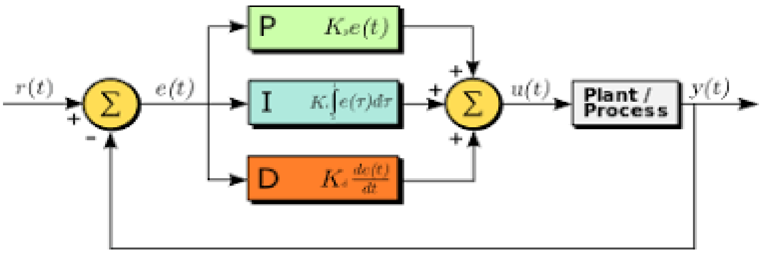

Fig 2.4: Block diagram of the PID controller

The "three term" functionalities are highlighted below. The terms K_p, K_i and K_d definitions are:

- **The proportional term:** providing an overall control action proportional to the error signal through the all pass gain factor.
- **The integral term:** reducing steady state errors through low frequency compensation by an integrator.
- **The derivative term:** improving transient response through high frequency compensation by a differentiator.

2.4.2 FUZZY LOGIC CONTROLLER (FLC)

A fuzzy design is based on the understanding of how to control a process. Then the obtained knowledge can be used to synthesize the controller. While the classical ideas of controlling, as for instance PID controller, usually use an analytical method to develop the controller, with Fuzzy the process of developing the controller exploits a natural language, becoming easier to set the controller for an one expert control engineer. In this case, it is more important to know how the plant works, and what the desired control is. Fuzzy controllers have four main components: “Rule-Base”, “Fuzzy Inference Mechanism”, “Fuzzification of crisp inputs” and “Defuzzification of fuzzy input”. All components are shown in figure 2.5

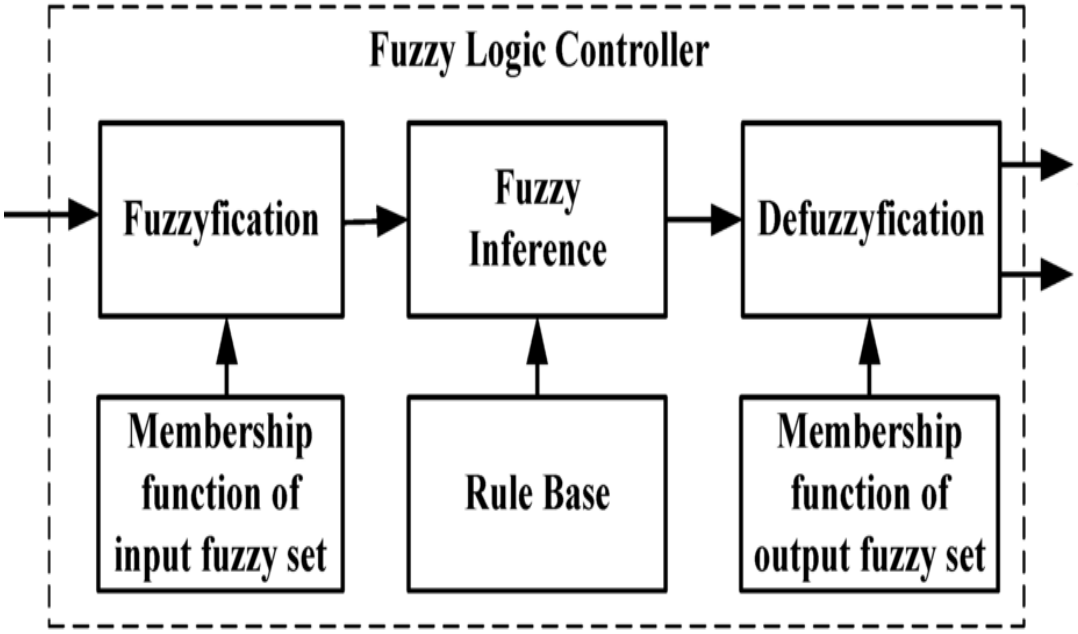

Fig 2.5: Block diagram of Fuzzy Logic Controller

Fuzzification: Fuzzification is the first step in the fuzzy inferencing process. This involves a domain transformation where crisp inputs are transformed into fuzzy inputs. Crisp inputs are those inputs which is measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.

Defuzzification: Defuzzification is the process of producing a quantifiable result in Crisp logic, given fuzzy sets and corresponding membership degrees. It is typically needed in fuzzy control systems.

Membership Functions and Fuzzy Rule base: The values of the variables are assigned in a form of linguistic values, which will be later assigned to numerical values. We will use seven linguistic values for the “Pitch error” input and for the output and only three linguistic values for the input change in Pitch error. In such a way we indicate the importance of each variable using one of these seven values. All the variables with seven values are assigned with one of the following linguistic values:

- "NB" negative big
- "NM" negative medium
- "NS" negative small
- "ZR" zero
- "PS" positive small
- "PM" positive medium
- "PB" positive large

The rule base for the fuzzy logic controller is the main part, rules are created by analyzing the pattern of error and change in error according to which the output of controller is decided. For two variable inputs in fuzzy inference system rules can be build with two types of commands i.e. OR and AND. If we use OR in making rules, the controller gives the output if any of the condition is true, for example "If error is NB. OR change in error is N, controller output is NB", this will give the controller output as NB if any of the conditions are true on the other hand if the AND is used in making the rules the controller gives output only if both the conditions are true, for example "If error is NB AND change in error is N, controller output is NB" this will give the controller output as NB only if both the conditions are true.

2.5 MATHEMATICAL MODELING

1. The helicopter is horizontal when the pitch angle equals 0.

2. The pitch angle increases positively, when the nose is moved upwards and the body rotates in the counter-clockwise (CCW) direction.

3. The yaw angle increases positively, when the body rotates in the clockwise (CW) direction.

4. If the pitch thrust force is positive $F_p > 0$, pitch increases.

5. If the yaw thrust force is positive, $F_y > 0$, yaw increases

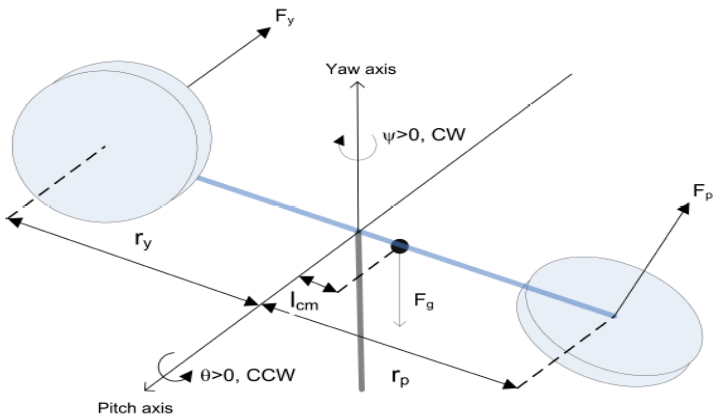

Fig 2.6: Simple Free Body Diagram of Two DOF Helicopter

2.5.1 State space model

The thrust forces a1cting on the pitch and yaw axes from the front and back motors are then defined. Using the Euler-Lagrange formula, the nonlinear equations of motion of the 2 DOF Helicopter systems is derived. These equations are linearized about zero and the linear state-space model (A, B, C, D) describing the voltage-to-angular joint position dynamics of the system is found. Given the state-space representation:

$$x = Ax + Bu \tag{2.6}$$

The state vector for the 2 DOF Helicopter is defined

$$y = Cx + Du \tag{2.7}$$

$$x^T = [\theta(t),\ \psi(t),\ \theta(\dot{t}),\ \psi(\dot{t})] \tag{2.8}$$

and the output vector is

$$y^T = [\theta(t),\ \psi(t)] \tag{2.9}$$

Where θ and ψ are the pitch and yaw angles, respectively. The corresponding helicopter state-space matrices (as derived in the Maple worksheet) are

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-B_p}{J_{Tp}} & 0 \\ 0 & 0 & 0 & \frac{-B_y}{J_{Ty}} \end{bmatrix} \tag{2.10}$$

$$B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{PP}}{J_{Tp}} & \frac{K_{py}}{J_{Tp}} \\ \frac{K_{yp}}{J_{Ty}} & \frac{K_{yy}}{J_{Ty}} \end{bmatrix} \tag{2.11}$$

$$J_{Tp} = J_{eq_{-p}} + m_{heli} l_{cm}^2 \tag{2.12(a)}$$

$$J_{Ty} = J_{eq_{-y}} + m_{heli} l_{cm}^2 \tag{2.12(b)}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.13}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2.14}$$

So, the final equations of the non linear model can be given by equations 2.15 and 2.16.

$$\theta = \frac{K_{pp}.V_{m\_p}}{J_{eq\_p}+m_{heli}.l_{cm}^2} + \frac{K_{py}.V_{m\_y}}{J_{eq\_p}+m_{heli}.l_{cm}^2}$$

$$-\frac{m_{heli}.g.\cos\theta(t).l_{cm}+B_p\left(\frac{d}{dt}\theta(t)\right)+m_{heli}.\left(\frac{d}{dt}\Psi(t)\right)^2.\sin\theta(t).\cos\theta(t).l_{cm}^2}{J_{eq\_p}+m_{heli}.l_{cm}^2} \tag{2.15}$$

$$\ddot{\Psi}=\frac{K_{yp}.V_{m\_p}}{J_{eq\_y}+m_{heli}.\cos\theta(t)^2\,l_{cm}^2}+\frac{K_{yy}.V_{m\_y}}{J_{eq\_y}+m_{heli}.\cos\theta(t)^2\,l_{cm}^2}$$

$$+\frac{2.m_{heli}.\sin\theta(t).\cos\theta(t).l_{cm}^2.\frac{d}{dt}\theta(t).\frac{d}{dt}\Psi(t)}{J_{eq\_y}+m_{heli}.\cos\theta(t)^2\,l_{cm}^2}$$

$$-\frac{B_y.\frac{d}{dt}\Psi(t)}{J_{eq\_y}+m_{heli}.\cos\theta(t)^2 l_{cm}^2} \tag{2.16}$$

2.5.2 <u>Controller model</u>

In this section a state-feedback controller is designed to regulate the elevation and travel angles of the 2 DOF Helicopter to desired positions. However, as will be shown, the control structure is basically linear proportional-integral derivative, i.e. PID, controller. The control gains are computed using the Linear-Quadratic Regulator algorithm [48].

The state-feedback controller entering the front motor, u_p , and the back motor, u_y, is defined

$$\begin{bmatrix} u_p \\ u_y \end{bmatrix} = K_{PD}(x_d - x) + V_i \;+\; \begin{bmatrix} u_{ff} \\ 0 \end{bmatrix} \tag{2.17}$$

with the proportional-derivative gain

$$K_{PD} = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} \end{bmatrix} \tag{2.18}$$

the desired state

$$x_d^T = [\theta_d \;\; \psi_d \;\; 0 \;\; 0\,] \tag{2.19}$$

the integral control

$$V_i = \begin{bmatrix} k_{1,5} \int(x_{d,1} - x_1)dt + k_{1,6} \int(x_{d,2} - x_2)dt \\ k_{2,5} \int(x_{d,1} - x_1)dt + k_{2,6} \int(x_{d,2} - x_2)dt \end{bmatrix} \quad (2.20)$$

and the nonlinear feed-forward control

$$u_{ff} = \frac{K_{ff}\, m_{heli} g l_{cm} \cos x_{d,1}}{K_{pp}} \quad (2.21)$$

2.5.3 Linear quadratic regulator

$$\dot{x} = Ax + Bu \quad (2.22)$$

$$x(0) = x_0 \quad (2.23)$$

$$y = Cx + Dy \quad (2.24)$$

The LQR problem for the system is to find a control input vector u:

$$u = -kx \quad (2.25)$$

To minimize the performance index J:

$$J = \int_0^\infty [(Cx + Du)^T\, Q_y (Cx + Du) + u^T R_u u]dt \quad (2.26)$$

$$= \int_0^\infty [x^T\,(C^T Q_y C)x + 2x^T(C^T Q_y D)u + u^T(D^T Q_y D + R_u)u]dt \quad (2.27)$$

$$= \int_0^\infty [(x^T\, Qx + 2x^T Nu + u^T Ru)\}dt \quad (2.28)$$

$$\int_0^\infty [F(u, x, t)dt \quad (2.29)$$

The control law has to be a function of x (t)

Pontryagin's maximum principle:

We can define Control problem as:

$$F = (u, x, t) + \lambda^T(Ax + Bu) \tag{2.30}$$

$$=(x^T Qx + 2x^T Nu + u^T Ru) + \lambda^T(Ax + Bu) \tag{2.31}$$

The Euler-Lagrange equation gives the necessary condition for a local minimum:

$$\frac{\partial H}{\partial x}=\dot{\lambda} = -(2Qx + 2Nu + A^T\lambda) \tag{2.32}$$

And optimal control equation is given as:

$$u^* = \frac{\partial H}{\partial u}=2N^T x + 2Ru + B^T\lambda) \tag{2.33}$$

Ricatti transformation to eliminate λ is assumed as:

$$\lambda = 2(Px) \tag{2.34}$$

Where P^T=P is Ricatti matrix

Now optimal control is

$$u^* = -R^{-1}(N^T x + B^T Px) \tag{2.35}$$

$$u^* = u_x{}^* \tag{2.36}$$

$$u^* = -K_x{}^* x \tag{2.37}$$

Optimal state feedback controller gain $K_x{}^*$ is given by

$$K_x{}^* = -R^{-1}(N^T + B^T P) \tag{2.38}$$

Ricatti Equation

Derivative of Ricatti transformation is

$$\dot{\lambda} = 2\dot{P}x + P\dot{x} \tag{2.39}$$

Using value of λ in Euler Lagrange equation

P matrix must satisfy the reduced form of the standard Riccati equation

$$\dot{P} + PA + A^{T}P - (PB + N)R^{-1}(N^{T} + B^{T}P) + Q = 0 \tag{2.40}$$

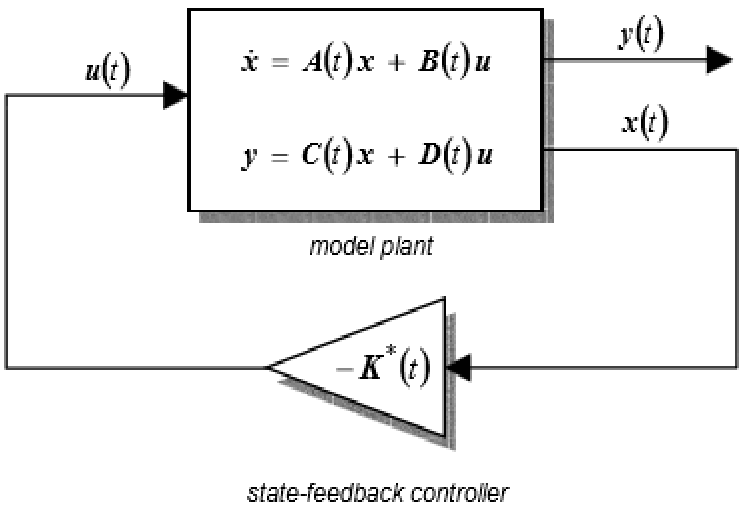

Fig.2.7: LQR control scheme for system

The control gains are computed using the Linear-Quadratic Regulator scheme. The system state is first augmented to include the integrals of the pitch and yaw states

$$x_i^T = [\theta \quad \psi \quad \dot{\theta} \quad \dot{\psi} \quad \int \theta dt \quad \int \lambda dt\,] \tag{2.41}$$

Using the feedback law

$$u = -Kx_i \tag{2.42}$$

the weighting matrices

$$Q = \begin{bmatrix} 200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 200 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix} \qquad (2.43)$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (2.44)$$

and the state-space matrices (A,B) found previously, the control gain

$$K = \begin{bmatrix} 18.9 & 1.98 & 7.48 & 1.53 & 7.03 & 0.770 \\ -2.22 & 19.4 & -0.45 & 11.9 & -0.770 & 7.03 \end{bmatrix} \qquad (2.45)$$

is calculated by minimizing cost function:

$$J = \int_0^\infty x_i^T \, Q x_i + u^T R u dt \qquad (2.46)$$

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1.3} & k_{1.4} & k_{1.5} & k_{1,6} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2.6} \end{bmatrix} \qquad (2.47)$$

CHAPTER 3

IMPLEMENTATION OF CONTROLLERS

3.1 INTRODUCTION

The two DOF model of helicopter is tested with different controllers. The controllers used are LQR+I, PID and Fuzzy. The results for every controller are tested on simulink MATLAB and on the real time system of two DOF helicopter model.

Figure 3.1 shows the simulation model of two DOF helicopter, it comprises of two controller blocks, one is for simulation in MATLAB and other is for real time system. The reference angles of pitch and yaw with unit radian are given from the top input block and the corresponding reference voltage is given from the below input block. The results are analyzed and the scopes and displays are in the simulation block named scopes. A control switch is provided in the simulation which controls the control algorithm i.e. 1 for LQR, 2 for LQR+I, 3 for pitch open loop and 4 for yaw open loop .

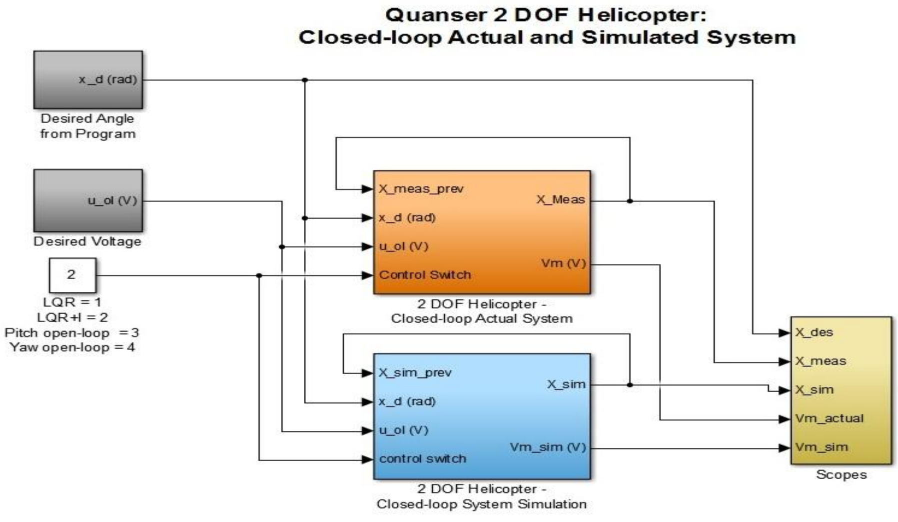

Fig 3.1: Two DOF helicopter model

3.2 LQR + I+ FF CONTROLLER

Figure 3.2 shows the subsystem of LQR+I+FF. The first block represents the feed forward control which is implemented only on pitch which is shown in figure 3.3. The output of feed forward block is the multiplication of pitch feed forward gain with cosine of pitch reference value.

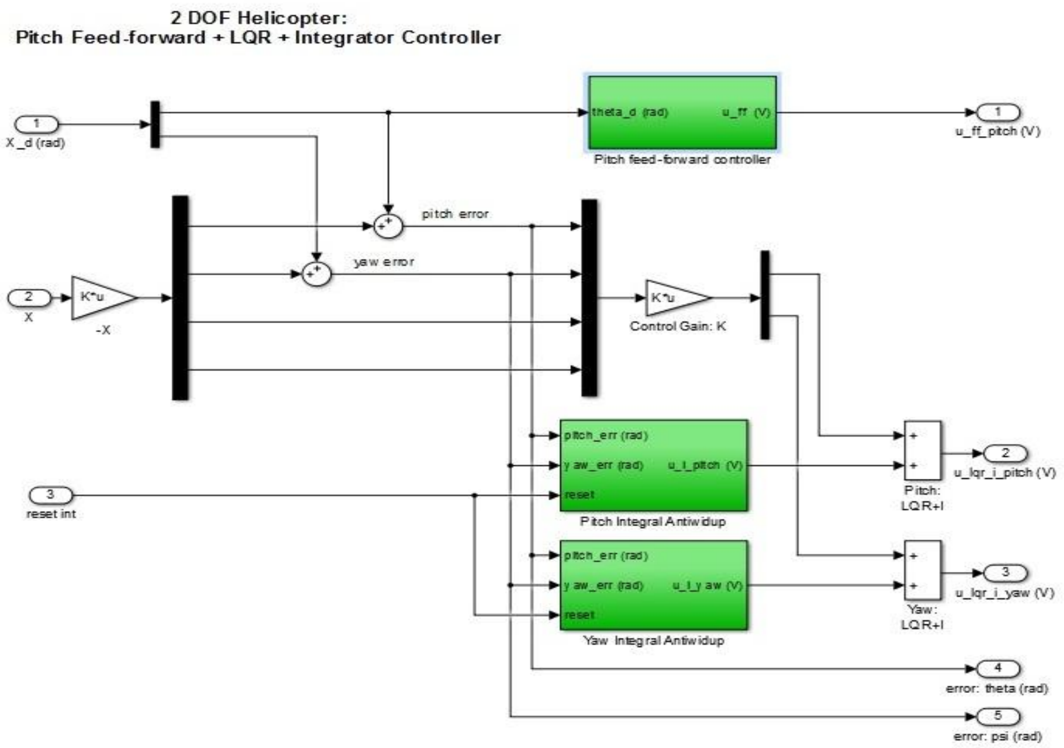

Fig 3.2: Model of LQR+FF+I Controller

Second part of figure 3.2 represents the Linear Quadratic Regulator. All the state variables are fed back. The reference pitch value and fed back pitch value is compared and the error in pitch is given as the first input, then the reference of yaw and feedback yaw is compared and given as second input, the third and fourth inputs are the change in pitch angle and change in yaw angle respectively. All of these inputs are then multiplied by LQR controller gain matrix which gives the two controller outputs i.e. yaw controller output and pitch controller output.

The third part of this simulation is the integral controller. The inputs are pitch error and yaw error which is given to pitch integral and yaw integral control and the outputs of these controllers are then added to LQR outputs as shown in figure 3.2.

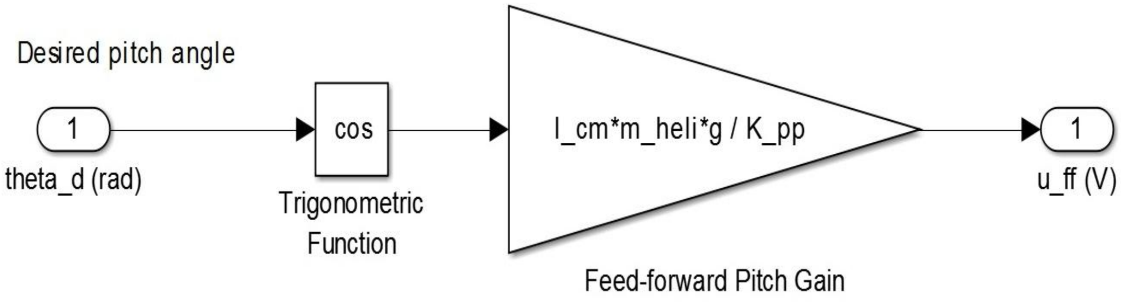

Fig 3.3: Feed Forward controller simulink model

These outputs are then given to non linear model as shown in figure 3.4 and non linear real time model as shown in figure 3.5. In figure 3.4 the designing is done according to the equations given in previous chapter. From these equations we get the symbols for yaw and symbol for pitch. Now these are integrated to get change in yaw and change in pitch and lastly by integrating these we get pitch and yaw angles.

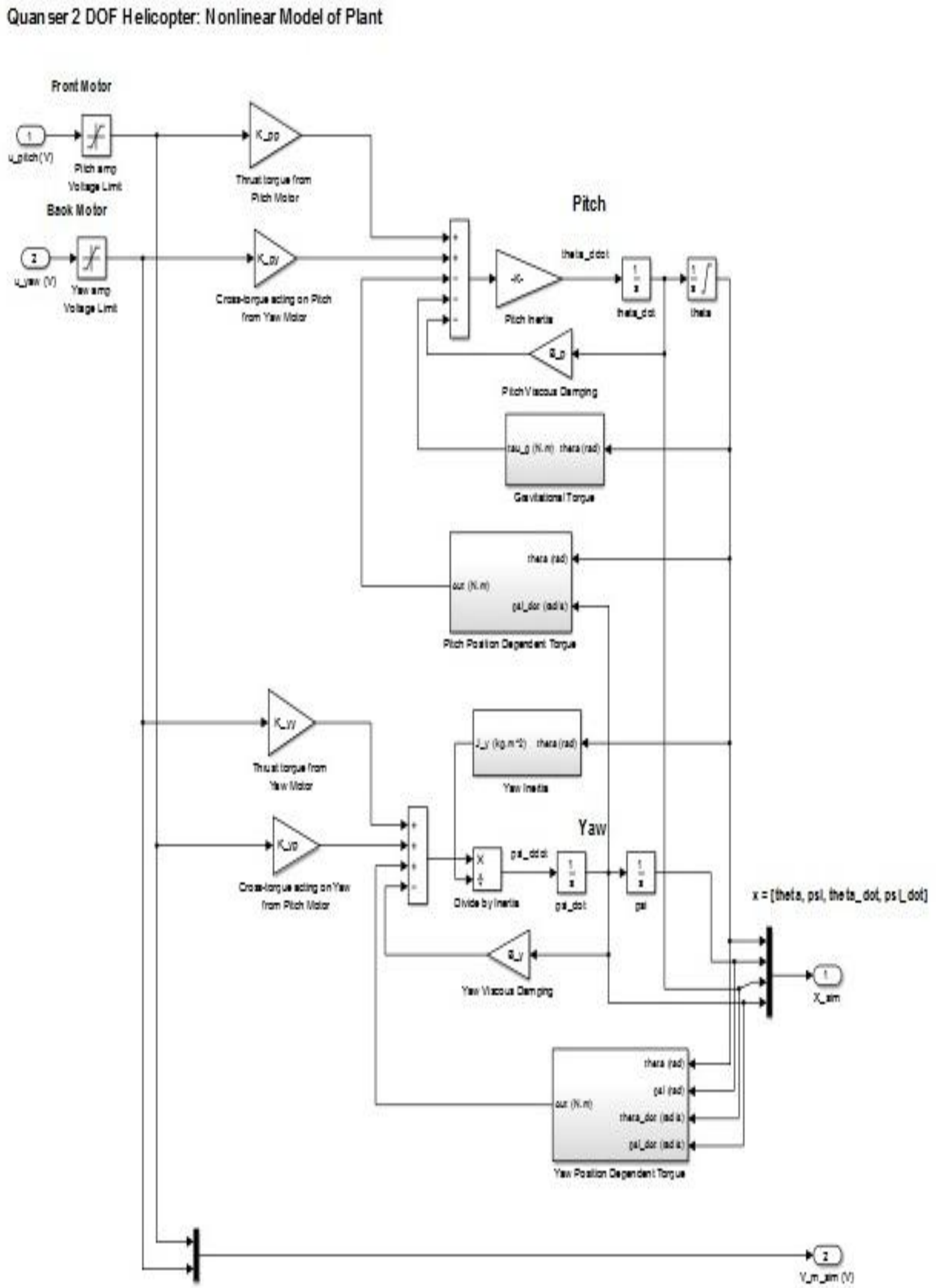

Fig 3.4: Non Linear model simulation modelling in simulink

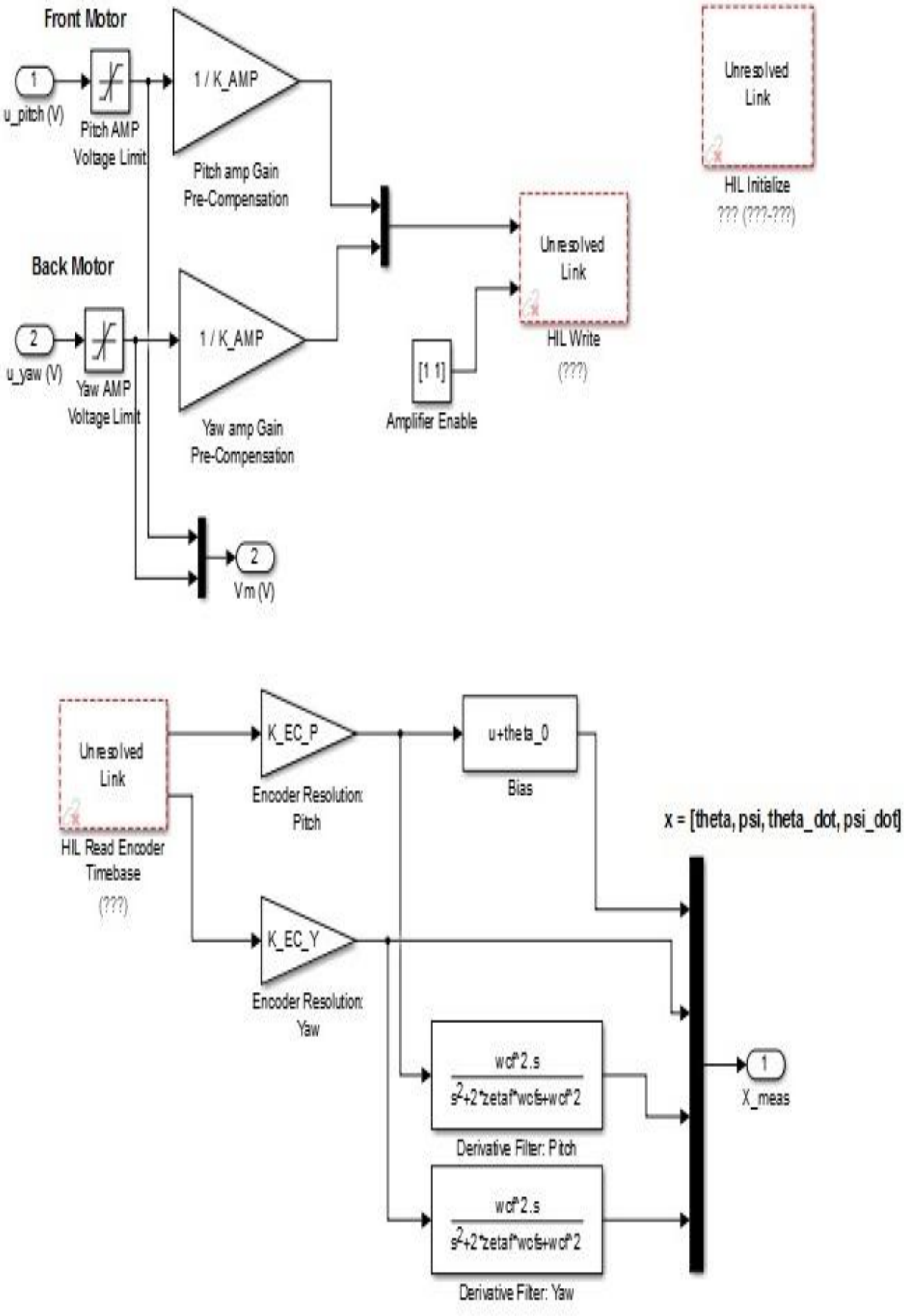

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Fig 3.5: Non linear real time model in simulink

3.3 PID CONTROLLER:

In PID controller the inputs are the change in pitch error and change in yaw error. The auto tune cannot be done because the model used in simulation is non linear model and the auto tune feature cannot be used in this case as this non linear model cannot be linearized. So the tuning is done by taking into consideration the effects of proportional, integral and differential control, which is explained in chapter 2. The simulation model showing the implementation of PID control along with LQR and feed forward is shown in figure 3.6.

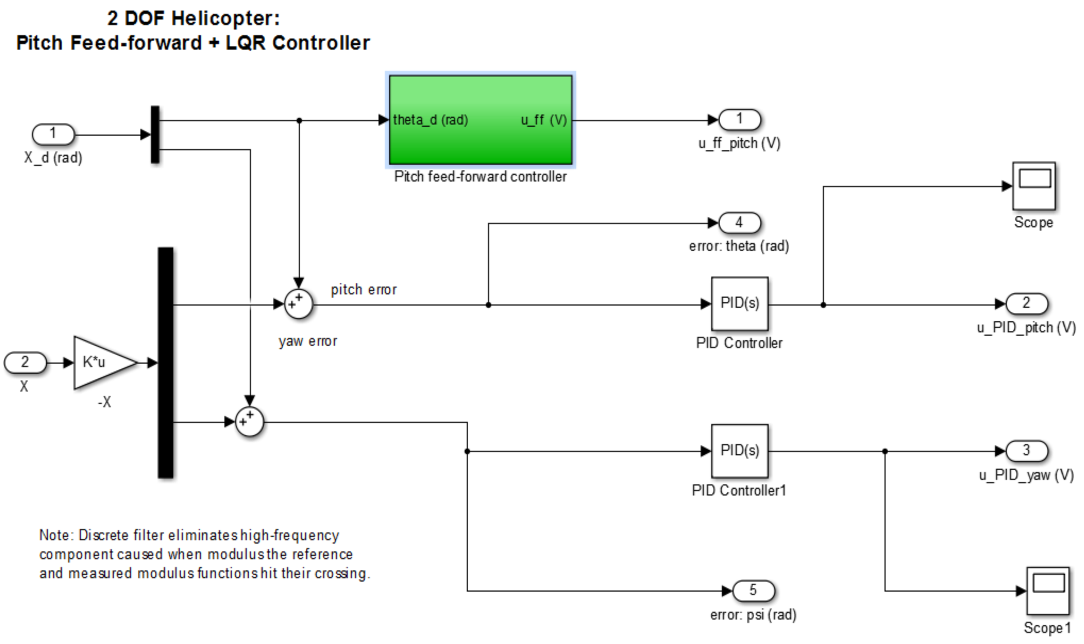

Fig 3.6: PID controller simulation model

The tuning for PID controller with which results are taken is given below:

<u>For Real Time System</u>

Pitch tuning:

Proportional Gain =0.08

Integral Gain =10

Differential Gain =0.0005

Yaw tuning:

Proportional Gain =5

Integral Gain =9

Differential Gain =3.8

For Simulation Model

Pitch tuning:

Proportional Gain =20

Integral Gain =1

Differential Gain =0.1

Yaw tuning:

Proportional Gain =60

Integral Gain =15

Differential Gain =0.1

3.4 FUZZY CONTROLLER:

LQR and integral controller is replaced with the fuzzy control system as shown in figure below. The input to the fuzzy controller are yaw error and change in yaw for yaw fuzzy controller and pitch error and change in pitch angle for fuzzy pitch controller. The simulation model with fuzzy logic controller is shown in figure 3.7

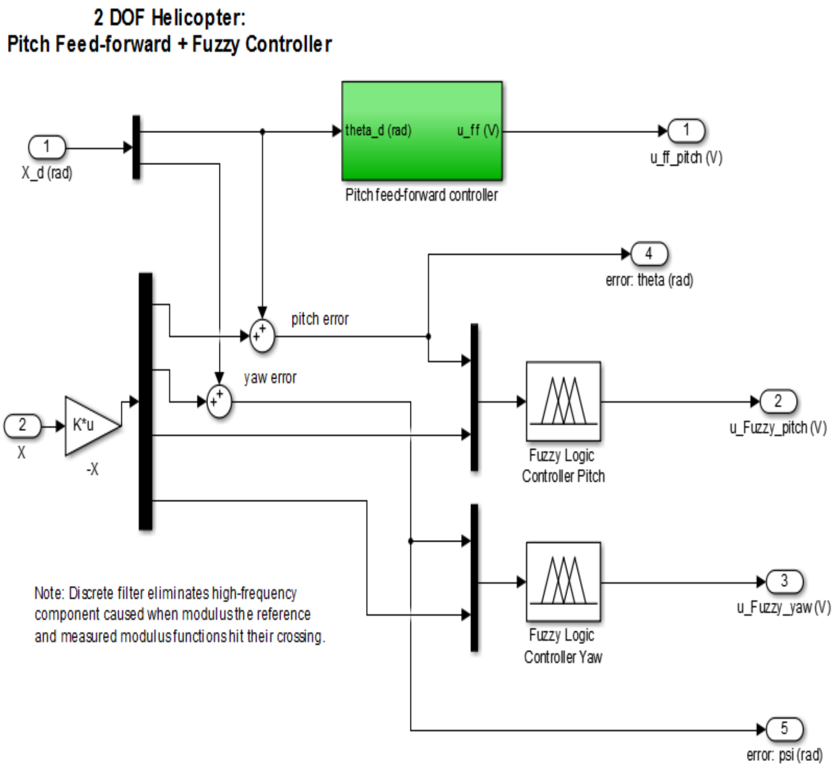

Fig 3.7: Fuzzy logic controller simulation model

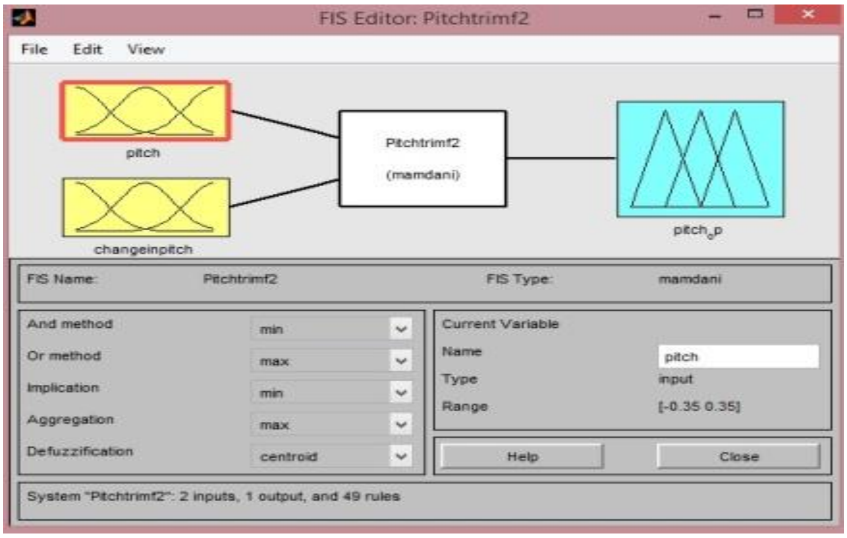

Fig 3.8(a): Fuzzy Inference System

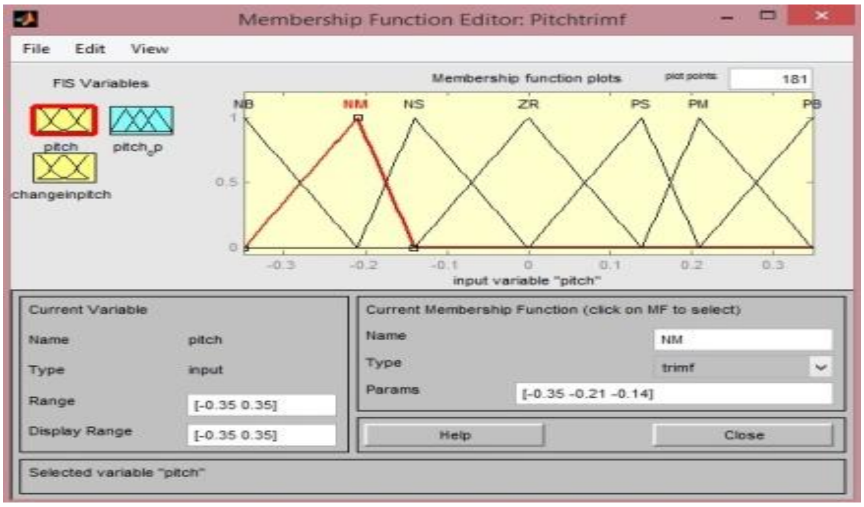

Fig 3.8(b): Membership function for pitch error with range -0.35 to 0.35

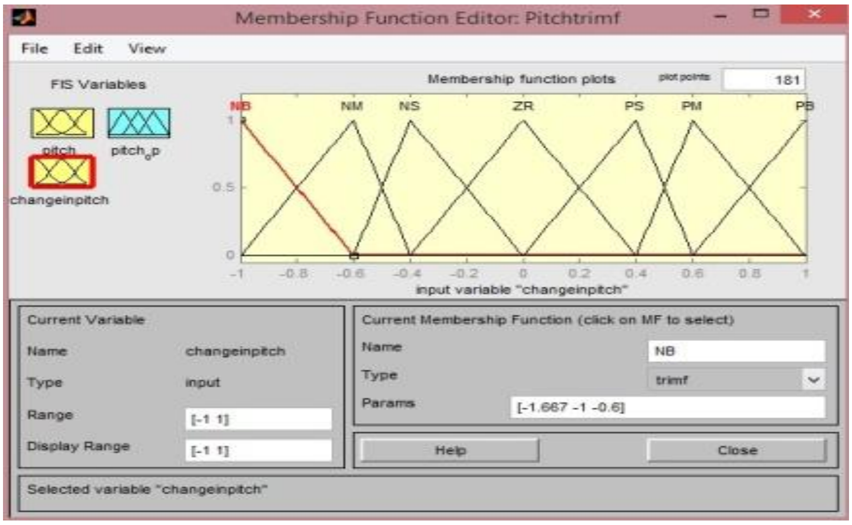

Fig 3.8(c): Membership function for change in pitch with range -1 to 1

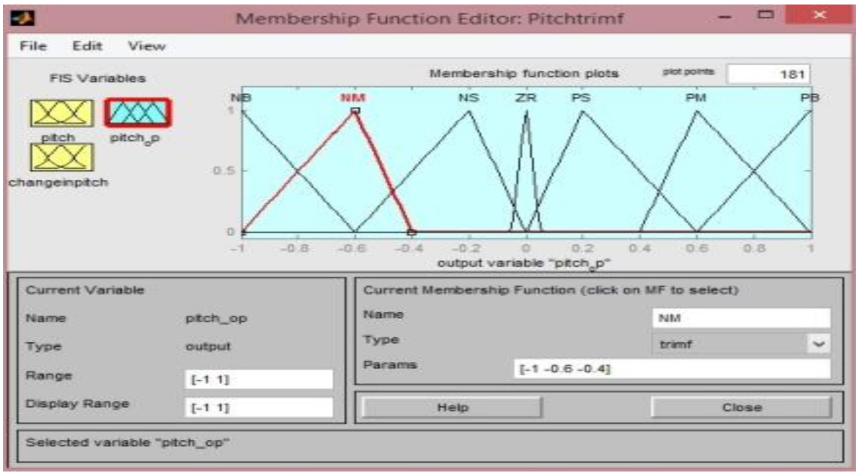

Fig 3.8(d): Membership function for output of controller

Figure 3.8(a) shows the fuzzy inference system, it is the masked system in which all the inputs outputs membership functions are defined along with their type and ranges, rule base is created in this system which is the essence of the fuzzy logic controller and finally the rule view and surface view can be analyzed by this system. Figure 3.8(b) and 3.8(c) shows the membership function for two inputs of the fuzzy inference system, the membership function chosen is the triangular membership function, for pitch error the range is defined -0.35 to 0.35 and for change in pitch angle the range is selected from -1 to 1, which was observed by analyzing the real time system model in MATLAB. Figure 3.8(d) shows the output of the controller with same membership function.

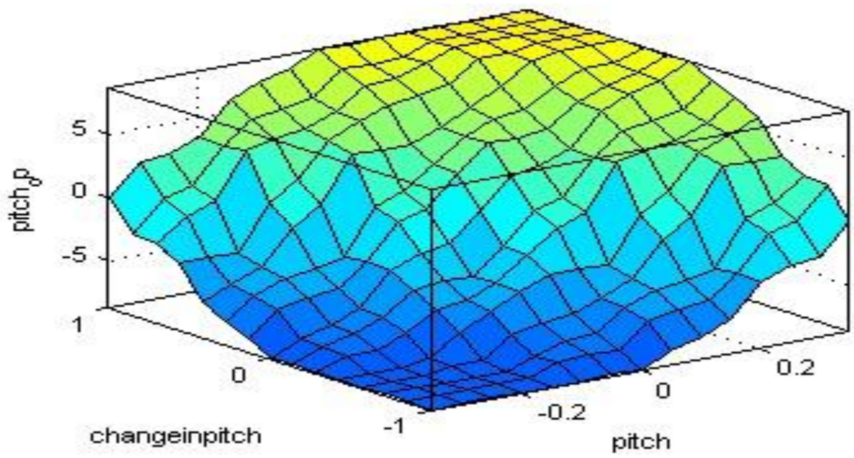

Fig 3.9: Surface view for pitch and yaw

Table 3.1: Rule base of the fuzzy inference system

E / $\dot{e}$	**NB**	**NM**	**NS**	**ZR**	**PS**	**PM**	**PB**
NB	NB	NB	NB	NB	NM	NS	ZR
NM	NB	NB	NB	NM	NS	ZR	PS
NS	NB	NB	NM	NS	ZR	PS	PM
ZR	NB	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PM	PB	PB
PM	NS	ZR	PS	PM	PB	PB	PB
PB	ZR	PS	PM	PB	PB	PB	PB

Above table shows the rule base of the fuzzy inference system. From real time system we can observe that if the pitch angle is negative then the motor controlling the pitch angle should give the high torque i.e. high voltage should be given to the motor which means that the controller output must be high. On the other hand if the pitch angle is positive or zero the controller output must be positive and zero respectively. So, according to this logic the rule base is defined and fed to the fuzzy inference system and the results for pitch and yaw controlling are obtained.

CHAPTER 4

RESULTS AND DISCUSSIONS

In this chapter the results for simulation and results are analyzed. The parameters for which the simulation and real time systems are analyzed for different controllers are rise time, settling time, peak overshoot, peak undershoot and steady state error.

Also, the results are taken for different non linearity such as dead-zone. In this chapter we have discussed the parameters of the results and comparison is done between them, then the results with nonlinearity are analyzed for real time system and simulation and finally comparative results are obtained for all the controllers in simulink MATLAB.

4.1 PARAMETER CALCULATION:

For the reference given -10 to 10 for pitch angle and yaw angle reference is given zero degrees, the given controllers are tested on real time system and simulation model. Figure 4.1 and 4.2 shows the output for pitch in real time system and simulation with three controllers that are LQR, PID and FLC respectively, and figure 4.4 and 4.5 shows the results for yaw for same controllers in real time and simulation with saturation non linearity. Similarly figure 4.7 and 4.8 shows the output for pitch in real time system and simulation with three controllers that are LQR, PID and FLC respectively, and figure 4.10 and 4.11 shows the results for yaw for same controllers in real time and simulation with deadzone non linearity.

4.1.1 PITCH OUTPUT WITH SATURATION NON LINEARITY

The results of pitch angle with saturation non linearity for different controllers are shown in the figures below and then the comparison of simulation with real time is done.

Simulation

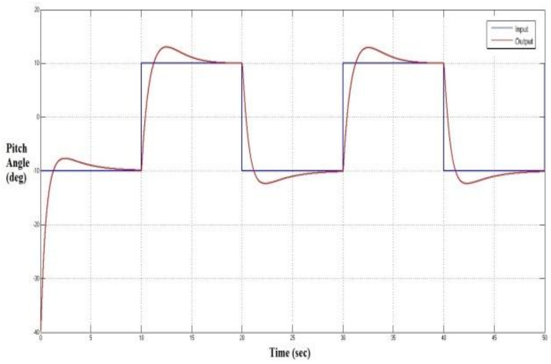

Fig 4.1(a): Pitch output with LQR controller

Real Time

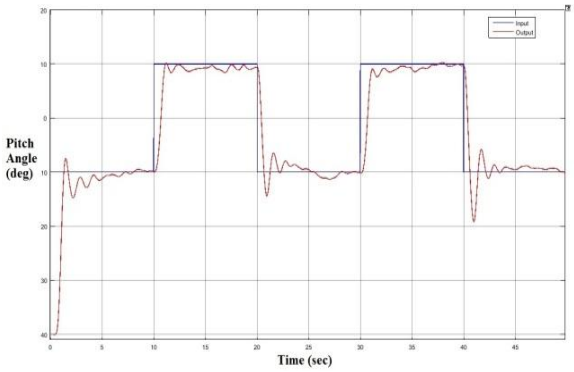

Fig 4.2(a): Pitch output with LQR controller

It can be observed from above results that in simulation the peak overshoot is greater than in the real time system.

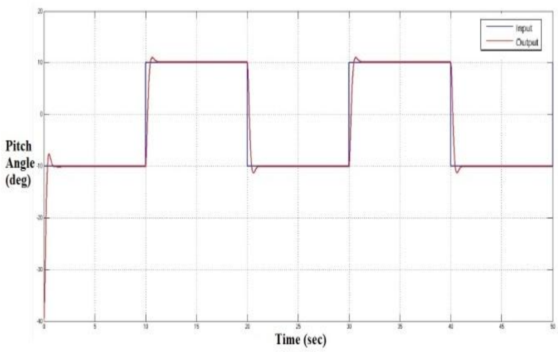

Fig 4.1(b): Pitch output with PID controller

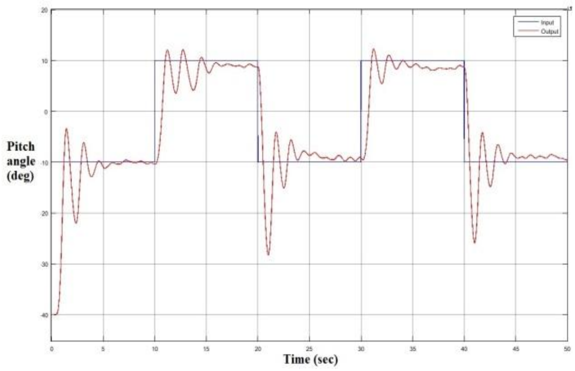

Fig 4.2(b): Pitch output with PID controller

It can be observed from above results that in simulation the vibrations are much more less than the real time system; also the overshoot is much larger in real time system as compared to simulation model.

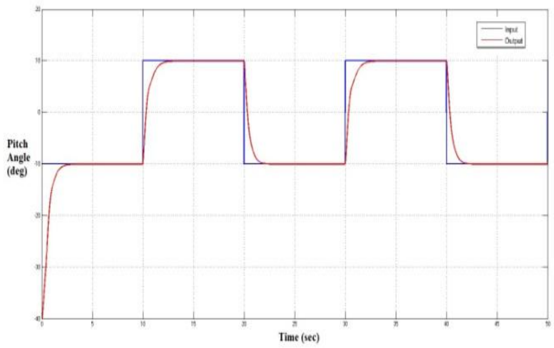

Fig 4.1(c): Pitch output with FLC controller

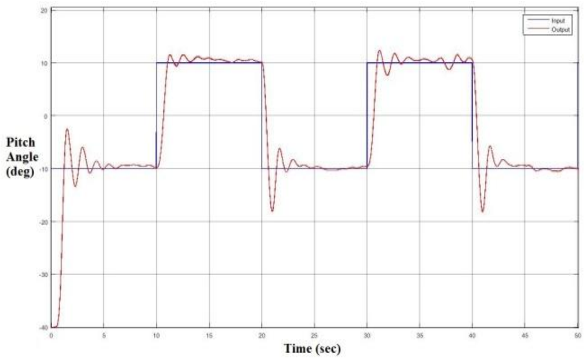

Fig 4.2(c): Pitch output with FLC controller

From above results of real time and simulation, it can be observed that simulation do not shows any overshoot and undershoot and also the steady state error is almost zero, on the other hand real time system have both overshoot and undershoot and also the steady state error is not negligible.

From figure 4.1(a), we can observe that in simulink model system have some overshoot and zero steady state error.

From figure 4.1(b) it can be observed that the system do not settles for the long time and even after settling persist vibrations, but the overshoot and rise time is satisfactory.

Figure 4.1(b) and 4.2(b) shows the response of the system with PID controller which is tuned with the parameters as shown below.

Proportional Gain (real) = 0.08 Proportional Gain (simulation) = 20

Integral Gain (real) = 10 Integral Gain (simulation) = 1

Differential Gain (real) = 0.0005 Differential Gain (simulation) = 0.1

The response has less steady state error as compared to LQR controlled system and also the settling time is reduced very much as compared to LQR controlled system, but the peak over shoot and undershoot values are increased significantly and the number of oscillations of system increased before it reaches its steady state. On the other hand if we compare the simulation models from figure 4.1(b) and 4.2(b), we can observe that overshoot, rise time and settling time are reduced significantly.

The Fuzzy Logic Controlled system response is shown in figure 4.3. It can be seen from the result that the oscillation before steady state are more if compared with LQR controlled system and less if compared with PID controlled system. The steady state error is reduced to almost zero which is significant in LQR and PID.

By comparing results in figure 4.1 and figure 4.2, we can say that the best results are obtained from fuzzy logic controller as it have the zero steady error and zero overshoots and under shoots.

The tables 4.1 and 4.2 below shows the pitch parameter comparison for three controllers for real time system and simulation model respectively.

Table 4.1: Time domain parameters of Pitch with LQR, PID and FLC on real time system with saturation non linearity

Parameters \ Controllers	LQR	PID	Fuzzy
Steady State Error(e_{ss})	0.6 degree	0.5 degrees	0.04 degrees
Settling time(t_s)	10 seconds	6.5 seconds	5 seconds
Peak Undershoot(U_p)	3.6 degree	10 degrees	2.3 degrees
Rise time (t_r)	1.2 seconds	1.15 seconds	1.2 seconds
Peak Overshoot (M_p)	2.8 degrees	4 degrees	6 degrees

The steady state error, settling time and peak undershoot are minimum in FLC, on the other hand rise time is less for PID and peak overshoot is minimum is minimum for LQR.

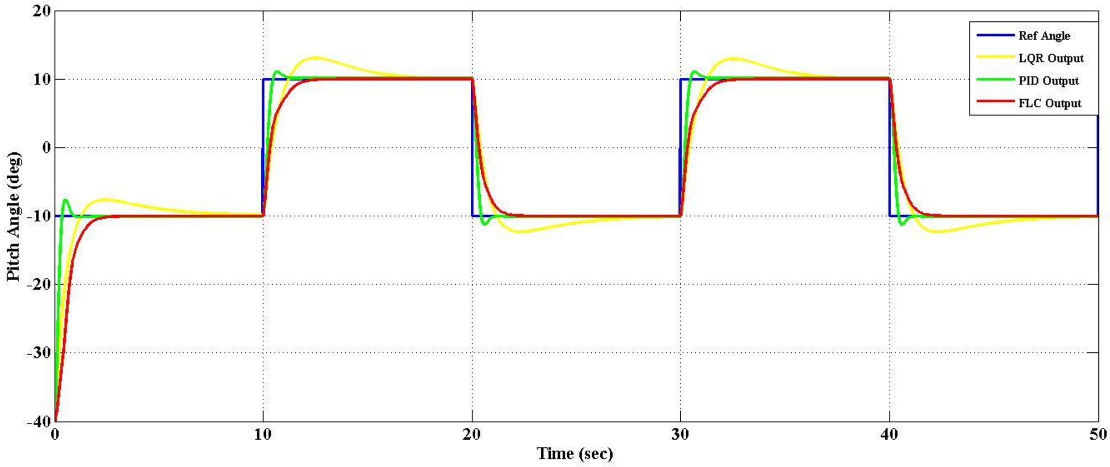

Fig 4.3: Combined result of Pitch with LQR, PID and FLC for calculation of time domain parameters on simulation model with saturation

Table 4.2: Time domain parameters of Pitch with LQR, PID and FLC on simulation model with saturation non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Steady State Error(e_{ss})	0 degree	0.03 degrees	0 degrees
Settling time(t_s)	6 seconds	0.7 seconds	2 seconds
Peak Undershoot(U_p)	0 degree	0.17 degrees	0 degrees
Rise time (t_r)	1.15 seconds	0.3 seconds	1.4 seconds
Peak Overshoot (M_p)	9.3 degrees	2.3 degrees	0 degrees

The steady state error, peak undershoot and overshoot are almost negligible for FLC, settling time and rise time are minimum for PID.

4.1.2 YAW OUTPUT WITH SATURATION NON LINEARITY

The results of yaw angle with saturation non linearity for different controllers are shown in the figures below and then the comparison of simulation with real is done

Simulation **Real Time**

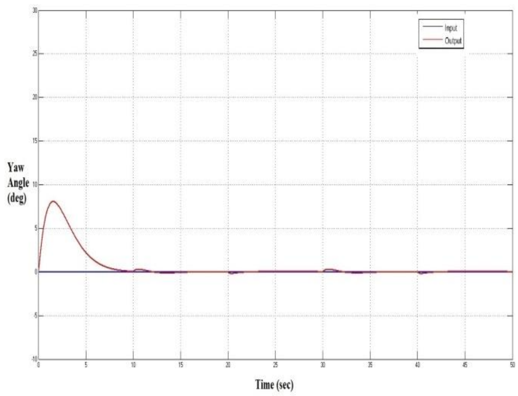

Fig 4.4(a): Yaw output with LQR controller

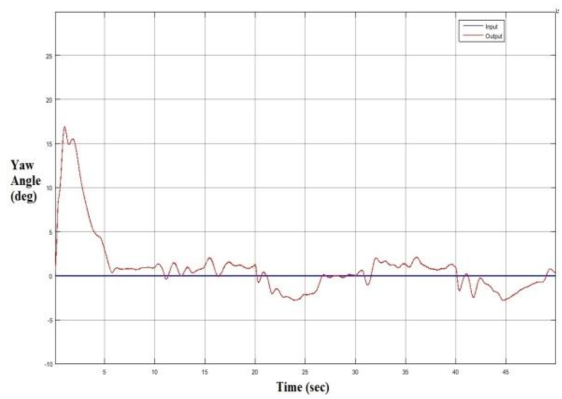

Fig 4.5(a): Yaw output with LQR controller

The steady state achieved in simulation is faster than in the real time system and also the disturbances are lot more in real time system than in simulation.

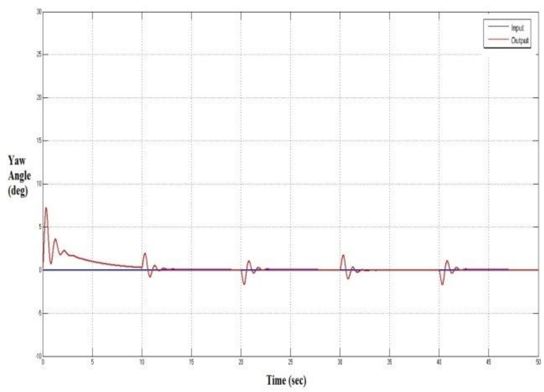

Fig 4.4(b): Yaw output with PID controller

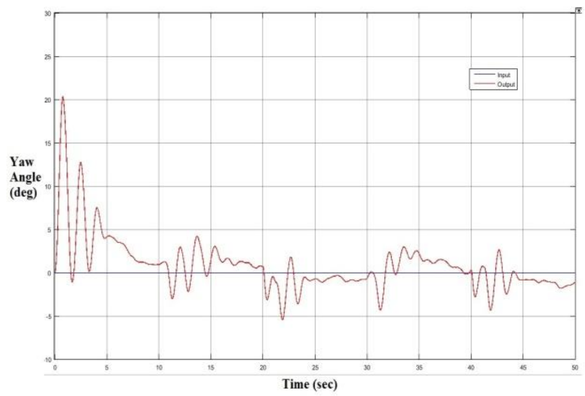

Fig 4.5(b): Yaw output with PID controller

From above result it can be observed that in the transient state the vibrations in real time are very high in real time system and in simulation these vibrations are less.

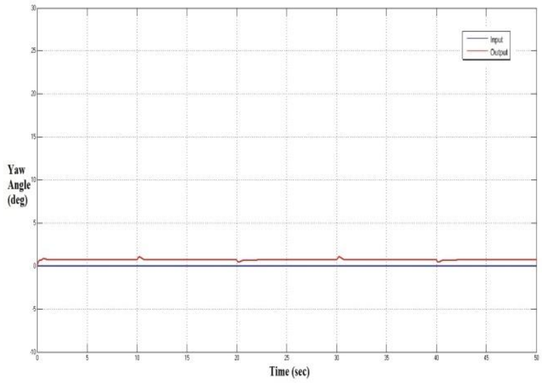

Fig 4.4(c): Yaw output with FLC controller

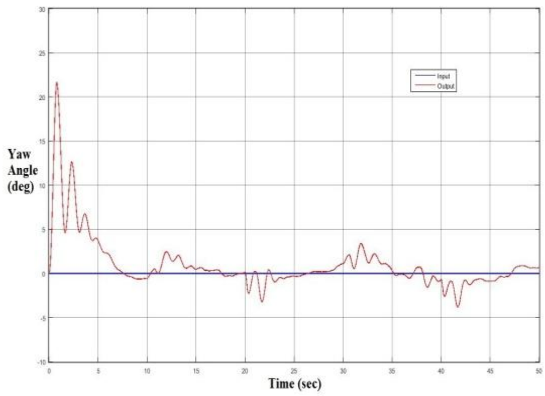

Fig 4.5(c): Yaw output with FLC controller

From above results with fuzzy logic controller it can be seen that in simulation the steady state error is almost constant with simulation but un real time system the vibrations are present.

The tables 4.3 and 4.4 below shows the yaw parameter comparison for three controllers for real time system and simulation model respectively.

Table 4.3: Time domain parameters of Yaw with LQR, PID and FLC on real time system with saturation non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Steady State error(e_{ss})	0.5 degrees	0.6 degrees	0.09 degrees
Settling time (t_s)	15 seconds	15 seconds	8 seconds

It can be seen from the above table that for the real time system the settling time for LQR controlled and PID controlled system is 15 seconds, which is reduced significantly by implementing Fuzzy Logic Controller to 8 seconds. Also the steady state error is also reduces from 0.5-0.6 degrees with LQR and PID controllers to 0.09 degrees with FLC.

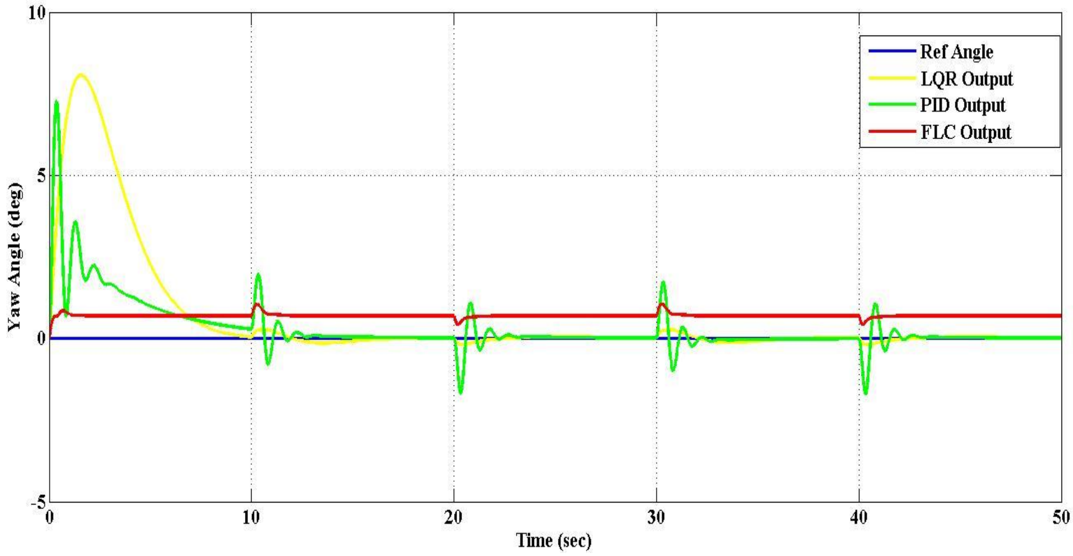

Fig 4.6: Combined result of Yaw with LQR, PID and FLC for calculation of time domain parameters on simulation model with saturation

Table 4.4: Time domain parameters of Yaw with LQR, PID and FLC on simulation model with saturation non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Steady State error(e_{ss})	0 degrees	0 degrees	0.69 degrees
Settling time (t_s)	7 seconds	12 seconds	1.1 seconds

In simulation, steady state error is almost zero for LQR and PID. The settling time of the system is reduced significantly to a value of 1.1 seconds with Fuzzy Logic Controller.

4.1.3 <u>PITCH OUTPUT WITH DEADZONE NON LINEARITY</u>

Now all the controllers tested above are with the non linearity saturation. All the controllers are tested with the dead zone non linearity to test the performance of the system with different non linearity. The results of pitch angle with deadzone non linearity for different controllers are shown in the figures below and then the comparison of simulation with real is done.

Simulation	**Real Time**

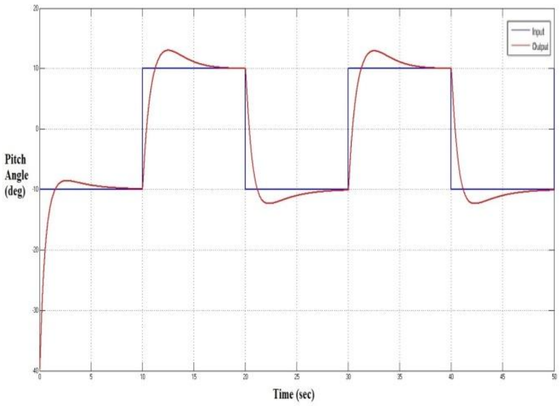

Fig 4.7(a): Pitch output with LQR controller

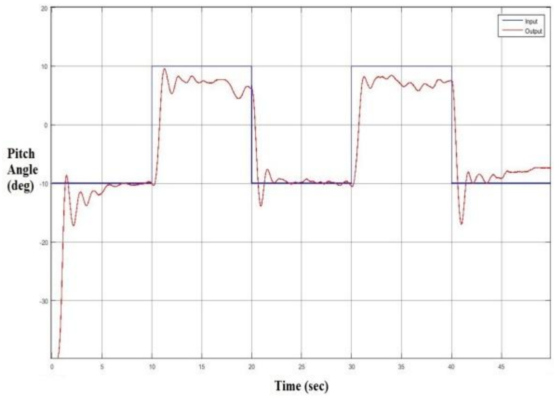

Fig 4.8(a): Pitch output with LQR controller

The steady state achieved in simulation is faster than in the real time system and also the disturbances are lot more in real time system than in simulation, also in real time the system do not achieve the set point on the other hand in simulation the response is very good.

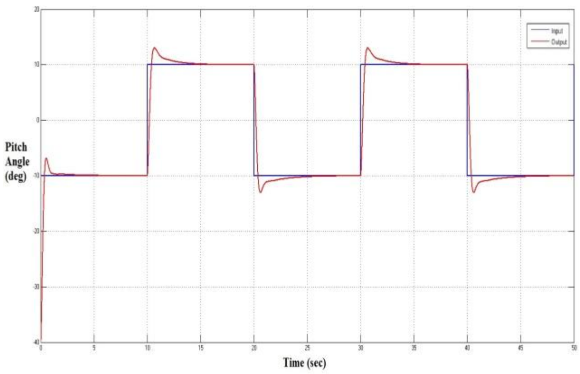

Fig 4.7(b): Pitch output with PID controller

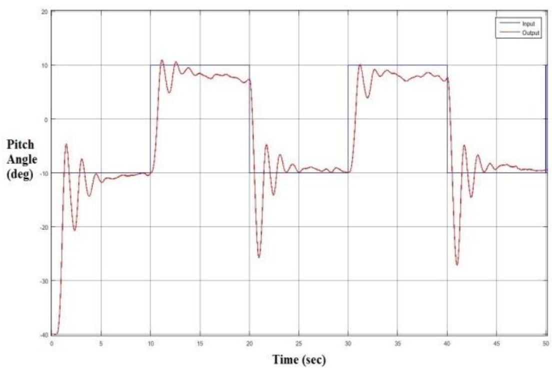

Fig 4.8(b): Pitch output with PID controller

From above result it can be observed that in the transient state the vibrations in real time are very high in real time system and in simulation these vibrations are less

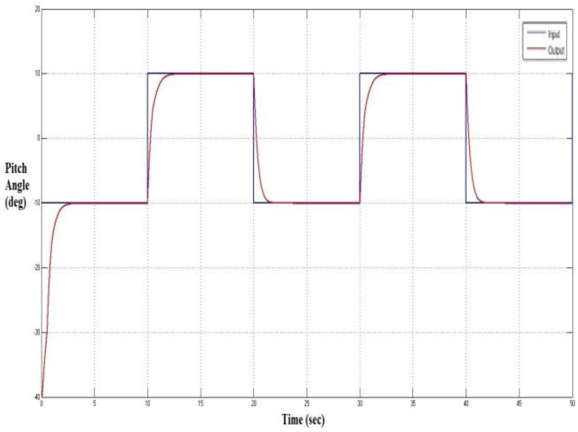

Fig 4.7(c): Pitch output with FLC controller

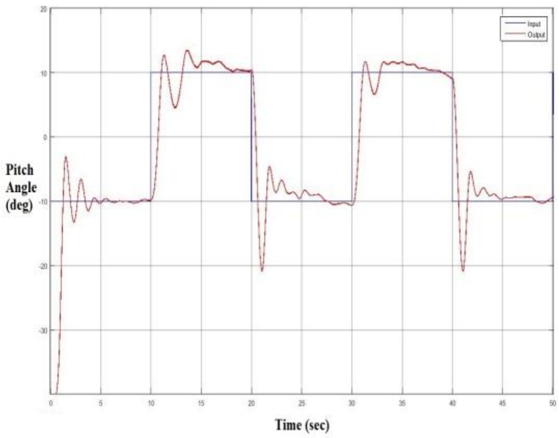

Fig 4.8(c): Pitch output with FLC controller

From above results with fuzzy logic controller it can be seen that in simulation the steady state error is almost constant with simulation but un real time system the vibrations are present, also the settling time is lot more less in simulation as compared to real time system.

The tables 4.5 and 4.6 below shows the pitch parameter comparison for three controllers for real time system and simulation model respectively.

Table 4.5: Time domain parameters of Pitch with LQR, PID and FLC on real time system with dead zone non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Steady State Error(e_{ss})	0.5 degree	0.5 degrees	0.05 degrees
Settling time(t_s)	5.5 seconds	7 seconds	5 seconds
Peak Undershoot(U_p)	7.5 degree	10 degrees	2.5 degrees
Rise time (t_r)	1.25 seconds	1.18 seconds	1.16 seconds
Peak Overshoot (M_p)	1 degrees	5 degrees	3.5 degrees

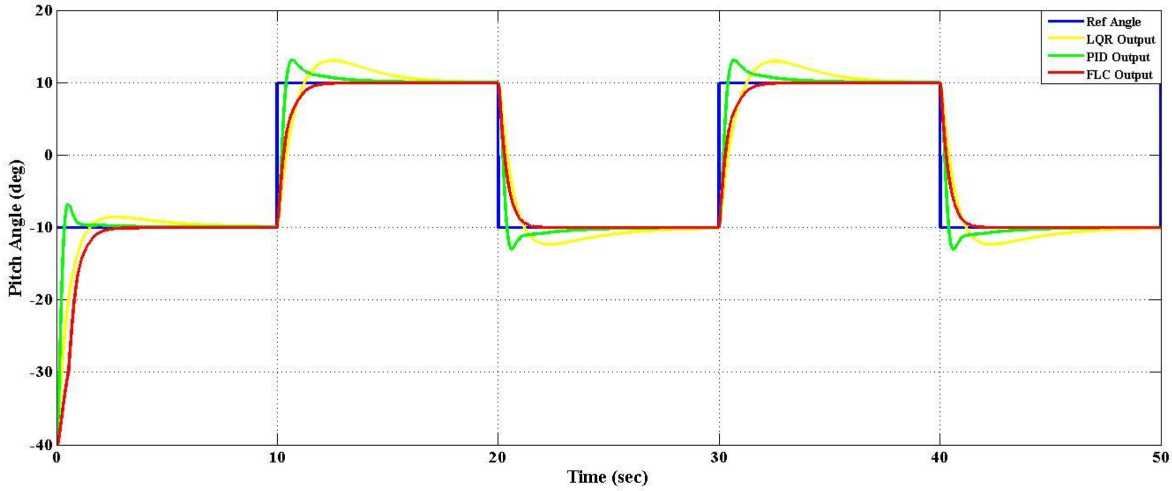

Fig 4.9: Combined result of Pitch with LQR, PID and FLC for calculation of time domain parameters on simulation model with deadzone

Table 4.6: Time domain parameters of Pitch with LQR, PID and FLC on simulation model with dead zone non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Steady State Error(e_{ss})	0 degree	0 degrees	0 degrees
Settling time(t_s)	5 seconds	1.15 seconds	2.4 seconds
Peak Undershoot(U_p)	0 degree	0 degrees	0 degrees
Rise time (t_r)	1.2 seconds	0.25 seconds	2 seconds
Peak Overshoot (M_p)	2 degrees	2 degrees	0 degrees

It can be observed from the results of pitch with dead zone that PID controlled system on real time persist maximum overshoot and under shoot, and fuzzy steady state error with FLC is minimum. On the other hand for simulation model the steady state error and undershoot is zero for all the controllers but the overshoot is almost zero for FLC controlled system, also rise time and settling time is minimum in PID.

4.1.4 <u>YAW OUTPUT WITH DEADZONE NON LINEARITY</u>

The results of yaw angle with deadzone non linearity for different controllers are shown in the figures below and then the comparison of simulation with real is done.

<u>Simulation</u> **<u>Real Time</u>**

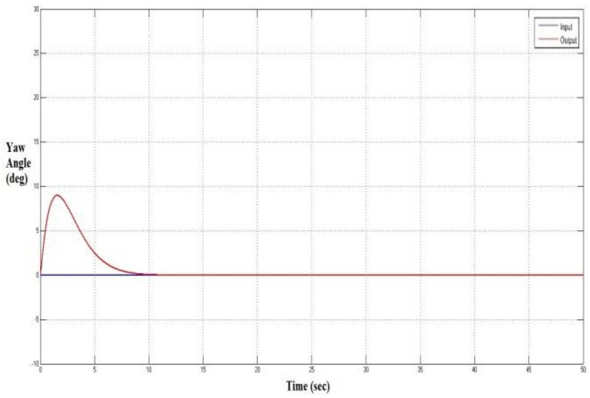

Fig 4.10(a): Yaw output with LQR controller

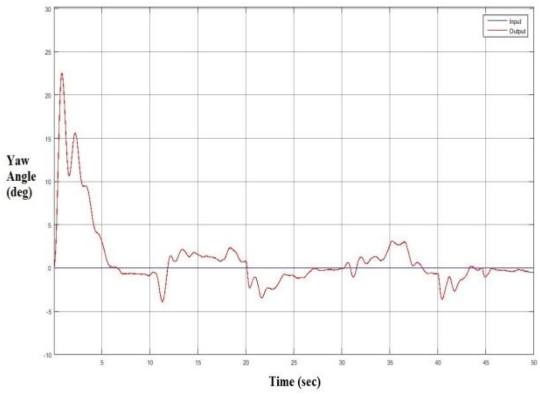

Fig 4.11(a): Yaw output with LQR controller

The steady state achieved in simulation is faster than in the real time system and also the disturbances are lot more in real time system than in simulation, also the steady state error is almost zero in simulation but noticeable in real time system.

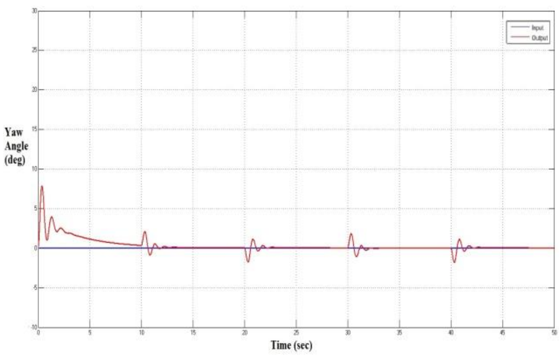

Fig 4.10(b): Yaw output with PID controller

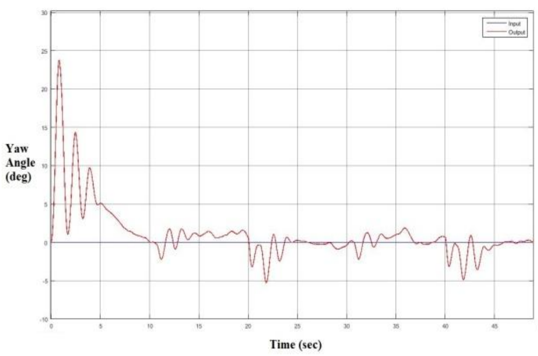

Fig 4.11(b): Yaw output with PID controller

From above result it can be observed that in the transient state the vibrations in real time are very high in real time system and in simulation these vibrations are less.

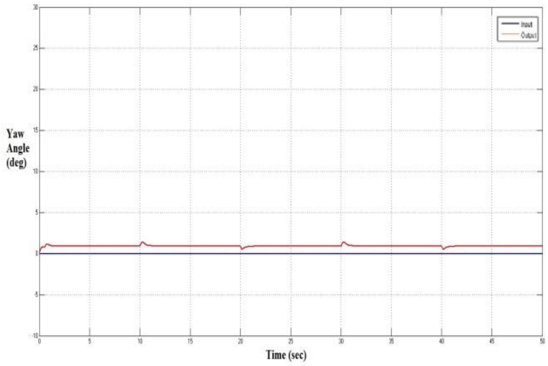

Fig 4.10(c): Yaw output with FLC controller

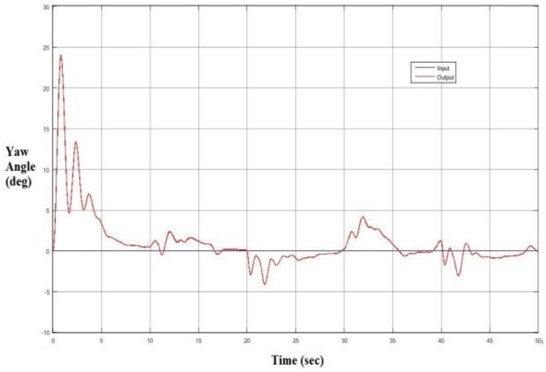

Fig 4.11(c): Yaw output with FLC controller

From above results with fuzzy logic controller it can be seen that in simulation the steady state error is almost constant with simulation but in real time system the vibrations are present.

The tables 4.7 and 4.8 below shows the yaw parameter comparison for three controllers for real time system and simulation model respectively.

Table 4.7: Time domain parameters of Yaw with LQR, PID and FLC on real time system with dead zone non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Settling time (t_s)	10 seconds	9 seconds	7 seconds
Steady State error(e_{ss})	4 degrees	5 degrees	4 degrees

It can be observed from the above table that for real time the results are almost similar for steady state error, the only parameter reduced is the settling time for FLC controlled system.

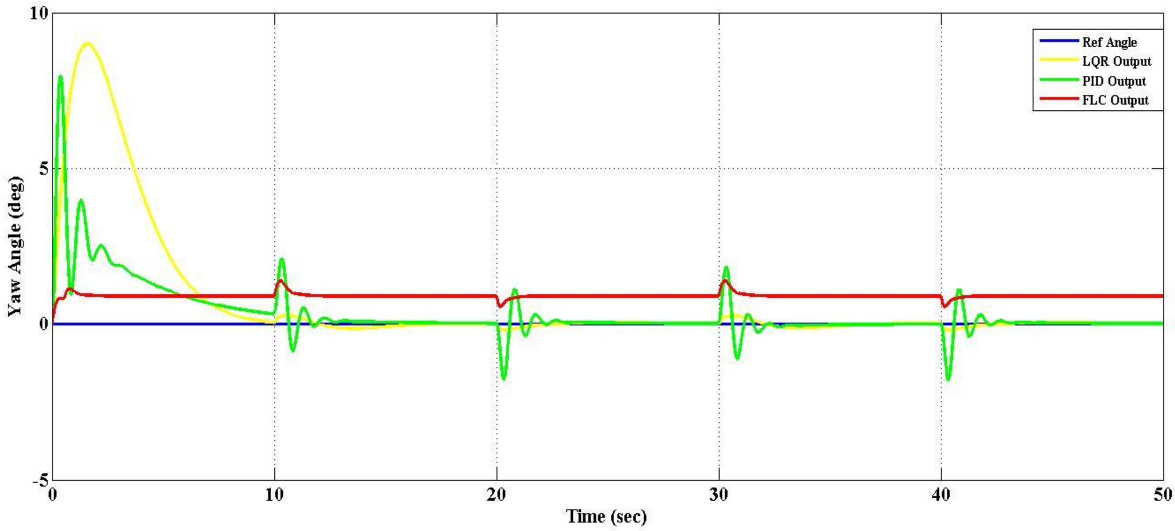

Fig 4.12: Combined result of Yaw with LQR, PID and FLC for calculation of time domain parameters on simulation model with dead zone

Table 4.8: Time domain parameters of Yaw with LQR, PID and FLC on real time system with dead zone non linearity

Controllers / Parameters	LQR	PID	Fuzzy
Settling time (t_s)	7.5 seconds	10 seconds	0.5 seconds
Steady State error(e_{ss})	0 degrees	0 degrees	1 degrees

For simulation the settling time is reduced significantly to value 0.5 seconds for FLC controlled system as compared to 7.5 seconds and 10 seconds for LQR and PID respectively.

CHAPTER 5

CONCLUSION AND FUTURE SCOPE OF WORK

5.1. CONCLUSION

The main objective of this thesis is to control the two parameters i.e. pitch angle and yaw angle with different type of controllers. Most of the systems in real world are non linear in nature, to control these type of systems a linearization technique is used to linearize the system, but this introduce the unavoidable errors in the system and the controlling done will not be accurate. The system used in this project is the 2 DOF helicopter model, the modeling of this system is studied along with the controllers used in the system. Feed-forward controller and Linear Quadratic Regulator (LQR) controller is used for controlling. A comparative study for three controllers is done on the system on simulation model in Simulink MATLAB as well as on the Hardware. Three types of controllers are implemented and the results for all the controllers are studied, the results for pitch and yaw are taken separately. The parameters used for the comparison of the results are rise time, settling time, steady state error etc. It can be concluded from this comparative study that the results obtained by Fuzzy Logic Controller are better than PID controlled and normal LQR+I controlled system. The controlling of pitch is improved in fuzzy significantly as compared to the original controller. All the time domain parameters i.e. rise time, settling time, peak overshoot, peak undershoots and settling time is improved as compared to the original system. The controlling of yaw is also improved in terms of settling time of the response.

5.2. FUTURE SCOPE OF WORK

The LQR, PID and FLC systems are successfully implemented on the 2 DOF system and it shows good results, these controllers can be implemented on the 3 DOF helicopter system which is a more complex system. The additional degree of freedom introduces more uncertainties in the system which does not allow better controlling with normal PID controller. Other control algorithms can also be implemented on the non linear model which is designed in this thesis for example, Neural Network, Sliding Mode Controller, Adaptive control algorithms and Model Predictive Control algorithms etc

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APPENDIX

MATLAB CODE FOR CONFIGURING PARAMETERS OF 2DOF SETUP

```
% SETUP_HELI_2D_CONFIGURATION
%
% SETUP_HELI_2D_CONFIGURATION sets and re
%
% Copyright (C) 2010 Quanser Consulting Inc.
% Quanser Consulting Inc.
%
%
function [ K_pp, K_yy, K_yp, K_py, J_eq_p, J_eq_y, B_eq_p, B_eq_y, m_heli, l_cm, g ] =
setup_heli_2d_configuration( )
%turns the model model parameters
% of the Quanser 2 DOF Helicopter plant.
%
% Gravitational Constant (m/s^2)
g = 9.81;
% Pitch and Yaw Motor Armature Resistance (Ohm)
R_m_p = 0.83;
R_m_y = 1.60;
% Pitch and Yaw Motor Current-Torque Constant (N.m/A)
K_t_p = 0.0182;
K_t_y = 0.0109;
% Pitch and Yaw Motor Voltage-Torque Constant (N.m/V)
K_yp = K_t_p / R_m_p;
K_py = K_t_y / R_m_y;
% Pitch and Yaw Viscous Damping Constant (N.m.s/rad)
B_eq_p = 0.8;  % tuned while running simulation and experiment in parallel
B_eq_y = 0.318; % identified as described in manual
% Mass of the Helicopter (kg)
m_heli = 1.3872;
% Mass of the Helicopter with Yoke (kg)
m_heli_case = 1.421;
% Distance from Pitch Pivot to Pitch/Front Motor and Yaw/Back Motor(m)
r_p = 7.75*0.0254;
r_y = (6+5/8)*0.0254;
% Pitch and Yaw Propeller Force-Thrust Constant found Experimentally (N/V)
K_f_p = 5*0.2074;
K_f_y = 3*0.1426;
% Pitch and Yaw Propeller Torque-Thrust Constant found Experimentally (N.m/V)
K_pp = K_f_p * r_p;
K_yy = K_f_y * r_y;
```

```
% Mass of Pitch/Front Motor and Yaw/Back Motor (kg)
m_motor_p = 0.292;
m_motor_y = 0.128;
% Mass of Pitch/Front and Yaw/Back Guard + Propeller(kg)
m_shield = 0.143 + 0.024;
% Total Mass of Pitch and Yaw Motor/Shield Assemblies (kg)
m_props = ( m_motor_p + m_motor_y + 2 * m_shield );
% Mass of Helicopter Body moving about Pitch Axis (kg)
m_body_p = m_heli - m_props;
% Mass of Helicopter Body moving about Yaw Axis (kg)
m_body_y = m_heli_case - m_props;
% Mass of Metal Shaft Moving about Yaw Axis (kg)
m_shaft = 0.151;
% Total Length of Helicopter Body (m)
L_body = 19*0.0254;
% Helicopter Center of Mass from Pivot along Pitch Axis (m)
l_cm = ( (m_motor_p + m_shield ) * r_p +  (m_motor_y + m_shield ) * r_y ) / ( m_props ) ;
% Length of Metal Shaft Moving about Yaw Axis through slip ring (m)
L_shaft = 11 * 0.0254;
% Pitch and Yaw Motor Rotor Moment of Inertia (kg.m^2)
J_m_p = 1.91e-6; %1.7070e-5;
J_m_y = 1.374e-4; %1.4471e-5;
% Moment of Inertia of Helicopter Body about its CM (kg.m^2)
J_body_p = m_body_p * L_body^2 / 12;
J_body_y = m_body_y * L_body^2 / 12;
% Moment of Inertia of Metal Shaft about Yaw Axis (kg.m^2)
J_shaft = m_shaft * L_shaft^2 / 3; %;0.0851;
% Moment of Inertia of Pitch Motor + Guard Assembly about Pivot (kg.m^2)
J_p = ( m_motor_p + m_shield ) * r_p^2;
% Moment of Inertia of Yaw Motor + Guard Assembly about Pivot (kg.m^2)
J_y = ( m_motor_y + m_shield ) * r_y^2;
% Equivalent Moment of Inertia about Pitch and Yaw Axis (kg.m^2)
J_eq_p = J_m_p + J_body_p + J_p + J_y;
J_eq_y = J_m_y + J_body_y + J_p + J_y + J_shaft;
%
% end of setup_heli_2d_configuration()
```

MATLAB CODE FOR STATE SPACE MODEL OF 2DOF SETUP

```
% Matlab equation file: "HELI_2D_ABCD_eqns.m"
% Open-Loop State-Space Matrices: A, B, C, and D
% for the Quanser 2 DOF Helicopter Experiment.

A( 1, 1 ) = 0;
A( 1, 2 ) = 0;
A( 1, 3 ) = 1;
```

```
A( 1, 4 ) = 0;
A( 2, 1 ) = 0;
A( 2, 2 ) = 0;
A( 2, 3 ) = 0;
A( 2, 4 ) = 1;
A( 3, 1 ) = 0;
A( 3, 2 ) = 0;
A( 3, 3 ) = -B_p/(J_eq_p+m_heli*l_cm^2);
A( 3, 4 ) = 0;
A( 4, 1 ) = 0;
A( 4, 2 ) = 0;
A( 4, 3 ) = 0;
A( 4, 4 ) = -B_y/(J_eq_y+m_heli*l_cm^2);

B( 1, 1 ) = 0;
B( 1, 2 ) = 0;
B( 2, 1 ) = 0;
B( 2, 2 ) = 0;
B( 3, 1 ) = K_pp/(J_eq_p+m_heli*l_cm^2);
B( 3, 2 ) = K_py/(J_eq_p+m_heli*l_cm^2);
B( 4, 1 ) = K_yp/(J_eq_y+m_heli*l_cm^2);
B( 4, 2 ) = K_yy/(J_eq_y+m_heli*l_cm^2);

C( 1, 1 ) = 1;
C( 1, 2 ) = 0;
C( 1, 3 ) = 0;
C( 1, 4 ) = 0;
C( 2, 1 ) = 0;
C( 2, 2 ) = 1;
C( 2, 3 ) = 0;
C( 2, 4 ) = 0;
C( 3, 1 ) = 0;
C( 3, 2 ) = 0;
C( 3, 3 ) = 1;
C( 3, 4 ) = 0;
C( 4, 1 ) = 0;
C( 4, 2 ) = 0;
C( 4, 3 ) = 0;
C( 4, 4 ) = 1;

D( 1, 1 ) = 0;
D( 1, 2 ) = 0;
D( 2, 1 ) = 0;
D( 2, 2 ) = 0;
D( 3, 1 ) = 0;
D( 3, 2 ) = 0;
D( 4, 1 ) = 0;
D( 4, 2 ) = 0;
```

Calculated LQR controller gain elements:

K = [14.1 V/rad 1.33 V/rad 7.33 V.s/rad 0.924 V.s/rad]
[-1.33 V/rad 14.1 V/rad -0.261 V.s/rad 7.99 V.s/rad]

Calculated LQR+I controller gain elements:

Ki = [18.9 V/rad 1.98 V/rad 7.49 V.s/rad 1.53 V.s/rad 7.03 V/(rad.s) 0.77 V/(rad.s)]
[-2.22 V/rad 19.4 V/rad -0.45 V.s/rad 11.9 V.s/rad -0.77 V/(rad.s) 7.03 V/(rad.s)]

****