DYNAMIC MODELLING OF A QUARTER CAR PASSIVE SUSPENSION SYSTEM AND OPTIMIZATION THROUGH GENETIC ALGORITHM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF THE DEGREE OF

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(COMPUTATIONAL DESIGN)

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SUBMITTED BY

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STUDENT'S DECLARATION

I, **Komal,** hereby certify that the work which is being presented in this thesis entitled **"Dynamic Modelling Of A Quarter Car Passive Suspension System And Optimization Through Genetic Algorithm"** is submitted in the partial fulfilment of the requirement for degree of **Master of Technology (Computational Design)** in Department of Mechanical Engineering at **Delhi Technological University** is an authentic record of my own work carried out under the supervision of**Prof.VikasRastogi.** The matter presented in this thesis has not been submitted in any other University/Institute for the award of Master of Technology Degree. Also, it has not been directly copied from any source without giving its proper reference.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Signature of Supervisor

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CERTIFICATE

This is to certify that this thesis report entitled, **"Dynamic Modelling Of A Quarter Car Passive Suspension System and Optimization Through Genetic Algorithm"** being submitted by **Komal (Roll No. 2K15/CDN/07)** at Delhi Technological University, Delhi for the award of the Degree of Master of Technology as per academic curriculum. It is a record of bonafide research work carried out by the student under my supervision and guidance, towards partial fulfilment of the requirement for the award of Master of Technology degree in Computational Design. The work is original as it has not been submitted earlier in part or full for any purpose before.

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ABSTRACT

The suspension system is essential to provide ride comfort and transmitted vibrations to human body in ergonomical range. It is mainly achieved by high damping at low frequencies and low damping at higher frequencies. For the improvement of handling performance of the vehicle, stiff suspension system should be used at all frequencies for reduced body attitude.

The present work deals with the analysis of the dynamic behaviour of a quarter car model using MATLAB/simulink®. In this work, two DOF system is solved analytically using eigen vector approach. The obtained mathematical model is converted into MATLAB code to create a function for optimization of ride comfort and ride height. The optimization tool is further used to optimize the multiobjective function that is coded in MATLAB through multiobjective optimization using Genetic Algorithm module. The results thus obtained are examined to select the parameters for the design analysis of the suspension system from the table.

The work includes simulation of quarter car model in four different cases involved in Simulink. The optimised parameters have been evaluated for the design statement using genetic algorithm.

Keywords: Optimization, Genetic algorithm, Simulink, Quarter car model

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CHAPTER 1

INTRODUCTION

The suspension system is one of the important part of any automotive system. The linkage between wheel axle and chassis is called automotive suspension system consists spring and damper. The function of suspension system is basically of comforting our ride and it has been used since ages starting from horse carriage which used leaf springs on wheels and now to all vehicles and further with expanding technology to complex algorithms. The suspension system supports the vehicle by isolating frame from road vibrations which gives directional stability. There are two types of suspension soft and stiff. Soft suspension if suspension is required for carrying more loads. Good handling requires a balance between soft and stiff suspension. However, these criteria subjectively depend on the purpose of the vehicle. For example, a sports car driver will accept a relatively hard ride as a compromise for high-speed handling and safe fast cornering. But the same ride would be intolerable for the passengers of a big saloon car.

1.1 MOTIVATION

The research in the vehicle dynamics has been an on-going study for decades, ever since the invention of automobiles. Engineers and researchers have been trying to fully understand the dynamic behavior of vehicles as they are subjected to different driving conditions, both moderate daily driving and extreme emergency maneuvers. They want to apply this finding to improve issues such as ride quality and vehicle handling stability, and to develop innovative designs that will improve vehicle operations. With the aid of fast computers to perform complicated design simulations and high speed electronics that can be used as controllers, new and innovative concepts have been tested and implemented into vehicles. Automotive companies are constantly improving on their chassis design and development through new technology. Examples are anti-lock braking systems and automatic traction systems. The system combines vehicle dynamics and electronics to improve handling quality of vehicle. They use a sensor to measure the wheel's rotational speed and a micro-controller to determine, in real time, whether slipping of the tyre is present or not. This results in full traction and braking under all road conditions, from dry asphalt to icy conditions. Another example of the benefit of combining vehicle dynamics with electronics is in controllable suspensions, such as those using semi-active damper. Semi-active dampers enable damping characteristics of the suspension system to be set by a feedback controller in real-time. This improves the ride quality of the vehicle on different types of road conditions.

A more advanced concept that is currently under research and development by automotive companies is an autonomous vehicle. This concept will enable the vehicle to get from one point to another without constant commands from the driver. The idea is to relieve the burden of vehicle control and operation from the driver and also to reduce the number of accidents associated with driver operating error. Tyre manufacturers also perform a variety of research on vehicle dynamics. They are interested in characterizing the performance of their tyres as a function of the tyre construction component. Their goal is to be able to predict or design tyres for any type of applications efficiently, and to reduce the cost associated with prototyping and testing. Their efforts require developing more accurate tyre models; specifically models that can predict how changing of the tyre affects the tyre performance the details of various suspensions are introduced in next subsections.

1.2 PASSIVE SUSPENSION SYSTEM

Passive suspensions systems are inexpensive, simple and provide satisfactory level of performance. However, the compromises between the above three conflicting demands have been fixed and it is not possible to enhance the overall performance at all load and speed conditions. Passive suspension system contains springs to store the energy and dampers to dissipate the energy. Both components are fixed at the design stage. Different types of springs and dampers might be used in design, but most suspensions in this class can be considered as a spring in parallel with a damper placed at each corner of the vehicle, and normally such a spring-damper unit is called a “strut”. In addition to the struts, some other elements or special geometrical arrangements are also used to increase the performance of passive systems. For example, additional roll springs (anti-roll bars) could be used to increase the stiffness to roll motion.

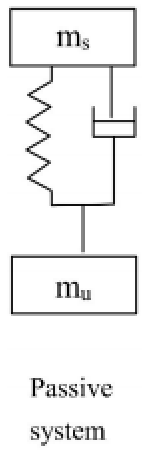


Figure 1.1 Passive Suspension System

Trailing arm could be implemented between the sprung mass and the wheel hub to reduce the dive and squat motion of the vehicle body during braking and acceleration. The Hydro elastic system interconnected the front and rear suspensions by hydraulic pipes to change the dynamic responses to pitch. In a self-energizing hydro pneumatic leveling system was presented which used the relative wheel-to-body movement to “pump” the car body up to achieve a desired ride height, so that soft springs can be employed to give a better ride.

1.3 SEMI ACTIVE SUSPENSION SYSTEM

In semi-active suspension, a variable damper or switchable dampers replace the conventional damper. It has the ability to change the damping characteristics according to body motion through mechanically changing orifices or fluid with adjustable viscosity. The control algorithm used in the design governs the amount of damping. Depending on the type of vehicle, a damper is chosen to make the vehicle perform best in its application. Ideally, the damper should isolate passengers from low-frequency road disturbances and absorb high-frequency road disturbances.

Passengers are best isolated from low-frequency disturbances when the damping is high. However, high damping provides poor high frequency absorption. Conversely, when the damping is low, the damper offers sufficient high frequency absorption, at the expense of low-frequency isolation. In other words, the damper will be set in soft damping characteristics, to have maximum comfort while traveling in a straight road and it will be set to hard while cornering for better stability. However, no energy is introduced into the system.

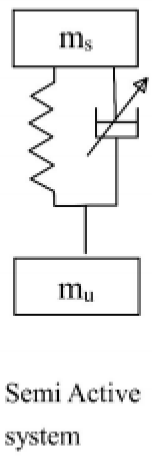


Figure 1.2 Semi Active Suspension System

1.4 ACTIVE SUSPENSION SYSTEM

The use of active suspension on road vehicles has been considered for many years. A large number of different arrangements from semi-active to fully active schemes have been investigated by many researchers. There has been interest in characterizing the degrees of freedom and constraints involved in active suspension design. Constraints on the achievable response have been investigated from "invariant points", transfer-function and energy/passivity point of view etc. A complete set of constraints was derived on the road and load disturbance response transfer functions. The choice of sensors needed to achieve these degrees of freedom was obtained for the quarter-car model. It was well proved that the road and load disturbance responses could not be adjusted independently for any passive suspension system. The fully active suspension can add power to the system. The idea behind fully active suspensions is that the force actuator is able to apply a force to the suspension in either bounce or rebound. This force is actively governed by the control scheme employed in this suspension. These suspension system can further be divided into two types, which are presented in next subsections.

1.4.1 Low Bandwidth Active Suspension System

This system is also known as slow – active or band limited system. In this, the actuator is placed in parallel or in series with the conventional passive system (i.e.) spring and damper. It aims to have control over the lower frequency range especially rattle space frequency. At higher frequencies the wheel-hop motion is controlled passively. As the conventional spring takes the static load of the vehicle, the amount of energy supplied to the actuator is fairly low and the actuator are more compact and easy to fit in existing vehicles. With these systems a significant reduction in body roll and pitch during maneuvers such as cornering and braking can be achieved. The schematic diagram of this type of suspension system is shown in Figure 1.3.

1.4.2 High Bandwidth Active Suspension System

It is also called fully active suspension system. The actuator alone is placed between the frame and the axle housing. Due to the entire weight of the car, the actuator has to create a large force to support the car. This force has to be adjusted frequently to keep the car in level while in motion and it consumes a large amount of energy.

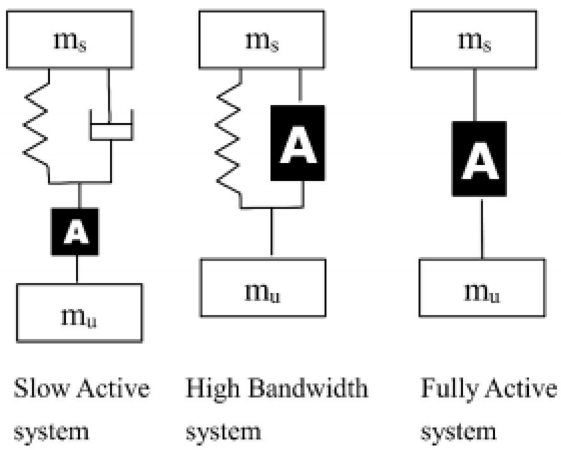


Figure 1.3 Active suspension system

The main advantage is that it is designed to improve suspension response at a large range of frequency including “Rattle Space” and “Tyre Hop” frequency. A fully active system will consume a significant amount of power and will require actuators with a relatively wide bandwidth. The schematic diagram is shown in Figure 1.3

1.5 SCOPE AND SPECIAL FEATURES OF THIS INVESTIGATION

It is a well-known fact that active suspension system plays a major role in isolating the vehicle vibration generated from the road disturbances and improving the ride comfort. For the past five decades or so, the scientific community has been constantly making attempts to quantify the performance improvements of active suspension system. Numerous investigators have modeled the system and numerically analyzed and simulated the various parameters by adopting reasonable assumptions. However, it is felt that there is a wide knowledge gap between the numerically analyzed data and experimental investigation. Though different models are available, still some more efforts are necessary to understand the performance improvement of the active suspension system. It is proposed to study the effect of stiffness ratio, suspension damping and tyre stiffness on passive and active suspension system by simulating the parameter using MATLAB.

In this present investigation, the quarter car model is considered for developing an integrated model. An experimental quarter car model including mechanical arrangement, hydraulic circuit, real-time digital embedded controller is fabricated. This model is used to study the effect of suspension parameters like mass, spring stiffness and the road disturbances. The performance improvement is quantified by measuring the sprung mass acceleration with the help of accelerometer. The main theme of the project is limited to study the ride comfort using quarter car model. However, the result obtained from this work can also be extended to a full car model in future. A full car model is based on the four identical quarter car models, which are coupled together by solid rods with respect to pitch and roll moment of inertia. For a full car model, the effect of acceleration/braking torque, influence of steering system etc can be included by incorporating suitable multiplication factor. The main objective is to develop a mathematical model for the passive and active suspension system to study the impact of suspension variable on ride comfort.

1.6 RESEARCH OBJECTIVE

The main objective of this study is

- To develop the mathematical model for the passive suspension system focused on performance.

- To study the quarter car model using MATLAB/Simulink model.
- To optimise the system using genetic algorithm in MATLAB through optimisation tool.

1.7 ORGANISATION OF THESIS

The chapters of the thesis are arranged in the following manner.

Chapter 1 presents the background of this project which includes the research objectives, motivation, and scope that is a frame line for this project. The primary aim of this chapter is to provide the reader with a basic idea of the work presented in the thesis.

Chapter 2 presents the literature survey performed in the field of suspension systems, their computational and experimental work and the algorithms involved in these literatures.

Chapter 3 demonstrates the mathematical modelling of the quarter car model. It contributes the equations involved in modal analysis; and depicting ride comfort and road holding.

Chapter 4 elaborates the computational modelling which involved MATLAB modules explained.

Chapter 5 provides the results obtained by the simulation study and discuss the performance of quarter car model before and after optimisation.

Chapter 6 concludes the thesis and provides suggestions for future work.

CHAPTER 2

LITERATURE SURVEY

In recent times the active suspension system is gaining popularity due to the increasing ride comfort but this system is not cost effective and used for systems, where high performance is required popularly in the field of racing, where at high velocity bump and droop affect the comfort significantly due to jerk. Thus for passenger cars the most economic system is passive system and from the literature survey, it has been concluded to optimise the parameters using genetic algorithm.

2.1 PASSIVE SUSPENSION SYSTEM

The passive system is most widely used system therefore many researchers have published research papers in optimising the sytem. It has been found in paper of Mitra et.al [1] is about the design of suspension system has been one of the challenging tasks for engineers. The main function of any suspension system is to reduce or eliminate the road excitations transmitted to vehicle body. An effort has been made in this paper for a passive suspension system by using an optimization technique called Genetic algorithm to absorb vibrations as per ISO 2631-1:1997 standards. The spring stiffness (ks), damping coefficient (cs), sprung mass (ms), unsprung mass (mu), tyre stiffness (kt) are optimized in such a way that ride comfort is increased .The quarter car and driver seat with the driver's body are simply modelled together as four degree of freedom (DOF) system by using SIMULINK for analyzing the ride comfort. In another by Vikranth and Kodati et.al [2] represents the double wishbone (DWB) and the MacPherson strut (MPS) suspension systems are commonly used independent suspensions in passenger cars. Their kinematics are complicated, and have not been analysed comprehensively in existing literature. This paper presents an analysis of the position kinematics of the complete spatial model of these suspension systems.

The presented solution is built upon two key elements: the use of Rodrigue's parameters to develop an algebraic set of equations representing the kinematics of the mechanisms, and the computation of Gröbner basis as a method of solving the resulting set of equations. It is found that the final univariate equation representing all the kinematic solutions for a given pair of steering and road-profile inputs, in the general case, is of 64 degree, for both the suspension mechanisms. It is also shown that in certain special cases, both the suspensions generate 28 solutions, instead of 64. Numerical accuracy of the solutions obtained is established by computing the residuals of the original set of kinematic constraint equations. The configurations of the mechanisms for the real solutions are depicted graphically. Finally, the responses of the suspensions to continuously varying steering and road-profile inputs are computed using a branch-tracking technique.

Kocsis et.al [3] performed the dynamic analysis of a beam on a block-and-tackle suspension system is accomplished using a continuum approach. The modal shape functions and the natural frequencies of the structure are derived in a dimensionless form for both slacked and stressed cables. A procedure is developed to handle the nonlinearity originated from the consecutive slacking and stressing of the suspension cable. Vibration analysis of the bilinear, multi-degree-of-freedom structure is accomplished for a vortex-shedding generated lift force and for a continuous pedestrian flow.

Ozcan et.al [4] wants to explain the optimum functional characteristics of suspension components, namely, linear/nonlinear spring and nonlinear damper characteristic functions are determined using simple lumped parameter models. A quarter car model is used to represent the front independent suspension, and a half car model is used to represent the rear solid axle suspension of a light commercial vehicle. The functional shapes of the suspension characteristics used in the optimisation process are based on typical shapes supplied by a car manufacturer. The complexity of a nonlinear function optimisation problem is reduced by scaling it up or down from the aforementioned shape in the optimisation process. The nonlinear optimised suspension characteristics are first obtained using lower complexity lumped parameter models. Then, the performance of the optimised suspension units are verified using the higher fidelity and more realistic carmaker model. An interactive software module is developed to ease the nonlinear suspension optimisation process using the Matlab graphical user interface tool.

In another paper of Nagina and Saeed et.al [5] which involves a relative study of three optimization methods, which are Hooke-Jeeves, Nelder-Mead, and Multi-

Directional Search methods, for design optimizing vehicle suspensions constructed on quarter vehicle model with different types of constrains. In optimization, three design norms are suspension working space, dynamic tire load and vertical vehicle acceleration. To execute design optimization five variables are nominated which are the tire stiffness, damping coefficient, sprung mass, spring stiffness and un sprung mass. It was resulted from the comparative study that the multidirectional search method is more reliable than Hooke-Jeeves method and Nelder-Mead method. The optimum results of the quarter car model were obtained by using MATLAB programming environment which demonstrated the effectiveness and applicability of the methods.

In the paper of Shirahatt et.al [6] the primary function of a vehicle suspension system is to isolate the road excitations experienced by the tyres from being transmitted to the passengers. In this paper, a suitable optimizing technique is applied at design stage to obtain the suspension parameters of a passive suspension and active suspension for a passenger car which satisfies the performance as per ISO 2631 standards. A number of objectives such as maximum bouncing acceleration of seat and sprung mass, root mean square (RMS) weighted acceleration of seat and sprung mass as per ISO2631 standards, jerk, suspension travel, road holding and tyre deflection are minimized subjected to a number of constraints. The constraints arise from the practical kinetic and comfortability considerations, such as limits of the maximum vertical acceleration of the passenger seat, tyre displacement and the suspension working space. The genetic algorithm (GA) is used to solve the problem and results were compared to those obtained by simulated annealing (SA) technique and found to yields similar performance measures. Both the passive and active suspension systems are compared in time domain analyses subjected to sinusoidal road input. Results show passenger bounce, passenger acceleration, and tyre displacement are reduced by 74.2%, 88.72% and 28.5% respectively, indicating active suspension system has better potential to improve both comfort and road holding.

Further by Likaj et.al [7] this paper deals with the optimal design and analysis of quarter car vehicle suspension system. These optimal designs are used the optimal parameters, which have been derived by comparison of two optimisation algorithms: Sequential Quadratic Program (SQP) and Genetic Algorithms (GA's), for a five chosen design parameters. The goal function is chosen to provide the possibility to emphasize three main objectives of vehicle suspensions; ride comfort, suspension travel and road holding. Fuzzy Logic Control (FLC) is considered to control active suspension for the optimal parameters derived by GA's, and the rule base can be tuned to improve each of the above objectives, while the main focus is to minimise the vertical vehicle body acceleration. It also deals with parametric analysis, state space modelling, Laplace Transform, Transfer Function, Stability, Controllability, Observability and many other important attributes to analyse quarter car vehicle model.

Koulocheris et.al [8] in this paper, the optimization of a heavy vehicle's suspension system were investigated setting different optimization targets. Conclusions have been made not only regarding the fitness functions but also for the optimization methods used in order to reach the optimum solution more accurately and with less computational time. The remarks regarding Case 3, discussed in the previous section, outline the importance of tire deflection being a part of the fitness function due to the superiority of Case 3 over Case 1. Due to its multi-objective character, Case 3 seems to outnumber Case 1, which is the most common main term of fitness functions in literature as far as the optimization of suspension systems is concerned. This could lead to the use of the tire deflection, as the main target in the optimization of suspension systems in combination with various existing methods of the literature mentioned in the introduction. Furthermore, the effectiveness of the hybrid algorithm proved promising in comparison with the other algorithms. They were able to find more satisfactory solutions in most cases and with less computational time than the other algorithms. The most important regarding the hybrid algorithms is the need of finding the balance between the use of its genetic and gradient based parts so as to gain more from the hybridization and help the problem to converge in less time. Further work is in progress to extend our research.

Jagtap et.al [9] the vehicle suspension design includes a number of compromises to provide good leveling of stability and ride comfort. Suspension system has to perform complexity requirements, which includes road holding and equality, driving pleasure, riding comfort to occupant. Riding pleasure depends on vertical acceleration, with main objective to minimize vertical acceleration. In this work, the geometric parameters of suspension system are optimized using Matlab as an optimization tool. The values of tire deflection reduced from 0.023 to 0.021 m and Seat acceleration reduced from 34.19 to 31.86 m/s^2.

Florin et.al [10] the purpose of a vehicle suspension system is to improve ride comfort and road handling. In current article is simulated and analyzed the handling and ride performance of a vehicle with passive suspension system, quarter car model with two degree of freedom. Since, the equations of the system cannot be solved mathematically has developed a scheme in Matlab Simulink that allows analyzing the behavior of the suspension. The schema that was created in Matlab Simulink, were compared with the State space model and the Transfer function. After completing the simulation scheme can be introduced excitation signals, this case a step signal.

Hassaan et.al [11] the objective of the paper is to investigate the dynamics of a quarter-car model with passive suspension and considering the driver-seat. The model is a 3 DOF one excited by the irregularities of the road pavement. The model dynamics are studied in terms of the frequency response of the driver using its transmissibility. The paper presents new technical terms used to assess the suspension parameters which are the accumulated and mean transmissibility. The driver frequency response is studied for road pavement of irregularities having harmonic components of frequencies up to 10 Hz. The natural frequencies of the system are calculated and the driver transmissibility is evaluated for suspension parameters in the range 2.85 to 11.4 kN/m for stiffness and 0.5 to 8 kNs/m for damping coefficient.

Liu et.al [12] two methods for parameters optimization of traditional passive suspension based on the invariant point theory are presented in this paper. Firstly, the two degree-of-freedom dynamical model for a quarter vehicle suspension is established. Then all the invariant points for the acceleration of vehicle body, suspension deformation, and tire displacement are obtained through the invariant equation associated with Laplace transform. Secondly, two optimization methods for the system parameters of passive suspension are presented, and the design objectives are to designate the invariant points as the local maximum points of the amplitude-frequency curves so as to improve the control performance of vibration isolation. At last, the control performances of the two improved passive suspension systems are compared with those of the original passive suspension, the active suspension, and the semi-active suspension by some performance indexes. The results show that the two optimization methods could greatly improve the control performance of passive suspension systems.

Tang and Guo et.al [13] in this paper, a five degree of freedom half body vehicle suspension system is developed and the road roughness intensity is modeled as a filtered white noise stochastic process. Genetic algorithm and neural network control are used to control the suspension system. The desired objective is proposed as the minimization of a multi-objective function formed by the combination of not only sprung mass acceleration, pitching acceleration, suspension travel and dynamic load, but also the passenger acceleration. With the aid of software Matlab/Simulink, the simulation model is achieved. Simulation results demonstrate that the proposed active suspension system proves to be effective in the ride comfort and drive stability enhancement of the suspension system. A mechanical dynamic model of the five degree of freedom half body of vehicle suspension system is also simulated and analyzed by using software Adams.

Chi et.al [14] this paper presents a comparative study of three optimization algorithms, namely Genetic Algorithms (GAs), Pattern Search Algorithm (PSA) and Sequential Quadratic Program (SQP), for the design optimization of vehicle suspensions based on a quarter-vehicle model. In the optimization, the three design criteria are vertical vehicle body acceleration, suspension working space, and dynamic tire load. To implement the design optimization, five parameters (sprung mass, un-sprung mass, suspension spring stiffness, suspension damping coefficient and tire stiffness) are selected as the design variables. The comparative study shows that the global search algorithm (GA) and the direct search algorithm (PSA) are more reliable than the gradient based local search algorithm (SOP). The numerical simulation results indicate that the design criteria are significantly improved through optimizing the selected design variables. The effect of vehicle speed and road irregularity on design variables for improving vehicle ride quality has been investigated. A potential design optimization approach to the vehicle speed and road irregularity dependent suspension design problem is recommended.

Nagarkar et.al [15] Suspension system has to perform complexity requirements, which includes road holding and equality, driving pleasure, riding comfort to occupant. Riding pleasure depends on vertical acceleration, with main objective to minimize vertical acceleration. The force transmitted by the deflection of tires to the unsprung mass is known as the Tire-Dynamic Force (TDF). The force should be considered while designing the suspension system. The TDF can cause vehicle instability and increase in the sprung mass acceleration. The objective of this research work is to minimize the TDF by optimum suspension design so that minimum vertical accelerations would be experienced by the passengers. To minimize the vertical acceleration, mathematical model of 2-DOF Quarter car model is considered for passive suspension system. Its equation of motion is formed with correlating objective function. Thus having objective function to, minimum weighted root mean square accelerations as per ISO 2631, viz. formulated for optimization. spring stiffness and damping coefficient are used as variables during optimization. The Genetic Algorithm is implemented as optimization tool with above mentioned objective function. The optimization results obtained were simulated and are compared with classical values. It is observed that the Seat acceleration, using optimized values, is reduced by 8.76% compared to the classical values. From experimental validation main objective function of Tire dynamic force value reduced from 2300 N to 2100 N, it is decreased by 8.69 % and Seat acceleration reduced from 34.9176 m/s2 to 32.0121 m/s2 after replacing the conventional by optimized strut. Thus provides improved ride comfort to occupant/driver.

Gupta et.al [16] the modeling the suspension of an automobile is of interest for many automotive and vibration engineers. Of importance for these engineers is the ride quality of the vehicle traversing over broken roads and control of body motion. When traveling over rough terrain, the vehicle exhibits bounce (up and down), pitch (rotation about the center of gravity along the vehicle's length) and roll (rotation about the center of gravity along the vehicle's width) motions. Optimization of vehicle ride and handling performance must meet many competing requirements. For example, vibration in the frequency range that causes driver discomfort needs to be minimized, which requires decreasing suspension stiffness. Yet the suspension deflection should stay within travel constraints, so suspension stiffness needs to be increased. The traditional practice of relying on test cars for suspension development is time consuming and costly. In this paper, we will demonstrate a simulation-led design approach, which reduces reliance on test vehicles and produces optimal results. The approach starts with the development of a fast and accurate vehicle model in Matlab® and Simulink® combined for testing the parameters, and concludes by automated optimization of suspension parameters using Genetic Algorithm, to meet performance requirements specified. This would produce more applicable results of industrial and commercial merit.

Shala et.al [17] in this paper it will be presented the comparison of two optimisation algorithms: Sequential Quadratic Program (SQP) and Genetic Algorithms (GAs) for the optimal design of quarter car vehicle suspensions. During the optimisation the three design criteria which have been used are; vertical vehicle body acceleration, dynamic tire load and suspension working space. For implementing a optimisation will be chosen five design parameters: sprung and un-sprung mass, spring stiffness, damping coefficient and tire stiffness. Through the simulation in Matlab it will be shown that GA is more powerful tool to find the global optimal point, without any restrictive requirements on the gradient and Hessian Matrix, while SQP has local convergence properties. In order to overcome the permanent conflict between vehicle comfort and vehicle handling under different riding conditions and speed, the main focus of this research will be on minimising the vertical vehicle body acceleration subjected to a suspension working space and the dynamic tire load. At the end of this paper, it will be shown the comparison between the simulation carried out with nominal and optimal values of design parameters.

2.2 SEMI ACTIVE SUSPENSION SYSTEM

Mori et.al [18] the paper is concerned with a fully adaptive semiactive control which can deal with uncertainties in both models of MR damper and suspension mechanism. The proposed approach consists of two adaptive control: One is an adaptive inverse control for compensating the nonlinear hysteresis dynamics of the MR damper, which can be realized by identifying a forward model of MR damper and then calculating the input voltage to MR damper to generate a reference damping force. It can also be realized directly by updating the inverse model of MR damper without identification of forward model, which works as an adaptive inverse controller. The other is an adaptive reference control which gives the desired damping force to match the seat dynamics to a specified reference dynamics even in the presence of uncertainties in the suspension system. Validity of the proposed algorithm is discussed in simulation studies.

Bourmistrova et.al [19] the suspension system design is a challenging task with multiple control parameters, complex (often conflicting) objectives and stochastic disturbances. It is essential to develop a design environment, in the form of a mathematical model, which will help engineering efforts, not only in algorithm design but also in the investigation of various research questions. This paper examines issues relevant to semi-active suspension control system design optimization. It also presents a numerical approach for such optimization. Evolutionary algorithms (EAs) are applied to the optimization of the control system parameters. EAs are computer-based techniques that mirror natural genetic evolution, and they have been found to be successful in application to a wide range of problems that are difficult to solve analytically. The algorithms use randomly chosen road surfaces as input to a quarter-car computer model and develop the design of a number of non-linear, semi-active suspension control systems. These are then compared using a fitness function as a measure. For a suspension system the goal of ride comfort conflicts with the restriction of staying within the limits of suspension travel, or rattle space (the distance between the car body and the tyre which is restricted to the suspension travel limit). Thus the EAs examined in this paper use a multiobjective fitness function which is a weighted sum of car body rate-of-change of acceleration and suspension travel. Two separate fitness measures are analysed and developed, and are combined in a weighted sum. Various different weights are used and the effect of the weighting is analysed. An EA was applied to the development of a semiactive suspension system using a computer simulation model. The modelled control system uses sensor measurements of car-body acceleration and suspension travel to provide indicators that will allow an EA to determine the best mix of strategies. The goal is to simultaneously maintain the suspension travel within the rattle space and to minimize car-body rate-of-change of acceleration. A number of car-suspension control algorithms mentioned in the literature are analysed using an EA, such as passive and skyhook controls as well as the "on-off skyhook control policy". Some minor variations on these designs are also analysed. The parameters developed in the EAs are compared and the corresponding fitness functions are also compared. The results are given below, indicating strengths and weakness of the various control strategies using the components of the fitness measure. The results are developed for verification and validation purposes. These indicated that the passive system was quite robust and that adapting the skyhook system to a semiactive system was counter productive for comfort and for keeping the suspension within the rattle space. It is also clear that genes covering more factors in combination are needed for a deeper investigation using this technique, and that it is necessary to go beyond the skyhook control. Experiments using a combination of factors, and using member-set functions, are currently being planned. Further work will use Pareto optimums. Pareto optimum measures compare fitness using a number of separate objectives simultaneously, and the various objectives can be compared independently. Experiments are being planned to investigate real road surfaces, especially the frequency profile of road noise. The simulation test bed is being extended to include active suspensions as well as more degrees of freedom.

Badri et.al [20] this paper deals with designing a robust fixed-order non-fragile dynamic output feedback controller for active suspension system of a quarter-car, by means of convex optimization and linear matrix inequalities (LMIs). Our purpose is to design a low-order controller that keeps the desired design specifications besides the simplicity of the implementation. The proposed controller is capable of asymptotically stabilizing the closed-loop system and developing H_∞ control, despite model uncertainties and nonlinear dynamics of the quarter-car as well as the norm bounded perturbations of controller parameters. Furthermore, controller parameters are prevented from taking very large and undesirable amounts through appropriate LMI constraints. Finally, a numerical example is presented to show the effectiveness of the proposed method by comparing it with similar works.

2.3 ACTIVE SUSPENSION SYSTEM

Zhou et.al [21] the objective of this work is to present a new two-acceleration-error-input (TAE!) proportional-integral-derivative (Pill) control strategy for active suspension. The novel strategy lies in the use of sprung mass acceleration and unsprung mass acceleration signals simultaneously, which are easily measured and obtained in engineering practice. Using a quarter-car model as an example, a TAEI Pill controller for active suspension is established and its control parameters are optimized based on the genetic algorithm (GA), in which the fitness function is a suspension quadratic performance index. Comparative simulation shows that the proposed TAEI Pill controller can achieve better comprehensive performance, stability, and robustness than a conventional single-acceleration-error-input (SAEI) Pill controller for the active suspension.

Huang et.al [22] this paper proposes adaptive control designs for vehicle active suspension systems with unknown nonlinear dynamics (e.g., nonlinear spring and piece-wise linear damper dynamics). An adaptive control is first proposed to stabilize the vertical vehicle displacement and thus to improve the ride comfort and to guarantee other suspension requirements (e.g., road holding and suspension space limitation) concerning the vehicle safety and mechanical constraints. An augmented neural network is developed to online compensate for the unknown nonlinearities, and a novel adaptive law is developed to estimate both NN weights and uncertain model parameters (e.g., sprung mass), where the parameter estimation error is used as a leakage term superimposed on the classical adaptations. To further improve the control performance and simplify the parameter tuning, a prescribed performance function (PPF) characterizing the error convergence rate, maximum overshoot and steady-state error is used to propose another adaptive control. The stability for the closed-loop system is proved and particular performance requirements are analyzed. Simulations are included to illustrate the effectiveness of the proposed control schemes.

Panchal et.al [23] in this paper, develop fuzzy logic controller to control active suspension system to minimize car body deflection. Also develop PID controller to control Active suspension system, also tune gain of PID controller using Genetic algorithm. By using all three methods, vehicle body deflection has been obtained & compare with each other. These comparisons display efficiency of FLC & GA-PID controller method.

Al-Mutar et.al [24] the objective of this paper is to design an efficient control scheme for car suspension system. The purpose of suspension system in automobiles is to improve more comfortable riding and good handling with road profile. A nonlinear hydraulic actuator is added to passive suspension system in parallel with damper. The Particles Swarm Optimization (PSO) is used to design a Fuzzy controller for active suspension system. The designed controller is applied for quarter car suspension system and result is compared with passive suspension system model. Simulation results show good performance for the designed controller.

Abdalla et.al [25] the objective of this paper is to design an efficient control scheme for car suspension system. The purpose of suspension system in vehicles is to get more comfortable riding and good handling with road vibrations. A nonlinear hydraulic actuator is connected to passive suspension system in parallel with damper. The Particles Swarm Optimization is used to tune a PID controller for active suspension system. The designed controller is applied for quarter car suspension system and result is compared with passive suspension system model and input road profile. Simulation results show good performance for the designed controller.

Shieh et.al [26] the aim of this paper is to integrate the artificial immune systems and adaptive fuzzy control for the automobile suspension system, which is regarded as a multiobjective optimization problem. Moreover, the fuzzy control rules and membership controls are then introduced for identification and memorization. It leads fast convergence in the search process. Afterwards, by using the diversity of the antibody group, trapping into local optimum can be avoided, and the system possesses a global search capacity and a faster local search for finding a global optimal solution. Experimental results show that the artificial immune system with the recognition and memory functions allows the system to rapidly converge and search for the global optimal approximate solutions.

Wang et.al [27] in this paper, the robust fault-tolerant (FT) H1 control problem of active suspension systems with finite-frequency constraint is investigated. A full-car model is employed in the controller design such that the heave, pitch and roll motions can be simultaneously controlled. Both the actuator faults and external disturbances are considered in the controller synthesis. As the human body is more sensitive to the vertical vibration in 4– 8 Hz, robust H1 control with this finite-frequency constraint is designed. Other performances such as suspension deflection and actuator saturation are also considered. As some of the states such as the sprung mass pitch and roll angles are hard to measure, a robust H1 dynamic output-feedback controller with fault tolerant ability is proposed. Simulation results show the performance of the proposed controller.

Ning et.al [28] in this paper, an innovative active seat suspension system for vehicles is presented. This seat suspension prototype is built with two low cost actuators each of which has one rotary motor and one gear reducer. A H1 controller with friction compensation is designed for the seat suspension control system where the friction is estimated and compensated based on the measurement of seat acceleration. This principal aim of this research was to control the low frequency vibration transferred or amplified by the vehicle (chassis) suspension, and to maintain the passivity of the seat suspension at high frequency (isolation vibration) while taking into consideration the trade-off between the active seat suspension cost and its high frequency performance. Sinusoidal excitations of 1–4.5 Hz were applied to test the active seat suspension both when controlled and when uncontrolled and this is compared with a well-tuned passive heavy duty vehicle seat suspension. The results indicate the effectiveness of the proposed control algorithm within the tested frequencies. Further tests were conducted using the excitations generated from a quarter-car model under bump and random road profiles. The bump road tests indicate the controlled active seat suspension has good transient response performance. The Power Spectral Density (PSD) method and ISO 2631-1 standards were applied to analyse the seat suspension's acceleration under random road conditions. Although some low magnitude and high frequency noise will inevitably be introduced by the active system, the weighted-frequency Root Mean Square (RMS) acceleration shows that this may not have a large effect on ride comfort. In fact, the ride comfort is improved from being an ‘a little uncomfortable’ to a ‘not uncomfortable’ level when compared with the well-tuned passive seat suspension. This low cost active seat suspension design and the proposed controller with the easily measured feedback signals are very practical for real applications.

Obaid et.al [29] this paper proposes a cascade control algorithm of an active suspension system (ASS) with hydraulic actuator dynamic for a quarter car model. The objective of designing a controller for the car suspension system is to improve the ride comfort while maintaining the constraints on to the suspension travel and tire deformation, which is subjected to different road profiles. The control algorithm is based on the fusion of robust control and computational intelligence techniques which consists of the inner loop controller for force tracking control of the hydraulic actuator model and the outer loop controller for disturbance rejection control. Particle swarm optimization (PSO) algorithm is employed to optimize the proportional-integral (PI) controller parameters for force tracking control of the hydraulic actuator model. Similarly, the PSO algorithm is utilized in the outer loop controller to search for the optimal values of the weighting matrices for the linear quadratic optimal control (LQR) such that the desired performance of the ASS is guaranteed. In comparison with the passive suspension system, the simulation results demonstrate the superiority of proposed PSO-based controller, where it significantly improved the ride comfort by maintaining the other constraints (the suspension travel, tire deflection, and control force) in their limits.

Shao et.al [30] the fault-tolerant fuzzy H_∞ control design approach for active suspension of in-wheel motor driven electric vehicles in the presence of sprung mass variation, actuator faults and control input constraints is proposed. The controller is designed based on the quarter-car active suspension model with a dynamic-damping-in-wheel-motor-driven-system, in which the suspended motor is operated as a dynamic absorber. The Takagi-Sugeno (T-S) fuzzy model is used to model this suspension with possible sprung mass variation. The parallel-distributed compensation (PDC) scheme is deployed to derive a fault-tolerant fuzzy controller for the T-S fuzzy suspension model. In order to reduce the motor wear caused by the dynamic force transmitted to the in-wheel motor, the dynamic force is taken as an additional controlled output besides the traditional optimization objectives such as sprung mass acceleration, suspension deflection and actuator saturation. The H_∞ performance of the proposed controller is derived as linear matrix inequalities (LMIs) comprising three equality constraints which are solved efficiently by means of MATLAB LMI Toolbox. The proposed controller is applied to an electric vehicle suspension and its effectiveness is demonstrated through computer simulation.

Kanarachos et.al [31] there is an increasing demand for vehicles suitable for both on and off road driving characterized by superior comfort and handling performance. This is problematic for most suspension systems because there is a trade off balance between vibration reduction, suspension travel, actuator effort, road holding capability, as well as noise and fatigue requirements. Only in the UK every 11 min a car is getting damaged because of potholes. In this paper, a method to design an intelligent suspension system with the objective to overcome the trade-off barrier using the smallest actuator is presented. An experts' based algorithm continuously monitors the road excitation in relation to the suspension travel and adapts accordingly the suspension system. It is shown that by applying genetic algorithm it is possible to optimally tune the system. However, the global optimum is hard to find due to the problem nonlinearity. A hybrid genetic algorithm that improves the probability of successfully finding the best design is presented. The simulation results show that the proposed intelligent system performs for – well known in the literature scenarios – better than others and remarkably this is achieved by reducing the actuator's size.

Rizvi et.al [32] the main objective of this paper is to develop improved robust control techniques for an active suspension system utilizing an improved mathematical model. For that purpose, Euler Lagrange equation is used to obtain a mathematical model for vehicle active suspension system. The dynamics of driver's seat are included to get a more appropriate model. Robust H1 controllers are designed for the system to minimize the effect of road disturbances on vehicle and passengers. The performance of active suspension system is determined by measuring the heave acceleration of driver's seat and rotational acceleration of vehicle around its center of gravity. Effectiveness of the proposed controllers is validated by simulation results.

2.4 MATHEMATICAL APPROACH

Kuo et.al [33] this paper describes a methodology to determine a set of optimum nonlinear system parameters for a passive suspension system (PSS) by using the evolutionary algorithms so that the best performance of the system can be achieved. The selected fitness functions are based on the desired performance such as sprung mass acceleration, suspension deflection, and tire deflection. A dynamic step-size operator is incorporated in the evolutionary algorithm for fine-tuning the mutation process. The simulation results show that the proposed method can provide better ride comfort and road holding ability in comparison with the commonly used passive suspension system.

Yu et.al [34] the primary function of a suspension system of a vehicle is to isolate the road excitations experienced by the tires from being transmitted to the passengers. In this project report, we formulate an optimal vehicle suspension design problem with the quarter-car vehicle dynamic model. The two objectives of the optimization are: 1. Minimize the maximum bouncing acceleration of the sprung mass 2. Minimize average suspension displacement subject to a number of constraints. The constraints arise from the practical kinetic and comfortability considerations, such as limits of the maximum vertical acceleration of the sprung mass and the suspension working space. In solving this problem, the genetic algorithms have consistently found near-optimal solutions within specified parameters ranges for several independent runs. This encourages us to extend the application of GA to other more complicated vehicle dynamics problems with full confidence.

2.5 EXPERIMENTAL INVESTIGATIONS

Marzbanrad et.al [35] this paper reported on an investigation to determine the spring and damper settings that ensured optimal ride comfort of vehicle in different speeds using design of experiment method (DOE). The extent to which the ride comfort optimal suspension settings vary for roads of different roughness and varying speeds and the levels of ride comfort that can be achieved, were addressed. Optimization was performed with the DOE method on a 7 DOF modelled in MATLAB software for speeds ranging from 60 to 90 km/h. Results indicated that optimization of suspension settings using the road and specified range of speed also improved the ride comfort on the same road at the different speeds. These settings also improved ride comfort for other roads at the optimization speed and other speeds, although not as much as when optimization has been done for the particular road. For improved ride comfort, damping generally has to be lower than the standard (compromised) setting, the rear spring as soft as possible and the front spring ranging from as soft as possible to stiffer depending on road and speed conditions. Ride comfort was most sensitive to a change in rear spring stiffness.

Mostaani et.al[36] This paper reports on an investigation to determine the spring and damper settings that will ensure optimal ride comfort of vehicle in different speeds using Design of Experiment Method (DOE). The extent to which the ride comfort optimal suspension settings vary for roads of different roughness and varying speeds and the levels of ride comfort that can be achieved, are addressed. Optimization is performed with the DOE method with 7 DOF modeled for speeds ranging from 60 to 90 km/h. Results indicate that optimization of suspension settings using different road rouphness and specified range of speed improve the ride comfort. It was found that ride comfort is most sensitive to a change in rear spring stiffness.

2.5 SUMMARY OF THE CHAPTER

In this chapter, the literature review has been represented from the various research paper studied. The topic wise discussion was represented for passive suspension, semi active suspension, active suspension, algorithms and experimental investigation. In next chapter the mathematical modelling is presented.

CHAPTER 3

MATHEMATICAL MODELLING

3.1 INTRODUCTION

This chapter includes the problem formulation using mathematical modelling. The physical model of the 2 DOF system has been converted into differential equations to generate analytic solution to obtain the necessary performance of the following in order to study its behaviour under certain given conditions.

3.2 MATHEMATICAL MODEL

Mathematical modelling is a tool used to analyse the quarter car suspension model. The model is based on the FBDs of the physical model.

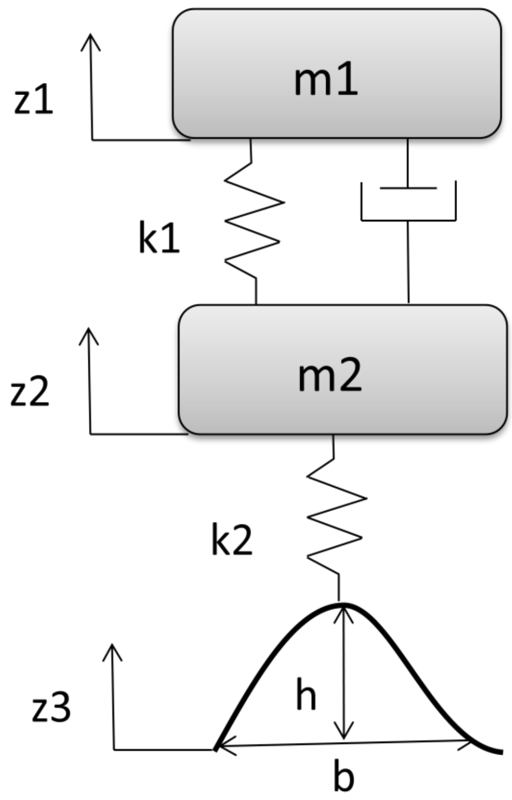


Figure 3.1 Quarter car model

Where, m1=Sprung mass, kg, m2=Unsprung mass, kg, k1=Spring Stiffness N/m, k2=Tire stiffness, N/m, z1=Displacement of sprung mass, m, z2=Displacement of unsprung mass, m, z3=Road excitation, m, h=Bump height, m, b=Bump width, m.

According to law of motion for the above FBD, ω(rad/sec) as the frequency of road excitation, the governing equations are:

$$\mathrm{m1} \times \ddot{\mathrm{z}}1 + \mathrm{c1} \times \left(\dot{\mathrm{z}}1 - \dot{\mathrm{z}}2\right) + \mathrm{k1} \times (\mathrm{z1} - \mathrm{z2}) + \mathrm{m2} \times \mathrm{g} - \mathrm{m1} \times \mathrm{g} = 0 \quad (3.1)$$

$$\mathrm{m2} \times \ddot{\mathrm{z}}2 - \mathrm{c1} \times \left(\dot{\mathrm{z}}1 - \dot{\mathrm{z}}2\right) - \mathrm{k1} \times (\mathrm{z1} - \mathrm{z2}) + \mathrm{k2} \times (\mathrm{z2} - \mathrm{z3}) + \mathrm{m2} \times \mathrm{g} = 0 \quad (3.2)$$

3.3 MODAL ANALYSIS

To find eigen values and eigen vectors equations of the multi DOF system, the equations are represented in matrix form.

$$\begin{bmatrix} \mathrm{m1} & 0 \\ 0 & \mathrm{m2} \end{bmatrix} \begin{bmatrix} \ddot{z}1 \\ \ddot{z}2 \end{bmatrix} + \begin{bmatrix} c1 & -c1 \\ -c1 & c1 \end{bmatrix} \begin{bmatrix} \dot{z}1 \\ \dot{z}2 \end{bmatrix} + \begin{bmatrix} \mathrm{k1} & -\mathrm{k1} \\ -\mathrm{k1} & \mathrm{k2} + \mathrm{k1} \end{bmatrix} \begin{bmatrix} z1 \\ z2 \end{bmatrix} = \begin{bmatrix} -\mathrm{m1} \times \mathrm{g} \\ \mathrm{k2} \times \mathrm{z3} - \mathrm{m1} \times \mathrm{g} \end{bmatrix} \quad (3.3)$$

To derive the objective function following assumptions are considered:

1. Vehicle is moving at a uniform velocity.
2. Damping coefficient of tire is negligible with respect to damping coefficient of damper.
3. Sprung mass is 10 times the unsprung mass.
4. Tire stiffness is 12 times the spring stiffness.
5. No dynamic weight transfer.
6. Road roughness not considered.
7. Tire stiffness is assumed to be constant.
8. Initial deflections and velocity at time t=0,is zero.

The various matrix may be expressed as

Mass matrix $$[\mathrm{M}] = \begin{bmatrix} \mathrm{m1} & 0 \\ 0 & \mathrm{m2} \end{bmatrix} \quad (3.4)$$

Stiffness matrix $$[K] = \begin{bmatrix} k1 & -k1 \\ -k1 & k2 + k1 \end{bmatrix} \tag{3.5}$$

Force matrix $$\begin{bmatrix} F1 \\ F2 \end{bmatrix} = \begin{bmatrix} -m1 \times g \\ -k2 \times z3 - m2 \times g \end{bmatrix} \tag{3.6}$$

Damping matrix $$[C] = \begin{bmatrix} c1 & -c1 \\ -c1 & c1 \end{bmatrix} \tag{3.7}$$

3.3.1 Eigen values (λ)

To find natural frequency, positive square roots of eigen values are determined. Corresponding mode shapes are determined using three variables as mass, spring stiffness and damping coefficient.

$$[D]=[M]^{-1}\times[K] \tag{3.8}$$

$$[D] = \begin{bmatrix} 1/m1 & 0 \\ 0 & 1/m2 \end{bmatrix} \times \begin{bmatrix} k1 & -k1 \\ -k1 & k2 + k1 \end{bmatrix}$$

$$[D] = \begin{bmatrix} k1/m1 & -k1/m1 \\ -k1/m1 & (k2 + k1)/m2 \end{bmatrix}$$

Finding the eigen values, determinant of $|D - \lambda I| = 0$,

$$|D - \lambda I| = \begin{vmatrix} \frac{k1}{m1} - \lambda & -k1/m1 \\ -k1/m1 & \frac{k2+k1}{m2} - \lambda \end{vmatrix} \tag{3.9}$$

After solving the quadratic equation,we get two eigen values:

$$\lambda_1=13(k1/m2) \qquad \lambda_2=0.9922(k1/m2) \tag{3.10}$$

corresponding natural frequencies are:

$$\omega_1=3.60\sqrt{\frac{k1}{m2}} \qquad \omega_2=0.303\sqrt{\frac{k1}{m2}} \tag{3.11}$$

where, ω_1 is natural frequency of unsprung mass and ω_2 is natural frequency of sprung mass.

The modal matix denotes the mode shapes or eigen vectors for the corresponding natural frequency mode shape is n-dimensional column vector of the form:

$$X_i = \begin{Bmatrix} X_{i1} \\ X_{i2} \\ \vdots \\ X_{in} \end{Bmatrix} \tag{3.12}$$

Substituting values of λ_1 and λ_2 in equation 3.13,

$$[D - \lambda I] \begin{Bmatrix} X1 \\ X2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.13}$$

The motion of a system where every point in a system executes harmonic motion with a particular natural frequency is called as principle mode shape.

$$\begin{Bmatrix} X1 \\ X2 \end{Bmatrix}_1 = p1 \times \begin{Bmatrix} 1 \\ 0.077 \end{Bmatrix} \text{ and } \begin{Bmatrix} X1 \\ X2 \end{Bmatrix}_2 = p2 \times \begin{Bmatrix} 1 \\ -129.077 \end{Bmatrix} \tag{3.14}$$

Where, p1 and p2 are proportionality constants.

Therefore, the modal matrix [U] can be defined as: $[U] = \left\{ \begin{Bmatrix} X1 \\ X2 \end{Bmatrix}_1 \quad \begin{Bmatrix} X1 \\ X2 \end{Bmatrix}_2 \right\}$

$$[U] = \begin{Bmatrix} 1 & 1 \\ 0.077 & -129.077 \end{Bmatrix} \times p \tag{3.15}$$

The proportional damping is assumed to calculate the corresponding damping ratios:

$$[C]=\alpha[M]+\beta[K] \tag{3.16}$$

Where; α, β are constants. This is known as proportional to either the mass M of the system, or the stiffness K of the system or both. Proportional damping is rather unique, since only one or two parameters α and β, appear to fully describe the complexity of damping, irrespective of the DOF. This is clearly not realistic. Hence, proportional damping is not a rule but rather the exception. Nonetheless the approximation of the proportional damping is useful since, mostly, damping is quite an elusive phenomenon, i.e., difficult to model and hard to measure but for a few DOFs.

The mass effect on damping is negligible. Hence, $\alpha=0$;

Therefore, $[C]=\beta[K]$, this is known as structural damping.

i.e. $\beta = \left(\frac{c1}{k1}\right)$

The damping ratio is define as, $\xi_i = \beta \times \left[\frac{\omega_i}{2}\right]$

Therefore,

$$\xi_1 = \left(\frac{c1}{k1}\right) \times \left(\frac{\omega_1}{2}\right) \text{ and } \xi_2 = \left(\frac{c1}{k1}\right) \times \left(\frac{\omega_2}{2}\right) \quad (3.17)$$

3.4 DECOUPLING OF EQUATIONS

This step makes the ODE(3.1-3.2) independent of each other, converts them into single variable and solves them individually to get their solutions.

To define the generalised mass M:

$$M_1 = \sum_{i=1}^{2} X_{i1}^{\ 2} \times m_{2i} \text{ and } M_2 = \sum_{i=1}^{2} X_{i2}^{\ 2} \times m_{2i} \quad (3.18)$$

Substituting X_{11}, X_{12}, X_{21}, X_{22}, m_2, m_1 from equation 3.14 and 3.4 in equation 3.17,

$M_1=1.06\times m_2$ and $M_2=166610.94\times m_2$

$$G_1 = \sum_{i=1}^{2} X_{i1} \times F_i \quad \text{and} \quad G_2 = \sum_{i=1}^{2} X_{i2} \times F_i \quad (3.19)$$

Substituting X_{11},X_{12},X_{21},X_{22},F_1,F_2 from equation 3.12 and 3.6 in equation 3.15 we get,

$G_1=0.929\times k1\times z3-98.86\times m2$ and $G_2=-1548.92\times k1\times z3+1168.15\times m2$

Where, g is acceleration due to gravity=9.81m/s^2

Now,

$$E_1=G_1/M_1 \quad \text{and} \quad E_2=G_2/M_2 \quad (3.20)$$

Substituting G_1,G_2,M_1,M_2 from equation 3.19 and 3.18 in 3.20, we get

$E_1=\frac{0.929\times k1\times z3-98.86\times m2}{1.06\times m2}$ and $E_2=\frac{-1548.92\times k1\times z3+1168.15\times m2}{166610.94\times m2}$

The general decoupled equation for a forced damped vibration system is given by:

$$\ddot{y}_1 + 2 \times \dot{y}_1 \times \xi_1 \times \omega_1 + \omega_1^2 \times y_1 = E_1 \quad (3.21)$$

$$\ddot{y}_2 + 2 \times \dot{y}_2 \times \xi_2 \times \omega_2 + \omega_2^2 \times y_2 = E_2 \quad (3.22)$$

To solve above equations, the general solutions is assume as,

$$y_1 = e^{(-\xi_1 \times \omega_1 \times t)} \times (A_1 \times \cos(\omega d_1 \times t) + B_1 \times \sin(\omega d_1 \times t)) + \frac{E_1}{12 \times k1 \times Mr} \quad (3.23)$$

$$y_2 = e^{(-\xi_2 \times \omega_2 \times t)} \times (A_2 \times \cos(\omega d_2 \times t) + B_2 \times \sin(\omega d_2 \times t)) + \frac{E_2}{k1 \times N} \quad (3.24)$$

Where,

$$Mr = \sqrt{(1 - r_1{}^2)^2 + (2 \times \xi_1 \times r_1)^2} \quad (3.25)$$

$$N = \sqrt{(1 - r_2{}^2)^2 + (2 \times \xi_2 \times r_2)^2} \quad (3.26)$$

$$r_1 = \left(\frac{\omega}{\omega_1}\right) \text{ and } r_2 = \left(\frac{\omega}{\omega_2}\right) \quad (3.27)$$

3.4.1 Equation of Road Holding

The equation of road holding (RH) is measured as relative displacement of unsprung mass with respect to road excitation.

$$\{x\}=[U]\{y\} \quad (3.28)$$

Putting value of matrix U from equation (3.15)

$$\begin{Bmatrix} x1 \\ x2 \end{Bmatrix} = \begin{Bmatrix} 1 & 1 \\ 0.077 & -129.077 \end{Bmatrix} \begin{Bmatrix} y1 \\ y2 \end{Bmatrix} \quad (3.29)$$

Where, x_1,x_2 are unsprung mass and sprung mass displacement as z_2 and z_1 shown in Figure 3.1.

Therefore,

$$z_2=x_1=y_1+y_2 \quad (3.30)$$

$$z_1=x_2=0.077 \times y_1-129.077 \times y_2 \quad (3.32)$$

$$RH=z_2-z_1 \quad (3.33)$$

The Ride Comfort for the system is formulated by double differentiating the expression z_1 obtained from the above equations.

3.5 SUMMARY OF THE CHAPTER

In this chapter, mathematical model of 2-DOF quarter car model using modal analysis. Next chapter will present the computational model of the 2-DOF quarter car using MATLAB and Simulink.

CHAPTER 4

COMPUTATIONAL MODELLING

4.1 INTRODUCTION

In this chapter, the details regarding the software and all the tools of the software have been explained briefly. It includes MATLAB coding details and Simulink model. The quarter car model has been created using mechanical tools provided in simulink. There are following assumptions considered for the modelling:

1. All components are considered rigid.
2. The elements of suspension system have linear characteristics.
3. The spring and damper are mass less.
4. The tire is assumed as spring and damping characteristics of tire is neglected.
5. Velocity of vehicle is assumed to be constant.
6. Road irregularity is bump type only, droops are not considered.

4.2 SIMULATION ENVIRONMENT

Simulink® is a 'block diagram environment for multidomain simulation and Model-Based Design'. It supports simulation, automatic code generation, and continuous test and verification of embedded systems.

Simulink provides a graphical editor, customizable block libraries, and solvers for modelling and simulating dynamic systems. It is integrated with MATLAB®, enabling you to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis.

4.2.1 MATLAB

MATLAB® is the 'easiest and most productive software for engineers and scientists'. Whether you're analysing data, developing algorithms, or creating models, MATLAB provides the environment that invites exploration and discovery. It combines a high-level language with a desktop environment tuned for iterative engineering and scientific workflows.

Key features of MATLAB are as follows:

- High-level language for scientific and engineering computing
- Desktop environment tuned for iterative exploration, design, and problem-solving
- Graphics for visualizing data and tools for creating custom plots
- Apps for curve fitting, data classification, signal analysis, and many other domain-specific tasks
- Add-on toolboxes for a wide range of engineering and scientific applications
- Tools for building applications with custom user interfaces
- Interfaces to C/C++, Java®, .NET, Python®, SQL, Hadoop®, and Microsoft® Excel®
- Royalty-free deployment options for sharing MATLAB programs with end users
- Build and package custom MATLAB apps and toolboxes to share with other MATLAB users.
- Create standalone executables to share with others who do not have MATLAB.
- Integrate with C/C++, Java, .NET, and Python. Call those languages directly from MATLAB, or package MATLAB algorithms and applications for deployment within web, enterprise, and production systems.
- Convert MATLAB algorithms to C, HDL, and PLC code to run on embedded devices.
- Deploy MATLAB code to run on production Hadoop systems.

4.2.2 MATLAB/SIMULINK®

- Graphical editor for building and managing hierarchical block diagrams
- Libraries of predefined blocks for modeling continuous-time and discrete-time systems

- Simulation engine with fixed-step and variable-step ODE solvers
- Scopes and data displays for viewing simulation results
- Project and data management tools for managing model files and data
- Model analysis tools for refining model architecture and increasing simulation speed
- MATLAB Function block for importing MATLAB algorithms into models
- Legacy Code Tool for importing C and C++ code into models

4.3 PHYSICAL MODELING IN SIMULINK

Physical modelling is a way of modelling and simulating systems that consist of real physical components. It employs a physical network approach, where Simscape™ blocks correspond to physical elements, such as pumps, motors, and op-amps. You join these blocks by lines corresponding to the physical connections that transmit power. This approach lets you describe the physical structure of a system, rather than the underlying mathematics.

Simscape Foundation libraries contain a comprehensive set of basic elements and building blocks, organized by domain. Connect these blocks together just as you would assemble a physical system. Use these blocks, along with the blocks from the add-on products, such as Simscape Electronics™ or Simscape Driveline™, to model multidomain physical systems.

4.3.1 Mechanical Models

Mechanical elements for rotational and translational motion, simple mechanisms, mechanical sensors and sources are inbuilt in Simulink library.

Mechanical libraries contain blocks for the mechanical rotational and mechanical translational domains, organized into rotational and translational elements, mechanisms, sources, and sensors. Connect these blocks together just as you would assemble a physical system. Use these blocks, along with the blocks from other Foundation libraries and the add-on products, to model multidomain physical systems.

Blocks used:

- Translational Spring

The Translational Spring block represents an ideal mechanical linear spring, described with the following equations:

$F=Kx$

$x=xinit+xR-xC$

$v=dx/dt$

The block positive direction is from port R to port C. This means that the force is positive if it acts in the direction from R to C. The block has the following ports:

R: Mechanical translational conserving port.

C: Mechanical translational conserving port.

- Translational damper

The Translational Damper block represents an ideal mechanical translational viscous damper, described with the following equations:

$F=Dv$

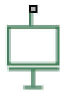
$v=vR-vC$

The block positive direction is from port R to port C. This means that the force is positive if it acts in the direction from R to C. The block has the following ports:

R=Mechanical translational conserving port associated with the damper rod.

C=Mechanical translational conserving port associated with the damper case.

- Mass

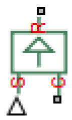


The Mass block represents an ideal mechanical translational mass, described with the following equation:

F=mdv/dt

The block has one mechanical translational conserving port. The block positive direction is from its port to the reference point. This means that the inertia force is positive if mass is accelerated in positive direction. The block has one mechanical translational conserving port, associated with the mass connection to the system.

- Ideal Translational Velocity Source



The Ideal Translational Velocity Source block represents an ideal source of velocity that generates velocity differential at its terminals proportional to the input physical signal. The source is ideal in a sense that it is assumed to be powerful enough to maintain specified velocity regardless of the force exerted on the system.

Connections R and C are mechanical translational conserving ports. Port S is a physical signal port, through which the control signal that drives the source is applied. The relative velocity (velocity differential) across the source is directly proportional to the signal at the control port S. The entire variety of Simulink® signal sources can be used to generate the desired velocity variation profile.

The block positive direction is from port R to port C. This means that the velocity is measured as $v = v_R – v_C$, where v_R, v_C are the absolute velocities at ports R and C, respectively, and force through the source is negative if it is acts from C to R. The power generated by the source is negative if the source delivers energy to port R.

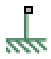
The block has the following ports:

R=Mechanical translational conserving port.

C=Mechanical translational conserving port associated with the source reference point (case).

S=Physical signal input port, through which the control signal that drives the source is applied.

- Mechanical Translational Reference



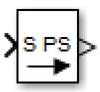
The Mechanical Translational Reference block represents a reference point, or frame, for all mechanical translational ports. All translational ports that are rigidly clamped to the frame (ground) must be connected to a Mechanical Translational Reference block. The block has one mechanical translational port.

- Solver Configuration

Each physical network represented by a connected Simscape™ block diagram requires solver settings information for simulation. The Solver Configuration block specifies the solver parameters that your model needs before you can begin simulation.

Each topologically distinct Simscape block diagram requires exactly one Solver Configuration block to be connected to it. The block has one conserving port. You can add this block anywhere on a physical network circuit by creating a branching point and connecting it to the only port of the Solver Configuration block.

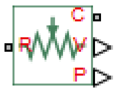
- Simulink-PS Converter



The Simulink-PS Converter block converts the input Simulink® signal into a physical signal. Use this block to connect Simulink sources or other Simulink blocks to the inputs of a Physical Network diagram.

Specify the desired unit as the Input signal unit parameter. The parameter value controls the unit of the physical signal at the output port of the block, which serves as the input signal for the Simscape™ physical network.

- Ideal Translational Motion Sensor



The Ideal Translational Motion Sensor block represents a device that converts an across variable measured between two mechanical translational nodes into a control signal proportional to velocity or position. You can specify the initial position (offset) as a block parameter.

The sensor is ideal since it does not account for inertia, friction, delays, energy consumption, and so on.

Connections R and C are mechanical translational conserving ports that connect the block to the nodes whose motion is being monitored. Connections V and P are physical signal output ports for velocity and position, respectively.

The block positive direction is from port R to port C. This means that the velocity is measured as $v = v_R - v_C$, where v_R, v_C are the absolute velocities at ports R and C, respectively. The block has the following ports:

R=Mechanical translational conserving port associated with the sensor positive probe.

C=Mechanical translational conserving port associated with the sensor negative (reference) probe.

V=Physical signal output port for velocity.

P=Physical signal output port for position.

4.4 OPTIMIZATION TOOL

In optimization tool in MATLAB fmincon solver is used to find a minimum of a constrained nonlinear multivariable function using the interior-point algorithm.

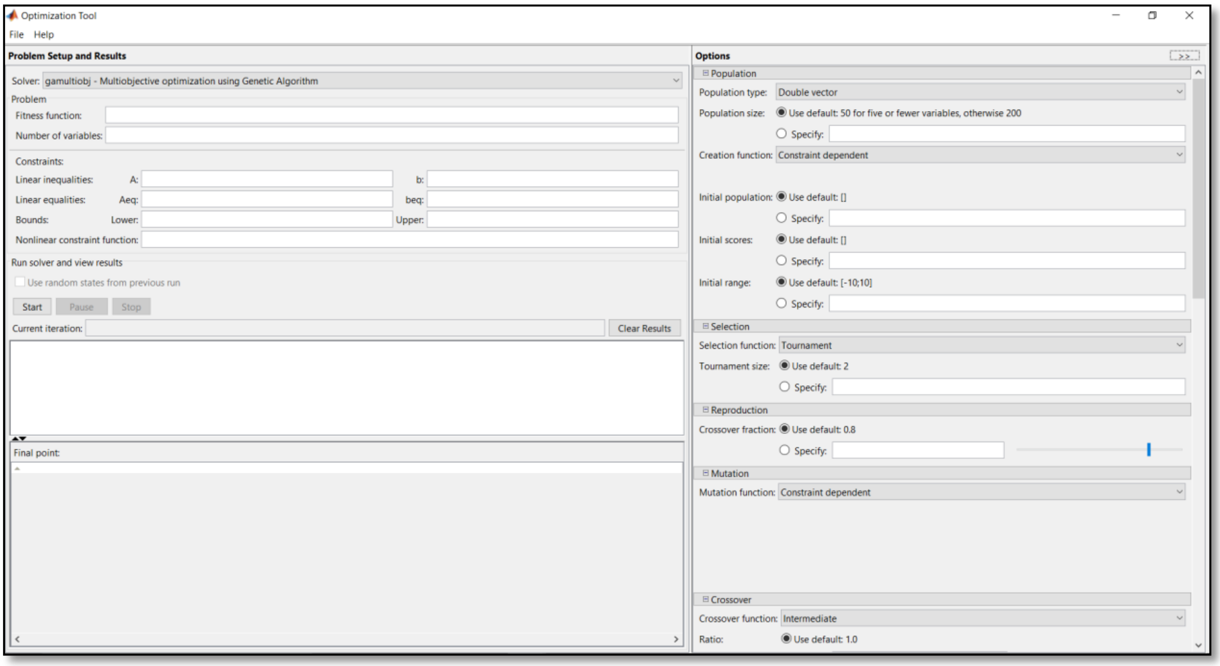


Figure 4.1 Optimization toolbox in MATLAB

Steps involved in optimization of the multiobjective problem:

1. Selecting the solver

There are various are pre-defined in the optimization toolbox. Some of the solvers are as follows

- Multiobjective goal attainment
- Single variable nonlinear mionimization with bounds
- Constrained nonlinear minimization
- Minimax optimization
- Unconstrained nonlinear minimization
- Unconstrained nonlinear minimization
- Semi variable nonlinear equation solving
- Genetic algorithm
- Multiobjective optimization using genetic algorithm
- Linear programming
- Nonlinear curve fitting
- Constrained linear least squares
- Nonlinear least squares

- Nonnegative linear least squares
- Pattern search
- Quadratic programming
- Simulated annealing algorithm

We have used multiobjective genetic algorithm solver as it is a global minimization algorithm.

2. Defining the problem

Fitness function (required) is the multiobjective (vector) function you want to minimize. You can specify the function as a function handle of the form @objfun, where objfun.m is a function file or as an anonymous function.

Number of variables is the required number of independent variables for the fitness function.

3. Constraints definition

At least one constraint is required to run the solver.

Linear inequalities of the form A*x = b are specified by the matrix A and the vector b.

Linear equalities of the form A_{eq}*x = b_{eq} are specified by the matrix A_{eq} and the vector b_{eq}.

4. Lower and upper bounds on the variables are defined, specified as vectors.

Nonlinear constraint function defines the nonlinear constraints. Specify the function as a function handle of the form @nonlcon, where nonlcon.m is a function file that returns the vectors c and ceq. The nonlinear equalities are of the form ceq = 0, and the nonlinear inequalities are of the form c = 0.

5. Run solver and view results

To run the solver, click the Start button.

When the algorithm terminates, the Run solver and view results window displays the status and results, including the reason the algorithm terminated. The Final point also updates to show the coordinates of the final point.

4.5 OPTIMISATION TECHNIQUE USING GENETIC ALGORITHM

Genetic Algorithms (GA) are direct, parallel, stochastic method for global search and optimization, which imitates the evolution of the living beings, described by Charles Darwin. GA is part of the group of Evolutionary Algorithms (EA). The evolutionary algorithms use the three main principles of the natural evolution: reproduction, natural selection and diversity of the species, maintained by the differences of each generation with the previous. Genetic Algorithm works with a set of individuals, representing possible solutions of the task. The selection principle is applied by using a criterion, giving an evaluation for the individual with respect to the desired solution. The best-suited individuals create the next generation. The large variety of problems in the engineering sphere, as well as in other fields, requires the usage of algorithms from different type, with different characteristics and settings.

Elements of GA

- **Chromosomes** During the division process of the human cells the chromatin (contained in the nucleus and built from DNA (deoxyribonucleic acid), proteins and RNA (ribonucleic acid)) become shorter and thicker and forms spiral strings – chromosomes. In these chromosomes are the genes, that carry the inherited cell information. Every gene codes particular protein and is independent factor of the genetic information, which determines the appearance of different peculiarities. For the genetic algorithms, the chromosomes represent set of genes, which code the independent variables. Every chromosome represents a solution of the given problem. Individual and vector of variables will be used as other words for chromosomes. From other hand, the genes could be Boolean, integers, floating point or string variables, as well as any combination of the above. A set of different chromosomes (individuals) forms a generation. By means of evolutionary operators, like selection, recombination and mutation an offspring population is created.
- **Selection** In the nature, the selection of individuals is performed by survival of the fittest. The more one individual is adapted to the environment - the bigger are its chances to survive and create an offspring and thus transfer its genes to the next population. In EA the selection of the best individuals is based on an

evaluation of fitness function or fitness functions. Examples for such fitness function are the sum of the square error between the wanted system response and the real one; the distance of the poles of the closed-loop system to the desired poles, etc. If the optimization problem is a minimization one, than individuals with small value of the fitness function will have bigger chances for recombination and respectively for generating offspring.

- **Crossover** In this section we will discuss some of the most popularly used crossover operators. It is to be noted that these crossover operators are very generic and the GA Designer might choose to implement a problem-specific crossover operator as well.

  One Point Crossover: In this one-point crossover, a random crossover point is selected and the tails of its two parents are swapped to get new off-springs.

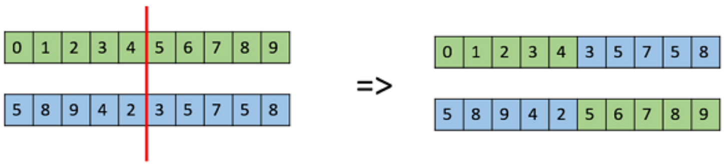


Figure 4.2 One Point Crossover

Multi Point Crossover: Multi point crossover is a generalization of the one-point crossover wherein alternating segments are swapped to get new off-springs.

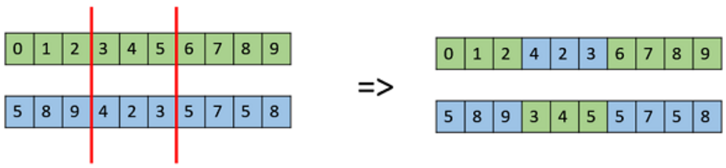


Figure 4.3 Multi Point Crossover

Uniform Crossover:In a uniform crossover, we don’t divide the chromosome into segments, rather we treat each gene separately. In this, we essentially flip a coin for each chromosome to decide whether or not it’ll be included in the off-spring. We can also bias the coin to one parent, to have more genetic material in the child from that parent.

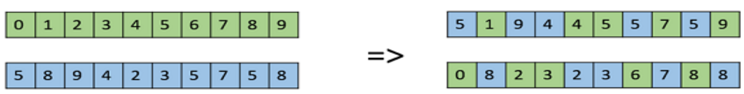


Figure 4.4 Uniform Point Crossover

- **Mutation** The newly created by means of selection and crossover population can be further applied to mutation. Mutation means, that some elements of the DNA are changed. Those changes are caused mainly by mistakes during the copy process of the parent's genes. In the terms of GA, mutation means random change of the value of a gene in the population

4.5.1 Scheme of the Evolutionary Algorithms

The EA holds a population of individuals (chromosomes), which evolve my means of selection and other operators like crossover and mutation. Every individual in the population gets an evaluation of its adaptation (fitness) to the environment. In the terms of optimization this means, that the function that is maximized or minimized is evaluated for every individual. The selection chooses the best gene combinations (individuals), which through crossover and mutation should drive to better solutions in the next population.

1. Generate initial population – in most of the algorithms the first generation is randomly generated, by selecting the genes of the chromosomes among the allowed alphabet for the gene. Because of the easier computational procedure it is accepted that all populations have the same number (N) of individuals.

2. Calculation of the values of the function that we want to minimize of maximize.

3. Check for termination of the algorithm – as in the most optimization algorithms, it is possible to stop the genetic optimization by: Value of the function – the value of the function of the best individual is within defined range around a set value. It is not recommended to use this criterion alone, because of the stochastic element in the search the procedure, the optimization might not finish within sensible time; · Maximal number of iterations – this is the most widely used stopping criteria. It guarantees that the algorithms will give some results within some time, whenever it has reached the extremum or not; · Stall generation – if within initially set number of iterations (generations) there is no improvement of the value of the fitness function of the best individual the algorithms stops.

4. Selection – between all individuals in the current population are chose those, who will continue and by means of crossover and mutation will produce offspring population. At this stage elitism could be used – the best n individuals are directly

transferred to the next generation. The elitism guarantees, that the value of the optimization function cannot get worst (once the extremum is reached it would be kept).

5. Crossover – the individuals chosen by selection recombine with each other and new individuals will be created. The aim is to get offspring individuals, that inherit the best possible combination of the characteristics (genes) of their parents.

6. Mutation – by means of random change of some of the genes, it is guaranteed that even if none of the individuals contain the necessary gene value for the general scheme of the evolutionary algorithms 8 extremum, it is still possible to reach the extremum.

7. New generation – the elite individuals chosen from the selection are combined with those who passed the crossover and mutation, and form the next generation.

4.6 SUMMARY OF THE CHAPTER

In this chapter, the software technique involved for solving the problem has been discussed. It includes details of MATLAB/Simulink® and optimisation toolbox. This chapter also includes the details of genetic algorithm. The next chapter discusses about the results of the multiobjective problem.

CHAPTER 5

RESULT AND DISCUSSIONS

5.1 INTRODUCTION

In the previous chapters, the methodology for simulation has been discussed, which included the literature review conducted under the observation of the fulfilment of the objective statement. The mathematical modelling and computational modelling has also been elaborated briefly for the quarter car suspension system.

In this chapter, the performance of two DOF system before and after optimisation will be discussed. The optimisation was a trade off between ride comfort and ride height, selected using pareto front in genetic optimisation module of MATLAB. The different performance curves are generated for sprung mass acceleration, velocity, displacement, same for the unsprung mass and maximum deflection for different cases involved in the convergence of the optimisation. The results obtained before and after optimisation have been compared.

5.2 ROAD PROFILE

The road profile selected for evaluation of the performance of the suspension system is a sinusoidal bump of amplitude 0.1m.

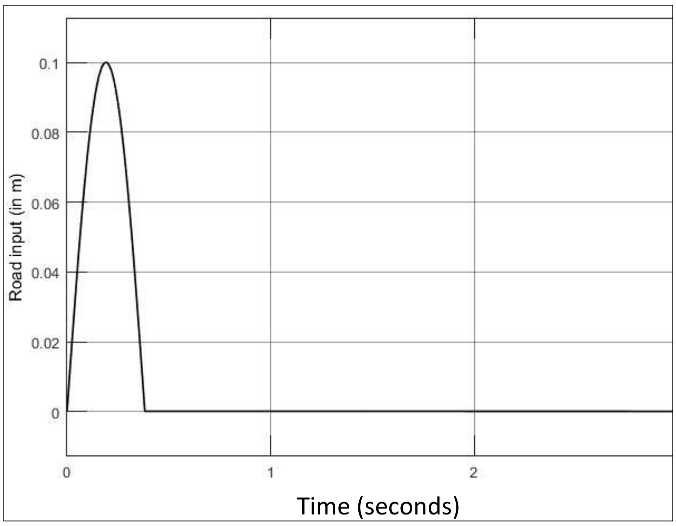


Figure 5.1 Road Input

This road input is fed in the simulink model, discussed in previous chapter, to the quarter car model.

5.3 PERFORMANCE OF QUARTER CAR MODEL AT INITIAL DESIGN CONDITIONS

The quarter car model was given an input of half sine bump. Simulation has been been carried out for the simulink model for quarter car model in the software environment of MATLAB®. The results have been displayed for sprung and unsprung mass acceleration, velocity, displacement and maximum deflection. For initial condition, parameters are m_1=210kg, m_2=21kg, k_1=12394N/m, k_2=148728N/m, c_1=1395Ns/m.

The quarter car model in simulink is processed for 5 seconds to obtain the following results. The body acceleration vs time plot for the above given condition is shown in Figure 5.2.

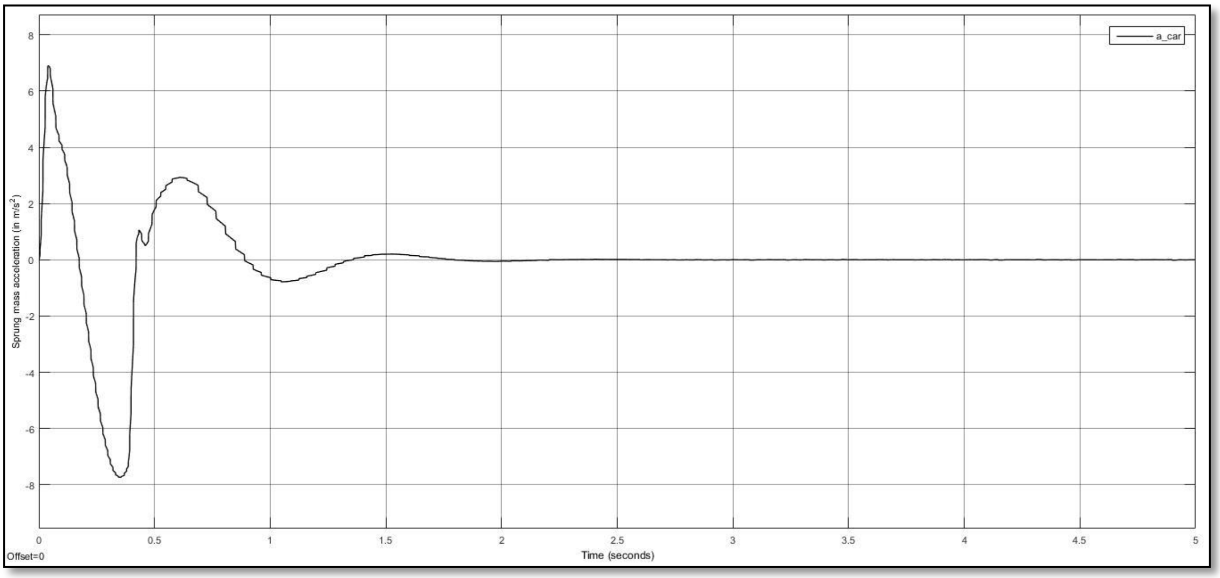


Figure 5.2 Sprung mass acceleration Vs Time for maximum RC

It is shown in Figure 5.2 that the body acceleration first peak is obtained at 0.041s with 6.894m/s^2 as the largest value of acceleration. The system settiling time is about 4.313s at which the acceleration reduced to 10^{-2} order. The rms value of acceleration is obtained as 2.569m/s^2.

It is evident that the the acceleration amplitude has been damped upto 2.157×10^{-2} m/s^2 in 2.4s.

The body velocity vs time plot has been shown in Figure 5.3.

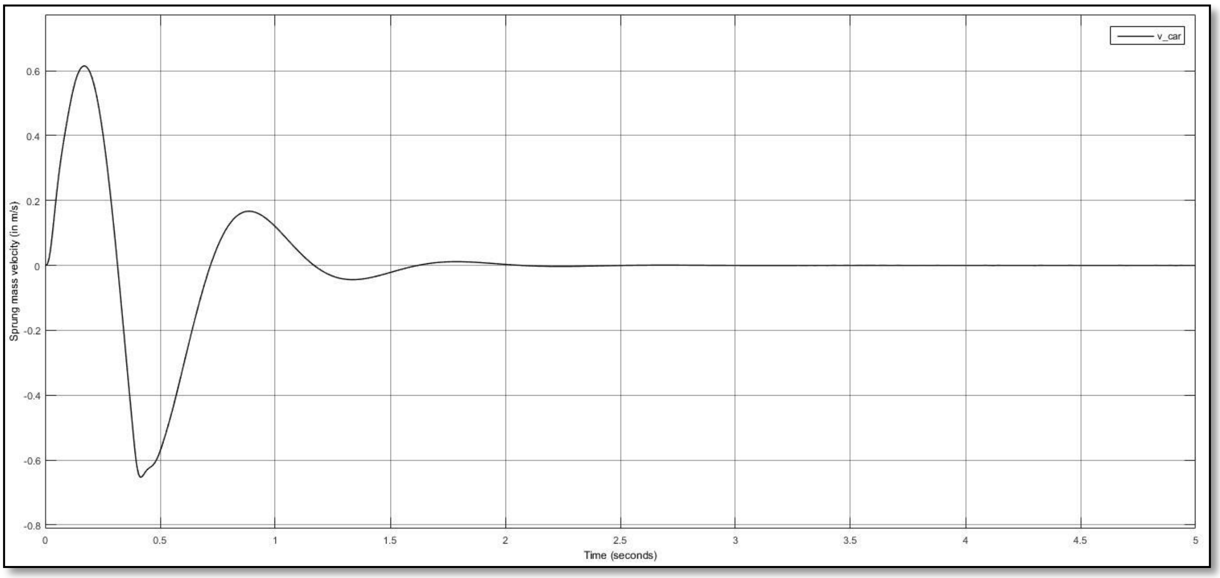


Figure 5.3 Sprung mass velocity Vs Time for maximum RC

It is shown in Figure 5.3 that the maximum value of velocity is 6.148×10^{-1}m/s at 0.17s and the amplitude get reduced to 5.82×10^{-4}m/s in 2.5s.

Figure 5.4 shows the body displacement vs time plot.

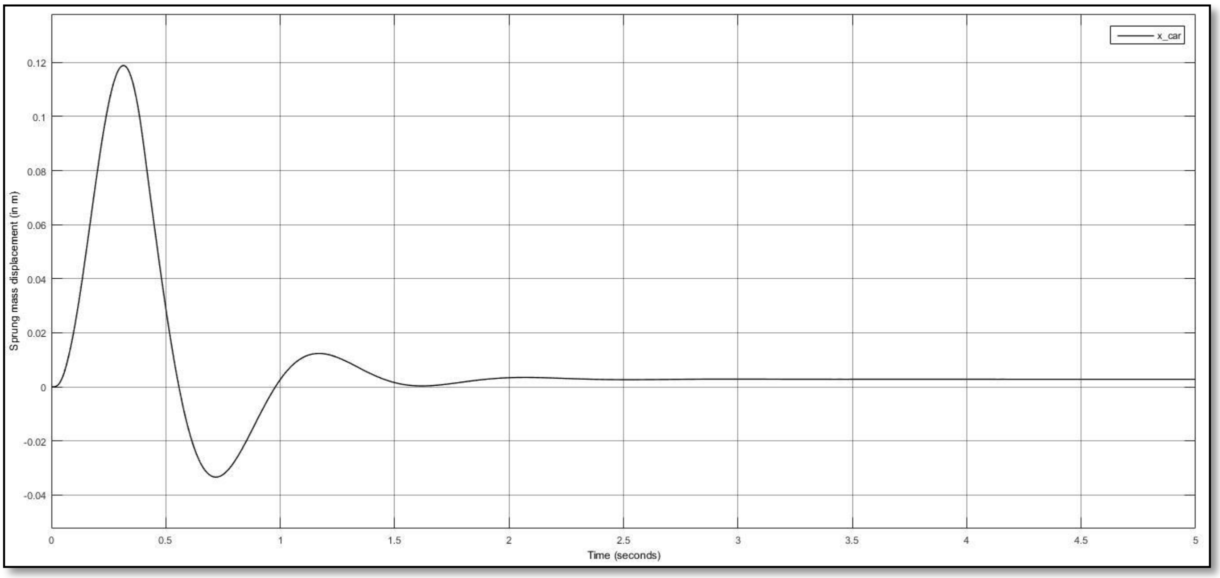


Figure 5.4 Sprung mass displacement Vs Time for maximum RC

It is shown in Figure 5.4 that the the body displacement varies from 1.189×10^{-1}m to 1.26×10^{-2}m in 0.853s.

The maximum deflection or the suspension travel is defined as the difference between body displacement and unsprung mass displacement (x_2-x_1). The plot for maximum deflection vs time is shown in Figure 5.5.

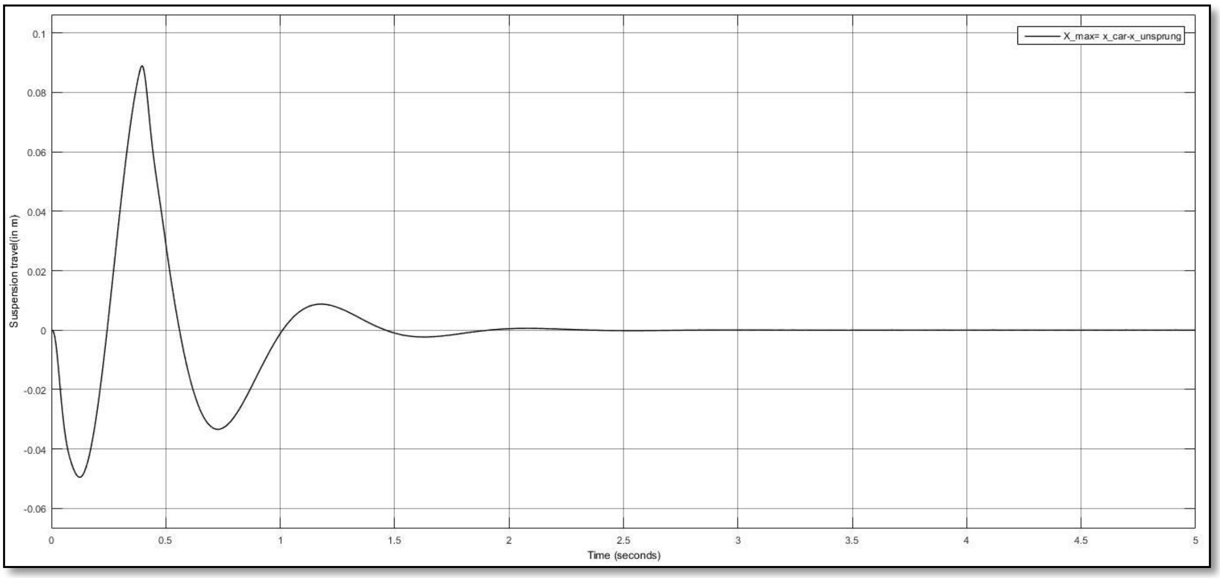


Figure 5.5 Maximum Travel Vs Time for maximum RC

The suspension travel amplitude varies from 8.89×10^{-2}m at 0.396s to 6.076×10^{-4}m at 2.084s.

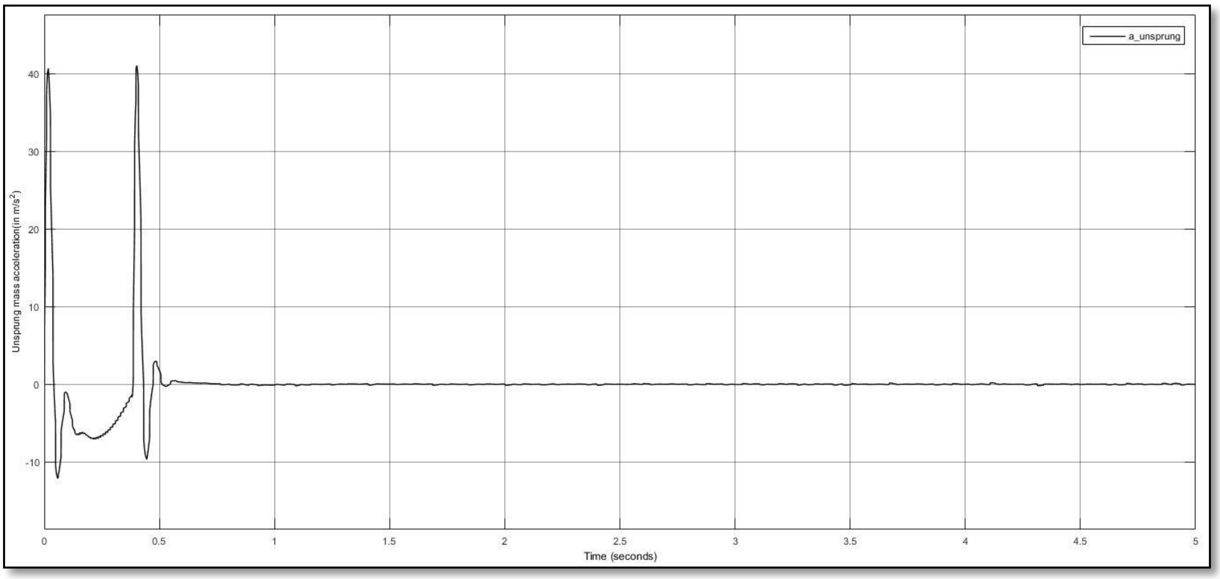


Figure 5.6 Unsprung mass acceleration Vs Time for maximum RC

The Figure 5.6 shows the variation of unsprung mass acceleration with time. It is also evident that the unsprung mass acceleration is very high compared to body acceleration.th maximum amplitude of unsprung mass acceleration is 4.096×10^{1}m/s^2 at .018s whether the body acceleration was 6.894m/s^2 at 0.041s, which shows the effective damping between the two stages.

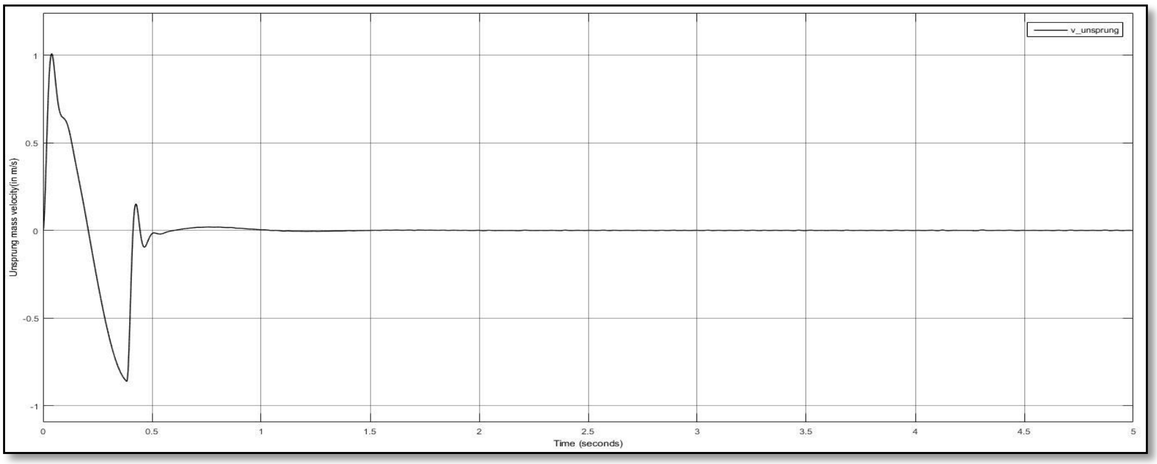


Figure 5.7 Unprung mass velocity Vs Time for maximum RC

The next plot is between unsprung mass velocity and time which shown in Figure 5.7.

Figure 5.8 is plotted between unsprung mass displacement and time. The unsprung mass displacement varies from 1.065×10^{-1}m at 0.208s to 2.192×10^{-3}m at 2.010s. The second peak is at -1.317×10^{-3}m at 0.602s. The third peak is obtained at 1.044s of amplitude equals to 3.947×10^{-3}.

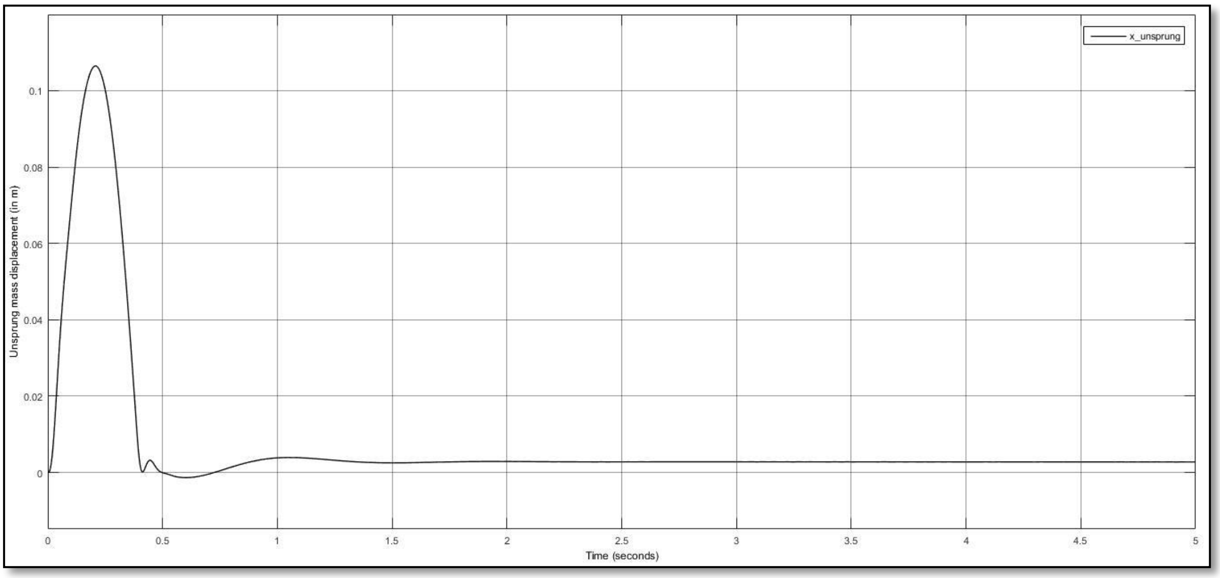


Figure 5.8 Unsprung mass displacement Vs Time for maximum RC

5.4 RESULTS OF OPTIMIZATION

The Figure 5.9 explains the variation of different parameters in the obsession of the Genetic Optimization. These graph shows the behaviour of the selection of population in the optimisation procedure and the range of parameters thus involved for the same.

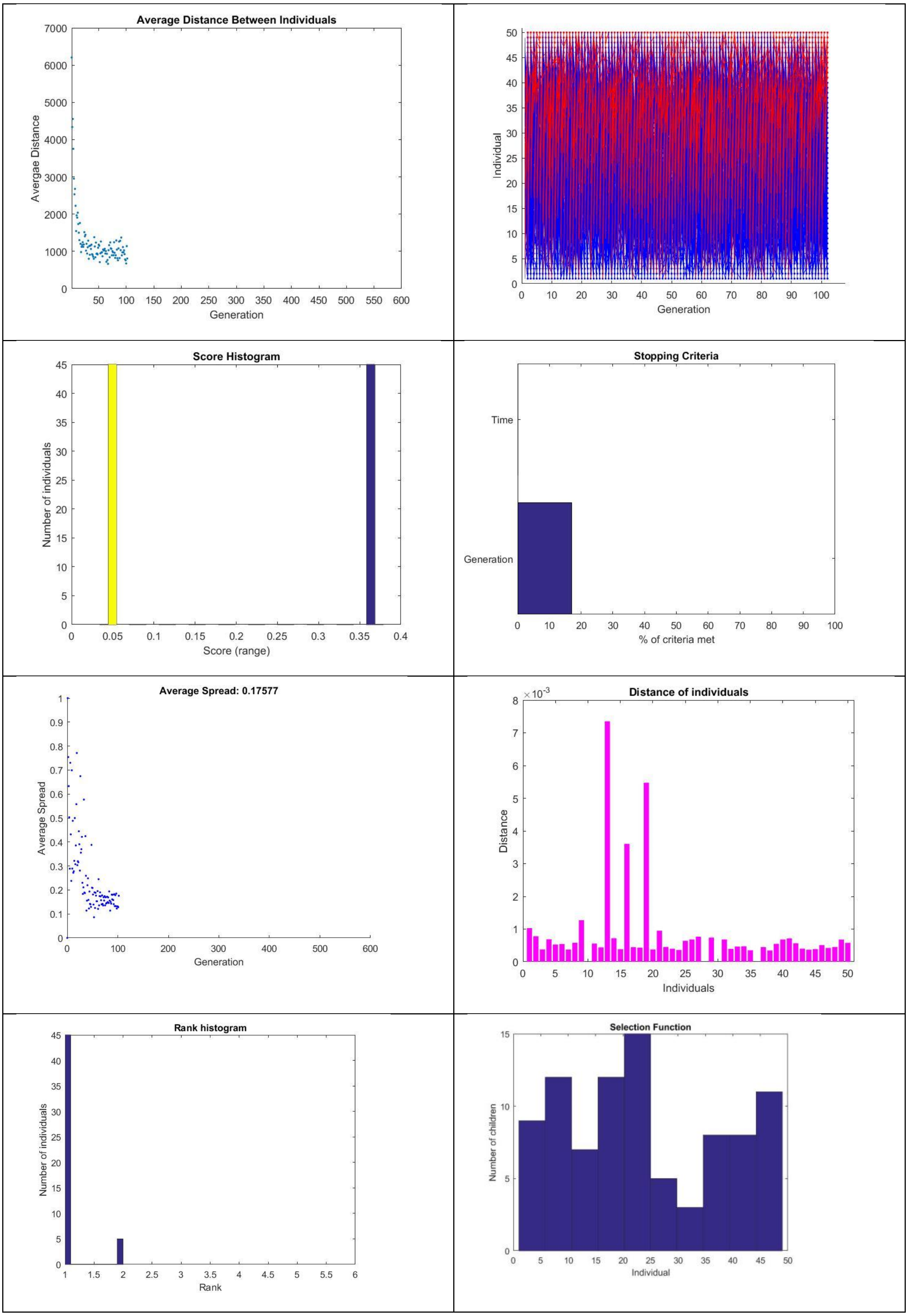


Figure 5.9 Genetic Algorithm response curves for multiobjective function system.

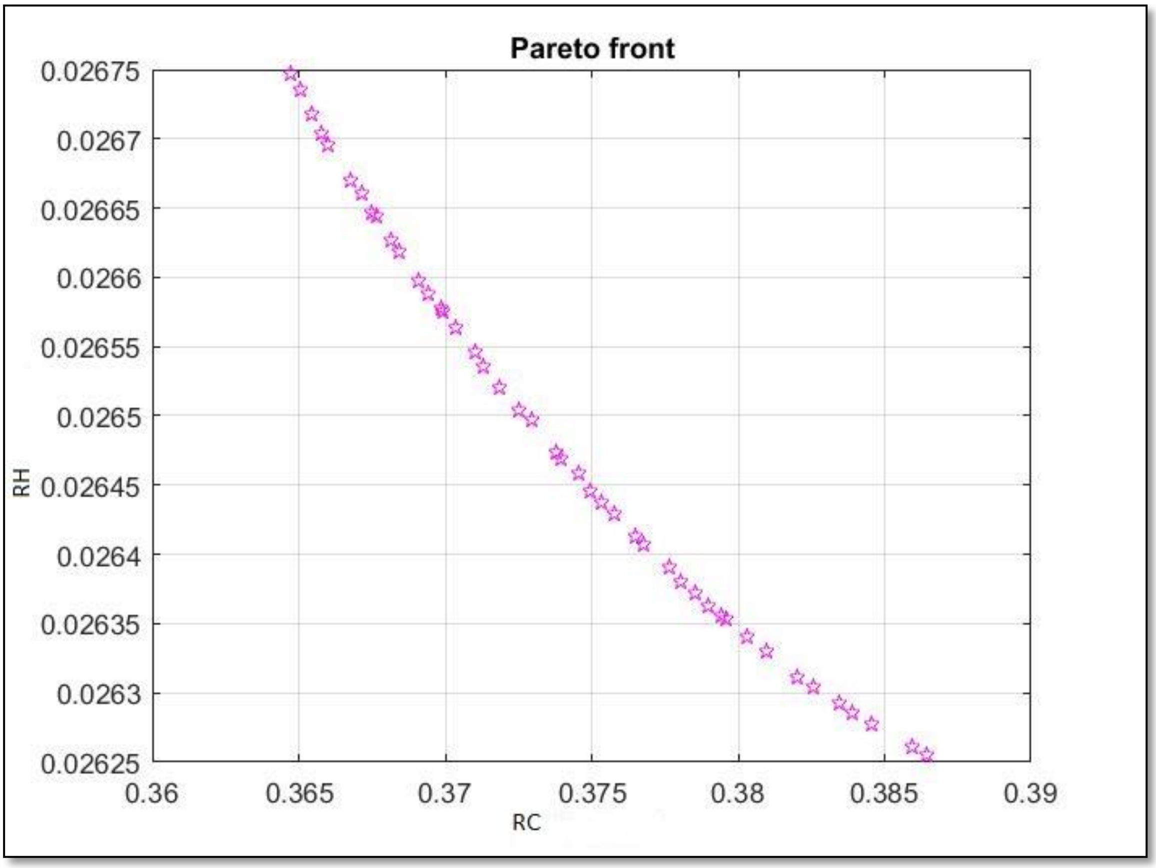


Figure 5.10 Pareto front for optimization of RC and RH

The multiobjective problem of optimizing RC and RH is performed using Genetic Algorithm multi objective module in MATLAB. The two objective functions RC and RH are to be maximised and minimised respectively. The variables taken for the process are m_1, m_2, k_1, k_2 and c_1. The default values of crossover probability, mutation and population size has been considered to obtain the Pareto front. The optimised state points are shown in Figure 5.10.

Now, considering the cases involved in reaching the optimised soution.

Csae I: For m_1=349.9 kg, m_2=34.9 kg, k_1=14727.2 N/m, k_2=176726.4 and c_1=1762.51 Ns/m.

Figure 5.11 shows the body accelration vs time plot for the above given parameters.

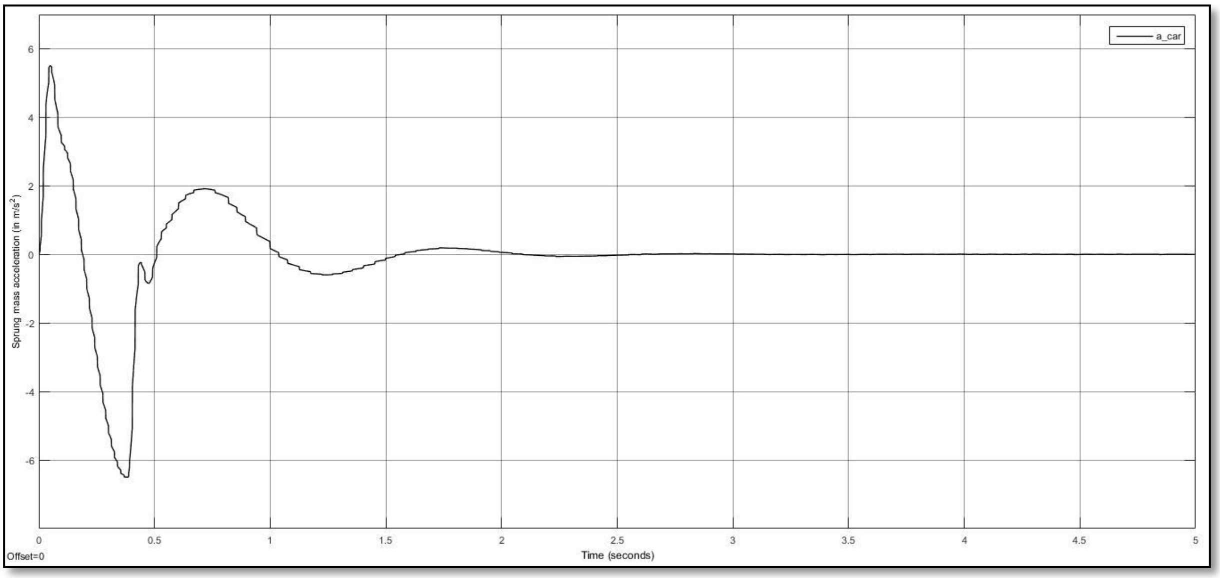


Figure 5.11 Sprung mass acceleration Vs Time for maximum RC

Here, for the above values of the governing parameter, the body acceleration amplitude has been reduced from 6.894m/s^2 to 5.496m/s^2, even the rms value is reduced from 2.56 to 2.05. The 10^{-2} order amplitude is attained in 2.8s which is better than 4.313 in the initial problem.

Figure 5.12 shows the plot between Sprung mass velocity and time for the optimised parameters mentioned above.

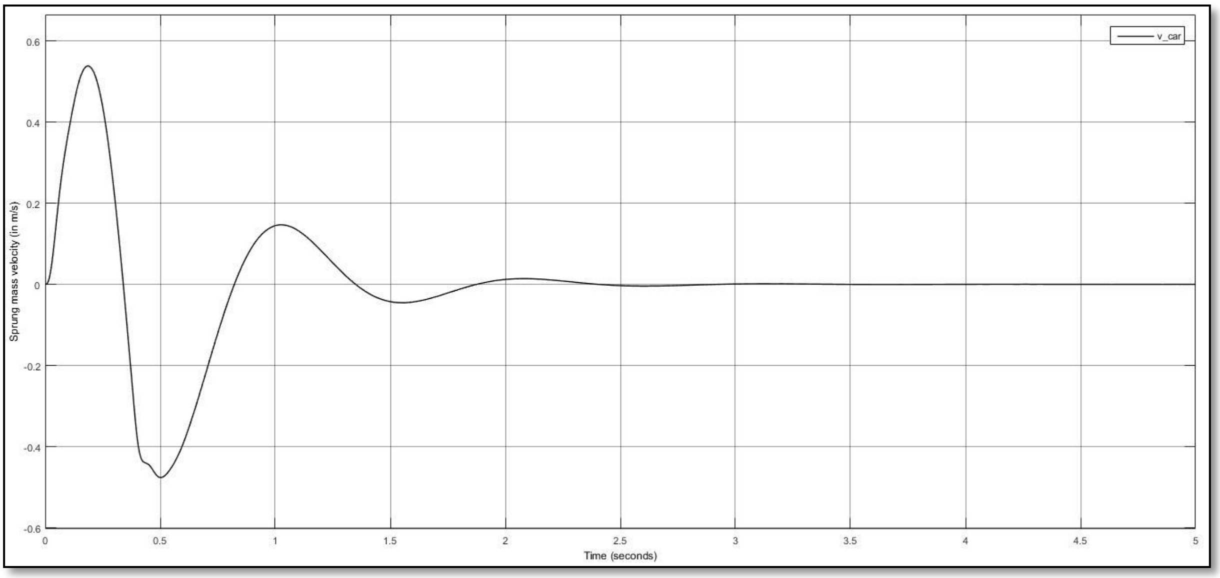


Figure 5.12 Sprung mass velocity Vs Time for maximum RC

The velocity of sprung mass has been reduced from 6.148×10^{-1}m/s to 5.382×10^{-1}m/s at 0.17s .

The Figure 5.13 represents the sprung mass displacement for the first case towards optimised solution.

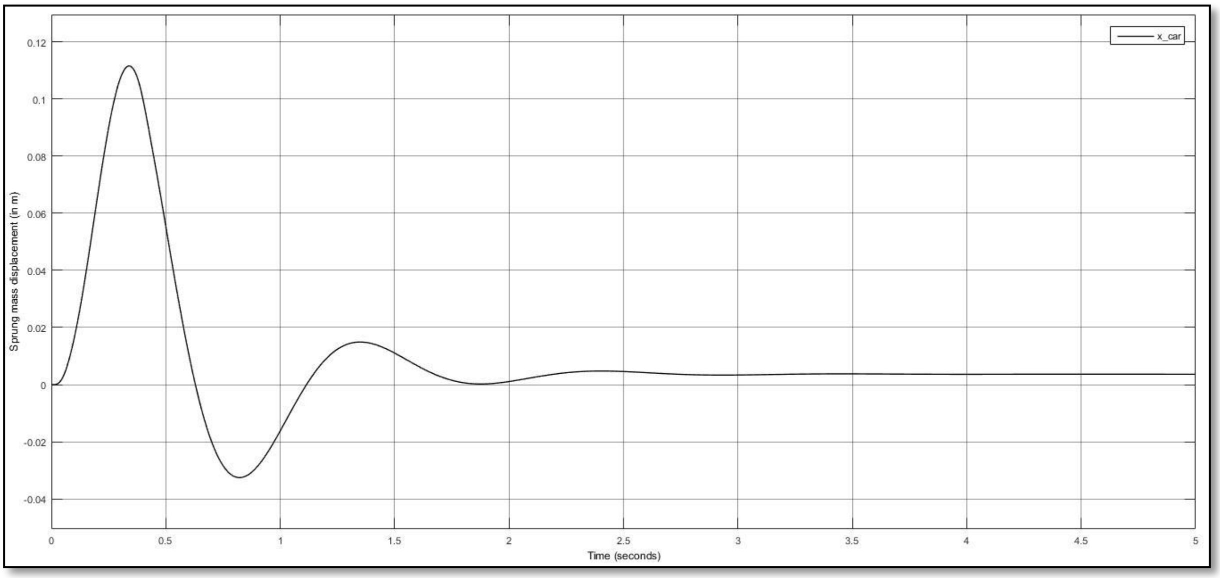


Figure 5.13 Sprung mass displacement Vs Time for maximum RC

The body displacement has been reduced to 1.15×10^{-1}m from 1.189×10^{-1} at 0.3s.

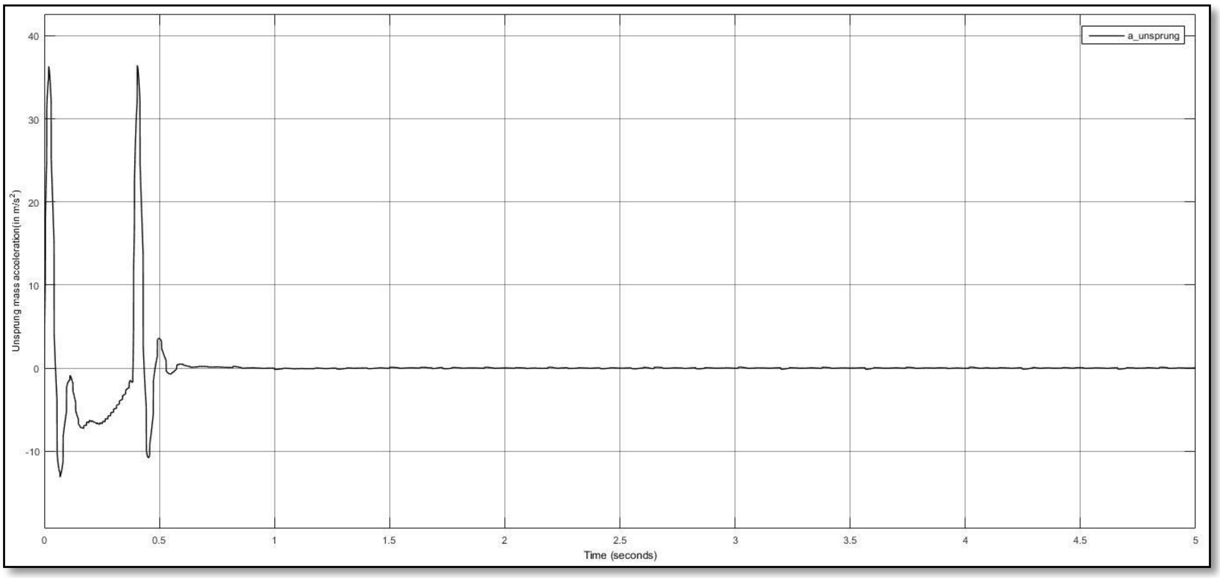


Figure 5.14 Unsprung mass acceleration Vs Time for maximum RC

Unprung mass acceleration has been decreased from 4.096×10^{1}m/s^{2} at 0.4s to 3.557×10^{1}m/s^{2} at 0.4s.

Figure 5.15 represents the unsprung velocity vs time plot which indicates the reduction of amplitude to the order of 10^{-2} in 0.7s instead of 0.9s in non-optimised results.

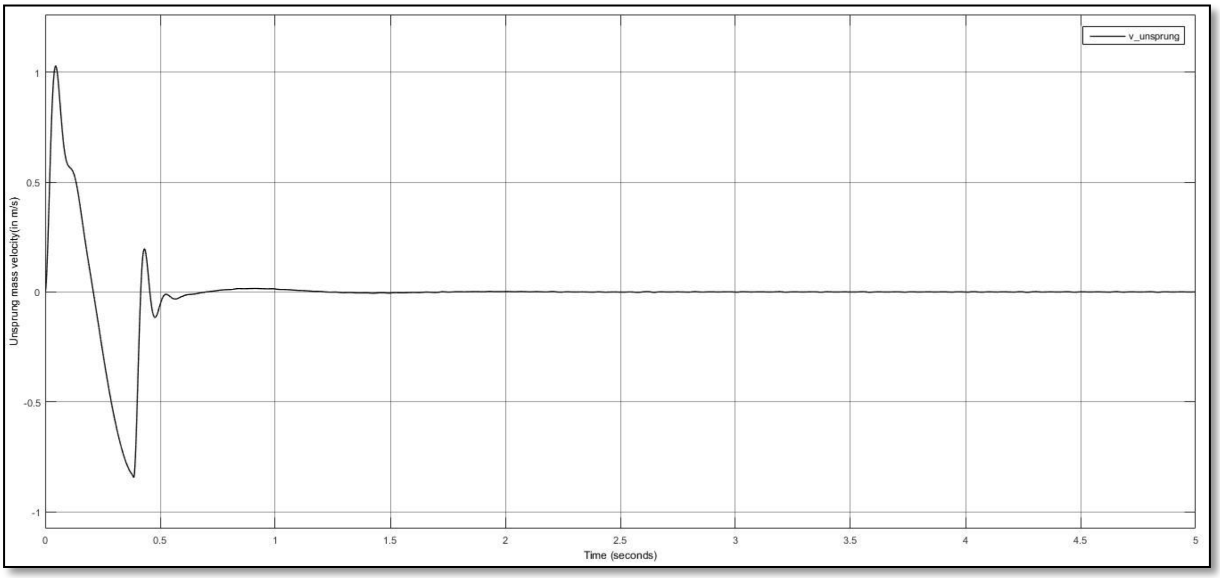


Figure 5.15 Unsprung mass velocity Vs Time for maximum RC

In the Figure 5.16 it is evident that the damping reduced the displacement amplitude to insignificant amount in 1.22s while it was 2.01s for non-optimised results.

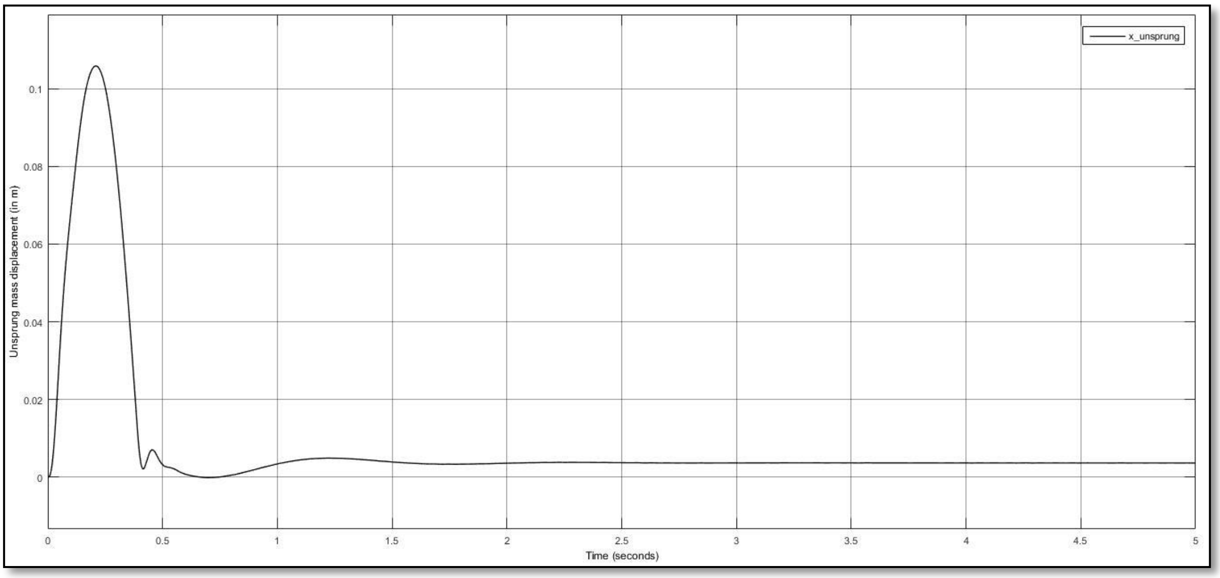


Figure 5.16 Sprung mass displacement Vs Time for maximum RC

The suspension travel has been increased from $8.89\times^{-2}$m to 9.428×10^{-2}m as shown in Figure 5.17.

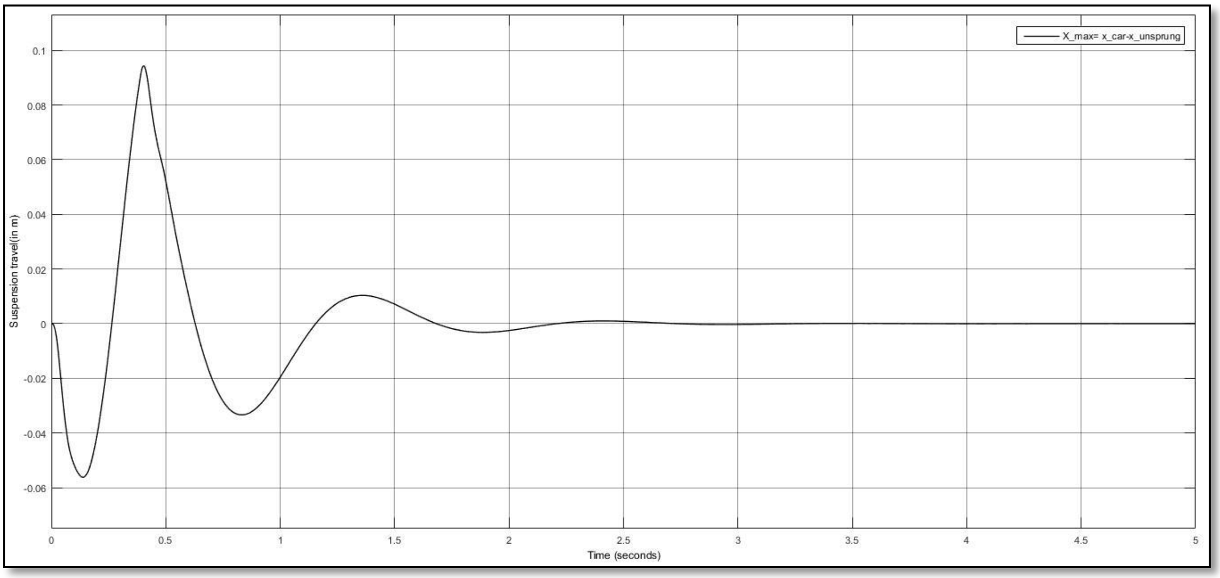


Figure 5.17 Maximum deflection Vs Time for maximum RC

Case II: For m_1=349.8 kg, m_2=34.9 kg, k_1=13366.17 N/m, k_2=160394.1 and c_1=1762.56 Ns/m.

In Figure 5.18 for the second case the optimised acceleration of sprung mass has been reduced to 5.496m/s^2 and the rms value to 2.056.

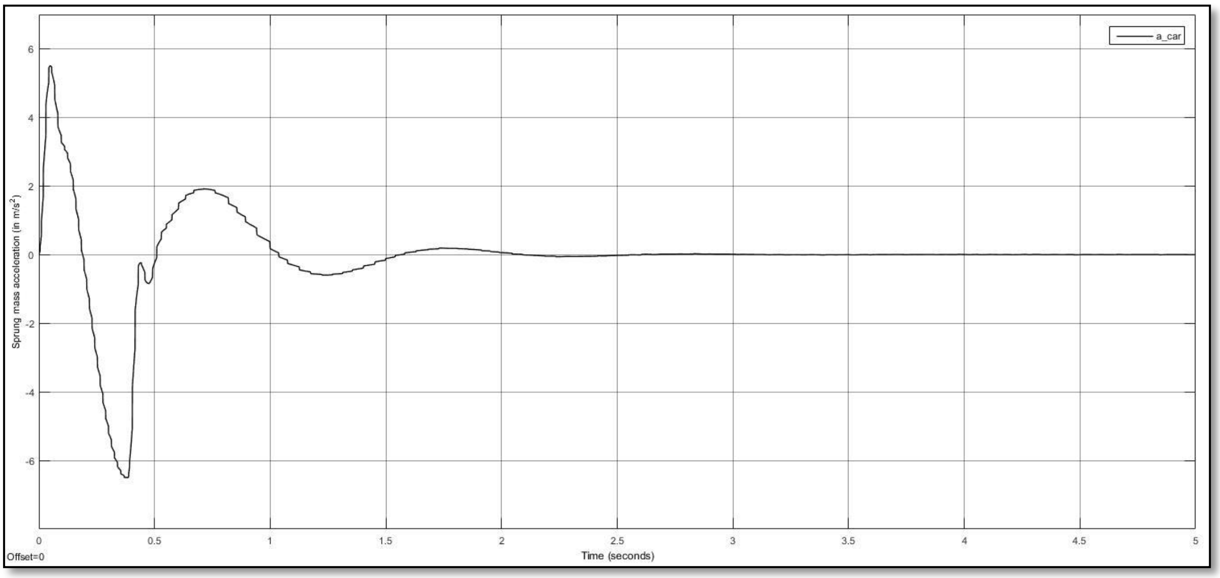


Figure 5.18 Sprung mass acceleration Vs Time for maximum RC

The sprung mass velocity is reduced to 5.382m/s as shown in Figure 5.19.

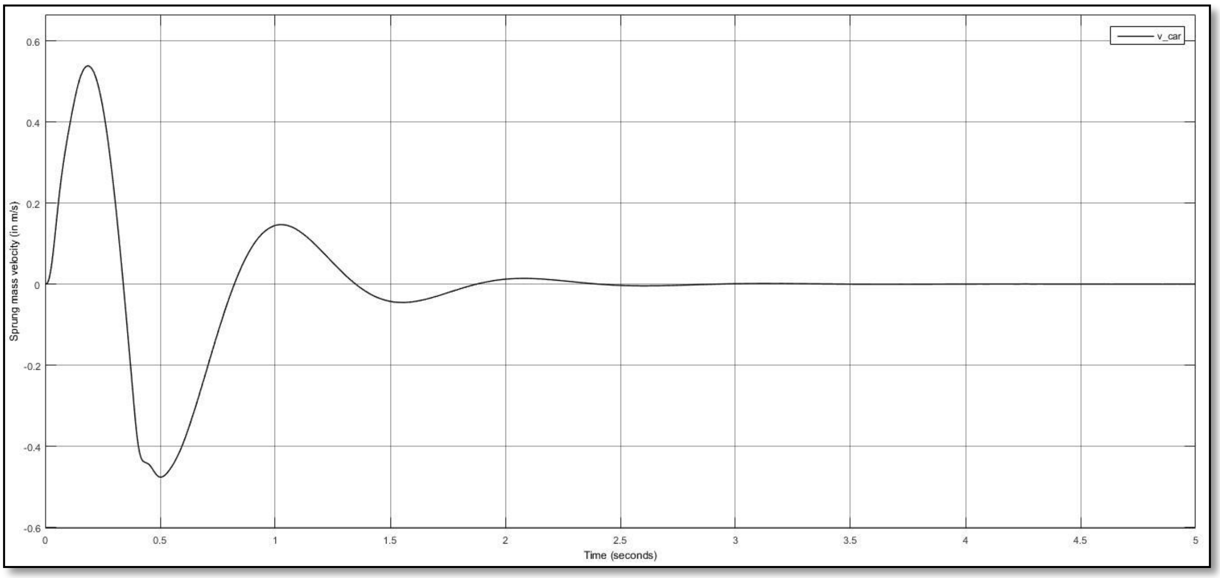


Figure 5.19 Sprung mass velocity Vs Time for maximum RC

In Figure 5.20, it is shown that sprung mass displacement is reduced to 0.115m.

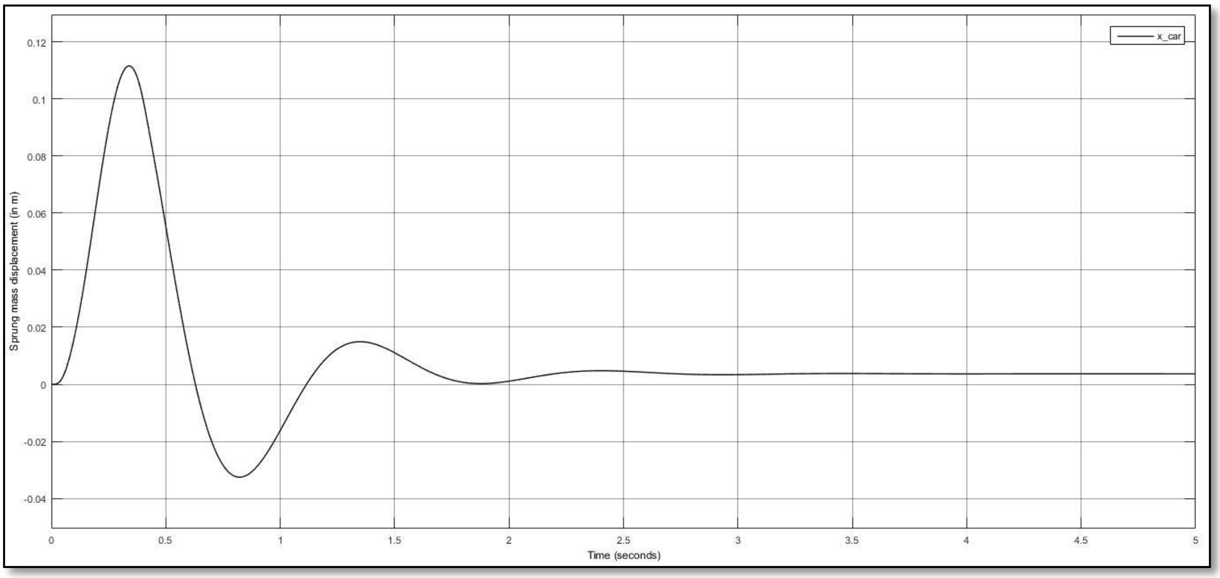


Figure 5.20 Sprung mass displacement Vs Time for maximum RC

The reduced value of unsprung mass acceleration is 3.642×10^1m/s^2 than that of 4.096×10^1m/s^2 in the non-optimised case as shown in Figure 5.21.

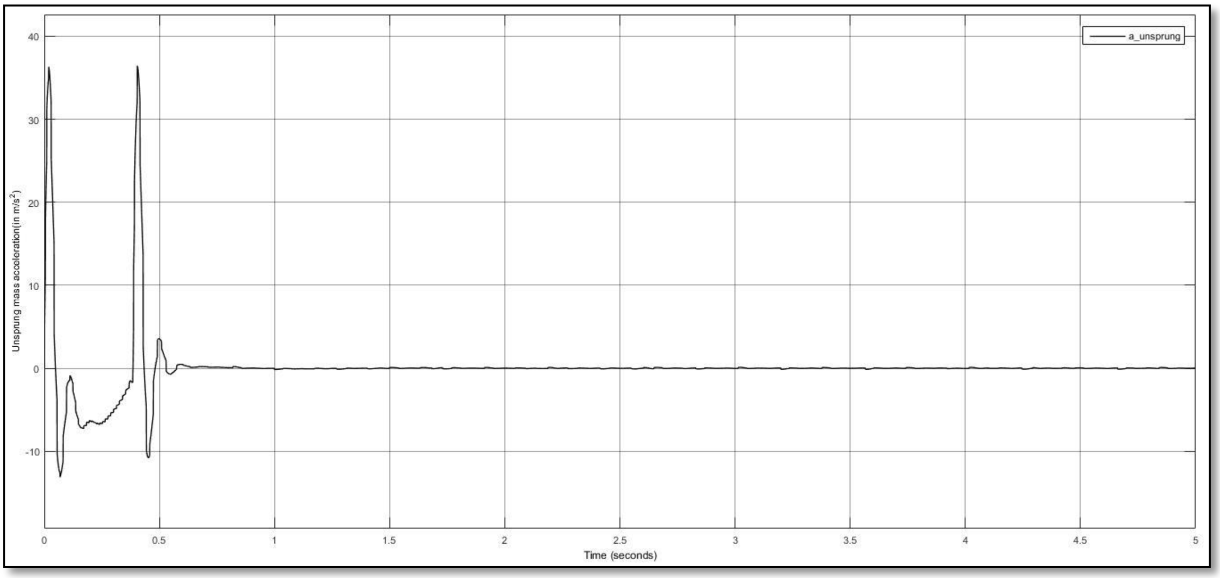


Figure 5.21 Unsprung mass acceleration Vs Time for maximum RC

Figure 5.22 shows the unsprung mass velocity vs time plot in which velocity amplitude has increased.

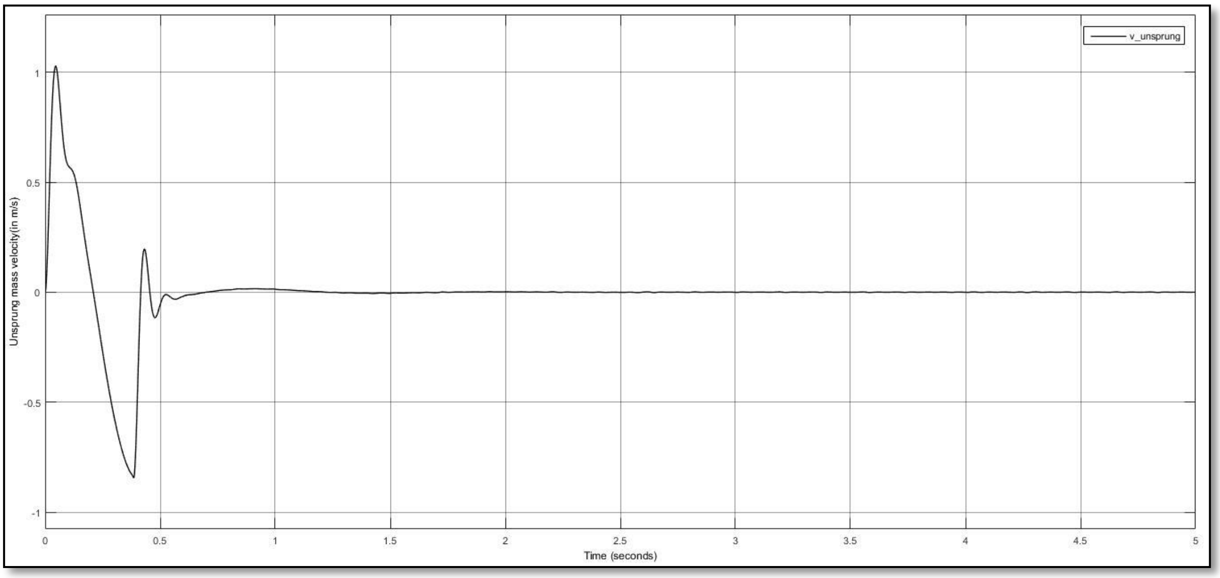


Figure 5.22 Unsprung mass velocity Vs Time for maximum RC

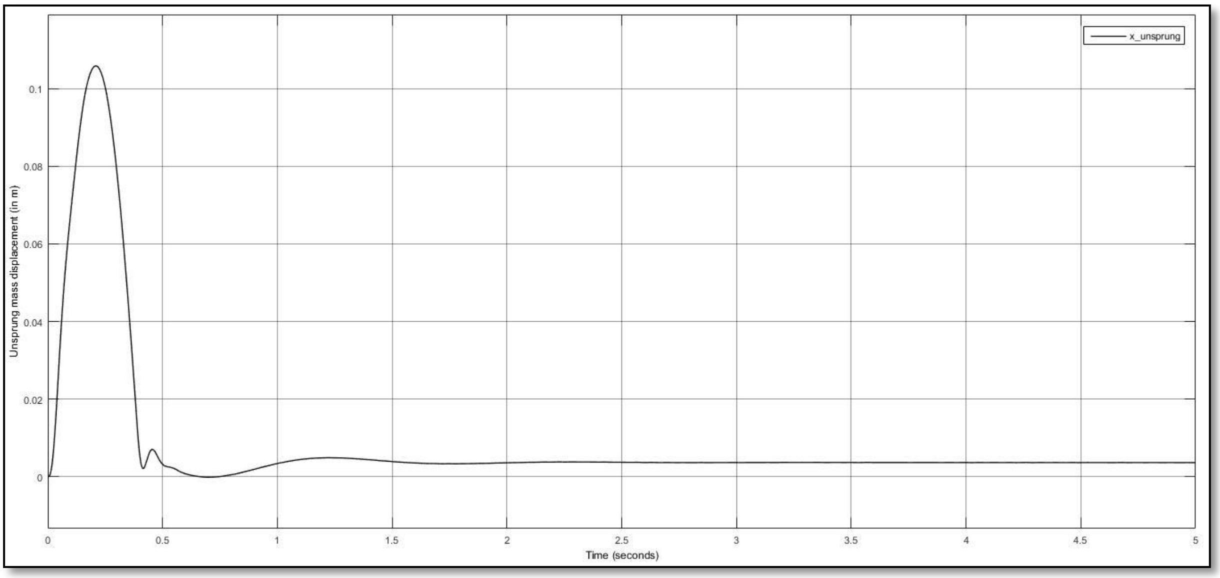


Figure 5.23 Unsprung mass displacement Vs Time for maximum RC

The maximum deflection has been increased from 8.893×10^{-2}m to 9.428×10^{-2}m as represented in Figure 5.24.

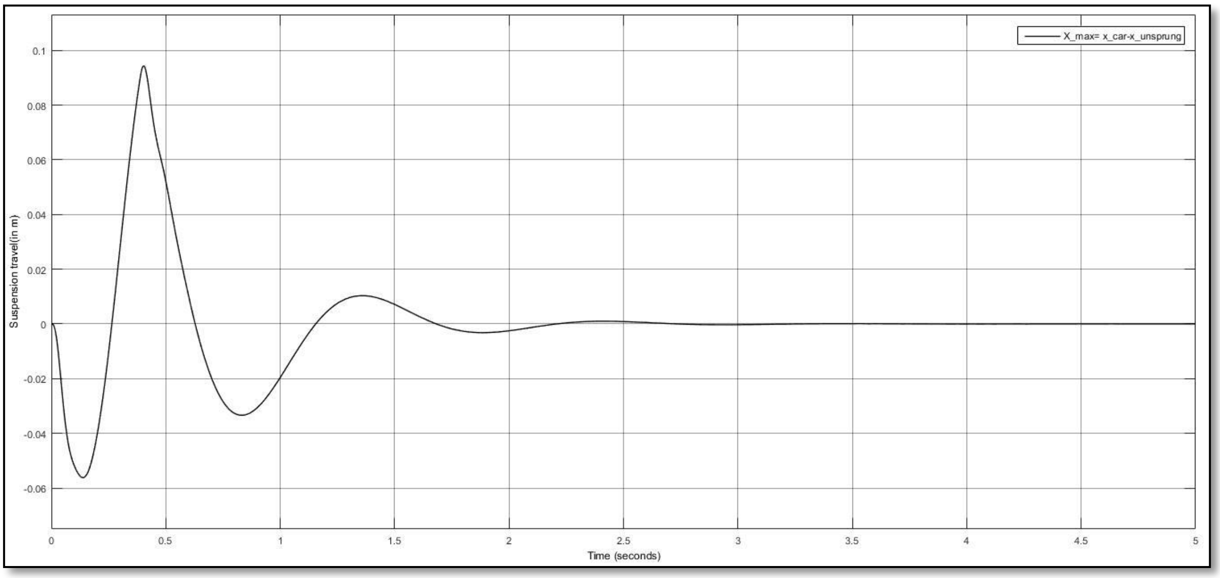


Figure 5.24 Maximum deflection Vs Time for maximum RC

Case III: For m_1=349.85 kg, m_2=34.9 kg, k_1=12354.91 N/m, k_2=148258.9 N/m and c_1=1762.51 Ns/m.

The Figure 5.25 shows the sprung mass acceleration of the final optimised system. It shows that the first peak amplitude has been reduced to 5.221m/s^2 from 6.894.in adition to this the rms value has also been reduced considerably from 2.569 to 1.873.

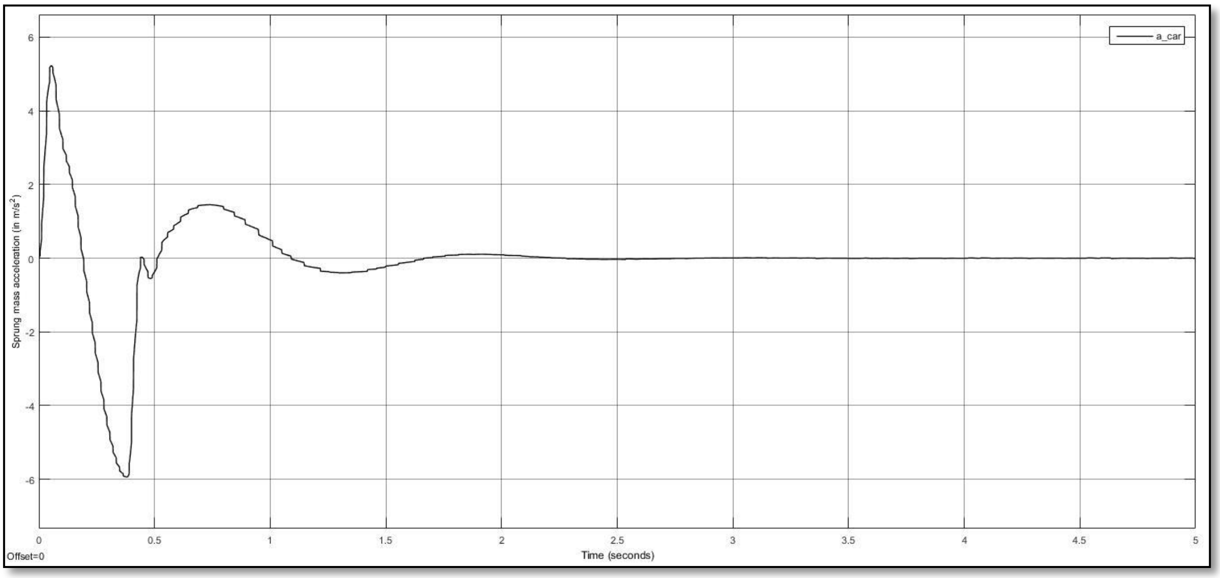


Figure 5.25 Sprung mass acceleration Vs Time for maximum RC

The sprung mass velocity is reduced to 5.107×10^{-1}m/s from 6.148×10^{-1}m/s represented in Figure 5.26.

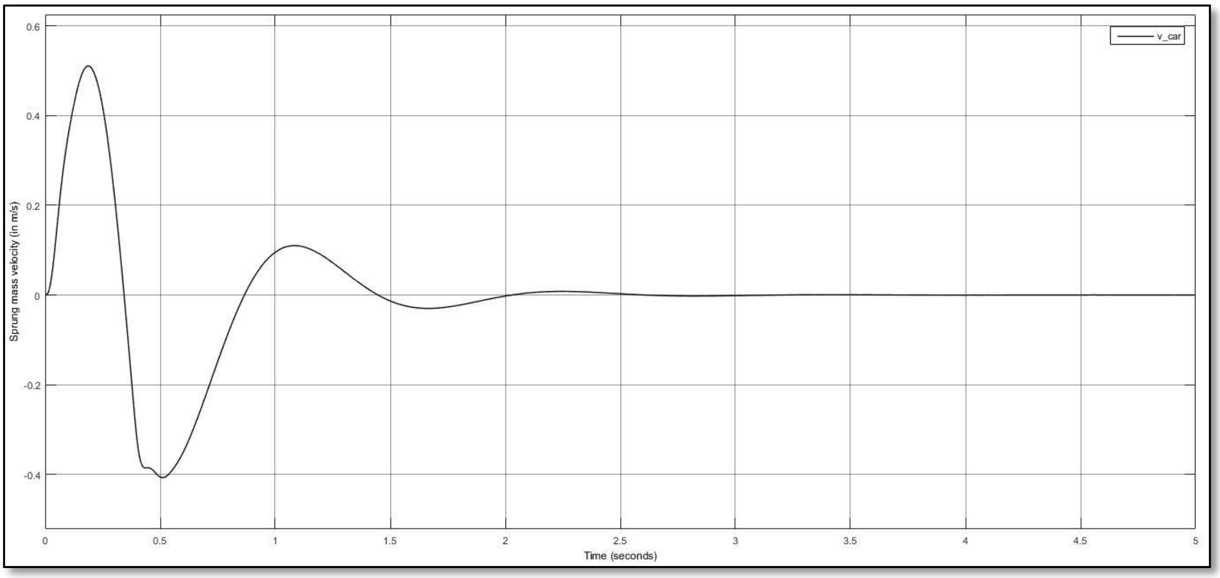


Figure 5.26 Sprung mass velocity Vs Time for maximum RC

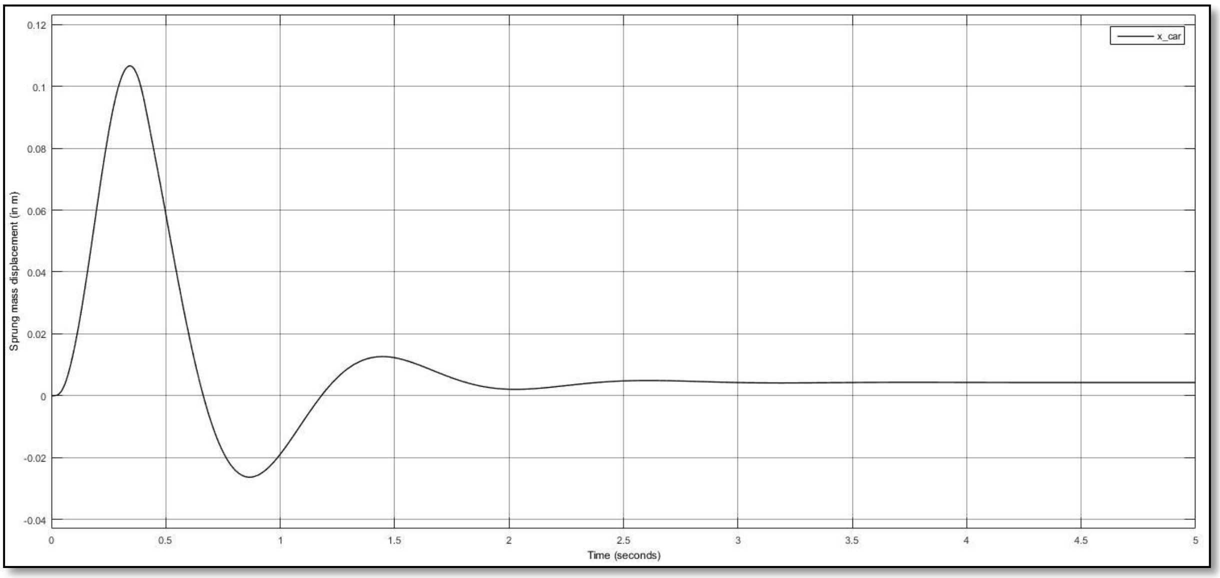


Figure 5.27 Sprung mass displacement Vs Time for maximum RC

In the Figure 5.27, the body displacement changed from 1.189×10^{-1}m to 1.066×10^{-1}m.

In Figure 5.28 the unsprung mass acceleration vs time ploy is shown and in comparison to the non-optimised problem its maximum amplitude has been reduced from $4.096\times10^{1}m/s^{2}$ to $3.212\times10^{1}m/s^{2}$.

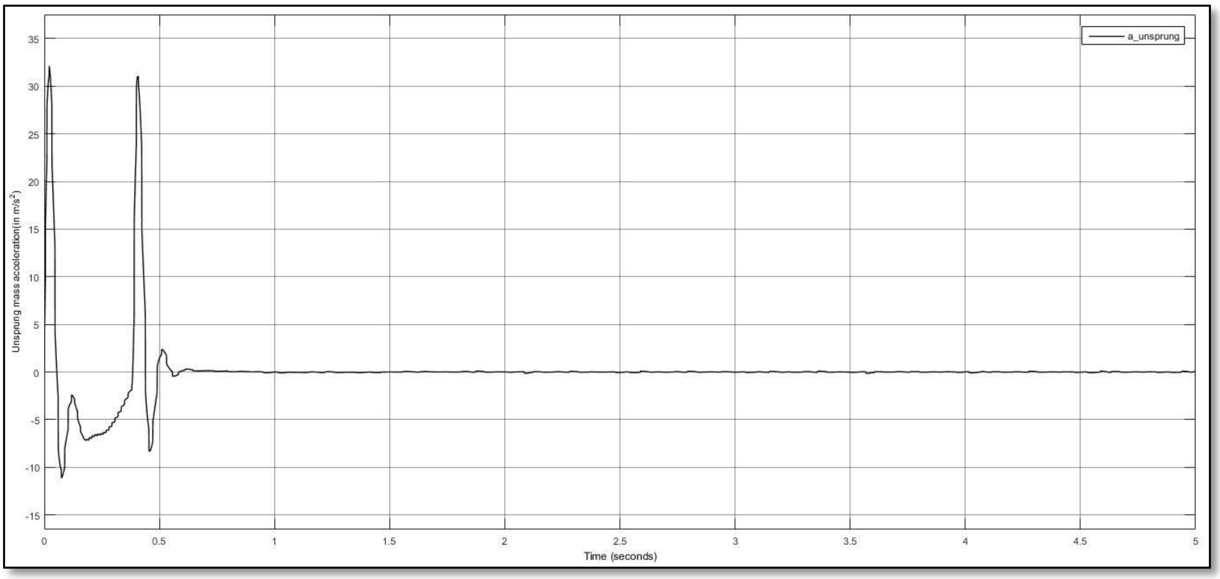


Figure 5.28 Unprung mass acceleration Vs Time for maximum RC

The first peak of the unsprung mass velocity is reduce to 1.003m/s to 1.008m/s represented in Figure 5.29.

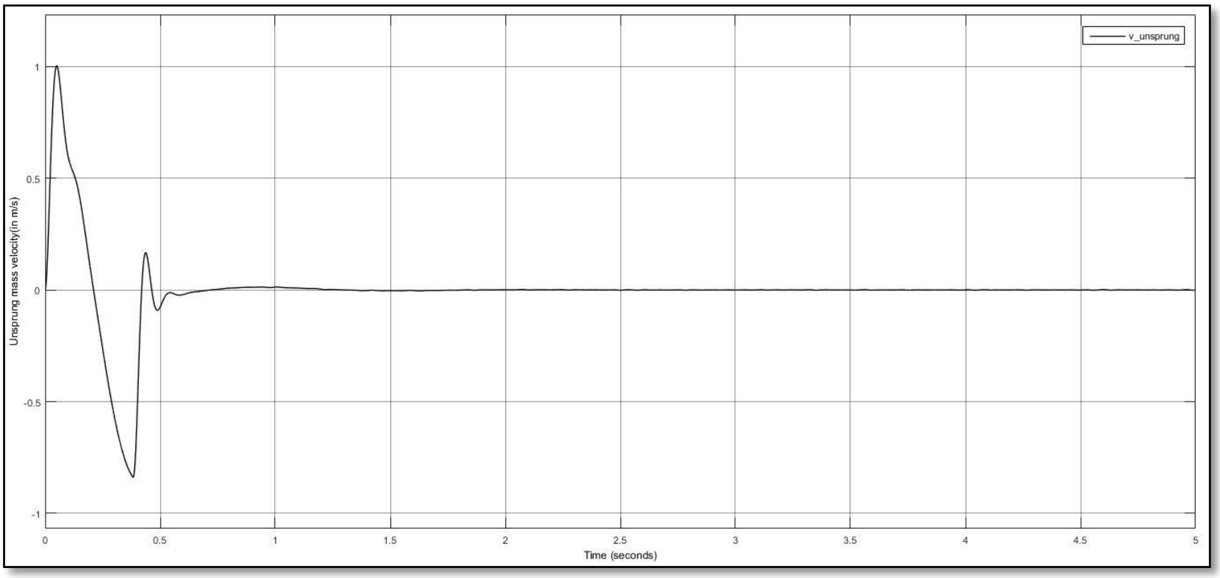


Figure 5.29 Unsprung mass velocity Vs Time for maximum RC

In Figure 5.30, it is showing the unsprung mass displacement variation under timeframe.

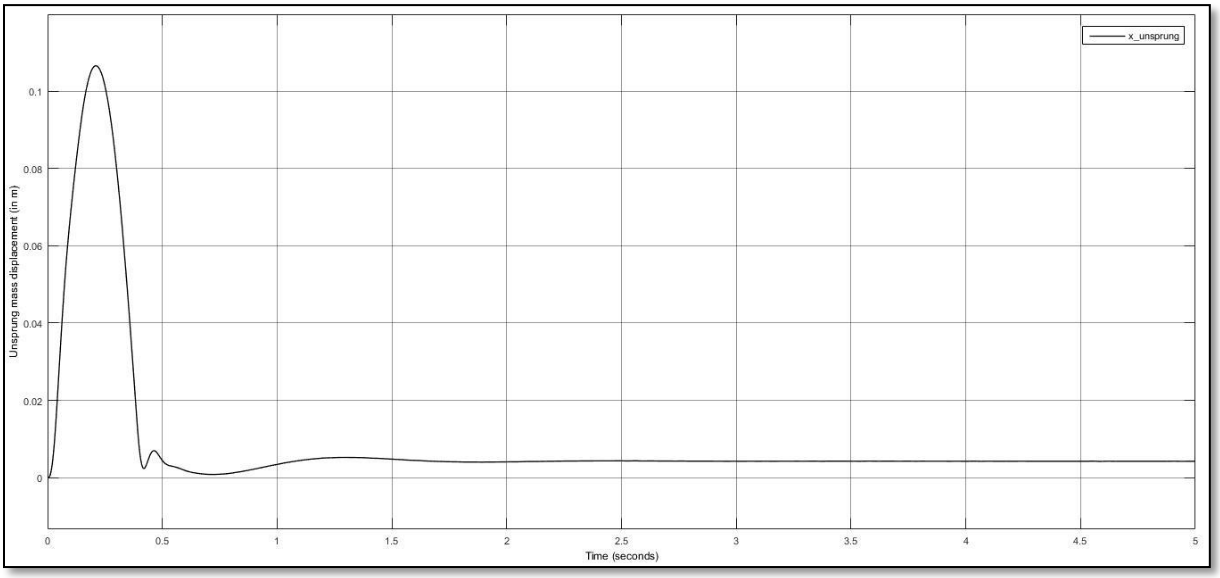


Figure 5.30 Unsprung mass displacement Vs Time for maximum RC

The suspension travel for the final optimise parameters is 9.047×10^{-2}m which was 8.89e-2m in the non-optimised case. It is represented in the Figure 5.31.

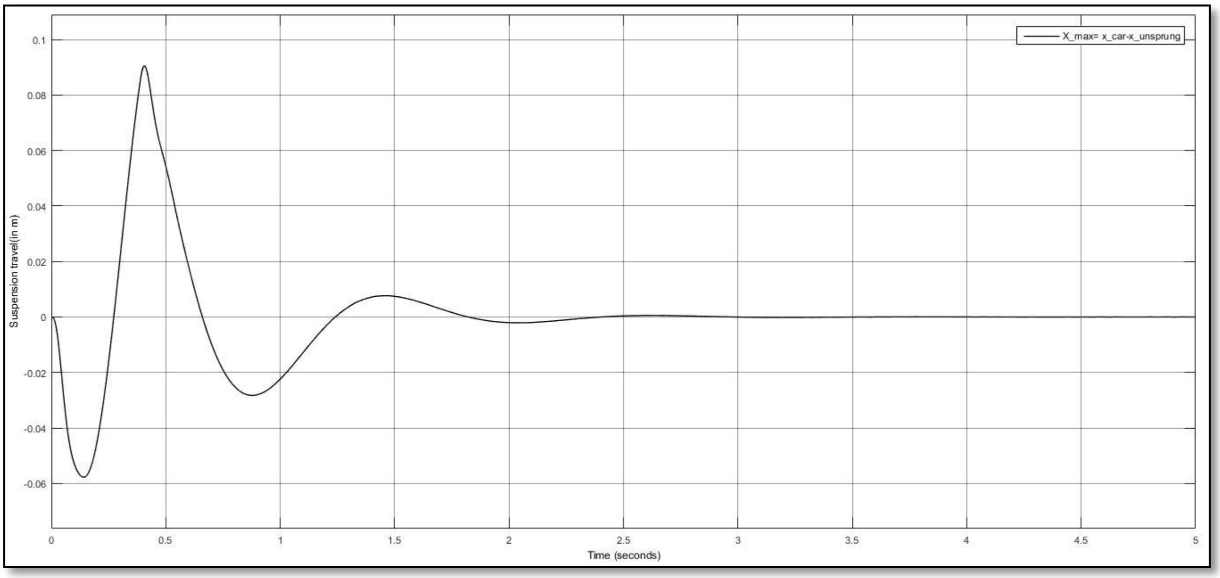


Figure 5.31 Maximum deflection Vs Time for maximum RC

5.6 SUMMARY OF RESULTS

The chapter has discussed the performance of different cases involved in optimisation. The performance of quarter car model has been evaluated for a half sine bump. The major findings of the results along with future scope will be presented in the next chapter.

CHAPTER 6
CONCLUSIONS AND FUTURE SCOPE

This dissertation work has been attempted to obtain the dynamic behaviour of passive suspension system for road vehicles through MATLAB and to evaluate the different parameters through Simulink and optimisation module. The model was subjected to half sine bump. The results have been scrutinised to evaluate the performance of quarter car model. The following conclusions are drawn in this work.

6.1 CONCLUSIONS

- The dynamic model of road vehicle had been constructed through MATLAB/Simulink environment.
- Vertical dynamics has been carried out for the quarter car model. A 2-DOF quarter car model is used for the analysis. Velocity input at the tire is given by considering half sine bump.
- The results are obtained for vertical acceleration, velocity, displacement of sprung and unsprung mass and suspension travel for the system.
- After optimisation the vertical body acceleration has been reduced to $5m/s^2$ from $6m/s^2$. Hence, the vibration are quickly diminished due to lesser magnitude.
- The suspension travel has been compromised for ride comfort comparatively.
- The optimised model is found to be damped within an interval of 2 seconds only to an order 10^{-2} magnitude.

6.2 FUTURE SCOPE

- The performance of quarter car model will be evaluated for semi-active and active system in future.
- The optimisation can be extended to full car model.
- The longitudinal and lateral dynamics can also be included.
- Other different road inputs can be considered for the performance evaluation of the system.

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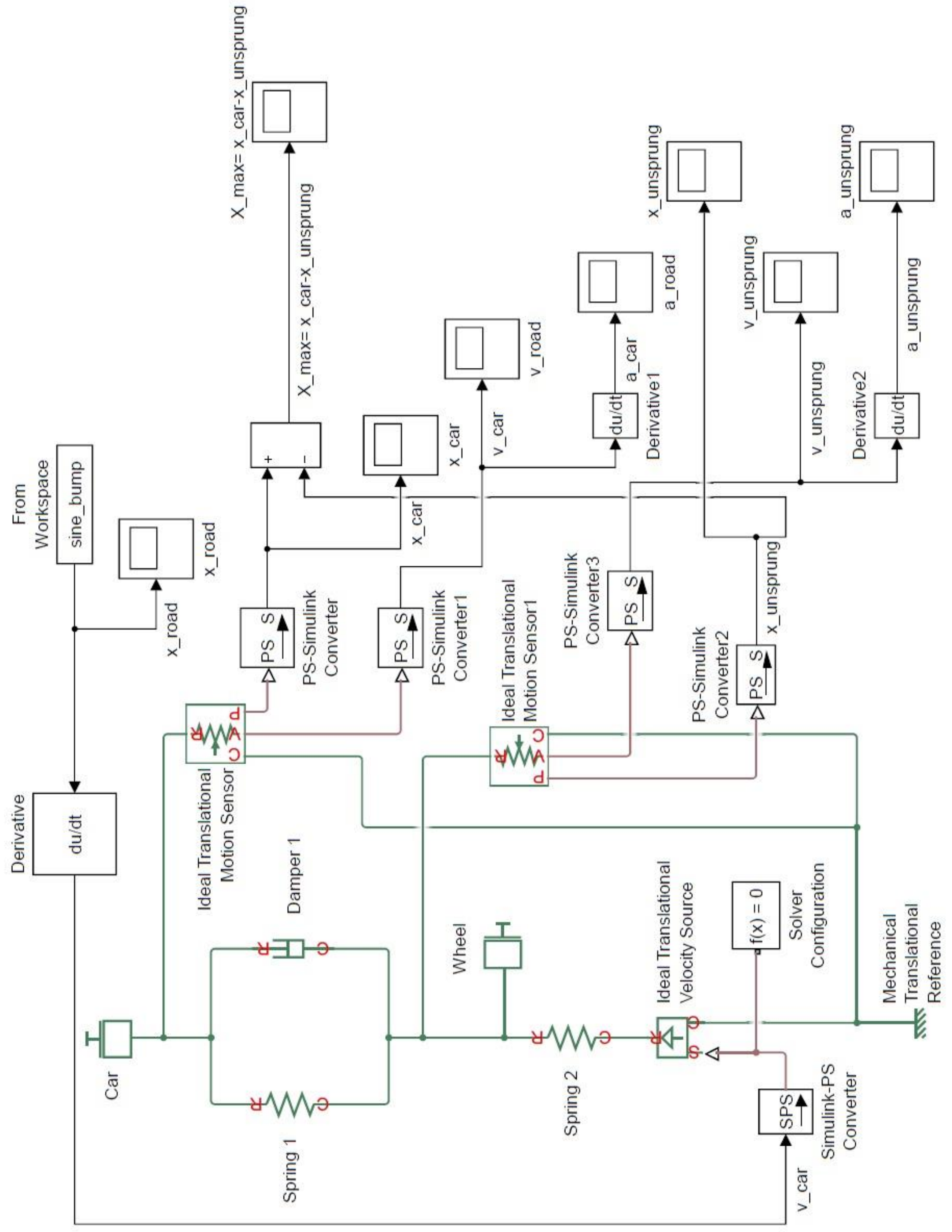
Appendix A


From Workspace
sine_bump
x_road
Derivative
du/dt
Ideal Translational Motion Sensor
PS-Simulink Converter
PS-Simulink Converter1
x_car
X_max= x_car-x_unsprung
v_car
v_road
Ideal Translational Motion Sensor1
PS-Simulink Converter3
PS-Simulink Converter2
x_unsprung
a_car
Derivative1
a_road
v_unsprung
Derivative2
a_unsprung
Damper 1
Car
Spring 1
Wheel
Spring 2
Ideal Translational Velocity Source
Solver Configuration
f(x) = 0
Mechanical Translational Reference
Simulink-PS Converter

Appendix B

The programming code in matlab for optimization in Genetic algorithm multiobjective module is linked as "@final", the function so defined will be fed in the system. The following is the MATLAB code:

****

```
func1tion CO=final(X)

%ENTER

% l-bump width

% m1=sprung mass

% m2=unsprung mass

% k1=spring stiffness

% k2=tire stiffness

% c1=damping c1oeffic1ient

% A=bump height

Syms m1 m2 k1 k2 c1 A v /;

m2=X(1);

k1=X(2);

c1=X(3);

l=0.3;

A=0.1;

m1=10*m2;

k2=12*k1;

v=50/18;

w=3.1416*v/l;

g=9.81;

M=[m1 0;0 m2];%mass matrix

K=[k1 -k1;-k1 k2+k1];%stiffness matrix

C=[c1 -c1;-c1 c1];%damping c1oeffic1ient matrix
```

```
D=K/M;%inverse(M)*K
eigenvalues=eig(D);
L=[eigenvalues(1,1);eigenvalues(2,1)];
X1=[1;-k1/(k2+k1-L(1,1)*m2)];
X2=[1;-k1/(k2+k1-L(2,1)*m2)];
zeta=[((L(1,1))^(1/2)*c1/(2*k1));((L(2,1))^(1/2)*c1/(2*(k1+k2)))];
U=[X1 X2];%MODAL MATRIX
%%M1=m2*(X1(1,1))^2+m1*(X1(2,1))^2;
%%M2=m2*(X2(1,1))^2+m1*(X2(2,1))^2;
%%K1=k2*(X1(1,1))^2+k1*(X1(2,1))^2;
%%K2=k2*(X2(1,1))^2+k1*(X2(2,1))^2;
p1=X1/max(X1);
p2=X2/max(X2);
M1=p1'*M*p1;
M2=p2'*M*p2;
K1=p1'*K*p1;
K2=p2'*K*p2;
r1=w/L(1,1)^(1/2);
r2=w/L(2,1)^(0.5);
%define z3 (sinusoidal)
%d=2;
t=40;
step_size=0.05;
n=t/step_size;
fori=0:n+1
t(i+1)=i*0.01;
if (t(i+1)>=0 && t(i+1)<=1/v)
z3(i+1)=A*sin((pi*v*t(i+1))/1);
```

```
else

z3(i+1)=0;

end

end

F1=-m1*g;

F2=k2*z3-m2*g;

G=[p1(1,1)*F1+p1(2,1)*F2;p2(1,1)*F1+p2(2,1)*F2];

E=[G(1,1)/M1;G(2,1)/M2];

wd1=L(1,1)^(1/2)*sqrt(1-(zeta(1,1))^2);

wd2=L(2,1)^(1/2)*sqrt(1-(zeta(2,1))^2);

Mr=sqrt((1-r1^2)^2+(2*zeta(1,1)*r1)^2);

N=sqrt((1-r2^2)^2+(2*zeta(2,1)*r2^2));

A1=-E(1,1)/(12*k1*Mr);

A2=-E(2,1)/(k1*N);

B1=(A1*zeta(1,1)*L(1,1)^(1/2)-A*w/M1/Mr)/wd1;

B2=(A2*zeta(2,1)*L(2,1)^(1/2)-k2*w*A/(M2*(k1)*N))/wd2;

%comfort objec1tive

y1=(exp(

zeta(1,1)*L(1,1)^(1/2)*t).*(A1*c1os(wd1*t)+B1*sin(wd1*t))+E(1,1)/(k2*M
r));

y2=(exp(-
zeta(2,1)*L(2,1)^(1/2)*t).*(A2*c1os(wd2*t)+B2*sin(wd2*t))+E(2,1)/(k1*N
));

z1=(U(1,1)*y1+U(1,2)*y2);

z2=U(2,1)*y1+U(2,2)*y2;

z2dot=diff(z2);

z2ddot=diff(z2dot);

RC=rm1(z2ddot);%ride c1omfort

RH=rm1(z1-z3);%ride height
```

```
CO(1)=RC1;

C1O(2)=RH;

end
```

****