

# **Facial Expression Analysis Using Difference Theoretic Texture Features**

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## **ABSTRACT**

Human beings are able to detect facial expressions without any effort or trouble while it is a tedious task for the computer. Therefore, only developing an automated system that is powerful and able to analyse the facial expressions is not the lone job. There are several other issues that need to be addressed: extracting face from an image, facial feature extraction which provides information about the facial expressions of the person. These systems need the capacity to perceive and understand emotions as communicated by the facial muscles. Seeing this automated analysis of non-verbal behaviour, many researchers have become interested in this field. An automated system that is able to perform all the above actions forms a major leap in achieving computer-like interaction.

This thesis deals and gives a solution to the above problem. An attempt has been made to provide an optimal and simple solution to achieve Facial expression recognition and an analysis has been made taking different algorithms and techniques and classifiers into account to get the best result. Not only, do we discuss about a good facial feature which would take account of every detail in the face but at the same a good classifier is needed to correctly classify the facial expression.

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## CERTIFICATE

This is to certify that the thesis entitled “**Facial Expression Analysis Using Difference Theoretic Texture Features**” submitted by **Gitin Kakkar (2K12/ISY/11 )** to the Delhi Technological University, New Delhi for the award of the degree of **Master of Technology** is a bonafide record of research work carried out by him under my supervision.

The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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## **CHAPTER 1:**

### **INTRODUCTION:**

Facial expressions are among the most universal forms of body language. Facial expressions are a result of various positions and motion of muscles. In other words, a facial expression is an emotional gesture. Facial expressions is one of the most natural and powerful means for humans to communicate with each other and understand the present mindset of the other person. They are a form of non-verbal communication. In sign languages also, facial expression play a major role. Many phrases that are used in sign languages take account into the various facial expressions.

#### **1.1 OBJECTIVE**

Facial Expression Recognition is in itself a tedious task to be performed. The primary objective of this dissertation is to provide an optimal algorithm for providing Facial Expression Recognition using Difference Theoretic Texture Features and an analysis has been done using various techniques. An attempt has been made to compare the results of Difference Theoretic Texture Features and Eavci Features for Facial Expression Recognition using the Cohn-Kanade database for Facial expressions.

Also, a comparison has been made as to which classifier is the best when working for Facial Expression recognition.

#### **1.2 HISTORY**

Taking account of history, Charles Darwin in 1872 worked on facial expressions which has a direct relationship to the work done today in automatic facial recognition. Darwin gave the basics of expressions of both humans and animals and established some general principles regarding them. After Darwin, some more important work was done on facial expressions by psychologist Paul Ekman and his colleagues since 1970. Around the 20<sup>th</sup> century, with such modernization and advances in the field of computer graphics, robotics and machine learning that computer professional are showing keen interests in the study of facial expressions.

The facial expressions have been primarily classified into six broad categories by Paul Ekman: Surprise, Sad, Fear, Anger, Disgust and Happy. All these emotions have different characteristics and have their own representation. The muscles of the face arrange themselves

accordingly and a particular emotion can be found out. The below given facial expressions are analysis of the techniques.

### **1.3 THE SIX BASIC EMOTIONS:**

1. **Surprise:** A surprise expression comes as a result of widening of eyes and an open mouth with eyebrows raising high. The expression surprise is very much close to fear. It is just an instantaneous reaction to something. This expression is likely to occur or happen when something unexpected occurs.
2. **Sad:** A sad expression is likely to come when a person hears a bad news or sees something that is disheartening. This expression is a result of upwardly slanted eyebrows and generally lips are also pouted downwards.
3. **Fear:** Fear may occur when a person is terrified to see certain events that are happening. The person obviously does not like what is happening. It happens when a person feels that he is in danger physically or mentally. Fear can be due to a present situation or due to something that a person has been hanging back from the past. A fear expression is generally marked with a widened or fully opened mouth with eyebrows slanted upwards. Though fear is very close to surprise but a surprised can be happy as well. So, surprise can be both positive and negative but fear is generally termed as negative expression.
4. **Anger:** An anger expression can be easily marked out from a person's face. It is one of the most distinguishable of the facial expressions. Anger is associated with irritating, frustrating and unpleasant behaviour of a person. An anger expression happens due to the tightening of the facial muscles. Eyebrows are squeezed together and eyelids become straight.
5. **Disgust:** A disgust expression appears on a person's face when he feels irritated on seeing what is happening. This expression is a result of eyebrows slanted downwards and sticking together, lips are contracted and the upper lip generally is slanted upwards. Due to this, this expression is termed very close to being sad but the difference is seen by gesture of the nose. The nose is contracted upwards. A disgust expression also appears when a person has smelled something bad.
6. **Happy:** Happiness is considered to be one of the most universal facial expressions carrying the same meaning across all cultures. This expression is indicated with a smile. A smile can be a simple smile where a person only widens his lips and feels relaxed or he can also exaggerate this more profoundly by opening of mouth with lips

widened and the teeth are clearly visible. This expression indicates that you are feeling good.

Facial expression analysis refers to computer systems that aim to automatically analyze and recognize facial expressions. Facial expression analysis has been an active research topic since the work of Darwin. Automatic facial expression is one of the most challenging and important problem and gives a pathway to many other applications as well. Computer facial expression analysis systems analyze facial expressions regardless of sex, culture, etc.

Facial expression analysis should be able to correctly recognize the expression of a person and also the extent of expression that the person is showing. For example, if a person is in fear, we should also make count of how much a person is in fear. He can be purely in fear or he can also show fear and some other emotion such as sad or a person's facial expression can be a complete mixture of emotions.

A very amount of ambiguity may arise among some facial expressions. For example, sad and disgust are two emotions which are very closely related to each other with both offering the same slanted eyebrows and the lips but here the middle part of the face, that is, the nose plays the part and helps in distinguishing both the expressions. In disgust, the nose is contracted but not in sad. With the help of this we know that taking into account only the upper part of the face, that is, eyes and eyebrows, and the lower part of the face, that is, the mouth will not to be sufficient to recognize the expression, the middle part is equally important.

#### **1.4 CHALLENGES FACED:**

Facial Expression Recognition is a tough task to be performed. A number of challenges were faced while achieving this task. One of the biggest challenged faced is that a good facial recognition is generally obtained only using images with high resolution. In real-world applications such as smart meeting and visual surveillance, the input images are often at low resolutions. Another challenge is that most expressions data that is collected by asking subjects to perform a series of expressions. These directed facial tasks may differ in appearance and timing from spontaneously occurring behaviour.

#### **1.5 THESIS ORGANIZATION:**

In this thesis, Chapter 1 provides an overview of Facial Expression Recognition and the basic six facial expressions and how the face muscles adapt and react to the mind's need of

changing the emotion. In Chapter, a review of the Difference Theoretic Texture Features has been done. The basis of this thesis has been completely based on recognising facial expressions using Difference Theoretic Texture Features. In Chapter 3, a literature review of facial expression recognition has been done. Various techniques have been considered by which various researchers have done facial expression recognition. Also, various feature extraction techniques have been discussed. In Chapter 4, we have put forth our proposed methodology and the proposed algorithm from which we have achieved facial expression recognition. In Chapter 5, we have discussed the results that we obtained after using the methodology discussed in chapter 4. Also, we have discussed about the various other techniques that could have been applied and how our technique achieves the best result.

## **CHAPTER 2:**

### **LITERATURE REVIEW:**

Humans does not feel a real challenge in recognising the facial expression of a person but getting reliable facial expression recognition is tough task. The process of Facial Expression Recognition is generally divided into three steps: Image Acquisition, Pre-processing, Feature Extraction and then Facial Classification and Post-processing.

In Image Acquisition, the facial image is given as input to determine the expression of the person concerned. It has to be kept in mind that this should be independent of the person. Images taken can be static or image sequences. An image sequence contains potentially more information than a still image, because the former also depicts the temporal characteristics of an expression[2]. Though monochromatic images are the most popular type of images used for facial expression recognition, coloured images may become popular in the future.

Pre-processing is done to remove noise and to normalize brightness. Also, segmentation and location of face in the image is done. This is a tough task to go through but many algorithms and techniques have been developed to do the same. Expressions are not prone to rotation, scaling and illumination. To eradicate this side-effect, the image is standardised prior to classification [2].

Feature extraction converts pixel data into a higher level representation of shape, motion, colour, texture of the face and its components [2]. The extracted feature has fewer dimensions than the original image but a note should be made the reduction in the dimension should retain the essential characteristics of the expression and image.

In Facial Classification, a classifier is selected and is used to get the best classification we are getting from the feature extraction keeping in mind our training and testing data. The most accurate class that we are getting, the image is allotted that class or expression.

Post-processing main motive is to improve recognition accuracy.

Different types of techniques have been used and put forth by researchers to obtain a high accuracy model for Facial Expression Recognition.

There can be a different type of Feature Extraction technique that is used or another classifier is giving better results. Some of the review of those techniques has been discussed below.

## **2.1 FACIAL EXPRESSION RECOGNITION USING FACIAL FEATURE REPRESENTATION (APPEARANCE BASED FEATURES):**

Appearance Based Facial Feature Representations use wrinkles, creases, furrows and textures of facial images to recognize facial expression [7]. As of today, appearance based methods have gathered a lot of attention of researchers and scholars and are now being highly used for Feature extraction because they do not rely on the facial components and on the contrary use a filter or a filter bank on the whole image to extract facial features. With Appearance Based methods, image filters such as Gabor wavelets are applied to either whole face or parts of image to extract the appearance changes of the face [3]. Appearance based methods are further categorised to static and dynamic appearance feature-based method. Dynamic appearance based methods exploit temporal changes while in static appearance based methods a single frame is used. [3].

Some examples of Appearance based approaches are Principal Component Analysis (PCA), Independent Component Analysis (ICA). There are also some local feature patterns that are used like Local Binary Patterns (LBP), Local Feature Analysis (LFA).

Principal Component Analysis is a way of identifying patterns in data and highlights the data in such a way so as to show the similarities and differences. Principal Component Analysis which is used in Feature extraction step is statistical method. PCA is widely used in data compression and analysis. There is also a modification of Principal Component Analysis that is the weighted Principal Component Analysis. It is based on multi-feature fusion. The high dimensional local self related features are extracted on the foundation of the facial expression images and then divided in the expression regions. Weighted Principal Component Analysis is useful in dimensionality reduction and feature extraction.

Local Binary Patterns (LBP) features were originally texture based features and are now used extensively on facial images. The advantage of using LBP features is that they are intolerant to illumination change and they are very easy to compute. LBP features can be derived very fast in a single scan through the raw image and lie in low-dimensional feature space, while still retaining discriminative facial information in a compact representation [3]. LBP generates feature representation by thresholding the gray value of the neighbour pixels with respect to gray value of the centre pixel of a local region. [4]. They provide robustness to the feature set. But, the performance of LBP degrades under the presence of random noise large

illumination variation. But, the performance of LBP features deteriorate when there is random noise in the image. To tackle this problem, MTP features can be used.

This method uses the median value of neighbouring cells around each pixel and quantizes the value into three different levels (-1,0,1) in order to generate local texture patterns. The database used to test MTP is Cohn-Kanade database and the classifier used is Support Vector Machine.

A filter which uses local neighbourhood median is more robust against the presence of noise in an image [6]. With this a multi-level gray-scale quantization increases the robustness against illumination variation [5]. The MTP operator is applied on face images and two encoded images are generated. The histogram generated from these two histograms is then merged to create a final histogram and this histogram is used for facial expression recognition.

Appearance based approaches process the pixel intensity of the whole image to obtain facial features [8]. But, the results for facial expression recognition depend upon the training system here. In order to increase the accuracy and performance of the results, compression techniques are used to reduce the dimensionality of the training system.

Gabor filters have also been proved to be very effective in doing the facial expression recognition because of its superior quality of multi-scale representation. According to the needs of special vision, it can adjust the spatial and frequency properties to face expression characteristic [13]. Gabor filter has good resolution both in spatial field and frequency field. The recognition for the six basic facial expressions was very much higher for Gabor features. The best performance was achieved using Gabor wavelet representation and Independent Component Analysis.

Active Appearance Model (AAM) and Lucas Kanade image alignment algorithms are used to align the basic facial images to obtain texture features [4]. Three kinds of texture features are used: texture features for the whole face, texture features of the upside face, texture features of the downside face. Texture features of the whole face are used as inputs of facial expression intensity recognition. Texture features of the upside face are used for inputs of upper face actions units recognition. Texture features of the downside face are used as inputs of lower face action units recognition.

The Active Appearance Model (AAM) contains a shape model and a texture model. In the aligning process of AAM, the Lucas Kanade algorithm is modified and for better efficiency inverse compositional algorithm is developed. Ekman developed the Facial Action Coding Systems (FACS) to recognize facial expressions by taking into consideration the contraction and relaxation of each muscle.

## **2.2 FACIAL EXPRESSION RECOGNITION USING GEOMETRIC FEATURE REPRESENTATION:**

The other type of facial feature extraction technique is Geometric Feature based methods. Geometric features represent the shape and locations of the facial components, which are extracted to form a feature vector that represents the face geometry [2]. Geometric based feature extraction techniques provide better performance than appearance based techniques in Action Unit recognition [2]. However, geometric based feature extraction technique or models usually require accurate and reliable facial feature detection and tracking which is difficult to obtain in many situations.

Geometric based feature extraction techniques exploit face model to describe facial features. Tracking of the features in low dimensionality domain is a tricky task. Researchers have used various classifiers and techniques. Padgett [9] recognized emotions using feed forward networks to train static images. Landabaso [10] described the adaptation of the technique to deal with unrestrained expression intervals in video sequences and took advantage of the symmetry of the expressions to analyze facial features [7]. Pardis [11] adopted Hidden Markov Models (HMM) to recognize facial images by using Facial Animation Parameters in long video sequences.

Pantic and Rothkrantz [14] have used the location and tracking of the face and its parts such as the frontal face and contours of the head, eyebrow region, eye region and mouth region. Essa and Pentland [15] have adopted the technique of locating the face by fitting a 3D mesh model of face geometry to a 2D face image.

In order to track and detect changes of facial components in facial images, Tian developed multi-state models to extract the geometric features. A three state lip model describes the lip state: open, closed and tightly closed. A two state model is used for the eyes to tell whether the eyes are open or closed. A one state model is used for brow and cheek. Some supporting geometric features like wrinkles and furrows are represented using two states i.e., present or



absent. After the face is detected in the image automatically, the contours and components of face are then adjusted manually.

### **2.3 VOLUME LBP FEATURES:**

Another type of feature extraction method is an extension to the Local Binary Pattern (LBP) features and that is the Volume Local Binary Pattern (VLBP) features. They are texture based features and very much helpful in facial expression recognition. Zhao [16] presented a method of recognising facial expressions using Volume Local Binary Patterns which are an extension to Local Binary Pattern features. They are widely used in texture analysis, combining motion and appearance [5]. The visual-features from human concept are not easy to be associated with texture information because modelling of texture retrieval cannot precisely bridge high-level human concepts to low-level texture features.

### **2.4 USING MACHINE LEARNING:**

Various machine learning methods are continuously used by researchers in the classification step to classify the facial expression. Some of them include Support Vector Machine (SVM), Neural Network, Linear Discriminant Analysis (LDA), Linear Programming Technique, Template Matching, Hidden Markov Models (HMM), etc.

In template matching, a template is created for each class of facial expression, then a classifier like nearest neighbour or k-nearest neighbour can be used to match the input image with the closest template [3]. In training, the histograms of facial expression images in a given class are averaged to generate a template for the present class. A chi-square statistic can then be computed for calculating the dissimilarity measure in histograms by:

$$\chi^2(S, M) = \sum_{i,j} \frac{(S_{i,j} - M_{i,j})^2}{(S_{i,j} + M_{i,j})} \quad \text{.....Eq (1)}$$

Weights can also be assigned to certain parts of the image since locally they contain very meaningful and important information such as areas near eyes and mouth. For example, the main expression features of face lie near the eyes and mouth. So, the following equation can be used:

$$\chi_w^2 (S, M) = \sum_{i,j} w_{ij} \frac{(S_{i,j} - M_{i,j})^2}{(S_{i,j} + M_{i,j})} \quad \dots\dots\dots \text{Eq (2)}$$

where S & M are two histograms and  $w_j$  is the weight for region j.

Another classifier that can be used is the Support Vector Machine (SVM). SVM performs an implicit mapping of data into a higher dimensional feature space and then finds a linear separating hyperplane with the maximal margin to separate data into this higher dimensional space [3]. SVM makes binary decisions so the multi-class classification can be achieved by classifying one class with all the other classes combined and then do the same for the other classes and it outputs the class with the largest output of binary classification.

Linear Discriminant Analysis is a supervised subspace learning technique which can be applied to facial expression recognition [3]. Facial deformations lie on lower dimensional subspaces [3]. LDA has been used extensively along with Local binary pattern features to recognize facial expressions.

Given a multi-dimensional data samples  $x_1, x_2, x_3, \dots, x_m$  in  $R^n$  that belongs to c classes, LDA finds a transformation matrix W that maps these m values to  $y_1, y_2, y_3, \dots, y_m$  in  $R^l$  ( $l \leq c$ ), where  $y_i = W^T x_i$ . The objective function of LDA is as follows:

$$\max_w \frac{w^T S_B w}{w^T S_W w} \quad \dots\dots\dots \text{Eq (3)}$$

$$S_B = \sum_{i=1}^c n_i (m^{(i)} - m)(m^{(i)} - m)^T \quad \dots\dots\dots \text{Eq (4)}$$

$$S_W = \sum_{i=1}^c \left( \sum_{j=1}^{n_i} n_i (x_j^{(i)} - m^{(i)})(x_j^{(i)} - m^{(i)})^T \right) \quad \dots\dots\dots \text{Eq (5)}$$

Where m is the mean of all samples and  $n_i$  is the number of samples in the  $i$ th class,  $x_j^{(i)}$  is the  $j$ th sample in the  $i$ th class,  $S_B$  is between-class scatter matrix and  $S_W$  is within-class scatter matrix.

## **2.5 INTEGRAL PROJECTION CURVE:**

With the help of Integral Projection Curve, an attempt is made to design a method that determines the effective areas of the face image. Therefore, at first, the effective areas of the face are extracted using Integral Projection Curve.

The most important area in a human face is the eyes, eyebrows and mouth. These areas are achieved using Integral Projection Curve. In the first step horizontal integral projection are applied to the binary image. The figure below shows the result of horizontal and vertical integral projection.

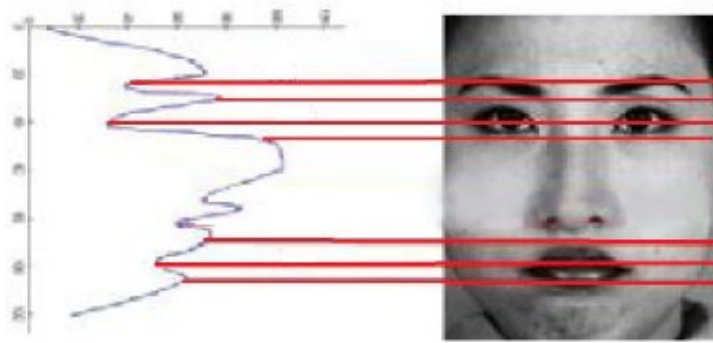


Fig 3.1. Horizontal Integral Projection Curve

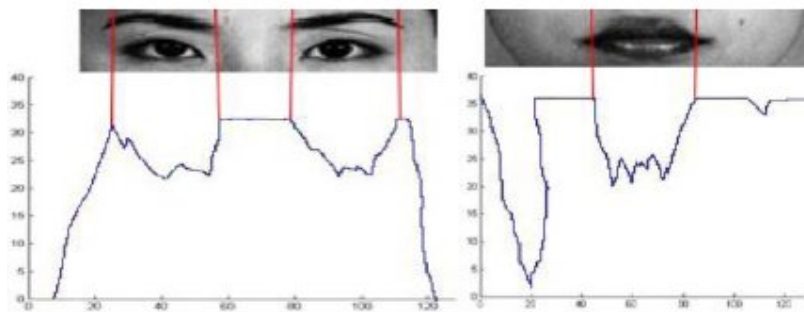


Fig 3.2. Vertical Integral Projection Curve

Let  $I(x,y)$  be a gray value of an image, the horizontal integral projection in intervals  $[y_1,y_2]$  and the vertical projections in interval  $[x_1,x_2]$  can be defined respectively as  $H(y)$  and  $V(x)$  as:

$$H(y) = \frac{1}{X_2 - X_1} \sum_{x=X_1}^{Y_2} I(x, y) \quad \text{.....Eq (6)}$$

$$V(x) = \frac{1}{Y_2 - Y_1} \sum_{y=Y_1}^{X_2} I(x, y) \quad \text{.....Eq (7)}$$

The horizontal projection indicates the X-axis of the eyes, eyebrows and mouth. Wave troughs are formed on the curves of the graphs since the image is black where there are eyes, eyebrows and mouths. After observing these wave troughs, we can locate the position of the eyes, eyebrows and mouths.

## **CHAPTER 2:**

### **REVIEW OF THE DIFFERENCE THEORETIC TEXTURE FEATURES:**

#### **3.1 INTRODUCTION:**

Difference Theoretic Texture Features are features which are independent of rotation, scale and illumination. These features taken into account both the local grey level differences and the global grey level differences of intensities in the image.

#### **3.2 REVIEW OF VARIOUS TEXTURE FEATURES:**

Texture can be evaluated as being fine, coarse or smooth; rippled, mottled, irregular or lineated. Texture contains important information about the structural arrangement of surfaces and their relationship to surrounding environment. Since textural properties of images appear to carry useful information for discrimination process, it is important to develop features for texture. Most of the other texture based features assume that the images have the same orientation and scale. It is noted that using these texture features against images that are scaled or rotated or have different illumination, the performance decreases. Features like Local Binary Pattern (LBP) features which use two histograms of two features namely, LBP and variance for rotation-invariant texture classification [1]. The drawback of LBP features is that while extracting the local features, the global image information and since VAR is continuous an additional quantization step is required and this requires a large number of training samples is required to get a higher efficiency [1].

#### **3.3 DIFFERENCE THEORETIC TEXTURE FEATURES:**

Difference Theoretic Texture Features are features which are derived from local and global grey level differences and their histograms which are invariant for a texture class. A very small assumption has been made while computing the difference theoretic features and that is that a certain correlation exists between the local pattern associated with a pixel and the actual pixel intensity. The local pattern is defined by the grey level differences of a pixel with its next immediate neighbour in the horizontal (H), vertical (V) and diagonal (D) directions taken separately to preserve the orientation information of the texture [1]. The global information regarding the pixel intensity is global contrast which is equal to the difference of pixel intensity with mean luminance of the image [1]. Higher the correlation between the local grey level difference and the global grey level differences in the image, more regular is

the texture. It clearly means that if there is a recurring pattern in the image, then a particular combination of local grey level differences and global grey level differences will appear every time the pattern occurs.

The difference theoretic features are a set of 11 features taken into account the horizontal, vertical and diagonal components. They uniquely characterize a texture irrespective of any scale, brightness and rotation changes. The feature set is given by:

D=[absdiff H, absdiff V, absdiff D, pdiff H, pdiff V, pdiff D, absY, pY, pdiff HY, pdiff VY, pdiff DY]

These features have been divided into 3 sets as follow:

**1. Computing the magnitude of local differences along with H, V, D directions:**

The absolute value of local grey differences between each pixel and its immediate neighbour is summed up and averaged over the horizontal, vertical and diagonal directions and the below feature set is obtained for each texture image:

[absdiff H, absdiff V, absdiff D]

A zero local grey level difference indicates the same grey level intensity and therefore zero information is received. The values of absdiff H, absdiff V, absdiff D is obtained as follow:

$$absdiffH = \frac{\sum_{i=1}^M \sum_{j=1}^N |f(i, j) - f(i, j+1)|}{M * N} \dots\dots\dots Eq (8)$$

$$absdiffV = \frac{\sum_{i=1}^M \sum_{j=1}^N |f(i, j) - f(i+1, j)|}{M * N} \dots\dots\dots Eq (9)$$

$$absdiffD = \frac{\sum_{i=1}^M \sum_{j=1}^N |f(i, j) - f(i+1, j+1)|}{M * N} \dots\dots\dots Eq (10)$$

**2. Computing the number of times a local difference repeats along with H, V and D directions:**

Greater the number of occurrences of a specific difference pattern along H, V and D directions, more regular the texture will be [1]. The mean probability of occurrence of patterns along the H,V and D directions is obtained by first plotting the histograms of the signed grey level differences along H,V and D directions for the entire image.

If h, v and d denote signed local grey level differences computed in all three directions, i.e., H, V and D, the total number of possible values for signed grey level differences for a 256-intensity image, with Q=128 bins for the histogram permits slight changes in the intensities because of noise or illumination effects.

The probability values  $p_h, p_v, p_d$  from h, v and d histograms are mapped back to each pixel in the image by matching the indices of the histogram with actual h, v and d values at the pixel and then these probabilities are averaged over all the pixels in the image [1]. The features that we get after this have the capability of discriminating the regular features from the irregular ones.

The values of pdiff H, pdiff V, pdiff D are obtained as follows:

$$pdiffH = \frac{\sum_{i=1}^M \sum_{j=1}^N p_h ((f(i, j) - f(i, j+1))_Q)}{M * N} \dots\dots\dots Eq (11)$$

$$pdiffV = \frac{\sum_{i=1}^M \sum_{j=1}^N p_v ((f(i, j) - f(i+1, j))_Q)}{M * N} \dots\dots\dots Eq (12)$$

$$pdiffD = \frac{\sum_{i=1}^M \sum_{j=1}^N p_d ((f(i, j) - f(i+1, j+1))_Q)}{M * N} \dots\dots\dots Eq (13)$$

**3. Measuring the Global Contrast of a Pixel with Mean Luminance of the Image and measuring the number of times it occurs in the image:**

The local pattern information that has been discussed in steps 1 & 2 does not take into account the intensity of the pixel. Therefore, the intensity of the pixel is not adding any value to the features of the image. The intensity of pixel has a very major role to play in the global image scenario. Taking the global image scenario into account is very important since the same local pattern can be achieved with different pixel intensities resulting in different types of textures. Therefore, to incorporate this global

image information, mean global contrast of the pixels in an image with respect to the mean luminance of the entire texture pattern  $absY$  is defined as below:

$$absY = \frac{\sum_{i=1}^M \sum_{j=1}^N |f(i, j) - \mu|}{M * N} \quad \dots\dots\dots Eq (14)$$

Where  $\mu$  is the mean luminance of the image and is given by:

$$\mu = \frac{1}{M * N} \sum_{i=1}^M \sum_{j=1}^N f(i, j) \quad \dots\dots\dots Eq (15)$$

If  $g$  represents the random variable for denoting the signed global differences in an image, then the mean frequency of occurrence of global differences in an image  $pY$ , is obtained by mapping back to each pixel the probability values  $p_g$  from the histogram of the global differences quantised to  $Q=128$  bins [1]. The probabilities are then averaged over all pixels as defined below:

$$pY = \frac{\sum_{i=1}^M \sum_{j=1}^N p_g \left( (f(i, j) - \mu)_Q \right)}{M * N} \quad \dots\dots\dots Eq (16)$$

**4. Computing the number of times a specific global contrast is associated with a specific local pattern in the H, V and D directions:**

In steps 1 & 2, local pattern information is computed which is very much useful in texture classification. In step 3, a pixel's intensity is given significant importance and is brought to the picture and is discussed about it. However, there are still some things which are needed to be evaluated and that is the number of times they occur together in the image. This, now takes the form of a joint probability of local differences and global contrasts averaged over the H, V and D directions. The extended feature subset now also contains  $pdiffHY$ ,  $pdiffVY$  and  $pdiffDY$  and they are defined as below:

$$pdiffHY = \frac{\sum_{i=1}^M \sum_{j=1}^N p_{hg} \left( (f(i, j) - f(i, j+1))_Q, (f(i, j) - \mu)_Q \right)}{M * N} \quad \dots\dots\dots Eq (17)$$



$$pdiffVY = \frac{\sum_{i=1}^M \sum_{j=1}^N p_{vg} \left( (f(i, j) - f(i+1, j))_Q, (f(i, j) - \mu)_Q \right)}{M * N} \dots\dots\dots Eq (18)$$

$$pdiffDY = \frac{\sum_{i=1}^M \sum_{j=1}^N p_{dg} \left( (f(i, j) - f(i+1, j+1))_Q, (f(i, j) - \mu)_Q \right)}{M * N} \dots\dots\dots Eq (19)$$

**3.4 TECHNIQUE USED:**

To calculate the difference theoretic features, firstly signed grey level differences of both local and global for each pixel in the image. Then, the histograms are generated for both signed local and global differences  $p_h, p_v, p_d, p_g$ . The joint histograms of  $p_{hg}, p_{vg}, p_{dg}$  are then generated. The probability values from the histograms to each pixel in the image based on the best match of histogram indices and actual pixel differences are found out. Lastly, the average of the absolute differences and the probabilities of the signed differences over all pixels in the image are found to form the 11-dimensional feature set.

The scale invariance is achieved by the averaging process. The use of difference operator solves the problem of illumination variations in the texture image since the luminance information is cancelled during the difference operation. The difference theoretic texture features are made rotation-invariant by assigning the maximum of the three absolute differences absdiff H, absdiff V and absdiff D as the new value of absdiff H and the next higher value is given to absdiff V and then to absdiff D. This ranking ensures that irrespective of the orientation of the image, ordering of three features will be kept the same during classification.

Therefore, the difference theoretic features are very robust features taking into account the absolute values of local and global grey level differences and the probability values derived from histograms of the signed grey level differences

## **CHAPTER 4:**

### **PROPOSED METHOD:**

#### **4.1 REASON FOR USING DIFFERENCE THEORETIC FEATURES:**

Difference Theoretic Texture Features are one of the most robust features set as these features take into account both the global and local grey level differences and also the probabilistic values derived from histogram are taken into account. These feature set have already been applied for scale, rotation and illumination-invariant texture classification with very high results. The accuracy achieved was 98.99% which was compared to other feature sets like LBPV which gave a lower accuracy.

#### **4.2 CLASSIFIER USED:**

The classifier earlier used was distance based classifier. The accuracy achieved was higher but a better accuracy came into picture when a modified distance based classifier was used. Here, after computing the distance, normalization of the values was done so that we can work in the range of 0 to 1. The results after using modified distance based classifier have been shown in the result section.

#### **4.3 TECHNIQUE USED:**

The data set considered by us is the Cohn-Kanade database which consists of various images of different people showing their different facial expressions. The facial expressions considered are surprise, sad, fear, anger, disgust and happy.

A total of 544 images are taken from the Cohn-Kanade database and taken in the ratio of 1:1, the image dataset is divided into training and testing dataset. As a result, there are 277 images in the training set and 277 images in the testing set. Images of 11 different people were taken in these 244 images.

The algorithm to facial expression analysis is divided into three steps: Face Acquisition, Feature Extraction and Facial Classification.

In Face Acquisition, the original image is taken and is cropped to get only the face of the person without including the background. This is done by the following method:

### **A) FACE ACQUISITION (CROPPING):**

An image is taken as input and the coordinates of the eyes are taken. Consider the coordinates to be (r1,c1) and (r2,c2) respectively.

$$\text{centre\_x} = \frac{(c1 + c2)}{2}$$

$$\text{centre\_y} = \frac{(r1 + r2)}{2}$$

$$\text{leftcorner\_x} = (\text{centre\_x} - (0.55 * 200))$$

$$\text{leftcorner\_y} = (\text{centre\_y} - (0.55 * 200))$$

$$\text{width} = (1.05 * 200)$$

$$\text{height} = (1.40 * \text{width})$$

Then, the image is cropped using the left corner coordinates and the width and height calculated.

### **B) FEATURE EXTRACTION:**

All images are resized to 256x256. All images are divided into three parts, that is, upper, middle and lower part and the difference theoretic texture features are computed for all images and for all three parts. The training and testing data sets are thus formed as each having three matrices of 277x11 dimensions.

### **C) FACIAL EXPRESSION CLASSIFICATION:**

1. An image is taken as input from testing data set. The difference theoretic features of three parts of this image are computed and are placed at the row next to the last row of the training matrix. So, the resultant training matrix is now 278x11 for every part.
2. Taking the upper part, the difference in the features of every row of training matrix is calculated with every other row present in the matrix. This would give us a three dimensional array where every matrix comprises of difference matrix between rows.
3. Now, we normalize the last row of all matrices (which contains test features) present in the three dimensional array (use of modified distance based classifier).
4. The result is that we get a new matrix which contains the normalized distance measure of the test features and is in 277x11 format.
5. All the 11 features are then summed up and we get a 277x1 sum array.

6. The process is repeated for middle and lower part and therefore gets two more  $277 \times 1$  sum arrays.
7. We then sum the values of all 3 parts and get a final sum matrix of  $277 \times 1$ .
8. The minimum value from this array is calculated and the image is checked and the expression of this image is assigned to the test image.
9. Steps 6 to Step 12 are repeated for all testing images.
10. The result was that a higher accuracy was achieved than using simple distance based classifier.
11. The algorithm was also tested for cross-validation.
12. The same algorithm was also made to run on Eavci features, the result of which all are shown in the result section.

#### **4.4 EXTENSION TO ABOVE ALGORITHM:**

Since the accuracy of the classified facial expression was high, it was considered to find how much is the extent of the emotion that the face is being classified into. To do this the top 10 minimum values are considered instead of only one minimum value. After this, the expression of the ten minimum values found is classified and a histogram is plotted against all emotions. It is noted that three regular patterns are followed:

1. **Pure Emotion:** This is the emotion that was completely recognized by all the values which shows that the person is showcasing only one emotion.
2. **Pure Emotion with Slight Other:** This is the emotion that is majorly classified to the primary emotion but also gives traces of a secondary emotion
3. **Mixed Emotion:** This is emotion where an image is showcasing multiple emotions and is showed here.

## **CHAPTER 5:**

### **DETAILED RESULTS AND DISCUSSION:**

In our analysis of Facial Expressions, we have taken for reference the Cohn-Kanade database of facial images. The Cohn-Kanade database consists of facial images of various people having different expressions. The link to Cohn-Kanade database is

“<http://www.consortium.ri.cmu.edu/ckagree/>”

The six basic expressions taken in this database are Surprise, Sad, Fear, Anger, Disgust and Happy. The images in this database have been provided in the form of video frames, i.e., if a person is making a happy face, the video frames of that expression would start as a neutral face of the person and then a slight change in every frame takes the expression of person from neutral to happy. Though Cohn-Kanade has provided the database of 123 different people containing various facial expressions, we have sought the permission of using 11 of those people who have the following Ids: S52, S55, S74, S106, S111, S113, S121, S124, S125, S130 and S132.

For every person's expression, we have taken the images from where the expression starts to appear on the face of the person and to maintain uniformity; we have assumed that this starts to happen after 50% of the video frames. Therefore, out of the total image frames, only those image frames are taken from which the expression starts to appear.

Therefore, a total of 544 images of these 11 people were taken from the Cohn-Kanade database. Out of these 544 images, 277 were taken as training data and the other 277 were taken as the testing data.

The programming language used for implementing the idea to demonstrate the working of Facial Expression Recognition through Difference Theoretic Texture Features is Matlab version 7.10.0(R2010a).

The system on which the program was run is Dell Inspiron 1545 with RAM 3GB and hard disk space of 320 GB.









The processor used was the Intel Core 2 Duo processor with 2.10 GHz.













The methodology (as discussed in Chapter 4) was adopted for both Difference Theoretic Features and the Eavci Features. We not only focus as to how many of the images are getting












correctly classified but we also focus on of what intensity the expression is being displayed and these intensities are compared for both Difference Theoretic Texture Features and Eavci Features.

**5.1 COMPARISON OF DIFFERENCE THEORETIC TEXTURE FEATURES AND E.AVCI FEATURES IN FACIAL EXPRESSION RECOGNITION**












TABLE 5.1












S.No	Diff Features						E. avci Features					
	1	2	3	4	5	6	1	2	3	4	5	6
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	5	3	2	0	0	0	5	1	4	0	0	0





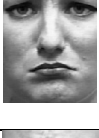
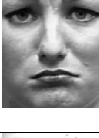





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	7	0	0	1	1	1	8	2	0	0	0	0
	7	0	0	0	2	1	10	0	0	0	0	0
	7	0	0	0	2	1	10	0	0	0	0	0
	7	0	0	0	2	1	10	0	0	0	0	0
	3	1	2	0	0	4	5	0	0	0	0	5
	4	1	3	0	0	2	5	0	0	0	0	5
	3	1	3	0	0	3	7	0	0	0	0	3
	3	1	2	0	0	4	9	0	0	0	0	1
	4	2	4	0	0	0	4	5	1	0	0	0









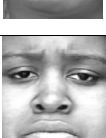

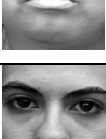
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	10	0	0	0	0	0	10	0	0	0	0	0
	10	0	0	0	0	0	10	0	0	0	0	0
	2	3	4	0	1	0	2	0	6	0	2	0
	3	0	6	0	0	1	3	0	6	0	1	0
	5	3	0	2	0	0	4	1	0	5	0	0
	5	5	0	0	0	0	10	0	0	0	0	0
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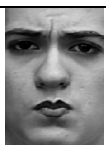
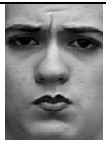
















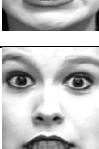

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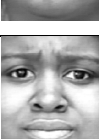
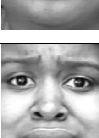


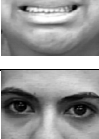
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	2	8	0	0	0	0	0	5	1	0	0	4











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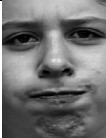









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	0	2	1	7	0	0	0	1	1	8	0	0












	0	2	1	7	0	0	0	3	0	7	0	0
	0	2	0	8	0	0	0	2	0	6	2	0
	0	1	0	9	0	0	0	0	0	9	1	0
	0	0	0	10	0	0	0	0	0	9	1	0
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	0	1	0	9	0	0	0	1	0	8	1	0
	0	0	0	10	0	0	0	1	0	9	0	0
	0	0	0	10	0	0	0	0	0	9	1	0
	0	1	0	9	0	0	0	1	0	9	0	0
	0	0	0	10	0	0	0	1	0	9	0	0
	0	2	1	3	0	4	0	1	1	5	0	3












	0	1	1	2	0	6	0	2	1	6	0	1
	0	1	0	2	3	4	0	3	3	4	0	0
	0	0	0	10	0	0	0	0	0	10	0	0
	0	0	0	10	0	0	0	0	0	10	0	0
	0	0	0	10	0	0	0	0	0	10	0	0
	0	1	0	9	0	0	0	0	0	10	0	0
	0	0	0	10	0	0	0	1	0	9	0	0
	0	0	0	10	0	0	0	2	0	8	0	0
	0	2	0	5	3	0	0	1	0	5	0	4
	0	3	0	5	2	0	0	1	0	5	0	4
	0	3	0	5	2	0	1	1	0	5	0	3

	0	3	0	5	2	0	1	1	0	5	0	3
	0	3	0	5	2	0	1	1	1	6	0	1
	0	0	1	4	3	2	1	3	1	1	4	0
	0	0	0	5	3	2	0	4	1	0	5	0
	0	1	0	6	3	0	0	2	2	1	5	0
	0	0	1	2	7	0	0	1	0	0	4	5
	0	0	1	2	6	1	0	1	3	1	4	1
	0	0	0	0	5	5	0	2	3	1	4	0
	0	0	0	0	5	5	0	2	3	0	4	1
	0	0	0	1	6	3	0	2	3	0	4	1
	0	1	1	5	3	0	0	2	1	4	3	0












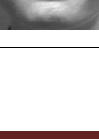
	0	3	0	3	3	1	0	3	0	3	3	1
	0	3	0	2	4	1	0	3	0	2	5	0
	0	2	0	1	4	3	0	3	0	2	4	1
	0	2	0	1	4	3	0	2	0	2	4	2
	0	1	0	2	5	2	0	0	0	0	5	5
	0	1	0	2	4	3	0	0	0	0	6	4
	0	1	0	2	4	3	0	0	0	0	7	3
	0	2	0	1	4	3	0	0	0	0	7	3
	0	2	0	0	2	6	0	0	0	0	2	8
	0	2	0	0	2	6	0	0	0	0	2	8
	0	2	0	0	2	6	0	0	0	0	3	7












	0	3	1	0	2	4	2	1	5	0	2	0
	0	3	2	0	2	3	2	0	5	0	2	1
	0	4	0	0	2	4	1	3	2	0	3	1
	0	3	0	4	3	0	0	5	0	0	3	2
	0	3	0	4	3	0	0	5	0	0	3	2
	0	3	0	4	3	0	0	5	0	0	3	2
	0	4	2	0	2	2	0	3	2	0	2	3
	0	3	2	0	4	1	0	2	1	0	4	3
	0	3	1	0	4	2	0	1	0	0	4	5
	0	2	0	0	3	5	0	1	0	0	4	5
	0	1	1	0	3	5	0	0	2	0	3	5












	0	1	1	0	3	5	0	0	2	0	3	5
	0	0	2	0	3	5	0	0	2	0	3	5
	0	1	1	0	3	5	0	0	2	0	3	5
	0	1	1	0	3	5	0	0	2	0	3	5
	0	1	1	0	3	5	0	0	2	0	3	5
	0	1	1	0	3	5	0	0	2	0	3	5
	0	1	1	0	3	5	0	0	2	0	3	5
	0	6	2	1	0	1	0	6	1	0	0	3
	0	0	0	0	0	10	0	1	0	0	0	9
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10











	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10

























	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	3	0	4	2	1	0	2	0	4	3	1
	0	2	0	0	4	4	0	1	0	2	2	5
	0	1	0	0	2	7	0	2	0	0	2	6
	0	1	0	0	3	6	0	1	0	0	3	6








	0	3	1	0	3	3	0	0	1	0	0	9
	0	2	1	0	3	4	0	0	1	0	0	9
	0	2	0	0	3	5	0	0	1	0	0	9
	0	2	1	0	2	5	0	0	1	0	0	9
	1	1	0	2	0	6	0	0	1	3	0	6
	0	0	0	2	2	6	0	1	1	2	1	5
	1	1	0	1	2	5	0	0	1	0	2	7
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10

	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	1	3	1	0	0	5	0	2	2	0	1	5
	1	3	1	0	0	5	0	2	2	0	1	5
	0	3	1	1	0	5	0	2	2	0	1	5
	0	3	1	1	0	5	0	2	2	0	1	5
	0	0	0	0	2	8	0	0	0	0	2	8
	0	0	0	0	2	8	0	0	0	0	2	8
	0	0	0	0	2	8	0	0	0	0	2	8
	0	0	0	0	2	8	0	0	0	0	2	8
	0	0	0	0	2	8	0	0	0	0	2	8

	0	0	0	0	2	8	0	0	0	0	1	9
	0	0	0	0	2	8	0	0	0	0	1	9
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	0	0	0	0	10	0	0	0	0	0	10
	0	1	1	0	2	6	0	2	2	0	1	5
	0	1	1	0	2	6	0	2	2	0	1	5

	1	1	1	0	2	5	0	2	2	0	1	5
	0	2	0	0	2	6	0	2	2	0	1	5
	0	1	2	0	2	5	0	2	2	0	1	5
	1	2	3	0	1	3	0	3	1	0	2	4
	0	2	0	0	2	6	0	0	2	1	0	7
	0	0	0	1	0	9	0	0	0	1	0	9
	0	0	0	1	0	9	0	0	0	1	0	9
	0	2	0	0	0	8	0	1	1	1	0	7
	0	2	0	0	0	8	0	1	0	2	0	7
	0	2	0	0	0	8	2	1	0	0	0	7
	0	3	0	0	0	7	1	1	0	1	0	7

	0	3	0	0	0	7	2	1	0	0	0	7
	0	2	0	0	0	8	1	1	0	1	0	7
	0	2	0	0	0	8	1	1	0	1	0	7
	0	0	0	2	0	8	0	4	0	0	3	3
	0	0	0	2	0	8	0	4	0	0	3	3
	0	0	0	3	0	7	0	3	0	0	3	4
	0	0	1	5	0	4	0	3	0	0	3	4
	0	0	4	0	0	6	0	1	2	0	0	7
	0	0	3	1	0	6	0	0	3	1	0	6
	0	0	3	1	0	6	0	0	3	1	0	6
	0	0	3	1	0	6	0	0	3	1	0	6

	0	0	3	1	0	6	0	0	3	1	0	6
	0	0	3	1	0	6	0	0	3	1	0	6
	0	1	0	0	4	5	0	3	1	0	1	5
	0	1	0	0	4	5	0	3	1	0	1	5
	0	2	0	0	3	5	0	2	0	0	3	5
	0	1	0	0	4	5	0	2	0	0	3	5
	0	2	0	0	3	5	0	2	0	0	3	5

#### Comparison in Intensities of DTTF and Eavci Features

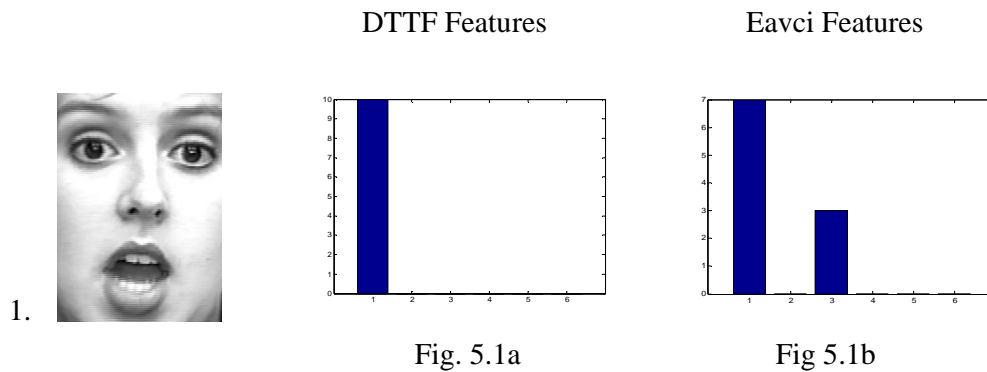
As discussed in chapter 4, we have divided the analysis of expressions into 3 parts: Pure Emotion, Pure Emotion with Slight Other and Mixed Emotion. We have taken the top 10 values of similarity measure which shows that the current image's expression is mostly similar to these 10 images and then the expression is categorised.

Below is shown the image numbers and to which of the three categories the image belonged to. Also, shown are the intensities of the facial emotions giving a comparison between the difference theoretic features and Eavci features with the help of histogram.

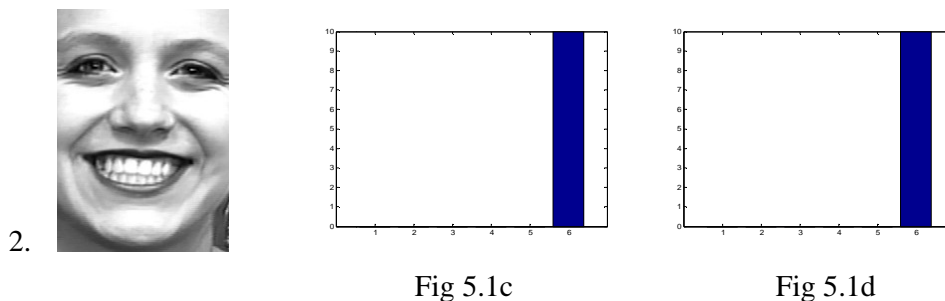
### 1. Pure Emotion:

The list of images which gave the result as pure emotion for Difference Theoretic Texture Features are:

1,2,3,4,5,6,7,22,24,25,32,33,34,35,36,37,38,39,40,41,60,61,62,63,78,130,131,132,133,134,135,136,137,138,142,143,144,145,146,147,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,224,225,226,227,228,229,234,235,236,237,238,239,240,241,242,243,244,245,246,254,255,256,257,258,261,262,263,264.



In the above image, the two histograms are created for Difference Theoretic Features and Eavci Features respectively. The original image consists of surprise emotion which is clearly demonstrated by the Difference Theoretic Texture Feature's histogram whereas Evaci Feature's histogram is demonstrating that image is a mixture of expression of surprise and fear.



Figures 5.3 and Fig 5.4 are able to correctly classify the emotion of the above image as Happy.



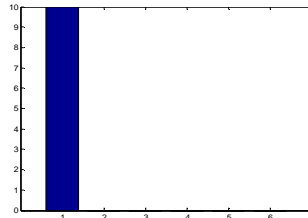
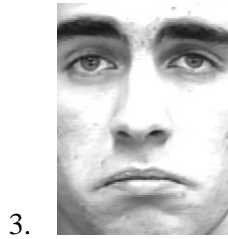


Fig 5.1e

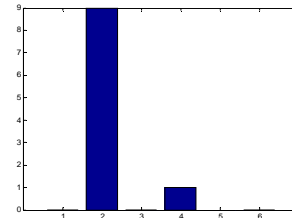


Fig 5.1f

In the above image, the person's expression can be clearly categorised as Sad which is correctly done by Difference Theoretic Texture Features whereas Eavci Features also categorizes the facial expression as Fear.

## 2. Pure with Slight Other Emotion:

The list of images which have been classified as pure emotion but also give traces of some other emotions have been shown below.

12,13,14,15,21,23,45,46,47,48,75,76,79,80,81,82,83,88,89,90,91,104,107,108,109,110,114,115,123,124,125,126,127,128,129,140,155,156,157,160,170,171,172,191,221,222,223,247,248,249,250,251,253,259,260,265,267,268,269,270,271,272.

DTTF Features

Eavci Features

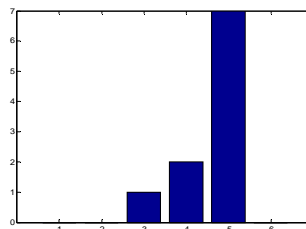


Fig. 5.2a

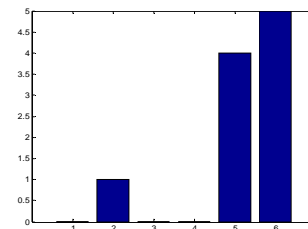


Fig 5.2b

By seeing the above image, we can say that the person is feeling disgusted and it is correctly classified by the Difference Theoretic Texture Features but they also classify it as anger and fear to a very small extent as compared to disgust which can also be deduced from the image. But, the Eavci features are classifying it majorly to be happy which in no context can be correct as clearly a happy gesture is not displayed by the person above.

2.

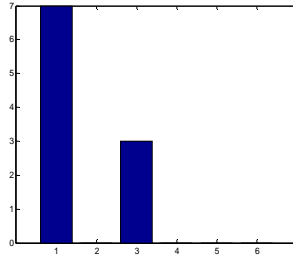
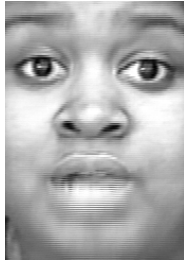


Fig 5.2c

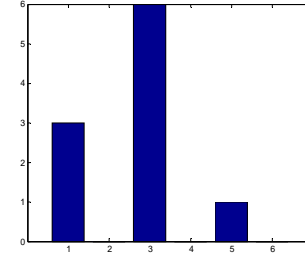


Fig. 5.2d

In the above image the woman is having a surprised expression primarily which is correctly classified by Difference Theoretic Texture Features and it also gives a small trace of fear whereas Eavci features give the primary expression as Fear which is found to be wrong.

3.

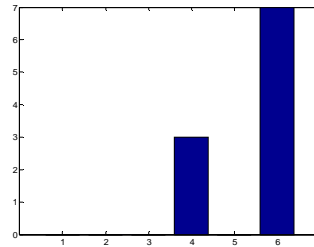


Fig. 5.2e

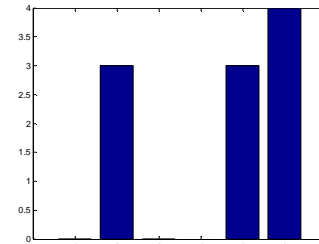


Fig 5.2f

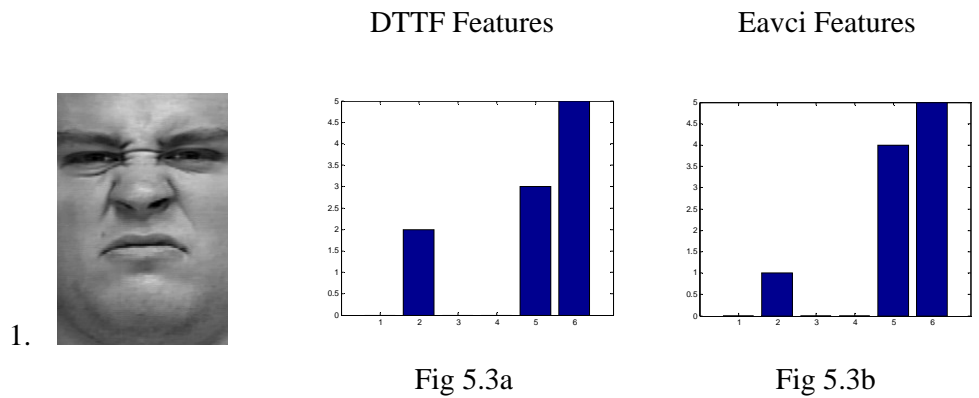
By the above image, we can understand that the facial expression of the person is Happy which is correctly classified by Difference Theoretic Texture Features but Eavci features classify it as having a mixed emotion containing happy disgust and sad which cannot be deduced from the image above.

### 3. Mixed Emotion:

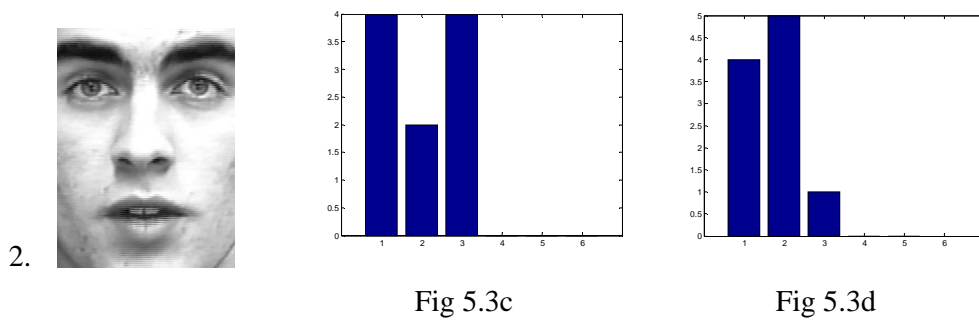
The list of images which have been classified as mixed emotions (which contains multiple emotion) have been shown below:

8,9,10,11,16,17,18,19,20,26,27,28,29,30,31,42,43,44,49,50,51,52,53,54,55,56,57,58,59,66,  
67,68,69,70,71,72,73,74,77,84,85,86,87,92,93,94,95,96,97,98,99,100,101,102,103,105,106,1  
11,112,113,114,116,117,118,119,120,121,122,139,141,148,149,150,151,152,153,154,158,15  
9,160,161,162,163,164,165,166,167,168,169,173,174,175,176,177,178,179,180,181,182,183,

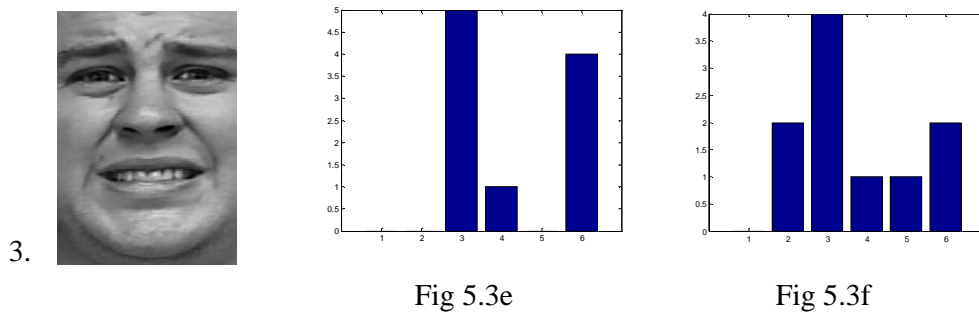
184,185,186,187,188,189,190,213,214,215,216,217,218,219,220,230,231,232,233,252,266,273,274,275,276,277.



In the above image, the facial expression deduced is disgusted which is classified by both Difference Theoretic Features and Eavci Features but in a mixed manner. Both features say that it contains disgusted, happy and sad expression.



In the above image, it can be said that the man is giving a surprise gesture which is correctly classified by Difference Theoretic Texture Features but also the facial expression is classified as Fear and also sad to some extent whereas the Eavci features are classifying the primary expression of person to be sad which is wrong.



By having a look at the above image, we can clearly say that the facial expression of the person is Fear and the primary expression is classified correctly by both Difference Theoretic Texture Features and Eavci features but the Eavci feature's mixed emotion result gives a variety of 4 more expressions as sad, anger, disgusted and happy which is wrong altogether.

### **5.2 DEDUCTION:**

From the above results we can deduce that Difference Theoretic Features work better than Eavci Features in classifying the facial expressions of people and also the intensities of the facial expression classified have better results with Difference Theoretic Features.

### **5.3 CONFUSION MATRICES OF IMAGES AFTER USING DIFFERENCE THEORETIC TEXTURE FEATURES AND E.AVCI FEATURES:**

A test was done taking 277 images as training data and other 277 as testing data out of the total 544 images considered. Every alternated image was taken in training data set and the other alternating sequence was taken in testing data set. The analysis was done for both Difference Theoretic Texture Features and Eavci features.

The overall accuracy achieved on a test of 277 images for Difference Theoretic Texture Features is **98.91%**. The breakup of all emotion set is given as follows:

TABLE 5.2

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	100%	0%	0%	0%	0%	0%
<b>Sad</b>	0%	100%	0%	0%	0%	0%
<b>Fear</b>	0%	0%	96.15%	3.85%	0%	0%
<b>Anger</b>	0%	0%	0%	100%	0%	0%
<b>Disgust</b>	0%	3.33%	0%	0%	96.67%	0%
<b>Happy</b>	1.05%	0%	0%	0%	0%	98.95%

#### Confusion Matrix for DTF Features

Using the same data set, we do the same analysis for Eavci Features. The overall accuracy achieved on a test of 277 images is **98.91%**. The breakup of all emotion set is given as follows:

TABLE 5.3

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	100%	0%	0%	0%	0%	0%
<b>Sad</b>	0%	100%	0%	0%	0%	0%
<b>Fear</b>	0%	0%	96.15%	3.85%	0%	0%
<b>Anger</b>	0%	0%	0%	100%	0%	0%
<b>Disgust</b>	0%	3.33%	0%	0%	96.67%	0%
<b>Happy</b>	1.05%	0%	0%	0%	0%	98.95%

#### Confusion Matrix for Eavci Features

Now, we perform the Cross-validation on the same set of images. The training and testing data sets are now interchanged with each other and same procedure is done.

Using the Difference Theoretic Texture Features, the overall accuracy achieved on a test of 277 images is **98.91%**. The breakup of all emotion set is given as follows:

TABLE 5.4

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	100%	0%	0%	0%	0%	0%
<b>Sad</b>	0%	100%	0%	0%	0%	0%
<b>Fear</b>	0%	0%	96.15%	3.85%	0%	0%
<b>Anger</b>	0%	0%	0%	100%	0%	0%
<b>Disgust</b>	0%	3.33%	0%	0%	96.67%	0%
<b>Happy</b>	1.05%	0%	0%	0%	0%	98.95%

#### Confusion Matrix for DTF Features(Cross-Validation)

Now, the same analysis is done for Eavci Features, the overall accuracy achieved on a test of 277 images is **98.55%**. The breakup is as follow:

TABLE 5.5

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
--	-----------------	------------	-------------	--------------	----------------	--------------

<b>Surprise</b>	97.77%	0%	2.23	0%	0%	0%
<b>Sad</b>	0%	100%	0%	0%	0%	0%
<b>Fear</b>	0%	4%	96%	0%	0%	0%
<b>Anger</b>	0%	0%	0%	100%	0%	0%
<b>Disgust</b>	0%	0%	0%	3.71%	96.29%	0%
<b>Happy</b>	0%	0%	0%	0%	1.03%	98.97%

Confusion Matrix for Eavci Features(Cross-Validation)

#### **5.4 INFERENCE:**

From the above results, we can understand the Difference Theoretic Features work better than Eavci Features as Eavci have lower accuracy than the Difference Theoretic Features.

#### **5.5 WHY PARTITIONING?:**

As discussed in Chapter 4, we divided the image into three parts. The main reason for partitioning the image is to give high relevance to all three main parts of the image i.e., eyes, nose and mouth. If we have considered the whole image and computed the Difference Theoretic Texture Features then the accuracy and performance would be lowered. The accuracy achieved using no partition procedure is 94.58% which is low as compared to our previous result of 98.91%. The breakup of this procedure is given below:

TABLE 5.6

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	90.90%	2.27%	0%	0%	2.27%	4.54%
<b>Sad</b>	0%	93.47%	0%	2.17%	0%	4.34%
<b>Fear</b>	0%	0%	88.46%	3.84%	0%	7.69%
<b>Anger</b>	0%	0%	2.77%	97.22%	0%	0%
<b>Disgust</b>	0%	3.33%	0%	3.33%	93.33%	0%
<b>Happy</b>	0%	0%	2.08%	0%	0%	97.92%

Confusion Matrix for DTF Features Without Partition

Comparing Table 5 with Table 4, it is clearly visible that individually also every emotion is being tested better when using the partitioning method.

### **5.6 COMPARISON OF VARIOUS CLASSIFIERS:**

The classifier used to obtain results in Table 1-4 is the modified nearest neighbour classifier. Various classifiers were tried to obtain a high accuracy for Facial Expression Recognition.

One of the techniques used to classify facial expressions was taking the city block distance. The city block distance is defined as the difference in the given values. Therefore, here the two expressions were subtracted from each other and it works on the principle that the similar expressions will have the nearly same set of features and therefore taking the difference of the two same expressions would give an absolute value close to zero. It was observed that an accuracy of 97.47% was achieved which is lower than our previous result. The breakup of the emotion set is shown below:

TABLE 5.7

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	88.63%	2.10%	2.10%	0%	0%	6.81%
<b>Sad</b>	2.17%	93.47%	0%	4.34%	0%	0%
<b>Fear</b>	0%	0%	96.15%	0%	0%	3.85%
<b>Anger</b>	0%	5.55%	2.77%	91.66%	0%	0%
<b>Disgust</b>	0%	13.33%	0%	0%	80%	6.67%
<b>Happy</b>	0%	0%	0%	1.06%	0%	98.94%

#### Confusion Matrix for DTF Features and City Block Distance Classifier

We also used the nearest neighbour classifier without any normalization. Nearest neighbour classifier uses Euclidean distance. An accuracy of 93.14% was obtained the breakup of emotion set is given as below:

TABLE 5.8

	<b>Surprise</b>	<b>Sad</b>	<b>Fear</b>	<b>Anger</b>	<b>Disgust</b>	<b>Happy</b>
<b>Surprise</b>	88.63%	2.10%	2.10%	0%	0%	6.81%
<b>Sad</b>	2.17%	93.47%	0%	4.34%	0%	0%

<b>Fear</b>	0%	0%	96.15%	0%	0%	3.85%
<b>Anger</b>	0%	5.55%	2.77%	91.66%	0%	0%
<b>Disgust</b>	0%	13.33%	0%	0%	80%	6.67%
<b>Happy</b>	0%	0%	0%	1.06%	0%	98.94%

Confusion Matrix for DTF Features using Nearest Neighbour classifier without Normalization

Table 8 clearly shows that the results have worsened if we take into account Table 1. Also, the overall accuracy has gone down from 98.91% to 93.14%.

Another classifier Support Vector Machine was taken into account. Support Vector Machine is only a two class classifier; therefore we can only check one emotion against all other emotions at a time. A similar analysis was done and the following results were obtained:

TABLE 5.9

Emotion	Accuracy
<b>Surprise</b>	96.75%
<b>Sad</b>	83.75%
<b>Fear</b>	90.61%
<b>Anger</b>	88.44%
<b>Disgust</b>	89.16%
<b>Happy</b>	86.28%

Confusion Matrix for DTF Features using SVM classifier

We have also considered the chi-statistic classifier which helps in classifying images as per the Eq. (1). The accuracy achieved was **83.03%** and the break up is given as follows:

TABLE 5.10

	Surprise	Sad	Fear	Anger	Disgust	Happy
Surprise	79.54%	0%	6.81%	0%	0%	13.63%
Sad	2.17%	76.08%	2.17%	4.34%	0%	15.24%
Fear	3.85%	3.85%	73.07%	3.85%	3.85%	11.53%



Anger	0%	2.77%	2.77%	88.89%	2.77%	2.77%
Disgust	6.67%	3.33%	0%	0%	73.33%	16.67%
Happy	1.05%	3.16%	2.10%	0%	2.10%	91.57%

Confusion Matrix for DTF Features using Chi-Statistic Classifier

After performing the Facial expression recognition by dividing the image into three parts, an attempt was made to perform further partitions. Therefore, the image was now divided into 6x7 sub-images as followed in [3] and the same technique followed and different weights are assigned to these images like the corner sub-images are weighted as 0 and the sub-images containing eyes and mouth as 4 and the sub-images like fore-head and nose as 2 and 3 respectively. The classifier used is the modified distance based classifier. An accuracy of **99.63%** was achieved but at the expense of very expensive computation. The break up is as follows:

TABLE 5.11

	Surprise	Sad	Fear	Anger	Disgust	Happy
Surprise	95.45%	0%	0%	0%	0%	4.55%
Sad	0%	100%	0%	0%	0%	0%
Fear	0%	0%	100%	0%	0%	0%
Anger	0%	0%	0%	100%	0%	0%
Disgust	0%	0%	0%	0%	100%	0%
Happy	0%	0%	0%	0%	0%	100%

Confusion Matrix for DTF Features using Modified Distance Based Classifier and Weighted Partitions

The method put up in [3] divides the image into 6x7 partitions and weights are given to sub-image's features as done above using chi-statistic classifier. An accuracy of **99.63%** was achieved but at the expense of very expensive computation. The break up is as follows:

TABLE 5.12

	Surprise	Sad	Fear	Anger	Disgust	Happy
Surprise	95.45%	0%	0%	0%	0%	4.55%
Sad	0%	100%	0%	0%	0%	0%

Fear	0%	0%	100%	0%	0%	0%
Anger	0%	0%	0%	100%	0%	0%
Disgust	0%	0%	0%	0%	100%	0%
Happy	0%	0%	0%	0%	0%	100%

Confusion Matrix for DTF Features using Chi-Statistic Classifier and  
Weighted Partitions

## CONCLUSION

Facial Expression Recognition can be done in a variety of ways using various types of features and classifiers. Using Difference Theoretic Texture Features have certainly provided a very high advantage to us as they take into account both the local and global grey level differences. These features help in finding the right texture even with a different scale or for a rotated image or even if there is a change in illumination. But, one more difficult task comes out as to which classifier to use when recognising Facial expressions. The new classifier explained known as the modified distance based classifier is found to give the best results when compared with very efficient classifiers like Support Vector Machine, city block distance, nearest neighbour classifier or chi-square statistic. Considering Table 5.11 and 5.12 also, modified distance based classifier does not fall behind the chi-statistic classifier. Also, it is very important to take a note that every part of a face provides a certain input to the facial expression it is demonstrating and it is very important to give importance to every part of the face.

Shan, McOwan tried to partition the image into various other images but the computation performed on a single image increases heavily and we tend to prefer methods with as much low computation as possible and therefore, partitioning the image into three parts and giving all parts equal importance using the modified distance based classifier. But, the modified distance based classifier and chi-statistic classifier give the exact same accuracy for the weighted partitioning method but not when image is divided into 3 parts.

## REFERENCES

- [1] Seba Susan, Madasu Hanmandlu, "Difference Theoretic Features set for scale-, illumination-, and rotation-invariant texture classification", *Image Processing, IET* (Vol 7, Issue: 8), pp. 725-732
- [2] Chibelushi, Claude C., and Fabrice Bourel. "Facial expression recognition: A brief tutorial overview." *CVonline: On-Line Compendium of Computer Vision* 9 (2003).
- [3] Shan, Caifeng, Shaogang Gong, and Peter W. McOwan. "Facial expression recognition based on local binary patterns: A comprehensive study." *Image and Vision Computing* 27.6 (2009),pp. 803-816.
- [4] Song, K. T., & Chien, S. C. (2012, October). Facial expression recognition based on mixture of basic expressions and intensities," In *Systems, Man, and Cybernetics (SMC)*", *2012 IEEE International Conference on* (pp. 3123-3128). IEEE.
- [5] Bashar, F., Khan, A., Ahmed, F., & Kabir, M. (2014, February). Robust facial expression recognition based on median ternary pattern (MTP). In *Electrical Information and Communication Technology (EICT), 2013 International Conference on* (pp. 1-5). IEEE.
- [6] R. C. Gonzalez, and R. E. Woods, *Digital Image Processing*, 3rd Ed, Pearson Education, 2009.
- [7] Sadeghi, H., Raie, A. A., & Mohammadi, M. R. (2013, September), "Facial expression recognition using geometric normalization and appearance representation". In *Machine Vision and Image Processing (MVIP), 2013 8th Iranian Conference on* (pp. 159-163). IEEE.
- [8] Zhang, H., Luo, S., & Yoshie, O. (2013, June). Facial expression recognition by analyzing features of conceptual regions. In *Computer and Information Science (ICIS), 2013 IEEE/ACIS 12th International Conference on* (pp. 529-534). IEEE.
- [9] C. Padgett and G. Cottrell, "Identifying emotion in static face images," In *Proc. 2nd Joint Symp. Neural Computation*, vol. 5, Univ. California, San Diego, pp. 91C101.
- [10] J. L. Landabaso, M. Pardis, and A. Bonafonte, "HMM recognition of expressions in unrestrained video intervals," In *Proc. ICASSP*, Hong Kong, China, 2003, pp. 197C200.
- [11] Avci, E. (2007). An expert system based on Wavelet Neural Network-Adaptive Norm Entropy for scale invariant texture classification. *Expert Systems with Applications*, 32(3), 919-926.
- [12] Facial Expression Recognition based on weighted principal component analysis and

support vector machines

based on MPEG-4 facial animation parameters,” In Proc. ICASSP, vol.4, Orlando, FL, 2002, pp. 3624C3627.

[13] Ou, J., Bai, X. B., Pei, Y., Ma, L., & Liu, W. (2010, January). Automatic facial expression recognition using Gabor filter and expression analysis. In *Computer Modeling and Simulation, 2010. ICCMS'10. Second International Conference on* (Vol. 2, pp. 215-218). IEEE.

[14] M. Pantic, L.J.M. Rothkrantz, "An Expert System for Multiple Emotional Classification of Facial Expressions", Proc. 11th IEEE Int. Conf. on Tools with Artificial Intelligence, pp. 113-120, 1999

[15] I.A. Essa, A.P. Pentland, "Coding, Analysis, Interpretation, and Recognition of Facial Expressions", IEEE Trans. Pattern Analysis and Machine Intelligence, Vol. 19, No. 7, pp. 757-763, 1997

[16] Wu, Y., Liu, H., & Zha, H. (2005, August). Modeling facial expression space for recognition. In *Intelligent Robots and Systems, 2005.(IROS 2005). 2005 IEEE/RSJ International Conference on* (pp. 1968-1973). IEEE.

[17] Song, K. T., & Chen, Y. W. (2011, November). A design for integrated face and facial expression recognition. In *IECON 2011-37th Annual Conference on IEEE Industrial Electronics Society* (pp. 4306-4311). IEEE.

[18] Cheon, Y., & Kim, D. (2008, December). A natural facial expression recognition using differential-AAM and k-NNS. In *Multimedia, 2008. ISM 2008. Tenth IEEE International Symposium on* (pp. 220-227). IEEE.

[19] Kotsia, I., & Pitas, I. (2007). Facial expression recognition in image sequences using geometric deformation features and support vector machines. *Image Processing, IEEE Transactions on*, 16(1), 172-187.

[20] Tian, Y. L., Kanade, T., & Cohn, J. F. (2005). Facial expression analysis. In *Handbook of face recognition* (pp. 247-275). Springer New York.

[21] Fasel, B., & Luettin, J. (2003). Automatic facial expression analysis: a survey. *Pattern Recognition*, 36(1), 259-275.

[22] Essa, I. A., & Pentland, A. P. (1997). Coding, analysis, interpretation, and recognition of facial expressions. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 19(7), 757-763.

[23] Gokturk, S. B., Bouguet, J. Y., Tomasi, C., & Girod, B. (2002, May). Model-based face tracking for view-independent facial expression recognition. In *Automatic Face and Gesture*

- Recognition, 2002. Proceedings. Fifth IEEE International Conference on* (pp. 287-293). IEEE.
- [24] Kotsia, I., & Pitas, I. (2005, July). Real time facial expression recognition from image sequences using support vector machines. In *Visual Communications and Image Processing 2005* (pp. 59602E-59602E). International Society for Optics and Photonics.
- [25] Kenji, M. A. S. E. (1991). Recognition of facial expression from optical flow. *IEICE TRANSACTIONS on Information and Systems*, 74(10), 3474-3483.
- [26] Aleksic, P. S., & Katsaggelos, A. K. (2006). Automatic facial expression recognition using facial animation parameters and multistream HMMs. *Information Forensics and Security, IEEE Transactions on*, 1(1), 3-11.
- [27] Lucey, P., Cohn, J. F., Kanade, T., Saragih, J., Ambadar, Z., & Matthews, I. (2010, June). The Extended Cohn-Kanade Dataset (CK+): A complete dataset for action unit and emotion-specified expression. In *Computer Vision and Pattern Recognition Workshops (CVPRW), 2010 IEEE Computer Society Conference on* (pp. 94-101). IEEE.
- [28] Yang, M. H., Kriegman, D., & Ahuja, N. (2002). Detecting faces in images: A survey. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 24(1), 34-58.
- [29] Lien, J. J., Kanade, T., Cohn, J. F., & Li, C. C. (1998, April). Automated facial expression recognition based on FACS action units. In *Automatic Face and Gesture Recognition, 1998. Proceedings. Third IEEE International Conference on* (pp. 390-395). IEEE.
- [30] Liu, S. S., Tian, Y. T., & Li, D. (2009, July). New research advances of facial expression recognition. In *Machine Learning and Cybernetics, 2009 International Conference on* (Vol. 2, pp. 1150-1155). IEEE.
- [31] Zhao, W., Chellappa, R., Phillips, P. J., & Rosenfeld, A. (2003). Face recognition: A literature survey. *Acm Computing Surveys (CSUR)*, 35(4), 399-458.
- [32] Esau, N., Wetzel, E., Kleinjohann, L., & Kleinjohann, B. (2007, July). Real-time facial expression recognition using a fuzzy emotion model. In *Fuzzy Systems Conference, 2007. FUZZ-IEEE 2007. IEEE International* (pp. 1-6). IEEE.
- [33] Krinidis, S., Buciu, I., & Pitas, I. (2003). Facial expression analysis and synthesis: A survey. *HCI'03*, 1432-1433.
- [34] De la Torre, F., & Cohn, J. F. (2011). Facial expression analysis. In *Visual Analysis of Humans* (pp. 377-409). Springer London.
- [35] Tian, Y., Kanade, T., & Cohn, J. F. (2011). Facial expression recognition. In *Handbook of face recognition* (pp. 487-519). Springer London.

- [36] Black, M. J., & Yacoob, Y. (1998). *U.S. Patent No. 5,774,591*. Washington, DC: U.S. Patent and Trademark Office.
- [37] Lajevardi, S. M., & Lech, M. (2008, December). Averaged Gabor filter features for facial expression recognition. In *Digital Image Computing: Techniques and Applications (DICTA), 2008* (pp. 71-76). IEEE.
- [38] Vretos, N., Nikolaidis, N., & Pitas, I. (2009, November). A model-based facial expression recognition algorithm using principal components analysis. In *Image Processing (ICIP), 2009 16th IEEE International Conference on* (pp. 3301-3304). IEEE.
- [39] Buenaposada, J. M., Muñoz, E., & Baumela, L. (2008). Recognising facial expressions in video sequences. *Pattern Analysis and Applications*, 11(1), 101-116.
- [40] Niu, Z., & Qiu, X. (2010, August). Facial expression recognition based on weighted principal component analysis and support vector machines. In *Advanced Computer Theory and Engineering (ICACTE), 2010 3rd International Conference on* (Vol. 3, pp. V3-174). IEEE.
- [41] Gang, L., Xiao-hua, L., Ji-Liu, Z., & Xiao-gang, G. (2009, August). Geometric feature based facial expression recognition using multiclass support vector machines. In *Granular Computing, 2009, GRC'09. IEEE International Conference on* (pp. 318-321). IEEE.
- [42] Wang, W., Chang, F., Zhao, J., & Chen, Z. (2010, July). Automatic facial expression recognition using local binary pattern. In *Intelligent Control and Automation (WCICA), 2010 8th World Congress on* (pp. 6375-6378). IEEE.
- [43] Bagherian, E., & Rahmat, R. W. O. (2008, August). Facial feature extraction for face recognition: a review. In *Information Technology, 2008. ITSIM 2008. International Symposium on* (Vol. 2, pp. 1-9). IEEE.
- [44] Koutlas, A., & Fotiadis, D. I. (2008, October). An automatic region based methodology for facial expression recognition. In *Systems, Man and Cybernetics, 2008. SMC 2008. IEEE International Conference on* (pp. 662-666). IEEE.
- [45] Jafri, R., & Arabnia, H. R. (2009), "A Survey of Face Recognition Techniques". *JIPS*, 5(2), 41-68.
- [46] Dubuisson, S., Davoine, F., & Masson, M. (2002), "A solution for facial expression representation and recognition". *Signal Processing: Image Communication*, 17(9), 657-673.
- [47] Pantic, M., & Rothkrantz, L. J. (2000). Expert system for automatic analysis of facial expressions. *Image and Vision Computing*, 18(11), 881-905.
- [48] Zeng, Z., Pantic, M., & Huang, T. S. (2009). Emotion recognition based on multimodal information. In *Affective Information Processing* (pp. 241-265). Springer London.

[49] Sun, X., Xu, H., Zhao, C., & Yang, J. (2008, September). Facial expression recognition based on histogram sequence of local Gabor binary patterns. In *Cybernetics and Intelligent Systems, 2008 IEEE Conference on* (pp. 158-163). IEEE.

[50] Pramadihanto, D., & YACHIDA, M. (2001),” Integrated person identification and expression recognition from facial images” *IEICE TRANSACTIONS on Information and Systems*, 84(7), 856-866.

[51] Patil, R. A., Sahula, V., & Mandal, A. S. (2012),”Features classification using support vector machine for a facial expression recognition system”, *Journal of Electronic Imaging*, 21(4), 043003-1.