# SYNOPSIS

The development in the field of Information Theory started rigorously as a branch of mathematics in 1948, when C. E. Shannon published a landmark paper *'The Mathematical Theory of Communication'* in the *'Bell System Technical Journal'*. Broadly speaking, the theory deals with the study of problems concerning any system. This includes information processing, information storage, information retrieval and decision making. In a narrow sense, theory deals with all theoretical problems connected with the transmission of information over communication channels. This includes the study of uncertainty measures and practical and economical methods of coding information for transmission.

A key feature of Shannon information theory is the term *'information'*. Information occurs only if there exists some a prior uncertainty and the amount of information obtained from an experiment/observation is the amount by which the uncertainty has been reduced. Shannon [36] conceived the statistical nature of the communication signal with that of the random variable $X = (X_1, X_2, X_3, ..., X_n)$ having probability distribution $P = (p_1, p_2, p_3, ..., p_n)$, and introduced a measure of information (or, uncertainty) as

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \ , \ 0 \leq p_i \leq 1, \ \sum_{i=1}^{i=n} p_i = 1, \tag{1}$$

associated with this experiment. This is also called the Shannon entropy measure. Shannon entropy satisfies a number of useful properties like non-negativity, continuity, symmetry, grouping, and additivity etc.

In case $X$ is a continuous random variable say, denoting the lifetime of a device with p.d.f. $f(x)$, then the measure of uncertainty associated with $X$ is given by

$$H(f) = -\int_0^\infty f(x) \log f(x) dx \ . \tag{2}$$

The measure $H(f)$ is called the differential entropy, refer to McEliece [29]. A huge literature devoted to the characterizations, generalizations and applications of the Shannon information measure is available, refer to Cover and Thomson [11], Aczel and Daroczy [2], Taneja [37] and Wells [44].

An additive generalization of the Shannon information measure was given by

Renyi's [35] as

$$H^{\alpha}(P) = \frac{1}{1-\alpha} \log \left\{ \sum_{i=1}^{n} p_i^{\alpha} \right\}, \;\; \alpha \neq 1, \; \alpha > 0 \; . \tag{3}$$

The continuous analogous to the measure (3) is

$$H^{\alpha}(f) = \frac{1}{1-\alpha} \log \left\{ \int_0^{\infty} f^{\alpha}(x) dx \right\}, \;\; \alpha \neq 1, \; \alpha > 0 \; . \tag{4}$$

A generalization of order $\alpha$ and type $\beta$ of the Shannon entropy (2) is the Verma's entropy [42] defined as

$$H_{\beta}^{\alpha}(f) = \frac{1}{\beta-\alpha} \log \left[ \int_0^{\infty} f^{\alpha+\beta-1}(x) dx \right], \; \beta - 1 < \alpha < \beta, \; \beta \geq 1. \tag{5}$$

With ever increasing applications, sub-additivity rather than additive is becoming an acceptable basis; in social and physical systems the additivity does not quite prevail. An important non-additive entropy measure by Havrda and Charvat [22] is

$$H_{\alpha}(f) = \frac{1}{2^{1-\alpha}-1} \left\{ \int_0^{\infty} f^{\alpha}(x) dx - 1 \right\}, \;\; \alpha \neq 1, \; \alpha > 0 \tag{6}$$

which finds applications in various disciplines, refer to Boghosian [10], Tsallis and Brigatti [41]. When $\alpha \to 1$, both measures (4) and (6) reduce to (2).

Kullback and Leibler [28] studied a measure of information from statistical aspects, involving two probability distributions associated with the same experiment. Thus, if $P = \{p_1, p_2, \ldots, p_n\}$ is probability distribution associated with the experiment $X = \{X_1, X_2, \ldots, X_n\}$ and $Q = \{q_1, q_2, \ldots, q_n\}$ is predicted (or, reference) distribution associated the same experiment, then Kullback's measure of relative information [28] is

$$H(P/Q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \; , \tag{7}$$

and Kerridge measure of inaccuracy [26] is given by

$$H(P;Q) = -\sum_{i=1}^{n} p_i \log q_i \; . \tag{8}$$

The corresponding information measures for relative information and inaccuracy in case of continuous random variable are given by

$$H(f/g) = \int_0^\infty f(x) \log \frac{f(x)}{g(x)} dx \ , \tag{9}$$

and

$$H(f;g) = -\int_0^\infty f(x) \log g(x) dx \ . \tag{10}$$

Shannon's entropy, Kullback-Leibler's relative information and Kerridge's inaccuracy are the three classical measures of information associated with one and two probability distributions. We observe that the measures of information, discrimination and inaccuracy are associated as

$$H(f) + H(f/g) = H(f;g),$$

that is, inaccuracy is the sum of entropy and discrimination. These three measures have found deep applications in the areas of information theory and statistics.

There is another class of information measures called the *weighted information measures*. We consider a system in which the importance of the different outcomes $X_i$'s depend upon the experimenter's goal, or upon some qualitative characteristic of the physical system taken into consideration. Thus in addition to the probability distribution $P = \{p_1, p_2, \dots, p_n\}$ associated with an experiment, we have $U = \{u_1, u_2, \dots, u_n\}$ a utility distribution which accounts for the qualitative aspects of the experiment depending upon the experimenter's goal.

Taking this into consideration, Belis and Guiasu [7] extended the concept of Shannon entropy to the systems with quantitative and qualitative characteristics and gave the 'useful' information measure

$$H(P;U) = -\sum_{i=1}^{n} u_i p_i \log p_i; \ u_i \geq 0, \tag{11}$$

where $u_i$ is the utility assigned with the outcome $X = X_i$. A quantitative-qualitative measure of relative information as suggested by Taneja and Tuteja [38] is given by

$$H(P/Q;U) = \sum_{i=1}^{n} u_i p_i \log \frac{p_i}{q_i} \ . \tag{12}$$

The utility $u_i$ of the outcome $x_i$ is independent of probability $p_i$ or $q_i$ and depends only on the qualitative characteristics of the physical system into account. In sequel to these measures, by considering the aspect analogous to (8), we have

$$H(P;U) + H(P/Q;U) = -\sum_{i=1}^{i=n} u_i p_i \log p_i + \sum_{i=1}^{n} u_i p_i \log \frac{p_i}{q_i}$$

$$= -\sum_{i=1}^{n} u_i p_i \log q_i = H(P,Q;U). \tag{13}$$

Taneja and Tuteja [39] defined (13) as the *quantitative qualitative measure of inaccuracy*. They have also characterized this measure using a set of axioms. This extends the concept of Kerridge inaccuracy [26] given by (8) to a system with qualitative concept. When the utilities are ignored, that is $u_i = 1$ for each $i$, then (11), (12) and (13) reduce respectively to measures of Shannon entropy [36], Kullback relative information [27], and Kerridge inaccuracy [26].

Generalizations of these 'useful' information measures and their applications to coding theory have been studied extensively, refer to Gurdial and Pessoa [21], Taneja [40], Taneja and Tuteja [38], Bhatia and Taneja [9], Jain and Srivastava [23], Kumar et al. [25] and Parkash et al. [31] .

If $X$ is the lifetime of a new unit, then Shannon entropy $H(f)$ can be applied to measure the associated uncertainty. However if the unit has already survived till age $t$, then $H(f)$ is no longer useful for measuring the uncertainty about the remaining lifetime of the unit. In such a situation, Ebrahimi [17] proposed another dynamic measure based on Shannon entropy, known as residual entropy, given by

$$H(f;t) = -\int_0^{\infty} f_t(x) \log f_t(x) dx \tag{14}$$

$$= -\int_t^{\infty} \frac{f(x)}{\bar{F}(t)} \log \frac{f(x)}{\bar{F}(t)} dx, \tag{15}$$

where $f_t(x)$ is the probability density function of the random variable $X_t = [X - t | X > t]$, the remaining lifetime of a unit of age t. Ebrahimi [17] showed that $H(f;t)$ uniquely determines the distribution function, and Rajesh and Nair [34] gave a similar result in the discrete case. Various results concerning $H(f;t)$ have been obtained by Asadi and Ebrahimi [3], Belzuence et al. [8], Asadi et al. [4], Nanda and Paul [30] and, Baig and Dar [6].

Ebrahimi and Kirmani [18, 19] considered the Kullback-Leibler discrimination information measure between two residual lifetime distributions, which is defined as

$$H(f/g;t) = \int_t^\infty \frac{f(x)}{\bar{F}(t)} \log\left(\frac{f(x)/\overline{F}(t)}{g(x)/\overline{G}(t)}\right) dx. \tag{16}$$

For each $t$, $t \geq 0$, $H(f/g;t)$ possesses all properties of the Kullback-Leibler information measure (9). Further Ebrahimi and Kirmani [18] have shown that the dynamic measure $H(f/g;t)$ is independent of $t$, if and only if the hazard rate functions are proportional, that is, $\lambda_G(x) = \beta\lambda_F(x),\ \beta > 0$.

Abraham and Sankran [1] introduced and studied the concept of Renyi's entropy for the residual lifetime distribution, which is defined as

$$H_\alpha(f;t) = \frac{1}{1-\alpha} \log\left\{\frac{\int_t^\infty f^\alpha(x)dx}{\bar{F}^\alpha(t)}\right\},\ \alpha \neq 1,\ \alpha > 0. \tag{17}$$

Asadi et al. [4] and Nanda and Paul [30] have obtained some characterization results for distributions based on the generalized residual entropy function.

In many realistic situation uncertainty is not necessarily related to the future but can also refer to the past. Suppose a system is found to be down at time $t$, then the uncertainty of the system's life relies on the past, that is, at which instant in $(0,t)$ the system has failed. The variable of interest in this case is $X_t$=$[t-X|X<t]$, known as the *inactivity time*.

Based on this idea, Di Crescenzo and Longobardi [13, 14] have studied measures of entropy and discrimination based on past entropy over $(0,\ t)$ given respectively as

$$H^*(f;t) = -\int_0^t \frac{f(x)}{F(t)} \log \frac{f(x)}{F(t)} dx \tag{18}$$

and

$$H^*(f/g;t) = \int_0^t \frac{f(x)}{F(t)} \log\left(\frac{f(x)/F(t)}{g(x)/G(t)}\right) dx. \tag{19}$$

We not that $H^*(f/g;t)$ is constant if and only if $X$ and $Y$ satisfy the proportional reversed hazard model (PRHM).

The concept of weighted distribution introduced by Rao [32] is widely used in statistics and other applications. Let $X$ be a non-negative continuous random variable with probability density function (p.d.f.) $f(x)$, and let $X^w$ be a weighted random variable corresponding to $X$ with weight function $w(x)$, which is positive for all value of $x \geq 0$. Then the probability density function $f^w(x)$ of the weighted random variable $X^w$ is given by

$$f^w(x) = \frac{w(x)f(x)}{E[w(X)]}, \quad 0 \leq x < \infty \tag{20}$$

with $0 < E[w(X)] < \infty$. Obviously $f^w(x) \geq 0$ and $\int_0^\infty f^w(x)dx = 1$.

When $w(x) = x$, $X^w$ is said to be a *length biased (or, a size biased) random variable* and the p.d.f. (20) in this case becomes

$$f^L(x) = \frac{xf(x)}{E[X]} \ . \tag{21}$$

The Shannon dynamic measures of entropy have been extended to the *length biased weighted residual entropy* and *length biased weighted past entropy*, defined respectively as

$$H^L(f,t) = -\int_t^\infty x\frac{f(x)}{\bar{F}(t)}\log\frac{f(x)}{\bar{F}(t)}dx \tag{22}$$

and

$$H^{*L}(f,t) = -\int_0^t x\frac{f(x)}{F(t)}\log\frac{f(x)}{F(t)}dx, \tag{23}$$

refer to Di Crescenzo and Longobardi [15].

Rao et al. [33] have pointed out some basic shortcomings in the Shannon differential entropy measure, like that : (1) It is based on the density of the random variable, which in general may or may not exists, (2) Shannon entropy of a discrete distribution is always non-negative, while the differential entropy of a continuous variable may take any value on the extended real line.

They proposed another measure of randomness called *Cumulative Residual Entropy (CRE)*, which is defined using distribution rather than density. The CRE of a positive random variable $X$ with distribution function $F(x)$ is

$$\xi(X) = -\int_0^\infty \bar{F}(x)\log\bar{F}(x)dx \ . \tag{24}$$

This measure parallels the well-known Shannon entropy but has the following advantages: (1) cumulative residual entropy has consistent definitions in both the continuous and discrete domains, (2) cumulative residual entropy is always non-negative, (3) cumulative residual entropy can be easily computed from sample data and these computations asymptotically converge to the true values. Wang and Vemuri [43] have obtained several properties of the measure (24) and have provided some applications of it in reliability engineering and computer vision.

Asadi and Zohrevand [5] have proposed a dynamic cumulative residual entropy and have obtained some of its properties. The cumulative residual entropy (CRE) for the residual lifetime distribution of a system with survival function $\overline{F_t}(x) = P(X - t > x | X > t) = \frac{\bar{F}(x+t)}{\bar{F}(t)}$, is given as

$$\xi(X;t) = -\int_t^{\infty} \frac{\bar{F}(x)}{\bar{F}(t)} \log \frac{\bar{F}(x)}{\bar{F}(t)} dx \ . \tag{25}$$

Analogous to the cumulative residual entropy (CRE) measure, Di Crescenzo and Longobardi [16] introduced and studied the cumulative entropy, defined as

$$\xi^*(X) = -\int_0^{\infty} F(x) \log F(x) dx. \tag{26}$$

A dynamic version of the cumulative entropy (26) given as

$$\xi^*(X;t) = -\int_0^{t} \frac{F(x)}{F(t)} \log \frac{F(x)}{F(t)} dx \ , \tag{27}$$

was also studied by Di Crescenzo and Longobardi [16].

While studying the characterization results in case of dynamic measures of inaccuracy two specific models, one proportional hazard model (PHM), refer to Cox [12], and secondly proportional reversed hazard model (PRHM), refer to Gupta et al. [20] are used.

In view of the above discussion, we were motivated to consider the dynamic entropy measures based on non-additive entropy, since non-additivity rather than additivity is more prevalent in many physical situations. Also, we found considerable interest in studying dynamic and weighted (length biased) dynamic inaccuracy measures, since this aspect was not explored much. Considering the importance of entropy measures based on distribution function over density function, we considered it worthwhile to study cumulative entropy measures based on generalized information measure, inaccuracy measure, and their dynamic versions.

The thesis comprises seven chapters including the last chapter on conclusion and further scope of work. The thesis has been organized as follows;

**Chapter 1** is introductory in nature presenting a brief account of the available literature and the various information measures proposed by the researchers. Some basic concepts of reliability, including that of proportional hazard model (PHM), proportional reversed hazard model (PRHM) and length biased model, have also been discussed.

In **Chapter 2**, we have considered Havrat and Charvat [22] measure of entropy which is a one parameter generalization of the Shannon entropy and is non-additive in nature. We have proposed a residual measure of entropy based on it and have proved a characterization theorem that the proposed measure determines the distribution function uniquely. Also we have characterized some specific probability distributions based on the proposed measure. The work reported in this chapter has been published in the papers entitled, **Non-additive Entropy Measure Based Residual Lifetime Distributions** in *JMI International Journal of Mathematical Sciences*, 2010, 1 (2), 1-9, and, **A Generalized Entropy- Based Residual Lifetime Distribution** in *International Journal of Biomathematics*, 2011, 4 (2), 171-184.

In **Chapter 3**, we have conceptualized the idea of dynamic measure of inaccuracy, both residual and past. In case of residual inaccuracy measure we have studied the characterization result using proportional hazard model; and in case of past inaccuracy measure we have studied this using proportional reversed hazard model. Also we have characterized some specific distributions based on these measures. The work reported in this chapter has been published in the papers entitled, **A Dynamic Measure of Inaccuracy Between Two Residual Lifetime Distributions** in *International Mathematical Forum*, 2009, 4 (25), 1213-1220, and, **A Dynamic Measure of Inaccuracy Between Two Past Lifetime Distributions** in *Metrika*, 2010, 74 (1), 1-10.

In **Chapter 4**, the results of Chapter 3 have been extended to weighted distributions, a concept of considerable importance as mentioned already.

Taking weights $w(x) = x$, we have introduced length biased measures of residual and past inaccuracies and have studied their respective characterization theorems, and other properties. The results reported in this chapter have been published in the papers entitled, **Length Biased Weighted Residual Inaccuracy Measure** in *Metron*, 2010, LXVIII (2), 153-160, and, **On Length Biased Dynamic Measure of Past Inaccuracy** in *Metrika*, 2012, 75 (1), 73-84. Also some results were presented at *International Conference in Mathematics and Applications* held in *Bangkok* on Dec. 19-21, 2009.

In **Chapter 5**, we have generalized the concept of cumulative residual entropy measure to one parameter and two parameters entropies, and have studied their dynamic versions and characterization results. The exponential, Pareto and finite range distribution, which are commonly used in reliability modeling, have been characterized in terms of generalized cumulative residual entropy measures. The work reported in this chapter has been published in the papers entitled, **On Dynamic Renyi Cumulative Residual Entropy Measure** in *Journal of Statistical Theory and Applications*, 2011, 10 (3), 491-500, and, **Some Characterization Results on Generalized Cumulative Residual Entropy Measure** in *Statistics and Probability Letters*, 2011, 81 (8), 72-77. Also some results were presented at *International Congress of Mathematicians (ICM)* held in *Hyderabad* on Aug. 19-27, 2010.

In **Chapter 6**, we have considered dynamic cumulative inaccuracy measures, both residual and past and have studied the characterization results respectively under proportional hazard model and proportional reversed hazard model. Also we have characterized certain specific probability distributions using relation between different reliability measure. It is expected that dynamic cumulative inaccuracy measures introduced will further extend the scope of study. The work reported in this chapter has been published in the paper entitled, **On Dynamic Cumulative Residual Inaccuracy Measure** in proceeding of the *World Congress on Engineering (WCE)*, held in *London* on July 4-6 2012, and, some results have been communicated for publication.

In **Chapter 7**, we have concluded the findings of the work carried out in this thesis and also have presented further scope of work. During the present investigation, several ideas have originated which have the potential to extend the study further. We can consider the proposed dynamic measures further for discrete cases, since practically discrete cases are suitable from application point of view. Further the discrete measure of the dynamic version proposed can possibly find wider applications in different area of interest. The work reported in this thesis can be extended to bivariate and multivariate domains. Also we can employ the concept of order statistics to the different dynamic measures reported in the thesis.

Following is the list of publications out of this thesis.

**Publications in Journals**

1. HC Taneja, Vikas Kumar and R. Srivastava, *A Dynamic Measure of Inaccuracy Between Two Residual Lifetime Distribution*, **International Mathematical Forum**, 2009, 4 (25), 1213-1220.

2. Vikas Kumar, HC Taneja and R. Srivastava, *Non-additive Entropy Measure Based Residual Lifetime Distributions*, **JMI International Journal of Mathematical Sciences**, 2010, 1 (2), 1 - 9.

3. Vikas Kumar, HC Taneja and R. Srivastava, *A Dynamic Measure of Inaccuracy Between Two Past Lifetime Distribution*, **Metrika**, 2010, 74 (1), 1-10.

4. Vikas Kumar, HC Taneja and R.Srivastava, *Length Biased Weighted Residual Inaccuracy Measure*, **Metron**, 2010, LXIII (2), 153-160.

5. Vikas Kumar and HC Taneja, *Some Characterization Results on Generalized Cumulative Residual Entropy Measure*, **Statistics Probability Letters**, 2011, 81 (8), 72-77.

6. Vikas Kumar, HC Taneja and R. Srivastava, *On Dynamic Renyi Cumulative Residual Entropy Measure*, **Journal of Statistical Theory and Applications**, 2011, 10 (3), 491-500.

7. Vikas Kumar and HC Taneja, *A Generalized Entropy Based Residual Lifetime Distributions*, **International Journal of Biomathematics**, 2011, 4 (2), 1-14.

8. Vikas Kumar and HC Taneja, *On Length Biased Dynamic Measure of Past Inaccuracy*, **Metrika**, 2012, 75 (1), 73-84.

**Papers in International Conferences**

**1.** Vikas Kumar and HC Taneja, *On weighted past inaccuracy measure.* Presented at International Conference in Mathematics and Applications (ICMA-MU) held at Bangkok, Dec. 17-21, 2009.

**2.** HC Taneja and Vikas Kumar, *Length biased weighted residual inaccuracy measure.* Presented at International Congress of Mathematicians (ICM) held at Hyderabad, August 19-27, 2010.

**3.** Vikas Kumar, HC Taneja and R. Srivastava, *On dynamic Renyi cumulative residual entropy measure.* Presented at International Congress of Mathematicians (ICM) held at Hyderabad, August 19-27, 2010.

**4.** HC Taneja and Vikas Kumar, *On dynamic cumulative residual inaccuracy measure.* Presented at World Congress of Engineering (WCE) held at London, U.K., July 06-08, 2011.

**5.** Vikas Kumar and HC Taneja, *Generalized dynamic cumulative residual entropy.* Presented at International Conference on Statistics, Probability and Related Areas held at Cochin University of Science and Technology, Kerla, Dec. 19-22, 2011 .

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