

# **DESIGNING OF MICROWAVE DIFFERENTIATOR AND INTEGRATOR USING DSP TECHNIQUE**

A Thesis submitted towards the partial fulfilment of the requirement for the  
award of the degree of

**Master of Technology**

**in**

**Microwave and Optical Communication**

Submitted by

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**2K11 / MOC / 13**

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JUNE 2013**

# CERTIFICATE

This is to certify that the thesis report entitled, "**Design of Microwave Differentiator and Integrator using DSP Technique**" being submitted by **Shafqat Abdullah Khan** to the *Department of Electronics and Communication Engineering and Applied Physics, Delhi Technological University* in partial fulfilment of the requirement for award of Master of Technology degree in *Microwave and Optical Communication* is a record of bona fide work carried out by him under the supervision and guidance of Dr. Priyanka Jain. The matter embodied in this report has not been submitted for the award of any other degree.

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# DECLARATION

I hereby declare that all the information in this document has been obtained and presented in accordance with academic rules and ethical conduct. This report is my own, unaided work. I have fully cited and referenced all material and results that are not original to this work. It is being submitted for the degree of Master of Technology in Engineering at the Delhi Technological University. It has not been submitted before for any degree or examination in any other university.

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# ABSTRACT

The differentiator and integrator have many advantages and are very useful signal processing tool which makes them suitable for a wide varieties of applications. Differentiator finds wide applications in many areas such as speech systems, image processing, radar and biomedical engineering. The integrator has been used extensively in many areas such as correlation estimation, waveform shaping and accumulator analysis. So far, the differentiator and integrator are mainly implemented in circuits for low-speed applications. Thus the implementation of these circuits for high frequency applications has been largely ignored.

In this thesis, simple and accurate formulations are presented to represent stable and optimized discrete-time infinite impulse response processes for both first order differentiators and integrator in the Z-domain. These formulations, in conjunction with the representations of transmission-line elements in the Z-domain, leads to transmission-line configuration that are eligible for wide-band microwave circuits.

Design simulations for digital differentiator and integrator are performed in Agilent SystemVue and MATLAB. In order to translate these circuits for high frequency application, T-parameter (chain scattering parameter) is employed. A detailed mathematical analysis of each design is analysed and corresponding Matlab code has been generated. The MATLAB simulations were compared with Agilent Advanced Design System (ADS) simulations in order to find the optimal design parameters required to arrive at an approximate solution. ADS simulations were used to accurately determine the final design.

The designed models are implemented using non-uniform microstrip lines in Agilent ADS and Agilent EMPro. Simulation results shows proposed models as good candidate for wide band microwave application.

To my parents...

# ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my project supervisor, Asst. Professor **Dr. Priyanka Jain**, for her supervision, invaluable guidance, motivation and support throughout the extent of the project. I have benefitted immensely from her wealth of knowledge.

I would also like to thank **Dr. AjeetKumar** (Physics Department),**Prof. Uma Nagiya** (EEE Department)and **Dr. Neeta Pandey** (ECE Department) for their precious suggestions, support and technical help during the course of this project.

I am indebted to **Prof. Rajiv Kapoor**, Head of Department of Electronics and Communication Engineering, Delhi Technological University for his support and encouragement in carrying out this project.

I wish to express my heart full thanks to **Prof. S.C. Sharma**, Head of Department of Applied Physics,Delhi Technological University for his support that helped me a lot in successful completion of this project.

I am also grateful to **Prof. P. B. Sharma**,Vice-Chancellor,Delhi Technological University for providing the research environment in the institute.

My gratitude is extended tomy colleagues and friendswho have not been mentioned here personally in making this project a success.

Last but not least, I take this opportunity to express my deepest thanks to my parents and my sister.Without their support, love and encouragement, it would not have been possible to pursue M.Tech. degree studies. I sincerely thank them.

Shafqat Abdullah Khan

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# LIST OF ABBREVIATIONS

ADS	:	Advanced Design System
$c$	:	Speed of light
DSP	:	Digital Signal Processing
$\epsilon_{reff}$	:	Effective relative permittivity
EDA	:	Electronics Design Automation
EMPro:		Electromagnetic Professional
$Err$	:	Error
$f$	:	Frequency
$f_s$	:	Sampling frequency
FEM	:	Finite element method
FIR	:	Finite Impulse Response
$h$	:	Height of dielectric substrate
H(z)	:	Discrete time System function
IIR	:	Infinite Impulse Response
$K$	:	Gain
$l$	:	Length of microstrip
$\lambda$	:	Wavelength
OCS	:	Open circuit Stub
$\Omega$	:	Normalized frequency
$S$	:	Scattering Parameter
SCS	:	Short circuit Stub
STL	:	Serial Transmission Line
$\tan\delta$	:	Loss tangent of dielectric or dissipation factor
$t$	:	Thickness
$\tau$	:	Time constant
T	:	Sampling Time interval
$\Gamma$	:	Reflection coefficient
TLS	:	Transmission line section
$w$	:	Width of microstrip
$z^{-1}$	:	unit sample delay
$Z_N$	:	Characteristic impedance of $N_{th}$ Section of microstrip line
$Z_0$	:	Characteristic impedance of microstrip line 50 Ohms

### 1.1 Overview

The differentiator and integrator have many advantages and are very useful signal processing tools which makes them suitable for a wide variety of applications. The differentiator is designed to determine and estimate time derivatives of a signal. It has been used extensively in many areas, such as image processing, speech systems, radar, sonar and many more. The velocity and acceleration can be computed from the position measurements using differentiator's digital control [1]. In applications such as biomedical engineering requires higher-order derivatives of biomedical data, especially at low-frequency range.

The integrator is an instrumental tool to estimate the time integral of measured signals. It has been used widely in many areas such as coherent detection, correlation estimation, accumulator analysis, and waveform shaping [2]. The integrator can also be employed to measure the delay times of microwave transistors [3] or it can be used to implement high-frequency active filters [4]. In other words, the integrator not only plays an important role in determining the inter-relation among various signals, but also detects the history of the signal itself. In the Fourier spectral analysis, the spectral of a measured signal is the output of an integrator that takes the time integration of the multiplication of the measured signal by harmonic signals [2].

Various Integrators and differentiator has been proposed [2, 5-8] in the study of discrete-time signal processing(DSP), however their magnitude response can only approximate that of ideal response for a fraction of full band nyquist frequency range. Among various techniques, trapezoidal rule and Simpson's rule in the Z-domain are two popular methods used for integrators. The trapezoidal-rule integrator produces a zero at the normalized frequency [2], while the Simpson-rule integrator yields a quasi-zero lying between dc and the normalizing frequency.

The existence of zeros causes the performance of these two integrator largely deviates from that of the ideal integrator. Therefore, both trapezoidal and Simpsons rule integrator are not adequate to be employed as a wide band integrator. To overcome the limitation, we proposed a new discrete-time integrator whose transfer function fits well with that of an ideal integrator.

The design procedure for integrator and differentiator as shown in flow chart figure 1.1 involves first obtaining a multivariable system function prototype and then optimizing its transfer function with reference to ideal, to obtain the wideband stable integrator and differentiator.

In this thesis, simple and accurate formulations are employed to represent stable and optimized discrete-time infinite impulse response processes for both first order differentiators and integrator in the Z-domain. These formulations, in conjunction with the representations of transmission-line elements in the Z-domain, leads to transmission-line configuration that are eligible for wide-band microwave circuits. In particular, many Z domain formats of transfer functions have been obtained to represent the characteristics of a differentiator and integrator.

The transmission line configuration can emulate the characteristics of the differentiator and integrator developed in a DSP study, meanwhile the operating frequency band of a differentiator and integrator is, thus, extended further into the microwave range. Both first order differentiators and integrator are implemented with microstrip transmission lines, of which the operating frequency is determined by the physical length of each line section. It is, therefore, plausible to design differentiators and integrator having operating frequencies larger than 10 GHz.

Computers are powerful tools for the microwave designers in performing arduous and error-prone calculations. Computers can be used as a tool to design many circuits and components faster and cheaper than conventional methods. To get high reliability standards, precisely characterised and less error to the designing circuits, Computer-Aided Design<sup>1</sup> (CAD) becomes a better simulator in microwave technology, and it is cannot be separated. By CAD, user will eliminate trial and error, laborious mathematical calculations and delaying in drafting. CAD also allows rapid evaluation of many existing circuits, safe a time and cost saving.

The design simulations for digital differentiator and integrator are performed in CAD software Agilent SystemVue and optimization is performed under MATLAB environment. A detailed mathematical analysis of each design was performed. Matlab code for optimization of design prototype was written. The designed models are implemented using non-uniform microstrip lines in Agilent ADS<sup>2</sup> and Agilent EMPro. Simulation results shows proposed models as good candidate for wide band microwave application.

The theory required for the design and implementation of the Microwave differentiator and integrator has been gathered from IEEE journals, relevant books<sup>3</sup>, and internet. The main constraint on the project has been time. This report describes the design simulation and implementation of 1<sup>st</sup> order microwave differentiator and integrator using microstrip transmission line.

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<sup>1</sup>Gupta, K. C., et al. (1981). Computer-Aided Design of Microwave Circuits, Artech.

<sup>2</sup>qthelp://ads.2011.10.app/doc/examples/S-Parameters\_of\_2-Port\_Terminated\_with\_Other\_Networks.html

<sup>3</sup>M. Radmanesh, Radio Frequency and Microwave Electronics, Prentice Hall, 2001



## 1.2 Design Flowchart

To be more systematic in designing process, the flow diagram must be obtained. The flow diagram can be used as a guide to the designer to obtain their strategy and getting a good result as required. The figure 1.1 depicts the design flow strategies used for obtaining optimized microwave differentiator and integrator circuits from discrete-time system function prototype.

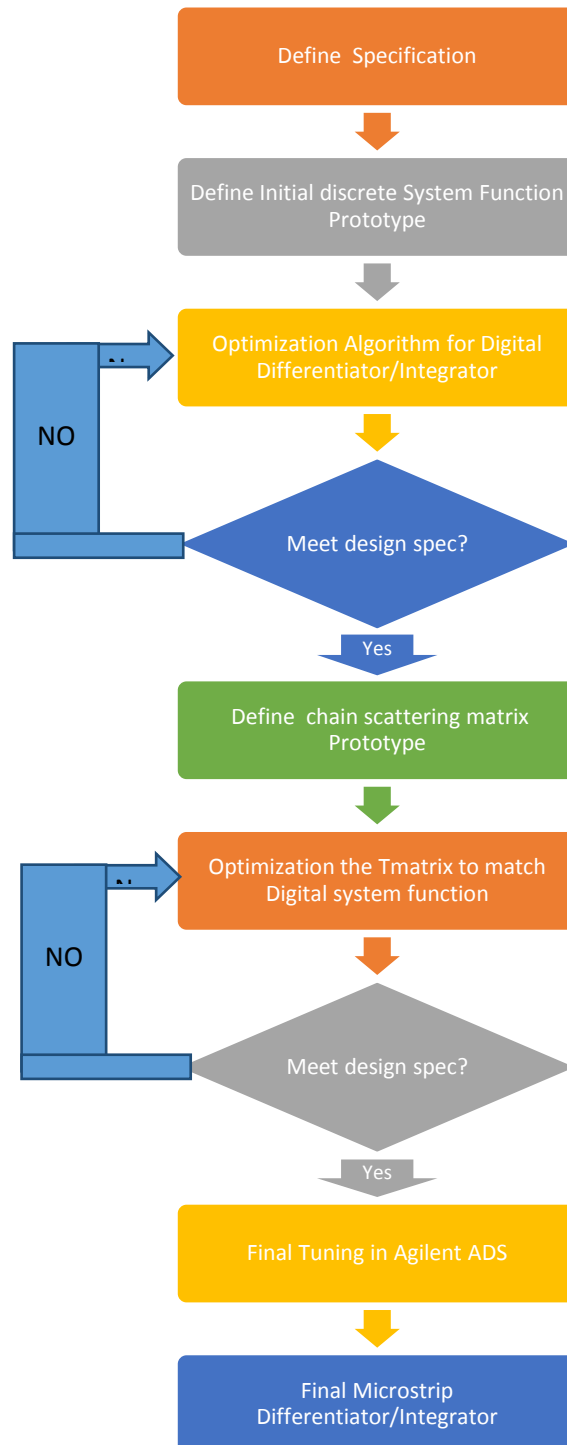


Figure.1.1: Flow chart of designing process for microwave differentiator and integrator

### 1.3 Motivation and Problem Statement

The motivation for project came while under taking course on DSP techniques. The circuit theory for differentiator, integrator and filter is very well developed in DSP. These devices are mainly implemented in circuits for low frequency application. Thus, so far the implementation of differentiator and integrator for high frequency application has been largely ignored. So an idea came to relate the DSP and microwave techniques by representations of transmission-line elements in the Z-domain for higher frequency circuit designing i.e. for microwave device implementation.

### 1.4 Goals/Scope of present work

The main goals of this project are to get acquainted with microwave circuit designing using CAD tool like Agilent Advanced Design System, SystemVue<sup>4</sup> and EMPro<sup>5</sup> software and the basics of DSP techniques. The present work consists of designing of 1<sup>st</sup> order microwave differentiator and integrator using microstrip transmission line technology. The scope of this project lies on designing and implementation of other microwave device components like differentiator and integrator with different time constant and microwave filters etc.

The next section describes the organization of chapters in the thesis.

### 1.5 Report Organization

The thesis report is divided into seven chapters, each having ample information for comprehending the concepts of this project.

*Chapter 1:* presents introduction to project, design flow chart, discusses the motivation and problem statement, goal and scope of present work.

*Chapter 2:* literature review and the theory involved in the research work of this project have been presented in this chapter. Here detail formulation of transfer function in Z-domain has been given which will be used in the rest of the thesis. In particular, many Z domain formats of the stable and optimized transfer functions have been obtained to represent the characteristics of a differentiator and integrator. The chain scattering parameter or T-parameter for serial transmission line, short stub and open stub are derived. The concept of time-constant is introduced. Finally, description of various types of optimization algorithm is introduced.

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<sup>4</sup>[qthelp://systemvue32.2011.10/doc/algorithm/Filter.html](http://qthelp://systemvue32.2011.10/doc/algorithm/Filter.html)

<sup>5</sup>[qthelp://empro.2011.07/doc/exportads/Exporting\\_ADS\\_Layouts\\_to\\_EMPro.html](http://qthelp://empro.2011.07/doc/exportads/Exporting_ADS_Layouts_to_EMPro.html)

*Chapter 3:* the detailed designing of stable and optimized digital differentiator and integrator system functions have been explained in this chapter. Here, the basic equations governing differentiators and integrators are derived from optimization algorithm. The designed system function is analysed in software tool like Agilent SystemVue. Implementation of direct form I and direct form II using MATLAB are realised.

*Chapter 4:* illustrates the thorough designing of microwave differentiators. Upon using the system functions designed in Chapter 3 corresponding microstrip line configuration is obtained and optimized to derive the characteristic impedance of microstrip sections. Software Simulation performed in Agilent ADS followed by exporting the designed prototype model to Agilent EMPro and optimization is performed in MATLAB environment. Finally, Plotting of magnitude response and group delay of final tuned differentiator are presented.

*Chapter 5:* describes the designing methodology of microwave integrator. Based on the digital integrator obtained in Chapter 3 the designing of optimized microwave integrators are presented. The designed integrator models are also analysed in Agilent EMPro. Lastly, magnitude response and group delay characteristic of designed integrators are presented.

*Chapter 6:* summarizes detailed results of simulation analysis.

The Final chapter of the thesis (Chapter 7) presents the conclusions and future aspects of this project. The significance and contribution of this work is summarized.

## Literature Review

### 2.1 Introduction

This chapter reviews several basic but important concepts that are necessary to comprehend the contents of this report. Here general formulation of transfer function in Z domain and at microwave frequency in terms of T-parameter (chain scattering) is discussed and equations are formed to link both techniques. The concept of time constant for differentiator and integrator is introduced.

### 2.2 Formulation of the Transfer Function in the Z domain

A DSP system (digital integrator and differentiator) can be defined with sets of recursive difference equations of the form(2.1)

$$y[n] = a_0x[n] + a_1x[n-1] + \dots + a_{M-1}x[n-(M-1)] - b_1y[n-1] + b_2y[n-2] - \dots - b_Ny[n-N] \quad 2.1$$

where,  $x(n)$  is the digital input and  $y(n)$  is the digital output of the filter. The parameters specify  $H(z)$ , the Z-transform of an impulse response  $h(n)$ . The output of IIR is the convolution of the input  $x(n)$  with  $h(n)$ .

The system function for linear, time-invariant, causal digital filter can be expressed in the Z-domain in the form [9]:

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^N b_i z^{-i}}{1 + \sum_{j=1}^N a_j z^{-j}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_M z^{-N}} = K \frac{\prod_{i=1}^N (z - z_i)}{\prod_{i=1}^N (z - p_i)} \quad 2.2$$

The coefficients of the polynomial are  $a_i$  and  $b_i$ . The zeros and poles of the factored form are  $z_i$  and  $p_i$ , respectively. The gain factor K is necessary for equivalence between the polynomial and factored forms. The order of  $H(z)$  is determined by N.

Not all filters defined by  $H(z)$  are feasible or implementable. Two properties of  $H(z)$  of concern to this thesis are

- A causal, linear, time invariant (LTI) system with system function  $H(z)$  is bounded input bounded output (BIBO) stable if and only if all the poles of  $H(z)$  lie inside the unit circle. ( $|p_i| < 1$ )
- A causal, stable, LTI system with system function  $H(z)$  is real if and only if all complex poles and zeros of  $H(z)$  have complex conjugate pairs or exist singularly on the real axis.

These properties should be enforced during the design and optimization process of an IIR filter.

For optimize digital IIR filters, a method for mapping a filter transfer function  $H_n(z)$  to an element  $x_n$  is needed. Two straightforward methods for doing this include mapping either the coefficients of the polynomial form of  $H_n(z)$  or the roots and gain of the factored form of  $H_n(z)$  to the vectors of  $x_n$ . While both of these options are mathematically equivalent, polynomial coefficients  $b_i$  and  $a_i$  can have several orders of magnitude of dynamic range necessitating a very large search space  $S$ . However, this is not the case for the factored version, thereby prompting its selection. Filter stability requires that all poles  $p_i$  of  $H_n(z)$  are inside the unit circle, thus, limiting the search space  $S$  for  $p_i$ . Placement of the zeros  $z_i$ , although not dictated by stability, can be restricted to the same region by imposing a minimum-phase requirement. Minimum-phase is accomplished by having all poles and zeros of  $H_n(z)$  inside the unit circle. This has the drawback of greatly restricting the possible phase response that can be achieved by the optimization algorithm, but this does not matter here since magnitude response is the only factor of optimization considered in this thesis.

Several design algorithms exist for finite-impulse-response (FIR) filters that enable relatively simple use of the filters in common practice. For example, the frequency weighted least squares (FWLS) technique enables an FIR filter of a given order to be designed according to a desired frequency response [10]. The resulting design equation has a closed form solution resulting in a very fast, accurate, and precise result. The design of infinite-impulse-response (IIR) filters, on the other hand, has shown to be more difficult. IIR filters are often preferred to FIR filters because of excellent magnitude response characteristics, especially when high attenuation or sharp transition bands are desired [11]. IIR filters typically meet a given set of specifications with a much lower filter order than a corresponding FIR filter and preserves impulse response and shape of frequency response, if there is no aliasing. The final digital filter design is independent of the sampling interval parameter  $T$ . The digital IIR filter design task can be approached like any other optimization problem.

## 2.2.1 Discrete-Time Differentiator

It is well known that the operation of a time derivative of a signal is represented by a complex-frequency variable  $s$  in the Laplace transform representation. Neglecting the loss factor, the complex-frequency variable  $s$  is equal to  $j\omega$ , i.e.  $s = j\omega$ , where,  $\omega$  is the signal angular frequency. As a result, a differentiator is a high-pass filter and the amplitude of its system function increases linearly as the signal frequency increases.

The frequency response of an ideal differentiator is given by

$$H_{\text{Diff}}(\omega) = K_d(j\omega) \quad 2.3$$

where,  $j = \sqrt{-1}$ ,  $K_d$  is the proportional constant of differentiator and  $\omega$  is the angular frequency in radians.

Various methods had been developed to design discrete finite impulsive response (FIR) and infinite impulsive response (IIR) differentiators [12]–[16]. Al-Alaoui [6] used Simpson's rule to develop a stable second-order recursive differentiator. Al-Alaoui [12] used interpolation method to develop a stable, minimum-phase digital differentiator. Pei and Shyu [15] used the eigen approach to design high order digital differentiators. In order to obtain low relative error, Kumar and Ohba [16] employed optimal method to develop digital differentiators which are maximally accurate at low frequencies. In order to develop a wide-band differentiator, Khan and Ohba [8] employed the central difference approximations of the derivative of a function to obtain a maximally linear differentiator. An important aspect of the previous investigation is that the exploration focused on the improvement of linearity over a wide frequency band.

### 2.2.2 Discrete-Time Integrator

The operation of a time integral of a signal is represented by an inverse of complex-frequency variable in the Laplace transform representation. Neglecting the loss factor, transfer function is equal to  $1/j\omega$ , where,  $\omega$  is the signal angular frequency. As a result, an integrator is a low-pass filter and the amplitude of its system function is inversely proportional to signal frequency increases.

The frequency response of an ideal integrator is given by

$$H_{\text{Int}}(\omega) = K_i/j\omega \quad 2.4$$

where,  $j = \sqrt{-1}$ ,  $K_i$  is the proportional constant of integrator and  $\omega$  is the angular frequency in radians.

Almost all the classical integrators are derived by taking Z-transform of the class of Newton-cotes interpolation formulas [17-18]. Newton-cotes interpolation formula is basically a technique of calculating a definite integral/curve by replacing that curve by a more integrable and

simpler curve, thus introducing some error in the equation, but still approximating the result to a great degree.

Few of the classical integrators that are worth mentioning are Trapezoidal Integrator, Simpson 1/3 Integrator, Simpson 3/8 Integrator and Boole's Integrator.

A new set of integrators has been developed in the recent past known as Recursive digital integrators. Several methods have been used for their design. This include putting different values of m and n for Newton-Cotes digital integrators, which is essentially arrived at by applying Z-transform to closed form Newton-Cotes integration formula[19], use of linear interpolation between the magnitude responses of the classical rectangular, trapezoidal and Simpson digital integrators [12]-[20] and use of linear programming approach in the design [21]. Every design has its own advantages and limitations. Newton-Cotes digital integrators [19], although applicable over a wideband and a maximum error margin of 6.5% compared to ideal analog integrators, are fit for use in higher frequency ranges. On the other hand, two integrator designs proposed by Al-Alaoui based on linear interpolation between magnitude responses of basic integrators [12]-[20] have lower error than Newton-Cotes Integrators but also have a lower operational bandwidth where the error margin is negligible. The design proposed by Papamarkos and Chamzas[21] based on the linear optimization techniques again suffer from a narrow bandwidth problem.

However none of the above stated integrators and differentiator are near the ideal integrator as far as the magnitude response is concerned. Chapter 3 introduces new designs of integrators and differentiator.

### 2.3 Time Constant of a Differentiator

A differentiator formed by an inverted operational amplifier and a serial capacitor-resistor circuit, where  $V_i(\omega)$  and  $V_o(\omega)$  are the input and output of the circuit, respectively. The transfer gain  $H(j\omega)$  of the differentiator in the frequency domain is defined as the ratio of  $V_o(\omega)$  to  $V_i(\omega)$  and is given as follows.

$$H(j\omega) = \frac{V_o(\omega)}{V_i(\omega)} = -j\omega RC \quad (2.17)$$

where  $\omega$  is the signal angular frequency, R the resistor and C the capacitor. Note that the transfer gain is linearly proportional to the angular frequency of the signal. In particular, the multiplication of C by R is the time constant of the circuit. As a result, the amplitude of transfer gain of a differentiator is equal to the multiplication of angular frequency by time constant. Therefore, we define the time constant of a differentiator as the ratio of transfer gain to signal angular frequency [22], which is expressed as follows

$$\tau_d = K_d = \frac{|H_{Diff}(j\omega)|}{\omega} \quad (2.18)$$

$$\tau_d = K_d = \frac{|H_{Diff}(e^{j\Omega})|}{\Omega}$$

The time constant determines the transient behaviour of an actual differentiator in the time domain consideration. As a consequence, it affects the frequency response of the differentiator. The frequency responses of the differentiator and integrator, formed by an operational amplifier and a resistor-capacitor (R-C) circuit given by (2.17). Equation (2) is valid also for microwave differentiators implemented by using transmission lines.

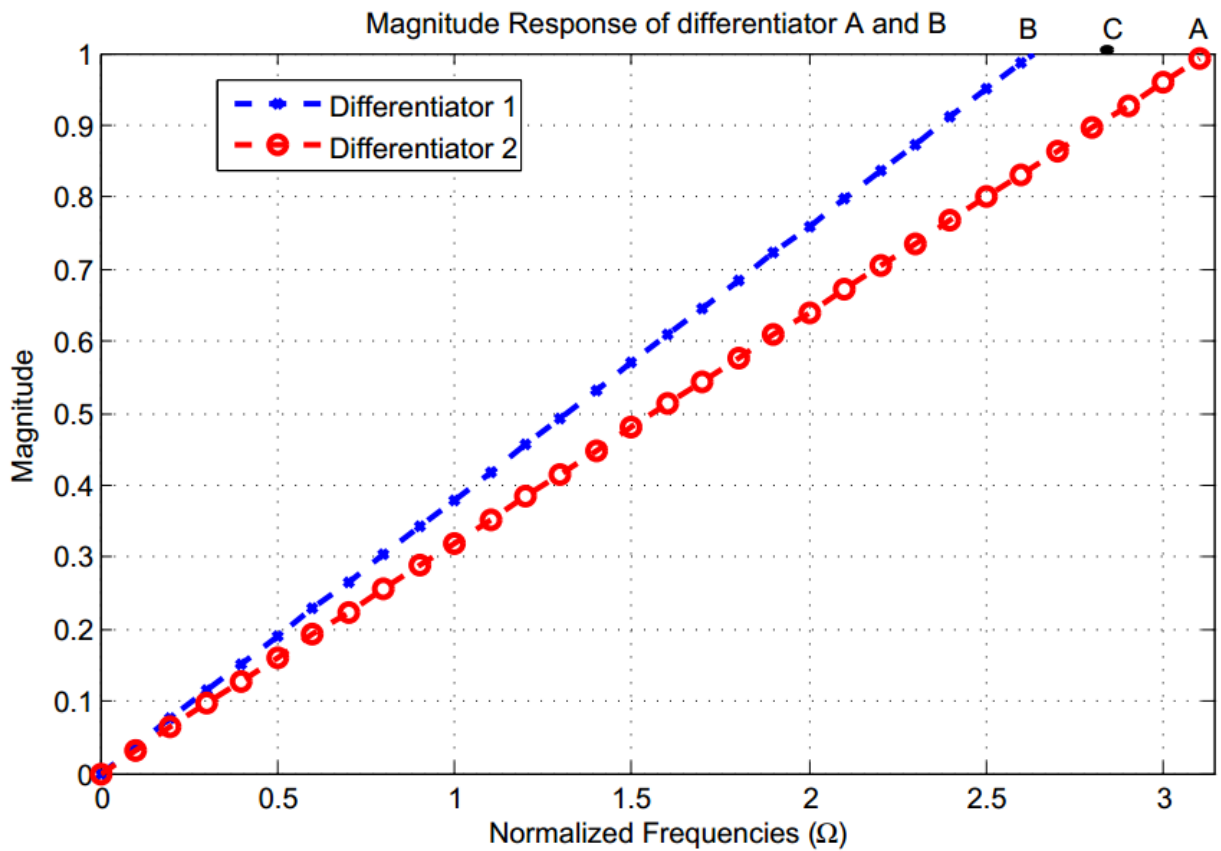


Figure.2.4 Magnitude response of the differentiator A and B

Figure.2.4 shows transfer functions  $H(j\omega)$  of two postulated A and B as a function of frequency, where the operation frequency bands of differentiators A and B are from DC to  $f_A$  and  $f_B$ , respectively, with  $f_A > f_B$ . Of course, an ideal differentiator has the operating frequency range extending from DC to infinity. Here, we limit our discussion to differentiators with finite frequency bands. We assume that the maximum value of transfer functions is unity. The selection of unity is due to the fact that the maximum transfer gain of transmission-line network is unity. The transfer



function of differentiator  $A$  increases linearly from DC to  $f_A$  and it reaches unity at  $f_A$ . The transfer function of differentiator  $B$  increases linearly from DC to  $f_B$  and remains at unity for  $f_B < f < f_A$ . For a signal having a spectral lying between  $f_B$  and  $f_A$ , differentiator  $B$  has no effect of signal differentiation on it. In fact, differentiator  $B$  behaves as an all-pass device for the signal. As a result, for a signal with a spectral extending from DC to  $f_A$ , the effect of differentiator  $B$  on such a signal is a combination of differentiation and all-pass processes. As is shown, the change rate of transfer function of differentiator  $B$  is greater than that of differentiator  $A$  for  $0 \leq f \leq f_B$ . On the other hand, differentiator  $A$  has a wider operating frequency band than differentiator  $B$ . To examine the physical significance of these two differentiators, we take a close look at points  $A$  and  $B$ , where two points are inter-section points caused by a constant frequency line  $f_c$  and transfer functions of differentiators  $A$  and  $B$ . Because the transfer gains of differentiator  $B$  is larger than that of differentiator  $A$ , (2) reveals that the time constant of differentiator  $B$  is larger than that of differentiator  $A$ . This indicates that differentiator  $B$  has a slower response than differentiator  $A$  in the time-domain consideration. However, since the transfer gain of differentiator  $B$  at the frequency  $f_c$  is larger than that of differentiator  $A$ , differentiator  $B$  produces larger output (in magnitude) than differentiator  $A$ . For the extremity that  $f_A$  approaches infinity, the time constant of the differentiator approximates zero. On the other hand, if  $f_B$  approaches zero, the differentiator behaves like an all-pass device. Both the time constant and amplitude response dictate the circuit behaviour of a microwave differentiator.

## 2.4 Time Constant of an Integrator

An integrator formed by an inverted operational amplifier and a serial resistor capacitor circuit, where  $V_i(\omega)$  and  $V_o(\omega)$  are the input and output of the circuit, respectively, and  $\omega$  is the signal angular frequency. The transfer function of the integrator in the frequency domain is defined as the ratio of  $V_o(\omega)$  to  $V_i(\omega)$  and is given as follows:

$$H(j\omega) = \frac{V_o(\omega)}{V_i(\omega)} = -\frac{1}{j\omega RC} \quad 2.19$$

Notice that the transfer function is inversely proportional to the angular frequency  $\omega$  of the signal. As a result, an integrator is treated as a low pass filter.

We define the time constant of an integrator as follows:

$$\tau_i = \frac{1}{K_i} = \frac{1}{|H_{int}(j\omega)|\omega} \quad 2.20$$

$$\tau_i = \frac{1}{K_i} = \frac{1}{|H_{int}(e^{j\Omega})|\Omega}$$

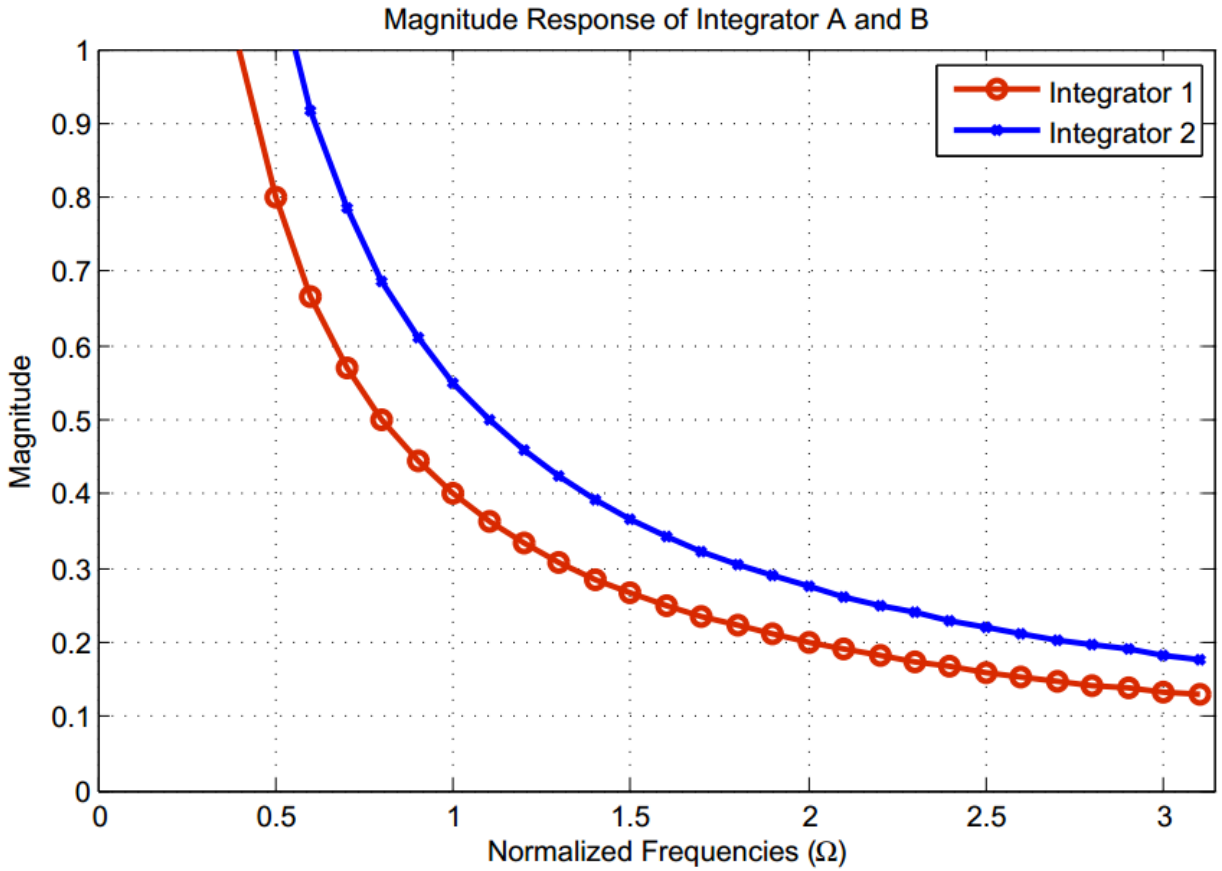


Figure.2.5 Magnitude response of proposed integrator with different time constant

The time-constant combined with the magnitude response describes the system behaviour of a digital integrator and differentiator. The time constant determines the transient behaviour of an integrator in the time-domain consideration. Equation (2) indicates that the multiplication of the transfer function by angular frequency of an integrator is a constant value. Therefore, in order to vary the time constant of the integrator, it is required to change its transfer function.

Such type of designs may be more powerful to design more accurate digital Proportional-Derivative (PD), Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers in control systems. Sometimes, it is also possible that the few designs of non-unity proportional constant may be more accurate compared to the existing designs of unity proportional constant. Therefore, the more accurate design of recursive digital differentiator and integrator for unity proportional constants may be derived by using the non-unity proportional constant design with a constant multiplier.

## 2.5 Formulation of the Transfer Function in terms of T-Parameter

The S matrix is a very convenient way to describe an n-port in terms of waves. It is very well adapted to measurements and simulations<sup>6</sup>. However, it is not well suited to for characterizing the response of a number of cascaded 2-ports. A very straightforward manner for the problem is possible with the T matrix (transfer matrix), which directly relates the waves on the input and on the output.

The transfer function of a cascaded network can be found by multiplying the chain scattering matrices of the components composing the network. The chain scattering parameters  $T_{mn}, m, n = 1,2$  of a two-port network are defined by assuming the waves  $V_1^+$  and  $V_2^-$  at port 1 in Fig. 2.2 are dependent variables, and the waves and at port 2 are independent variables[23]. The T matrix (transfer matrix), which directly relates the waves on the input and on the output, is defined as:

$$\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix} \quad 2.5$$

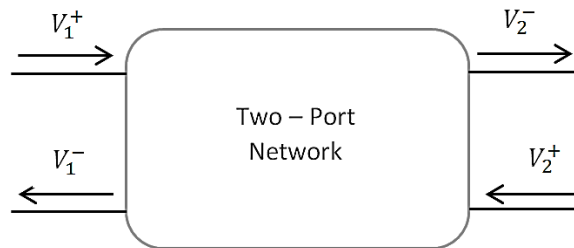


Figure.2.1. Two-port network

As the transmission matrix (T matrix) simply links the in and outgoing waves in a way different from the S matrix, one may convert the matrix elements mutually. The chain scattering matrix can be found from the scattering matrix in the following way [24-26]:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{21} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix} \quad 2.6$$

Let the length of all stubs and transmission-line sections be  $l = \lambda_o/4$ , where  $\lambda_o$  is the wavelength of the lines at the normalizing angular frequency  $\omega_o$ . In other words, the electrical length of all components is  $90^\circ$  at the normalizing frequency.

<sup>6</sup>[www.microwaves101.com/encyclopedia/sparameters.cfm](http://www.microwaves101.com/encyclopedia/sparameters.cfm)

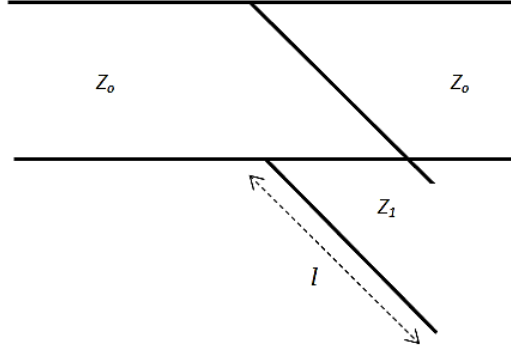


Figure.2.2 Open-circuited stub

Given the impedance of an open-circuited stub to be  $Z_1$ , shown in Fig. 2, we find that its chain scattering parameters are as follows:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{o.c.} = \begin{bmatrix} 1 + j \frac{Z_0}{2Z_1} \tan(\beta l) & j \frac{Z_0}{2Z_1} \tan(\beta l) \\ -j \frac{Z_0}{2Z_1} \tan(\beta l) & 1 - j \frac{Z_0}{2Z_1} \tan(\beta l) \end{bmatrix} \quad 2.7$$

where,  $Z_0$  is the reference characteristic impedance and is the propagation constant. The reference planes for both ports are at the intersection of the stub and the reference transmission line.

Let  $\omega$  be the angular frequency and be the propagation delay caused by the length. All the terms  $j \tan(\beta l) = j \tan(\omega\tau)$  can be represented in a new form by using  $D^{-1} = e^{j\omega\tau}$ , which can be considered as a unit of delay, i.e.,

$$j \tan(\omega\tau) = \frac{e^{j\omega\tau} - e^{-j\omega\tau}}{e^{j\omega\tau} + e^{-j\omega\tau}} = \frac{D - D^{-1}}{D + D^{-1}} \quad 2.18$$

Consequently, we have

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{o.c.} = \frac{1}{1 + D^{-2}} \begin{bmatrix} (1 + a) + (1 - a)D^{-2} & a - aD^{-2} \\ -a + aD^{-2} & (1 - a) + (1 + a)D^{-2} \end{bmatrix} \quad 2.9$$

Where,  $a = Z_0/2Z_1$

If the stub is short circuited, its chain scattering parameters can be expressed as follows:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{s.c.} = \frac{1}{1 - D^{-2}} \begin{bmatrix} (1 + a) - (1 - a)D^{-2} & a + aD^{-2} \\ -a - aD^{-2} & (1 - a) - (1 + a)D^{-2} \end{bmatrix} \quad 2.10$$

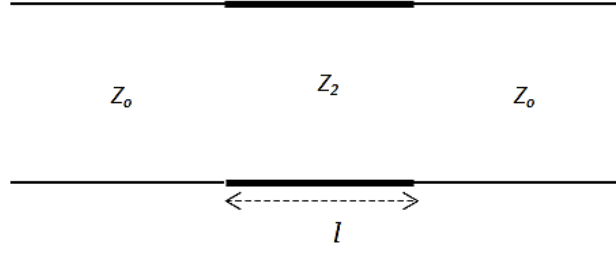


Figure.2.3. Transmission-line section

By the same token, the chain scattering parameters of a transmission line section with impedance  $Z_2$ , shown in Fig. 2.3, can be converted to functions in the  $Z$  domain as follows:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{T.L.S.} = \frac{1}{D^{-1}(1-\Gamma^2)} \begin{bmatrix} 1-\Gamma^2 D^{-2} & -(\Gamma-\Gamma D^{-2}) \\ \Gamma-\Gamma D^{-2} & -\Gamma^2 + D^{-2} \end{bmatrix} \quad 2.12$$

where,  $\Gamma_m = \frac{Z_2 - Z_1}{Z_2 + Z_1}$ . The reference planes for both ports are at two connecting points between  $Z_2$  and  $Z_0$ .

By cascading open-circuited/short-circuited stubs and transmission line sections to form a network, the overall chain scattering matrix of the network can be found by the multiplications of the chain scattering matrix of each component, i.e.,

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{Network} = \prod_{i=1}^N \begin{bmatrix} T_{11}^i & T_{12}^i \\ T_{21}^i & T_{22}^i \end{bmatrix} \quad 2.12$$

where,  $N$  is the number of the components, and  $T_{11}^i, T_{12}^i, T_{21}^i, T_{22}^i$  and are the matrix elements representing the  $i_{th}$  component.

Assume the network is composed of  $L$  open-circuited stubs,  $K$  short-circuited stubs, and  $M$  transmission-line sections. The fact that the numerators of all the matrix elements in (7) - (9) have the form of  $\alpha_0 + \alpha_1 D^{-2}$  ( $\alpha_0$  and  $\alpha_1$  are real numbers) leads to the following:

$$T_{11}(D) = \frac{\sum_{i=0}^N \alpha_i D^{-2i}}{\prod_{k=1}^K (1 + D^{-2}) \prod_{l=1}^L (1 - D^{-2}) \prod_{m=1}^M (D^{-1}(1 - \Gamma_m^2))} \quad 2.13$$

where all  $\alpha_i$ 's are real numbers and can be determined by the characteristic impedances of both stubs and transmission-line sections. In addition, the term  $(I + D^{-2})$  comes from each open circuited stub, the term  $(I - D^{-2})$  comes from each short-circuited stub, and  $D^{-1}(1 + \Gamma_m^2)$  comes from the  $m_{th}$  transmission-line section.

When the output port of the network uses matched termination, we have  $V_2^+ = 0$  in Fig. 1. The transfer function, denoted as  $T(D)$ , can then be obtained by the inverse of  $T_{11}(D)$ , i.e.,

$$T(D) = \frac{V_2^-}{V_1^+} \Big|_{V_2^+ = 0} = \frac{1}{T_{11}(D)} = \frac{\prod_{k=1}^K (1 + D^{-2}) \prod_{l=1}^L (1 - D^{-2}) \prod_{m=1}^M (D^{-1}(1 - \Gamma_m^2))}{\sum_{i=0}^N \alpha_i D^{-2i}} \quad 2.14$$

To make (12) in a form proper for the design purpose, we set  $z = D^{-2}$ , which corresponds a scaling by two on the frequency axis. The transfer function is then modified as follows:

$$T(z) = T(D) \Big|_{z=D^{-2}} = \frac{z^{-M/2} \prod_{k=1}^K (1 + z^{-1}) \prod_{l=1}^L (1 - z^{-1})}{\sum_{i=0}^N A_i z^{-i}} \quad 2.15$$

where,  $A_i = \frac{\alpha_i}{\prod_{m=1}^M (D^{-1}(1 - \Gamma_m^2))}$  are functions of the characteristic impedances of both stubs and

transmission-line sections. Equation (13) reveals that  $T(z)$  has zeros at  $z = -1$  (or the normalizing frequency  $\omega_o$ ), which are contributed by the open-circuited stubs, and zeros at dc, which are contributed by the short-circuited stubs. If the zeros contributed from the stubs are removed from  $T(z)$ , the remaining part of the transfer function is recognized as an AR process multiplied by a term of  $z^{-M/2}$  corresponding to some delay. We express the AR process with the function and we get

$$T_{AR}(z) = \frac{1}{\sum_{i=0}^N A_i z^{-i}} \quad 2.16$$

Since the frequency response of the AR process is uniquely determined by the coefficients  $A_i$  and these coefficients are determined by the characteristic impedances of both stubs and transmission-line sections, we could adjust the impedances of these components so that  $T_{AR}(z)$  approximates a proposed AR process.

Transmission line Configuration	T- parameter
Serial Transmission line	$\frac{1}{z^{-1/2}(1-\Gamma^2)} \begin{bmatrix} 1-\Gamma^2 z^{-1} & -(\Gamma-\Gamma z^{-1}) \\ \Gamma-\Gamma z^{-1} & -\Gamma^2+z^{-1} \end{bmatrix}$ where, $\Gamma_m = \frac{Z_2-Z_1}{Z_2+Z_1}$
Shunt-Short stub	$\frac{1}{1-z^{-1}} \begin{bmatrix} (1+a)-(1-a)z^{-1} & a+az^{-1} \\ -a-az^{-1} & (1-a)-(1+a)z^{-1} \end{bmatrix}$ where, $a = \frac{Z_0}{2Z_1}$
Shunt-Open stub	$\frac{1}{1+z^{-1}} \begin{bmatrix} (1+a)+(1-a)z^{-1} & a-az^{-1} \\ -a+az^{-1} & (1-a)+(1+a)z^{-1} \end{bmatrix}$ where, $a = \frac{Z_0}{2Z_1}$

Table I: Basic Transmission-Line Element's Chain Scattering-Parameter Matrices

## 2.6 Optimization Techniques

Optimization techniques<sup>7</sup> are used to find a set of design parameters,  $x = \{x_1, x_2, \dots, x_n\}$ , that can in some way be defined as optimal. In a simple case this might be the minimization or maximization of some system characteristic that is dependent on  $x$ . In a more advanced formulation the objective function,  $f(x)$ , to be minimized or maximized, might be subject to constraints in the form of equality constraints,  $G_i(x) = 0$  ( $i = 1, \dots, m$ ); inequality constraints,  $G_i(x) \leq 0$  ( $i = m_e + 1, \dots, m$ ); and/or parameter bounds,  $x_l, x_u$ .

A General Problem (GP) description is stated as

$$\min_x f(x),$$

subject to

$$G_i(x) = 0 \quad i = 1, \dots, m_e$$

$$G_i(x) \leq 0 \quad i = m_e + 1 \dots m;$$

where  $x$  is the vector of length  $n$  design parameters,  $f(x)$  is the objective function, which returns a scalar value, and the vector function  $G(x)$  returns a vector of length  $m$  containing the values of the equality and inequality constraints evaluated at  $x$ .

There are a number of optimization techniques present. Selection of an optimization algorithm depends upon type of problem. The non-traditional techniques are:

### 2.6.1 Stimulated Annealing

<sup>7</sup><http://www.mathworks.com/help/optim/ug/optimization-theory-overview.html>

Simulated annealing is a generic probabilistic metaheuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. It is often used when the search space is discrete (e.g., all tours that visit a given set of cities). For certain problems, simulated annealing may be more efficient than exhaustive enumeration provided that the goal is merely to find an acceptably good solution in a fixed amount of time, rather than the best possible solution [27].

### **2.6.2 Genetic algorithm**

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued [28].

### **2.6.3 Pattern Search**

A pattern is a set of vectors  $\{v_i\}$  that the pattern search algorithm uses to determine which points to search at each iteration. The set  $\{v_i\}$  is defined by the number of independent variables in the objective function,  $N$ , and the positive basis set. Two commonly used positive basis sets in pattern search algorithms are the maximal basis, with  $2N$  vectors, and the minimal basis, with  $N+1$  vectors. [29] Pattern search (PS) is a family of numerical optimization methods that do not require the gradient of the problem to be optimized. Hence PS can be used on functions that are not continuous or differentiable.

In this thesis, we have used pattern search optimization due to its fast convergence.

The next chapter describes the details of designing and analysis of stable and optimized discrete-time differentiators and integrators.



## Chapter 3

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### **Designing of Discrete Time Differentiators and Integrators**

The focus of this chapter is the designing of digital differentiator and integrator. Here, the basic equations governing differentiators and integrators are derived from optimization algorithm. The designed system function is analysed in software tool like Agilent SystemVue. Implementation of direct form I and direct form II using MATLAB are realised.

### 3.1 Introduction

A digital IIR filter can be defined with sets of recursive difference equations of the form

$$y[n] = a_0x[n] + a_1x[n-1] + \dots + a_{M-1}x[n-(M-1)] - b_1y[n-1] + b_2y[n-2] - \dots - b_Ny[n-N] \quad 3.1$$

where  $x[n]$  is the digital input and  $y[n]$  is the digital output of the filter. The parameters specify  $H(z)$ , the Z-transform of an impulse response  $h(n)$ . The output of IIR is the convolution of the input with  $h(n)$ .

The system function for linear, time-invariant, causal digital filter can be expressed in the Z-domain in the form:

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{i=0}^N b_i z^{-i}}{1 + \sum_{j=1}^N a_j z^{-j}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_M z^{-N}} = K \frac{\prod_{i=1}^N (z - z_i)}{\prod_{i=1}^N (z - p_i)} \quad 3.2$$

The coefficients of the polynomial are  $a_i$  and  $b_i$ . The zeros and poles of the factored form are  $z_i$  and  $p_i$ , respectively.

For optimization of the discrete time system function prototype, an error function is defined as:

$$Err = Min \left[ \int_0^{\pi} (H_{Ideal}(z) - H_{New}(z))^2 d\Omega \right] \quad 3.3$$

Error function is basically an objective function for optimization algorithm, which is calculated by integrating the error between ideal response and proposed system function response. Sampling period 'T' of the filter is normalised to unity (i.e. T = 1) for the frequency plots and Nyquist frequency is taken as  $\pi$  radians. The integration is performed over normalized frequency (frequency vector)  $\Omega$  over the range of 0 to  $\pi$ .

### 3.1 Digital Differentiator

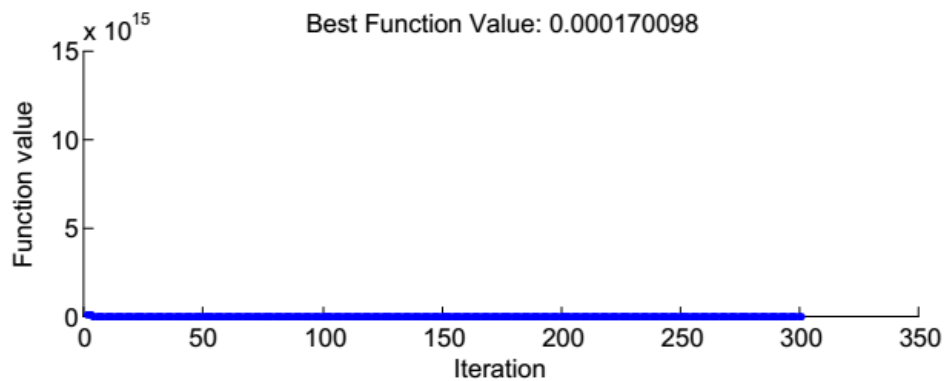
#### 3.1.1 Design of Stable Digital Differentiator:

An Infinite impulse response system function is selected for designing first order discrete time differentiator. The proposed digital differentiator prototype is given as:

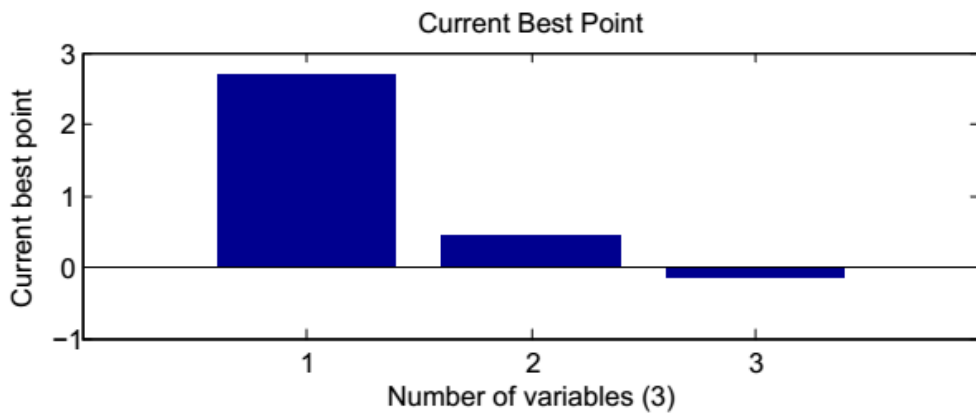
$$H_{Diff}(z) = \frac{1 - z^{-1}}{x_1 + x_2 z^{-1} + x_3 z^{-2}} \quad 3.4$$

The defined system function (3.3) has a zero at  $z = 1$  i.e. at DC value. The zero location is selected in order to correlate the discrete time system function with transfer function of microstrip. The poles of system function are variables determined by multivariable optimization algorithm scheme<sup>8</sup>. The pole location is selected between the range of  $0 \leq z \leq 1$ . So that system function obtained after optimization is stable.

The analysis software is inserted into an optimisation loop and used to calculate responses for successive sets of parameters, which are then compared to the desired response until the designs are met its requirements. Upon using optimization algorithm using objective function defined as (3); the error between ideal and proposed differentiator is given by:  $Err = 1.7 \times 10^{-4}$ .



**Figure 3.1:** Optimization process Err function plot v/s iteration using Pattern search



<sup>8</sup>Optimization Toolbox™ User's Guide, Matlab, 2012b

Figure 3.2: Optimized variable coefficient best point plot result using Pattern search

The optimized value of system function coefficient obtained by pattern search optimization is tabulated in Table 3.1.

First order discrete time differentiator		
S.No.	Coefficient	Value
1	$x_1$	2.687
2	$x_2$	0.429
3	$x_3$	-0.134

Table.3.1: Optimized coefficient value of first order differentiator

System function obtained for differentiator obtained after substituting the optimized coefficient is given by:

$$H_{Diff\_New}(z) = \frac{1 - z^{-1}}{2.687 + 0.429z^{-1} - 0.1340z^{-2}} \quad 3.5$$

### 3.1.2 Frequency Response of Digital Differentiator Using Matlab:

Frequency response of discrete time differentiator can be obtained by substituting in (3.4) with the following relation:

$$z = e^{j\Omega} \quad 3.6$$

where,  $\Omega$  is the frequency angel and  $0 \leq \Omega \leq \pi$ .

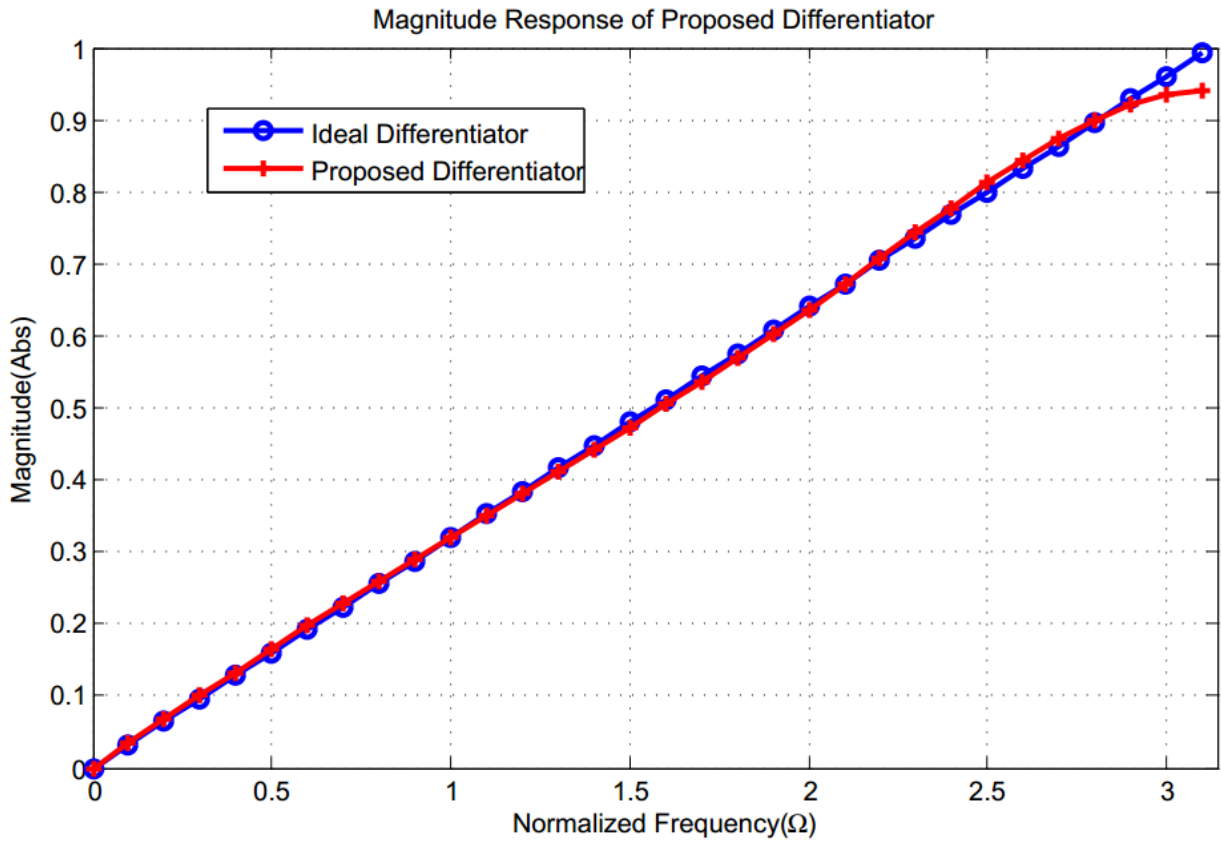


Figure.3.3: Frequency response of proposed digital differentiator and ideal response for sampling frequency normalised to  $\pi$  radian

The ideal differentiator is assumed to have precisely linear amplitude response for all frequency as shown in figure3.3. The proposed differentiator (3.5) is proper to be adopted as the system function of a wide-band differentiator

### 3.1.3 SystemVue Simulation of Designed Digital Differentiator:

For extensive analysis of given filter function under other design criteria such as stability, impulse response, step response and group delay is also carried out in Agilent SystemVue.

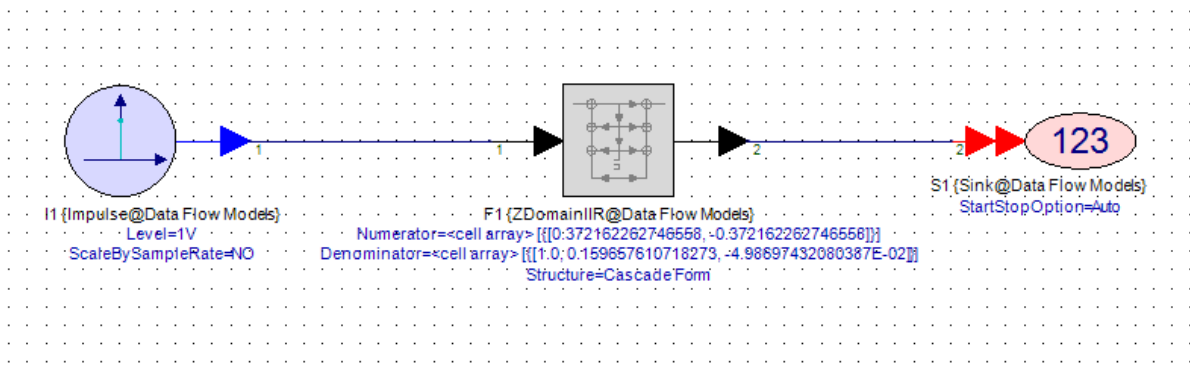


Figure.3.4: Schematic of digital differentiator in SystemVue

### 3.1.4 Frequency and Pole Zero plot in SystemVue:

The frequency response obtain from SystemVue are found to be in agreement with MATLAB response. The pole zero plot is shown in figure.3.5. The poles obtained are  $p_1 = 0.1573$  and  $p_2 = -0.3170$  which shows the stability of designed discrete time filter.

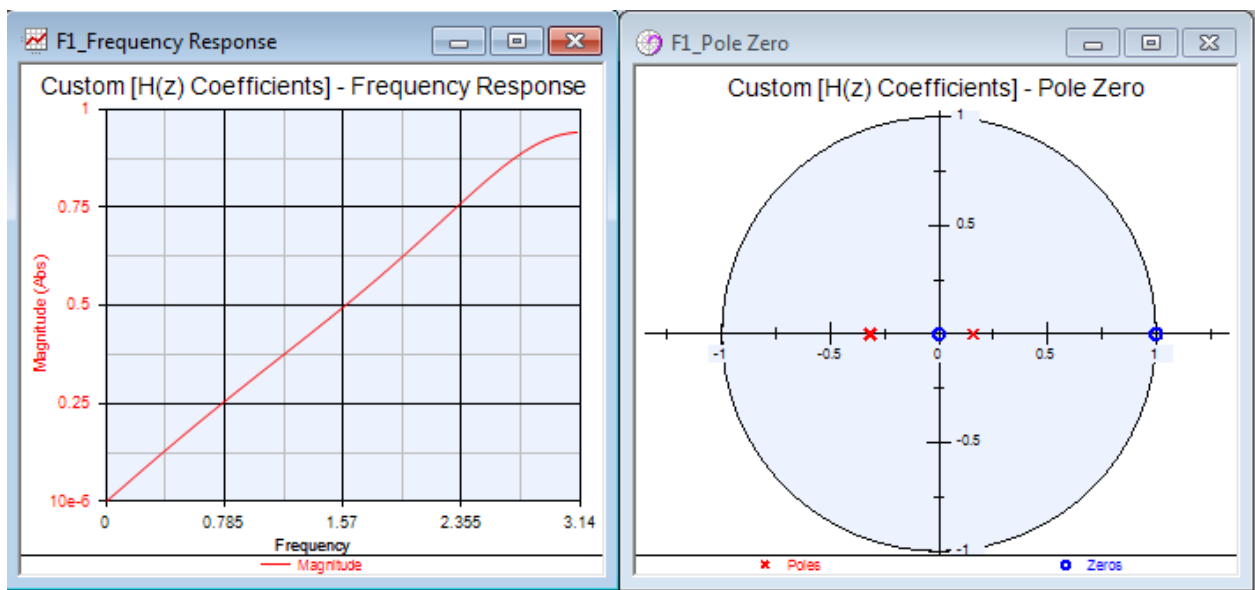


Figure.3.5: Frequency and stable Pole zero plot of proposed digital differentiator in SystemVue

### 3.1.5 Impulse Response:

The impulse response and step response to proposed differentiator is shown in figure 3.6

Designing is data type floating point filter that can be used for HDL code generation for implementation in a FPGA

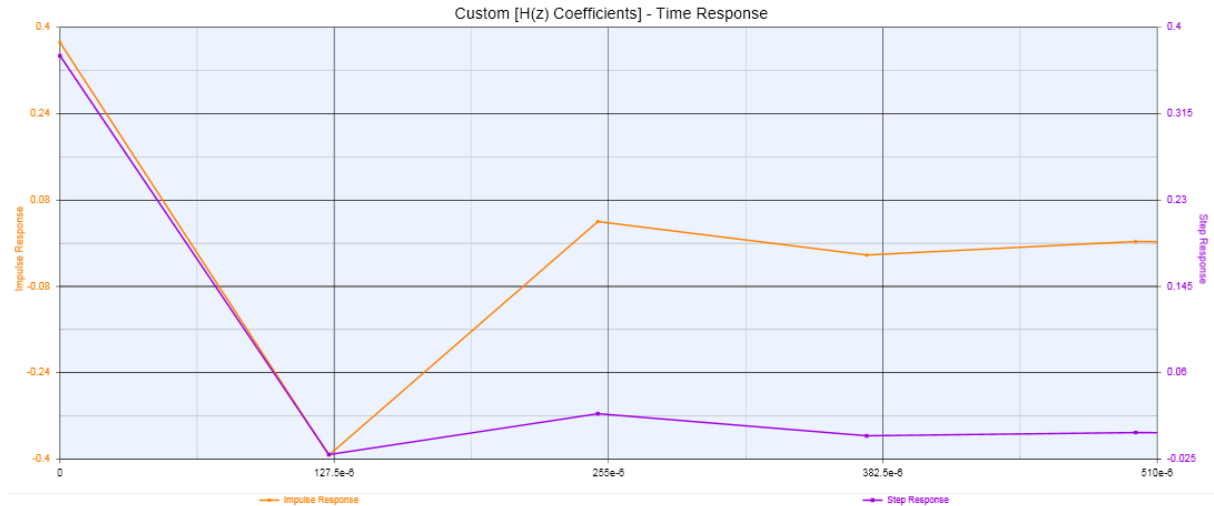


Figure.3.6: Impulse response and step response of designed digital differentiator

### 3.1.6 Implementation Direct form realization of digital differentiator:

The filter (differentiator) obtained can be realised in either the direct form I or direct form II. We prefer to realize a circuit that would require a minimum number of unit delays that is equal to the order of the filter. A realization that contains the minimum number of delays is defined as canonical realization.

## Filter Realizations

Once we have obtained the transfer function of an FIR or IIR filter that approximates the desired specifications in the frequency domain or the time domain, our next step is to investigate as many filter structures as possible, before we decide on the optimal or suboptimal algorithm for actual implementation or application. A given transfer function can be realized by several structures or what we will call “circuits,” and they are all equivalent in the sense that they realize the same transfer function under the assumption that the coefficients of the transfer function have infinite precision. But in reality, the algorithms for implementing the transfer function in hardware depend on the filter structure chosen to realize the transfer function. We must also remember that the real hardware has a finite number of bits representing the coefficients of the filter as well as the values of the input signal at the input. The internal signals at the input of multipliers and the signals at the output of the multipliers and adders also are represented by a finite number of bits. The effect of rounding or truncation in the addition and multiplications of signal values depends on, for example, the type of representation of binary numbers, whether they are in fixed form or floating form, or whether they are in sign magnitude or two-complementary form. The effects of all these finite

values for the number of bits used in hardware implementation is commonly called “finite word length effects,”

It is true that a real hardware can be programmed to implement a large number of algorithms, by storing the data that represent the input signals and coefficients of the filter in a memory. But remember that it can implement an algorithm only in the time domain, whereas programming it to find the frequency response is only a simulation. Three algorithms in the time domain that are the recursive algorithm, convolution sum, and the FFT algorithm. It is the difference equations describing these algorithms that have to be implemented by real digital hardware.[30]

But when the two algorithms have to be programmed and implemented by hardware devices, the results would be very different and the accuracy of the resulting output, the speed of the execution, and the throughput, and other factors would depend not only on the finite word length but also on so many other factors, including the architecture of the DSP chip, program instructions per cycle, and dynamic range of the input signal.

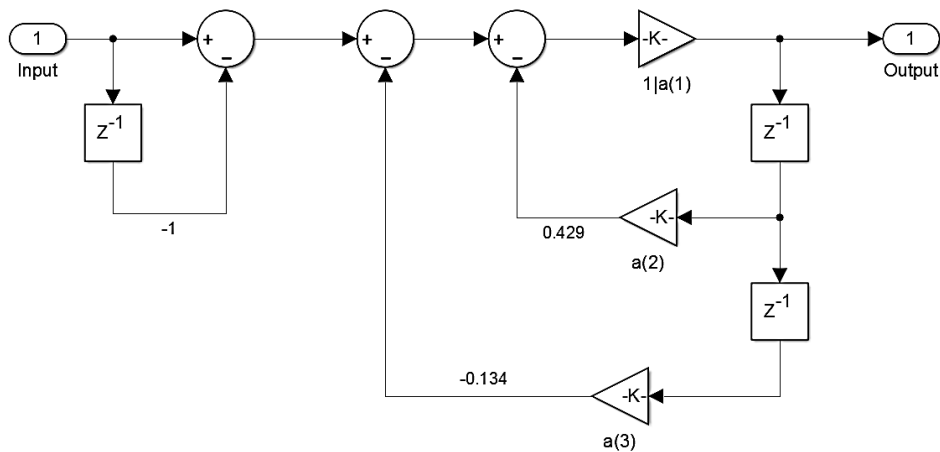


Figure.3.7 Direct form I realization of digital differentiator by MATLAB

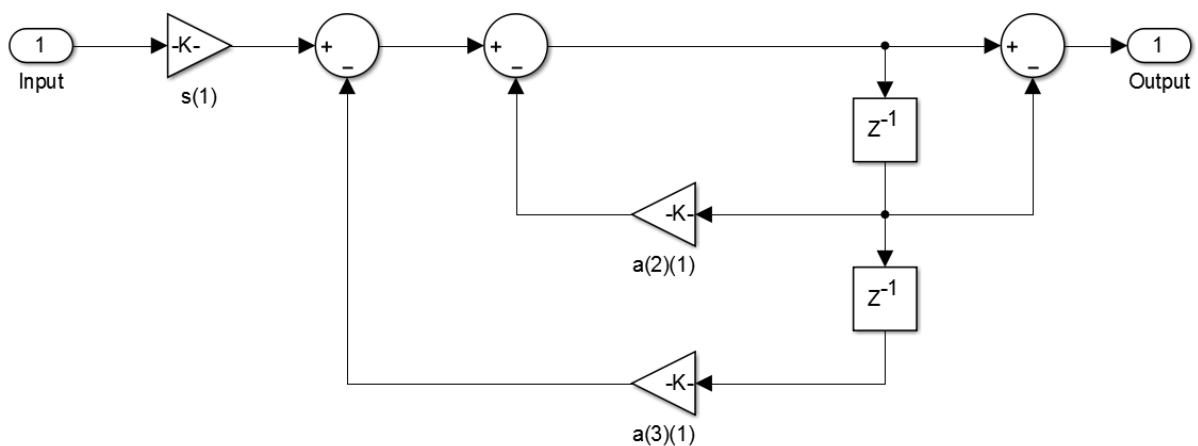




Figure.3.8 Direct form II realization of digital differentiator by MATLAB

The above differentiator can be implemented on hardware but there is practical limitation of these circuits. In order to make differentiator to work in microwave frequency. Microstrip is suitable candidate for these frequency applications.

### 3.2 Designing of Different Time Constant Differentiator

An Infinite impulse response system function is selected for designing first order discrete time differentiator. The proposed digital differentiator prototype is given as:

$$H_{Diff}(z) = \frac{(1 - z^{-1})}{x_1 + x_2 z^{-1} + x_3 z^{-2} + x_4 z^{-3} + x_5 z^{-4}} \quad 3.7$$

This System function has a zero at  $z = 1$  i.e. at DC value. The zero location is selected in order to correlate the discrete time system function with transfer function of microstrip. The poles of system function are variables determined by multivariable optimization algorithm scheme. The pole location is selected between the range of  $0 \leq z \leq 1$ . So that system function obtained after optimization is stable.

Upon using optimization, the Error between Ideal and Proposed Differentiator is given as  $Err = 6.7 \times 10^{-7}$

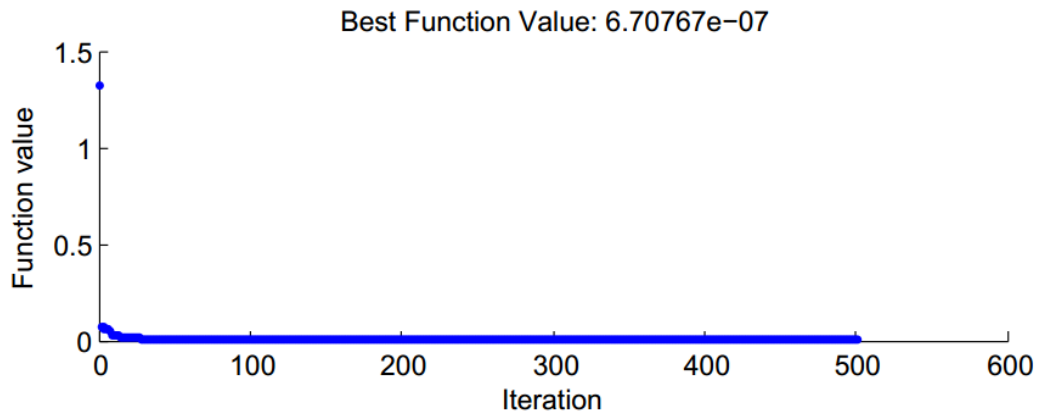


Figure.3.11: Optimization process Err function plot v/s iteration using Pattern search

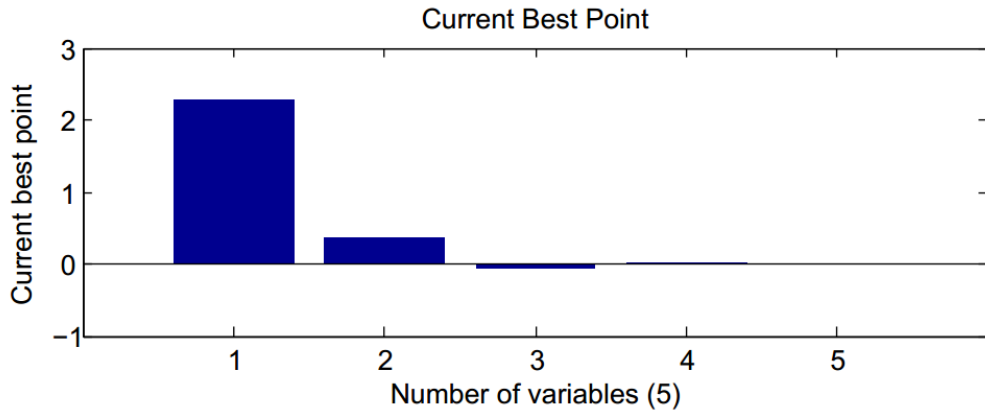


Figure.3.12: Optimized variable coefficient best point plot result using Pattern search

The optimized value of system function array coefficient obtained by pattern search optimization is tabulated in Table 3.1.

First order discrete time differentiator		
S. No.	Coefficient	Value
1	$x_1$	2.289
2	$x_2$	0.391
3	$x_3$	-0.056
4	$x_4$	0.02
5	$x_5$	-0.07

Table.3.2: Optimized coefficient value of first order differentiator

System function obtained for differentiator obtained after substituting the optimized coefficient is given by:

$$H_{Diff\_New}(z) = \frac{1 - z^{-1}}{2.289 + 0.391z^{-1} - 0.056z^{-2} + 0.02z^{-3} - 0.07z^{-4}} \quad 3.8$$

### 3.2.2 Frequency Response of Digital Differentiator Using Matlab:

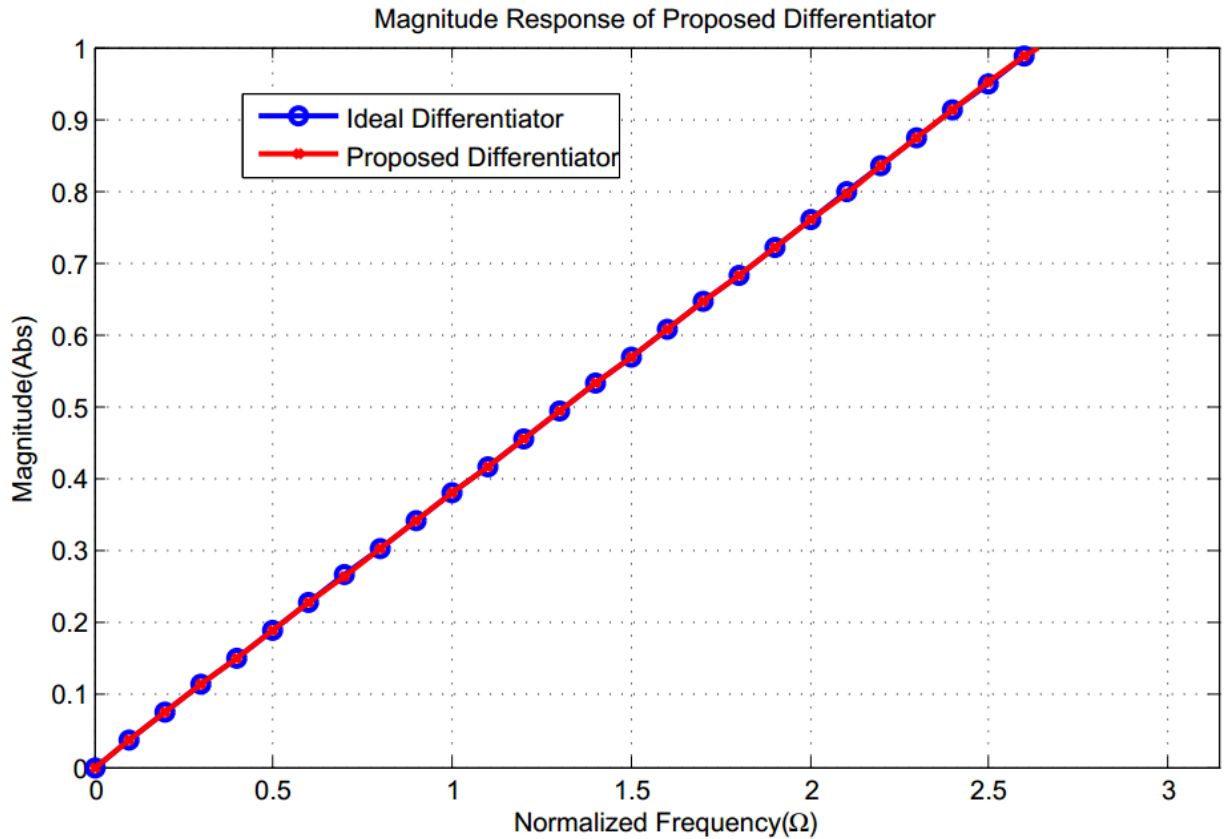


Figure.3.13: Frequency response of proposed differentiator for sampling frequency normalised to  $\pi$  radian

If a differentiator is implemented by using transmission line, the maximum value of the transfer function  $H_{int}(z)$  as 1 for the frequency range  $2.7 \leq \Omega \leq \pi$ . The rest part of the transfer function in the range  $0 \leq \Omega \leq 2.7$  satisfies equation 3.8. Under such circumstance, the circuit thus obtained behaves as a differentiator over the frequency range  $0 \leq \Omega \leq 2.7$ .

### 3.1.3 SystemVue Simulation of Designed Digital Differentiator

For extensive analysis of given filter function under other design criteria such as stability, impulse response, step response and group delay is also carried out in Agilent SystemVue.

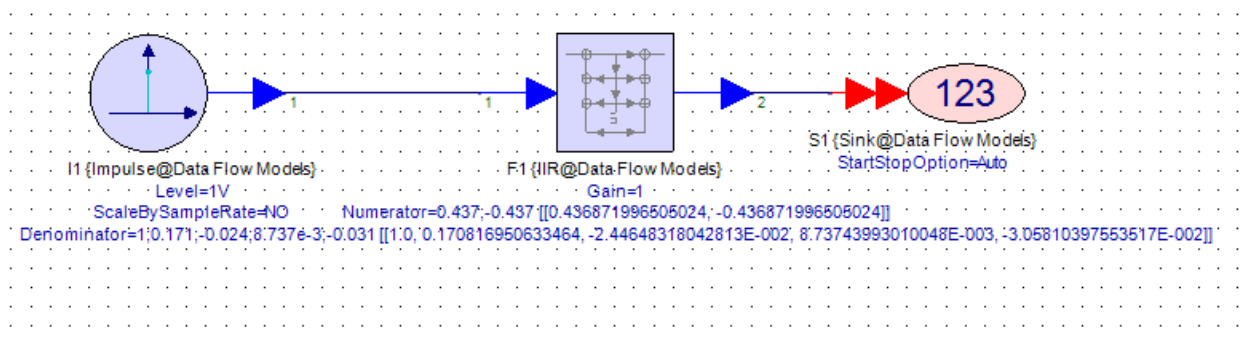


Figure.3.15: Schematic of digital differentiator in SystemVue

### 3.2.4 Frequency and Pole Zero plot in SystemVue

The frequency response obtain from SystemVue are found to be in agreement with MATLAB response. The pole zero plot is shown in figure.3.5. The poles obtained are  $p_1 = -0.4998$ ,  $p_2 = 0.3820$ ,  $p_3 = -0.0265 + 0.3994j$  and  $p_4 = -0.0265 - 0.3994j$  which shows the stability of designed discrete time filter.

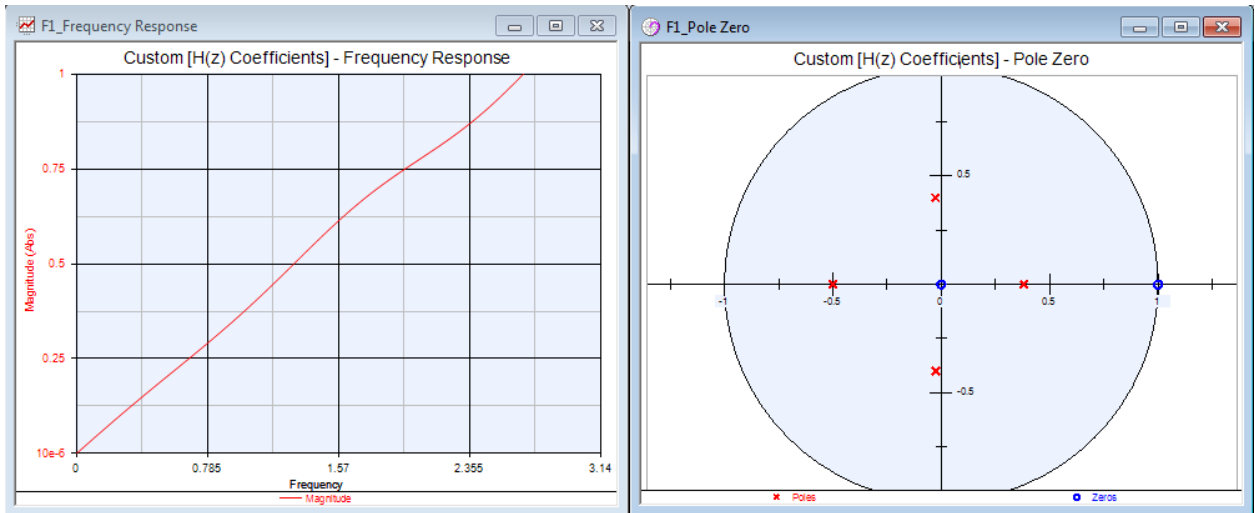


Figure3.16: (a) Frequency response of proposed digital integrator, (b) Pole-Zero Plot using SystemVue

### 3.1.5 Impulse Response:

The impulse response and step response to proposed differentiator is shown in figure 3.6

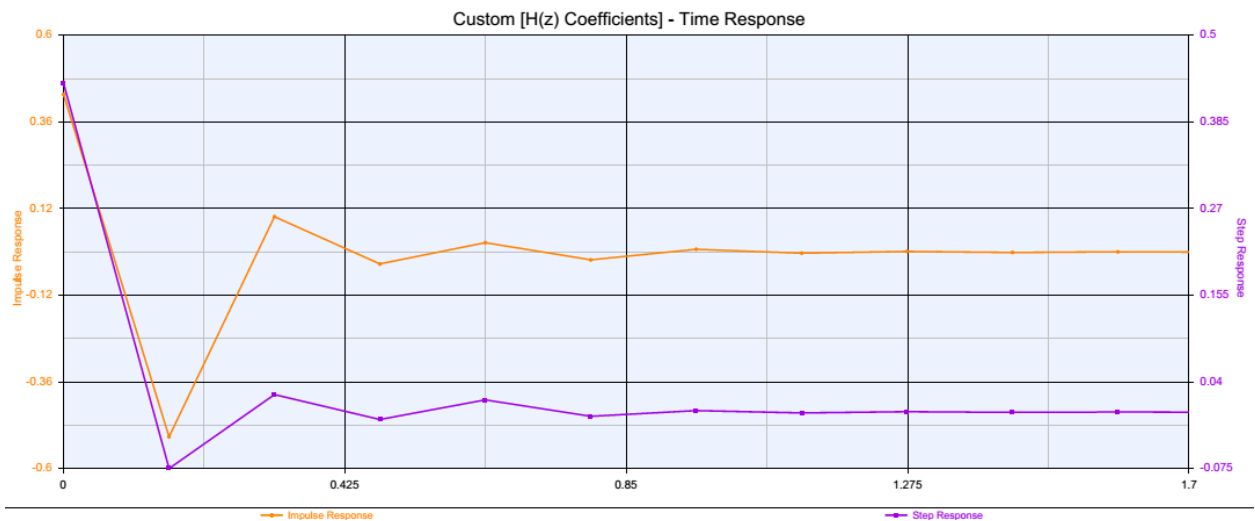


Figure.3.6: Impulse response and step response of designed digital differentiator

### 3.1.6 Implementation Direct form realization of digital differentiator:

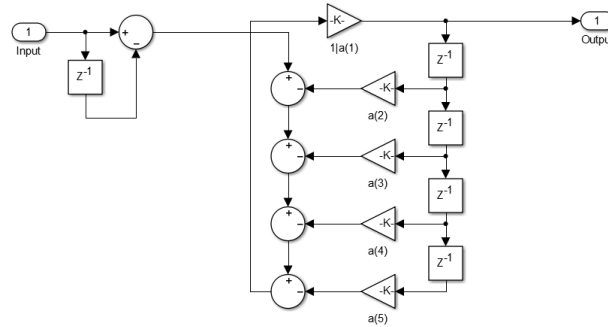


Figure.3.17: Direct form I realization of digital integrator by MATLAB

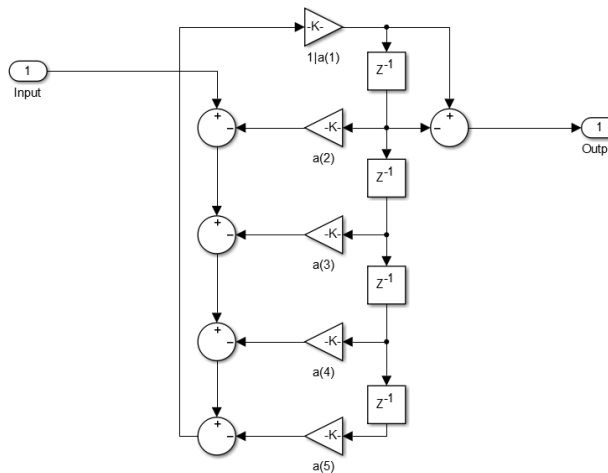


Figure.3.18: Direct form II realization of digital differentiator by MATLAB

## 3.2 Digital Integrator

An Infinite impulse response system function is selected for designing first order discrete time integrator. The proposed digital integrator prototype is given as:

$$H_{int}(z) = \frac{1}{x_1 + x_2 z^{-1} + x_3 z^{-2} + x_4 z^{-3} + x_5 z^{-4} + x_6 z^{-5} + x_7 z^{-6}} \quad 3.9$$

This System function has a zero at  $z = 1$  i.e. at DC value. The poles of system function are variables determined by multivariable optimization algorithm scheme. The pole location is selected between the range of  $0 \leq z \leq 1$ . So that system function obtained after optimization is stable.

Upon using optimization, the Error between Ideal and Proposed integrator is given by:  $Err = 6.18 \times 10^{-6}$ .

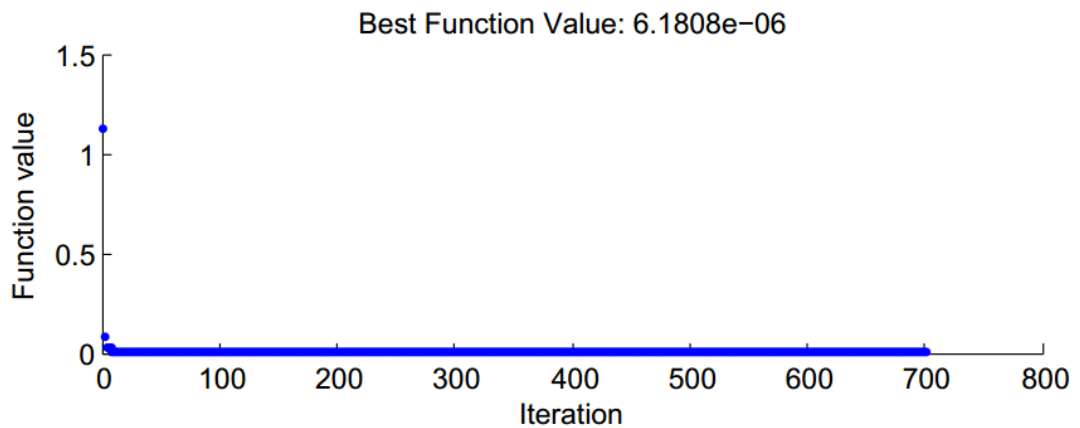


Figure.3.11: Optimization process Err function plot v/s iteration using Pattern search

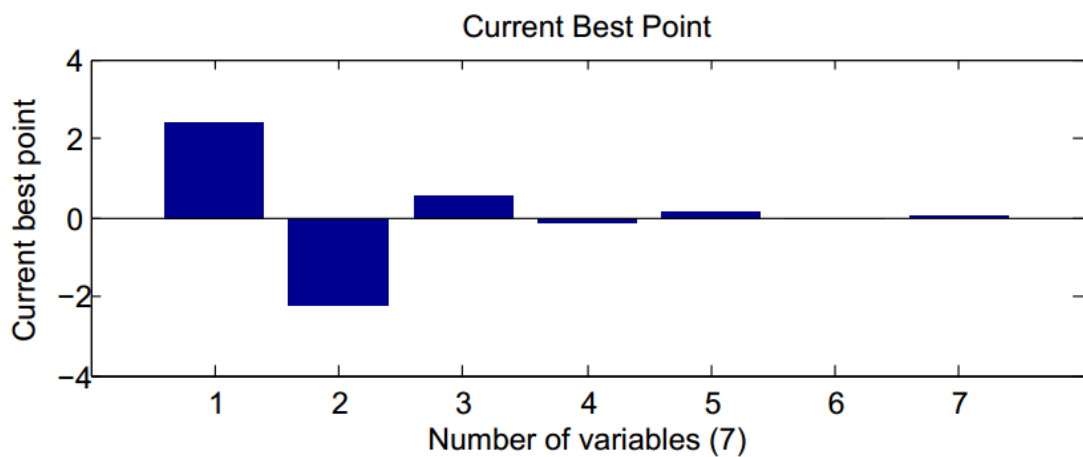


Figure.3.12: Optimized variable coefficient best point plot result using Pattern search

The optimized value of system function coefficient obtained by pattern search optimization is tabulated in Table 3.3.

First order discrete time differentiator		
S. No.	Coefficient	Value
1	$x_1$	2.378

2	$x_2$	-2.221
3	$x_3$	0.524
4	$x_4$	-0.156
5	$x_5$	0.123
6	$x_6$	-0.036
7	$x_7$	0.036

Table.3.3: Optimized coefficient value of first order differentiator

System function obtained for integrator obtained after substituting the optimized coefficient is given by:

$$H_{int}(z) = \frac{1}{2.378 - 2.221z^{-1} + 0.524z^{-2} - 0.156z^{-3} + 0.123z^{-4} - 0.036z^{-5} + 0.036z^{-6}} \quad 3.10$$

### 3.1.2 Frequency Response of Digital Integrator Using Matlab

The ideal integrator is assumed to have amplitude response inversely proportional to all frequency as shown in figure3.13. The proposed integrator (3.10) is proper to be adopted as the system function of a wide-band integrator

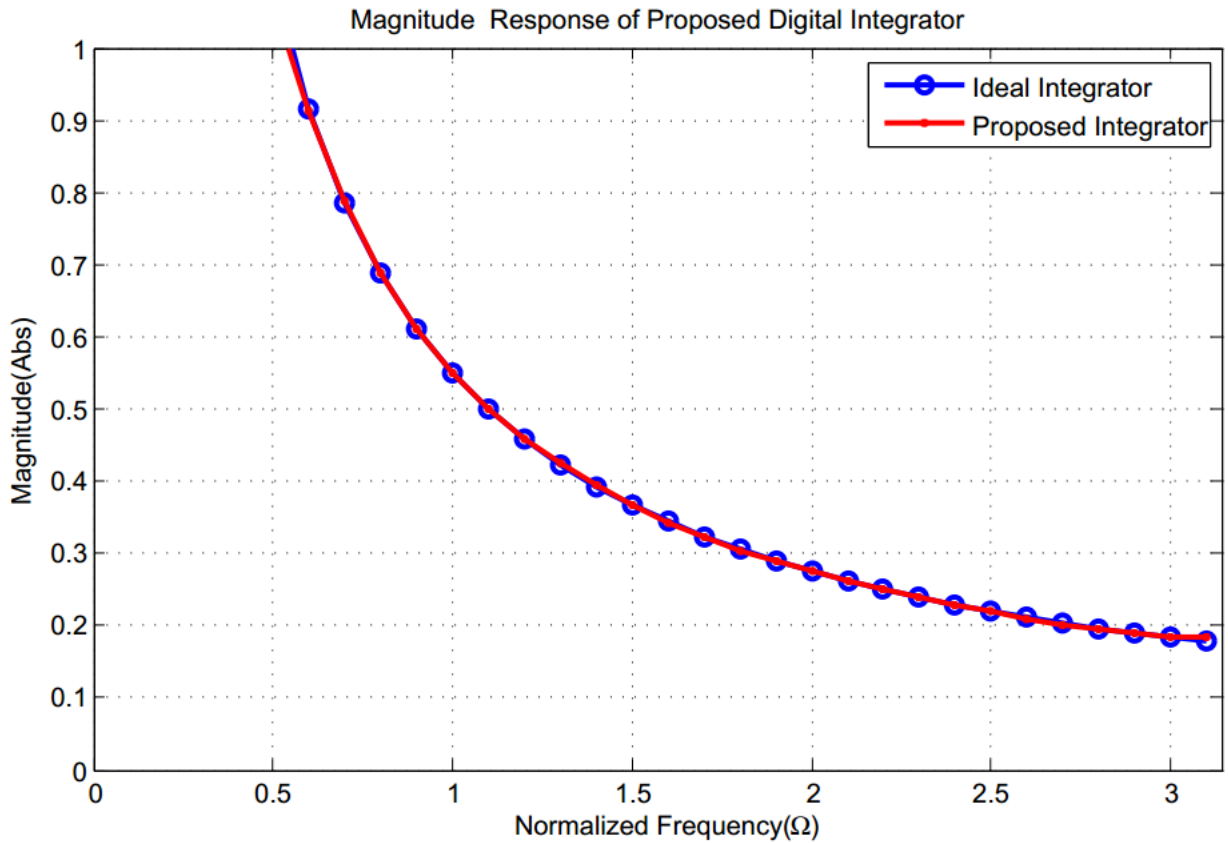


Figure.3.13: Frequency response of proposed integrator for sampling frequency normalised to  $\pi$  radian

If an integrator is implemented by using transmission line, the maximum value of the transfer function  $H_{int}(z)$  as 1 for the frequency range  $0 \leq \Omega \leq 0.6$ . The rest part of the transfer function in the range  $0.5 \leq \Omega \leq \pi$  satisfies equation 3.10. Under such circumstance, the circuit thus obtained behaves as an integrator over the frequency range  $0.5 \leq \Omega \leq \pi$ .

### 3.1.3 SystemVue Simulation of Designed Digital Integrator

For extensive analysis of given filter function under other design criteria such as stability, impulse response, step response and group delay is also carried out in Agilent SystemVue.

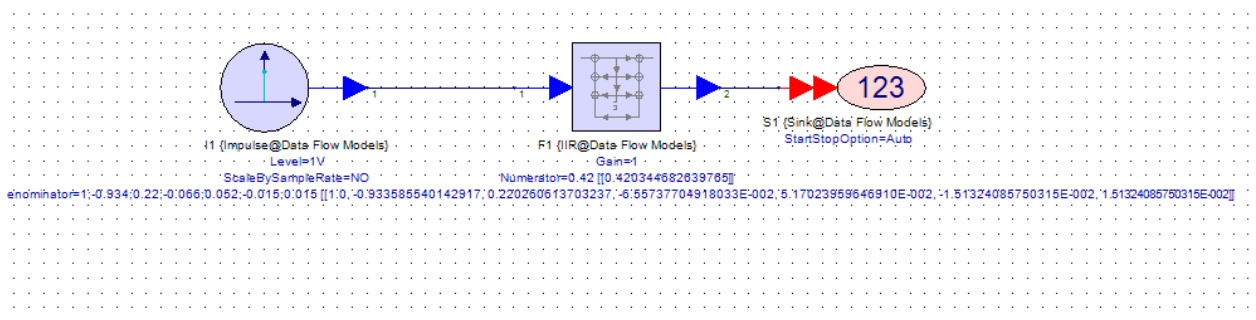




Figure.3.15: Schematic of digital integrator in SystemVue

### 3.2.4 Frequency and Pole Zero plot in SystemVue

The frequency response obtain from SystemVue are found to be in agreement with MATLAB response. The pole zero plot is shown in figure.3.5. The poles obtained are  $p_1 = 0.6416 + 0.1870j$ ,  $p_2 = 0.6416 - 0.1870j$ ,  $p_3 = -0.2974 + 0.3109i$ ,  $p_4 = -0.2974 - 0.3109i$ ,  $p_5 = 0.1226 + 0.4099j$  and  $p_6 = 0.1226 - 0.4099j$  which shows the stability of designed discrete time filter.

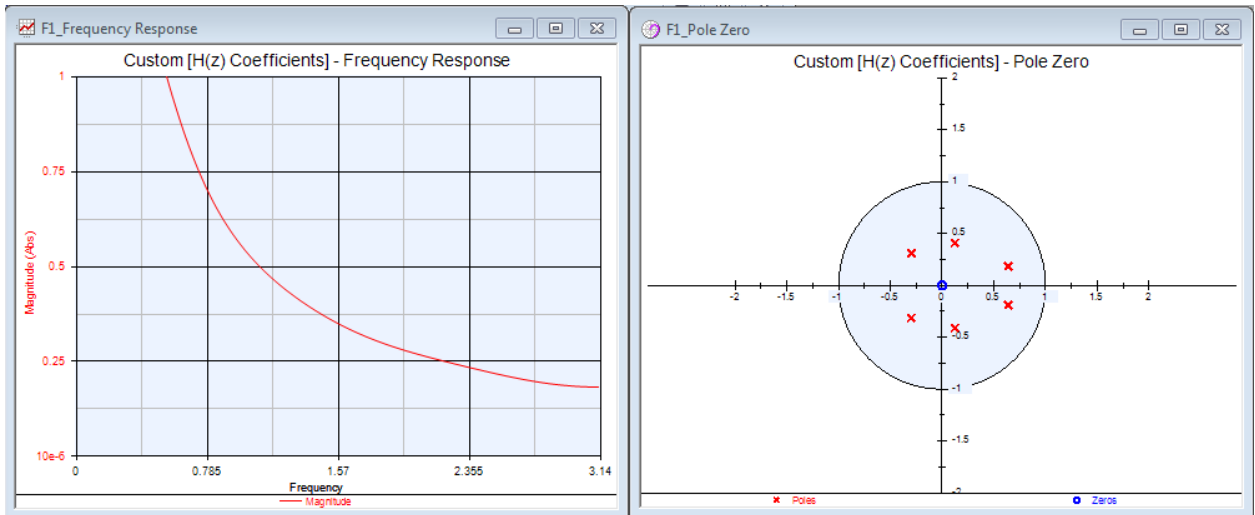


Figure3.16: (a) Frequency - response of proposed digital integrator, (b) Pole-Zero Plot using SystemVue

### 3.1.5 Impulse Response:

The impulse response and step response to proposed integrator is shown in figure 3.6

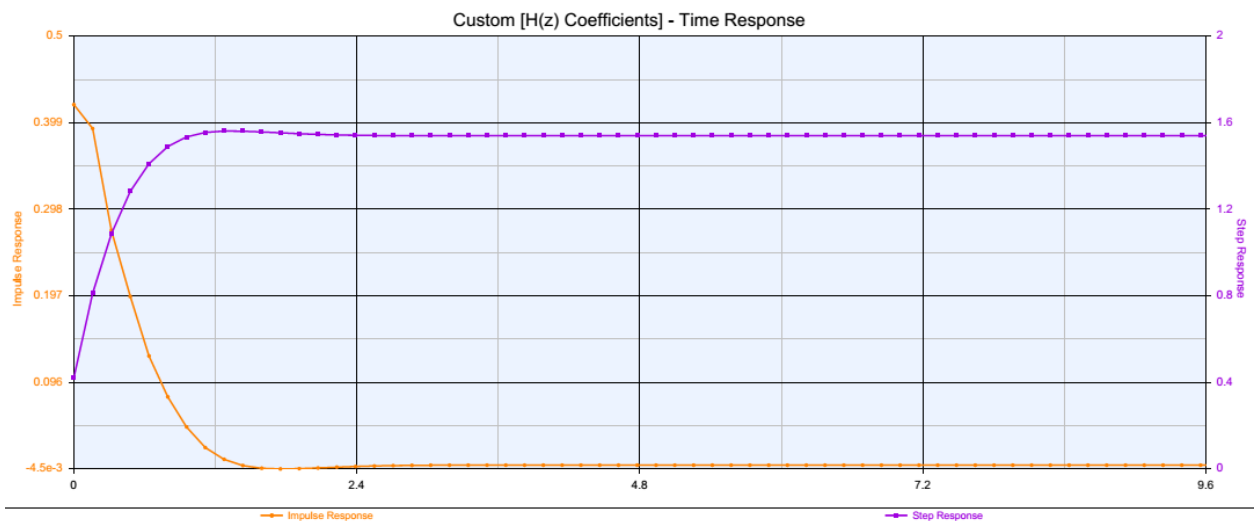


Figure.3.6: Impulse response and step response of designed digital integrator

### 3.1.6 Implementation Direct form realization of digital integrator:

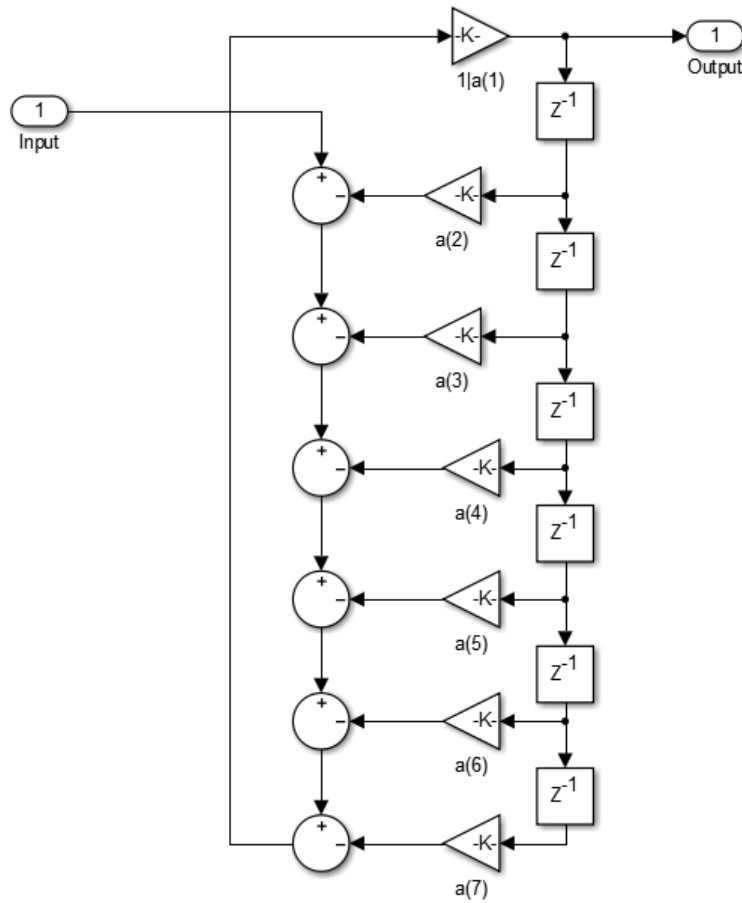
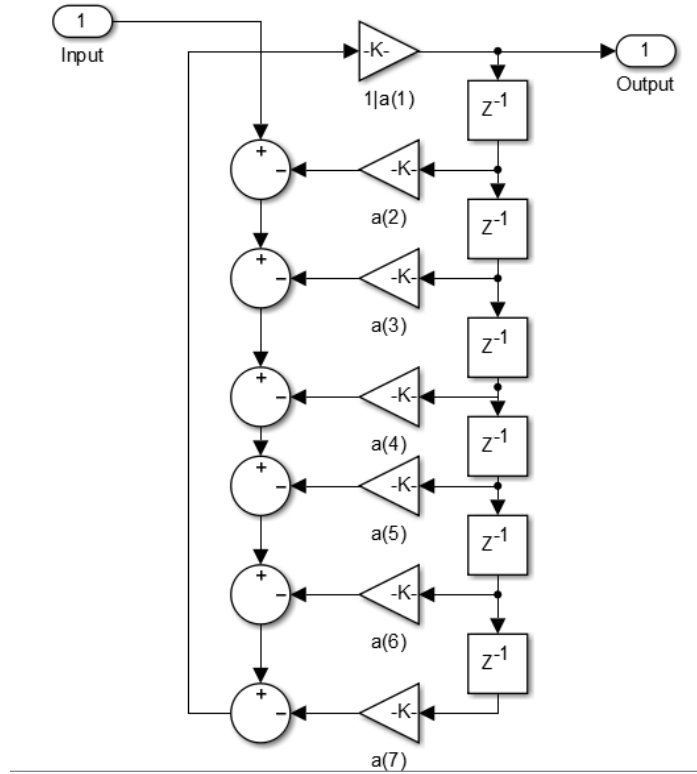


Figure.3.17: Direct form 1 realization of digital integrator by MATLAB



*Figure.3.18: Direct form II realization of digital integrator by MATLAB*

### 3.2 Digital Integrator: Eight Section

An Infinite impulse response system function is selected for designing first order discrete time integrator. The proposed digital integrator prototype is given as:

$$H_{int}(z) = \frac{1}{x_1 + x_2 z^{-1} + x_3 z^{-2} + x_4 z^{-3} + x_5 z^{-4} + x_6 z^{-5} + x_7 z^{-6} + x_8 z^{-7} + x_9 z^{-8}} \quad 3.11$$

This System function has a zero at  $z = 1$  i.e. at DC value. The poles of system function are variables determined by multivariable optimization algorithm scheme. The pole location is selected between the range of  $0 \leq z \leq 1$ . So that system function obtained after optimization is stable.

Upon using optimization, the Error between Ideal and Proposed integrator is given by:  $Err = 2.94 \times 10^{-6}$

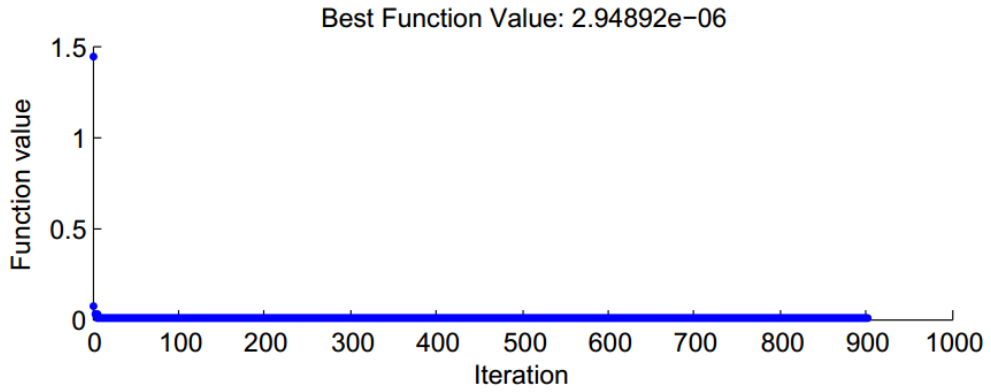


Figure.3.11: Optimization process Err function plot v/s iteration using Pattern search

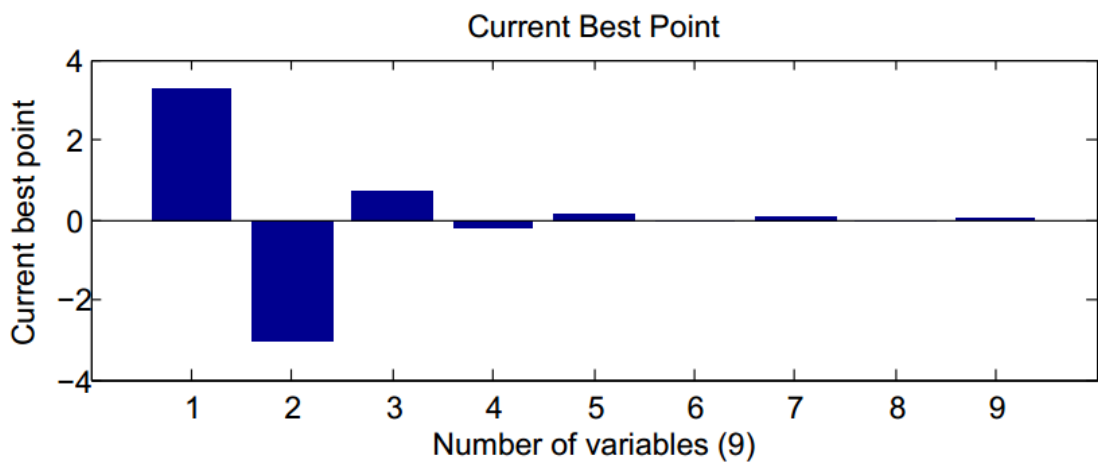


Figure.3.12: Optimized variable coefficient best point plot result using Pattern search

The optimized value of system function coefficient obtained by pattern search optimization is tabulated in Table 3.4.

First order discrete time integrator		
S. No.	Coefficient	Value
1	$x_1$	3.273
2	$x_2$	-3.044
3	$x_3$	0.721
4	$x_4$	-0.195
5	$x_5$	0.17
6	$x_6$	-0.033
7	$x_7$	0.072
8	$x_8$	-0.014
9	$x_9$	0.033

Table.3.1: Optimized coefficient value of first order integrator

$$H_{int}(z) = \frac{1}{3.273 - 3.044z^{-1} + 0.721z^{-2} - 0.195z^{-3} + 0.17z^{-4} - 0.033z^{-5} + 0.072z^{-6} - 0.014z^{-7} + 0.033z^{-8}} \quad 3.12$$

### 3.1.2 Frequency Response of Digital Integrator Using Matlab:

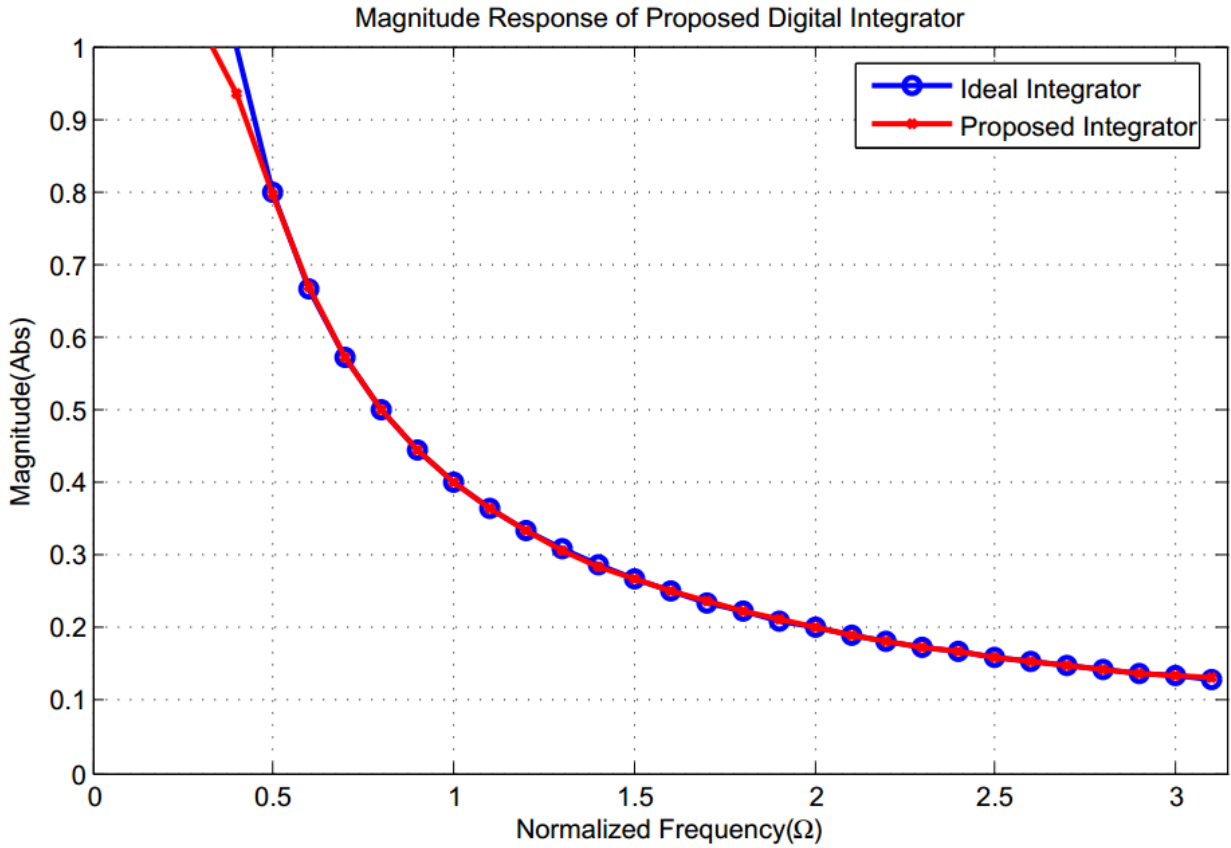


Figure.3.13: Frequency response of proposed integrator for sampling frequency normalised to  $\pi$  radian

If an integrator is implemented by using transmission line, the maximum value of the transfer function  $H_{int}(z)$  as 1 for the frequency range  $0 \leq \Omega \leq 0.1\pi$ . The rest part of the transfer function in the range  $0.1\pi \leq \Omega \leq \pi$  satisfies equation 3.4. Under such circumstance, the circuit thus obtained behaves as an integrator over the frequency range  $0.1\pi \leq \Omega \leq \pi$ .

### 3.1.3 SystemVue Simulation of Designed Digital Integrator

For extensive analysis of given filter function under other design criteria such as stability, impulse response, step response and group delay is also carried out in Agilent SystemVue.

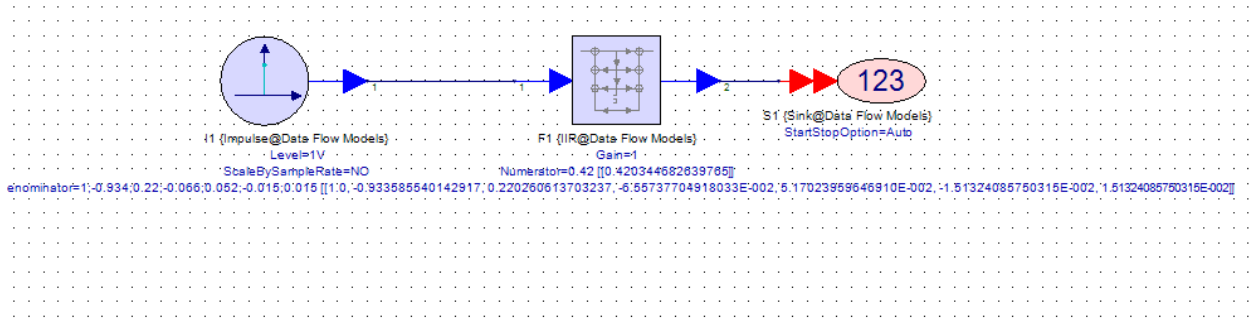


Figure.3.15: Schematic of digital integrator in SystemVue

### 3.2.4 Frequency and Pole Zero plot in SystemVue

The frequency response obtain from SystemVue are found to be in agreement with MATLAB response. The pole zero plot is shown in figure.3.5. The poles obtained are  $p_1 = 0.7111 + 0.2314j$ ,  $p_2 = 0.7111 - 0.2314j$ ,  $p_3 = 0.2826 + 0.4477j$ ,  $p_4 = 0.2826 - 0.4477j$ ,  $p_5 = -0.4289 + 0.2705i$ ,  $p_6 = -0.4289 - 0.2705i$ ,  $p_7 = -0.0999 + 0.4900i$  and  $p_8 = -0.0999 + 0.4900i$  which shows the stability of designed discrete time filter.

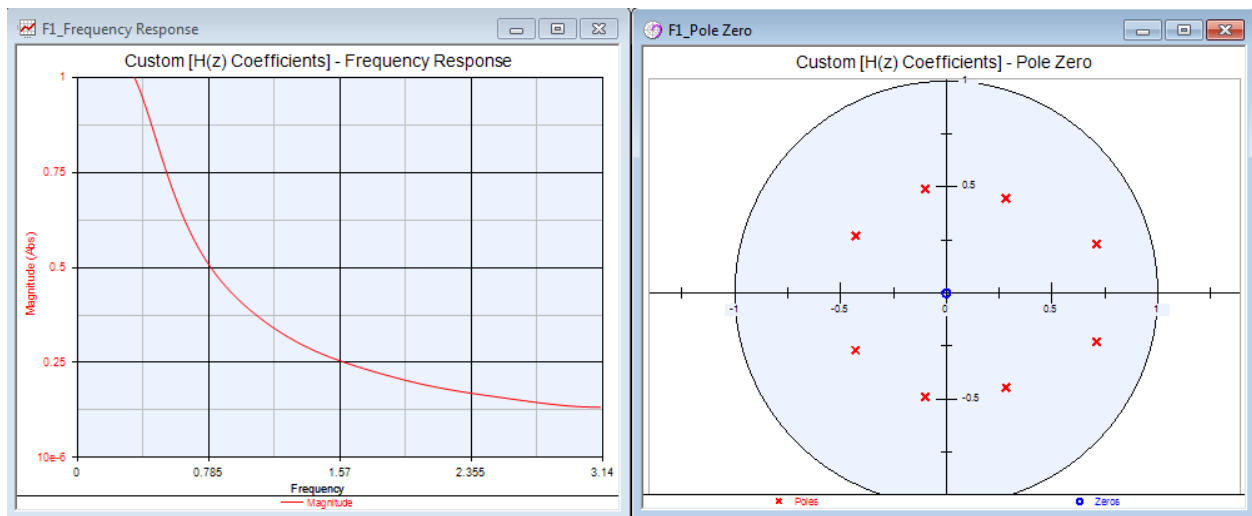
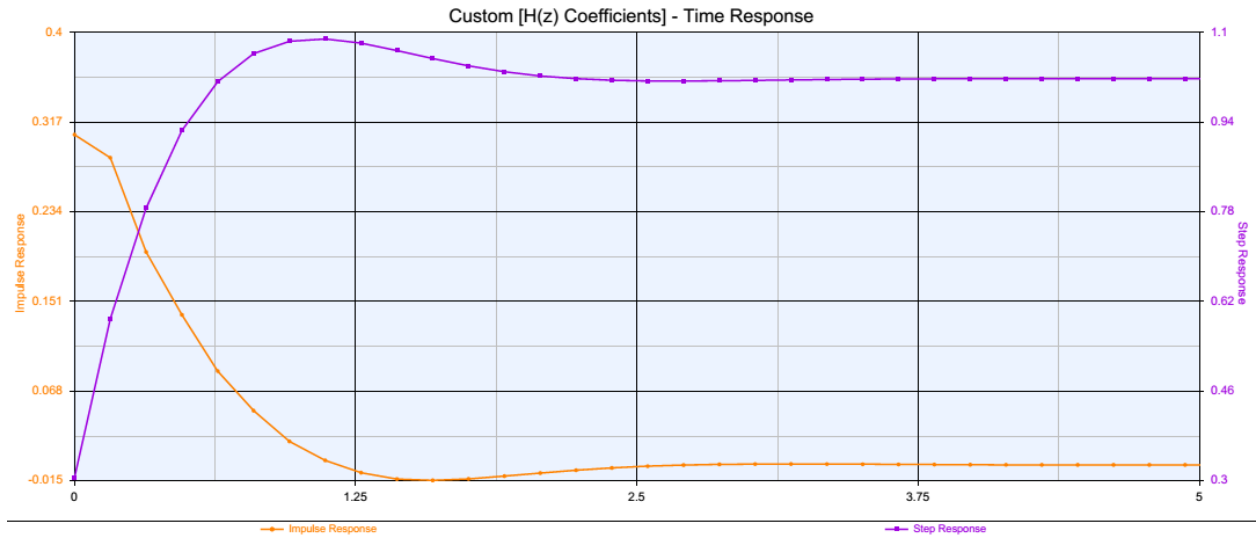


Figure3.16: (a) Frequency response of proposed digital integrator, (b) Pole-Zero Plot using SystemVue

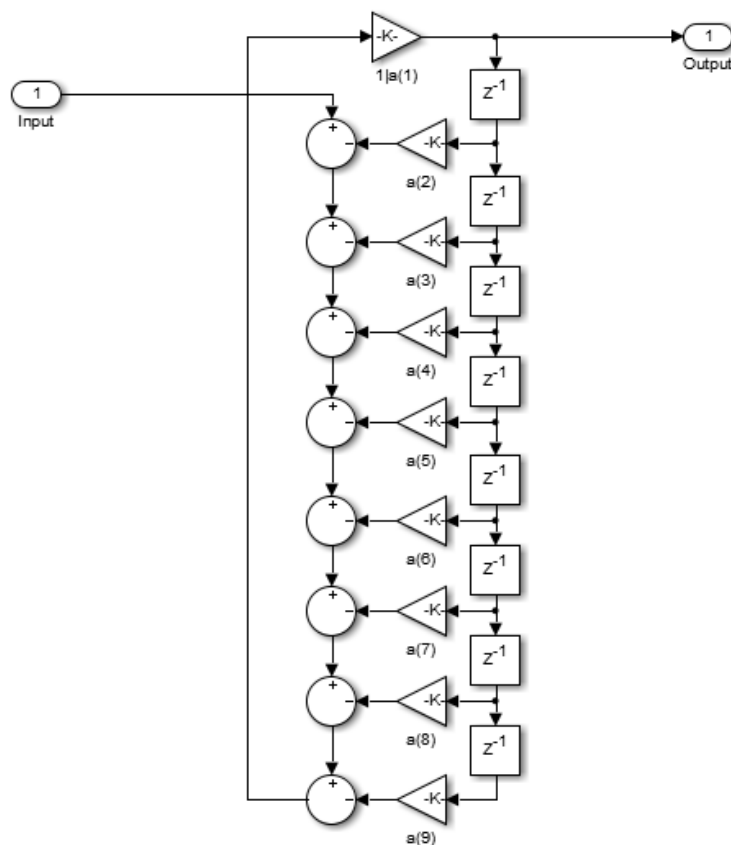
### 3.1.5 Impulse Response:

The impulse response and step response to proposed integrator is shown in figure 3.6



*Figure.3.6: Impulse response and step response of designed digital integrator*

### 3.1.6 Implementation Direct form realization of digital integrator:



*Figure.3.18: Direct form II realization of digital integrator by MATLAB*

The designed digital differentiators and integrators in this chapter are used as reference for designing of microwave differentiator and integrator in the consecutive chapters 4 and 5.

## Designing of Microstrip Differentiators

### 4.1 Introduction

In this chapter designing methodology of microwave differentiator is discussed. Based on the digital differentiator transfer functions obtained in Chapter 3 the designing of optimized microwave differentiators are presented. Here, two different differentiator models with different time constant have been designed. In particular, the time constant is proposed to characterize the performance of the differentiator and it serves as an important factor that determines the magnitude response of a differentiator. The designed differentiator models are also analysed in Agilent EMPro software tool. Lastly, performance characteristic such as magnitude response and group delay characteristic of designed differentiator are presented.

### 4.2 First Order Microwave Differentiator

For designing a differentiator that have the operating frequencies up to 10GHz using microstrip configuration, a network consist of a short stub and a serial transmission lines is used. It has been shown (Table 1) that zero occurring on the unit circle  $|z| = 1$  can be implemented by using shunted transmission-line elements. The number of sections and configuration of microstrip is determined by the optimization process that involves the curve fitting of transfer function of transmission line to the amplitude response of the digital differentiator (3.5) which represents a good approximation of an ideal differentiator in Z-domain.

For obtaining the transfer function of the  $T(z)$  of the network (2.15) MATLAB program [Appendix 1] is used.

The system function obtained using MATLAB is given as:

$$T_{11}(z) = \frac{(1-\Gamma_2^2)(1-z^{-1})}{-1 - a_1 - a_1\Gamma_2^2 + (1-a_1 + a_1\Gamma_2^2 + \Gamma_2^2)z^{-1} + (-\Gamma_2^2 + a_1\Gamma_2^2 + a_1\Gamma_2)z^{-2}} \quad 4.1$$

$$\Gamma_2 = \frac{(z_1+z_0)}{(z_1-z_0)} \quad 4.2$$

Where,  $a_1$  and  $\Gamma_2$  are constant and reflection coefficient respectively, are the design variables to be optimized.



The system function (4.1) obtained for transmission line network is optimized with respect to digital differentiator system function obtained in (4). The optimization gives the value of optimized  $a_1$  and  $\Gamma_2$  coefficients.

### 4.2.1 Optimization in MATLAB

The error function to be minimized using pattern search optimization is defined using (3.2). Upon using optimization algorithm, the error between ideal and proposed differentiator is equals to  $Err = 1.7 \times 10^{-4}$ .

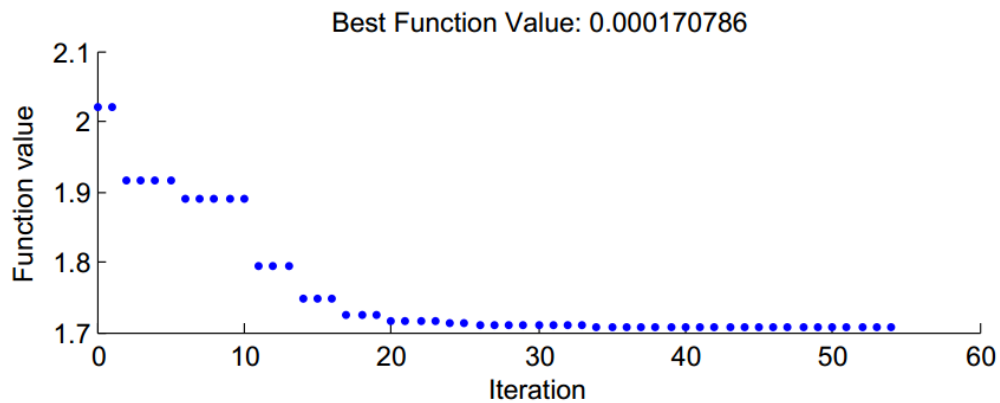


Figure.4.1: Optimization process Err function plot v/s iteration using Pattern search

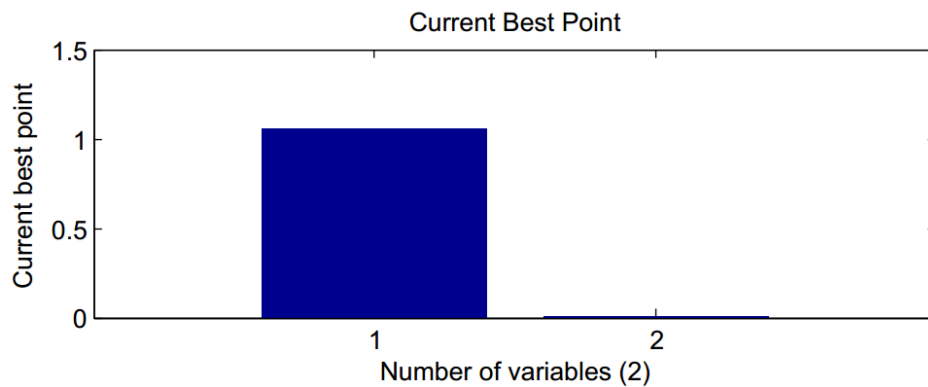


Figure.4.2: Optimized variable coefficient best point plot result using Pattern search

The value of designed variable of system function (5.1) obtained by pattern search optimization algorithm using MATLAB is tabulated in Table 4.1.

S. No.	Design variables	Value
1	$a_1$	1.049
2	$\Gamma_2$	0.043

Table 4.1 Optimized value of design variables obtained using pattern search algorithm

## 4.2.2 Agilent Line Calc:

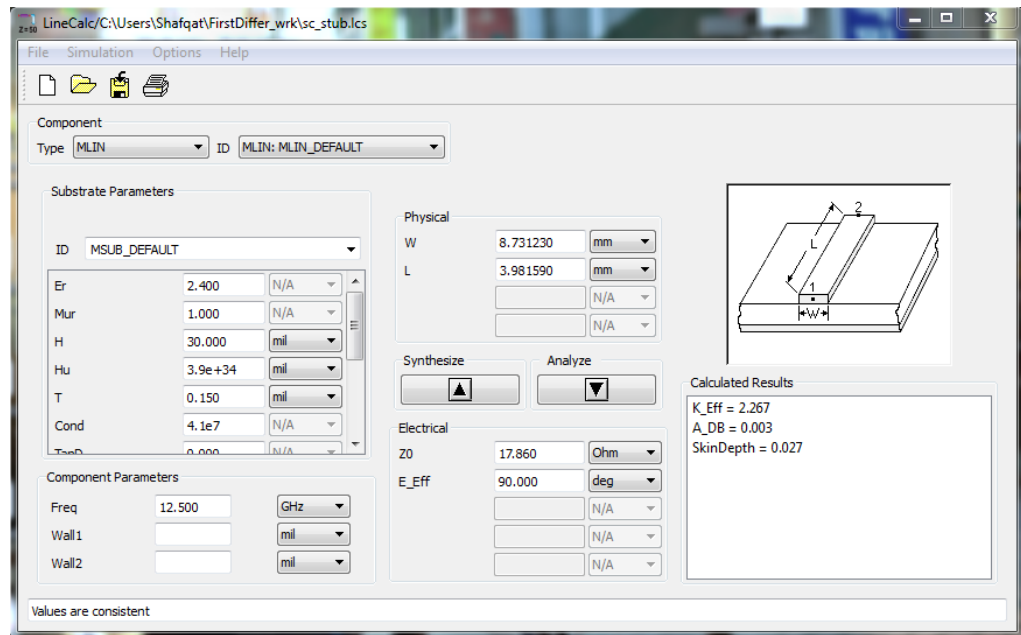


Figure.4.1 Agilent ADS LineCalc

Corresponding to designing variables  $a_1$  and  $\Gamma_2$  values as mentioned in table 4.1, the value of characteristic impedance of each sections of transmission line is calculated using (4.2) and the values of width and length of each section of microstrip is obtained from Agilent Line calc tool as shown in figure 4.1. The RT/duroid® 5870 is used as dielectric substrate having a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . Table 4.2 tabulates the dimensions of each transmission line section.

First order discrete time differentiator				
S.No.	Coefficient	Value	Width(mm)	Length(mm)
1	$Z_1$	47.62 $\Omega$	2.402	4.773
2	$Z_2$	54.54 $\Omega$	1.952	2.713
3	$Z_0$	50 $\Omega$	2.249	4.17

Table.4.2: Serial Transmission Line parameter from Line Calc Optimized coefficient value of first order differentiator

## 4.2.3 Schematic of microwave differentiator using Agilent ADS:

Figure 4.4 represents the schematic for microwave differentiator using Agilent ADS<sup>9</sup>. To implement the shunt transmission-line stub having a characteristic impedance of 23.81  $\Omega$ , a parallel configuration i.e., the equivalent microstrips are placed symmetrically as shown in figure 4.4 is used. The transmission line TL2 and TL5 represents short stub and TL6 represents serial transmission line. The microstrip line named as TL3 and TL4 represents the 50 $\Omega$  characteristic impedance transmission line. S\_Parameter palette is used for S parameter analysis of proposed differentiator. The simulation is performed with linear sweep of frequency from DC to 10GHz.

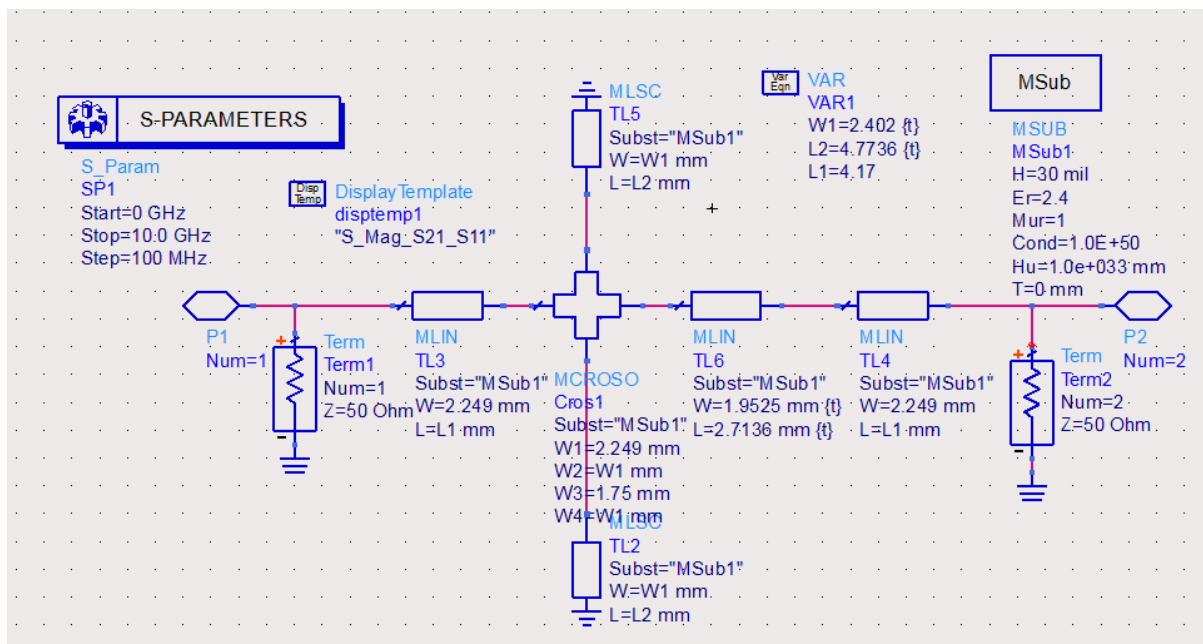


Figure.4.4:Agilent ADS Schematic of 1<sup>st</sup> Order Differentiator

### 4.1.3 Substrate definition of microstrip :

A substrate in EM simulation describes the media where a circuit exists. To demonstrate the proposed design methodology, the microwave differentiator is simulated using microstrip line build on a RT/duroid® 5870 substrate with a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The figure 4.5 shows substrate for microstrip in Agilent ADS software.

<sup>9</sup>[www.ads.2011.10.gs/doc/adstour/ADS\\_Design\\_Environment.html#ADSDesignEnvironmentADSMMainWindow](http://www.ads.2011.10.gs/doc/adstour/ADS_Design_Environment.html#ADSDesignEnvironmentADSMMainWindow)



Figure.4.5: Substrate Definition

#### 4.1.4 Layout diagram of differentiator in ADS:

Based on the schematic diagram (section 5.2.2) the layout for differentiator is generated in Agilent ADS. For characterization of designed differentiator meshing is performed and analysis is done in FEM solver. To generate an electromagnetic field solution from which S-parameters can be computed, FEM Simulator employs the finite element method. In general, the finite element method divides the full problem space into thousands of smaller regions and represents the field in each sub-region (element) with a local function. In FEM Simulator, the geometric model is automatically divided into a large number of tetrahedra, where a single tetrahedron is formed by four equilateral triangles. Figure 4.6 shows the layout structure of microstrip differentiator after meshing operation.

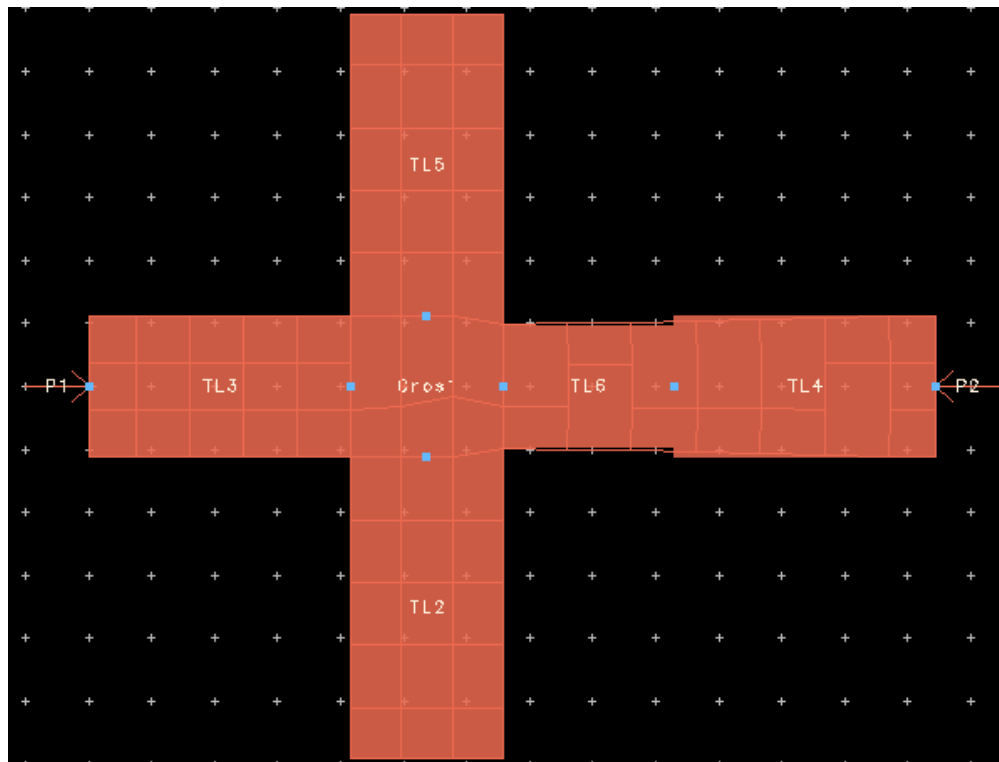
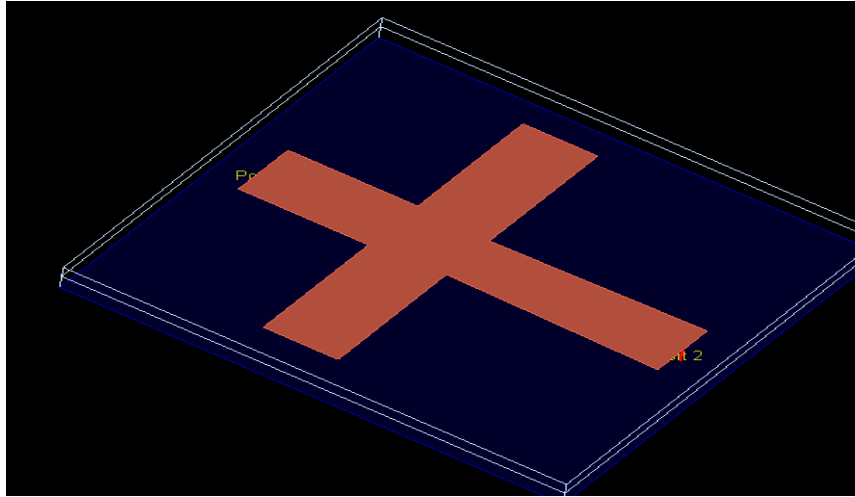


Figure.4.6: Layout diagram of 1<sup>st</sup> order differentiator

### 4.1.5 3D EMPro layout:

Figure 4.7 represents the final two section microstrip differentiator prototype build in Agilent EMPro software.



*Figure.4.7: 3D Agilent EMPro Preview*

### 4.1.6 Magnitude Response of Ideal and Proposed Differentiator:

For characterization of designed differentiator, reflection coefficient  $S_{11}$  and transmission coefficient  $S_{21}$  parameters have been plotted together with ideal differentiator response to show to good degree of agreement between proposed and ideal differentiator. In can be seen that the designed microwave differentiatorfollow well the ideal differentiator characteristic between the frequency range DC to 9.2 GHz.

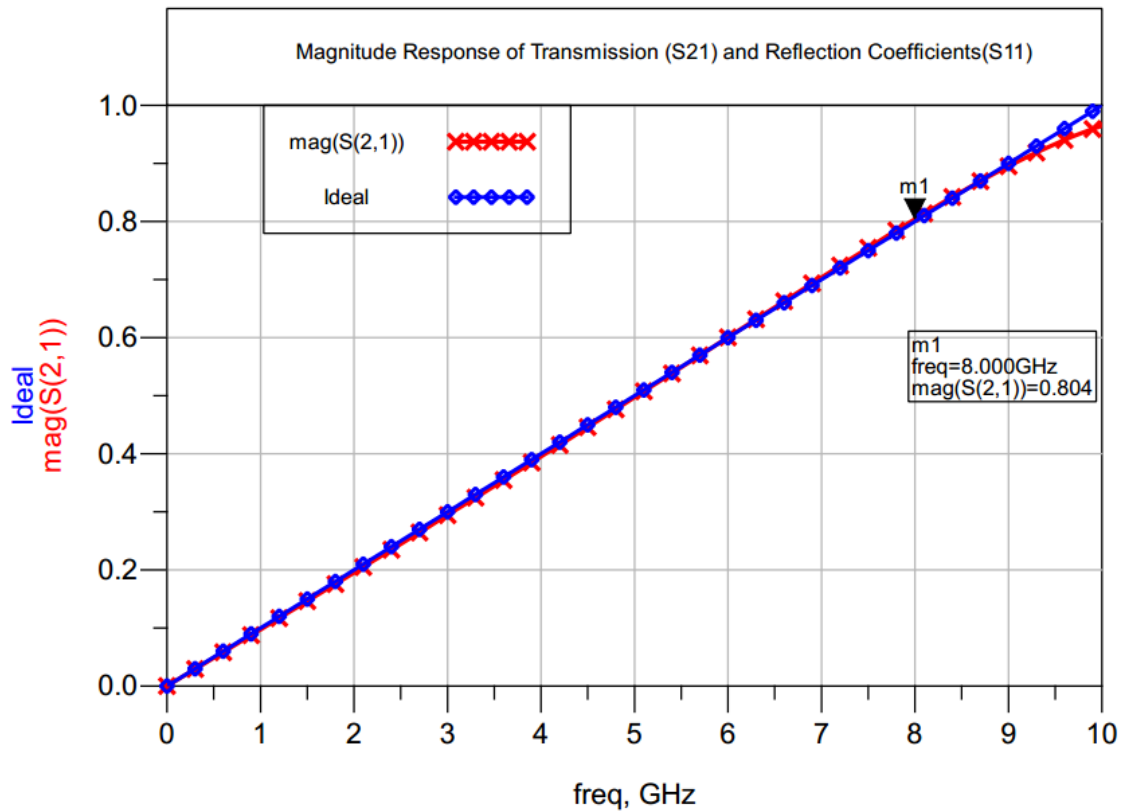


Figure.4.8: Frequency Response of 1<sup>st</sup> order differentiator and ideal differentiator by momentum simulation

### 4.1.7 Group Delay:

Group delay is a measure of the time delay of the amplitude envelopes of the various sinusoidal components of a signal through a device under test, and is a function of frequency for each component. The non-linear phase delay results in a non-constant group delay, for different frequencies it will be different propagation time as show in figure 4.9. It has a constant Group delay. The time constant of designed integrator is xyz sec.

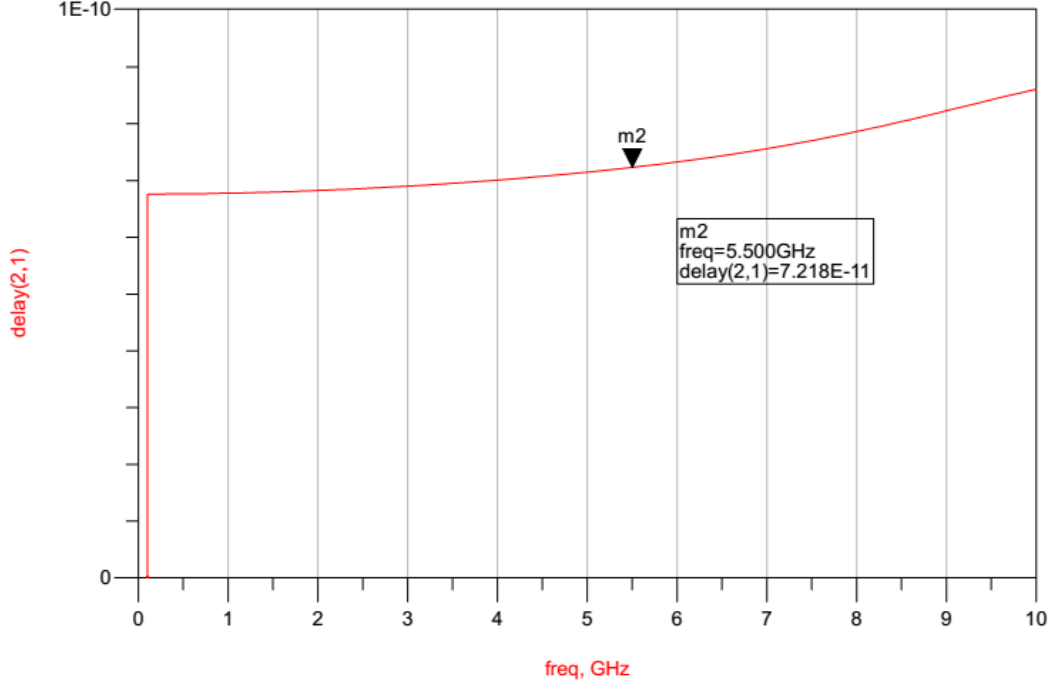


Figure.4.9:ADS simulation plot of group delay versus the frequency in GHz

## 4.2 First order Differentiator with different time constant

For designing a differentiator that have the operating frequencies up to 10GHz using microstrip configuration, a network consist of a short stub and a serial transmission lines is used. It has been shown (Table 1) that zero occurring on the unit circle  $|z| = 1$  can be implemented by using shunted transmission-line elements. The number of sections of integrators is determined by the optimization process that involves the curve fitting of transfer function of transmission line to the amplitude response of the digital differentiator (3.8) which represents a good approximation of an ideal differentiator.

For obtaining the transfer function of the  $T(z)$  of the network (3.12) MATLAB program [Appendix 1] is used.

The system function obtained using MATLAB is given as:

$$T_{11} = \frac{(1-\Gamma_2^2)(1-z^{-1})}{\alpha_1 + (\alpha_2)z^{-1} + (\alpha_3)z^{-2} + (\alpha_4)z^{-3} + (\alpha_5)z^{-4}} \quad 4.3$$

$$Num = 1 - \Gamma_2^2 - \Gamma_3^2 - \Gamma_4^2 + \Gamma_3^2\Gamma_4^2 + \Gamma_2^2\Gamma_3^2 + \Gamma_2^2\Gamma_4^2 - \Gamma_2^2\Gamma_3^2\Gamma_4^2$$

$$\alpha_1 = 1 + C_1 - \Gamma_2\Gamma_3 - \Gamma_3\Gamma_4 + C_1\Gamma_2 - C_1\Gamma_2\Gamma_3 - C_1\Gamma_3\Gamma_4 - C_1\Gamma_2^2\Gamma_3 + \Gamma_2\Gamma_3^2\Gamma_4 + C_1\Gamma_2^2\Gamma_3\Gamma_4 - C_1\Gamma_2\Gamma_3\Gamma_4 + C_1\Gamma_2\Gamma_3^2\Gamma_4$$

$$\alpha_2 = 1 - C_1 - 3\Gamma_2\Gamma_3 + \Gamma_2\Gamma_4 - 3\Gamma_3\Gamma_4 + C_1\Gamma_2^2 + C_1\Gamma_3^2 + C_1\Gamma_4^2 + \Gamma_2^2 + \Gamma_3^2 + \Gamma_4^2 - C_1\Gamma_3 - C_1\Gamma_2\Gamma_3 + C_1\Gamma_2\Gamma_4$$

$$-C_1\Gamma_3\Gamma_4 + C_1\Gamma_2\Gamma_3^2 + C_1\Gamma_2\Gamma_4^2 - C_1\Gamma_4\Gamma_2^2 + C_1\Gamma_4\Gamma_3^2 - \Gamma_2\Gamma_4^2\Gamma_3 + 3\Gamma_2\Gamma_3^2\Gamma_4 - \Gamma_2^2\Gamma_3\Gamma_4 - C_1\Gamma_2^2\Gamma_3\Gamma_4^2 - 2C_1\Gamma_2\Gamma_3\Gamma_4$$

$$-C_1\Gamma_2\Gamma_3\Gamma_4^2 + C_1\Gamma_2\Gamma_4\Gamma_3^2 - C_1\Gamma_3\Gamma_4\Gamma_2^2$$

Where,  $a_1$ ,  $\Gamma_2$ ,  $\Gamma_3$  and  $\Gamma_4$  are constant and reflection coefficient defined in table(2) respectively, are the design variables to be optimized.

The system function (4.3) obtained for transmission line network is optimized with respect to digital differentiator system function obtained in (3.8). The optimization gives the value of optimized  $a_1, \Gamma_2, \Gamma_3$  and  $\Gamma_4$  coefficients.

### 4.2.1 Optimization in MATLAB

The error function to be minimized using pattern search optimization is defined using (3.3). Upon using optimization algorithm the error between ideal and proposed differentiator is given as  $Err = 6.0 \times 10^{-3}$ . The figure 4.10 presents the progress of error function minimization with respect to increasing iteration. The optimized values of design variables i.e.  $a_1, \Gamma_2, \Gamma_3$  and  $\Gamma_4$  are plotted in figure 4.11.

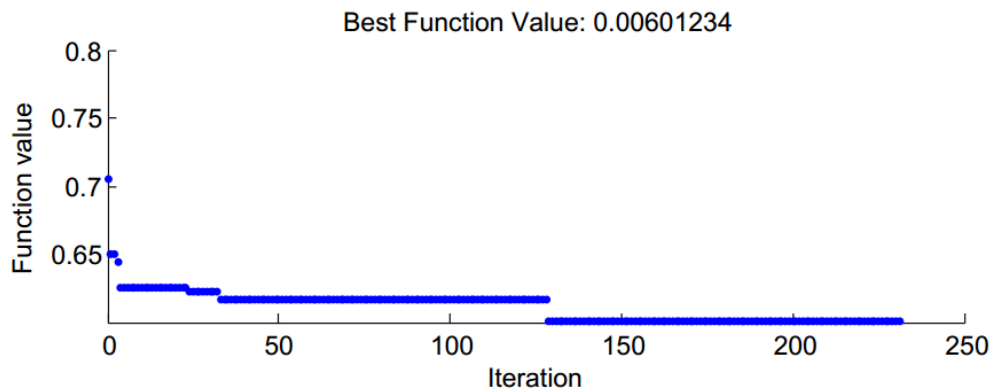


Figure.4.10: Optimization process Err function plot v/s iteration using Pattern search

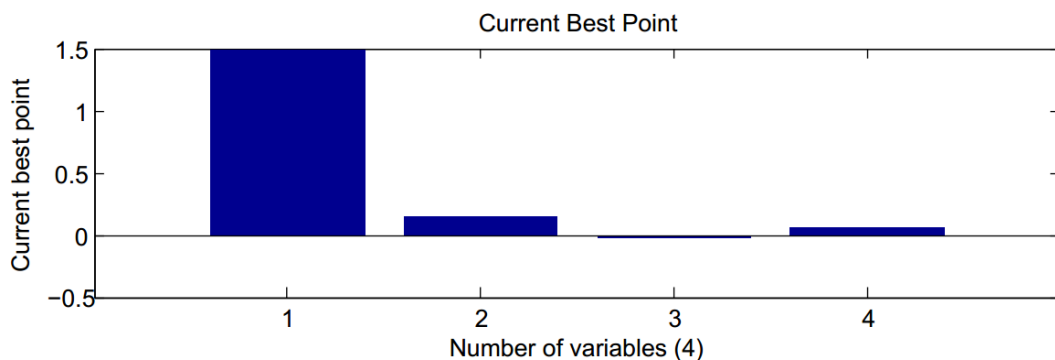




Figure.4.11: Optimized variable coefficient best point plot result using Pattern search

The optimized value of design variables of system function obtained by pattern search optimization algorithm using MATLAB is tabulated in Table 4.3.

S.No.	Design variables	Value
1	$a_1$	1.450
2	$\Gamma_2$	0.164
3	$\Gamma_3$	-0.062
4	$\Gamma_4$	0.015

Table 4.3 Optimized value of reflection coefficients obtained using pattern search algorithm

Corresponding to reflection coefficients values as mentioned in table 4.3, the values of characteristic impedance of each section of transmission line can be calculated using (5.4). For implementation of microstrip integrator, RT/duroid® 5870 is used as dielectric substrate having a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The resultant value of the value of width and length of each section of microstrip is tabulated in table 4.4.

Serial Transmission Line					
S.No.	Configuration	Characteristic Impedance	Value( $\Omega$ )	Width(mm) $w$	Length(mm) $l$
1	SCL	$Z_1$	34.323	3.813	5.712
2	STL	$Z_2$	69.716	1.2964	2.080
3	STL	$Z_3$	44.135	2.6488	2.3176
4	STL	$Z_4$	51.534	2.1325	3.7113
5	TL	$Z_o$	50.000	2.249	4.170

Table.4.4: Serial Transmission Line parameter from Line Calc Optimized coefficient value of first order differentiator

## 4.2.2 Schematic of microwave differentiator using Agilent ADS

Figure 4.12 represents the schematic for microwave differentiator using Agilent ADS. To implement the shunt transmission-line stub having a characteristic impedance of  $17.16 \Omega$ , a parallel configuration i.e., the equivalent microstrips are placed symmetrically. The transmission line TL2 and TL5 represents short stub and TL6, TL7 and TL8 represents serial transmission line. The

microstrip line named as TL3 and TL4 represents the 50Ω characteristic impedance transmission line. S\_Parameter palette is used for S parameter analysis of proposed differentiator. The simulation is performed with linear sweep of frequency from DC to 10GHz.

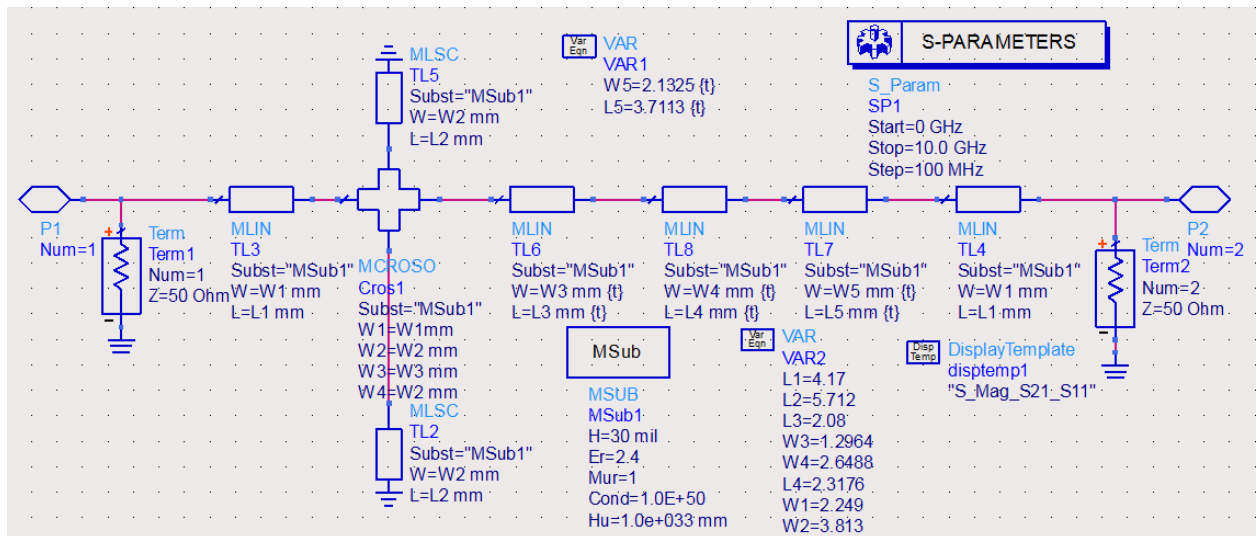


Figure.4.12 Schematic of 1<sup>st</sup> Order Differentiator with time constant()

### 4.2.3 Substrate:

A substrate in EM simulation describes the media where a circuit exists. To demonstrate the proposed design methodology, the microwave integrator is simulated using microstrip line is build on a RT/duroid® 5870 substrate with a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The below diagram shows substrate defining in Agilent ADS software.

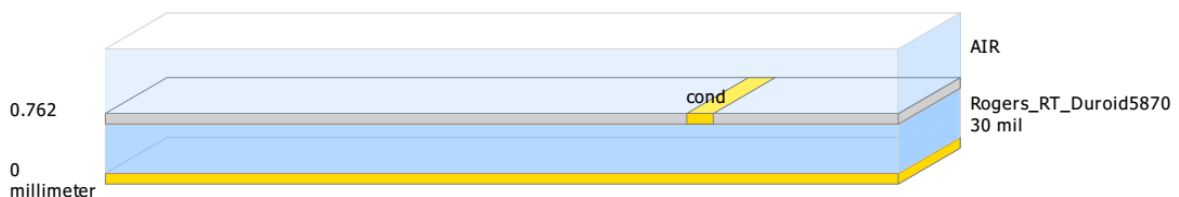
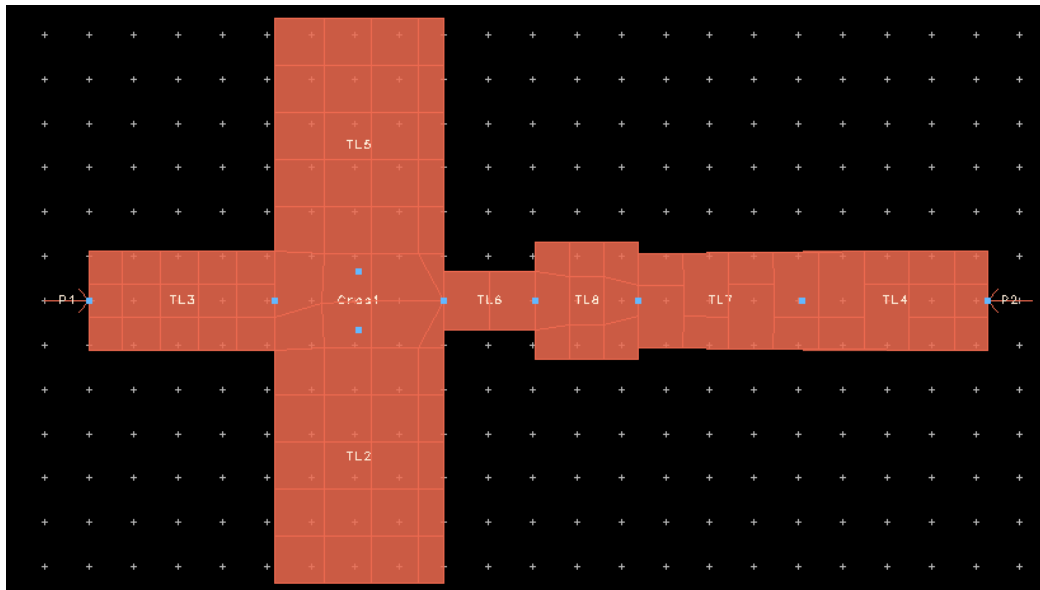


Figure.4.13:RT/duroid® 5870 Substrate

### 4.2.4 Layout diagram of differentiator in ADS:

Based on the schematic diagram (section 4.2.2) the layout of differentiator is generated as shown in figure 4.14. For characterization of designed differentiator meshing is performed and analysis is done in FEM solver. To generate an electromagnetic field solution from which S-parameters can be

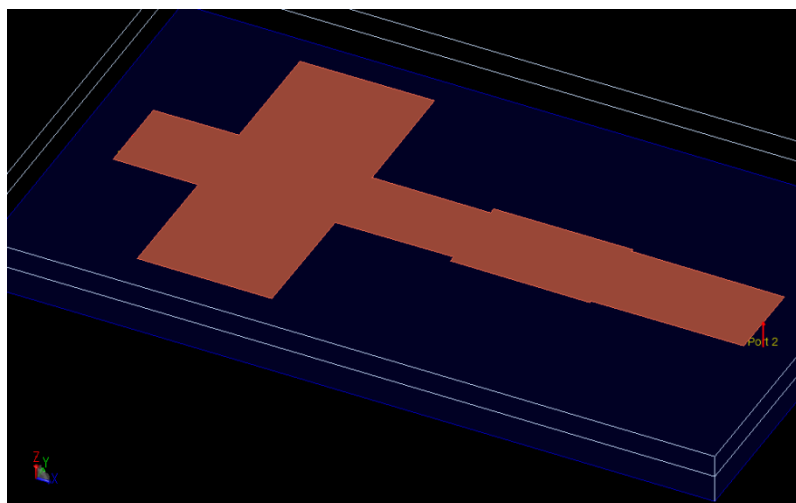
computed, FEM Simulator employs the finite element method. In general, the finite element method divides the full problem space into thousands of smaller regions and represents the field in each sub-region (element) with a local function.



*Figure.4.14: Layout diagram of 1<sup>st</sup> order differentiator*

#### **4.2.5 Agilent EMPro layout:**

Figure 4.15 represents the final microstrip differentiator prototype build in Agilent EMPro software.



*Figure.4.15: Agilent EMPro Layout diagram of first order differentiator*

## 4.2.6 Magnitude Response of Ideal and Proposed Differentiator:

For characterization of designed differentiator,  $S_{11}$  and  $S_{21}$  parameters have been plotted together with ideal differentiator response to show to good degree of agreement between proposed and ideal differentiator. It can be seen that the designed microwave differentiator follows well the ideal differentiator characteristic between the frequency range DC to 8 GHz. The time constant of designed integrator is xyz sec.

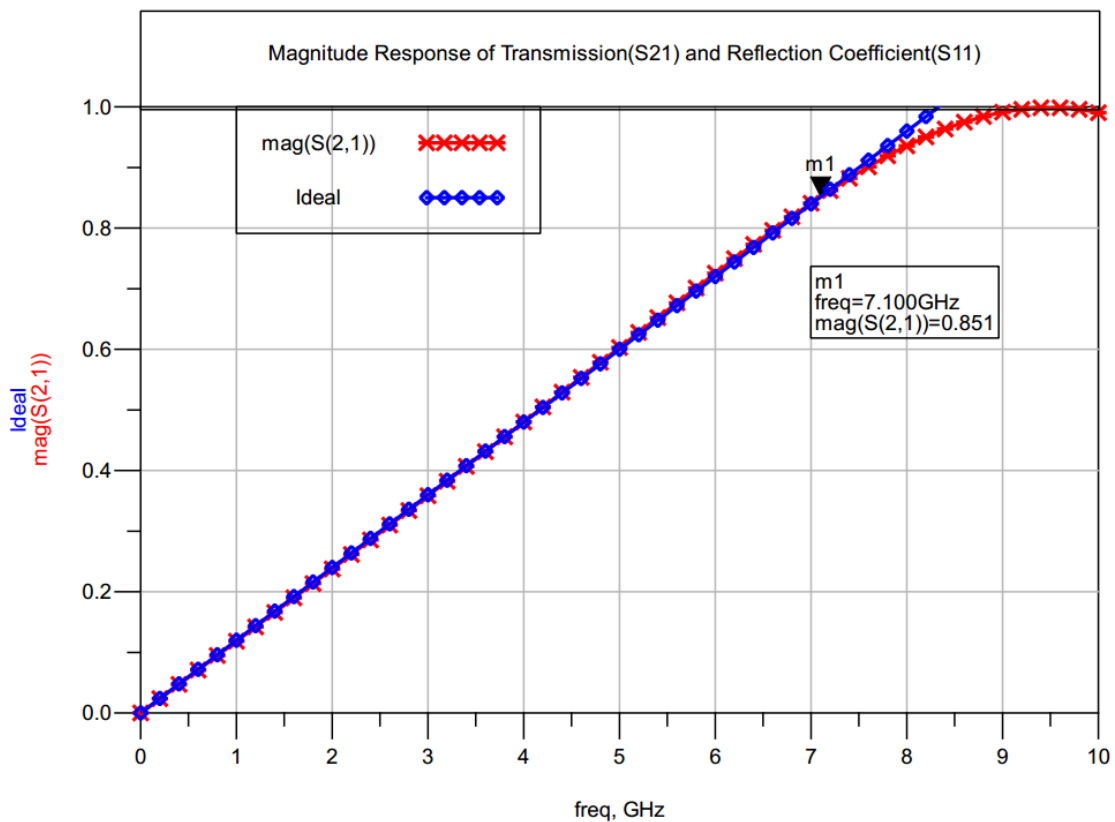


Figure.4.16: Frequency Response of the proposed 1<sup>st</sup> order differentiator and ideal differentiator

## 4.2.7 Group Delay time for designed differentiator

Group delay is a measure of phase distortion. Group delay is the actual transit time of a signal through a device under test as a function of frequency. From figure 4.17 it can be seen that group delay time varies slightly under acceptable range of tolerance. The value of group delay time from port 1 to port 2 is  $1.05 \times 10^{-10}$  sec. The time constant of designed integrator is xyz

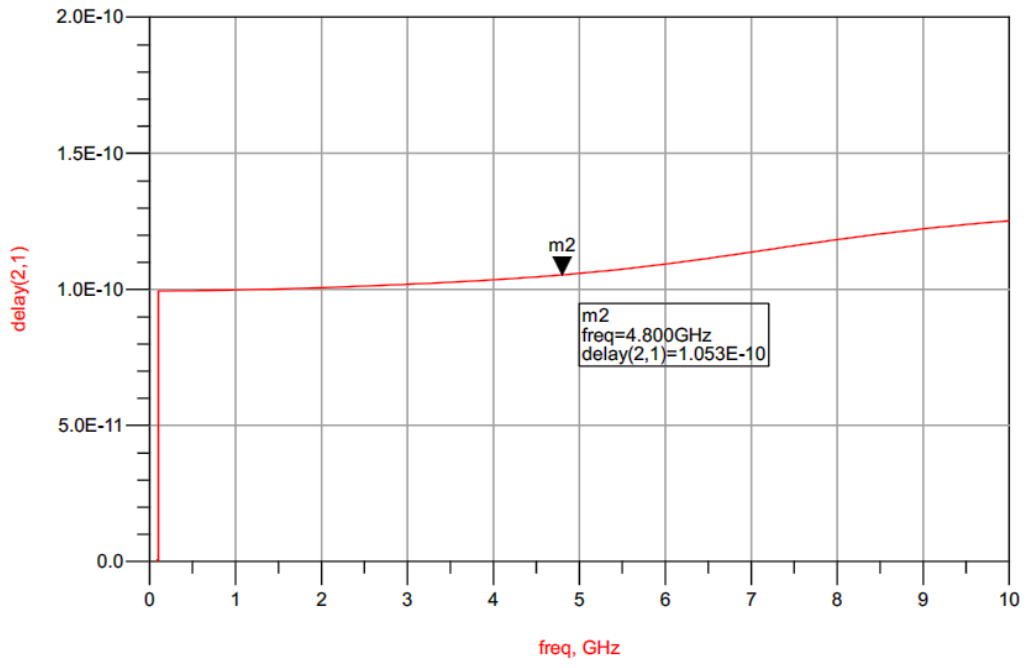


Figure4.17: ADS simulation plot of group delay versus the frequency in GHz

## Designing of Microstrip Integrator

### 5.1 Introduction

This chapter illustrates the synthesis methodology of microwave integrator. Based on the digital integrator system functions obtained in Chapter 3 the designing of optimized microwave integrators are presented. Here, two different integrators models with different time constant have been designed. In particular, the time constant is proposed to characterize the performance of the integrator and it serves as an important factor that determines the amplitude response of an integrator. The designed integrator models are also analysed in Agilent EMPro software tool. Lastly, performance characteristic such as magnitude response and group delay characteristic of designed integrators are presented.

### 5.2 First Order Microwave Integrator

For designing an integrator that have the operating frequencies up to 10GHz using microstrip configuration, a network consist of six sections of serial transmission lines used. It is to be noted that if a network obtained by cascading four section of serial transmission line, there is practical limitation of a maximum frequency operation of 5GHz. The number of sections or device length of microstrip network determines the time constant and frequency response of an integrator. The number of sections of integrators is determined by the optimization process that involves the curve fitting of transfer function of transmission line to the amplitude response of the digital integrator (3.5) which represents a good approximation of an ideal integrator.

For obtaining the transfer function of the  $T(z)$  of the network (2.15) MATLAB program [Appendix 1] is used.

The system function obtained using MATLAB is given as

$$T_{11} = \frac{\beta}{\alpha_1 + \alpha_2 z^{-1} + \alpha_3 z^{-2} + \alpha_4 z^{-3} + \alpha_5 z^{-4} + \alpha_6 z^{-5} + \alpha_7 z^{-6}} \quad 5.1$$

$$\Gamma_n = \frac{(Z_n - Z_0)}{(Z_n + Z_0)} \quad 5.2$$

where,  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6,$  and  $\alpha_7$  coefficients are functions of reflection coefficient of each serial transmission lines given by  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$  and  $\Gamma_6$  respectively. The design variables  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$  and  $\Gamma_6$  to be optimized. The system function (5.1) obtained for six sections transmission line network is optimized with respect to digital integrator system function obtained in (3.5).

### 5.2.1 Optimization in MATLAB

The error function(3.3) to be minimized is performed using pattern search optimization. Upon using optimization algorithm, the error between ideal and proposed integrator is given as  $Err = 2.2 \times 10^{-3}$ .

The figure 5.1 presents the progress of error function minimization with respect to increasing iteration. The optimized values of design variables i.e.  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$  and  $\Gamma_6$  are plotted in figure 5.2.

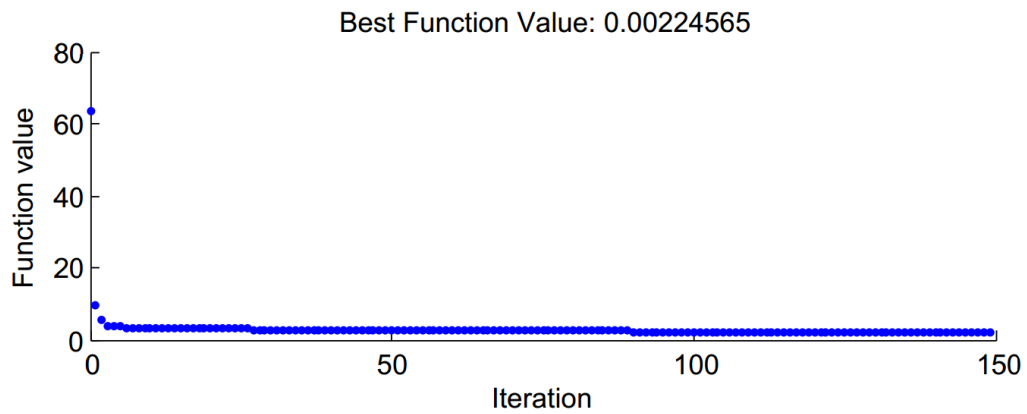


Figure.5.1: Optimization process error function plot versus iteration using Pattern search

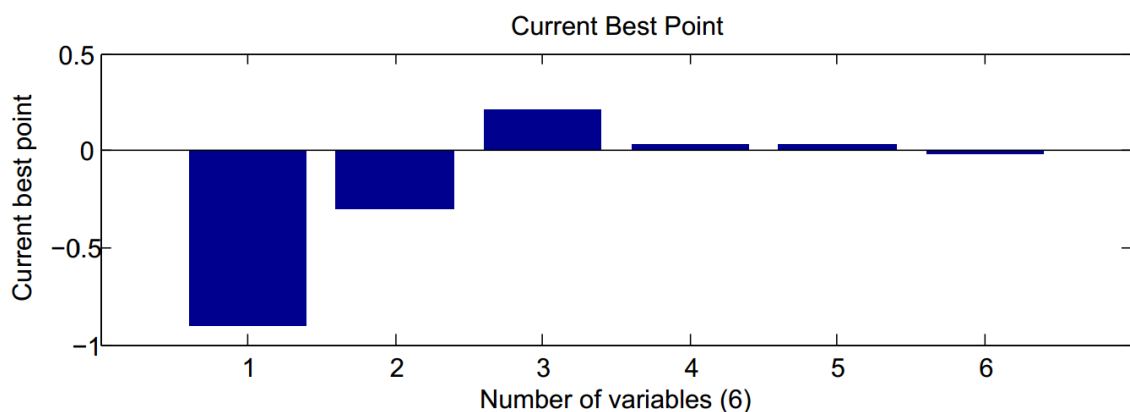


Figure.5.2: Optimized variable coefficient best point plot results using Pattern search

The value of designed variables of system function (5.1) obtained by pattern search optimization algorithm using MATLAB is tabulated in Table 5.1.

S.No.	Reflection coefficient	Value
1	$\Gamma_1$	-0.700
2	$\Gamma_2$	-0.321
3	$\Gamma_3$	0.365
4	$\Gamma_4$	0.022
5	$\Gamma_5$	0.039
6	$\Gamma_6$	-0.021

Table 5.1 Optimized values of reflection coefficients obtained using pattern search algorithm

Corresponding to reflection coefficients values as mentioned in table 5.1, the value of characteristic impedance of each sections of transmission line can be calculated using (5.2). For implementation of microstrip integrator, RT/duroid® 5870 is used as dielectric substrate having a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The resultant value of width and length of each sections of microstrip is calculated using Line Calc tool of Agilent ADS and are tabulated in table 5.2.

Serial Transmission Line				
S.No.	Characteristic Impedance $Z_N$	Value( $\Omega$ )	Width(mm) $w$	Length(mm) $l$
1	$Z_1$	8.807	19.474	3.446
2	$Z_2$	25.657	5.564	2.520
3	$Z_3$	107.464	0.530	4.900
4	$Z_4$	52.212	2.090	2.660
5	$Z_5$	54.147	1.975	7.751
6	$Z_6$	47.913	2.381	5.245
7	$Z_0$	50.000	2.238	5.237

Table.5.2: Serial Transmission Line parameter from Line Calc Optimized coefficient value of first order integrator

## 5.2.2 Schematic of microwave integrator using Agilent ADS:



Figure 5.3 represents the schematic for microwave integrator using Agilent ADS. Six sections of microstrip transmission line named as TL1 through TL6 are used to design integrator. The microstrip line named as TL8 and TL9 represents the 50Ω characteristic impedance transmission line. Microstrip substrate properties are defined under MSUB palette. S\_Parameter palette is used for S-parameter analysis of proposed wideband microwave integrator. The simulation is performed with linear sweep of frequency from DC to 10GHz.

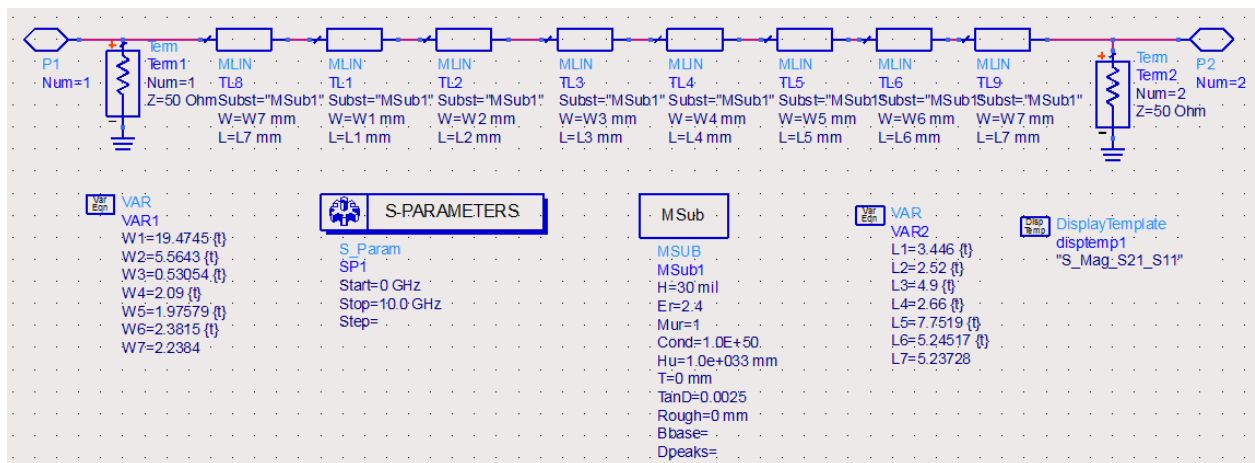


Figure.5.3:Agilent ADS Schematic of six element transmission line 1<sup>st</sup> Order Integrator

### 5.2.3 Substrate definition of microstrip:

A substrate in EM simulation describes the media where a circuit exists. To demonstrate the proposed design methodology, the microwave integrator is simulated using microstrip line build on a RT/duroid® 5870 substrate with a thickness of 30mil (0.762mm), loss tangent of 0.0025 and relative dielectric constant of  $\epsilon = 2.4$ . The figure 5.6 represents the substrate definition for microstrip under Agilent ADS software.



Figure.5.6: RT/duroid® 5870 Substrate

### 5.2.4 Layout diagram of six element integrator in ADS:

Based on the schematic diagram (section 5.2.2) the layout for six element integrator is generated in Agilent ADS. For characterization of designed integrator meshing is performed and analysis is done in FEM simulation. An FEM simulation mesh is a part of the entire 3D problem domain, which is divided into a set of tetrahedra (or cells). The pattern of cells is based on the geometry of a layout so each layout has a unique mesh calculated for it. The mesh is then applied to the geometry to compute the electric fields within each cell. It also helps to identify any coupling effects in the layout during simulation. From these calculations, S-parameters are then calculated for the layout.

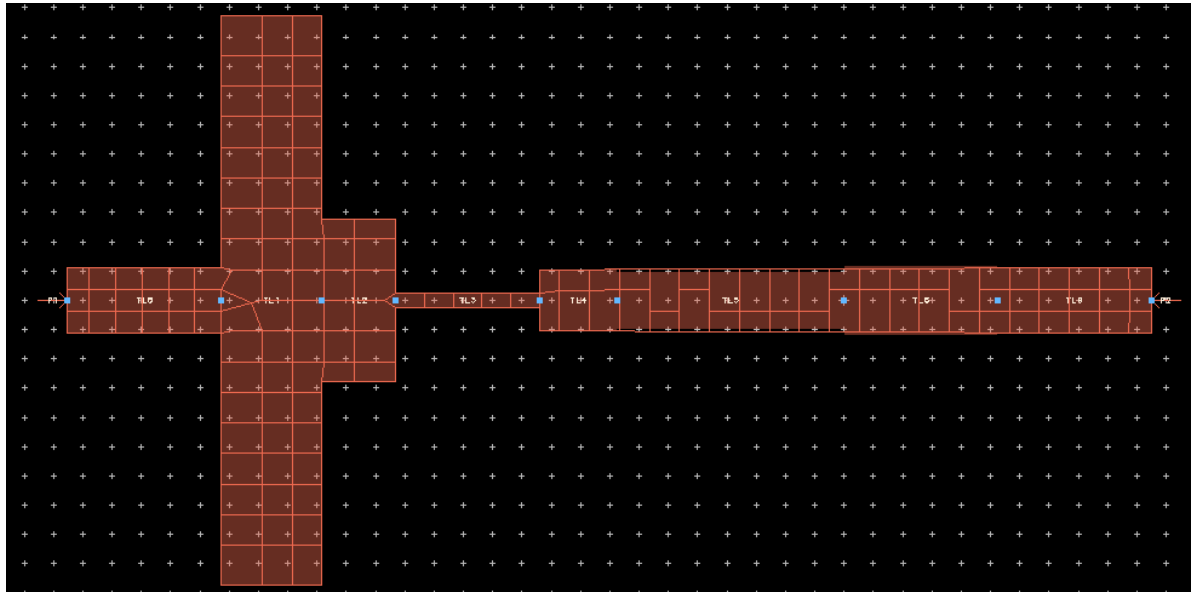


Figure.5.4: Layout diagram of 1<sup>st</sup> order integrator with six sections

### 5.2.5 Agilent EMPro layout:

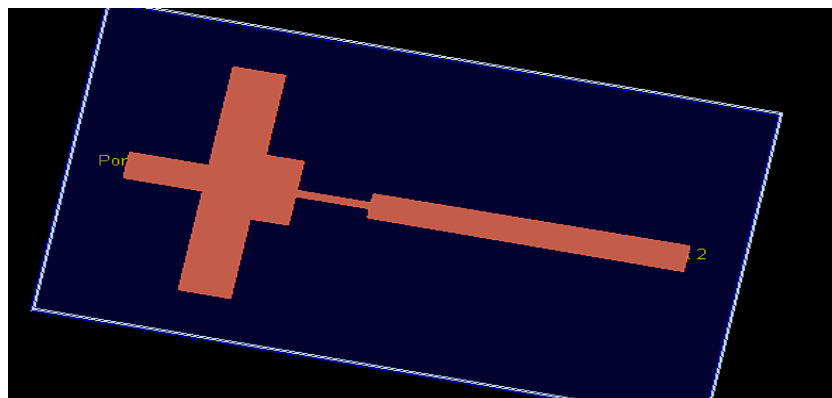


Figure.5.5: Agilent EMPro layout of first order integrator

Figure 5.5 represents the final six section microstrip integrator prototype build in Agilent EMPro software.

## 5.2.6 Magnitude Response of Ideal and Proposed Integrator:

For characterization of designed integrator, reflection coefficient  $S_{11}$  and transmission coefficient  $S_{21}$  parameters have been plotted together with ideal integrator response to show to good degree of agreement between proposed and ideal integrator. It can be seen that the designed microwave integrator follows well the ideal integrator characteristic between the frequency range 2.5GHz to 10GHz.

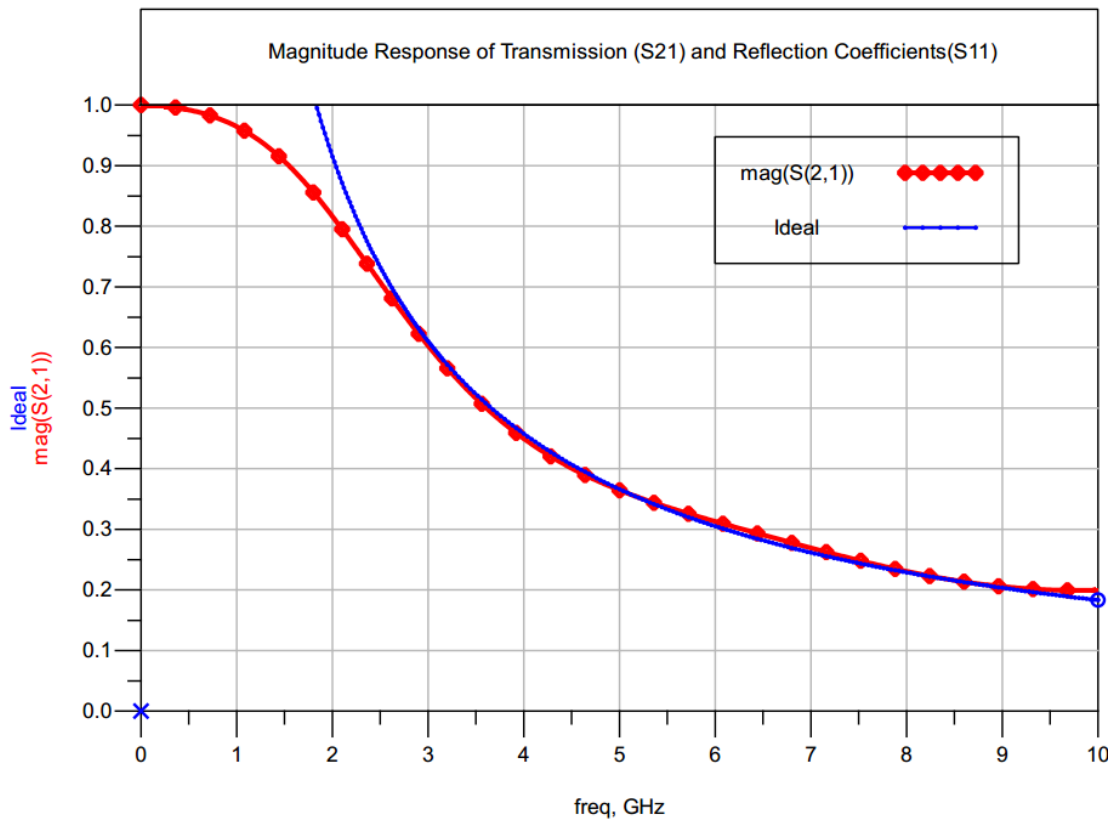


Figure.5.7: Frequency Response of the proposed 1<sup>st</sup> order integrator and ideal integrator

## 5.2.7 Group delay time for designed integrator

Group delay is a measure of phase distortion. Group delay is the actual transit time of a signal through a device under test as a function of frequency. From figure 5.8 it can be seen that group delay time vary slightly under acceptable range of tolerance. The value of group delay time from port 1 to port 2 is found to be equals to  $1.63 \times 10^{-10}$  sec. The time constant of designed integrator is xyz sec.

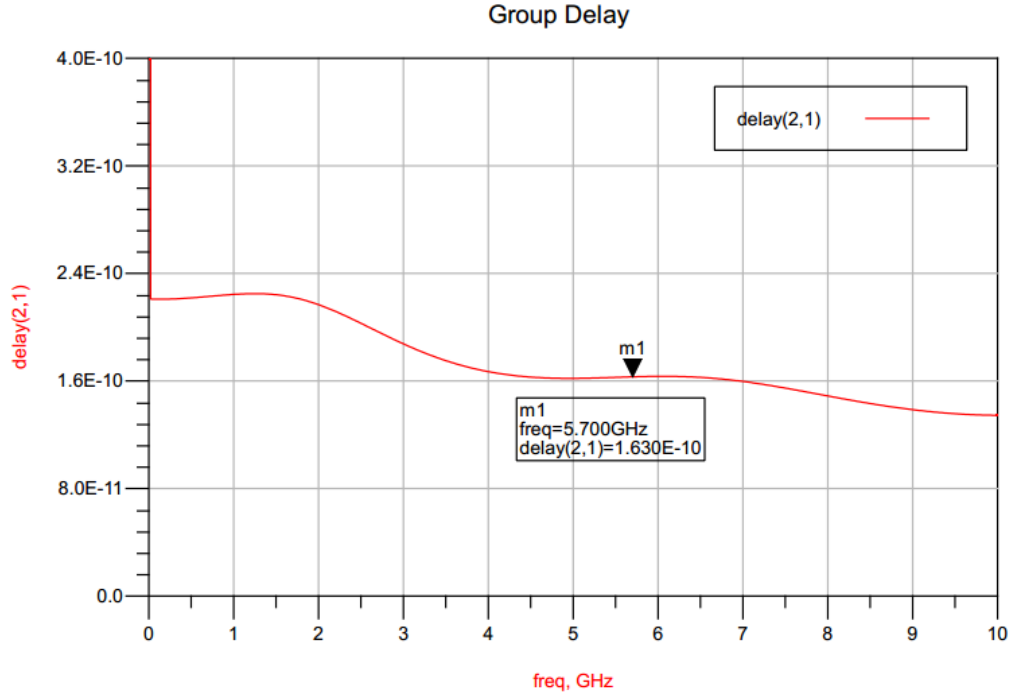


Figure.5.8: ADS simulation plot of group delay versus the frequency in GHz

### 5.3 Integrator with different time constant

For designing integrator that have the operating frequencies up to 10GHz using microstrip configuration, a network consist of eight sections of serial transmission lines is used. The number of sections of integrators is determined by the optimization process that involves the curve fitting of transfer function of transmission line to the amplitude response of the digital integrator which represents a good approximation of an ideal integrator.

For obtaining the transfer function of the  $T(z)$  of the network (3.12) MATLAB program [Appendix 1] is used.

The system function obtained using MATLAB is given as:

$$T_{11} = \frac{\beta}{\alpha_1 + \alpha_2 z^{-1} + \alpha_3 z^{-2} + \alpha_4 z^{-3} + \alpha_5 z^{-4} + \alpha_6 z^{-5} + \alpha_7 z^{-6} + \alpha_8 z^{-7} + \alpha_9 z^{-8}} \quad 5.3$$

$$\Gamma_n = \frac{(Z_n - Z_0)}{(Z_n + Z_0)} \quad 5.4$$

Where,  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$  and  $\alpha_9$  coefficients are functions of reflection coefficient of each serial transmission line given by  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6, \Gamma_7, \Gamma_8$  and  $\Gamma_9$  respectively, The design variables

$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6, \Gamma_7, \Gamma_8$  and  $\Gamma_9$  to be optimized. The system function (5.3) obtained for transmission line network is optimized with respect to digital integrator system function obtained in (3.12).

### 5.3.1 Optimization in MATLAB

The error function to be minimized using pattern search optimization is defined using (3.3). Upon using optimization algorithm, the error between ideal and proposed integrator is given as  $Err = 6.8 \times 10^{-3}$ . The figure 5.1 presents the progress of error function minimization with respect to increasing iteration. The optimized values of design variables i.e.  $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5$  and  $\Gamma_6$  are plotted in figure 5.2.

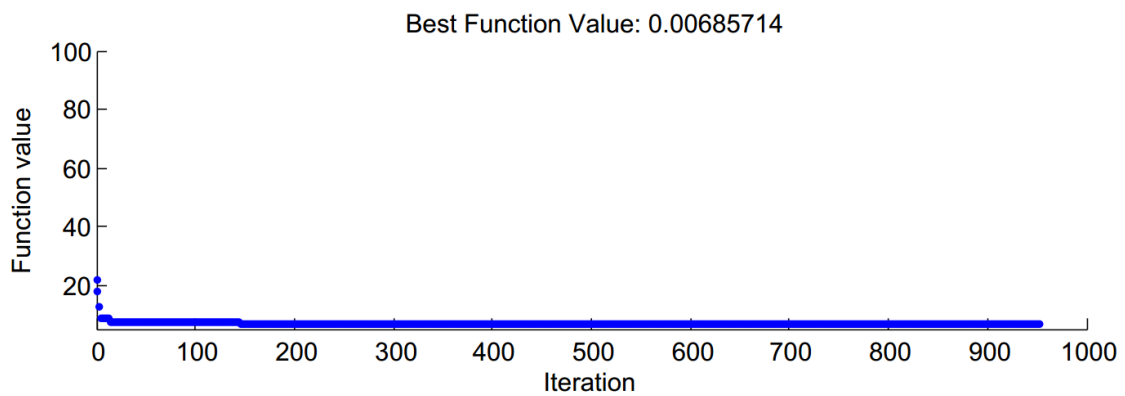


Figure.5.9: Optimization process Err function plot v/s iteration using Pattern search

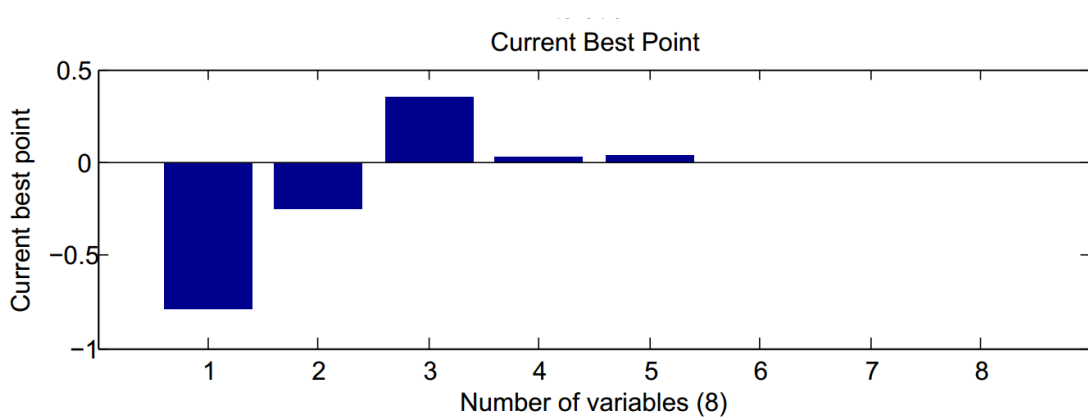


Figure.5.10: Optimized variable coefficient best point plot result using Pattern search

The value of designed variables of system function (5.3) obtained by pattern search optimization algorithm using MATLAB is tabulated in Table 5.3.

S. No.	Reflection coefficient	Value
--------	------------------------	-------

1	$\Gamma_1$	-0.757
2	$\Gamma_2$	-0.300
3	$\Gamma_3$	0.380
4	$\Gamma_4$	0.021
5	$\Gamma_5$	0.039
6	$\Gamma_6$	-0.001
7	$\Gamma_7$	-0.001
8	$\Gamma_8$	-0.001

Table 5.3 Optimized values of reflection coefficients obtained using pattern search algorithm

Corresponding to reflection coefficients values as mentioned in table 5.3, the values of characteristic impedance of each section of transmission line can be calculated using (5.4). For implementation of microstrip integrator, RT/duroid® 5870 is used as dielectric substrate having a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The resultant value of width and length of each sections of microstrip is calculated using Line Calc tool of Agilent ADS and are tabulated in table 5.4.

Serial Transmission Line				
S.No.	Characteristic Impedance	Value( $\Omega$ )	Width(mm) $w$	Length(mm) $l$
1	$Z_1$	6.903	25.400	3.446
2	$Z_2$	26.888	5.244	2.520
3	$Z_3$	109.636	0.505	4.900
4	$Z_4$	52.212	2.090	2.660
5	$Z_5$	54.147	1.975	7.751
6	$Z_6$	49.900	2.381	5.245
7	$Z_7$	49.900	2.381	5.245
8	$Z_8$	49.900	2.381	5.245
9	$Z_0$	50.000	2.238	5.237

Table.5.4: Serial Transmission Line parameter from Line Calc Optimized coefficient value of first order integrator

### 5.3.2 Schematic of microwave integrator using Agilent ADS:

Figure 5.11 represents the schematic for microwave integrator using Agilent ADS. Eight sections of microstrip transmission line named as TL1 through TL8 are used to design integrator. The microstrip line named as TL9 and TL10 represents the 50Ω characteristic impedance transmission line. S\_Parameter palette is used for S parameter analysis of proposed integrator. The simulation is performed with linear sweep of frequency from DC to 10GHz.

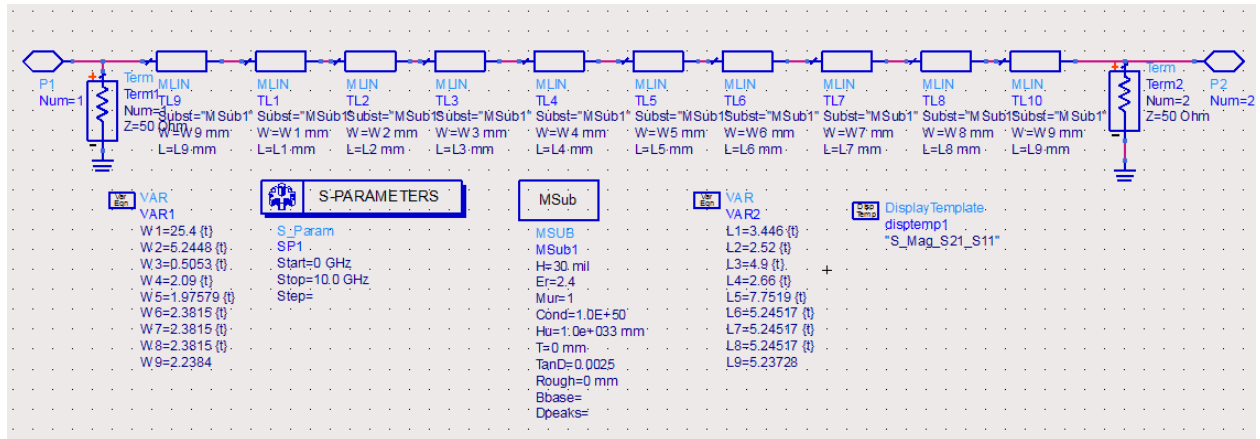


Figure.5.11:Agilent ADS Schematic of eight sections 1<sup>st</sup> Order Integrator

### 5.3.3 Substrate definition of microstrip:

A substrate in EM simulation describes the media where a circuit exists. To demonstrate the proposed design methodology, the microwave integrator is simulated using microstrip line is build on a RT/duroid® 5870 substrate with a thickness of 30mil (0.762mm) and relative dielectric constant of  $\epsilon = 2.4$ . The below diagram shows substrate defining in Agilent ADS software.

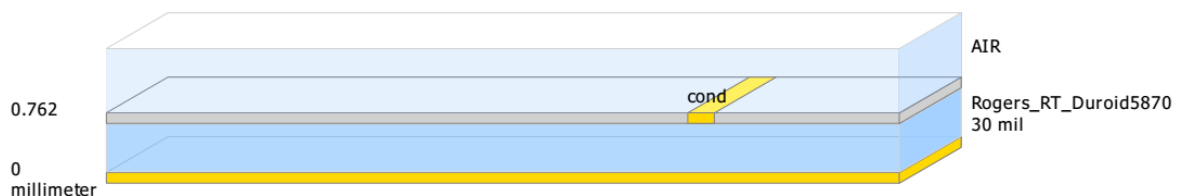


Figure.5.6:RT/duroid® 5870 Substrate

### 5.3.4 Layout diagram of eight element integrator in ADS:

Based on the schematic diagram (section 5.3.2) the layout of eight element integrator is generated as shown in figure 5.12. For characterization of designed integrator meshing is performed and analysis is done in FEM solver. To generate an electromagnetic field solution from which S-

parameters can be computed, FEM Simulator employs the finite element method. In general, the finite element method divides the full problem space into thousands of smaller regions and represents the field in each sub-region (element) with a local function.

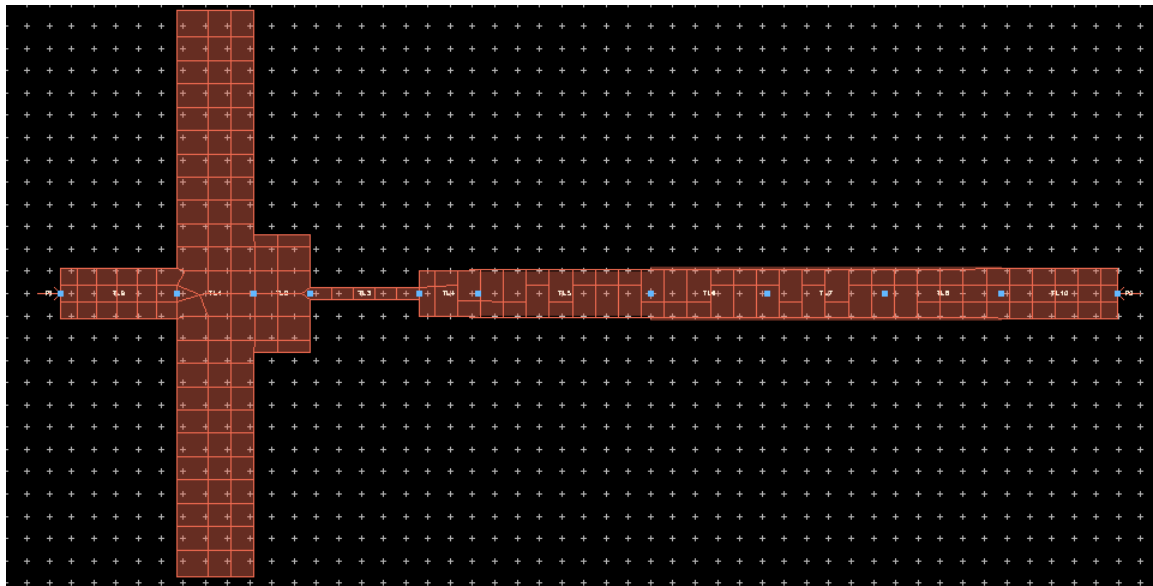


Figure.5.12: Layout diagram of 1<sup>st</sup> order differentiator with eight sections

### 5.3.5 Agilent EMPro layout

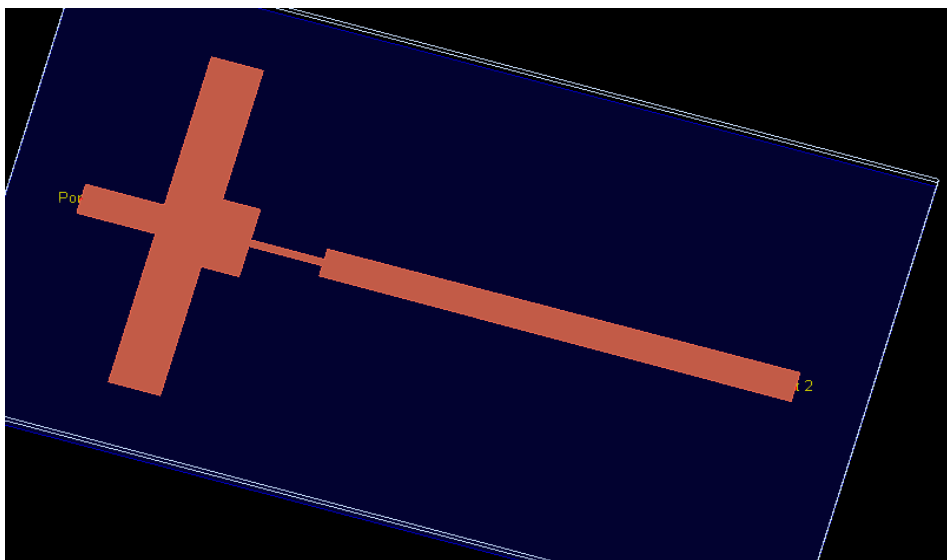


Figure.5.13 Agilent EMPro layout of first order integrator

Figure 5.13 represents the final eight section microstrip integrator prototype build in Agilent EMPro software.



### 5.3.6 Magnitude Response of Ideal and Proposed Differentiator

For characterization of designed integrator,  $S_{11}$  and  $S_{21}$  parameters have been plotted together with ideal integrator response to show to good degree of agreement between proposed and ideal integrator. In can be seen that the designed microwave integrator follow well the ideal integrator characteristic between the frequency range 2GHz to 10GHz. The time constant of designed integrator is xyz sec.

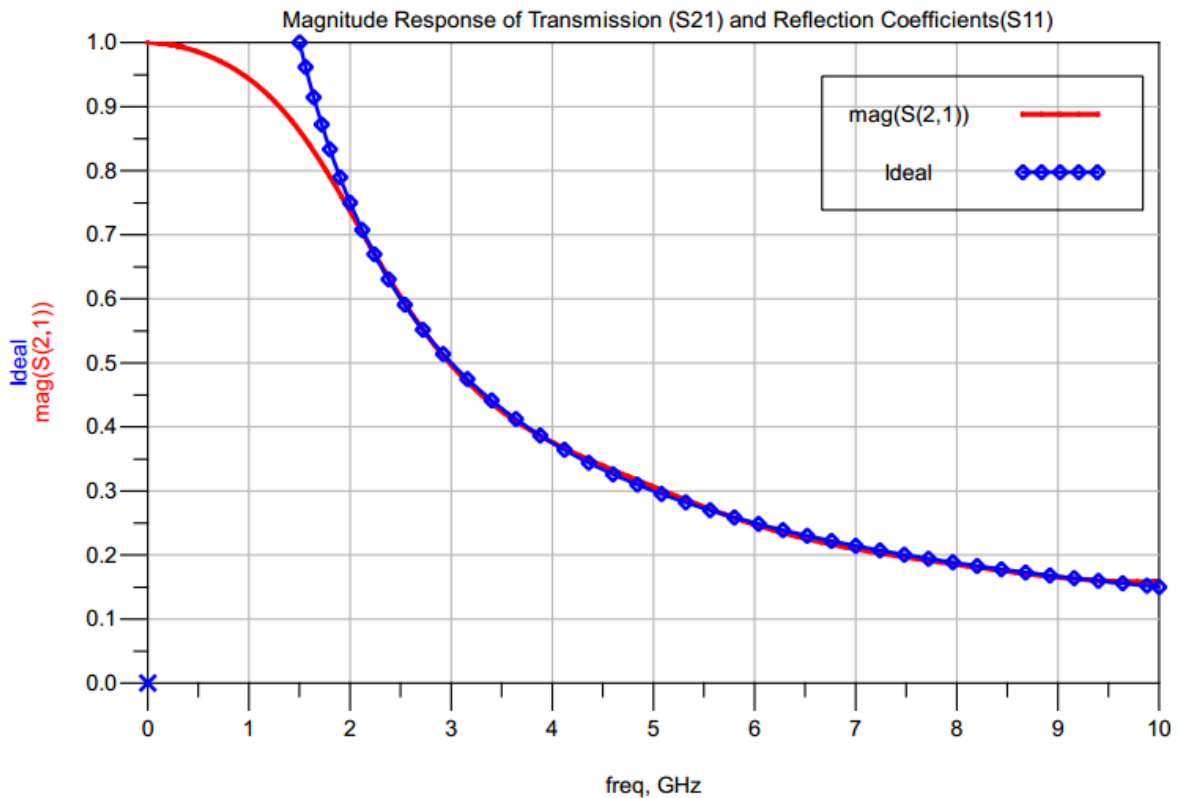


Figure.5.15 Frequency Response of the proposed 1<sup>st</sup> order integrator and ideal integrator

### 5.3.7 Group delay time for designed integrator

Group delay is a measure of phase distortion. Group delay is the actual transit time of a signal through a device under test as a function of frequency. From figure 5.16 it can be seen that group delay time vary slightly under acceptable range of tolerance. The value of group delay time from port 1 to port 2 is  $2.17 \times 10^{-10}$  sec. The time constant of designed integrator is xyz sec.

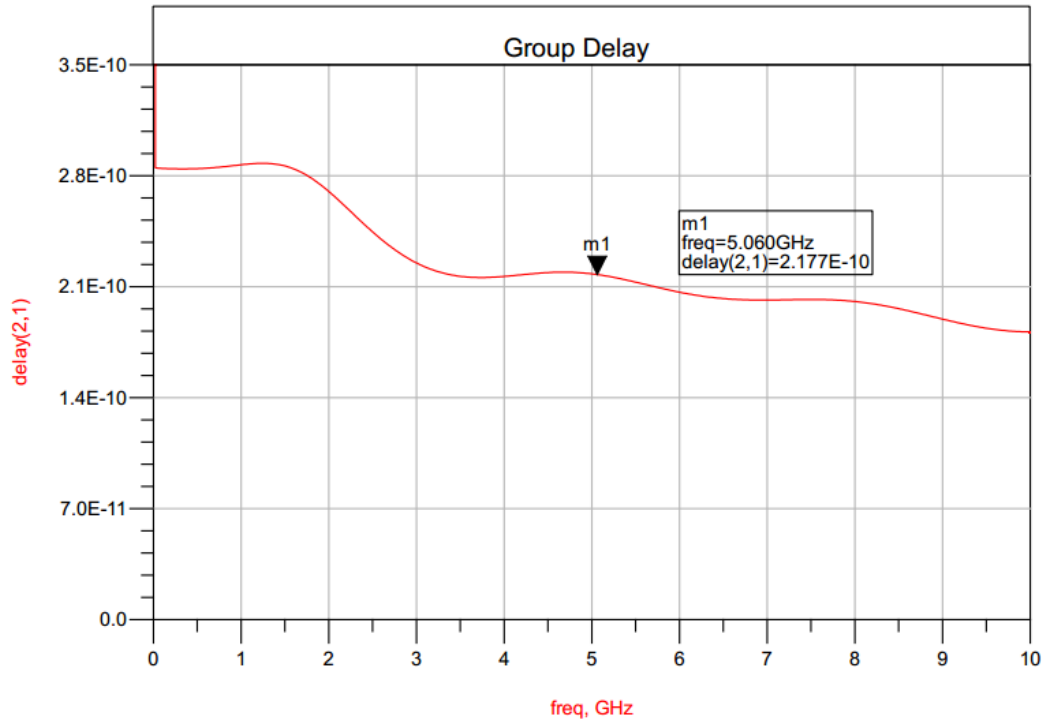


Figure.5.16: ADS simulation plot of group delay versus the frequency in GHz

# Chapter 6

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## Results

- The design of accurate digital integrator and differentiator has been presented. Thus the designed Integrator and differentiator are of high accuracy with an error of order  $10^{-5}$ . The low order of the differentiator makes it suitable for real-time applications. It approximates an ideal differentiator in the passband region with accuracy comparable to that obtained by higher order filters. The proposed method is fast and eases the design complexity of wideband digital integrator and differentiator
- It has been shown that zeros occurring on the unit circle  $|z| = 1$  can be implemented by using shunted transmission-line elements.
- From fig. , we came across conclusion that better linearity in amplitude response comes when  $d = 0.166$ . The relative error is less than 1% (or -40dB) when  $0 \leq \Omega \leq 0.8\pi$ . The bilinear transformation, when is one, has a good linearity when the normalized frequency is less than  $0.3\pi$ . So, bilinear transformation is improper to be adopted as the system function of a wide-band differentiator.
- The magnitude response of simulated differentiator extends good linearity agreement till 0.8 of full band normalised frequency, which corresponds to 10GHz.
- For designed integrator, is a good candidate for integration operation up to the maximum operating frequency of 10GHz. Except for the lower frequency band, amplitude response are in good agreement with the ideal response.
- For 4 elements frequency response is till 5 GHz so length also increases frequency response.

### 6.1 Conclusions

The objective of this work has been to

- Simple and accurate formulations have been employed to represent both first and second-order differentiators in the Z-domain.
- In particular, the Z-domain representations of scattering characteristics of equal length non-uniform transmission lines facilitate the implementation of discrete domain differentiators in the microwave frequency range. These differentiators have been implemented by using microstrip transmission lines.
- It is possible that many other circuits developed in DSP studies can also be implemented by using transmission lines for microwave applications.
- The mathematical analysis was done for each design and verified by writing a MATLAB code. The effects due to conductor and dielectric losses were ignored in the code. ADS was used to accurately arrive at the final design.

### 6.2 Future Scope of present work

By using different configuration of serial, short, and open stub Transmission line we can implement many other circuits developed in DSP studies. The future extension of this project shall be:

- Implementation of higher order integrators and differentiator, variable time constant differentiator, variable time constant differentiator [5-8] and
- Different types of filters such as Low pass, high pass, bandpass and bandstop filter for high frequency application [9].

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```

Tsc(c, z) = (1/(1-z))*[(1+c)-(1-c)*z c+c*z ; -c-c*z (1-c)-(1+c)*z];
    w = subs(Tsc,c,bhh);
    f = collect(w(c,z));
case 3
% Z(i) = 50
% T = (Z(i)-Zo)/(Z(i)+Zo);
bb = Tvar(i);
syms Tz
% Ttls = sym(zeros(2,2));
Ttls(T,z) = (1/(1-T^2))*[1-z*T^2 -(T-T*z);(T-T*z) (-T^2 + z)];
    w = subs(Ttls,T,bb);
    f = collect(w(T,z));
otherwise
fprintf(' %d is an invalid value! Select values from 1 or 2 or 3 only \n',choice);
end
Tnetwork = Tnetwork*f;
end
Qnw = collect(1/Tnetwork(1,1));
pretty(Qnw) % gives same result as in num den output
S(z) = Qnw;
[numexpr, denexpr] = numden(sym(S));%// 'sym' makes sure that S is symbolic
%// Extract numerator coefficients
[numcoef, numpow] = coeffs(expand(numexpr), z);
num = rot90(sym(sym2poly(sum(numpow))), 2);
num(num ~= 0) = coeffs(expand(numexpr), z); % repre terms as 1 z^-1 z^-2 form

%// Extract denominator coefficients
[dencoef, denpow] = coeffs(expand(denexpr), z);
den = rot90(sym(sym2poly(sum(denpow))), 2);
den(den ~= 0) = coeffs(expand(denexpr), z); % % repre terms as 1 z^-1 z^-2 form

%%
syms C1T2;
% x = [C1 T2];
den1(C1,T2) = den(1)/num(1);
den2(C1,T2) = den(2)/num(1);
den3(C1,T2) = den(3)/num(1);
%p = [2.687 0.429 -0.134];
%fval = (den1 - p(1))+(den2 - p(2))+(den3 - p(3));

%%
%function Err = ObjDiff(x)
optimization()

% function fval = optimization(C1,T2)

[x,fval] = patternsearch(@optimization,[1.5 0.1]);
% end

```

## Appendix 2

### MATLAB Code for objective function

(1)

```
function E = Obj_Diff_Firs_FourTL(x)
f = @(w)(0.38*w - (abs((1 - exp(-1j*w))./(x(1)+x(2)*exp(-1j*w) + ...
x(3)*exp(-2j*w) + x(4)*exp(-3j*w)+ x(5)*exp(-4j*w))))).^2;
E = integral(f,0,pi-0.1);
```

(2)

```
function Err = optdiffmain(Z)
C1 = 25./Z(1);
C3 = 25./Z(3); C4 = 25./Z(4);
T2 = (Z(2) - 50)./(Z(2) + 50);
b0 = (C1 + C3 + C4 + C1*C3 + C1*C4 + C1*T2 + C3*T2 + C4*T2 + ...
2*C1*C3*T2 + 2*C1*C4*T2 + C1*C3*T2^2 + C1*C4*T2^2 + 1)./T2;
b1 = (- C1 - C3 - C4 - C1*T2^2 - C3*T2^2 - C4*T2^2 - T2^2 - ...
2*C1*T2 - 2*C3*T2 - 2*C4*T2 - 3)./T2;
b2 = (C1*T2^2 - C3 - C4 - C1 + C3*T2^2 + C4*T2^2 + 3*T2^2 - ...
2*C1*C3 - 2*C1*C4 - 4*C1*C3*T2 - 4*C1*C4*T2 - 2*C1*C3*T2^2 - ...
2*C1*C4*T2^2 + 3)./T2;
b3 = (C1 + C3 + C4 + C1*T2^2 + C3*T2^2 + C4*T2^2 - 3*T2^2 + ...
2*C1*T2 + 2*C3*T2 + 2*C4*T2 - 1)./T2;
Err = (5.75 - b0).^2 + (-3.84 - b1).^2 + (-1.7528 - b2).^2 + ...
(-0.15517 - b3).^2;
```

(3)

```
function E = Obj_Integ_First(x)
f = @(w)(0.33./w - (abs((1-exp(-1j*w))./(x(1)+x(2)*exp(-1j*w) + x(3)*exp(-2j*w) + x(4)*exp(-3j*w))))).^2;
%f = @(w) (0.33./w - abs( (1-exp(-1j*w))./(exp(-1j*w)+x(1)).*(exp(-1j*w)+x(2)).*(exp(-1j*w)+x(3)) ) ) );
E = integral(f,0.314,pi);
```

```
% plotting of Integrator
clear all
clc
w = 0.1:0.05:pi;
%% Ideal
Hideal = 1./w;
plot(w,Hideal);
axis([0 pi 0 pi])
xlabel('Frequency')
ylabel('Magnitude')
grid
hold on
%% My proposed Pattern search Paper
S = [3.632 1.22 -0.361 -0.03 -4.799 2.151 1.662];
Hmy = abs((S(1)+S(2)*exp(-1j*w)+S(3)*exp(-2j*w)+S(4)*exp(-3j*w))./(1+S(5)*exp(-1j*w)+S(6)*exp(-
2j*w)+S(7)*exp(-3j*w)));
```

```

plot(w,Hideal,w,Hmy,'r');
%%
% S = [0.8657 0.5895 0.0543 -0.4975 -0.5025];
S = [0.866 0.589 0.05 -0.499 -0.501];
% S = [0.241 1.247 0.985 -0.316 -1.251]
Hupad = abs((S(1)+S(2)*exp(-1j*w)+S(3)*exp(-3j*w))./(1+S(4)*exp(-1j*w)+S(5)*exp(-2j*w)));
plot(w,Hupad,'r');

%%
S = [3.632 1.22 -0.361 -0.03 -4.799 2.151 1.662];
Hmy = abs((S(1)+S(2)*exp(-1j*w)+S(3)*exp(-2j*w)+S(4)*exp(-3j*w))./(1+S(5)*exp(-1j*w)+S(6)*exp(-2j*w)+S(7)*exp(-3j*w)));
plot(w,Hmy,'r');

%% Menakshi
Hmmk = abs((0.3815*(exp(1j*w) + 2.285).*(exp(2j*w) - 0.247*exp(1j*w) + 0.04543))./(exp(2j*w).*(exp(1j*w) - 1)));
plot(w,Hmmk,'*')
%% Ngo
Hngo = abs((exp(1j*w) + 2.368).*(exp(1j*w) - 0.2167*exp(1j*0.9427)).*(exp(1j*w) - 0.2167*exp(-1j*0.9427))./(2.7925*exp(2j*w).*(exp(1j*w)-1)));
plot(w,Hngo,'x');

%% Al-Aloui
Hal = abs((0.8416*exp(2j*w) + 0.6635*exp(1j*w) + 0.0499 )./(exp(2j*w) - 0.4438*exp(1j*w) -0.5549));
plot(w,Hal,'o')
%% Upadhyay
Hupad = abs((0.8657*(1 + 0.681*exp(-1j*w) + 0.0628*exp(-2j*w)))./(1 - 0.4975*exp(-1j*w) - 0.5025*exp(-2j*w)));
plot(w,Hupad,'+');
legend('Ideal','My','MMK','Ngo','Al-Aloui','Upad')

%%
                                ERROR Calculation

%% My proposed Pattern search best
S = [3.632 1.22 -0.361 -0.03 -4.799 2.151 1.662];
Err_My = @(x) 1./x - abs((S(1)+S(2)*exp(-1j*x)+S(3)*exp(-2j*x)+S(4)*exp(-3j*x))./(1+S(5)*exp(-1j*x)+S(6)*exp(-2j*x)+S(7)*exp(-3j*x)));
Q = quad(Err_My,0.1,pi) % 2.9210e-04(0.1,pi), 2.9210e-04(0.5 to pi) -1.7772e-04(1 to pi)

%% Menakshi
Err_Menak = @(x) 1./x - abs((0.3815*(exp(1j*x) + 2.285).*(exp(2j*x) - 0.247*exp(1j*x) + 0.04543))./(exp(2j*x).*(exp(1j*x) - 1)));
Q = quad(Err_Menak,0.1,pi) % -0.0232(0.1 to pi) -0.0224(0.5 to pi)

```

```

%% Ngo
Err_Ngo = @(x) 1./x - abs((exp(1j*x) + 2.368).*(exp(1j*x) - 0.2167*exp(1j*0.9427)).*(exp(1j*x) -
0.2167*exp(-1j*0.9427))./(2.7925*exp(2j*x).*(exp(1j*x)-1)));
Q = quad(Err_Ngo,0.1,pi) % 0.1066

%% Al-Aloui
Err_Al = @(x) 1./x - abs((0.8416*exp(2j*x) + 0.6635*exp(1j*x) + 0.0499 )./(exp(2j*x) - 0.4438*exp(1j*x) -
0.5549));
Q = quad(Err_Al,0.1,pi) % 0.0263

%% Upadhayay
Err_Upad = @(x) 1./x - abs((0.8657*(1 + 0.681*exp(-1j*x) + 0.0628*exp(-2j*x)))./(1 - 0.4975*exp(-1j*x) -
0.5025*exp(-2j*x)));
Q = quad(Err_Upad,0.1,pi)

%%          ERROR PLOT
%% Error Plot
%hold off
Err1 = ((Hideal - Hmy)./Hideal).^2;
Err2 = ((Hideal - Hmmk)./Hideal).^2;
Err3 = (Hideal - Hngo);
Err4 = (Hideal - Hal);
Err5 = (Hideal - Hupad);
plot(w,Err1)
xlabel('Frequency')
ylabel('Magnitude')
grid
hold on
plot(w,Err2,'r')
plot(w,Err3,'g')
plot(w,Err4,'')
legend('My','MMK','NGO','AL-Aloui')

```

## Objective Fuction for First order differentiator Optmization

```
function Err = myfirstdiff(S)
%w = 0:0.2:pi;
y = @(w) abs(w - abs((1 - exp(-1j*w))./(S(1) + S(2)*exp(-1j*w) )));
Err = quad(y,0,pi-0.4);
```

### Appendix 3

#### MATLAB Command Window

Written by © Shafqat A. KHAN on 10 march 2013

This Program calculate Transmission Line System function  
 Enter the number of Open Stub Line K = 0  
 Enter the number of Short Stub Line L = 1  
 Enter the number of Series Transmission Line M = 1

The Total number of network element N is 2

Choose from following :

1 --> Open Stub

2 --> Short Stub

3 --> Serial Tx. Line

choose the 1th element --> 2

choose the 2th element --> 3

$$\frac{(1 - T_2^2) z^2 + T_2^2 - 1}{(C_1 T_2^2 - T_2^2 + C_1 T_2) z^2 + (C_1 T_2^2 - C_1 + T_2^2 + 1) z - C_1 - C_1 T_2 - 1}$$

>>

### Appendix

TABLE VI  
 EQUATIONS SHOWING THE CONVERSIONS BETWEEN *S* AND *T* PARAMETERS

$S_{11} = \frac{T_{21}}{T_{11}}$	$T_{11} = \frac{1}{S_{21}}$
$S_{12} = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{11}}$	$T_{12} = \frac{-S_{22}}{S_{21}}$
$S_{21} = \frac{1}{T_{11}}$	$T_{21} = \frac{S_{11}}{S_{21}}$
$S_{22} = \frac{-T_{12}}{T_{11}}$	$T_{22} = \frac{S_{12}S_{21} - S_{11}S_{22}}{S_{21}}$

*title('The Sine of 0 to 2\pi')*

*xlabel('t = 0 to 2\pi')*

*ylabel('sin(t)')*

*text(3\*pi/4,sin(3\*pi/4),...*

*\leftarrow sin(t) = .707',...*

*'FontSize',16)*

*text(pi,sin(pi),'\leftarrow sin(t) = 0',...*

*'FontSize',16)*

*text(5\*pi/4,sin(5\*pi/4),'sin(t) = -.707\rightarrow',...*

*'HorizontalAlignment','right',...*

*'FontSize',16)*