STUDY OF COSMOLOGICAL MODELS IN MODIFIED THEORIES OF GRAVITATION

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in

MATHEMATICS

by

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DECLARATION

I declare that the research work reported in this thesis entitled "**Study of Cosmological Models in Modified Theories of Gravitation**" for the award of the degree of *Doctor of Philosophy in Mathematics* has been carried out by me under the supervision of *Dr. Chandra Prakash Singh*, Department of Applied Mathematics, Delhi Technological University, Delhi, India.

The research work embodied in this thesis, except where otherwise indicated, is my original research. This thesis has not been submitted by me earlier in part or full to any other University or Institute for the award of any degree or diploma. This thesis does not contain other person's data, graphs or other information, unless specifically acknowledged.

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CERTIFICATE

This is to certify that the thesis entitled **"Study of Cosmological Models in Modified Theories of Gravitation"** submitted by **Mr. Vijay Singh** in the Department of Applied Mathematics, Delhi Technological University, Delhi, India for the award of degree of *Doctor of Philosophy in Mathematics*, is a record of bonafide research work carried out by him under my supervision.

To the best of my knowledge the work reported in this thesis is original and has not been submitted to any other Institution or University in any form for the award of any degree or diploma.

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Dedicated to My Parents

Shri Niranjan Singh

&

Smt. Devki Devi

Contents

Preface

In recent years, cosmology has become a precision science after many observational and experimental data coming from numerous projects such as type Ia supernova (SNe Ia), Cosmic Microwave Background (CMB), Large Scale Structure (LSS), gravitational lensing, Wilkinson Microwave Anisotropy Probe (WMAP), Baryon Acoustic Oscillations (BAO), Sloan Digital Sky Survey (SDSS), PLANCK, etc. These observations indicate that the expansion of the Universe at later stage is in an accelerated phase. The measurement of CMB and galaxy spectrum also suggest that about two third of the critical energy density in a spatially flat isotropic Universe, seems to be stored in a form of unknown component known as 'dark energy' whose properties are still mysterious. This confronts the fundamental theories with great challenges and also makes the research on this problem a major endeavour in modern astrophysics and cosmology. Many cosmological models have been proposed to explain the accelerating Universe. The modification of Einstein's general theory of relativity is one of the attempt to discuss the dark energy phenomena. In the past two decades, a number of modified theories such as higher derivative theory, Gauss-Bonnet $f(G)$ theory, $f(R)$ theories, $f(T)$ theory, $f(R,T)$ gravity etc. have been proposed to explain the current epoch of cosmic acceleration.

Nowadays, the study of dark energy models are of great interest in such modified theories of gravitation. This thesis is devoted to investigate both isotropic Friedmann-Robertson-Wlaker (FRW) and anisotropic Bianchi cosmological models in some of the modified theories of gravitation, namely, higher derivative (HD) theory, $f(R)$ theory and $f(R,T)$ gravity by taking various matter contents such as perfect fluid, minimally coupled quintessence or phantom scalar field, and normal or phantom tachyonic fields. A theoretical approach has been followed to understand the cosmic acceleration in the framework of these modified theories of gravitation. Some exact solutions with perfect fluid and scalar fields for known history of evolution of the Universe have been found in HD theory of gravity. In HD theory, the possibility of a singularity free model (emergent Universe) has also been discussed with quintessence and phantom scalar fields. The reconstruction of $f(R)$ and $f(R,T)$ has also been done for some known history of evolution of the Universe in isotropic and anisotropic space-time. In $f(R,T)$ gravity, we have also proposed a non-singular power-law model with the particle creation phenomena.

The characteristics of the dynamical evolution of each cosmological model have been performed. A number of viability criteria such as existence of exact real solutions, stability criteria, compatibility with cosmological observations etc. have been carried out for each cosmological model. It has been shown that the inclusion of both types of matter (baryonic and exotic), opens an enormous dynamical complexity of possible evolutional paths of the Universe. It leads to a new and natural possibility of an unified description of the cosmological evolutions (an inflationary epoch, a radiation-dominated phase, a matter-domination era and finally the present accelerated expansion) in some of the cases.

The thesis entitled **"Study of Cosmological Models in Modified Theories of Gravitation"** comprises nine chapters. The bibliography and the list of publications have been given at the end of the thesis.

Chapter 1 is introductory in nature which gives a short review of the past and present understanding about the Universe through the important equations. In this chapter, a survey of the literature has been made and the principle problems plagued in cosmology have been described. Some important cosmological parameters, which describe the physical and geometric properties of the Universe, have also been introduced. The alternatives of dark energy and some modified gravitational theories which are related to the thesis have been introduced briefly. The purpose of this chapter is to provide the motivation of the work carried out in the thesis.

Chapter 2 explores the dynamics of the Universe in FRW models containing a perfect fluid and a minimally coupled scalar field with scalar potential in HD theory of gravity. The exact cosmological solutions for flat, closed and open models have been obtained by assuming the scalar potential and the scale factor as functions of the scalar field. A number of evolutionary phases have been discussed which are physically interesting for the description of the early and present-day Universe. The result of this chapter has been published as a research paper entitled “FRW Models With a Perfect Fluid and a Scalar Field in Higher Derivative Theory", *Modern Physics Letter A* **26,** 1495 (2011).

In **Chapter 3**, the exact solutions have been found by assuming the power-law expansion of the scale factor for flat FRW model. The expression for scalar field potential has been obtained and the properties of scalar field and other physical parameters have been discussed in detail. We have noticed some new results which are different from previous chapter. This chapter is based on a research paper entitled “Power-Law Expansion and Scalar Field Cosmology in Higher Derivative Theory", published in *International Journal of Theoretical Physics* **51,** 1889 (2012).

Chapter 4 deals with the possibility of the emergent Universe (EU) filled with a scalar (or tachyonic) field of normal or phantom form, minimally coupled to gravity in a spatially homogeneous and isotropic flat FRW model in the framework of HD theory. The stability of the solutions and their physical behaviors have been discussed in detail. The content of this chapter has been published as a research paper entitled “Emergent Universe with Scalar (Or Tachyonic) Field in Higher Derivative Theory", in *Astrophysics and Space Science* **339,** 101 (2012).

In **chapter 5**, we have studied $f(R)$ theory of gravity in a locally-rotationally-symmetric Bianchi I anisotropic space-time model with a perfect fluid. A functional form of $f(R)$ has been reconstructed by assuming the constant deceleration parameter and the shear scalar proportional to the expansion scalar. We have discussed the stability of the reconstructed functional form of $f(R)$. The work presented in this chapter comprises the results of a research paper entitled “Functional Form of $F(R)$ with Power-Law Expansion in Anisotropic Model", published in *Astrophysics and Space Science* **346,** 285 (2013).

In **chapter 6,** a particular form of $f(R,T) = R + 2f(T)$ have been reconstructed for de Sitter and power-law models within the framework of a flat FRW space-time. The gravitational field equations have been considered with two fluid sources, one is perfect fluid and other is due to the $f(R,T)$ gravity, the later one has been considered as an exotic fluid. The behaviour of both the models have been discussed through the equation of state (EoS) parameters of exotic matter and effective fluid. Both the models exhibit a rich behaviour of the early and late-time evolution of the Universe. This chapter is based on a research paper entitled “Reconstruction of Modified $f(R,T)$ Gravity with Perfect Fluid Cosmological Models", published in

General Relativity and Gravitation **46,** 1696 (2014).

In **chapter 7,** we have reconstructed the $f(R,T)$ gravity with the normal or phantom scalar field for a flat FRW model. We have first explored a model where the potential is a constant and the Universe evolves as a de Sitter expansion. We have also explored another model where the scalar field potential and the scale factor evolve exponentially with the scalar field. We have also compared our results with the recent observational data for the later model. It has been found that some values of parameters are consistent with SNe Ia and $H(z)$+SNe Ia data to describe accelerated expansion only whereas some give both decelerated and accelerated expansions with $H(z)$, WMAP7 and WMAP7+BAO+$H(z)$ observational data. The result of this chapter has been published as a research paper entitled “Modified $f(R,T)$ Gravity Theory and Scalar Field Cosmology", in *Astrophysics and Space Science* **355,** 2183 (2014).

Chapter 8 is devoted to study the theoretical and observational consequences of thermodynamics of an open system in a flat FRW model which allows creation of matter within the framework of $f(R,T)$ theory. The simplest model $f(R,T) = R+2f(T)$ with “gamma-law" equation of state $p=(\gamma-1)\rho$ has been assumed to obtain the exact solution. A power-law expansion model has been proposed by considering the natural phenomenological particle creation rate $\psi = 3\beta nH$, where β is a pure number of the order of unity, n is the particle number density and H is the Hubble parameter. Some kinematic tests such as lookback time, luminosity distance, proper distance, angular diameter *versus* redshift have been discussed in detail to observe the role of particle creation in early and late time evolution of the Universe. This chapter comprises the result of a research paper entitled “Friedmann Cosmology with Matter Creation in Modified $f(R,T)$ Gravity", published in *International Journal of Theoretical Physics*, DOI 10.1007/s10773-015-2767-z (2015).

Finally, the summary of the results and the future perspectives of the work have been reported in chapter 9. The bibliography and list of author's publications have been given at the end of the thesis.

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Chapter 1

Introduction

This introductory chapter gives a short review of the basic mathematical equations which govern the evolution of the Universe. This chapter also describes several issues related to gravitation and cosmology, namely, problems related to the early inflation and the late-time cosmic acceleration. Some modified theories of gravity are briefly introduced which explain the dark energy and dark matter phenomena. The foundation of this chapter provides the motivation of the work carried out in this thesis.

Cosmology is the study of the origin and evolution of the entire Universe on the large scale of space and time. The study of cosmology depends crucially on our understanding of the gravitational interaction. Our understanding of the Universe has increased dramatically in recent years - both observationally and theoretically. In recent years, cosmology has become a prominent branch of science after many astronomical observations. The most crucial observation is the accelerated expansion of the Universe. Let us begin the introduction with the concept of space-time geometry of the Universe.

1.1 Space-time geometry

Modern cosmology began nearly 100 years ago with the development of Einstein's general theory of relativity (GTR) in 1917 [1]. General relativity is a theory of space-time and gravitation, which is widely accepted as a fundamental theory to describe the geometrical properties of space-time. According to GTR, gravity is the geometry of the four-dimensional curved space-time. A space-time geometry is described by a line-element giving the space-time distance between any two nearby points. The coordinates of the four-dimensional space-time are (x^0, x^1, x^2, x^3), where $x^0 = t$ is a time coordinate and x^1, x^2, x^3 are space coordinates. We use the Greek index to denote an arbitrary space-time coordinate, x^μ, where μ can have any values $0, 1, 2, 3$. The line-element, ds^2 between the points separated by coordinate intervals, dx^μ, is given in the tensorial form as

$$ds^2 = \sum_{\mu,\, \nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu. \tag{1.1.1}$$

We use the Einstein's summation convention rule. Here, the coefficients $g_{\mu\nu}$ are the functions of space-time coordinates x^μ, subject to the restriction $g = \mid g_{\mu\nu} \mid \neq 0$. The quantities $g_{\mu\nu}$ are the components of a covariant symmetric tensor of rank two, called metric tensor. They have, in principle, the dimension of the distance squared. In an orthogonal coordinate system the coordinate lines are everywhere orthogonal to each other and the metric is then diagonal. In this thesis we use only orthogonal coordinate systems. Here, $dx^0 = dt$, $dx^1 = dx$, $dx^2 = dy$ and $dx^3 = dz$. The line-element (1.1.1) represents the curved geometry. Thus, according to GTR, the space is curved in a gravitational field and the geometry of space in gravitational field is Riemannian. The contravariant metric tensor $g^{\mu\nu}$ is defined as

$$g^{\mu\nu} = \frac{\text{cofactor of } g_{\mu\nu} \text{ in } g}{g}. \tag{1.1.2}$$

The metric tensor $g^{\mu\nu}$ is also symmetric tensor of rank two. This tensor is reciprocal of $g_{\mu\nu}$, is called conjugate metric tensor. Throughout the thesis the summation convention is used with Greek indices running from 0 to 3 and geometrized units are used. A 'space-like convention' for the metric has been adopted in the thesis such that when it is diagonalised, it has signature $(-,+,+,+)$.

1.2 Homogeneous and isotropic metric

The simplest assumption for building the standard cosmological models in the framework of GTR is the *cosmological principle (CP)*. Our Universe contains gravitationally clustered matter in galaxies, and unclustered energy. According to CP, galaxies are uniformly distributed and Universe looks likes a uniform density cloud of dust on a very large scale. The CP is related to two precise mathematical properties of the Universe. According to CP, the Universe at a very large scale $(\gg 100\,Mpc)$ is homogeneous and isotropic[1] at each instant of cosmic time. This means that the curvature of space-time must be same everywhere and into every direction, but it may change along the time-axis. Then the Universe has to be maximally symmetric as far as three dimensionally space is concern.

On the large scale the simplest example of a homogeneous and isotropic line-element is described by

$$ds^2 = -c^2dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{1.2.1}$$

where $a(t)$ is a function of time coordinate t, which is related to the expansion (possible contraction) of the Universe. The time-dependent factor $a(t)$ is called *scale factor*. Here, c is the velocity of light in vacuum. The line-element (1.2.1) is called the flat *Robertson-Walker metric* (RW). It is called the *Friedmann-Robertson-Walker* (FRW) metric when the scale factor obeys the Einstein's field equations (see, section 1.4).

In spherical coordinates where $x^0 = ct$, $x^1 = r\sin\theta\ \cos\varphi$, $x^2 = r\sin\theta\ \sin\varphi$, $x^3 = r\cos\theta$, the line-element (1.2.1) can be written as [2]

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right], \tag{1.2.2}$$

where k is a constant describing the curvature of the spatial sections and having values -1, 0 and $+1$ for open, flat and closed Universes, respectively, and $0 \le \theta \le \pi$ and $0 \le \varphi \le 2\pi$. The FRW model (1.2.2) describes the time evolution of a homogeneous and isotropic Universe that gets larger in time as $a(t)$ increases and smaller as $a(t)$ decreases. All the information about the evolution of the Universe is contained in this one function determined by the Einstein's field equations. The coordinates $(ct,\ r,\ \theta,\ \varphi)$ of FRW metric are *comoving coordinates* [2]. I have worked with FRW line-element (1.2.2) in a large part of this thesis.

[1]The Universe is homogeneous means the space has same metric properties at all points and isotropic means the space has same measures in all directions. This is said to be the Cosmological Principle (CP) which leads to the requirement that the Universe is both homogeneous and isotropic at large scale. The homogeneity and isotropy are symmetries of space and not of space-time.

[2]Comoving means that the coordinate system follows the expansion of space, so that the space coordinates of objects which do not move remains the same.

1.3 Homogeneous and anisotropic metric

The observational data of Cosmic Microwave Background (CMB) [3] and Wilkinson Microwave Anisotropy Probe (WMAP) [4] admit the existence of anisotropy which gains a lot of interest. It is supposed that the CMB anisotropies at small angular scales are the base for the formation of discrete structures. The theoretical arguments also support the existence of an anisotropic phase that approaches to an isotropic phase in late time evolution. Amongst the various families of homogeneous but anisotropic[3] geometries, the most well-known are the Bianchi type I–IX space-time line elements [5]. These homogeneous and anisotropic line elements play a significant role to describe the behavior of the early stages of the evolution of the Universe. Unlike to isotropic FRW space-time metric, Bianchi type models have different scale factors in each direction, which introduces the anisotropy in the system. The simplest example of a homogeneous and anisotropic model is the Bianchi type-I (B-I), which is more general than flat FRW line-element. The line-element of the B-I model is described by

$$ds^2 = -c^2dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1.3.1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the scale factors, called directional scale factors in the direction of coordinate axes. If $A = B = C$, the line-element $(1.3.1)$ reduces to flat RW model $(1.2.1)$, and if $A \neq B = C$ (or $A = B \neq C$ or $A = C \neq B$), the line-element $(1.3.1)$ is said to be the locally-rotationally-symmetric (LRS) B-I model whereas $A \neq B \neq C$ gives totally anisotropic B-I model.

1.4 Gravitational action and Einstein's field equations

The general theory of relativity is described by a gravitational action known as the Einstein-Hilbert (EH) action. The action is assumed to be a function of the metric, connected by Levi-Civita connection, which is of second order in its derivatives of metric. The simplest EH action for gravity with the inclusion of matter fields, which

[3] spatial sections are directional dependent.

yield the Einstein's field equations, is given by [2]

$$S = \int \left(\frac{1}{2\kappa} R + \mathscr{L}_m \right) \sqrt{-g}\, d^4x, \tag{1.4.1}$$

where $R = g_{\mu\nu} R^{\mu\nu}$ (here $R^{\mu\nu}$ is Riemann curvature tensor) is the Ricci scalar curvature, $\mathscr{L}_m$ is the matter Lagrangian density of any matter fields, and $\kappa = 8\pi G c^{-4}$, where G is Newton's gravitational constant. The reduced Plank mass can be defined by $M_p^{-2} = 8\pi G$.

Varying the action (1.4.1) with respect to the metric tensor $g_{\mu\nu}$, the Einstein's field equations which couple the geometry of the Universe with the matter content, are given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{\kappa}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathscr{L}_m)}{\delta g^{\mu\nu}} = \kappa T_{\mu\nu}, \tag{1.4.2}$$

where $R_{\mu\nu}$ and $T_{\mu\nu}$ are the Ricci tensor and the energy-momentum tensor, respectively. The solution of Einstein's field equations describes the evolution of the Universe because it describes the whole of space-time.

In Einstein's theory, the matter content is described by *energy-momentum tensor*, also called *stress-energy tensor* $T_{\mu\nu}$, which carries the information of energy density, momentum density, pressure and stress. The energy-momentum tensor for a perfect fluid (frictionless continuous matter) is given by

$$T_{\mu\nu} = (\rho c^2 + p) u_\mu u_\nu + p g_{\mu\nu}, \tag{1.4.3}$$

where ρc^2 is the energy density, p is the pressure of the perfect fluid and u^μ is the four-velocity vector such that $u_\mu u^\nu = -1$. The distribution of matter content of the Universe, i.e, $T_{\mu\nu}$, is only a function of t not of θ and φ due to spatial homogeneity.

The Universe not only has non-relativistic matter, but it also has electromagnetic radiation, dark matter and dark energy. The fraction of energy distribution of visible galaxies and gas clouds is estimated to be about 4 to 5% in the present Universe. About 25% contribution comes from dark matter which is suppose to be clustered or clumped around galaxies. The Universe is suppose to have 70% matter in the form of dark energy. In section 1.9, we shall discuss about dark energy in more detail.

1.5 Friedmann's equations

In general theory of relativity, the cosmic evolution of the Universe is described by a cosmological model which requires three ingredients:

1. The cosmological principle, which leads to the FRW line-element described by Eq. (1.2.2).

2. Weyl's postulate, which requires the energy-momentum tensor as defined in Eq. (1.4.3) for perfect fluid.

3. The Einstein's field equations (1.4.2).

In the comoving coordinate system, the above three ingredients lead to the following two independent equations.

$$\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho, \tag{1.5.1}$$

$$\frac{2\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2} = -\frac{4\pi G}{c^2}p, \tag{1.5.2}$$

where a dot denotes differentiation with respect to cosmic time t. The equations (1.5.1) and (1.5.2) are called the *Friedmann equations*, were first derived by Alexander Friedmann in 1922 [6] which describe the evolution of the Universe.

Equation (1.5.2) can be simplified as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho c^2+3p). \tag{1.5.3}$$

From (1.5.1) and (1.5.3), we get

$$pd(a^3) = -d(\rho c^2 a^3), \tag{1.5.4}$$

which is the law of conservation of energy and analogous to the conventional conservation equation

$$pdV = -dE, \tag{1.5.5}$$

where $V = a^3$ is the volume and $E = \rho V$ is the total mass-energy in volume V. The Eq. (1.5.5) is said to be the *first law of thermodynamics*.

Equation (1.5.4) or (1.5.5) tells that the work done by the pressure in any change in volume equals loss of energy inside. Equation (1.5.4) can also be obtained by conservation of energy-momentum tensor $\nabla^{\nu}T_{\mu\nu}$ (here, ∇_{ν} is the covariant derivative) which is a consequence of contracted Bianchi identities. Equation (1.5.4) can be rewritten as

$$d(\rho c^2) = -3(\rho c^2 + p)\frac{\dot{a}}{a}. \tag{1.5.6}$$

Equations (1.5.1) and (1.5.2) have three unknowns functions, namely, $a(t)$, $\rho(t)c^2$ and $p(t)$. We need one more relation to solve them completely. This is provided by *equation of state* (EoS), which relates energy density ρc^2 to the pressure p by an equation

$$p = (\gamma - 1)\rho c^2, \tag{1.5.7}$$

where γ is an EoS parameter. In general relativity, γ is treated as a constant and its value lies in the range $0 \leq \gamma \leq 2$.

1.6 Einstein's modified field equations

Friedmann [6] solved the Einstein's field equations for FRW line-element and found a non-static solution. But, to achieve a static Universe, Einstein modified his field equation (1.4.2) by introducing an additional term Λ which is known as *cosmological constant*.

The EH action (1.4.1) including the cosmological constant term Λ is modified to [2]

$$S = \int \left[\frac{1}{2\kappa}(R - 2\Lambda) + \mathscr{L}_m\right]\sqrt{-g}\, d^4x, \tag{1.6.1}$$

which yields the modified Einstein's field equations as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{1.6.2}$$

Here, Λ has dimension of inverse length squared.

In comoving coordinate system, the Friedmann equations with cosmological constant from (1.6.2) for metric (1.2.2) and energy-momentum tensor (1.4.3) lead to the following equations:

$$\frac{\dot{a}^2}{a^2}+\frac{kc^2}{a^2}-\frac{\Lambda}{3} = \frac{8\pi G}{3}\rho, \tag{1.6.3}$$

$$\frac{\ddot{a}}{a}-\frac{\Lambda}{3} = -\frac{4\pi G}{3c^2}(\rho c^2+3p). \tag{1.6.4}$$

The Λ-term was abandoned by Einstein himself after Edwin Hubble's discovery of expanding Universe in 1929 [7]. However, it appeared again in 1998 with the discovery of the accelerating Universe [8–12]. In section 1.9, we shall discuss this in detail.

1.7 Some cosmological parameters

Let us introduce some theoretical and observational cosmological parameters which are frequently used in this thesis.

1.7.1 Hubble parameter

During 1920-1930, it was discovered that the Universe was composed of a vast collection of galaxies, each assembling our own Milky Way. When a galaxy is observed at visible wavelengths, there is a wavelength shift of a light signal which is the difference between the wavelength λ_{em} emitted by source and the wave length λ_{rec} received by the observer. This change in wavelength is defined through the redshift z, which is given by

$$z=\frac{\lambda_{rec}-\lambda_{em}}{\lambda_{em}}. \tag{1.7.1}$$

For non-relativistic motion, it can be stated as

$$z=\frac{\triangle\lambda}{\lambda}\approx\frac{v}{c}, \tag{1.7.2}$$

where v is the receding velocity between source and observer.

In 1929, Edwin Hubble [7] found that the redshift z was proportional to the distance d of the light-emitting galaxy, that is,

$$z=\frac{H_0}{c}d, \tag{1.7.3}$$

where H_0 is the *Hubble constant* which gives the recession speed per unit separation between the receiving and emitting galaxies. From (1.7.2) and (1.7.3), we get

$$v = H_0 d, \tag{1.7.4}$$

Equation (1.7.4) states that the speed of receding galaxies is proportional to the separation between them which is known as *Hubble's law*. In an expanding Universe, H_0 is taken to be positive. It also determines the expansion rate of the Universe. The accurate value of H_0 is still unknown. The current value of Hubble constant is $H_0 = (69.32 \pm 0.80)$ $km/s/Mpc$ which has been found by combined data of WMAP+CMB+BAO+H(z) [13]. The subscript '0' stands for the present epoch $H_0 = H(t_0)$. Although usually quoted in units of $km/s/Mpc$, the Hubble's law (1.7.4) shows that H_0 has the dimension of inverse time, $t_H = H_0^{-1}$ which is called the *Hubble time*. It is used to determine the age of the expanding Universe since the Big-Bang occurred. The present value of Hubble time has been observed to be $t_H = 13.77$ GYr (1 $GYr = 10^9$ $years$ = 1 $Billion\ years$).

Due to the presence of matter and energy in the Universe, the Hubble constant is not expected to be a constant with respect to time. The gravitational attraction between matter and energy slows down the expansion, which leads to a decreasing expansion rate $H(t)$, i.e., a *decelerating Universe*. Therefore, the Hubble parameter parameterizes the expansion rate of the Universe and is defined by

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \tag{1.7.5}$$

The time-varying Hubble parameter (1.7.5) measures the rate of change of the scale factor $a(t)$ and provides a way to link the observations with a proposed model using the scale factor. It is to be noted that we can expect the constant expansion rate throughout its history, $H(t) = H_0$ only in a empty space.

1.7.2 Critical density

It is useful to express the mass density in terms of Hubble constant H. The critical density (ρ_c) in terms of Hubble parameter of the Universe is defined as

$$\rho_c = \frac{3H^2}{8\pi G}\,. \tag{1.7.6}$$

Since the Hubble constant is a function of time, the critical density also evolves with time. One may compute the present value of the critical density from the known value of H_0. It is to be noted that for the present Hubble constant $H_0 = (69.32 \pm 0.80)\ km/s/Mpc$, the present critical density has the value $\rho_c = (0.86 \pm 0.04) \times 10^{-29} gm/cm^3$.

1.7.3 Density parameter

The density parameter (Ω) determines the spatial geometry of our Universe. It is the ratio of the true (actual) density of the Universe at a given time to the critical density at that time, that is,

$$\Omega = \frac{\rho}{\rho_c}\,. \tag{1.7.7}$$

A closed, flat and open Universe correspond to $\Omega > 1$, $\Omega = 1$ and $\Omega < 1$, respectively. Observations have shown that the present Universe is very close to a spatially flat geometry ($\Omega \simeq 1$).

The total mass of the Universe is divided into two categories: baryonic[4], which may be luminous or non-luminous, and dark matter[5], which has only weak interaction. Therefore, the density parameter for total mass is given by

$$\Omega_m = \Omega_B + \Omega_{DM}. \tag{1.7.8}$$

The Friedmann equation (1.6.3) can be written in terms of the present values of density parameter as

$$\Omega \equiv \Omega_m + \Omega_k + \Omega_\Lambda = 1, \tag{1.7.9}$$

where $\Omega_m = \frac{\rho_m}{\rho_c}$, $\Omega_k = -\frac{kc^2}{a^2 H_0^2}$, and $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$.

1.7.4 Deceleration parameter

An important observational quantity is the deceleration parameter (DP). It measures the rate at which the expansion of the Universe is changing with time in

[4]Ordinary matter made of baryons (protons, neutrons) and electrons is referred to baryonic matter. Baryonic matter can clump to form atoms and molecules. Luminous matter (shining star) is baryonic matter. The inter stellar or intergalactic gas are non-luminous baryonic matter.

[5]Dark matter is made of exotic particles which have no electromagnetic interaction. The relativistic particles are the hot dark matter whereas the non-relativistic particles are said to be cold dark matter. They have different distinct effects in the formation of galaxies.

terms of the scale factor. It is denoted by q and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \,. \tag{1.7.10}$$

The sign of q characterizes the accelerating or decelerating nature of the Universe. The positive sign of q corresponds to decelerating model whereas the negative sign indicates an accelerating model.

1.8 Phases of the Universe

It is assumed that the history of the Universe is divided into four main different phases:

1. The pre-matter phase in which the matter had a density nearly equal to planck density and pressure $p = -\rho$.

2. The radiation-dominated phase where the matter was at a very high temperature.

3. The matter-dominated phase where matter in the galaxies is well approximated by a pressureless gas.

4. The present accelerating phase where some unknown matter with negative pressure is dominating.

Usually the field equations are solved and analyzed separately for these different epochs where the different kinds of matter are non-interacting. Using (1.5.7) in (1.5.6), we find

$$\rho = \rho(t_0)\left[\frac{a(t_0)}{a(t)}\right]^{-3\gamma}, \tag{1.8.1}$$

where t_0 is the present instant of time. We observe that the matter density in different phases is determined by the scale factor which is obtained from Eqs. (1.6.3) and (1.8.1) for a flat FRW model $(k = 0)$ and $\Lambda = 0$ as

$$a(t) = a(t_0)\left(\frac{t}{t_0}\right)^{\frac{2}{3\gamma}}, \qquad (\gamma \neq 0). \tag{1.8.2}$$

In case of $\gamma = 0$, one obtains

$$a(t) = a(t_0)\exp\left[\left(\frac{8\pi G\rho(t_0)}{3}\right)^{1/2} t\right]. \tag{1.8.3}$$

Let us discuss the early inflationary, radiation-dominated, matter-dominated and present accelerating phases in brief.

1.8.1 Inflationary phase

The inflationary phase as proposed by Alan Guth [14] in 1981, is basically a short period of rapid expansion in the very early Universe, at the end of which the description of the standard Big-Bang model is applied. Inflation is the most convincing explanation for the flatness, isotropy and homogeneity of the observed Universe. This phase not only resolves the flatness and horizon problems, but also explains a nearly flat spectrum of temperature anisotropies observed in cosmic microwave background (CMB). The inflationary scenario actually means a period of phase transition which is controlled by a scalar field [15]. The scalar field may contribute to the negative pressure and once the phase transition is over, the scalar field decays away and the inflationary expansion terminates. We shall discuss more about scalar field in section 1.10.1.

The inflationary phase of the Universe corresponds to $\gamma = 2/3$. Therefore, the relation between the energy density and pressure in (1.5.7) for $\gamma = 2/3$ is given by $p = -\rho c^2/3$. If $p = -\rho c^2/3$, then from (1.5.4), i.e., $d(\rho c^2 a^3) = -\rho c^2 d(a^3)/3$ implies

$$\rho \propto a^{-2}. \tag{1.8.4}$$

From Friedmann equation (1.5.1) for $k = 0$, we get $\dot{a}^2 \propto const.$ which gives $a \propto t$. Therefore, the expansion of the Universe is linear which is said to be the marginal inflation. Similarly, for $\gamma = 0$ we get $\rho = const.$ and $a \propto exp\sqrt{H_0}t$, which shows exponential expansion of the scale factor.

1.8.2 Radiation-dominated phase

In the early Universe, the expansion took place mainly due to relativistic particles, which describe the radiation-dominated era. During this period the Universe is filled with isotropic black body radiation due to which the rate of expansion of the Universe slows down. Indeed, there are stringent observational bounds on the abundances of light elements, such as deuterium, helium and lithium, which require that Big-Bang Nucleosynthesis (BBN), the production of nuclei other than hydrogen, takes place during radiation-domination [16].

The radiation-dominated phase of the evolution of the Universe corresponds to $\gamma = 4/3$. Therefore, Eq. (1.5.7) gives $p = \rho c^2/3$ for diffuse radiation in thermal equilibrium. If $p = \rho c^2/3$, then Eq. (1.5.4) implies

$$\rho \propto a^{-4}. \tag{1.8.5}$$

From Friedmann equation (1.5.1) for $k = 0$, we get $\dot{a}^2 \propto a^{-2}$ which gives $a \propto t^{1/2}$.

At present the fraction of radiation in the Universe is about 10^{-5} but Eq. (1.8.5) shows that as the radiation density goes at a^{-4} it would have been dominated when 'a' was small. As the Universe expanded it cooled and various light nuclei were formed. At a later time, neutral atoms were formed. At this stage the radiation became decoupled from matter and the radiation-dominated era was entered into matter-dominated era.

1.8.3 Matter-dominated phase

After a phase transition, radiation is decoupled from the matter and the Universe became matter-dominated as we observe today. Since the temperature of the Universe has fallen to around 3000 K, most of the particles have non-relativistic velocities ($v \ll c$). Therefore, during this phase the Universe is assumed to be filled with incoherent matter (dust) that uniformly occupies in the space exerting zero pressure ($p = 0$).

If $k = 0$ and $p = 0$, then (1.5.4) implies

$$\rho \propto a^{-3}. \tag{1.8.6}$$

From Friedmann's equation (1.5.1) for $k = 0$ it follows that $\dot{a}^2 \propto a^{-1}$ which gives $a \propto t^{2/3}$.

The transition, from radiation-dominated phase to matter-domination Universe, comes naturally since the matter energy density is inversely proportional to the volume and, therefore, proportional to a^{-3}, whereas the radiation energy density is directly proportional to a^{-4} and therefore it decreases more faster than the matter energy density with the evolution of the Universe.

There are some intermediate phases, for example, (i) Zel'dovich or stiff-matter phase where $\gamma = 2$, $p = \rho c^2$. This gives $\rho \propto a^{-6}$ and $a \propto t^{1/3}$. (ii) Vacuum Universe where $p = 0$ and $\rho = 0$ for any values of γ. This gives $a = const$. These two phases also describe several important phenomena of the evolution of Universe.

1.8.4 Accelerating phase

In the past two decades, cosmology has shown tremendous progress through observational/ experimental data from numerous projects and accurate theoretical concepts. Therefore, cosmology has become a precision science to understand the early and late-time evolution of the Universe. The rapid development in observational cosmology which started during late 1990s shows that the Universe passes two phases of cosmic acceleration: The first cosmic accelerated phase which is known as the inflationary phase is believed to have occurred prior to the radiation-dominated phase as we have already discussed in section 1.8.1.

In the early 1990s, one thing was very clear about the expansion of the Universe. According to the theoretical point of view, the expansion of the Universe after the inflationary phase had to be slow. But in 1998, the Hubble Space Telescope (HST) [8] observations of very distant supernovae showed that the expansion of the Universe has not been slowing down due to gravity, as everyone thought, it has been accelerating. This second cosmic accelerated phase which is known as the late-time cosmic acceleration, is believed to have started after the matter-dominated phase. This transition from decelerating phase to the accelerating phase has been confirmed by a number of observations such as the measurements of SNe Ia [9], CMB [3], Large Scale Structures (LSS) [17], Wilkinson Microwave Anisotropy Probe (WMAP) [13], Baryon Acoustic Oscillations (BAO) [18] and very recent Planck Collaboration [19].

Theorists have suggested three sorts of explanations for this acceleration: it may be a result of cosmological constant; there may be some strange kind of energy-fluid that filled space and the last possibility may be that there is something wrong with Einstein's theory of gravity and a new theory could include some kind of field that creates this cosmic acceleration. Theorists still don't know which one is correct to explain this acceleration, but they have given the name to this unknown phenomena known as *dark energy*. [20]. let us discuss all the three possibilities one by one in next sections.

1.9 The standard ΛCDM model

The quest to understand the dynamics of evolution of the Universe, the modern cosmology requires two outstanding concepts [21]: (i) the matter which does not interact with the electromagnetic force - known as *dark matter* (DM), and (ii) the unknown form energy, tends to increase the rate of expansion of the Universe, known as dark energy (DE). DE and DM, detectable only because of their effect on the visible matter around them. DE makes up over roughly 70 % of all the energy in the Universe, DM is about 25 % and rest 5 % is the visible part [4, 13, 17, 19, 21–23].

The DE is a hypothetical type of energy that fills most of the space which accelerates the expansion of the Universe. The first and simplest explanation for DE is that it may be the property of space itself. Albert Einstein was the first person to realize that the empty space is not empty but it may possess its own energy. Because this energy is a property of space itself, more of this energy would appear due to the existence of more space. Due to this result, this form of energy would cause the rapid expansion of the Universe.

The investigations on DE have shown that its properties are very close to that of a cosmological constant [10, 24–26]. Therefore, the cosmological constant has been reconsidered as a prime candidate to represent the unknown energy density of space which is responsible for cosmic acceleration [11, 12, 20, 24, 27]. In fact, the concept of DE and the physics of accelerating Universe appears to be inherent in the cosmological constant term of Einstein's field equations. If the Λ-term is moved to the right-hand side of the Einstein's field equations (1.6.2) considering the Λ as a part of the matter content then the cosmological constant can be formulated to be equivalent to the vacuum energy [27]. Therefore, the Einstein's field equations (1.6.2) now can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\left(T_{\mu\nu} + T_{\mu\nu}^{vac.}\right), \tag{1.9.1}$$

where the additional contribution $T_{\mu\nu}^{vac.}$ is called 'vacuum contribution' which is given by

$$T_{\mu\nu}^{vac.} = -\frac{\Lambda}{\kappa} g_{\mu\nu}. \tag{1.9.2}$$

The energy-momentum tensor for $T_{\mu\nu}^{vac.}$ is given by

$$T_{\mu\nu}^{vac.} = (\rho_{vac.}c^2 + p_{vac.})u_\mu u_\nu + p_{vac.}g_{\mu\nu}, \tag{1.9.3}$$

where

$$\rho_{vac.}c^2 = \frac{\Lambda}{\kappa}, \text{ and } p_{vac.} = -\rho_{vac.}c^2. \tag{1.9.4}$$

If Λ is positive, $\rho_{vac.}$ is positive and $p_{vac.}$ is negative which provides the repulsion for accelerate the expansion of the Universe.

The FRW metric with cosmological constant leads to the *Lambda-cold dark matter (ΛCDM) model* which has been referred as 'standard model' of cosmology. The ΛCDM model is also known as *concordance model* [28] which fits with the observations of SNe Ia, CMB, LSS, WMAP etc. with a remarkable agreement. Within the framework of the standard model the Universe starts from an initial singularity known as the "Big-Bang". Most of the theories in modern cosmology are based on the concept of the Big-Bang and its variants. The Big-Bang model has been outstandingly successful in describing the evolution of the Universe.

Despite of outstanding features, the ΛCDM model faces some serious problems. They are cosmological constant problem [29, 30] which is also known as *the fine-tuning problem* [12, 24], flatness and horizon problems [2], coincidence problem [12, 24], monopole problem [2], singularity problem etc. [2, 31, 32]. The early inflationary phase successfully addresses the flatness and horizon problems [2, 14, 31, 32]. But the problems of dark matter and dark energy are the most serious and are of current interest in cosmology and astrophysics [20, 21]. The increasing difficulties with the ΛCDM model inspired several cosmologists to compel the investigation of some alternatives of cosmological constant.

1.10 Alternatives

There are basically two theoretical approaches to explain DE: (i) the dynamical energy fluids which fill all of space, and (ii) the modification of Einstein's GR. Let us discuss about the dynamical dark energy and some of the modified theories of gravity.

1.10.1 Dynamical dark energy

Fine-tuning and coincidence problems associated with the cosmological constant have led to the search of dynamical DE models [33]. A phenomenological solution of these problems is to consider a time dependent cosmological term [34,35]. One of the simplest and probably the most common candidate of dynamical DE is the *'quintessence'* [36–44]. The concept of quintessence basically uses a *scalar particle field* [15,45]. The motivation of the interest in scalar field cosmologies is the unified characteristic of scalar fields. Historically, the scalar fields are used as the responsible agents for inflation [46]; to seed the primordial perturbation for the structure formation during an early inflationary epoch; and as the candidate for cold dark matter, responsible for the formation of the actual cosmological structure [47]. Due to remarkable qualitative similarity between the present DE and primordial DE that derived inflation in the early Universe, inflationary models based on scalar fields have also been applied for the description of the late-time cosmic acceleration [21,26,33,39–41,43,44]. Therefore, the scalar field cosmological models have acquired a great popularity in recent decades. Earlier studies have come with a non-minimally coupled scalar field [44,48,49]. Since the energy density of a scalar field should come to dominate over other components in the Universe in late-time only, therefore, these models face the cosmic coincidence problem. Later on, in order to alleviate this problem, many coupled scalar field models [39, 40, 50–52] have been considered, in which matter and DE scale in the same way with time.

The outcomes from different observational data [53–57] also show a possibility of the existence of some strange kind of fields in the Universe such as *phantom field* as proposed by Caldwell [58] having negative kinetic energy [59, 60]. Some other candidates of such dynamical DE are quintom (a combination of quintessence and phantom scalar fields) [61], tachyonic field [52, 62, 63], k-essence [52,64,65], Chaplygin gas [66,67] etc. Nowadays, it is a common issue to make the use of such exotic matters as the responsible agent to describe the late-time acceleration of the Universe [33, 43, 52, 62, 67–69]. I herewith introduce briefly those exotic matters which are related to my thesis work.

A time-dependent cosmological constant as well as the quintessence can be modeled as the energy of a slowly evolving cosmic scalar field ϕ with an appropriate self-interacting scalar potential $V(\phi)$ [33, 43, 52, 69]. Formally, one get phantom by switching the sign of kinetic energy of the Lagrangian of standard quintessence scalar field [52, 58, 60, 70, 71]. Therefore, the matter Lagrangian of a quintessence or phantom scalar field minimally coupled to the gravity is given by [33, 43, 45]

$$\mathscr{L}_\phi = \frac{1}{2}\varepsilon\nabla^\sigma\phi\nabla_\sigma\phi - V(\phi), \tag{1.10.1}$$

where $\varepsilon = \pm 1$ correspond to quintessence and phantom models, respectively.

The general EH action for a minimally coupled quintessence or phantom scalar field in the EH frame is given as

$$S = \int \left(\frac{1}{2\kappa}R + \frac{1}{2}\varepsilon\nabla^\sigma\phi\nabla_\sigma\phi - V(\phi)\right)\sqrt{-g}\, d^4x. \tag{1.10.2}$$

The energy-momentum tensor of quintessence or phantom scalar field, $T^{(\phi)}_{\mu\nu}$ is defined as

$$T^{(\phi)}_{\mu\nu} = \varepsilon\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\varepsilon\nabla^\sigma\phi\nabla_\sigma\phi + V(\phi)\right]. \tag{1.10.3}$$

The scalar (phantom) fields obey the Klein-Gordon equation

$$g^{\mu\nu}\varepsilon\nabla_\mu\nabla_\nu\phi + \frac{\partial V(\phi)}{\partial\phi} = 0. \tag{1.10.4}$$

Assuming that the scalar (phantom) field evolve in an isotropic and homogenous space-time and ϕ as a function of time alone, the energy density ρ_ϕ and pressure p_ϕ of scalar field are respectively given by

$$\rho_\phi = \frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi), \tag{1.10.5}$$

$$p_\phi = \frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi). \tag{1.10.6}$$

The Klein-Gordon equation (1.10.4) in an isotropic and homogenous space-time reduces to

$$\varepsilon\ddot{\phi} + 3H\varepsilon\dot{\phi} + V'(\phi) = 0\,, \tag{1.10.7}$$

where a prime denotes the derivative with respect to the argument.

The pressure and energy density of these scalar fields are connected by a relation $p_\phi = \omega_\phi\rho_\phi$, known as equation of state (EoS) of scalar field. Here, ω_ϕ is known as the EoS parameter. Therefore, ω_ϕ for quintessence or phantom is given as

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi)}. \tag{1.10.8}$$

It is well known from observational data that $-1 < \omega_\phi < -1/3$ gives the region for quintessence, $\omega_\phi = -1$ corresponds to the cosmological constant and $\omega_\phi < -1$ represents phantom region [70]. Although, due to a number of problems related to the phantom matter such as negative kinetic energy, violation of energy conditions [72], the problem of stability (ghost) [52,73] and future curvature Big-Rip singularity [69,74], it does not seem such matter fields to be quite realistic alternative of DE. But the studies of dynamical properties of the phantom field are still going on to resolve these problems in alternative approaches [52,68,69,71,73–78]. It is clear that there needs more and better data to decide between DE possibilities.

Another possibility is a tachyonic field. The gravitational Lagrangian of tachyonic field is given by [62,63,79–81]

$$\mathscr{L}_\psi = -V(\psi)\sqrt{1-\varepsilon\nabla_\mu\psi\nabla^\nu\psi}, \tag{1.10.9}$$

where $V(\psi)$ is relevant tachyonic potential of tachyonic field ψ. Here, $\varepsilon = \pm 1$ correspond to normal and phantom tachyonic fields, respectively.

The EH action for a tachyonic field is given by

$$S = \int \left(\frac{1}{2\kappa}R - V(\psi)\sqrt{1-\varepsilon\nabla_\sigma\psi\nabla^\sigma\psi} \right) \sqrt{-g}d^4x. \tag{1.10.10}$$

The energy-momentum tensor of tachyonic field, $T^{(\psi)}_{\mu\nu}$) is defined as

$$T^{(\psi)}_{\mu\nu} = V(\psi)\left[\frac{\varepsilon\nabla_\mu\psi\nabla^\nu\psi}{\sqrt{1-\varepsilon\nabla_\mu\psi\nabla^\nu\psi}} + g_{\mu\nu}\sqrt{1-\varepsilon\nabla_\sigma\psi\nabla^\sigma\psi} \right]. \tag{1.10.11}$$

In a homogenous and isotropic space-time, the energy density and pressure of tachyonic field become

$$\rho_\psi = \frac{V(\psi)}{\sqrt{1-\varepsilon\dot{\psi}^2}}, \tag{1.10.12}$$

$$p_\psi = -V(\psi)\sqrt{1-\varepsilon\dot{\psi}^2}. \tag{1.10.13}$$

Consequently, the EoS parameter ω_ψ for tachyonic field has the expression

$$\omega_\psi = \frac{p_\psi}{\rho_\psi} = -1 + \varepsilon \dot{\psi}^2. \tag{1.10.14}$$

It can be seen that $\omega_\psi > -1$ or < -1 according to normal tachyon $(\varepsilon = +1)$ or phantom tachyon $(\varepsilon = -1)$. One can observe from (1.10.14) that as the kinetic term tends to zero, the model approaches to the cosmological constant model, i.e, $\omega_\psi = -1$.

The energy conservation equation for tachyonic field for a homogenous and isotropic space-time usually has the form

$$\frac{\ddot{\psi}}{1-\varepsilon\dot{\psi}^2} + 3\varepsilon H \dot{\psi} = -\frac{V'(\psi)}{V(\psi)}. \tag{1.10.15}$$

where a prime denotes derivative with respect to ψ.

The exotic matter cosmologies form an interesting set of models of the Universe which support the prediction of recent observational data [33, 41, 43, 51]. Therefore, the mathematical and physical properties of such exotic matters deserve further studies. Keeping in view that the scalar fields or tachyonic fields play an important role in explaining the early and late-time cosmic acceleration, one of the motivation of my research work in this thesis is to study FRW models with a perfect fluid, a quintessence scalar field, phantom scalar field and tachyonic (normal or phantom) field.

The potential $V(\phi)$ is not known and one must assume the specific form as a function of the scalar field ϕ. There has been many such proposals available of this potential like power-law, exponential, zero, constant potentials etc [15, 38, 46, 79–88]. Hence, it is of interest to understand the early inflation and late-time acceleration of the Universe with scalar fields along with the various form of scalar potentials. The purpose of the work is to emphasize that the scalar (quintessence or phantom) fields with a suitable potential, and tacyonic field may have important cosmological consequences in explaining the early and late-time evolution of the Universe in modified theories of gravitation (see, subsection 1.10.2).

Undoubtedly, the DE models are the most popular explanation of the current epoch of the accelerating Universe, but they do not seem to be as well motivated theoretically as one would desire [12, 54, 68, 70, 72–74, 80, 89]. Therefore, the mystery is continued with the existence and nature of such exotic matters. Also, it is still a challenging task to construct viable scaling models which give rise to a matter-dominated phase followed by an accelerating phase [90]. In the absence of an evidence for the existence of DE, there leaves a space to explore other possible ways to alleviate the most crucial problem of cosmic acceleration. The modified theories of gravitation, which are the modification of Einstein's GR, have been proposed to explain such cosmic acceleration. In the next subsection let us discuss some of the modified theories in detail.

1.10.2 Modified theories of gravity

The idea of an alternative theory to Einstein's GR is not new. It is worth mentioning that it took only four years after the introduction of GR to start questioning its unique status among gravitational theories. Weyl [91] in 1919, and Eddington [92] in 1923 extended GR to incorporate a broader and more unified theory to describe the evolution of the Universe. During early 1970s, there were various modified theories of GR in existence. Notable examples are Weyl's scale independent theory [91], Eddington's theory of connections [92], Brans Dicke's scalar-tensor theory [93], the higher dimensional theories of Kaluza [94] and Klein [95], and many others [96–98].

The attention in modified theories of gravity has increased at the end of 20th century due to the combined motivation coming from cosmology, astrophysics and high-energy physics [99, 100]. The possibility that the modification in GR at galactic and cosmological scales can replace DM and/or DE, has become an active area of research in recent years [101–110]. At present, there exist a numerous proposals which are the modification in some way of EH gravitational action of Einstein's GR, namely, $f(R)$ theories [100,101,103,111,112], Gauss-Bonnet $f(G)$ theory [113,114], Brane World gravity [115], Horava-Lifshitz gravity [116,117] and $f(T)$ theory [118]. However, none of these solve the mysteries of the Universe thoroughly [119]. The modified gravity theories have already given qualitative answers to a number of fundamental questions including DE, DM and late-time cosmic acceleration. Therefore, there is still a resurgence of interest in these theories to seek the answer of several cosmological problems. The attractive features of modified theories of gravity are [100, 102, 103]:

1. Modified theories of gravity provide a very promising gravitational alternative to DE.
2. They present very natural unification of the early-time inflation and late-time cosmic acceleration.
3. They quite naturally describe the transition from decelerated to accelerated expansion of the Universe.
4. They naturally describe the transition from non-phantom phase to phantom phase without introducing any exotic matter.
5. The effective DE dominance may be assisted by the modification of gravity. Hence, the coincidence problem may be resolved.

Let us briefly introduce some of the modified gravity theories which are related to the thesis work.

1.10.3 Higher derivative gravity

In the beginning of 1960's, it was observed that EH action of GR was not renormalized and therefore it could not be conventionally quantized. In 1962, Utiyama [120], and Utiyama and DeWitt [121] showed that renormalization at one-loop demands that the EH action must be supplemented by higher order curvature terms. Motivated by this result, first studies including higher order curvature terms in the EH action came during 1969-1971 [122–124]. In 1977, Stelle [125] showed that higher order actions are indeed renormalizable but not unitary. Finally, Starobinsky [126] in 1980, successfully constructed the first internally self-consistent cosmological model replacing R by $R+\lambda R^2$ plus some small non-local terms, emerging a (quasi-) de Sitter (latter dubbed inflationary) stage in the early Universe and a graceful exit to the subsequent radiation-dominated phase followed by matter-dominated epoch. Adding a squared scalar curvature term R^2 in EH action gives the modified gravitational Lagrangian ($\mathscr{L}=R+\lambda R^2$, where $\lambda>0$ is a coupling constant), known as **higher derivative (HD) theory**.

The EH action for HD theory by adding an additional term λR^2 with matter lagrangian $\mathscr{L}_m$, is given as [126]

$$S=\int\left[\frac{1}{2\kappa}\left(R+\lambda R^2\right)+\mathscr{L}_m\right]\sqrt{-g}\,d^4x\,, \tag{1.10.16}$$

It can be observed that the standard EH action of GR (1.4.1) is recovered by taking $\lambda = 0$.

The variation of action (1.10.16) with respect to the metric tensor, $g_{\mu\nu}$ yield the following set of field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \lambda \left[2R(R_{\mu\nu} - \frac{1}{4}\, g_{\mu\nu} R) + 2(\nabla_\mu \nabla_\nu - g_{\mu\nu}\, \Box) R \right] = \kappa T_{\mu\nu} \,, \qquad (1.10.17)$$

where $\Box \equiv \nabla_\mu \nabla^\mu$ is the covariant differential operator which is also known as d'Alembert operator.

The HD theory has a number of good features. Initially, the additional term R^2 in the EH action was added to regularize ultraviolet divergences [121]. Later on, it was applied to cosmology to obtain a bouncing model of the Universe, and consequently avoiding the singularity at the Big-Bang [126, 127]. The various structure and attributes of HD theory were further elaborated in subsequent works [128, 129]. It is well known that Starobinsky's [126] inflationary model of the early Universe has been remarkably successful. An interesting feature of HD theory is that the inflation emerges in a most direct manner without using any fictitious exotic matter [130].

After the discovery of accelerating Universe and due to the remarkable quantitative analogy between the properties of primordial DE (responsible for inflation in the early Universe) and the present DE, it was thought that the origin of DE might also be explained by some sub-leading gravitational terms which become relevant as the curvature decreases at late times. In this analogy, the field equations can be recast in a way that the higher order corrections are written as an energy-momentum tensor of the geometrical origin describing an effective source term on the right hand side of the standard Einstein's field equations. In this scenario, the cosmic acceleration can be shown as a result from such a new geometrical contribution to the whole cosmic energy density budget [131, 132]. The presence of higher order terms in the gravitational sector even may be understood as the introduction of an effective fluid which is not restricted to hold the usual energy conditions [99]. Therefore, they may also lead to the cosmological constant, quintessence or phantom at late times, without introducing the exotic matters with strange properties (like negative kinetic energy) or with complicated potentials [101]).

In the first such DE model of higher order correction, Capozziello [131] proposed that the cosmic speed-up can be explained simply by the fact that some sub-dominant terms like $1/R$ may become essential at small curvature. But modifying gravity in such a manner was proven inconsistency with the experimental data. The GR is a very robust and well tested theory, and it has been found that even a slight modification often leads it to the matter instabilities [133, 134] and the propagation of ghosts [135]. Therefore, when introducing additional terms into the gravitational action, one must be careful to respect the success of GR in both the low and high curvature regimes, to ensure that the new model agrees with all known observational tests of gravity. Therefore, Nojiri and Odintsov [136] introduced a model containing a particular combination of $1/R$ and R^2 terms which not only produces late-time cosmic acceleration but also passes the solar system constraints successfully. Another approach with negative and positive power terms was suggested in a ref. [137] where the positive power terms would dominate on small scales while the negative power terms dominate on large cosmic scales. Finally, it is concluded that the positive powers of R in EH action produce early inflationary epoch whereas the negative powers serve as effective DE admitting late-time acceleration of the Universe [100, 138, 139].

Some authors have shown that, in R^m gravity [132, 139], it is possible to have a transient matter-dominated decelerated expansion phase, followed by a smooth transition to a DE era which drives the cosmological acceleration. In particular, for $m = 2$, the R^m gravity reduces to the HD gravity. The cosmological models with perfect in HD theory have extensively been studied to obtain viable cosmological scenario of the early and late-time evolution of the Universe [140–147]. Inspired by these works, I have also discussed some FRW cosmological models in HD theory in this thesis with the perfect fluid, quintessence, phantom and tachyonic fields, which could explain the history of evolution of the Universe.

1.10.4 The modified $f(R)$ theory

Among the generalization of geometrically modified gravity [102], the most successful and widely accepted are $f(R)$ theories [100–109, 111, 112]. The intention of introducing $f(R)$ theories was that one may obtain a gravitational alternative to the conventional description of DE. In fact, the metric variation in EH action of $f(R)$ gravity introduces an additional scalar degree of freedom which leads to an accelerated expansion of the Universe at late-time, induced by the Ricci scalar. One may call this “curvature DE" or “dark gravity" [101, 106–109]. The $f(R)$ theories not only describe DE but also provide a very natural unification of the sequence of cosmological evolutionary phases [105, 136, 148, 149].

The $f(R)$ theory of gravity, first proposed by Buchdahl [122], is one of the generalization of the higher order gravity theories in which R is replaced by an arbitrary function $f(R)$ of Ricci scalar curvature in EH action [100,101,111,112]. The gravitational action for $f(R)$ gravity with matter Lagrangian $\mathscr{L}_m$ is given by [100]

$$S=\int\left[\frac{1}{2\kappa}f(R)+\mathscr{L}_m\right]\sqrt{-g}\,d^4x. \tag{1.10.18}$$

The trivial case of ΛCDM model corresponds to $f(R)=R-2\Lambda$. The field equations by varying the action (1.10.18) with respect to metric tensor $g_{\mu\nu}$ are obtained as

$$f'(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box\right)f'(R)=\kappa T_{\mu\nu}, \tag{1.10.19}$$

where a prime denotes the derivative with respect to the argument. The $f(R)$ function significantly encapsulates some of the basic characteristics of higher-order gravity theories [100, 101]. Many works on $f(R)$ theories are also available in the literature addressing the other well-known issues such as DE, DM and accelerating Universe [101,131,132,136]. solar system test [133], stability [134], singularity problem [150], etc. However, most of these earlier attempts remained unsuccessful due to non-viability (inconsistency with the observations) [133–135, 151] or due to practically indistinguishable from the standard ΛCDM model [90, 234]. Later on, a large number of viable $f(R)$ gravity models have been proposed in the literature [99–104, 111, 112, 131, 132, 136, 138, 139, 148, 149, 152]. Recently, it has observed that the $f(R)$ gravity models pass all known observational local tests [105, 153]. However, almost of these considerations have primarily investigated in a spatially isotropic space-time. But the theoretical studies and the outcomes from various observational data which support the existence of anisotropic phase, lead to consider the models of the Universe in anisotropic space-time. Therefore, since past few years some authors have also started working to explore the features of $f(R)$ gravity in anisotropic background [154–162].

One of the interesting issue in cosmology is the reconstruction of modified theories of gravity (see ref. [105] for recent review). The general scheme for the reconstruction of modified gravity from any realistic FRW cosmology was proposed by Nojiri et al. [163]. In reconstruction schemes it is assumed that the expansion history is known exactly and one inverts the field equations to deduce what class of modified theory gives rise to the desired model [164]. In reconstruction of $f(R)$ also, the proposal have come to find analytical solutions for some known functional form of $f(R)$. The ordering of this approach can also be reversed, that is, for a known scale factor, one may reconstruct functional form of $f(R)$ which yields such scale factors as solutions [164, 165]. Therefore, following this reverse approach, it is of interest to reconstruct cosmological models in modified theories of gravity.

In this thesis one chapter is devoted for the reconstruction of a functional form of $f(R)$ with power-law expansion in a locally-rotationally-symmetric (LRS) Bianchi I anisotropic model filled with the perfect fluid.

1.10.5 The modified $f(R,T)$ theory

Even though one decides that the modification of the gravitational theory is a way to overcome the problem of DE and accelerating Universe but it is not an easy task because there may be a numerous way to deviate from GR. In fact, the EH action in GR has a additive structure in Ricci scalar R and matter Lagrangian $\mathscr{L}_m$, both of which have very different conceptual levels without any interaction between them. However, there is no any fundamental guiding principle for considering the matter and geometry to be additive. Moreover, a more generalised EH action requires a general coupling between matter and geometry. The idea of non-minimal coupling between matter and geometry was first considered by Goenner [166] in 1984. In 2007, Bertolami et al. [167] proposed a maximal extension of EH action by introducing an explicit coupling of arbitrary function of R and $\mathscr{L}_m$. In 2008, Harko [168] extended it to the case of arbitrary coupling between R and $\mathscr{L}_m$. These theories came to be known as the $f(R,\mathscr{L}_m)$ gravity theories [168–172].

Poplawski [173] implemented a particular application of $f(R,\mathscr{L}_m)$ gravity based on the principle of least action in a relativistically covariant model of interacting DE. They have assumed that an interaction between baryonic matter and DE may be consider as a time-dependent cosmological constant. In addition, a variable cosmological constant must depend only on relativistic invariants for preserving the general covariance of the field equations. It is to be noted that the choice $\Lambda = \Lambda(R)$ is equivalent to the class of $f(R)$ theories of gravity. Poplawski [173] in his application of $f(R,\mathscr{L}_m)$ proposed the cosmological constant as a function of the trace of energy-momentum tensor, i.e., $T = g^{\mu\nu}T_{\mu\nu}$.

In 2011, following Poplawski [173], Harko et al. [174] have proposed a general non-minimal coupling between matter and geometry in the framework of an effective gravitational Lagrangian consisting an arbitrary function of R and T, and introduced $f(R,T)$ gravitational theory. The EH action for $f(R,T)$ gravity is given as

$$S = \int \left[\frac{1}{2\kappa}f(R,T) + \mathscr{L}_m\right]\sqrt{-g}\, d^4x. \tag{1.10.20}$$

The field equations of $f(R,T)$ gravity by varying the action (1.10.20) with respect to metric tensor have the form

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T) = \kappa T_{\mu\nu} - f_T(R,T)(T_{\mu\nu} + \ominus_{\mu\nu}), \tag{1.10.21}$$

where f_R and f_T denote the derivatives of $f(R,T)$ with respect to R and T, respectively and $\ominus_{\mu\nu}$ is defined by

$$\ominus_{\mu\nu} \equiv g^{ij}\frac{\delta T_{ij}}{\delta g^{\mu\nu}}, \qquad i,j = 0,1,2,3. \tag{1.10.22}$$

The authors argued that the justification of choosing T as an argument for the Lagrangian is from exotic imperfect fluids or quantum effects (conformal anomaly). In addition, the new matter and time-dependent terms in the gravitational field equations play the role of an effective cosmological constant. They also suggested that due to the coupling of matter and geometry, $f(R,T)$ gravity depends on a source term representing the variation of the matter stress-energy tensor with respect to the metric. They also obtained a general expression for the source term as a function of the matter Lagrangian $\mathscr{L}_m$. A strange behavior of $f(R,T)$ gravity is that the covariant divergence of the stress-energy tensor does not vanish. As a consequence, the equations of motion show the presence of an extra-force acting on the test particles and the motion are generally non-geodesic [167]. The authors have applied this theory to analyse the Newtonian limit of the equations of motion and provided a constraint on the magnitude of the extra acceleration by investigating the perihelion precession of Mercury. Therefore, the $f(R,T)$ gravity also has a promising feature that an extra acceleration is always present due to the coupling between matter and geometry. This extra acceleration in $f(R,T)$

gravity results not only from geometrical contribution but also from the matter content. These interesting features of $f(R,T)$ gravity have attracted many theorists to explore its various features and applications for resolving several issues of current interest in cosmology and astrophysics.

Jamil et al. [175] have found, that the first law of black hole thermodynamics is violated in this new gravitational theory. Sharif and Zubair [176] have discussed a non-equilibrium thermodynamics by taking two forms of the energy-momentum tensor of dark components, which endorses second law of thermodynamics both in phantom and non-phantom phases. Azizi [177] has examined the possibility of wormhole geometry in the context of $f(R,T)$ gravity. Alvarenga et al. [178] have paid special attention on $f(R,T) = R + 2f(T)$ assuming special function $f(T)$ showing energy conditions can be satisfied for suitable input parameters. Alvarenga et al. [179] have studied the evolution of scalar cosmological perturbations in the background of metric formalism in $f(R,T)$ theory, assuming a specific model that guarantees the standard continuity equation. They obtained the complete set of differential equations for the matter density perturbations and showed that for general $f(R,T)$ Lagrangian the quasi-static approximation leads to very different results as compared to the ΛCDM cosmology. However, most of the works have been carried out in an isotropic and homogenous FRW background [176–184]. Some authors have also explored $f(R,T)$ theory in anisotropic space-time [185–189].

The reconstruction of cosmological models in $f(R,T)$ theory have been made by several authors [175,181,184,190,191]. Thus, there is a lot of scope to investigate a general class of $f(R,T)$ gravity models to describe early and late-time evolution of the Universe. I have reconstructed $f(R,T)$ gravity for de Sitter and power-law models with perfect fluid within the framework of a flat FRW space-time. I have also reconstructed scalar field cosmological models for constant and exponential potentials in $f(R,T)$ gravity in flat FRW space-time. I have also studied the theoretical and observational consequences of thermodynamics of an open system which allow matter creation in $f(R,T)$ theory within the framework of a flat FRW model.

In light of the discussion mentioned in sections 1.1–1.10, I have discussed some of the modified gravity theories with perfect fluid and scalar field within the framework of FRW and anisotropic models. The actual work has been presented in chapters 2–8 based on the motivation acknowledged so far. The abstract at the beginning of a chapter gives a brief outlines of the work carried out in that chapter. The sum up of the findings have been accumulated in the concluding section at the end of each chapter. A brief summary and future scope of the research work carried out in the thesis have been mentioned in chapter 9. The thesis ends with the bibliography and the list of publications.

Chapter 2

FRW models in higher derivative theory

In this chapter[1] we study FRW models containing a perfect fluid and a scalar field minimally coupled to gravity with self-interacting potential in HD theory. We assume the scalar potential and scale factor as functions of the scalar field to obtain the exact solution of the field equations. We explore the cosmological solutions for flat, closed and open models, which are physically interesting for the description of the whole cosmological evolution. The objective of this chapter is to explore the effects of higher order terms in the evolution of the Universe in the presence of a scalar field and a perfect fluid.

2.1 Introduction

In past few decades, the scalar field cosmological models have acquired a great popularity due to their explanation of early inflationary phase [15, 36, 45, 83, 84, 192,193], and late-time cosmic acceleration [39,41–44,48,80,88,145,194]. Some authors have also considered some other matter sources with scalar field. Ellis and Madsen [193] have considered a FRW model with a minimally coupled scalar field and a perfect fluid in the form of radiation. Chimento and Jakubi [195] have studied scalar field cosmologies with a perfect fluid in Robertson-Walker metric. Sen and Banerjee [196] have obtained an exact cosmological solution for (FRW) metric with a scalar field along with a potential in the presence of a causal viscous

[1]The result of this chapter has been published in a research paper entitled "FRW models with a perfect fluid and a scalar field in higher derivative theory", in *Modern Physics Letter A* **26** 1495–1507 (2011).

fluid. Coley and Goliath [197] have investigated self-similar spherically symmetric cosmological solutions with a perfect fluid and a scalar field. In many cosmological models based on scalar field ϕ, the potential function $V(\phi)$ is related to the evolution of the Universe [15, 38, 46, 79, 82–88]. Barrow and Saich [15] assumed that the kinetic and potential terms for the scalar field are proportional to each other. A common functional form for the self-interacting potential is Liouville form (an exponentially dependence upon the scalar field) [38, 46, 83, 84, 86]. The possible cosmological role of exponential potential as a means of driving a period of cosmological inflation and late-time acceleration has been investigated in the literature [38, 46, 82–88, 145]. Models with an exponential scalar field potential also arise naturally in alternative theories of gravity [85].

The dynamics of HD cosmology is also directly related to inflationary models of scalar field with scalar potential. Kofman et al. [198] claimed that the combined action of the R^2 term and the scalar field ϕ might lead to double inflation, i.e., two consecutive inflationary stages separated by a power-law expansion. There has been considerable interest in scalar fields with the exponential potential in HD gravity. Several authors have discussed the viable cosmological models with a variety of energy contents in HD theory [141–147]. It is therefore worthwhile to explore HD gravity models containing a scalar field with exponential scalar potential and a perfect fluid, which exhibit the evolution of the Universe from an inflationary scenario at early time followed by radiation- and matter-dominated eras, respectively to the present accelerated phase.

The motivation of this chapter is to examine the dynamics of the expansion of the Universe in FRW models with a scalar field and a perfect fluid in HD theory. We investigate the exact cosmological solutions for the flat, closed and open FRW models by considering the scalar potential and the scale factor exponentially varying with scalar field. In particular, we focus to examine whether the HD theory could be responsible for deriving the late-time accelerated expansion of the Universe.

2.2 Gravitational action and the field equations

The EH action (1.10.16) of HD theory of gravity constituting a perfect fluid described by the matter Lagrangian $\mathscr{L}_m$ and a scalar field ϕ with scalar potential

$V(\phi)$, minimally coupled to gravity in the units $\kappa = 1 = c$ modifies as [141, 146]

$$S = \int \left[\frac{1}{2}\,\left(R + \lambda R^2\right) + \mathscr{L}_m + \frac{1}{2}\,\nabla^\sigma \phi\,\nabla_\sigma \phi - V(\phi)\right]\sqrt{-g}\,d^4x\,. \tag{2.2.1}$$

The variation of action (2.2.1) with respect to the metric tensor, $g_{\mu\nu}$ yields the following set of field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \lambda\left[2R(R_{\mu\nu} - \frac{1}{4}\,g_{\mu\nu}R) + 2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\,\Box)R\right] = T_{\mu\nu}\,, \tag{2.2.2}$$

where $T_{\mu\nu}$ is the effective energy-momentum tensor of a perfect fluid and a scalar field, given as

$$T_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{(\phi)}_{\mu\nu}, \tag{2.2.3}$$

where $T^{(m)}_{\mu\nu}$ is the energy-momentum tensor of the perfect fluid, which is given by

$$T^{(m)}_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}, \tag{2.2.4}$$

where ρ_m and p_m are the energy density and pressure of the perfect fluid. The energy-momentum tensor (1.10.3) for a minimally coupled quintessence ($\varepsilon = 1$) scalar field ϕ with self interacting potential $V(\phi)$, takes the form

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}\nabla^\sigma\phi\nabla_\sigma\phi + V(\phi)\right]. \tag{2.2.5}$$

We assume that the perfect fluid and the scalar field are non-interacting which leads to the following separate energy-conservation laws

$$\nabla^\nu T^{(m)}_{\mu\nu} = 0 = \nabla^\nu T^{(\phi)}_{\mu\nu}. \tag{2.2.6}$$

Consequently, $\nabla^\nu T^{(m)}_{\mu\nu} = 0$ leads to the conservation equation (1.5.6), which can be written as

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \tag{2.2.7}$$

and $\nabla^\nu T^{(\phi)}_{\mu\nu} = 0$ yields the Klein-Gordon equation (1.10.7), which becomes

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{2.2.8}$$

We consider FRW models whose metric is given by Eq. (1.2.2). The Ricci scalar curvature for this metric becomes

$$R = -6\left[\dot{H} + 2H^2 + \frac{k}{a^2}\right]. \tag{2.2.9}$$

With the energy-momentum tensor (2.2.3) and for FRW metric (1.2.2), the field equations (2.2.2) yield

$$3H^2 + 3\frac{k}{a^2} - 18\,\lambda\left[2\ddot{H}H - \dot{H}^2 + 6\dot{H}H^2 - \frac{2kH^2}{a^2} + \frac{k^2}{a^4}\right] = \rho_m + \rho_\phi, \tag{2.2.10}$$

$$\begin{aligned} 2\dot{H} + 3H^2 + \frac{k}{a^2} &- 6\lambda\left[2\dddot{H} + 12\ddot{H}H + 18\dot{H}H^2 + 9\dot{H}^2\right] \\ &+ 6\lambda\frac{k}{a^2}\left[4\dot{H} + 2H^2 + \frac{k}{a^2}\right] = -p_m - p_\phi, \end{aligned} \tag{2.2.11}$$

where ρ_ϕ and p_ϕ are given by (1.10.5) and (1.10.6) which can be obtained from (2.2.5).

2.3 Solution of the field equations

We observe from Eqs. (2.2.7), (2.2.8), (2.2.10) and (2.2.11) that three of these four equations are independent. So we have three equations with five unknowns, namely, a, ρ_m, p_m, ϕ and V. It follows that one needs to provide two more relations in order to construct a definite cosmological scenario. We make two assumptions, one is for the scalar field potential and another is for the scale factor to find the exact solution of the field equations.

The models with an exponential scalar field potential arise naturally in alternative theories of gravity (such as scalar-tensor theories) [85] and are of particular interest since such theories occur as the low-energy limit in supergravity theories. A number of authors have studied scalar field cosmologies with an exponential potential within GR [38, 82, 84, 86, 196]. Therefore, we assume the exponential potential of the form [46, 196]

$$V(\phi) = V_0\, e^{-\beta\phi}\,, \tag{2.3.1}$$

where V_0 and $\beta(>0)$ are constants. The parameter β has the dimension of inverse of mass as ϕ has the dimension of mass.

In our second assumption, we consider the scale factor which evolves exponentially with the scalar field [196, 197], that is,

$$a = a_0\, e^{\alpha\phi}\ , \tag{2.3.2}$$

where α is a constant and a_0 is the proportionality constant representing the present value of the scale factor.

Using (2.3.1) and (2.3.2), the conservation equation of scalar field, i.e., (2.2.8) can be rewritten as

$$\frac{d}{dt}\left(\dot{\phi}^2 e^{6\alpha\phi}\right) = K_1 \frac{d}{dt}\left(e^{\phi(6\alpha-\beta)}\right), \tag{2.3.3}$$

where $K_1 = \frac{2V_0\beta}{6\alpha-\beta}$.

Integrating (2.3.3), we get

$$\dot{\phi}^2 = K_1 e^{-\alpha\phi}, \tag{2.3.4}$$

where the integration constant is taken to be zero for simplicity. The real solution exists provided $K_1 > 0$, i.e., $\beta < 6\alpha$.

Further integration of (2.3.4), gives

$$\phi = \frac{2}{\beta}\log\left(\frac{\phi_0\beta}{2} \pm \frac{\beta\sqrt{K_1}}{2}\, t\right), \tag{2.3.5}$$

where ϕ_0 is a constant of integration.

Using (2.3.5) into (2.3.2), one gets

$$a = a_0\left(\frac{\phi_0\beta}{2} \pm \frac{\beta\sqrt{K_1}}{2}\, t\right)^{\frac{2\alpha}{\beta}}. \tag{2.3.6}$$

Since we are living in an expanding Universe, therefore, we consider the positive sign within the bracket and by suitable choice of origin, we take $\phi_0 = 0$, therefore, Eqs. (2.3.5) and (2.3.6) take the form

$$\phi(t) = \frac{2}{\beta}\log\left(\frac{\beta\sqrt{K_1}}{2}\, t\right), \tag{2.3.7}$$

and

$$a(t) = a_\star\, t^{\frac{2\alpha}{\beta}}\ , \tag{2.3.8}$$

where $a_\star = a_0\left(\beta\sqrt{K_1}/2\right)^{\frac{2\alpha}{\beta}}$.

Equation (2.3.8) shows the power-law expansion of the scale factor with time. Thus, the two assumptions made in Eqs. (2.3.1) and (2.3.2) with the conservation equation of scalar field naturally lead to the power-law expansion of the Universe.

Now, the scalar potential (2.3.1) takes the form

$$V = \frac{4V_0}{\beta^2 K_1}\frac{1}{t^2}. \tag{2.3.9}$$

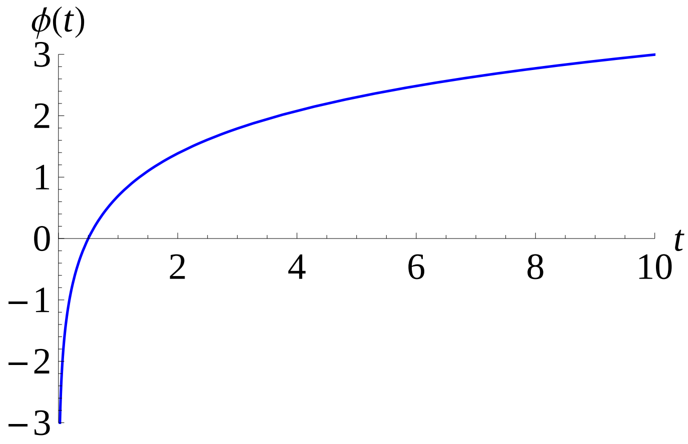

Figure 2.1: $\phi(t)$ *vs.* t with $\beta = 2$, $\alpha = 1/2$ and $V_0 = 1$.

From (2.3.7) we observe that the scalar field ϕ increases with time, which is shown in fig. 2.1, whereas the scalar potential $V(\phi)$ obtained in Eq. (2.3.9) is a decreasing function of time and tends to zero as $t \to \infty$.

The energy density and pressure of quintessence ($\varepsilon = 1$) scalar field defined in Eqs. (1.10.5) and (1.10.6), respectively give

$$\rho_\phi = \frac{12\alpha}{\beta^3 t^2}, \tag{2.3.10}$$

$$p_\phi = \frac{4(\beta - 3\alpha)}{\beta^3 t^2}. \tag{2.3.11}$$

Consequently, the EoS parameter corresponding to scalar field which is defined in Eq. (1.10.8), gives

$$\omega_\phi = \frac{\beta}{3\alpha} - 1, \tag{2.3.12}$$

which is constant. Thus, the value of ω_ϕ depends on α and β.

The Hubble and deceleration parameters which are defined in Eqs. (1.7.5) and

(1.7.10), respectively give

$$H = \frac{2\alpha}{\beta}\frac{1}{t}, \tag{2.3.13}$$

$$q = \frac{\beta}{2\alpha} - 1. \tag{2.3.14}$$

Equation (2.3.14) shows that the deceleration parameter is constant which tells that the Universe decelerates for $\beta/2\alpha > 1$, inflates marginally for $\beta/2\alpha = 1$ and accelerates for $\beta/2\alpha < 1$.

Now we seek the cosmological solutions for flat, closed and open models, respectively, in the upcoming sections.

2.3.1 Solution for flat model ($k = 0$)

Using (2.3.7), (2.3.8) and (2.3.9) into (2.2.10) and (2.2.11), we obtain

$$\rho_m = 12\left[\left(\frac{\alpha}{\beta}\right)^2 - \frac{1}{\beta^2}\left(\frac{\alpha}{\beta}\right)\right]\frac{1}{t^2} + 216\,\lambda\left[4\left(\frac{\alpha}{\beta}\right)^3 - \left(\frac{\alpha}{\beta}\right)^2\right]\frac{1}{t^4}, \tag{2.3.15}$$

$$\begin{aligned} p_m = & -12\left(1-\frac{\beta}{3\alpha}\right)\left[\left(\frac{\alpha}{\beta}\right)^2 - \frac{1}{\beta^2}\left(\frac{\alpha}{\beta}\right)\right]\frac{1}{t^2} \\ & -72\,\lambda\left[2\left(\frac{\alpha}{\beta}\right) - 11\left(\frac{\alpha}{\beta}\right)^2 + 12\left(\frac{\alpha}{\beta}\right)^3\right]\frac{1}{t^4}. \end{aligned} \tag{2.3.16}$$

It can be seen from the above expressions that the energy density and pressure are determined by a coupling parameter λ of HD theory. The Universe starts with higher energy density as compare to standard cosmology based on GR. The additional terms due to HD theory decrease faster than other terms in these two physical quantities.

The total cosmological density parameter (Ω_T) of effective matter is

$$\Omega_T = \Omega_m + \Omega_\phi = \frac{\rho_m + \rho_\phi}{3H^2}, \tag{2.3.17}$$

where, $\Omega_m = \rho_m/3H^2$ is the density parameter for the perfect fluid and $\Omega_\phi = \rho_\phi/3H^2$ is the density parameter for the scalar field. Inserting the values of ρ_m, ρ_ϕ and H in Eq. (2.3.17), we get the expression for effective density parameter as

$$\Omega_T = 1 + 18\lambda\left(\frac{4\alpha}{\beta} - 1\right)\frac{1}{t^2}, \tag{2.3.18}$$

which shows that the Universe was curved during early stages of its evolution and becomes flat as $t \to \infty$ in HD theory of gravity. In absence of HD theory ($\lambda = 0$) the Universe becomes flat ($\Omega_T = 1$) throughout its evolution. If $4\alpha < \beta$ the Universe becomes open ($\Omega_T < 1$) whereas it is closed ($\Omega_T > 1$) for $4\alpha > \beta$. In case of $4\alpha = \beta$, the Universe becomes flat even in HD theory of gravity.

Now, keeping in view the standard cosmological evolutionary phases, which have been discussed in section 1.8, we assume different relations between the constants α and β to study these phases.

Case (i) Solution for $\alpha = \beta/2$:

In this case, the scale factor evolves as $a(t) \sim t$, which is similar to the standard inflationary phase. The energy density ρ_m and pressure p_m of the perfect fluid become

$$\rho_m = 3\left(1-\frac{2}{\beta^2}\right)\frac{1}{t^2}+54\lambda\frac{1}{t^4}, \quad (\beta > \sqrt{2}), \tag{2.3.19}$$

$$p_m = -\left(1-\frac{2}{\beta^2}\right)\frac{1}{t^2}+18\lambda\frac{1}{t^4}. \tag{2.3.20}$$

The energy density and pressure are determined by a coupling parameter λ of HD theory. The deceleration parameter gives $q = 0$ which shows the coasting cosmology, that is, the marginal inflation. From Eq. (2.3.18), the density parameter has the value $\Omega_T = (1+18\lambda/t^2)$ showing that the Universe was curved during early stages of its evolution and becomes flat, i.e., $\Omega_T = 1$ at late times. For $\lambda = 0$, we get $p_m = -\rho_m/3$, which is the EoS of the standard inflationary phase in GR. Thus, the model asymptotically tends to the usual inflationary Universe in late-time where the slow-roll approximation is not valid at the beginning.

Case (ii) Solution for $\alpha = \beta/4$:

In this case, the scale factor evolves as $a(t) \sim t^{1/2}$ which shows the behavior of the radiation-dominated phase. The energy density and pressure of the perfect fluid are given by

$$\rho_m = \frac{3}{4}\left(1-\frac{4}{\beta^2}\right)\frac{1}{t^2}, \quad (\beta > 2), \tag{2.3.21}$$

$$p_m = \frac{1}{4}\left(1-\frac{4}{\beta^2}\right)\frac{1}{t^2}. \tag{2.3.22}$$

From (2.3.21) and (2.3.22), we get $p_m = \rho_m/3$, which is the EoS of the standard radiation-dominated phase in GR. The energy density and pressure are independent of HD term. From Eq. (2.3.18) we get $(\Omega_T = 1)$, which shows that the Universe remains flat throughout the evolution during this phase. The deceleration parameter is $q = 1$ and hence the Universe expands with decelerated rate. Thus, we find that the solution in this case is similar to the usual radiation-dominated phase of GR, which we get even if $\lambda \neq 0$. Therefore, it is evident that the presence of HD term does not affect the cosmological evolution in radiation-dominated phase and remains same as that obtained in GR.

Case (iii) Solution for $\alpha = \beta/3$:

In this case, the scale factor evolves as $a(t) \sim t^{2/3}$, which is similar to the standard matter-dominated era of GR. The solutions of ρ_m and p_m are given by

$$\rho_m = \frac{4}{3}\left(1 - \frac{3}{\beta^2}\right)\frac{1}{t^2} + 8\lambda\frac{1}{t^4}\,, \quad (\beta > \sqrt{3})\,, \tag{2.3.23}$$

$$p_m = \lambda\frac{8}{t^4}\,. \tag{2.3.24}$$

In this case, both ρ_m and p_m are determined by coupling parameter of HD gravity, which are higher in the early evolution of the Universe and tend to zero as $t \to \infty$. From Eq. (2.3.18), $\Omega_T = (1 + 6\lambda/t^2)$, which shows that the Universe was curved during early stages of its evolution. The Universe becomes flat, i.e., $\Omega_T = 1$ at late-time of evolution. The deceleration parameter has the value $q = 0.5$, which shows that the Universe expands with decelerated rate. The physical behavior of the model in this case is similar to the standard matter-dominated phase of GR. In the absence of HD theory, the solution reduces exactly similar to the matter-dominated phase $(p_m = 0)$ of Einstein's GR.

Case (iv) Solution for $\alpha = 2\beta/3$:

In this case, the scale factor evolves as $a(t) \sim t^{4/3}$, which is a rapid power-law expansion. The energy and pressure have the expressions

$$\rho_m = \frac{8}{3}\left(2-\frac{3}{\beta^2}\right)\frac{1}{t^2}+160\lambda\frac{1}{t^4}, \quad (\beta > \sqrt{3/2}), \tag{2.3.25}$$

$$p_m = -\frac{4}{3}\left(2-\frac{3}{\beta^2}\right)\frac{1}{t^2}. \tag{2.3.26}$$

In this case, the energy density depends on the coupling parameter of HD gravity. However, the pressure is independent of HD term and is negative. The density parameter has the value $\Omega_T = (1+30\lambda/t^2)$, which shows the curved Universe in its early stages of evolution and flat, i.e., $\Omega_T = 1$ at late-time. It is also observed that $q = -0.25$, hence, the solution obtained in this case describes an accelerating Universe which is compatible with the recent observations. In the absence of λ -term, the solutions (2.3.25) and (2.3.26) give a relation $p_m = -\rho_m/2$, which reveals the case of quintessence phase $(\omega_m < -1/3)$ of the accelerating Universe. It is pointed out that the higher order correction disappears at late-time and it is only the hypothetical fluid having negative pressure, which is responsible for giving rise to the accelerated expansion of the Universe in this case.

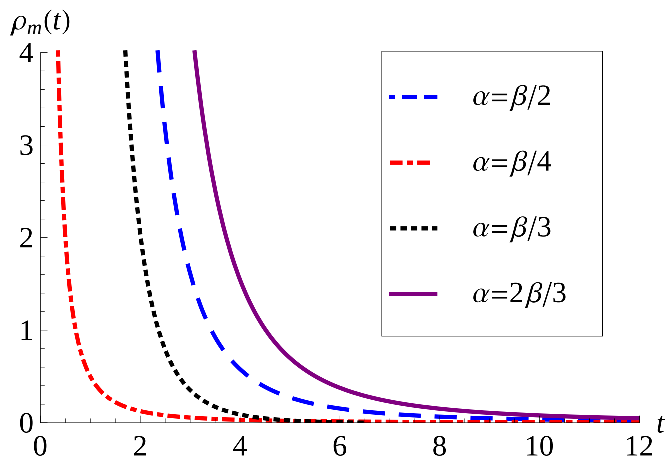

Figure 2.2: ρ_m *vs.* t with $\beta = 2\sqrt{3}$ and $\lambda = 2$.

Fig. 2.2 plots energy density (ρ_m) *verses* cosmic time (t) for various relation between α and β. We observe that the energy density decreases gradually with time for $\alpha = \beta/2$ (case(i): inflation), $\alpha = \beta/3$ (case(iii): matter) and $\alpha = 2\beta/3$ (case(iv): acceleration) as compared to $\alpha = \beta/4$ (case(ii): radiation). The energy density is determined by the extent of coupling parameter of HD theory. It decreases fast in case of $\alpha = \beta/4$ (case(ii): radiation) as compared to other three cases. In radiation-dominated era, HD theory does not affect the behavior of the energy density and it remains same as in Einstein gravity. In all cases the energy density tends to zero in late times. We find that the conservation equation (2.2.7) is identically satisfied in each case. The model has a singularity at $t = 0$ in all phases.

Fig. 2.3 plots density parameter Ω_T *verses* cosmic time t. We observe that

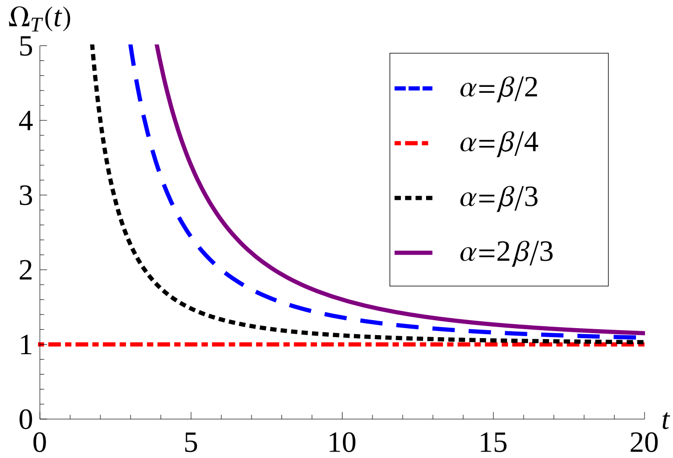

Figure 2.3: Ω_T *vs.* t with $\lambda = 2$.

the Universe shows curve during its early stages of evolution during inflationary ($\alpha = \beta/2$), matter-dominated ($\alpha = \beta/3$) and accelerated ($\alpha = 2\beta/3$) phases of the Universe in HD theory. The Universe becomes flat at late-time in all phases. However, in radiation-dominated phase ($\alpha = \beta/4$) we find a flat Universe throughout its evolution. The HD theory does not affect the behavior of density parameter in this case and it remains same ($\Omega_T = 1$) as in Einstein's GR.

2.3.2 Solution for closed and open models ($k = \pm 1$)

In this case, the conservation equation of the perfect fluid (2.2.7) and the wave equation of the scalar field (2.2.8) remain unaltered. We take the same assumptions given in Eqs. (2.3.1) and (2.3.2) to solve the field equations (2.2.10) and (2.2.11) along with Eq. (2.2.8). The solutions for the scalar field, scale factor and scalar potential remain same as given by Eqs. (2.3.7), (2.3.8) and (2.3.9), respectively but the energy density and pressure of the perfect fluid have the following expressions.

$$
\begin{aligned}
\rho_m = \;& 12 \left[\left(\frac{\alpha}{\beta}\right)^2 - \frac{1}{\beta^{\,2}}\left(\frac{\alpha}{\beta}\right)\right]\frac{1}{t^2} + 216\,\lambda\left[4\left(\frac{\alpha}{\beta}\right)^3 - \left(\frac{\alpha}{\beta}\right)^2\right]\frac{1}{t^4} \\
+ \;& \frac{3k}{a_\star^2 t^{4\,\alpha/\beta}} + 18\,\lambda\left[\frac{8k}{a_\star^2}\left(\frac{\alpha}{\beta}\right)^2\frac{1}{t^2} - \left(\frac{k}{a_\star^2}\right)^2\frac{1}{t^{\,4\alpha/\beta}}\right]\frac{1}{t^{4\alpha/\beta}},
\end{aligned} \tag{2.3.27}
$$

$$
\begin{aligned}
p_m = \;& -\; 12\left(1-\frac{\beta}{3\alpha}\right)\left[\left(\frac{\alpha}{\beta}\right)^2 - \frac{1}{\beta^{\,2}}\left(\frac{\alpha}{\beta}\right)\right]\frac{1}{t^2} - \frac{k}{a_\star^2}\frac{1}{t^{\,4\alpha/\beta}} \\
& -\; 72\,\lambda\left[2\left(\frac{\alpha}{\beta}\right) - 11\left(\frac{\alpha}{\beta}\right)^2 + 12\left(\frac{\alpha}{\beta}\right)^3\right]\frac{1}{t^4} \\
& +\; 6\,\lambda\left[\frac{8k}{a_\star^2}\left\{\frac{\alpha}{\beta} - \left(\frac{\alpha}{\beta}\right)^2\right\}\frac{1}{t^2} - \left(\frac{k}{a_\star^2}\right)^2\frac{1}{t^{\,4\alpha/\beta}}\right]\frac{1}{t^{\,4\alpha/\beta}}\,.
\end{aligned} \tag{2.3.28}
$$

The energy density and pressure are determined by a coupling parameter λ of HD theory. The energy density tends to infinity at $t=0$ and becomes zero as $t\to\infty$. The model has a singularity at $t=0$.

It is very difficult to find the conditions for positivity of energy density for general values of constants. For a very special case where $\alpha=\beta/2$, a simple analysis has been observed. In this case, we get $a(t)\sim t$ and $q=0$, which shows the coasting cosmology or marginal inflation. The energy density and pressure are respectively, given by

$$
\rho_m = 3\left[1-\frac{2}{\beta^2}+\frac{k}{a_\star^2}\right]\frac{1}{t^2} + 18\lambda\left[3+\frac{2k}{a_\star^2} - \left(\frac{k}{a_\star^2}\right)^2\right]\frac{1}{t^4}\,, \tag{2.3.29}
$$

and

$$
p_m = -\left[1-\frac{2}{\beta^2}+\frac{k}{a_\star^2}\right]\frac{1}{t^2} + 6\lambda\left[3+\frac{2k}{a_\star^2} - \left(\frac{k}{a_\star^2}\right)^2\right]\frac{1}{t^4}\,. \tag{2.3.30}
$$

The above solutions identically satisfy the conservation equation (2.2.7). In the absence of HD term ($\lambda=0$), we get $p_m=-\rho_m/3$, which is the standard inflationary phase of GR. Thus, we observe that this particular solution shows the behavior of the inflationary phase of the Universe.

One may observe that the solutions in other cases as discussed in Sec. 2.3.1 admit the similar behavior of the radiation-dominated, matter-dominated and accelerating phases of the Universe.

2.4 Conclusion

In this chapter, we have investigated flat, open and closed FRW cosmological models with a perfect fluid and a minimally coupled scalar field with scalar potential in HD theory of gravitation. We have assumed the scalar potential and scale factor as exponential functions of scalar field to find the exact solutions of the field equations. We have explored some interesting solutions exclusively for various phases of the Universe, viz., inflationary phase, radiation-dominated phase, matter-dominated phase and accelerating phase of the Universe. The physical relevance of each model has been discussed through some physical quantities and cosmological parameters under certain constraints of constants.

We have found that the assumptions of $V(\phi)$ and a as exponential functions of ϕ give power-law expansion. The scaler field increases with time whereas the scalar potential decreases and tends to zero at late-time. The power-law scale factor gives constant values of deceleration parameter and EoS parameter. We have first explored a flat FRW model in HD theory. It has been observed that the solution in radiation-dominated phase in HD theory is similar to GR. Thus, the presence of HD theory does not affect the behavior of the Universe in radiation-dominated phase. However, in inflationary, matter-dominated and accelerated phases, the physical parameters are determined by a coupling parameter λ. We have observed that the higher gravity correction disappears at late-time and the solution tend asymptotically to the usual inflationary, matter-dominated and accelerating phases in GR. The physical behavior of energy density *versus* time has been shown in fig. 2.2 for all these phases. In each case the energy-density is decreasing function of t and it becomes infinite at $t = 0$ but tends to zero as $t \to \infty$. Therefore, each model has singularity at $t = 0$.

We have also noted an interesting new solution, which admits an accelerating quintessence Universe with $p_m = -\rho_m/2$ and $q = -0.25$. Hence, it is consistent with the recent observations. It has been noted that the acceleration is caused by the scalar field only which provides negative pressure to give rise to the accelerated expansion of the Universe at late-time. The pressure, in this case, does not involve higher order correction. Therefore, we conclude that the late-time cosmic acceleration is caused by the hypothetical fluid (scalar field) in our model. The HD theory is not responsible for late-time acceleration.

We have also discussed the density parameter in each phase. Its behavior has shown in fig. 2.3 It has been observed that the Universe was curved during early times and becomes flat at late-time evolution. However, in radiation-dominated phase, the Universe remains flat throughout its evolution.

We have also presented the solution for closed and open FRW models in HD theory and discussed the solution for a very special case which shows the coasting cosmology. The conservation Eq. is identically satisfied in all phases of evolution of the Universe which supports the consistency of the solution in HD theory. The parameter λ when set equal to zero, the results obtained by Sen and Banerjee [196] for perfect fluid and scalar field may be recovered.

Chapter 3

Scalar field cosmology in higher derivative theory

In this chapter[1] we examine the dynamics of expansion of the Universe in a flat FRW model containing a perfect fluid and a scalar field with scalar potential in HD theory of gravity. We reconstruct the scalar field potential by assuming a power-law expansion of the scale factor. A number of evolutionary phases of the Universe including the present accelerating phase are studied. The properties of scalar field and the other physical parameters are discussed in detail. It is observed that HD term could hardly be a candidate to describe the observed accelerated expansion of the Universe. It is only the hypothetical fluid, which provides the late-time acceleration. It is also noted that HD theory does not affect the evolution in radiation-dominated phase of scalar field cosmology.

3.1 Introduction

In scalar field cosmology, the usual approach to build a dynamical cosmological model is to solve the Einstein's equations for a given potential $V(\phi)$ of the scalar field ϕ. However, a convincing and unambiguous expression for $V(\phi)$ is still lacking [85,87]. It is due to the fact that there is no underlying principle which uniquely

[1]The result of this chapter is based on a research paper entitled "Power-law expansion and scalar field cosmology in higher derivative theory", published in *International Journal of Theoretical Physics* **51** 1889–1900 (2012).

specifies the potential for the scalar field. The technique for reconstruction of the potentials for scalar fields reproducing a given cosmological scenario has attracted the attention of many researchers for a long time [32,46,85,87,88,193]. Some of these were based on a new particle physics and gravitational theories [32]. Others were postulated ad hoc to obtain the desired evolution of the scale factor [193]. For instance, one can have power-law or exponential inflation consistent with the model with an arbitrary potential which may drive inflation and late-time acceleration of the Universe [38, 46, 82–88, 145, 199]. In previous chapter we have also considered this usual approach by assuming scalar field potential and scale factor varying exponentially with scalar field which lead to the power-law expansion of the scalar factor with time [196].

Using a technique different from the usual approach, Ellis and Madsen [193] introduced a new scheme to obtain the expressions for the scalar potential for a given evolution of the Universe in the framework of Einstein gravity. The authors considered a FRW model containing a minimally coupled classical scalar field and a perfect fluid in the form of non-interacting radiation to find suitable potentials for different inflationary models in an elegant way. They solved the field equations for the scalar field ϕ for a variety of given scale factors of the Universe for which the required potentials were then derived. Following the similar technique of Ellis and Madsen [193], Paul [146] have obtained some new and interesting scalar potentials for interacting scalar field in HD theory for some known behaviours of the Universe such as de Sitter expansion and power-law inflation.

In the present chapter, we study a flat FRW model filled with a minimally coupled scalar field and a perfect fluid in HD theory. Following a technique to determine potential similar to that used by Ellis and Madsen [193] in Einstein gravity and Paul [146] in HD gravity, we also explore a specific expression for the scalar field and the corresponding scalar potential by assuming a power-law expansion of the scale factor. We study the behaviour of the scalar field and the corresponding potential in the early and late-time evolution of the Universe through EoS parameter of the scalar field.

3.2 Model and the field equations

We consider the case of a spatially flat ($k=0$) FRW Universe, described by the line-element (1.2.2)

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]. \tag{3.2.1}$$

The Ricci scalar for (3.2.1), is given by

$$R = -6\left[\dot{H} + 2H^2\right]. \tag{3.2.2}$$

For the metric (3.2.1) and the energy-momentum tensor (2.2.3), the field equations of HD gravity (2.2.2) for ($k=0$), yield

$$3H^2 - 18\lambda\left[2\ddot{H}H - \dot{H}^2 + 6\dot{H}H^2\right] = \rho_m + \rho_\phi\,, \tag{3.2.3}$$

$$2\dot{H} + 3H^2 - 6\lambda\left[2\dddot{H} + 12\ddot{H}H + 18\dot{H}H^2 + 9\dot{H}^2\right] = -p_m - p_\phi. \tag{3.2.4}$$

In present study, we again assume that both the matters (perfect fluid and scalar field) do not interact each other. Therefore, we use separate energy-conservation laws as defined in Eqs. (2.2.7) and (2.2.8), respectively.

3.3 Solution of the field equations

In chapter 2, we have assumed that the scalar field potential and the scale factor evolve exponentially with the scalar field. In this chapter, we are interested to find the form of the scalar field potential in HD theory. In reconstruction scheme of scalar field cosmologies, the scalar field potential is required a known evolution of the Universe in the framework of the concerned theory. We have seen that the two assumptions made in chapter 2, naturally lead to the power-law expansion of the scale factor with time [196]. The considerable importance of exact power-law solution is the representation of all possible cosmological evolutions [200]. Therefore, in first assumption, we consider the power-law expansion of the scale factor in its usual form, i.e.,

$$a = a_0\left(\frac{t}{t_0}\right)^n, \tag{3.3.1}$$

where a_0 is a positive constant and $n \geq 0$, which determines the expansion of the scale factor in the different phases of evolution of the Universe. Eq. (3.3.1) gives accelerated expansion when $n > 1$. The speed is $\dot{a} = na/t$ and acceleration is $\ddot{a} = n(n-1)a/t^2$. Here a_0 and t_0 are the present values of a and t.

The deceleration parameter (1.7.10) for (3.3.1) gives

$$q = \frac{1-n}{n}, \tag{3.3.2}$$

with $0 < n < 1$ for decelerated expansion, $n > 1$ for accelerated expansion and $n = 1$ corresponds to the marginal inflation of the Universe.

The Hubble parameter (1.7.5) for power-law expansion (3.3.1), is given by

$$H = \frac{n}{t} \,. \tag{3.3.3}$$

Using (3.3.3) in (3.2.2), we have $R \propto t^{-2}$, and consequently, $R^2 \propto t^{-4}$.

In the second assumption, we consider the perfect fluid EoS (1.5.7), which can be taken as

$$p_m = \omega_m \rho_m, \qquad \text{where } \ \omega_m = \gamma - 1. \tag{3.3.4}$$

Using Eqs. (3.3.4) and (3.3.3), the conservation equation of the perfect fluid, i.e., Eq. (2.2.7) readily integrates to obtain

$$\rho_m = \rho_{m0}\, t^{-3n(1+\omega_m)} \,, \tag{3.3.5}$$

where, $\rho_{m0} = c_0(t_o^n/a_o)^{3(1+\omega_m)}$ and c_0 is a constant of integration. For $\rho_{m0} > 0$ we must have $c_0 > 0$.

Now, Eq. (3.2.3) can be rewritten as

$$\rho_\phi = 3H^2 - 18\,\lambda[2H\ddot{H} - \dot{H}^2 + 6\dot{H}H^2] - \rho_m \,. \tag{3.3.6}$$

Using of (3.3.3) and (3.3.5), (3.3.6) gives

$$\rho_\phi = \frac{3n^2}{t^2} - \frac{\rho_{m0}}{t^{3n(1+\omega_m)}} + \frac{54\,\lambda\, n^2(2n-1)}{t^4} \,. \tag{3.3.7}$$

The energy conservation equation of the scalar field (2.2.8) can be written as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0, \tag{3.3.8}$$

which gives

$$\dot{\phi}^2 = -\frac{1}{3H}\frac{d\rho_\phi}{dt} \,. \tag{3.3.9}$$

Using Eqs. (3.3.3) and (3.3.7) into the above equation, we obtain the kinetic term in terms of t

$$\dot{\phi}^2 = \frac{2n}{t^2} - \frac{(1+\omega_m)\rho_{m0}}{t^{3n(1+\omega_m)}} + \frac{72\,\lambda\,n\,(2n-1)}{t^4}\,, \tag{3.3.10}$$

which should not be negative for the model to be consistent, and therefore, it must has the restriction

$$\frac{2n}{t^2} - \frac{(1+\omega_m)\rho_{m0}}{t^{2n(1+\omega_m)}} + \frac{72\lambda n(2n-1)}{t^4} \geq 0\,. \tag{3.3.11}$$

Using (3.3.7) and (3.3.10) into (1.10.5), the scalar field potential is given by

$$V(t) = \frac{n(3n-1)}{t^2} + \frac{[(1+\omega_m)-2]\rho_{m0}}{2\,t^{\,3n(1+\omega_m)}} + \frac{18\,\lambda\,n\,(2n-1)(3n-2)}{t^4}\,, \tag{3.3.12}$$

which represents the potential as a function of time t. We observe that the kinetic term and the scalar potential have the term of $1/t^4$ due to HD gravity. The Einstein's solutions may be recovered for $n = 1/2$ or $\lambda = 0$.

On integrating Eq. (3.3.10), one can find the scalar field as a function of time. Inverting the time parameter as a function of ϕ and substituting the obtained relation into Eq. (3.3.12), one arrives to the uniquely reconstructed potential $V(\phi)$.

Using (3.3.10) and (3.3.12), the time-dependent pressure of the scalar field is extracted from (1.10.6), as

$$p_\phi = \frac{n(2-3n)}{t^2} + \frac{18\,\lambda\,n(2n-1)(4-3n)}{t^4} - \frac{((1+\omega_m)-1)\rho_{m0}}{t^{3n(1+\omega_m)}}\,. \tag{3.3.13}$$

Thus, we straight forward reach to the time evolution of EoS parameter corresponding to the scalar field

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dfrac{n(2-3n)}{t^2} + \dfrac{18\,\lambda\,n(2n-1)(4-3n)}{t^4} - \dfrac{[(1+\omega_m)-1]\rho_{m0}}{t^{3n(1+\omega_m)}}}{\dfrac{3n^2}{t^2} - \dfrac{\rho_{m0}}{t^{3n(1+\omega_m)}} + \dfrac{54\,\lambda\,n^2(2n-1)}{t^4}}\,. \tag{3.3.14}$$

3.4 Evolution of the Universe

It is very difficult to integrate Eq. (3.3.10) for $\phi(t)$, in general. Since the evolution of the Universe is usually divided in several phases, during each of them some kind of assumptions are made to simplify the system of equations. Therefore, it is reasonable to explore the above solutions for the standard values of EoS parameter ω_m of the perfect fluid and corresponding value of parameter n in power-law. Let us find some solutions and discuss their consistency in different phases of the Universe in the following subsections.

3.4.1 Solution with $\omega_m = -1/3$ and $n = 1$

In this case, we get $p_m = -\rho_m/3$ and the scale factor varies as a linear expansion, i.e., $a \sim t$, which is the case of inflationary phase. The energy density varies as inverse of cosmic time t, i.e., $\rho_m \sim 1/t^2$. The deceleration parameter has the value $q = 0$, which implies the 'coasting cosmology or marginal inflationary phase' of the early Universe.

In this case, Eq. (3.3.10) becomes

$$\dot{\phi}^2(t) = 2\left(1 - \frac{\rho_{m0}}{3}\right)\frac{1}{t^2} + \frac{72\,\lambda}{t^4}\,, \quad (\rho_{m0} \leq 3) \tag{3.4.1}$$

which, on integration, it gives

$$\phi(t) - \phi_1 = \sqrt{D_1}\left[\log\left\{2D_1 t\left(1 + \sqrt{1 + \frac{72\,\lambda}{D_1 t^2}}\right)\right\} - \sqrt{1 + \frac{72\,\lambda}{Bt^2}}\right]\,, \tag{3.4.2}$$

where ϕ_1 is a constant of integration and $D_1 = 2(1 - \rho_{m0}/3)$. We consider here and thereafter only positive sign without loss of generality.

From Eq. (3.3.12), the scalar potential takes the form

$$V(t) = 2\left(1 - \frac{\rho_{m0}}{3}\right)\frac{1}{t^2} + \frac{18\,\lambda}{t^4}. \tag{3.4.3}$$

From (3.4.2) and (3.4.3), we observe that the kinetic term $\dot{\phi}^2(t)$ and the potential function $V(t)$ decrease from large values to zero during the evolution of the Universe. The kinetic term and the scalar potential have the same expression ($V = \dot{\phi}^2$) in the absence of HD theory ($\lambda = 0$).

Equations (3.3.7) and (3.3.13) give the scalar field energy density and pressure, respectively as

$$\rho_\phi = 3\left(1-\frac{\rho_{m0}}{3}\right)\frac{1}{t^2}+\frac{54\,\lambda}{t^4}\,, \tag{3.4.4}$$

$$p_\phi = -\left(1-\frac{\rho_{m0}}{3}\right)\frac{1}{t^2}+\frac{18\,\lambda}{t^4}\,, \tag{3.4.5}$$

which show that both ρ_ϕ and p_ϕ decrease with time.

Equation (3.3.14) gives

$$\omega_\phi = \frac{-\left(1-\frac{\rho_{m0}}{3}\right)\frac{1}{t^2}+\frac{18\,\lambda}{t^4}}{3\left(1-\frac{\rho_{m0}}{3}\right)\frac{1}{t^2}+\frac{54\,\lambda}{t^4}}. \tag{3.4.6}$$

It is observed that $\omega_\phi = 1/3$ when $t \to 0$ or $\rho_{mo} = 3$ and if $t \to \infty$ or $\lambda = 0$ we have $\omega_\phi = -1/3$. We also observe that ω_ϕ makes smooth transition from $\omega_\phi = 1/3$ to $\omega_\phi = -1/3$, which is shown in fig 3.4. Thus, $\omega_m = \omega_\phi$ at late-time expansion of the Universe.

3.4.2 Solution with $\omega_m = 1/3$ and $n = 1/2$

In this case, we have $p_m = \rho_m/3$ and $a \sim t^{1/2}$, which is the usual radiation-dominated phase. The behavior of energy density of perfect fluid is similar to that of a radiation-dominated phase and varies as $\rho_m \sim 1/t^2$. The deceleration parameter $q = 1$ and hence the Universe expands with decelerated rate.

In this case, Eq. (3.3.10) becomes

$$\dot{\phi}^2(t) = \left(1-\frac{4\rho_{m0}}{3}\right)\frac{1}{t^2}\,, \quad \left(\rho_{m0} < \frac{3}{4}\right)\,, \tag{3.4.7}$$

which on integration, we get

$$\phi(t)-\phi_2 = \sqrt{1-\frac{4\rho_{m0}}{3}}\;\log t\,. \tag{3.4.8}$$

where ϕ_2 is a constant of integration.

The scalar potential has the form

$$V(t) = \frac{1}{4}\left(1-\frac{4\rho_{m0}}{3}\right)\frac{1}{t^2}\,. \tag{3.4.9}$$

It is to be noted that both the scalar field and the scalar potential are independent of HD term, hence it is evident that the presence of HD term does not affect the behavior of the model in radiation-dominated phase. Therefore, the cosmological evolution remains same as in radiation-dominated phase of Einstein's gravity.

Inverting Eq. (3.4.8), we find

$$t(\phi) = \exp\left[\frac{1}{\sqrt{1-\frac{4\rho_{m0}}{3}}}(\phi(t)-\phi_2)\right]. \tag{3.4.10}$$

Hence, Eq. (3.4.9) becomes

$$V(\phi) = \frac{3}{4}\left(1-\frac{4\rho_{m0}}{3}\right)\exp\left[-2\sqrt{\frac{3}{3-4\rho_{m0}}}\Big(\phi(t)-\phi_2\Big)\right]. \tag{3.4.11}$$

The above expression shows that the scalar potential decreases exponentially with the scalar field as shown in fig. 3.1.

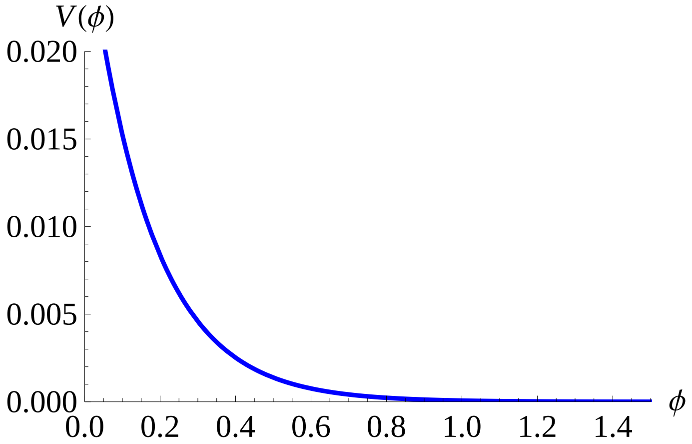

Figure 3.1: Scalar potential $V(\phi)$ *vs.* scalar field ϕ with $\phi_2 = 0$, and $\rho_{m0} = 2/3$.

If $3(1-4\rho_{m0}/3)/4 = V_0$, $1/\sqrt{V_0} = \beta$ and $\phi_2 = 0$ then $V(\phi) = V_0 e^{-\beta\phi}$, thus, the exponential potential (2.3.1) considered in chapter 2 is recovered. Similarly, the scale factor assumed in (2.3.2) of the form $a = a_0 e^{\alpha\phi}$ may be recovered.

From Eqs. (3.3.7) and (3.3.13), the scalar field density and pressure are respectively become

$$\rho_\phi = \left(\frac{3}{4}-\rho_{m0}\right)\frac{1}{t^2}, \tag{3.4.12}$$

$$p_\phi = \frac{1}{3}\left(\frac{3}{4}-\rho_{m0}\right)\frac{1}{t^2}. \tag{3.4.13}$$

From these equations the EoS parameter of scalar field and perfect fluid have the same constant value, i.e., $\omega_\phi = 1/3 = \omega_m$ at all times during the radiation epoch, hence the behavior of scalar field density and pressure are similar to radiation era as observed in GR. The Universe decelerates throughout the evolution during radiation-dominated phase even if $\lambda \neq 0$ in HD theory. For $\rho_{m0} = 3/4$, the scalar field cosmology vanishes and the solutions reduce to the perfect fluid model in this case.

3.4.3 Solution with $\omega_m = 0$ and $n = 2/3$

For these values of ω_m and n, we get $p_m = 0$ and $a \sim t^{2/3}$ respectively, which corresponds to the matter-dominated phase. The deceleration parameter has the value $q = 0.5$, which shows that the Universe expands with decelerated rate. The energy density varies as $\rho_m \sim 1/t^2$.

The kinetic term is given by

$$\dot{\phi}^2(t) = \left(\frac{4}{3} - \rho_{m0}\right)\frac{1}{t^2} + \frac{16\,\lambda}{t^4}\,, \quad \left(\rho_{m0} < \frac{4}{3}\right)\,. \tag{3.4.14}$$

Integrating Eq. (3.4.14), we obtain

$$\phi(t) - \phi_3 = \sqrt{D_2}\left[\log\left\{2D_2 t\left(1 + \sqrt{1 + \frac{16\,\lambda}{D_2 t^2}}\right)\right\} - \sqrt{1 + \frac{16\,\lambda}{D_2 t^2}}\right]\,, \tag{3.4.15}$$

where ϕ_3 is a constant of integration and $D_2 = (4 - 3\rho_{m0})/3$.

The scalar potential becomes

$$V(t) = \left(\frac{4 - 3\rho_{m0}}{6}\right)\frac{1}{t^2}\,. \tag{3.4.16}$$

It is to be noted that the scalar field ϕ contains the coupling parameter λ but the scalar potential $V(t)$ is independent of HD term. Therefore, the scalar potential is similar to the Einstein's gravity during the matter-dominated era.

In this case, the scalar field density and pressure are respectively given as

$$\rho_\phi = \left(\frac{4}{3} - \rho_{m0}\right)\frac{1}{t^2} + \frac{8\,\lambda}{t^4}\,, \tag{3.4.17}$$

$$p_\phi = \frac{8\,\lambda}{t^4}\,. \tag{3.4.18}$$

Both the quantities, ρ_ϕ and p_ϕ are decreasing function of time.

The EoS parameter of scalar field is given by

$$\omega_\phi = \frac{\dfrac{8\,\lambda}{t^4}}{\left(\dfrac{4}{3}-\rho_{m0}\right)\dfrac{1}{t^2}+\dfrac{8\,\lambda}{t^4}}\,. \tag{3.4.19}$$

As $t \to 0$ or $\rho_{mo} = 4/3$, we get $\omega_\phi = 1$, i.e., the scalar field represents the stiff matter $(p_\phi = \rho_\phi)$. As $t \to \infty$ or $\lambda = 0$ we have $\omega_\phi = 0$, which is equivalent to the EoS of matter-dominated phase. This shows that the scalar field acts like stiff matter at early time but in late-time it behaves as pressureless dust in matter-dominated phase. The scalar field does not inflate the Universe during the matter-dominated era even in HD theory.

3.4.4 Solution with $\omega_m = -1/2$ and $n = 4/3$

In this case, we get $p_m = -\rho_m/2$, $a \sim t^{4/3}$ and $\rho_m \sim 1/t^2$. The deceleration parameter $q = -0.25$ which reveals the case of quintessence phase of the accelerating Universe.
Eq. (3.3.10) gives

$$\dot{\phi}^2(t) = \left(\frac{8}{3}-\frac{\rho_{m0}}{2}\right)\frac{1}{t^2}+\frac{160\,\lambda}{t^4}\,, \quad \left(\rho_{m0} < \frac{16}{3}\right)\,, \tag{3.4.20}$$

which on integration, we get

$$\phi(t)-\phi_4 = \sqrt{D_3}\left[\log\left\{2D_3t\left(1+\sqrt{1+\frac{160\,\lambda}{D_3t^2}}\right)\right\}-\sqrt{1+\frac{160\,\lambda}{D_3t^2}}\right]\,, \tag{3.4.21}$$

where ϕ_4 is the integration constant and $D_3 = (16-3\rho_{m0})/6$.

From Eq. (3.3.12), we obtain

$$V(t) = \left(4-\frac{3\rho_{m0}}{4}\right)\frac{1}{t^2}+\frac{80\,\lambda}{t^4}. \tag{3.4.22}$$

From Eqs. (3.4.21) and (3.4.22), it is evident that the scalar field and the scalar potential depend on the coupling parameter associated with the HD term. The solutions obtained here describe an accelerating Universe which is compatible with the recent observations.

Eqs. (3.3.7) and (3.3.13) give

$$\rho_\phi = \left(\frac{16}{3}-\rho_{m0}\right)\frac{1}{t^2}+\frac{160\,\lambda}{t^4}\,, \tag{3.4.23}$$

$$p_\phi = -\frac{1}{2}\left(\frac{16}{3}-\rho_{mo}\right)\frac{1}{t^2}\,. \tag{3.4.24}$$

Therefore, the EoS parameter of scalar field evolves as

$$\omega_\phi = \frac{-\frac{1}{2}\left(\frac{16}{3}-\rho_{mo}\right)\frac{1}{t^2}}{\left(\frac{16}{3}-\rho_{m0}\right)\frac{1}{t^2}+\frac{160\,\lambda}{t^4}}\,. \tag{3.4.25}$$

If $t\to 0$ or $\rho_{m0}=16/3$ we get $\omega_\phi=0$, and if $t\to\infty$ or $\lambda=0$, we get $\omega_\phi=-1/2$, i.e., the quintessence model. The Universe is matter-dominated at early time and quintessence-dominated at late-time. The scalar field and the perfect fluid have the same behavior at late-time. The pressure is negative and does not contain HD term which provides the repulsion to accelerate the Universe in late times. Since R^2 correction tends to zero as $t\to\infty$, therefore, it is only the scalar field contribution that causes the accelerated expansion of the Universe. Thus, HD theory is not responsible for late-time acceleration in th present model.

Figs. 3.2 and 3.3 plot scalar field $\phi(t)$ *verses* time and scalar potential $V(t)$ *verses* time, respectively, for above discussed different phases of the Universe for some particular values of parameters.

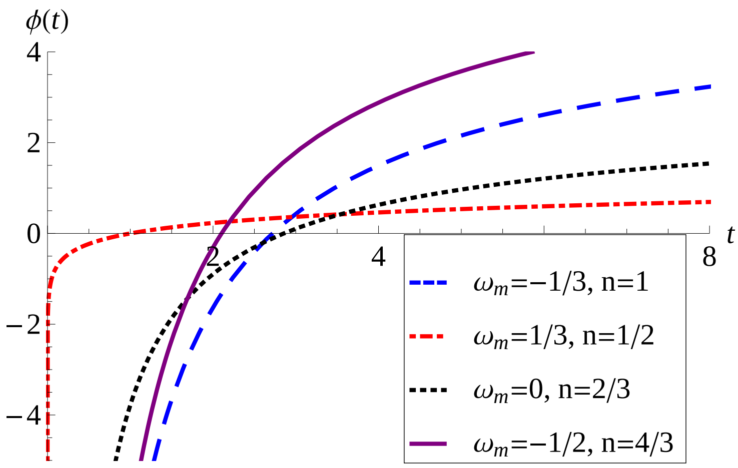

Figure 3.2: $\phi(t)$ *vs.* t with $\phi_1=\phi_2=\phi_3=\phi_4=0$, $\lambda=2$ and $\rho_{m0}=2/3$.

In fig. 3.2, we observe that the scalar field increases with decelerated rate during all phases. However, it grows faster in radiation-dominated phase due to the absence of HD term.

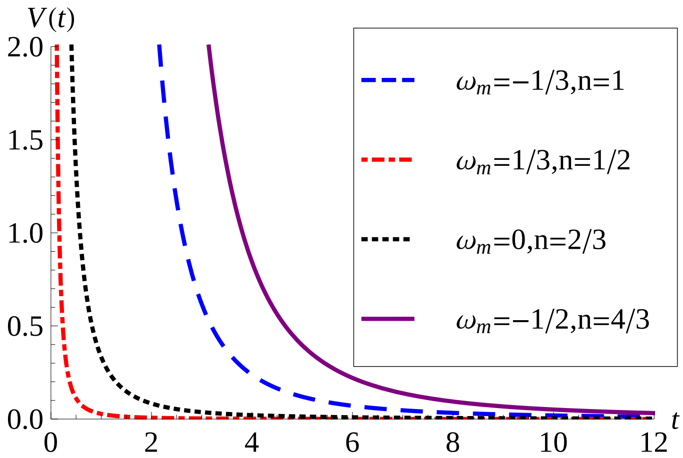

Figure 3.3: $V(t)$ *vs.* t with $\lambda = 2$ and $\rho_{m0} = 2/3$.

In fig. 3.3, the scalar field potential decreases slowly with time in inflationary, matter-dominated and accelerated phases in compare to the radiation-dominated phase where it shows the graph same as in GR. It decreases fast in radiation-dominated phase due to the absence of HD term.

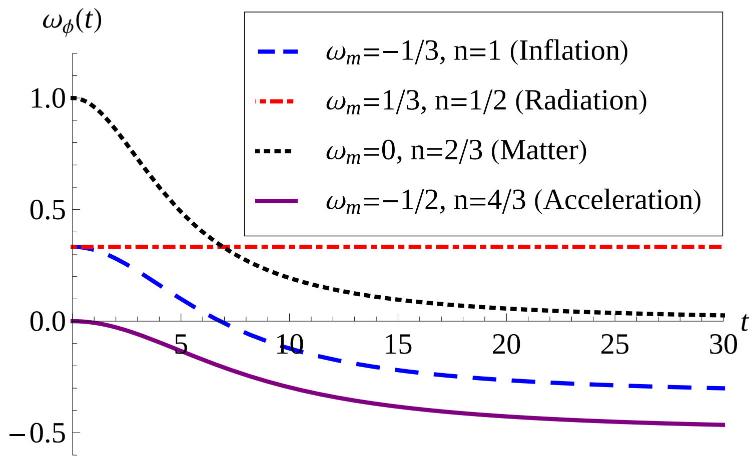

Figure 3.4: $\omega_\phi(t)$ *vs.* t with $\lambda = 2$ and $\rho_{m0} = 2/3$.

Fig. 3.4 plots $\omega_\phi(t)$ *verses* t for certain values of the arbitrary constants for different phases of the Universe. It is clear that $\omega_\phi = \omega_m$ as $t \to \infty$ in all cases as discussed above since the $R^2 \to 0$ at late-time.

3.5 Conclusion

In this chapter, we have studied a flat FRW model containing R^2 terms in the curvature, a self-interacting scalar field and a perfect fluid. Exact solutions for scalar field density, scalar potential and some other physical parameters have been determined under the assumptions of power-law expansion of the scale factor. We have observed that the power-law cosmology is compatible with the observations since it gives a constant deceleration parameter to describe the decelerated and accelerated phases of the Universe.

The field equations have been solved exactly for specific values of EoS parameter of perfect fluid in different phases of evolution of the Universe. We have noted that the physical quantities contain higher order terms in inflationary, matter-dominated and accelerated phases whereas HD theory does not affect the behavior of radiation-dominated era. In radiation phase we have obtained the solution similar to Einstein's gravity. In radiation era, it has been possible to reconstruct the scalar field potential $V(\phi)$ as a function of ϕ of the form $V(\phi) = V_0 e^{-\beta_0 \phi}$ which is similar to that we have assumed in chapter 2. The scale factor of the form $a = a_0 e^{\alpha \phi}$, which we have assumed in chapter 2 may also be recovered in radiation phase. The model has Big-Bang singularity at $t = 0$.

The Universe decelerates ($q < 0$) during radiation- and matter-dominated phases even in the presence of HD terms. In case of $\omega_m = 1/2$ and $n = 4/3$, it has been observed that HD theory is not responsible for late-time acceleration of the Universe. It is only the hypothetical fluid (scalar field), which gives rise to the accelerated expansion at late-time where the higher gravity correction disappears. The scalar field acts as quintessence at late-time of evolution of the Universe. Thus, HD theory is useful to study the Universe in early time of its evolution.

It is to be noted that if the HD term is removed, i.e., $\lambda = 0$, the solutions obtained here are similar to that studied by Ellis and Madsen [193] in the Einstein's gravity. In the absence of the perfect fluid, the solutions reduces to the solutions that obtained by Paul [146] in HD theory.

Chapter 4

Emergent Universe with exotic matter in higher derivative theory

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In this chapter[1] we explore the possibility of the emergent Universe filled with a scalar or tachyonic field (quintessence and phantom) minimally coupled to gravity in HD theory within the framework of a spatially homogeneous and isotropic flat FRW space-time. We obtain the exact solution of gravitational field equations and observe that the emergent Universe is not possible with quintessence scalar and tachyonic fields but it exists with phantom scalar and tachyonic fields in HD gravity. The models have no time-like singularity and admits an ever accelerating Universe.

4.1 Introduction

It is well known that the standard cosmological model is plagued Big-Bang singularity that lurks at the origin of the cosmic expansion. This undigestible feature of an initial singularity is a state of infinite energy density and pressure where the laws of physics break down. So a fundamental cosmological shortcoming arises naturally: what was there before the stage of classical expansion of the "Big-Bang" ? To elude this serious problem several solutions have been proposed.

[1]The content of this chapter in the form of research paper entitled "Emergent Universe with scalar (or tachyonic) field in higher derivative theory", has been published in *Astrophysics and Space Science* **339** 101–109 (2012).

Some arguments were put forward for several inflationary cosmologies that were past-eternal. An alternative is the possibility of a fetus Universe which existed as an eternal “seed" before sprouting into the macroscopic Universe. Harrison [201] found an exact solution of a radiation-dominated closed Universe with a positive cosmological constant having similar properties. In Harrison's mechanism, the Universe originates from an Einstein static state with a radius determined by the value of Λ, before entering into a endless period of de Sitter expansion.

Later on, an interesting model of this scenario was presented by Ellis and Maartens [202] for a spatially closed space-time filled with ordinary matter and minimally coupled scalar field, which was also past asymptotic to an Einstein static Universe with a radius determined by kinetic energy of scalar field. Ellis et al. [203] proposed a closed Universe in which there was no beginning of time, and therefore, no time-like singularity, which also effectively avoids any quantum regime by staying large enough at all times. Their model also relies on a minimally coupled scalar field ϕ with a special form of interacting potential $V(\phi)$ which is asymptotically flat as $\phi \to -\infty$, but reaches at maximum as $\phi \to 0$, signaling the beginning of de Sitter inflationary phase. The de Sitter inflation naturally comes to an end as the scalar field starts oscillating around the minimum of its potential, before entering into the next phases of standard hot Big-Bang expansion. In this way the search of singularity free Universe in the context of classical GR led to the development of so called **“Emergent Universe (EU)"**.

The salient features of EU are summarized in refs. [204] as follows.

1. The most recognizable description of EU is that this scenario replaces the Big-Bang singularity by an Einstein static phase occurring over an infinite time in the past.

2. Consequently, it is ever existing and hance there is no Big-Bang singularity.

3. There is no quantum era because the initial static state can be chosen to have a radius larger than the Planck scale.

4. There is no horizon problem since the Universe is always large enough so that the classical description of isotropy and homogeneity may be adequate.

5. The Universe may contain exotic matter so that the energy conditions may be violated.

6. The Universe may accommodate late-time acceleration as well.

A viable cosmological model should exhibit an inflationary phase in the early time and an accelerating phase at late-time, the EU scenario is also promising from this perspective of offering early and late-time dynamics of the Universe in an unified manner. It is to be noted that the focal point of such unification lies in the choice of either the EoS for matter or on the scalar field dynamics through the different choices of scalar field potentials [88, 205]. A general framework of EU scenario, containing a polytropic fluid with a non-linear EoS of the form $p = \gamma_1 \rho - \gamma_2 \sqrt{\rho}$, where γ_1 and γ_2 are constants, has been proposed by Mukherjee et al. [206] for a flat model in GR. Paul et al. [207, 208] have found the range of the permissible values for the parameters γ_1 and γ_2 needed for EU, using observational data from $H(z)$ and BAO. Recently, Marra et al. [209] have imposed tight bounds on these constraints from Planck 2013 observational data.

Most of the EU scenarios are expected to be dominated by some energy component that violates strong energy condition (SEC) to attain the transition from the static phase to inflationary phase. Therefore, the Universe must include some exotic matter sources. It has also been shown that the matter which violates the SEC, allows the cyclic (oscillating and non-singular) Universe [210]. One way to get a transition from a decelerating phase to an accelerating phase in a flat Universe is to violate the weak energy condition (WEC). It seems that similar solutions should also appear for the phantom matter which violates WEC and SEC [58, 68, 68, 71, 74, 75, 152, 211, 212]. Debnath [213] has studied the behavior of different stages of the evolution of EU filled with normal matter and a phantom field (or tachyonic field) in GR.

Phantom matter may also arise in higher order theories of gravity [214]. Therefore, many authors have been realized the EU scenario in modified theories of gravitation. Campo et al. [215] have studied EU in the context of a self-interacting Jordan-Brans-Dicke theory. Banerjee et al. [216, 217] have discussed EU models in Brane-World scenario. Beesham et al. have studied nonlinear sigma model of EU with (exact global phantonical solution) [218], and with dark sector fields (a chiral cosmological model) [219]. Mukerji and Chakraborty [220], and Paul and Ghose [221] independently developed EU scenario in Einstein-Gauss-Bonnet theory. Mukerji and Chakraborty [222] have considered the FRW cosmological model of EU in Horav̆a gravity. Debnath and Chakraborty [223], in Brane World scenario and Chakraborty and Debnath [224] in anisotropic Universe have examined EU and found that this scenario could be realized fairly well. Recently, Chervon et al. [225] have analysed chiral cosmological fields in Einstein-Gauss-Bonnet gravity.

Murlryne et al. [226] have pointed out that the scalar potential for scalar field chosen by Ellis et al. [203] is similar to what one obtains from a modified gravitational action with a polynomial Lagrangian, $R+\lambda R^2$ [126, 227–231]. This concept motivates us to examine EU in $R+\lambda R^2$ gravity by taking scalar and tachyonic fields with their corresponding potentials.

In the present chapter, we seek the possibility of EU scenario in HD theory for a flat FRW model with quintessence or phantom scalar field or tachoynic field (normal or phantom) minimally coupled to gravity with corresponding potentials.

4.2 Quintessence and phantom scalar fields models

4.2.1 Gravitational action and the field equations

The gravitational action for HD theory of gravity with quintessence or phantom scalar field ϕ, minimally coupled to gravity in the units of $8\pi G = 1 = c$, is given by

$$S = \int \left[\frac{1}{2}\left(R+\lambda R^2\right) + \frac{1}{2}\,\varepsilon\phi_{,\mu}\,\phi^{,\mu} + V(\phi)\right]\sqrt{-g}\,d^4x\,, \qquad (4.2.1)$$

where $\varepsilon = \pm 1$ correspond to normal and phantom scalar field, respectively.

Variation of action (4.2.1) with respect to the metric tensor, $g_{\mu\nu}$ leads to the following field equations

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R + \lambda\left[2R(R_{\mu\nu} - \frac{1}{4}\,g_{\mu\nu}R) + 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)R\right] = T^{(\phi)}_{\mu\nu}\,, \qquad (4.2.2)$$

where $T^{(\phi)}_{\mu\nu}$ is the energy-momentum tensor of quintessence or phantom scalar field, given in Eq. (1.10.3).

We consider a homogeneous and isotropic flat FRW model which is given by the metric (3.2.1). The field equations (4.2.2) with energy-momentum tensor (1.10.3) for the line-element (3.2.1), yield

$$
\begin{aligned}
3H^2-18\lambda\left[2\ddot{H}H-\dot{H}^2+6\dot{H}H^2\right] &= \frac{1}{2}\varepsilon\dot{\phi}^2+V(\phi)\,, && (4.2.3)\\
2\dot{H}+3H^2-6\lambda\left[2\dddot{H}+12\ddot{H}H+18\dot{H}H^2+9\dot{H}^2\right] &= -\left[\frac{1}{2}\varepsilon\dot{\phi}^2-V(\phi)\right]. && (4.2.4)
\end{aligned}
$$

We also consider the Klein-Gordon equation (1.10.7) for scalar field and the EoS parameter ω_ϕ for scalar field which is given by Eq. (1.10.8).

Now, we confine our attention towards EU. Mathematically, the scale factor of FRW metric does not vanish in EU models and usually has the form [202,213,215]

$$a=a_0\left(\beta_1+e^{\alpha_1 t}\right)^n\,, \tag{4.2.5}$$

where a_0, β_1, α_1 and n are positive constants. Accordingly, the energy density, pressure and other physical quantities do not diverge at any stage of evolution of the Universe. For the above form of scale factor, the Hubble parameter (1.7.5) and its derivatives are obtained as

$$
\begin{aligned}
H &= \frac{n\alpha_1 e^{\alpha_1 t}}{\beta_1+e^{\alpha_1 t}},\\
\dot{H} &= \frac{n\beta_1\alpha_1^2 e^{\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2},\\
\ddot{H} &= \frac{n\beta_1\alpha_1^3 e^{\alpha_1 t}(\beta_1-e^{\alpha_1 t})}{(\beta_1+e^{\alpha_1 t})^3},\\
\dddot{H} &= -\frac{n\beta_1\alpha_1^4(5e^{3\alpha_1 t}-2\beta_1 e^{2\alpha_1 t}-\beta_1^2 e^{\alpha_1 t})}{(\beta_1+e^{\alpha_1 t})^4}.
\end{aligned} \tag{4.2.6}
$$

Here, H and $\dot{H}$ are positive definite but $\ddot{H}$ and $\dddot{H}$ change their sign at $t=(\log\beta_1)/\alpha_1$ and $t=\left[\log\beta_1(1+\sqrt{6})/5\right]/\alpha_1$, respectively. All the four tend to zero as $t\to-\infty$ whereas the model becomes a de Sitter Universe as $t\to\infty$.

The above specific form of Hubble parameter satisfies a first order differential equation given by

$$\dot{H}=\alpha_1 H-\frac{1}{n}H^2. \tag{4.2.7}$$

The deceleration parameter q defined in (1.7.10), gives

$$q=-1-\frac{\beta_1}{ne^{\alpha_1 t}}, \tag{4.2.8}$$

which shows that q is a function of t. As $t\to-\infty$, $q\to-\infty$ and as $t\to\infty$, q asymptotically tends to -1. Thus, $q<0$ throughout the evolution of the Universe and hence the scale factor of the form (4.2.5) always exhibits an ever accelerating Universe.

On solving (4.2.3) and (4.2.4) by use of (4.2.6), we obtain

$$\varepsilon\dot{\phi}^2 = -\frac{2n\beta_1\alpha_1^2 e^{\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2} - \frac{12n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[(3n-1)e^{3\alpha_1 t}-(9n-4)\beta_1 e^{2\alpha_1 t}-\beta_1^2 e^{\alpha_1 t}\right], \quad (4.2.9)$$

and

$$\begin{aligned} V(t) &= \frac{n\alpha_1^2(3ne^{2\alpha_1 t}+\beta_1 e^{\alpha_1 t})}{(\beta_1+e^{\alpha_1 t})^2} - \frac{6n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4} \\ &\times \left[(18n^2-9n+1)e^{3\alpha_1 t}+4(3n-1)\beta_1 e^{2\alpha_1 t}+\beta_1^2 e^{\alpha_1 t}\right]. \quad (4.2.10) \end{aligned}$$

These are the expressions of kinetic energy of quintessence or phantom scalar field and scalar potential, respectively. It is very difficult to analyse the behaviors of these physical quantities due to complicated expressions. Therefore, we examine the possibility of existence of EU scenario through some graphical representation for quintessence and phantom scalar fields, respectively, in the following subsections.

4.2.2 Quintessence scalar field model

For quintessence scalar field ($\varepsilon=+1$), Eq. (4.2.9) can be read as

$$\dot{\phi}^2 = -\frac{2n\beta_1\alpha_1^2 e^{\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2} - \frac{12n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[(3n-1)e^{3\alpha_1 t}-(9n-4)\beta_1 e^{2\alpha_1 t}-\beta_1^2 e^{\alpha_1 t}\right]. \quad (4.2.11)$$

We observe that $\dot{\phi}^2$ is negative for some particular values of parameters during

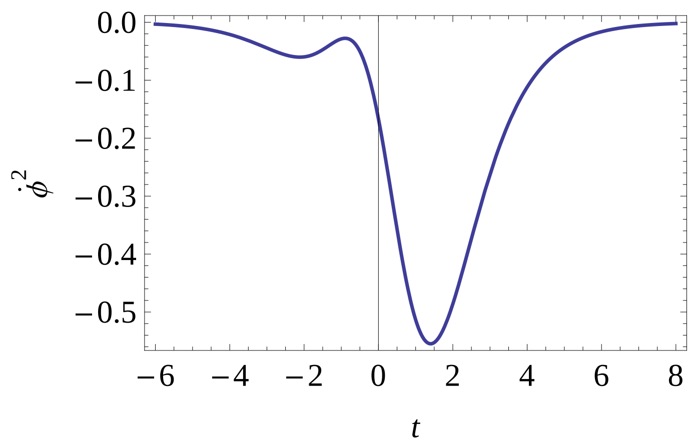

Figure 4.1: $\dot{\phi}^2(t)$ *versus* t for $\beta_1=1$, $\alpha_1=1$, $n=4/3$, and $\lambda=1/12$.

the evolution of the Universe as shown in fig. 4.1. One may observe this behavior of kinetic energy term for any set of parameters. Hence, ϕ is imaginary for quintessence scalar field. Since the kinetic term $(\dot{\phi}^2/2)$ is negative, therefore, EU is not possible for flat Universe with quintessence scalar field in HD theory as the energy density of any matter can not be negative.

4.2.3 Phantom scalar field model

In case of phantom scalar field ($\varepsilon = -1$), Eq. (4.2.9) takes the form

$$\dot{\phi}^2 = \frac{2n\beta_1\alpha_1^2 e^{\alpha_1 t}}{(\beta_1 + e^{\alpha_1 t})^2} + \frac{12n\lambda\beta_1\alpha_1^4}{(\beta_1 + e^{\alpha_1 t})^4}\left[(3n-1)e^{3\alpha_1 t} - (9n-4)\beta_1 e^{2\alpha_1 t} - \beta_1^2 e^{\alpha_1 t}\right]. \quad (4.2.12)$$

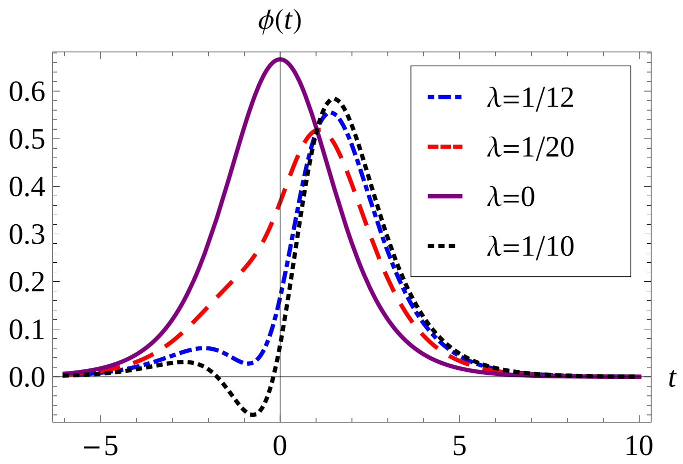

Figure 4.2: $\dot{\phi}^2(t)$ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

The variation of $\dot{\phi}^2$ with t for different values of λ and some particular values of other parameters is shown in fig. 4.2. It is observed that $\dot{\phi}^2$ is positive for small values of λ at any time t starting with zero at infinite past. It increases to a maximum value during the early time, goes to maximum finite value and then starts decreasing and tends to zero at late-time. We find that $\dot{\phi}^2$ increases or decreases sharply in GR ($\lambda = 0$) whereas it increases or decreases gradually for some values of $\lambda = 1/20$ and $\lambda = 1/12$. It is to be noted that the solutions are stable for small values of λ for any time t. However, the solution is unstable for $\lambda = 1/10$, i.e., large values of λ as shown in fig. 4.2. It is not possible to find out the range of instability of cosmic time due to the complicated expression in Eq. (4.2.12).

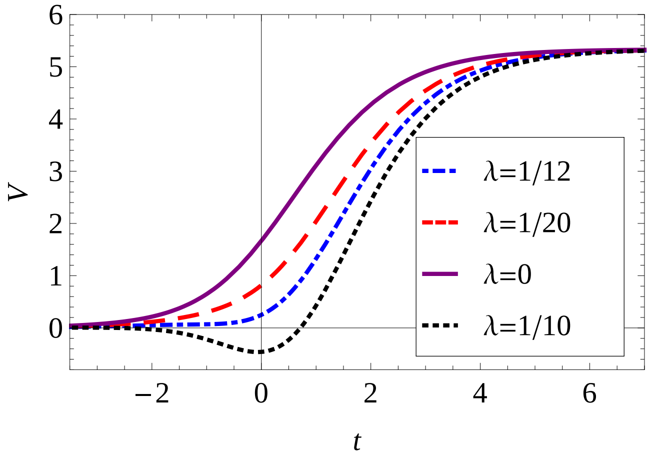

Figure 4.3: $V(t)$ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

The scalar potential $V(t)$, given in (4.2.10) is independent of ε, therefore, it remains same for quintessence and phantom scalar field. Now, it is very difficult to express the phantom field ϕ in closed form, so the potential V can not be expressed in terms of ϕ explicitly. In fig. 4.3 the graph of scalar potential with time for some particular values of parameters shows that $V(t)$ grows from zero at infinite past for different values of λ and becomes flat as $t \to \infty$. Therefore, the phantom field rolls to the maximum of its potential and then settles to a constant value in late-time of evolution of the Universe. It is to be noted that $V(t)$ grows slowly with the evolution of the Universe due to HD term as compare to GR ($\lambda = 0$). We have also observed that the scalar field density is zero at infinite past which increases with time and attains a maximum value in late-time of the evolution. The model

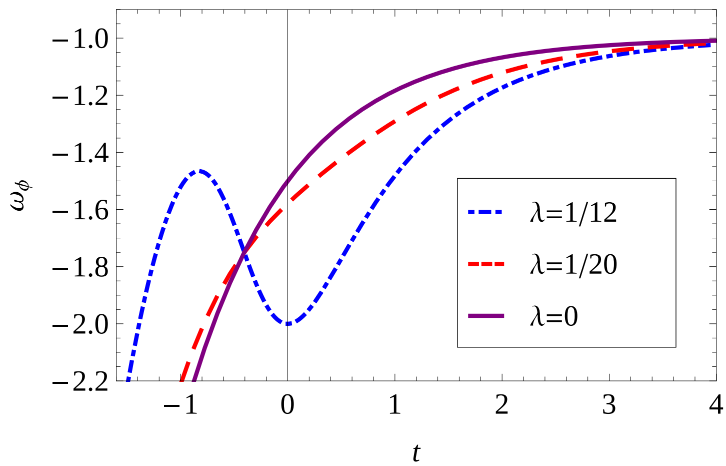

Figure 4.4: ω_ϕ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

has no time-like singularity at infinite past. It eventually evolves into an inflationary phase and accelerates throughout the evolution of the Universe.

From fig. 4.4 we can see that EoS parameter starts from phantom phase and it approaches to -1 in late-time. We observe that the stability of the solutions depend on the coupling parameter λ, i.e., the solutions are stable for small values of λ whereas unstable for large values of λ. We conclude that EU is possible with phantom scalar field for a flat FRW model in HD theory.

4.2.4 Particular solution

Since the field equations in HD theory permit the emergent solution (4.2.5) where the Hubble parameter satisfies a first order differential equation (4.2.7). Let us consider a particular solution by taking the particular values $\alpha_1 = 1/\sqrt{6\lambda}$ and $n = 2/3$ in Eq. (4.2.7). For these particular values, Eq. (4.2.7) becomes [144]

$$\dot{H} = \frac{1}{\sqrt{6\lambda}} H - \frac{3}{2} H^2 . \tag{4.2.13}$$

The scale factor (4.2.5) now has the form

$$a = a_0 \left[\beta_1 + e^{\frac{1}{\sqrt{6\lambda}} t} \right]^{\frac{2}{3}} . \tag{4.2.14}$$

The field equations (4.2.3) and (4.2.4) yield

$$\varepsilon \dot{\phi}^2 = -\frac{4\beta_1}{9\lambda} \frac{e^{3\sqrt{\frac{1}{6\lambda}}\, t}}{\left(\beta_1 + e^{\frac{1}{\sqrt{6\lambda}}\, t} \right)^4} , \tag{4.2.15}$$

and

$$V(t) = \frac{2}{9\lambda} \frac{e^{3\sqrt{\frac{1}{6\lambda}}\, t}}{\left(\beta_1 + e^{\frac{1}{\sqrt{6\lambda}}\, t} \right)^3} . \tag{4.2.16}$$

In case of quintessence scalar field ($\varepsilon = 1$), it is clear from (4.2.15) that $\dot{\phi}^2$ is negative throughout the evolution of the Universe for any positive values of λ, hence ϕ becomes imaginary. But for phantom scalar field ($\varepsilon = -1$), $\dot{\phi}^2$ is positive throughout the evolution with negative kinetic term as expected. Taking $\varepsilon = -1$

and considering the positive sign of $\dot{\phi}$, we integrate Eq. (4.2.15) to get

$$\begin{aligned}\phi(t) &= 2\,e^{-\frac{3}{2}\sqrt{\frac{1}{6\lambda}}\,t}\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)\sqrt{\frac{2\lambda e^{3\sqrt{\frac{1}{6\lambda}}\,t}}{3\beta_1\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)}}\\ &\times\left[\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)tan^{-1}\left(\frac{e^{\frac{1}{2\sqrt{6\lambda}}\,t}}{\sqrt{\beta_1}}\right)-\sqrt{\beta_1}\,e^{\frac{1}{2\sqrt{6\lambda}}\,t}\right], \qquad (4.2.17)\end{aligned}$$

where the integration constant is taken to be zero for simplicity. It is very difficult to express the phantom potential V in terms of ϕ explicitly. Therefore, we plot

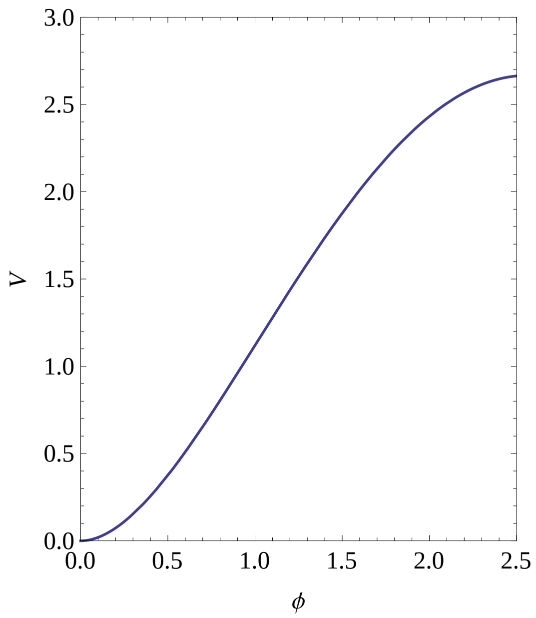

Figure 4.5: V *versus* ϕ for $\beta_1 = 1$ and $\lambda = 1/12$.

V against ϕ for some particulars values of λ and β_1 as shown in fig. 4.5. From figure, it is to be seen that V always increases as ϕ increases from zero at infinite past to a flat potential at late-time.

The energy density ρ_ϕ and pressure p_ϕ for phantom scalar field given in (1.10.5) and (1.10.6), respectively, gives

$$\rho_\phi = \frac{2}{9\lambda}\frac{e^{\frac{4}{\sqrt{6\lambda}}\,t}}{\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)^4}, \qquad (4.2.18)$$

$$p_\phi = -\frac{2}{9\lambda}\frac{e^{3\sqrt{\frac{1}{6\lambda}}\,t}\left(2\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)}{\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}\,t}\right)^4}. \qquad (4.2.19)$$

We observe that all the physical parameters contain only HD terms in this particular model. We observe that ρ_ϕ grows with time whereas p_ϕ stays negative during the evolution of the Universe. The dominance energy condition (DEC), $\rho \geq |p|$, violates in late times. The model has no time-like singularity at any stage of its evolution.

The value of EoS parameter is given by

$$\omega_\phi = -1 - 2\beta_1 e^{-\frac{1}{\sqrt{6\lambda}}t}, \tag{4.2.20}$$

which remains less than -1 for any values of t but attains to -1 as $t \to \infty$. The Universe expands exponentially and accelerates throughout its evolution. It is to be noted that this particular model is stable for any value of λ for any time t. Hence, EU is possible with phantom scalar field for any positive value of λ in this particular model.

4.3 Tachyonic (normal and phantom) field models

4.3.1 Action and the field equations

The gravitational action for HD theory of gravity with a tachyonic field ψ, minimally coupled to gravity in the units $8\pi G = 1 = c$, is given as

$$S = \int \left[\frac{1}{2} \left(R + \lambda R^2 \right) + V(\psi)\sqrt{1 - \varepsilon \nabla_\sigma \psi \nabla^\sigma \psi} \right] \sqrt{-g}\, d^4x\,, \tag{4.3.1}$$

where $V(\psi)$ is relevant tachyonic potential of tachyonic field. Here, $\varepsilon = \pm 1$ correspond to normal and phantom tachyonic fields, respectively.

The energy-momentum tensor $T_{\mu\nu}^{(\psi)}$ for tachyonic field is given by Eq. (1.10.11). The field equations (1.10.17) of HD gravity with energy-momentum tensor (1.10.11) for a flat FRW line-element (3.2.1) in HD theory, yield

$$3H^2 - 18\lambda \left[2\ddot{H}H - \dot{H}^2 + 6\dot{H}H^2 \right] = \frac{V(\psi)}{\sqrt{1 - \varepsilon \dot{\psi}^2}}\,, \tag{4.3.2}$$

$$2\dot{H} + 3H^2 - 6\lambda \left[2\dddot{H} + 12\ddot{H}H + 18\dot{H}H^2 + 9\dot{H}^2 \right] = V(\psi)\sqrt{1 - \varepsilon \dot{\psi}^2}. \tag{4.3.3}$$

Solving (4.3.2) and (4.3.3), we get

$$\varepsilon\dot{\psi}^2 = \frac{-2\dot{H}+12\lambda\left[\dddot{H}+3\ddot{H}H+6\dot{H}^2\right]}{\left[3H^2-18\lambda\{2\ddot{H}H+6\dot{H}H^2-\dot{H}^2\}\right]}\,, \tag{4.3.4}$$

and

$$\begin{aligned} V(\psi) &= \sqrt{3H^2-18\lambda\left(2\ddot{H}H+6\dot{H}H^2-\dot{H}^2\right)} \\ &\quad \times\sqrt{2\dot{H}+3H^2-6\lambda\left(2\dddot{H}+12\ddot{H}H+18\dot{H}H^2+9\dot{H}^2\right)}. \end{aligned} \tag{4.3.5}$$

Now, we examine the possibility of EU for normal and phantom tachyonic fields, respectively, in the following subsections.

4.3.2 Normal tachyonic field model

From Eqs. (4.3.4) and (4.3.5), by use of (4.2.6), we get the following expressions for kinetic term of normal tachyonic field ($\varepsilon=+1$) and tachyonic potential, respectively

$$\dot{\psi}^2 = \frac{\dfrac{-2n\beta_1\alpha_1^2e^{\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2}-\dfrac{12n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[(3n-1)e^{3\alpha_1 t}-(9n-4)e^{2\alpha_1 t}-\beta_1^2e^{\alpha_1 t}\right]}{\dfrac{3n^2\alpha_1^2e^{2\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2}-\dfrac{18n^2\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[2(3n-1)e^{3\alpha_1 t}+\beta_1e^{2\alpha_1 t}\right]}\,, \tag{4.3.6}$$

$$\begin{aligned} V(t) &= \left[\frac{3n^2\alpha_1^2e^{2\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2}-\frac{18n^2\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\{2(3n-1)e^{3\alpha_1 t}+\beta_1e^{2\alpha_1 t}\}\right]^{\frac{1}{2}} \\ \times &\left[\frac{n\alpha_1^2(3ne^{2\alpha_1 t}+2\beta_1e^{\alpha_1 t})}{(\beta_1+e^{\alpha_1 t})^2}-\frac{6n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\{n_0e^{3\alpha_1 t}+(21n-8)\beta_1e^{2\alpha_1 t}+2\beta_1^2e^{\alpha_1 t}\}\right]^{\frac{1}{2}}. \end{aligned} \tag{4.3.7}$$

where $n_0=2(9n^2-6n+1)$. Again, it is very difficult to draw conclusion from above complicated expressions. Therefore, we observe from graphical representation that $\dot{\psi}^2$ is negative for any set of values of parameters, therefore, ψ becomes imaginary. Hence, EU does not exist with normal tachyonic field in HD theory.

4.3.3 Phantom tachyonic field model

In case of phantom tachyonic field ($\varepsilon = -1$), the kinetic term of tachyonic field ($\dot{\psi}^2$) takes the form

$$\dot{\psi}^2 = \frac{\frac{-2n\beta_1\alpha_1^2 e^{\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2} - \frac{12n\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[(3n-1)e^{3\alpha_1 t}-(9n-4)e^{2\alpha_1 t}-\beta_1^2 e^{\alpha_1 t}\right]}{\frac{-3n^2\alpha_1^2 e^{2\alpha_1 t}}{(\beta_1+e^{\alpha_1 t})^2} + \frac{18n^2\lambda\beta_1\alpha_1^4}{(\beta_1+e^{\alpha_1 t})^4}\left[2(3n-1)e^{3\alpha_1 t}+\beta_1 e^{2\alpha_1 t}\right]}. \quad (4.3.8)$$

Fig. 4.6 shows that $\dot{\psi}^2$ is positive throughout the evolution of the Universe for

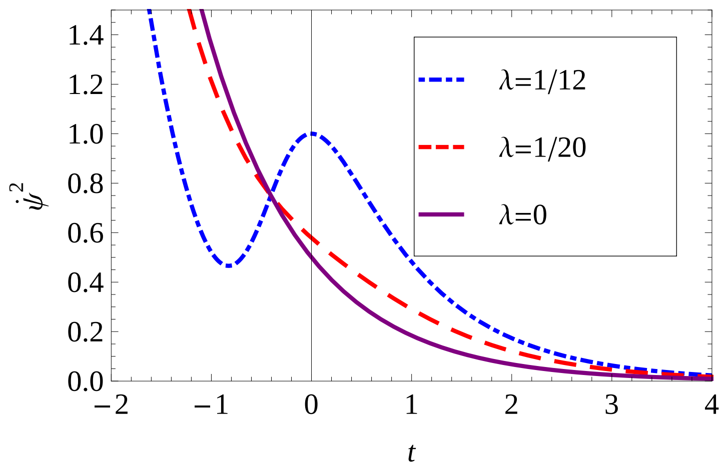

Figure 4.6: $\dot{\psi}^2(t)$ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

small values of λ. It is infinite at infinite past, which decreases with time and tends to zero at late-time. The kinetic term $\dot{\psi}^2$ decreases sharply in Einstein gravity ($\lambda = 0$) as compared to HD theory.

The tachyonic potential has the same expression (4.3.7) since it is independent

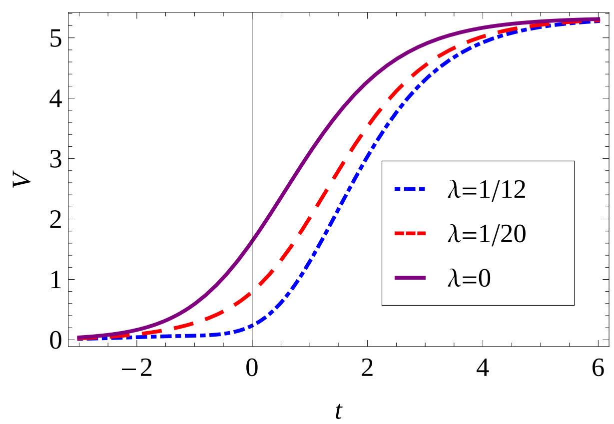

Figure 4.7: $V(t)$ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

of ε. It increases from zero at infinite past to a maximum constant value in late times as we can see in fig. 4.7.

We have also observed that the energy density for tachyonic field is positive throughout the evolution. It is zero at infinite past, which increases with time and finally attains a finite maximum value at late-time. The model has no time-like singularity at any time. Therefore, EU can be described with phantom tachyonic field in HD theory for small values of λ.

The variation of EoS parameter ω_ψ with time is shown in fig. 4.8. It is to be

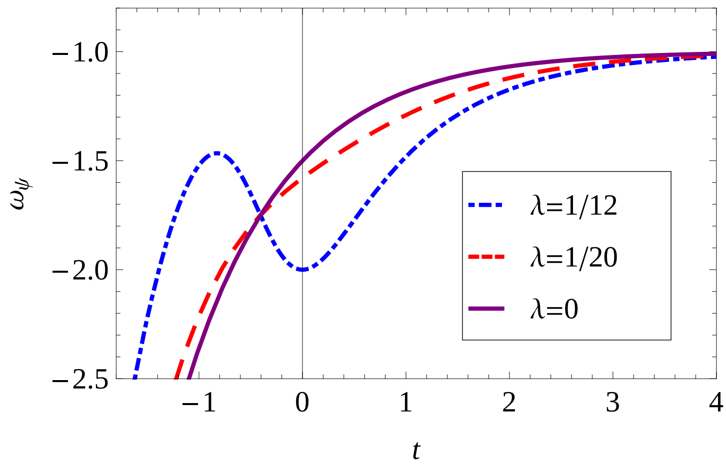

Figure 4.8: ω_ψ *versus* t for $\beta_1 = 1$, $\alpha_1 = 1$ and $n = 4/3$.

seen that $\omega_\psi < -1$ during the evolution of the Universe and settles into a state of $\omega_\psi = -1$ at late-time. The solutions are stable only for small values of coupling parameter λ of HD theory. We conclude that the stability of the solution depends on the coupling parameter λ, i.e., the solutions are stable for small values of λ whereas unstable for large values of λ. We conclude that EU is possible with phantom tachyonic field for a flat FRW model in HD theory for small values of λ. The Universe accelerates throughout its evolution.

4.3.4 Particular solution

We solve the field equations (4.3.2) and (4.3.3) again for the same particular values of α_1 and n, i.e., $\alpha_1 = 1/\sqrt{6\lambda}$ and $n = 2/3$ as described in Sec. 4.2.4. Considering (4.2.13) and (4.2.14), the field equations (4.3.2) and (4.3.3) yield

$$\varepsilon \dot{\psi}^2 = -2\beta_1 \, e^{-\frac{1}{\sqrt{6\lambda}} t} \, , \tag{4.3.9}$$

and

$$V(t)=\frac{2}{9\lambda}\frac{\sqrt{e^{\frac{7}{\sqrt{6\lambda}}t}\left(2\beta_1+e^{\frac{1}{\sqrt{6\lambda}}t}\right)}}{\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}t}\right)^4}\,. \tag{4.3.10}$$

We see that $\dot{\psi}^2$ is negative for normal tachyonic field ($\varepsilon=1$) for any positive value of λ, but it is positive for phantom tachyonic field ($\varepsilon=-1$). Therefore, we find the solution for phantom tachyonic field. For $\varepsilon=-1$ and considering the positive sign of $\dot{\psi}$ we integrate (4.3.9) to get

$$\psi(t)=-\sqrt{48\lambda\beta_1\,e^{-\frac{1}{\sqrt{6\lambda}}t}}\,, \tag{4.3.11}$$

where the integration constant is taken to be zero for simplicity.

The potential function V can be expressed in terms of ψ as

$$V(\psi)=\frac{2\sqrt{\left(\frac{48\lambda\beta_1}{\psi^2}\right)^7\left(2\beta_1+\frac{48\lambda\beta_1}{\psi^2}\right)}}{9\lambda\left(\beta_1+\frac{48\lambda\beta_1}{\psi^2}\right)}. \tag{4.3.12}$$

Fig. 4.9 plots the graph between V and ψ which shows that V increases with ψ.

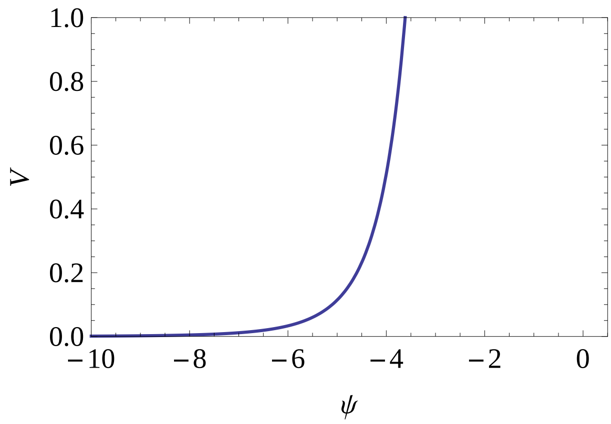

Figure 4.9: V *versus* ψ for $\beta_1=1$ and $\lambda=1/12$.

The energy density ρ_ψ and pressure p_ψ for phantom ($\varepsilon=-1$) tachyonic field given in (1.10.12) (1.10.13), respectively give

$$\rho_\psi=\frac{V(\psi)}{\sqrt{1+\dot{\psi}^2}}=\frac{2}{9\lambda}\frac{\sqrt{e^{\frac{7}{\sqrt{6\lambda}}t}\left(2\beta_1+e^{\frac{1}{\sqrt{6\lambda}}t}\right)}}{\left(\beta_1+e^{\frac{1}{\sqrt{6\lambda}}t}\right)^4\sqrt{\left(1+2\beta_1 e^{-\frac{1}{\sqrt{6\lambda}}t}\right)}}\,, \tag{4.3.13}$$

$$p_\psi = -V(\psi)\sqrt{1+\dot{\psi}^2} = -\frac{2}{9\lambda}\frac{\sqrt{e^{\frac{7}{\sqrt{6\lambda}}t}\left(\beta_1 + e^{\frac{1}{\sqrt{6\lambda}}t}\right)\left(1+2\beta_1 e^{-\frac{1}{\sqrt{6\lambda}}t}\right)}}{\left(\beta_1 + e^{\frac{1}{\sqrt{6\lambda}}t}\right)^4}. \tag{4.3.14}$$

We find that ρ_ψ grows with time and becomes constant in late times whereas the pressure is always negative. The model has no time-like singularity.

From Eqs. (1.10.14) and (4.3.9), the EoS parameter takes the form

$$\omega_\psi = -1 - 2\beta_1 e^{-\frac{1}{\sqrt{6\lambda}}t}, \tag{4.3.15}$$

which always stays less than -1 during the evolution of the Universe but attains to -1 at late-time. The solutions are stable for any value of λ for any time t. We can say that EU is possible with phantom tachyonic field for flat FRW model in HD theory for any positive values of λ in this particular case.

4.4 Conclusion

In this chapter, we have examined the possibility of EU scenario with scalar field (quintessence and phantom) and tachyonic field (normal or phantom) for a flat FRW model in HD theory of gravity. It is well known that the kinetic term must be positive in case of quintessence scalar or normal tachyonic field in most conventional models. We have found that EU is not possible for quintessence scalar and normal tachyonic field in HD theory due to the negative kinetic term. However, EU is possible with phantom scalar and phantom tachyonic fields. The summary of each phantom model is as follows:

In phantom scalar field model, we have observed that $\dot{\phi}^2$ increases from zero at infinite past, attains a finite maximum value at some finite time and then decreases to zero in late times. We have obtained the scalar potential which increases with time and attains a maximum finite value at late-time evolution of the Universe. Thus, a phantom field rolls to the maximum of its potential with positive potential energy where $\omega_\phi < -1$ during the evolution, but settles into a state with $\omega_\phi = -1$ at late-time. In general, the stability of the solution depends on the constraints of the arbitrary parameters. The solutions are stable for small values of coupling parameter λ of HD theory whereas unstable for large values of λ. We have also discussed a particular solution in subsection 4.2.4 where the model is stable for all positive values of λ.

In phantom tachyonic field model, the $\dot{\psi}^2$-term decreases from infinity at infinite past to zero as $t \to \infty$. It remains positive during the evolution for small values of λ. The tachyonic potential increases from zero at infinite past to a flat potential in late times. The stability of the solution can be described in same manner as discussed in the case of phantom scalar field. We have also discussed a particular solution in subsection 4.3.4 which is stable for any positive value of λ. We have observed that $\omega_\psi < -1$ during the evolution of the Universe which settles into a state with $\omega_\psi = -1$ at late-time.

We have also noticed the behavior of the energy densities in each model which are zero at infinite past and grow with time and attain a maximum finite value at late-time. The models eventually evolve from a finite size in the infinite past into an inflationary stage and accelerate throughout the evolution. The models have no time-like singularity at any time. The coupling parameter λ of HD theory affects the evolution of EU.

In concluding remark, we would like to mention that EU scenario with phantom models arising from scalar and tachyon fields can successfully implemented in HD theory.

Chapter 5

The modified *f(R)* gravity in anisotropic model

In this chapter[1], we study the generalization of HD theory, i.e., $f(R)$ theory of gravity in a locally-rotationally-symmetric anisotropic Bianchi I space-time model in the presence of perfect fluid. A functional form of $f(R)$ is reconstructed from the field equations by assuming the constant deceleration parameter and the shear scalar proportional to the expansion scalar. Exact cosmological solutions of the modified Einstein field equations are obtained by using reconstructed functional form of $f(R)$, which shows the decelerated phase of the Universe. We also discuss the stability of the solution that holds good for decelerated model.

5.1 Introduction

As we have mentioned in section 1.10.4 that the functional form of $f(R)$ accommodating transition from deceleration to acceleration can be reconstructed using the realistic expansion history of the Universe. However, the weak point of so developed reconstruction schemes is that the final function of $f(R)$ usually possesses some polynomial in the positive and negative powers of scalar curvature. On the same time the viable models on $f(R)$ theories have strongly non-linear structure.

[1]The work presented in this chapter comprises the results of a research paper entitled “Functional form of $f(R)$ with power-law expansion in anisotropic model", published in *Astrophysics and Space Science* **346** 285–289 (2013).

Despite the success of cosmology based on $f(R)$ gravity, there is no general criteria in the literature to gauge its viability (consistency with the experimental data). The reconstruction of viable cosmological models of $f(R)$ theories has become a debate during the past few years. This debate began from ref. [90] in which the author suggested that $f(R)$ theories that behave as a power of R at large or small R, are not cosmologically viable because they lead to the wrong expansion history of the Universe, viz., $a \sim t^{1/2}$ instead of $a \sim t^{2/3}$ during the matter-dominated era. This result was challenged in refs. [163, 232, 233], which demonstrate that a wide class of $f(R)$ gravity models describing matter-dominated and present accelerating phases can be reconstructed by means of observational data. The debate was continued in ref. [234] in which a detailed phenomenological analysis of the cosmic evolution of $f(R)$ theories was presented. Finally, a numerous simple conditions for a phenomenological $f(R)$ gravity to be viable were accumulated in refs. [235–237].

A rather narrow class of $f(R)$ has been found in an inverse-power-law in refs. [238, 239] and in an exponential-law in ref. [236] which can satisfy the first four viability conditions and even partially the fifth one listed in refs. [235, 236]. Thus, the debate ended with the conclusion that some specific functional forms of $f(R)$ may be perfectly viable in different contexts. A huge class of such viable functional forms of $f(R)$ gravity has been reconstructed in many dynamical DE models during past decade [136, 164, 165, 232, 233, 240–243], being compatible with cosmological or astrophysical test [244, 245] and local gravity (solar system) constraints [237,246–251]. Recently, some $f(R)$ gravity models have been reconstructed which pass all known observational local test [153].

In reconstruction schemes of cosmological models, the proposal have usually come to find analytical solutions for some known functional form of $f(R)$. The ordering of this approach can also be reversed. In the reverse process, it is assumed that the expansion history of the Universe is known exactly and one may invert the field equations to deduce what class of modified theory gives rise to a desired model [164]. Moreover, for a known scale factor one may construct functional form of $f(R)$ which yields such scale factors as solutions [165, 252]. In this chapter, we have also reconstructed a functional form of $f(R)$ by following this reverse approach.

On the other hand, most of the considerations in $f(R)$ gravity have mainly investigated in a spatially flat homogeneous and isotropic space-time described by FRW metric. The theoretical studies and the observational data which support the existence of an anisotropic phase, lead to consider the models of the Universe in anisotropic background. Many authors have explored the features of $f(R)$ gravity for anisotropic models [154–162]. The studies of the possible effects of anisotropic Universe in the early time make the Bianchi type I model as a prime alternative.

In this chapter, we reconstruct a functional form of $f(R)$ for a known scale factor in a locally-rotationally-symmetric Bianchi-I space-time with a perfect fluid. We obtain the exact cosmological solutions using the reconstructed functional form of $f(R)$ and discuss the viability of the model. We take the viability constraints that exist in the literature to analyze the stability of the obtained functional form.

5.2 The LRS Bianchi I model

Let us consider a homogenous and anisotropic locally-rotationally-symmetric (LRS) Bianchi type-I line-element which is given by

$$ds^2 = -dt^2 + \mathrm{A}^2 dx^2 + \mathrm{B}^2(dy^2 + dz^2), \tag{5.2.1}$$

where the metric coefficients A and B are the directional scale factors in an anisotropic background and are functions of cosmic time t only.

The average scale factor is defined as

$$\mathsf{a} = (\mathrm{AB}^2)^{\frac{1}{3}}. \tag{5.2.2}$$

The rates of the expansion along x, y, and z-axes are given by

$$H_x = \frac{\dot{\mathrm{A}}}{\mathrm{A}}, \;\; H_y = H_z = \frac{\dot{\mathrm{B}}}{\mathrm{B}}, \tag{5.2.3}$$

where an over dot denotes ordinary derivative with respect to cosmic time t. The average Hubble parameter (average expansion rate), which is the generalization of the Hubble parameter in an isotropic case, H is given as

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{3}\left(\frac{\dot{\mathrm{A}}}{\mathrm{A}} + 2\frac{\dot{\mathrm{B}}}{\mathrm{B}}\right). \tag{5.2.4}$$

The anisotropy parameter $\mathscr{A}$, the expansion scalar ϑ, and the shear scalar σ are respectively defined as

$$\mathscr{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{5.2.5}$$

$$\vartheta = 3H = u^i_{;i} = \frac{\dot{\mathrm{A}}}{\mathrm{A}} + 2\frac{\dot{\mathrm{B}}}{\mathrm{B}}, \tag{5.2.6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{\mathrm{A}}}{\mathrm{A}} - \frac{\dot{\mathrm{B}}}{\mathrm{B}}\right)^2, \tag{5.2.7}$$

where

$$\sigma^1_1 = \frac{2}{3}\left(\frac{\dot{\mathrm{A}}}{\mathrm{A}} - \frac{\dot{\mathrm{B}}}{\mathrm{B}}\right),$$
$$\sigma^2_2 = \sigma^3_3 = -\frac{1}{3}\left(\frac{\dot{\mathrm{A}}}{\mathrm{A}} - \frac{\dot{\mathrm{B}}}{\mathrm{B}}\right), \ \ \sigma^4_4 = 0, \tag{5.2.8}$$

The scalar curvature for the metric (5.2.1) has the form

$$R = -2\left(\frac{\ddot{\mathrm{A}}}{\mathrm{A}} + 2\frac{\ddot{\mathrm{B}}}{\mathrm{B}} + 2\frac{\dot{\mathrm{A}}\dot{\mathrm{B}}}{\mathrm{AB}} + \frac{\dot{\mathrm{B}}^2}{\mathrm{B}^2}\right). \tag{5.2.9}$$

5.3 The action and the field equations in $f(R)$ gravity

The gravitational action for $f(R)$ theory of gravity (1.10.18) coupled with matter Lagrangian in the units $16\pi G = 1 = c$, reduces to the following form [100,103,165]

$$S = \int [f(R) + \mathscr{L}_m]\sqrt{-g}\, d^4x, \tag{5.3.1}$$

where $\mathscr{L}_m$ is the Lagrangian density.

The field equations are obtained by varying the action (5.3.1) with respect to metric tensor $g_{\mu\nu}$, which are given by

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)F(R) = T_{\mu\nu}, \tag{5.3.2}$$

where $F(R) = f'(R)$. A prime denotes derivative with respect to argument and $T_{\mu\nu}$ is the energy-momentum tensor of perfect fluid defined in (2.2.4). Hence, we take $\rho_m \sim \rho$ and $p_m \sim p$ for the energy density and pressure of the perfect fluid.

Using metric (5.2.1) and energy-momentum tensor (2.2.4) into the field equations

(5.3.2), we obtain the following system of equations

$$\left(\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}\right)F+\frac{1}{2}f-\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\dot{F} = -\rho, \tag{5.3.3}$$

$$\left(\frac{\ddot{A}}{A}+2\frac{\dot{A}\dot{B}}{AB}\right)F+\frac{1}{2}f-\ddot{F}-2\frac{\dot{B}}{B}\dot{F} = p, \tag{5.3.4}$$

$$\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)F+\frac{1}{2}f-\ddot{F}-\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{F} = p. \tag{5.3.5}$$

From (5.3.4) and (5.3.5), we get

$$\frac{\dot{F}}{F}=-\frac{\left(\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}\right)}{\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)}. \tag{5.3.6}$$

On integration of (5.3.6), we obtain

$$F=\frac{F_0}{B(\dot{A}B-\dot{B}A)}, \text{ provided } A\neq bB, \tag{5.3.7}$$

where F_0 is a constant of integration and b is a positive real number.

5.4 Reconstruction of $f(R)$

For any physically relevant model, the Hubble parameter and the deceleration parameter are the most important observational quantities in cosmology. Berman [253]; and Berman and Gomide [254] proposed a law of variation for Hubble parameter in FRW model that yields a constant value of deceleration parameter and a power-law and exponential forms of scale factor. In recent references [255–260] this assumption has been generalized in anisotropic models. According to this assumption let us consider a constant deceleration parameter, that is,

$$q=m-1, \tag{5.4.1}$$

where $m(\geq 0)$ is a constant. It is to be noted that this form of deceleration parameter is same as given in Eq. (3.3.2) for $m=1/n$. The Universe decelerates for $m>1$, accelerates for $m<1$ and $m=1$ gives marginal inflation.

In the present anisotropic model, the assumption (5.4.1) yields

$$\mathsf{a}=(AB^2)^{\frac{1}{3}}=(a_0+a_1t)^{\frac{1}{m}}, \qquad m\neq 0, \tag{5.4.2}$$

where a_0 and a_1 are positive constants of integration. For a power-law expansion (5.4.2), we must have $m > 0$.

In view of anisotropy of the space-time, we assume that shear scalar (σ) is proportional to the expansion scalar (ϑ), which lead to the following relation between the metric coefficients [261]

$$\mathrm{A} = \mathrm{B}^{\tau}, \tag{5.4.3}$$

where $\tau > 1$ is a constant. For sake of simplicity, we take the proportionally constant as unity.

Using (5.4.2) and (5.4.3), we get the metric coefficients as

$$\mathrm{A} = (a_0 + a_1 t)^{\frac{3\tau}{m(\tau+2)}}, \tag{5.4.4}$$

$$\mathrm{B} = (a_0 + a_1 t)^{\frac{3}{m(\tau+2)}}. \tag{5.4.5}$$

Using (5.4.4) and (5.4.5) in (5.2.9), the expression for the Ricci scalar becomes

$$R(t) = -2R_0 t^{-2}, \tag{5.4.6}$$

where $R_0 = \frac{9\{(\tau+2)+3\}-3m(\tau+2)^2}{m^2(\tau+2)^2}$.

Using the above solutions into (5.3.7), we find $f'(R)$ in terms of R as

$$F(R) = f'(R) = \frac{2^{\frac{m-3}{2m}} F_0(\tau+2)}{3(\tau-1)} \left[m\sqrt{-\frac{R_0}{R}} \right]^{\frac{m-3}{m}}. \tag{5.4.7}$$

We observe that for a real valued solution of $f(R)$, R and R_0 must be of opposite sign. On integration of (5.4.7), we get

$$f(R) = \frac{2^{\frac{3(m-1)}{2m}} F_0(\tau+2)mR}{3(\tau-1)(m+3)} \left[m\sqrt{-\frac{R_0}{R}} \right]^{\frac{m-3}{m}} + f_0, \tag{5.4.8}$$

where f_0 is a constant of integration. On imposing $f(0) = 0$ [236], we find $f_0 = 0$, therefore, Eq. (5.4.8) gives the required functional form of $f(R)$

$$f(R) = \frac{2^{\frac{3(m-1)}{2m}} F_0(\tau+2)mR}{3(\tau-1)(m+3)} \left[m\sqrt{-\frac{R_0}{R}} \right]^{\frac{m-3}{m}}, \tag{5.4.9}$$

which is basically of the form $f(R) \propto R^{\delta}$, where $\delta = \frac{m+3}{2m} > 0$ as $m > 0$. For $m = 1$,

$f(R) \propto R^2$ and $q = 0$ which corresponds to the marginal inflationary model. For $m = 3$, we have $f(R) \propto R$ and $q = 2$ which shows the decelerated phase of GR. It is also clear that the power of R, i.e., δ contains only m and is independent of τ.

The energy density and pressure in terms of cosmic time t are given as

$$\rho = 6\left[\frac{m(\tau+2)^2 - 3\{\tau(\tau+2)+3\}}{(\tau+2)(\tau-1)(m+3)}\right]\frac{1}{(mt)^{\frac{m+3}{m}}}, \tag{5.4.10}$$

$$p = 2\left[\frac{m(\tau+2)^2 - 3\{\tau(\tau+2)+3\}}{(\tau+2)(\tau-1)(m+3)}\right]\frac{m}{(mt)^{\frac{m+3}{m}}}. \tag{5.4.11}$$

For reality of the model, the energy density must be positive. Therefore, we have $m(\tau+2)^2 - 3\{\tau(\tau+2)+3\} > 0$, as $\tau > 1$. The energy density and pressure decrease with time and tend to zero for large t.

Equations (5.4.10) and (5.4.11) give

$$\rho + p = 2\left[\frac{m(\tau+2)^2 - 3\{\tau(\tau+2)+3\}}{(\tau+2)(\tau-1)}\right]\frac{1}{(mt)^{\frac{n+3}{m}}}, \tag{5.4.12}$$

which shows that null energy condition (NEC) is satisfied for $\tau > 1$.

The EoS parameter $\omega = p/\rho$, gives

$$\omega = \frac{m}{3}, \tag{5.4.13}$$

which is positive throughout the evolution of the Universe as $m > 0$.

From (5.4.1) and (5.4.13), we have the following linear relation between q and ω

$$q = 3\omega - 1, \tag{5.4.14}$$

which is constant. This shows that the functional form of $f(R)$ constructed in (5.4.9) may describe decelerated phases of the Universe for $\omega > 1/3$ and accelerated phase for $\omega < 1/3$. But we observe that the constraints does not support for an accelerating Universe as the energy density is negative for those values of parameters. The positive EoS parameter also evidences that the functional form of $f(R)$, obtained in (5.4.9) is not suitable to describe an accelerated expansion of the Universe. Therefore, keeping in view of the positivity of energy density we find the following constraints under which the model decelerates.

$$2 < m < 3 \text{ and } 1 < \tau < \frac{3-2m}{m-3} + 3\sqrt{\frac{m-2}{(m-3)^2}}, \tag{5.4.15}$$

$$\text{or} \qquad m \geq 3 \text{ and } \tau > 1. \tag{5.4.16}$$

5.5 Stability analysis

In this section, we study the stability of the foresaid model. The conditions for the cosmological viability of $f(R)$ models have been derived in ref. [235, 236]. Among the consistency requirements listed in refs. [237], an acceptable functional form of $f(R)$ must satisfy following classical and quantum stability in the region of R.

$$f'(R) > 0, \tag{5.5.1}$$

$$f''(R) > 0. \tag{5.5.2}$$

The first condition requires that gravity is attractive and the graviton is not a ghost. It was found that its violation during the evolution of a FRW background, results in the immediate loss of homogeneity and isotropy and render a strong space-like anisotropic curvature singularity [227, 262]. However, we have considered anisotropic model sustaining preservation of homogeneity.

Starobinsky [228, 229] followed the above viability criteria when he constructed his inflationary models. However, in the case of $f(R)$ gravity models of present DE, the necessity to keep it valid for all values of R during the matter- and radiation-dominated stages in order to avoid the Ostrogradski instability [151] and Dolgov-Kawasaki instability [134] which have been realized rather recently [234,237]. The requirement of the above two stability criteria are particularly important to give rise to a saddle matter era followed by a late-time cosmic acceleration. The cosmologically viable $f(R)$ models need to be close to the ΛCDM model in the deep matter era, but the deviation from it becomes important around the late stage of the matter era. In addition, a weak ("sudden") curvature singularity forms generically if $f''(R) = 0$ for a finite value R. Some examples of such viable models were presented in refs. [263].

One may observe that the stability condition (5.5.1) always holds as $\tau > 1$. To obtain the constraints which satisfy (5.5.2), we differentiate second order (5.4.7) with respect to R, to get

$$f''(R) = \frac{2^{-\frac{(m+3)}{2m}} F_0(\tau+2)m(m-3)R_0}{3(\tau-1)R^2} \left[m\sqrt{-\frac{R_0}{R}} \right]^{-\frac{m+3}{m}}. \tag{5.5.3}$$

Hence, $f''(R) > 0$ gives

$$m > 3 \text{ and } \tau > 1. \tag{5.5.4}$$

Thus, we find that out of two constraints obtained in (5.4.15) and (5.4.16), only the constraint (5.4.16) favors the stability of the solution and it is feasible to describe the decelerated phase of evolution of the Universe. Hence, the functional form of $f(R)$, reconstructed in (5.4.9) is only suitable to describe the decelerated Universe. It may be noted that the anisotropic Bianchi models represent cosmos in its early stages of evolution of the Universe.

5.6 Conclusion

In this chapter, we have studied $f(R)$ theory of gravity in LRS Bianchi-I anisotropic space-time. We have assumed a constant deceleration parameter and a proportionality relation between shear scalar and scalar expansion to reconstruct an exact form of $f(R)$. Using the reconstructed functional form of $f(R)$, the corresponding various cosmological parameters have been obtained. The EoS parameter has a constant value, i.e., $\omega = m/3$, which is positive as $m > 0$. This allows to describe the decelerated phases of the Universe. The deceleration parameter, q and EoS parameter, ω have a linear relation, i.e., $q = 3\omega - 1$. We have obtained some specific constraints on parameters which also permits to describe only the decelerated phases of evolution the Universe.

We have also analyzed the stability of the reconstructed functional form of $f(R)$ in section 5.5. It has been observed that it is completely stable but only for the decelerated phases of the Universe. It may be noted that even though the $f(R)$ gravity describes an early-time inflation and late-time acceleration, but the results obtained in this chapter shows that the $f(R)$ theory gravity in anisotropic models is also suitable to describe the evolution of the Universe in decelerated phases.

Chapter 6

Reconstruction of *f(R,T)* gravity with perfect fluid

In present chapter[1], we present the cosmological viability of reconstruction of the modified $f(R,T)$ gravity in a flat FRW model. A functional form of $f(R,T) = R+2f(T)$ is chosen for the reconstruction. The gravitational field equations contain two non-interacting fluid sources, one is perfect fluid and other is due to modified $f(R,T)$ gravity which is to be considered as an exotic fluid. Two known forms of scale factor (de Sitter and power-law) are considered for the explicit and successful reconstruction. In de Sitter solution, the $f(R,T)$ fluid behaves as phantom dark energy when the usual matter (perfect fluid) shows the behavior between decelerated phase to accelerated phase. In the absence of usual matter it behaves as a cosmological constant. In case of power-law cosmology two different cases are discussed and analyzed the behavior of different phases of the Universe accordingly through the equation of state and density parameters.

6.1 Introduction

As discussed in section 1.10.5, $f(R,T)$ gravity theory, proposed by Harko et al. [174], is a new class of modified theory which includes an arbitrary function

[1]This chapter is based on a published research paper entitled "Reconstruction of modified $f(R,T)$ gravity with perfect fluid cosmological models", in *General Relativity and Gravitation* **46** 1696 (2014).

of Ricci scalar R and trace of energy-momentum tensor $T_{\mu\nu}$ in the EH action. The justification of choosing T as an argument for the Lagrangian is from exotic imperfect fluids or quantum effects. Harko et al. [174] have argued that due to the coupling of matter and geometry, this gravity model depends on a source term, representing the variation of the matter-stress energy with respect to the metric. In modified $f(R,T)$ theory, the cosmic acceleration is not only derived from geometric contribution but also from matter content. The corresponding field equations in $f(R,T)$ gravity have been derived in metric formalism for several particular cases.

Jamil et al. [175] have reconstructed cosmological models in the framework of $f(R,T)$ gravity, which reproduce dust fluid ΛCDM model, Einstein static Universe, de Sitter Universe and phantom-non-phantom era. They have also reconstructed different models by including Chaplygin gas and minimally coupled scalar field with some specific forms of $f(R,T)$, which are compatible with the recent observational data of BAO for low redshifts $z<2$. Chakraborty [181] has shown that a part of an arbitrary function of $f(R,T)$ can be determined by taking into account of the conservation of stress-energy tensor. Houndjo [190] has developed the cosmological reconstruction of the form $f(R,T)=f_1(R)+f_2(T)$ using auxiliary scalar field with two known examples of scale factor corresponding to an accelerating Universe, describing a transition from matter-dominated phase to late-time accelerated phase. Houndjo and Piatella [191] have numerically reconstructed $f(R,T)$ holographic DE and DM models which are able to reproduce the same expansion history generated in GR. Houndjo et al. [182] have investigated $f(R,T)$ gravity models which reproduce four types of future finite-time singularities. Pasqua et al. [184] have considered modified holographic Ricci dark energy (MHRDE) model with a particular form of $f(R,T)=\lambda_1 R+\lambda_2 T$, for which EoS parameter ω approaches from quintessence to cosmological constant. The deceleration parameter q passes from decelerated to accelerated phase at a red-shift $z\approx 0.2$. Therefore, these works on $f(R,T)$ theory of gravity have motivated us to develop different schemes for the reconstruction of $f(R,T)$.

In this chapter, we have explored various general forms of the function $f(T)$ for a particular form $f(R,T)=R+2f(T)$ and examined their contribution in the evolution of the Universe. A flat FRW space-time cosmological model with the perfect fluid has been assumed for the reconstruction of $f(R,T)$. We have derived the explicit forms of $f(R,T)$ by considering two known forms (de Sitter and power-law) of expansion history of the Universe. It has been observed that the reconstructed $f(R,T)$ gravity is capable to reproduce accelerated Universe.

6.2 Fundamental formalism of *f(R,T)* gravity

Let us start with the gravitational action of $f(R,T)$ theory [174] given in (1.10.20). In the units of $8\pi G = 1 = c$, it can be written as

$$S = \int \left[\frac{1}{2}f(R,T) + \mathscr{L}_m\right]\sqrt{-g}\, d^4x, \tag{6.2.1}$$

where $\mathscr{L}_m$ corresponds to the matter Lagrangian density.

As usual the energy-momentum tensor $T^{(m)}_{\mu\nu}$ is defined by

$$T^{(m)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathscr{L}_m)}{\delta g^{\mu\nu}}, \tag{6.2.2}$$

and its trace $T = g^{\mu\nu}T^{(m)}_{\mu\nu}$.

The field equations of $f(R,T)$ gravity by varying the action (6.2.1) with respect to metric tensor have the form

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T) = T^{(m)}_{\mu\nu} - f_T(R,T)(T^{(m)}_{\mu\nu} + \ominus_{\mu\nu}), \tag{6.2.3}$$

where the symbols have their usual meanings. Here $\ominus_{\mu\nu}$ is defined in Eq. (1.10.22).

Since the field equations of $f(R,T)$ theory depend on $\ominus_{\mu\nu}$, i.e., on the physical nature of the matter source. Therefore, a number of models corresponding to various forms of $f(R,T)$ may be generated depending on the nature of the matter source. We consider that the matter Lagrangian density $\mathscr{L}_m$ depends on the metric tensor components $g_{\mu\nu}$, not on its derivatives. Therefore, the energy-momentum tensor of matter given by Eq. (6.2.2), simplifies to

$$T^{(m)}_{\mu\nu} = g_{\mu\nu}\mathscr{L}_m - 2\frac{\partial \mathscr{L}_m}{\partial g^{\mu\nu}}. \tag{6.2.4}$$

Using (7.2.3) into (1.10.22), one gets

$$\ominus_{\mu\nu} = -2T^{(m)}_{\mu\nu} + g_{\mu\nu}\mathscr{L}_m - 2g^{ij}\frac{\partial^2 \mathscr{L}_m}{\partial g^{\mu\nu}\partial g^{ij}}. \tag{6.2.5}$$

Now, the equations in $f(R,T)$ gravity are much complicated even for FRW metric as compared to GR. Therefore, it is very difficult to reconstruct a general form of $f(R,T)$ or to solve the field equations, in general. Therefore, most of the works in $f(R,T)$ gravity have been carried out by assuming a number of suitable forms of $f(R,T)$, such as $f(R,T)=R+\lambda f(T)$, $f(R,T)=R+2f(T)$, $f(R,T)=\lambda_3 f_1(R)+\lambda_4 f_2(T)$, where $f_1(R)$ and $f_2(T)$ are arbitrary functions of R and T, and λ_3 and λ_4 are real constants, $f(R,T)=R\,f(T)$, etc., [174, 175, 181, 182, 190, 191]. In the present work we are interested to reconstruct the following particular form [174, 178].

$$f(R,T)=R+2f(T). \tag{6.2.6}$$

This assumption is particularly interesting choice which modifies the EH action of GR by adding a function of T. The term $2f(T)$ in the gravitational action modifies the gravitational interaction between matter and curvature. It is to be noted that the above form of $f(R,T)$ has also been discussed by Jamil et al. [175] by assuming R as a constant. But we consider R as an arbitrary function in the present work.

Using (6.2.6), one can re-write (6.2.3) as

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=T^{(m)}_{\mu\nu}-2(T^{(m)}_{\mu\nu}+\ominus_{\mu\nu})f'(T)+f(T)g_{\mu\nu}, \tag{6.2.7}$$

where a prime stands for derivative of $f(T)$ with respect to T. On comparing (6.2.7) with Einstein's field equations, we find that the gravitational field equations (6.2.7) can be recast in such a form that the higher order corrections coming both from the geometry and from matter-geometry coupling. It provides an energy-momentum tensor of geometrical and matter origin, describing an effective source term on right hand side of (6.2.7) .

The main issue now arises of the matter content in the Universe through the assumption of energy-momentum tensor, and consequently on the matter Lagrangian $\mathscr{L}_m$ and the trace of the energy-momentum tensor. In the following section, we consider the perfect fluid as the only matter content in a flat FRW model for the reconstruction of $f(R,T)$ from known expansion history of Universe using the field equation (6.2.7).

6.3 Reconstruction of *f(R,T)* in perfect fluid cosmology

We consider a spatially isotropic and homogenous flat FRW model described by metric (3.2.1). We assume the matter content as a perfect fluid with the energy-momentum tensor given by Eq. (2.2.4). The trace T of (2.2.4) gives

$$T = \rho_m - 3p_m. \tag{6.3.1}$$

The definition of the matter Lagrangian for a perfect fluid is not unique. In order to be consistent with the variation of the energy-momentum tensor (2.2.4) with respect to metric, $\mathscr{L}_m = -p_m$ is assumed [178]. Consequently, the second variation of the matter Lagrangian in (6.2.5) becomes zero. Thus, with this assumption the tensor $\ominus_{\mu\nu}$, defined in (6.2.5), gives

$$\ominus_{\mu\nu} = -2T^{(m)}_{\mu\nu} - p_m g_{\mu\nu}. \tag{6.3.2}$$

Using (2.2.4) and (6.3.2), the field equations (6.2.7) for the line-element (3.2.1), yield

$$\begin{aligned} 3H^2 &= \rho_m + 2(\rho_m + p_m)f'(T) + f(T), & (6.3.3)\\ 2\dot{H} + 3H^2 &= -p_m + f(T), & (6.3.4) \end{aligned}$$

which are the Friedmann equations for the present model.

We consider that Eqs. (6.3.3) and (6.3.4) consist of a non-interacting two fluids system. One is usual perfect fluid matter whose energy-momentum tensor is given by (2.2.4) and the other is due to modified $f(R,T)$ gravity which considered as an exotic matter having energy density and pressure

$$\begin{aligned} \rho_d &= 2(\rho_m + p_m)f'(T) + f(T), & (6.3.5)\\ p_d &= -f(T), & (6.3.6) \end{aligned}$$

respectively, which are related by EoS $p_d = \omega_d \rho_d$, where ω_d is the EoS parameter corresponds to exotic matter.

Using EoS (3.3.4) into (6.3.1), we get

$$\rho_m = \frac{T}{1-3\omega_m}, \qquad \omega_m \neq \frac{1}{3}. \tag{6.3.7}$$

The classical known history of expansion of the Universe (some form of scale factor) can be used for explicit and successful reconstruction of some versions of $f(R)$ theories [264]. The existence of such solutions is particulary relevant because they represent all possible cosmological evolution. In what follows we consider two known scale factors of the expansion history of the Universe for explicit and successful reconstruction of $f(R,T)$ gravity.

6.3.1 Solution with de Sitter expansion

As well known that de Sitter solution is one of the most important cosmological solution which describes the late-time accelerated phase as well as the inflationary expansion epoch of the Universe. The expansion history in de Sitter Universe is governed by exponential expansion of the scale factor

$$a(t) = a_0 e^{H_0 t}, \tag{6.3.8}$$

where a_0 and H_0 are positive constants. The Hubble parameter for exponential expansion of the scale factor (6.3.8) has a constant value, i.e.,

$$H(t) = H_0. \tag{6.3.9}$$

Using (6.3.7) and (6.3.9) into (6.3.3) and (6.3.4), and simplifying, we obtain

$$(1+\omega_m)\left[2f'(T)+1\right]T = 0, \tag{6.3.10}$$

where $\omega_m \neq -1$. The solution of (6.3.10) is

$$f(T) = T_0 - \frac{T}{2}, \tag{6.3.11}$$

where T_0 is a constant of integration. Hence, the functional form of $f(R,T)$ as assumed in (6.2.6), has the form

$$f(R,T) = R + 2T_0 - T. \tag{6.3.12}$$

Thus, in principle, any cosmology expressed as (6.3.8) can be reconstructed by the specific form of $f(R,T)$ gravity given by (6.3.12). Let us discuss some physical significance of this form of $f(R,T)$.

Using (6.3.11) into (6.3.3) and (6.3.4), we obtain the energy density and pressure of the perfect fluid in terms of T as

$$\rho_m = 3(T_0 - 3H_0^2) - \frac{T}{2}, \tag{6.3.13}$$

$$p_m = T_0 - 3H_0^2 - \frac{T}{2}. \tag{6.3.14}$$

From (6.3.7) and (6.3.13), we get

$$T = \frac{2(T_0 - 3H_0^2)(1 - 3\omega_m)}{1 - \omega_m}. \tag{6.3.15}$$

By use of (6.3.15), the energy density (6.3.13) and pressure (6.3.14), become

$$\rho_m = \frac{2(T_0 - 3H_0^2)}{1 - \omega_m}, \tag{6.3.16}$$

$$p_m = \frac{2(T_0 - 3H_0^2)\omega_m}{1 - \omega_m}, \tag{6.3.17}$$

which are constants. For a realistic model the energy density must be positive, therefore, we have $T_0 \geq 3H_0^2$ as $-1 < \omega_m < 1$.

The energy density and pressure of the exotic matter defined in (6.3.5) and (6.3.6), respectively, give

$$\rho_d = \frac{2T_0 + 3(\omega_m - 3)H_0^2}{\omega_m - 1}, \tag{6.3.18}$$

$$p_d = \frac{2T_0\omega_m + 3(1 - 3\omega_m)H_0^2}{\omega_m - 1}, \tag{6.3.19}$$

which are also constants. In this case, for a positive energy density, we must have $T_0 \leq \frac{3}{2}(3 - \omega_m)H_0^2$.

The EoS parameter ω_d is given by

$$\omega_d = \frac{2T_0\omega_m + 3(1 - 3\omega_m)H_0^2}{2T_0 + 3(\omega_m - 3)H_0^2}. \tag{6.3.20}$$

One may observe that $\omega_d < -1$ whenever $-1 < \omega_m < 1$, which shows that the matter contribution due to $f(R,T)$ gravity behaves as phantom DE.

The density parameter corresponding to exotic matter ($\Omega_d = \rho_d/3H^2$), is given by

$$\Omega_d = \frac{2T_0 + 3(\omega_m - 3)H_0^2}{3H_0^2(\omega_m - 1)}, \qquad (6.3.21)$$

which shows that $\Omega_d < 1$, i.e., an open model in phantom DE ($\omega_d < -1$) whereas it becomes flat ($\Omega_d = 1$) when the exotic matter behaves like a cosmological constant ($\omega_d = -1$) in the absence of the perfect fluid. It is also to be noted that the closed model of the Universe is not possible throughout its evolution in present study.

The effective energy density (the sum of energy densities of perfect fluid and exotic matter) and its effective pressure have the same constant values

$$\rho_{eff} = 3H_0^2 = p_{eff}, \qquad (6.3.22)$$

which gives the effective EoS parameter $\omega_{eff} = 1$. Hence, the effective fluid behaves as stiff matter in this model. We also find that $\Omega_{eff} = 1$, i.e., the model becomes flat due to the effect of both matter sources.

In a particular case, when $3H_0^2 = T_0$ where ρ_m and p_m both vanish, we get $T = 0$ and hence $f(T) = 3H_0^2$, which is constant. The EoS parameter has the value $\omega_d = -1$, which shows that the matter due to $f(R,T)$ gravity behaves like a cosmological constant in the absence of perfect fluid.

6.3.2 Solution with Power-law expansion

The power-law expansion is widely accepted model to explain the expansion history of evolution of the Universe. We consider the power-law expansion of the scale factor which is assumed in Eq. (3.3.1) with $t_0 = 1$. The deceleration parameter in this scenario is given by Eq. (3.3.2), that is, $q = 1/n - 1$. As $n \geq 0$ is required in power-law cosmology, hence $q \geq -1$. The Hubble parameter for this form of scale factor is given by Eq. (3.3.3). Using (6.3.7) and (3.3.3) into (6.3.3) and (6.3.4), we get

$$2(3n-2)(1+\omega_m)Tf'(T) - 2(1-3\omega_m)f(T) + [3n(1+\omega_m) - 2]T = 0. \qquad (6.3.23)$$

In what follows, we discuss the possible solutions of the above Eq. (6.3.23) for $\omega_m \neq -1$.

Case (i) When $n \neq 2/3$:

In this case, the solution of (6.3.23) is obtained as

$$f(T) = \beta_2 T + T_1 T^{\alpha_2}. \tag{6.3.24}$$

Here, T_1 is a constant of integration, $\alpha_2 = \frac{1-3\omega_m}{(3n-2)(1+\omega_m)}$ and $\beta_2 = \frac{2-3n(1+\omega_m)}{2[(3n(1+\omega_m)+(\omega_m-3))]}$, where $n \neq \frac{3-\omega_m}{3(1+\omega_m)}$.

The function $f(R,T)$ takes the form

$$f(R,T) = R + 2(\beta_2 T + T_1 T^{\alpha_2}). \tag{6.3.25}$$

Using (6.3.24) the energy density and pressure of exotic matter in terms of T are respectively given by

$$\rho_d = \left(\frac{\beta_2(3-\omega_m)}{1-3\omega_m}\right) T + \left(\frac{3n}{3n-2}\right) T_1 T^{\alpha_2}, \tag{6.3.26}$$

$$p_d = -(\beta_2 T + T_1 T^{\alpha_2}). \tag{6.3.27}$$

From (6.3.26) and (6.3.27), the EoS parameter in terms of T may be written as

$$\omega_d = -\frac{(\beta_2 T + T_1 T^{\alpha_2})}{\left(\frac{\beta_2(3-\omega_m)}{1-3\omega_m}\right) T + \left(\frac{3n}{3n-2}\right) T_1 T^{\alpha_2}}. \tag{6.3.28}$$

One can observe that it is too difficult to express ω_d in terms of t, in general. However, it can be seen the behavior of ω_d for some particular values of ω_m and n, lying in between $-1 < \omega_m < 1$, for examples, when $\omega_m = -1/3$ and $n = 1$ $(q = 0)$, we get $\omega_d = -1/3$, and $\omega_d = -1/2$ for $\omega_m = -1/2$ and $n = 4/3$ $(q = -0.25)$. Hence, ω_d of exotic matter in these two cases are equal to ω_m of perfect fluid.

In case of dust matter ($\omega_m = 0$), ω_d in (6.3.28) for a particular choice of $n = 5/6$, reduces to

$$\omega_d = -\left(\frac{1+2T_1 T}{3+10T_1 T}\right). \tag{6.3.29}$$

Let us consider the energy density and pressure of the effective matter for the solution of (6.3.25), which in terms of T are respectively, given by

$$\rho_{eff} = \left[\frac{1+\beta_2(3-\omega_m)}{1-3\omega_m}\right] T + \left(\frac{3n}{3n-2}\right) T_1 T^{\alpha_2}, \tag{6.3.30}$$

$$p_{eff} = -\left(\beta_2 - \frac{\omega_m}{1-3\omega_m}\right) T - T_1 T^{\alpha_2}. \tag{6.3.31}$$

Substituting the values of β_2 and α_2 in the above Eqs. (6.3.30) and (6.3.31) and simplifying, we get the effective EoS parameter as

$$\omega_{eff} = \frac{(2-3n)}{3n}, \tag{6.3.32}$$

which is constant and depends only on n. This relation between ω_{eff} and n describes the evolution of the Universe.

Also, using (6.3.24) into (6.3.4) for these particular values $\omega_m = 0$ and $n = 5/6$, we get an equation of T in terms of t as

$$12T_1T^2 + 6T = 5t^{-2}. \tag{6.3.33}$$

Therefore, for a suitable physical solution of T in terms of t where ρ_m and ρ_d must be positive, we can express ω_d in terms t. A graph between ω_d and t is shown in fig. 6.1, which shows that ω_d starts from -0.20 and approaches to -0.33.

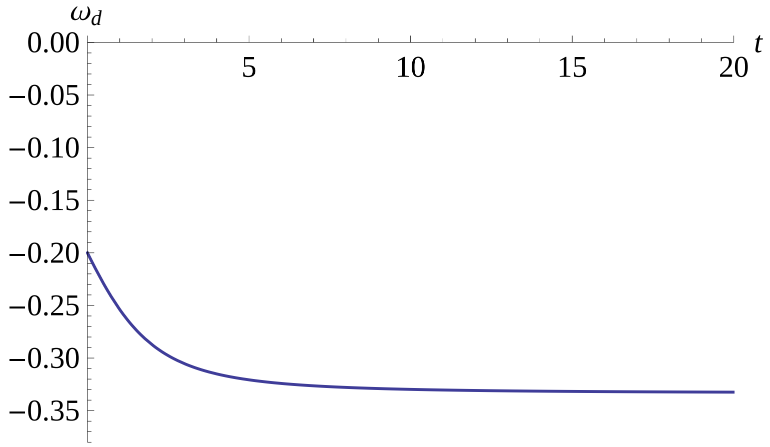

Figure 6.1: ω_d *versus* t with $T_1 = 1$, $\omega_m = 0$ and $n = 5/6$.

The density parameter corresponding to the exotic matter for $\omega_m = 0$ and $n = 5/6$ is given by

$$\Omega_d = \frac{(3+10T_1T)T}{6H^2}. \tag{6.3.34}$$

Using (6.3.33), a graph between Ω_d and t is plotted in fig. 6.2, which shows that $\Omega_d < 1$, i.e., an open model.

The effective density parameter for $\omega_m = 0$ and $n = 5/6$ can be obtained as

$$\Omega_{eff} = \frac{5}{6}\frac{(1+2T_1T)T}{H^2}. \tag{6.3.35}$$

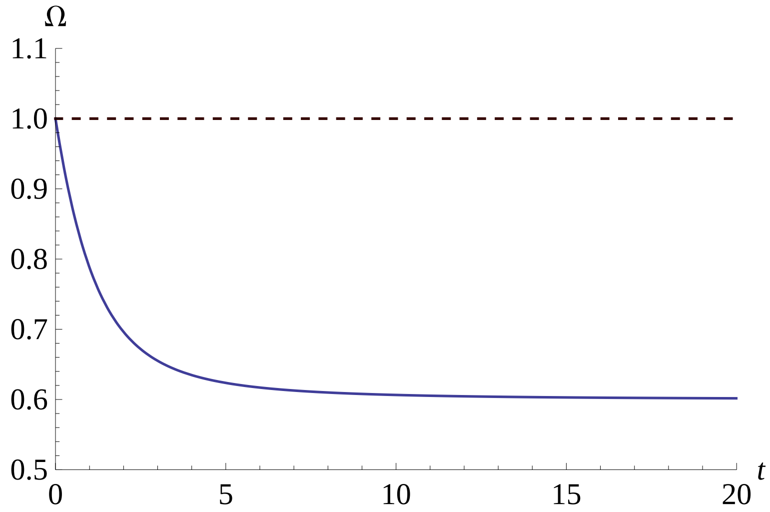

Figure 6.2: Ω_d (solid line) and Ω_{eff} (dashed line) *versus* t with $T_1 = 1$, $\omega_m = 0$ and $n = 5/6$.

Using (6.3.33), we find that $\Omega_{eff} = 1$ as shown in fig. 6.2 and hence this particular dust model is effectively flat.

The solution obtained in (6.3.24) is valid only for $n \neq \frac{3-\omega_m}{3(1+\omega_m)}$. Let us consider the case when $n = \frac{3-\omega_m}{3(1+\omega_m)}$. In this case, the solution of (6.3.23) is given by

$$f(T) = \frac{\omega_m - 1}{2(1-3\omega_m)} T \ln T + T_2 T, \tag{6.3.36}$$

where T_2 is a constant of integration. The function $f(R,T)$ becomes

$$f(R,T) = R + \frac{\omega_m - 1}{(1-3\omega_m)} T \ln T + 2T_2 T. \tag{6.3.37}$$

Case (ii) When $n = 2/3$:

In this case, the solution of (6.3.23) is

$$f(T) = \frac{\omega_m}{1-3\omega_m} T, \tag{6.3.38}$$

and hence

$$f(R,T) = R + \frac{2\omega_m}{1-3\omega_m} T. \tag{6.3.39}$$

The energy density and pressure of exotic matter in terms of T are given by

$$\rho_d = \frac{\omega_m(3-\omega_m)T}{(1-3\omega)^2}, \tag{6.3.40}$$

$$p_d = -\frac{\omega_m T}{1-3\omega_m}. \tag{6.3.41}$$

The EoS parameter corresponding to exotic matter gives

$$\omega_d = \frac{(1-3\omega_m)}{\omega_m - 3}. \tag{6.3.42}$$

We observe that $-1 < \omega_d < 1$ as $-1 < \omega_m < 1$, therefore, the exotic matter also behaves similar to perfect fluid.

We also find

$$\rho_{eff} = \frac{1-\omega_m^2}{(1-3\omega_m)^2}, \tag{6.3.43}$$

$$p_{eff} = 0. \tag{6.3.44}$$

Consequently, the effective EoS parameter $\omega_{eff} = 0$. Thus, the effective matter behaves as dust.

It is to be noted that for dust matter ($\omega_m = 0$), $\rho_d = 0 = p_d$ and hence the modified $f(R,T)$ theory reduces to Einstein's GR.

6.4 Conclusion

In this chapter, we have reconstructed the modified $f(R,T)$ gravity with the perfect fluid in a flat FRW model. The gravitational field equations have been reconstructed for a particular form of $f(R,T) = R + 2f(T)$. It has been found that the field equations are equivalent to Einstein's field equations with the effective energy-momentum tensor containing a sum of the usual matter and the exotic matter due to $f(R,T)$ gravity. Using two known forms of the scale factor (de Sitter and power-law) of cosmic history, we have obtained the exact form of $f(R,T)$ in terms of T. We have examined the features of the solutions for the reconstructed functions $f(R,T)$ in both models. The reconstructed functional form of $f(R,T)$ gives the DE epoch in both models which have been analyzed through EoS parameters.

In de Sitter model we have found that the exotic fluid due to modified gravity shows the character of phantom DE ($\omega_d < -1$) when the usual matter behaves between decelerated phase to accelerated phase ($-1 < \omega_m < 1$). In the absence of perfect fluid ($\rho_m = 0 = p_m$), the exotic matter behaves like a cosmological constant ($\omega_d = -1$). We have also found that the model becomes open ($\Omega_d < 1$) when exotic matter shows the characteristic of phantom DE and becomes flat ($\Omega_d = 1$)

when it behaves like a cosmological constant. The effective matter shows the behavior of stiff matter in this model which is effectively flat $\Omega_{eff} = 1$.

In power-law cosmological model we have reconstructed the form of $f(R,T)$ for $\omega_m \neq -1$ when $n \neq 2/3$ and $n = 2/3$, and have obtained the physical quantities. In case of $n \neq 2/3$, the EoS parameters of exotic matter have the same values for some particular EoS parameters of usual perfect fluid. We have found a relation $\omega_{eff} = (2-3n)/3n$, which describes the expansion history of the Universe. In case of dust matter $(\omega_m = 0)$ with $n = 5/6$, the EoS parameter of exotic matter lies between $-0.33 < \omega_d < 0.20$, and the model is open ($\Omega_d < 1$). In this particular case, the effective density parameter shows a flat model. In case of $n = 2/3$, we have observed that $-1 < \omega_d < 1$ as $-1 < \omega_m < 1$, therefore, the exotic matter also behaves similar to perfect fluid. It is to be noted that for dust matter ($\omega_m = 0$), $\rho_d = 0 = p_d$ and hence the modified $f(R,T)$ theory reduces to Einstein's GR. We have also found that $\rho_{eff} = (1-\omega_m^2)/(1-3\omega_m)^2$ and $p_{eff} = 0$, which implies $\omega_{eff} = 0$. Hence, the effective matter also behaves as dust.

In summary, we have reconstructed the model with a suitable choice of the form $f(R,T) = R + 2f(T)$, where R is considered as variable and have analyzed that it is possible to explain the DE phenomena through the reconstructed forms of $f(R,T)$ for de Sitter and power-law models with the perfect fluid in a flat FRW space-time.

Chapter 7

Scalar field cosmology in *f(R,T)* gravity

In this chapter[1], we reconstruct the functional form of $f(R,T) = R + 2f(T)$ in scalar field cosmology with two known history of expansion of the Universe for a flat FR-W model. The Universe is assumed to be filled with two non-interacting matter sources, one is quintessence or phantom scalar field minimally coupled to gravity with self interacting scalar potential and other is the matter contribution due to $f(R,T)$ gravity. We first explore a model where the potential is constant and the Universe evolves as a de Sitter expansion. This model is found to be compatible with phantom scalar field only. In second model, we consider the same forms of scalar potential and scale factor assumed in chapter 2, i.e., both evolving exponentially with scalar field. This model is found to be compatible with quintessence scalar field only. We also compare our results for this model with the recent observational data and find that some values of parameters are consistent with SNe Ia and $H(z)$+SNe Ia data to describe accelerated expansion only whereas some give both decelerated and accelerated expansions consistent with $H(z)$, WMAP7 and WMAP7+BAO+$H(z)$ observational data.

7.1 Introduction

The scalar fields drive rapid expansion during inflationary scenario in the early Universe [15, 36, 45, 46, 83, 84, 192, 193] whereas they could be responsible for

[1]The results of this chapter have been published in a research paper entitled "Modified $f(R,T)$ gravity theory and scalar field cosmology", in *Astrophysics and Space Science* **355** 2183–2192 (2014).

present accelerating Universe in various models of DE [21, 33, 39–44, 48, 50, 51, 68, 71, 80, 88, 145, 146]. Recently, Harko et al. [194] have presented several exact cosmological solutions with scalar field. Therefore, the theoretical and observational investigation of scalar field models is an essential task in cosmology.

The purpose of the present chapter is to study the scalar field cosmological models in $f(R,T)$ theory within the framework of a flat FRW metric. We have assumed that the Universe contains two matter sources, one is quintessence or phantom scalar field minimally coupled to gravity with self interacting scalar potential and another one is the matter contribution due to $f(R,T)$ gravity. We have reconstructed the same particular form of $f(R,T) = R + 2f(T)$ in scalar field cosmology which has been reconstructed in previous chapter with the perfect fluid. We have considered same assumptions of previous chapter, namely, de Sitter and power-law cosmological model to describe the expansion history of the Universe. However, in de Sitter model we have consider a flat potential whereas the power-law model is obtained by the assumptions of the scale factor and scalar potential as assumed in chapter 2, i.e., both evolving exponentially with scalar field.

The recent results from Planck Collaboration [19] and first Panoramic Survey Telescope and Rapid Response System [265] motivate to concentrate specially on the value of EoS parameter. The principle tool to find the most precise value of EoS parameter at present is the combination of the most mature, well-studied and robust probes of DE: SNe Ia, BAO and CMB. We have obtained the exact value of the parameters in the exponential potential model using the constraints of power-law cosmology from the existing observational data of $H(z)$, SNe Ia, $H(z)$+SNe Ia [266], WMAP7 and WMAP7+BAO+$H(z)$ [267].

7.2 Scalar field cosmology in *f(R,T)* theory of gravity

We consider a minimally coupled scalar field ϕ in $f(R,T)$ gravity which is given by

$$S = \int \left[\frac{1}{2} f(R,T) + \mathscr{L}_\phi\right] \sqrt{-g}\, d^4x, \tag{7.2.1}$$

where $\mathscr{L}_\phi$ corresponds to the matter Lagrangian of scalar field (1.10.1).

The energy-momentum tensor of matter source is given by

$$T_{\mu\nu}^{(\phi)} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathscr{L}_\phi)}{\delta g^{\mu\nu}}, \tag{7.2.2}$$

where the trace is defined by $T = g^{\mu\nu} T_{\mu\nu}^{(\phi)}$. Equation (7.2.2) gives

$$T_{\mu\nu}^{(\phi)} = g_{\mu\nu}\mathscr{L}_\phi - 2\frac{\partial \mathscr{L}_\phi}{\partial g^{\mu\nu}}. \tag{7.2.3}$$

Variation of action (7.2.1) with respect to metric tensor $g_{\mu\nu}$ results the field equations of $f(R,T)$ gravity (6.2.3). Using the simplest particular form $f(R,T) = R+2f(T)$, Eq. (6.2.3) gives

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}^{(\phi)} - 2(T_{\mu\nu}^{(\phi)} + \ominus_{\mu\nu})f'(T) + f(T)g_{\mu\nu}, \tag{7.2.4}$$

where

$$\ominus_{\mu\nu} = -2T_{\mu\nu}^{(\phi)} + g_{\mu\nu}\mathscr{L}_\phi - 2g^{ij}\frac{\partial^2 \mathscr{L}_\phi}{\partial g^{\mu\nu}\partial g^{ij}}. \tag{7.2.5}$$

We consider a spatially isotropic and homogenous flat FRW line-element (3.2.1) and the energy-momentum tensor (1.10.3). The trace of $T_{\mu\nu}^{(\phi)}$ is given by

$$T = -\varepsilon\dot{\phi}^2 + 4V(\phi). \tag{7.2.6}$$

In an isotropic and homogenous space-time the matter Lagrangian of scalar field has the form

$$\mathscr{L}_\phi = -\left(\frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi)\right). \tag{7.2.7}$$

Using (7.2.7) into (7.2.5), the tensor $\ominus_{\mu\nu}$ becomes

$$\ominus_{\mu\nu} = -2T_{\mu\nu}^{(\phi)} - g_{\mu\nu}\left(\frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi)\right). \tag{7.2.8}$$

Jamil et al. [175] have reconstructed the form of $f(R,T) = R+2f(T)$ by treating R as a constant. In the present work, we consider R as a function of cosmic time t and try to find a general form of $f(T)$ for known scale factors.

With the energy-momentum tensor (1.10.3) and using (7.2.8), the field equations

(7.2.4) for flat FRW metric (3.2.1), yield

$$3H^2 = \rho_\phi + \rho_f, \tag{7.2.9}$$

$$2\dot{H} + 3H^2 = -\left(p_\phi + p_f\right), \tag{7.2.10}$$

where ρ_ϕ and p_ϕ are the energy density and pressure of quintessence or phantom scalar field given by Eqs. (1.10.5) and (1.10.6), respectively. The EoS parameter for quintessence or phantom scalar field is defined in Eq. (1.10.8). Here, ρ_f and p_f are the energy density and pressure of the matter contribution due to the modified gravity, given by

$$\rho_f = 2\varepsilon\dot{\phi}^2 f'(T) + f(T), \tag{7.2.11}$$

$$p_f = -f(T). \tag{7.2.12}$$

We consider that the scalar field and matter contribution due to $f(R,T)$ are non-interacting and together represent an effective matter. The EoS parameter of the matter due to $f(R,T)$ is defined by $\omega_f = p_f/\rho_f$. We also consider that the scalar field (quintessence or phantom) obeys the Klein-Gordon equation which is given by Eq. (1.10.7).

Now, we reconstruct the functional form of $f(R,T)$ considered in (6.2.6) in the following subsections, for two known history of evolution of the Universe.

7.2.1 Model with constant potential

Let us explore a model with constant potential, i.e., $V(\phi) = V_0$, and the Universe behaves like a de Sitter model with an exponential expansion given by Eq. (6.3.8), which gives a constant value of Hubble parameter, i.e., (6.3.9)

Using (7.2.6), (1.10.5), (7.2.11), and (6.3.9) into (7.2.9), we obtain

$$2(4V_0 - T)f'(T) + f(T) + \frac{1}{2}(4V_0 - T) + V_0 - 3H_0^2 = 0, \tag{7.2.13}$$

which gives

$$f(T) = 3H_0^2 + V_0 - \frac{T}{2} + T_2\sqrt{2(T - 4V_0)}, \tag{7.2.14}$$

where T_2 is a constant of integration. For real solution we must have $T - 4V_0 > 0$ and one may observe from (7.2.6) that $T - 4V_0 > 0$ if $\varepsilon = -1$. Therefore, this model is compatible with phantom scalar field only. Hence, we shall use $\varepsilon = -1$ in the further discussion of the present model.

The functional form of $f(R,T)$ assumed in (6.2.6) takes the form

$$f(R,T) = R + 2(3H_0^2 + V_0) - T + 2T_2\sqrt{2(T - 4V_0)}. \quad (7.2.15)$$

Now, let us discuss the behaviors of some physical quantities and cosmological parameters of the present model.

By use of (6.3.9), the Klein-Gordon equation (1.10.7), gives

$$\phi(t) = \phi_1 - \frac{\phi_0 e^{-3H_0 t}}{3H_0}, \quad (7.2.16)$$

where ϕ_0 and ϕ_1 are constants of integration. We observe that the scalar field ϕ increases with time and becomes constant as $t \to \infty$. Therefore, the kinetic term of phantom scalar field $-\dot{\phi}^2/2$ also increases with time which dominates over scalar field potential and generates negative pressure for late-time acceleration.

The energy density and pressure of phantom scalar field are respectively given by

$$\rho_\phi = V_0 - \frac{1}{2}\phi_0^2 e^{-6H_0 t}, \quad (7.2.17)$$

$$p_\phi = -\left(V_0 + \frac{1}{2}\phi_0^2 e^{-6H_0 t}\right). \quad (7.2.18)$$

For reality of any cosmological model where the energy density must be positive, i.e., $\rho_\phi > 0$, we must have $\phi_0 < \sqrt{2V_0}$. We observe that ρ_ϕ starts with a finite value $V_0 - (\phi_0^2/2)$ at $t = 0$, which shows that the model avoids the initial singularity. It increases with time and approaches to a constant value V_0 as $t \to \infty$. We also observe that the pressure is always negative, which increases during the evolution and becomes constant ($-V_0$) at late-time. Thus, we can say that the potential dominates over the kinetic energy of phantom scalar field at late-time. The EoS parameter ω_ϕ starts from $\omega_\phi < -1$ and approaches to $\omega_\phi = -1$ as $t \to \infty$. Therefore, we conclude that the model describes the behavior of a phantom scalar field cosmology during the evolution of the Universe and of a cosmological constant at late-time.

The energy density and pressure of matter due to $f(R,T)$ gravity are respectively given by

$$\rho_f = 3H_0^2 - V_0 + \frac{1}{2}\phi_0^2 e^{-6H_0 t}, \tag{7.2.19}$$

$$p_f = -(3H_0^2 - V_0) + \frac{1}{2}\phi_0^2 e^{-6H_0 t} - \sqrt{2\phi_0^2 T_2 e^{-3H_0 t}}. \tag{7.2.20}$$

Again, we must have $V_0 \leq 3H_0^2$ for reality of the model. We observe that ρ_f starts from a finite constant value $3H_0^2 - V_0 + (\phi_0^2/2)$, which also asserts that the model is non-singular. It decreases with time and approaches to a positive constant value $3H_0^2 - V_0$ as $t \to \infty$. The pressure p_f remains always negative and increases with time.

Fig. 7.1 plots the graph of ω_f *versus* time t for a negative value of T_2 and different values of potential. From this figure we see that the matter due to $f(R,T)$

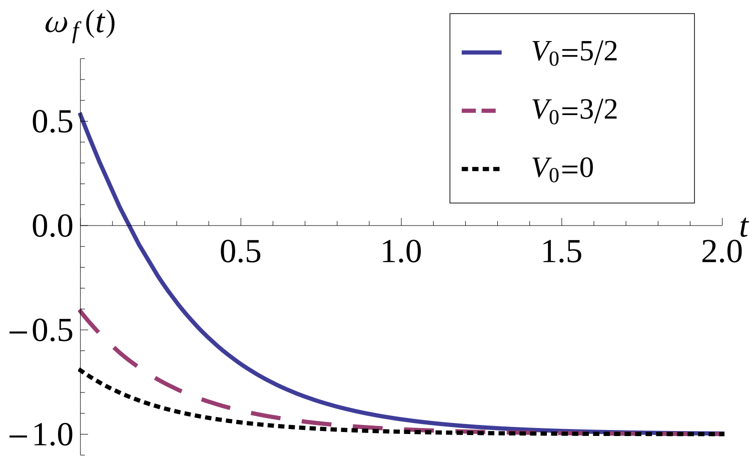

Figure 7.1: ω_f *versus* t with $T_2 = -1$, $H_0 = 1$ and $\phi_0 = 0.5$

gravity exhibits a wide variety of early time behaviors with different set of parameters which are consistent with the constraint $V_0 < 3H_0^2$. One may see that ω_f changes its sign from positive to negative for large values of potential whereas it remains negative for small values of potential and even for zero potential. It means that the matter due to $f(R,T)$ gravity exhibits the transition from ordinary matter to a normal scalar field (quintessence) for some set of parameters during the evolution of the Universe and like a cosmological constant at late-time. Therefore, the matter due to $f(R,T)$ provides transition from decelerated to accelerated phase for some set of parameters. However, the Universe accelerates throughout the evolution for some other set of parameters as we can see in the figure for small values of potential.

The behavior of ω_f for a positive value of T_2 with different values of potential under the constraint $V_0 < 3H_0^2$ is shown in fig. 7.2.

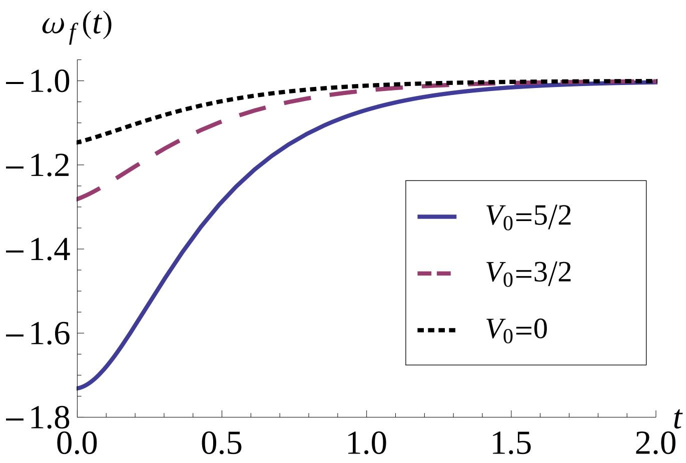

Figure 7.2: ω_f *versus* t with $T_2 = 1$, $H_0 = 1$ and $\phi_0 = 0.5$

The figure shows that the matter due to $f(R,T)$ gravity always behaves like phantom matter ($\omega_f < -1$) during the evolution of the Universe and cosmological constant at late-time. Therefore, we can say that the functional form of $f(R,T)$ obtained in (7.2.15) with a positive coefficient T_2 accelerates the Universe throughout its evolution. Let us consider a particular case where $V_0 = 3H_0^2$. The EoS param-

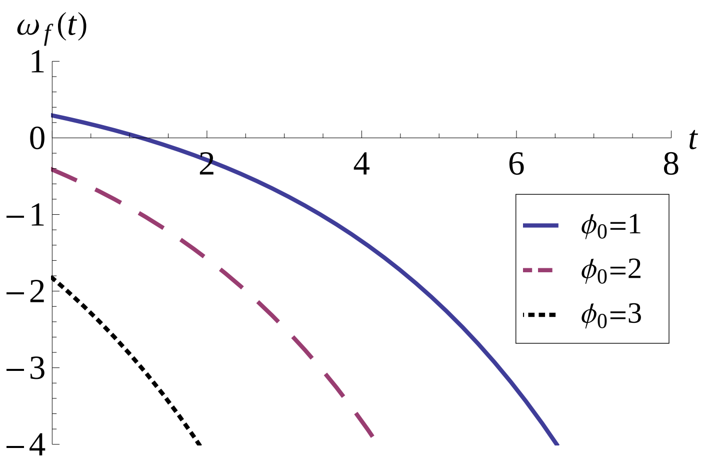

Figure 7.3: ω_f *versus* t with $T_2 = 1$ and $H_0 = 0.1$

eter ω_f for this particular case reduces to

$$\omega_f = 1 - \frac{2\sqrt{2}T_2}{\phi_0} e^{3H_0 t}, \qquad (7.2.21)$$

which is free from scalar field potential but depends on ϕ_0. From the above expression it is clear that $\omega_f \leq 1$ for $T_2 \geq 0$ and $\omega_f > 1$ for $T_2 < 0$. The behavior of ω_f

in this particular case with a positive value of T_2 and for some different values of ϕ_0 is shown in fig. 7.3.

The EoS parameter shows a transition from a positive to negative value for small values of ϕ_0 whereas it gives negative value for sufficiently large values of ϕ_0. Hence, the matter due to $f(R,T)$ for some sets of parameters give the transition from ordinary matter to phantom matter crossing the phantom dividing line whereas it always behaves like phantom matter for some other set of parameters.

Note that instead of ω_f if we consider the effective EoS parameter ω_{eff} in this particular model, then

$$w_{eff} = \frac{p_\phi + p_f}{\rho_\phi + \rho_f} = -1 - \frac{\sqrt{2\phi_0^2}T_2 e^{-3H_0 t}}{3H_0^2}, \tag{7.2.22}$$

which is also independent of scalar field potential. Fig. 7.4 plots ω_{eff} *versus* time t for some negative values of T_1 with a particular set of H_0 and ϕ_0. The effective EoS

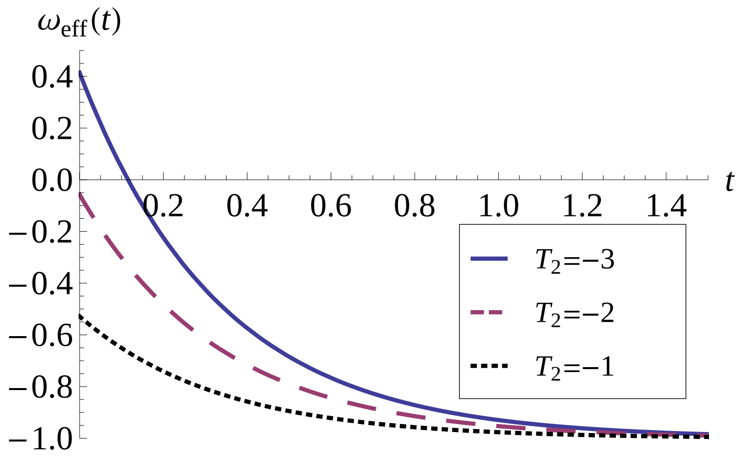

Figure 7.4: ω_{eff} *versus* t with $H_0 = 1$, $\phi_0 = 1$ and with some negative values of T_2

parameter ω_{eff} makes transition from some positive to negative values for some particular set of parameters. However, it remains negative $(-1 < \omega_{eff} < 0)$ for some other set of parameters but behaves like a cosmological constant at late-time always. Therefore, we conclude that this model describes transition from decelerated to accelerated phase for some set of parameters whereas it exhibits acceleration throughout the evolution for some other set of parameters.

The behavior of ω_{eff} for some positive values of T_2 and a particular set of values of H_0 and ϕ_0 is shown in fig 7.5. It is clear that the effective matter behaves as phantom matter during the evolution of the Universe and cosmological constant at

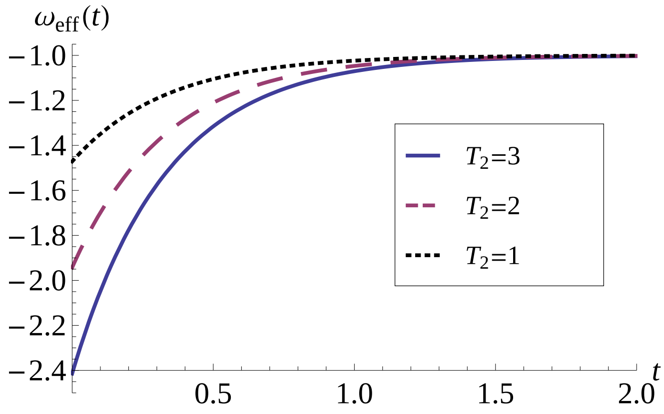

Figure 7.5: ω_{eff} *versus* t with $H_0 = 1$, $\phi_0 = 1$ and with some positive values of T_2

late-time for positive coefficients T_2 in (7.2.15). It is also to be noted that $\omega_{eff} = -1$ for $T_2 = 0$.

We also observe that the effective matter for different choices of T_2 behaves similar to matter due to $f(R,T)$ gravity for different choices of V_0 (compare figs. 7.2 *versus* 7.4, and *versus* 7.5). The large or small potential and the large negative or small positive values of T_2 affect the evolution of the Universe in similar manner.

7.2.2 Model with exponential potential

In chapter 2, we have assumed that the scalar potential and scale factor evolve exponentially with scalar field. We reconsider both these assumptions for the present model which are given by Eqs. (2.3.1) and (2.3.2), respectively. The Klein-Gordon equation (1.10.7) of normal or phantom scalar field for these two assumptions can be rewritten as

$$\frac{d}{dt}\left(\dot{\phi}^2 e^{6\alpha\phi}\right) = K_2 \frac{d}{dt}\left(e^{\phi(6\alpha-\beta)}\right), \tag{7.2.23}$$

where $K_2 = \frac{2V_0\beta}{(6\alpha-\beta)\varepsilon}$. This equation on integration gives

$$\dot{\phi}^2 = K_2 e^{-\beta\phi}, \tag{7.2.24}$$

where the integration constant is taken zero for the sake of simplicity. On integration (7.2.24), one obtains

$$\phi(t) = \frac{2}{\beta}\, ln\left(\frac{\phi_0\beta}{2} \pm \frac{\beta\sqrt{K_2}}{2}t\right), \;\; \beta \neq 0, \tag{7.2.25}$$

where ϕ_0 is a constant of integration. The real solution exists for $K_2 > 0$, i.e., if $(6\alpha-\beta) > 0$ for $\varepsilon = 1$ (quintessence scalar field) and if $(6\alpha-\beta) < 0$ for $\varepsilon = -1$ (phantom scalar field).

The scale factor (2.3.2) takes the form

$$a(t) = a_0\left(\frac{\phi_0\beta}{2} \pm \frac{\beta\sqrt{K_2}}{2}t\right)^{\frac{2\alpha}{\beta}}. \tag{7.2.26}$$

Let us assume that the Universe originates as $a(0) = 0$, which implies $\phi_0 = 0$. Therefore, Eqs. (7.2.25) and (7.2.26) for an expanding Universe respectively become

$$\phi(t) = \frac{2}{\beta}\, ln\left(\frac{\beta\sqrt{K_2}}{2}t\right), \tag{7.2.27}$$

$$a(t) = a_\star t^{\frac{2\alpha}{\beta}}, \tag{7.2.28}$$

where $a_\star = a_0\left(\beta\sqrt{K_2}/2\right)^{2\alpha/\beta}$. Equation (7.2.27) shows that the scalar field $\phi(t)$ increases with time. The later expression of scale factor describes the power-law expansion of the Universe.

The scalar potential $V(\phi)$ and the trace T, respectively, take the form

$$V(t) = \frac{2\varepsilon(6\alpha-\beta)}{\beta^3}\frac{1}{t^2}, \tag{7.2.29}$$

$$T = \frac{12\varepsilon(4\alpha-\beta)}{\beta^3}\frac{1}{t^2}. \tag{7.2.30}$$

Equation (7.2.29) shows that the scalar potential $V(t)$ decreases with time and tends to zero as $t \to \infty$.

The energy density (1.10.5) and pressure (1.10.6) of scalar field (quintessence or phantom) are respectively given by

$$\rho_\phi = \frac{12\varepsilon\alpha}{\beta^3 t^2}, \tag{7.2.31}$$

$$p_\phi = \frac{4\varepsilon(\beta-3\alpha)}{\beta^3 t^2}. \tag{7.2.32}$$

For reality of the model the energy density must be positive, i.e., $\rho_\phi > 0$, which is possible if $\varepsilon = 1$. Therefore, the assumptions (2.3.1) and (2.3.2) are suitable to describe quintessence model only. The EoS parameter ω_ϕ, Hubble parameter H, and deceleration parameter q, come out with the same values as given by Eqs. (2.3.12), (2.3.13), and (2.3.14), respectively. It is to noted that all the three parameters are free from ε, therefore, these cosmological parameter remain same for quintessence and phantom scalar field for both assumptions (2.3.1) and (2.3.2). The EoS parameter (2.3.12) shows that the scalar field has the behavior of quintessence for $\beta > 3\alpha$ and phantom for $\beta < 3\alpha$. However, for reality of the model we shall discuss the solution for quintessence field only in rest of our discussion.

From (2.3.12) and (2.3.14), we have

$$q = \frac{1+3\omega_\phi}{2}, \tag{7.2.33}$$

which is the well known relation between deceleration parameter and EoS parameter for a standard flat FRW cosmological model.

From (2.3.13) and (2.3.14), we get

$$H(t) = \frac{1}{(1+q)t}. \tag{7.2.34}$$

To analyse the behavior of the model, let us express the scale factor in terms of redshift z which is given by

$$a(z) = \frac{a_0}{1+z}. \tag{7.2.35}$$

From (7.2.26) and (7.2.35), one obtains

$$t = \frac{t_0}{(1+z)^{\frac{\beta}{2\alpha}}}, \tag{7.2.36}$$

where $t_0 = 2/\beta\sqrt{K_2}$. Using the above relation we can expect the z-dependence of all the relevant quantities of the scenario at hand, which can then be confronted by the data.

In particular, the Hubble parameter in terms of z can be written as

$$H(z) = H_0(1+z)^{1+q}, \tag{7.2.37}$$

where $H_0 = \alpha^2\sqrt{K}_2/\beta$. The above equation shows that the evolution of the Universe governs by the parameters H_0 and q.

Some authors have constrained on parameters H_0 and q for the power-law cos-

Table 7.1: Values of $\beta/2\alpha$ and corresponding evolution of Universe

Data	**q**	$\frac{1}{1+q}$	$\frac{\beta}{2\alpha}$	**Nature of** $\frac{\beta}{2\alpha}$ **with error bars**	**Expansion of Universe**
$H(z)$	$-0.04^{+0.05}_{-0.05}$		$-0.96^{+0.05}_{-0.05}$	>1 (with +ve error)	Decelerated
				<1 (with -ve error)	Accelerated
SNe Ia	$-0.36^{+0.05}_{-0.05}$		$-0.64^{+0.05}_{-0.05}$	<1 (with +ve error)	Accelerated
				<1 (with -ve error)	Accelerated
$H(z)$+SNe Ia	$-0.21^{+0.04}_{-0.04}$		$-0.79^{+0.04}_{-0.04}$	<1 (with +ve error)	Accelerated
				<1 (with -ve error)	Accelerated
WMAP7		$-0.99^{+0.04}_{-0.04}$	$1.01^{-0.04}_{+0.04}$	<1 (with -ve error)	Accelerated
				>1 (with +ve error)	Decelerated
BAO+$H(z)$ +WMAP7		$-0.99^{+0.02}_{-0.02}$	$1.01^{-0.02}_{+0.02}$	<1 (with -ve error)	Accelerated
				>1 (with +ve error)	Decelerated

mology. Kumar [268] has constrained the parameters H_0 and q using 14 points of $H(z)$ data and 557 data points of SNe Ia observations. Gumjudpai [266] has constrained on parameters H_0 and $1/1+q$ for WMAP7 and WMAP7+BAO+$H(z)$ data sets. Recently, Rani et al. [267] have also re-constrained the parameters of ref. [268] for the latest 29 points of $H(z)$ data and 580 data points from Union 2.1 SNe Ia observations. Since in the present model we have three parameters, namely α, β, and H_0, therefore, it is not possible to constraint these parameters separately. However, using the values of q [267] and $1/1+q$ [266], we find the values of $\beta/2\alpha$ and analyze the corresponding evolution (decelerated or decelerated) of the Universe through the theoretical prediction of deceleration parameter $q=\beta/2\alpha-1$, i.e., $\beta/2\alpha>1$ or <1 as given in Table 7.1. We observe that our model is best fitted with SNe Ia and $H(z)$+SNe Ia for accelerated Universe including error bars whereas decelerated and accelerated models are well agreed with others observational data. We shall use these constraints in finding the nature of EoS parameters of matter contribution due to $f(R,T)$ gravity and effective matter of scalar field and matter due to $f(R,T)$ gravity in the forthcoming discussion.

Using (1.10.5), (7.2.11), (7.2.27), (7.2.28), (7.2.29) and (7.2.30), Eq. (7.2.9) for quintessence scalar field ($\varepsilon=1$) can be written as

$$2\beta T f'(T)+3(4\alpha-\beta)f(T)-3\alpha(\alpha\beta-1)T=0, \tag{7.2.38}$$

which is integrated to give

$$f(T)=\frac{3\alpha(\alpha\beta-1)}{12\alpha-\beta}T+T_3T^{-\frac{3(4\alpha-\beta)}{2\beta}}, \tag{7.2.39}$$

where T_3 is a constant of integration.

The function $f(R,T)$ considered in (6.2.6) takes the form

$$f(R,T) = R + \frac{6\alpha(\alpha\beta-1)}{12\alpha-\beta}T + 2T_3 T^{-\frac{3(4\alpha-\beta)}{2\beta}}. \tag{7.2.40}$$

The energy density and pressure of the matter due to $f(R,T)$ gravity respectively have expressions

$$\rho_f = \frac{12\alpha(\alpha\beta-1)}{\beta^3}\frac{1}{t^2}, \tag{7.2.41}$$

$$p_f = -\frac{36\alpha(4\alpha-\beta)(\alpha\beta-1)}{\beta^3(12\alpha-\beta)}\frac{1}{t^2} - T_3\left[\frac{12(4\alpha-\beta)}{\beta^3}\frac{1}{t^2}\right]^{-\frac{3(4\alpha-\beta)}{2\beta}}. \tag{7.2.42}$$

For $\rho_f \geq 0$, i.e., for real model we must have $(\alpha\beta-1) \geq 0$. We observe that the EoS parameter of matter due to $f(R,T)$ fluid is compatible to describe decelerated model for negative values of T_3 whereas positive values of T_3 are feasible to describe accelerating Universe.

Let us examine the behavior of EoS parameter ω_f under the constraints for which our model fits with the observations without including error bars. Fig. 7.6 plots the graph between EoS parameter ω_f and t for decelerated ($T_3 > 0$) model for some values of α and β which satisfy the constraint from WMAP7 and WMAP7+BAO+$H(z)$ given in Table 7.1. The figure shows that the EoS parameter

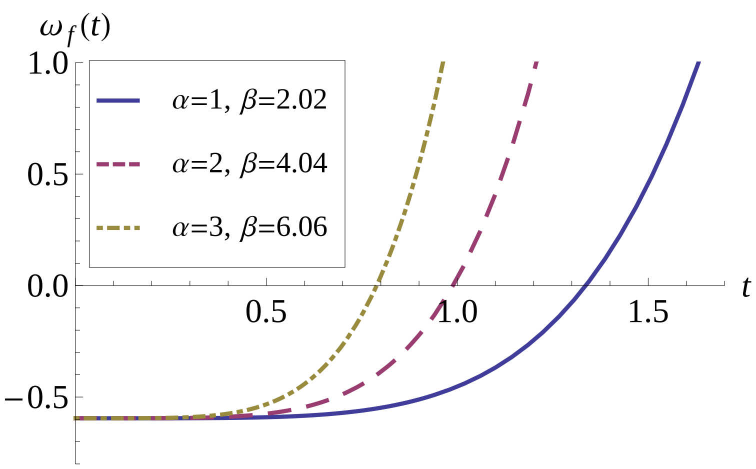

Figure 7.6: ω_f *versus* t for $T_3 = -1$ and some consistent values of α and β with WMAP7 and WMAP7+BAO+$H(z)$ observations

starts from $\omega_f = -0.6$ irrespective of sets of parameters and approaches to the positive values, which shows that the model transits from early inflationary phase to decelerated phase.

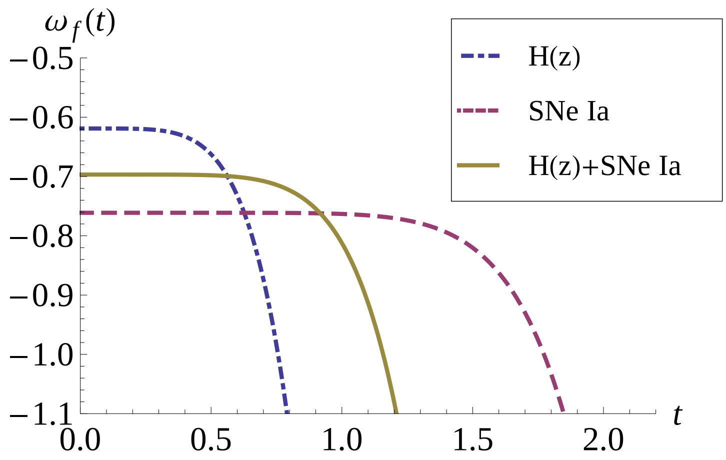

Figure 7.7: ω_f *versus* t for $T_3 = 1$, $\alpha = 1$, and some consistent values $\beta = 1.92$ ($H(z)$), $\beta = 1.28$ (SNe Ia) and $\beta = 1.58$ ($H(z)$+SNe Ia)

Fig. 7.7 plots ω_f *versus* t for accelerated ($T_3 < 0$) model with $\alpha = 1$ and different values of β which satisfy the constraints from $H(z)$ ($\beta = 1.92$), SNe Ia ($\beta = 1.28$) and joint $H(z)$+SNe Ia ($\beta = 1.58$). The graph clearly indicate that ω_f varies from $-1 < \omega_f < 0$ to $\omega_f < -1$. Hence, the model enters from quintessence to phantom phase in this case.

The effective energy density and pressure are respectively given by

$$\rho_{eff} = \frac{12\alpha^2}{\beta^2}\frac{1}{t^2}, \tag{7.2.43}$$

$$p_{eff} = -\frac{4(3\alpha-\beta)}{\beta^3}\frac{1}{t^2} - \frac{36\alpha(4\alpha-\beta)(\alpha\beta-1)}{\beta^3(12\alpha-\beta)}\frac{1}{t^2} - T_2\left[\frac{12(4\alpha-\beta)}{\beta^3}\frac{1}{t^2}\right]^{-\frac{3(4\alpha-\beta)}{2\beta}} \tag{7.2.44}$$

One may observe that the effective EoS parameter exhibits the same behavior as ω_f. Thus, the model successfully describes the transition from early inflationary phase to matter-dominated era as well as the late-time acceleration of the Universe exhibiting transition from quintessence to phantom phase.

7.3 Conclusion

In this chapter, we have studied modified $f(R,T)$ gravity with quintessence and phantom scalar field in a flat FRW model. We have considered a particular form $f(R,T) = R + 2f(T)$, which leads to the field equations equivalent to the Einstein's field equations with an effective energy-momentum tensor containing a sum of quintessence or phantom scalar field and the matter contribution due to $f(R,T)$ gravity. We have reconstructed this form of $f(R,T)$ with two well-known scale factors along with the constant and exponential scalar potentials, respectively.

First we have explored a model where the potential of the scalar field is constant and the Universe evolves as de Sitter exponential expansion. In the second model, we have considered the scalar field potential and scale factor evolve exponentially with the scalar field (quintessence or phantom), which lead to the power-law expansion of the Universe. The behavior of each model has been examined through the deceleration parameter, the EoS parameters of matter contribution of $f(R,T)$ gravity (ω_f) and the effective EoS parameter (ω_{eff}) of matter due to $f(R,T)$ and scalar field. We have compared the later model with the observational results of $H(z)$, SNe Ia, $H(z)$+SNe Ia, WMAP7 and $H(z)$+BAO+WMAP7. The summary of the results of both the models are as follows.

In the first model where we have considered de Sitter expansion and a flat potential, it has been found that the solutions are compatible with the phantom scalar field only. The model is free from Big-Bang singularity and exhibits a wide variety of early time behaviors of evolution of the Universe with different sets of parameters. The model ultimately describes the behaviour of cosmological constant at late-time as expected. We have noted from fig. 7.1 that for negative value of T_2, ω_f changes its sign from positive to negative for large values of potential whereas it remains negative for small values of potential even for zero potential. It means that the model exhibits transition from ordinary matter to quintessence during early times and ultimately a cosmological constant at late-time. Therefore, the matter due to $f(R,T)$ describes the evolution from decelerated to accelerated phase for some set of parameters. However, the Universe accelerates throughout the evolution for some other set of parameters. In fig. 7.2 for positive values of T_1, we have observed that the matter due to $f(R,T)$ gravity always behaves like a phantom matter ($\omega_f < -1$) during the evolution of the Universe and becomes cosmological constant at late-time. Therefore, the Universe accelerates throughout the evolution for any set of parameters with positive values of T_1. The effective matter also behaves in a similar manner as the matter due to $f(R,T)$ as shown in figures 7.4 and 7.5.

In the second model, where we have assumed scalar potential and scale factor evolve exponentially with the scalar field, it has been found that the model is compatible with the quintessence scalar field only. We have already seen in chapter 2 that these two assumptions lead to the power-law expansion of the Universe which encounters a Big-Bang singularity at $t = 0$. The scalar field increases whereas the scalar potential decreases with time and tends to zero as $t \to \infty$. We have also computed the value of $\beta/2\alpha$ with error bars using existing observational data from $H(z)$, SNe Ia, $H(z)$+SNe Ia, WMAP7 and WMAP7+BAO+$H(z)$ available on power-law cosmology as given in Table 7.1. The behavior of the model has been shown by EoS parameter due to $f(R,T)$ matter for the parameters which satisfy the observational constraints. The model exhibits decelerated as well as accelerated Universe. The decelerated model describes the early Universe and shows transition from inflationary to decelerated phase as shown in fig. 7.6. The model shows acceleration at late-time and transits from quintessence to phantom phase (see fig. 7.7). It is to be noted that these figures have been plotted excluding the error bars. One may observe that the effective EoS parameter shows the same behavior as ω_f. The values of $\beta/2\alpha$ from SNe Ia and H(z)+SNe Ia observations are well agreed for accelerating Universe including error bars whereas decelerated and accelerated nature of the Universe have been observed with others observational data.

In concluding remark we can say that both the models are suitable to describe a wide variety of early and late-time evolution of the Universe in $f(R,T)$ theory, some of which give inflation, quintessence or phantom phase in their respective EoS.

Chapter 8

Matter creation in modified *f(R,T)* gravity

This chapter[1] deals with the theoretical and observational consequences of thermodynamics of open systems which allow matter creation in modified $f(R,T)$ gravity within the framework of a flat FRW model. The simplest functional form of $f(R,T) = R + 2f(T)$ and the 'gamma-law' EoS, i.e., $p = (\gamma - 1)\rho$ are assumed to obtain the exact solution. A power-law expansion model has proposed by considering the natural phenomenological particle creation rate $\Gamma = 3\beta_0\eta H$, where β_0 is a pure number of the order of unity and η is the particle number density. A Big-Rip singularity is observed for $\gamma < 0$ which describes the phantom cosmology. We observe that the accelerated expansion of the Universe is driven by the particle creation. Some kinematic tests such as lookback time, luminosity distance, proper distance, angular diameter versus *redshift have discussed in detail to observe the role of particle creation in early and late-time evolution of the Universe.*

8.1 Introduction

In the context of early Universe, the standard ΛCDM model presents several theoretical and observational difficulties, such as the singularity problem [2, 31,

[1]This chapter comprises the results of a paper entitled "Friedmann cosmology with matter creation in modified $f(R,T)$ gravity", *International Journal of Theoretical Physics*, **DOI 10.1007/s10773-015-2767-z** (2015).

32, 269], reheating during the inflationary epoch [270], confliction between the age of the Universe and the age of the oldest stars in globular clusters (age problem) [271], the entropy problem [272] etc. The introduction of an inflationary phase derived by a scalar field or by HD theory ($R+\lambda R^2$) resolves the flatness and horizon problems together with the entropy problem [2, 14, 31, 32]. The emergent Universe scenario resolves the issue of singularity problem [202]. But the age confliction [273] is not an isolated complication, it comes with another serious trouble that is structure formation through gravitational amplification of small primeval density perturbation. The issues related to the early Universe open the door of investigations of many alternative theories [91–93, 96–98].

Among the ways to resolve the problems of early Universe, Dirac's large number hypothesis [274] inspired a class of new cosmology named particle creation [275]. The steady state model introduced by Bondi and Gold [276] on the foundation of perfect cosmological principle (PCP) also asserts the continuous generation of matter in the Universe. Hoyle [277] and Narlikar [278] have independently proposed a creation field theory and studied the matter creation during the evolution of the Universe. Tryon [279] and Fomin [280] in their individual work have proposed a theoretical concept of the creation of the Universe as a vacuum fluctuation. Brout et al. [281, 282] have builded a strong foundation of simultaneous creation of matter and curvature from a quantum fluctuation of the Minkowskian space-time vacuum.

Later on, Gunzig et al. [283] and Prigogine et al. [284] have established the theoretical scenario of matter creation in the framework of cosmology. They showed that the second law of thermodynamics might be modified to accommodate the flow of energy from gravitational field to the matter field, resulting in the creation of particles and consequently entropy. Their work might suggest that at the expense of the gravitational field, particle creation takes place as an irreversible process constrained by the usual requirements of the non-equilibrium thermodynamics, however, the reverse process (matter destruction) thermodynamically forbidden. Calváo et al. [285] have extended this new theoretical concept of matter creation under adiabatic conditions. The further results were generalized by Lima and Germano [286] through a contravariant formulation allowing specific entropy variation as usually expected for non-equilibrium process in fluids. Lima and Alcaniz [287], and Alcaniz and Lima [288] have investigated observational consequences of FRW models driven by adiabatic matter creation through some kinematic tests. Singh and Beesham [289,290], and Singh [291,292] have studied early Universe in FRW cosmology with particle creation through some kinematic tests.

After the discovery of accelerating Universe the particle creation theory has reconsidered to explain it and favourable results have been obtained. Zimdahl et al. [293] and Qiang et al. [294] have tested some models with adiabatic particle creation which are consistent with SNe Ia data. The theoretical formulation of continuous creation of matter in the Universe may reinterpret several predictions of the standard Big-Bang cosmology.

On the other hand, a number of pioneer concepts of modifying GR have been proposed to reconcile the problems related to late-time Universe particularly cosmic acceleration [102]. As mentioned in chapter 6 that the interesting features of $f(R,T)$ gravity have attracted many researchers for resolving several issues of current interest in cosmology and astrophysics [175–191]. Since the cosmic acceleration in $f(R,T)$ gravity results not only from geometrical effect but also from the matter contribution. The negative pressure due to particle creation also might play the role of exotic matter component being responsible for late-time cosmic acceleration. Therefore, these two similar concepts motivate us to study how the particle creation phenomena and $f(R,T)$ gravity together affect the early and late-time evolution of the Universe.

In this chapter, we investigate the theoretical and observational implication of particle creation in modified $f(R,T)$ theory in a flat FRW model. Exact cosmological solutions are obtained by assuming the suitable form of $f(R,T) = R+2f(T)$, EoS of perfect fluid, and particle creation rate. We also study some kinematic tests to explain the physical significance of particle creation during early and late-time evolution of the Universe.

8.2 Theory of particle creation

If we regard the whole Universe as a closed thermodynamical system in which the number of the particles in a given volume is constant then the laws of thermodynamics have the form

$$d(\rho_m V) = dQ - p_m dV, \tag{8.2.1}$$

and

$$\mathrm{T}dS = d(\rho_m V) + p_m dV, \tag{8.2.2}$$

where $\rho_m,\ p_m,\ V,\ \mathrm{T}$ and S are the energy density, thermodynamical pressure, volume, temperature and entropy, respectively. Here, dQ is the heat received by the system during time dt. From (8.2.1) and (8.2.2), the entropy production is given by

$$\mathrm{T}dS = dQ. \tag{8.2.3}$$

Eq. (8.2.3) shows that for a closed adiabatic system ($dQ = 0$) the entropy remains stationary, *i.e.*, $dS = 0$. However, if we treat the Universe as an open thermodynamic system allowing irreversible matter creation from the energy of the gravitational field, we can account for entropy production right from the beginning, and the second law of thermodynamics is also incorporated into the evolutionary equations in a more meaningful way. In such situation the number of particles N in a given volume V is not to be a constant but is time dependent. Therefore, Eq. (8.2.1) modifies as

$$d(\rho_m V) = dQ - p_m dV + (h/\eta)\, d(\eta V), \tag{8.2.4}$$

where $N = \eta V$, η is the particle number density and $h = (\rho_m + p_m)$ is the enthalpy per unit volume of the system. In case of adiabatic system, i.e., $dQ = 0$, Eq. (8.2.4) for an open system reduces to

$$d(\rho_m V) + p_m dV = (h/\eta)\, d(\eta V). \tag{8.2.5}$$

We see that in such a system the thermal energy is received due to the change of the number of particles. In cosmology, this change may be considered as a transformation of energy from gravitational field to the matter.

In the context of an open system, Eq. (8.2.5) can be rewritten as

$$d(\rho_m V) = -(p_m + p_c)dV, \tag{8.2.6}$$

where

$$p_c = -(h/\eta)(dN/dV). \tag{8.2.7}$$

Equation (8.2.6) suggests that the creation of matter in an open thermodynamic system corresponds to a supplementary pressure p_c, which must be considered as a part of the cosmological pressure entering into the Einstein field equations (decaying of matter leads to a positive decay pressure) and is equivalent to adding the term p_c given by (8.2.7) to the thermodynamic pressure p_m. It is to be noted that p_c is negative or zero depending on the presence or absence of particle creation, respectively.

Since the increment in entropy for an adiabatic system is only caused by creation of matter, therefore, the entropy is an extensive property of the system. In present scenario, S is proportional to the number of particles included in the system. Therefore, the entropy change dS from (8.2.2) and (8.2.5) for an open system becomes

$$\mathrm{T}dS = (h/\eta)\, d(nV) - \Upsilon\, d(\eta V) = (\mathrm{T}S/N)dN \Rightarrow \frac{dS}{S} = \frac{dN}{N}, \tag{8.2.8}$$

where Υ is the chemical potential given by $\Upsilon = (h - \mathrm{T}s)/\eta$, here $s = S/V$ is entropy per unit volume. Since the second law of thermodynamics is a fundamental law in physics, the presence or absence of particle creation can not affect it. This law basically requires $dS \geq 0$, consequently, Eq. (8.2.8) gives

$$dN \geq 0. \tag{8.2.9}$$

The above inequality implies that the space-time can produce matter whereas the reverse process is thermodynamically not admissible.

The purpose of this entire formulation is to modify the usual energy-momentum conservation law in an open thermodynamical system, which leads to the explicit use of a balance equation for the number density of the particles created, in addition to Einstein's field equations.

The particle flux vector is given by

$$N^{\upsilon} = \eta u^{\upsilon}, \tag{8.2.10}$$

and N^{υ} is assumed to satisfy the balance equation [285, 295]

$$N^{\upsilon}_{;\upsilon} = \Gamma, \tag{8.2.11}$$

where the function Γ denotes a source term of particle creation which is positive or negative depending on whether there is production or annihilation of particles. In standard cosmology Γ is usually assumed to be zero.

In the presence of a gravitational particle source, the balance equation (8.2.11) for the particle flux becomes

$$\dot{\eta} + 3\eta H = \Gamma. \tag{8.2.12}$$

Thus, the creation pressure p_c depends on the particle creation rate, and for adiabatic matter creation, (8.2.7) takes the form [285]

$$p_c = -\frac{(\rho_m + p_m)}{3\eta H}\,\Gamma. \tag{8.2.13}$$

Therefore, Eq. (8.2.13) shows that p_c is negative for $\Gamma > 0$, which can help to derive the era of accelerated cosmic expansion.

8.3 Model and the field equations

We consider the gravitational action for $f(R,T)$ modified theory of gravity [48] in the units $G = 1 = c$

$$I = \frac{1}{8\pi}\int\left[\frac{f(R,T)}{2} + \mathscr{L}_m\right]\sqrt{-g}\,d^4x, \tag{8.3.1}$$

where the other symbols have their usual meaning.

The equations of motion by varying the action (8.3.1) with respect to metric tensor become

$$\begin{aligned} f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} \quad &+ \quad (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T) \\ &= \quad 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\ominus_{\mu\nu}, \end{aligned} \tag{8.3.2}$$

where $\ominus_{\mu\nu}$ is given by (6.2.5).

We assume a functional form of $f(R,T) = R + 2f(T)$. Consequently, the gravitational field equations (8.3.3) become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} - 2(T_{\mu\nu} + \ominus_{\mu\nu})f'(T) + f(T)g_{\mu\nu}. \tag{8.3.3}$$

In the formalism of particle creation, the second law of thermodynamics naturally leads to the modification of energy-momentum tensor with an additional creation pressure depending on the creation rate of the particles. In the presence of particle creation, the energy-momentum tensor of perfect fluid (2.2.4) modifies as

$$T_{\mu\nu}^{(m)} = (\rho_m + p_m + p_c)u_\mu u_\nu - (p_m + p_c)g_{\mu\nu}. \tag{8.3.4}$$

The trace of energy-momentum tensor (8.3.4), gives

$$T = \rho_m - 3(p_m + p_c). \tag{8.3.5}$$

We treat the scalar invariant $\mathscr{L}_m$ as the effective pressure of the perfect fluid matter and pressure originated by creation of particles. Therefore, the matter Lagrangian may be assumed as $\mathscr{L}_m = -(p_m + p_c)$. Therefore, Eq. (6.2.5) becomes

$$\ominus_{\mu\nu} = -2T_{\mu\nu}^{(m)} - (p_m + p_c)g_{\mu\nu}. \tag{8.3.6}$$

In view of (8.3.6), the field equations (8.3.3) give

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}^{(m)} + 2\left[T_{\mu\nu}^{(m)} + g_{\mu\nu}(p_m + p_c)\right]f'(T) + g_{\mu\nu}f(T). \tag{8.3.7}$$

We consider a homogenous and isotropic flat FRW model as given in Eq. (3.2.1). The field equations (8.3.7) for a fluid endowed with matter creation (8.3.4) in the background of a flat FRW metric (3.2.1), yield

$$3H^2 = 8\pi\rho_m + 2(\rho_m + p_m + p_c)f'(T) + f(T), \tag{8.3.8}$$

$$2\dot{H} + 3H^2 = -8\pi(p_m + p_c) + f(T). \tag{8.3.9}$$

8.4 Solution of the field equations

The field equations (8.3.8) and (8.3.9) have five unknowns, namely, H, ρ_m, p_m, p_c and $f(T)$. Therefore, one needs three more relations in order to construct a definite cosmological scenario.

In first choice, we consider a particular function $f(T)$ in (6.2.6) [174]

$$f(T) = \lambda T, \tag{8.4.1}$$

where λ is a constant. With this assumption, the field equations (8.3.8) and (8.3.9), respectively, reduce to the form

$$3H^2 = (8\pi + 3\lambda)\rho_m - \lambda(p_m + p_c), \tag{8.4.2}$$

$$2\dot{H} + 3H^2 = -(8\pi + 3\lambda)(p_m + p_c) + \lambda\rho_m. \tag{8.4.3}$$

In order to obtain the exact solution of these field equations, we assume two more additional relations: the equation of state of the perfect fluid and the matter creation rate $\Gamma(t)$. In the cosmological domain, the former usually has the form $p_m = (\gamma - 1)\rho_m$ as given in Eq. (3.3.4).

Using (3.3.4) into (8.4.2) and (8.4.3), and simplifying, we get a single evolution equation for H:

$$2\dot{H} + (8\pi + 2\lambda)(\gamma\rho_m + p_c) = 0. \tag{8.4.4}$$

For the last assumption, we confine our attention to the simple phenomenological expression for the matter creation rate [272]

$$\Gamma(t) = 3\beta_0\eta H, \tag{8.4.5}$$

where the parameter β_0 lies in the interval $(0,1)$ and is assumed to be a constant.

Using (3.3.4) and (8.4.5) into (8.2.13), we have

$$p_c = -\beta_0\gamma\rho_m, \tag{8.4.6}$$

Putting (3.3.4) and (8.4.6) into (8.4.2), we obtain

$$\rho_m = \frac{3H^2}{8\pi + 4\lambda - \gamma\lambda(1-\beta_0)}. \tag{8.4.7}$$

Substituting (8.4.6) and (8.4.7) into (8.4.4), we get

$$\dot{H} + \frac{3}{2}\frac{\gamma(8\pi + 2\lambda)(1-\beta_0)}{[8\pi + 4\lambda - \gamma\lambda(1-\beta_0)]}H^2 = 0. \tag{8.4.8}$$

The solution of (8.4.8) for $\gamma \neq 0$ is given by

$$H(t) = \left(C_1 + \frac{3}{2}\frac{\gamma(8\pi + 2\lambda)(1-\beta_0)}{[8\pi + 4\lambda - (1-\beta_0)\gamma\lambda]}\,t\right)^{-1}, \tag{8.4.9}$$

where C_1 is an integration constant. For $\gamma = 0$, the well known de Sitter scale factor $a(t) = a_0\, e^{H_0 t}$ is obtained.

From Eq. (8.4.9) we find the following expression for the scale factor

$$a(t) = C_2 \left(C_1 + \frac{3}{2} K_3 \gamma\, t \right)^{\frac{2}{3K_3\gamma}}, \tag{8.4.10}$$

where C_2 is a new integration constant and $K_3 = \frac{(8\pi+2\lambda)(1-\beta_0)}{8\pi+4\lambda-\gamma\lambda(1-\beta_0)}$.

The above scale factor may be rewritten as

$$a(t) = a_0 \left(1 + \frac{3}{2} K_3 \gamma H_0 (t - t_0) \right)^{\frac{2}{3K_3\gamma}}, \tag{8.4.11}$$

where $H = H_0 > 0$ at $t = t_0$. The subscript '0' refers to the present values of parameters. Since $0 \leq \gamma \leq 2$, we must have $K_3 > 0$ for expansion of the Universe. Also, $K_3 > 0$ implies $\lambda > 0$ as $0 \leq \beta_0 < 1$.

The model avoids the initial singularity but encounters a Big-Rip singularity at a finite value of cosmic time $t_{br} = t - t_0 = -2/3H_0 K_3 \gamma$ for $\gamma < 0$. Thus, the model may describe the phantom cosmology for $\gamma < 0$. If one choose $t_0 = 2H_0^{-1}/3K_3\gamma$ then (8.4.11) takes the familiar form of power-law expansion of the Universe, *i.e.*,

$$a(t) = a_0 \left(\frac{3}{2} K_3 \gamma H_0\, t \right)^{\frac{2}{3K_3\gamma}}. \tag{8.4.12}$$

If $\lambda = 0 = \beta_0$, (8.4.10) and (8.4.12) reduce to the well-known expressions of power-law expansion of scale factor for a flat FRW model in GR.

By the use of (8.4.11) we obtain the energy density of matter, particle creation pressure and the particle number density as functions of the scale factor a, which respectively, have the following forms

$$\rho_m = \rho_0 \left(\frac{a_0}{a} \right)^{3K_3\gamma}, \tag{8.4.13}$$

$$p_c = -\beta_0 \gamma \rho_0 \left(\frac{a_0}{a} \right)^{3K_3\gamma}, \tag{8.4.14}$$

$$\eta = \eta_0 \left(\frac{a_0}{a} \right)^{3(1-\beta_0)}, \tag{8.4.15}$$

where $\rho_0 = 3H_0^2/[8\pi + 4\lambda - \gamma\lambda(1-\beta_0)]$ is the present value of energy density. Here, n_0 is the present value of particle number density for any values of β_0. The above results show that the transition from one phase to other phase, in the course of expansion, happens exactly as in the standard cosmological model.

The number of particles N in a given volume V is given by

$$N = N_0 \left(\frac{a}{a_0}\right)^{3\beta_0}, \tag{8.4.16}$$

which shows that N increases with time. If $\beta_0 = 0$, N would remain constant throughout the evolution of the Universe and we would recover the standard FRW model of the Universe in $f(R,T)$ theory. Again, from Eq. (8.2.8), $S = S_0(N/N_0)$, the entropy in terms of scale factor is

$$S = S_0 \left(\frac{a}{a_0}\right)^{3\beta_0}. \tag{8.4.17}$$

The deceleration parameter (1.7.10), gives

$$q = -1 + \frac{3\gamma K_3}{2} = \left[\frac{3\gamma}{2}\frac{(8\pi+2\lambda)(1-\beta_0)}{[8\pi+4\lambda-(1-\beta_0)\gamma\lambda]} - 1\right]. \tag{8.4.18}$$

which shows that q is independent of cosmic time t. Therefore, q may be positive or negative for a given set of values of β_0 and λ. We know that the Universe accelerates for $q < 0$, therefore, the value of A must be $0 < K < 2/3\gamma$ for an accelerated Universe. As expected, the above solutions reduce to the standard FRW model of GR for $\beta_0 = 0$ and $\lambda = 0$ and for all values of γ.

In what follows, we study the role of $f(R,T)$ gravity and particle creation in the early and late-time evolution of the Universe.

Case (i) $\gamma = \frac{2}{3}$:

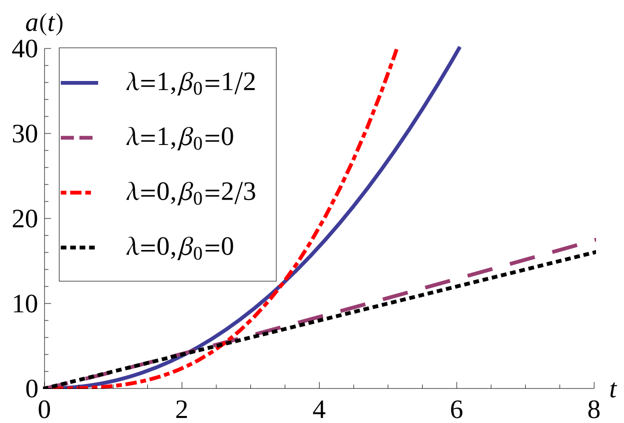

Figure 8.1: Scale factor as a function of time for $\gamma = \frac{2}{3}$ and some selected values of λ and β_0

Fig. 8.1 plots the scale factor *versus* time for $\gamma = 2/3$ and some selected values of λ and β_0. We observe that if $\beta_0 = 0$, $q < 0$ for all $\lambda > 0$, therefore, we find that the Universe accelerates in $f(R,T)$ gravity without particle creation. Similarly, if $\lambda = 0$, *i.e.*, in the absence of $f(T)$, $q = -\beta_0 < 0$ for any values of $\beta_0 > 0$. Thus, the acceleration occurs due to particle creation. The rate of expansion increases more rapidly for non-zero values of β_0 and λ. It is to be noted that if $\lambda = 0 = \beta_0$ then the marginal inflationary phase of GR is recovered, *i.e.*, $a \sim t$ and $q = 0$.

Case (ii) $\gamma = \frac{4}{3}$:

In this case, if $\beta_0 = 0$ and $\lambda > 0$, we have $q > 0$. This shows that the Universe decelerates in the absence of particle creation. If $\lambda = 0$ then $q \geq 0$ for $0 < \beta_0 \leq 1/2$, and $q < 0$ for $1/2 < \beta_0 < 1$. Therefore, in the absence of $f(R,T)$ gravity, the Universe decelerates or accelerates due to particle creation depending on the rate of creation. However, if $\lambda \neq 0$ and $\beta_0 \neq 0$, the deceleration or acceleration of the Universe depend on the following constrains, respectively:

$$0 < \beta_0 \leq \frac{1}{4},\ \lambda > 0\ \ or\ \ \frac{1}{4} < \beta_0 < \frac{1}{2},\ 0 < \lambda < \frac{6\pi - 12\pi\beta_0}{-1+4\beta_0}, \tag{8.4.19}$$

$$\frac{1}{4} < \beta_0 \leq \frac{1}{2},\ \lambda > \frac{6\pi - 12\pi\beta_0}{-1+4\beta_0}\ \ or\ \ \frac{1}{2} < \beta_0 < 1,\ \lambda > 0. \tag{8.4.20}$$

The behavior of scale factor *versus* time is shown in fig. 8.2 for some selected

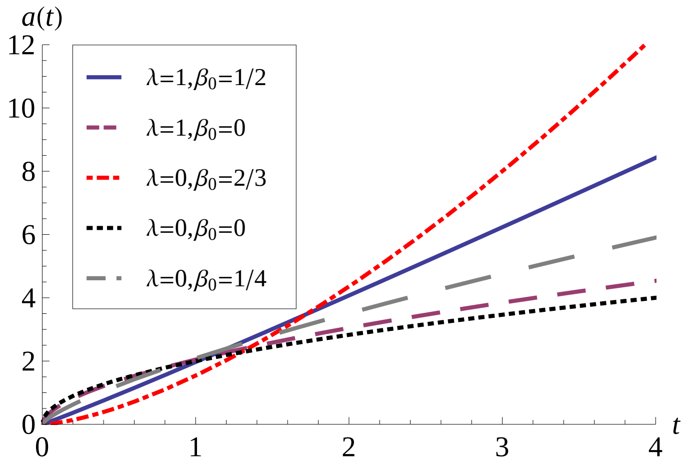

Figure 8.2: Scale factor as a function of time for $\gamma = \frac{4}{3}$ and some selected values of λ and β_0

values of λ and β_0. The figure shows that the Universe accelerates faster due to higher particle creation rate. For $\lambda = 0 = \beta_0$, we have $a \sim t^{1/2}$ and $q = 1$, which is the radiation-dominated phase of GR.

Case (iii) $\gamma = 1$:

In this case, the Universe expands with decelerated rate as $q > 0$ for $\beta_0 = 0$ and $\lambda > 0$. Fig. 8.3 plots graph between scale factor and time for some selected values

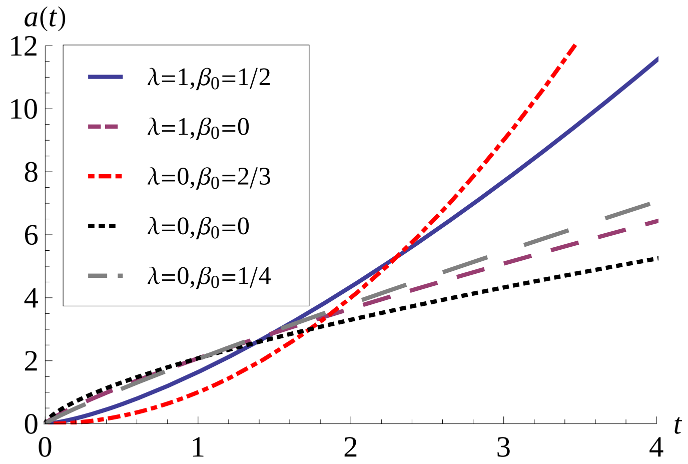

Figure 8.3: Scale factor as a function of time for $\gamma = 1$ and some selected values of λ and β_0

of λ and β_0. For $\lambda = 0$, we have $q > 0$ for $0 < \beta_0 < 1/3$, and $q < 0$ for $1/3 < \beta_0 < 1$. The critical case ($\beta_0 = 1/3$, $q = 0$), describes the coasting cosmology. For $\lambda \neq 0$ and $\beta_0 \neq 0$, the model decelerates or accelerates under the following constraints:

$$0 < \beta_0 < \frac{1}{3},\ 0 < \lambda < \frac{\pi - 3\pi\beta_0}{\beta_0}, \tag{8.4.21}$$

$$0 < \beta_0 \leq \frac{1}{3},\ \lambda > \frac{\pi - 3\pi\beta_0}{\beta_0} \ \ or\ \ \frac{1}{3} < \beta_0 < 1,\ \lambda > 0, \tag{8.4.22}$$

respectively. For $\lambda = 0 = \beta_0$, we have $a \sim t^{2/3}$ and $q = 1/2$, as expected, *i.e.*, the model reduces to standard the matter-dominated era of GR.

Case (iv) $\gamma = \frac{1}{2}$:

In this case, if $\lambda = 0 = \beta_0$, $a \sim t^{4/3}$ and $q = -1/4$, which corresponds to the present accelerated phase of the Universe of the standard FRW Universe in GR. Since the Universe accelerates even in absence of both $f(T)$ and particle creation, therefore, the contribution of $f(R,T)$ gravity or particle creation just enhance the rate of acceleration of the Universe. Fig. 8.4 plots the dynamics of scale factor *versus* t, which is similar to case (i).

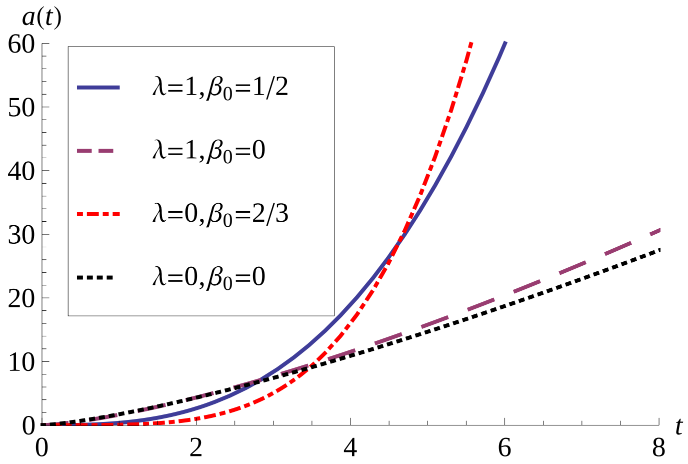

Figure 8.4: Scale factor as a function of time for $\gamma = \frac{1}{2}$ and some selected values of λ and β_0

8.5 Kinematic tests

Now, we derive some kinematic relations for the model.

8.5.1 The density parameter

The density parameter (1.7.7) gives

$$\Omega_m = \frac{8\pi}{8\pi + 4\lambda - (1-\beta_0)\gamma\lambda}. \tag{8.5.1}$$

Therefore, it is clear that $\Omega_m < 1$ for all values of $0 \le \gamma \le 2$, $0 < \beta_0 < 1$ and $\lambda > 0$. Hence the Universe is negatively curved. In the absence of both λ and β_0, we have $\Omega_m = 1$ for all γ, *i.e.*, the flat model of GR is recovered.

8.5.2 Lookback time-redshift

The lookback time $\Delta t = t_0 - t(z)$, is the difference between the age of the Universe at the present time $z = 0$ and the age of the Universe when a particular light ray at redshift z was emitted.

For a given redshift z, the scale factor $a(z)$ is related to a_0 by Eq. (7.2.35). From (8.4.12) and (7.2.35), the cosmic time in terms of redshift is given by

$$t(z) = \frac{2H_0^{-1}}{3\gamma K_3}(1+z)^{-\frac{3\gamma K_3}{2}}. \tag{8.5.2}$$

Consequently, we have

$$t_0 - t(z) = \frac{2H_0^{-1}}{3\gamma K_3}\left[1-(1+z)^{-\frac{3\gamma K_3}{2}}\right]. \tag{8.5.3}$$

Fig. 8.5 plots lookback time *versus* redshift for $\gamma = 1$ and some selected values

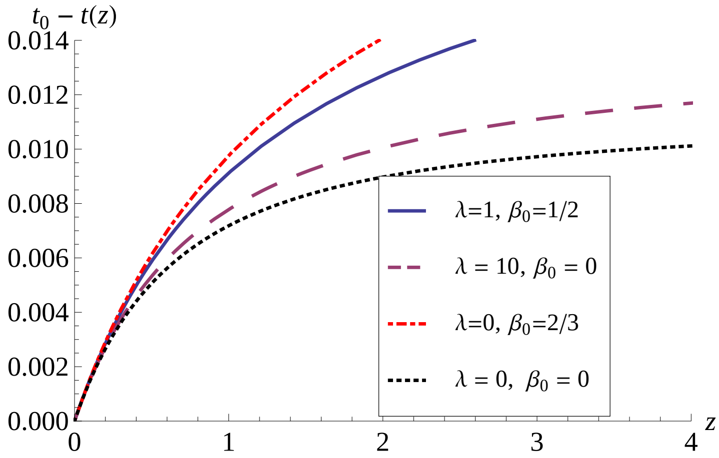

Figure 8.5: Lookback time *versus* redshift for $\gamma = 1$, $H_0 = 60$ and some selected values of λ and β_0

of λ and β_0. All models coincide for lower redshift since they follow the same behavior. The graph shows that the lookback time increases for higher values of β_0. Thus, the Universe with larger matter creation rate is older.

For small values of redshift, (8.5.3) becomes

$$H_0\,(t_0 - t(z)) = z - \left(1 + \frac{3\gamma K_3}{2}\right) z^2 + \cdots. \tag{8.5.4}$$

Taking $\lim z \to \infty$ in (8.5.3), the present age of the Universe is

$$t_0 = \frac{2H_0^{-1}}{3\gamma K_3} = \frac{H_0^{-1}}{1+q}. \tag{8.5.5}$$

Thus, the age of the Universe depends on both parameters β_0 and λ.

8.5.3 Proper distance-redshift

The proper distance between the source and observer is defined as $d(z) = a_0 r(z)$, where $r(z)$ is the radial distance of the object at the time of light emission, given as

$$r(z)=\int_t^{t_0}\frac{dt}{a(t)}=\frac{H_0^{-1}}{a_0\left(\frac{3\gamma K_3}{2}-1\right)}\left[1-(1+z)^{-\left(\frac{3\gamma K_3}{2}-1\right)}\right]. \tag{8.5.6}$$

Consequently, the proper distance becomes

$$d(z)=\frac{H_0^{-1}}{\left(\frac{3\gamma K_3}{2}-1\right)}\left[1-(1+z)^{-\left(\frac{3\gamma K_3}{2}-1\right)}\right]. \tag{8.5.7}$$

The proper distance as a function of redshift for some selected values of β_0 and λ is displayed in fig. 8.6. We observe that the $f(T)$ contribution in $f(R,T)$ and

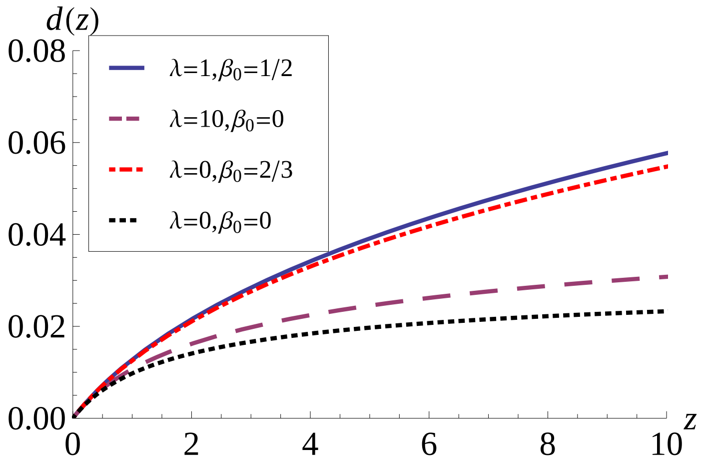

Figure 8.6: Proper distance *versus* redshift for $\gamma=1$, $H_0=60$ and some selected values of λ and β_0

particle creation give rise to proper distance.

Equation (8.5.7) can be rewritten as

$$H_0 d(z)=z-\frac{3\gamma K_3}{4}z^2+\cdots. \tag{8.5.8}$$

From (8.5.7), it is observed that the distance dz is maximum at $z\rightarrow\infty$. Hence,

$$H_0 d(z\rightarrow\infty)=\frac{1}{\frac{3\gamma K_3}{2}-1}=\frac{1}{q}. \tag{8.5.9}$$

8.5.4 Luminosity distance-redshift

The best-known way to trace the evolution of the Universe observationally is to look into the redshift-luminosity distance relation. The luminosity distance d_l is defined by the relation $d_l^2=\frac{l}{4\pi L}$, where l is the luminosity of the object and L is the measured flux from the object. In standard FRW cosmology it is defined in terms of redshift as

$$d_l = a_0(1+z)r(z) = (1+z)d(z). \tag{8.5.10}$$

From (8.5.7) and (8.5.10), we get

$$d_l H_0 = \frac{1}{\left(\frac{3\gamma K_3}{2} - 1\right)} \left[(1+z) - (1+z)^{-\left(\frac{3\gamma K_3}{2} - 2\right)}\right]. \tag{8.5.11}$$

The graph between Luminosity distance and redshift for some selected values of β_0 and λ is plotted in fig. 8.7. One may observe that the luminosity distance

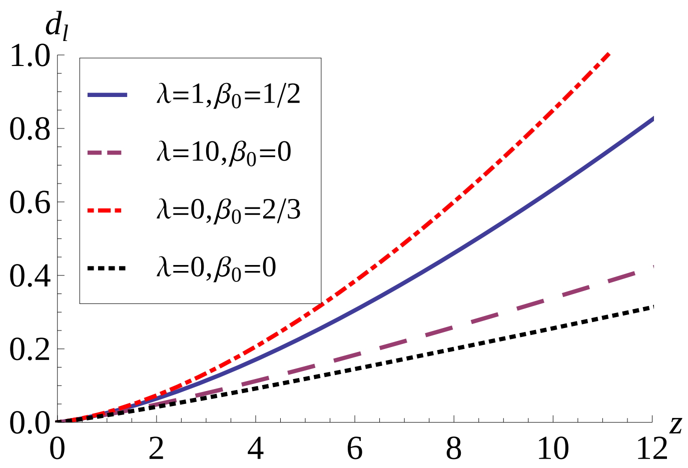

Figure 8.7: Luminosity distance *versus* redshift for $\gamma = 1$, $H_0 = 60$ and some selected values of λ and β_0

corresponding to any specific value of redshift rises due to $f(R,T)$ gravity and particle creation.

Expanding (8.5.11) for small z, we find

$$H_0 d_l = z - \frac{1}{2}\left(\frac{3\gamma K_3}{2} - 2\right) z^2 + \cdots. \tag{8.5.12}$$

As expected, we find the same behavior for different models at $z \ll 1$ and the possible difference in behaviors for different models come at large redshift $(z \gg 1)$. In fig. 8.7 we observe that all curves start with the linear Hubble's law $(z = d_l H_0)$ for small z, but only the curve for $q = 1$, *i.e.*, $\beta_0 = 0 = \lambda$ stays linear all the way. We also note that for the small redshift the luminosity distance is larger for small values of q.

8.5.5 Angular diameter distance-redshift

The angular diameter distance d_A is the ratio of physical transverse size of an object to its angular size (in radians). In terms of z, it is given by

$$d_A = \frac{d(z)}{1+z} = \frac{d_l}{(1+z)^2}. \tag{8.5.13}$$

Using (8.5.7), we have

$$H_0 d_A = \frac{1}{\left(\frac{3\gamma K_3}{2} - 1\right)} \left[(1+z)^{-1} - (1+z)^{-\frac{3\gamma K_3}{2}}\right]. \tag{8.5.14}$$

In fig. 8.8 we plot the angular diameter distance *versus* redshift for some selected

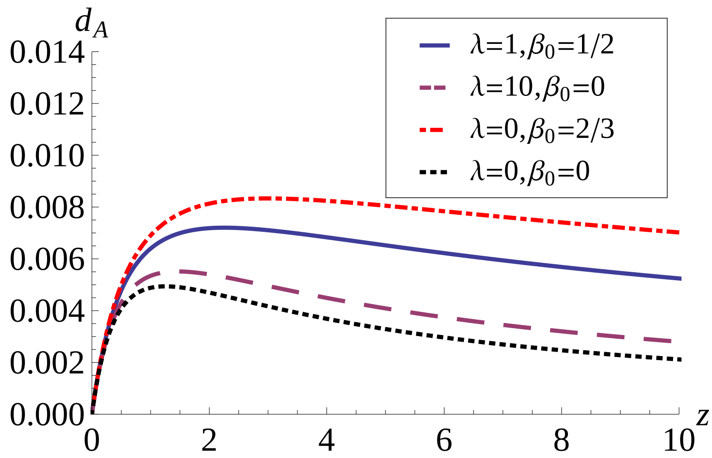

Figure 8.8: Angular diameter distance *versus* redshift for $\gamma = 1$, $H_0 = 60$ and some selected values of λ and β_0

values of β_0 and λ. The graph shows that the $f(R,T)$ gravity and particle creation enhance the angular distance. The angular diameter distance initially increases with increasing z and eventually begins to decrease.

On expanding (8.5.14), we get

$$H_0 d_A = z + \left[1 - \frac{\left(\frac{3\gamma K_3}{2}+1\right)\left(\frac{3\gamma K_3}{2}+2\right)}{2\left(\frac{3\gamma K_3}{2}-1\right)}\right] z^2 + \cdots. \tag{8.5.15}$$

Thus, the angular diameter shows linear behavior up to first approximation whereas it has quadratic up to second approximation.

8.6 Conclusion

In this chapter, we have studied a flat FRW cosmological model described by an open thermodynamic system including particle creation at the expense of gravitational field in $f(R,T)$ theory of gravity. We have obtained exact solutions for the scale factor and various physical quantities by assuming a suitable form of $f(R,T) = R + 2f(T)$ and 'gamma-law' equation of state. The model exhibits non-singular power-law expansion of the Universe for $0 \leq \gamma \leq 2$. The model exhibits Big-Rip singularity at some finite time for $\gamma < 0$ (phantom matter). The dynamics of the scale factor and other physical quantities have been examined through some graphical representations in various phases of evolution of the Universe.

It has been observed that the scale factor evolves with decelerated and accelerated rate depending upon the contribution of particle creation and the coupling parameter λ of $f(R,T)$ theory. The Universe accelerates in early inflationary and late-time accelerated phase even in the absence of $f(R,T)$ gravity and without creation of particles. The presence of $f(R,T)$ contribution and the creation of particles just enhance the expansion rate in these two phases. The Universe decelerates in radiation- and matter-dominated phases even in $f(R,T)$ gravity without of particle creation. However, in GR (absence of $f(R,T)$ gravity) the Universe may accelerates in these two phases for higher creation rate of particles.

The energy density and effective pressure always decrease with time and both tend to zero in late-time for $0 \leq \gamma \leq 2$. The number of particles increase with time in all the phases. The number of particles in the absence of particle production remain constant throughout the evolution of the Universe, which is quit obvious. The deceleration parameter has been found having a constant value, which also describes both decelerated and accelerated Universe under some constraints on other parameters. The density parameter shows that the model becomes open in the presence of particle creation.

We have also discussed some observational consequences of the model through some kinematic tests such as lookback time, proper distance, luminosity distance and angular diameter distance with respect to redshift. The results for the cosmological tests have been found to be compatible with the recent observations. These tests have been found to be affected by λ and β_0. The Universe with particle creation is always older than the standard cosmological model. The model of

Lima et al. [16] may be recovered for $\lambda = 0$.

In summary, we have studied a cosmological model with particle creation in $f(R,T)$ gravity theory to understand early behavior of the Universe and the present accelerating phase. We have found that the negative pressure due to the matter creation may play the role of DE to derive the accelerated expansion of the Universe in $f(R,T)$ theory. We may expect that the process of particle creation is also an ingredient which accounts the sudden change in the evolution of the Universe from deceleration to acceleration. The changes introduced by the particle creation process, provide reasonable observational results. The new fact justifying the present work is that we have considered the thermodynamics approach for which particle creation is at the expense of the gravitational field. One may find that the particle creation changes the predictions of standard cosmology, thereby alleviating the problem of reconciling observations with the inflating scenario.

Chapter 9

Summary and future scope of the work

In this thesis we have analyzed some alternatives to the standard cosmological model to explain the evolution of the Universe specially late-time cosmic acceleration. We have studied cosmological models in $R+\lambda R^2+\mathscr{L}_m$ theory with perfect fluid and exotic matter in FRW and anisotropic space-times. It has been noted that the cosmological evolution could be fairly explain in HD theory. However the late-time cosmic acceleration is caused by the exotic matter. We have observed that HD theory is not responsible for the late-time acceleration. These models have been encountered Big-Bang singularity. Therefore, we have also explored the emergent Universe with some exotic matters in the framework of HD theory. It has been found that the emergent Universe is not possible with quintessence scalar and normal tachyonic fields but it exists with phantom scalar and phantom tachyonic fields in HD gravity. The models have no time-like singularity at infinite past and admits an ever accelerating Universe. We have also extended our work to $f(R)$ theory of gravity in the presence of perfect fluid. A functional form of $f(R)$ has been reconstructed in LRS Bianchi I space-time which shows the decelerated phase of the Universe.

Theoretically, the $R+\lambda R^2+\mathscr{L}_m$ theory and $f(R)+\mathscr{L}_m$ theory have additive structure of geometry and matter in EH action. Therefore, we have studied another modified gravity theory, i.e., $f(R,T)$ theory, which has a non-minimal coupling between matter and geometry. We have reconstructed a functional form of $f(R,T) = R+2f(T)$ with a perfect fluid for de Sitter and power-law models in a flat FRW model. It has been observed that the reconstructed forms of $f(R,T)$ successfully explain the candidates of DE, i.e., quintessence, phantom and cosmological constant. We have also reconstructed this form of $f(R,T) = R + 2f(T)$ gravity with quintessence and phantom scalar field for constant and exponential scalar potential. The constant potential model has been found compatible with the phantom scalar field only whereas exponential potential model has been found compatible with quintessence scalar field. Both the models successfully address the various decelerated and accelerated phases of evolution of the Universe. The exponential potential model leads to the power-law expansion of the Universe. We have also compared our results for this model with some observational constraints. It has been observed that this model is compatible with $H(z)$, SNe Ia, $H(z)$+SNe Ia, WMAP7 and WMAP7+BAO+$H(z)$ observational constraints available on power-law cosmology.

We have also studied a cosmological model with particle creation in $f(R,T)$ theory to understand early behavior of the Universe and its present accelerating expansion. It has been found that the process of particle creation is also an ingredient which accounts the sudden change in the evolution of the Universe from decelerated phase to accelerated phase. Thus, the higher order gravity theories, $f(R)$ theory and $f(R,T)$ theory of gravitation have their own significance to understand the evolution of the Universe.

The predictions of the $f(R,T)$ gravity could lead to some major differences in several problems of current interest in cosmology and astrophysics. The study of these phenomena may also provide some specific signatures and effects, which could distinguish and discriminate between various gravitational models. So far, a serious shortcoming of $f(R,T)$ theory is the non-conservation of the energy-momentum tensor. An interesting question is the possibility of the conservation of the energy-momentum tensor in this theory. This feature has first undertaken by Chakraborty [181]. The author has shown that a part of an arbitrary function of $f(R,T)$ can be determined by taking into account the conservation of stress-energy tensor. Later on, Alvarenga and collaborators [179] have constructed $f(R,T)$ gravity models where they consistently ensured the conservation of the energy-momentum tensor. The authors have investigated the dynamics of scalar perturbation within the obtained model and have shown that their results are very different from the concordance ΛCDM model. The result obtained in that paper is quite reasonable due to the choice of the ordinary matter content and the determination of the integration constant. Thus, there is a need to explore $f(R,T)$ gravity taking into the consideration of conservation equation. Also, there has not been paid the full attention to study the density contrast evolution in $f(R,T)$ theories. One may expect more encouraging work in future on $f(R,T)$ theory to understand the mysterious dark side of our Universe.

Considerable knowledge can be gained from the theoretical point of view, as the study of alternatives helps to understand and clarify the properties of the standard paradigm. Therefore, various more generalised modified theories of gravity are being developed to explain the accelerated expansion of the Universe and other phenomena. However it is very far from clear which class of modified theories will finally prevail. Studying the phenomenological implications of alternative models and comparing them with the observational data is a decent way to work in cosmology. Therefore, the validity and viability of these theories have still to be subjected to many theoretical and experimental tests. Hence, very serious reconsideration as well as more precise and complete observational data are requested in order to have the answer to the fundamental question: what is the gravitation theory which governs the expansion of our Universe ?

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List of Publications

1. C. P. Singh and **Vijay Singh**; *FRW Models with a perfect fluid and a scalar field in higher derivative theory*, Mod. Phys. Lett. A **26**, 1495–1507 (2011).

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