

A Major Project Report
On

**FINITE ELEMENT METHOD BASED COMPARATIVE STUDY
OF SEEPAGE IN EARTHEN DAM**

Submitted in Partial Fulfilment for the Award of the Degree of

MASTER OF TECHNOLOGY

IN

CIVIL ENGINEERING

With Specialisation

in

GEOTECHNICAL ENGINEERING

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July 2014



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CERTIFICATE

This is to certify that the project report entitled “**Finite Element Method Based Comparative Study of Seepage in Earthen Dam**” is a bonafide record of work carried out by Mr. Waqar Ahmad Khan (2K12/GTE/19), under my guidance and supervision, during the session 2014 in partial fulfillment of the requirement for the degree of Master of Technology (Geotechnical Engineering) from Delhi Technological University, Delhi.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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DECLARATION

I, hereby declare that the work embodied in the dissertation entitled “**Finite Element Method Based Comparative Study of Seepage in Earthen Dam**” in partial fulfilment for the award of degree of Master of Technology in “Geotechnical Engineering”, is an original piece of work carried out by me under the supervision of Prof. K.C. Tiwari, Department of Civil Engineering, Delhi Technological University. The matter of this work either full or in part have not been submitted to any other institution or University for the award of any other Diploma or Degree or any other purpose what so ever.

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Abstract

Dams are the structures that are used for the storage of water. Its failure may greatly affect the surrounding locality. Therefore, it is necessary to investigate the causes of failure and use preservative measures by proper designing. Among various causes, one of the major cause of failure is seepage. Seepage is an important issue that needs to be considered before designing of an Embankment Dam. Seepage through the body of the Embankment Dam adversely affects the stability of the Dam

In this thesis, Analysis of seepage is carried out using Finite Element Method. Results obtained are then compared with Analytical Method. In this study, parametric study was carried out with height and slope of the embankment being the variable. Firstly, Analysis was carried out using Finite Element Method, then seepage discharge was calculated numerically using Analytical Method and the results obtained was compared. The result obtained using Finite Element method is about 12% greater than the result obtained using Analytical Method. It's likely that the result obtained using Finite Element Method is more realistic because it has greater accuracy due to large discretization of structure and difference in result is due to difference in method.

CHAPTER 1

INTRODUCTION

1.1 Introduction

The objective of this thesis is to analyse seepage through earthen embankment dam using Finite Element Method (FEM) and then validating the result with theoretical method like Analytical Method. FEM is a tool which implements hefty calculations to solve a large number of engineering problems.

In Finite Element Method (FEM), the whole structure is subdivided into small elements called Meshing. The behaviour of each element is analysed and calculated individually. All the elements are then added to give the overall behaviour of the structure.

Objectives of this thesis is to understand elementary principles of two dimensional flows through soil media. This understanding has lot of use for the problems involving seepage flow through soil medium which are often faced in the design of engineering structures.

The problem of two dimensional flow can be of two types,

1. Confined flow
2. Unconfined flow

These problems in geotechnical engineering are required to be studied to meet the following requirements:

1. To calculate amount of seepage flow
2. To find distribution of seepage pressure and uplift pressure
3. To verify piping tendency leading to instability

1.2 Objective

Seepage flow through soil media is a complex phenomenon and its analysis involves understanding of basic principles of two dimensional flows through soil

media because seepage flow through soil media and around impermeable boundaries have a bearing on the design of engineering structures.

The objective of this thesis is to carry out a Finite Element Method (FEM) based comparative study of seepage through embankment dam and then validating the results with certain other analytical methods.

1.3 Purpose of the study

The purpose of this study is to draw attention to critical failure modes associated with seepage and the important aspects of seepage control measures in the design of a new embankment dam or in the evaluation and/or modification of an existing embankment dam, and to present recommended practices for analysis of seepage issues and design of seepage control features.

Seepage analysis is a means to:

- (a) Estimate basic seepage-related issues that may influence a dam;
- (b) Forecast seepage in an existing or new embankment and its foundation;
- (c) Evaluate the usefulness of various seepage control features;
- (d) Provide numerical estimations, as well as general understandings, for design of the seepage control features.

1.4 Applicability

The regulation and methods are applicable to the analysis of seepage issues associated to embankment dams and their foundations. Regulation and discussion are provided on the types of failure modes that may occur due to the effect of seepage, data needed for the evaluation of seepage problems, procedures for analysing seepage problems, various seepage removal methods, and certain considerations for seepage monitoring.

1.5 Data Requirement

Basically five important parameters are required by the FEM based software for the analysis of seepage flow through embankment dam. These five parameters define the material properties and therefore explains how porous the material is, its grain sizes are distribution, the hydraulic conductivity of the saturated material and the amount of water left in the material after the free water has been drained away.

These five parameters are-

- 1) Porosity
- 2) Coefficient of Permeability (K)
- 3) Diameter at 10% passing (D_{10})
- 4) Diameter at 60% passing (D_{60})
- 5) Residual Water Content

CHAPTER 2
LITERATURE
REVIEW

Gao (2005) studied problem related to seepage and stability for Hwang Bejang Earthen Dam. He said that filters which are designed with empirical method are not entirely trustable. **Aljeyri (2009)** studied two dimensional behavior of Earthen Dam using Ansys. In his study, it was assumed that, none impervious layer behind layer are exist and downstream seepage is influenced by each change of two impervious layers which are concluded dams. **Kratutich (2004)** numerically studied for no stationary of free surface at Earthen Dams. One the main reason for the unsuccessful behavior of Dam is seepage force and water percolation during flooding condition. He concluded from his work that seepage and thermal distribution are similar type and hence hydraulic Analysis are to be done with thermal method using Ansys software. **Dr. Karjani (2009)** studied behavior of maroon dam with the help of Geo-Studio software. He evaluated flow net at stable condition; slope stability factor at steady seepage and after rapid drawdown and minimum coefficient at critical situation at dynamic and static condition has been calculated. Finally, he investigated the dynamic behaviour of Maroon Dam and two different reactions against possible quacks.

2.1 Seepage ^[12]

Seepage is the flow of water under the gravitational forces in a permeable medium. Flow of water occurs from a point of high head to a point of low head. The flow is usually laminar.

The route taken by water particle is signified by a flow line. Infinite number of flow lines can be drawn but for convenience only few are drawn. At any certain point on the flow line, the total head will be same. The line joining the points of equal total

head is called as Equipotential line. As flow always takes place along the steepest hydraulic gradient, the Equipotential line crosses the flow line at right angle. The equipotential line and flow line are together known as Flow net. The flow net gives clear view about path taken by water particle and head variation along the path.

Seepage analysis is important in detecting internal erosion and designing of drainage structure for all types of structures including dam to control threats such as slides and flooding (Jackson 1997). Excessive seepage through the foundation of the dam causes failure of the structure.

Generally seepage is considered two dimensional for a homogeneous and isotropic soil with respect to permeability (R.F.Craig) while determining seepage in homogeneous and isotropic soil.

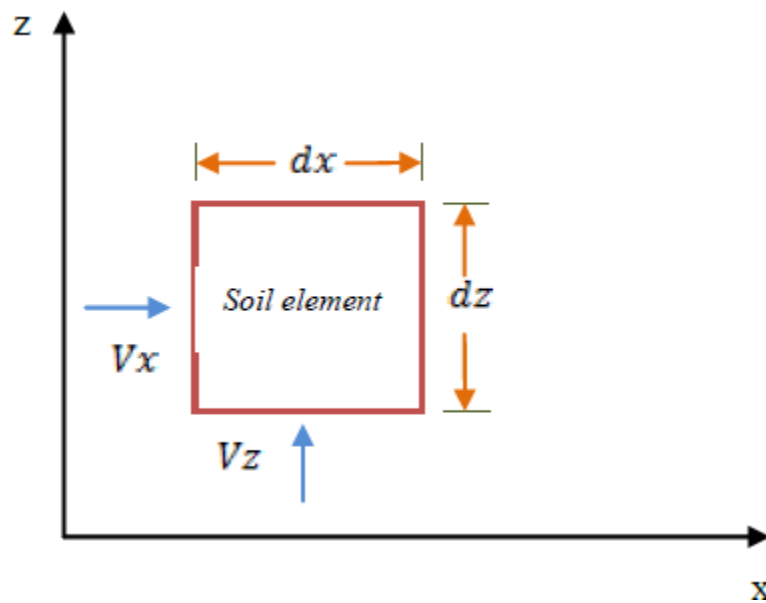


Figure 2.1: Two Dimensional Flow^[20]

Let us consider an element of soil of size dx by dz through which flow is taking place. Let the velocity at inlet and outlet faces be v_x and $(v_x + \frac{\partial v_x}{\partial x} * dx)$ in x -direction and v_z and $(v_z + \frac{\partial v_z}{\partial z} * dz)$ in z -direction.

The total volume of water which is entering the soil element per unit time will be given by

$$V_x dz + v_z dx$$

The total volume of water which is leaving the soil element per unit time will be given by

$$(v_x + \frac{\partial v_x}{\partial x} * dx) dz + (v_z + \frac{\partial v_z}{\partial z} * dz) dx$$

As the flow is steady and the soil is incompressible, the discharge entering the element is equal to the discharge leaving the element.

$$V_x dz + v_z dx = (v_x + \frac{\partial v_x}{\partial x} * dx) dz + (v_z + \frac{\partial v_z}{\partial z} * dz) dx$$

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) = 0 \quad \text{----- (1)}$$

Let 'h' be the total head at any point. The horizontal and vertical components of hydraulic gradients are

$$i_x = - \frac{\partial h}{\partial x}$$

$$i_z = - \frac{\partial h}{\partial z}$$

Using Darcy's Law

$$V = ki$$

$$V_x = -k_x \frac{\partial h}{\partial x}$$

$$V_z = -k_z \frac{\partial h}{\partial z}$$

Substituting this in equation (1) we get,

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial z^2} = 0$$

For Isotropic soil $k_x = k_z$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \text{----- (2)}$$

This is the Laplace Equation in terms of head. Similarly it can be represented in terms of velocity potential ($\Phi = -kh$) as,

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{----- (3)}$$

This is Laplace Equation in terms of Velocity Potential. Laplace Equation can be solved if the boundary conditions at the inlet and exit are known. The equation represents two families of curves which are orthogonal to each other. One family represents the flow lines along which the flow takes place. The other family represents the equipotential lines along which the potential or total head is constant. The graphical representation of Laplace equation is called Flow net.

The flow net can be obtained by any of the following methods-

- (1) Graphical Method
- (2) Electrical Analogy Method
- (3) Soil Model
- (4) Plastic Model
- (5) Solution of Laplace Equation

2.2 Flow Net in Earth Dam with a Horizontal Filter ^[12]

Seepage through an earth dam is a case of unconfined seepage in which the upper boundary of flow net is not known. In such case it becomes necessary to first locate the upper boundary before a flow net can be drawn.

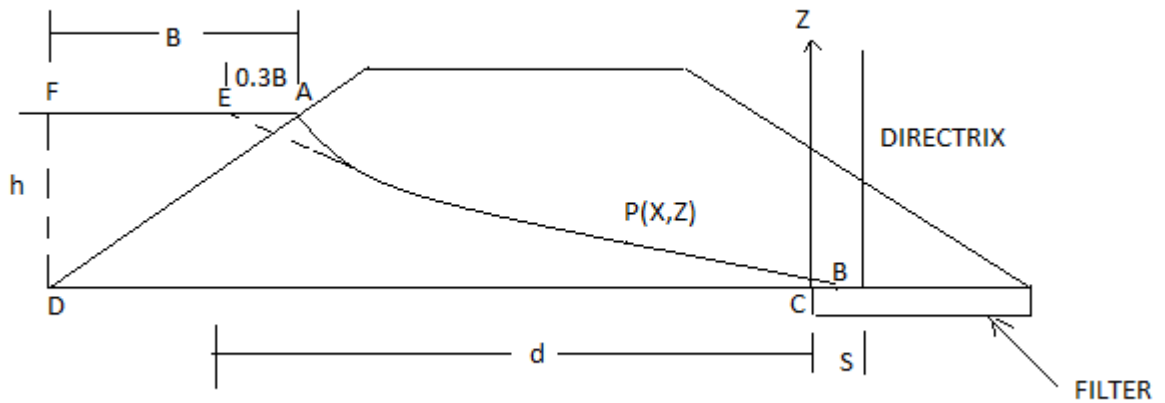


Figure 2.2: Earth dam with a horizontal filter

Consider a homogeneous earthen dam with horizontal filter at downstream toe lying on impervious foundation. The horizontal filter starts at point C. The impermeable boundary CD is a flow line which forms the lower boundary of the flow net. The upstream face AD is an equipotential line because the total head at all the points on this surface is equal to h . The discharge face CB is the equipotential line of zero potential. Thus three hydraulic boundary conditions are known. The fourth boundary of the flow net is the top flow line AB, which is not known in the beginning. Below the line AB, the soil is saturated and the pressure everywhere on the line AB is atmospheric. The line AB is known as phreatic line or seepage line. Once the phreatic line has been located, the flow net can be drawn by the usual method.

Kozeny studied the problem using the method of conformal transformation.

The boundary condition for the flow region ABCD are as under-

Equipotential line AD $\Phi = -kh$

Equipotential line BC $\Phi = 0$

Flow line DC $\Psi = 0$

Flow line AB $\Psi = q$

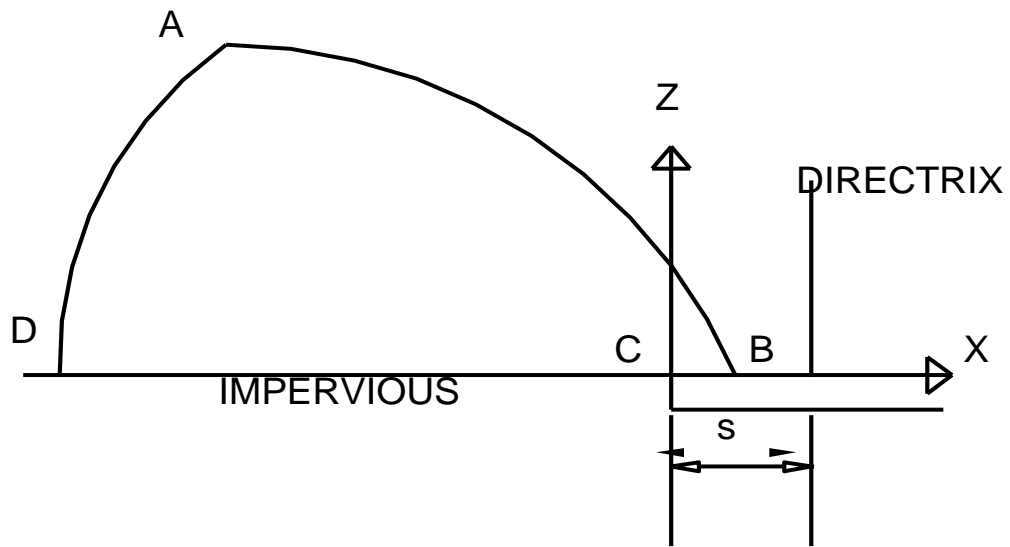


Figure 2.3: Kozeny's Solution

Kozeny's solution represents a family of confocal parabolas of flow lines and equipotential lines. The equation of Kozeny's basic parabola AB with C as focus as well as origin, is

$$x = \frac{1}{2} \left(\frac{q}{k} - \frac{k}{q} z^2 \right)$$

Kozeny's conditions are not entirely fulfilled by any practical earth dam. However, an earth dam with a horizontal drainage approximates the conditions at exit. An inconsistency occurs due to the fact that the upstream equipotential line in an actual earth dam is a plane surface and not a parabola as assumed by Kozeny. Casagrande (1940) recommended that the seepage line in actual dams can also be taken as basic parabola, provided the starting point for parabola is taken at point E, such that $AE = 0.3 AF$. The distance AF is the projection of the upstream slope on the

water surface. The coordinates of the phreatic line can be determined using above equation. The origin is at C, which is also the focus.

Substituting $z = 0$ in above equation, the value of 'x' is given by,

$$x_o = \frac{1}{2} \left(\frac{q}{k} \right) = \frac{q}{2k}$$

$$q = 2kx_o$$

The distance $2x_o$ between the focus and the directrix is known as focal distance (s).

Thus

$$q = ks$$

Substituting the value of q

$$x = \frac{1}{2} \left(\frac{ks}{k} - \frac{k}{ks} z^2 \right) = \frac{s}{2} - \frac{z^2}{2s}$$

$$s^2 - 2xs - z^2 = 0$$

If x is taken positive towards left of F, the above equation becomes

$$s^2 + 2xs - z^2 = 0$$

The value of 's' can be determined using the coordinates of the starting point E.

Substituting $x = d$ and $z = h$ in above equation

$$s^2 + 2ds - h^2 = 0$$

$$s = \sqrt{d^2 + h^2} - d$$

An entrance correction is required for the phreatic line obtained by the above procedure. The flow line must start at point A should be perpendicular to the upstream

face which is the equipotential line. Once the phreatic line has been drawn, the flow net can be completed using various methods.

To determine the discharge through the body of the earth dam, let us consider the flow passing through the section PQ. From Darcy's law, discharge per unit length is given by

$$q = k i A$$

$$q = k \frac{dz}{dx} (z)$$

$$z = (2xs + s^2)^{1/2}$$

$$\frac{dz}{dx} = s / (2xs + s^2)^{1/2}$$

$$q = k s$$

This is the simple equation which gives approximate discharge through the body of the dam.

2.3 Seepage Through Earth Dam With Discharge Angle Less Than 30° [12]

If the angle β is less than 30°, point S at which seepage line becomes tangential to the downstream face can be obtained using Schaffernack's method. It is assumed that part CS of the seepage line is a straight line. A tangent at point S coincides over the length CS with the seepage line.

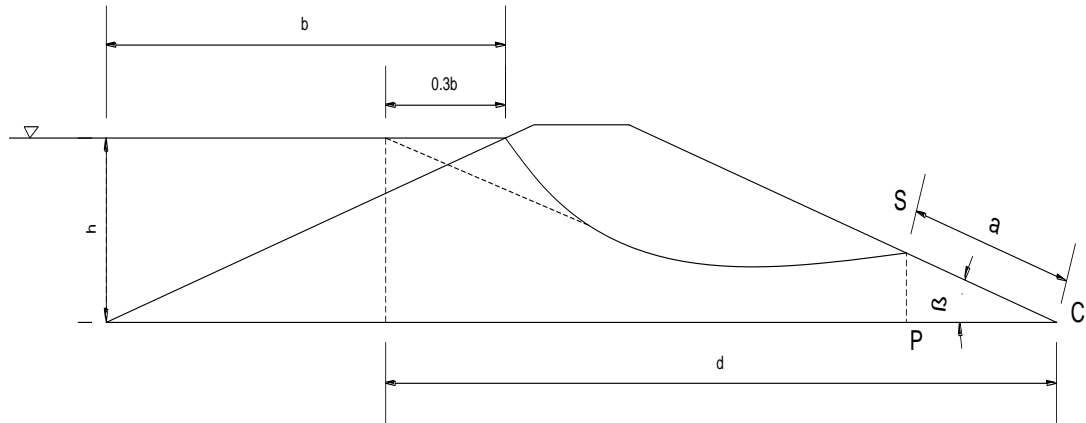


Figure 2.4: Earth Dam with discharge angle less than 30°

$$q = k z \frac{dz}{dx}$$

$$\frac{dz}{dx} = i = \tan \beta$$

$$z = \text{Distance SP} = a \sin \beta$$

where, $a = SC$

$$q = k (a \sin \beta) \tan \beta$$

Combining above two equations

$$k z \frac{dz}{dx} = k (a \sin \beta) \tan \beta$$

$$z dz = a \sin \beta \tan \beta dx$$

Integrating between $x = a \cos \beta$ to $x = d$, and between $z = a \sin \beta$ to h

$$\int_{a \sin \beta}^h z dz = a \sin \beta \tan \beta \int_{a \cos \beta}^d dx$$

$$\frac{1}{2} (h^2 - a^2 \sin^2 \beta) = a \sin \beta \tan \beta (d - a \cos \beta)$$

$$h^2 - a^2 \sin^2 \beta = 2a \sin \beta \tan \beta (d - a \cos \beta)$$

$$a^2 \cos \beta - 2 ad + \frac{h h \cos \beta}{\sin \beta \sin \beta} = 0$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2}$$

Once the value of 'a' has been determined, the discharge can be determined.

2.4 Seepage Through Earth Dam With Discharge Angle Greater Than 30° But Less Than 60° [12]

Casagrande suggested that the actual hydraulic gradient for discharge angle greater than 30° is given by-

$$i = \frac{dz}{ds}$$

where distance 's' is measured along the curve.

Based on this assumption, the discharge expression can be written as-

$$q = k \frac{dz}{ds} z$$

where, z = Distance SP = a sin β

$$\frac{dz}{ds} = \sin \beta$$

Therefore,

$$q = k a \sin^2 \beta$$

Equating both equations we get,

$$k \frac{dz}{ds} z = k a \sin^2 \beta$$

$$z dz = a \sin^2 \beta ds$$

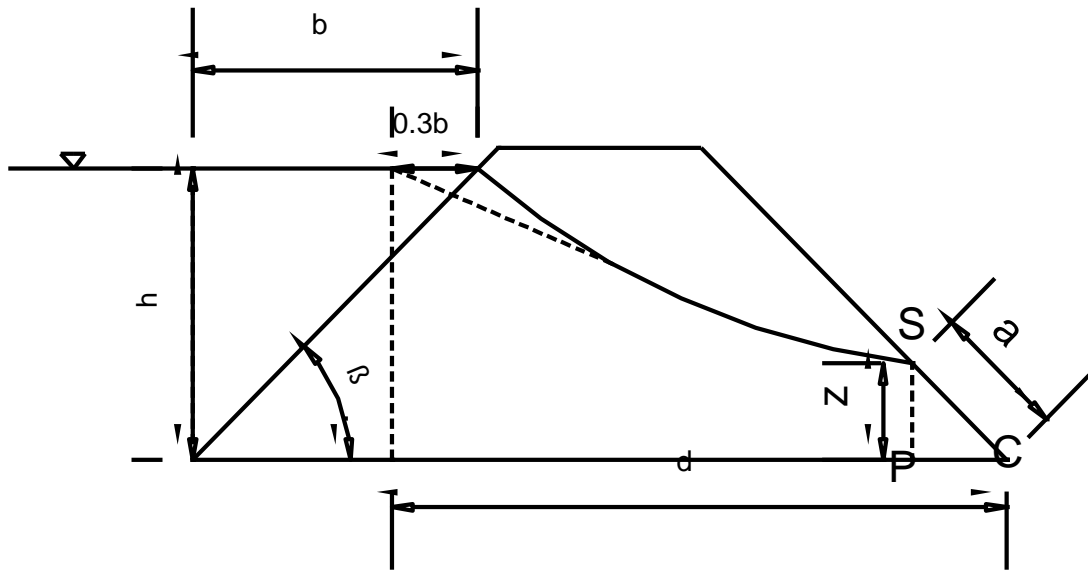


Figure 2.5: Earth Dam with discharge angle more than 30°

Integrating both sides we get,

$$\int_{a \sin \beta}^h z \, dz = a \int_a^S \sin 2 \beta \, ds$$

$$\frac{1}{2} (h^2 - a^2 \sin^2 \beta) = a \sin^2 \beta (S - a)$$

$$a^2 - 2aS + (h^2 / \sin^2 \beta) = 0$$

$$a = S - \sqrt{(S)^2 + \left(\frac{h}{\sin \beta}\right)^2}$$

$$S = \sqrt{d^2 + h^2}$$

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 + (h \cot \beta)^2}$$

Once the value of 'a' has been determined, the discharge can be obtained using the relation-

$$q = k a \sin^2 \beta$$

CHAPTER 3

THEORY

3.1 Theory

This chapter illustrates the theory behind water flow and a short description of the Finite Element Method, followed by the explanation of the methodology used in the software in order to calculate water flow through a porous material.

3.2 Method

In this dissertation, Finite Element Method (FEM) is used for the analysis of seepage discharge in embankment dam. And also validating the result by Analytical method.

3.3 Water Flow ^[20]

For the water to flow from one point to other point, there must exist a pressure difference or potential. Another condition for the flow of water is availability of interconnected pores. The ability of porous material to allow fluids to allow flow through its pores is called the permeability.

The difference in potential over the length is called Hydraulic Gradient. It is the gradient that is the driving force of the water.

$$i = dH/dL$$

dH – hydraulic potential

dL – Length of the sample

The flow rate of water per unit area, v , is defined as the volume of water, Q , over the section area of the sample, A , and time.

$$v = \frac{Q}{A * t}$$

According to Darcy's law the coefficient of permeability can be calculated with the following equation:

$$k = \frac{v}{i}$$

The coefficient of permeability is only valid as long as the flow is laminar, which means that the flow occurs parallel with no disruption between the streamlines. Turbulent flow on the other hand is more complex and develops in higher flow rates. When the hydraulic gradient reaches a certain level the curve bends off, which means that the flow rate stops increasing linearly with the gradient, see Figure 3.1. The reason is that the flow in the pores no longer is laminar, but turbulent, which implies that Darcy's law is no longer valid.

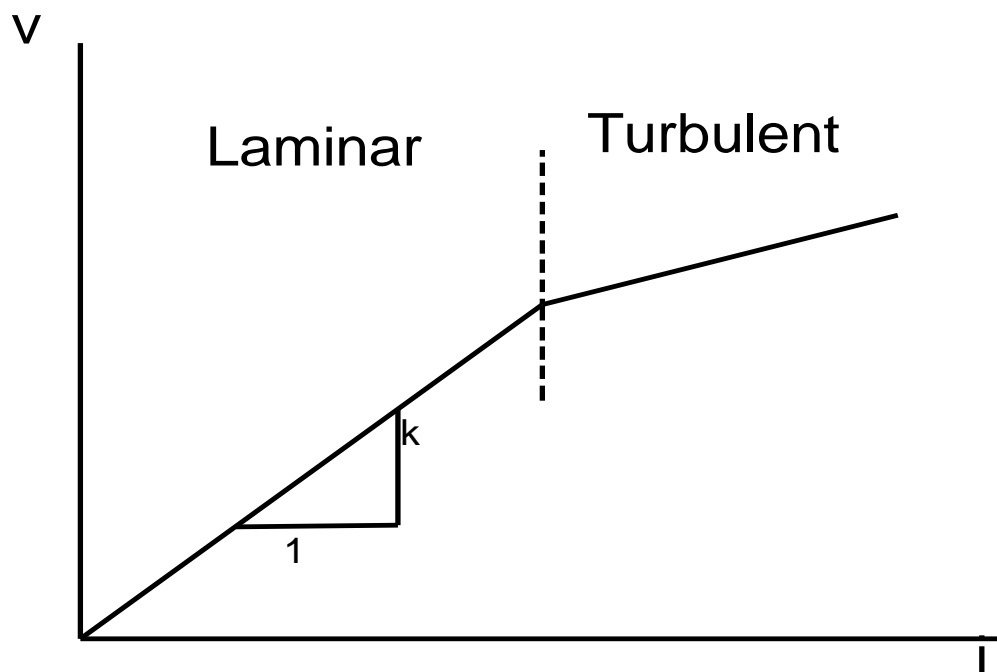


Figure 3.1: The graph indicates where Darcy's Law is valid

Table 3.1: Typical values for permeability of different types of soil are as given [20]-

Type of Soil	K [cm/s]
Gravel	>1
Sand	$1 - 10^{-3}$
Silt	$10^{-3} - 10^{-6}$
Moraine	$10^{-4} - 10^{-7}$
Clay	$10^{-6} - 10^{-9}$

3.4 Methods of analysis ^[20]

Seepage can be basically analyzed using two methods:

3.4.1 Analytical Method ^[20]

3.4.1.1 Khosla Theory ^[20]

This theory is used to determine uplift pressure at the key points in a weir or barrage. In this method a combined weir or barrage unit is split up into a number of simple forms of known analytical solution.

3.4.1.2 Graphical Method ^[20]

Some of the seepage problems can be determined with the help of charts and graphs available. A most widely used graphical method for the assessment the location of the phreatic surface within an embankment was developed by Casagrande. The most widely used graphical method for seepage evaluation is the use of flow nets.

The flow net is a graphical representation of hydraulic potentials and flow direction. Flow nets are used for the assessment of pore pressure, hydraulic gradient, and discharge quantity.

3.4.2 Finite Element Method (FEM) ^[20]

3.4.2.1 SEEP/W ^[20]

SEEP/W is the seepage analysis program currently used by the Geotechnical Engineerings. SEEP/W is a 2-D, finite element software program for examining

ground water and excess pore-water pressure dissipation problems in a porous media. The comprehensive nature of the program enables analyses ranging from simple, saturated, steady state problems to sophisticated, saturated and unsaturated, time dependent problems. Good quality output graphics allow a visual display of equipotential lines and flow paths, and contours can be plotted for a number of properties/results such as pore pressures, seepage velocities, and gradients. As with most seepage analysis programs, computations include flow quantities and uplift pressures at user-selected locations in the model.

3.4.2.2 FLAC ^[20]

Fast Lagrangian Analysis of Continua (FLAC) is a 2-D, explicit finite difference program that can model a number of different engineering applications. Although it is most typically used within Reclamation for analysis of seismic deformations, it can also be used for seepage analyses. As a 2-D program, however, there appears to be little benefit in using it over a simpler program like SEEP/W. FLAC may be useful in modelling pore pressure effects on stability of an embankment. Since Reclamation has had little experience in using FLAC for seepage analyses, analysts should consult the program's user manual for information on potential applicability and whether it is the best tool for the job.

3.4.2.3 FRACMAN ^[20]

FRACMAN is a program that models fracture networks in rock and, thus, permits the simulation of flow through fractured bedrock, as opposed to equivalent porous media models. Reclamation has had relatively limited experience with it. Obviously, a lot of geologic information is required in order to develop a reasonable model of the fractured/jointed bedrock system. The U.S. Geological Survey has more

experience with the program and was contracted to model seepage through the bedrock foundation at Horsetooth Dam.

3.4.2.4 Boundary Integral Equation (BIE) ^[20]

The boundary element method has been used in Reclamation for solving seepage related boundary value problems. It is an effective, efficient, and accurate method compared to other numerical methods discussed in this chapter. In this method, only the boundary of the flow region is discretized; thus a 2-D problem is reduced to a 1-D problem. The computer programs BIE2DCP and BIE2DCS are available in Reclamation for seepage analysis in zoned anisotropic medium.

3.5 Numerical Modelling ^[20]

The Finite Element Method, FEM, is the numerical model used in GeoStudio SEEP/W. The FEM is a powerful tool performing heavy calculations to solve a number of engineering problems. It is commonly used in geotechnical and structural engineering.

The main principle behind the FEM is to subdivide the structure into smaller elements which is called discretization or meshing. The behaviour of each element is described approximately. Each elements will be sub parts of the main structure and the behaviour of each and one for the elements will be calculated and then added together.

Like every modern FEM program, SEEP/W consist of a pre-processor where the user gives the input parameters e.g. geometry, material properties and boundary conditions etc. It is in the pre- processor that the meshing occurs. The pre-processor delivers the necessary information to the part of the program that performs the

calculations and the results are displayed in the post-processor where the user can see the outcome of the problem.

In order to succeed with numerical simulation it is important to choose adequate size of the elements. If too coarse elements are used the solution will not be accurate enough and if too small elements are used huge compute power will be needed. The same issues are valid for the time integration step, long time steps will give inaccurate values and small time steps will generate huge amount of data.

SEEP/W is a FEM software designed for calculation of water flow in both saturated and unsaturated porous material. A mental picture of how the steps are connected in SEEP/W.

shows a simplification of how the solution approach is designed.

(1) First, a geometry is created by the user, where the embankment profile is drawn which gives a graphical representation of the model. The meshing is also performed in this step, the user chooses the shape and the size of the elements.

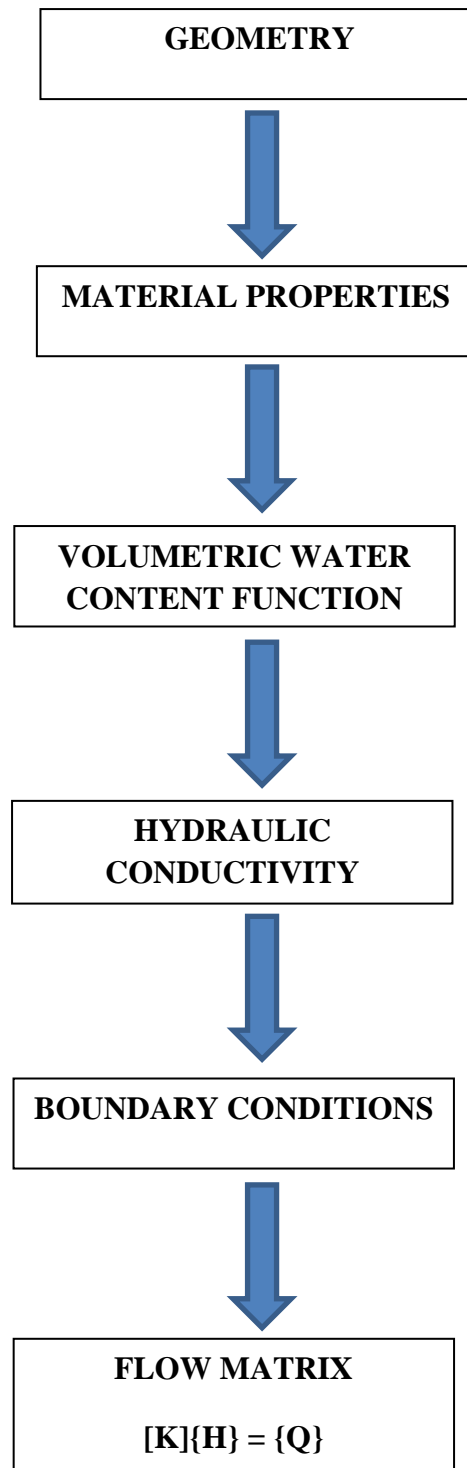
(2) In the second step we define the type and properties of soil mass. Parameters such as porosity and grain size is defined.

(3) In the third step, the program creates volumetric water content function, which shows how much water the material contains.

(4) In the fourth step, hydraulic conductivity function is defined. This function defines the conductivity of the material based on water content.

(5) In the fifth step, we have to define the Boundary condition i.e. zero pressure, total hydraulic head, etc.

(6) Using the data given from point (1) to (4), [K]- matrix is generated.



3.5.1 Volumetric Water Content Function ^[20]

Soil consists of soil solid and voids. In the fully saturated condition, all the voids are occupied by the water and no air void exists and in such case volumetric water content is equal to the porosity of the soil.

$$\Theta_w = nS \quad \text{----- (1)}$$

Where,

S = Degree of saturation

n = Porosity

Θ_w = Volumetric water content

In the case of unsaturated soil, water content within the pores depends upon the difference between air pressure and water pressure ($U_a - U_w$). This difference is called Matric Suction. The volumetric water content function describes how the portion of the pores filled with water changes with matric pressure.

3.5.2 Hydraulic Conductivity Function ^[20]

Transportation of water and air takes place through the pores of the soil. The soil may be partially saturated or fully saturated. In fully saturated soil all the pores are completely filled with water while in partially saturated soil, pores are filled by both air and water. Air filled pores are non-conductive and does not transport water. This means that flow rate will decrease in case of partially saturated soil as compared to fully saturated soil because water takes longer path in the case of partially saturated soil.

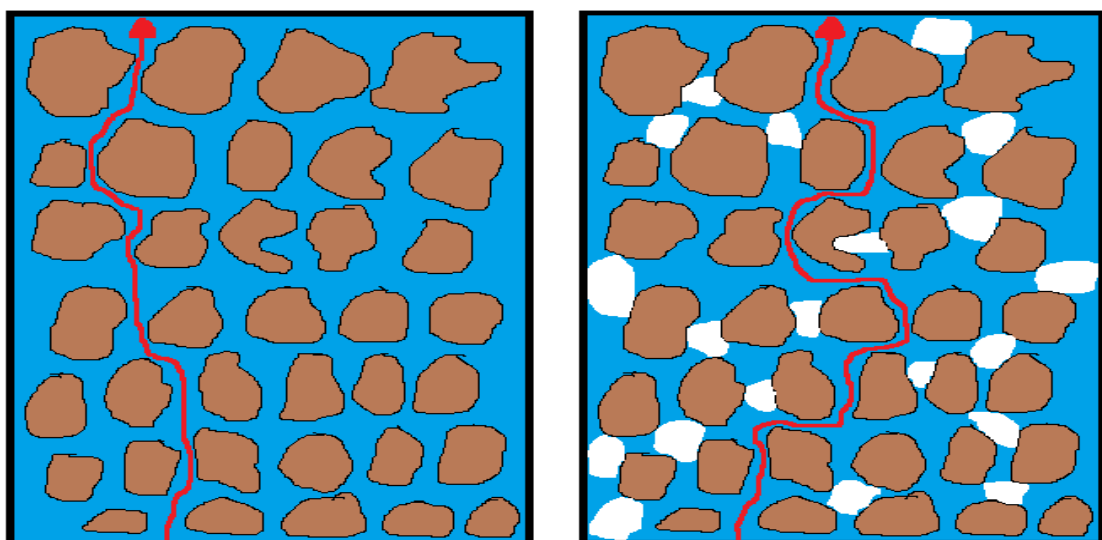


Figure 3.3: Fully saturated soil (left) and partially saturated soil (right) ^[20]

The conductivity decreases with the increase in air within the pores of the soil until the soil reaches residual water content stage. The capability of water to flow through the soil depends upon the amount of water within the soil and volumetric water content function.

In SEEP/W there are three different methods of estimation in order to establish the shape of the hydraulic conductivity function. The method used in the calculations for this analysis is the approach suggested by van Genuchten.

$$k_w = k_s \frac{[1 - (a\Psi^{n-1})(1 + (a\Psi^n)^{-m})]^2}{((1 + a\Psi^n)^{\frac{m}{2}})} \quad \text{----- (2)}$$

Where,

k_w = Conductivity for specified water content

k_s = saturated hydraulic conductivity

a, n, m = Curve fitting parameters

$n = 1/(1-m)$

Ψ = Required suction range

In the above equation, a and m are the two parameters that are required for estimating conductivity. The best point for estimating the parameter is half way between the saturated water content and residual water content. The slope of the function at this point is calculated using following equation.

$$S_p = \frac{1}{(\theta_s - \theta_r)} \left| \frac{d\theta_p}{d(\log \Psi_p)} \right| \quad \text{----- (3)}$$

Where,

θ_s = Saturated volumetric water content

θ_r = Residual volumetric water content

θ_p = Volumetric water content at half way point

Ψ_p = Matric suction at half way point

a and m are estimated using following relations-

$$a = \frac{1}{\psi} (2^{\frac{1}{m}} - 1)^{(1-m)}$$

For $0 < S_p < 1$

$$m = 1 - \exp(-0.8 S_p)$$

For $S_p > 1$

$$m = 1 - \frac{0.5755}{S_p} + \frac{0.1}{S_p^2}$$

3.5.3 Water Flow Matrix ^[20]

Flow matrix consist of partial differential water flow equation

$$\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y}) + Q = \frac{\partial \theta}{\partial t} \quad \text{----- (4)}$$

Where,

Q = Flux (Rate of flow)

H = Total Head

θ = Volumetric water content

t = Time

k_x = Coefficient of permeability in x-direction

k_y = Coefficient of permeability in y-direction

In the above equation, Q is the flow going in and $\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x})$ and $\frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y})$ is the flow leaving in x- and y-direction respectively over a small time interval $\frac{\partial \theta}{\partial t}$.

However, during steady state condition there is no water storage, hence the equation becomes,

$$\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y}) + Q = 0 \quad \text{----- (5)}$$

The program code in SEEP/W is written in a way that the total stress (σ) remains constant, i.e. no loading or unloading takes place for the soil elements. It is also assumed that the pore-air pressure is constant at atmospheric pressure. This means that the difference between the total stress and pore air pressure is constant and causes no change of the volumetric water content. The volumetric water content is only dependent on the difference between pore air pressure (u_a) and pore water pressure (u_w). As mentioned before, the pore-air pressure is kept constant, which results in the change of volumetric water content being dependent on the pore water pressure change, according to:

$$\partial\theta = m_w \partial u_w \quad \text{----- (6)}$$

m_w = Slope of water storage curve

The total hydraulic head is defined as-

$$H = (u_w / \gamma_w) + y \quad \text{----- (7)}$$

Where,

y = Elevation

u_w = Pore water pressure

γ_w = Unit weight of water

Using this in the above equation we get,

$$\partial\theta = m_w \gamma_w (H - y) \quad \text{----- (8)}$$

Therefore,

$$\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y}) + Q = m_w \gamma_w \frac{\partial(H-y)}{\partial t} \quad \text{----- (9)}$$

For a soil element at an elevation, , the elevation remains constant and the dependency disappears resulting in the following equation which is used in the SEEP/W FEM calculations.

$$\frac{\partial}{\partial x} (k_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial H}{\partial y}) + Q = m_w \gamma_w \frac{\partial H}{\partial t} \quad \text{----- (10)}$$

3.6 Dams ^[18]

Dams are barriers constructed across a river to hold and store water. It is constructed to make the optimum use of the available supply of water in a stream. More than 52% of the world's dams are located in China, 16% in the United States, and 6% in Japan.

3.6.1 Function of The Dam ^[18]

Main functions of the Dams are:

- (a) Storage of Water
- (b) Controlling of Flood
- (c) For Irrigation Work
- (d) Water supply
- (e) Power Generation

3.6.2 Types of Dam ^[18]

Dams are categorized according to the materials of construction and structure type. The dams which are categorized on the basis of the structure are gravity dams, buttress dams, arch dams, and embankment dams. Embankment dams are further classified as Embankment Earthfill dams and Embankment Rockfill dams. Dams that are categorized according to the materials of construction are masonry dams, filling dams, both masonry and filling dams, and framed dams. Masonry dams are further classified as stone and brick dams, concrete dams, reinforced concrete dams, and prestressed concrete dams. The most common type of dam is embankment earthfill dams. The following summarize structure types of dams.

3.6.2.1 Gravity Dam

A gravity dam depends on its own weight for stability and is usually straight in plan although sometimes slightly curved. It looks like a retaining wall, set across a river.

3.6.2.2 Arch Dam

Arch dams transmit almost all the horizontal thrust of the water behind them to the abutments by arch action and they have thinner cross-sections than gravity dams.

3.6.2.3 Buttress Dam

Buttress dams are dams in which the face is held up by a series of supports. Buttress dams can take many forms. The face may be flat or curved. A buttress dam is supported by a series of buttress walls, set at right angles to the dam on the downstream side. There are several types of buttress dams, the most important ones are flat-slab and multiple-arch buttress dams.

3.6.2.4 Embankment Dam

Embankment Dams are of two types such as Earthfill Dam and Rockfill Dam. An earthfill dam is made up partly or entirely of pervious material which consists of fine particles usually clay, or a mixture of clay and silt or a mixture of clay, silt and gravel. They are principally constructed from available excavation material. The dam is built up with rather flat slopes. Fine, impervious material of an earthfill dam occupies a relatively small part of the structure, it is known as the core. The core is located either in a central position or in a sloping position upstream of the center. Most new earthfill dams are rock fill type dams, which can be further classified as homogenous, zoned, or diaphragm. Homogenous earthfill dams are composed of only one kind of material, besides the slope protection material. The material used must be

impervious enough to provide an appropriate water barrier and the slope must be reasonably flat for stability. It is more common today to build modified homogeneous sections in which pervious materials are placed to control steeper slopes. When pervious material is used in order to drain the material three methods are used. Rockfill toe, horizontal drainage blanket, inclined filter drain with a horizontal drainage blanket.

The main body of a rockfill dam consists of a mass of dumped rock, which is allowed to take its own angle of repose. A rockfill dam consists of rock of all sizes to provide stability and an impervious core membrane. Membranes include an upstream facing of impervious soil, a concrete slab, asphaltic concrete paving, steel plates, and other impervious soil.

3.7 Seepage In Earthfill Embankment ^[18]

An earthfill dam's body prevents the flow of water from dam's back to downstream. However, with the most impermeable materials used in the dam's body, some amount of water seeps into dam's body and goes out from downstream of body slope until it meets an impermeable barrier. So if the water level at the upstream side is rapidly lowered, the water-soaked material may become unstable. This has to be considered in the design of earthfill dams. Earthfill dams are usually designed pervious, and some seepage flow through the dam body must be expected.

Seepage flow which occurs in the earthfill dam's body has a top surface which is called as phreatic surface or zero pressure curve. There is a pore water pressure under the phreatic line. According to the analysis, value of pore water pressure depends on the type tightness degree, humidity, and impermeability of soil, and load on soil etc. Pore water pressure decreases the shearing resistance of earth mass. If the rate of pore water pressure drop resulting from seepage exceeds the resistance of a

soil particle to motion, that particle will tend to move. This results in piping, the removal of the finer particles from the dam's body. Piping usually occurs near the downstream toe of a dam when seepage is excessive (Linsley and Franzini, 1964). According to these reasons for stability of dam the level of seepage flow especially phreatic line must be limited. In this thesis, there are measurements results for determination of seepage flow using piezometers, in this thesis in a later section there are model results which are obtained according to these piezometric measurement results.

In addition, seepage in the dam's body is important due to two reasons. First one is that, phreatic line cuts downstream slab. The higher cutting of the dam slab because of phreatic line is the more dangerous condition for the slab, because the soil under that point will be saturated, when the soil saturation increases, pore water pressure increases too and due to the quantity of saturation, collapse probability increases. That makes the body of dam unstable. Second reason is maximum reservoir position that contains the body's maximum saturation degree is the most critical condition for the downstream slab's stability after the construction. The most critical condition for upstream slab's stability is the sudden drop in the water level in the reservoir. That makes the body of dam unstable.

CHAPTER 4

FEM

4.1 Introduction ^[19]

Process of dividing the problem area into a corresponding number of smaller units is called Discretization. Once the problem domain is discretized, solution for each smaller domain or unit is obtained. Finally, all the unit solutions are added to give solution of entire domain. Hence, the solution is obtained from the approach known as ‘Going from part to Whole.’ Discrete points considered in the domain are called Nodes and the smaller domain or units considered is called Element. Element and Nodes together constitute Mesh. Fineness of Mesh increases the accuracy of the solution but takes longer time. The number of unknown parameters at each Node determines the Degree of Freedom.

4.1.1 Working Of FEM ^[19]

In Engineering problems of continuum in nature, the field variables (such as Displacement, Potential, Pressure, Velocity and Temperature) possesses infinitely many values because it is a continuous function.

Hence, the problem becomes one with an infinite number of unknowns. Discretization procedure changes the problem into finite number of unknowns by dividing the problem into elements. Then the unknown is expressed in terms of assumed approximating functions within each element.

Approximating function (or Interpolating functions) are defined in terms of values of the field variables at specified node or nodal points.

Nodal values of the field variable and interpolating function for the element within the element defines the concerned field variable behaviour. The nodal values

of the field variable becomes the new unknown for the finite element representation of the problem.

Once the Nodal unknowns are obtained, the interpolation functions defines the field variable throughout the element of the problem. Solution of individual elements are added to give solution for entire problem. The degree of accuracy depends upon the size and number of elements and the kind of interpolation function used.

4.1.2 Two-Dimensional Problem ^[19]

Consider a two-dimensional steady flow through a homogeneous isotropic earth dam resting on an impermeable formation. The physical problem includes the ‘potential’ distribution in the domain of the dam due to the differential heads at u/s and d/s and determination of the free surface position in the dam.

$H_1 = U/s$ head value

$H_2 = D/s$ head value

The problem can be mathematically represented by the Laplace equation as

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Where, $h(x,y)$ is the Head or Potential.

To get a solution, firstly a free surface is chosen and dam domain is discretized into triangular elements using connecting nodes.

Here the unknown variable is the head or ‘potential’ (h) at various nodes.

The unknown variable h is approximated by the two dimensional interpolation or shape function assembled over the triangular element at the node.

Then, equation governing the behaviour of the problem is derived.

Finally, the potential is obtained by solving the approximated system of equations after introducing u/s and d/s head values.

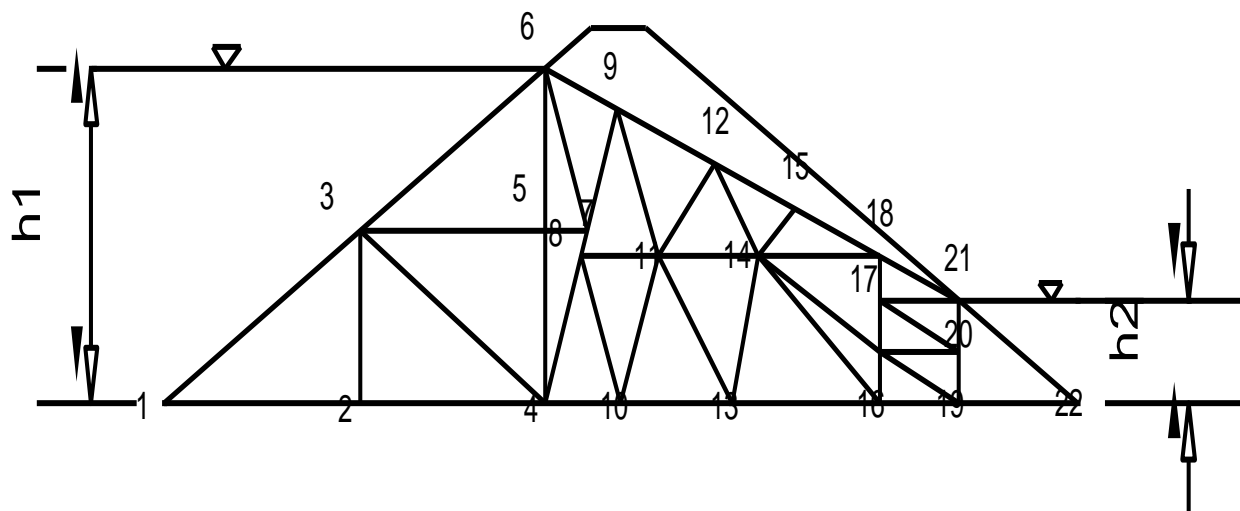


Figure 4.1: Earth Dam discretization using two dimensional finite elements

4.1.3 Merits Of FEM ^[19]

- (1) Modelling of complex geometries and irregular shape is easier.
- (2) Boundary condition can be easily incorporated.
- (3) Different types of material can be easily accommodated in modelling from element to element or within elements.
- (4) Heterogeneous, Anisotropic, Non-linear and Time dependent problem can be dealt.
- (5) FEM can be coupled with CAD programs.

(6) Accuracy can be controlled.

(7) Possible to interpret the method in physical terms.

4.1.4 Demerits Of FEM ^[19]

(1) Numerical solution is obtained one time for a specific problem case only.

(2) There is no advantage of flexibility and generalization unlike analytical solution.

(3) Large amount of input data required.

(4) Poor selection of element type or discretization may lead to faulty results.

4.2 General Steps of FEM ^[19]

FEM procedure may vary depending upon the problem and approach but the general steps remains nearly same. Based on available data, a mathematical model is developed defining the geometry of problem, material property, assumptions and simplifications used, governing equation, boundary and initial conditions.

Step 1: Discretize and select Element types

Body is subdivided into small bodies called Finite Elements. Points at which primary unknowns are required to be evaluated are called 'Nodes' and the interfaces between elements are called Nodal Lines (or planes or surface). Number of unknowns at a node is termed as 'Nodal degree of freedom (DOF).'

Step 2: Select Approximation Function

This step involves choosing a pattern or shape for the distribution of the unknown quantity 'u' within each element.

Unknown quantity can be displacement for stress-analysis problem, Temperature in heat flow problem, Fluid pressure and Velocity for fluid flow problem.

Approximation function is defined within the element using the nodal values of the element. Linear, Quadratic and Cubic polynomial are frequently used function because they are simple to work with Finite Element Formulation. Trigonometric series can also be used. For an N-node element, approximate function can be expressed as-

$$U = N_1u_1 + N_2u_2 + \dots + N_nu_n$$

Where, u_1, u_2, \dots, u_n are unknown values at the nodal points and N_1, N_2, \dots, N_n are the interpolation function or Shape function.

Step 3: Define the gradient of the unknown quantity and constitutive relationship

These relationships are necessary for deriving the equation for each finite element.

For one dimensional flow through porous media-

$$\text{Fluid Gradient} = g_x = \frac{dh}{dx}$$

Constitutive relation-

$$V_x = -k_{xx}g_x \text{ (Darcy's Law)}$$

Where,

K_{xx} = Coefficient of Permeability

V_x = Velocity

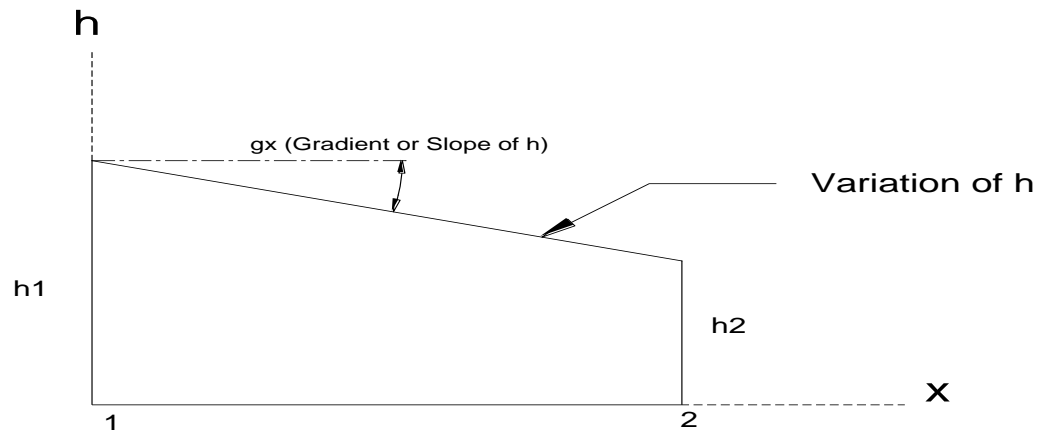


Figure 4.2: One Dimensional flow through porous media

Step 4: Derive Element Equations

In this step, equation governing the behaviour of typical finite element are obtained by using available laws and principles. These equation shows relationship between nodal DOF and Nodal forcing parameters. The relationship can be written as-

$$[K^e] \{q^e\} = \{f^e\}$$

Where,

$[K^e]$ = Element Property Matrix or Element Stiffness Matrix

$\{q^e\}$ = Element Vector of unknown DOF

$\{f^e\}$ = Vector of element nodal forcing parameters

Element equation is derived in this step

Step 5: Assemble Element equations to obtain Total or Global equation and introduce Boundary Condition

Derive Element equation for all other elements. Then, element equations are added together using method of superposition to obtain Global or Total equation for

the entire body. The process of superposition is called ‘Assembling.’ The assembled equation can be written in the matrix form as-

$$[K] \{q\} = \{F\}$$

Where,

$[K]$ = Assembled (Global) Stiffness Matrix (Assembly of $[K^e]$)

$\{q\}$ = Global vector of Nodal unknowns (Assembly of $\{q^e\}$)

$\{F\}$ = Global vector of Nodal Forcing parameters (Assembly of $\{f^e\}$)

The above equation indicates the capabilities of a body to withstand applied forces. To evaluate the capability (performance) of a body, certain Boundary Condition (B.C.) need to be imposed. B.C. are the physical constraints or supports that must exist so that the structure or body is not mobile.

Step 6: Solve for the Unknown DOF (Primary unknowns)

The assembled equations (after the modification of Boundary Conditions) are solved for the q 's by using Gauss elimination or iterative method. The q 's are called the primary unknowns because they are the first quantity determined using FEM.

Step 7: Solve for Secondary quantities

Once the primary quantities are known, the relationship defined in step 3 can be used to find secondary quantities. In case of fluid flow problem secondary quantity is velocity and discharge.

Step 8: Interpret the result

Step 7 and 8 are essentially post processing part of finite element analysis. Usually, a tabulated or graphical presentation of result helps in making the design/analysis decision.

4.3 Approaches Used In FEM ^[19]

4.3.1 Energy Approach

Principle of virtual work, the principle of minimum potential energy and Castigliano's theorem are frequently used to derive equations of elements used in stress analysis problems.

Here principle of minimum potential energy will be considered. Principle of minimum energy is based on finding the equilibrium state of the body or structure associated with stationary values of a scalar quantity assumed by loaded bodies. This scalar quantity is referred as 'functional.'

For example- Let $f(x)$ be a function of variable x and π be the function defined such that $\pi = \pi[f(x)]$. Then π is a function of function f .

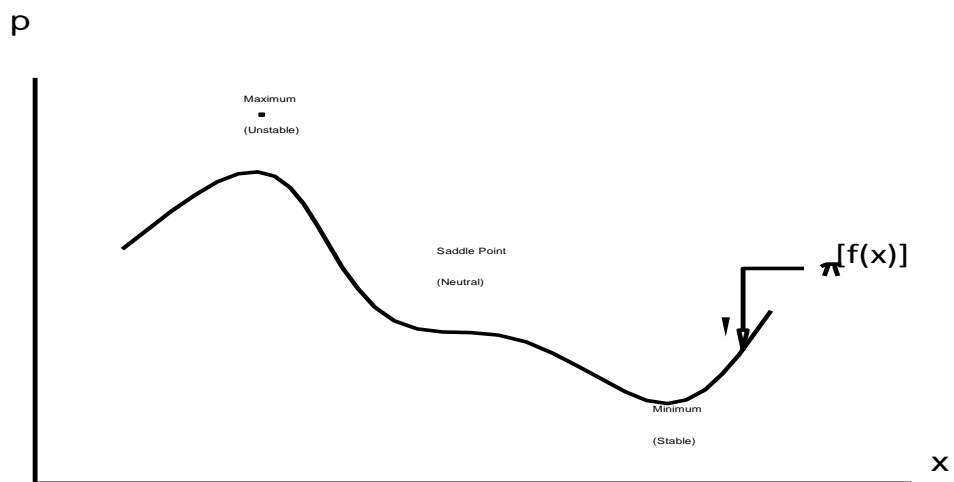


Figure 4.3: Function Example

Stationary values are given by-

$$\frac{d\pi}{dx} = 0$$

In the structural analysis problem, π is the potential energy of the body or structure. FEM can be applied to any problem if the scalar function π is available.

Consider a three spring system with four nodes such that external forces acting on nodes are F_1 F_2 F_3 and F_4 respectively and spring stiffness coefficient is K_1 K_2 and K_3 . Potential energy for this elastic system is –

$$\pi = \frac{1}{2} K [\delta_1 - \delta_2]^2$$

$$\frac{\partial \pi}{\partial \delta_1} = k\delta_1 - k\delta_2$$

$$\frac{\partial \pi}{\partial \delta_2} = -k\delta_1 + k\delta_2$$

Stiffness Matrix can be written as –

$$[K^e] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

4.3.2 Weighted Residual Approach

This method is a technique for obtaining approximate solutions to linear and non- linear partial differential equation.

The domain considered is first discretized. Then, general behaviour of dependent field variable is assumed so as to approximately satisfy the given differential equation.

Initially approximation is applied over each element then substituted in original Differential Equation. Approximation results in same error called ‘Residual’ which is required to be removed. After substituting certain Boundary Conditions we get system of equations which is then solved.

This approach is advantageous because it makes it possible to use FEM to problems where no function is available.

Our Aim is to find Approximate Functional representation for the field variable h governed by the differential equation-

$$L(h) - g(x,y) = 0 \quad \text{----- (1)}$$

In the domain Ω bounded by surface Γ with appropriate Boundary Conditions.

Function g is a known function of the independent variables. Finite Element procedure using method of weighted residual is used. Commonly used weighted residual technique is Galerkin’s Method.

In the first step, the unknown exact solution h is approximated by approximating function

$$h = \hat{h} = \sum_{i=1}^m N_i D_i \quad \text{----- (2)}$$

Here,

N_i is an assumed function

D_i are the unknown parameters or functions

Since, \hat{h} is an approximated function, it will not satisfy equation (1) when it is substituted in eq. (1). Hence,

$$L(\hat{h}) - g(x,y) = R_1 \quad \text{----- (3)}$$

Here, R_1 is the Residue or Error that results due to \hat{h} .

The m unknown D_i are determined in such a way that the error R_1 over the entire solution vanishes or becomes very small. This is achieved by using m linearly independent weighing function Ψ_i . Hence,

$$\int [L(\hat{h}) - g(x,y)] \Psi_i d\Omega = \int R_1 \Psi_i d\Omega \quad \text{----- (4)}$$

$$i = 1, 2, 3, \dots, m$$

After this approximation, the residual approaches to zero in weighted sense.

According to Galerkin's Method, $\Psi_i = N_i$ for $i = 1, 2, 3, \dots, m$. Thus, equation (4) can be written as-

$$\int [L(\hat{h}) - g(x,y)] N_i d\Omega = 0 \quad \text{----- (5)}$$

As the domain is discretized into elements and nodes, initially the weighted Residual Technique based on Galerkin's approach can be formulated for an element and then assembled for the entire domain.

N_i are the interpolation function, $N_i^{(e)}$ defined over the element and D_i are undetermined parameters which may be the nodal values of the field variable or its derivatives.

$$\int [L(h^{(e)}) - g(x,y)^{(e)}] N_i^{(e)} d\Omega^{(e)} = 0 \quad \text{----- (6)}$$

$$i = 1, 2, 3, \dots, r$$

Here, superscript (e) refers to an element $\mathbf{h}^{(e)} = [\mathbf{N}^{(e)}] \{\mathbf{h}\}^{(e)}$, $g(x,y)^{(e)}$ is the forcing function defined over element (e) and r is number of unknown parameters assigned to the element.

4.4 Interpolation Function ^[19]

Functions used to represent behaviour of a field variable within an element are called Interpolation Function or Shape Function or Approximating Function.

Depending upon problem dimension, polynomial in one, two or three independent variables may be used in the Interpolation function.

One Independent Spatial variable:

A general complete n th order polynomial may be written for one dimensional analysis as-

$$P_n(x) = \sum_{i=0}^n a_i x^i \quad \text{----- (7)}$$

Where,

$P_n(x)$ = n th order polynomial

a_i (i = 0, 1, 2n) = Coefficient of x^i

Example:

$$P_1(x) = a_0 + a_1x$$

$$P_2(x) = a_0 + a_1x + a_2x^2$$

$$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

Two Independent Spatial Variable:

A general polynomial for two-dimensional analysis may be written as-

$$P_{mn}(x,y) = \sum_{i=0}^m \sum_{j=0}^n a_{ij} x^i y^j \quad \text{----- (8)}$$

Example:

$$P_{11}(x,y) = a_{00} + a_{10}x + a_{01}y$$

$$P_{22}(x,y) = P_{11}(x,y) + a_{11}xy + a_{20}x^2 + a_{02}y^2$$

4.4.1 Two Dimensional Element

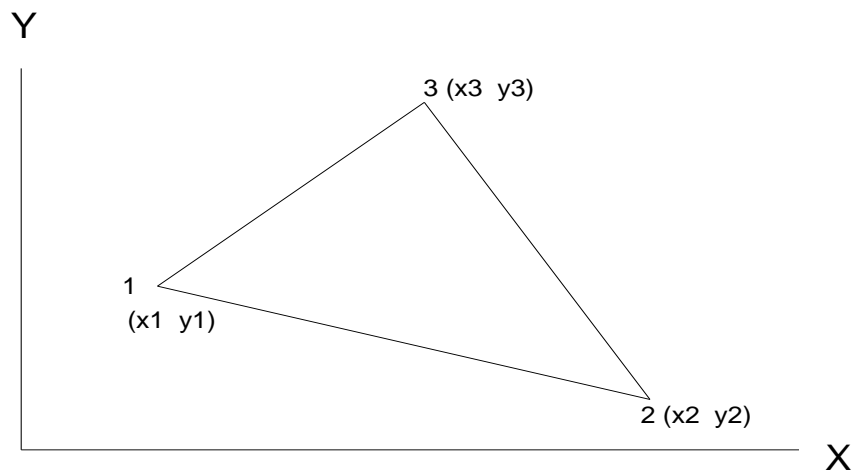
Triangular Element: Linear Interpolation Function in Cartesian Co-ordinates

Triangular Element is used when the field variable is a function of two independent variable x and y.

$$\Phi = \Phi(x,y) = \alpha_1 + \alpha_2x + \alpha_3y \quad \text{----- (9)}$$

Where,

$\alpha_1 \alpha_2 \alpha_3$ are the coefficient to be evaluated



Let,

$$\Phi = \Phi(x,y) = [1 \ x \ y] \{\alpha\}$$

$$\{\Phi\} = [P] \{\alpha\} \quad \text{----- (10)}$$

Where,

$$[P] = [1 \ x \ y]$$

$$\{\alpha\} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

$$\{\Phi\} = \begin{Bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} \quad \text{----- (11)}$$

$$\{\Phi\} = [G] \{\alpha\}$$

$$\{\alpha\} = [G]^{-1} \{\Phi\} \quad \text{----- (12)}$$

Combining Equation (10) and (12)

$$\Phi = [P] [G]^{-1} \{\Phi\} \quad \text{----- (13)}$$

$$\Phi = [N] \{\Phi\} \quad \text{----- (14)}$$

Here, $[N] = [P] [G]^{-1}$

$$[G] = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

$$[G]^{-1} = \frac{1}{2A} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad \text{----- (15)}$$

$$2A = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = 2 \text{ (Area of Element)} = a_1 + a_2 + a_3 \quad \text{----- (16)}$$

Also,

$$a_1 = x_2y_3 - x_3y_2$$

$$a_2 = x_3y_1 - x_1y_3$$

$$a_3 = x_1y_2 - x_2y_1$$

$$b_1 = y_2 - y_3$$

$$b_2 = y_3 - y_1$$

$$b_3 = y_1 - y_2$$

$$c_1 = x_3 - x_2$$

$$c_2 = x_1 - x_3$$

$$c_3 = x_2 - x_1$$

$$[N] = \frac{1}{2A} [1 \ x \ y] \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

Therefore,

$$N_1 = N_1(x, y) = \frac{1}{2A} (a_1 + b_1x + c_1y)$$

$$N_2 = N_2(x, y) = \frac{1}{2A} (a_2 + b_2x + c_2y)$$

$$N_3 = N_3(x, y) = \frac{1}{2A} (a_3 + b_3x + c_3y)$$

Variation of N_i ($i = 1, 2, 3$) is such that N_i at node i is 1 and zero at other end.

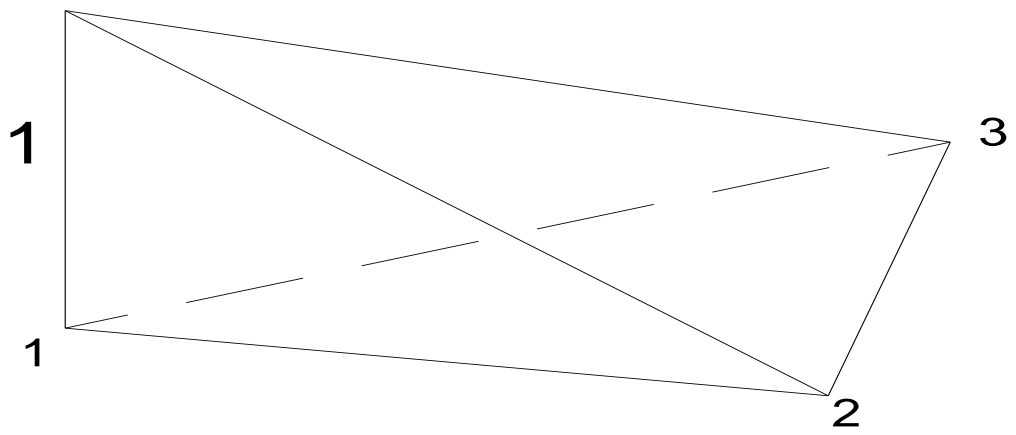


Figure 4.4: Variation of N_1

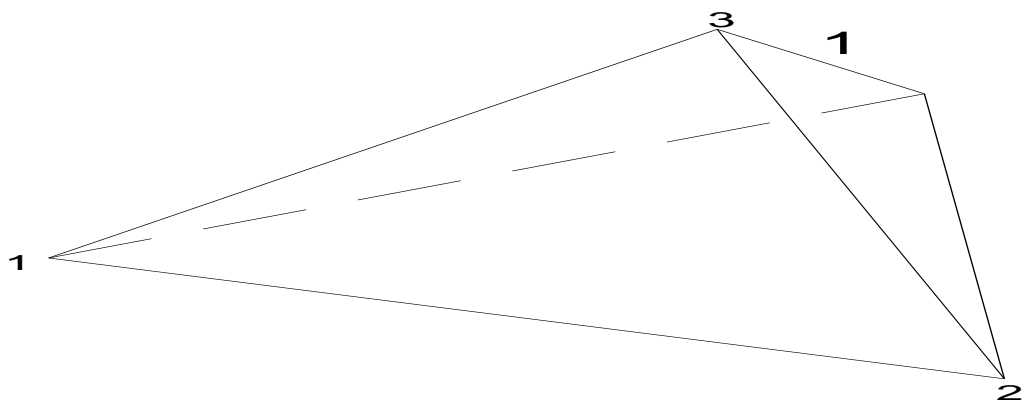


Figure 4.5: Variation of N_2

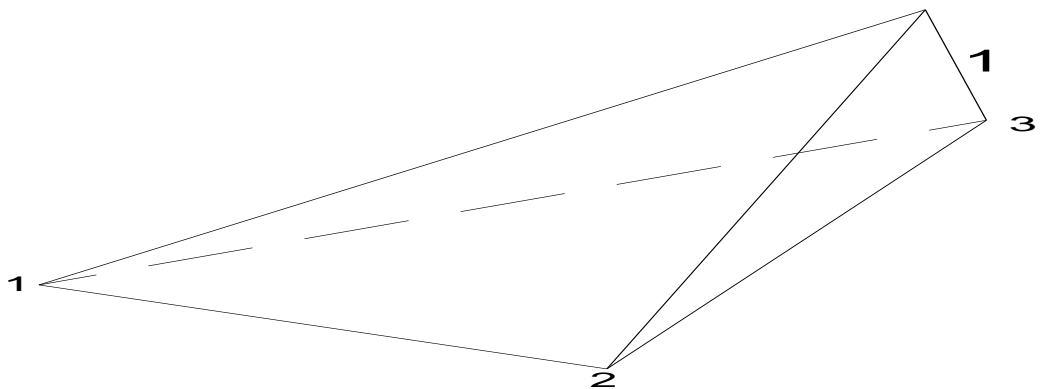


Figure 4.6: Variation of N_3

4.5 Two Dimensional Finite Element Analysis ^[19]

At steady state, the governing differential equation for these problems can be written as –

$$K_x \frac{\partial^2 \Phi}{\partial x^2} + k_y \frac{\partial^2 \Phi}{\partial y^2} + Q = 0 \quad \text{----- (1)}$$

The primary unknown field Φ is function of (X, Y)

Equation (1) can be solved by modelling the problem with two-dimensional Finite Elements. The Φ can be approximated as-

$$\Phi = h + p/w$$

Where,

h = Elevation (or Fluid Head)

p = Pore Pressure

w = Specific weight of the fluid (water)

The velocity head term $\frac{v^2}{2g}$ has been omitted from above equation.

The two-dimensional seepage problem can be categorised into-

- (i) Unconfined Seepage Problem
- (ii) Confined Seepage Problem

4.5.1 Unconfined Seepage problem

Seepage through Earth dam can be classified as Unconfined Seepage. Unconfined seepage is distinguished from the confined seepage by the presence of a free or phreatic surface, the potential Φ equals the fluid head measured from a datum.

Moreover, there cannot be flow in the direction normal to the free surface (flow cannot occur through an impervious bed or, $\frac{\partial \Phi}{\partial y} = 0$ at impervious bed).

Mathematically these two equations can be written as-

$$\Phi = h$$

$$\frac{\partial \Phi}{\partial y} = 0$$

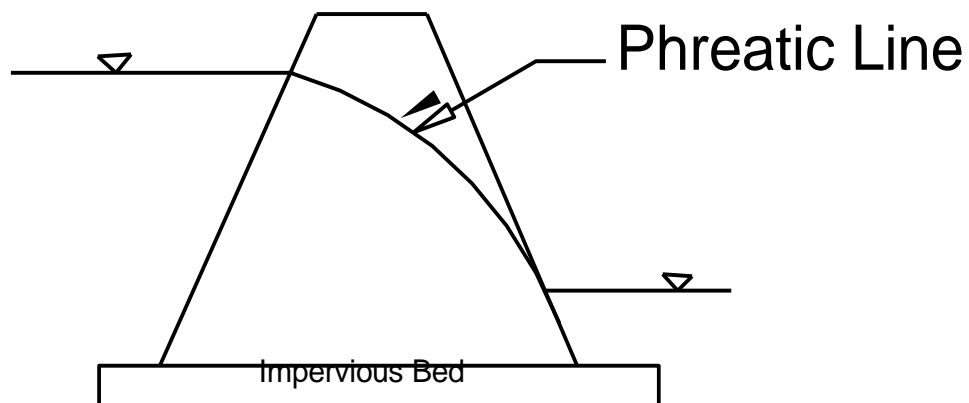


Figure 4.7: Dam Profile

4.5.2 Confined Seepage Problem

In this case, flow occurs in the absence of free surface through a saturated media subjected to prescribed Boundary condition (usually Φ). The confined steady

state seepage requires a linear analysis for solution. The porous saturated media can be modelled by employing a variety of two-dimensional elements. A typical Constant Strain Triangular (CST) element is used in discretization.

4.6 Steps for Two Dimensional Seepage Analysis ^[19]

4.6.1 Discretize and select element type

A typical discretized i th element is as shown in figure. The element has three nodes. At each node there is one primary unknown fluid potential Φ . Thus, the nodal DOF for each element are Φ_1 , Φ_2 and Φ_3 at node 1, 2 and 3 respectively. Let the nodal coordinates in X-Y plane be defined as (X_1, Y_1) , (X_2, Y_2) and (X_3, Y_3) .

4.6.2 Select Approximation Function

$$\Phi(x, y) = \sum_i \sum_j d_{ij} x^i y^j \quad \text{----- (1)}$$

$$\Phi(x, y) = d_{00} + d_{10}X + d_{01}Y \quad \text{----- (2)}$$

$$\Phi(x, y) = d_0 + d_1X + d_2Y \quad \text{----- (3)}$$

Application of above equation at node 1, 2 and 3 results in –

$$\Phi_1 = d_0 + d_1X_1 + d_2Y_1$$

$$\Phi_2 = d_0 + d_1X_2 + d_2Y_2 \quad \text{----- (4)}$$

$$\Phi_3 = d_0 + d_1X_3 + d_2Y_3$$

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix}$$

$$\Rightarrow \{q^e\} = [P] \{d\}$$

$$\Rightarrow \{d\} = [P]^{-1} \{q^e\}$$

$$\Rightarrow \{d\} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = \frac{1}{|P|} \begin{bmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{bmatrix} \{q^e\} \quad \text{----- (5)}$$

$$a_1 = X_3 - X_2$$

$$a_2 = X_1 - X_3$$

$$a_3 = X_2 - X_1$$

$$b_1 = Y_2 - Y_3$$

$$b_2 = Y_3 - Y_1$$

$$b_3 = Y_1 - Y_2$$

$$c_1 = X_2 Y_3 - Y_2 X_3$$

$$c_2 = X_3 Y_1 - Y_3 X_1$$

$$c_3 = X_1 Y_2 - Y_1 X_2$$

$$|P| = c_1 + c_2 + c_3 \quad \text{----- (6)}$$

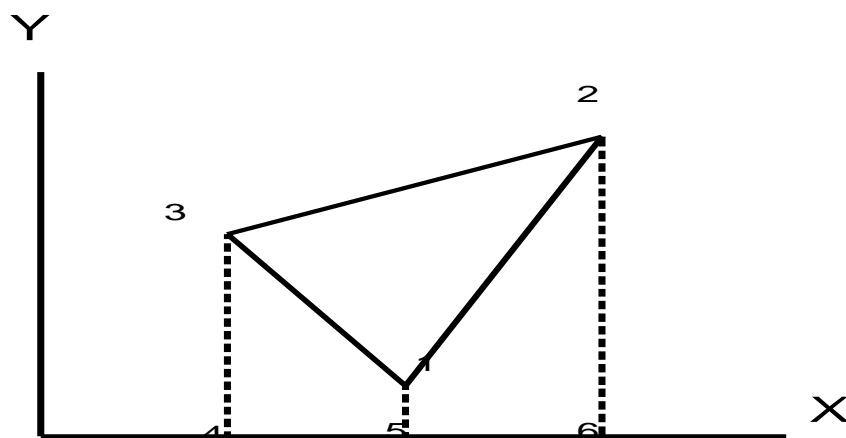


Figure 4.8: Area Element

Area (A) of the element = Area of trap. 2346 – Area of trap. 1345 – Area of trap. 2156

$$A = \frac{1}{2} (c_1 + c_2 + c_3) \quad \text{----- (7)}$$

$$A = \frac{1}{2} |P|$$

$$\Rightarrow |P| = 2A \quad \text{----- (8)}$$

Using $|P| = 2A$ in eq (5) we get,

$$\begin{Bmatrix} d_0 \\ d_1 \\ d_2 \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{bmatrix} \begin{Bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{Bmatrix} \quad \text{----- (9)}$$

$$d_0 = \frac{1}{2A} [\Phi_1 c_1 + \Phi_2 c_2 + \Phi_3 c_3]$$

$$d_1 = \frac{1}{2A} [\Phi_1 b_1 + \Phi_2 b_2 + \Phi_3 b_3] \quad \text{----- (10)}$$

$$d_2 = \frac{1}{2A} [\Phi_1 a_1 + \Phi_2 a_2 + \Phi_3 a_3]$$

Substitute equation (10) in equation (9)

$$\Phi (X, Y) = N_1 \Phi_1 + N_2 \Phi_2 + N_3 \Phi_3$$

$$\Phi (X, Y) = [N] \{q^e\}$$

Where,

$$N_1 = \frac{1}{2A} [c_1 + b_1 X + a_1 Y]$$

$$N_2 = \frac{1}{2A} [c_2 + b_2 X + a_2 Y]$$

$$N_3 = \frac{1}{2A} [c_3 + b_3 X + a_3 Y]$$

$$[N] = [N_1 \quad N_2 \quad N_3]$$

N_1, N_2 and N_3 are called the Shape Function for the CST and $[N]$ denotes the shape function matrix (N_i are also called Local Area Co-ordinates).

4.6.3 Define constitutive law and gradients

Gradients of unknown fields-

Let ϵ_x and ϵ_y be the gradients (slopes) of Φ in the X and Y direction respectively.

$$\epsilon_x = \frac{\partial \Phi}{\partial x} \quad \text{and} \quad \epsilon_y = \frac{\partial \Phi}{\partial y} \quad \text{----- (11)}$$

From equation (6) and equation (11) we get,

$$\epsilon_x = d_1 \quad \text{and} \quad \epsilon_y = d_2 \quad \text{----- (12)}$$

Thus the gradient (strains) are constant over the element and hence a 3 node triangular element is referred to as Constant Strain Triangle (CST) element.

By substituting equation (12) into (11)

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \end{Bmatrix} = [B] \{q^e\} \quad \text{----- (13)}$$

Where,

$$[B] = \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{bmatrix} \quad \text{----- (14)}$$

Constitutive Relations (Darcy's Law)

$$V_x = -k_x \frac{\partial \Phi}{\partial x} = -\sigma_x$$

$$\Rightarrow \sigma_x = k_x \epsilon_x \quad \text{----- (15)}$$

$$V_y = -k_y \frac{\partial \phi}{\partial y} = -\sigma_y$$

$$\Rightarrow \sigma_y = k_y \epsilon_y \quad \text{----- (16)}$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} = \begin{Bmatrix} k_x \epsilon_x \\ k_y \epsilon_y \end{Bmatrix} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \end{Bmatrix}$$

$$\{\sigma\} = [D] \{\epsilon\} \quad \text{----- (17)}$$

$$\{\sigma\} = [D] [B] \{q^e\} \quad \text{----- (18)}$$

Where,

$$[D] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \quad \text{----- (19)}$$

4.6.4 Derive Element Equation

Energy method is applied to derive element equation. Application of the energy method requires scalar function Π_p to be defined. The Π_p for seepage analysis is given by-

$$\Pi_p = \frac{1}{2} \int_V \{\sigma\}^T \{\epsilon\} dV - \{q^e\}^T \{f^e\} \quad \text{----- (20)}$$

$$\{f^e\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \text{Nodal fluid flux vector} \quad \text{----- (21)}$$

$$\begin{aligned} \Pi_p &= \frac{1}{2} \int_V \{q^e\}^T [B]^T [D] [B] \{q^e\} dV - \{q^e\}^T \{f^e\} \\ &= \frac{1}{2} \{q^e\}^T [K^e] \{q^e\} - \{q^e\}^T \{f^e\} \quad \text{----- (22)} \end{aligned}$$

Where,

$$[K^e] = \int_V [B]^T [D] [B] dA \quad \text{----- (23)}$$

= Element Permeability Matrix

$$[K^e] \{q^e\} = \{f^e\} \quad \text{----- (24)}$$

[B] and [D] are constant matrix

Therefore equation (24) can be written as

$$[K^e] = [B]^T [D] [B] \int_V dA$$

$$= A [B]^T [D] [B]$$

$$= A \frac{1}{2A} \begin{bmatrix} b_1 & a_1 \\ b_2 & a_2 \\ b_3 & a_3 \end{bmatrix} \begin{bmatrix} kx & 0 \\ 0 & ky \end{bmatrix} \frac{1}{2A} \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{bmatrix}$$

$$= \frac{1}{4A} \begin{bmatrix} b_1 & a_1 \\ b_2 & a_2 \\ b_3 & a_3 \end{bmatrix} \begin{bmatrix} kx & 0 \\ 0 & ky \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{bmatrix}$$

CHAPTER 5
IMPLEMENTATION
USING FEM AND
ANALYTICAL
METHOD

5.1 Result using Finite Element Method

In this research, parametric study was carried out using Finite Element Method. Seepage discharge is evaluated through the body of the embankment using this software. Steady state analysis is carried out considering the embankment to be homogeneous. The base of the embankment is considered to be impermeable such that no seepage occurs below the base of the embankment. The variables in this study are height and slope of the upstream and downstream face. Height of the embankment is kept as 10m, 15m and 20m. For each height, slope is varied from 1:1, 1:2 and 1:3. Dam profile is drawn and proper boundary conditions are implemented before solving the problem using the software.

5.1.1 Height = 10 m and Slope = 1:1

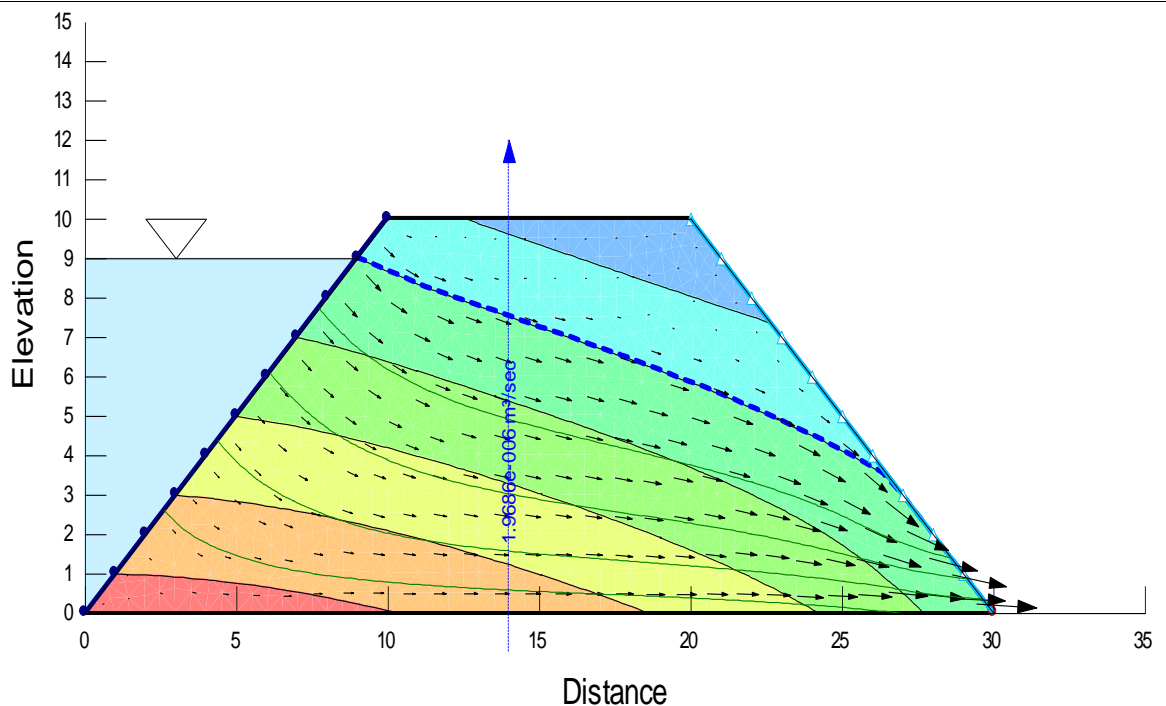


Figure 5.1.1: Profile of the Embankment having filter with seepage line

D/s Fluxes from seepage face

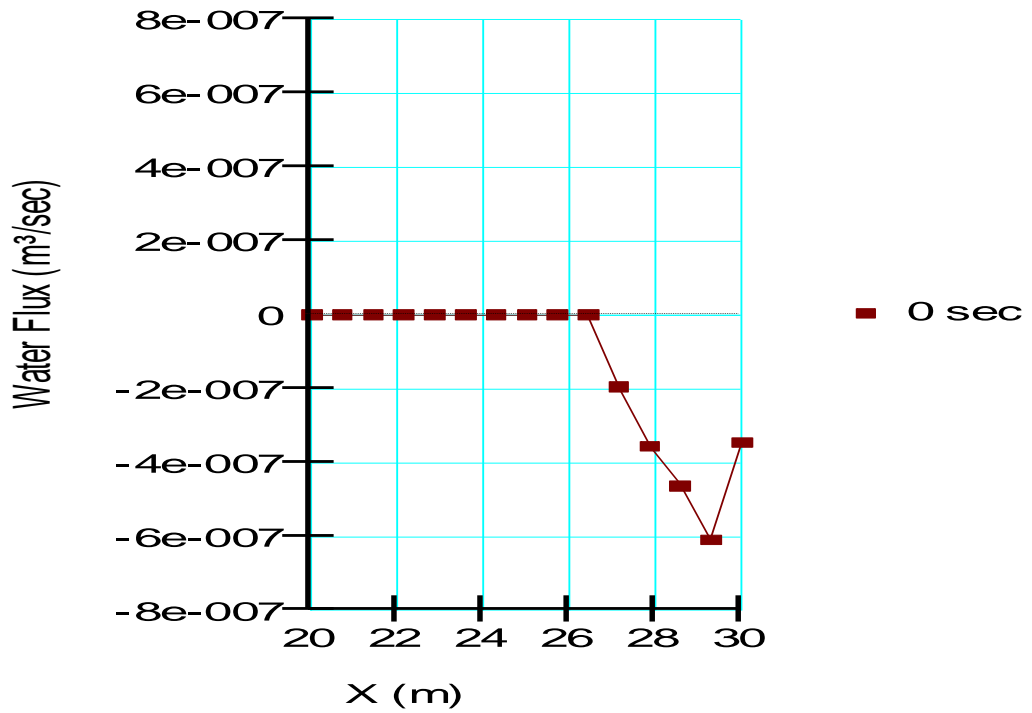


Figure 5.1.2: Variation of Flux with horizontal distance

Pressure Head Profile

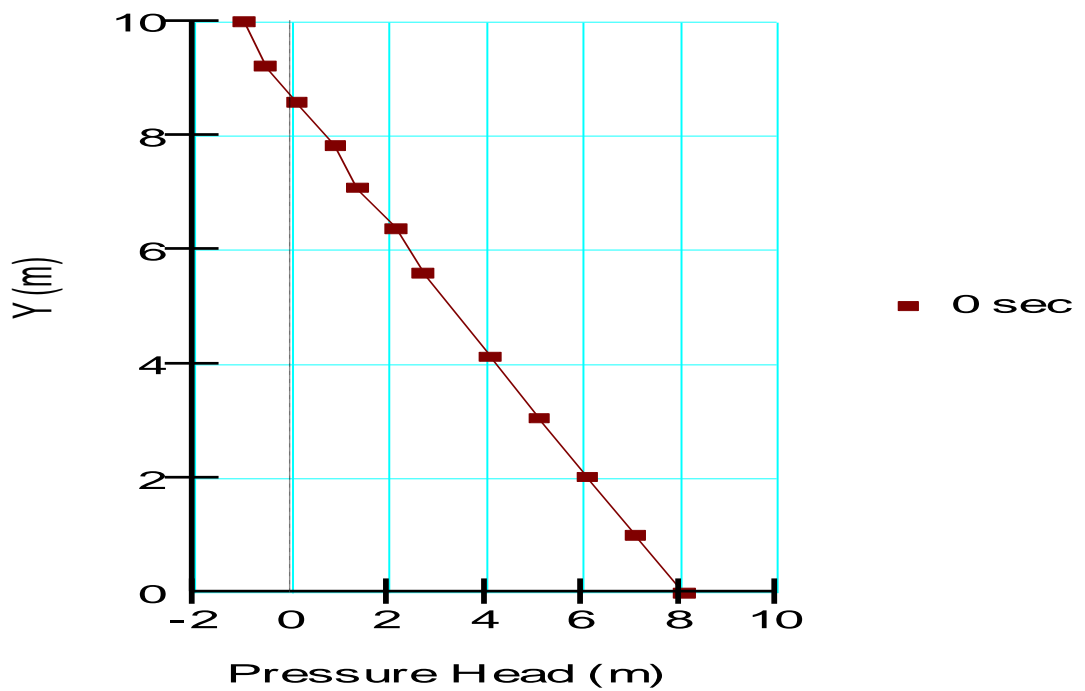


Figure 5.1.3: Variation of Pressure Head with Elevation

5.1.1b With Drainage Filter

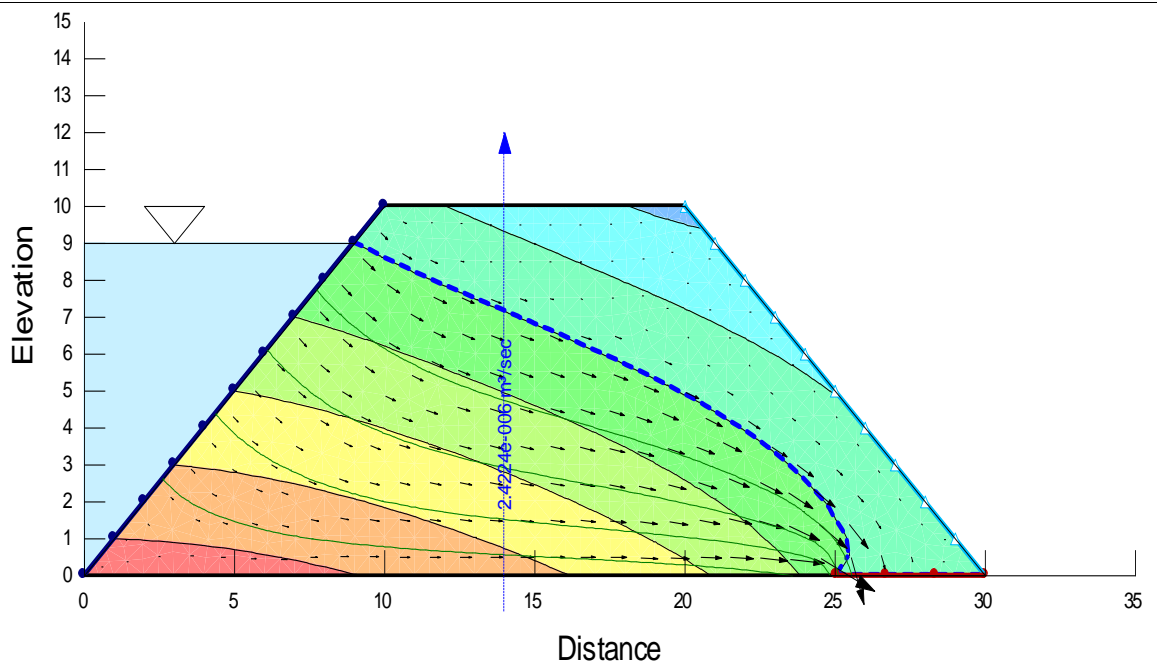


Figure 5.1.4: Profile of the Embankment having filter with seepage line

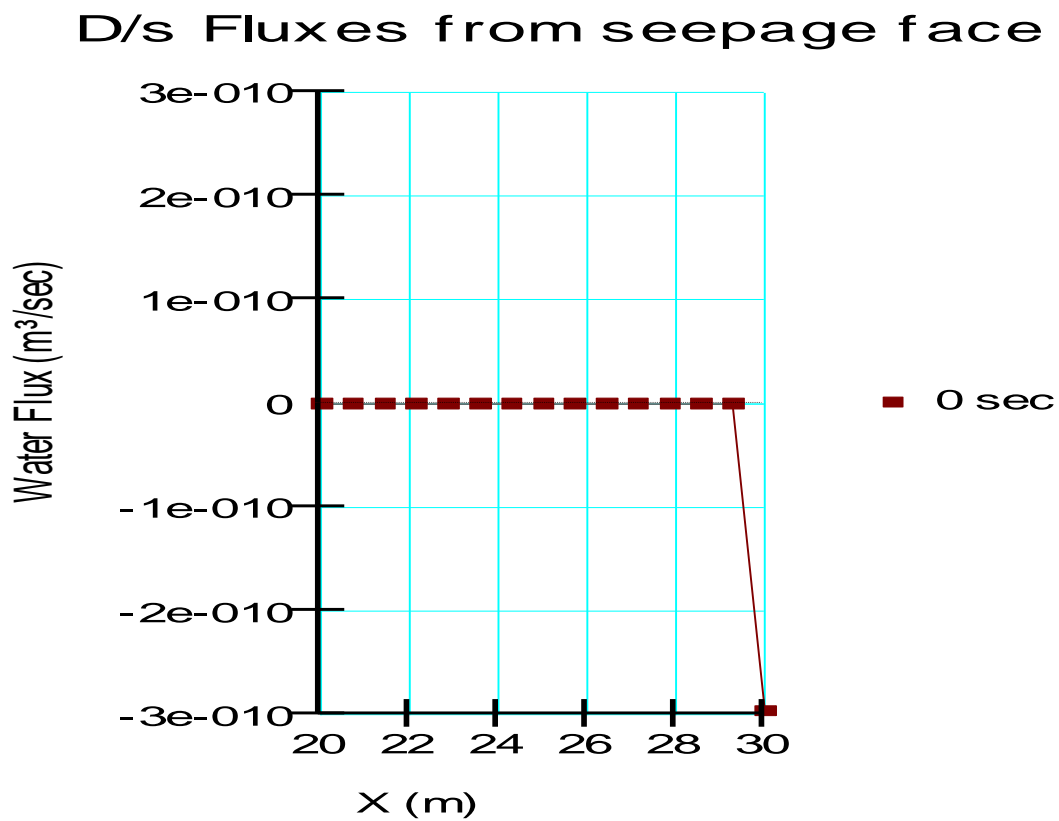


Figure 5.1.5: Variation of Flux with horizontal distance

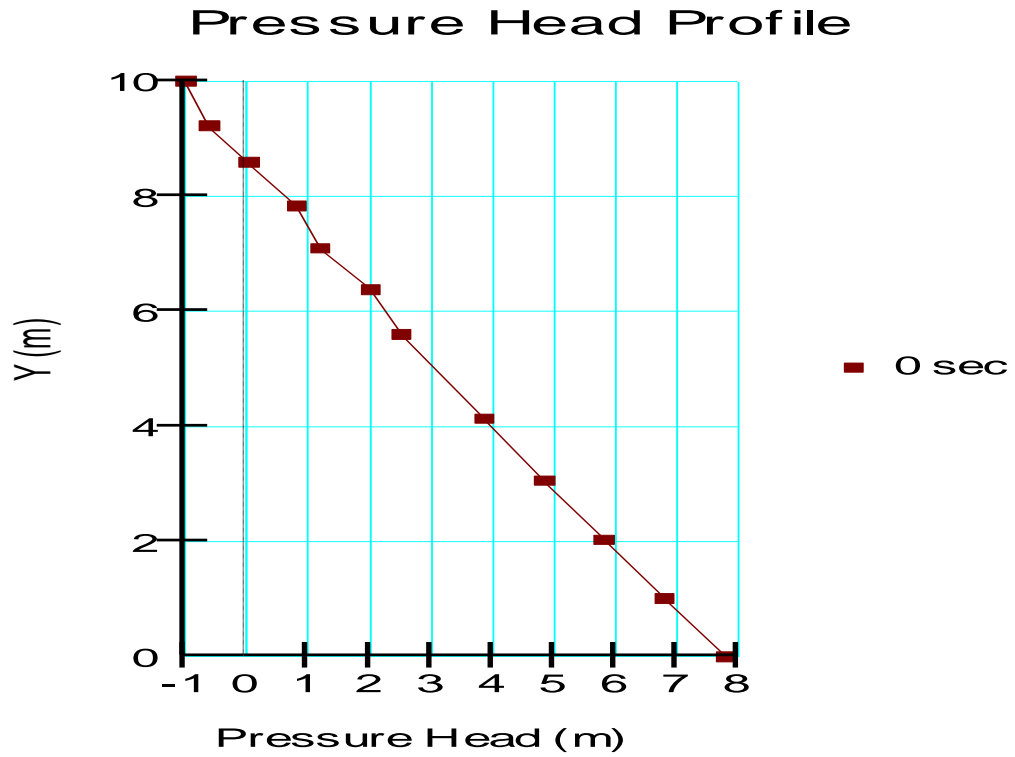


Figure 5.1.6: Variation of Pressure Head with Elevation

5.1.2 Height = 10 m and Slope = 1:2

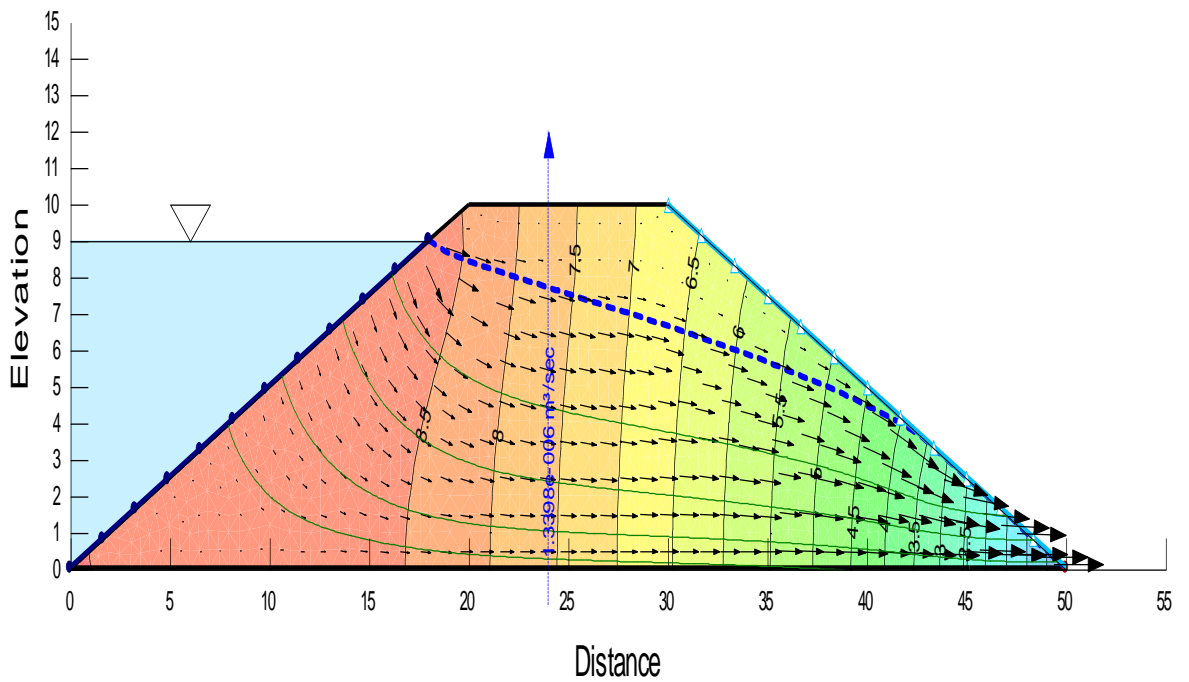


Figure 5.1.7: Profile of the Embankment having filter with seepage line

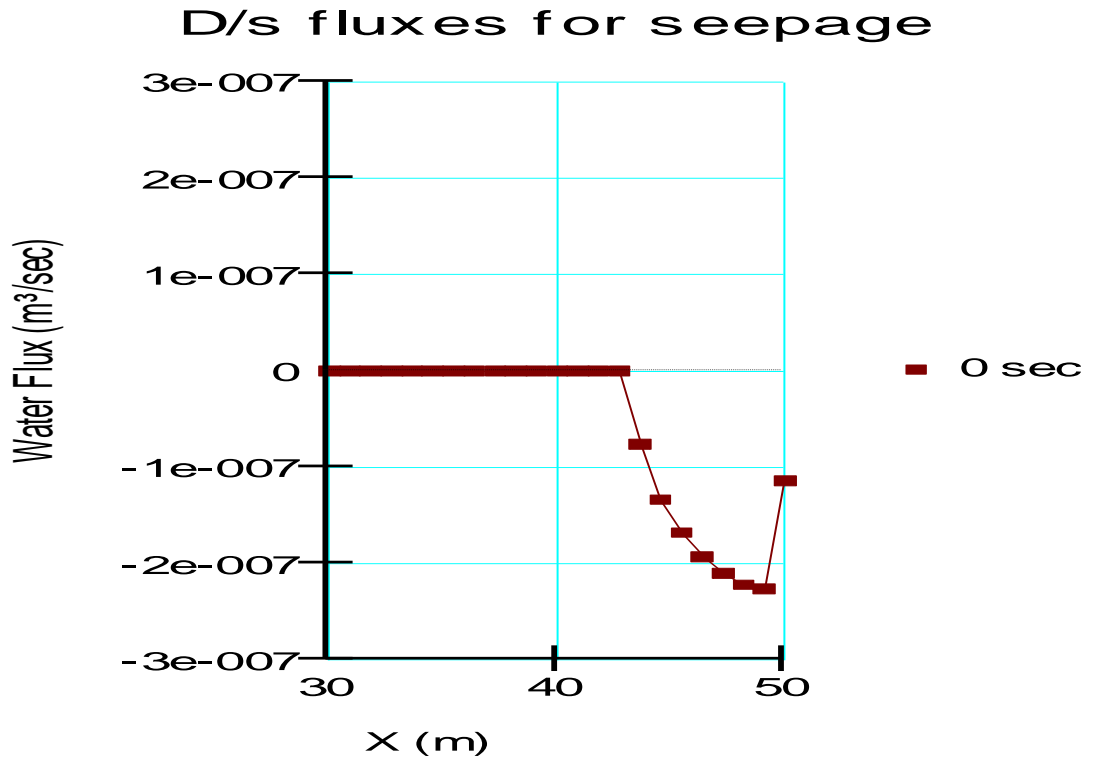


Figure 5.1.8: Variation of Flux with horizontal distance

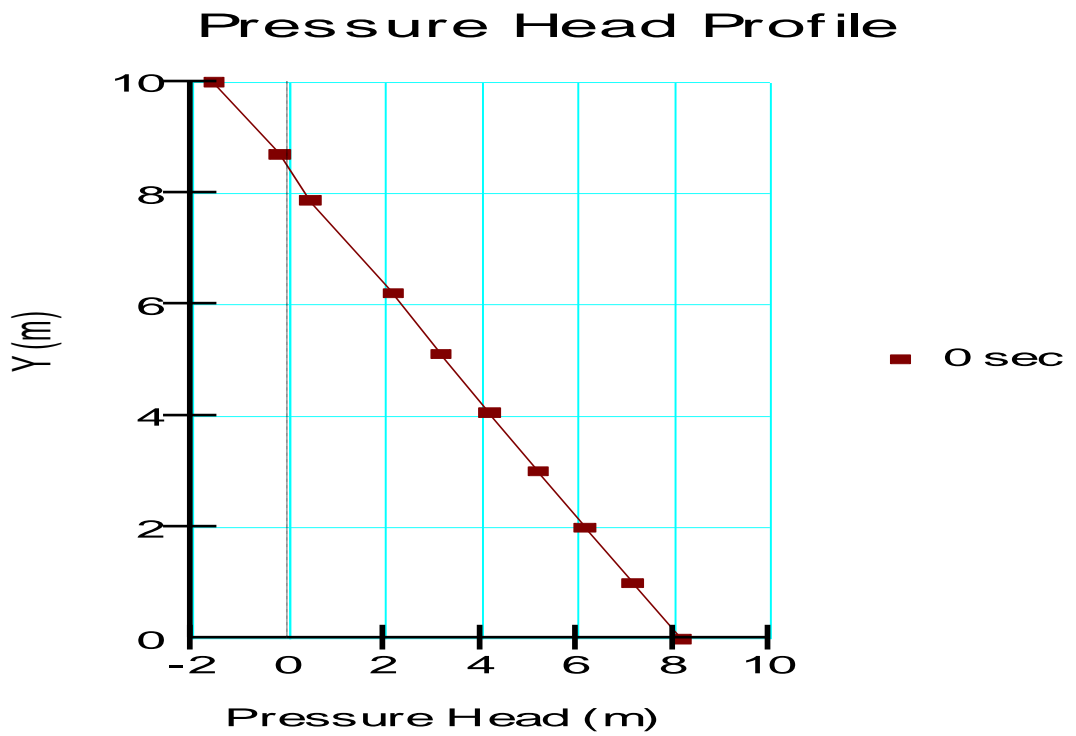


Figure 5.1.9: Variation of Pressure Head with Elevation

5.1.2b With Drainage Filter

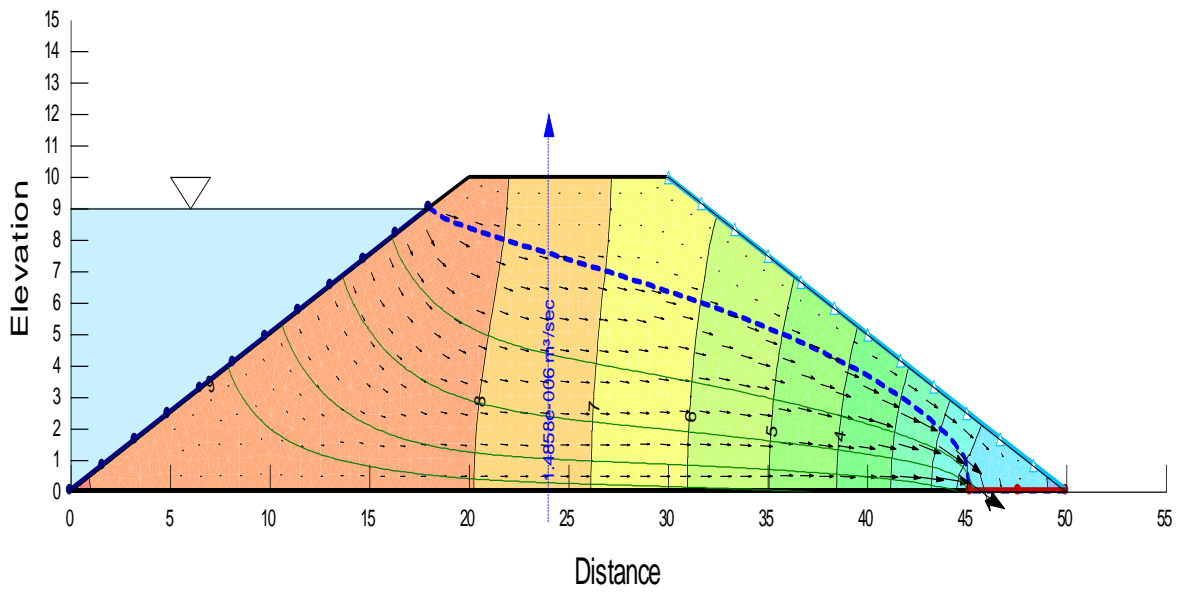


Figure 5.1.10: Profile of the Embankment having filter with seepage line

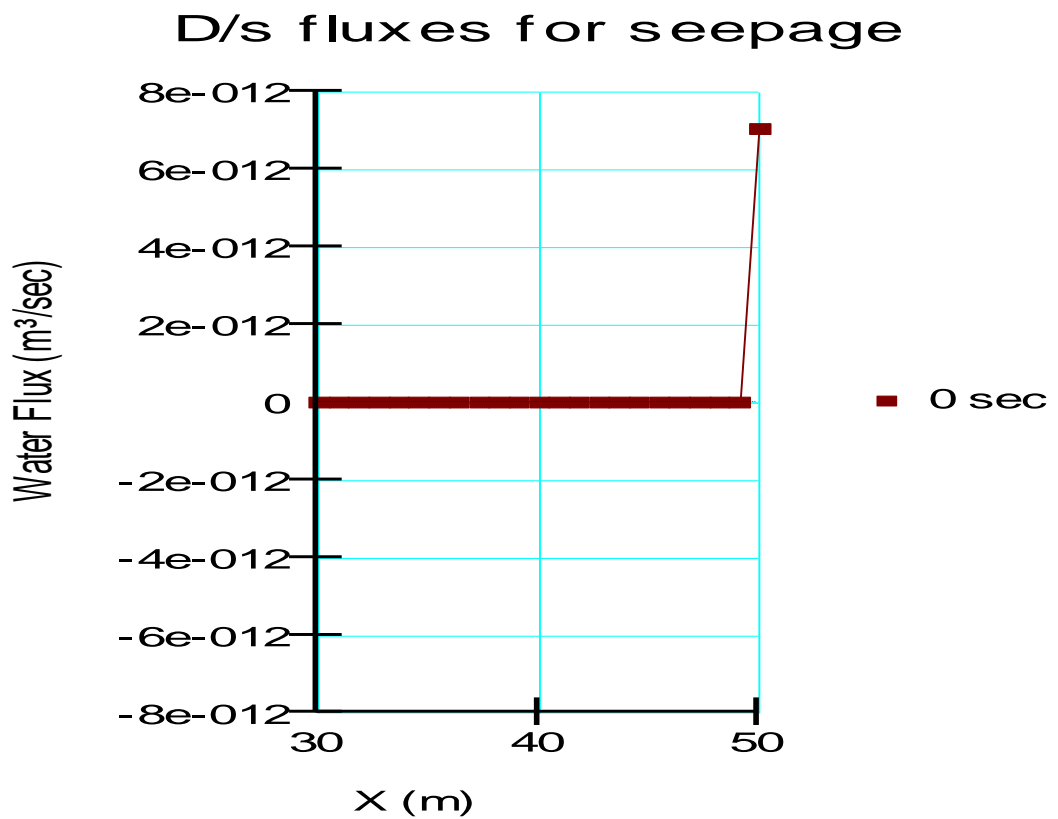


Figure 5.1.11: Variation of Flux with horizontal distance

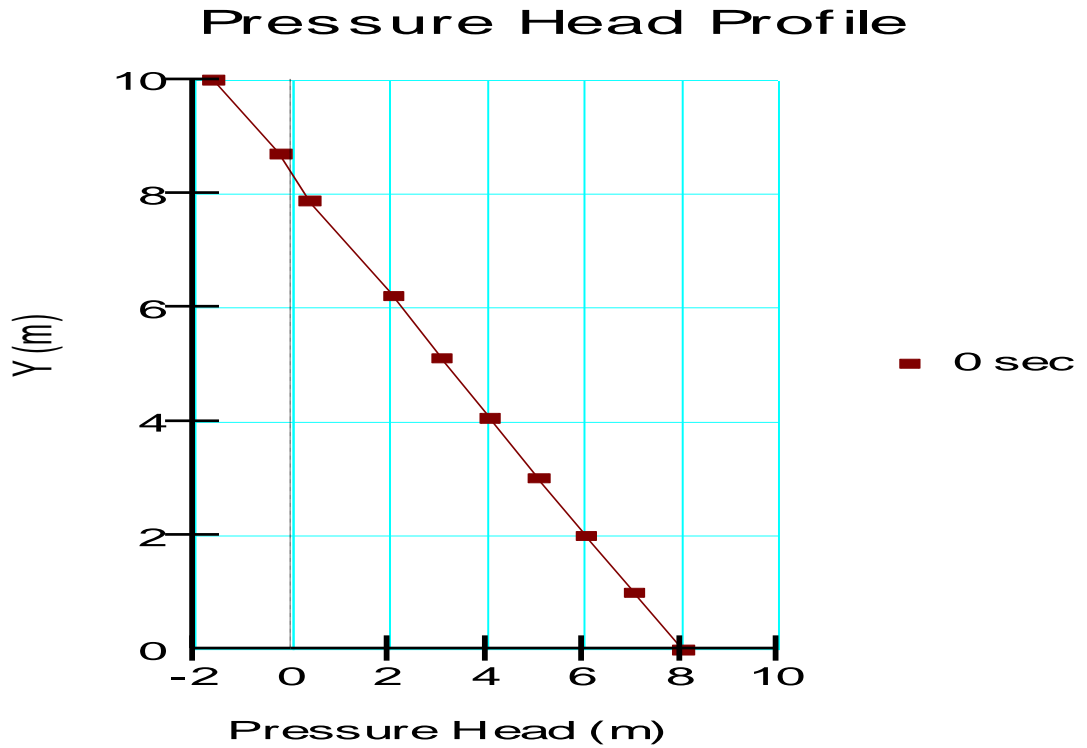


Figure 5.1.12: Variation of Pressure Head with Elevation

5.1.3 Height = 10 m and Slope = 1:3

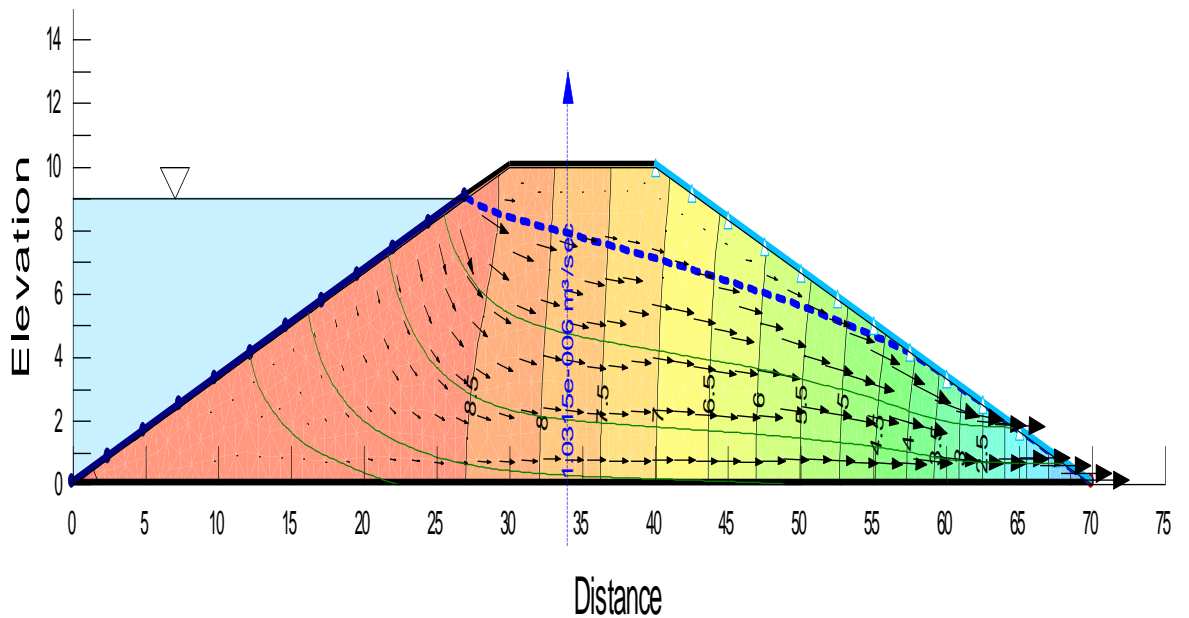


Figure 5.1.13: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

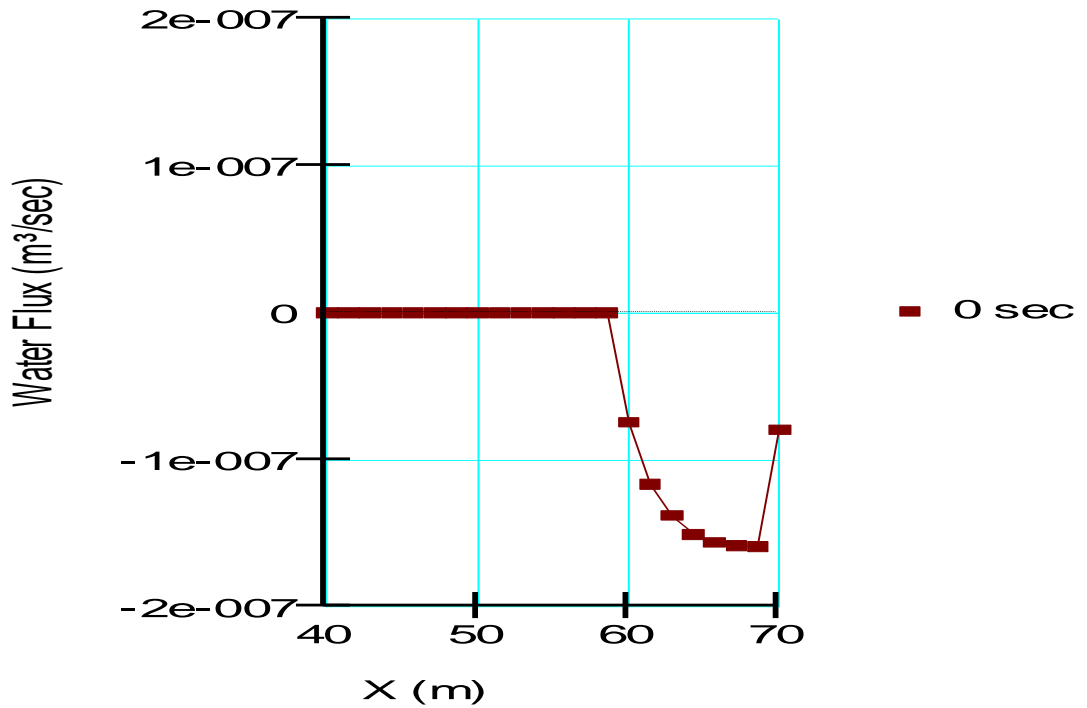


Figure 5.1.14: Variation of Flux with horizontal distance

Pressure Head Profile

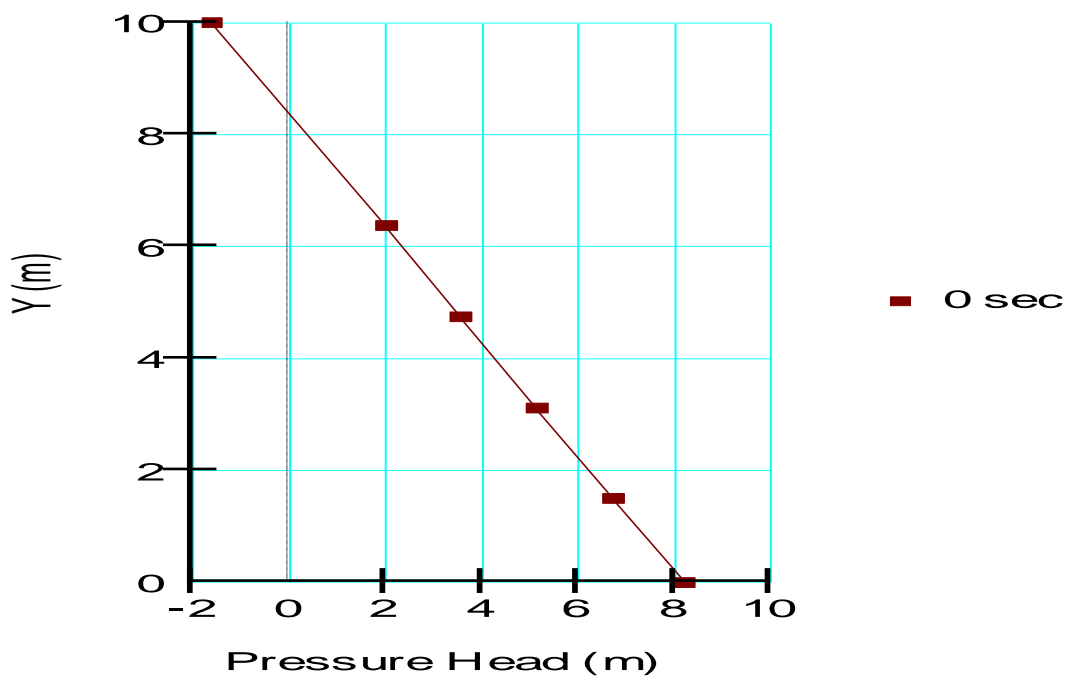


Figure 5.1.15: Variation of Pressure Head with Elevation

5.1.3b With Drainage Filter

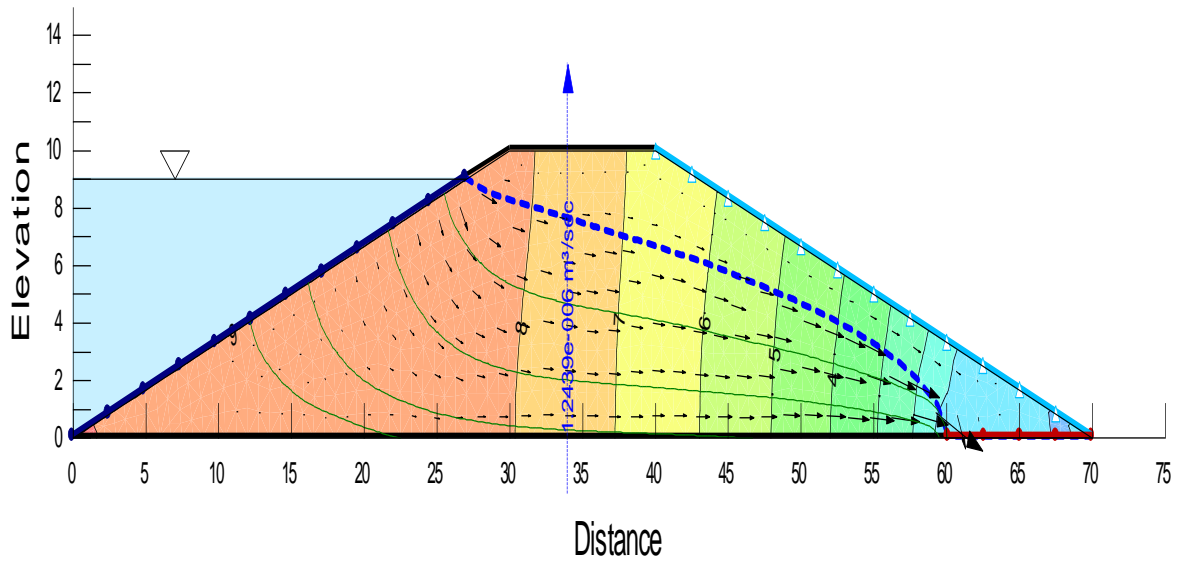


Figure 5.1.16: Profile of the Embankment having filter with seepage line

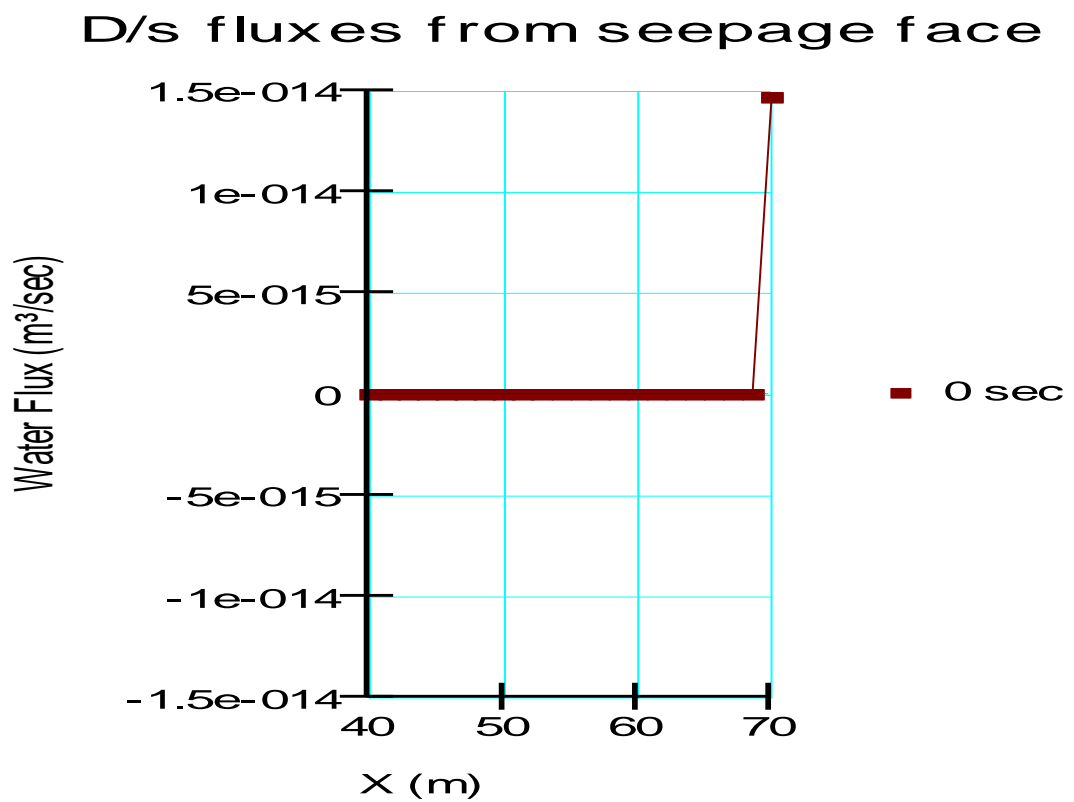


Figure 5.1.17: Variation of Flux with horizontal distance

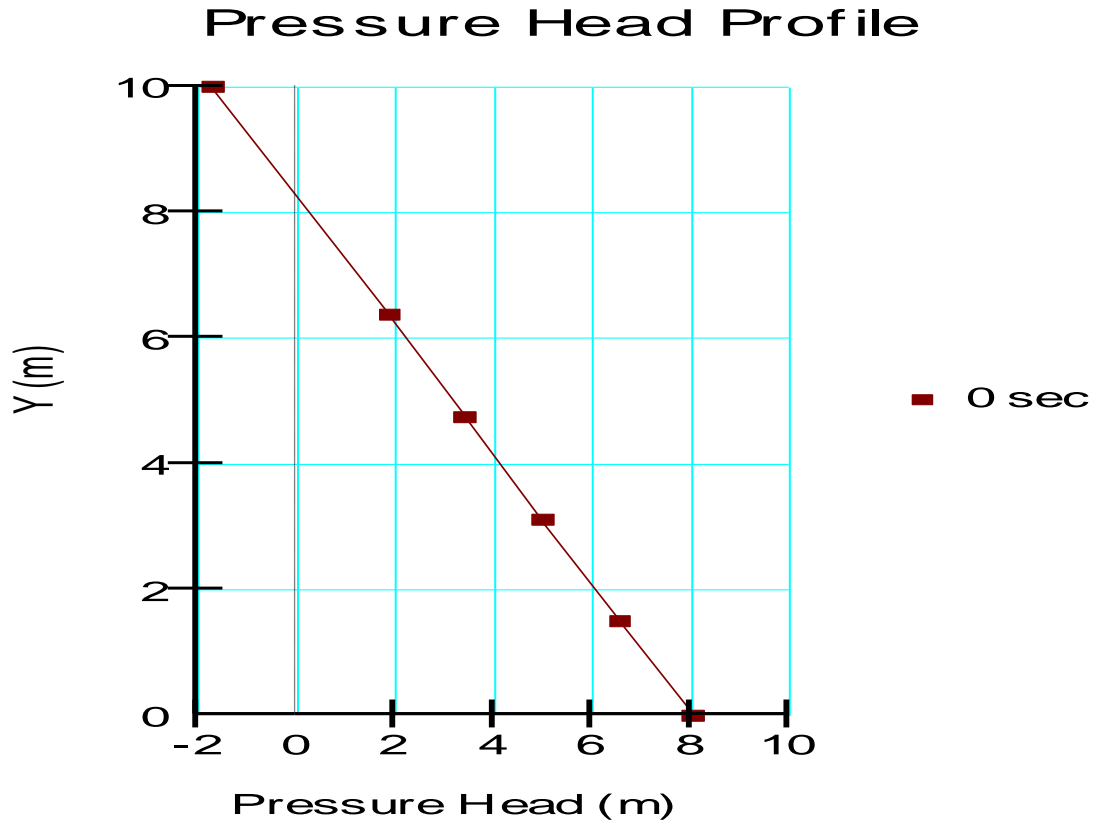


Figure 5.1.18: Variation of Pressure Head with Elevation

5.1.4 Height = 15 m and Slope = 1:1

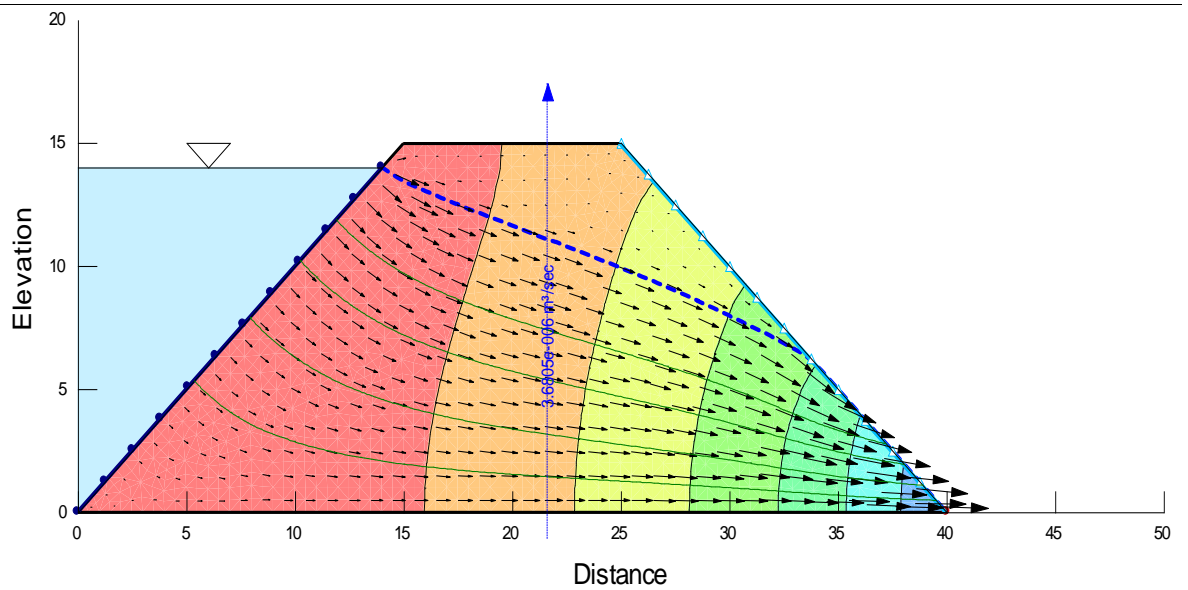


Figure 5.1.19: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

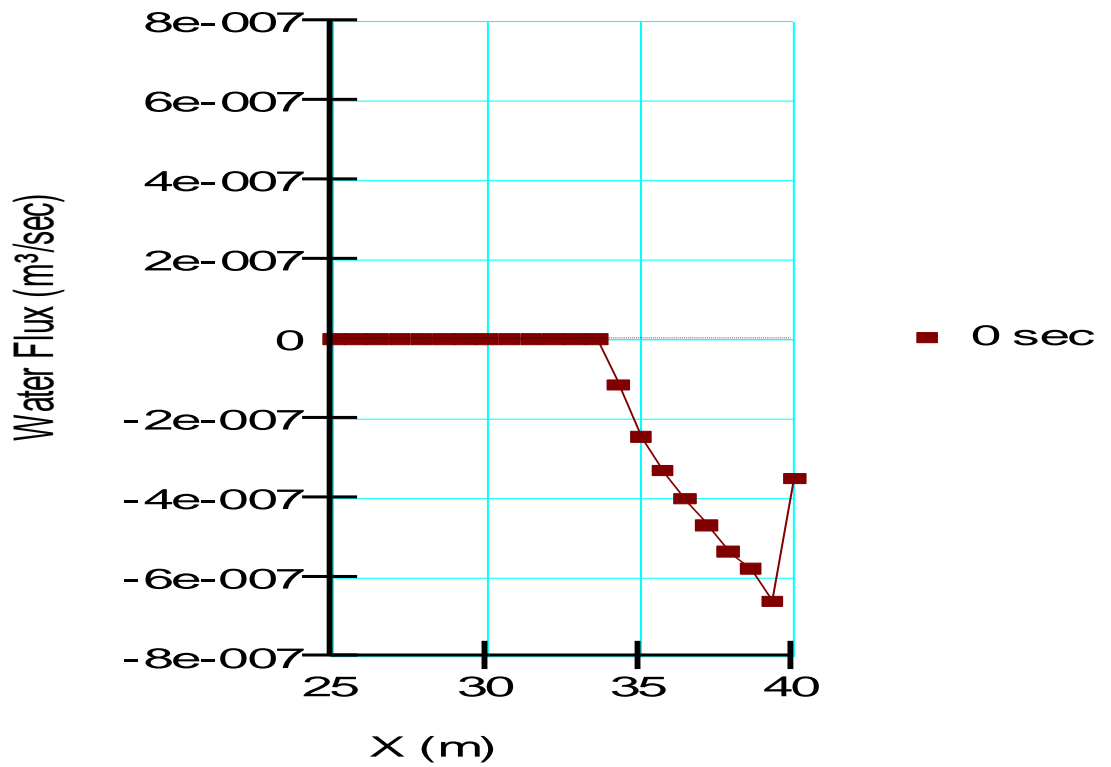


Figure 5.1.20: Variation of Flux with horizontal distance

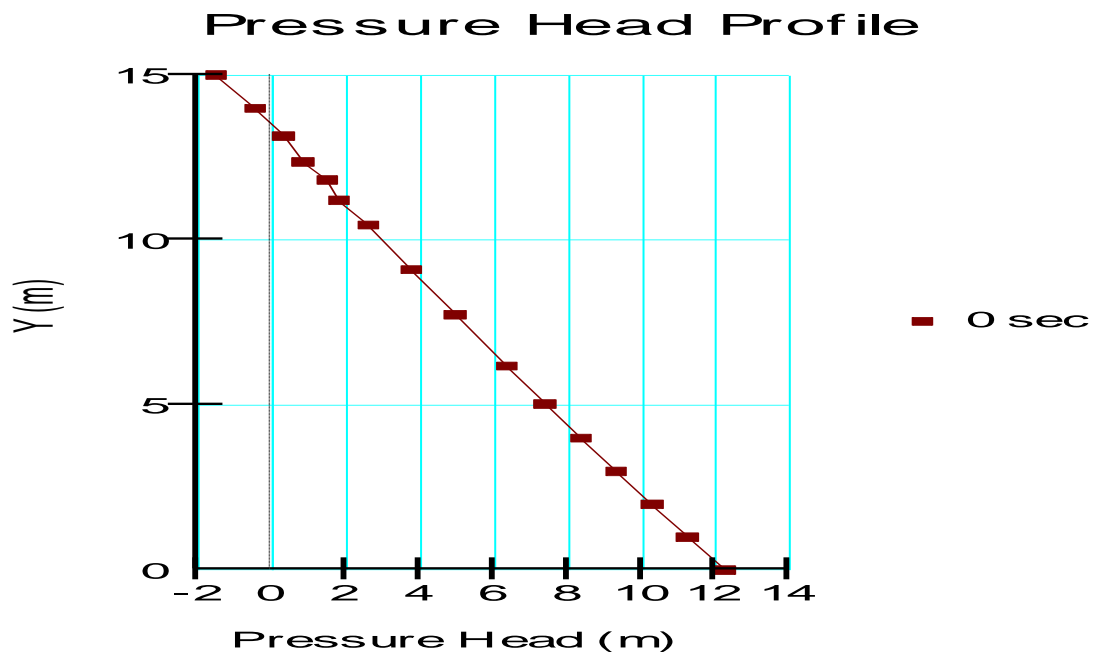


Figure 5.1.21: Variation of Pressure Head with Elevation

5.1.4b With Blanket Drain

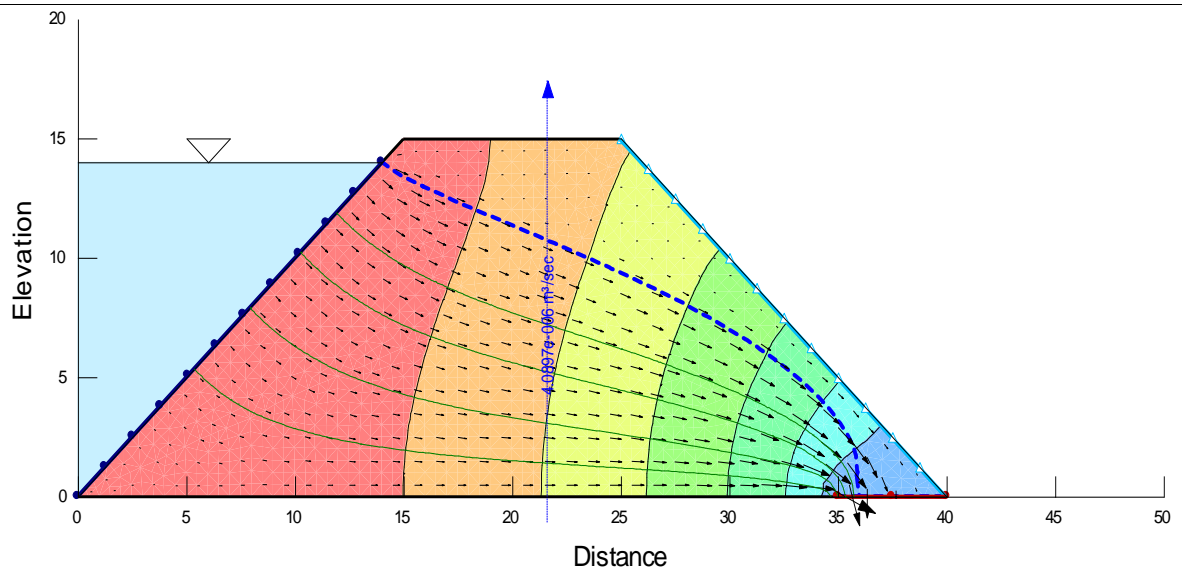


Figure 5.1.22: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

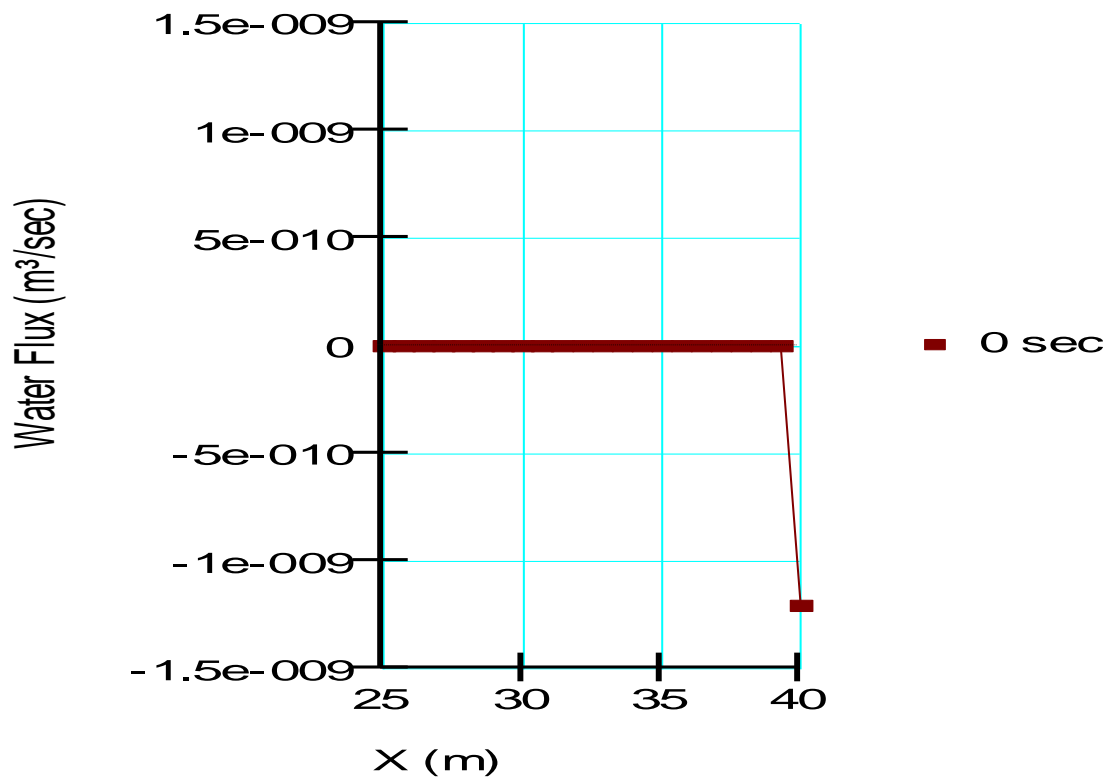


Figure 5.1.23: Variation of Flux with horizontal distance

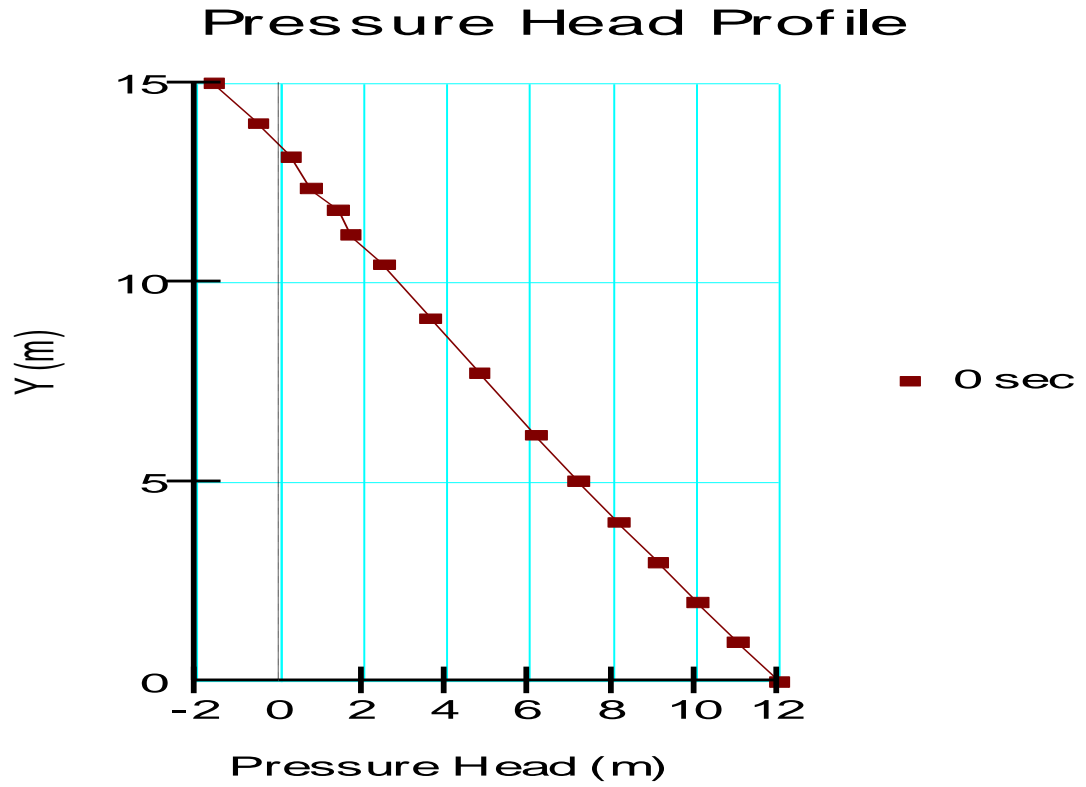


Figure 5.1.24: Variation of Pressure Head with Elevation

5.1.5 Height = 15 m and Slope = 1:2

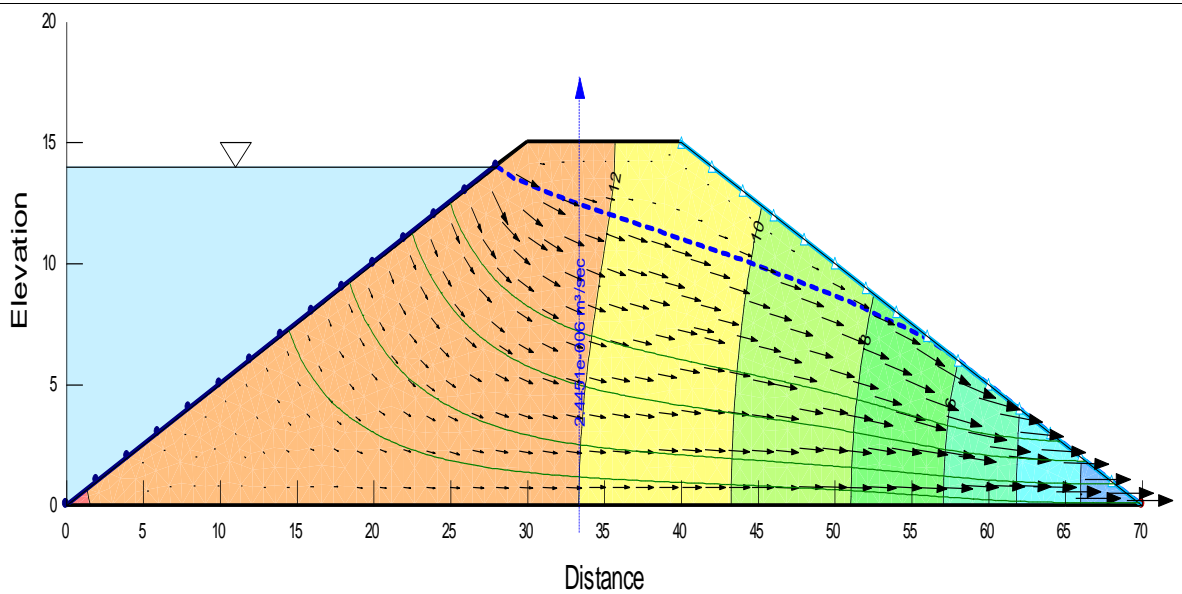


Figure 5.1.25: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

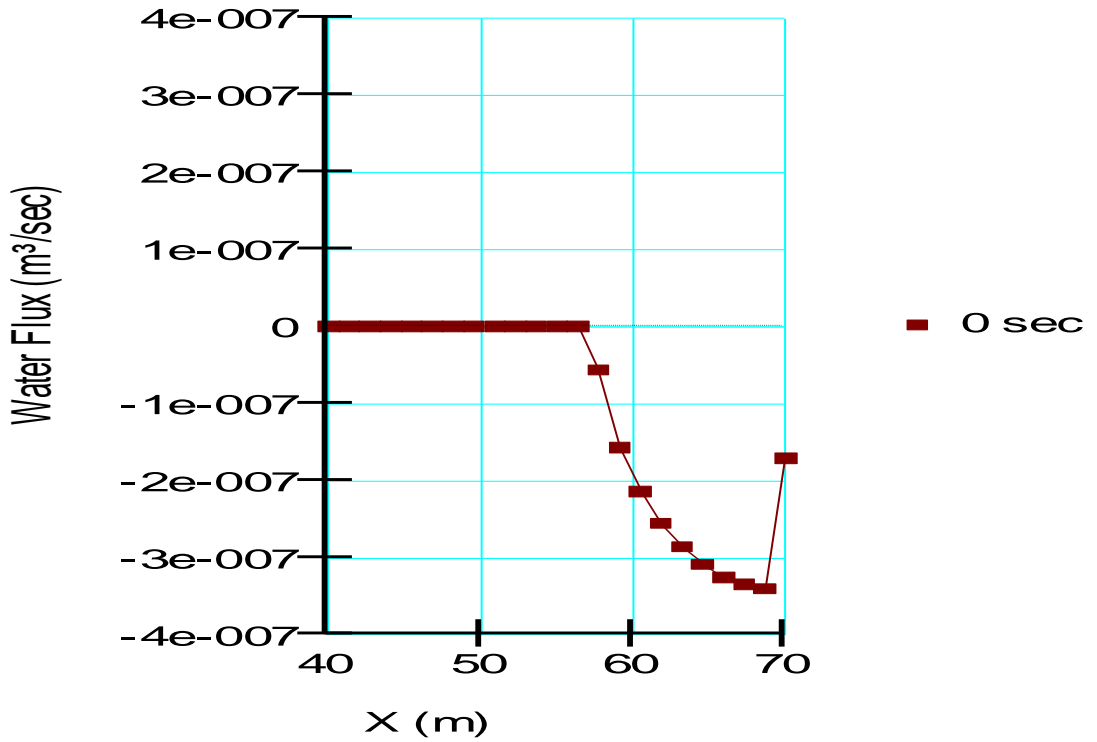


Figure 5.1.26: Variation of Flux with horizontal distance

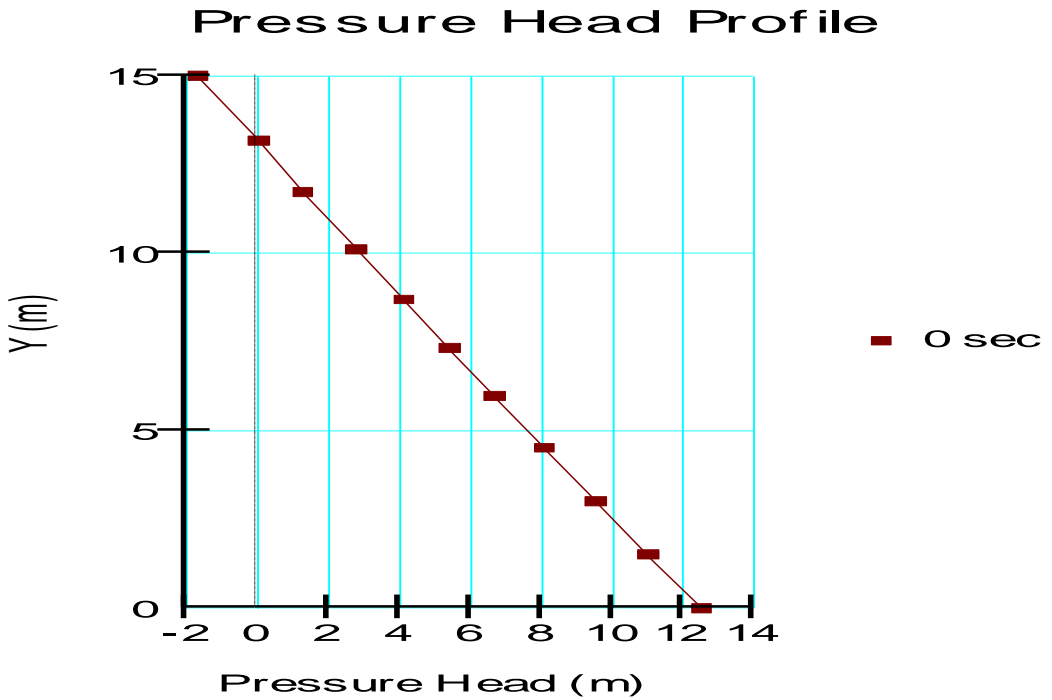


Figure 5.1.27: Variation of Pressure Head with Elevation

5.1.5b With Drainage Filter

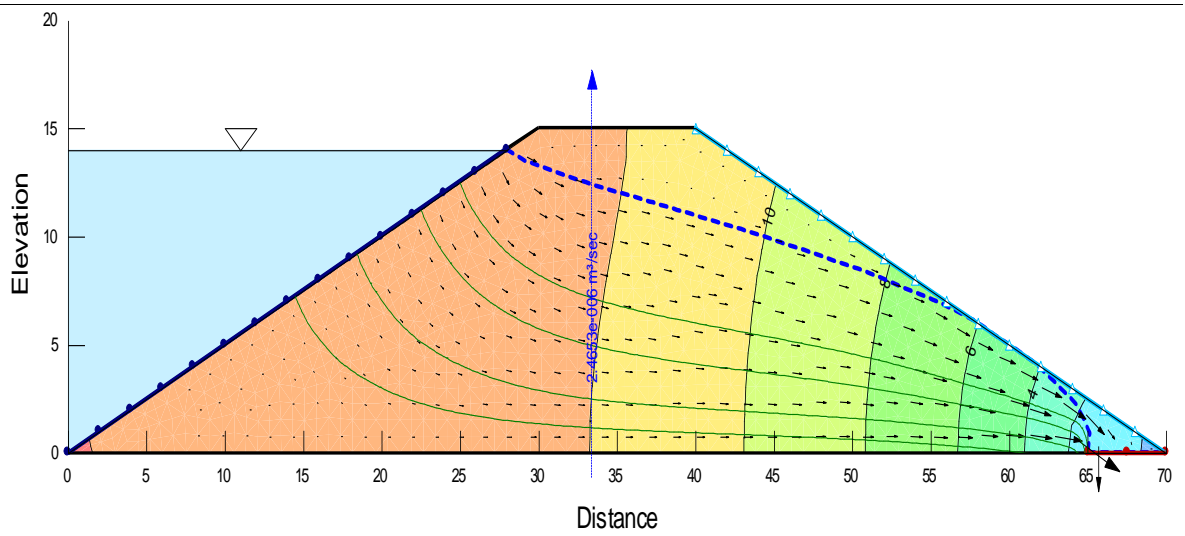


Figure 5.1.28: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

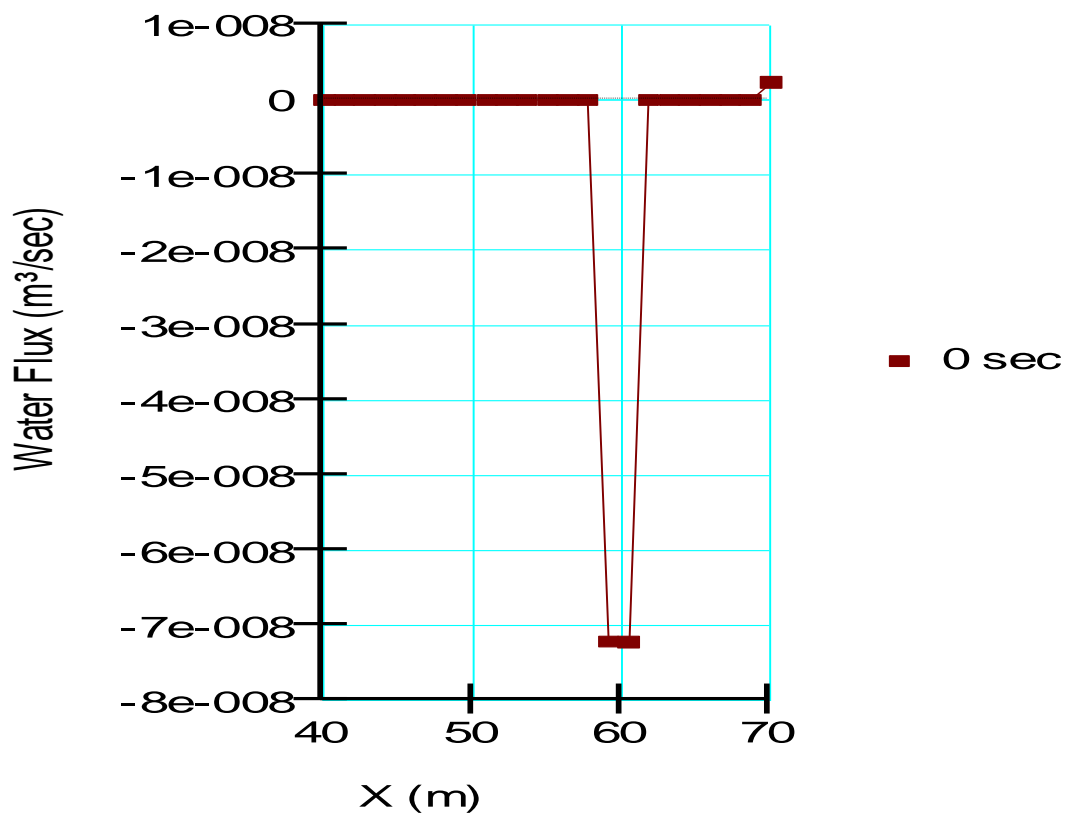


Figure 5.1.29: Variation of Flux with horizontal distance

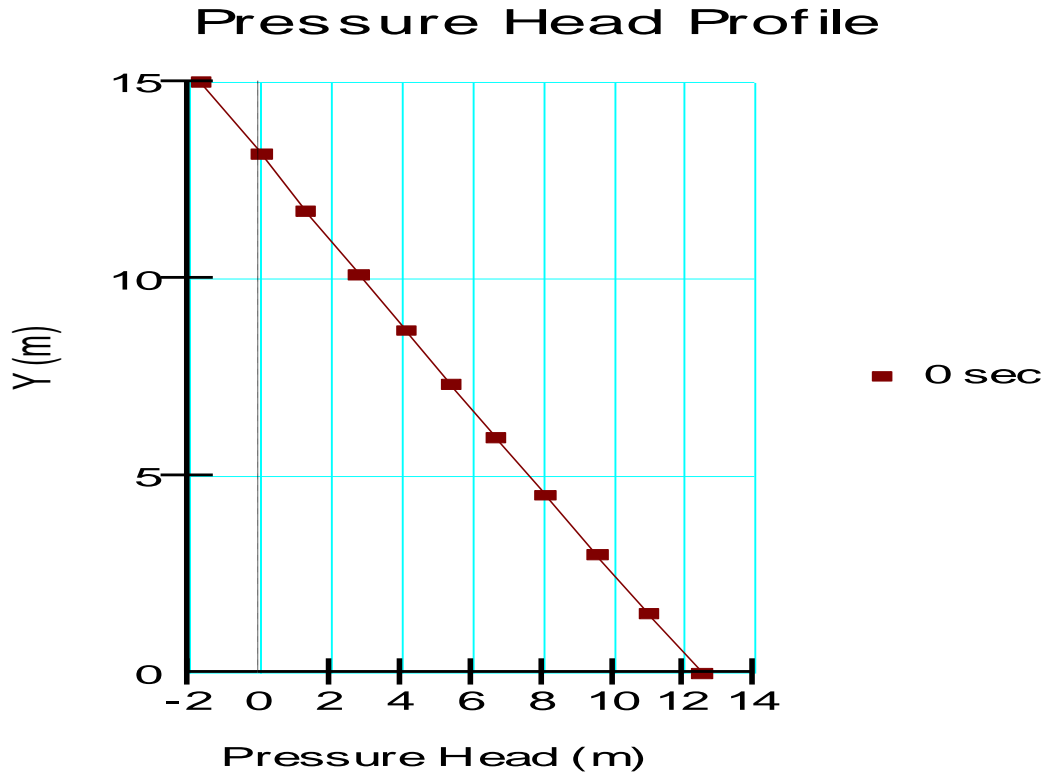


Figure 5.1.30: Variation of Pressure Head with Elevation

5.1.6 Height = 15 m and Slope = 1:3

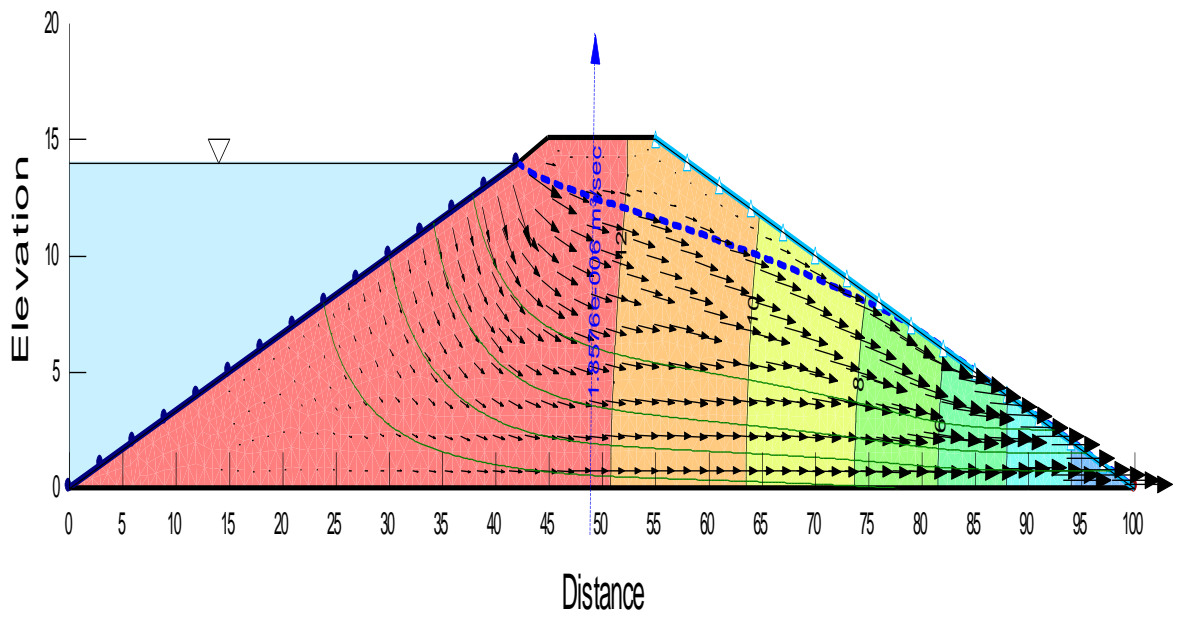


Figure 5.1.31: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

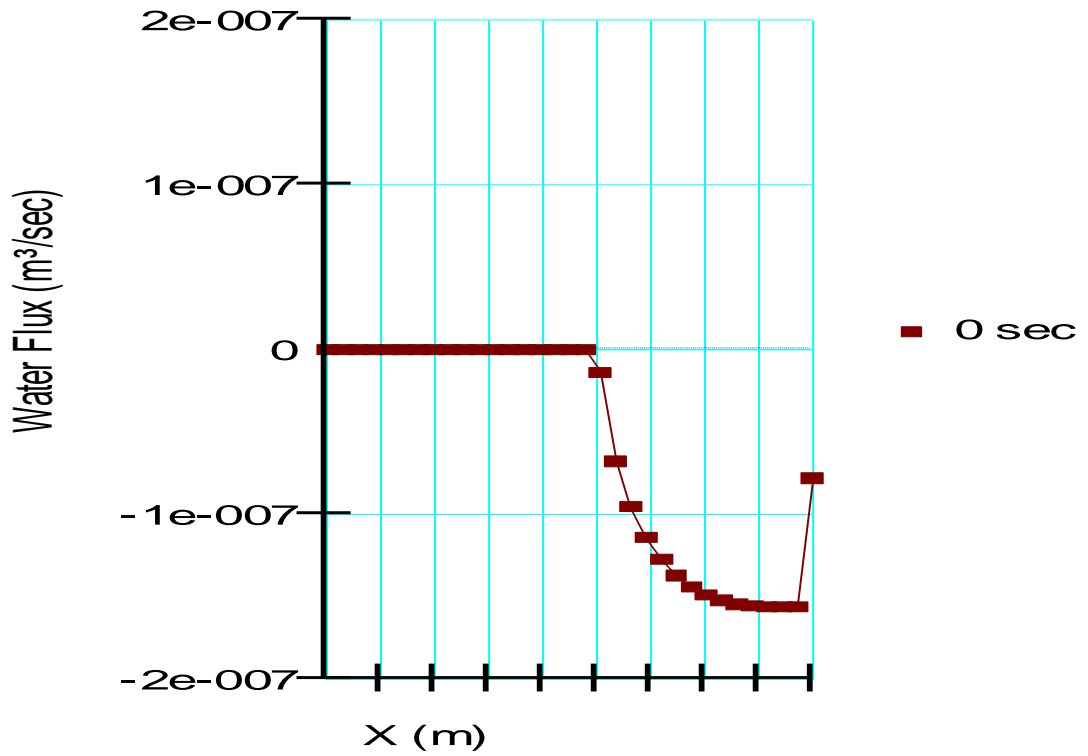


Figure 5.1.32: Variation of Flux with horizontal distance

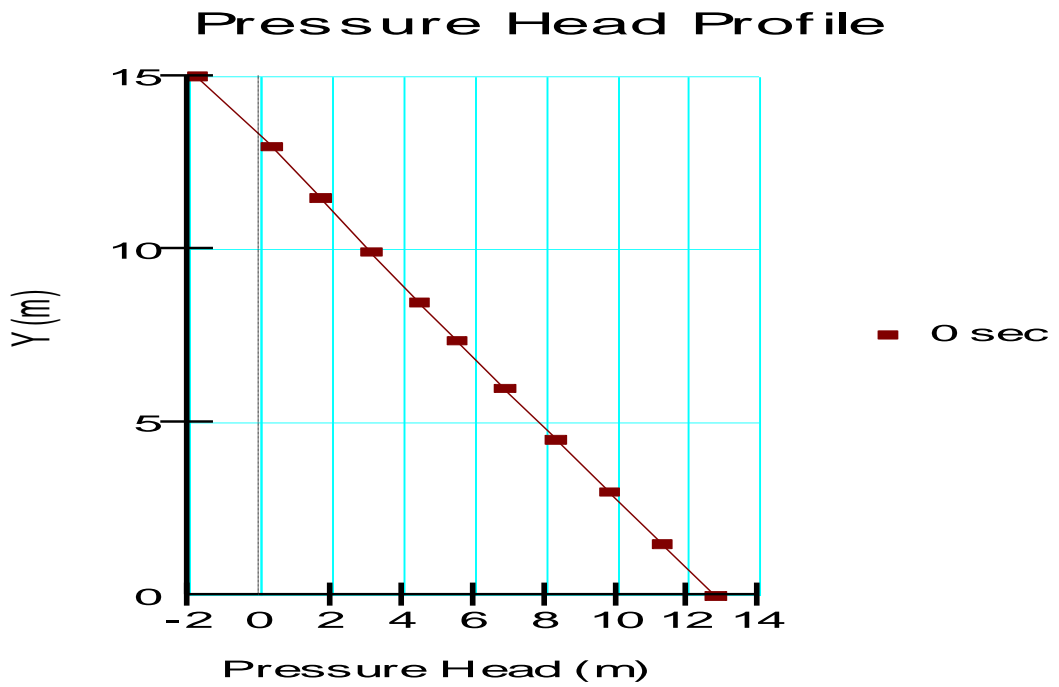


Figure 5.1.33: Variation of Pressure Head with Elevation

5.1.6b With Drainage Filter

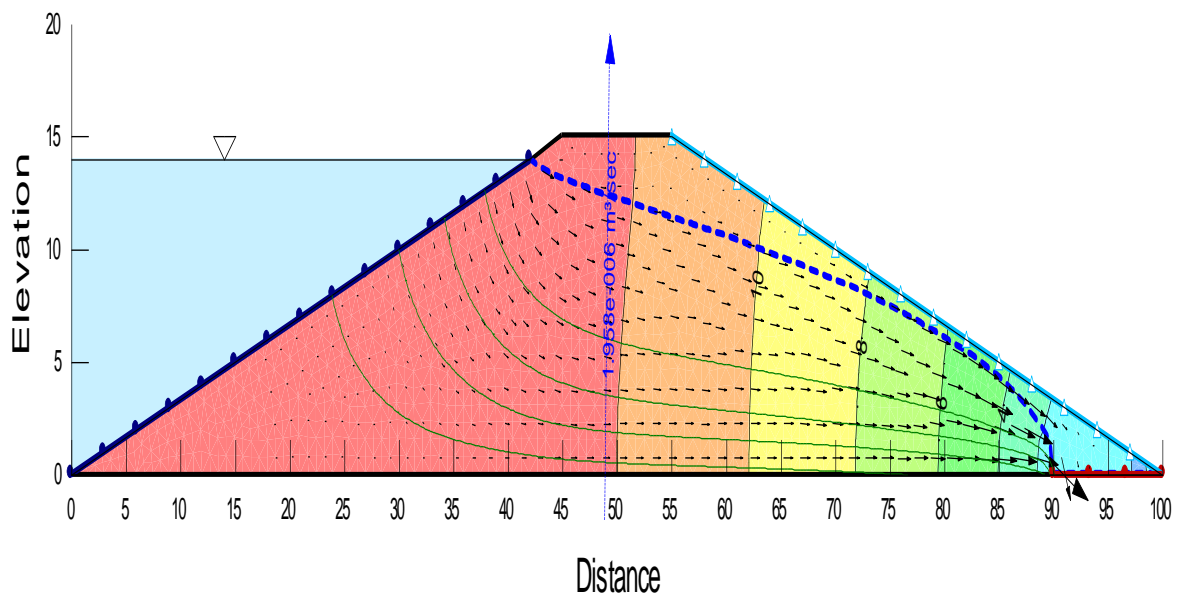


Figure 5.1.34: Profile of the Embankment having filter with seepage line

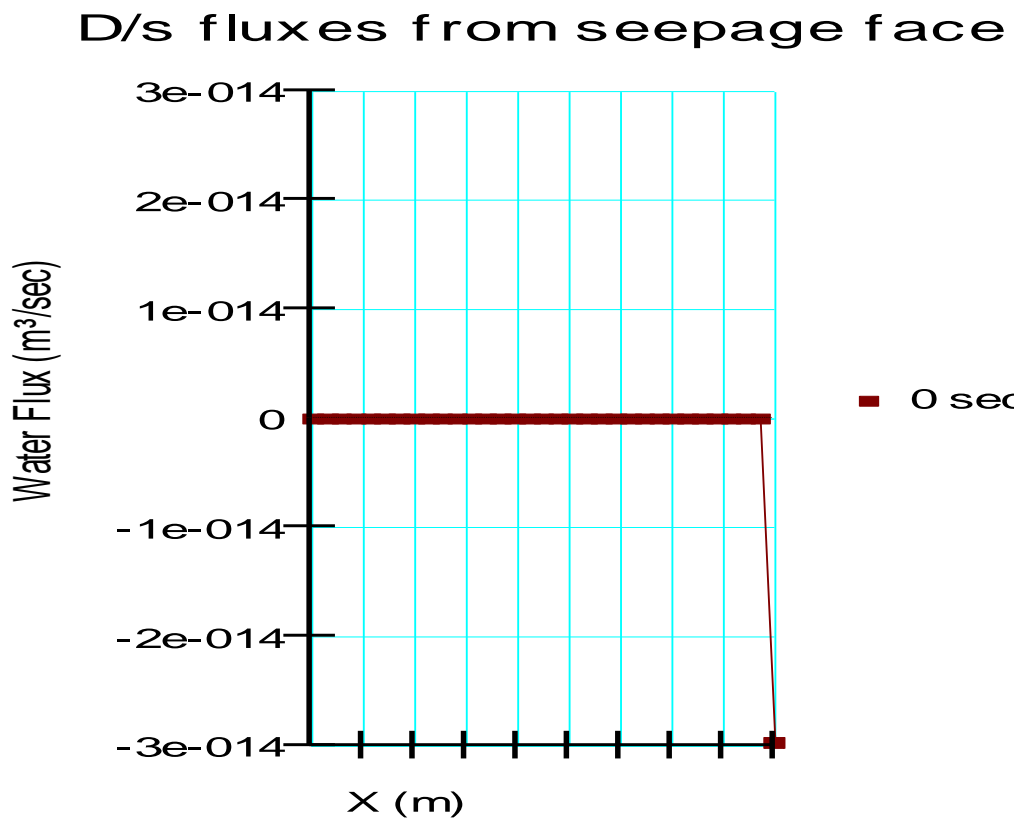


Figure 5.1.35: Variation of Flux with horizontal distance

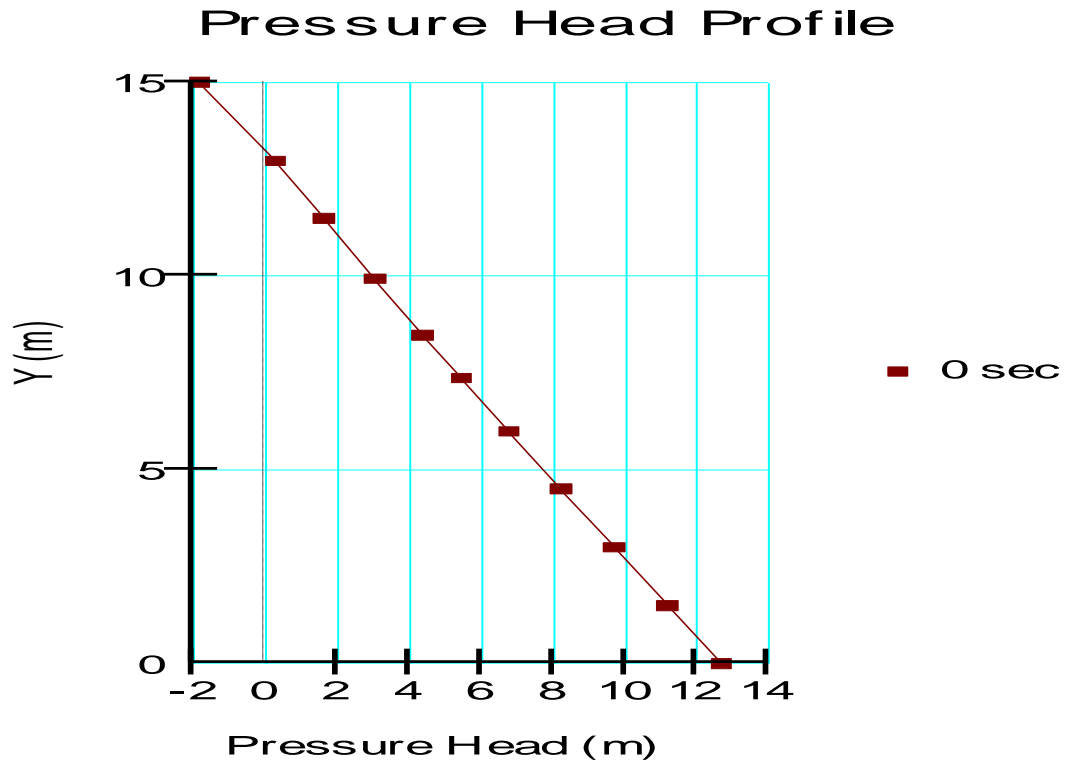


Figure 5.1.36: Variation of Pressure Head with Elevation

5.1.7 Height = 20 m and Slope = 1:1

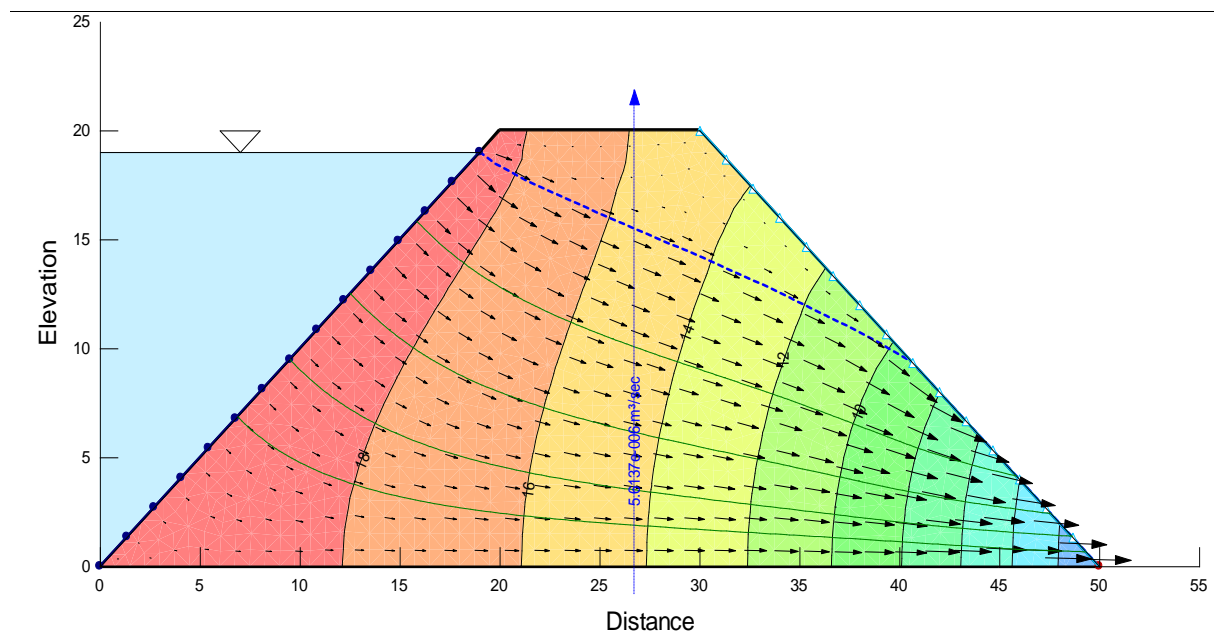


Figure 5.1.37: Profile of the Embankment with seepage line

D/s Fluxes from seepage face

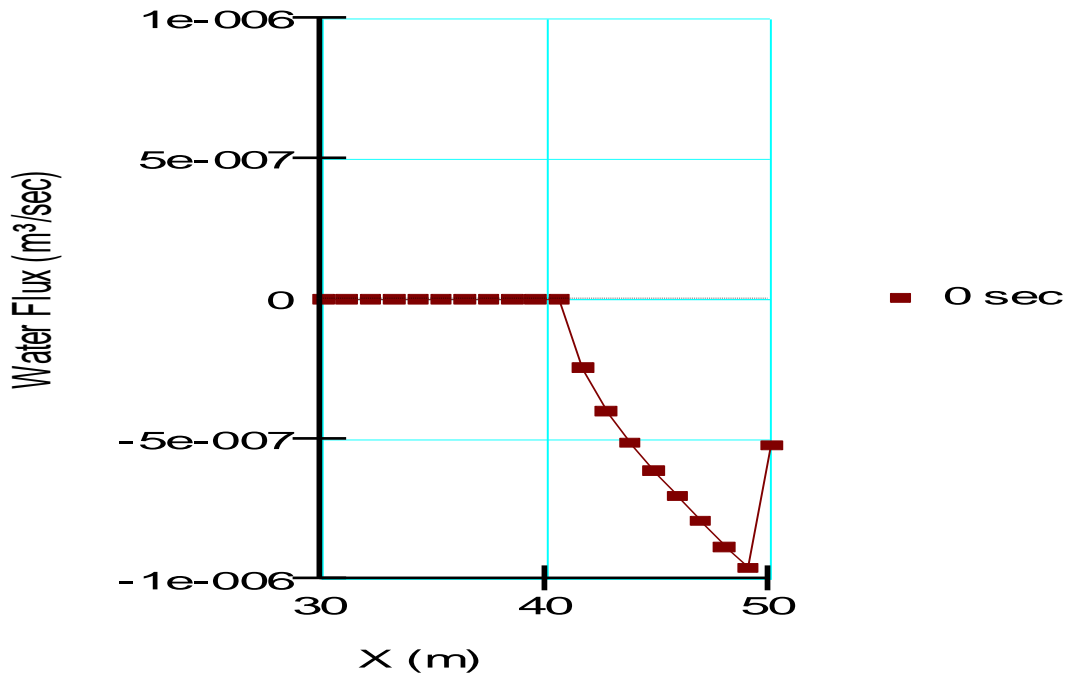


Figure 5.1.38: Variation of Flux with horizontal distance

Pressure Head Profile

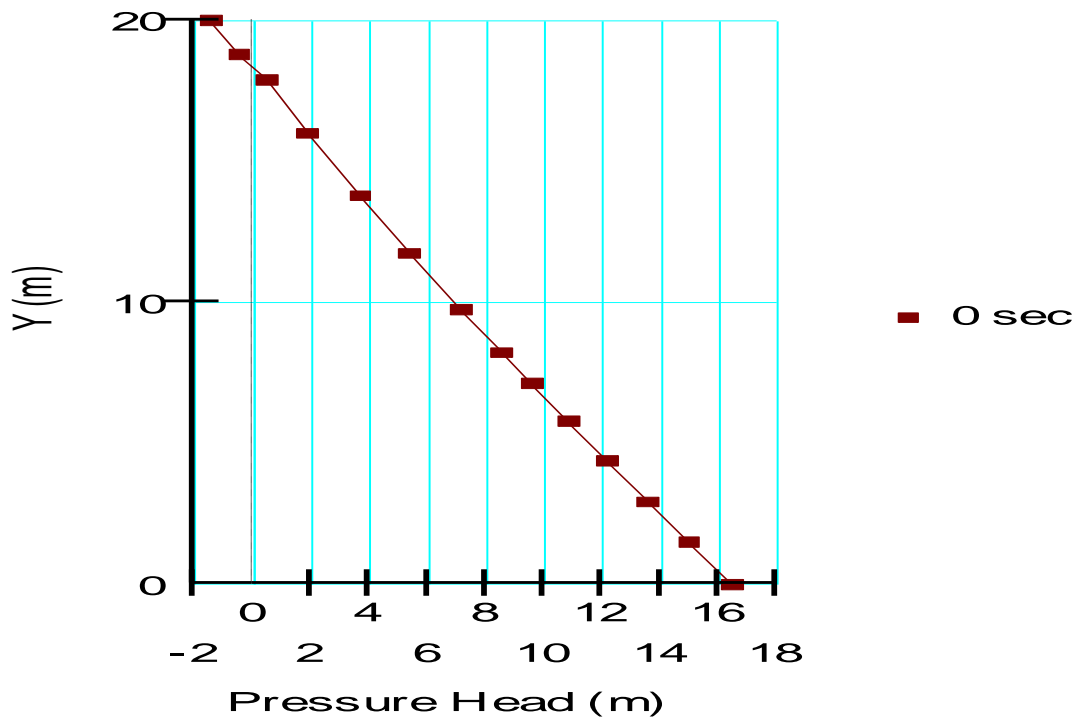


Figure 5.1.39: Variation of Pressure Head with Elevation

5.1.7b With Blanket Drain

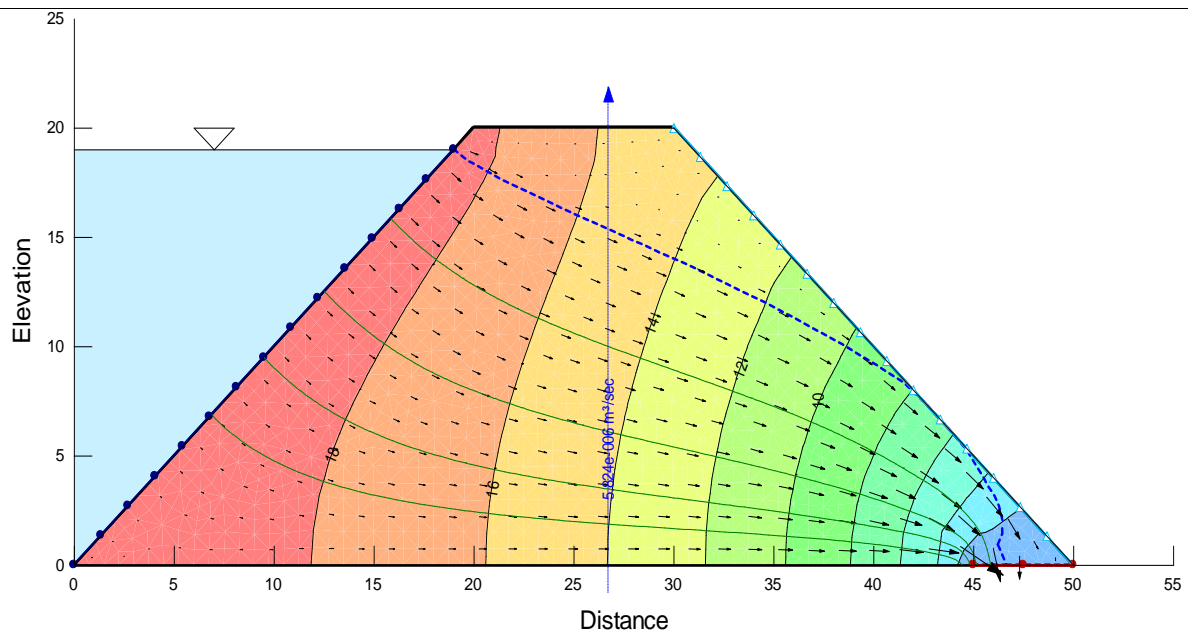


Figure 5.1.40: Profile of the Embankment having filter with seepage line

D/s Fluxes from seepage face

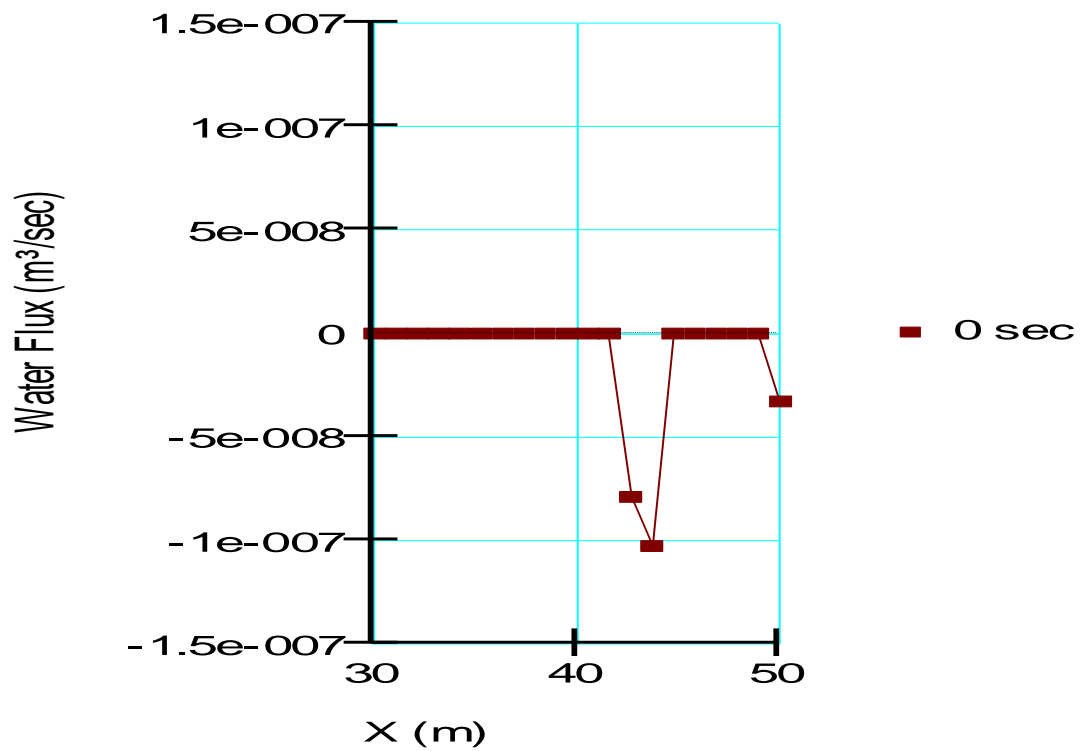


Figure 5.1.41: Variation of Flux with horizontal distance

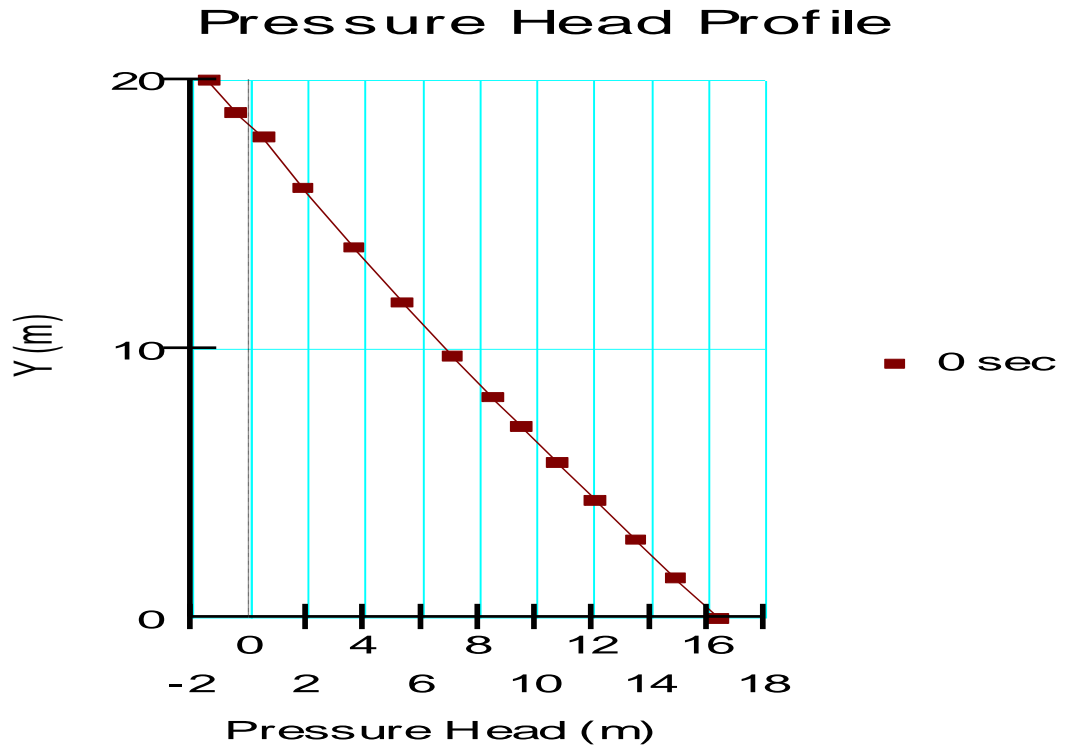


Figure 5.1.42: Variation of Pressure Head with Elevation

5.1.8 Height = 20 m and Slope = 1:2

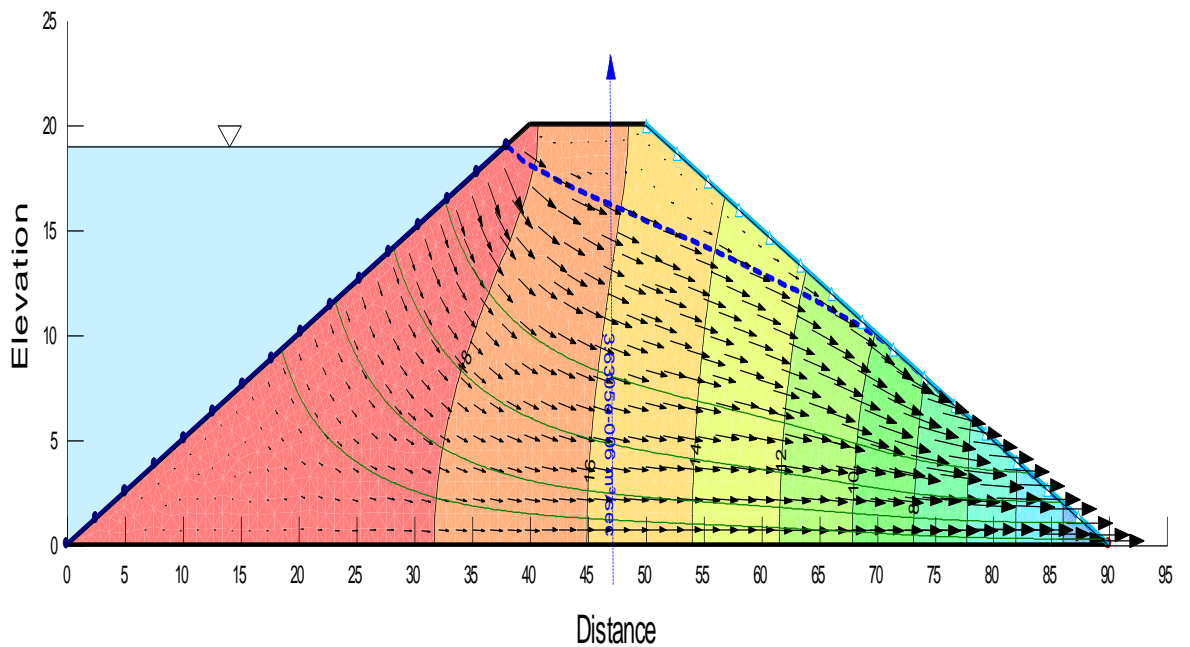


Figure 5.1.43: Profile of the Embankment having filter with seepage line

D/s Fluxes from seepage face

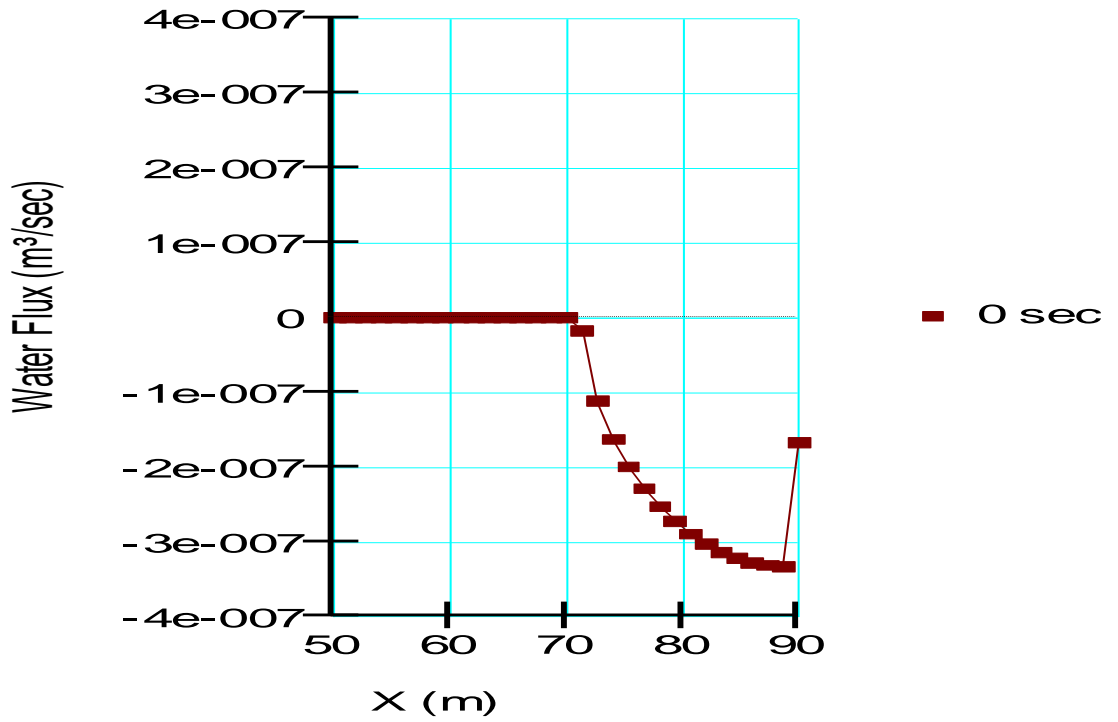


Figure 5.1.44: Variation of Flux with horizontal distance

Pressure Head Profile

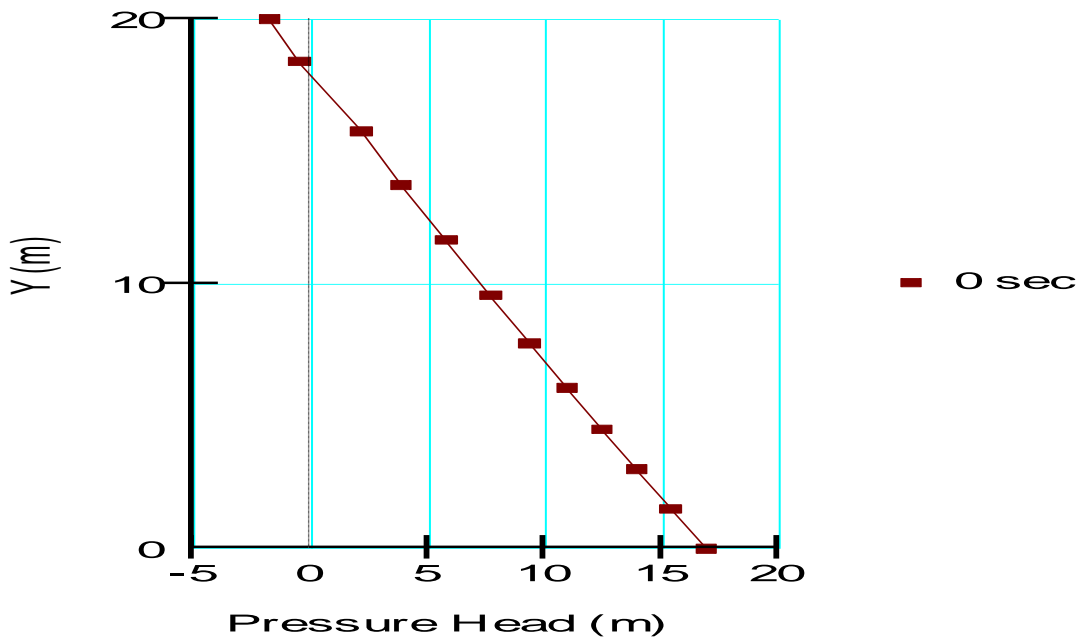


Figure 5.1.45: Variation of Pressure Head with Elevation

5.1.8b With Drainage Filter

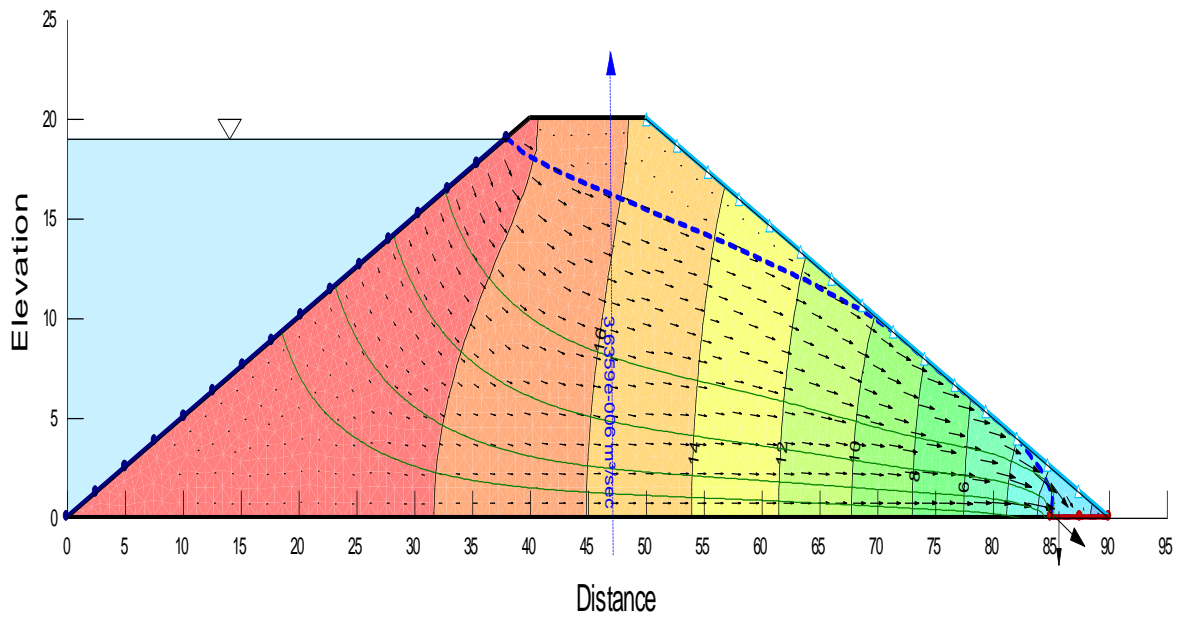


Figure 5.1.46: Profile of the Embankment having filter with seepage line

D/s Fluxes from seepage face

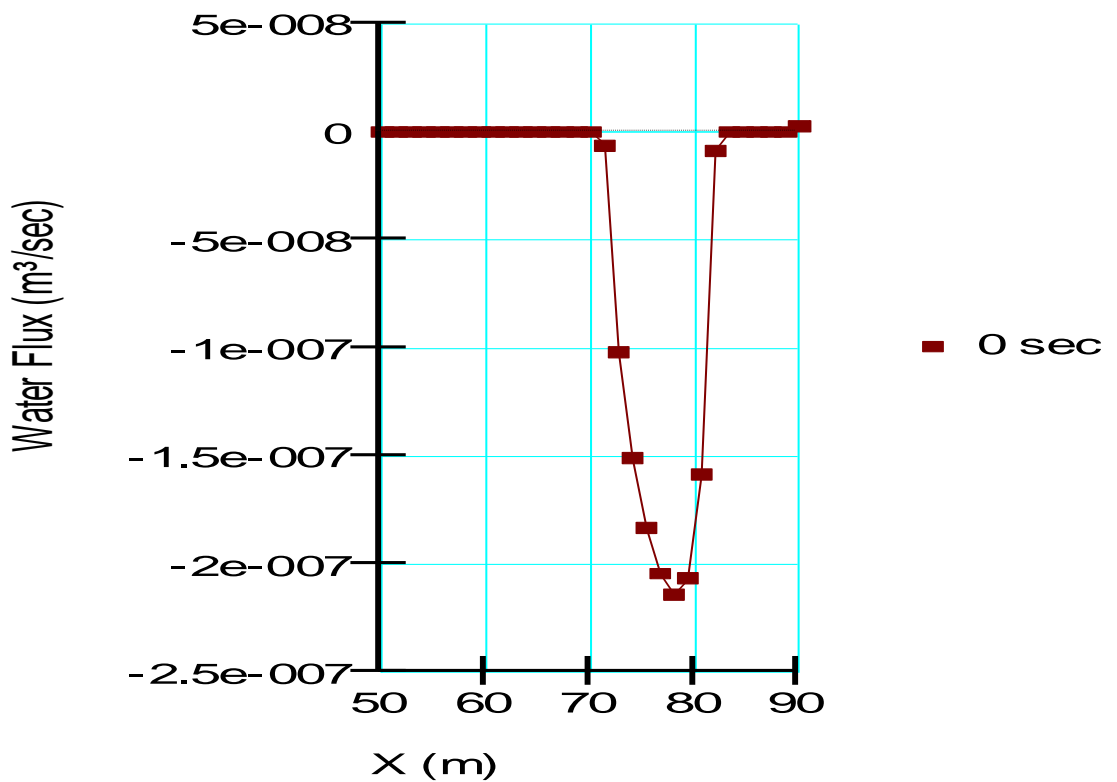


Figure 5.1.47: Variation of Flux with horizontal distance

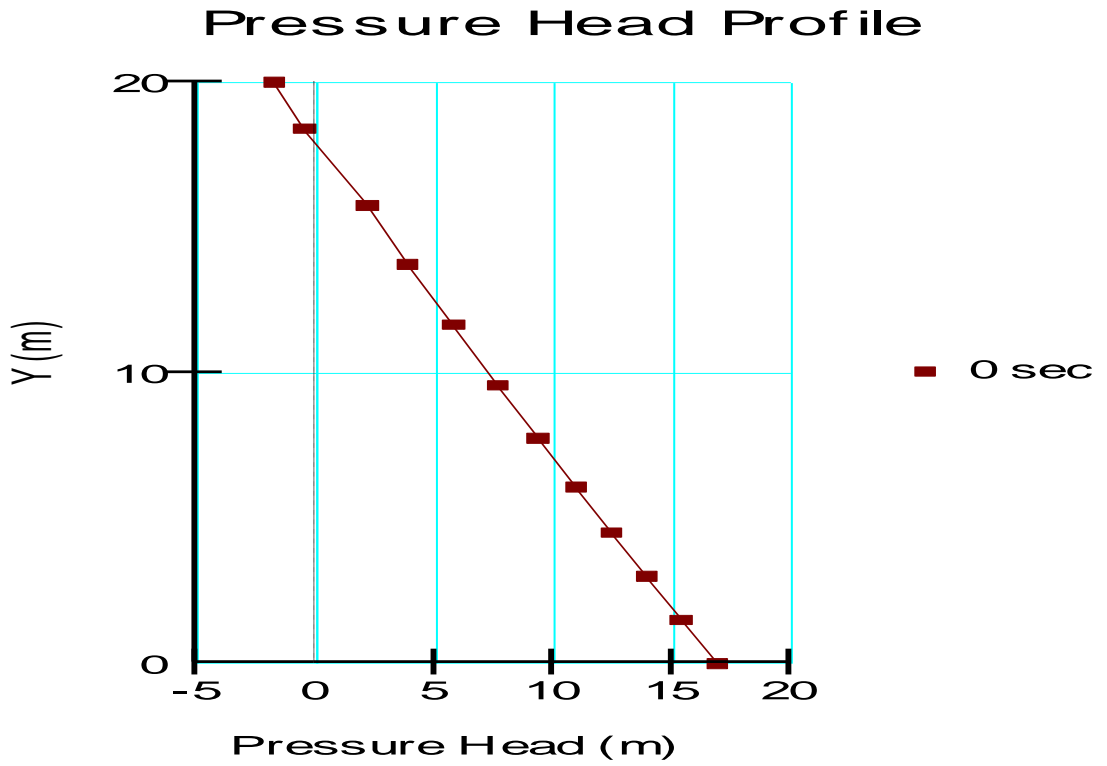


Figure 5.148: Variation of Pressure Head with Elevation

5.1.9 Height = 20 m and Slope = 1:3

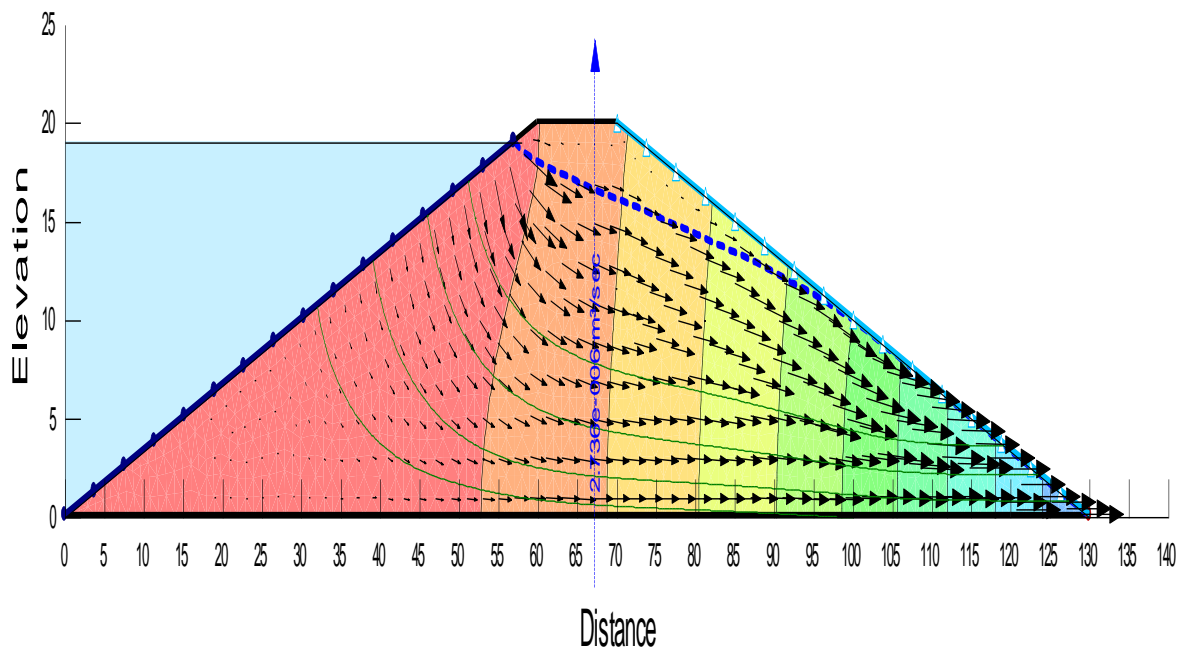


Figure 5.149: Profile of the Embankment having filter with seepage line

D/s fluxes from seepage face

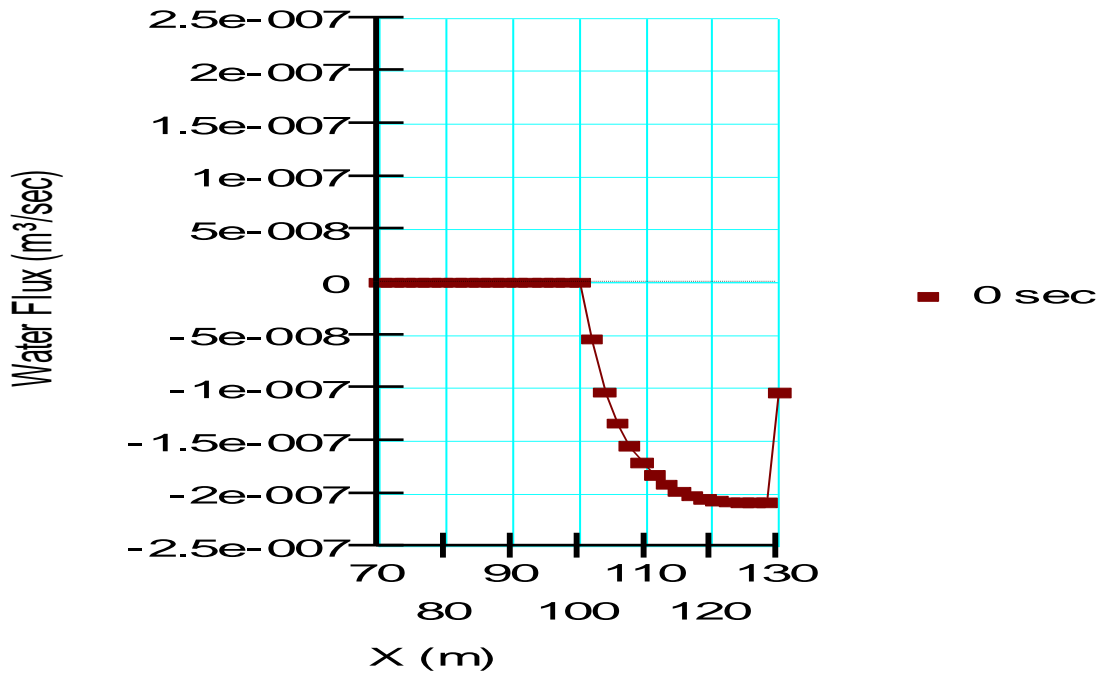


Figure 5.1.50: Variation of Flux with horizontal distance

Pressure Head Profile

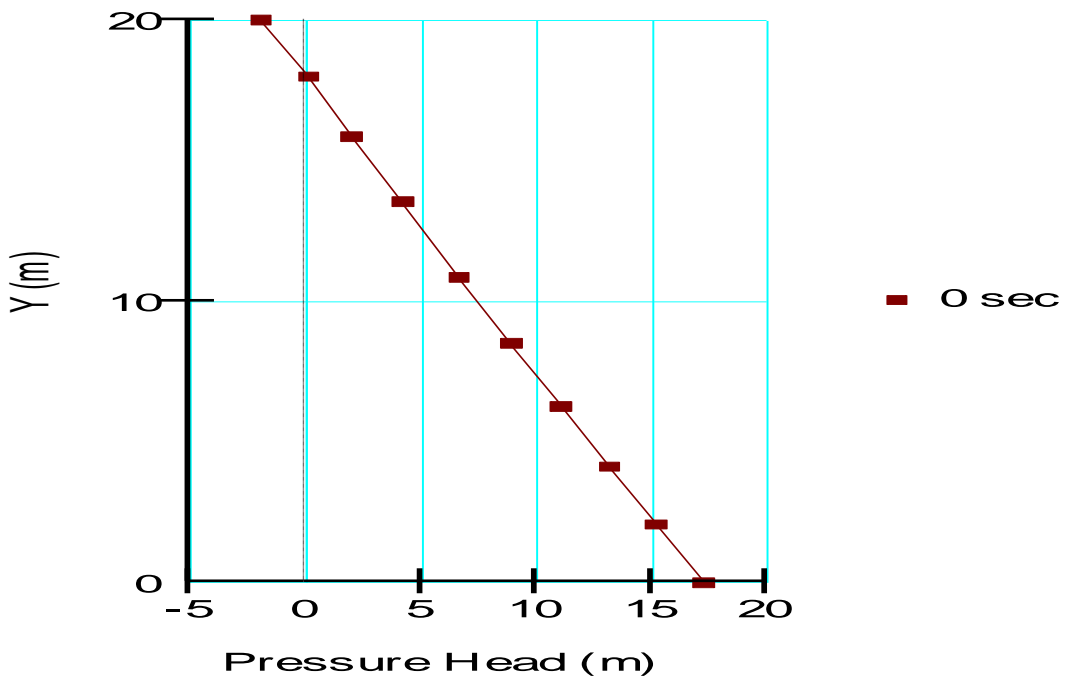


Figure 5.1.51: Variation of Pressure Head with Elevation

5.1.9b With Drainage Filter

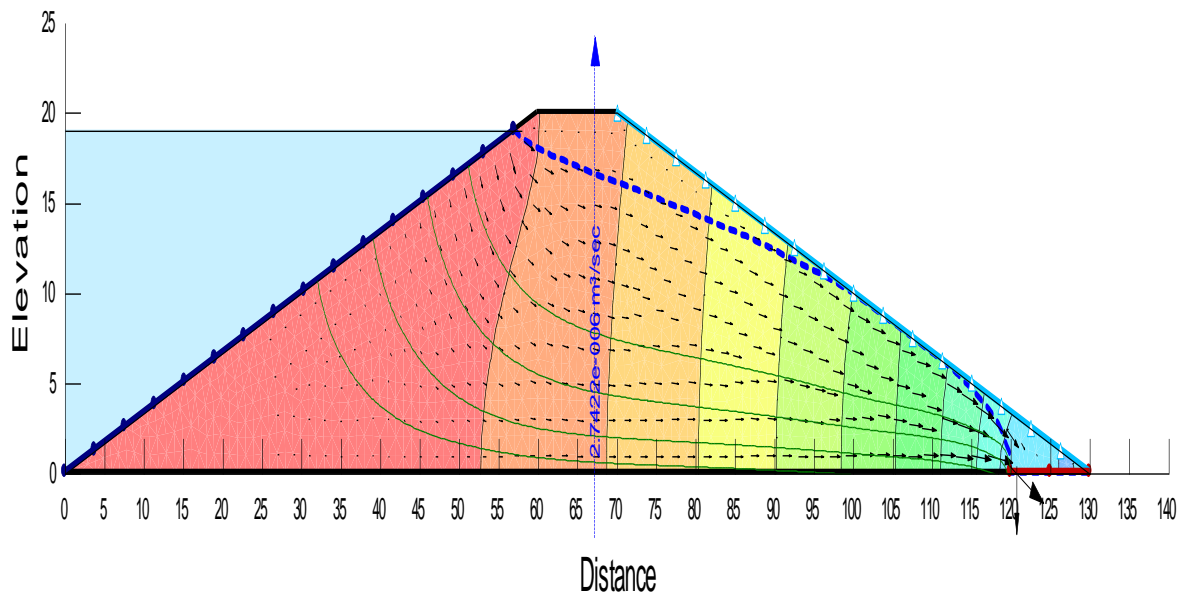


Figure 5.1.52: Profile of the Embankment having filter with seepage line

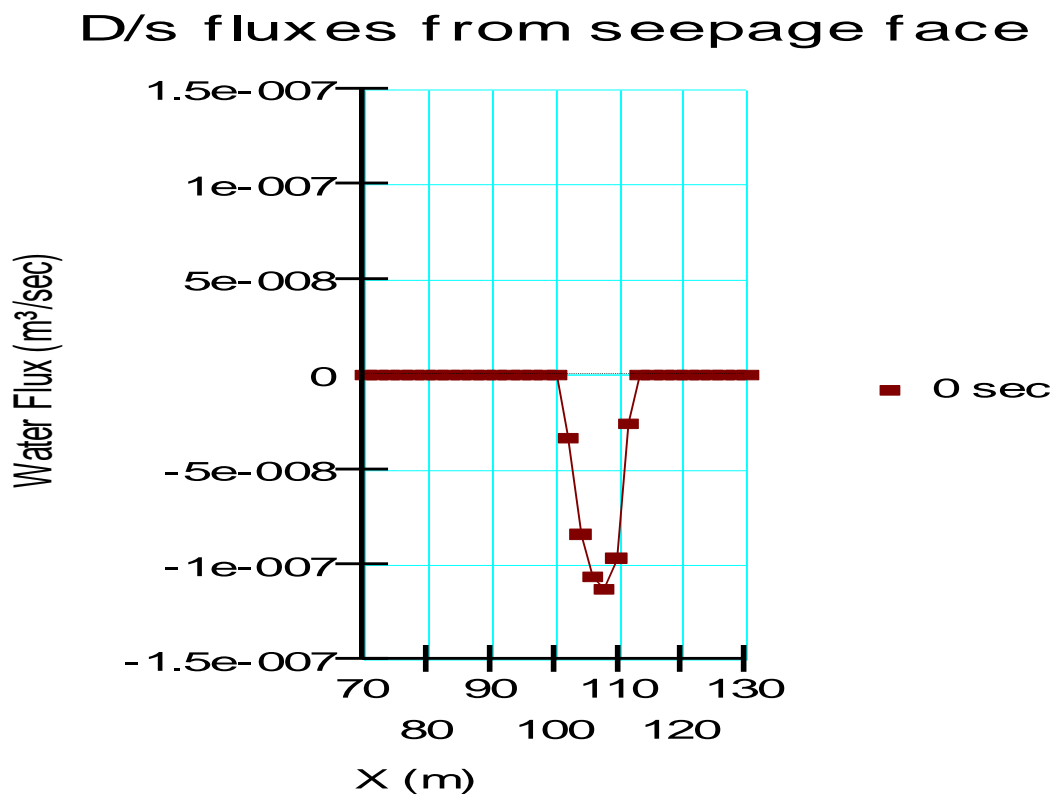


Figure 5.1.53: Variation of Flux with horizontal distance

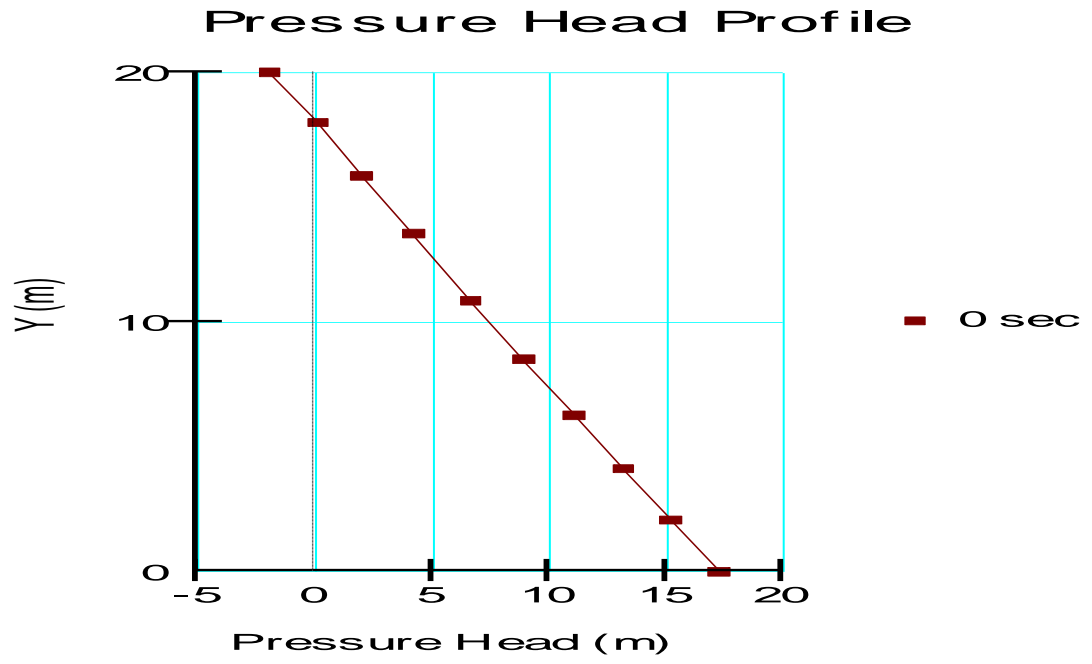


Figure 5.1.54: Variation of Pressure Head with Elevation

**Table 5.1: Variation of Discharge with Slope and Height of Embankment
(Without Drainage Filter)**

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (Without Filter) (q)
1	10 m	9 m	1:1	1.9686e-006
2	10 m	9 m	1:2	1.3398e-006
3	10 m	9 m	1:3	1.0315e-006
4	15 m	14 m	1:1	3.6805e-006
5	15 m	14 m	1:2	2.4451e-006
6	15 m	14 m	1:3	1.8576e-006
7	20 m	19 m	1:1	5.6137e-006
8	20 m	19 m	1:2	3.6305e-006
9	20 m	19 m	1:3	2.7360e-006

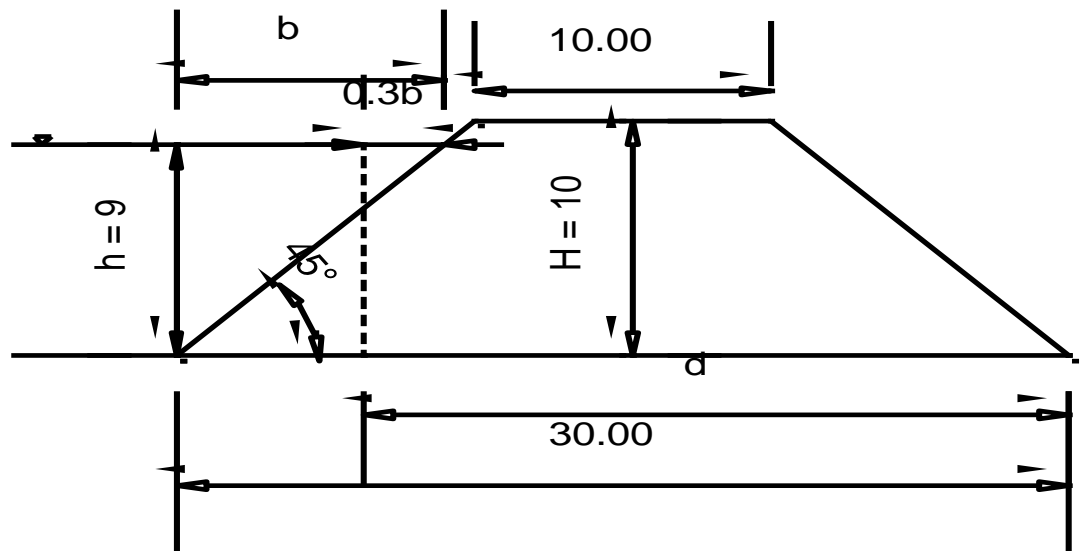
**Table 5.2: Variation of Discharge with Slope and Height of Embankment
(With Drainage Filter)**

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (With Filter) (q)
1	10 m	9 m	1:1	2.4224e-006
2	10 m	9 m	1:2	1.4858e-006
3	10 m	9 m	1:3	1.2439e-006
4	15 m	14 m	1:1	4.0897e-006
5	15 m	14 m	1:2	2.4653e-006
6	15 m	14 m	1:3	1.9580e-006
7	20 m	19 m	1:1	5.8240e-006
8	20 m	19 m	1:2	3.6359e-006
9	20 m	19 m	1:3	2.7422e-006

5.2 Result using Analytical Method

Analytical method is basically a graphical method which uses the theory of parabola to determine the seepage through the body of the embankment dam. The result obtained by this method is little lower than that obtained using Finite Element Method due to lesser accuracy and assumptions made in this theory. However, there is not much difference in the result obtained by this method.

5.2.1 Height (H) = 10 m and Slope = 1:1 (Without Drainage Filter)



Height of water level (h) = 9 m

Therefore,

$$\tan \beta = (1/1)$$

$$\text{or, } \beta = 45^\circ > 30^\circ$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 45^\circ = 0.707$$

$$i = 0.707$$

$$b = (1 \times 9) = 9 \text{ m}$$

$$d = 10 + 10 + 1 + (0.3 \times 9)$$

$$d = 23.7 \text{ m}$$

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 + (h \cot \beta)^2}$$

$$a = \sqrt{23.7^2 + 9^2} - \sqrt{23.7^2 + (9 \cot 45)^2}$$

$$a = 3.43 \text{ m}$$

Discharge is given as,

$$q = k \times a \times \sin^2 \beta$$

$$q = (1 \times 10^{-6}) \times 3.43 \times \sin^2 45$$

$$q = 1.715 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.1b With Drainage Filter

$$\text{Height of Reservoir (h)} = 9 \text{ m}$$

Therefore,

$$b = 1 \times 9 = 9 \text{ m}$$

$$d = (10 - 5) + 10 + (1 \times 1) + (0.3 \times 9)$$

$$d = 18.7 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{18.7^2 + 9^2} - 18.7$$

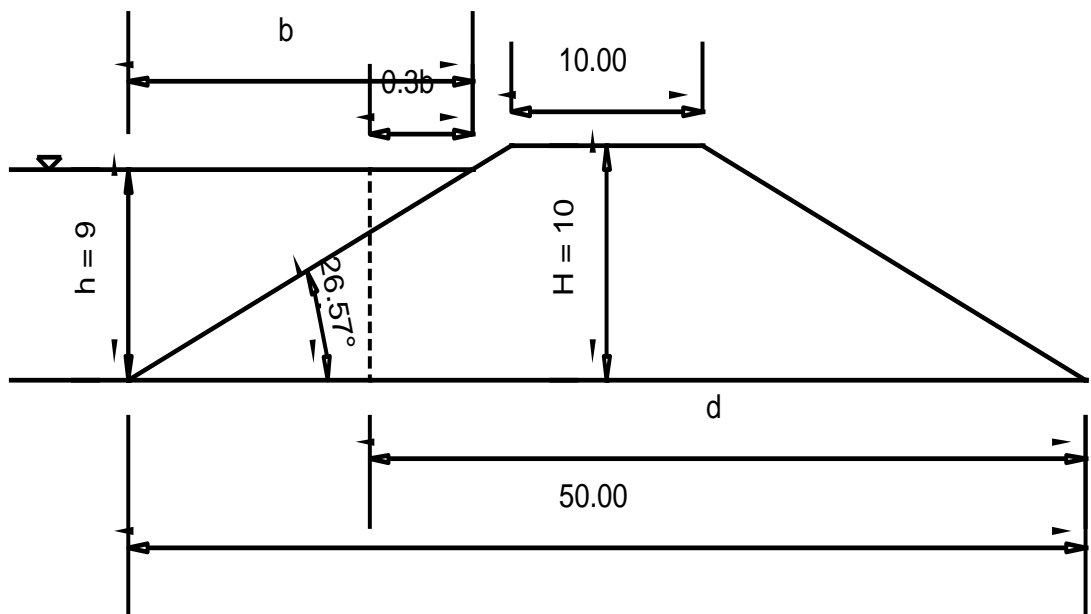
$$s = 2.053 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 2.053$$

$$q = 2.053 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.2 Height (H) = 10 m and Slope = 1:2 (Without Drainage Filter)



Height of water level (h) = 9 m

Therefore,

$$\tan \beta = (1/3)$$

$$\text{or, } \beta = 26.56^\circ < 30^\circ$$

$$b = (h \times 2)$$

$$b = 18 \text{ m}$$

$$d = (10 \times 2) + 10 + (1 \times 2) + (0.3 \times 18)$$

$$d = 37.4 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 26.56 = 0.447$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{37.4}{\cos 26.56} - \sqrt{\left(\frac{37.4}{\cos 26.56}\right)^2 - \left(\frac{9}{\sin 26.56}\right)^2}$$

$$a = 5.16 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times e-006) (5.16 \times \sin 26.56) \tan 26.56$$

$$q = 1.152 \times e-006 \text{ m}^3/\text{s}$$

5.2.2b Without Drainage Filter

$$\text{Height of Reservoir (h)} = 9 \text{ m}$$

Therefore,

$$b = 2 \times 9 = 18 \text{ m}$$

$$d = (20 - 10) + 10 + (1 \times 2) + (0.3 \times 18)$$

$$d = 27.4 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{27.4^2 + 9^2} - 27.4$$

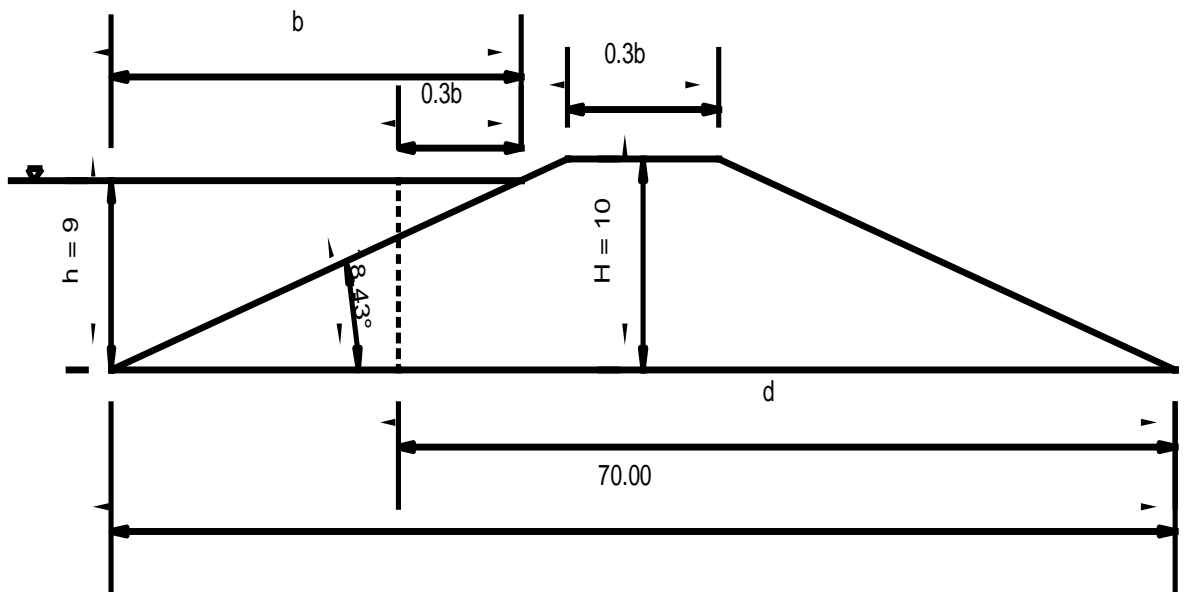
$$s = 1.440 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 1.440$$

$$q = 1.440 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.3 Height (H) = 10 m and Slope = 1:3 (Without Drainage Filter)



Height of water level (h) = 9 m

Therefore,

$$\tan \beta = (1/3)$$

$$\text{or, } \beta = 18.43^\circ < 30^\circ$$

$$b = (h \times 3)$$

$$b = 27 \text{ m}$$

$$d = (10 \times 3) + 10 + (1 \times 3) + (0.3 \times 27)$$

$$d = 51.1 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 18.43 = 0.316$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{51.1}{\cos 18.43} - \sqrt{\left(\frac{51.1}{\cos 18.43}\right)^2 - \left(\frac{9}{\sin 18.43}\right)^2}$$

$$a = 8.14 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times e-006) (8.14 \times \sin 18.43) \tan 18.43$$

$$q = 0.857 \times e-006 \text{ m}^3/\text{s}$$

5.2.3b With Drainage Filter

$$\text{Height of Reservoir (h)} = 9 \text{ m}$$

$$b = 3 \times 9 = 27 \text{ m}$$

$$d = (30 - 10) + 10 + (1 \times 3) + (0.3 \times 27)$$

$$d = 41.1 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{41.1^2 + 9^2} - 41.1$$

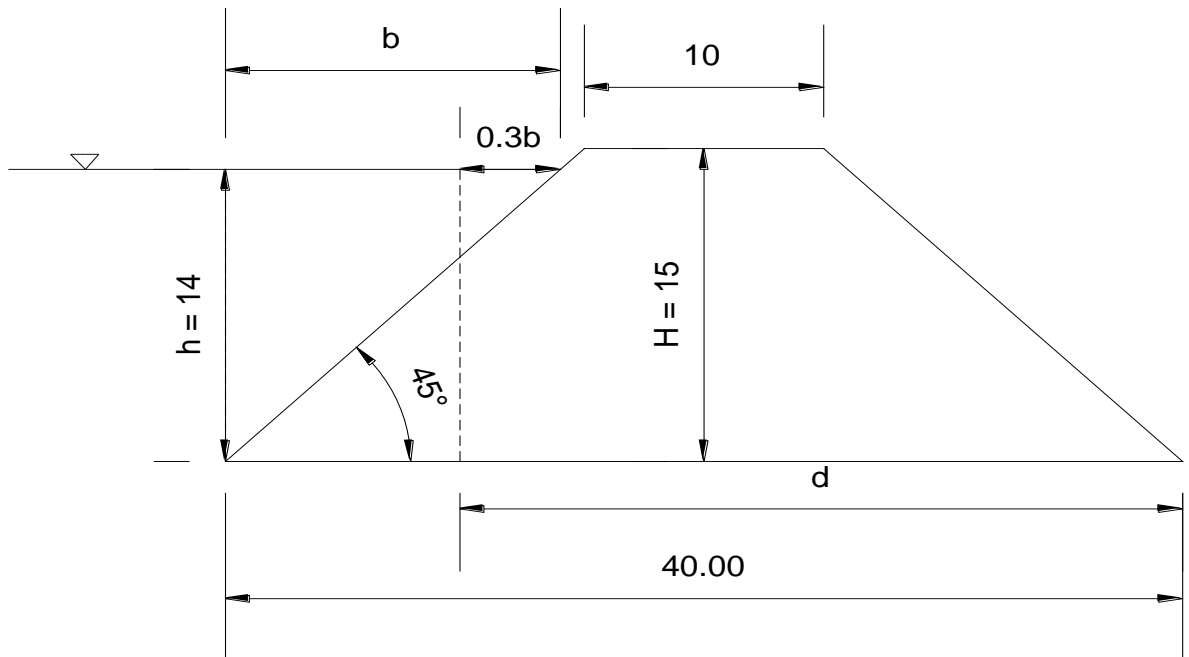
$$s = 0.974 \text{ m}$$

$$q = k s$$

$$q = (1 \times e-006) \times 0.974$$

$$q = 0.974 \times e-006 \text{ m}^3/\text{s}$$

5.2.4 Height (H) = 15 m and Slope = 1:1 (Without Drainage Filter)



Height of water level (h) = 14 m

Therefore,

$$\tan \beta = (1/1)$$

$$\text{or, } \beta = 45^\circ > 30^\circ$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 45^\circ = 0.707$$

$$i = 0.707$$

$$b = (1 \times 14) = 14 \text{ m}$$

$$d = 15 + 10 + 1 + (0.3 \times 14)$$

$$d = 30.2 \text{ m}$$

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 + (h \cot \beta)^2}$$

$$a = \sqrt{30.2^2 + 14^2} - \sqrt{30.2^2 + (14 \cot 45)^2}$$

$$a = 6.53 \text{ m}$$

Discharge is given as,

$$q = k \times a \times \sin^2 \beta$$

$$q = (1 \times 10^{-6}) \times 6.53 \times \sin^2 45$$

$$q = 3.265 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.4b With Drainage Filter

Height of Reservoir (h) = 14 m

$$b = 1 \times 14 = 14 \text{ m}$$

$$d = (15 - 5) + 10 + (1 \times 1) + (0.3 \times 14)$$

$$d = 25.2 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{25.2^2 + 14^2} - 25.2$$

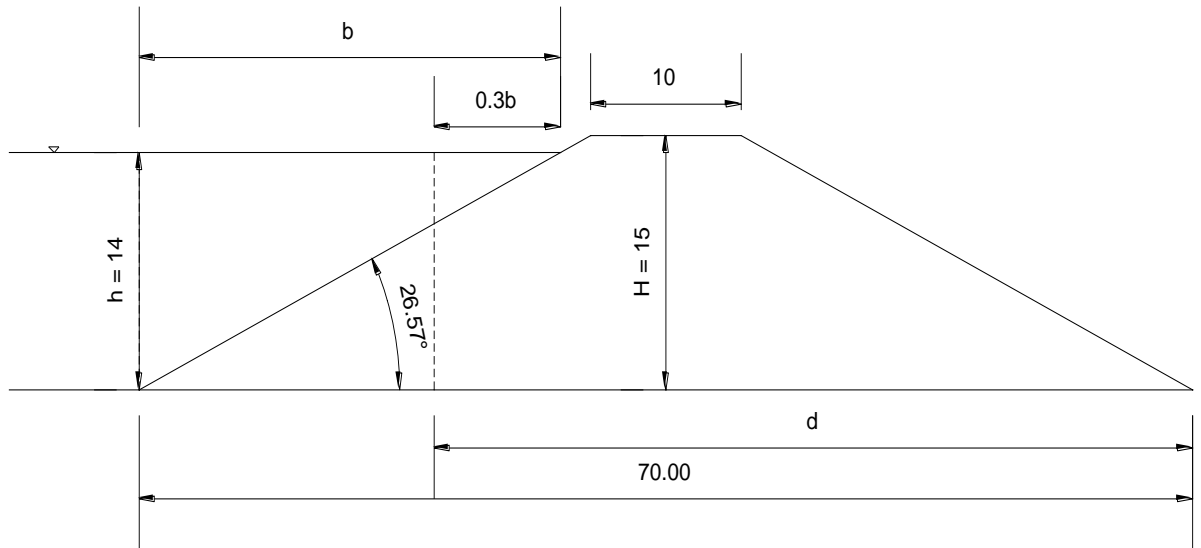
$$s = 3.627 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 3.627$$

$$q = 3.627 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.5 Height (H) = 15 m and Slope = 1:2 (Without Drainage Filter)



Height of water level (h) = 14 m

Therefore,

$$\tan \beta = (1/2)$$

$$\text{or, } \beta = 26.56^\circ < 30^\circ$$

$$b = (h \times 2)$$

$$b = 28 \text{ m}$$

$$d = (15 \times 2) + 10 + (1 \times 2) + (0.3 \times 28)$$

$$d = 50.4 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 26.56 = 0.447$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{50.4}{\cos 26.56} - \sqrt{\left(\frac{50.4}{\cos 26.56}\right)^2 - \left(\frac{14}{\sin 26.56}\right)^2}$$

$$a = 9.50 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times 10^{-6}) (9.50 \times \sin 26.56) \tan 26.56$$

$$q = 2.123 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.5b With Drainage Filter

$$\text{Height of Reservoir (h)} = 14 \text{ m}$$

$$b = 2 \times 14 = 28 \text{ m}$$

$$d = (30 - 10) + 10 + (1 \times 2) + (0.3 \times 28)$$

$$d = 40.4 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{40.4^2 + 14^2} - 40.4$$

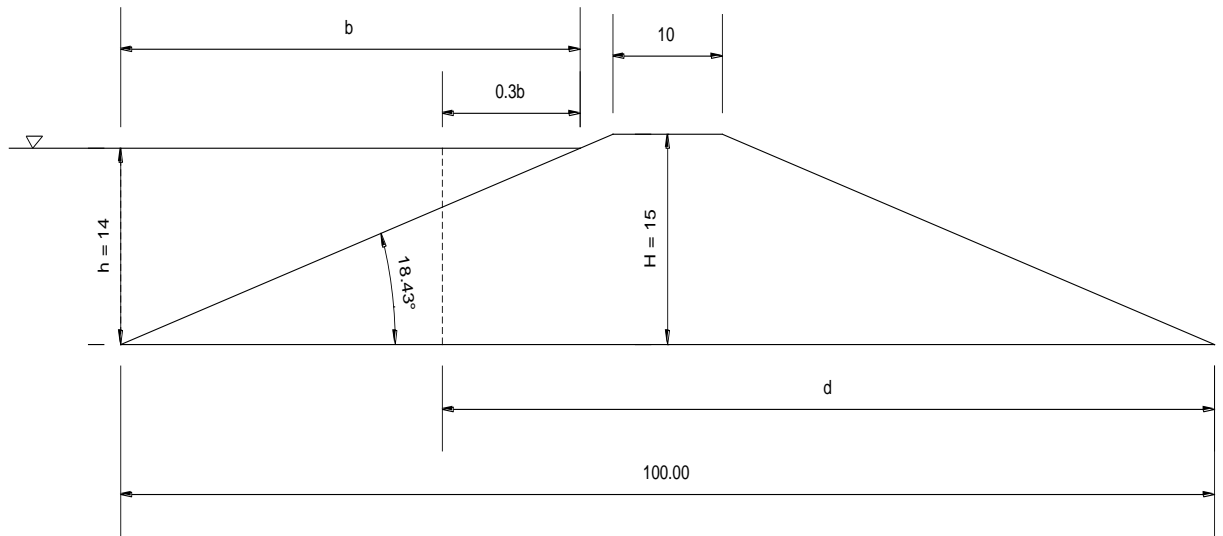
$$s = 2.357 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 2.357$$

$$q = 2.357 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.6 Height (H) = 15 m and Slope = 1:3 (Without Drainage Filter)



Height of water level (h) = 14 m

Therefore,

$$\tan \beta = (1/3)$$

$$\text{or, } \beta = 18.43^\circ < 30^\circ$$

$$b = (h \times 3)$$

$$b = 42 \text{ m}$$

$$d = (15 \times 3) + 10 + (1 \times 3) + (0.3 \times 42)$$

$$d = 70.6 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 18.43 = 0.316$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{70.6}{\cos 18.43} - \sqrt{\left(\frac{70.6}{\cos 18.43}\right)^2 - \left(\frac{14}{\sin 18.43}\right)^2}$$

$$a = 14.61 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times 10^{-6}) (14.61 \times \sin 18.43) \tan 18.43$$

$$q = 1.53 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.6b With Drainage Filter

Height of Reservoir (h) = 14 m

Solution:

$$b = 3 \times 14 = 42 \text{ m}$$

$$d = (45 - 10) + 10 + (1 \times 3) + (0.3 \times 42)$$

$$d = 60.6 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{60.6^2 + 14^2} - 60.6$$

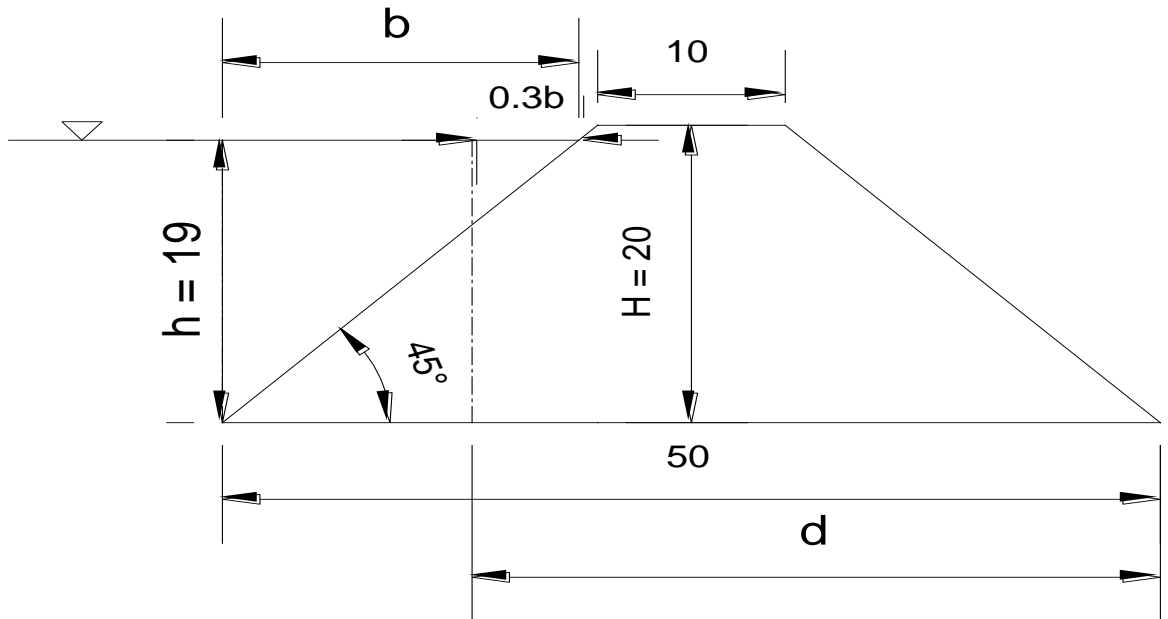
$$s = 1.596 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 1.596$$

$$q = 1.596 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.7 Height (H) = 20m and Slope = 1:1 (Without Drainage Filter)



Height of water level (h) = 19 m

Therefore,

$$\tan \beta = (1/1)$$

$$\text{or, } \beta = 45^\circ > 30^\circ$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 45^\circ = 0.707$$

$$i = 0.707$$

$$b = (1 \times 19) = 19 \text{ m}$$

$$d = 20 + 10 + 1 + (0.3 \times 19)$$

$$d = 36.7 \text{ m}$$

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 + (h \cot \beta)^2}$$

$$a = \sqrt{36.7^2 + 19^2} - \sqrt{36.7^2 + (19 \cot 45)^2}$$

$$a = 9.93 \text{ m}$$

Discharge is given as,

$$q = k \times a \times \sin^2 \beta$$

$$q = (1 \times 10^{-6}) \times 9.93 \times \sin^2 45$$

$$q = 4.965 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.7b With Drainage Filter

Height of Reservoir (h) = 19 m

$$b = 1 \times 19 = 19 \text{ m}$$

$$d = (20 - 5) + 10 + (1 \times 1) + (0.3 \times 19)$$

$$d = 31.7 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{31.7^2 + 19^2} - 31.7$$

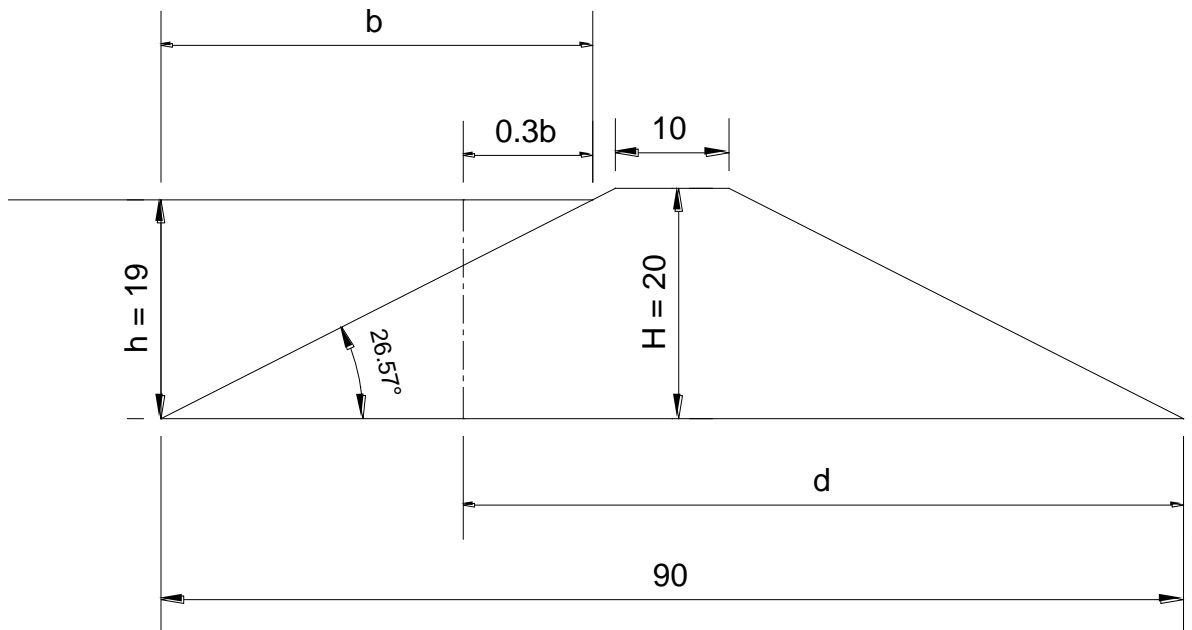
$$s = 5.26 \text{ m}$$

$$q = k s$$

$$q = (1 \times 10^{-6}) \times 5.26$$

$$q = 5.26 \times 10^{-6} \text{ m}^3/\text{s}$$

5.2.8 Height (H) = 20m and Slope = 1:2 (Without Drainage Filter)



Height of water level (h) = 19 m

Therefore,

$$\tan \beta = (1/2)$$

$$\text{or, } \beta = 26.56^\circ < 30^\circ$$

$$b = (h \times 2)$$

$$b = 38 \text{ m}$$

$$d = (20 \times 2) + 10 + (1 \times 2) + (0.3 \times 38)$$

$$d = 63.4 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 26.56 = 0.447$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{63.4}{\cos 26.56} - \sqrt{\left(\frac{63.4}{\cos 26.56}\right)^2 - \left(\frac{19}{\sin 26.56}\right)^2}$$

$$a = 14.15 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times e-006) (14.15 \times \sin 26.56) \tan 26.56$$

$$q = 3.16 \times e-006 \text{ m}^3/\text{s}$$

5.2.8b With Drainage Filter

$$\text{Height of Reservoir (h)} = 19 \text{ m}$$

$$b = 2 \times 19 = 38 \text{ m}$$

$$d = (40 - 10) + 10 + (1 \times 2) + (0.3 \times 38)$$

$$d = 53.4 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{53.4^2 + 19^2} - 53.4$$

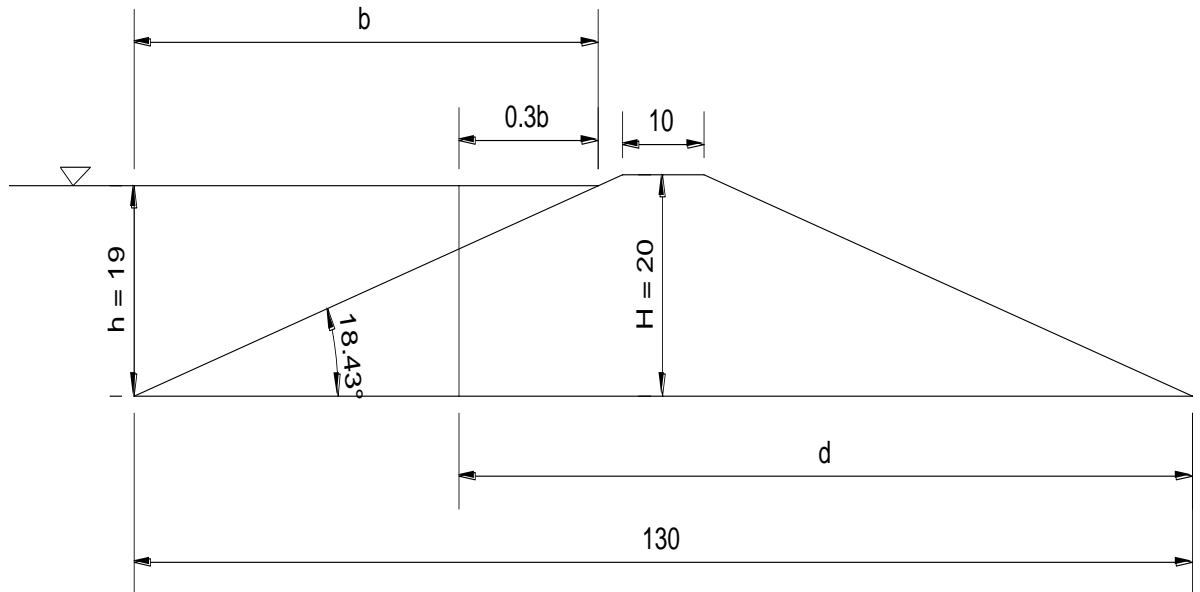
$$s = 3.28 \text{ m}$$

$$q = k s$$

$$q = (1 \times e-006) \times 3.28$$

$$q = 3.28 \times e-006 \text{ m}^3/\text{s}$$

5.2.9 Height (H) = 20m and Slope = 1:3 (Without Drainage Filter)



Height of water level (h) = 19 m

Therefore,

$$\tan \beta = (1/3)$$

$$\text{or, } \beta = 18.43^\circ < 30^\circ$$

$$b = (h \times 3)$$

$$b = 57 \text{ m}$$

$$d = (20 \times 3) + 10 + (1 \times 3) + (0.3 \times 57)$$

$$d = 90.1 \text{ m}$$

$$i = \frac{dz}{ds} = \sin \beta = \sin 18.43 = 0.316$$

$$a = \frac{d}{\cos \beta} - \sqrt{\left(\frac{d}{\cos \beta}\right)^2 - \left(\frac{h}{\sin \beta}\right)^2}$$

$$a = \frac{90.1}{\cos 18.43} - \sqrt{\left(\frac{90.1}{\cos 18.43}\right)^2 - \left(\frac{19}{\sin 18.43}\right)^2}$$

$$a = 9.93 \text{ m}$$

$$q = k (a \sin \beta) \tan \beta$$

$$q = (1 \times e-006) (9.93 \times \sin 18.43) \tan 18.43$$

$$q = 1.046 \times e-006 \text{ m}^3/\text{s}$$

5.2.9b With Drainage Filter

$$\text{Height of Reservoir (h)} = 19 \text{ m}$$

$$b = 3 \times 19 = 57 \text{ m}$$

$$d = (60 - 10) + 10 + (1 \times 3) + (0.3 \times 57)$$

$$d = 80.1 \text{ m}$$

$$s = \sqrt{d^2 + h^2} - d$$

$$s = \sqrt{80.1^2 + 19^2} - 80.1$$

$$s = 2.22 \text{ m}$$

$$q = k s$$

$$q = (1 \times e-006) \times 2.22$$

$$q = 2.22 \times e-006 \text{ m}^3/\text{s}$$

Table 5.3: Discharge through embankment without drainage filter using Analytical Method

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (q) m³/s
1	10 m	9 m	1:1	1.715 e-006
2	10 m	9 m	1:2	1.152 e-006
3	10 m	9 m	1:3	0.857 e-006
4	15 m	14 m	1:1	3.265 e-006
5	15 m	14 m	1:2	2.123 e-006
6	15 m	14 m	1:3	1.530 e-006
7	20 m	19 m	1:1	4.965 e-006
8	20 m	19 m	1:2	3.160 e-006
9	20 m	19 m	1:3	2.046 e-006

Table 5.4: Discharge through Embankment with drainage filter using Analytical Method

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (q) m³/s
1	10 m	9 m	1:1	2.053 e-006
2	10 m	9 m	1:2	1.440 e-006
3	10 m	9 m	1:3	0.974 e-006
4	15 m	14 m	1:1	3.627 e-006
5	15 m	14 m	1:2	2.357 e-006
6	15 m	14 m	1:3	1.596 e-006
7	20 m	19 m	1:1	5.260 e-006
8	20 m	19 m	1:2	3.280 e-006
9	20 m	19 m	1:3	2.220 e-006

CHAPTER 6
RESULTS
AND
DISCUSSION

6.1 Comparison of results obtained using FEM

From the results obtained, it is observed that with the decrease in slope angle of upstream face, there is significant decrease in the seepage discharge value occurring through the body of the embankment dam. This is due to the reason that with the increase in the width of the embankment, pressure difference remains same but the seepage path increases. Thus, seepage discharge decreases due to longer path. Along with this, it is also observed that with the increase in the height of the embankment, seepage value increases due to greater difference in pressure at upstream and downstream face.

Table 6.1: Comparison between Discharge with and without Filter using FEM

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (Without Filter) (q) m ³ /s	Discharge (With Filter) (q) m ³ /s
1	10 m	9 m	1:1	1.9686e-006	2.4224e-006
2	10 m	9 m	1:2	1.3398e-006	1.4858e-006
3	10 m	9 m	1:3	1.0315e-006	1.2439e-006
4	15 m	14 m	1:1	3.6805e-006	4.0897e-006
5	15 m	14 m	1:2	2.4451e-006	2.4653e-006
6	15 m	14 m	1:3	1.8576e-006	1.9580e-006
7	20 m	19 m	1:1	5.6137e-006	5.8240e-006
8	20 m	19 m	1:2	3.6305e-006	3.6359e-006
9	20 m	19 m	1:3	2.7360e-006	2.7422e-006

6.11 Discharge v/s Slope

A graph is plotted between discharge and slope of the embankment keeping the height constant. It is observed that for 10m height, discharge increases with the provision of filter. However, this effect reduces with the increase in height and for 20m there is very minor difference in discharge for filtered and non-filtered earthen embankment dam. Also, the discharge reduces with the decrease in slope angle of embankment face.

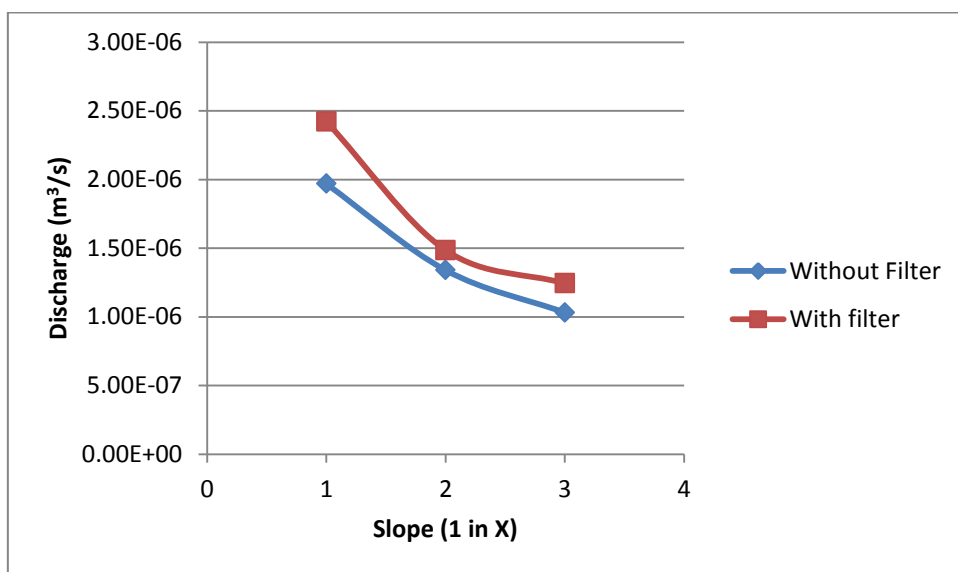


Figure 6.1: For 10m height

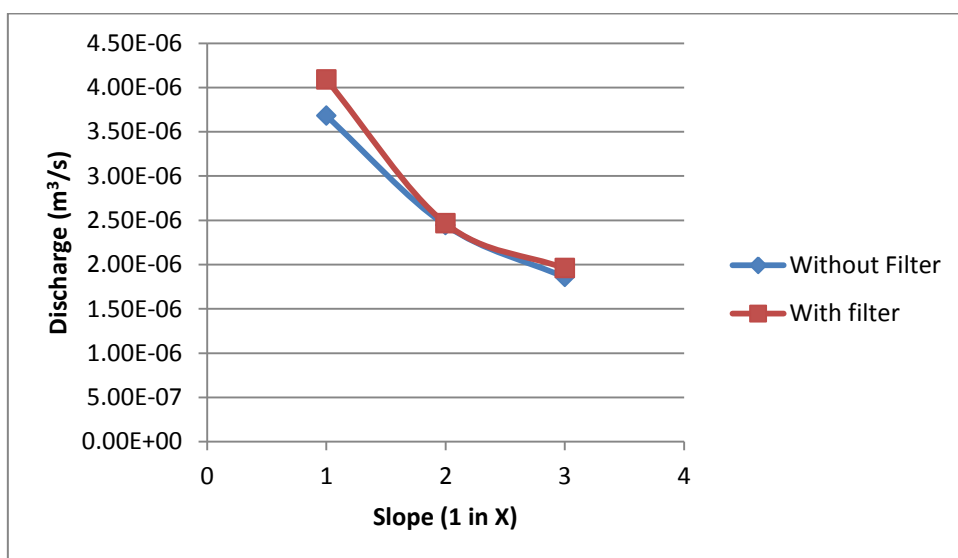


Figure 6.2: For 15m height

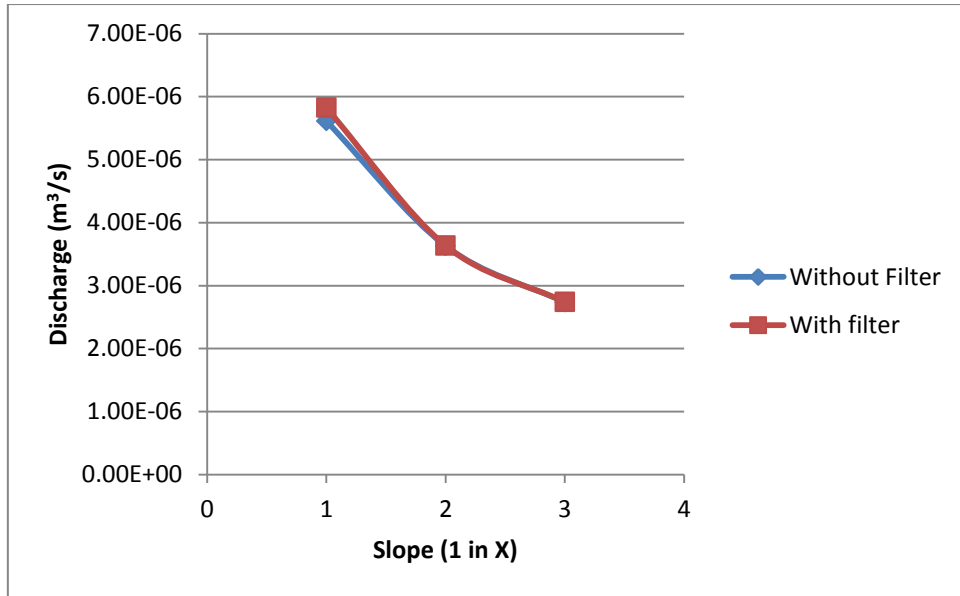


Figure 6.3: For 20m height

6.12 Discharge v/s Height

A graph is plotted between discharge and height of the embankment keeping the slope constant. It is observed that discharge obtained for filtered embankment is greater than non-filtered embankment and increases linearly with the increase in height.

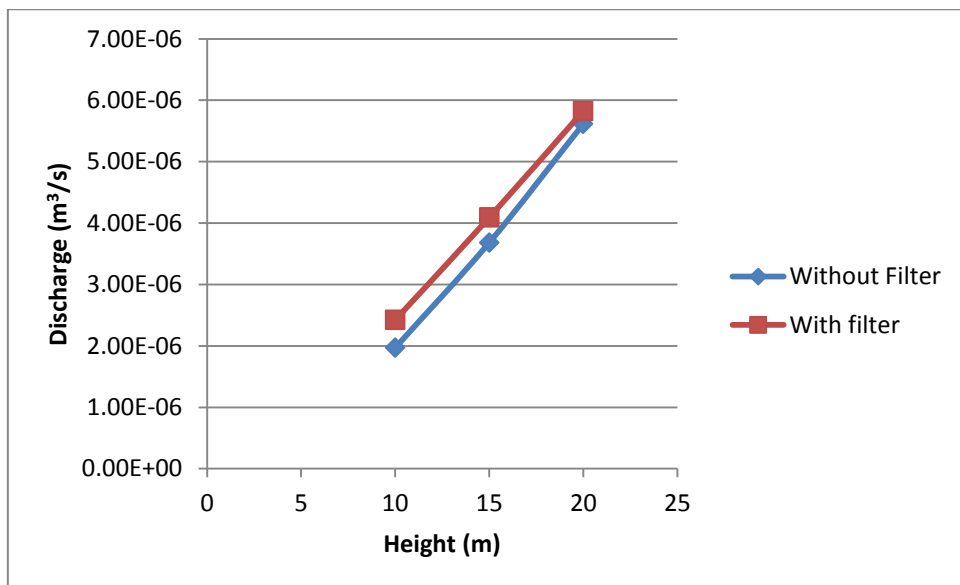


Figure 6.4: For 1:1 Slope

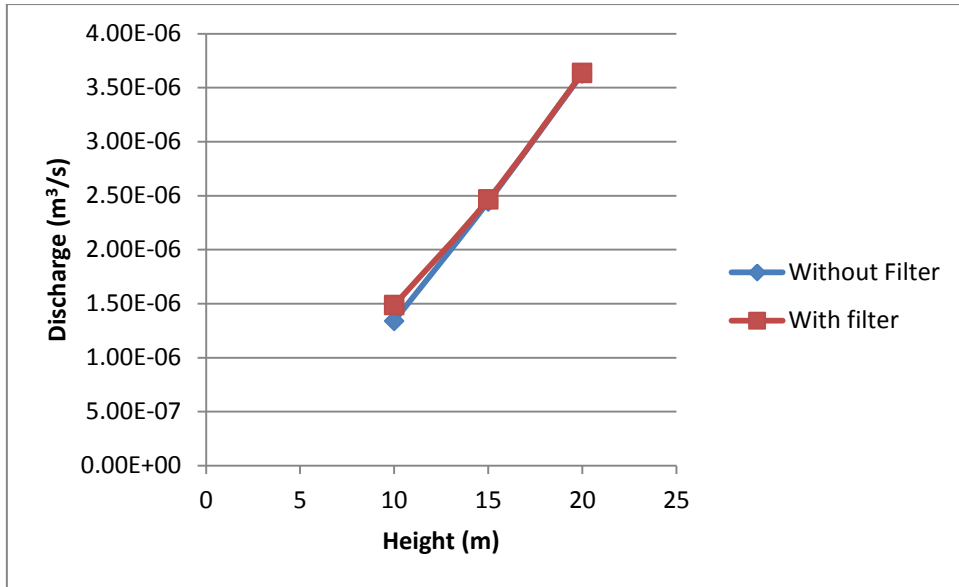


Figure 6.5: For 1:2 Slope

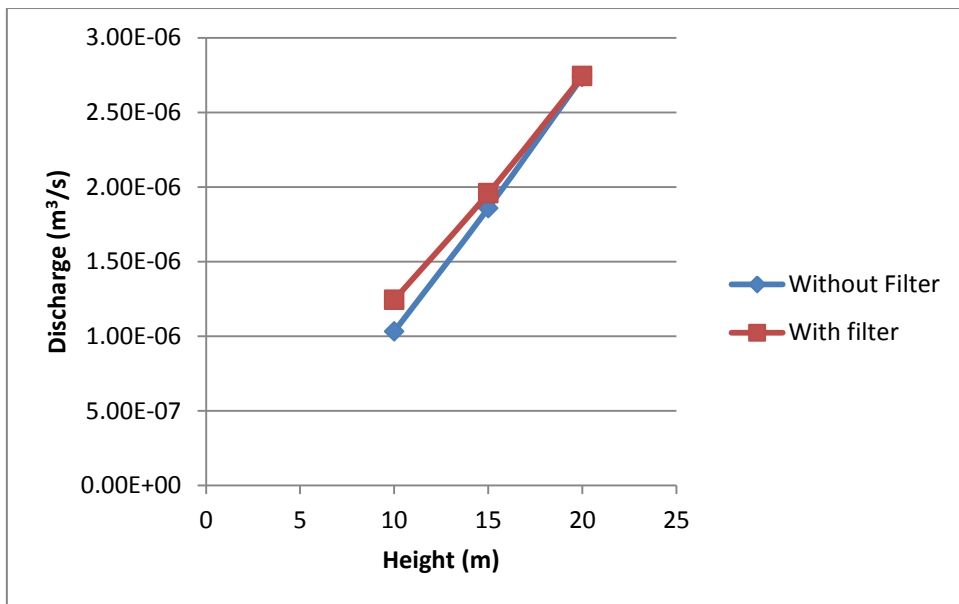


Figure 6.6: For 1:3 Slope

6.2 Comparison of results obtained using Analytical Method

A similar type of result was obtained in the case of Analytical method as that of Finite Element Method. Same type of trend is obtained when a graph is plotted between Discharge v/s Slope and Discharge v/s Height. Discharge decreases with the decrease in slope angle of upstream face. And discharge increases with the increase in height of the embankment.

Table 6.2: Comparison between Discharge with and without Filter using Analytical Method

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (Without Filter) (q) m ³ /s	Discharge (With Filter) (q) m ³ /s
1	10 m	9 m	1:1	1.715e-006	2.053e-006
2	10 m	9 m	1:2	1.152e-006	1.440e-006
3	10 m	9 m	1:3	0.857e-006	0.974e-006
4	15 m	14 m	1:1	3.265e-006	3.627e-006
5	15 m	14 m	1:2	2.123e-006	2.357e-006
6	15 m	14 m	1:3	1.530e-006	1.596e-006
7	20 m	19 m	1:1	4.965e-006	5.260e-006
8	20 m	19 m	1:2	3.160e-006	3.280e-006
9	20 m	19 m	1:3	2.046e-006	2.220e-006

6.21 Discharge v/s Slope

A graph is plotted between discharge and slope of the embankment keeping the height constant. It is observed that for 10m height, discharge increases with the provision of filter. However, this effect reduces with the increase in height and for 20m there is very minor difference in discharge for filtered and non-filtered earthen embankment dam. Also, the discharge reduces linearly with the decrease in slope angle of embankment face.

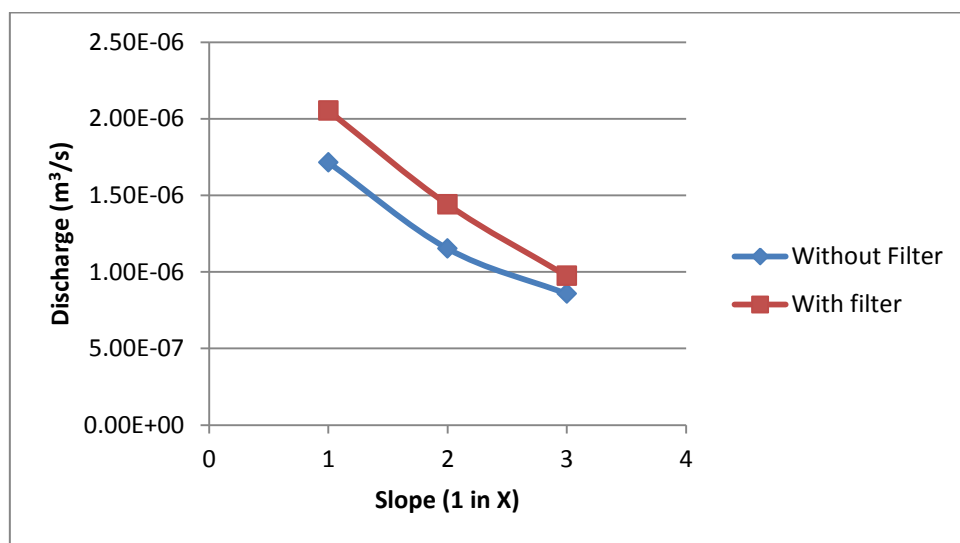


Figure 6.7: For 10m height

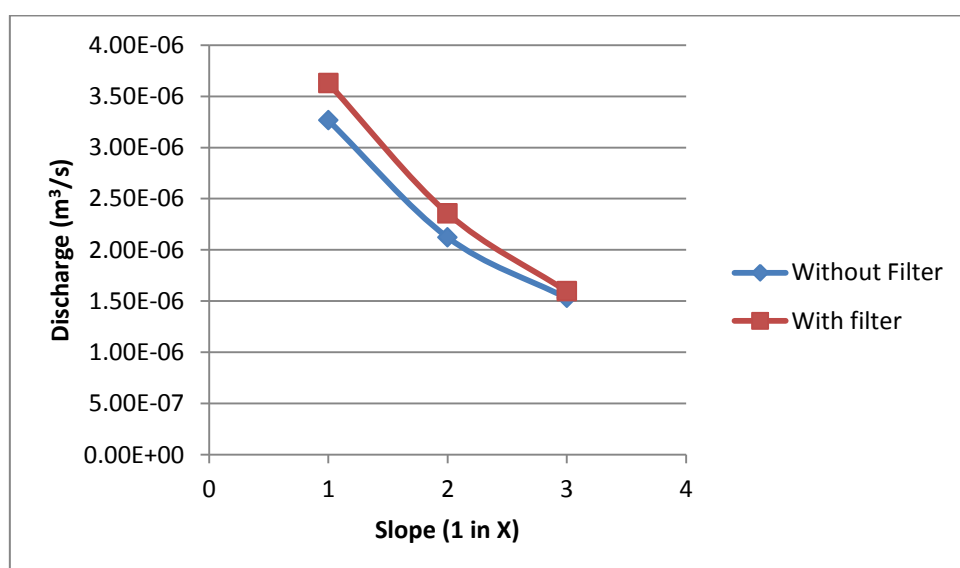


Figure 6.8: For 15m height

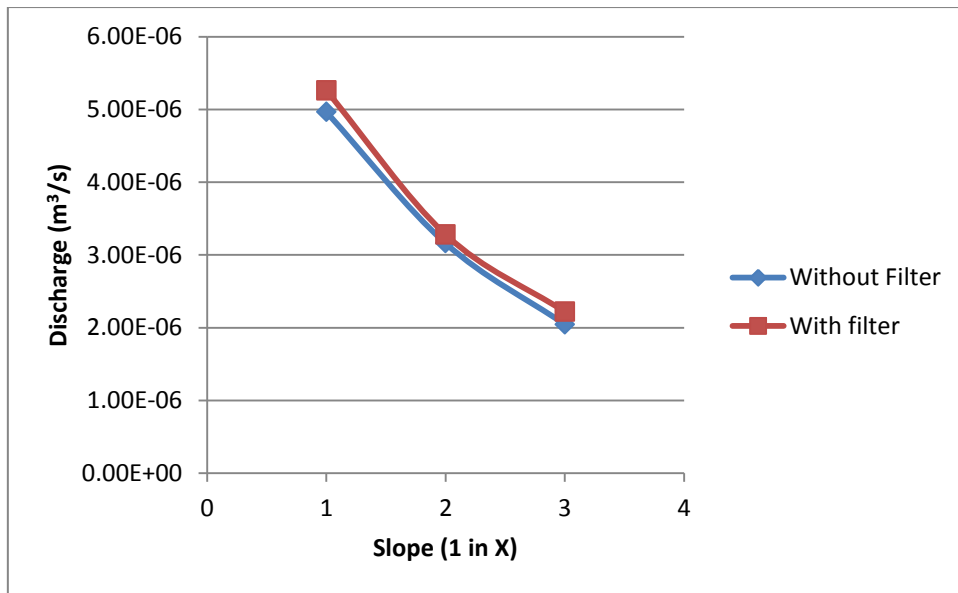


Figure 6.9: For 20m height

6.22 Discharge v/s Height

A graph is plotted between discharge and height of the embankment keeping the slope constant. It is observed that discharge obtained for filtered embankment is greater than non-filtered embankment and increases linearly with the increase in height.

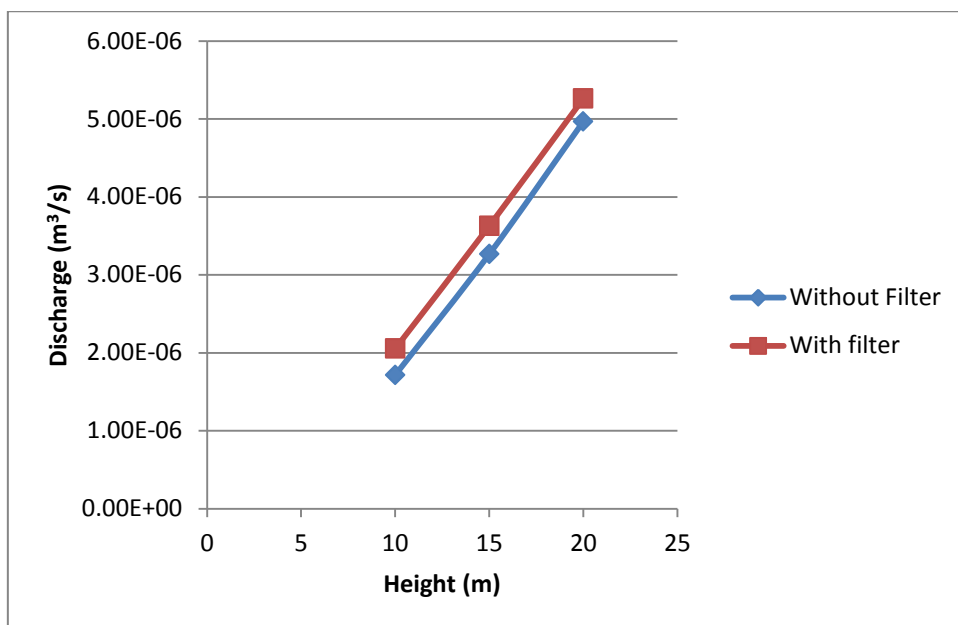


Figure 6.10: For 1:1 Slope

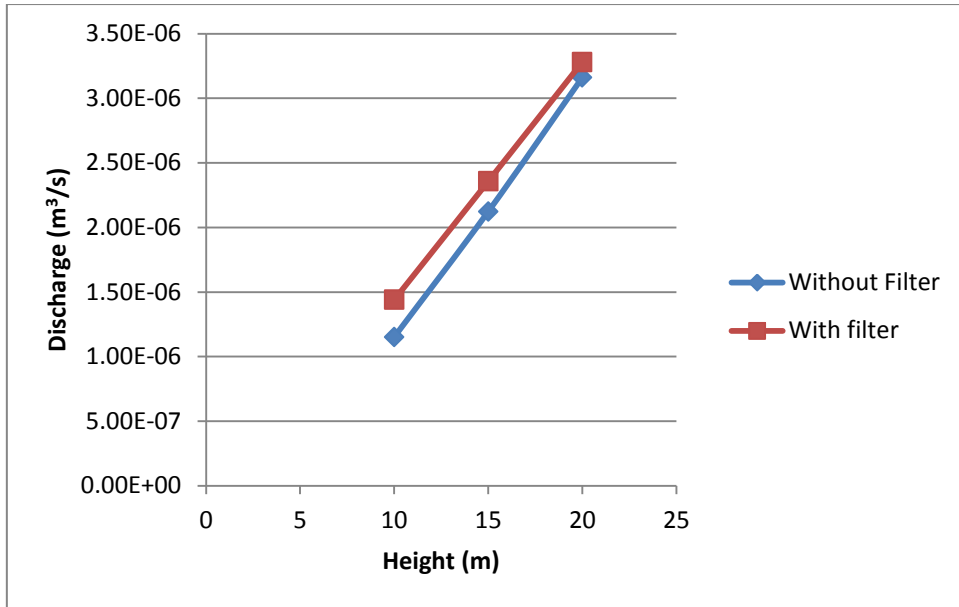


Figure 6.11: For 1:2 Slope

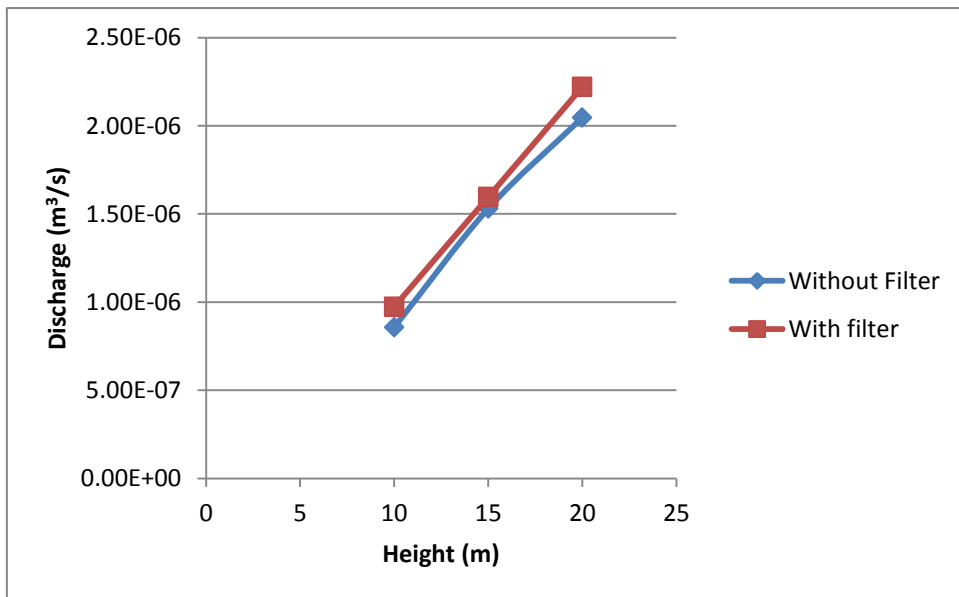


Figure 6.12: For 1:3 Slope

6.3 Comparison between FEM and Analytical Method

6.3.1 Without Drainage Blanket

It was observed that the result obtained by Finite Element Method is somewhat greater the result obtained by Analytical Method. The reason for this may be due to the difference in method.

Table 6.3: Comparison of result between FEM and Analytical Method (Without Blanket Drain)

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (SEEP/W) (q) m ³ /s	Discharge (Analytical Method) (q) m ³ /s	Difference (%)
1	10 m	9 m	1:1	1.9686e-006	1.715e-006	12.76
2	10 m	9 m	1:2	1.3398e-006	1.152e-006	14.01
3	10 m	9 m	1:3	1.0315e-006	0.857e-006	16.91
4	15 m	14 m	1:1	3.6805e-006	3.265e-006	11.28
5	15 m	14 m	1:2	2.4451e-006	2.123e-006	13.17
6	15 m	14 m	1:3	1.8576e-006	1.530e-006	17.61
7	20 m	19 m	1:1	5.6137e-006	4.965e-006	11.55
8	20 m	19 m	1:2	3.6305e-006	3.160e-006	12.95
9	20 m	19 m	1:3	2.7360e-006	2.046e-006	25.21

6.3.1.1 Discharge v/s Slope

Discharge v/s Slope graph obtained shows that the seepage discharge obtained using Finite Element Method is greater than the discharge value obtained using Analytical Method. Seepage discharge decreases with the decrease in the slope angle of embankment.

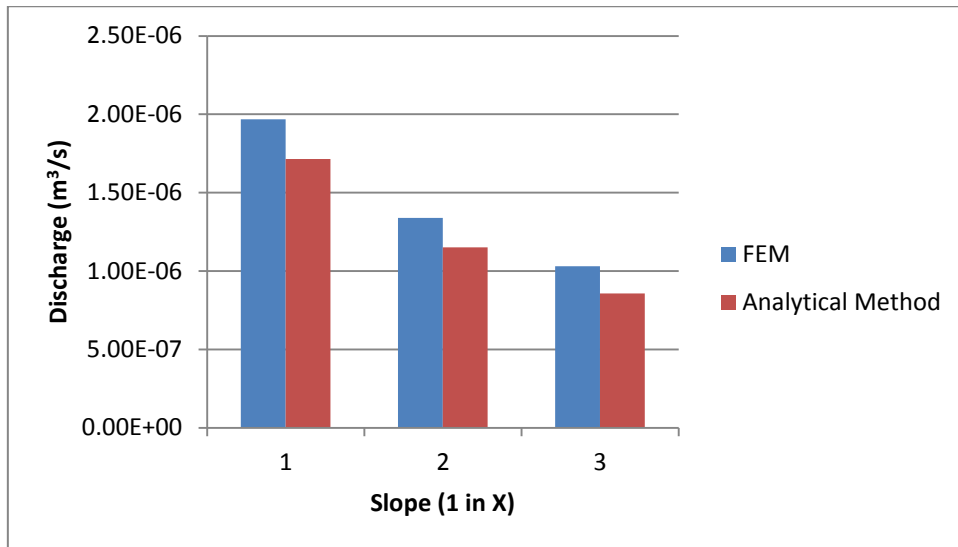


Figure 6.13: For 10m height

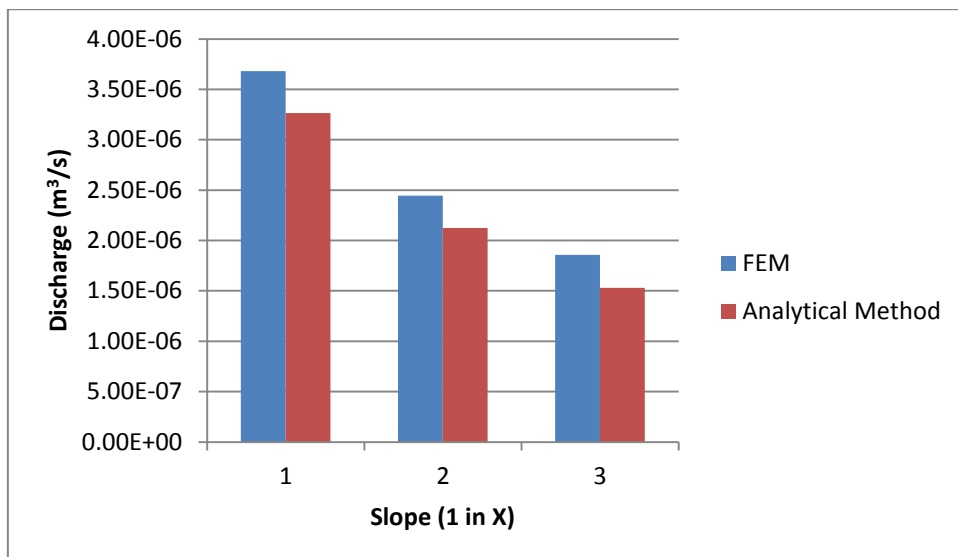


Figure 6.14: For 15m height

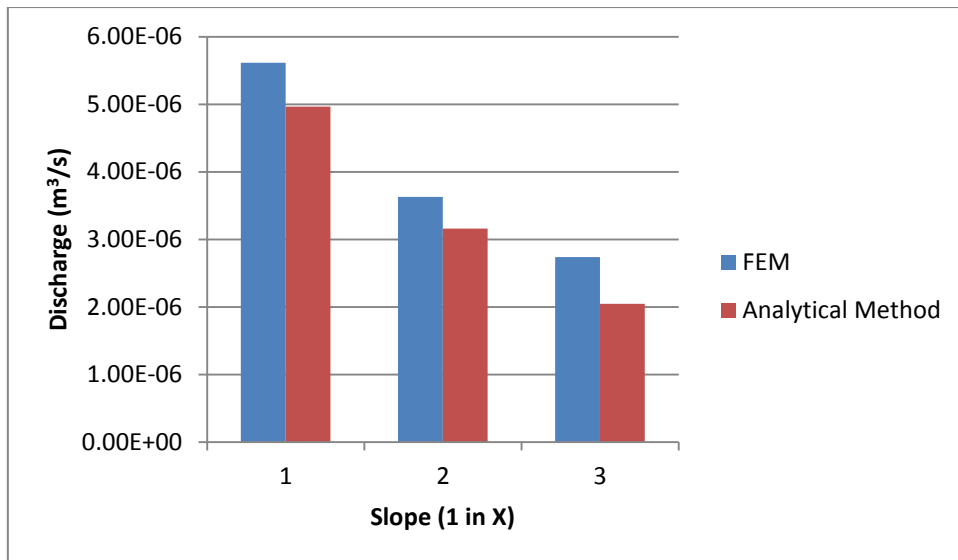


Figure 6.15: For 20m height

6.3.1.2 Discharge v/s Height

Discharge v/s Height graph obtained shows that the seepage discharge obtained using Finite Element Method is greater than the discharge value obtained using Analytical Method. Seepage discharge increases with the increase in the height of the embankment.

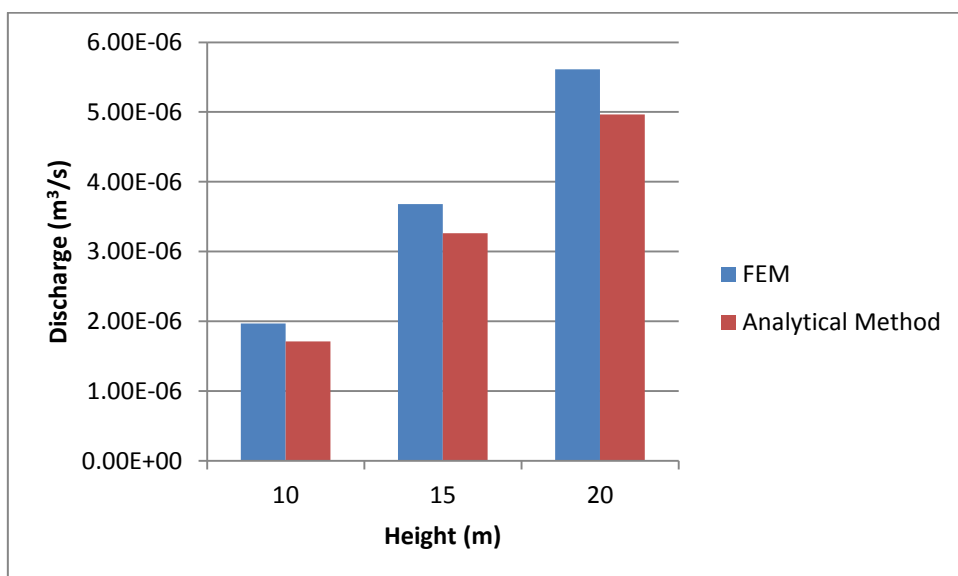


Figure 6.16: For 1:1 Slope

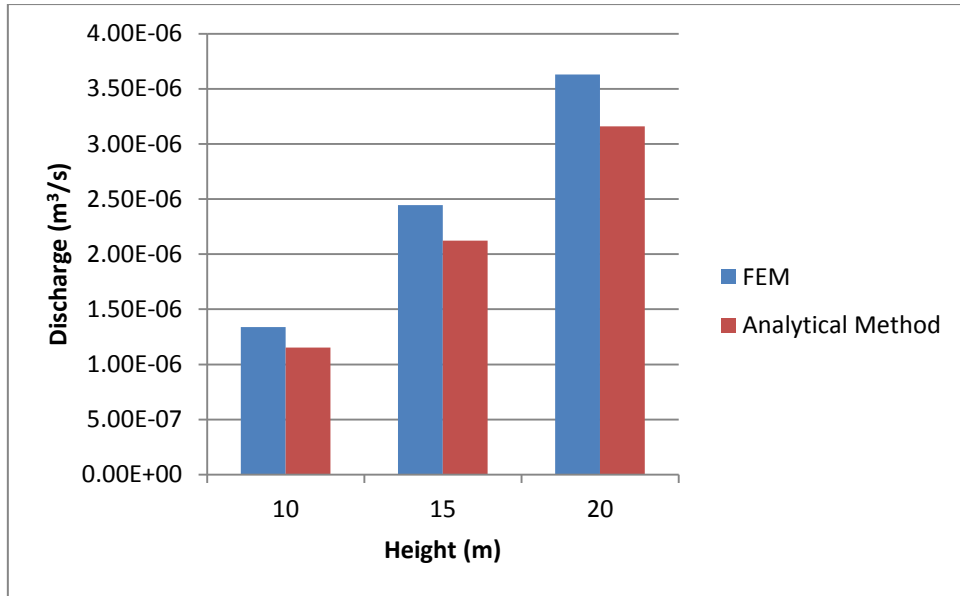


Figure 6.17: For 1:2 Slope

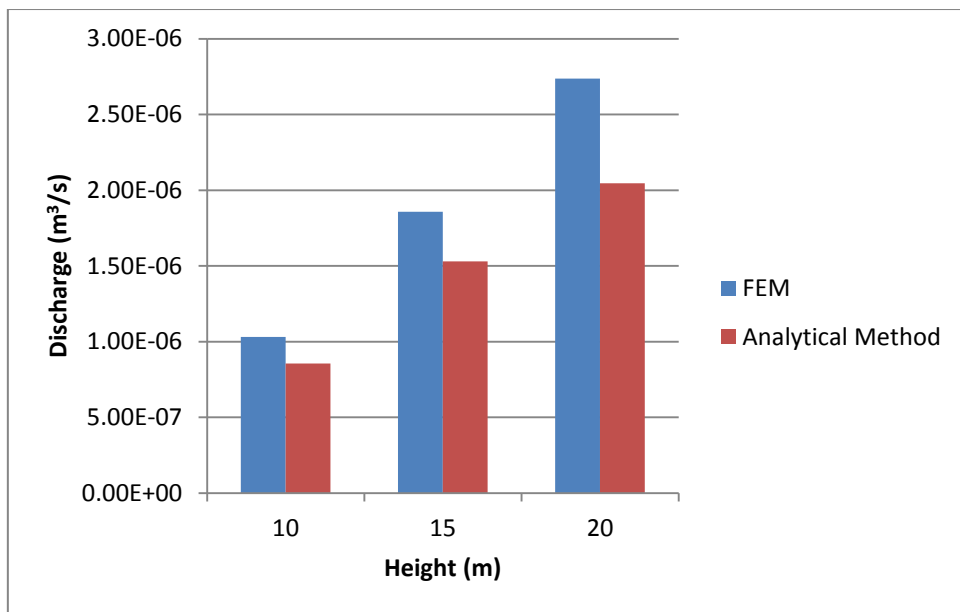


Figure 6.18: For 1:3 Slope

6.3.2 With Blanket Drain

A similar type of result is obtained in case too. Discharge obtained from Finite Element Method is some what greater than the result obtained by Analytical Method.

However, the difference in result is lesser with average variation being 12.5%.

Table 6.4: Comparison of result between SEEP/W and Analytical Method (With Blanket Drain)

S. No.	Height of Embankment (H)	Height of Reservoir Level (h)	Slope (S)	Discharge (SEEP/W) (q) m ³ /s	Discharge (Analytical Method) (q) m ³ /s	Difference (%)
1	10 m	9 m	1:1	2.4224e-006	2.053e-006	15.24
2	10 m	9 m	1:2	1.4858e-006	1.440e-006	3.08
3	10 m	9 m	1:3	1.2439e-006	0.974e-006	21.69
4	15 m	14 m	1:1	4.0897e-006	3.627e-006	11.31
5	15 m	14 m	1:2	2.4653e-006	2.357e-006	4.39
6	15 m	14 m	1:3	1.9580e-006	1.596e-006	18.48
7	20 m	19 m	1:1	5.8240e-006	5.260e-006	9.68
8	20 m	19 m	1:2	3.6359e-006	3.280e-006	9.78
9	20 m	19 m	1:3	2.7422e-006	2.220e-006	19.04

6.3.2.1 Discharge v/s Slope

Discharge v/s Slope graph obtained shows that the seepage discharge obtained using Finite Element Method is greater than the discharge value obtained using Analytical Method. Seepage discharge decreases with the decrease in the slope angle of embankment.

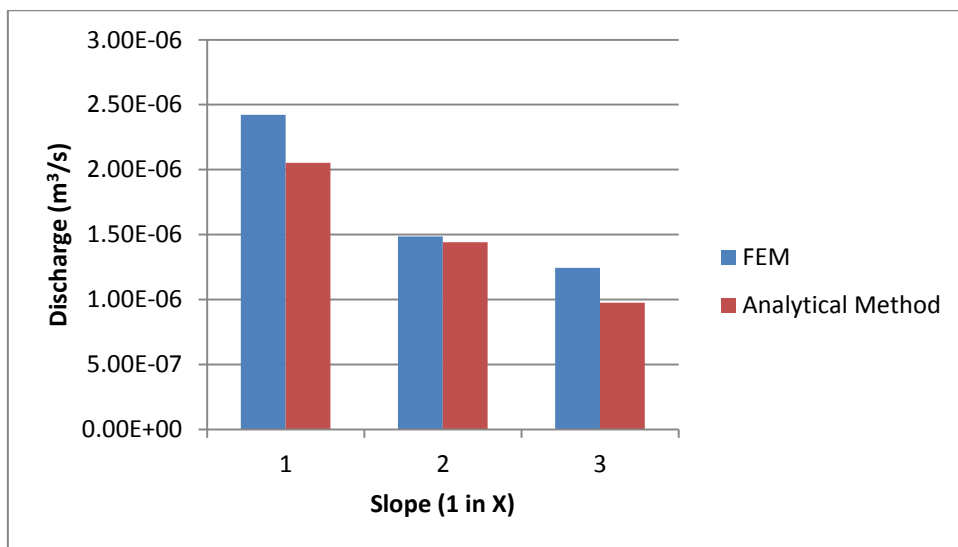


Figure 6.19: For 10m height

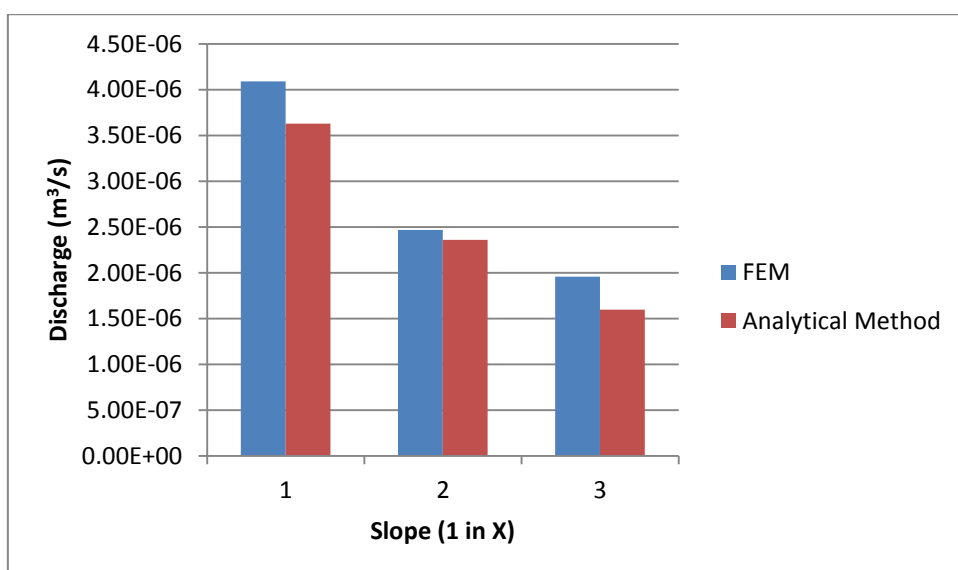


Figure 6.20: For 15m height

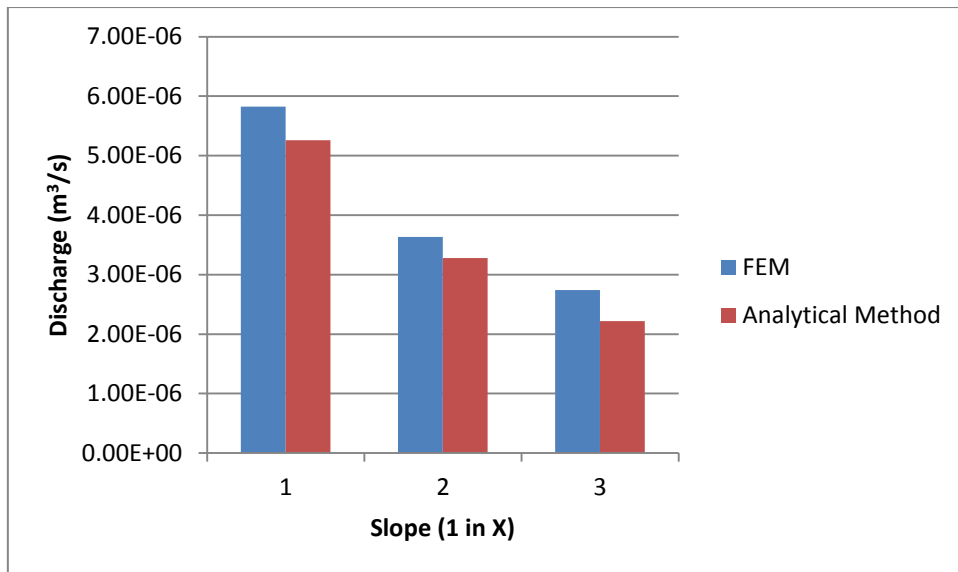


Figure 6.21: For 20m height

6.3.2.2 Discharge v/s Height

Discharge v/s Height graph obtained shows that the seepage discharge obtained using Finite Element Method is greater than the discharge value obtained using Analytical Method. Seepage discharge increases with the increase in the height of the embankment.

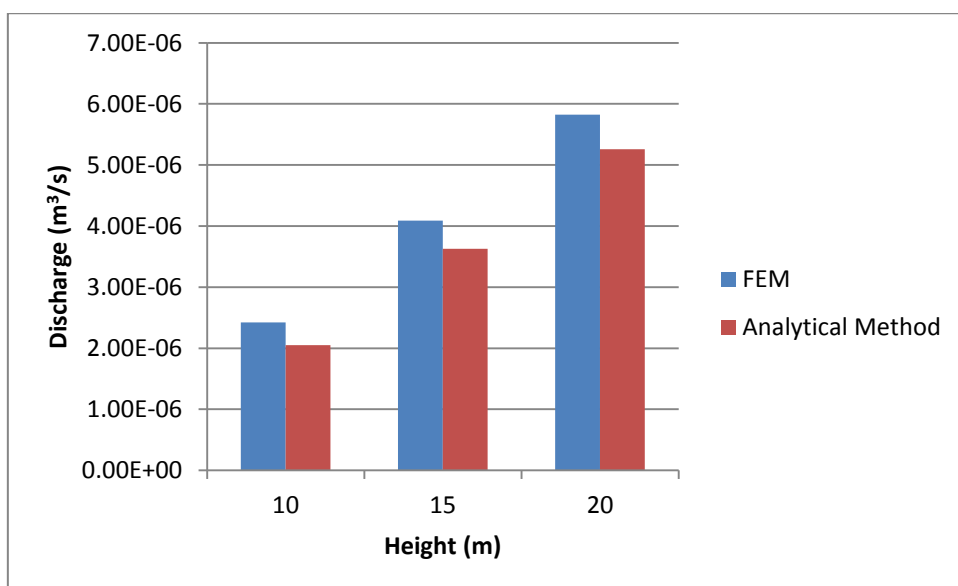


Figure 6.22: For 1:1 Slope

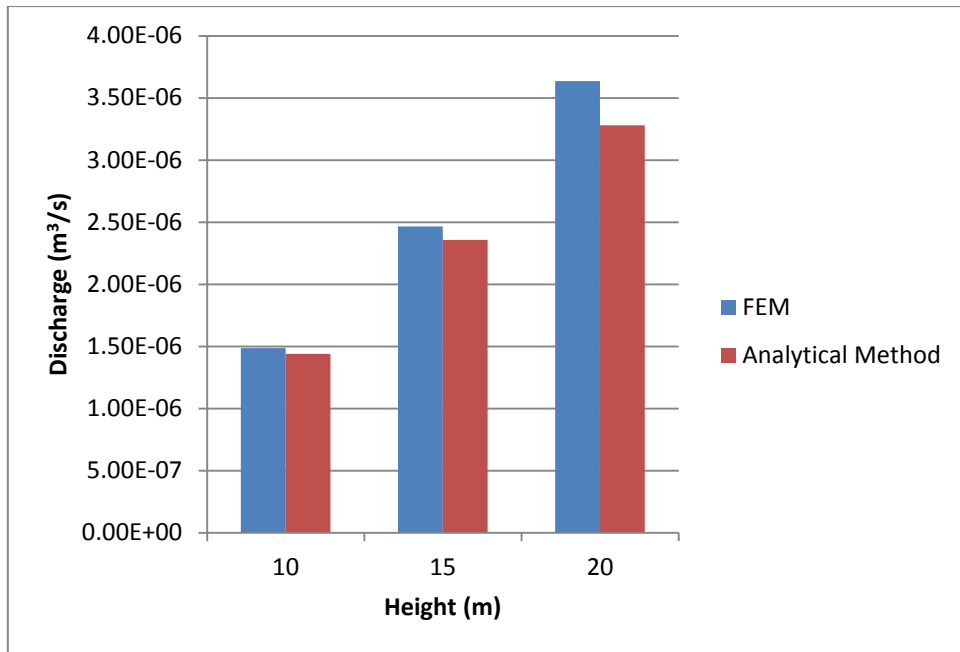


Figure 6.23: For 1:2 Slope

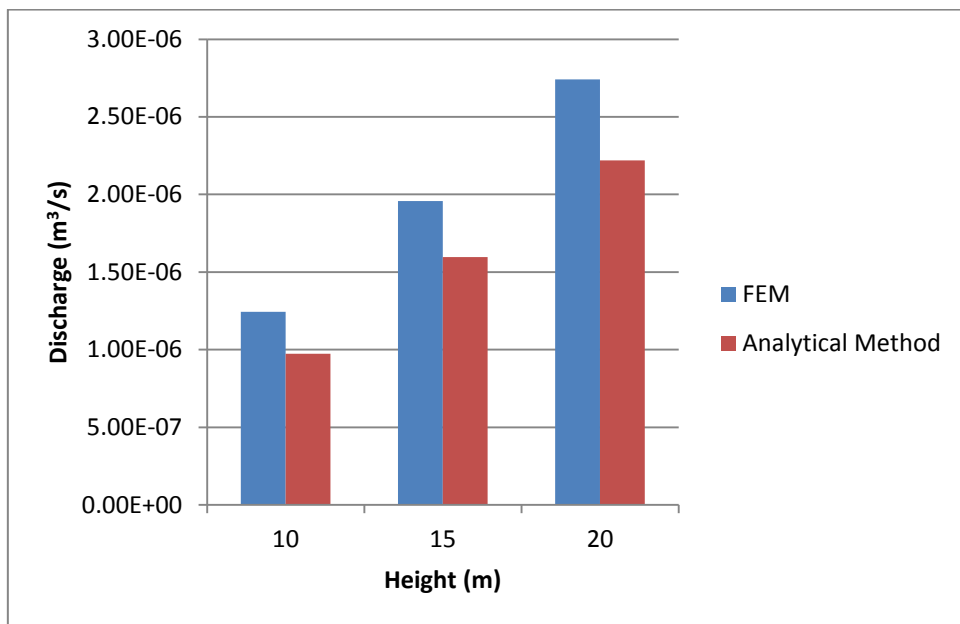


Figure 6.24: For 1:3 Slope

CHAPTER 7

SUMMARY AND CONCLUSION

In this Dissertation, seepage analysis was carried out using Finite Element Method and Analytical method. Parametric Study was carried out using both the methods. The result obtained using FEM was then compared with the result obtained from Analytical method. The conclusions of the report are as follow:

- The solution of seepage problem using Finite Element Method is compared with the solution obtained by Analytical Method. It is observed that the result is satisfactory with the maximum deviation of 21.69% and minimum deviation of 3.08%. The average deviation of the result is approximately 10% which is satisfactory.
- With the increase in the slope of the embankment, there is considerable decrease in the value of seepage discharge.
- For the same slope, there is increase in seepage discharge with the increase in the height of the embankment.
- With the provision of Drainage blanket at the toe of the Embankment Dam, there is slight increase in the value of seepage discharge.
- Effect of drainage blanket is more for lesser slope and less for higher slope with height being kept constant.

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APPENDIX

Problem 1

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File Information

Created By: Waqar
Last Edited By: Waqar
Revision Number: 2
File Version: 8.2
Tool Version: 8.12.3.7901
Date: 12-06-2014
Time: 23:42:53
File Name: Problem 1.gsz
Directory: C:\Users\sony\Desktop\seepw\
Last Solved Date: 12-06-2014
Last Solved Time: 23:42:55

Project Settings

Length(L) Units: meters
Time(t) Units: Seconds
Force(F) Units: kN
Pressure(p) Units: kPa
Mass(M) Units: g
Mass Flux Units: g/sec
Unit Weight of Water: 9.807 kN/m³
View: 2D
Element Thickness: 1

Analysis Settings

Problem 1

Description: Embankment with 1:1 slope, ht 20m

Kind: SEEP/W

Method: Steady-State

Settings

Include Air Flow: No

Control

Apply Runoff: Yes

Convergence

Maximum Number of Iterations: 500

Minimum Pressure Head Difference: 0.005

Significant Digits: 2

Max # of Reviews: 10

Hydraulic Under-Relaxation Criteria

Under-Relaxation Initial Rate: 1

Under-Relaxation Min. Rate: 0.1

Under-Relaxation Reduction Rate: 0.65

Under-Relaxation Iterations: 10

Equation Solver: Direct

Time

Starting Time: 0 sec

Duration: 0 sec

Ending Time: 0 sec

Materials

Embankment

Model: Saturated / Unsaturated

Hydraulic

K-Function: Embankment conductivity

Ky'/Kx' Ratio: 1

Rotation: 0°

Boundary Conditions

Zero Pressure

Type: Pressure Head 0

Review: No

Potential Seepage Face

Type: Total Flux (Q) 0

Review: Yes

Reservoir Head = 19 m

Type: Head (H) 19

Review: No

Flux Sections

Flux Section 1

Coordinates

Coordinate: (26.732955, -1.2784091) m

Coordinate: (26.732955, 21.875) m

K Functions

Embankment conductivity

Model: Hyd K Data Point Function

Function: X-Conductivity vs. Pore-Water Pressure

Curve Fit to Data: 100 %

Segment Curvature: 100 %

K-Saturation: 1e-006

Data Points: Matric Suction (kPa), X-Conductivity (m/sec)

Data Point: (2, 1e-006)

Data Point: (4.7824308, 3.6854683e-007)

Data Point: (10.701103, 1.5235938e-007)

Data Point: (100, 1e-008)

Estimation Properties

Hyd. K-Function Estimation Method: Van Genuchten Function

Hydraulic K Sat: 0 m/sec

Residual Water Content: 0 m³/m³

Maximum: 1,000

Minimum: 0.01

Num. Points: 20

Points

	X (m)	Y (m)	Hydraulic Boundary
Point 1	0	0	
Point 2	20	20	
Point 3	30	20	
Point 4	50	0	Zero Pressure
Point 5	19	19	

Lines

	Start Point	End Point	Hydraulic Boundary
Line 1	2	3	
Line 2	3	4	Potential Seepage Face
Line 3	4	1	
Line 4	1	5	Reservoir Head = 19 m
Line 5	5	2	

Regions

	Material	Points	Area (m ²)
Region 1	Embankment	1,5,2,3,4	600