Reliability analysis using FORM and SORM

A Dissertation submitted towards the partial fulfilment of
the requirement for the award of degree of

Master of Technology in
Structural Engineering

Submitted by
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JUNE 2016

DELHI TECHNOLOGICAL UNIVERSITY

Established by Govt. of Delhi vide Act 6 of 2009

(*Formerly Delhi College of Engineering)*

SHAHBAD DAULATPUR, BAWANA ROAD, DELHI-110042

DEPARTMENT OF CIVIL ENGINEERING

CANDIDATE'S DECLARATION

I do hereby declare that the work presented in this report entitled "Reliability analysis using FORM and SORM based on IS:800-2007" in partial fulfilment of the requirement for the award of degree of "Master of Technology" in structural engineering, submitted in the Department of Civil Engineering, Delhi Technological University, is an authentic record of my own work under the supervision of Mr. G.P. Awadhiya, Associate Professor, Department of Civil Engineering.

I have not submitted this matter for award of any other degree or diploma.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ACKNOWLEDGEMENT

Any accomplishment require the effort of many people and this work is no exception. I appreciate the contribution and support, which various individuals have provided for the successful completion of this study. It may not be possible to mantion all by name but the following were singled out for their exceptional contribution.

I would like to express my gratitude to **Mr. G. P. Awadhiya,** Associate Professor, Department of Civil Engineering, Delhi Technological University, New Delhi, who has graciously provided me his valuable time whenever I required his assistance. His counselling, supervision and suggestions were always encouraging and it motivated me to complete the job at hand. He will always be regarded as a great mentor for me.

I am deeply grateful to **Prof. Narendra Dev,** Head of Department, Department of Civil Engineering, Delhi Technological University for his support and encouragement in carrying out this project.

I am grateful to **Prof Armen Der Kiureghian,** Department of Civil and Environmental Engineering, University of California, Berkeley for his grateful response.

I would like to express my heartiest thank to my friends, for constant support and motivation.

Last but not the least I thank my parents and family, for everything I am and will be in future. It's their unspoken prayers and affection that keep me moving forward.

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ABSTRACT

A comprehensive analysis is presented for the scrutiny of structural reliability with incomplete probability information. The performance of a structure is assessed by its safety, serviceability, and economy. The information about input variables is never certain, precise, and complete. In the presence of uncertainties, the absolute safety of structures is impossible due to the unpredictability of variables involved. In the conventional deterministic and analysis and design methods, it is assumed that all parameters (loads, strengths of materials, etc.) are not subjected to probabilistic variations. The safety factors provided in the existing codes and standards, primarily based on practice, judgment, and experience may not be adequate or economical. This technique is built on the first-order Taylor series approximation of the performance function which is linearized at the mean values of random variables. A better approximation by the second order terms is required for highly non-linear limit states. This is often termed as Second Order Reliability Method (SORM), unlike FORM the expression is considered till second order terms. A beam and column analysis is performed manually as well as with the help of softwares ETABS and COMRELin this project. Beam is analyzed for shear while the column is analyzed for axial force and bi-axial bending. The IS codes failure criteria's have been used to get to the limiting equations which is then used to calculate the reliability of the components.

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CHAPTER 1
INTRODUCTION

1.1. General

The evaluation of the safety of structures is a task of much importance. The safety of structures depends on the resistance, R, of the structure and the action, S, (load or load effect) on the structure. The action is the function of loads (live loads, wind loads, etc.), which are random variables. Similarly, the resistance or response of structure depends on the physical properties of material, and the geometric properties of structure which are also subjected to statistical variation, and are probabilistic. There is a need for a lucid approach to the evaluation of structural safety, taking into account these random variations.

Using probabilistic approach, there is a possibility of obtaining uniform reliability (uniform performance in structures under different design situations) which may probably lead to optimized designs. Data obtained to asses strength of materials from measurements and experiments exhibit an intrinsic variability, and need to be manipulated so that it presents to the engineer some useful information, and hence call for a statistical and probabilistic approach.

Some software is also available these days to analyze the reliability of existing structures. One of them is COMREL, which has been used in this project in calculating the reliability of given analysis problem.

1.2. Objective and scope of study

Following are the foremost objectives of the present study-

- To study the different methods of probability analysis.
- To study in detail the reliability methods which are used in the analysis of structures.
- To analyze the reliability of beams and columns based on Indian Standard codes using different methods of reliability analysis and having different probability distribution curves.

1.3. Organization of Report

This report is organised into eight chapters. First introductory section i.e. Chapter 1 presents the background, objectives, and methodology of the project in which the basic requirements of a reliable design is discussed along with the failures and weakness of the existing design system. It has been tried to build a consensus on the weaknesses of existing methods and need of new reliability methods. Chapter 2 discusses in detail about the paper studied and literature reviewed for the project. Chapter 3 presents a brief review of random vibration analysis and discusses different issues on the design methodologies already existing along with some basic terms of statistics used in the analysis of the problem. Chapter 4 studies the concept and need of structural reliability and methods used to calculate structural reliability index. A special case for normal random variables is also discussed separately in this chapter. Chapter 5 then presents a useful insight of two of the important methods of structural reliability analysis, the first order reliability analysis method (FORM) and second order reliability analysis method (SORM). It also focuses on the concept of linear limit state functions. Chapter 6 is incorporated for the knowledge about the softwares used in calculating the reliability of a structure. The softwares component reliability (COMREL) and ETABS has been explained in detail as that has been used extensively in this project. Chapter 7 contains a real-time analysis of an I-shaped beam on the Hasofer and Lind's method of structural reliability analysis and FORM and SORM analysis of the same in the software. It also contains a reliability problem of a corner column of a six-story building subjected to time-variant earthquake loading with limiting equztion takem from steel code, IS:800-2007. Finally in chapter 8 conclusions future scope are summarized and important points deduced from this analysis has been recorded.

CHAPTER 2

LITRATURE REVIEW

The concept of structural reliability analysis and design has been in the interest of a lot of scholars and researchers from quite long. Different approaches, analysis and design methodologies have been devised and worked upon subsequently. During the course of this project, guidance have been taken from some of the renowned scholars in this field. Review of their papers have been explained briefly.

R Ranganathan [1], engineering decisions must be made in the presence of uncertainties which are invariably present in practice. In the presence of uncertainties in the various parameters encountered in analysis and design, achievement of absolute safety is impossible. It is now more than 25 years since it was proposed that a rational criterion for the safety of structures is its reliability or probability of survival. In structural reliability, the probability of failure (which is taken as one minus reliability) is taken as a quantitative measure of structural safety. Probabilistic concepts are used in reliability analysis and design of structures. Using structural reliability theory, the level of reliability of existing structures (structures designed by existing structural standards) can be evaluated. It can also be used for developing a design criterion, that is, calibrating codes and developing partial safety factors, the use of which will result in design with an accepted level of reliability. Structural reliability has been applied to many decision making problems, such as development of partial safety factors, establishing inspection criteria, taking suitable decisions for improving the capability of existing structures, development of maintenance schedule etc.., in the field of engineering.

Armen Der Kurighian [2], presented the geometry of random vibrations and solutions by FORM and SORM. The geometry of the random vibration problems in the space of standard normal random variables obtained from the discretization of the input process is explored. For linear systems subjected to the Gaussian excitation, the problems of curiosity are characterized by modest geometric forms, such as half spaces, vectors, planes, ellipsoids, and wedges. For non-Gaussian responses, the problems of interest are characterized by non-linear geometric forms. Approximate solutions for such problems are obtained by use of the first- and second-order reliability methods (FORM and SORM). This article offers a new outlook on random vibration problems and an approximate method for their solution. Examples involving a response to non-Gaussian excitation and out-crossing of a vector process from a non-linear domain are used to demonstrate the approach. The main objectives of these examples are to demonstrate the applicability of FORM and SORM and to examine their accuracy.

A. Der kiureghian, and P.-L. Liu [3], A comprehensive framework is set forth for the scrutiny of structural reliability with incomplete probability information. Under specified requirements of operability, invariance, simplicity, and consistency, a method is developed to unite in the reliability analysis incomplete probability info on random variables, including partial joint distributions, bounds, marginal distributions, and moments. The method is consistent with the philosophy of Ditlevsen's generalized reliability index and complements existing second-moment and full-distribution structural reliability theories. Consistent with Ditlevsen's notion of the generalized reliability index, under incomplete probability information, we seek a formal distribution model for *X* and a transformation *T(-)* such that *Y* is standard normal. As ground rules for selection of this transformation and distribution, the following requirements are stipulated:

1. Consistency - The distribution model for *X* shall satisfy the rules of probability and be consistent with the available information.

2. Invariance - The reliability index, β, shall be invariant with respect to all conjointly consistent formulations of the transformation or the distribution model.

3. Operability - The distribution model shall apply to random numbers of basic variables and be capable of combining any and all available information.

4. Simplicity - The strength needed for computing the reliability index shall be appropriate with the quality of information accessible.

O. Ditlevesen [5], A crucial property of any degree of structural reliability should be comparativeness. With this point in mind, this paper discusses some well-known versions of so-called reliability indices. Such reliability indices, defined by use of second-moment information, have been used for the last decade, specifically within safety code committee work. This paper defines a generalized second-moment reliability index that fulfills some few fundamental established rules and principles of simplicity. Reliability index is precisely defined to be used when no great quality information is accessible to the engineer other than limit state surface and a second-moment representation for the set of basic variables of the structural problem.

C. Boyer & A. Beakou & M. Lemaire [11], Traditional design methods use global safety factors to take into account the uncertainties in manufacturing, loads, materials properties... Their values have been established after many years of experiments and calibration by judgement, but they are not suited to new materials with particular features. Fibre-reinforced composite materials are charac- terized by their exceptitonally low weight-strength ratio but also by considerable scatters of mechanical properties and many failure modes. For these materials, the choice of safety factors cannot be only based on poor or inexistent experiments. Therefore, their calibration, by means of reliability methods, is one way to determine the new values for reliable optimized structure design. A method to determine safety factors equivalent to those used in metallic structures is presented. The author does not, however, take account of the particular properties of composites such as anisotropy, ultimate limit states. In this paper, second moment reliability methods which allow us to consider these characteriistic features are used.

CHAPTER 3
BRIEF REVIEW OF RANDOM VIBRATION ANALYSIS

3.1 Introduction

3.1.1 General

When the excitation function applied to a structure having an irregular shape that is described indirectly by statistical means, we called it a random vibration. Such a function is usually described discrete or continues function of existing frequencies, in a manner similar to the description of the function by Fourier series. A random variable is a variable that takes on numerical values according to a chance process. Random variables are generally of two types-

The discrete random variable is a countable values i.e. number of coursed selected by the students in the university.

Continuous random variable, all values in intervals like (0,1) i.e. height of randomly selected adult in the range of 1 to 2 meter.

In structural dynamics, the random excitations are more often encountered are either acoustic pressure or motion transmitted through the foundation, both of these types of loadings are generated by explosion occurring in the vicinity of the structure. Common sources of these type of explosions are construction work and mining.

Another type of loading such as earthquake excitation may also be considered the random function of time. In this case, the structural response id obtained in probabilistic terms using random vibration theory. The main characteristic of such random function is that its instantaneous value cannot be predicted in a deterministic sense. The discretion and analysis or random process are established in a probabilistic sense for which it is necessary to use tools provided by the theory of statistics.[9] As we know that experiments are conducted in civil engineering to ascertain a lot of parameters such as cube strength on-site, wind load values, earthquake magnitudes, soil properties, etc. The results obtained shows that though much care has been instituted to maintain the uniformity of the experiments, the results of two similar experiments are seldom same. Also the experiments such as variation of live load intensity with time and space, variation of yearly maximum wind speed in a city, variation in column depth, variation of concrete strength of cube casted on-site with the same grade of concrete, etc. have led to the conclusion that though in real practice we consider them to be constant they are always variable with varying degree of variations. Having accepted that there is bound to be some pattern of variations inherent in all observed data, it is now the problem for an engineer to take decisions based on that data. This problem often lead to conservative decisions which may not always result in economic designs. However, in many problems the variations are too large to be overlooked and hence call for a statistical or probabilistic approach. A basic notion of statistics is the notion of variation. It is the science of making decisions on incomplete information; that is drawing conclusions from the observed data.

3.1.2 Mean and Variance

Sample mean of a random variable is a measure of the central tendency (central value). This is by far the best statistic to numerically summarize a distribution and the center of gravity of data.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Where $x_1, x_2, x_3, \ldots, x_n$ is the sequence of observed value.

The variability or the dispersion of the data set is also a significant characteristic of the data set. This dispersion may be described by the sample variance s^2 given by

$$s^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

3.2 Probability density function and cumulative density function

Let there be a record of a random function $x(t)$. The values between x_1 & x_2 is taken and then measure the corresponding time intervals Δt_i. The ratio is given by

$$P(x_1 \leq x \leq x_2) = \frac{\Delta t_1 + \Delta t_2 + \cdots \Delta t_n}{T}$$

Moreover, calculated for the entire record length T, is the probability of x having the value between x_1 & x_2 at any selected time t_i during the random process.

Similarly, the probability of *x(t)* being smaller than a value of *x* can be expressed as

$$P(x) = P[x(t) < x] = \lim_{T\to\infty} \sum_i \Delta t_i$$

Where the time delta are now those for which the function *x(t)* as a value smaller than the specified *x*. The function *P(x)* in the equation is known as the cumulative distribution function of the random function *x(t)*. This function is plotted in the figure as a function of *x*. The cumulative distribution function is a monotonically increasing function for which

$$P(-\infty) = 0, 0 \leq P(x) \leq 1, P(\infty) = 1$$

Now the probability that the value of the random variable is smaller than the value $x+\Delta x$ is denoted by P($x+\Delta x$) and that *x(t)* takes value between x and $x+\Delta x$ is P($x+\Delta x$)-*P(x)*

This allows us to define the probability density function as

$$p(x) = \lim_{\Delta x\to 0} \frac{P(x-\Delta x) - P(x)}{\Delta x} = \frac{dP(x)}{dx}$$

Thus, the probability density function *p(x)* is represented geometrically by the slope of the cumulative probability function *P(x)*. The functions $p(x)$ and $P(x)$ are shown in that figure. From equation, we conclude that the probability that a random variable *x(t)* has a value between *x* and $x+\Delta x$ is given by *p(x)dx*, where *p(x)* is the probability density junction.

$$P(x \leq x \leq x + \Delta x) = \int_{x}^{x+\Delta x} p(x)dx$$

3.2.1 Some useful probability distributions

In this section, some probability distributions of continuous random variable and their properties, which are used in practical applications mostly, are presented briefly.

3.2.1.1 Normal (Gaussian) Distribution

the normal or Gaussian probability density function of a random variable X is the one mostly used in practice. It is defined in general as

$$p(x) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{\frac{\frac{1}{2}(x-\bar{x})^2}{\sigma^2}}$$

in which $\bar{x}$ & σ^2 are respectively the mean and standard variation of X. the corresponding CDF is calculated from:

$$P(x) = \int_{-\infty}^{\infty} p(\epsilon)d\epsilon = \emptyset\left(\frac{x-\bar{x}}{\sigma}\right)$$

Where $\emptyset(-)$ is called as the standard normal distribution function and its PDF is denoted by $\varphi(-)$, which are defined:

Standard Normal PDF $\varphi(x) = \frac{1}{\sqrt{2\pi}} \int e^{-x^2/2}$

Standard Normal CDF $\emptyset(x) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} e^{-u^2/2} du$

If multivariate normal variables are involved in a process, then a multivariate normal PDF will be required. In this case, a vector process is used, and the multivariate normal PDF is stated as,

Multivariate Normal PDF $p(x) = \left(\frac{1}{2\pi}\right)^{\frac{p}{2}} \frac{1}{\sqrt{|\rho|}} e^{-\frac{x^2}{2}}$

where $\bar{X}$ is a vector of p-dimensional random variable, $\bar{x}$ is a vector of their realizations and χ^2 is a scalar calculated from the product

Scalar: $\chi^2 = (\bar{x} - \bar{m})^T \rho^{-1} (\bar{x} - \bar{m})$

in which $\bar{m}$ is a vector of mean values and ρ is the covariance matrix of xbar and modrow in eq denotes the determinant of ρ

These definitions are written:

Vector of multivariate random variable: $\tilde{X} = \{X_1, X_2 \dots\dots\dots\dots X_n\}^T$

Vector of realizations of $\tilde{X}$: $\tilde{X} = \{x_1, x_2 \dots\dots\dots\dots x_n\}^T$

Vector of mean values of $\tilde{X}$: $\tilde{m} = \{m_1, m_2 \dots\dots\dots\dots m_n\}^T$

The covariance matrix ρ is defined as

Covariance matrix of $\tilde{X}$: $\rho = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$

As it is seen from equation, the diagonal terms of this matrix are the variances of the random variable X_i, for uncorrelated random variable, the off diagonal terms will be zero and the matrix becomes diagonal.

3.2.1.2 Lognormal Distribution

One other commonly used distribution in practice is the logarithmic distribution. If the random variable *X* has a normal distribution with a mean and variance, m_x and σ_2, then the random variable. $Y = e^x$ is said to be a log normally distributed. It is written as exponential function of X: $Y = e^X \; and \; X = \ln Y$

Using eq the PDF of the random variable $Y = e^x$, can be obtained as written

Lognormal PDF: $p(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\frac{1}{y}e^{-\frac{1}{2}\left(\frac{\ln y - mx}{\sigma_x}\right)^2}$ for$(y>0)$

In the region of (y<0), PDF of the random variable will be zero, i.e. $f_y(y)=0$ for $(y=0)$.

The mean and variance of an lognormal random variable, are calculated from

Mean of the random variable $Y = e^x$: $m_y = e^{m_x}e^{\frac{\sigma_x^2}{2}}$

Variance of the random variable $Y = e^x$: $\sigma_y^2 = m_y^2\left(e^{\frac{\sigma_x^2}{2}} - 1\right)$

If m_y and σ_y are given, then the variance and mean of *X* are calculated from the following statements

$$\sigma_x^2 = \ln\left[1 + \left(\frac{\sigma_Y}{\mu_Y}\right)^2\right]$$

And

$$\mu = \left(\ln \mu_y - \frac{\sigma_x^2}{2}\right)$$

3.2.1.3 Gamma Distribution

The gamma distribution represents the sum of r independent exponentially distributed random variable, and random variable that always take positive values. It's PDF and CDF functions are defined as written

Gamma Dist., PDF: $f_x(x) = \frac{\lambda}{\Gamma(r)}(\lambda x)^{r-1}e^{-\lambda x}$ if *(x≥0, λ≥0)*

Gamma Dist., CDF: $1 - \sum_{k=o}^{r-1}\frac{1}{k!}(\lambda x)^k(\lambda x)^k$ if *(x≥0, λ≥0)*

In which Γ(.) represents a gamma function which is defined as:

Gamma function: $\Gamma(x) = \int_0^\infty e^{-u}u^{(x-1)}du$

The mean and variance of the gamma distribution are calculated to be

Mean: $m_x = \frac{r}{\lambda}$

And Variance: $\sigma_x^2 = \frac{r}{\lambda^2}$

The parameters r and λ are respectively the shape and scale parameters of the distribution. For different values of r and λ, different type of distribution are obtained. When (r=1) it gives the exponential distribution. If (r<1), then the distribution is exponentially shaped and asymptotic to both horizontal and vertical axes. If (r>1), its shape is unimodal and skewed with the mode equals $(x_m = (r-1)/\gamma)$. the skewness reduces with increasing value of r as it is seen from the coefficient of skewness, $\left(\gamma_1 = \frac{2}{\sqrt{r}}\right)$. If (*r=s/2)* and (*γ=1/2*), then the gamma distribution becomes the χ^2 distribution with s degree of freedom. In engineering applications, gamma distributions occur frequently in models of failure analysis, for rainfall studies since the variables are always positive and the results are unbalanced.

3.2.1.4 Rayleigh Distribution

The Rayleigh distribution is used as a probability model describing the distribution of wind speed over a 1-year period. It is often used for the probability model of the absolute value of components of random variables, both with zero mean and variance equal to σ^2 and of we define a function $Z = \sqrt{X^2 + Y^2}$ then this function has a Rayleigh distribution with parameter σ. It also describes the probability distribution of maxima of a narrow band random process with normal distribution. The PDF and CDF of the Rayleigh distribution is given as

Rayleigh PDF: $$f_x(x) = f(x) = \begin{cases} \frac{x}{\sigma^2} \exp(-\frac{x^2}{2\sigma^2}, & x \geq 0 \\ 0, x < 0 \end{cases}$$

Rayliegh CDF: $$f_x(x) = 1 - exp\left(-\frac{x^2}{2\sigma^2}\right)$$

In which σ is the only parameter of the distribution, which is equal to the standard deviation of the random variables X and Y with normal distributions and zero mean. The mean and variance of the Rayleigh distribution are calculated to be

Mean: $$m_x = \sigma\sqrt{\frac{\pi}{2}}$$

Variance: $$\sigma_x^2 = 2\sigma^2\left(1 - \frac{\pi}{4}\right)$$

3.2.1.5 Gumbel Distribution

The Gumbel distribution is usually used to model the distribution of the maximum, or the minimum, of some samples of various distributions. It can be used to determine the probability that an extreme event, such as an earthquake, flood or another natural disaster, will occur. The Gumbel distribution is also termed as the extreme value type I distribution. It has two forms as one is for extreme maximum (extreme value largest I) and one is for extreme minimum (extreme value smallest I), which are respectively defined below.

Gumbel (EV largest-I): $$f_x(x) = \alpha e^{-\alpha(x-\beta)} e^{-\exp(-\alpha(x-\beta))}$$

$$F_x(x) = e^{-\exp(-\alpha(x-\beta))} \text{ for } (-\infty < x < \infty)$$

Gumbel (EV smallest-I) $$f_x(x) = \alpha e^{-\alpha(x-\beta)} e^{-\exp(-\alpha(x-\beta))}$$

$$F_x(x) = 1 - e^{-\exp(-\alpha(x-\beta))} \text{ for } (-\infty < x < \infty)$$

In which β is the location parameter and α is the scale parameter, which is defined *(α > 0)*. The Gumbel distribution supports the range of outcomes of the random variable. X between $(-\infty < x < \infty)$. The means and variances of both largest-I and smallest-I distributions are calculated from

Mean: $m_x = \beta + \frac{0.57722156649}{\alpha}$ (largest-I)

$m_x = \beta - \frac{0.57722156649}{\alpha}$ (smallest-I)

Variance: $\sigma_x^2 = \frac{\pi^2}{6\alpha^2}$ (largest-I and smallest-I)

The value (0.57722156649) in above equations is the Euler's constant.

CHAPTER 4
STRUCTURAL RELIABILITY

4.1 Introduction

The performance of a structure is assessed by its safety, serviceability, and economy. The information about input variables is never certain, precise, and complete. The source of uncertainties maybe

a) Inherent randomness, i.e. physical uncertainty

b) Limited information i.e. statistical uncertainty,

c) Imperfect knowledge, i.e. modal uncertainty,

d) Gross errors

In the presence of uncertainties, the absolute safety of structures is impossible due to the unpredictability of loads on the structure during its life, in place material strengths, and human errors; structural idealizations in forming the mathematical model of the structure to predict its response or behavior; and the limitation in numerical methods. In the conventional deterministic and analysis and design methods, it is assumed that all parameters (loads, strengths of materials, etc.) are not subjected to probabilistic variations. The safety factors provided in the existing codes and standards, primarily based on practice, judgment, and experience may not be adequate or economical.

The concept of reliability is applied to many fields and has been interpreted in many ways. The most common definition, and accepted by all, of reliability, is in another word the probability of an item performing its anticipated function over a given period under the operating conditions encountered. It can also be explained as the probability that a structure will not attain each specified limit (flexure or shear or torsion or deflection criteria) during a specified reference period (life of structure). For convenience, the reliability R_0 is defined in terms of the probability of failure, p_f, which is taken as

$$R_0 = 1 - P_f$$

4.2 Levels of reliability methods

There are different stages or level of reliability analysis, which can be used in any design methodology subject to the importance of the structure. The term '**level**' is described by the extent of information about the problem that is used and delivered. The methods of safety analysis suggested currently for the attainment of a given limit state can be assembled under four basic “levels” (namely level IV, III, II, and I) depending upon the degree of sophistication smeared to the treatment of the several problems.

4.2.1 In the **level I** methods, the probabilistic characteristic of a given numerical problem is taken into account by inserting the safety analysis appropriate “characteristic values” of the random variables, considered as fractile of an already defined order of the statistical distributions concerned. These characteristic values are then associated with partial safety factors that should be inferred from probabilistic considerations so as to ensure suitable levels of reliability in the design. In this method, the reliability of the design diverge from the target value, and the objective is to minimize such an error. Load & Resistance Factor Design(LRFD) method is an example in this category.

4.2.2 Reliability methods, which employ two values of each uncertain parameter (i.e., mean and variance), complemented with an amount of the correlation between parameters, are categorized as **level II** methods.

4.2.3 **Level III** methods include a complete analysis of the problem and also include integration of multidimensional joint probability density function of random variables stretched over the safety domain. Reliability is expressed in terms of appropriate safety indices, eg- reliability index(β) and failure probabilities.

4.2.4 **Level IV** methods are suitable for structures that are of foremost economic importance, involve the principles of engineering economic analysis and design under uncertainty, and consider costs and benefits of construction, maintenance, repair, magnitudes of failure, and interest on capital, etc. Foundations for sensitive projects like nuclear power projects, transmission towers, highway bridges, are suitable objects of level IV design.

4.3 Computation of structural reliability

For structural components and systems, first of all, no relevant failure data are available. Secondly, failure occurs significantly rarer, and thirdly, the mechanism behind failures is different. Structural failure occurs not predominantly due to elderly processes but additionally due to the effect of severe events, such as extreme winds, snowfall, earthquake, or combinations. For the reliability assessment, it is consequently necessary to consider the influences acting from outside i.e. loads and influences acting from the inside i.e. resistances individually. It is thus necessary to establish probabilistic models for loads and resistances including all available information about the statistical characteristics of the parameters influencing these. Such information is such as data regarding the earthquakes, experimental results of concrete compression strength, etc.

4.3.1 Basic method

The basic structural reliability problem contemplates only one load effect (*S)* which is resisted by one resistance (*R)*. Each is described by a well-known probability density function, f_S ( ) and f_R( ) respectively. It is essential that *R* and *S* be expressed in the same units. For ease, but without loss of generality, only the safety of a structural element will be measured here and as usual, that structural element will be considered to have failed if its resistance *R* is less than the resultant stress *S* acting on it. The probability p_f of failure of the structural element can be stated in any of the following ways,

$$p_f = P(R \leq S)$$

$$= P(R - S \leq 0)$$

$$= P\left({R}/{S} \leq 1 \right)$$

$$= P(lnR - lnS \leq 0)$$

or, in general

$$= P(G(R,S) \leq 0)$$

Where *G( )* is designated the *limit state function* and the probability of failure is similar with the probability of limit state violation. General density functions f_S and f_R for S and *R* separately are shown in Figure 2 along with the joint (bivariate) density function $f_{RS}(r,s)$.

For any infinitesimally small element ($\Delta r \Delta s$), the latter signifies the probability that *R* takes a value in between *r* and $r+\Delta r$ and *S* value in between *s* and $s+\Delta s$ as Δr and Δs each approaches zero. In Figure 2, the Equations above are represented by the hatched failure domain *D*, so that the probability of failure becomes:

$$p_f = P(R - S \leq 0) = \iint_D f_{RS}(r,s)drds$$

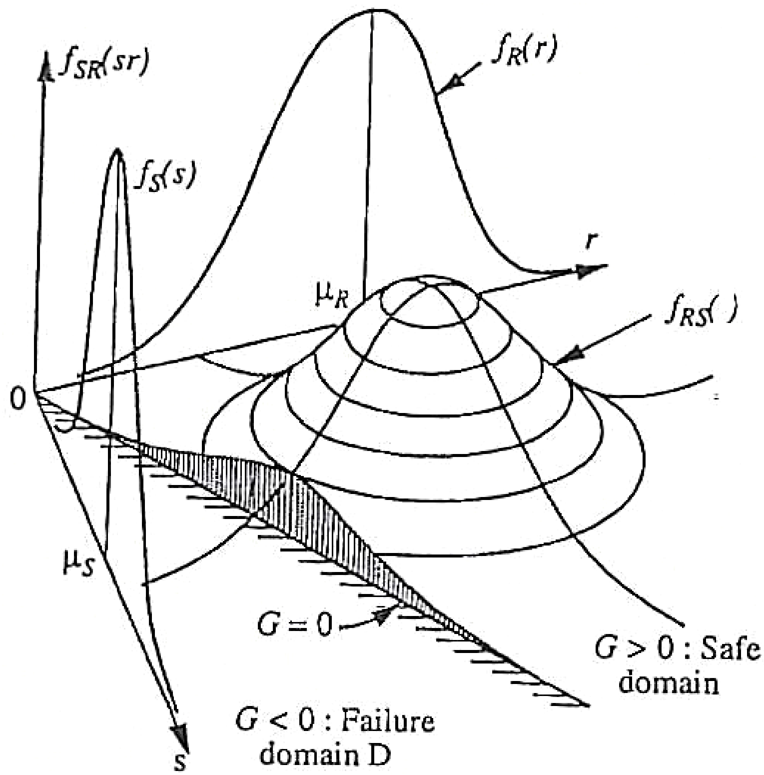

Fig. 1 -Joint density function $f_{RS}(r,s)$, marginal density functions $f_R(r)$ and $f_S(s)$ and failure domain *D*

When *R* and *S* are independent,

$$f_{RS}(rs) = f_R(r)f_S(s)$$

moreover, equation for probability of failures then becomes:

$$p_f = P(R - S \leq 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{s\geq r} f_R(r)f_S(s)drds = \int_{-\infty}^{\infty} F_R(x)f_s(x)dx$$

This can be termed as a convolution integral with significance readily explained by reference to Figure 2. $F_R(x)$ is probability that $R \leq x$ or probability that actual resistance *R* of the structure is less than some value *x*. Let this represent a failure of the structure. The term $f_S(x)$ represents the probability that load effect S acting in the member obtains a value between x & $x+\Delta x$ in the limit as $\Delta x \rightarrow 0$. Now, considering all possible values of x, or by taking the integral over all x, the total probability of failure is obtained. This is also seen in Figure 3 where the density functions $F_R(x)$ and $f_S(x)$ have been drawn along the same axis.

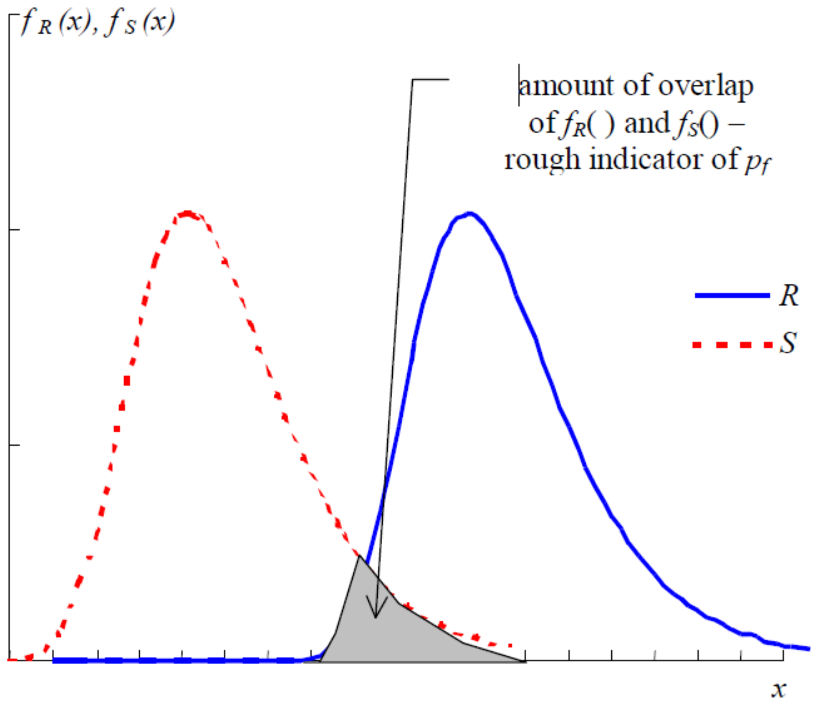

Fig. 2 - Basic *R-S* problem: $f_R(\ )\ f_S(\ )$ representation

4.3.2 Special case of normal random variables

For a few distributions of R and S, it is possible to integrate the convolution integral (2) analytically. One notable example is when both are normal random variables with means μ_R and μ_S and variances σ_R^2 and σ_S^2 respectively. The safety margin $Z=R-S$ then has a mean and variance given by well-known rules for addition of normal random variables:

$$\mu_Z = \mu_R - \mu_s$$

$$\sigma_Z^2 = \sigma_R^2 + \sigma_S^2$$

Equation for probability of failure then becomes

$$p_f = P(R - S \le 0) = P(Z \le 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right)$$

Where $\Phi(\ )$ is the standard normal distribution function (zero mean and unit variance). The random variable $Z = R\text{-}S$ is shown in Figure, in which the failure region $Z \leq 0$ is shown shaded. Using equations above, it follows that

$$P_f = \Phi\left[\frac{-(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)^{\frac{1}{2}}}\right] = \Phi(-\beta)$$

Where, $\beta = {}^{\mu_Z}/_{\sigma_Z}$ is defined as ***reliability (safety) index***.

If either of the standard deviations σ_R and σ_S or both are increased, the term in square brackets in above equation will become smaller and hence, P_f will increase. Similarly, if the difference between the mean of the load effect and the mean of the resistance is reduced, P_f increases. These observations may also be deduced from Figure 5, taking the amount of overlap of $f_R(\ )$ and $f_S()$ as a rough indicator of P_f.

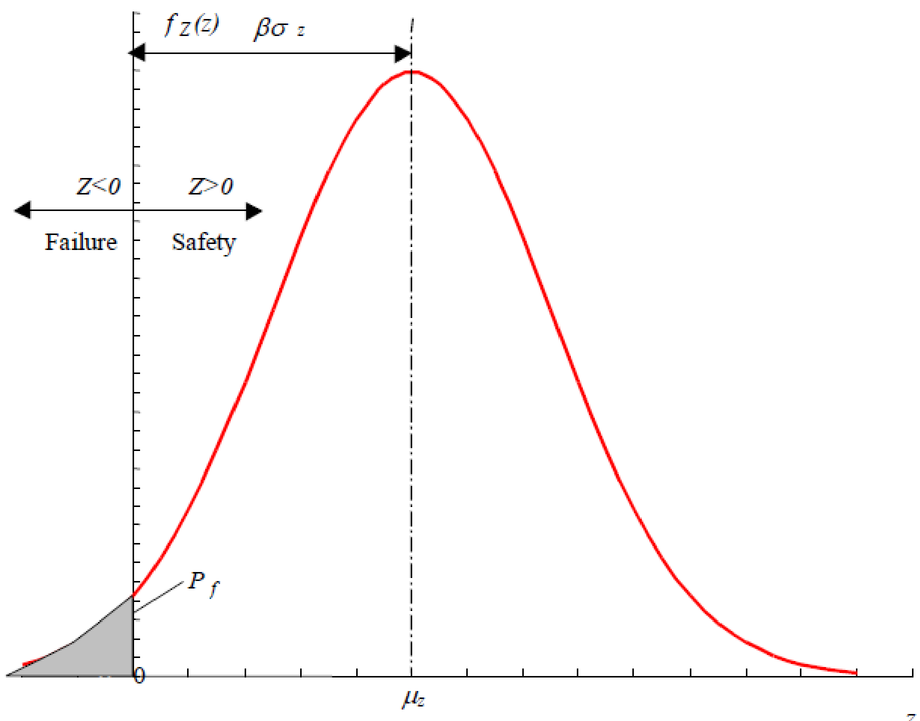

Fig. 3 - Distribution of safety margin $Z = R – S$

4.4 Computation of structural reliability

4.4.1 Reduced Variables

It is expedient to transform all random variables to their "standard form" which is a nondimensional form of the variables. For basic variables R and Q, the standard forms can be expressed as

$$Z_R = \frac{R - \mu_R}{\sigma_R}$$

$$Z_Q = \frac{R - \mu_Q}{\sigma_Q}$$

The variables Z_R and Z_Q, are called *reduced variables.* By reorganizing Equation no.1, the resistance R and the load Q can be expressed in terms of the reduced variables as follows:

$$R = \mu_R + Z_R\sigma_R$$

$$Q = \mu_Q + Z_Q\sigma_Q$$

The limit state function is $g(R, Q) = R - Q$. Which can be stated in terms of the reduced variables by using Eqs.2. The result is

$$g(Z_R, Z_Q) = \mu_R + Z_R\sigma_R - \mu_Q - Z_Q\sigma_Q = (\mu_R - \mu_Q) + Z_R\sigma_R - Z_Q\sigma_Q$$

For any definite value of $g(Z_R, Z_Q)$, Equation above represents a straight line in the space of reduced variables Z_R and Z_Q. The line corresponding to $g(Z_R, Z_Q) = 0$ separates the safe and failure domain in the space of reduced variables. The loads Q and resistances R are sometimes indicated in terms of capacity C and demand D as well in literature.

4.4.2 Definition of the Reliability Index

A form of the reliability index is defined as the inverse of the coefficient of variation. The reliability index is the perpendicular or shortest distance from the origin of reduced variables to the failure point or design point as illustrated in Figure 3, line $g(Z_R, Z_Q) = 0$. This definition was presented by Hasofer and Lind. Using geometry we can determine the reliability index (shortest distance) from the following formula

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

where β is the inverse of coefficient of variation of function $g(R, Q) = R - Q$. if R and Q are uncorrelated for normally distributed random variables R and Q, then the reliability index is related to probability of failure by

$$\beta = -\phi^{-1}(P_f) \ \ or \ \ P_f = \phi(-\beta)$$

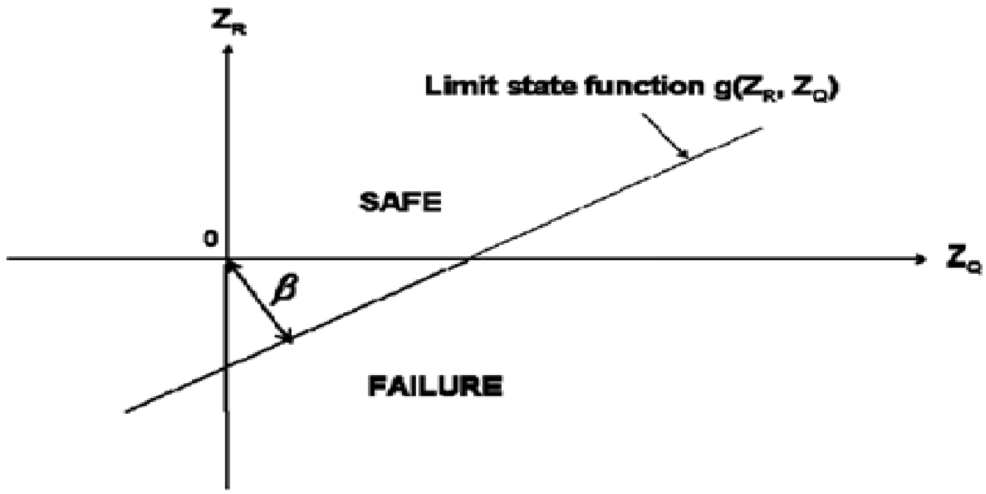

Fig. 4 - Reliability index defined as the shortest distance in the space of reduced variables

CHAPTER 5
FORM and SORM

5.1 First order reliability method (FORM)

This technique is built on the first-order Taylor series approximation of the performance function which is linearized at the mean values of random variables. It is also known as mean value first-order second moment (MVFOSM) method; It uses only second-moment statistics (mean & variance) of the random variables. Initially, Cornell (1969) used the simple two variable approaches. On the basic assumption that the resulting probability of Z is a normal distribution, he defined the reliability index using the ratio of the expected value of Z upon its standard deviation. Cornell reliability index (β_c) is absolute value of ordinate of the point conforming to Z = 0 on standardized normal probability plot as shown in Figure 4 and equation

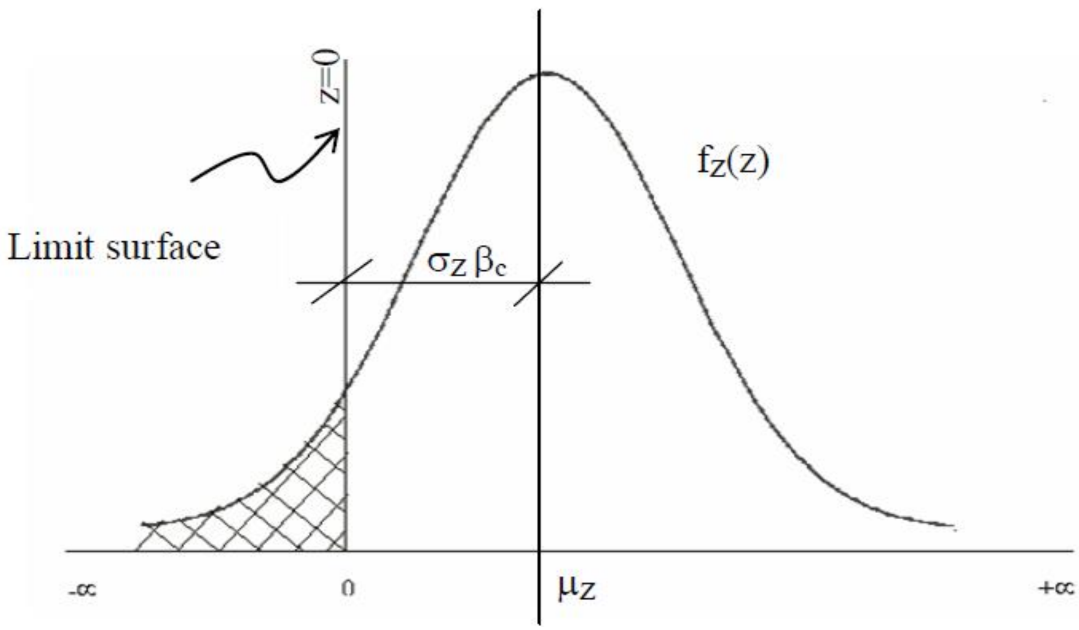

Fig. 5 – definition of limit state and reliability index

$$\beta_c = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

Alternatively, if joint probability density function $f_X(x)$ is identified for the multi variable case, then probability of failure p_f is given by

$$p_f = \int_L f_X(x)dX$$

Where L is the domain of X such that $g(X)<0$.

Generally, the above integral cannot be solved analytically. Therefore, an approximation is obtained by FORM approach. In this approach, the general case is approximated to an ideal condition where X is a vector of independent Gaussian variables having zero mean and unit standard deviation, and where $g(X)$ is a linear function. The probability of failure p_f is then

$$p_f = P(g(X) < 0) = P\left(\sum_{i=1}^{n} \alpha_i X_i - \beta < 0\right) = \phi(-\beta)$$

Where α_i is direction cosine of random variable X_i, β is the distance between origin and the hyperplane $g(X)=0$, n is the number of basic random variables X, and Φ is standard normal distribution function.

The above formulations can now be generalized for many random variables denoted by vector X. Let performance function is given as

$$Z= g(X) = g\ (X_1, X_2....X_n)$$

Using the Taylor series expansion, the performance function about mean value is given by the equation

$$Z = g(\mu_X) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(X_i - \mu_{X_i}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 g}{\partial \mathrm{X}_i\, \partial X_j}(X_i - \mu_{X_i})\left(X_j - \mu_{X_j}\right) + \cdots$$

Where derivatives are calculated at mean values of random variables $(X_1, X_2 ... X_n)$ and μ_{X_i} is the mean value of X_i. Trimming the series in linear terms, the first order mean and variance of Z can be obtained as

$$\mu_Z \approx g\left(\mu_{X_1}, \mu_{X_2, \ldots\ldots\ldots\ldots} \mu_{X_n}\right)$$

And,

$$\sigma_Z^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} var\left(X_i, X_j\right)$$

Where $var(X_i, X_j)$ is covariance of X_i and X_j. If variances are uncorrelated, then the variance for *z* is given as

$$\sigma_Z^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 var(X_i)$$

The reliability index can then be calculated by taking ratio of mean (μ_z) and standard deviation of Z (σ_z) as

$$\beta = \frac{\mu_Z}{\sigma_Z}$$

5.1.1 Hasofer and Lind's method

Let the failure function g be function of independent basic variables $X_1, X_2, \ldots X_n$, $i.e.\ g(X_1, X_2, \ldots \ldots X_n)$. The basic variables are then normalized using the relationship

$$Z_i = \frac{X_i - \mu_i}{\sigma_i} \quad i = 1,2,3, \ldots \ldots, n.$$

Where, $\mu_i = \mu_{X_i}$ and $\sigma_i = \sigma_{X_i}$.

In the Z coordinate system, the failure system is a function of Z_i. Using above equation in the failure function and equating it to zero, the failure surface equation is written in the normalized coordinate system. This failure surface also divides the design sample space into two regions, safe and failure. Because of the normalization of the basic variables, $\mu_{Z_i} = 0$ and $\sigma_{Z_i} = 1$.

It is to be noted that the z-coordinate system has a rotational symmetry with respect to the standard deviation and the origin will usually lie in the safe region. It is also to be noted that as the failure surface $g(z_1, z_2)$ moves away from the origin, the reliability, g(z)>0, increases and as it moves closer to the origin, reliability decreases as shown in figure 8. Hence, the position of failure surface with respect to origin in the normalized coordinate system determines the measure of reliability.

Hasofer and Lind defined the reliability index β as the shortest distance from the origin O to the failure surface in normalized coordinate system. Now β is related to the failure surface and not to the failure functions. The safety measure obtained is invariant to the failure function, since equivalent failure functions will result in same failure surface. The reliability index $\beta = \frac{\mu_M}{\sigma_M}$ can be used when the failure function is linear function of basic variables.

For a nonlinear failure surface, the shortest distance of the origin (in normalized coordinate system) to the failure surface is not unique as in the case of linear failure surface. The computation of probability of failure involves numerical integration. The tangent plane to the design point may be used to approximate the value of β. If the failure surface is concave towards the origin, the approximation will be on the safer side, while for the surface convex towards the origin it will be on the unsafe side.

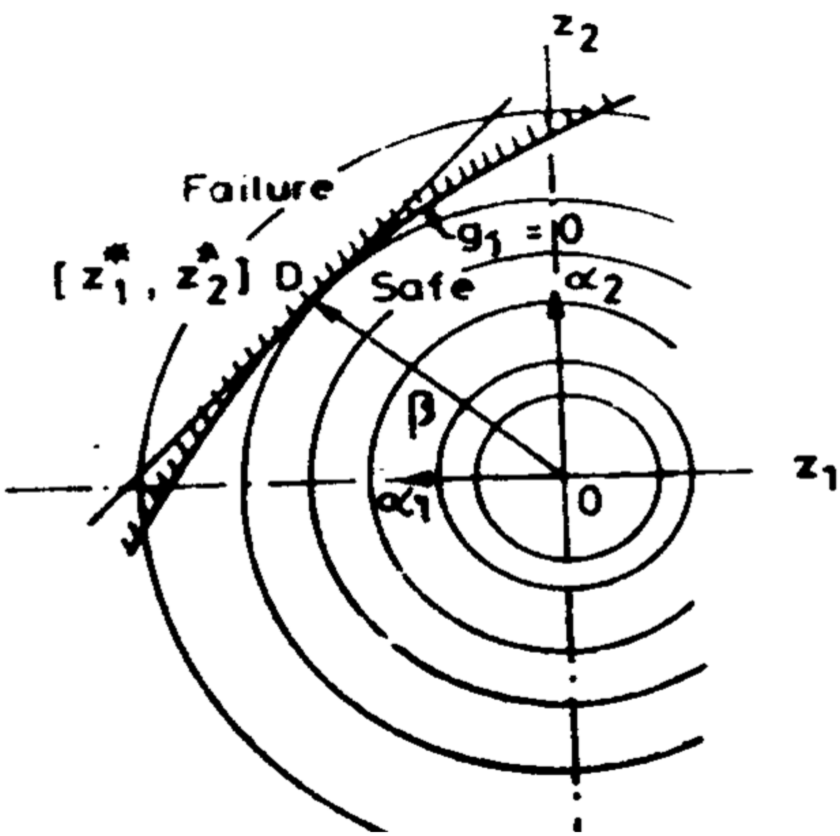

Fig. 6 – Formulation of safety analysis in normalized coordinates.

5.2 Second order reliability method (SORM)

In above part, a first order approximation for $g(X)$ was discussed for various combinations of the type of random variables to calculate reliability index. But in reality, the limit states are greatly non-linear in standard normal space and hence a first order approximation may contribute significant error in reliability index evaluation. Thus, a better approximation by the second order terms is required for highly non-linear limit states. This is often termed as Second Order Reliability Method (SORM). Revisiting the Taylor's series,

$$g(X_1, X_2, \dots, X_n) = g(x_1^*, x_2^*, \dots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(x_i - x_i^*) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 y}{\partial X_i \, \partial X_j}(x_i - x_i^*)(x_j - x_j^*) + \cdots$$

One can observe that unlike FORM the expression is considered till second order terms. Using this higher order expansion, Breitung (1984) suggested an asymptotic approximation as shown in Eq. below for estimating the probability of failure based on the β estimated in FORM,

$$p_{f_{SORM}} \approx \phi(-\beta) \prod_{i=1}^{n-1} \left(1 + \beta_{K_i}\right)^{-\frac{1}{2}}$$

where K_i is the main curvatures of the limit state surface at design point. It is computed through a series of steps as explained below

Initial orthogonal matrix T_0 is evaluated from the direction cosines evaluated as explained in FORM under Rackwitz algorithm

$$T_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{bmatrix}$$

Further, $T_0 = [\{t_{01}\}, \{t_{02}\}, \dots, \{t_{0n}\}]^t$ is modified using Gram-Schmidt orthogonal procedure as

$$t_k = t_{0k} - \sum_{i=k+1}^{n} \frac{t_i t_{0k}^t}{t_i t_i^t} t_i$$

Where, t_k is the row vectors of revised orthogonal matrix $T = [\{t_1\}, \{t_2\}, \dots, \{t_n\}]^t$ and k varies from n, $n-1$, $n-2$, …, 2, 1. Suffix t, i.e. $(.)^t$ denotes transpose of corresponding vector or matrix. The rotation matrix in produced by normalizing these vectors to unit

Orthogonal transformation of random variables X into Y as shown in above eqn. is evaluated using orthogonal matrix T (also known as rotation matrix)

$$Y = TX$$

Again using orthogonal matrix T, another matrix A is calculated as

$$A = [a_{ij}] = \frac{(THT^t)_{ij}}{\|G^*\|} \qquad i, j = 1,2, \dots, n-1$$

Where, H represents the double derivative matrix (or Hessian matrix) of limit state in standard normal space at design point.

Further, the last row and last column in A matrix and last row in the Y matrix are eliminated to consider a factor that last variable y_n coincides with β computed in FORM. Thus, limit state is expressed as

$$y_n = \beta + \frac{1}{2} y^t A y$$

Now, the size of the coefficient matrix A is reduced to $(n-1) \times (n-1)$ and main curvatures K_i are given by computing eigen values of matrix A.

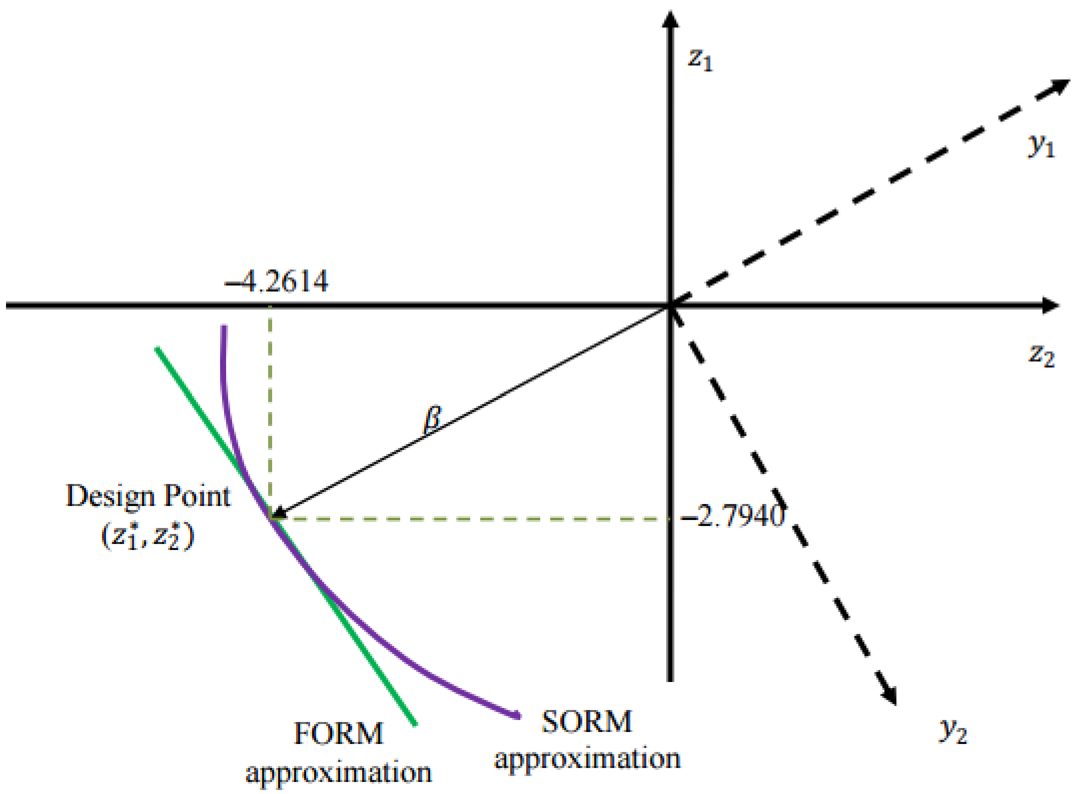

Fig. 7 - Difference between FORM and SORM design point approximation

CHAPTER 6

BRIEF REVIEW OF SOFTWARES

6.1 Component reliability (COMREL)

COMREL is a relatively new software which performs time-invariant reliability analysis of individual failure modes based on advanced FORM/SORM methodology. A lot of different algorithms to find the most likely failure point (β-point) are implemented including a gradient-free algorithm for non-differentiable failure criteria (state functions). Alternative computational options are MVFO(Mean Value First Order), Monte Carlo simulation, Spherical Sampling, Adaptive Sampling, several Importance Sampling schemes and Subset Simulation. COMREL can also deal with arbitrary dependence structures in stochastic model (Nataf-models, Rosenblatt, and Hermite). The full set of stochastic models offered by STATREL is supported (44 models), and it can be inserted either in parameter form or in terms of first two moments and additional parameters if required. The models can be truncated, and new user-defined models can also be inserted. The user can make distribution parameters dependent on other variables, functions, and parameters. Dependencies can also be labeled in terms of correlations when this is academically admissible. The increased flexibility in stochastic modeling certainly is one of the strengths of COMREL and SYSREL.

In COMREL several failure conditions can also be defined in one work. State functions can be either called from external programs or easily implemented in the Graphical User Interface. State functions can also be specified in normal mathematical representation. Names for parameters and variables can be chosen at will and are automatically transmitted to the stochastic model and vice versa. Some important constants are inbuilt and predefined. Built-in functions include all hyperbolic, trigonometric, logarithmic, elementary and some special functions like Bessel and Gamma functions, Gaussian distribution function and its inverse. Several alternatives for differentiation, numerical integration, and root finding are available along with test functions and comparative operators. Auxiliary user-defined functions, as well as reference functions, can also be defined.

6.2 ETABS

ETABS is a program for static, nonlinear, dynamic and linear analysis, and the design of building systems. From a systematic standpoint, multistorey buildings constitute a very distinct class of structures and therefore deserve distinct treatment. The concept of special programs for building structures was familiarized over 40 years ago and lead to the development of the TABS series of computer programs.

The innovative and ground-breaking new ETABS is the ultimate integrated software suite for the structural analysis and design of buildings. Combining 40 years of incessant research and development, this latest ETABS bids unmatched 3D object based demonstrating and visualization tools, very fast linear and nonlinear analytical power, refined and comprehensive design competencies for a wide-range of materials, and astute graphic displays, schematic drawings, and reports that allow users to decipher quickly and easily and apprehend analysis and design results.

From the beginning of design commencement through the production of schematic drawings, it integrates every aspect of engineering design process. Intuitive drawing commands allow for the swift and speedy generation of floor and elevation framing. AutoCAD drawings can be converted straight into ETABS models or used as prototypes onto which ETABS objects may be overlaid. The state-of-the-art SAPFire solver allows extremely large and multifaceted models to be rapidly analyzed and provisions nonlinear modeling techniques such as time effects (e.g., creep and shrinkage) and construction sequencing.

The numerical solution, input and output techniques of this software are specifically designed to take benefit of the unique numerical and physical characteristics related to building type structures. As a result, this analysis and design tool further execution throughput, data preparation, and output interpretation.

The need for special purpose programs has never been more apparent as Structural Engineers put non-linear dynamic analysis into run-through and use the greater computer power accessible today to create larger analytical models.

Over the past decades, it has several mega-projects to its credit and has established itself as the standard of the industry. This software is clearly recognized as the utmost practical and efficient tool for the dynamic and static analysis of shear wall buildings and multistorey frames. ETABS is also capable of performing time variant earthquake analysis such as response spectrum analysis, time history analysis, etc.

CHAPTER 7

PROBLEM STUDIED FOR RELIABILITY

7.1 Reliability of beam problem in shear

The problem involves determining the reliability index of a simply supported I beam at the limit state of shear. The beam is exposed to a load Q(point load) at mid span. It is given that:

$\mu_Q = 4000$ N

$\sigma_Q = 1000$ N

$\mu_{fs} = 95$ N/mm^2

$\sigma_{fs} = 10$ N/mm^2

$\sigma_d = 2.5$ mm

$\frac{d}{tw} = 40$

$\mu_d = 50$ mm

Where d is depth of the beam, t_w is the thickness of the web; f_s is the shear strength of the material. The coefficient of variation of t_w is negligible.

7.1.1 Manual calculation

As we know,

Maximum shear force =.Q/2

It is assumed that the web resists the whole shear. The beam fails in shear if

$$f_s t_w d - \frac{Q}{2} \leq 0$$

hence, the failure surface equation is

$$g(X) = f_s t_w d - \frac{Q}{2} = 0$$

As variation in t_w is negligible, t_w is considered as deterministic

let,

$$z_1 = \frac{(fs - \mu fs)}{\sigma fs}$$

$$z_2 = \frac{(d - \mu d)}{\sigma d}$$

$$z_3 = \frac{(Q - \mu Q)}{\sigma Q}$$

Substituting them in the equation for g(X) = 0 we get

$$g_1(z) = t_w(\sigma_{fs} z_1 + \mu_{fs})(\sigma_d z_2 + \mu_d) - \frac{1}{2}\sigma_Q z_3 - \frac{\mu Q}{2} = 0$$

Substituting the given data, we have

$$g_1(z) = 625z_1 + 296.88z_2 + 31.25z_1z_2 - 500z_3 + 3937.5 = 0$$

At design point we know that, $z_i = \beta\alpha_i$

$$g_1(z) = 625\ \beta\alpha_i + 296.88\ \beta\alpha_1 + 31.25\ \beta^2\alpha_1\ \alpha_2 - 500\ \beta\alpha_3 + 3937.5 = 0$$

Taking partial derivative of $g_1(z)$,

$$\left(\frac{\partial g1}{\partial z1}\right)^* = (\ 625 + 31.25z_2\ )^*$$

$$= (625 + 31.25\ \beta\alpha_2)^*$$

$$\left(\frac{\partial g1}{\partial z2}\right)^* = (296.88 + 31.25z_1)^*$$

$$= (296.88 + 31.25\ \beta\alpha_1)^*$$

$$\left(\frac{\partial g1}{\partial z3}\right)^* = -500$$

Start with

$$\beta = 6,\ \alpha_1 = -0.58,\ \alpha_2 = -0.58 \text{ and } \alpha_3 = +0.58$$

Using these in equations above we have,

$$\beta = \frac{-3937.5}{625(-0.58)+296.88(-0.58)+31.25(6)(-0.58)(-0.58)-500(0.58)}$$

$$= 5.17$$

Using equation

$$\alpha_1 = -\frac{1}{K}\left(\frac{\partial g1}{\partial zi}\right)^*$$

$$\alpha_1 = -\frac{1}{K}[625 + 31.25(5.17)(-0.58)] = -\frac{531.29}{K}$$

$$\alpha_2 = -\frac{1}{K}[296.88 + 31.25(5.17)(-0.58)] = -\frac{203.18}{K}$$

$$\alpha_3 = -\frac{1}{K}[-500] = \frac{500}{K}$$

$$K^2 = (-531.29)^2 + (-203.18)^2 + (500)^2$$

$$= 573551.17$$

$$K = 757.33$$

Hence,

$$\alpha_1 = -\frac{531.29}{757.33} = -0.702$$

$$\alpha_2 = -\frac{203.18}{757.33} = -0.263$$

$$\alpha_3 = \frac{500}{757.33} = 0.66$$

With these new values of β, α_1, α_2 and α_3, the cycle is repeated till β converges to the minimum. Summarized results are given in the following table:

Variable	Iteration					
	Start	1	2	3	4	5
B	6	5.17	4.82	4.805	4.798	4.796
α_1	-0.58	-0.702	-0.738	-0.740	-0.740	-0.741
α_2	-0.58	-0.263	-0.241	-0.237	-0.235	-0.234
α_3	0.58	0.660	0.640	0.635	0.630	0.629

Hence,

The solution is reliability index (β) = 4.796

Also, the design point is $z^* = (\beta\, \alpha_1, \beta\, \alpha_2, \beta\, \alpha_3)$

7.1.2 Solved using COMREL

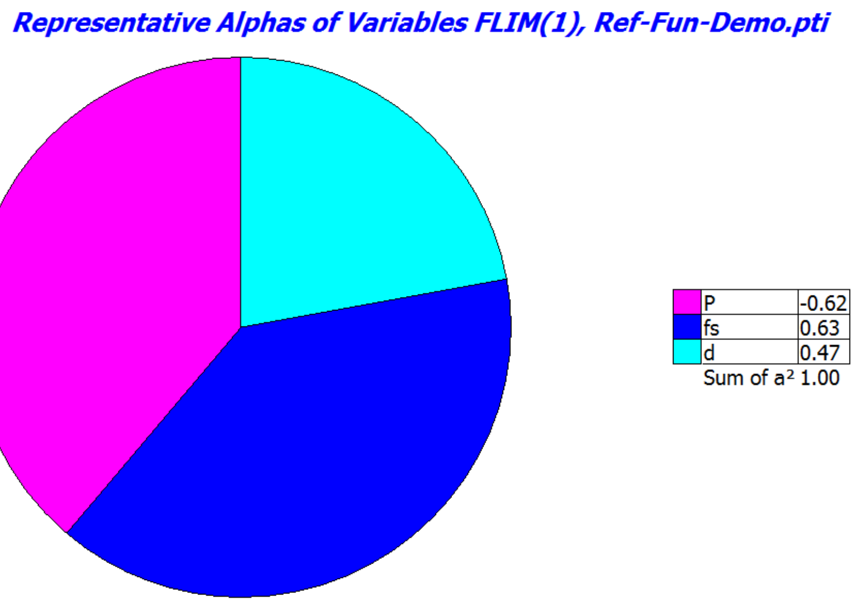

Fig. 8 - α values obtained for all the three variables at design point

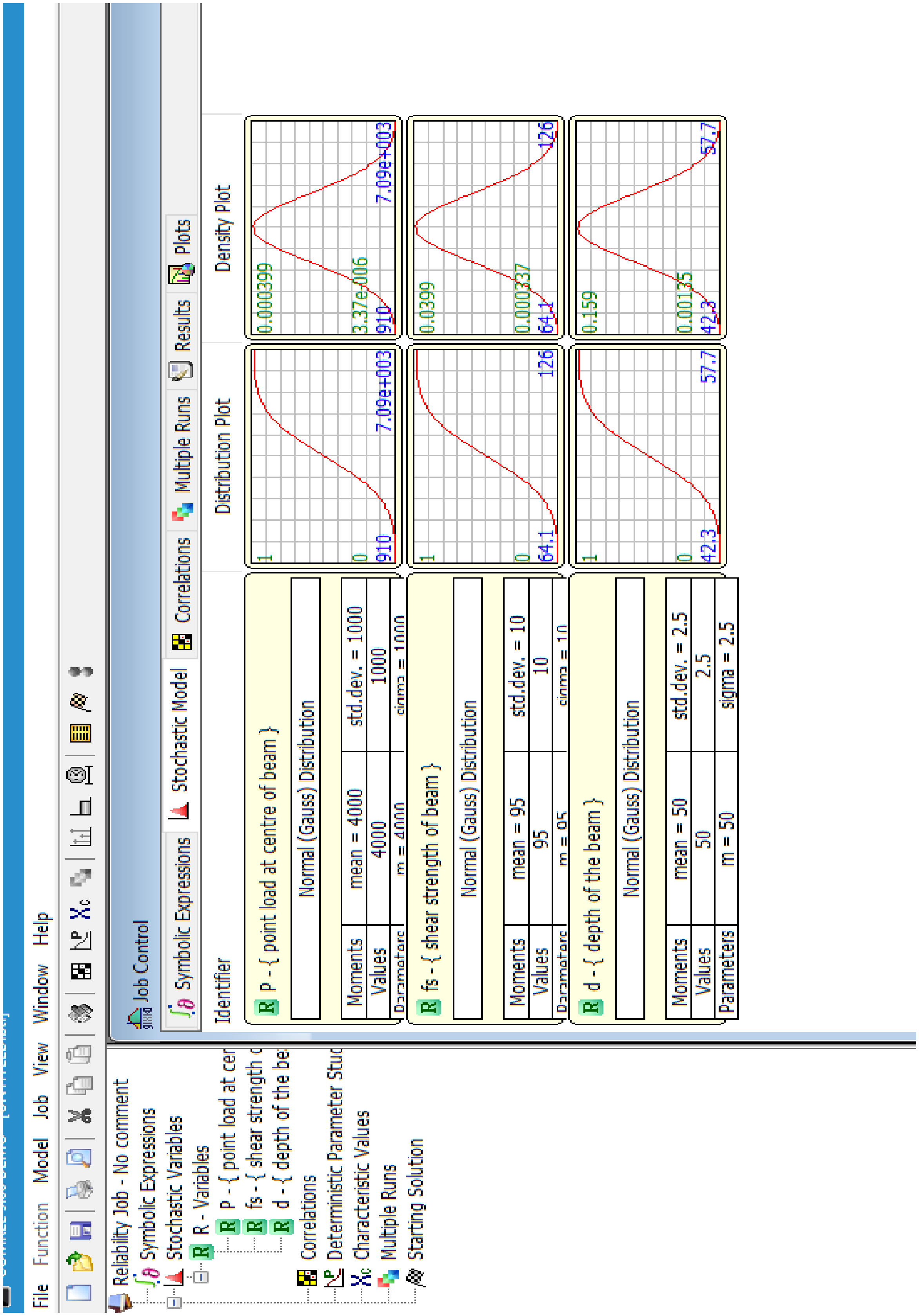

Fig. 9 - Input values of the beam problem along with type of distribution

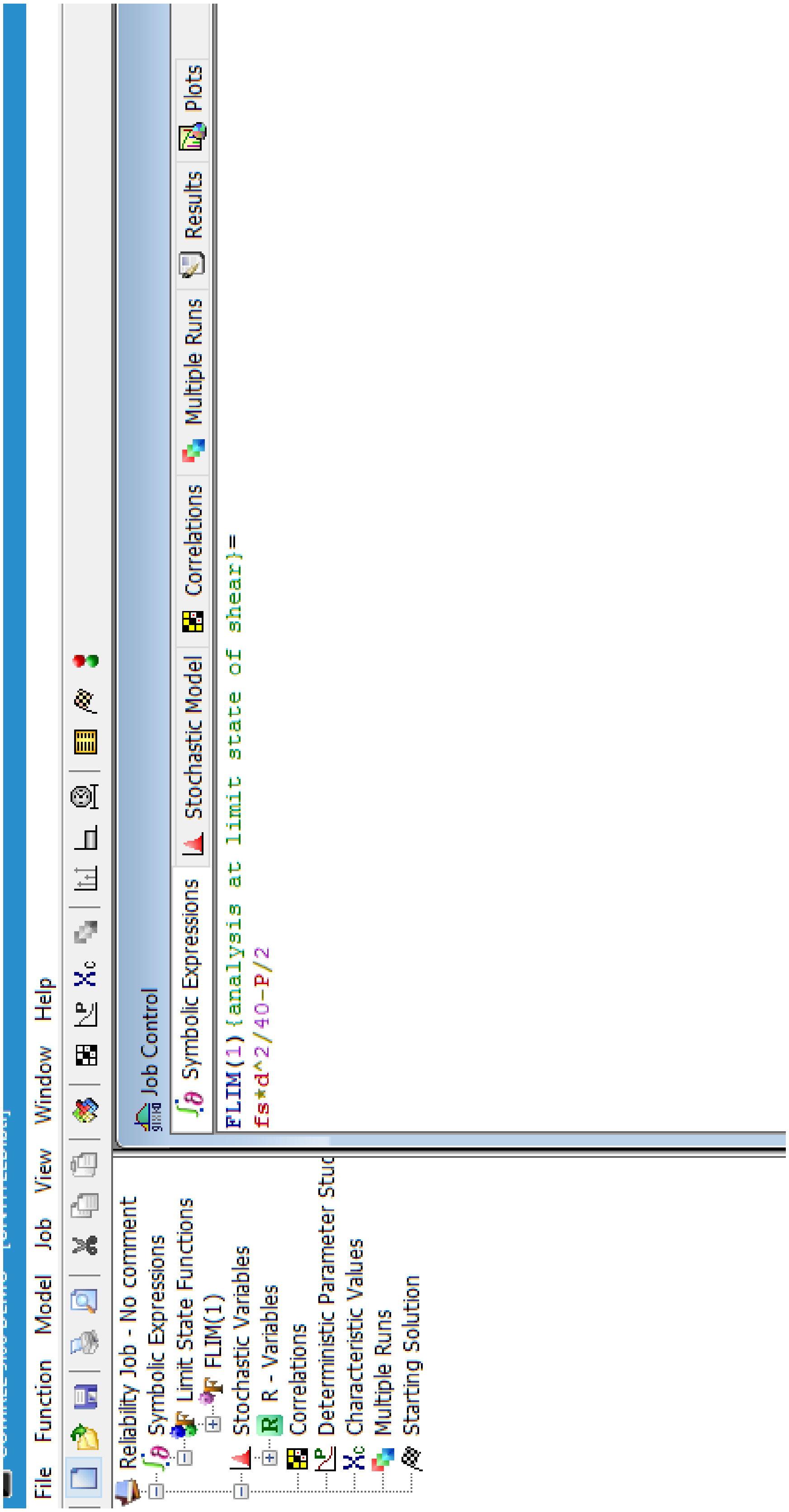

Fig. 10 - Limit state equation formulated in the software

```
------------------------------------------------------------------------------
  Job name ............ :                                       Ref-Fun-Demo
  Failure criterion no. :      1
  Comment : No commen
  Transformation type   : Rosenblatt
  Optimization algorithm: RFLS
------------------------------------------------------------------------------
------------------------------------------------------------------------------
 Iteration No.  1; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) =  0.1080     ; BETA =      0.0000; BETA/||U||=     0.0000
  Multipl.=   31.22    ; Step-length=    1.0000; State Func.calls:    5
------------------------------------------------------------------------------
 Iteration No.  2; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) = -0.5250E-03; BETA =      3.8955; BETA/||U||=     0.8796
  Multipl.=   43.71    ; Step-length=    1.0000; State Func.calls:    9
------------------------------------------------------------------------------
 Iteration No.  3; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) = -0.3040E-04; BETA =      4.4279; BETA/||U||=     1.0006
  Multipl.=   43.50    ; Step-length=    1.0000; State Func.calls:   13
------------------------------------------------------------------------------
 Iteration No.  4; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) = -0.1795E-05; BETA =      4.4253; BETA/||U||=     1.0000
  Multipl.=   43.44    ; Step-length=    1.0000; State Func.calls:   17
------------------------------------------------------------------------------
 Iteration No.  5; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) = -0.1061E-06; BETA =      4.4252; BETA/||U||=     1.0000
  Multipl.=   43.43    ; Step-length=    1.0000; State Func.calls:   21
------------------------------------------------------------------------------
 Iteration No.  6; CPU-seconds(cumulative):     0.000
  Scaled St.F(U) = -0.6279E-08; BETA =      4.4252; BETA/||U||=     1.0000
  Multipl.=   43.43    ; Step-length=    1.0000; State Func.calls:   25

 FORM-beta=   4.425; SORM-beta=   4.409; beta(Sampling)=   --    (IER=   0)
 FORM-Pf=  4.82E-06; SORM-Pf=  5.19E-06; Pf(Sampling)=    --

            --------- Statistics after COMREL-TI ---------
             State Function calls                =       35
             State Funct. gradient evaluations =     6
             Total computation time (CPU-secs.)=     0.02
             The error indicator (IER) was =    0
            **********************************************

Reliability analysis is finished
```

Fig. 11 - β and p_f obtained using both FORM and SORM

7.2 Reliability of column problem under earthquake loading as per IS:800-2007

A corner steel column has been considered from a six-storey steel building frame which is subjected to earthquake loading. The data for earthquake loading has been taken from the very famous El Centro earthquake which is widely used for earthquake analysis by the researchers.

Time history analysis has been performed in ETABS to get to the axial load and biaxial bending moments acting on the column. The table obtained in ETABS is then exported to MS Excel for study. It has been attached at the end of this report. The value of mean and standard deviations are then calculated in MS Excel using the formulae inbuilt in the software. The value of mean and standard deviation for the yield strength steel of grade 250 has been taken from a result published for the same by IIT Bombay. The column failure criteria has been taken from the steel code practiced in India IS:800-2007 for axial loading and biaxial bending which is given as:

$$\left(\frac{M_y}{M_{ndy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{ndz}}\right)^{\alpha_2} \leq 1.0$$

Where,

$M_y, M_z =$ factored applied moments about the minor and major axis of cross-section, respectively;

$M_{ndy}, M_{ndz} =$ design reduced flexural strength under combined axial force and the respective uniaxial moment acting alone;

$\alpha_1, \alpha_2 =$ constants obtained from a table in IS:800-2007

$n = \frac{N}{N_d}$,

$N =$ factored applied axial force (tension, T or compression, P);

$N_d =$ design strength in compression due to yielding given by $N_d = \frac{A_g f_y}{\gamma}$;

$A_g =$ gross area of the cross-section;

for standard I or H sections –

$$M_{ndy}, M_{ndz} = 1.56 M_{dy/dz}(1-n)(n+0.6)$$

Where,

$M_{dy}, M_{dz} = \beta_b Z_p f_{bd}$

$\beta_b = 1.0$ for plastic and compact sections.

$Z_p =$ plastic section modulus with respect to extreme compression fibre.

$f_{bd} =$ design bending compressive stress, obtained as below

$f_{bd} = {\chi_{LT} f_y}/{\gamma_{m0}}$

Where,

γ_{m0} = Partial safety factor against yield stress and buckling

χ_{LT} = Bending stress reduction factor to account for lateral torsional buckling, given by:

$$\chi_{LT} = \frac{1}{\{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\}} \leq 1.0$$

$$\phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

Where,

$\alpha_{LT} = 0.49$ for welded steel section

$$\lambda_{LT} = \sqrt{\beta_b Z_p f_y / M_{cr}}$$

M_{cr} = elastic critical moment,

The above equations are used to formulate the limiting state equation of failure in COMREL. The analysis has been performed for different PDFs such as normal, lognormal, and Gumbel max. and then optimization of reliability is achieved by using specific PDFs for specific input variables. The FORM and SORM analysis have been performed and reliability and probability of failure have been calculated from both the methods.

The input values, intermediate steps, and results captured from different softwares used are shown in figures below:

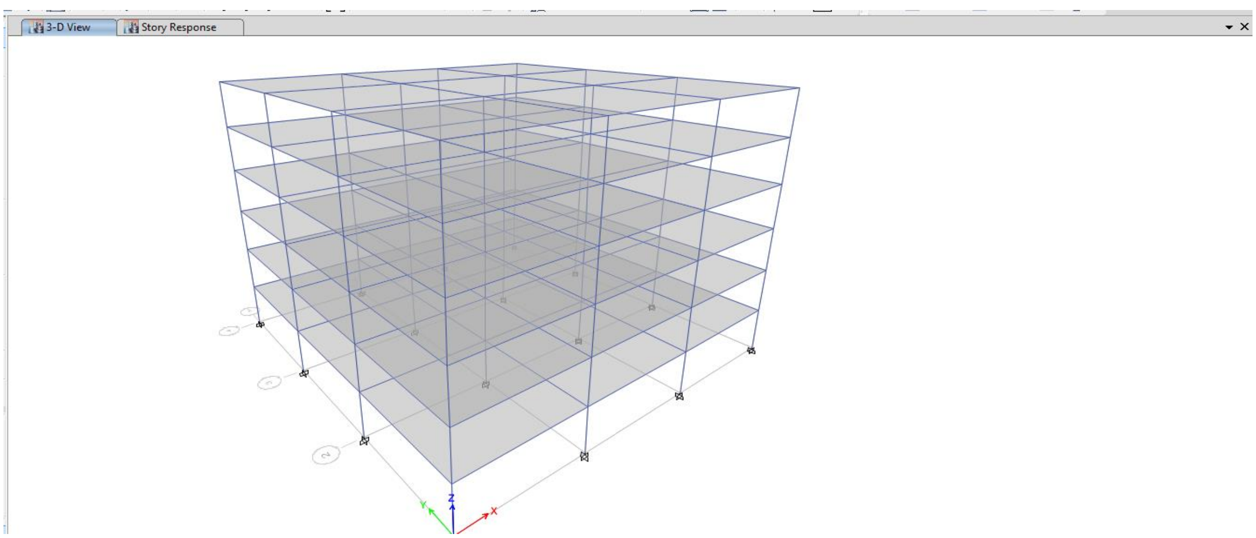

Fig. 12 - Six storey steel model for analysis in ETABS

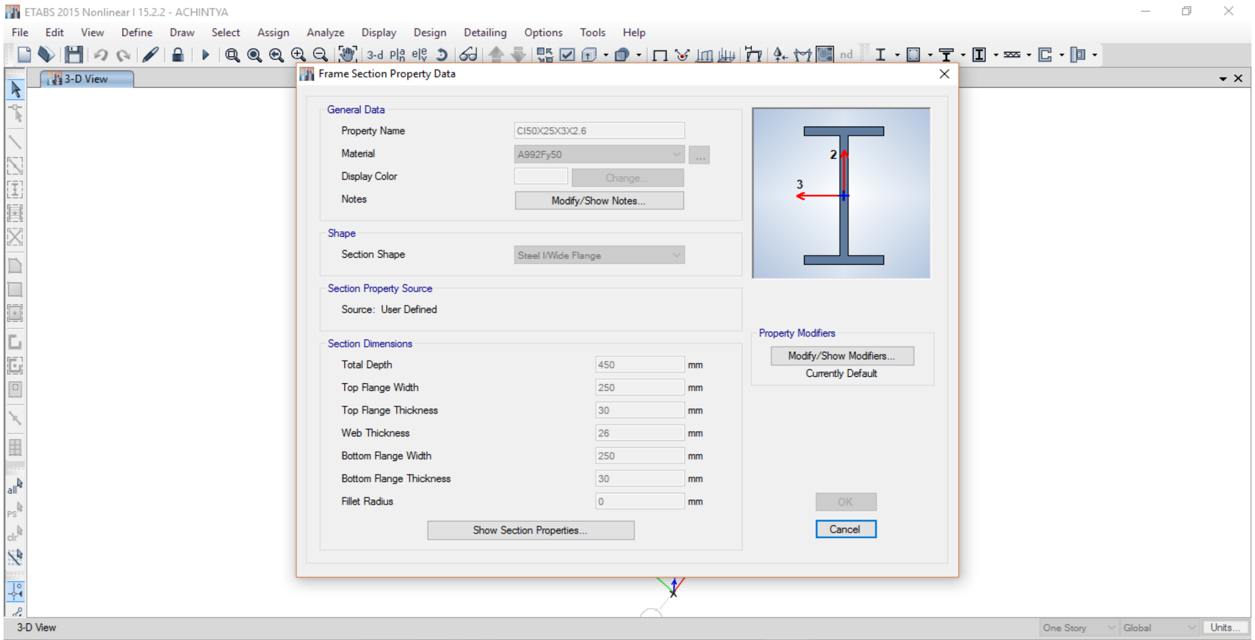

Fig. 13 - Column specifications used for design and analysis

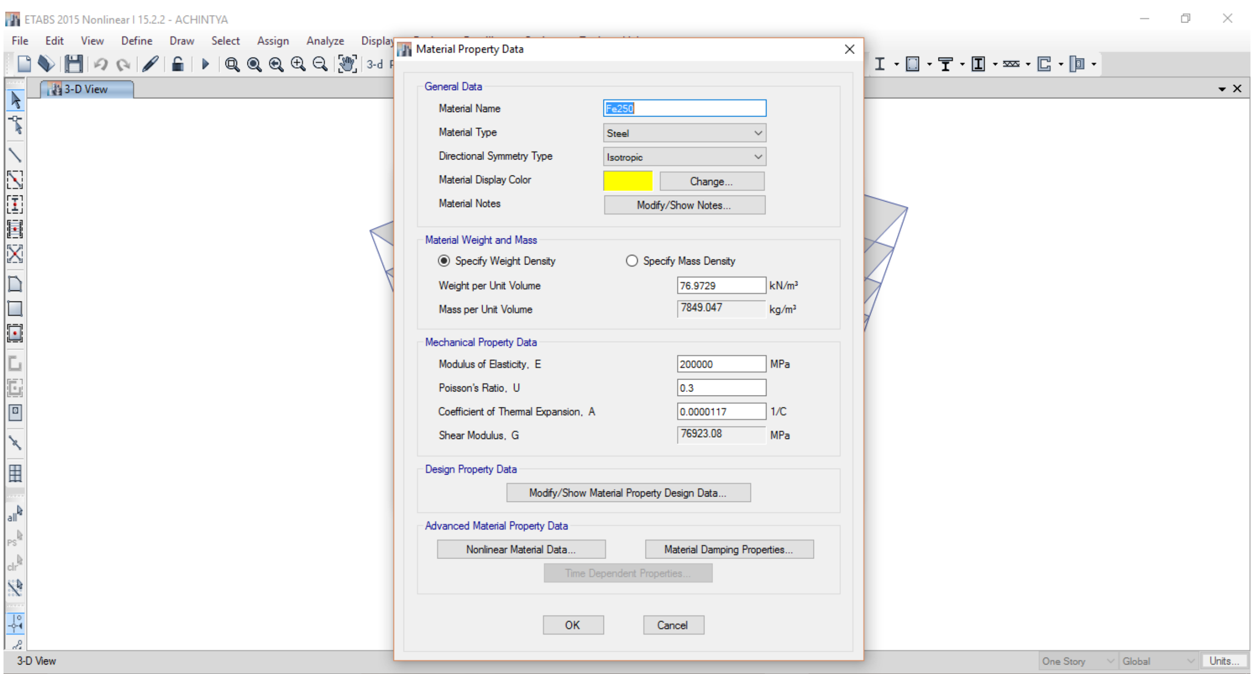

Fig. 14 - Specifications of rolled steel used

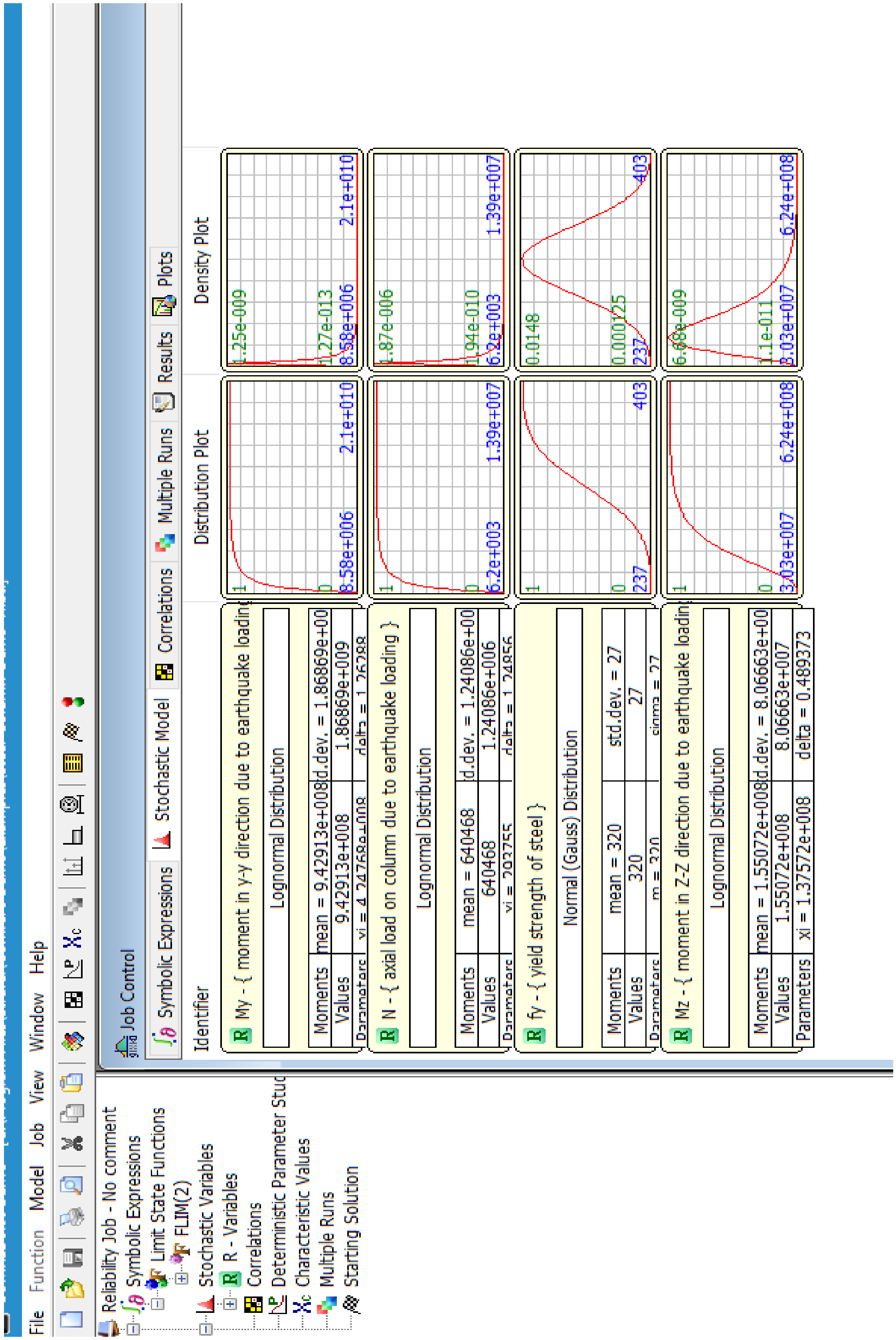

Fig. 15 – Input variables for column reliability

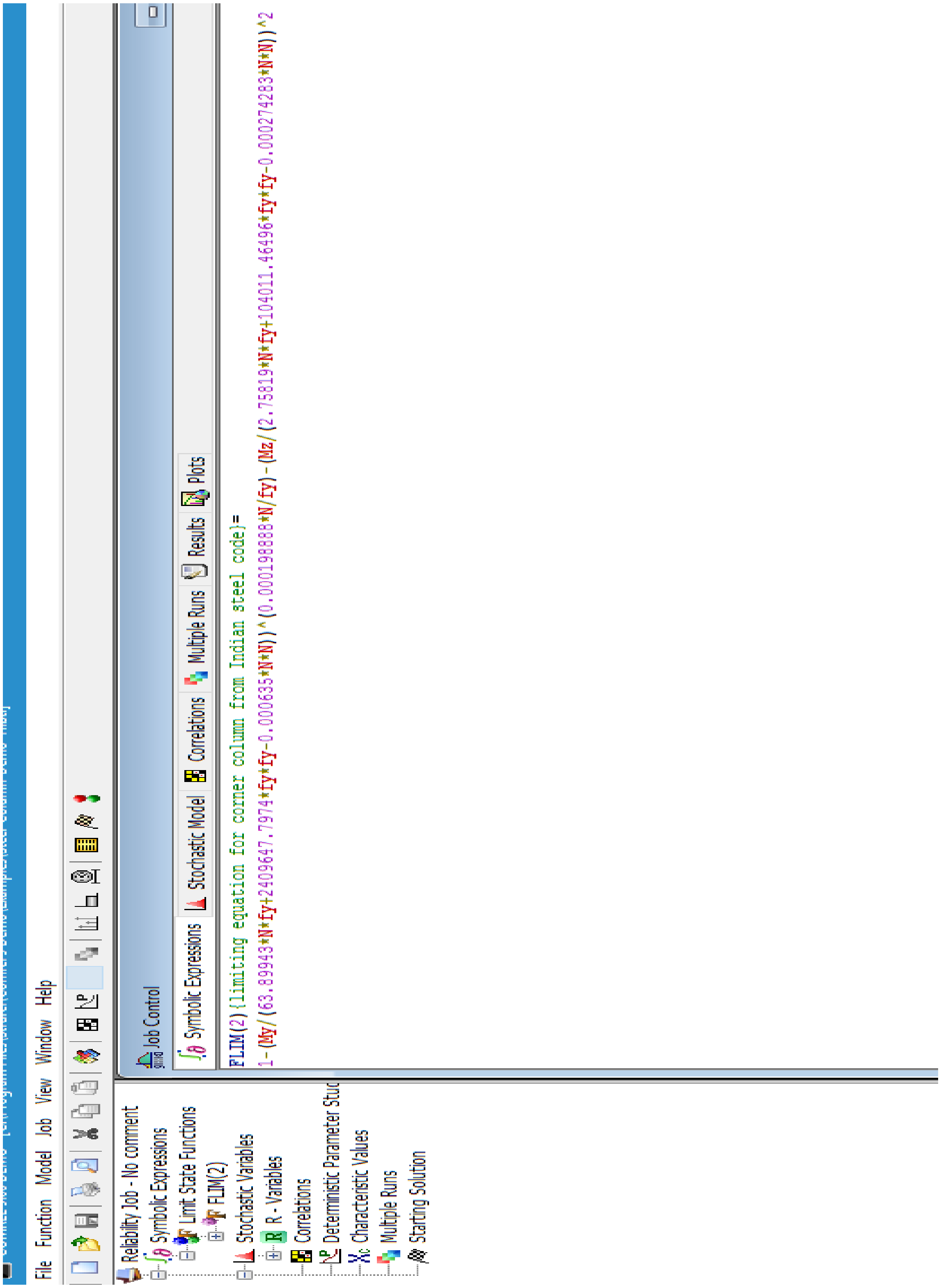

Fig. 16 - Limit state equation of corner column formulated in COMREL

Values of reliability (β) and probability of failure (p_f) using both FORM and SORM methods obtained from the software COMREL has been directly imported here:

```
*********************************************
----------- Comrel-TI (Version 9) ------------
---- (c) Copyright: RCP GmbH (1989-2015) -----
*********************************************


------------------------------------------------------------------------------
Job name reliability analysis of column:                Steel-Column-Demo-TI
Failure criterion no. :      2
Comment : limiting equation for corner column from Indian steel code
Transformation type    : Rosenblatt
Optimization algorithm: RFLS
------------------------------------------------------------------------------
------------------------------------------------------------------------------
Iteration No.  1; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.2340    ; BETA =      0.0000; BETA/||U||=      0.0000
Multipl.=   4.451    ; Step-length=    1.0000; State Func.calls:    6
------------------------------------------------------------------------------
Iteration No.  2; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.1482    ; BETA =      1.4918; BETA/||U||=      0.7943
Multipl.=   17.33    ; Step-length=    0.4484; State Func.calls:   12
------------------------------------------------------------------------------
Iteration No.  3; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.1008    ; BETA =      1.8781; BETA/||U||=      0.8556
Multipl.=   30.41    ; Step-length=    0.3813; State Func.calls:   18
------------------------------------------------------------------------------
Iteration No.  4; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.7194E-01; BETA =      2.1951; BETA/||U||=      0.8892
Multipl.=   48.74    ; Step-length=    0.3352; State Func.calls:   24
------------------------------------------------------------------------------
Iteration No.  5; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.5313E-01; BETA =      2.4685; BETA/||U||=      0.9103
Multipl.=   73.40    ; Step-length=    0.3018; State Func.calls:   30
------------------------------------------------------------------------------
Iteration No.  6; CPU-seconds(cumulative):     0.000
Scaled St.F(U) =  0.4026E-01; BETA =      2.7118; BETA/||U||=      0.9246
Multipl.=   105.7    ; Step-length=    0.2765; State Func.calls:   36
------------------------------------------------------------------------------
Iteration No.  7; CPU-seconds(cumulative):     0.016
Scaled St.F(U) =  0.3112E-01; BETA =      2.9329; BETA/||U||=      0.9349
Multipl.=   147.2    ; Step-length=    0.2568; State Func.calls:   42
------------------------------------------------------------------------------
Iteration No.  8; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.2444E-01; BETA =      3.1369; BETA/||U||=      0.9427
Multipl.=   199.6    ; Step-length=    0.2410; State Func.calls:   48
------------------------------------------------------------------------------
Iteration No.  9; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.1945E-01; BETA =      3.3274; BETA/||U||=      0.9488
Multipl.=   264.9    ; Step-length=    0.2279; State Func.calls:   54
------------------------------------------------------------------------------
Iteration No. 10; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.1565E-01; BETA =      3.5068; BETA/||U||=      0.9537
Multipl.=   345.6    ; Step-length=    0.2171; State Func.calls:   60
------------------------------------------------------------------------------
Iteration No. 11; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.1271E-01; BETA =      3.6768; BETA/||U||=      0.9577
Multipl.=   444.3    ; Step-length=    0.2079; State Func.calls:   66
------------------------------------------------------------------------------
Iteration No. 12; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.1040E-01; BETA =      3.8388; BETA/||U||=      0.9611
Multipl.=   564.0    ; Step-length=    0.1999; State Func.calls:   72
------------------------------------------------------------------------------
```

```
Iteration No. 13; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.8566E-02; BETA =      3.9939; BETA/||U||=      0.9640
Multipl.=   708.1    ; Step-length=    0.1931; State Func.calls:   78
-------------------------------------------------------------------------------
Iteration No. 14; CPU-seconds(cumulative):     0.031
Scaled St.F(U) =  0.7099E-02; BETA =      4.1429; BETA/||U||=      0.9664
Multipl.=   880.3    ; Step-length=    0.1872; State Func.calls:   84
-------------------------------------------------------------------------------
Iteration No. 15; CPU-seconds(cumulative):     0.047
Scaled St.F(U) =  0.5913E-02; BETA =      4.2864; BETA/||U||=      0.9685
Multipl.=   1085.    ; Step-length=    0.1821; State Func.calls:   90
-------------------------------------------------------------------------------
Iteration No. 16; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.4945E-02; BETA =      4.4250; BETA/||U||=      0.9704
Multipl.=   1326.    ; Step-length=    0.1779; State Func.calls:   96
-------------------------------------------------------------------------------
Iteration No. 17; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.4149E-02; BETA =      4.5590; BETA/||U||=      0.9720
Multipl.=   1610.    ; Step-length=    0.1745; State Func.calls:  102
-------------------------------------------------------------------------------
Iteration No. 18; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.3488E-02; BETA =      4.6891; BETA/||U||=      0.9733
Multipl.=   1941.    ; Step-length=    0.1722; State Func.calls:  108
-------------------------------------------------------------------------------
Iteration No. 19; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.2935E-02; BETA =      4.8156; BETA/||U||=      0.9744
Multipl.=   2327.    ; Step-length=    0.1712; State Func.calls:  114
-------------------------------------------------------------------------------
Iteration No. 20; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.2465E-02; BETA =      4.9392; BETA/||U||=      0.9751
Multipl.=   2775.    ; Step-length=    0.1723; State Func.calls:  120
-------------------------------------------------------------------------------
Iteration No. 21; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.2059E-02; BETA =      5.0606; BETA/||U||=      0.9753
Multipl.=   3296.    ; Step-length=    0.1771; State Func.calls:  126
-------------------------------------------------------------------------------
Iteration No. 22; CPU-seconds(cumulative):     0.062
Scaled St.F(U) =  0.1697E-02; BETA =      5.1812; BETA/||U||=      0.9747
Multipl.=   3906.    ; Step-length=    0.1889; State Func.calls:  132
-------------------------------------------------------------------------------
Iteration No. 23; CPU-seconds(cumulative):     0.078
Scaled St.F(U) =  0.1352E-02; BETA =      5.3035; BETA/||U||=      0.9723
Multipl.=   4633.    ; Step-length=    0.2185; State Func.calls:  138
-------------------------------------------------------------------------------
Iteration No. 24; CPU-seconds(cumulative):     0.078
Scaled St.F(U) =  0.9510E-03; BETA =      5.4331; BETA/||U||=      0.9633
Multipl.=   5542.    ; Step-length=    0.3190; State Func.calls:  144
-------------------------------------------------------------------------------
Iteration No. 25; CPU-seconds(cumulative):     0.078
Scaled St.F(U) = -0.1309E-03; BETA =      5.5868; BETA/||U||=      0.9131
Multipl.=   6846.    ; Step-length=    1.0000; State Func.calls:  149
-------------------------------------------------------------------------------
Iteration No. 26; CPU-seconds(cumulative):     0.078
Scaled St.F(U) = -0.2159E-04; BETA =      5.3296; BETA/||U||=      0.9272
Multipl.=   6755.    ; Step-length=    0.2655; State Func.calls:  155
-------------------------------------------------------------------------------
Iteration No. 27; CPU-seconds(cumulative):     0.078
Scaled St.F(U) =  0.1011E-03; BETA =      5.4324; BETA/||U||=      1.0013
Multipl.=   3592.    ; Step-length=    1.0000; State Func.calls:  160
-------------------------------------------------------------------------------
Iteration No. 28; CPU-seconds(cumulative):     0.078
Scaled St.F(U) =  0.3111E-05; BETA =      5.4248; BETA/||U||=      0.9987
Multipl.=   778.5    ; Step-length=    1.0000; State Func.calls:  165
-------------------------------------------------------------------------------
Iteration No. 29; CPU-seconds(cumulative):     0.094
Scaled St.F(U) = -0.1149E-06; BETA =      5.4320; BETA/||U||=      1.0000
Multipl.=   806.3    ; Step-length=    1.0000; State Func.calls:  170
-------------------------------------------------------------------------------
Iteration No. 30; CPU-seconds(cumulative):     0.094
Scaled St.F(U) = -0.4148E-09; BETA =      5.4322; BETA/||U||=      1.0000
Multipl.=   785.8    ; Step-length=    1.0000; State Func.calls:  175
```

```
--------------------------------------------------------------------------------
Iteration No. 31; CPU-seconds(cumulative):     0.094
Scaled St.F(U) = -0.1286E-10; BETA =      5.4322; BETA/||U||=      1.0000
Multipl.=   784.7    ; Step-length=    1.0000; State Func.calls:  180

FORM-beta=   5.432; SORM-beta=   5.402; beta(Sampling)=   --    (IER=   0)
FORM-Pf=  2.79E-08; SORM-Pf=  3.30E-08; Pf(Sampling)=    --


               --------- Statistics after COMREL-TI ---------
                State Function calls                =      195
                State Funct. gradient evaluations =    31
                Total computation time (CPU-secs.)=     0.11
                The error indicator (IER) was =     0
               *********************************************

Reliability analysis is finished
```

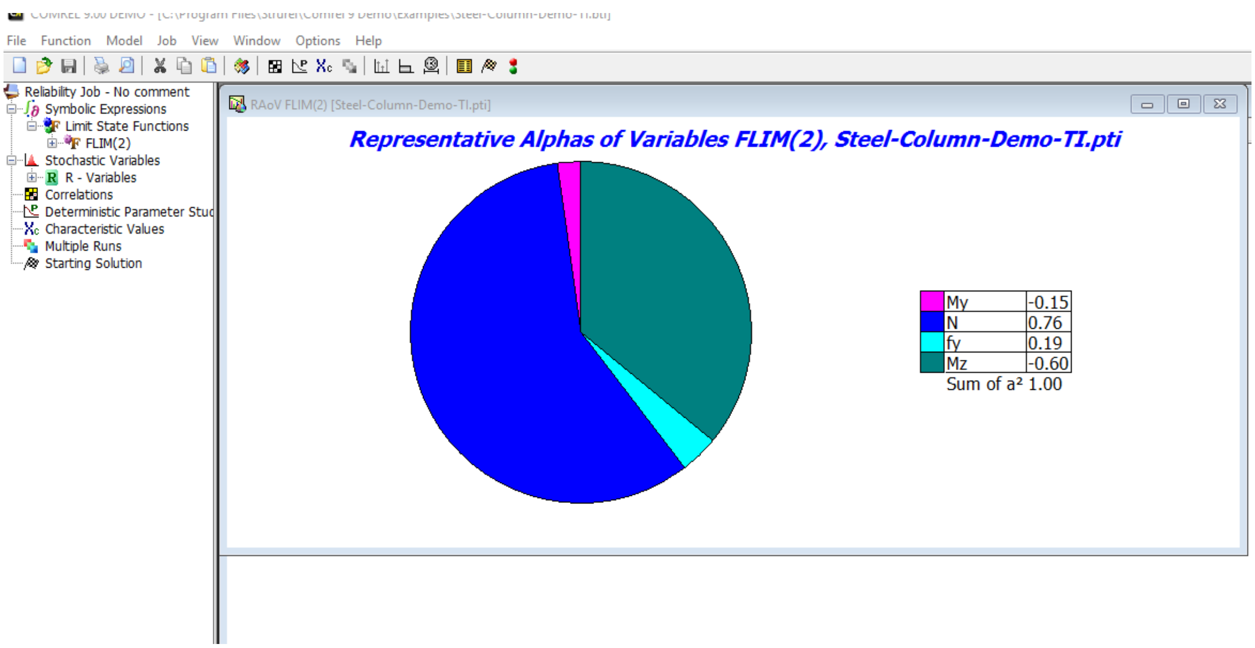

Fig. 17 – Alfa (α) values of the variables obtained at design point.

The result obtained above has been reproduced in the table below:

Table	FORM	SORM
Reliability index (β)	5.432	5.402
Probability of failure (p_f)	2.97 x 10^{-8}	3.30 x 10^{-8}

CHAPTER 8
SUMMARY AND CONCLUSION

8.1 Conclusion

The required objective of reliability analysis of a corner column of the six-storey building was successfully conducted and important results were obtained. Some of the key conclusions which can be taken away from this study are listed as follows:

- The small difference between these estimates is an indication that, in the space of the standard normal variables **z**, the limit-state surface is nearly flat in the neighborhood of the design point.
- Further, while the surface remains closed in the **z** space, only the region close to the design point makes a dominant contribution.
- Evidently, the rotationally symmetric domain in the **x** space results in a closed but strongly skewed domain in the **z** space
- The reliability analysis method which includes determination of reliability index is a suitable and more advanced method of checking the failure status of a building or its components.
- The reliability of widely used design methods such as working stress method, limit state method, ultimate strength method, etc. is questionable during significant earthquakes such as one came at El Centro, California.
- The value of reliability index obtained through either FORM or SORM both converses to similar values with SORM result being a bit more on the safer side or more conservative.
- This method of reliability analysis and then design based on this reliability index obtained can greatly reduce both the risk of failure as well as the cost of construction of structures.

8.2 Future scope

The method of reliability analysis and design has many future scopes, particularly in the construction of important structures such as hospitals, fire stations, high-rise structures, etc. The FORM and SORM analysis can also be updated and reviewed as at times they converse to local design points instead of global design points. The software COMREL can help greatly in calculating reliability and doing other work on different methods being researched in this field by saving the time taken in solving equations manually and performing calculations. This project’s data for reliability index can be used to study the efficiency of the design method elaborated in IS:800-2007 for compression members under combined loading. The whole area of study is quite new particularly in India hence has a lot of future scope in Indian conditions.

Appendix

TABLE	TIME HISTOERY ANALYSIS RESULT					
Time	**Axial Force**		**Moment Z-Z**		**Moment Y-Y**	
sec	kN	N	kN-m	N-mm	kN-m	N-mm
0	0	0	0	0	0	0
0.1	42.2979	42297.9	-4.9076	-4907600	-133.3	-133299600
0.2	160.9776	160977.6	-11.1128	-11112800	-235.507	-235507300
0.3	-281.402	-281402	-19.2406	-19240600	-424.308	-424307800
0.4	-384.5428	-384543	-23.4029	-23402900	-577.417	-577417100
0.5	-425.7043	-425704	-7.9964	-7996400	-597.271	-597270800
0.6	-374.8696	-374870	-17.3278	-17327800	-579.715	-579715100
0.7	-306.2507	-306251	-17.8687	-17868700	-488.718	-488718200
0.8	-260.0898	-260090	-2.1883	-2188300	-404.687	-404686800
0.9	-289.5495	-289550	33.3557	33355700	-488.895	-488894600
1	-377.0504	-377050	107.9763	107976300	-589.815	-589815200
1.1	-491.3442	-491344	163.4287	163428700	-754.376	-754375700
1.2	-606.6635	-606664	92.5238	92523800	-910.42	-910420000
1.3	-667.8402	-667840	58.0998	58099800	-977.914	-977913600
1.4	-652.5814	-652581	45.5038	45503800	-1006.65	-1006645000
1.5	-604.378	-604378	61.5188	61518800	-948.772	-948771500
1.6	-594.887	-594887	213.7543	213754300	-933.191	-933190600
1.7	-734.2212	-734221	35.7547	35754700	-1179.04	-1179038600
1.8	-865.2007	-865201	-169.4876	-169487600	-1287.9	-1287904300
1.9	-968.3652	-968365	-445.6601	-445660100	-1548.51	-1548506600
2	-1110.7875	-1110788	-309.2985	-309298500	-1587.09	-1587090700
2.1	-1126.6876	-1126688	167.6669	167666900	-1739.09	-1739087500
2.2	-1141.4269	-1141427	445.0612	445061200	-1796.99	-1796988900
2.3	-1156.3558	-1156356	427.6232	427623200	-1797.94	-1797935500
2.4	-1135.7605	-1135761	153.9744	153974400	-1709.7	-1709703100
2.5	-1210.6371	-1210637	-421.6932	-421693200	-1880.93	-1880932700
2.6	-1272.9335	-1272934	-536.9966	-536996600	-1993.89	-1993894700
2.7	-1350.8903	-1350890	-693.3893	-693389300	-2067.3	-2067295900
2.8	-1452.5682	-1452568	-366.4755	-366475500	-2222.25	-2222254000
2.9	-1560.0209	-1560021	72.6537	72653700	-2348.14	-2348137200
3	-1621.0469	-1621047	313.8347	313834700	-2511.42	-2511421800
3.1	-1638.7073	-1638707	547.3732	547373200	-2567.62	-2567621900
3.2	-1613.4082	-1613408	291.2422	291242200	-2350.38	-2350382500
3.3	-1639.5898	-1639590	150.4231	150423100	-2592.25	-2592246400
3.4	-1696.8305	-1696831	-212.3025	-212302500	-2649.57	-2649569700
3.5	-1822.6456	-1822646	-361.2204	-361220400	-2828.34	-2828343600
3.6	-2010.5316	-2010532	-320.7203	-320720300	-3012.57	-3012568900
3.7	-2107.5459	-2107546	-208.7621	-208762100	-3178.94	-3178942900

3.8	-2157.7743	-2157774	138.5138	138513800	-3410.78	-3410778800
3.9	-2227.278	-2227278	298.4683	298468300	-3424.11	-3424113400
4	-2193.9211	-2193921	455.284	455284000	-3249.87	-3249867200
4.1	-2224.44	-2224440	266.7045	266704500	-3457.62	-3457623800
4.2	-2235.5516	-2235552	63.2802	63280200	-3438.7	-3438702700
4.3	-2232.0924	-2232092	-42.1094	-42109400	-3380.62	-3380623900
4.4	-2269.4067	-2269407	-104.8275	-104827500	-3506.81	-3506807900
4.5	-2372.2996	-2372300	8.1091	8109100	-3614.72	-3614718400
4.6	-2438.9954	-2438995	-43.5973	-43597300	-3807.58	-3807576000
4.7	-2496.0696	-2496070	-120.1021	-120102100	-3912.78	-3912780300
4.8	-2495.5893	-2495589	-50.0944	-50094400	-3610.79	-3610794500
4.9	-2506.0016	-2506002	47.773	47773000	-3967.65	-3967647500
5	-2536.6798	-2536680	220.5618	220561800	-3876.91	-3876906800
5.1	-2511.2247	-2511225	121.8803	121880300	-3807.03	-3807032200
5.2	-2517.8946	-2517895	-128.6226	-128622600	-3893.77	-3893774800
5.3	-2595.5891	-2595589	-421.4613	-421461300	-3908.95	-3908952400
5.4	-2619.2875	-2619288	-494.4155	-494415500	-4158.65	-4158646400
5.5	-2694.8233	-2694823	-364.6173	-364617300	-4185.86	-4185860500
5.6	-2681.8709	-2681871	-22.4247	-22424700	-3913.76	-3913759900
5.7	-2698.4005	-2698401	352.4967	352496700	-4246.8	-4246795800
5.8	-2720.8292	-2720829	575.6435	575643500	-4161.59	-4161590000
5.9	-2702.5611	-2702561	477.7788	477778800	-4120.08	-4120076600
6	-2725.7527	-2725753	148.9918	148991800	-4162.26	-4162263800
6.1	-2778.1075	-2778108	-259.4453	-259445300	-4228.1	-4228097500
6.2	-2804.5885	-2804589	-467.9608	-467960800	-4421.43	-4421428800
6.3	-2871.5858	-2871586	-383.9338	-383933800	-4505.06	-4505064600
6.4	-2896.0277	-2896028	-44.7368	-44736800	-4233.41	-4233412000
6.5	-2965.2556	-2965256	268.7288	268728800	-4633.8	-4633804100
6.6	-3008.7556	-3008756	416.6197	416619700	-4618.44	-4618443400
6.7	-3002.1239	-3002124	294.9337	294933700	-4542.15	-4542149500
6.8	-3004.0708	-3004071	8.6059	8605900	-4620.19	-4620188600
6.9	-3055.1421	-3055142	-264.4421	-264442100	-4633.63	-4633631900
7	-3064.8057	-3064806	-365.6765	-365676500	-4829.39	-4829390100
7.1	-3120.4173	-3120417	-196.134	-196134000	-4882.19	-4882194600
7.2	-3122.0302	-3122030	49.1036	49103600	-4562.24	-4562239400
7.3	-3180.1292	-3180129	279.2853	279285300	-5000.41	-5000413000
7.4	-3232.1446	-3232145	272.483	272483000	-4955	-4955000900
7.5	-3234.7247	-3234725	123.7155	123715500	-4914.96	-4914964600
7.6	-3274.6124	-3274612	-39.7565	-39756500	-5025.08	-5025082100
7.7	-3356.3499	-3356350	-193.0674	-193067400	-5082.3	-5082300600
7.8	-3370.0824	-3370082	-161.3377	-161337700	-5318.1	-5318103800
7.9	-3428.1857	-3428186	-18.9514	-18951400	-5357.83	-5357825300
8	-3415.2602	-3415260	143.1615	143161500	-4980.82	-4980819200

8.1	-3469.0266	-3469027	220.9558	220955800	-5471.79	-5471794600
8.2	-3547.4132	-3547413	159.7042	159704200	-5450.7	-5450701300
8.3	-3593.6463	-3593646	-41.772	-41772000	-5459.98	-5459984500
8.4	-3660.9281	-3660928	-208.61	-208610000	-5571.35	-5571347000
8.5	-3707.296	-3707296	-236.3392	-236339200	-5622.71	-5622705900
8.6	-3695.4987	-3695499	-125.0052	-125005200	-5850.29	-5850290000
8.7	-3762.5246	-3762525	65.703	65703000	-5862.3	-5862301000
8.8	-3740.5335	-3740534	115.4296	115429600	-5481.87	-5481868200
8.9	-3822.7956	-3822796	116.1475	116147500	-6007.88	-6007883700
9	-3919.4693	-3919469	54.5287	54528700	-5989.08	-5989081000
9.1	-3949.6814	-3949681	-74.0161	-74016100	-6022.39	-6022388100
9.2	-4030.5067	-4030507	-162.5157	-162515700	-6167.5	-6167500500
9.3	-4142.314	-4142314	-165.4477	-165447700	-6277.42	-6277423700
9.4	-4147.6359	-4147636	-133.0775	-133077500	-6488.98	-6488984400
9.5	-4143.0211	-4143021	23.9793	23979300	-6482.14	-6482139800
9.6	-4059.2067	-4059207	120.3303	120330300	-5950	-5950001800
9.7	-4098.8596	-4098860	163.7408	163740800	-6486.5	-6486500500
9.8	-4210.738	-4210738	149.3201	149320100	-6474.26	-6474264400
9.9	-4291.7668	-4291767	62.1077	62107700	-6483.21	-6483210200
10	-4366.4159	-4366416	-84.2042	-84204200	-6677.74	-6677743400
10.1	-4450.6566	-4450657	-102.0061	-102006100	-6740.46	-6740459500
10.2	-4431.0549	-4431055	-117.2758	-117275800	-6966.78	-6966783900
10.3	-4446.7497	-4446750	4.1191	4119100	-6964.95	-6964948500
10.4	-4373.8239	-4373824	109.8043	109804300	-6385.22	-6385223300
10.5	-4403.7467	-4403747	87.5294	87529400	-6970.82	-6970818900
10.6	-4504.3345	-4504335	146.9015	146901500	-6891.12	-6891119500
10.7	-4536.9655	-4536966	-48.3475	-48347500	-6896.96	-6896959300
10.8	-4609.1835	-4609184	-138.2131	-138213100	-7065.47	-7065469900
10.9	-4716.8477	-4716848	-184.2434	-184243400	-7122.4	-7122399600
11	-4717.1626	-4717163	-131.5885	-131588500	-7442.08	-7442083500
11.1	-4764.8903	-4764890	28.4344	28434400	-7403.37	-7403368400
11.2	-4666.5858	-4666586	120.1819	120181900	-6849.32	-6849316400
11.3	-4717.0856	-4717086	140.5488	140548800	-7474.2	-7474199900
11.4	-4860.7092	-4860709	36.6081	36608100	-7439.53	-7439534800
11.5	-4944.625	-4944625	-176.534	-176534000	-7524.22	-7524216000
11.6	-3535.8288	-3535829	-210.0488	-210048800	-3049.22	-3049224500
11.7	756.9219	756921.9	-81.6419	-81641900	-147.651	-147651200
11.8	3623.0538	3623054	230.8251	230825100	4984.138	4984138200
11.9	4422.0293	4422029	392.9131	392913100	5855.315	5855315400
12	2647.9545	2647955	273.6835	273683500	2803.127	2803127100
12.1	-908.4116	-908412	-62.0842	-62084200	-466.012	-466011900
12.2	-3188.4791	-3188479	-455.9757	-455975700	-4686.46	-4686461000
12.3	-3821.7979	-3821798	-605.5705	-605570500	-4474.45	-4474445600

12.4	-1923.0287	-1923029	-489.3961	-489396100	-2486.71	-2486712700
12.5	912.3403	912340.3	-171.4193	-171419300	950.5879	950587900
12.6	2875.9837	2875984	145.4904	145490400	3980.191	3980190500
12.7	3210.7785	3210779	250.0279	250027900	3741.656	3741656400
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12.9	-915.8209	-915821	-147.9976	-147997600	-1271.98	-1271982700
13	-2596.1491	-2596149	-399.8145	-399814500	-3339.39	-3339389500
13.1	-2640.5476	-2640548	-481.0768	-481076800	-3215.64	-3215642000
13.2	-1045.2718	-1045272	-363.7672	-363767200	-1499.26	-1499256800
13.3	929.6029	929602.9	-116.5017	-116501700	1331.696	1331695900
13.4	2304.7586	2304759	87.7595	87759500	2835.922	2835921800
13.5	2159.467	2159467	145.4531	145453100	2760.322	2760322000
13.6	769.5756	769575.6	27.7815	27781500	1023.911	1023911400
13.7	-938.1415	-938142	-183.0056	-183005600	-1271.8	-1271799700
13.8	-2013.1834	-2013183	-350.1434	-350143400	-2474.8	-2474801500
13.9	-1768.5832	-1768583	-387.2253	-387225300	-2308.4	-2308402700
14	-544.3267	-544327	-274.6198	-274619800	-664.036	-664036300
14.1	923.7722	923772.2	-95.0408	-95040800	1181.17	1181169500
14.2	1739.5934	1739593	42.2573	42257300	2177.423	2177423100
14.3	1450.3597	1450360	61.4957	61495700	1884.288	1884287600
14.4	357.9849	357984.9	-43.1405	-43140500	417.8603	417860300
14.5	-884.5168	-884517	-196.4464	-196446400	-1101.05	-1101048400
14.6	-1496.3433	-1496343	-307.8771	-307877100	-1905.79	-1905786400
14.7	-1185.4176	-1185418	-313.5438	-313543800	-1516.27	-1516270800
14.8	-208.1802	-208180	-218.2112	-218211200	-245.378	-245378400
14.9	827.9294	827929.4	-87.0575	-87057500	1031.59	1031590400
15	1284.5937	1284594	2.0765	2076500	1647.075	1647074900
15.1	961.2297	961229.7	-1.8429	-1842900	1214.162	1214162300
15.2	93.3097	93309.7	-88.0664	-88066400	117.0614	117061400
15.3	-763.1282	-763128	-199.9731	-199973100	-961.865	-961865300
15.4	-1100.3825	-1100383	-270.1983	-270198300	-1407.95	-1407949800
15.5	-771.4791	-771479	-259.832	-259832000	-971.012	-971012100
15.6	-8.801	-8801	-182.1828	-182182800	-16.808	-16808000
15.7	696.4203	696420.3	-87.2645	-87264500	886.0598	886059800
15.8	939.0268	939026.8	-32.6076	-32607600	1195.25	1195250300
15.9	612.5745	612574.5	-47.2176	-47217600	773.6554	773655400
16	-51.4518	-51451.8	-116.8422	-116842200	-60.468	-60468000
16.1	-630.8391	-630839	-196.7994	-196799400	-805.136	-805135800
16.2	-797.3039	-797304	-238.8474	-238847400	-1011.09	-1011093600
16.3	-481.2511	-481251	-221.5189	-221518900	-611.088	-611087900
16.4	93.2759	93275.9	-159.4725	-159472500	116.7194	116719400
16.5	567.5489	567548.9	-92.5366	-92536600	723.4082	723408200
16.6	673.2828	673282.8	-60.6256	-60625600	853.1019	853101900

16.7	373.8308	373830.8	-79.5464	-79546400	476.3022	476302200
16.8	-121.2175	-121218	-134.4545	-134454500	-154.362	-154362400
16.9	-507.102	-507102	-190.2003	-190200300	-644.734	-644733600
17	-565.5615	-565562	-213.9683	-213968300	-717.388	-717388400
17.1	-286.5317	-286532	-194.3353	-194335300	-365.183	-365182500
17.2	138.5496	138549.6	-146.0739	-146073900	176.9294	176929400
17.3	450.0141	450014.1	-99.8651	-99865100	571.2292	571229200
17.4	472.7203	472720.3	-82.6251	-82625100	600.4787	600478700
17.5	215.9113	215911.3	-102.2763	-102276300	274.7	274700000
17.6	-147.6763	-147676	-144.4415	-144441500	-188.182	-188182000
17.7	-396.7973	-396797	-182.5576	-182557600	-503.536	-503536300
17.8	-393.1976	-393198	-194.5979	-194597900	-499.851	-499851100
17.9	-159.0823	-159082	-175.4522	-175452200	-201.99	-201990300
18	150.503	150503	-138.8051	-138805100	191.3127	191312700
18.1	347.8307	347830.7	-107.5392	-107539200	441.6136	441613600
18.2	325.3947	325394.7	-99.594	-99594000	413.6543	413654300
18.3	113.6875	113687.5	-117.865	-117865000	144.1972	144197200
18.4	-148.6224	-148622	-149.565	-149565000	-188.69	-188689500
18.5	-303.2814	-303281	-175.0494	-175049400	-385.27	-385270200
18.6	-267.8068	-267807	-179.8158	-179815800	-340.308	-340307600
18.7	-77.7792	-77779.2	-162.6591	-162659100	-98.6683	-98668300
18.8	143.3669	143366.9	-135.3646	-135364600	181.9961	181996100
18.9	263.1202	263120.2	-114.7393	-114739300	334.3437	334343700
19	219.0888	219088.8	-112.3997	-112399700	278.2941	278294100
19.1	49.7104	49710.4	-128.3022	-128302200	63.1231	63123100
19.2	-135.8157	-135816	-151.695	-151695000	-172.474	-172473800
19.3	-227.1823	-227182	-168.2579	-168257900	-288.672	-288672200
19.4	-178.0581	-178058	-168.7794	-168779400	-226.142	-226141900
19.5	-28.0708	-28070.8	-154.198	-154198000	-35.6889	-35688900
19.6	126.8114	126811.4	-134.243	-134243000	161.0958	161095800
19.7	195.2294	195229.4	-121.0568	-121056800	248.0339	248033900
19.8	143.6715	143671.5	-121.866	-121866000	182.4809	182480900
19.9	11.6577	11657.7	-135.1119	-135111900	14.8414	14841400
20	-116.9939	-116994	-152.0545	-152054500	-148.646	-148645700
20.1	-166.9878	-166988	-162.4498	-162449800	-212.127	-212127200
20.2	-115.0027	-115003	-160.6967	-160696700	-146.089	-146088700
20.3	0.5431	543.1	-148.763	-148763000	0.6755	675500
20.4	106.841	106841	-134.4459	-134445900	135.7431	135743100
20.5	142.1688	142168.8	-126.3439	-126343900	180.5916	180591600
20.6	91.2284	91228.4	-128.7362	-128736200	115.9001	115900100
20.7	-9.3762	-9376.2	-139.4082	-139408200	-11.9114	-11911400
20.8	-96.7058	-96705.8	-151.4493	-151449300	-122.856	-122856300
20.9	-120.4771	-120477	-157.6781	-157678100	-153.041	-153041100

21	-71.6213	-71621.3	-154.884	-154884000	-90.9925	-90992500
21.1	15.5394	15539.4	-145.4045	-145404500	19.7459	19745900
21.2	86.8448	86844.8	-135.3267	-135326700	110.3221	110322100
21.3	101.6182	101618.2	-130.6181	-130618100	129.0902	129090200
21.4	55.5441	55544.1	-133.631	-133631000	70.5643	70564300
21.5	-19.6062	-19606.2	-141.9993	-141999300	-24.911	-24911000
21.6	-77.438	-77438	-150.3917	-150391700	-98.3707	-98370700
21.7	-85.3041	-85304.1	-153.8755	-153875500	-108.369	-108368600
21.8	-42.4433	-42443.3	-150.7821	-150782100	-53.9181	-53918100
21.9	22.0467	22046.7	-143.4372	-143437200	28.0084	28008400
22	68.6053	68605.3	-136.4849	-136484900	87.1514	87151400
22.1	71.2591	71259.1	-133.98	-133980000	90.5265	90526500
22.2	31.8407	31840.7	-137.0512	-137051200	40.4478	40447800
22.3	-23.2438	-23243.8	-143.4632	-143463200	-29.5275	-29527500
22.4	-60.4186	-60418.6	-149.1905	-149190500	-76.7529	-76752900
22.5	-59.2242	-59224.2	-150.9205	-150920500	-75.2367	-75236700
22.6	-23.325	-23325	-147.9453	-147945300	-29.6301	-29630100
22.7	23.5082	23508.2	-142.3763	-142376300	29.863	29863000
22.8	52.9131	52913.1	-137.6862	-137686200	67.219	67219000
22.9	48.959	48959	-136.563	-136563000	62.1954	62195400
23	16.544	16544	-139.3914	-139391400	21.0165	21016500
23.1	-23.0893	-23089.3	-144.2043	-144204300	-29.3313	-29331300
23.2	-46.0965	-46096.5	-148.0204	-148020400	-58.5596	-58559600
23.3	-40.2437	-40243.7	-148.6744	-148674400	-51.1236	-51123600
23.4	-11.1971	-11197.1	-146.026	-146026000	-14.2244	-14224400
23.5	22.186	22186	-141.8864	-141886400	28.1841	28184100
23.6	39.9565	39956.5	-138.8032	-138803200	50.7592	50759200
23.7	32.8786	32878.6	-138.5065	-138506500	41.7674	41767400
23.8	7.029	7029	-140.9556	-140955600	8.9296	8929600
23.9	-20.9549	-20954.9	-144.4993	-144499300	-26.6203	-26620300
24	-34.4664	-34466.4	-146.9709	-146970900	-43.7847	-43784700
24.1	-26.684	-26684	-147.0008	-147000800	-33.8982	-33898200
24.2	-3.824	-3824	-144.7599	-144759900	-4.8579	-4857900
24.3	19.518	19518	-141.7404	-141740400	24.795	24795000
24.4	29.5903	29590.3	-139.7765	-139776500	37.5902	37590200
24.5	21.499	21499	-139.9408	-139940800	27.3114	27311400
24.6	1.4005	1400.5	-141.9725	-141972500	1.7792	1779200
24.7	-17.9693	-17969.3	-144.5335	-144533500	-22.8275	-22827500
24.8	-25.2859	-25285.9	-146.0782	-146078200	-32.1221	-32122100
24.9	-17.1808	-17180.8	-145.7772	-145777200	-21.8258	-21825800
25	0.3932	393.2	-143.9501	-143950100	0.4996	499600
25.1	16.3796	16379.6	-141.7881	-141788100	20.8079	20807900
25.2	21.5079	21507.9	-140.5876	-140587600	27.3227	27322700

25.3	13.6032	13603.2	-140.9798	-140979800	17.281	17281000
25.4	-1.6838	-1683.8	-142.6112	-142611200	-2.1391	-2139100
25.5	-14.8014	-14801.4	-144.4275	-144427500	-18.8031	-18803100
25.6	-18.2097	-18209.7	-145.3473	-145347300	-23.1329	-23132900
25.7	-10.6557	-10655.7	-144.8992	-144899200	-13.5366	-13536600
25.8	2.576	2576	-143.4522	-143452200	3.2724	3272400
25.9	13.2727	13272.7	-141.9336	-141933600	16.8611	16861100
26	15.3452	15345.2	-141.2413	-141241300	19.4939	19493900
26.1	8.2416	8241.6	-141.7183	-141718300	10.4698	10469800
26.2	-3.156	-3156	-142.9941	-142994100	-4.0093	-4009300
26.3	-11.8195	-11819.5	-144.2573	-144257300	-15.015	-15015000
26.4	-12.8695	-12869.5	-144.7666	-144766600	-16.3489	-16348900
26.5	-6.2769	-6276.9	-144.281	-144281000	-7.974	-7974000
26.6	3.4947	3494.7	-143.1626	-143162600	4.4396	4439600
26.7	10.4587	10458.7	-142.1174	-142117400	13.2863	13286300
26.8	10.74	10740	-141.754	-141754000	13.6437	13643700
26.9	4.6893	4689.3	-142.2332	-142233200	5.9571	5957100
27	-3.6496	-3649.6	-143.2084	-143208400	-4.6363	-4636300
27.1	-9.2004	-9200.4	-144.0683	-144068300	-11.6878	-11687800
27.2	-8.9169	-8916.9	-144.3165	-144316500	-11.3277	-11327700
27.3	-3.4162	-3416.2	-143.8544	-143854400	-4.3398	-4339800
27.4	3.6671	3667.1	-143.0083	-143008300	4.6586	4658600
27.5	8.049	8049	-142.3051	-142305100	10.2251	10225100
27.6	7.3634	7363.4	-142.147	-142147000	9.3541	9354100
27.7	2.4043	2404.3	-142.5847	-142584700	3.0543	3054300
27.8	-3.5846	-3584.6	-143.3151	-143315100	-4.5537	-4553700
27.9	-7.005	-7005	-143.8864	-143886400	-8.8989	-8898900
28	-6.0456	-6045.6	-143.9752	-143975200	-7.68	-7680000
28.1	-1.6081	-1608.1	-143.5665	-143566500	-2.0428	-2042800
28.2	3.4317	3431.7	-142.939	-142939000	4.3595	4359500
28.3	6.0661	6066.1	-142.4781	-142478100	7.7061	7706100
28.4	4.9329	4932.9	-142.442	-142442000	6.2666	6266600
28.5	0.9889	988.9	-142.819	-142819000	1.2563	1256300
28.6	-3.2317	-3231.7	-143.3556	-143355600	-4.1054	-4105400
28.7	-5.2277	-5227.7	-143.7243	-143724300	-6.641	-6641000
28.8	-3.998	-3998	-143.7214	-143721400	-5.0789	-5078900
28.9	-0.5142	-514.2	-143.3772	-143377200	-0.6533	-653300
29	3.0028	3002.8	-142.9204	-142920400	3.8146	3814600
29.1	4.484	4484	-142.628	-142628000	5.6963	5696300
29.2	3.2163	3216.3	-142.6593	-142659300	4.0858	4085800
29.3	0.1567	156.7	-142.9707	-142970700	0.199	199000
29.4	-2.7588	-2758.8	-143.3578	-143357800	-3.5046	-3504600
29.5	-3.8282	-3828.2	-143.5872	-143587200	-4.8632	-4863200

29.6	-2.5658	-2565.8	-143.5362	-143536200	-3.2595	-3259500
29.7	0.1067	106.7	-143.2566	-143256600	0.1355	135500
29.8	2.5102	2510.2	-142.9302	-142930200	3.1888	3188800
29.9	3.2533	3253.3	-142.7524	-142752400	4.1328	4132800
30	2.0275	2027.5	-142.8163	-142816300	2.5757	2575700
30.1	-0.2949	-294.9	-143.0656	-143065600	-0.3746	-374600
30.2	-2.2648	-2264.8	-143.3396	-143339600	-2.8771	-2877100
30.3	-2.7519	-2751.9	-143.4753	-143475300	-3.4959	-3495900
30.4	-1.5845	-1584.5	-143.4036	-143403600	-2.0129	-2012900
30.5	0.4237	423.7	-143.1828	-143182800	0.5383	538300
30.6	2.028	2028	-142.954	-142954000	2.5762	2576200
30.7	2.3168	2316.8	-142.8524	-142852400	2.9432	2943200
30.8	1.2221	1222.1	-142.9277	-142927700	1.5526	1552600
30.9	-0.5061	-506.1	-143.1222	-143122200	-0.643	-643000
31	-1.8036	-1803.6	-143.3123	-143312300	-2.2912	-2291200
31.1	-1.9412	-1941.2	-143.3866	-143386600	-2.466	-2466000
31.2	-0.9276	-927.6	-143.3105	-143310500	-1.1784	-1178400
31.3	0.5527	552.7	-143.1402	-143140200	0.7022	702200
31.4	1.5941	1594.1	-142.9832	-142983200	2.025	2025000
31.5	1.6184	1618.4	-142.9305	-142930500	2.0559	2055900
31.6	0.6899	689.9	-143.0052	-143005200	0.8765	876500
31.7	-0.5722	-572.2	-143.1535	-143153500	-0.7269	-726900
31.8	-1.4007	-1400.7	-143.2826	-143282600	-1.7794	-1779400
31.9	-1.3423	-1342.3	-143.3181	-143318100	-1.7052	-1705200
32	-0.4997	-499.7	-143.2464	-143246400	-0.6348	-634800
32.1	0.5715	571.5	-143.1179	-143117900	0.726	726000
32.2	1.2241	1224.1	-143.0125	-143012500	1.5551	1555100
32.3	1.1072	1107.2	-142.9903	-142990300	1.4065	1406500
32.4	0.3487	348.7	-143.058	-143058000	0.443	443000
32.5	-0.5561	-556.1	-143.1688	-143168800	-0.7064	-706400
32.6	-1.0643	-1064.3	-143.2543	-143254300	-1.3521	-1352100
32.7	-0.908	-908	-143.2662	-143266200	-1.1534	-1153400
32.8	-0.2302	-230.2	-143.2032	-143203200	-0.2925	-292500
32.9	0.5305	530.5	-143.1081	-143108100	0.6739	673900
33	0.9208	920.8	-143.0392	-143039200	1.1697	1169700
33.1	0.7399	739.9	-143.0351	-143035100	0.94	940000
33.2	0.1383	138.3	-143.0931	-143093100	0.1757	175700
33.3	-0.4981	-498.1	-143.1743	-143174300	-0.6328	-632800
33.4	-0.7928	-792.8	-143.2293	-143229300	-1.0071	-1007100
33.5	-0.5989	-598.9	-143.2277	-143227700	-0.7608	-760800
33.6	-0.068	-68	-143.1749	-143174900	-0.0864	-86400
33.7	0.4618	461.8	-143.1058	-143105800	0.5866	586600
33.8	0.6794	679.4	-143.0623	-143062300	0.863	863000

33.9	0.481	481	-143.068	-143068000	0.611	611000
34	0.0153	15.3	-143.1157	-143115700	0.0195	19500
34.1	-0.4234	-423.4	-143.1742	-143174200	-0.5379	-537900
34.2	-0.5795	-579.5	-143.2082	-143208200	-0.7361	-736100
34.3	-0.383	-383	-143.1997	-143199700	-0.4866	-486600
34.4	0.0233	23.3	-143.1569	-143156900	0.0296	29600
34.5	0.3846	384.6	-143.1077	-143107700	0.4885	488500
34.6	0.492	492	-143.0814	-143081400	0.625	625000
34.7	0.3021	302.1	-143.0917	-143091700	0.3837	383700
34.8	-0.0507	-50.7	-143.1298	-143129800	-0.0644	-64400
34.9	-0.3464	-346.4	-143.1711	-143171100	-0.4401	-440100
35	-0.4158	-415.8	-143.1911	-143191100	-0.5282	-528200
35.1	-0.2355	-235.5	-143.1797	-143179700	-0.2992	-299200
35.2	0.0692	69.2	-143.146	-143146000	0.088	88000
35.3	0.3098	309.8	-143.1116	-143111600	0.3935	393500
35.4	0.3497	349.7	-143.0967	-143096700	0.4443	444300
35.5	0.1811	181.1	-143.1085	-143108500	0.2301	230100
35.6	-0.0809	-80.9	-143.1382	-143138200	-0.1028	-102800
35.7	-0.2752	-275.2	-143.1668	-143166800	-0.3495	-349500
35.8	-0.2927	-292.7	-143.1776	-143177600	-0.3719	-371900
35.9	-0.137	-137	-143.1657	-143165700	-0.174	-174000
36	0.0872	87.2	-143.1398	-143139800	0.1108	110800
36.1	0.2429	242.9	-143.1162	-143116200	0.3086	308600
36.2	0.2438	243.8	-143.1086	-143108600	0.3097	309700
36.3	0.1014	101.4	-143.1202	-143120200	0.1288	128800
36.4	-0.0896	-89.6	-143.1427	-143142700	-0.1138	-113800
36.5	-0.2132	-213.2	-143.1621	-143162100	-0.2709	-270900
36.6	-0.202	-202	-143.1672	-143167200	-0.2566	-256600
36.7	-0.073	-73	-143.156	-143156000	-0.0927	-92700
36.8	0.089	89	-143.1365	-143136500	0.113	113000
36.9	0.1861	186.1	-143.1207	-143120700	0.2365	236500
37	0.1664	166.4	-143.1177	-143117700	0.2114	211400
37.1	0.0505	50.5	-143.1281	-143128100	0.0641	64100
37.2	-0.0862	-86.2	-143.1449	-143144900	-0.1095	-109500
37.3	-0.1617	-161.7	-143.1577	-143157700	-0.2054	-205400
37.4	-0.1363	-136.3	-143.1593	-143159300	-0.1732	-173200
37.5	-0.0328	-32.8	-143.1496	-143149600	-0.0417	-41700
37.6	0.082	82	-143.1352	-143135200	0.1041	104100
37.7	0.1397	139.7	-143.1249	-143124900	0.1775	177500
37.8	0.111	111	-143.1244	-143124400	0.141	141000
37.9	0.0192	19.2	-143.1334	-143133400	0.0244	24400
38	-0.0767	-76.7	-143.1457	-143145700	-0.0975	-97500
38.1	-0.1202	-120.2	-143.1539	-143153900	-0.1527	-152700

38.2	-0.0897	-89.7	-143.1534	-143153400	-0.1139	-113900
38.3	-0.0088	-8.8	-143.1453	-143145300	-0.0112	-11200
38.4	0.071	71	-143.1349	-143134900	0.0902	90200
38.5	0.1029	102.9	-143.1284	-143128400	0.1307	130700
38.6	0.0719	71.9	-143.1294	-143129400	0.0914	91400
38.7	0.0011	1.1	-143.1367	-143136700	0.0013	1300
38.8	-0.065	-65	-143.1456	-143145600	-0.0825	-82500
38.9	-0.0877	-87.7	-143.1506	-143150600	-0.1114	-111400
39	-0.0572	-57.2	-143.1492	-143149200	-0.0726	-72600
39.1	0.0046	4.6	-143.1427	-143142700	0.0058	5800
39.2	0.0589	58.9	-143.1352	-143135200	0.0748	74800
39.3	0.0744	74.4	-143.1313	-143131300	0.0945	94500
39.4	0.045	45	-143.133	-143133000	0.0571	57100
39.5	-0.0086	-8.6	-143.1388	-143138800	-0.0109	-10900
39.6	-0.053	-53	-143.1451	-143145100	-0.0673	-67300
39.7	-0.0628	-62.8	-143.148	-143148000	-0.0798	-79800
39.8	-0.035	-35	-143.1462	-143146200	-0.0444	-44400
39.9	0.0113	11.3	-143.1411	-143141100	0.0143	14300
40	0.0473	47.3	-143.1359	-143135900	0.0601	60100
40.1	0.0528	52.8	-143.1337	-143133700	0.0671	67100
40.2	0.0268	26.8	-143.1355	-143135500	0.0341	34100
40.3	-0.0129	-12.9	-143.1401	-143140100	-0.0164	-16400
40.4	-0.042	-42	-143.1444	-143144400	-0.0533	-53300
40.5	-0.0441	-44.1	-143.1459	-143145900	-0.0561	-56100
40.6	-0.0202	-20.2	-143.1441	-143144100	-0.0257	-25700
40.7	0.0137	13.7	-143.1401	-143140100	0.0175	17500
40.8	0.037	37	-143.1366	-143136600	0.047	47000
40.9	0.0367	36.7	-143.1355	-143135500	0.0467	46700
41	0.0149	14.9	-143.1373	-143137300	0.0189	18900
41.1	-0.014	-14	-143.1407	-143140700	-0.0178	-17800
41.2	-0.0324	-32.4	-143.1436	-143143600	-0.0412	-41200
41.3	-0.0304	-30.4	-143.1444	-143144400	-0.0386	-38600
41.4	-0.0106	-10.6	-143.1426	-143142600	-0.0135	-13500
41.5	0.0138	13.8	-143.1397	-143139700	0.0176	17600
41.6	0.0283	28.3	-143.1373	-143137300	0.0359	35900
41.7	0.025	25	-143.1369	-143136900	0.0318	31800
41.8	0.0073	7.3	-143.1385	-143138500	0.0093	9300
41.9	-0.0134	-13.4	-143.141	-143141000	-0.017	-17000
42	-0.0246	-24.6	-143.143	-143143000	-0.0312	-31200
42.1	-0.0205	-20.5	-143.1432	-143143200	-0.026	-26000
42.2	-0.0047	-4.7	-143.1417	-143141700	-0.0059	-5900
42.3	0.0127	12.7	-143.1395	-143139500	0.0161	16100
42.4	0.0212	21.2	-143.1379	-143137900	0.0269	26900

42.5	0.0166	16.6	-143.1379	-143137900	0.0211	21100
42.6	0.0026	2.6	-143.1393	-143139300	0.0034	3400
42.7	-0.0118	-11.8	-143.1411	-143141100	-0.015	-15000
42.8	-0.0182	-18.2	-143.1424	-143142400	-0.0231	-23100
42.9	-0.0134	-13.4	-143.1423	-143142300	-0.0171	-17100
43	-0.0011	-1.1	-143.141	-143141000	-0.0014	-1400
43.1	0.0109	10.9	-143.1394	-143139400	0.0139	13900
43.2	0.0156	15.6	-143.1385	-143138500	0.0198	19800
43.3	0.0107	10.7	-143.1387	-143138700	0.0137	13700
43.4	-0.00003267	-0.03267	-143.1398	-143139800	-4.2E-05	-41.51
43.5	-0.01	-10	-143.1411	-143141100	-0.0127	-12700
43.6	-0.0133	-13.3	-143.1419	-143141900	-0.0169	-16900
43.7	-0.0085	-8.5	-143.1416	-143141600	-0.0108	-10800
43.8	0.0009	0.9	-143.1406	-143140600	0.0011	1100
43.9	0.009	9	-143.1395	-143139500	0.0115	11500
44	0.0112	11.2	-143.1389	-143138900	0.0143	14300
44.1	0.0067	6.7	-143.1392	-143139200	0.0085	8500
44.2	-0.0014	-1.4	-143.1401	-143140100	-0.0018	-1800
44.3	-0.0081	-8.1	-143.141	-143141000	-0.0103	-10300
44.4	-0.0095	-9.5	-143.1415	-143141500	-0.0121	-12100
44.5	-0.0052	-5.2	-143.1412	-143141200	-0.0066	-6600
44.6	0.0018	1.8	-143.1404	-143140400	0.0023	2300
44.7	0.0072	7.2	-143.1396	-143139600	0.0092	9200
44.8	0.008	8	-143.1393	-143139300	0.0101	10100
44.9	0.004	4	-143.1396	-143139600	0.005	5000
45	-0.002	-2	-143.1403	-143140300	-0.0026	-2600
45.1	-0.0064	-6.4	-143.1409	-143140900	-0.0081	-8100
45.2	-0.0067	-6.7	-143.1411	-143141100	-0.0085	-8500
45.3	-0.003	-3	-143.1409	-143140900	-0.0038	-3800
45.4	0.0022	2.2	-143.1403	-143140300	0.0027	2700
45.5	0.0056	5.6	-143.1397	-143139700	0.0072	7200
45.6	0.0055	5.5	-143.1396	-143139600	0.007	7000
45.7	0.0022	2.2	-143.1398	-143139800	0.0028	2800
45.8	-0.0022	-2.2	-143.1404	-143140400	-0.0028	-2800
45.9	-0.0049	-4.9	-143.1408	-143140800	-0.0063	-6300
46	-0.0046	-4.6	-143.1409	-143140900	-0.0058	-5800
46.1	-0.0015	-1.5	-143.1406	-143140600	-0.002	-2000
46.2	0.0022	2.2	-143.1402	-143140200	0.0027	2700
46.3	0.0043	4.3	-143.1398	-143139800	0.0055	5500
46.4	0.0038	3.8	-143.1398	-143139800	0.0048	4800
46.5	0.0011	1.1	-143.14	-143140000	0.0013	1300
46.6	-0.0021	-2.1	-143.1404	-143140400	-0.0026	-2600
46.7	-0.0037	-3.7	-143.1407	-143140700	-0.0047	-4700

46.8	-0.0031	-3.1	-143.1407	-143140700	-0.0039	-3900
46.9	-0.0007	-0.7	-143.1405	-143140500	-0.0008	-800
47	0.002	2	-143.1402	-143140200	0.0025	2500
47.1	0.0032	3.2	-143.1399	-143139900	0.0041	4100
47.2	0.0025	2.5	-143.1399	-143139900	0.0032	3200
47.3	0.0004	0.4	-143.1401	-143140100	0.0005	500
47.4	-0.0018	-1.8	-143.1404	-143140400	-0.0023	-2300
47.5	-0.0028	-2.8	-143.1406	-143140600	-0.0035	-3500
47.6	-0.002	-2	-143.1406	-143140600	-0.0026	-2600
47.7	-0.0001	-0.1	-143.1404	-143140400	-0.0002	-200
47.8	0.0017	1.7	-143.1402	-143140200	0.0021	2100
47.9	0.0024	2.4	-143.14	-143140000	0.003	3000
48	0.0016	1.6	-143.14	-143140000	0.002	2000
48.1	-0.00003402	-0.03402	-143.1402	-143140200	-4.3E-05	-43.22
48.2	-0.0015	-1.5	-143.1404	-143140400	-0.0019	-1900
48.3	-0.002	-2	-143.1405	-143140500	-0.0026	-2600
48.4	-0.0013	-1.3	-143.1405	-143140500	-0.0016	-1600
48.5	0.0002	0.2	-143.1403	-143140300	0.0002	200
48.6	0.0014	1.4	-143.1402	-143140200	0.0018	1800
48.7	0.0017	1.7	-143.1401	-143140100	0.0022	2200
48.8	0.001	1	-143.1401	-143140100	0.0013	1300
48.9	-0.0002	-0.2	-143.1403	-143140300	-0.0003	-300
49	-0.0012	-1.2	-143.1404	-143140400	-0.0016	-1600
49.1	-0.0014	-1.4	-143.1405	-143140500	-0.0018	-1800
49.2	-0.0008	-0.8	-143.1404	-143140400	-0.001	-1000
49.3	0.0003	0.3	-143.1403	-143140300	0.0004	400
49.4	0.0011	1.1	-143.1402	-143140200	0.0014	1400
49.5	0.0012	1.2	-143.1401	-143140100	0.0015	1500
49.6	0.0006	0.6	-143.1402	-143140200	0.0007	700
49.7	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
49.8	-0.001	-1	-143.1404	-143140400	-0.0012	-1200
49.9	-0.001	-1	-143.1404	-143140400	-0.0013	-1300
50	-0.0004	-0.4	-143.1404	-143140400	-0.0006	-600
50.1	0.0003	0.3	-143.1403	-143140300	0.0004	400
50.2	0.0009	0.9	-143.1402	-143140200	0.0011	1100
50.3	0.0008	0.8	-143.1402	-143140200	0.0011	1100
50.4	0.0003	0.3	-143.1402	-143140200	0.0004	400
50.5	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
50.6	-0.0008	-0.8	-143.1404	-143140400	-0.001	-1000
50.7	-0.0007	-0.7	-143.1404	-143140400	-0.0009	-900
50.8	-0.0002	-0.2	-143.1403	-143140300	-0.0003	-300
50.9	0.0003	0.3	-143.1403	-143140300	0.0004	400
51	0.0007	0.7	-143.1402	-143140200	0.0008	800

51.1	0.0006	0.6	-143.1402	-143140200	0.0007	700
51.2	0.0002	0.2	-143.1403	-143140300	0.0002	200
51.3	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
51.4	-0.0006	-0.6	-143.1404	-143140400	-0.0007	-700
51.5	-0.0005	-0.5	-143.1404	-143140400	-0.0006	-600
51.6	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
51.7	0.0003	0.3	-143.1403	-143140300	0.0004	400
51.8	0.0005	0.5	-143.1402	-143140200	0.0006	600
51.9	0.0004	0.4	-143.1402	-143140200	0.0005	500
52	0.00004856	0.04856	-143.1403	-143140300	0.0001	100
52.1	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
52.2	-0.0004	-0.4	-143.1403	-143140300	-0.0005	-500
52.3	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
52.4	-0.0000151	-0.0151	-143.1403	-143140300	-1.9E-05	-19.18
52.5	0.0003	0.3	-143.1403	-143140300	0.0003	300
52.6	0.0004	0.4	-143.1403	-143140300	0.0005	500
52.7	0.0002	0.2	-143.1403	-143140300	0.0003	300
52.8	-0.000009551	-0.00955	-143.1403	-143140300	-1.2E-05	-12.13
52.9	-0.0002	-0.2	-143.1403	-143140300	-0.0003	-300
53	-0.0003	-0.3	-143.1403	-143140300	-0.0004	-400
53.1	-0.0002	-0.2	-143.1403	-143140300	-0.0002	-200
53.2	0.00002718	0.02718	-143.1403	-143140300	3.45E-05	34.53
53.3	0.0002	0.2	-143.1403	-143140300	0.0003	300
53.4	0.0003	0.3	-143.1403	-143140300	0.0003	300
53.5	0.0001	0.1	-143.1403	-143140300	0.0002	200
53.6	-0.00003926	-0.03926	-143.1403	-143140300	-5E-05	-49.87
53.7	-0.0002	-0.2	-143.1403	-143140300	-0.0002	-200
53.8	-0.0002	-0.2	-143.1403	-143140300	-0.0003	-300
53.9	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
54	0.000047	0.047	-143.1403	-143140300	0.0001	100
54.1	0.0002	0.2	-143.1403	-143140300	0.0002	200
54.2	0.0002	0.2	-143.1403	-143140300	0.0002	200
54.3	0.0001	0.1	-143.1403	-143140300	0.0001	100
54.4	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
54.5	-0.0001	-0.1	-143.1403	-143140300	-0.0002	-200
54.6	-0.0002	-0.2	-143.1403	-143140300	-0.0002	-200
54.7	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
54.8	0.0001	0.1	-143.1403	-143140300	0.0001	100
54.9	0.0001	0.1	-143.1403	-143140300	0.0002	200
55	0.0001	0.1	-143.1403	-143140300	0.0002	200
55.1	0.00004679	0.04679	-143.1403	-143140300	0.0001	100
55.2	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
55.3	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100

55.4	-0.0001	-0.1	-143.1403	-143140300	-0.0001	-100
55.5	0.00003268	0.03268	-143.1403	-143140300	-4.2E-05	-41.52
55.6	0.0001	0.1	-143.1403	-143140300	0.0001	100
55.7	0.0001	0.1	-143.1403	-143140300	0.0001	100
55.8	0.0001	0.1	-143.1403	-143140300	0.0001	100
55.9	0.0000216	0.0216	-143.1403	-143140300	2.74E-05	27.44
56	0.00004942	0.04942	-143.1403	-143140300	-0.0001	-100
56.1	0.0001	0.1	-143.1403	-143140300	-0.0001	-100
56.2	0.0001	0.1	-143.1403	-143140300	-0.0001	-100
56.3	0.00001301	0.01301	-143.1403	-143140300	-1.7E-05	-16.52
56.4	0.00004642	0.04642	-143.1403	-143140300	0.0001	100
56.5	0.0001	0.1	-143.1403	-143140300	0.0001	100
56.6	0.0001	0.1	-143.1403	-143140300	0.0001	100
56.7	0.000006436	0.006436	-143.1403	-143140300	8.18E-06	8.176
56.8	0.00004304	0.04304	-143.1403	-143140300	-0.0001	-100
56.9	0.0001	0.1	-143.1403	-143140300	-0.0001	-100
57	0.00004495	0.04495	-143.1403	-143140300	-0.0001	-100
57.1	0.000001504	0.001504	-143.1403	-143140300	-1.9E-06	-1.911
57.2	0.00003947	0.03947	-143.1403	-143140300	0.0001	100
57.3	0.0001	0.1	-143.1403	-143140300	0.0001	100
57.4	0.0000358	0.0358	-143.1403	-143140300	4.55E-05	45.48
57.5	0.000002112	0.002112	-143.1403	-143140300	-2.7E-06	-2.683
57.6	0.00003586	0.03586	-143.1403	-143140300	-4.6E-05	-45.55
57.7	0.00004593	0.04593	-143.1403	-143140300	-0.0001	-100
57.8	0.00002824	0.02824	-143.1403	-143140300	-3.6E-05	-35.87
57.9	0.000004679	0.004679	-143.1403	-143140300	5.94E-06	5.944
58	0.00003231	0.03231	-143.1403	-143140300	4.1E-05	41.04
58.1	0.00003882	0.03882	-143.1403	-143140300	4.93E-05	49.31
58.2	0.00002202	0.02202	-143.1403	-143140300	2.8E-05	27.97
58.3	0.000006419	0.006419	-143.1403	-143140300	-8.2E-06	-8.154
58.4	0.00002889	0.02889	-143.1403	-143140300	-3.7E-05	-36.7
58.5	0.00003265	0.03265	-143.1403	-143140300	-4.1E-05	-41.48
58.6	0.00001694	0.01694	-143.1403	-143140300	-2.2E-05	-21.52
58.7	0.000007514	0.007514	-143.1403	-143140300	9.55E-06	9.546
58.8	0.00002567	0.02567	-143.1403	-143140300	3.26E-05	32.6
58.9	0.00002734	0.02734	-143.1403	-143140300	3.47E-05	34.73
59	0.00001282	0.01282	-143.1403	-143140300	1.63E-05	16.28
59.1	0.000008113	0.008113	-143.1403	-143140300	-1E-05	-10.31
59.2	0.00002266	0.02266	-143.1403	-143140300	-2.9E-05	-28.79
59.3	0.00002277	0.02277	-143.1403	-143140300	-2.9E-05	-28.92
59.4	0.000009492	0.009492	-143.1403	-143140300	-1.2E-05	-12.06
59.5	0.000008337	0.008337	-143.1403	-143140300	1.06E-05	10.59
59.6	0.00001989	0.01989	-143.1403	-143140300	2.53E-05	25.27

59.7	0.00001887	0.01887	-143.1403	-143140300	2.4E-05	23.97
59.8	0.000006836	0.006836	-143.1403	-143140300	8.68E-06	8.684
59.9	0.000008283	0.008283	-143.1403	-143140300	-1.1E-05	-10.52
60	0.00001737	0.01737	-143.1403	-143140300	-2.2E-05	-22.06