OPTIMIZATION OF THE SYSTEM AVAILABILITY THROUGH CONDITION BASED MAINTENANCE

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By

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CERTIFICATE

This is to certify that the thesis entitled "**Optimization of the System Availability through Condition Based Maintenance**" submitted by **Nikhil (2K14/PIE/23)**, during the session 2014-2016 for the award of M.Tech degree of Delhi Technological University, Delhi is absolutely based upon his work done under our supervision and guidance and that neither this thesis nor any part of it has been submitted for any degree/diploma or any other academic award.

The assistance and help received during the course of investigation have been fully acknowledged. We wish him good luck in future.

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DECLARATION

I, Nikhil hereby certify that the work which is being presented in this thesis entitled "Optimization of the System Availability through Condition Based Maintenance", is submitted, and in the partial fulfillment of the requirements for degree of Master of Technology at Delhi Technological University is an authentic record of my own work carried under the supervision of Dr. Pravin Kumar and Dr. Girish Kumar. I have not submitted the matter embodied in this seminar for the award of any other degree or diploma also it has not been directly copied from any source without giving its proper reference.

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ABSTRACT

This project deals with the optimization of the system availability through condition based maintenance using MARKOV analysis. The conventional techniques such as reliability block diagram, fault tree analysis and reliability graphs are of no use when there comes repairs and other dependencies. The MARKOV approach is considered for modeling as it can incorporate repair and other dependency features in the model. Generally, mechanical systems deteriorate gradually before they fail catastrophically. So it would be more appropriate, if multi state degradation is considered in the modeling; inspection is introduced in the model to monitor the condition and to take appropriate maintenance or repair action. Repair action depends upon the system state at particular interval. Here three modes of repair actions that are nominal repair, intermediate repair and major repair are taken. Repair rates can make system work in a prescribed manner, but cannot fix it always to initial stage or new state. If it is degraded to some lower state then it may be possible to recover it exactly, but at a highly degraded state it is almost impossible to fix it to a new state, therefore in such situations other repairs options like nominal repair just to make it work for some more time are taken. It has also noticed that by changing inspection time, availability varies. Therefore, the analysis was carried out to determine inspection interval when the availability becomes maximum using Markov approach.

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ABBREVIATIONS

CBM: Condition based maintenance

MTTF: Mean time to failure

SCADA: Supervisory control and data acquisition

TBM: Time-based maintenance

CM: Corrective maintenance

RBD: Reliability block diagram

DD: Dependence diagram

MSF: Modelling System Failures

FTD: fault tree diagrams

FFT: Fast Fourier Transformation

QMO: Quantitative maintenance optimization

ANN: Artificial neural network

ATM: Advanced terotechnological model

EUT: Eindhoven university of technology model

AHP: Analytic hierarchy process

LIST OF SYMBOLS

Symbols	Description
λ	Denotes the failure rate; the rate at which a system fails.
μ	Denotes the rate of repair; the rate at which the system is being
	repaired.
\mathbf{N}_1	Initial state of the system where health is at its best.
O ₁	The minor degraded state to which our system goes after some use.
O_2	The intermediate degraded state to which our system goes after
	prolonged use.
O ₃	The major degraded state to which our system goes after prolonged
	use.
Fr	The state of Random failure where the system goes after sudden
	adverse situations viz. mishandling, natural disasters.
λ_{N1I}	State of inspection in which the components goes from the state N_1 .
λ_{O1I}	State of inspection in which the components goes from the state O_1 .
λ_{O2I}	State of inspection in which the components goes from the state O_2 .
λ_{O3I}	State of inspection in which the components goes from the state O_3 .

- μ_{n1} The state of Minor repair where the system goes to the state N₁; if found faulty after inspection.
- μ_{n2} The state of Minor repair where the system goes to the state O₁; if found faulty after inspection.
- μ_{H1} The state of higher repair where the system goes to the state O₁; if found faulty after inspection.
- μ_{a1} The state of intermediate repair where the system goes to the state O_1 ; if found faulty after inspection.
- μ_{H2} The state of higher repair where the system goes to the state N₁; if found faulty after inspection.
- μ_{n4} The state of intermediate repair where the system goes to the state O₂; if found faulty after inspection.
- μ_{a2} The state of intermediate repair where the system goes to the state O₂; if found faulty after inspection.
- μ_{H3} The state of intermediate repair where the system goes to the state O₁; if found faulty after inspection.
- λ_{N101} Denotes the rate at which the system will go from N₁ to O₁.
- λ_{O1O2} Denotes the rate at which the system will go from O_1 to O_2 .
- λ_{O2O3} Denotes the rate at which the system will go from O₂ to O₃.

λ_{N1I}	Denotes the rate at which the system will go from N_1 to I.
λ_{In1}	Denotes the rate at which the system will go from I to n1.
λ_{O1I}	Denotes the rate at which the system will go from O_1 to I.
λ_{In2}	Denotes the rate at which the system will go from I to n2.
$\lambda_{\rm IH1}$	Denotes the rate at which the system will go from I to H_1 .
λ_{O2I}	Denotes the rate at which the system will go from O_2 to I.
λ_{Ia1}	Denotes the rate at which the system will go from I to a1.
λ_{IH2}	Denotes the rate at which the system will go from I to H_2 .
λ_{In4}	Denotes the rate at which the system will go from I to n_4 .
λ_{O3I}	Denotes the rate at which the system will go from O_3 to I.
λ_{IH3}	Denotes the rate at which the system will go from Ito H_3 .
λ_{Ia2}	Denotes the rate at which the system will go from I to a_2 .
λ_{N1Fr}	Denotes the rate at which the system will go from N_1 to Fr.
λ_{O1Fr}	Denotes the rate at which the system will go from O_1 to Fr.
λ_{O2Fr}	Denotes the rate at which the system will go from O_2 to Fr.
λ_{O3Fr}	Denotes the rate at which the system will go from O_3 to Fr.

- μ_{n1N1} Denotes the rate at which the system will be restored from the repair state n_1 to state N_1 .
- μ_{n2O1} Denotes the rate at which the system will be restored from the repair state n_2 to state O_1 .
- μ_{H1N1} Denotes the rate at which the system will be restored from the repair state H₁ to state N₁.
- μ_{a101} Denotes the rate at which the system will be restored from the repair state a1 to state O₁.
- μ_{H2N1} Denotes the rate at which the system will be restored from the repair state H₂ to state N₁.
- μ_{n43O2} Denotes the rate at which the system will be restored from the repair state n4 to state O₂.
- μ_{a2O2} Denotes the rate at which the system will be restored from the repair state a_2 to state O_2 .

CHAPTER 1

INTRODUCTION

Modern mechanical systems are the integral part of various industries such as refineries, power plant, and oil manufacturing systems, etc. These are intended to perform the specific function for the time; the plant is being operated which is not possible without having proper maintenance policies. If not followed with given safety requirements, it can cause risk to the personnel, public or the environment. But, it is unfortunate that a threat of deteriorating process involved in components or machine parts is always present, therefore maintenance measures are to be followed to control the deterioration of the machine components and also to increase the performance of the system during its life time. For this purpose various decisions are to be taken for inspection to assess health of the system and to achieve the maximum benefit from the system after employing appropriate maintenance. Maintenance is important to make the system work at least for the time it is intended to work. These days proper maintenance actions are necessary for the industries to make the system work in a prescribed manner. Maintenance eventually increases system reliability and availability.

Availability has gained importance in recent years and is a part of process specification. It is also becoming an integral part of product design, life cycle costing, inventory management and maintenance system decision making (Ebeling, 1997). Maintenance does help in increasing the system availability. A mechanical system is not a single unit system, but consists of units which work together to perform desired task. While working together, it results in wear out of the mating components which cause system failure. Failure is due to wear, fracture or fatigue or sometimes due to manufacturing errors also that causes random failure. Mechanical system generally deteriorates as the time passes and hence if inspection is not done, it will not be possible to find out when it will fail. Therefore, inspection is carried out to the system as its condition starts deteriorating. Maintenance actions cannot make the system as new as it was, but can provide it a healthier state so that it may work properly.

Traditionally, maintenance used in power plants was breakdown maintenance that is also called firefighting maintenance or failure maintenance or corrective maintenance. Breakdown maintenance can only be carried out until a failure of component or machine occurs. It is best suited only to non-critical areas where the capital costs are small also the consequences of failure are feeble and there is no safety risk. Under the breakdown maintenance the failure shall be identified quickly and the repair will be quick. Unfortunately, Breakdown maintenance is widely accepted by default in power plants and therefore, becomes costly. It is important for a power system to work as per the requirements, but due to the unpredictable nature of failure of components, it is not possible to give a continuous supply to the consumers. In the era of higher competition and to fulfill continuous demand, scheduled maintenance was thus introduced in power plants. Therefore, maintenance was planned and performed periodically to avoid any possible failures. This is also known as preventive scheduled maintenance, a maintenance strategy regardless of the healthy status of a machine component. Preventive maintenance then adopted by many power plants and it certainly improved the reliability and safety levels of generating in most of the cases. However, with increase in development and technology machine components are getting convoluted, requiring higher reliability and quality. Maintenance at fixed interval often raises the cost involved in power system. Also, due to lack of knowledge of accurate failure rate of machines, it is almost hard to define the effective inspection intervals. The cost associated with preventive maintenance in the competitive market, thus does not make it the most effective strategy.

More efficient maintenance techniques such as condition based maintenance (CBM) are being implemented to handle the situation (Jardine *et al.*, 2006). Condition monitoring sense the condition of the system by checking its vibration level, the oil checks etc. CBM is a maintenance program, which takes maintenance actions after getting the information through condition monitoring. Generally, maintenance policies are time based, but it has made for maintenance policy to be based on its condition. Therefore, in this work maintenance activities are performed only after looking at the abnormality of the system and hence to avoid any unnecessary maintenance tasks. If CBM is implemented properly then maintenance cost associated with scheduled preventive maintenance approach can be reduced. The benefit of CBM includes reduced machine downtime, increase in production rate and most efficient maintenance work with lower involvement of the cost (Williams *et al.*, 1994; Yang, 2003).

CBM can be optimum maintenance scheme using the information collected till the decision making time of the component. Condition information is important for CBM. Information is composed of delivery quality, monitored information, working environment and maintenance history. The regular process of CBM is to estimate the condition of component by various kinds of specific data collected through online monitoring (Fig 1.1). CBM consists of three steps;

- **Data acquisition** It is basically the information collecting step about the system health in which it currently is.
- **Data processing** It is information handling step. In this step, the collected information is analyze and data received is properly understood.
- **Maintenance decision-making** In this step, maintenance policies are implemented after taking accurate decisions.



Figure 1.1 Steps in CBM program.

Traditionally, reliability block diagram, fault tree analysis and reliability graph were used for reliability and availability analysis. These techniques are simple and exact, but because of their static nature complex systems which include repairs sequences and non-exponential probability distributions cannot be solved with these techniques. Therefore, advancement to such techniques is Markov approach.

In this work, degraded system model with various repairs rates, inspection, random failures and failures is developed and analysis is carried out to determine inspection interval when the availability will be maximum using Markov approach.

CHAPTER 2

LITERATURE REVIEW

2.1 Review

It has been emphasized that maintenance activities should be decided on the basis of actual conditions of systems under CBM policies. CBM policies have gained great attention, and the study of mathematical models for CBM is growing in popularity since the 1990s. Both corrective and preventive maintenances bring the system to an "as good as new" stage. Using Markov chains, closed-form analytical solutions for the performance measures of the model are suggested by Amari and McLaughlin (2004), Chen *et al.* (2003), Coolen and Dekker (1995) .The authors presented algorithms to find the optimal model parameters that maximize the system availability. Moreover, the authors considered a deteriorating process for a multi-state system. The objective of these models is to determine the states for which the unit is replaced in order to gain the minimum expected maintenance cost.

Technique for condition monitoring in which wear is taken as a parameter was proposed by Pellegrin (1992). Wear accumulates over time and monitoring inspections are performed at chosen times to monitor and measure the cumulative wear. If past measurements of wear are available up to the present, and the component is still active, the decision problem is to choose an appropriate time for the next inspection based upon the condition information obtained to date. Be'renguer *et al.* (2000) proposed a system, which is following random deterioration, its condition is monitored through perfect inspections and when it exceed from a defined level L then declared failed and corrective maintenance is carried out. Various researchers developed system that deteriorates gradually and stochastically, and the analytical model of the proposed condition based inspection policy for such a system have been suggested by Grall *et al.* (2005),

Dieulle *et al.* (2003) and Castanier *et al.* (2002). Considering failure and repair rates constant authors applied markov approach on the system.

A Grall *et al.* (2001) proposed a condition based maintenance policy for stochastically deteriorating systems. In this paper, analytic modeling of condition based inspection and replacement policy for stochastically and continuous deteriorating system is done. Decision variables taken as inspection schedule and replacement threshold are both considered and finally a model is built to calculate the associated cost. Castanier *et al.* (2004) proposed a condition-based maintenance policy with non-periodic inspections for a two-unit series system. Each unit is subject to gradual deterioration and is monitored by sequential non-periodic inspections. Every inspection or replacement entails a set-up cost and a component-specific unit cost, but if actions on the two components are combined, the set-up cost is charged only once.

Banks *et al.* (2006) presented a review on system reliability and condition based maintenance. The authors also proposed the concept of replacing an investment in increased inherent reliability with system health monitoring. Zhong *et al.* (2006) proposed condition based maintenance system of hydroelectric generating unit. The authors used a three levels system which are field level subsystems, unit level subsystem and enterprise level which includes SCADA (supervisory control and data acquisition) and maintenance system via field bus and intranet. Andrew *et al.* (2006) focused on a review on machinery diagnostics and prognostics implementing condition-based maintenance and hence attempted to summarize and review the recent research and developments in diagnostics and prognostics of mechanical systems implementing CBM with emphasis on models, algorithms and technologies for data processing and maintenance decision-making. Ling Wang *et al.* (2007) proposed a condition-based order-replacement policy for a single-unit system. The concerned system deteriorates stochastically and gradually, and is

inspected periodically. Under the proposed policy, both the preventive replacement and the spare order are decided based on the observed deterioration level of the system. Therefore, the decision variables for this order-replacement problem include the inspection interval, the ordering threshold, and the preventive replacement threshold.

Tian (2009) suggested an artificial neural network based method for predicting the remaining useful life of equipment subjected to condition monitoring. The author also introduced an approach to deal with the multi-objective condition based maintenance optimization problem. Zhang et al. (2009) studied a hybrid prognostics and health management approach for condition based maintenance. The authors proposed physics of failure approach and categories data driven approach for prognostics and health management application. Yongji et al. (2009) proposed a remote condition-based maintenance for web-enabled robotic system. Mathematical modeling of system availability has been derived in order to account for other failures that might occur in the subsystems of the robot. Compared to the schedule based maintenance strategies, the proposed approach shows great potential for improving overall production efficiency, while reducing the cost of maintenance. The current trends in industry include an integration of information and knowledge-base network with a manufacturing system, which coined a new term, emanufacturing. From the perspective of e-manufacturing any production equipment and its control functions do not exist alone, instead becoming a part of the holistic operation system with distant monitoring, remote quality control, and fault diagnostic capabilities.

Li *et al.* (2009) focused on performance of gas turbine prognostic for condition-based maintenance. On the basis of historical health information remaining useful life of gas turbine engines were calculated. A combined regression techniques, including both linear and quadratic models, was proposed to predict the remaining useful life of gas turbine engines. A statistic

compatibility check is used to determine the transition point from a linear regression to a quadratic regression. The developed prognostic approach has been applied to a model gas turbine engine similar to rolls-royce industrial gas turbine AVON 1535 implemented with compressor degradation over time. Wang et al. (2009) introduced a condition based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure. The authors considered deteriorating systems with a number of identical units. A scalar random variable was taken to describe the deterioration level of each unit .The authors also presented a simulation model for the development of system operation within the proposed policy. Wu et al. (2010) proposed an online adaptive condition based maintenance method for mechanical systems. The authors developed method based on a subtype of neutral network technique also known as selforganizing map. Also, the authors concluded that self-adaptive training algorithms will be activated if any unknown conditions are found in which neural classifier is not valid to update the map .So, continuous purge prototype on map is not required and it also provide excellent visualization capability which is useful for condition monitoring activity. Rausch et al. (2010) proposed joint production and spare part inventory control strategy driven by condition based maintenance. The authors considered two step heuristic approach and the computational algorithms were used as tool in account to perform maintenance on degraded systems while controlling the spare part inventories.

Zhigang *et al.* (2011) focused their study on condition based maintenance optimization for wind power generation systems under continuous monitoring. Condition monitoring information was collected from wind turbine components, condition based maintenance (CBM) strategy can be used to reduce the operation and maintenance costs of wind power generation systems. The existing CBM methods for wind power generation systems deal with wind turbine components separately in which maintenance decisions are made on individual components rather than the whole system. However, a wind farm generally consists of multiple wind turbines; and each wind turbine has multiple components including main bearing, gearbox, generator, etc. There are economic dependencies among wind turbines and their components. The proposed maintenance policy is defined by two failure probability threshold values at the wind turbine level. Abeer et al. (2011) proposed condition based maintenance of wind turbine systems considering different turbine types. The authors suggested consideration of different types of wind turbines in a wind farm, taking into account the economic dependency among different turbines and their components. A CBM technique is developed by introducing different CBM decision making threshold values for different types of turbines. Cabyono et al. (2011) proposed condition assessment of 500/150KV power transformer based on condition based maintenance. Tian et al. (2011) proposed condition based maintenance optimization for multi component system using proportional hazards model. The authors also developed an algorithm for the cost evaluation of multi component using CBM policy. Carr and Wang (2011) proposed an approximate methodology using extended kalman filtering and condition monitoring information to recursively establish a conditional probability density function for the residual life of a component. Yanbin et al. (2012) presented a model on post vacuum circuit breaker using condition based maintenance. The authors built a model based on rough set and support vector machine similar to the real conditions to solve the problems in maintenance for vacuum circuit breaker. Also, the authors conducted a research with about the data of 100 box type substation in distributing network of one power supply company to prove the high accuracy of the prescribed method. Khatab et al. (2012) proposed a condition based maintenance approach for availability optimization. The considered system was subjected to stochastic degradations, but continuously

monitored. Rosmaini et al. (2012) focused on an overview of time-based and condition-based maintenance in industrial application. The authors presented an overview of two maintenance techniques which are time-based maintenance (TBM) and condition-based maintenance (CBM). The authors also discussed how the TBM and CBM techniques work toward maintenance decision making. Recent research articles covering the application of each technique were reviewed. The authors also compared the challenges of implementing each technique from a practical point of view, focusing on the issues of required data determination and collection, data analysis, and decision making. Van et al. (2012) presented a CBM model in which both perfect and imperfect maintenance actions are considered for a deteriorating system. In this model, the condition of the system is periodically assessed as per remaining useful life (RUL) basedinspection policy. Kabir et al. (2014) presented a review on offshore wind turbine fault detection and recent development in condition monitoring based maintenance system. The authors suggested continuous monitoring of the critical parts as efficient and tend to improve reliability of wind turbine. Li et al. (2014) proposed CBM optimization of an aircraft assembly process considering multiple objectives. Onchis et al. (2014) suggested time-frequency methods for condition based maintenance and for modal analysis.

Wang *et al.* (2015) suggested a condition based maintenance approach to an optimal maintenance strategy considering equipment imperfect maintenance model. The authors developed a union method of the system condition based maintenance and equipment imperfect maintenance model. Zhang *et al.* (2015) suggested decision making methods of condition based maintenance. The authors concluded that the decision-making methods based on the optimum maintenance occasion, the condition threshold and classification of the technical condition are

different in theoretical foundation and have different decision-making process. Tsang (2015) presented based decision making tools for CBM.

2.2 Research gap

Although plenty of work has been reported in the literature on condition based maintenance, but the modeling on system availability assessment is not much reported. Complex systems where time is also a function is not introduced. Constant repair and failure rates are assumed while working on system using Markov approach. Realistic results cannot be achieved if using constant repair or failure rates. The realistic results imply that the obtained value of the system availability is near to the expected value. Therefore, not yielding of realistic results means the obtained value is far away from the logical or expected value; it is also clear from the literature review that authors worked on unrealistic values and parameter like maintenance cost is likely to be reduced. CBM is suggested by many authors, but its effect on availability is not much introduced. So system model on availability analysis has been developed in this project work and a trial is made to improve the system availability.

2.3 Research problem and objectives

Based on the above, research problem is formulated, along with the research objectives in this section.

2.3.1 Problem formulation

Literature review and discussion in this chapter have helped in identifying the research gaps, leading to the formulation of the research problem. It is quite clear that the availability assessment of deteriorating mechanical system is difficult with the existing models and

methodologies. This is mainly due to the assumptions in developing models and their inability to model complexities

2.3.2 Research objectives

The following are the research objectives in developing availability modeling of deteriorating mechanical system.

- To develop an analytical framework for transient availability.
- To incorporate select maintenance policy of condition based maintenance.

CHAPTER 3

AVAILABILITY AND SYSTEM MAINTENANCE

3.1 Reliability and availability measures

In this section various measures related to reliability and availability are described.

3.1.1 Availability

It can be defined as the degree to which equipment is in a specified operable and performable state for a defined function, when the function is called for. To define it in simpler way, it is a discrete proportion of time for which the equipment is in functioning state. Availability of a repairable system can be defined as the probability that the system is operating at a specified time. Ebeling, (1997) defined availability as a probability that a component or system is performing its required function at a given point in time or over a stated period of time when operated and maintained in a prescribed manner. Availability is measure of the degree of a system which is in the operable and committable state at the start of mission when the mission is called for at an unknown random point in time. The gain of a productive system is directly proportional to its availability. As a measure of performance criterion, the study of availability measures has a significant role in improving the effectiveness of repairable systems.

$$A = \frac{E \text{ [Uptime]}}{E[\text{Uptime]} + E[\text{Downtime]}}$$

3.1.2 Reliability

It can be defined in many ways such as, the probability that the function will be performed by a specified unit under defined conditions in a specified period of time, or capability of a system or equipment to perform the specific function, or the resistance of a failure of system or equipment,

or the notion that the specified thing is healthy for the job, and finally in few aspects it can also be defined as ability to fail without catastrophic outcomes.

The following graph is called as 'Bathtub' curve which is used in the concept of reliability to explain the pattern of failure rate versus time. The first phase is decreasing failure rates popularly known as 'infant mortality', the second phase is constant failure rates popularly known as 'random failures' and the third phase is an increasing failure rates popularly known as wear out failures.



Figure 3.1. Bathtub curve.

3.1.3*Maintainability*

Maintainability is defined as the probability that a failed system can be made operable in a specified interval of downtime. The downtime consists of the time it takes to discover that a failure exists, identify the problem, acquire the appropriate tools and parts, and perform the necessary maintenance actions. Therefore, downtime is a function of the failure detection time,

repair time, administrative time, and the logistics time connected with the repair cycle. The maintainability function describes probabilistically how long a system remains in a failed state. From the definitions of reliability and maintainability, it is clear that the reliability considers only the failure behaviors of the system and the maintainability considers only the effects of maintenance actions. With increasing complexity and the resulting high operational and maintenance costs, greater emphasis has been placed on reducing system maintenance while improving reliability. Therefore, a measure that considers both the failure behaviors and the effects of maintenance actions is more appropriate for measuring the performance of a repairable system. In this regard, availability, which is a combined measure of reliability and maintainability, has received wide acceptance as a measure of performance of maintained systems.

3.2 Maintenance

Today, in the modern industry, equipment and machinery is a very important part of the total productive effort than was the case years ago. The efficiencies of production function solely depends on the functional reliability of the production facilities, which are nothing but a package or combination of land, building, plants, tools, equipment and services utilized in the plant such as material handling, power plant, water supply and fire-fighting facilities etc. All these facilities are subjected to wear and tear so a proper attention should be given to protect them from undue wear as well as wear and tear by elapse of time or by the frequency of their use. A proper attention means lubrication, cleaning, timely inspection and systematic maintenance. It cannot be said that breakdown will not occur when there is maintenance system in operation. Such a system however, minimizes the costly breakdowns. The word 'maintenance' does not mean repairs but maintenance really means to keep up and not only to repair when it breaks down. The emphasis should be on maintenance rather than on repair. Machinery must be lined and leveled,

wearing surfaces must be examined and replaced and oiling schedules must be laid down at regular intervals. A machine in good operating condition subjected to regular inspection and adjustment will continue to produce quality products for a long time. Thus, it can be said that maintenance is responsible for the smooth and efficient working of industrial plant and helps in improving the productivity. It also helps in keeping the components in a state of maximum efficiency with economy.

3.2.1 Types of maintenance

Maintenance is related to profitability through equipment output and equipment running cost. Maintenance work raises the level of equipment performance and availability, but at the same time it adds to running cost .The objective of an industrial maintenance department should be the achievement of the optimum balance between these effects that is the balance which minimizes the department's contribution to profitability.

Maintenance is required to make sure that the components continue to perform the functions for which they were designed. The basic objectives of the maintenance activity are to deploy the minimum resources required to ensure that components perform their intended functions properly, to ensure system reliability and to recover from breakdowns. In general terms, the equipment which is not well maintained and fails periodically experiences lack of precision and hence, tends to produce defects. More often than not such equipment drives manufacturing processes out of control. A process that is out of control produces defective products and therefore increases the production cost which amounts to less profitability which endangers the survival of the organization (Ben-Daya and Duffuaa 1995).

3.2.2 Planned maintenance

Planned maintenance is a system where repairs are made to maintain a machine before it fails. Scheduled repairs allow operators to feel confident in the safety of their equipment operation, schedule the work crew and resource allocation around the repairs, and order parts in leisure. Breakdown of a machine does not occur in a planned manner, but maintenance work can be planned in well advance. Planned maintenance involves the inspection of all plants and equipment according to pre-determined schedule in order to overhaul, service, lubricate or repair before actual breakdown in service occurs. It is also known as scheduled maintenance or productive maintenance. Planned maintenance includes systems, which bring equipment in for repairs every given time period as preventive maintenance.

3.2.2.1 Preventive maintenance

This concept was introduced in 1951, which is a kind of a physical check up of the equipment to prevent equipment breakdown and prolong equipment service life. Preventive Maintenance (PM) comprises of maintenance activities that are undertaken after a specified period of time or amount of machine use. During this phase, the maintenance function is established and time based maintenance (TBM) activities are generally accepted .This type of maintenance relies on the estimated probability that the equipment will breakdown or experience deterioration in performance in the specified interval. The preventive work undertaken may include equipment lubrication, cleaning, parts replacement, tightening, and adjustment. The production equipment may also be inspected for signs of deterioration during preventive maintenance work. It attempts to prevent any probable failures/breakdowns resulting in production stoppages. It refers to maintenance action performed to keep or retain a machine or asset in a satisfactory operating condition through periodic inspections, lubrications, replacements etc. Preventive maintenance consists of scheduled activities performed to reduce the number of system failures, thus reducing unplanned system downtime. The objectives of PM program includes: minimizing the maintenance cost, minimizing the number of unexpected breakdowns in the system so that financial and safety loses can be restrained. It also increasing the productive life of all equipments and it helps in promoting better safety and health of the work force

Condition based PM is sometimes referred to as predictive maintenance estimate, through diagnostic tools and measurements, when a part is near failure and should be replaced. Advancement in modern sensors and data processing technologies promote the development of such strategies. Time-based PM is effective mainly for deteriorating systems with increasing failure rates and is performed according to the age of the system regardless of its condition.

3.2.2.2 Corrective maintenance

This is a system, introduced in 1957, in which the concept to prevent equipment failures is further expanded to be applied to the improvement of equipment, so that the equipment failure can be eliminated (improving the reliability) and the equipment can be easily maintained (improving equipment maintainability). The primary difference between corrective and preventive maintenance is that a problem must exist before corrective actions are taken. The purpose of corrective maintenance is improving equipment reliability, maintainability, and safety, design weaknesses (material, shapes), existing equipment undergoes structural reform, to reduce deterioration and failures, and to aim at maintenance-free equipment. Maintenance information, obtained from Corrective Maintenance (CM), is useful for maintenance prevention for the next equipment and improvement of existing manufacturing facilities. It is important to form setups to provide the feedback of maintenance information. A maintenance work carried out to restore the equipment/machine to a satisfactory condition after the failure has occurred. Corrective maintenance is required not only when the asset/machine item fails but also when indicated by condition based criteria.

(CM) is performed in reaction to failure and comprises of activities that are required to restore a system to an operating level after a known or suspected failure has occurred. CM can include any or all of the following steps: localization, isolation, disassembly, interchange, reassembly, alignment, and checkout. No activity will be taken as planned or scheduled while the system is still functioning. In this case, the cost of the activity increases when the unplanned system failures increases. CM generally reflects the philosophy "if it isn't broken, don't fix it".

3.2.3 Unplanned Maintenance

It is an operation carried out without any prior planning. Generally, it is very urgent in nature. The unplanned maintenance is emergent in nature in view of the fact that here the recovery time is the most important factor in order to minimize the consequences of serious breakdown.

3.2.3.1 *Opportunistic Maintenance*

If anyone equipment/component fails then the system is under corrective maintenance and in addition to this identify any other component whether it is degraded or not. If it is degraded then we are having the opportunity to carry out maintenance in order to restore the component from degraded state to operative state.

Opportunistic Maintenance (OM), (Nicolai and Dekker 2008) is not a maintenance policy for a single component, but for a collection of components in a production line or plant. Opportunistic maintenance aims to create efficiency for a maintenance crew, by combining various maintenance activities on different plant components. Although the goal of opportunistic maintenance is to reduce maintenance costs, it may also impact plant availability. If maintenance of a component implies plant shutdown, then plant availability may be better served by combining maintenance activities on several components

OM can be defined as a systematic method of collecting, investigating, preplanning, and publishing a set of proposed maintenance tasks and acting on them when there is an unscheduled failure or repair "opportunity". Opportunistic maintenance can be thought of as a modification of the run-to-fail maintenance management philosophy. Generally, there are two main purposes for applying opportunistic maintenance:

- To extend equipment lifetime or at least the mean time to the next failure whose repair may be costly. It is expected that this maintenance policy can reduce the frequency of service interruption and the many undesirable consequences of such interruption, and
- To take advantage of the resources, efforts and time already dedicated to the maintenance of other parts in the system in order to cut cost.

3.2.3.2 Condition Based Maintenance

Condition based maintenance (CBM) is concerned with extracting information from machines to indicate their condition and hence to enable them to be operated and maintained with safety and economy .It is a maintenance program that recommends maintenance actions based on information collected through condition monitoring. CBM attempts to avoid unnecessary maintenance tasks by taking maintenance actions only when there is evidence of abnormal behavior of a physical asset. A CBM program, if properly established and effectively implemented, can reduce maintenance cost by reducing the number of unnecessary scheduled preventive maintenance operations.

CBM is a maintenance policy for equipment components, which is based on monitoring the operating condition of the component .When applicable; CBM allows just-in-time maintenance for the component, because maintenance can be done just before the component fails. CBM is a preventive maintenance policy in which the incipient faults can be identified before their

occurrence. Eliminating sudden failures and performing maintenance as it is needed make this policy very attractive and may justify the initial investment and further use of this policy.

Condition based maintenance is thus to carry out preventive maintenance at some irregular intervals as per the actual condition of the machine. Its main function is to provide the knowledge of machine condition, and the possible are of change which is essential to the operation of this method. The knowledge of the condition of the machine can be obtained by selecting a suitable parameter for measuring deterioration, and recording its values of intervals.

CBM is also becoming popular in power sectors due to its important role in detecting potential modes of failures. The use of CBM technique will help improving plant reliability, availability and reduce downtime cost. It is also noticed that in some cases, use of CBM leads to identify any hidden defect from which random failure may occur. Thus, implementing CBM, defect can be identified and further corrective actions may be taken.

3.2.4 Condition monitoring methods

The aim of condition monitoring is to obtain an indication of condition of machine, to be operated and maintained with safety and economy. The indication of machine condition can be at two levels:

The indication if a problem exists.

- > The definition of what problem it is.
- This is not new in engineers, or plant head, they have always used as per their own senses to obtain a general indication of machine condition. The simple sensual methods followed by engineers in charge are:
- Sight: Leaks, smoke, indicating overheating.
- Smell: Leaks, overheating.
- Hearing: Abnormal noise, indicating some malfunction.
- Feel: Abnormal vibrations, indicating some malfunctions.

So, these sensual feelings gave them idea for the condition of the machine. But, the main drawback is there may be error include in personal opinion. For the elimination of these errors, a better solution is to be found for use of simple instrumentation to give numerical readings. The obtained values are comparable to the data from the machine manufactures for normal operation and also to the previous values obtained.

Simple instrumentation is usually fitted to most of the machines, and they give numerical indication of the existence of the problem in the machine. Few examples of simple instrumentation are pressure gauges, temperature indicators, tachometers and ammeters. Others sensors like warning lights and emergency alarms set to operate at certain values can also indicate that a problem exists. The recognition of the value of machine condition monitoring has resulted in the development of special instrumentation which can give precise and more accurate indication of existence of problem in a machine. In many cases, the instrumentation often used with appropriate techniques can give a clear definition of what is the actual problem within the machine. Few examples of special instruments, which can detect the existence of problem in machines, are:

- Vibration measuring instruments attached to the machine casing.
- Thermographic cameras to give thermal map of machine casing
- Shaft position indicators giving static and dynamic, radial and axial movements.

In spite of large number of techniques and instrumentations, there are only few basic methods of condition monitoring that are available for condition monitoring, these are mentioned in the following subsections.

3.2.4.1 Visual monitoring

Machine components are visually examined to determine their condition. Visual inspection of components is one of the simplest and most familiar methods of assessing their condition. Its advantage is that the inspecting engineer receives an immediate and direct indication of the component condition without the need for any processing results. But, having a disadvantage that only stationary component and other components which can be seen clearly are inspected. Thus, visual inspection has a limited scope as invisible failures are not be detectable by using this technique.

3.2.4.2 Performance monitoring

How well the component is working gives idea for the performance monitoring. Monitoring the performance of machines and the behavior of components, to discover how well they are performing their intended functions is one of the most direct methods of monitoring their condition.

Advantages of performance monitoring

- Provided the measurements taken are accurate, any major deviations in performance from standard values must be relevant to the existence of some real problem and spurious readings are hardly possible.
- Performance monitoring can give a direct indication of machine efficiency as well as some indication of impending failure. It can therefore make a direct contribution to effect machine operation, by guiding the selection of operating conditions. Also, it can often be readily integrated into machine systems.

Disadvantages of performance monitoring

- Monitoring a complete machine in terms of its performance tends to be a rather insensitive method of detecting incipient machine failure, since many serious components defects become quite advanced before they produce a measureable effect on machine performance.
- To obtain measurements, the machine usually has to run at standard conditions. This may be inconvenient or give rise to errors.

3.2.4.3 Component behavior monitoring

The majority of component behavior monitoring can best be carried out by visual monitoring, vibration monitoring and wear debris monitoring. However, there are few other techniques used for recognizing the essential function of the component, and then monitoring it to see how effectively it is carried out this function. The function of component like seals (static and dynamic) is to prevent leakage. The abnormal behavior that is the leakage can be easily detected by smell, visual inspection, sniffing and noise etc. The intended function of bearings is to locate shafts as required, but with minimum of resistance to rotation. Abnormality measurement of shaft location is done by proximity transducers. In components like brakes or clutches, periodic wear measurement is used and also temperature measurement to detect general overheating.

3.2.4.4 *Vibration monitoring*

Amount and the nature of vibration produced by the machine or component gives the idea of vibration monitoring. Vibrations are much more easily applied to rotating machinery than to reciprocating machinery. In rotating components, such as motors, fans and pumps, vibration analysis is capable of detecting failure more specifically by frequency distribution. All rotating equipment produces ultrasonic or acoustic vibration regardless of the state of lubrication (Smith,

1989). Vibration analysis is used for monitoring the failure behavior of rotating equipment such as the wheels and bearings of the gearbox, generator bearings, main shaft and bearings of the turbine. The nature and magnitude of vibrations of a particular item of machinery can provide valuable information about its mechanical condition. Vibrations arise from cyclic excitation forces within the machine. These forces can be inherent in the design of the machine, or can be due to propagation of some defect. Since no machine structure is perfectly rigid, components of machine will flex and move cyclically in response to these excitation forces, machine noise is generated by the action of some vibrating or cyclically impacting component causing compression waves to be set up and transmitted through the surrounding media, which can either solid, liquid or gas. These compression waves when experienced by human ear result in the phenomenon called sound, which can be detected using acoustic apparatus such as microphone. Thus, vibration and noise both originate from the same source and both can contain valuable information about the condition of machine. However, it is generally agreed that for condition monitoring applications, vibration measurements are more reliable and decipher than noise measurements.

The principle of vibration monitoring to detect incipient faults as illustrated in Figure 3.2. Vibration monitoring involves using sensors. The sensors employed depend on the frequency range of the equipment to be monitored. Low frequency range equipment requires position transducers, middle frequencies require velocity sensors and high frequency requires accelerometers. Appropriate vibration sensors are mounted rigidly on the components to register the local motion.



Figure 3.2 Fault detection mode

Using this technique it is simple to successfully identify the current status of machines and many upcoming troubles by predicting the condition of machines with industrial equipment and especially with rotating machinery. There are three types of vibration transducers which are installed on the appropriate location of the machine to measure different types of vibration parameters (displacement, velocity and acceleration). As shown in the Table 3.1, each type of transducers is responded to different vibration parameters.

Table 3.1

Name	Sensitive to the parameters
Proximity Probe	Displacement
Velocity Probe	Velocity
Accelerometer	Acceleration

Types of transducers with their measuring parameters.

3.2.4.5 Wear debris monitoring

The machine surface if producing any debris gives idea for wear debris monitoring. Analysis of debris is done by lubricating oil. Wear debris monitoring involves the monitoring of machine lubricants for the presence of wear debris. This debris is generated by wear processes at the

relatively moving surfaces of load carrying machine components such as bearings, gears, pistons etc. by monitoring the quantity an nature of wear debris, it is possible to obtain an indication of the condition of the various machine components which are in contact with the lubricant. For many failures of lubricated surfaces which are progressive in nature, rather than sudden, the method also enables a useful advanced warning of failure to be obtained. The components like rolling bearings, gear teeth; cams and tappets etc. are usually wear out due to ferrous particles of various shapes and sizes, but mainly because of chunky particles around 1mm in size arising from surface fatigue. Piston rings, cylinders and splines are failed because of ferrous flakes less than 150 μ m acoss; and fine iron or iron oxide particles. Bolted and riveted joints can be failed due to fretting debris consisting of fine iron oxide particles. The methods of wear debris can be direct detection methods, debris collection method or lubricant sample analysis. In direct detection methods wear debris in the lubricant is detected in the machine by arranging for the oil to flow through a device which is sensitive to the presence of debris. Debris collection methods are associated with collection of the wear debris in a device, fitted to the machine which is convenient to remove, so that the debris can be extracted for examination. While in lubricant sample analysis, a sample of lubricant is extracted from the machine and analysed for wear debris contamination. These methods are normally used to monitor the condition of the components lubricated by circulatory oil system.

3.2.4.6 Oil analysis

Oil analysis includes fluid property analysis (fluid viscosity, additive level, oxidation properties and specific gravity), fluid contamination analysis (moisture, metallic particles, coolant and air) and wear debris analysis. Fluid property and contamination analysis is used to analysis the quality of oil so that we can change the oil when it is necessary. These analyses indicate the condition of the oil itself .Wear debris analysis is becoming a common use of condition monitoring techniques to give insights into equipment's operating condition (health) by analyzing the content of debris in the lubrication and hydraulic oil samples. The analysis is based on understanding wear particle types, wear particle size, wear particle shape and concentration of wear particles in the lubrication oil. A procedure of oil analysis is shown on Figure 3.3 below.



Figure 3.3 Procedure of oil analysis.

3.2.4.7 *Temperature Monitoring*

Temperature monitoring techniques are mostly used in different high temperature and speed environment, usually heavy industries and industrial purpose tools. Temperature sensor consists of two metals which are joined together, and when exposed to some temperature difference generates a thermoelectric voltage which is dependent on the specific material properties and temperature difference. Bearing related defects generate some excessive heat in the rotating apparatus of machines. Continuous monitoring of temperature difference of a housing, casing or lubricant is one of the effortless methods for fault identification into machines by temperature analysis. Machinery defects generate excessive heat which results in deviation from the allowable temperature range into rotating components. Monitoring temperature of bearing housing is not very efficient for huge industrial machinery scenario as small deviation into a temperature becomes quite hard to detect.

Infrared thermography is a two dimensional real time condition monitoring techniques used to assess the operating condition of a machine by measuring the relative temperature or intensity of emitted radiation (measured as a form of temperature). As we see on Figure 3.4, the infrared energy emitted from the object is received by a non-contact remote device and converted it to electrical signal. This signal is amplified and further processed into a visible image and displayed on the monitor as a thermograph. Then after, the temperature calculation will be performed. Thermography is often used for monitoring electronic and electric components and identifying failure. The technique is only applied off-line, and often involves visual interpretation of hot spots that arise due to bad contact or a system fault. At present the technique is not particularly well-established for online CM, but cameras and diagnostic software that are suitable for on-line process monitoring are starting to become available. Infrared cameras have been used to visualize variations in blade surface temperature and can effectively indicate cracks as well as places threatened by damage. In the longer term, this might be applicable to generators and power electronics too. Pulsed thermography can be employed for the structural evaluation of blades, but due to the bulky equipment involved this is not a standard methodology amongst turbine operators.

Thermographs are used to analyze patterns of heat gain or loss. Infrared analysis is an effective predictive maintenance tool, because mechanical or electrical breakdowns are often preceded or

accompanied by changes in operating temperatures. This information can be particularly important in electrical machinery where circuits and connections may show no visible signs of deterioration before a failure. Condition monitoring of the equipment plays a vital role in up keeping the health and availability in delivering the quality product. Infrared thermography is an essential tool to assess the healthiness of the machine from time to time in process plants and enhances reliability of the plant.



Figure 3.4 Thermography condition monitoring technique.

3.2.4.8 Ultrasonic monitoring

Ultrasonic condition monitoring techniques is a simple method used to detect mechanical equipment failures, such as bearing failure, lack of lubrication, over lubrication, early wear, leakage, etc. by translating a high frequency machine sound to a low frequency signal (Bandes, 2009). Therefore, operators or inspectors can hear the sound of equipment defects .Once ultrasonic signal from the machine during its normal operating condition is registered as a benchmark the future reading will be compared to this value to determine the status of the machine. Using advanced digital technology, the operator can read the ultrasonic level meter

level overtime or recording the signal which is received from ultrasonic receiver sensors installed on the machine and analyzes it with specialized software.

3.2.5 Selecting methods for monitoring

A machine fails because of the failure of one or more critical components. Consequently the most sensitive methods of monitoring machine condition work by detecting the symptoms of single component failure, since greatest degree of deviation from the normal conditions shall be concentrated in these symptoms. A measurement is needed which is particularly sensitive to machine condition in order to:

- give the longest lead time to failure
- compensate for the effect of variation between successive readings, that is typical of any experimental measurement.
- reduce the effect of any external factors which might interfere with the operation of the monitoring method

The selection of an appropriate method for monitoring the condition of machine must therefore be based on a consideration of which of its components are likely to fail, and in what way. Possible methods for monitoring these components failure can be considered, and in many cases it will be it will be possible to select a single method which can detect all the possible failures in a machine with the acceptable efficiency. Any method which monitors the machine as a whole, such as performance measurement, can potentially detect any import a problem, but tends to be relatively insensitive in terms of:

- lead time failure
- the definition of the exact nature of the problem.

Overall there are scope of matching methods for the monitoring of machines and components. In a machine there are stationary components as well as moving components. In any machine changes can be detected by its performance measurement, overall levels of vibration and noise. Further detects in stationary parts and moving parts can also be detected with the help of latest development in the monitoring system.

- **Stationary components**: Stationary components are those which are either permanently fixed, or monitored only during the period at rest. Various methods are used for finding the defects in stationary components such as Boroscopes, Radiography, Acoustic emission, Resonance change, Strain gauges, Brittle coats and crack detection etc.
- Moving components: Reciprocating and rotating components are known to be the moving components. Defects can be detected by analyzing the vibration and noise produced by the machine. Following are the possible general monitoring method which can be applied to various machine components.

Table 3.2

Component to be monitored		General monitoring method			
	-	Visual	Behaviour	Vibration	Wear debris
Station	nary components				
1.	Casings	Yes	Yes	No	No
2.	Mountings and	Yes	No	Yes	No
	foundations				
3.	Tanks and containers	Yes	Yes	No	No
4.	Pressure vessels	Yes	Yes	No	No
5.	Heat exchangers	Yes	Yes	No	No
6.	Stator blades	Yes	No	No	No
7.	Screens and	Yes	Yes	No	No
	separators.				
Rotati	ng components				
1.	Shafts	No	No	Yes	Yes
2.	Machine rotors	No	No	Yes	No
3.	Turbine blades	Yes	Yes	Yes	No
4.	Impellors and propellors	Yes	Yes	Yes	No
5.	Wheels	Yes	No	Yes	No
6.	Gears	Yes	No	Yes	Yes
7.	Chain drives	No	No	Yes	Yes
8.	Flexible couplings	Yes	No	Yes	Yes
9.	Pulleys and belts	Yes	No	Yes	No
	. Governors	No	Yes	No	Yes
Recipr	ocating components				
1.	Pistons	No	No	No	Yes
2.	Linkages and levers	Yes	No	No	Yes
3.	Cams and tappets	No	No	Yes	Yes
4.	Valves	No	Yes	Yes	Yes
5.	Cables and chains	Yes	No	No	Yes

Monitoring methods for various components.

6.	Bellows	Yes	Yes	No	No
7.	Diaphragms	Yes	Yes	No	No
8.	Springs	Yes	No	Yes	Yes
	Guides and slides	Yes	No	Yes	Yes
	. Splines	Yes	No	No	Yes
10	- ~Piilies		1.0	1.0	
Frictio	on components				
	Brakes	Yes	Yes	No	No
2.	Clutches	Yes	Yes	No	Yes
3.	Vibration dampers	No	Yes	yes	Yes
Bearin					
1.	Plain	No	Yes	Yes	Yes
2.	Rolling	No	No	Yes	Yes
	Flexure	Yes	Yes	No	No
Seals					
1.	Lip	No	Yes	No	No
2.	Mechanical	No	Yes	No	No
3.	Packed glands	No	Yes	No	No
4.	Windback	No	Yes	No	Yes
5.	Labyrinth	No	Yes	Yes	Yes
6.	Piston ring	No	Yes	No	Yes
Wear 1	resistant surfaces				
	Hard	Yes	No	No	No
	Elastic	Yes	No	No	No
			110	1.0	1.0
Manuf	facturing tools				
1.	Cutting tools	Yes	Yes	No	No
2.	Metal working tools	Yes	Yes	No	No
3.	Casting and	Yes	Yes	No	No
	moulding dies				
Worki	ng fluids				
	Hydraulic	Yes	No	No	No
2.	•	Yes	Yes	No	No
	transfer				
3.	Lubricants	Yes	No	No	No
					I

CHAPTER 4

AVAILABILITY ASSESSMENT APPROACHES

Availability is an important metric used to assess the performance of repairable systems, accounting for both the reliability and maintainability properties of a component or system. In this chapter reliability and availability modeling and analysis are described. The conventional techniques such as Reliability block diagram and Faulty tree diagram with their limitations are discussed. Finally, the modern technique Markov approach is explained.

4.1 Conventional techniques for reliability and availability modeling

4.1.1 Reliability block diagrams

A reliability block diagram is a diagrammatic method for showing how component reliability contributes to the success or failure of a complex system. RBD is also known as dependence diagram (DD).

An RBD is drawn as a series of blocks connected in parallel or series configuration. Each block represents a component of the system with a failure rate in parallel or series configuration Parallel paths are redundant, meaning that all of the parallel paths must fail for the parallel network to fail. By contrast, any failure along a series path causes the entire series path to fail. RBD of two components in series is shown in Figure 4.1



Figure 4.1 RBD of two components in series.

4.1.2 Fault tree analysis

Fault tree diagrams are logic block diagrams that display the state of a system (top event) in terms of the states of its components (basic events). FTA is used to find the root-causes of an undesirable event in order to determine a solution. It can also be used to estimate consequences

and probabilities of occurrence of a top-event (Huggett *et al.* 2003). Like reliability block diagrams (RBDs), fault tree diagrams are a graphical design technique, and as such provide an alternative methodology to RBDs. An FTD is built top-down and in term of events rather than blocks. It uses a graphic "model" of the pathways within a system that can lead to a foreseeable, undesirable loss event (or a failure). The pathways connect contributory events and conditions, using standard logic symbols (AND, OR, etc). The basic constructs in a fault tree diagram are gates and events, where the events have an identical meaning as a block in an RBD and the gates are the conditions.

Fault trees have traditionally been used to analyze fixed probabilities (i.e., each event that composes the tree has a fixed probability of occurring) while RBDs may include time-varying distributions for the blocks' success or failure, as well as for other properties such as repair/restoration distributions.

Fault trees are built using gates and events (blocks). The two most commonly used gates in a fault tree are the AND and OR gates. As an example, consider two events (called input events) that can lead to another event (called the output events). If the occurrence of either input event causes the output event to occur, then these input events are connected using an OR gate. Alternatively, if both input events must occur in order for the output event to occur, then they are connected by an AND gate. As a visualization example, consider the simple case of a system composed of two components, A and B, where a failure of either component causes system failure. Suppose A and B are connected in series as shown in Figure 4.2, failure of either component causes system failure.



The fault tree diagram for this system includes two input events connected to an OR gate which is the output event or the "top event". If the top event is system failure and the two input events are component failures, then this fault tree indicates that the failure of A or B causes the system to fail.



Figure 4.3 OR gate connected in series system.

4.1.3 Limitations of RBD and FTD

Techniques like RBD and FTD are simple in nature but are static. Complex systems incorporating repair sequences and non-exponential probability distributions cannot be realistically solved with these techniques.

Markov approach is advancement to such techniques as it provides the capability to introduce repair in the system. Markov approach encompasses mainly two concepts. The "state" of the system and the "transitions" in the system from operating to non-operating and vice versa.

4.2 Markov approach

A Markov model needs identification of possible states of the system, their transition paths, and the rate parameters of the transitions. Each state represents the different condition of the system. The transition from one state to another state occurs, with failure and repair rate exponentially distributed. It is a widely used technique for many applications, including evaluation of reliability and availability performance. The work related to application of Markov approach is reported in the following paragraphs.

Sahner and Trivedi (1986) proposed hierarchical modeling using the Markov approach for a complex system to deal with the problem of state space explosion. The authors suggested a mechanism for decomposition and aggregation based on functional similarity. The proposed approach allows for both combinatorial and Markov models, and can analyze each model to produce a distribution function. Kim and Park (1994) proposed system reliability based on Markov model for a phased mission. Pukite and Pukite (1998) in their book presented various modeling and analysis techniques for reliability, maintainability, availability, safety and supportability of complex computer systems that included sub-classes of Markovian approaches, Petri net, Monte Carlo simulation. The authors also listed advantages and limitations of each modeling technique, with special emphasis on Markov modeling. Xie *et al.* (2000) investigated the use of exponential distribution as an approximation to weibull distribution for reliability and maintainability studies. The proposed framework addressed optimal maintenance in respect of time and spare allocation.

Vulpe and Carausu (2004) proposed stochastic evaluation of the availability of subsystems by Markov and Semi-Markov models. Ajah *et al.* (2006) introduced hierarchical Markov based reliability and availability modeling for energy and industrial systems. The authors suggested decomposition of the reliability/availability problems in three levels (components, units and system) and aggregation based on functional and structural similarities. The proposed methodology reduced the problem of state space explosion problem for large systems. Carter and Malerich (2007) studied impact of the exponential repair assumption on reliability assessment. The authors observed that the exponential repair assumption inflated system reliability. Guo and Yang (2008) presented a methodology for the automatic creation of Markov models for reliability evaluation of safety instrumented systems. Andrews (2009) reviewed the state-of-the-art techniques, including the Markov approach for system reliability evaluation. The author also discussed the likely applications in the context of the recent advances in the assessment techniques.

Welte (2009) presented an approach, with gamma distribution transformed to a Markov Process (MP), with sequence of states having exponentially distributed sojourn times. The approximation of the gamma distribution into exponential distribution yielded good results. Some of the recent work of the researchers on reliability and availability modeling considered features such as imperfect repairs, common cause failure, human error, etc. and used Markov approach, which included Hajeeh and Jabsheh (2009).

For any system, a Markov model consists of a list of the possible states of that system, the possible transition paths between those states, and the rate parameters of those transitions. In reliability analysis the transitions usually consist of failures and repairs. When representing a Markov model graphically, each state is usually depicted as a "bubble", with arrows denoting the transition paths between states. For example in single component that has just two states: healthy and failed as shown in Figure 4.4.



Figure 4.4 Two states healthy and failed.

The symbol λ denotes the rate parameter of the transition from State 1 to State 2. In addition, we denote by $P_j(t)$ the probability of the system being in State j at time t. If the device is known to be healthy at some initial time t = 0, the initial probabilities of the two states are $P_1(0) = 1$ and $P_2(0) = 0$. Thereafter the probability of State 1 decreases at the constant rate λ , which means that if the system is in State 1 at any given time, the probability of making the transition to State 2 during the next increment of time dt is λdt . Therefore, the overall probability that the transition from State 1 to State 2 will occur during a specific incremental interval of time dt is given by multiplying (1) the probability of being in State 1 at the beginning of that interval, and (2) the probability of the transition during an interval dt given that it was in State 1 at the beginning of that increment. This represents the incremental change dP₁ in probability of State 1 at any given time, so we have the fundamental relation:

$$dP_1 = -(P_1)(\lambda dt)$$

Dividing both sides by dt we get

$$dP_1/dt = -\lambda P_1$$

This signifies that a transition path from a given state to any other state reduces the probability of the source state at a rate equal to the transition rate parameter λ multiplied by the current probability of the state. Now, since the total probability of both states must equal 1, it follows that the probability of State 2 must increase at the same rate that the probability of State 1 is decreasing. Thus the equations for this simple model are

$$dP_1/dt = -\lambda P_1$$

$$dP_2/dt = \lambda P_1$$

Also $P_1 + P_2 = 1$

The solution of these equations, with the initial conditions $P_1(0) = 1$ and $P_2(0) = 0$, is

$$P_1(t) = e^{-\lambda t}$$
 and $P_2(t) = 1 - e^{-\lambda t}$

The form of this solution explains why transitions with constant rates are sometimes called "exponential transitions", because the transition times are exponentially distributed. Also, it's clear that the total probability of all the states is conserved. Probability simply "flows" from one state to another.

A two unit system is taken here for understanding, in which the components are added as a parallel system as shown in Figure 4.5. This system will do intended function unless unit one or unit is working.



Figure 4.5 RBD of parallel system

This system can be in one of four possible states.

- Both units healthy (state 0)
- Unit 1 failed but unit 2 healthy (state 1)
- Unit 2 failed but unit 1 healthy (state 2)
- Both units failed (state 3)

This two unit model can be depicted as in Figure 4.6



Figure 4.6 State diagram for two parallel system.

On the basis of the above figure the equations are as:

 $(\lambda_1 + \lambda_2)P_0 = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3$ $\lambda_1 P_0 = (\lambda_2 + \mu_1)P_1$ $\lambda_2 P_0 = (\lambda_1 + \mu_2)P_2$ $\lambda_2 P_1 + \lambda_1 P_2 = \mu_3 P_3$

These differential equations can be solved by Ranga-Kutta using MATLAB and the availability of the system is obtained by adding the operating state.

Markov models offer significant advantages over other reliability modeling techniques, some of these advantages are:

- Simplistic modeling approach: the models are simple to generate although they do require a more complicated mathematical approach.
- Redundancy management techniques: system reconfiguration required by failure is easily incorporated in the model.

- Coverage: covered and uncovered failures of components are mutually exclusive events.
 These are not easily modeled using classical techniques, but are readily handled by the Markov mathematics.
- Complex systems: many simplifying techniques exist which allow the modeling of complex systems.

4.3 Maintenance optimization

Maintenance optimization is a process which attempts to balance maintenance requirements and the resources used to carry out maintenance program. A maintenance strategy which seems to be optimal and appropriate now may not be optimal in the future due to desultory nature of input variables like failure behavior, components cost and interest rate etc. As maintenance optimization is a continuous process rather than one off procedure so requires periodic evaluation of performance to improve the past techniques. Basically, there are two approaches to maintenance optimization that are qualitative and quantitative.

Arthur (2005) and Scarf (1997) observed that qualitative maintenance optimization is often clouded with subjective opinion and experience, and further suggest the utilization of quantitative methods to optimize the maintenance activities of physical assets. Quantitative maintenance optimization (QMO) techniques employ a mathematical model in which both the cost and benefits of maintenance are quantified and an optimum balance between both is obtained (Dekker, 1996). There are a number of QMO techniques in the field of applied mathematics and operational research such as Markov Chains and Analytical Hierarchy Processes (AHP) (Chiang *et al.* 2001).

The main purpose of maintenance optimization is to determine the most cost effective maintenance strategy. This strategy should provide the best possible balance between direct maintenance costs (labour, materials, administration) and the consequences or penalty of not performing maintenance as required (i.e. labour, materials, administration, loss of production and anticipated profit, etc) without prejudice to Health, Safety and Environmental factors. Evidently, carrying out maintenance activities such as inspection, preventative maintenance, and replacement of components more frequently, increases the direct cost of maintenance. Thus, the risk exposure or the consequences of not performing maintenance activities as required, reduces. There are two common approaches to maintenance optimization; qualitative and quantitative.

4.3.1 Quantitative Maintenance Optimization

Quantitative maintenance optimization (QMO) techniques employ a mathematical model in which both costs and benefits of maintenance are quantified and an optimum balance between both is obtained (Dekker, 1996). There are a number of QMO techniques in the field of Applied Mathematics and Operational Research, for example, Markov Chains and Analytical hierarchy processes (Chiang and Yuan, 2001); Genetic Algorithms (Tsai et al. 2001), etc. Furthermore, Arthur (2005) observed that quantitative maintenance optimization can be clouded through the rigorous data demands of mathematical modeling and these same models require data that is often unavailable.

4.3.2 Bayesian approach

A fully Bayesian i.e. subjective approach towards straightforward means of presenting uncertainty related to future events to decision makers in the context of an inspection maintenance decision problem has also been optimally discussed (Apeland and Scarf, 2003). This approach is in contrast with the classical probabilistic approach that assumes the existence of true probabilities and probability distributions.

4.3.3 *Mixed integer linear programming*

Goel et al. (2003) presented a new mathematical model i.e. Mixed Integer Linear Programming (MILP) for the integrated design, production and maintenance planning for a multi-process plant. A reliability allocation model at the design stage is coupled with the existing optimization framework to identify the optimal size and initial reliability for each unit of equipment at the design stage. In contrast to earlier approaches, which focus mainly on deriving an effective maintenance policy at the operational stage, the proposed integrated approach provides a designer with an opportunity to improve the operational availability at the design stage itself.

4.3.4 Maintenance approach using fuzzy multiple criteria and linguistic approaches

Al-Najjar and Alsyouf (2003) assess and select most informative (efficient) maintenance approach using fuzzy MCDM evaluation methodology. Triantaphyllou *et al.* (1997) earlier reported similar approach. Mechefske and Wang (2003, 2001) have used a fuzzy linguistic approach to achieve subjective assessments of maintenance strategies and practices in an objective manner. Swanson (2003) has applied Galbraith's information processing model to study how the maintenance function applies different strategies to cope with the environmental complexity. Pieri et al. (2002) have presented a knowledge-based decision support system, Materially per Apparecchiature de Impiariti Chemiei for maintenance of a chemical plant.

4.3.5 Simulation and Markovian probabilistic models

Chen and Popova (2002) and Barata et al. (2002) use Monte Carlo simulation to determine optimum maintenance policy (i.e. minimizing total service cost) and for modeling of continuously monitored deteriorating systems. A simulation model (Sarker and Haque, 2000) has also been developed to reduce maintenance and inventory costs for a manufacturing system with stochastic item failure, replacement and order lead times. Balakrishnan (1992) demonstrates application of simulation models to evaluate maintenance policies (i.e. selected out of opportunistic, failure and block) for an automated production line in a steel rolling mill.

4.3.6 Golden section search

Keifer (1953) recommend the golden section search for one dimensional search for one dimensional search. The golden section search is a technique for finding the extremum (minimum or maximum) of a strictly unimodal function by successively narrowing the range of values inside which the extremum is known to exist. The golden ratio (φ) is a constant whose value is given by the expression.

$$\varphi = \frac{\sqrt{5} - 1}{2} = 0.618033988\dots$$

The golden section search is based on the following algorithm.

Step 1: Determine the initial end point x_{low} and x_{up} with a preliminary analysis, such that the maxima lie between low and up.

Step 2: Determine two intermediate points x_1 and x_2 such that; $x_1 = x_{low} + (x_{up} - x_{low})^* \phi$ and $x_2 = x_{up} - (x_{up} - x_{low})^* \phi$.

Step 3: If availability at x1 is greater than availability at x₂ then; $x_{low} = x_2$, $x_2 = x_1$ and $x_1 = x_{low} + (x_{up} - x_{low})^* \varphi$.

Step 4: If availability at x_2 is greater than availability at x_1 then; $x_{up} = x_1$, $x_1 = x_2$ and $x_2 = x_{up} - (x_{up} - x_{low})^* \phi$.

Step 5: Repeat Step 3 and Step 4 till desired level of accuracy of availability is obtained.

CHAPTER 5

SYSTEM MODELLING

5.1 System description

Turbine blade is the component which is taken under consideration for the study. Blades of steam turbine are very critical components in power plants. The blade converts linear motion into rotary motion of the turbine. A turbine blade is the individual component which makes up the turbine section of a gas turbine. The blades are responsible for extracting energy from the high temperature, high pressure gas produced by the combustor. The turbine blades are often the limiting component of gas turbines.

To survive in the difficult environment, turbine blades often use exotic materials like super alloys and many different methods of cooling, such as internal air channels, boundary layer cooling, and thermal barrier coatings. The blade fatigue failure is one of the major sources of outages in any steam turbines and gas turbines which are due to high dynamic stresses caused by blade vibration and resonance within the operating range of machinery. Turbine blades are subjected to very strenuous environments inside a gas turbine. They face high temperatures, high stresses, and a potential environment of high vibration. If blades of turbine fail, eventually the power plant will shut down. This will cause long time current failure and economic loss. Therefore, it became necessary to reduce the failure chances of blades so that reliability and hence availability can be increased.

Blade failure may initiate due to number of known phenomenon which having the ability to develop a situation that can make the unit shut. The most common mechanism which can cause failure are free or forced vibrations which are caused by transient operating conditions and also, because of transported and accumulated corrosive ions during working. Therefore its condition based on vibration levels gives the idea to consider repairs actions. Various sensors are used to get the values of blade degradation based on vibrations.

Weibull analysis is appropriate for modeling component life time. Life time estimates for the blade are made by reliability experts from reliability experts (Weber 1996) using the experience and the data available in gas turbine industry. Gas turbine blades have shape factor in between 0.9 to 2.7 depending upon the mode of failure. The characteristic life varies between 10,000 hours and 160,000 hours depending upon stress induced in their very high temperature environment.

5.2 Modeling aspects

In this section various aspects of system states are described.

5.2.1 System degradation

When the important parameters of a system gradually worsen if left unattended, the process leads to deterioration failure. For example, wear, dimensional changes with time, effect of contamination, and deleterious environmental factors such as; temperature, voltage, and radiations. CBM can be initiated according to the state of a degrading system, that is monitored directly or indirectly through vibration, temperature, fluid particulate, or any other characteristic measure, that describes the system state. Once the system degradation characteristic crosses a specified failure threshold, the maintenance activity is initiated. CBM updates the knowledge of the system failure time, and provides a means to determine inspection and maintenance activities as needed (Barabady 2005).

For most of the systems, it is possible to measure the degradation by sensing such as vibration, power, performance, etc. Degradation modeling is a critical and challenging aspect of the implementation of a CBM program. Typically, degradation model measurements traverse upward or downward toward a failure threshold, and the system is considered failed at the time

when the measurement crosses a predetermined failure threshold. The failure mechanisms for the system must be understood so that an appropriate degradation model is developed and employed in practice. Typically, degradation phenomena are characterized by a linear, convex or concave degradation path. Markov chains represent a discrete time and discrete state stochastic process, and are utilized to describe state transitions mathematically. Markov chain methods are applied to CBM policies to establish the relationship between state transitions for a degradation process. The degradation phenomenon is defined according to discrete states, modeled with Markov chains. To utilize Markov methods, multiple states have to be identified, which can be challenging to define in practice and arbitrary in many instances. Along with the state definitions, the Markov methods require transition probabilities between states, difficult to be determined in practice. Incorrectly or arbitrary defining states and transition probabilities can negate the value of the maintenance policy.

Degradation is not sudden but gradual. Whenever a system is in working it is always degrading with time. Firstly, the system is in new state then with time it starts degrading. The system with gradual degradation is considered. In addition, random failure is also considered which will stop the working of the system and hence can cause a major break down. The repair rates are defined as per the condition of the system.

At state N_1 the component is new. After some time it starts to degrade and go to degraded stage O_1 and finally to O_3 . At stage O_3 major repair is applied which bring the system/ component to second stage O_1 or third stage O_2 by applying imperfect repair. Finally, the system goes to F that is failure state. No repair action is allowed at this stage as the system is too old and replacement is a better option.



Figure 5.1 Component degradation.

5.2.2 Inspection

To check the system's health and maintain it so that it works inspection is necessary. After inspection decision for what kind of repair action is required to make the system work and when system is working properly no need to apply any maintenance action or repair. Inspection is of two types:

- Online inspection
- Offline inspection.

In online inspection the system is in running mode and hence availability is more as no breakdown occur. In offline inspection the system is stopped to inspect it that adds downtime. In the current system model offline inspection is considered. Refer, Figure 5.2, the system state during inspection is designated as I.



Figure 5.2 Model with inspection states.

5.2.3 Incorporate repair

The decision whether to undertake maintenance or maintain the status quo, is taken based on the condition of the system. In case the system health is normal and does not show deterioration from the CM metric check, the decision is 'no repair' and the system is allowed to operate. Otherwise, an appropriate maintenance action such as: minimal, intermediate or major repair is carried out to bring back the system to its current, previous better or the initial state respectively. In this system model, Figure 5.5, minimal repair is considered for all the degraded states, i.e n_1 , n_2 and n_4 .while major repair is considered for all degraded states except the first degraded state and intermediate repair is considered at all degraded states, except the first and second degraded state. Also, the third degraded state onwards, the number of imperfect states at each subsequent state increases because the imperfect repair can lead to a better degraded state i.e., in between the initial state and current degraded state. This means, it may be more than one state. It is well known that a minimal repair takes, in general, the least time, while the imperfect repair takes more time than the minimal repair, but a lesser time than a major repair. Moreover, the time of repair will be higher, if the repair is carried out at a highly degraded state. The lognormal distribution, being more appropriate for repair of mechanical systems, is employed in the model (Dennis, 2003). However, the distribution parameter value will be decided based on plant experience and in consultation with the vendor. After applying inspection if any fault is there we take repair action as per the condition of the component. In our model we have taken three kinds of repairs namely minor repair, Intermediate repair and major repair. On the basis of condition we apply the repairs as the component at stage N₁ needs no major repair as its probability of failure is low so minor repair is considered. Further at some degraded stage O_1 probability of failure slightly increases so depending repair action is to be considered, minor repair if stage O₁ is to be achieved and major repair if stage N_1 is to be achieved. Further moving on to some more

degraded stage O_2 any of three repair action can be considered depending on what stage should be achieved and lastly at stage O3 probability of failure increases rapidly so decision should be made about intermediate or higher repair. Although at this stage new or N_1 stage is difficult to achieve even if higher repair action is considered. Considering all these aspects a system model is developed. Here N_1 is a new component. With time it degrades to O_1 , O_2 , O_3 and finally failure. I is the inspection we are going to do in states N_1 , O_1 , O_2 and O_3 and finally failure where no need for inspection. After inspection we see that how that particular state is affected and what kind of repair does it need to carry on working. Repair rates which are considered are:

- n_i is nominal repair
- H_i is major repair
- a_i is intermediate repair

Fr is random failure as a component can be fail at any stage due to some manufacturing defects, human error or even natural disaster. Depending on offline or online inspection the values can vary. In this system we are following offline inspection so, I is taken as inspection in the system at each degraded stage

5.2.4 Random failure

Random failure is defined as the condition in which the system fails due to some random causes. These random causes can be anything from natural calamity to human error. Random failures can also occur due to voltage fluctuations, manufacturing defects, problem in system components, etc. States associated with random failure are shown in Figure 5.3.



Figure 5.3 Random failure.

On the basis of these assumptions system model is developed refer Figure 5.4. In this system model, three types of maintenance are: minor maintenance, maintenance on the basis of the requirement intermediate maintenance and major maintenance.

In stage N_1 , our system is new, thus maintenance is not much needed. Therefore we have kept the probability for our system to undergo minor maintenance to be 0.2 and the probability that system would go back to the stage N_1 without any maintenance to be 0.8.

Similarly in stage N_2 as our system is in continuous working state, it deteriorates and thus its efficiency decreases and the need to repair it or maintain it increases as compared to the system in stage N_1 . Due to this reason we have decreased the probability that the system would go back to stage O_2 without any repair from 0.8 to 0.6 and the probability that the system would require maintenance has been increased from 0.2 to 0.4.

Now, when our system moves from stage O_2 to O_3 , it deteriorates further giving rise to the need to repair it in order to increase its availability. Therefore the probability is that system requires minor repair or intermediate repair or major repair or no repair has been altered again. The probability that the system would require minor, major or intermediate repair is 0.2 and the probability that the system would go back to stage O_3 without any repair has been changed to 0.4. Finally the last stage before failure is O_4 , deterioration level at this stage is high therefore no probability for the system to go back to stage O_4 . Now only intermediate and major repair actions can be taken. So now probability for system to take intermediate repair is 0.5 and the probability that system would require major repair is 0.4.



Figure 5.4 System model.

5.3 Mathematical modeling

After developing the model as above, we will now obtain the solution using analytical approach (Markov Analysis). The first equation tells the transition of first component with its degraded, inspection and repair states. λ represents the failure rate and μ represents the repair rate. Similarly for all the states the transition is shown by respective states.

5.3.1 Notations

 $\lambda i j$:Failure rate of component where "i" is the initial state of the component and "j" is the final state of component.

 μ ij :Repair rate of component where "i" is the initial state of the component and "j" is the final state of component.

yi : The probability of the system to be in the state "i" at time t.

5.3.2 Equations for Markov Analysis of CBM Model

Following the methodology as per Section 4.2 the following set of equations are derived for the system, Figure 5.4.

$$\frac{d\rho_{N_1}}{dt} = -\lambda_{N_1O_1}\rho_{N_1}(t) - \lambda_{N_1I}\rho_{N_1}(t) - \lambda_{N_1Fr}\rho_{N_1}(t) + \mu_{IN_1}\rho_{I}(t) + \mu_{n_1N_1}\rho_{n_1}(t) + \mu_{H_1N_1}\rho_{H_1}(t) + \mu_{H_2N_1}\rho_{H_2}(t) + \mu_{F_rN_1}\rho_{Fr}(t)$$

$$\frac{d\rho_{o_1}}{dt} = -\lambda_{o_1I}\rho_{o_1}(t) - \lambda_{o_1O_2}\rho_{o_1}(t) - \lambda_{o_1Fr}\rho_{o_1}(t) + \mu_{IO_1}\rho_{I}(t) + \mu_{n_2O_1}\rho_{n_2}(t) + \mu_{N_1O_1}\rho_{N_1}(t) + \mu_{H_3O_1}\rho_{H_3}(t) + \mu_{a_1O_1}\rho_{a_1}(t)$$

$$\frac{d\rho_{o_2}}{dt} = -\lambda_{o_2I_3}\rho_{o_2}(t) - \lambda_{o_2Fr}\rho_{o_2}(t) - \lambda_{o_2Fr}\rho_{o_2}(t) + \mu_{I_3O_2}\rho_{I_3}(t) + \mu_{n_2O_2}\rho_{n_2}(t) + \mu_{O_1O_2}\rho_{O_1}(t) + \mu_{a_2O_2}\rho_{a_2}(t) - \lambda_{O_2O_3}\rho_{O_3}(t)$$

$$\frac{d\rho_{O_3}}{dt} = -\lambda_{O_3I}\rho_{O_3}(t) - \lambda_{O_3Fr}\rho_{O_3}(t) + \mu_{O_2O_3}\rho_{O_2}(t)$$

$$\frac{d\rho_{I}}{dt} = -\mu_{IN_{1}}\rho_{I}(t) - \lambda_{In_{1}}\rho_{I}(t) + \lambda_{N_{1}I}\rho_{N_{1}}(t)$$

$$\frac{d\rho_{I}}{dt} = -\mu_{IO_{1}}\rho_{I}(t) + \lambda_{O_{1}I}\rho_{O_{1}}(t) - \lambda_{In_{2}}\rho_{I}(t) - \lambda_{I_{H_{1}}}\rho_{I}(t)$$

$$\frac{d\rho_{I}}{dt} = -\lambda_{Ia_{1}}\rho_{I}(t) - \lambda_{IH_{2}}\rho_{I}(t) - \lambda_{In_{2}}\rho_{I}(t) - \mu_{IO_{2}}\rho_{I}(t) + \lambda_{O_{2}I}\rho_{O_{2}}(t)$$
$$\frac{d\rho_{I}}{dt} = -\lambda_{Ia_{2}}\rho_{I}(t) - \lambda_{IH_{3}}\rho_{I}(t) + \lambda_{O3I}\rho_{O_{3}}(t)$$

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$$\begin{aligned} \frac{d\rho n_{1}}{dt} &= -\mu_{n_{1}N_{1}}\rho_{n_{1}}(t) + \lambda_{In_{1}}\rho_{I}(t) \\ \frac{d\rho n_{2}}{dt} &= -\mu_{n_{2}O_{1}}\rho_{n_{2}}(t) + \lambda_{In_{2}}\rho_{I}(t) \\ \frac{d\rho n_{1}}{dt} &= -\mu_{H_{1}N_{1}}\rho_{H_{1}}(t) + \lambda_{IH_{1}}\rho_{I}(t) \\ \frac{d\rho a_{1}}{dt} &= -\mu_{a_{1}O_{1}}\rho_{a_{1}}(t) + \lambda_{Ia_{1}}\rho_{I}(t) \\ \frac{d\rho n_{4}}{dt} &= -\mu_{H_{2}N_{1}}\rho_{H_{2}}(t) + \lambda_{IH_{2}}\rho_{I}(t) \\ \frac{d\rho n_{4}}{dt} &= -\mu_{n_{4}O_{2}}\rho_{n_{4}}(t) + \lambda_{In_{4}}\rho_{I}(t) \\ \frac{d\rho n_{2}}{dt} &= -\mu_{a_{2}O_{2}}\rho_{a_{2}}(t) + \lambda_{Ia_{2}}\rho_{I}(t) \\ \frac{d\rho n_{3}}{dt} &= -\mu_{H_{3}N_{1}}\rho_{H_{3}}(t) + \lambda_{IH_{3}}\rho_{I}(t) \end{aligned}$$

5.3.3 Solution for the Markov model

Considering these values a program on Matlab is developed. Working hours of turbine blade are taken 12000 hours using Weibull data. The following distribution parameters are taken for various states as shown in Table 5.1.

The sets of differential equations for each model is simultaneously solved with initial conditions $y_1(0)=1$, $y_2(0)=0$, $y_3(0)=0$, $y_4(0)=0$, $y_5(0)=0$, $y_6(0)=0$, $y_7(0)=0$, $y_8(0)=0$, $y_9(0)=0$, $y_{10}(0)=0$, $y_{11}(0)=0$, $y_{12}(0)=0$, $y_{13}(0)=0$, $y_{14}(0)=0$, $y_{15}(0)=0$, $y_{16}(0)=0$, $y_{17}(0)=0$, $y_{18}(0)$ and for a required mission time T. The availability (A) of the system at the end of the mission time is given by A = $y_1(t)+y_2(t)+y_3+y_4(t)$.

Table 5.1

S.no.	Transition	Parameter	Values
1	N ₁ O ₁	λ _{N101} 0.00013333	
2	N ₁ Fr	λ_{N1Fr}	0.00005
3	N ₁ I	λ_{N1I}	Ι
4	IN ₁	μIN_1	0.08
5	n ₁ N ₁	$\mu n_1 N_1$	0.01
6	H ₁ N ₁	$\mu H_1 N_1$	0.002
7	H ₂ N ₁	$\mu H_2 N_1$	0.001666
8	FrN ₁	μFrN_1	0.001666
10	O ₁ Fr	$\lambda O_1 Fr$	0.00002
11	O ₁ O ₂	$\lambda O_1 O_2$	0.0003333
12	O ₁ I	$\lambda O_1 I$	Ι
13	IO ₁	μIO ₁	0.03
14	n ₂ O ₁	$\mu n_2 O_1$	0.005
15	H ₃ O ₁	μH_3O_1	0.002
16	a ₁ O ₁	μa ₁ O ₁	0.0025
18	O ₂ I	$\lambda O_2 I$	Ι
19	O ₂ O ₃	$\lambda O_2 O_3$	0.0006666
20	O ₂ Fr	λO2Fr	0.00002
21	IO ₂	μIO ₂	0.0133333
22	n4O ₂	$\mu n_4 O_2$	0.005
23	a ₂ O ₂	$\mu a_2 O_2$	0.0025

CBM distribution parameters for various repairs, inspection interval, random failure and failure.
25	O ₃ F	$\lambda O_3 F$	0.001
26	O ₃ I	λO ₃ I	Ι
27	O ₃ Fr	λO ₃ Fr	0.00002
28	In1	λInl	0.02
29	In ₂	λIn_2	0.01
30	IH ₁	λIH_1	0.01
31	Ia ₁	λIa_1	0.006666
32	IH ₂	λIH_2	0.006666
33	In ₄	λIn_4	0.006666
34	Ia ₂	λIa_2	0.1251
35	IH ₃	λIH ₃	0.01

Availability vs inspection time interval thus plotted. The Figure 5.5, shows that after 100 hours of working if component is inspected this will give minimum value to availability. At stage O1 which is second degraded stage a check is necessary. At around 1900 hours if inspected this will improve availability of the whole system. After this interval graph is decreasing so as the availability. And at 12000 hours which is considered as component life time hour the availability is having a minimum value again. So the component must be inspected neither too early nor too late. An optimum value will fall in between which will give rise to maximum availability.



Figure 5.5 Availability Vs Inspection interval.

The particular values of availability and inspection interval is shown in Table 5.2. It is clear that at 1900 hours availability is having maximum value which is 0.7300.

Golden section method is now applied to check the optimized value of inspection interval and availability, refer section 4.3.6. From Table 5.2 we see that system availability increases as we proceed from inspection interval Ti = 100 hours to Ti = 1900 hours and then decreases. Hence the inspection interval for optimal availability must be between Ti = 1500 hours to Ti = 2500 hours.

Table 5.2

Availability vs inspection interval.

Inspection interval (hours)	Availability
100	0.5810
500	0.6520
1000	0.6770
1500	0.7205
1900	0.7300
2000	0.7275
4000	0.6704
6000	0.6333
8000	0.6205
10000	0.5906
12000	0.5812
	100 500 1000 1000 1500 1900 2000 4000 6000 8000 10000

Choosing the initial end points as 1500 and 2500 hours and applying golden search technique, we find the inspection inverval for the optimal availability as:

Ti = 1813 hours



Figure 5.6 Result of Golden section search

Figure 5.6 shows graphical representation of the golden section. The corresponding optimal system availability is:

 $A_{optimal} = 0.7313$

CHAPTER 6

CONCLUSION

In this project, system availability model considering multi stage degradation, condition based maintenance and random failure is developed. The system model is solved analytically by MARKOV approach.

- As far as frequency of inspection is concerned at stage N1, less frequent inspection should be done as the health of the component is very good and unnecessary inspection will only lead to time wastage and reducing our component availability.
- At stage O1, the inspection should be done too frequently either as here too the health of the component is fairly good.
- At stage O2, inspection work should be done quite frequently as the health of the component has deteriorated and frequent inspection would readily provide us information about its degradation so we can undertake necessary repair actions.
- At stage O3 as system is about to fail so inspection should be frequent then other stages and necessary actions should be taken.

6.1 Scope for future work

The studies and projects which were studied before were moreover theoretical and lacking the real life factors which affect a component during its working life span. Our modeling includes the real life factors of the component working life cycle thus make it closer to reality. Also it makes the model more practical, thereby ensuring the results which are to be obtained from its analysis will give a more accurate and confirmed to the actual observed values depicting a true behavior, free from errors.

After developing the system model the analytical solution is solved by using matlab software and inspection interval is to be determined which will give a maximum value to the availability. A trial is to be produced to increase the system availability by changing the inspection time in the matlab programme and the inspection time at which system availability is maximum is to be figure out.

Various factors such as consideration of multi stage degradation, periodic inspection, condition based maintenance and random failure have made our model quiet close to a real one, hence, in future when such a study is taken up, a decent acceptable base model is already available in the form of this model.

In many industries, still, not much attention is paid to the above considered factors, this analysis shows how the individual parameters can contribute significantly to the enhanced availability. Hence, it can be an initiation in this regard for many firms to analyze the parameters discussed here and improve the availability of the component(s) and thereby, that of the overall system.

In this project work, only the availability analysis part is focused and discussed hence, there lies a scope for extending it for cost analysis. Also, we have considered a one component system. So this analysis can be considered for multi components attached in series or parallel. Hence no analysis is done to optimize the performance of the component; so here lies a scope to undertake this work. This Project work has been based on the calculations by Markov Approach which is applicable for exponential distribution. Therefore undertaking this analysis work can be done for other kinds of distributions too.

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APPENDIX

Functiondy=cbm(t,y,L12,L418,L19,I15,M101,M121,M141,M51,M91,L29,I29,L23,M62,M112, M172,M132,L34,I37,M153,M163,M73,L49,I48,L510,L612,L611,I26,L715,L714,L713,L817,L8 16,L39,L62);

L12 = 1/7500; L19 = 1/50000; I15 = 1/500; M101 = 1/100; M121 = 1/600; M141 = 1/600; M51 = 0.8/10; M91 = 1/600; L23 = 1/3000; M62 = 0.6/20; M112 = 1/200; M172 = 1/450; M132 = 1/400; L29 = 1/50000; L34 = 1/1500;

I37 = 1/200; M153 = 1/200; M163 = 1/400; M73 = 0.4/30; L49 = 1/50000; I48 = 1/200; L510 = 0.2/10; L612 = 0.2/20; L611 = 0.2/20;

I26 = 1/200; L715 =0.2/30; L714 = 0.2/30; L713 = 0.2/30; L817 = 0.4/40; L816 = 0.5/40; L39 = 1/50000; L418 = 1/1500;

dy = zeros(18,1); % a column vector

dy(1) = (-L12 - L19 - I15)*y(1) + M101*y(10) + M121*y(12) + M141*y(14) + M51*y(5) + M91*y(9);

dy(2) = (-L29 - I26 - L23)*y(2) + M62*y(6) + M112*y(11) + M172*y(17) + L12*y(1) + M132*y(13);

dy(3) = (-L39-L34-I37)*y(3) + M73*y(7) + L23*y(2) + M153*y(15) + M163*y(16);

dy(4) = (-L49-I48-L418)*y(4)+L34*y(3);

dy(5) = (-L510-M51)*y(5)+I15*y(1);

dy(6) = (-L612 - L611 - M62)*y(6) + I26*y(2);

dy(7) = (-L715 - L714 - L713 - M73)*y(7) + I37*y(3);

dy(8) = (-L817 - L816)*y(8) + I48*y(4);

dy(9) = (-M91+L39)*y(9)+L49*y(4)+L19*y(1)+L29*y(2);

dy(10) = -M101*y(10)+L510*y(5);

dy(11) = -M112*y(11)+L611*y(6);

```
dy(12) = -M121*y(12)+L612*y(6);
```

```
dy(13) = L713*y(7) - M132*y(13);
```

```
dy(14) = -M141*y(14)+L714*y(7);
```

```
dy(15) = -M153*y(15) + L715*y(7);
```

```
dy(16) = -M163*y(16) + L816*y(8);
```

```
dy(17) = -M172*y(17)+L817*y(8);
```

```
dy(18) = L418*y(4);
```

Golden section algorithm

```
x = 1500:2500;
0.403038787878788
figure(1)
plot(x,f(x))
xlow = 1600;
xup = 1900;
goldenratio = (sqrt(5) - 1)/2;
d = goldenratio*(xup - xlow);
x1 = xlow + d;
x^2 = xup - d;
plot(x1,f(x1), '*k')
plot (x2,f(x2), '*m')
plot (xlow,f(xlow), '*r')
plot (xup,f(xup), '*g')
hold off
figure(2)
plot (x,f(x),b')
hold on
for i = 0:100
  f(x2)
  f(x1)
if (f(x2) > f(x1))
70 | Page
```

```
xup = x1;
    x1 = x2;
    d = goldenratio*(xup - xlow);
    x2 = xup - d;
  else
if(f(x1)>f(x2))
       xlow = x2;
       x^2 = x^1;
       d = goldenratio * (xup - xlow);
       x1 = xlow + d;
else
end
end
plot(xlow,f(xlow),'*r')
plot(xup,f(xup),'*g')
end
hold off
display (xlow)
display(f(xlow))
display (xup)
display (f(xup))
```

.