A Dissertation
on

DIGITAL MODULATION CLASSIFICATION

submitted in partial fulfillment of the requirements for the award of degree of

Master of Technology

in

Signal Processing & Digital Design

Submitted By

Pranay Chandel

(Roll no.:-2K12/SPD/13)

Under the guidance of

Dr. Sudipta Majumdar
(Assistant Professor)

Department of Electronics and Communication Engineering
Delhi Technological University
(Formerly Delhi College of Engineering)
Bawana Road, Delhi-110042

(2013-2014)

CERTIFICATE

DELHI TECHNOLOGICAL UNIVERSITY,
(Formerly Delhi College of Engineering)
BAWANA ROAD, DELHI – 110042
Department of Electronics & Communication Engineering

This is to certify that the report entitled ***"Digital Modulation Classification"*** submitted by Pranay Chandel, Roll. No. *2K12/SPD/13*, in partial fulfillment for the award of degree of Master of Technology in Signal Processing & Digital Design at **Delhi Technological University, Delhi**, is a bonafide record of student's own work carried out by him under my supervision and guidance in the academic session 2013-14. The matter embodied in dissertation has not been submitted for the award of any other degree or certificate in this or any other university or institute.

(Pranay Chandel)
2K12/SPD/13
M.Tech (SPDD)

COUNTERSIGNED

(Dr. Sudipta Majumdar)
ASSISTANT PROFESSOR
Department of Electronics & Communication Engineering
Delhi Technological University,
(Formerly Delhi College of Engineering)
Delhi -110042

ACKNOWLEDGEMENT

I express my sincere gratitude to my guide **Dr. Sudipta Majumdar** for giving valuable during the course of the investigation, for her ever encouraging and moral support. Her enormous knowledge and investigation always helped me unconditionally to solve various problems. I would like to thank her for introducing me with the problem and providing valuable advice throughout the course of work. I truly admire her depth of knowledge and strong dedication to students and research that has made her one of the most successful professors ever. Her mastery of any topic is amazing, but yet she is such a humble and down to earth person. I am glam that I was given opportunity to work with her. She surely brings out the best in her students.

I am greatly thankful to **Prof. Rajiv Kapoor, Professor and Head,** Department of Electronics & Communication Engineering, entire faculty and staff of electronics & Communication Engineering for their, continuous support, encouragement and inspiration in the execution of this "**thesis"** work.

I would like to thanks my parents who bestowed upon me their grace and were source of my inspiration and encouragement.

I am thankful to almighty god his grace and always with me whenever I felt lonely.

Pranay Chandel

ABSTRACT

The modulation recognition plays an important role in various civilian and military applications. In blind environments, an effective recognition algorithm is needed which should be able to discriminate between digital communication signals. Many researches have worked on the problem of modulation recognition of digitally modulated signals. Many of the algorithms developed need a priori information of a few parameters and thus crops up a space for a class of algorithms that are independent of any information of the signal. This novelty presents a strong case for a leap forward towards blind recognition of the modulation. A feature based algorithm extracts discriminatory features for data representation amongst different modulation schemes and then makes a decision based on thresholds. The first of the algorithm discussed is essentially a sequence of steps which employs statistical parameters such as fourth order cumulants, variance of the centered normalized signal amplitude, instantaneous phase, the zero-crossing sequence shape (ZCSS) and power spectral density as features for classification by employing opportunely set thresholds. The other algorithm approaches the classification problem as multiresolution analysis and comes up with the idea of employing wavelet transform of the signal. The variance of the magnitude of the signal wavelet transform (WT) after peak removal is compared against a set threshold and classification decision is made. It is prudent to mention that a digitally modulated waveform is cyclostationary and exhibits transients whenever a symbol undergoes a transition. The difference is thus exploited for modulation classification.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

INTRODUCTION

Modulation techniques revolutionized the communication systems as it was perceived and increased its impact by sheer multitude of its capability and capacity. Modulation provides a communication system with robustness to withstand noisy environments. It provides it the much needed flexibility in terms of techniques to be employed and the much needed security.

1.1 Motivation

This study deals with the problem of modulation characterization in the presence of varying noise and varying carrier frequency. Signal interception and modulation recognition is extremely important when spoken in context of military intelligence or keeping a check on unlicensed transmitters. Modulation classification is an intermediate step between signal detection and demodulation and plays a key role in military and civilian applications alike. At the heart of an electronic surveillance system lies an intelligent receiver which is busy classifying the incoming signals on basis of its parameters in presence of impediments like noise, effects of multipath fading etc. to name a few. Amongst various methods in practice the prime focus of this study is the ones using Wavelet transform and the other one using statistical parameters to validate the a particular modulation.

An efficient system would follow an algorithm which doesn’t need any *a priori* knowledge of a modulated signal and still plays a pivotal role in signal verification and signal identification. The modulation classification algorithms has been three pronged. First, where only analog communications signals have been classified. Second, for digital modulations and third where a few schemes for both digital and analog modulated signals have been classified. A bottom up approach further reveals that the classification algorithm are either decision theory based approach or a feature matching based approach.

The first approach (decision theory) a maximum likelihood criterion is applied to the signal itself or to the transform of the signal which yields either a likelihood ratio or a likelihood functions. The classification is then made by comparing the result to a threshold. These formulation are computationally complex although it gives an optimal solution as misclassification probability is taken into account. The second approach dwells on one or more features extracted from a signal. This approach (features based) is easy to implement and does return a near optimal performance.

1.1.2 Classification Algorithms Overview

Although analog communication signals are not within the ambit of this study however a few algorithms are discussed. Nandi and Azzouz [1] based their algorithm on four features for classification of AM, DSB, LSB, USB, FM, vestigial-sideband (VSB) and combined AM-FM modulation signals. The four features being, $\gamma_{\max}$, σ_{ap} , σ_{dp} & P , here ($P = \left|P_L - P_U\right| / \left(P_L - P_U\right)$) $\gamma_{\max}$ is the maximum value of the estimated power spectral density (PSD) of the normalized centralized instantaneous amplitude of the input. σ_{ap} is the standard deviation of the absolute centralized instantaneous phase. σ_{dp} is the standard deviation of the centralized instantaneous phase. P is the received signal's power in the frequency range. The signals are then chronologically classified.

The cyclostationarity of AM, DSB, SSB, CW and noise signals was exploited Seaman and Braun [2]. For a received signal the cyclic spectral density (CSD) is estimated and used for classification. The devised approach is validated by different figures for CSDs [2] which are perceptible for a human eye, but is intrinsically an arduous for a machine to recognize a CSD pattern.

1.1.3 Digital Modulation Classification

The digital modulation classification is based on assumptions of equi-probability of transmitted symbols and the symbols being independently & identically distributed. This leads to both the decision theory approach and feature based approach applicable towards modulation classification. During the course of the study some common underlying assumptions were made. **(A1)**, the impulse response of the transmission filter or the pulse-shaping function was a standard unit pulse defined for a duration T which is same as the symbol period and is reciprocal to the symbol rate. **(A2)**, the communication channel does not introduce amplitude distortion except the additive white Gaussian noise (AWGN). **(A3)** the symbol rate of the received signal and carrier frequency is either known in advance or can be calculated by the algorithm itself.

A phase based approach (phase as a feature) was employed in [3] and classified CW, PSK2 and PSK4 based on the instantaneous phases. From an analytic signal, the instantaneous phase sequence was extracted (including phase unwrapping), and then the first-difference of the instantaneous phase (also called the delta-phases) are calculated. In the next step mean value of the delta phases is removed, and the modified delta-phases are then integrated, resulting in another phase sequence. The phase sequence thus obtained in the previous step is processed by using a filter that computes the absolute difference of samples separated by L samples (where L is a design factor). The classification is made by comparing the histogram of the filter outputs with two thresholds. This algorithm works with assumption A1 & A3, and the received signal is over-sampled with respect to the symbol rate as well as the carrier frequency.

MPSK classifier as developed in [4] first forms the instantaneous phase's histogram with N bins as the estimate of the phase PDF followed by N-point discrete Fourier transform (DFT) of the histogram. On the basis of the maximum DFT magnitude amongst the DFT bins of interest the classification is made.

A zero crossing based approach is introduced in [5]-[6] to classify CW, MPSK and MFSK signals. The variance of ZC intervals is shown to increase as the number of the sub-carrier frequencies increases. The variance of ZC intervals is analyzed, and then an ML criterion is employed to discriminate between MFSK and MPSK. For a signal classified as MFSK, the number of hills in the ZC interval histogram gives the estimate of the level of modulation in MFSK signal ($N_F \leq 2^D$, where $M = 2^D$). For MPSK, the estimation of its alphabet size is accomplished in a way similar to [7], and the phase histogram is employed to find the level of modulation. The algorithm has some serious limitations. The variance of ZC interval is a function of SNR, carrier frequency and frequency deviation (in MFSK). This algorithm gives an expression to handle first two but is non-committal on handling of frequency deviation especially in blind modulation classification. Further the detection of inter symbol transients in events of low SNR *a priori* knowledge of level of modulation and frequency deviation is required which is not covered in the algorithm thus leads to an implementation dead lock.

Another method in [8] proposed to classify ASK2, ASK4, PSK2, PSK4, PSK8, FSK2, FSK4, FSK8, QAM16, QAM64 and some orthogonal frequency division multiplexing (OFDM) signals. The fourth-order cumulants as described in [9]-[10] are adopted as the feature for discriminating between OFDM and non-OFDM signals. The feature $\gamma_{\max}$ as in [1] is employed to discriminate between amplitude modulations (i.e.,MASK & QAM) and phase modulations (i.e., MPSK & MFSK). A simple figure of merit which is nothing but the mean of normalized-centralized magnitudes is compared to a threshold to discriminate between MASK and QAM. The discrimination between MPSK and MFSK is ZCSS (zero crossing sequence shape) as in [5]-[6] and [11]. The alphabet size of MPSK is determined as in [7], and of MFSK is estimated by counting the hills in the ZC interval histogram, and that of MASK and QAM are estimated by counting the hills in the histogram of normalized-centralized magnitudes. In OFDM signals, the modulation type is recognized by comparing the estimated symbol rate and PSD with that of existing standards.

In cumulants/moments based approaches some statistic of the received signal or its transforms as is taken to be the classification feature. The different modulation formats will correspond to different feature value range which is then compared to a threshold and a classification decision is made. An exact expressions of the moments of the instantaneous phase of MPSK signals was found in [13] and was consequently observed that the even order moments are monotonic functions of M, which could be chosen to be the feature for classifying MPSK signals. Further the central limit theorem shows that the estimated moment is shown to be normally distributed. Then the decision thresholds are determined accordingly.

In addition to the features $\gamma_{\max}$, σ_{ap}, σ_{dp} of [7], two new features namely the standard deviation of the absolute instantaneous amplitude σ_{aa}, and the standard deviation of the absolute normalized-centralized instantaneous frequency σ_{af} are introduced in [14] for classifying ASK2, ASK4, PSK2, PSK4, FSK2 and FSK4.

In another technique [26] after dividing the received signal into short time segments a second order autoregressive (AR) model is employed to estimate the frequency and bandwidth of each segment. The FSK and non-FSK signals are then differentiated on the basis of the standard deviation of the frequencies as estimated from the segments. Further if the signal is classified as non-FSK the standard deviation of the bandwidths is used to discriminate between CW and PSK.

Wavelet based approach in [15] thrives on the fact that the magnitude of the amplitude of Haar wavelet transform |HWT| is a staircase function for MFSK, with M distinct DC levels with spikes at the instant of symbol transition. For MPSK the |HWT| gives DC with spikes at the instants of symbol transitions. The spikes are then removed by median filtering the |HWT| magnitudes and the variance is then calculated to discriminate between MFSK and MPSK. The classification threshold is then determined. The number of modulation levels, M, of an MPSK signal is determined counting the histogram of HWT magnitude peaks. In case of PSK the signal is classified as M-ary if the histogram has M/2 to M-1 number of peaks, and as M-ary FSK if the histogram has M/2 to M number of peaks. But the fact persist that for a machine to count no of peaks is no trivial task. In [16] the method as presented above has been modified and approach is also extended to include QAM signal as well. Now the |HWT| of the received signal and the |HWT| of amplitude normalized received signal are employed in tandem for signal classification. In variably the wavelet based methods require a high sampling rate.

1.2 Scope of this Project

The scope of this project is to present a chronological details of modulation identification algorithms as employed in intelligent receivers. With the phenomenal advancement of in field of digital electronics and communications the trend is heavily lopsided towards the use digital communications instead of analog communications. It is assumed that the PDF of the symbols of each modulation type is known in advance. Further the transmitted symbols are assumed to be independent identically distributed (i.i.d.) with equi-probability of occurrence. The algorithm we discuss both can find out symbol rate and carrier frequency itself. In the subsequent paragraphs modulation identification techniques which are essentially feature based are discussed briefly. The first of the techniques discussed is essentially a series of tests which a signal is required to go through and then compared against a threshold and a classification decision/ discrimination decision amongst different class of modulations is made at every instance. On the other hand the second technique exploits the property of wavelets for location of transients in both frequency and time domain by virtue of scaling of wavelets over the entire range of the signal.

1.3 Modulation Classification Method Based on ZCSS and Statistical Parameters

A tree structure for the classification method is as shown in the Fig 1. This [8]-[10] method in itself follows different procedures at each step to ascertain a type of modulation and distinguish between one another. As a first step a single carrier and a multiple carrier signal is distinguished by using fourth order cumulants [9]-[10], and thus obviating the running the multiple carrier test on single carrier signals. Then in order to differentiate between amplitude and angle modulated signals an approximation of power spectral density is used as a feature [17] -[18].

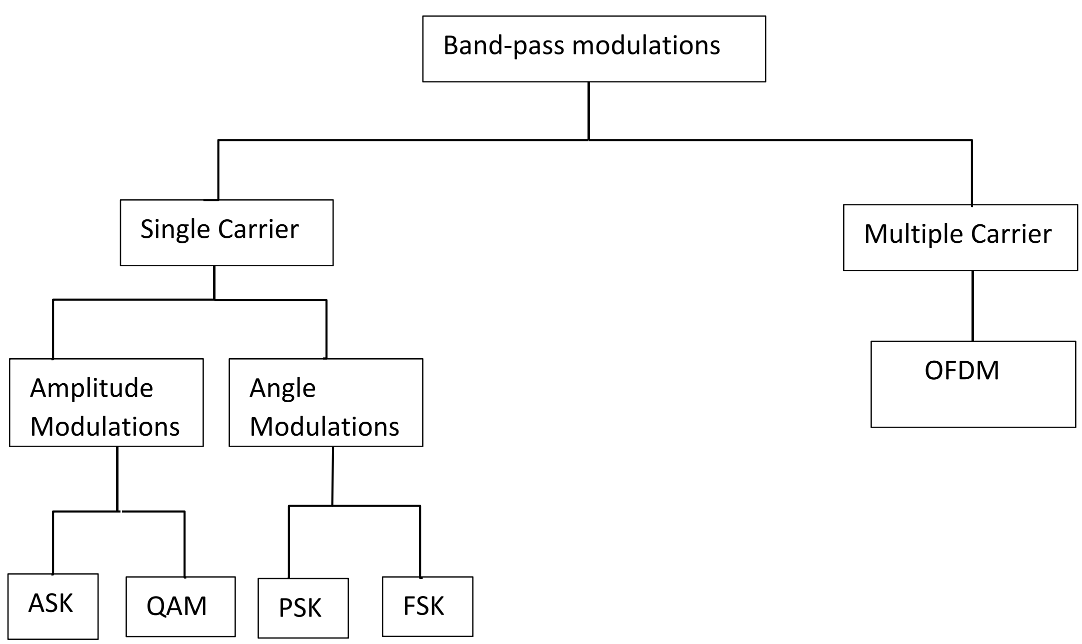


Fig. 1.1. Structure for classification method.

Amongst angle modulated signals FSK/PSK classifications is made using the ZCSS-based technique presented in [12]-[13] & [19]-[21]. The OFDM modulation schemes are classified by virtue of cyclic extensions added to a particular standard [22]-[23].

1.4 Wavelet Transform Based Modulation Classification

A wavelet transform based modulation identification algorithm computes the wavelet transform of the received modulation signal where the choice of wavelet varies as per user. The magnitude of the thus computed wavelet transform is median filtered to remove peaks as in [15]-[16]. The variance is then calculated of the filtered output and is compared against a threshold and thus the signal is classified as PSK or FSK as shown in the Fig 1.2.

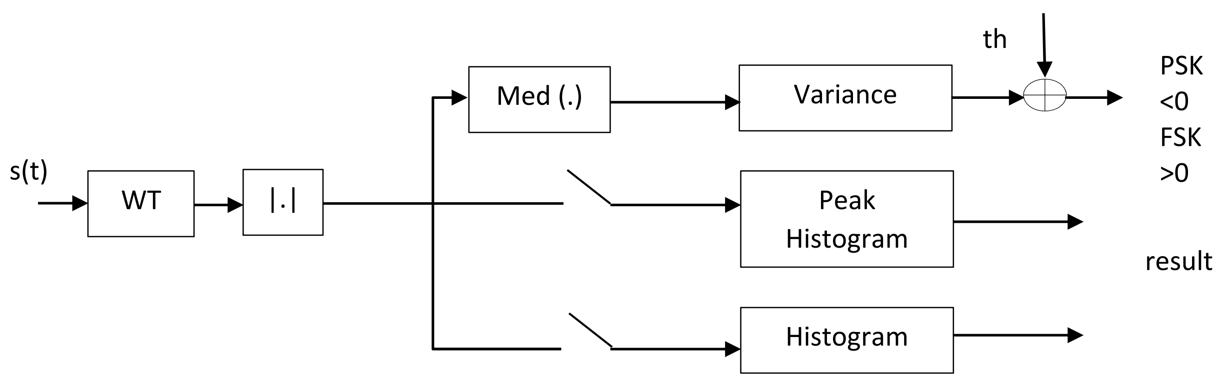


Fig. 1.2 A Wavelet transform based modulation identifier

Once the classification decision is taken the in favor of either of the modulation scheme the level of modulation is found by computing respective histograms.

1.5 Organization of Thesis

Chapter 2: The chapter deals with statistical parameters identified for several of modulation schemes which can be employed to discriminate modulated signals amongst two classes. The tests based on evaluation of the aforesaid parameters are applied on the received signal. The results would essentially vary depending upon the innate modulation scheme of the received signal. ZCSS (zero crossing sequence shape) technique which is based on the zero crossings of the signal is employed to amplify the inherent differences amongst different in modulation schemes.

Chapter 3: The wavelet transform is a potent tool to capture transients when considering the multi resolution analysis. This property of wavelet transform is capitalized upon in tracking transients within a modulated signal be it phase, frequency or the amplitude.

Chapter 4: It analyzes the results of the simulations & results, advantages and disadvantages of the algorithms are discussed.

Chapter 5: In this chapter we dwell upon the future scope of work and a way forward to envisage a framework which encompasses all modulations and obviates the operator to make a classification decision.

Note: All the plots and tables as shown in this thesis were obtained during multiple simulation runs of the methods/ algorithms for different parameters. It is pertinent to mention that these plots and tables have not been copied / illegally acquired from any source whatsoever.

Chapter 2

MODULATION CLASSIFICATION BASED ON HIGHER ORDER STATISTICS AND ZCSS

The modulation classifications intrinsically differ in the manner the data is embedded in a carrier signal. Thus the modulated signal would also differ. These differences are thus reflected in various statistical parameters which are computed for a known modulated signal. Based on the knowledge of the feature set of known signals under different conditions such as variable noise, carrier signal power etc. a classification decision can be made for an unknown signal.

2.1 Mathematical Representation Modulation Schemes

The general model used for modulated signals with nomenclature during the course of this discussion is as shown below [24].

$$s(t) = x(t) + n(t) \tag{2.1}$$

Where $x(t)$ depends on the modulation type and $n(t)$ is additive white Gaussian noise.

$$x_{ASK}(t) = A\,\mathrm{Re}[\sum_k A_k e^{j2\pi f_c t} g(t - kT_s)] \tag{2.1a}$$

$$A_k = 2i - M - 1; i = 1, 2, ...M$$

$$x_{PSK}(t) = A\,\mathrm{Re}[\sum_k C_k e^{j2\pi f_c t} g(t - kT_s)] \tag{2.1b}$$

$$C_k = e^{j2\pi i/M}; i = 0, 1, 2, ...M - 1$$

$$x_{FSK}(t) = A\,\mathrm{Re}[\sum_k e^{j2\pi(f_c + \vartriangle f_k)t} g(t - kT_s)\ ] \tag{2.1c}$$

$$\vartriangle f_k = [i - (\frac{M-1}{2})]\vartriangle f; i = 1, 2, ...M$$

$$x_{QAM}(t) = A\,\mathrm{Re}\left[\sum_k C_k e^{j2\pi f_c t} g(t - kT_s)\ \right] \tag{2.1d}$$

$$C_k = a_k + jb_k;\quad a_k,\ b_k = \left[\pm 1, \pm 2, ...\pm M^{\frac{1}{2}} - 1\right]$$

$$x_{OFDM}(t) = A\,\mathrm{Re}\left[\sum_k \sum_{n=0}^{N_p - 1} C_{n,k} e^{j2\pi n \vartriangle f t}\ \right] \tag{2.1e}$$

$$C_{n,k} \in C; E\left\{C_{n,k}\right\} = 0$$

C_k , $\vartriangle f_k$, $C_{n,k}$, A_k map the transmitted symbols; T_s is the symbol period, f_c is the carrier frequency and the function $g(t)$ is a finite energy signal with T_s duration.

2.2 SC/MC Selection – Fourth Order Cumulants

To begin with it is prudent to ascertain the number of carriers involved in the modulation technique as shown in Fig.1. A modular structure obviates running tests meant for multiple carrier (MC) signaling techniques as OFDM on single carrier (SC) signaling techniques and vice versa, thus saving time and enhancing system efficiency.

A sample of an OFDM signal comprises a great number of i.i.d. random variables by virtue of which, by applying central limit theorem (CLT) the amplitude distribution may be approximated as a Gaussian. However this is not the case for a single carrier signal which reduces the MC/SC test to a normality test. Thus a fourth order cumulants test is defined for separating MC and SC signal [9]-[10]. The c_r^4 calculation is done to obtain a **'c'** vector estimate whose elements are defined as $\widehat{c}_{4s}(\eta)$.

$$\widehat{c}_{4s}(\eta) = \frac{1}{N_o}\sum_{i=0}^{N_o-1-\eta} s^2(i)s^2(i+\eta) - \left(\frac{1}{N_o}\sum_{i=0}^{N_o-1} s^2(i)\right)^2 - 2\left(\frac{1}{N_o}\sum_{i=0}^{N_o-1-\eta} s(i)s(i+\eta)\right)^2 \quad (2.2)$$

$$\eta = 0,.............\left\lceil N_o^{0.4}\right\rceil - 1$$

Where N_o is number of samples in the acquired signal, $\eta \in \left[0, 1.5N_s\right]$, N_s is number of samples corresponding the sample period. Finally the following inequality is to be verified [8].

$d_{G,4} = c^T c < \tau_c$, where τ_c is threshold

Consequently the threshold value of $d_{G,4}$ = -3dB permits classification between SC ($d_{G,4} > -3dB$) and MC ($d_{G,4} < -3dB$).

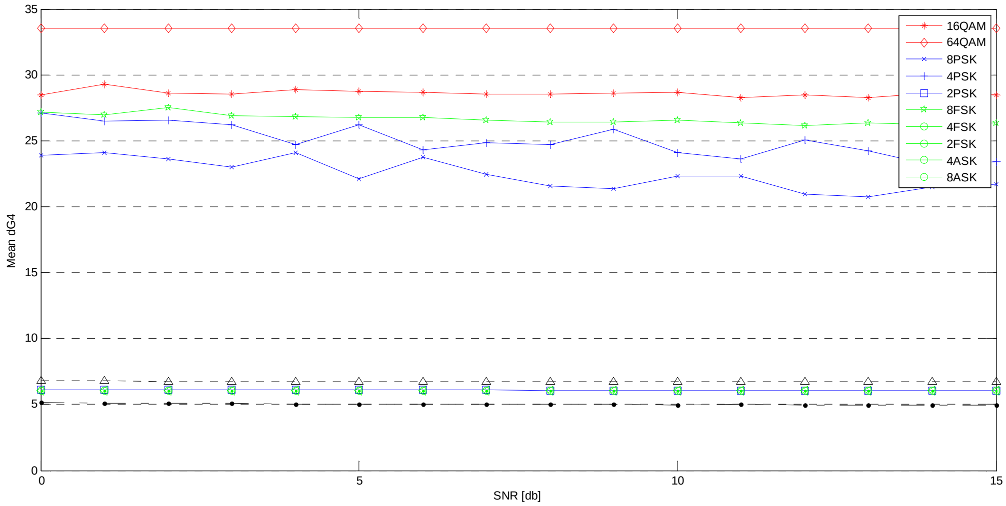


Fig 2.1 Trend for dG4 vs SNR for SC signals

2.3 Amplitude Modulation Test

The single carrier classified signal is now required to be discriminated between angle modulated signals and amplitude modulated signals. The instantaneous amplitude of the signal is evaluated from analytic representation of the signal $(s_+(t))$ [17].

$$s_+(t) = s(t) + j\hat{s}(t) \tag{2.3}$$

here $\hat{s}(t)$ is Hilbert transform of the original signal. The instantaneous amplitude of the signal is thus calculated as

$$a(t) = \sqrt{s(t)^2 + \hat{s}(t)^2} \tag{2.3a}$$

The sampled signal is the centered and normalized giving a sequence which is independent of channel gain.

$$a_{cn}[k] = \frac{\mathrm{a[k]}}{\mathrm{m_a}} - 1 \qquad \textit{where } m_a \textit{ is mean of } \mathrm{a[k]} \tag{2.3b}$$

The classification is carried out by evaluating γ_m [18]. It can be seen as approximation of power spectral density (PSD) of the signal.

$$\gamma_m = \max\left|DFT(a_{cn}[k])\right|^2 / N_o \tag{2.3c}$$

Here N_o is total number of samples of the received signal. Angle modulated signals have been observed to have $\gamma_m < 1$ so the threshold after considering SNR and different modulation schemes is kept at $\tau_m < 1.5$.

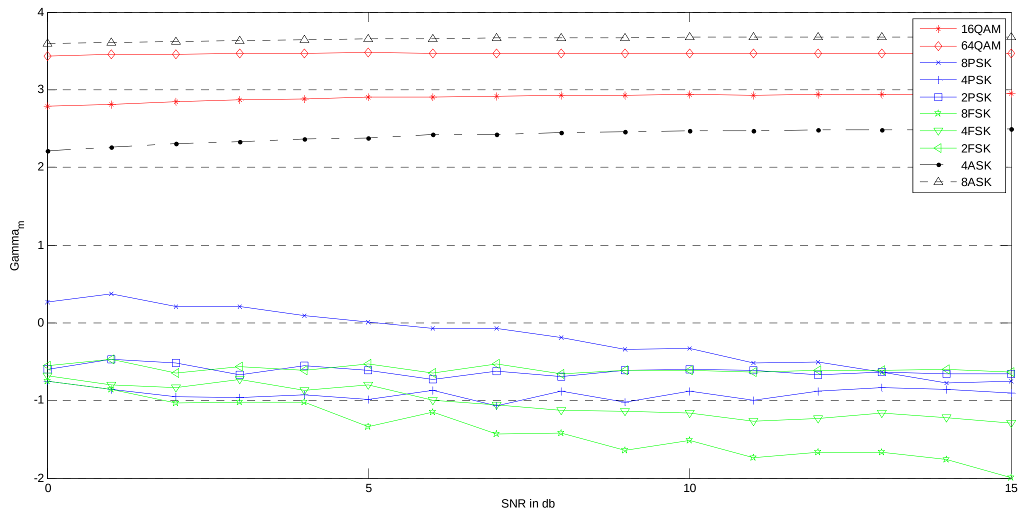


Fig 2.2 Trend for γ_m vs SNR for SC signals

2.4 ASK vs QAM Test

The sequence $a_{cn}[k]$ is used to differentiate between ASK & QAM signals. The pdf of ASK modulated signal is centered around 0.5 while for QAM it is not symmetric and has lower mean value. A simple figure of merit enables to differentiate between the two.

$$m_{aa} = E\{|a_{cn}|\}$$
$$\approx \frac{1}{N_o}\sum_k |a_{cn}| \tag{2.4}$$

Thus the signal is classified ASK if $m_{aa} > \tau_a$ where $\tau_a < 0.5$

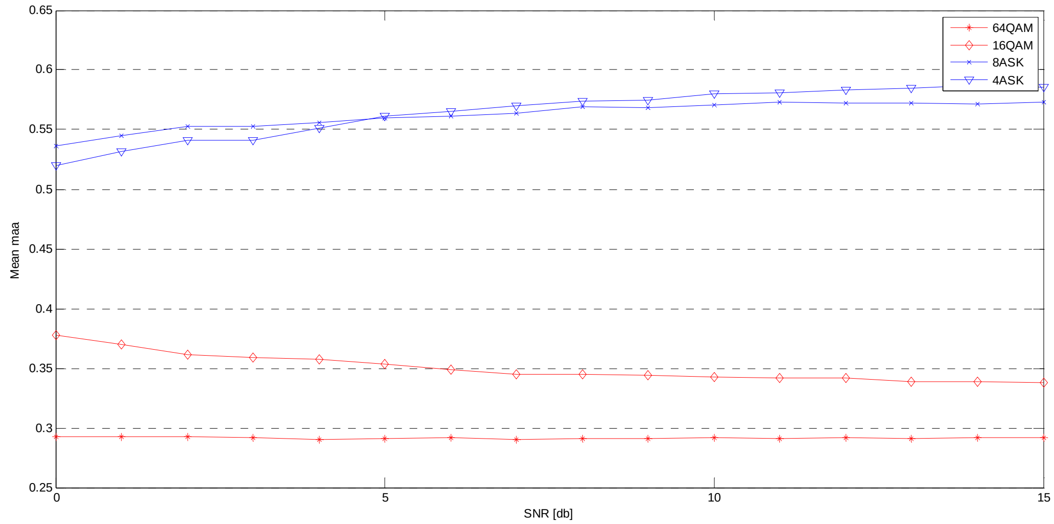


Fig 2.3 Figure of merit between ASK & QAM

2.5 FSK vs PSK Test

The classification between FSK and PSK is made by using zero crossing sequence shape (ZCSS) [5]-[6] & [12]. First the time tags of the zero crossing points of the modulated signal are recorded to form sequence $\{x(i)\}$. First of the other two sequences which are required to extract phase and frequency information is $y(i)$ and is called as the zero crossing interval sequence.

$$y(i) = x(i+1) - x(i) \qquad i = 1,2,............N-1 \qquad (2.5)$$

The second sequence is zero crossing interval difference sequence, given as

$$z(i) = y(i+1) - y(i) \qquad i = 1,2,............N-2 \qquad (2.5a)$$

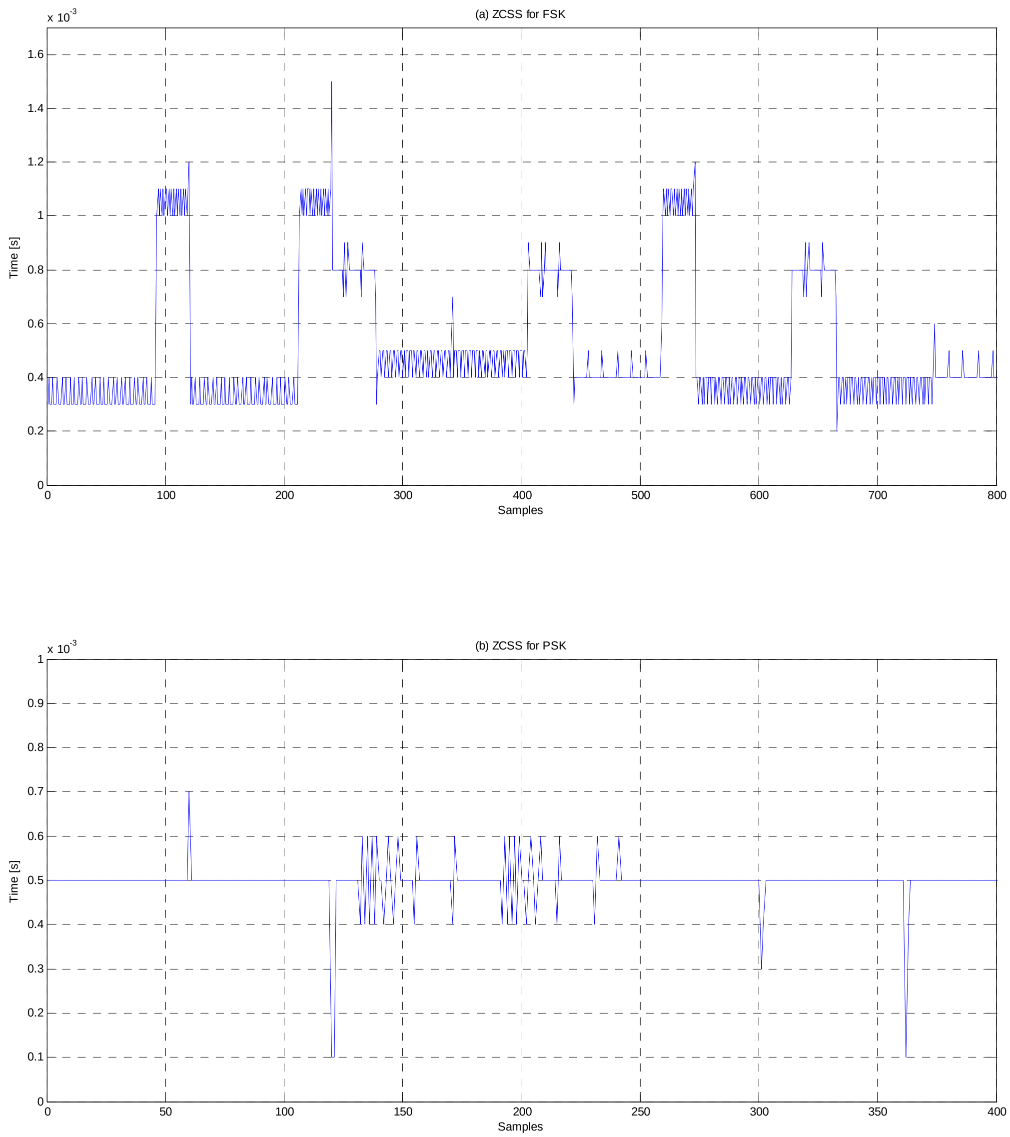


Fig 2.4 (a) Shows ZCSS for FSK, (b) Shows ZCSS for PSK

The classification between FSK/PSK can clearly be made by analyzing the shape of the ZCSS.

2.5.1 Carrier Frequency and CNR Estimation

From the dense portion of the density histogram of $\{z(i)\}$ new sequence $\{z_a(i)\}$ is formed and variance σ_{za}^2 is ascertained. A new sequence $\{y_a(i)\}$ is so formed by discarding those $y(i)$ samples whose $|z(i+1)| > 3.034\sigma_{za}^2$. The length of the resultant sequence is denoted as N_y. The carrier frequency and CNR can be found by using these equations.

$$f_c = \frac{N_y}{2\sum_{i=1}^{N_y} y_a(i)} \qquad (2.5b)$$

$$\gamma = \frac{1}{\left(2\pi f \sigma_{za}\right)^2}\left[3 + 4\rho\left(\frac{1}{2f_c}\right) + \rho\left(\frac{1}{f_c}\right)\right] \qquad (2.5c)$$

Here ρ is normalized autocorrelation function of noise.

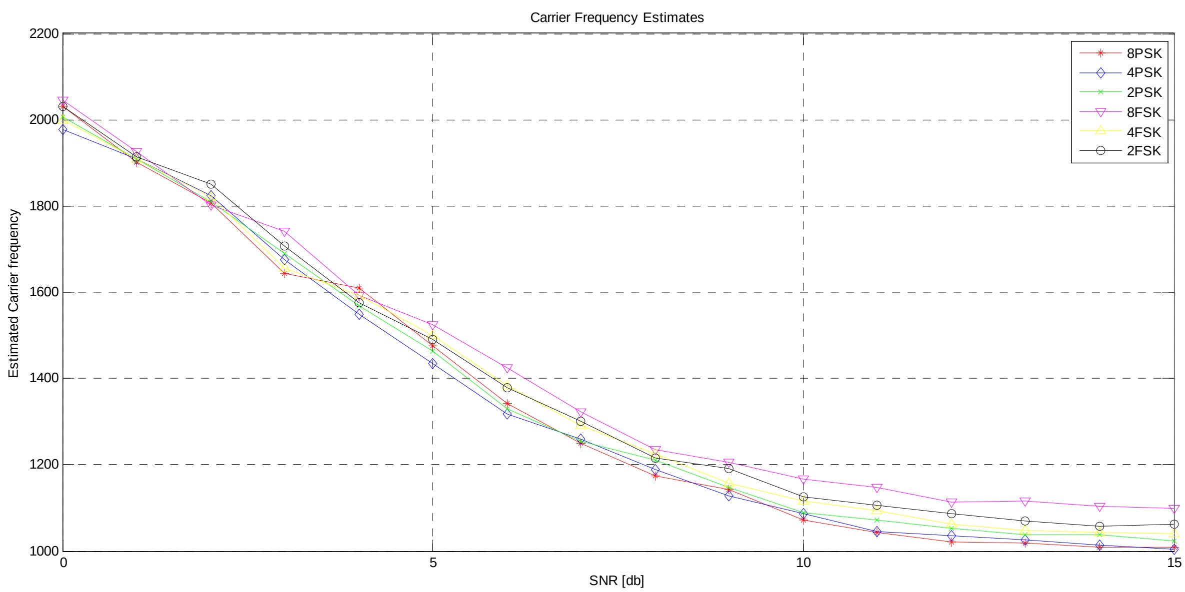


Fig 2.5 Carrier frequency estimates for PSK & FSK for carrier signal with unit power

2.6 Modulation Level Estimate of SC Modulations -PSK

The modulation level estimate of SC modulation is case of PSK modulation is ascertained by comparing the measured histogram of phase deviation with the theoretical one as in [8]. This method is not very effective and a better way out is by the process of phase deviation [4] histogram. The signal of interest is obtained and analytic representation of the signal is derived. Since the received signal is a modulated signal thus comprises of a carrier signal. The carrier is thus required to be removed by complex mixing and the phase samples are extracted. Since the carrier frequency has already been calculated as above the complex mixing is just the multiplication of the carrier signal and the signal from a local oscillator with frequency equal to the carrier frequency. The method in [4] assumes that the carrier frequency is known in advance. The output is then low pass filtered and what remains of the signal is the phase samples and the added noise (AWGN). The phase samples thus obtained are required to make a phase histogram with ‘N’ bins. The ‘N’ points of the histogram are thus operated upon by DFT.

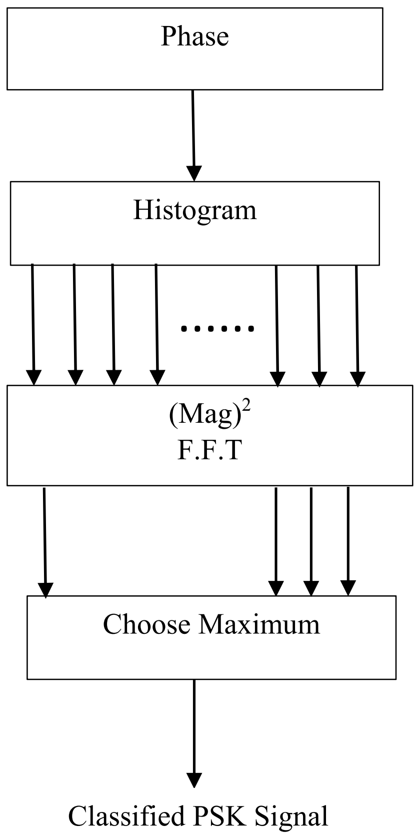


Fig 2.6 Algorithmic Description of Classification of PSK Signal Using Phase Histogram

The classification decision is made on the basis of the maximum DFT magnitude for the bins of interest. N is kept at least four times the highest symbol number.

$$\alpha_n \in MAX[D(\alpha_n)] \tag{2.6}$$

α_n is the number of states in the nth PSK signal e.g. if 1,2,4 & 8 PSK is to be classified then the bins 1,2,4,8 are examined and whichever bin returns maximum magnitude the signal is classified to be as that PSK.

2.6.1 FSK

For FSK the histogram of frequencies is calculated and the number of different levels is counted. However for a signal classified as FSK a DFT of the signal itself tells the level of modulations.

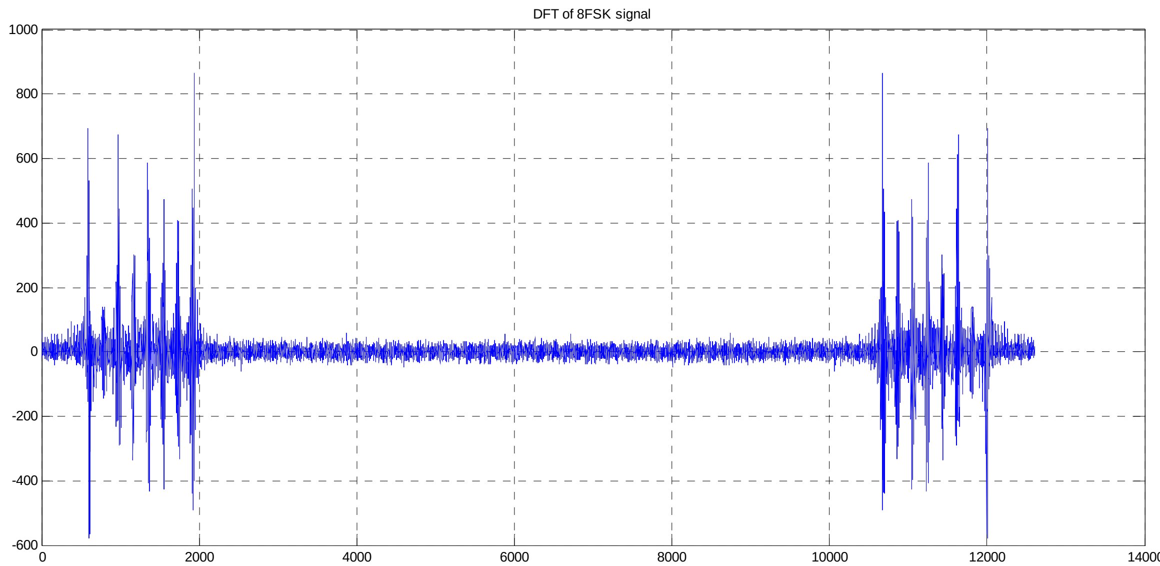


Fig 2.7 DFT of 8FSK signal

2.6.2 ASK & QAM

For ASK and QAM modulations the histogram of is computed from the centered and normalized amplitude sequence as derived in equation 2.3b.

2.7 OFDM Signal Identification

OFDM transmission has two grave issues in particular. One being the problem of inter carrier interference which occurs due to loss of orthogonality amongst its many subcarriers. Second is the inter symbol interference, and both are a cumulative effect of transmission over dispersive channels. A scheme of adding cyclic extension where a part of the transmitted symbol is either added to front or to the tail of the next symbol is formulated. The duration of this cyclic extension prefix/suffix becomes a characterizing parameter for modulation identification.

$$r_{ss}(m) = E\{s(k)s(k+m)\} = \begin{cases} \sigma_s^2 + \sigma_n^2, & m = 0 \\ \sigma_s^2 P\{k \in I\}, & m = N_s \\ 0, & elsewhere \end{cases} \tag{2.7}$$

Where σ_s^2 is signal variance and σ_n^2 is noise variance

Then the cyclic extension percentage of the total symbol duration is estimated as

$$\frac{r_{ss}(N_s)}{r_{ss}(0)} = P\{k \in I\}\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} = \frac{\widehat{T}_{ce}}{\widehat{T}_{ce} + \widehat{T}_s}\frac{SNR}{SNR+1} \tag{2.7b}$$

$\widehat{T}_{ce}$ & $\widehat{T}_s$ are the estimated cyclic extension duration and estimated overall symbol duration respectively.

Thus the modulation classification can be done by comparing the $\widehat{T}_{ce}$ & $\widehat{T}_s$ with the values required for a particular standard.

Chapter 3

MODULATION CLASSIFICATION USING WAVELET TRANSFORM

This chapter gives a brief description of digital modulation techniques and signal model used for simulation. It also goes on to discuss the wavelet transform [15]-[16] technique used for modulation classification.

3.1 Digital Modulation

A general model for a modulated signal is taken as shown where $x(t)$ the modulated signal which

$$x(t) = s(t) + n(t) \tag{3.1}$$

$$s(t) = \tilde{s}(t)e^{j(\omega_c k + \theta_c)}$$

depends on modulation type and $n(t)$ is additive white Gaussian noise. The signal models thus has been used i.e. $\tilde{s}(t)$ are [24] as given below.

$$\tilde{s}_{PSK}(t) = \sqrt{S}\sum_i e^{j\phi_i} u_T(t - iT) \qquad (3.1a)$$

$$\phi_i \in \left\{\frac{2\pi}{M}(m-1)\right\}_{m=1}^{M}$$

$$\tilde{s}_{FSK}(t) = \sqrt{S}\sum_i e^{j(\omega_i + \phi_i)} u_T(t - iT) \qquad (3.1a)$$

$$\omega_c \in \{\omega_1, \omega_2, \omega_3, ...\omega_M\} \qquad \phi_i \in \{0, 2\pi\}$$

$$\tilde{s}_{QAM}(t) = \sum_{i=1}^{N}(A_i + B_i)u_T(t - iT) \qquad (3.1c)$$

$$A_i, B_i \in \{2m - 1 - M, m = 1, 2, 3.........., M\}$$

Here $s(t)$ is modulated complex waveform ω_c is intermediate frequency after down conversion θ_c is the carrier phase and u_T is unit height rectangular function with support [*0, T*].

3.2 Wavelet Transform

Wavelet transform is a powerful tool to for analyzing non-stationary signals. The continuous wavelet transform (CWT) of signal $s(t)$ is given as

$$CWT(a,\tau) = \int s(t)\psi_a^*(t)dt \tag{3.2}$$

$$= \frac{1}{\sqrt{a}}\int s(t)\psi^*\left(\frac{t-\tau}{a}\right)dt \tag{3.2a}$$

where a is the scale, τ is the translation and the superscript $*$ denotes the complex conjugate. The baby wavelets $\psi_a(t)$ are time shifted and scaled versions of the mother wavelet $\psi(t)$. The wavelet means a small wave which refers to its small size or the finite length is thus compactly supported and wave implies that it is oscillatory in nature.

3.2.1 Conditions for Transient Detection

For a function $\psi(t)$ to best detect the transients in function $f(t,\gamma(t))$ on account of change in parameter $\gamma(t)$ it must satisfy following conditions [25].

C1: If no transients occur, WT transform should yield a constant output

$$CWT(a,\tau/\gamma) = \kappa(\text{constant}) \tag{3.2b}$$

C2: If the wavelets covers the transients, and $\gamma(t)$ changes at time T the CWT should yield an output which is distinguishable from a constant value as yielded in the equation above.

C3: The best choice of a wavelet would have the difference between the CWT and the constant $\kappa(a)$ as given below maximized

$$D = \left|CWT(a,T) - \kappa(a)\right| \tag{3.2c}$$

For digital implementation, the integral in eqn. 2.2a can be written as

$$WT(a,n) = \frac{1}{\sqrt{a}}\sum_k s(k)\psi^*\left(\frac{k-n}{a}\right) \tag{3.2d}$$

3.3 The HAAR Wavelets

The discrete time Haar wavelets can be given as

$$\frac{1}{\sqrt{a}}\psi\left(\frac{k}{a}\right)=\begin{cases}1/\sqrt{a} & \text{when } k = -a/2, -a/2+1,\dots\dots\dots\dots, -1\\ -1/\sqrt{a} & \text{when } k = 0,1\dots\dots\dots\dots\dots, a/2+1\\ 0 & \text{otherwise}\end{cases} \tag{3.3}$$

Now let us consider a sampled PSK symbol within $(i-1)T < k < (i+1)T$. The phase change is denoted at $k = iT$.

$$s(k)=\begin{cases}\sqrt{S}e^{i(\omega_c k+\theta_c)}e^{i\phi} & (i-1)T \le k \le iT\\ \sqrt{S}e^{i(\omega_c k+\theta_c)}e^{i(\phi_i+\alpha)} & iT \le k \le (i+1)T\end{cases} \tag{3.3a}$$

When $(i-1)T + a/2 \le n \le iT - a/2$ i.e. wavelet is within the symbol period, it doesn't cover the phase transition the transform equation becomes

$$WT_P(a,n)=\sqrt{\frac{S}{a}}\left\{\sum_{k=n-a/2}^{n-1} e^{i(\omega_c k+\theta_c+\phi_i)} - \sum_{k=n}^{n+a/2-1} e^{i(\omega_c k+\theta_c+\phi_i)}\right\} \tag{3.3b}$$

Which can be solved using summation formula of geometric series. Taking magnitude we get

$$\left|WT_P(a,n)\right| = 2\sqrt{\frac{S}{a}}\left|\frac{\sin^2(\omega_c a/4)}{\sin(\omega_c/2)}\right| \tag{3.3c}$$

Which is clearly independent of 'n'. At an instant when $n=iT$, i.e. wavelet covers the phase change the equation for a wavelet transform becomes

$$WT_P(a,n)=\sqrt{\frac{S}{a}}\left\{\sum_{k=n-a/2}^{n-1} e^{i(\omega_c k+\theta_c+\phi_i)} - \sum_{k=n}^{n+a/2-1} e^{i(\omega_c k+\theta_c+\phi_i+\alpha)}\right\} \tag{3.3d}$$

$$\left|WT_P(a,n)\right| = 2\sqrt{\frac{S}{a}}\left|\frac{\sin(\omega_c a/4)\sin(\omega_c a/4+\alpha/2)}{\sin(\omega_c/2)}\right| \tag{3.3e}$$

It clearly indicates the change in magnitude because of a phase change.

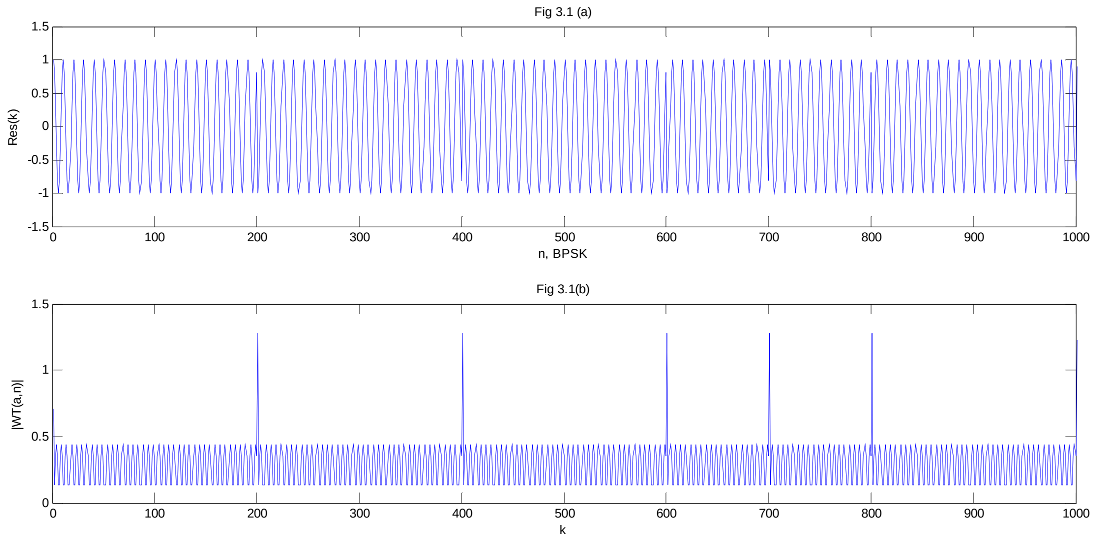


Fig. 3.1(a) BPSK modulated signal and Fig 3.1(b) Haar wavelet magnitudes for BPSK.

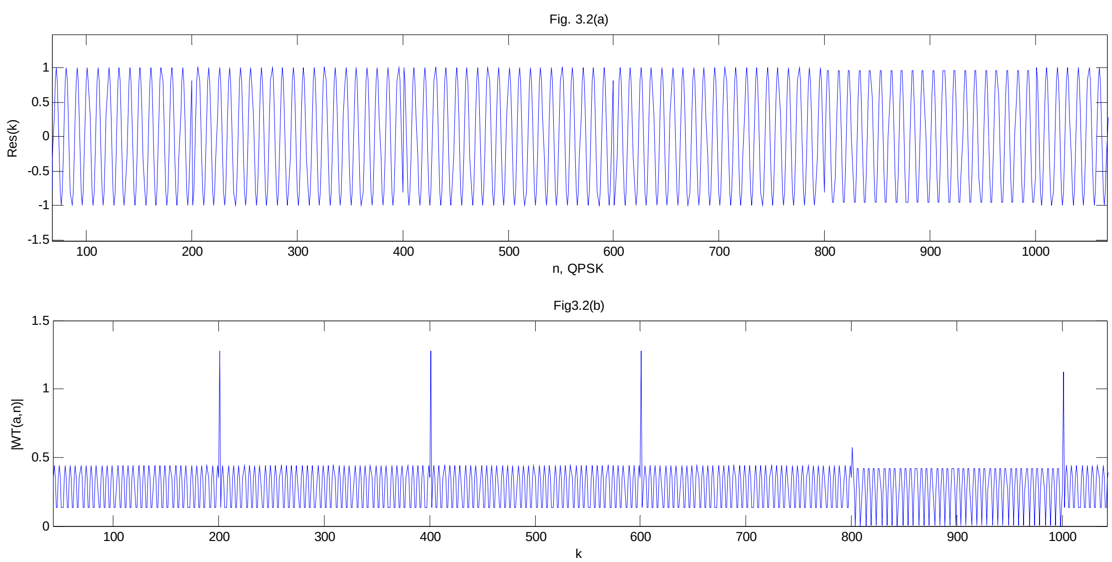


Fig 3.2(a) QPSK modulated signal, Fig. 3.2(b) |HWT| for QPSK.

The difference in the peaks of wavelet magnitudes are attributed to the different phase changes in BPSK and QPSK signals i.e. different PSK types will give different set of peaks. Another point worth noting is that the DC level is the same in the magnitude plot as the frequency remains constant throughout, it is only the phase of the signal which undergoes the transition. Proceeding as in case of a PSK signal the wavelet transform of a FSK modulated signal can be calculated.

The following figures bring out the difference between |HWT| magnitudes of FSK and PSK signals.

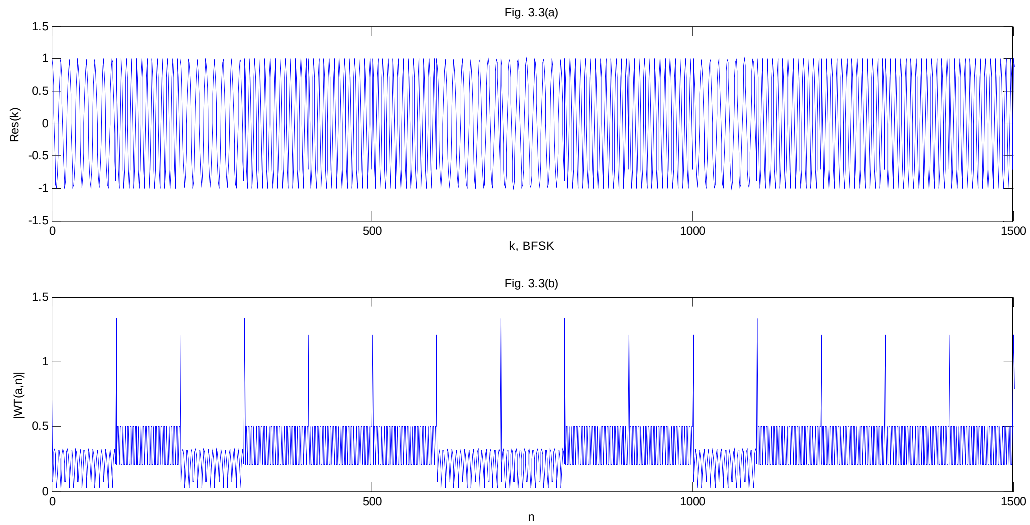


Fig. 3.3(a) BFSK modulated signal and Fig. 3.3(b) |HWT| for BFSK.

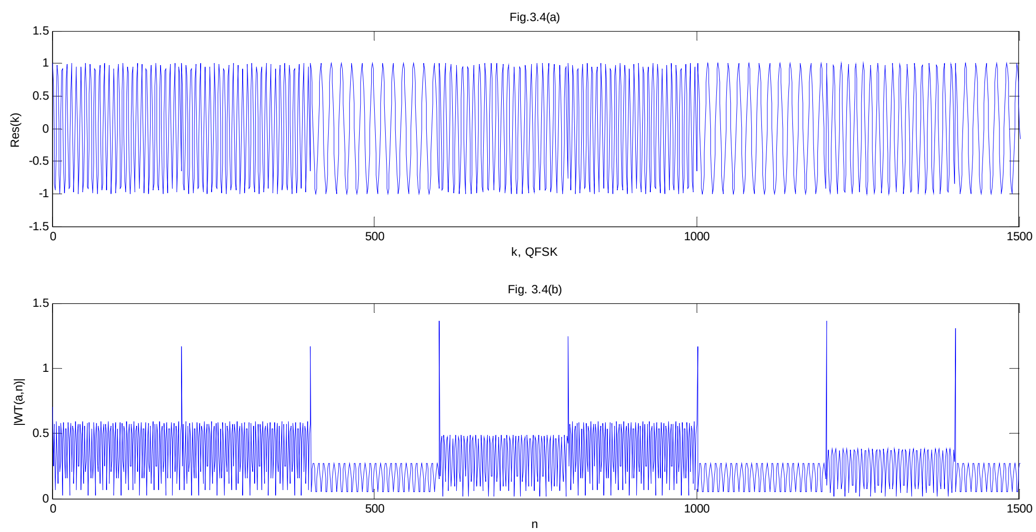


Fig. 3.4(a) QFSK modulated signal and Fig. 3.4(b) |HWT| for QFSK.

The plot of Haar wavelet magnitude for FSK signal resembles a multistep function with levels equal to the number of modulation frequencies. M-ary FSK can be identified determining the number of distinct DC levels in the |HWT| of FSK signal. Another approach [16] evaluates the |HWT| of amplitude normalized signals.

Amplitude normalization and corresponding amplitude normalized functions are of the form

$$\bar{s}(t) = \frac{\tilde{s}(t)}{|\tilde{s}(t)|} \tag{3.3f}$$

$$\bar{s}_{FSK}(t) = \sum_{i=1}^{N} e^{i(\omega_i k+\theta_i)} u_T(t - iT)$$

$$\bar{s}_{PSK}(t) = \sum_{i=1}^{N} e^{j\phi_i} u_T(t - iT)$$

$$\bar{s}_{QAM}(t) = \sum_{i=1}^{N} e^{j(\arctan\frac{B_i}{A_i})} u_T(t - iT)$$

Amplitude normalization does not affect FSK and PSK as these two schemes are independent of amplitude variations. However in QAM amplitude variations would disappear because of amplitude normalization. Thus QAM has a constant |HWT| with peaks which arise due to phase change.

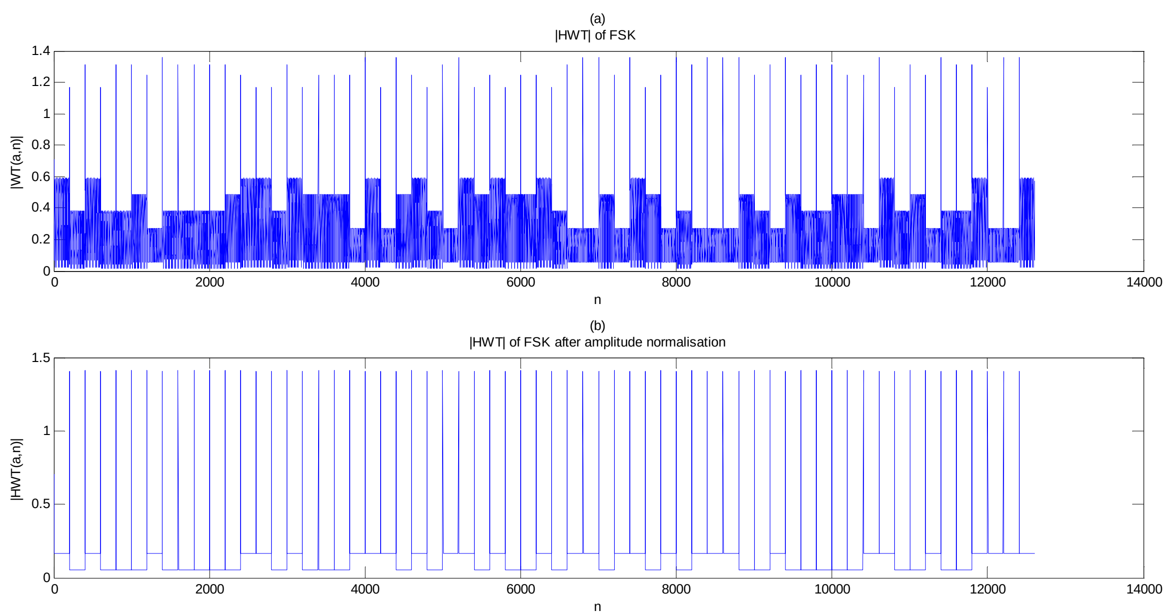


Fig. 3.5 |HWT| (a) of FSK and (b) |HWT| of FSK with amplitude normalization.

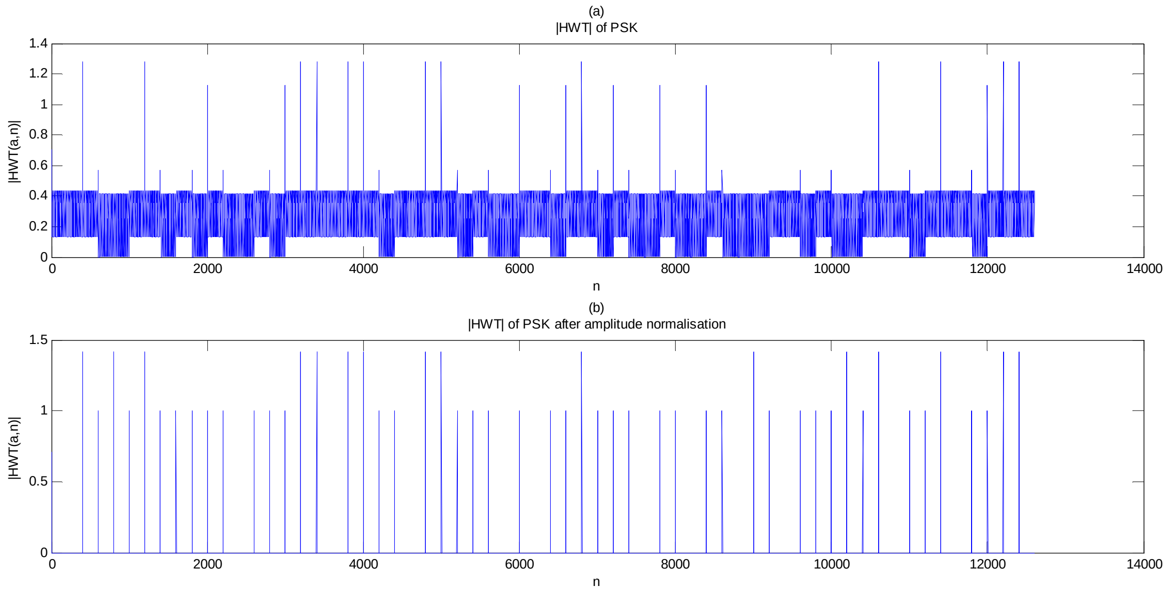


Fig. 3.6 |HWT| (a) of PSK and (b) |HWT| of PSK with amplitude normalization.

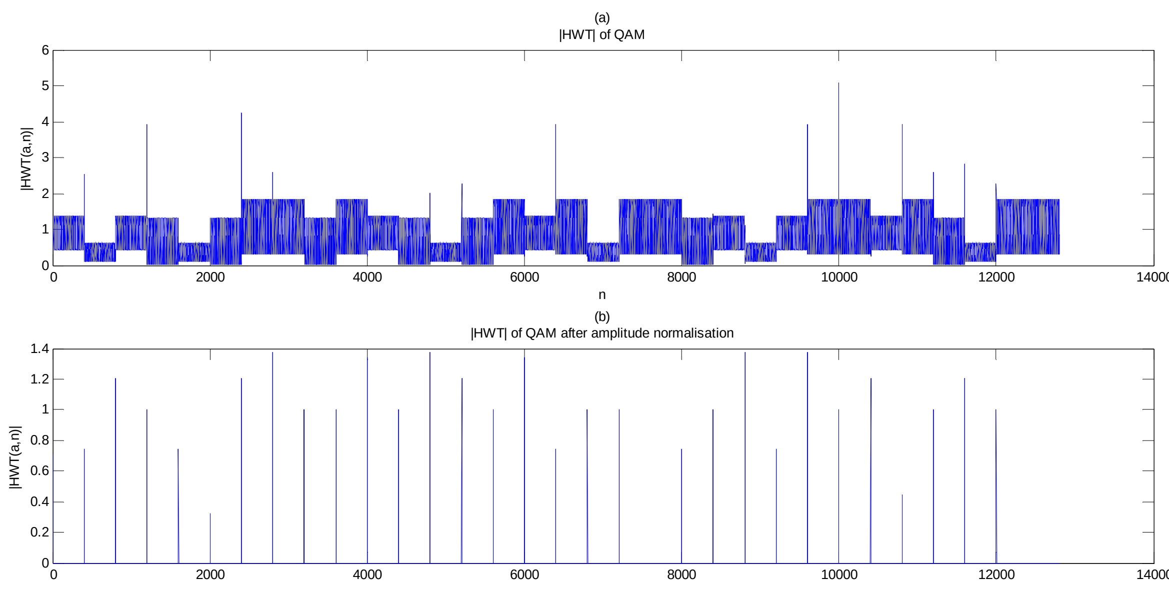


Fig. 3.7 |HWT| (a) of QAM and (b) |HWT| of QAM with amplitude normalization.

3.4 PSK vs FSK vs QAM

Ignoring the peaks of |HWT| of PSK is a constant or has a one DC level whereas the FSK continues to be a multistep function. The variance of a constant is zero while for a multistep function it is larger than zero. Thus a discriminating feature is the variance of |HWT| with and without amplitude normalization after removing peaks by median filtering.

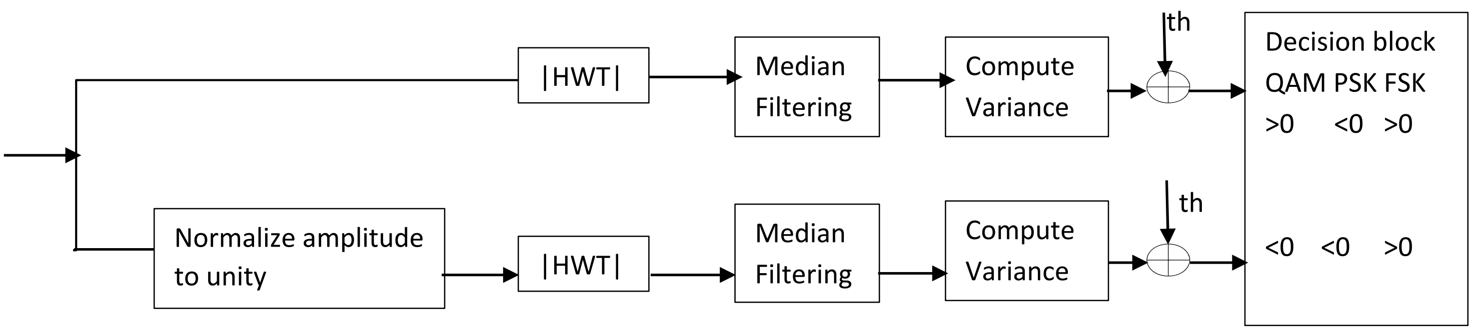


Fig. 3.8 Block diagram for the classifier

The variance is calculated for PSK as the theoretical variance of the median filter output is unknown as it depends on unknown modulation frequencies on the other hand for PSK it can be calculated as its ideal output is constant with some random noise added to it. For a given median filter output the variance is calculated as

$$\Lambda = \frac{1}{N_a}\sum_{n=0}^{N_a-1} v(a,n)^2 - \left\{\frac{1}{N_a}\sum_{n=0}^{N_a-1} v(a,n)\right\}^2 \tag{3.4}$$

3.5 Symbol Time Estimation (PSK)

The symbol time estimation procedure exploits the periodicity of transients. The WT magnitude is auto correlated which in effects reduces the noise and the peaks due to the transients become more apparent. The first step towards the estimation of estimation of symbol time is by evaluating the autocorrelation function.

$$R(a,l) = \frac{1}{N}\sum_{n=0}^{N-|l|-1} \left\{|WT_P(a,n)| - |\tilde{WT}_P(a)|\right\}\left\{|WT_P(a,n+l)| - |\tilde{WT}_P(a)|\right\} \tag{3.5}$$

$$where, |\tilde{WT}_P(a)| = \frac{1}{N}\sum_{n=0}^{N-|l|-1} |WT_P(a,n)|$$

N is total number of the WT magnitudes available.

The peaks to qualify to be the peaks due the transients in the modulation signal must exceed in their magnitude to a threshold. The threshold is set proportional to magnitude variance and is given as

$$TH(l) = \mu\sqrt{\frac{N-|l|}{N}}R(a,0) \qquad (3.5a)$$

Here μ is a positive constant to control the probability of false peak detection. Clearly the threshold is lag dependent *'l'* as the no of samples taken to calculate the autocorrelation depends on '*l*'.

After checking each |WT| magnitude against the set threshold and the peaks thus segregated the difference between the successive peaks is generated. The mode of the histogram of this new sequence gives the symbol time estimate. The same method as discussed above is employed for FSK as well.

3.6 Level of Modulation in M-ary PSK and M-ary FSK

The signal is further classified as M-ary PSK if the histogram of the WT peaks has *M/2* to *M-1* different peaks, and as M-ary FSK if the histogram has *M/2* to *M* different peaks.

Chapter 4

SIMULATIONS & RESULTS

All simulation have been carried out in MATLAB (R2009a). The results are enumerated in succeeding paragraphs.

4.1 Simulation Parameters

The methods employed have been tested for varying SNR. The modulated signals were characterized as per the models discussed in previous sections. The carrier frequency 'f_c' , sampling frequency 'f_s' the symbol rate was taken to be 1kHz, 10kHz and 100Hz respectively. The frequency deviations for Mary-FSK modulation has been kept at 0.5 f_c for 2FSK, 0.25 f_c for 4FSK and at 0.8 f_c for 8 FSK. The SNR for all the modulation schemes has been varied from 0 to 15 dB in order to ascertain and validate the results put forth. The number of samples for a signal taken are to the tune of 12600, with number of bits 'N' over 100.

4.2 ZCSS Based Methods

The method has returned quite accurate estimation of some important parameters and the carrier frequency estimations. Following are the carrier frequency estimates for varying signal power (3/2, 5/2, 7/2)

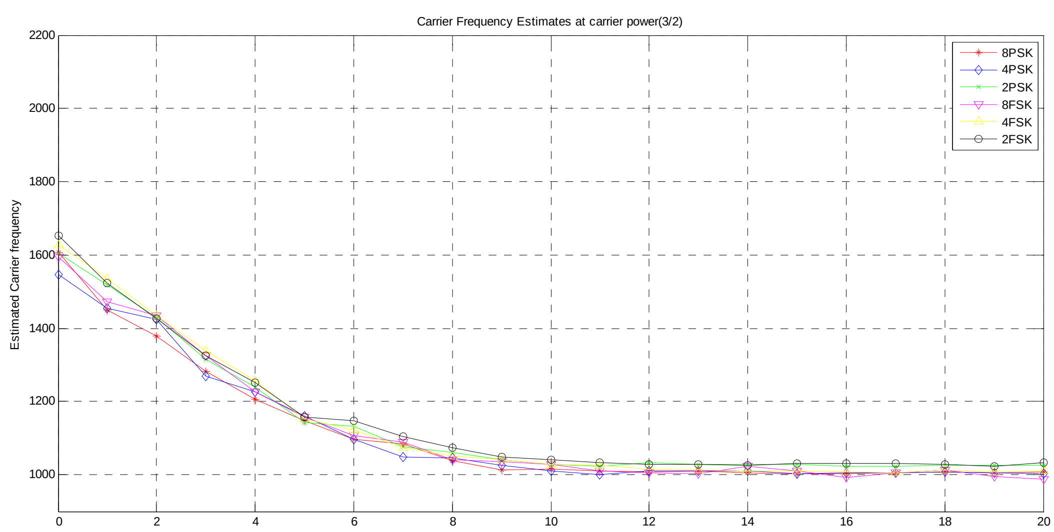


Fig 4.1 Carrier frequency estimates by ZCSS method for Carrier power (3/2)

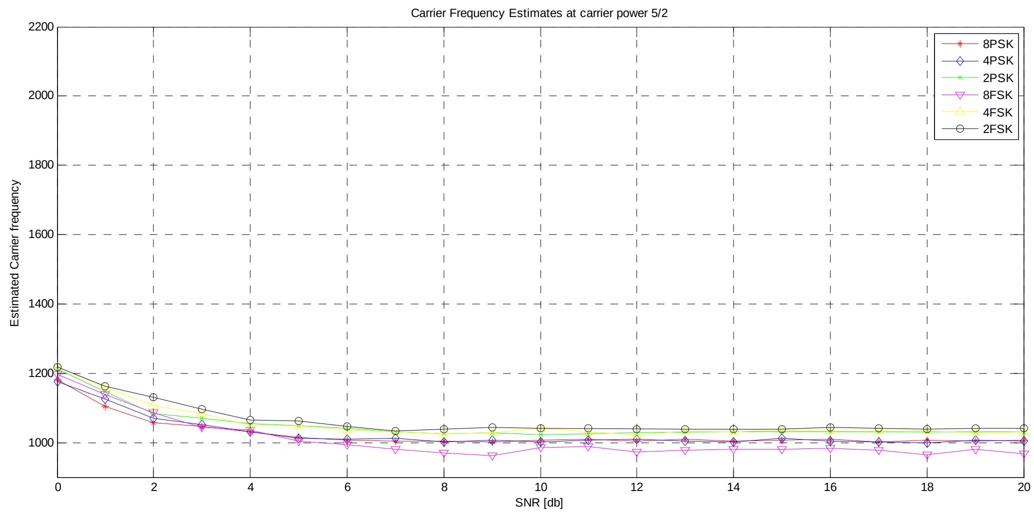


Fig 4.2 Carrier frequency estimates by ZCSS method for Carrier power (5/2)

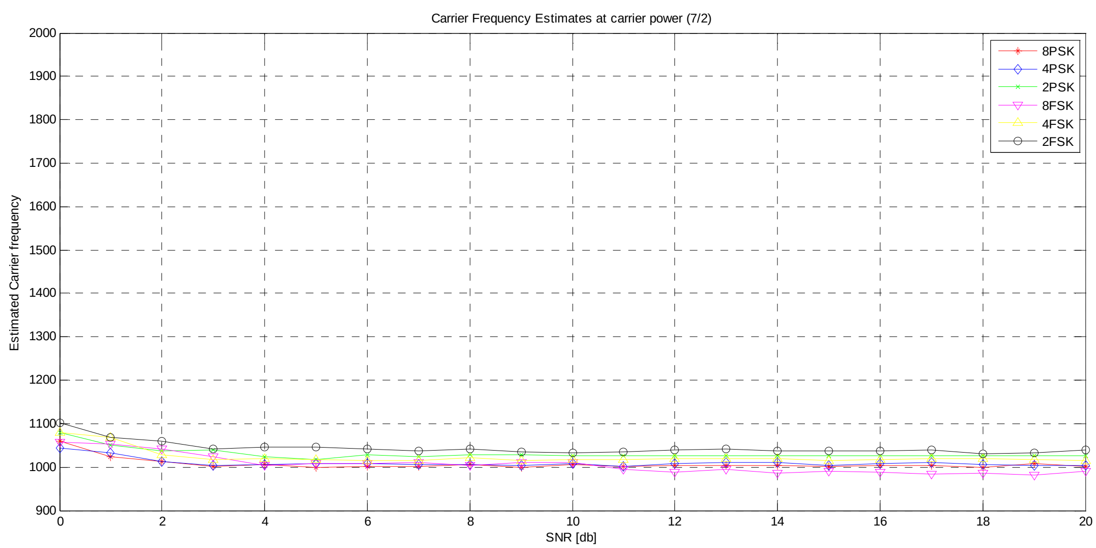


Fig 4.3 Carrier frequency estimates by ZCSS method for Carrier power (7/2)

Below are tabulated the frequency estimates for PSK and FSK signals as calculated using ZCSS method for carrier power as unity. It is worth mentioning here that the accuracy of the frequency estimates increases with a signal of higher power & SNR and the same has been validated by using the above mentioned method.

SNR[dB]	2PSK	4PSK	8PSK	2FSK	4FSK	8FSK
0	1995.8	2016.4	2012.7	1093.8	1065.9	1996.6
4	1536.8	1534.5	1538.9	1042.9	1016.6	1565.6
8	1179.4	1194.1	1160.1	1035.1	1009.8	1211.5
12	1044.2	1017.1	1028.8	1040.0	1006.9	1038.9
16	1029.2	1004.6	1005.0	1039.3	1005.5	982.8
20	1022.5	1004.7	1005.7	1041.2	1007.0	973.5

TABLE I

FREQUENCY ESTIMATES FOR CARRIER SIGNAL WITH UNIT POWER

4.2.1 SC Modulation Level Estimates -PSK

For PSK modulation level estimates the method of phase deviations [4] was employed and the results were accurate. The details of the method are explained thoroughly in chapter 2. The assumption which this method requires is that the carrier frequency is estimated correctly. The results thus obtained are tabulated as below.

SNR[dB]	2PSK	4PSK	8PSK
0	100	100	100
5	100	100	100
15	100	100	100
20	100	100	100

TABLE II

SUCCESS PERCENTAGE OF CLASSIFICATION OF PSK SIGNALS USING PHASE DEVIATION METHOD

4.2.2 FSK

A zero-crossing interval histogram is obtained from the sample distribution where number of hills in the histogram, N_F represents the number of states. As $N_F \leq 2^D$, the modulation type MFSK is reported with $M = 2^D$. However it is generally not very intuitive to classify based on the histogram of frequencies instead FFT of the signal may be used.

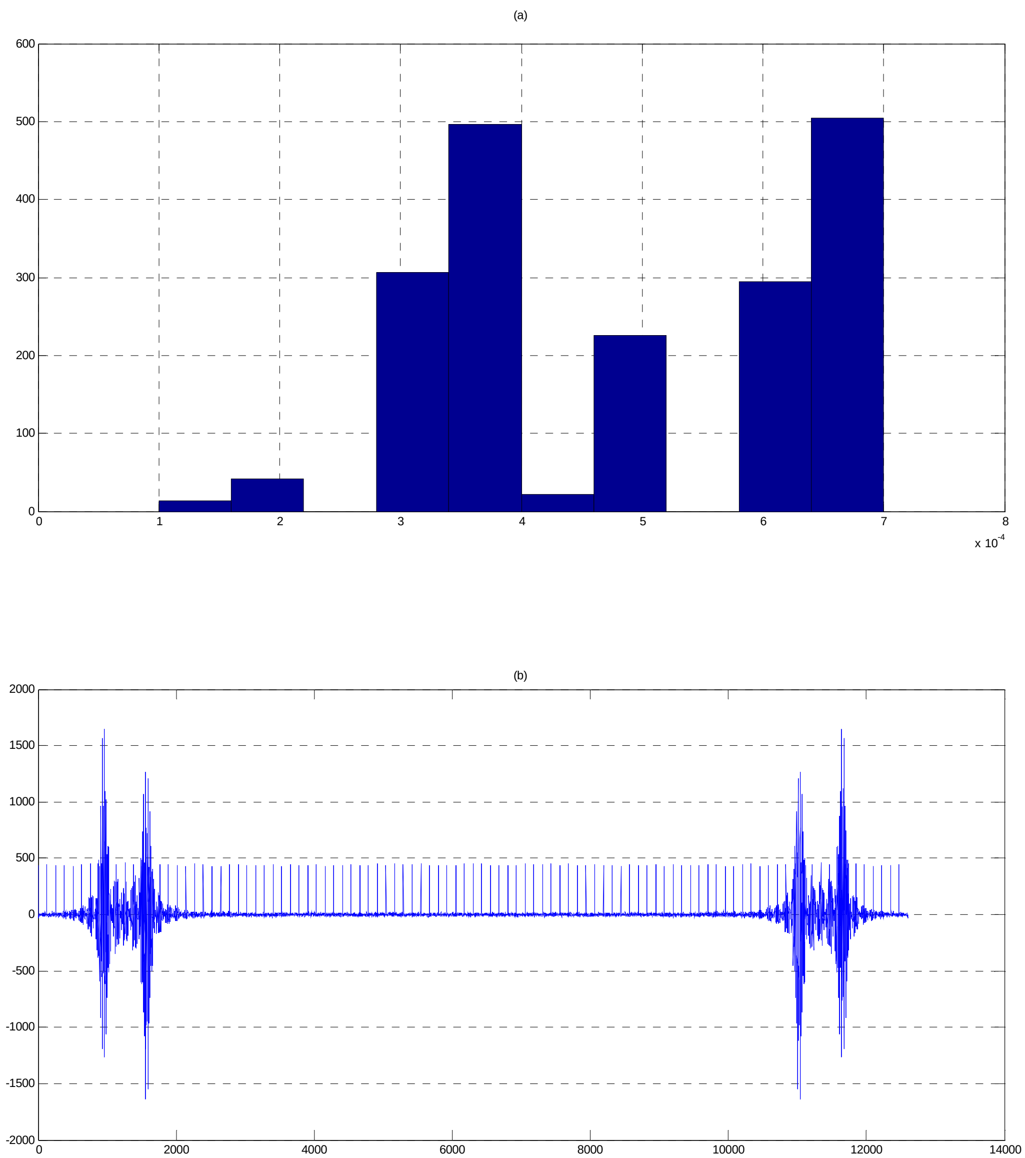
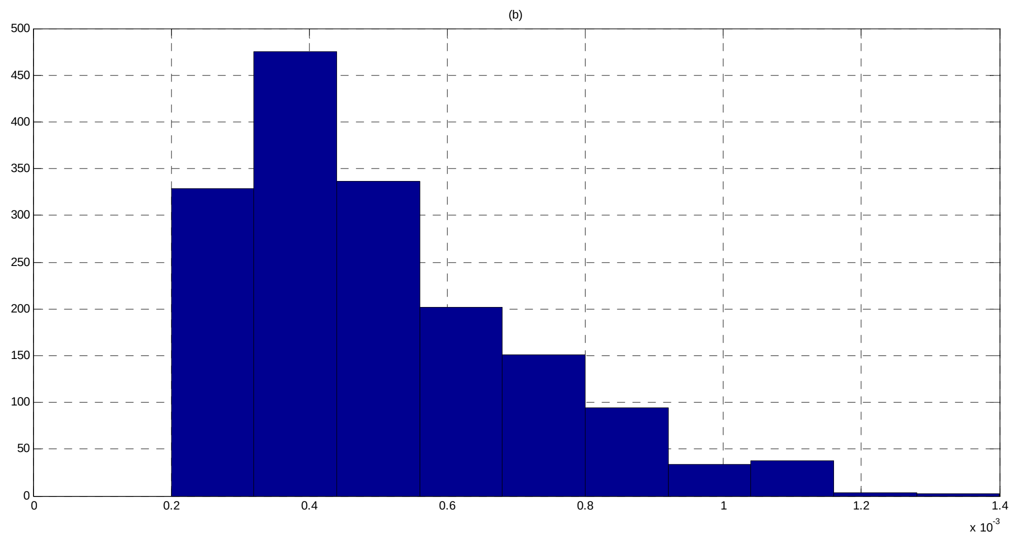


Fig 4.4, 2FSK signal (a) Histogram of frequencies (b) DFT for the signal



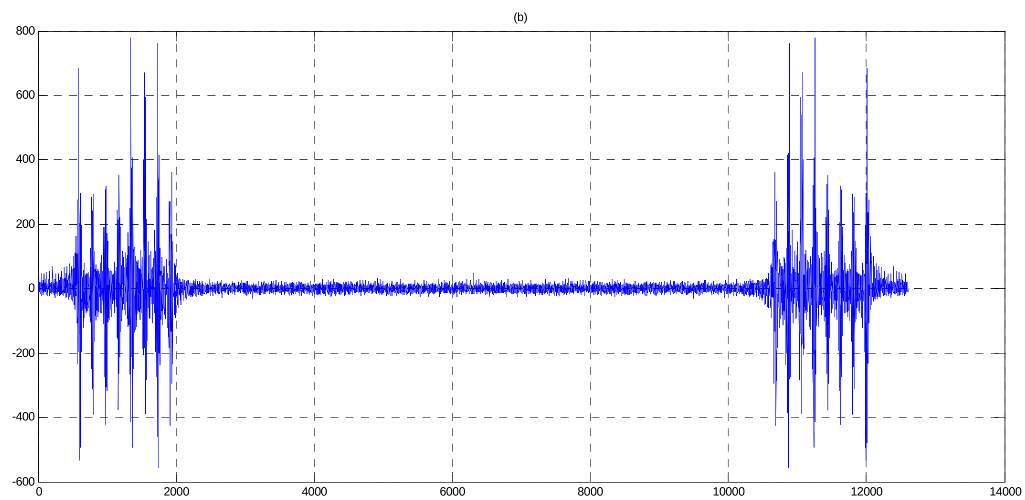


Fig 4.5, 8FSK signal (a) Histogram of frequencies (b) DFT for the signal

4.2.2 QAM & ASK

The level of modulation for QAM and ASK is found by counting the hills in the histogram obtained from the normalized centered amplitude sequence as shown below.

$$a_{cn}[k] = \frac{\mathrm{a[k]}}{\mathrm{m_a}} - 1 \qquad \textit{where } m_a \textit{ is mean of } \mathrm{a[k]}$$

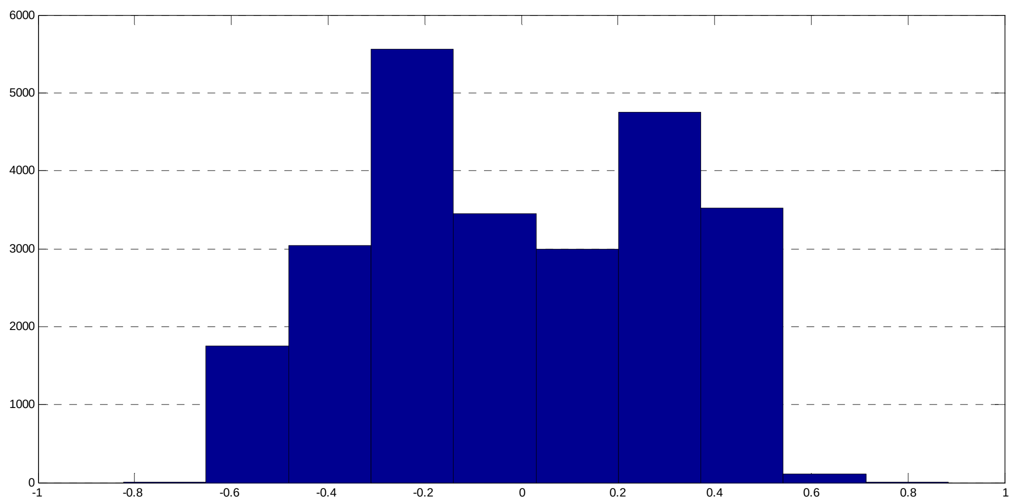


Fig 4.6 Histogram for normalized centered amplitude of a 64 QAM signal

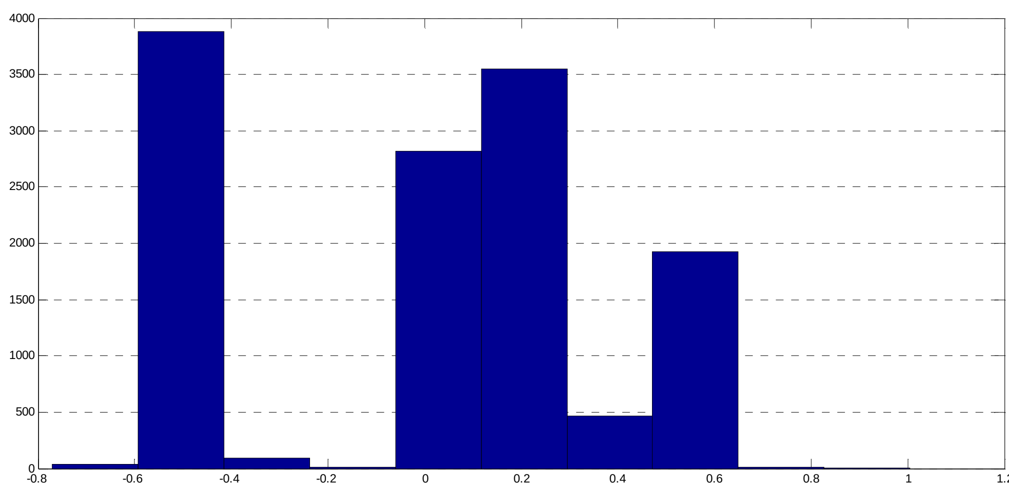


Fig 4.7 Histogram for normalized centered amplitude of a 16 QAM signal

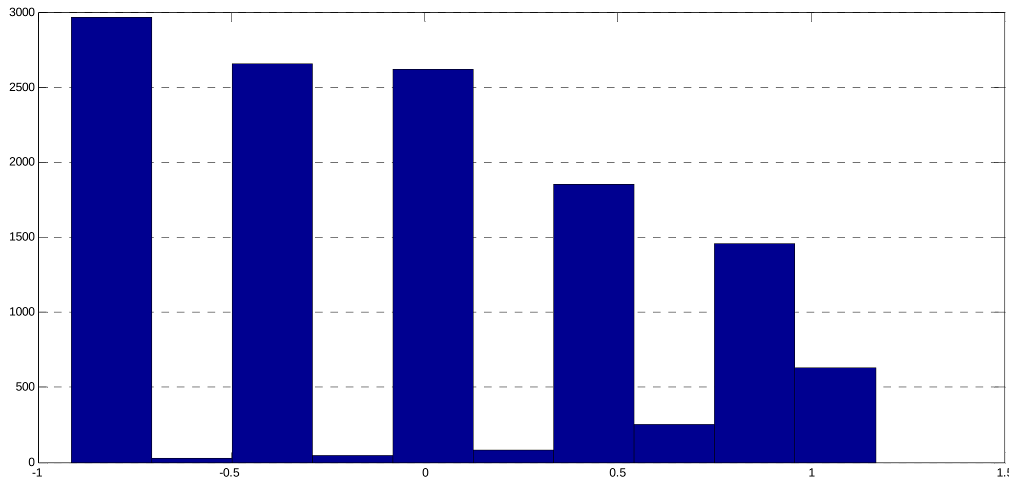


Fig 4.8 Histogram for normalized centered amplitude of ASK8 signal

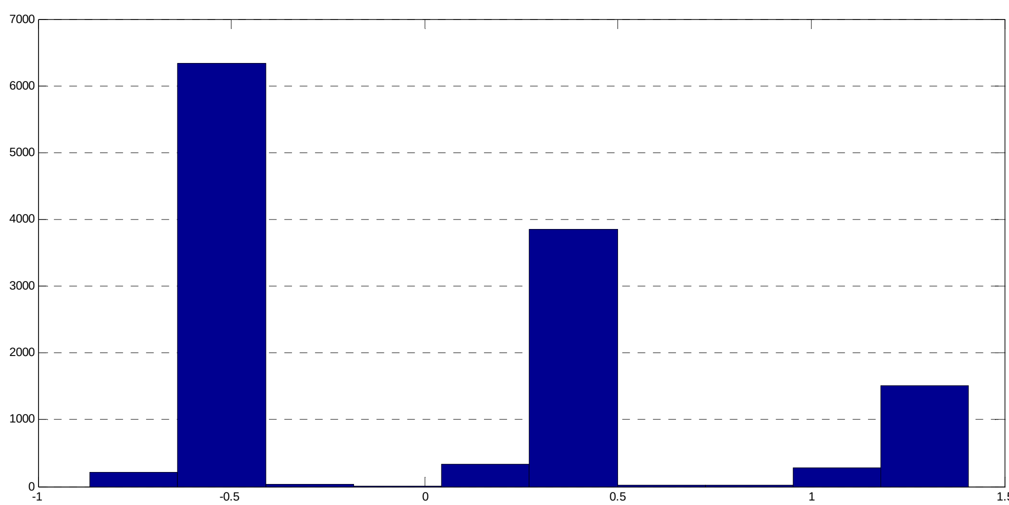


Fig 4.9 Histogram for normalized centered amplitude of ASK4 signal

4.3 Wavelet Transform Based Method

The wavelet based approach in [15]-[16] calculates the variance of the median filtered output of the |HWT| magnitudes thus obtained and classifies to separate FSK, PSK. The QAM can also be discriminated against by observing the |HWT| of the normalized amplitudes.

Variance	**Modulation scheme**		
	QAM	**PSK**	**FSK**
Variance without Normalization (Va)	0.1276	0.0024	0.0141
Variance with amplitude normalization (Van)	0.3150×10^{-35}	0	0.0030

TABLE III

VARIANCE OF MEDIAN FILTERED |HWT| WITH AND WITHOUT NORMALISATION

The median filter is of length 5, with all the signal parameters remaining the same as mentioned in section 4.1. The modulation schemes considered her are 16QAM, PSK4 & FSK4. The values of variances are calculated and the threshold is selected. The decision block then on basis of the difference between the threshold and the variance classifies the modulation type.

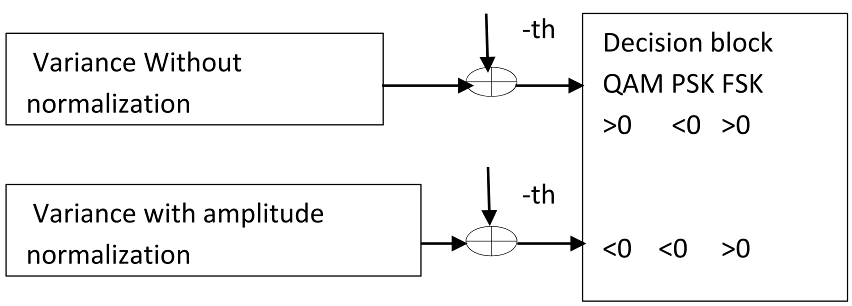


Fig. 4.10 Decision block to classify a received signal on basis of variance

The modulation levels in Mary-PSK is determined by either finding the DFT of phase histogram by matching the histogram of |HWT| peaks with theoretical PDF for different values of M.

4.3.1 Classification as M-ary PSK and M-ary FSK

The histogram of the peaks at the time of transients gives the modulation level for FSK and PSK. If the number of peaks is between *M/2* to *M-1* the signal is classified as M-ary PSK and if the number of peaks is between *M/2* to *M* the signal is classified as M-ary FSK. The results thus found are enumerated as below.

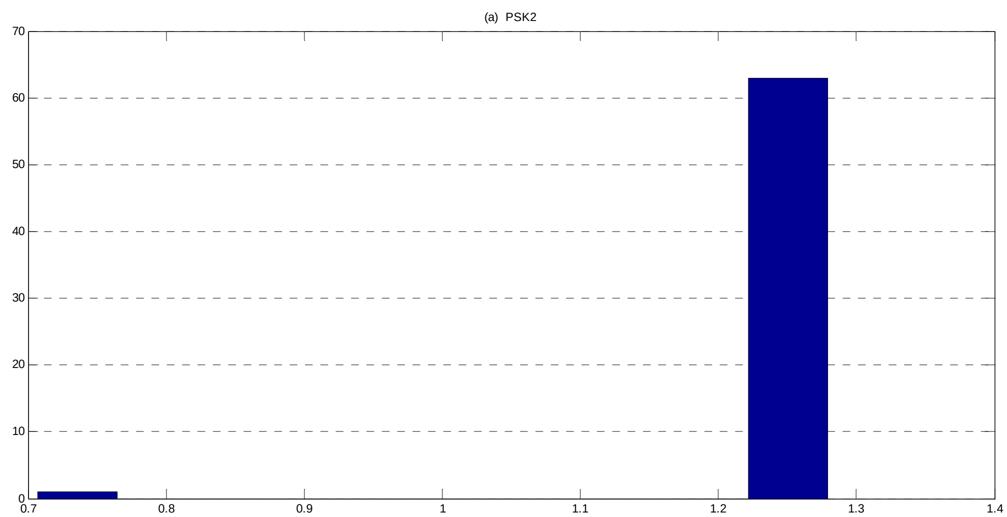


Fig. 4.11(a) Histogram of peaks for PSK2 signal

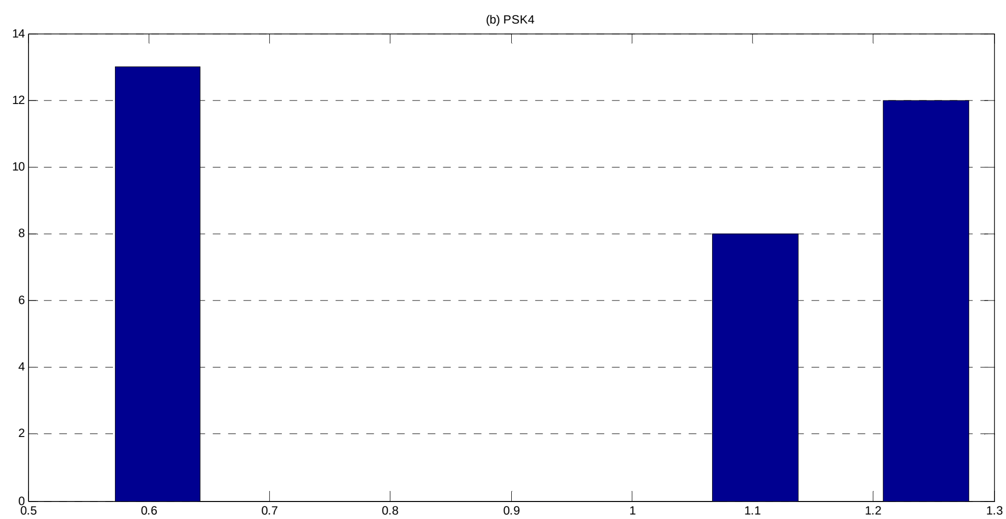


Fig. 4.11(b) Histogram of peaks for PSK4 signal

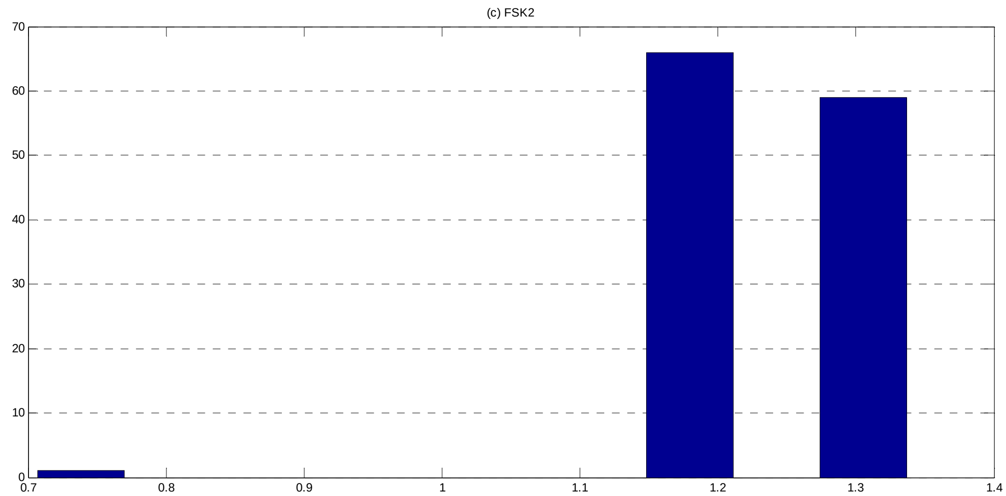


Fig. 4.11(c) Histogram of peaks for FSK2 signal

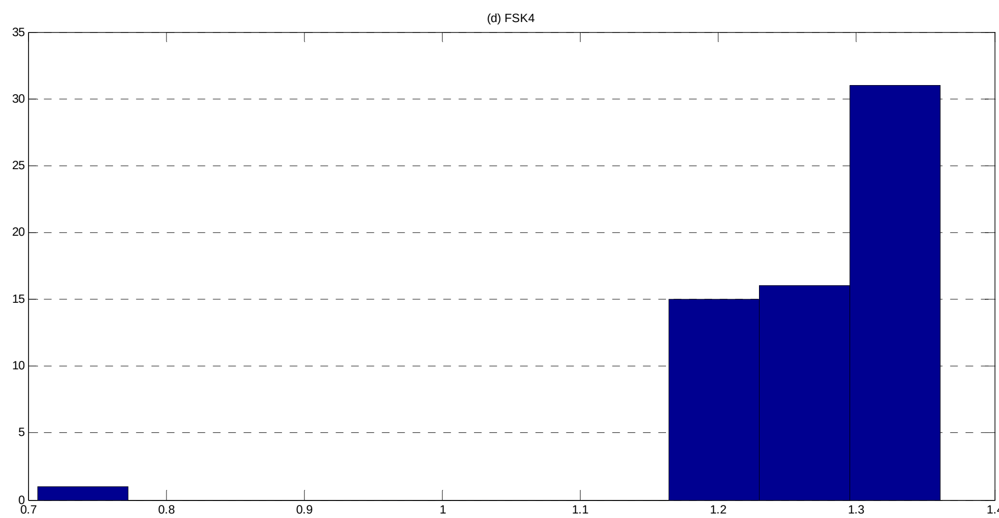


Fig. 4.11(d) Histogram of peaks for FSK4 signal

Chapter 5

FUTURE SCOPE OF WORK

With the advent technologies such as realizable software defined radios which can dynamically adapt to communication channels the digital modulation classification is a promising research area. It has opened up avenues for cutting edge research where requirement is of real time classification algorithms**.**

5.1 Future Scope

In blind environments where the classifier is supposed to classify without any prior knowledge the two important aspects that are needed to be taken care of are carrier frequency and symbol time. The two approaches discussed perform favorably for different SNR conditions but advancement is required in field of a comprehensive algorithm to be present which classifies all possible modulation with no a priori information available. Further to identify OFDM based modulation rather than comparison with standard framework is required. In wavelet transform methods it is an inherent requirement of the system to oversample the data. Further no concrete method has been shown to decide on about the scale of wavelet transform to be employed. The wavelet transform can give more accurate estimations of instantaneous parameters which needs to be explored further. The methods still would rely on a human interface as the recognition is based on the shape of some sequence or the about counting the peaks in a histogram. The use of wavelet transform can give more accurate estimations of instantaneous parameters which can be explored further.

It is therefore imperative that efforts be made in the directions of obviating the human element and comprehensive framework be designed reducing dependence of a human operator and thus increasing efficiency in real time environments.

References

1. A. K. Nandi and E. E. Azzouz, "Automatic Analogue Modulation Recognition", *Signal Processing*, Vol. 46, No. 2, pp. 211-222, Oct. 1995.

2. B. Seaman and R. M. Braun, "Using Cyclostationarity in the Modulation Classification of Analogue Signals", *Proceedings of the 1998 South African Symposium on Communications and Signal Processing,* pp. 261-266, Sept. 1998.

3. R. J. Mammore, R. J. Rothaker, and C. I. Podilchuk, "Estimation of Carrier Frequency, Modulation Type and Bit Rate of an Unknown Modulated Signal", *Proceedings of the 1987 IEEE International Conference on Communications,* Vol. 2, pp. 1006-1012, June 1987

4. P. C. Sapiano, J. D. Martin, and R. J. Holbeche, "Classification of PSK Signals Using the DFT of Phase Histogram", *Proceedings of the 1995 International Conference on Acoustics, Speech, and Signal Processing,* Vol. 3, pp. 1868- 1871, May 1995.

5. S. Z. Hsue and S. S. Soliman, "Automatic Modulation Using Zero Crossing,*" Proc. Inst. Electr. Eng.*, vol. 137, no. 6, pp. 459–464, Dec. 1990.

6. S. Z. Hsue and S. S. Soliman, "Automatic Modulation Recognition of Digitally Modulated Signals," *Proceedings of the 1989 IEEE Military Communications Conference,* Vol. 3, pp. 645-649, Oct. 1989.

7. F. F. Liedtke, "Computer Simulation of an Automatic Classification Procedure for Digitally Modulated Communication Signals with Unknown Parameters," *Signal Processing,* Vol. 6, No. 4, pp. 311-323, Aug. 1984.

8. D. Grimaldi, A. Palumbo, and S. Rapuano, "An Automatic Digital Modulation Classifier for Measurement on Telecommunication Networks", *IEEE transactions in instrumentation and measurement*, Vol, 56, No. 5, October 2007.

9. W. Akmouche, "Detection of Multicarrier Modulations Using 4th-Order Cumulants", *Proceedings of the 1999 IEEE Military Communications Conference,* Vol. 1, pp. 432-436, Nov. 1999.

10. G. B. Giannakis and M. K. Tsatsanis, "Time-Domain Tests for Gaussianity and Time-Reversibility," *IEEE Trans. Signal Process*, vol. 42, no. 12, pp. 3460–3472, Dec. 1994.

11. P. Marchand et al., “Multiple Hypothesis Modulation Classification Based on Cyclic Cumulants of Different Orders,” *in Proc. IEEE Int. Conf. Acoust., Speech Signal Process.*, 1998, vol. 4, pp. 2157–2160.

12. D. Grimaldi, A. Palumbo, and S. Rapuano, “Automatic Modulation Classification and Measurement of Digitally Modulated Signals,” *in Proc. 11th IMEKOTC-4 Symp. Trends Elect. Meas. Instrum.*, 2001, vol. I, pp. 112–116.

13. S. S. Soliman and S.-Z. Hsue, "Signal Classification Using Statistical Moments," *IEEE Transactions on Communications*, Vol. 40, No. 5, pp. 908-916, May 1992.

14. A. K. Nandi and E. E. Azzouz, “Automatic Identification of Digital Modulations Types,” *Elsivier signal processing*, 47(1995) 55-69, 1995.

15. K. C. Ho, W. Prokopiw, and Y. T. Chan, “Modulation Identification of Digital Signals by Wavelet Transform,” in *Proc. MILCOM*, 1998, pp. 325–330.

16. Liang Hong, and K. C. Ho, “Identification of Digital Modulation Types Using Wavelet Transform”, *Communications Conference Proceedings, 1999. MILCOM 1999. IEEE* Volume 1, 31 Oct.-3 Nov. 1999, pp. 427 – 431.

17. K. R. Farrel and R. J. Mammone, “Modulation Classification Using a Neural Tree Network,” in *Proc. MILCOM*, 1993, vol. 3, pp. 1028–1032.

18. A. K. Nandi and E. E. Azzouz, “Procedure For Automatic Recognition of Analogue and Digital Modulations,” *Proc. Inst. Electr. Eng.—Communication*, vol. 143, no. 5, pp. 259–266, Oct. 1996.

19. B. M. Sadler and S. D. Casey, “Frequency Estimation via Sparse Zero Crossings,” *in Proc. ICASSP*, May 1996, vol. 5, pp. 2990–2993.

20. P. Daponte, D. Grimaldi, A. Molinaro, and Y. D. Sergeyev, “An Algorithm for Finding the Zero Crossing of Time Signals with Lipschitzean Derivatives,” *Measurement*, vol. 16, no. 1, pp. 37–49, Sep. 1995.

21. P. Daponte, D. Grimaldi, A. Molinaro, and Y. D. Sergeyev, “Fast Detection of the First Zero-Crossing in a Measurement Signal Set,” *Measurement*, vol. 19, no. 1, pp. 29–39, Jun. 1996.

22. J. J. van de Beek, P. Ödling, S. K. Wilson, and P. O. Börjesson, “Orthogonal Frequency-Division Multiplexing,” *in Review of Radio Science 1996–1999. London, U.K.: Int. Union Radio Sci. (URSI)*, Oxford Univ. Press, 1999.

23. J. J. van de Beek,M. Sandell, and P. O. Börjesson, "ML Estimation of Time and Frequency Offset in OFDM Systems," *IEEE Trans. Signal Process.*, vol. 45, no. 7, pp. 1800–1805, Jul. 1997.

24. J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.

25. Q. Jin, K.M. Wong and Q. WU, "Optimum Wavelet Design for Transient Detection", Proceedings of IEEE seventh SP workshop on *Statistical signal and array processing*, Qubec city Canada, 1994, pp. 255-258.

26. Assaleh, K. Farrell, and R. J. Mammone, "A New Method of Modulation Classification for Digitally Modulated Signals," *Proceedings of the 1992 IEEE Military Communications Conference*, Vol. 12, pp. 712-716, Oct. 1992.

27. W. Wei and J. M. Mendel, "A New Maximum-Likelihood Method for Modulation Classification," *in Proc. ASILOMAR-29,* Nov. 1995, vol. 2, pp. 1132–1136.

28. H. V. Trees, *Detection, Estimation and Modulation Theory*, New York, Wiley, 1968.

29. S. M. Kay, *Fundamentals of statistics and signal processing*, Vol. II: Detection theory, Prentice Hall, 1998.

30. P.Prakasam, M. Madheswaran, "Digital Modulation Identification Model Using Wavelet Transform and Statistical Parameters", *Journal of Computer Systems, Networks and Communications*, 8 Pages, 2008.

31. L. Hong and K. C. Ho, "BPSK and QPSK Modulation Classification with Unknown Signal Level," *Proceedings of the 2000 IEEE Military Communications Conference*, Vol. 2, pp. 976-980, Oct. 2000.