A
Dissertation on

"COMPARATIVE STUDY OF EMG SIGNAL DENOISING TECHNIQUES"

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Submitted by

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CERTIFICATE

This is to certify that the report entitled "Comparative Study of EMG Signal Denoising Techniques" which is submitted by Abhishek, Roll no. 2K12/SPD/01 in partial fulfillment for the award of the degree of Master of Technology in Signal Processing and Digital Design at Delhi Technological University, Delhi is a bonafide record of his work under my supervision in the academic session 2013-2014. This is also certified that the matter given in this report has not been submitted for the award of any degree in any other university.

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Sometimes one cannot express his regards to someone only by words for there motivation, guidance and blessings towards you. Some works are there, which alone cannot be successfully possible. You need someone's support , encouragement to complete the work successfully. Just as one cannot clap from a single hand, you have to join both your hands for clapping.

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I would like to thank my classmates and all the Faculty members, staff members of the DTU for there support and help.

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ABSTRACT

Electromyogram signals provides a major solution in detecting the abnormalities such as cervical pain, fatigue etc. with the body muscles. Various diseases and abnormalities that are associated with the skeletal muscles can be treated based upon the EMG signal. Based upon correct and uncontaminated signal, correct prescription about the muscles can be made by an Medical Practitioner.

Sometimes because of presence of some artifacts like Power Line Noise, Electrocardiogram noise, motion artifact, instrument noise, cross talk etc distorts the signal, thus sometimes a correct diagnosis may not be provided.

To cater to these artifacts various methods have been proposed in recent past. Many methods have provided good efficiency in removal of these artifacts. Sometimes because of weak nature (sometimes low SNR) of EMG and presence of artifact sometimes makes it difficult to detect the presence of the EMG signal in the body.

It is therefore a very essential task to remove the artifacts that are present in the EMG signal, so that a correct signal study can be made. Based upon some practical difficulties and major sources of noise such as Power Line Interference and Electrocardiogram artifact, a comparative study of some previously proposed methods has been done in this work.

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LIST OF ABBREVATIONS

EMG	Electromyography
SEMG	Surface Electromyography
ECG	Electrocardiogram
PLI	Power Line Interference
EMD	Empirical Mode Decomposition
EEMD	Ensemble Empirical Mode Decomposition
AP	Action Potential
MUAP	Motor Unit Action Potential
MU	Motor Unit
IMF	Intrinsic Mode Function
ANC	Adaptive Noise Cancellation
EMGdi	Diaphragm Electromyogram
IC	Independent Components
ICA	Independent Component Analysis
NADA	Noise Assisted Data Analysis
SD	Sum of difference
MPA	Matching Pursuit Algorithm
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
RLS	Recursive Least Squares
SNR	Signal To Noise Ratio
SER	Signal To Error Ratio
TP	Total Power
MSE	Mean Square Error
AMP_CO	Adaptive Matching Pursuit with Cosine Packet Dictionary

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CHAPTER 1

INTRODUCTION

1.1 ELECTROMYOGRAPHY

Electromyography is a technique of measuring the Electromyogram (EMG) signals [10,26,27,28,29,30]. EMG is basically a study of the electrical signals produced by the skeletal muscles which is under consideration. EMG signal gives the electrical activity of the muscles of humans / any living organisms. It is the study of structure and function of the muscles of living beings.

The muscle under consideration on being activated, produces electrical potential which got recorded by the electromyography. Electromyography is the technique used for performing EMG signal study. The study of electromyography provide a platform of understanding the structural, functional and behavior of the human body under normal conditions and under some medical problems (Diseases).

As EMG signal is a very descriptive tool for the identification of the muscular problems in living beings, it becomes very important to proper collect it and to infer proper diagnostics to patients based upon the collected signal. It provides information of the strength, effectiveness and healthiness of the muscles. It provides information about weakness, fatigue, and other abnormalities of the muscle basically it provides information about the correct movement of the human muscle.

So for proper prescription it is necessary to collect, correct signal and to remove any unwanted things. EMG proved to be an very efficient technique in identifying various abnormalities and diseases such:-

(i) Neuromuscular diseases
(ii) Cervical pain
(iii) Myopathic problems
(iv) Neuropathic
(v) Or some other injury in muscles etc.

In identifying the Neuropathic diseases, the EMG signals shows some basic characteristics. However they may differ from person to person. Some of these basic characteristics are as follows:-

(i) Considering the action potential in case of neuropathic problems, action potential duration may increase.

(ii) In normal conditions, from few muscles-fibers to several thousands muscles- fibers may innervate under motor unit but in case of neuropathic problem, these fibers may get increased, resulting in increase in amplitude of the Action potentials (AP).

(iii) Motor unit may get decrease in these cases.

Fig 1.1

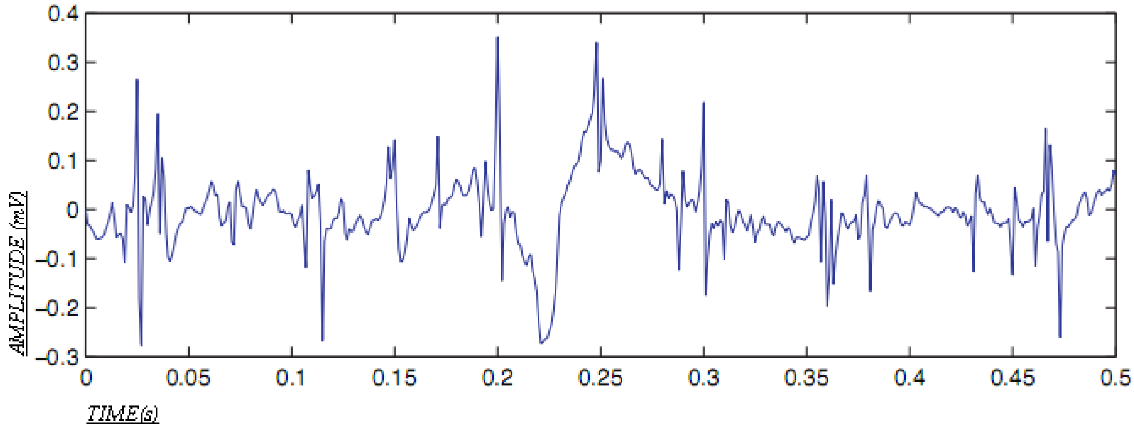

Fig 1.1 shows a clean EMG signal[12]

However as said above the factor may vary from person to person, but there are some general conditions one may refer while studying diseases through study of EMG signals. Also while diagnosis Myopathic diseases; following characteristics of EMG can be taken into account.

These characteristics are as follows:-

(i) The number of the motor units of the muscle may got decreased. However this may not happen in every person. These characteristics happen in very few cases.

(ii) Considering the action potential, as contradictory to Neuropathic diseases, the duration in case of the Myopathic diseases decreases.

(iii) The ratio of the, area to amplitude of action potential (AP) decreases.

These are some general considerations one can adopt. However several other than above mentioned problems exists. The electromyography can also be used for identifying the pain / fatigue in the muscles. As discussed above by studying the changes that occur in the action potential the amount of fatigue can be predicted and thus with the help of these changes, proper prescription by the Doctor or any other Medical Practitioner can be given to the patient.

After studying the nature of EMG signals, the practitioner can tell the condition of the muscles. So it is very important and necessary that the signal should be free from any kind of distortion / noise, so that a correct prescription can be made. Based upon that the reading of the EMG, muscles activity can be predicted that whether the muscles is satisfactory or it needs medication. Taking this into account, a risk should not be taken with the patient. One should be very careful while taking the EMG signal of the patients.

So if some changes in signal characteristics can be seen in the signal than one must see the concerned person. Some of these changes based on which fatigue can be stated are:-

(i) Increase in duration of Action potential (AP).

(ii) Increase in amplitude of the AP.

(iii) Increase in mean-absolute-value of the signal may occur.

Now by studying the above changes / or practical changes that may occur in the normal/ healthy EMG signal, a proper cure can be given to the patient.

EMG signal is basically an electrical activity that is produced by the skeletal muscle. Skeletal muscles are basically a kind of striated muscle. Skeletal muscles are control by the nervous system of the human body. These muscles are made up of the muscle fibers/cells.

In the contraction or movement of the muscles, motor unit plays an ultimate role. This unit can be considered as the basic function unit of this muscle movement. The motor unit basically have an Motor neuron. A motor neuron is an nerve cell. Originating from the central part of the brain and helps in controlling the muscles movement. The motor unit innervates many a fibers (muscle fibers).

Action Potential is a very small/short event during which the potential of the cell membrane changes (falls and rises). This happens during the depolarization point during which the channels performs opening / closing operations. They in this interval allows the exchange of ions. Thus changes the potential of the cell membrane.

As at the stage when the action potential of motor nerve, reaches some depolarization point, then only the muscles contracts. This point is basically responsible for the generation of the EM field. Collectively the muscle action potential is known as the MUAP. i.e. The motor unit action potential.

Therefore the EMG signal may be defined as the collective addition/summation of the MUAP's.

Any portion of muscle may contain [30] approximately 20-50 MU's (Motor Units) which in turn contain fibers ranging from few hundreds (100) to many a thousands of fibers. This basically depends on the type of the muscle we are considering, a small muscle (let say which control fine movements) contain few fibers per MU (app. nearly ten or sometimes even more less than this).

Before measuring / recording the signal skin should be prepared for this.

1.2 Preparation of the skin

Before placing the electrode on the skin it should be cleaned. This cleaning can be done in many ways such, for making a good contact between the skin surface and the electrode surface, excess of hair can be removed by trimming, shaving. Other than these steps skin can be cleaned by some abrasive material, also alcohol can be used for removing dirt etc.

1.3 EMG Measurement

Usually EMG can be measured by two types of methods:-

(i) Intramuscular electromyography

(ii) Surface electromyography

(i) **Intramuscular electromyography**:- In this technique the signal is measured with the help of needles. These needled are inserted in the muscle under consideration. Some of the advantages of this technique are as follows:-

ADVANTAGES

(a) They have the capability to perform on muscles which reside deeply in skin.

(b) They can perform on even very small muscles present in the skin.

(c) Cross talk problem is less

(d) A very specific area of the test electrode.

With the above given advantages some disadvantages of this technique are also there:-

DISADVANTAGES

(a) As the needle are inserted inside the muscles, the cause pain to the patient which can last too many days.

(b) Difficulty in finding the same place where the needle has inserted previously.

Apart from this intramuscular methods, another method known as the surface electromyography may also be used.

Surface electromyography:- this method is quite different from the previous one. In this method, the electrode are placed directly on the skin surface, where the recordings has to be taken. This method has several advantages.

ADVANTAGES

(a) As in case with the intramuscular EMG (needle EMG), this method does not insert the needle / electrode in the skin, rather electrodes are placed on the surface. So, in this technique, the pain to the patient has been minimized upto a very large extent.

(b) No need of a doctor or any other special medical practitioner to put the electrodes, electrodes can be applied easily.

However apart from these advantages this method suffers from few disadvantages:-

DISADVANTAGES

(a) Surface electrodes has an large pick up area, resulting in problem of the cross-talk.

(b) May not provide efficient recording from deep buried muscles.

1.4 ARTIFACTS IN EMG SIGNAL

During the recording of the EMG signal, some artifacts / noises may crop up which effect the quality and efficiency of the signal. Some of the artifacts which effect the signal are as follows:-

(i) Cross talk

(ii) Motion artifact

(iii) Power line interference

(iv) ECG artifact

(i) **Cross talk**:- sometimes while recording the EMG signals, cross talk effect may also crop up. While cross talk is more pronounced in surface electromyography. As in case of surface electromyography, the electrode have a large pick up area. And because of that they pick's up the active potential from near by muscles which then cause the cross talk problem.

However by choosing proper distance b/w electrodes and proper electrode size so that they pick up only the required potential, can reduce this effect upto a certain extent.

(ii) **Motion artifact**:- Motion artifact is also one of the artifact which effect the emg signal. Motion artifact occurs when there is a motion that exist b/w the electrode surface and the skin (where the electrode is applied). The frequency for this kind of artifact exist between 1-10hz. However this kind of artifact can be removed by roper skin treatment / cleaning and abrashing before applying the electrode.

(iii) **Power line interference**:- power line noise is one of the major interference which effect the EMG signal. One of the cause of PLI(power line interference) is the mutual inductance between the conductors. Suppose that, a current is flowing through the conducting wire, then it may generate the magnetic flux and because of this induced magnetic flux in turn generate current in the nearby wire i.e. we can also say that the difference b/w the current(stray current) which is flowing through the cable and through the patient and between the electrode impedance. The frequency range of PLI is 60 Hz and may be sometimes its harmonics. Several methods are there for power line noise removal. Methods such as high pass filtering, infinite impulse response notch filter[12], adaptive algorithm using LMS and matching pursuit[22] are there.

(iv) **ECG Noise**:- ECG noise is one of the most interfering noise present in the recorded electromyography near to the heart. For instance in case of trapezius muscles, neck muscle, or trunk muscles etc. however many other artifact can be removed by some standard filtering procedure. But in case of this artifact one should be more careful, this is because of the fact that sometimes only high pass filtering does not solve the purpose very well.

In case of ECG artifact the EMG signal and the ECG signal at some point in amplitude and frequency overlap each other. For ECG artifact removal , techniques such as ICA and adapted filter[7] and another technique which uses ANC using RLS algorithm[16] proves efficient. However many other have also been applied previously which also proves efficient.

CHAPTER 2

LITERATURE SURVEY

In today's world EMG has become a very popular technique / diagnostic tool for the purpose of identification of various diseases or problems related with muscles. As on the basis of the EMG signal a prescription can be made by the medical practitioner. So a correct and artifact free signal is needed. Various techniques have been proposed by several authors.

S. Yacoub and K. Raoof proposes "Power line interference rejection from surface electromyography signal using an adaptive algorithm" in 2008[22]. In this method, a Least Mean Square (LMS) adaptive structure for the purpose of estimation of PLI as an reference signal. Again for improving another the output given by LMS in the previous step. Then, another algorithm called Matching Pursuit has been applied for the purpose of improvement in the estimation of PLI. Using both techniques PLI has been removed from the EMG signal efficiently.

Xu Zhang and Ping Zhou proposes "Filtering of surface EMG using Ensemble empirical mode decomposition on 2013[11]. This method, EEMD is applied for the reduction of PLI, WGN (white Gaussian noise), and for BW(baseline wondering) artifacts. The PLI noise has been reduced efficiently. This method makes use of the repetitive application of the EMD in ensemble sense. Also EEMD Method overcome the Mode mixing effect that crop up during the EMD method.

Yun Li, Xiang Chen, Xu Zhang and Ping Zhou proposes " ECG artifact removal from EMG recordings using Independent Component Analysis and Adapted filter" in 2013[7]. In this technique ICA and an Adaptive filter with RLS algorithm has been used. The purpose of using ICA is to extract the Independent Components (IC) from these extracted components ECG is then can be used as an reference signal. The ECG then is estimated by using the IC's by the Adaptive FIR filter having RLS as an tool for updating the filter coefficients.

Baratta RV, Solomonow M, Zhou B-H, Zhu M proposed "Method to reduce the variability of EMG power spectrum estimates" in 1998[31]. In this, a technique has been proposed for the reduction of PLI(power line noise) and also the system noise. The variability that present in the Electromyogram power density spectrum variable. The phase as well as the amplitude of the PLI has been estimated from the EMG recording (Clean segment). However while recording the EMG, at any stage if there is some change in the phase or amplitude takes place than this method will not work

A. Bartolo, C Roberts, R.R. Dzwonczyk, E. Goldman in 1996[3]. Proposes an "Analysis of diaphragm EMG signals: comparison of gating vs. subtraction for removal of ECG contamination. This method compares gating method and subtraction for the purpose of removal of electrocardiogram artifacts present in Diaphragm EMG signals. This method removes the portion of the EMGdi (Diaphragm Electromyogram) signal which, in terms of amplitude gets overlapped with the QRS complex of the ECG signal. Thus a degradation of the signal may take place.

C. Marque, C. Bisch, R. Dantas, S. Elayoubi, V. Brosse, C. Perot in 2005[4]. Proposes "Adaptive filtering for ECG rejection from surface EMG recordings. While analysis SEMG signals for evaluation of fatigue, sometimes SEMG may got corrupted with ECG noise. For reduction of ECG noise from EMG LMS algorithm has been applied. But still sometimes ECG artifacts residual may get present as, the signal for reference has been directly taken using band pass filter from the noisy contaminated signal.

Jacek Piskorowski in 2013[12] proposes "Time efficient removal of power line noise from EMG signals using IIR notch filters with non-zero initial conditions". In this a technique has been proposed for removing PLI. A vector projection is used to find the initial conditions for the notch filter. These initial conditions are used to reduce the transients that may crop up during the processing of EMG signals through the Notch filter. A good efficiency in removing the ECG artifact from EMG signal has been achieved.

G. Lu, J.S. Brittain, P. Holland, J. Yianni, A.L. Green, J.F. Stein, T.Z. Aziz, S. Wang in 2009[16] proposed a method "Removing ECG noise from surface EMG signals using adaptive filtering". In this an ANC (Adaptive noise cancellation) filter is used for updating weights, of the coefficients vector, thus minimizing the cost function, which is the objective of the method. This method utilizes the concept of Adaptive filter and thus adjusts the coefficients accordingly to the input signal. A good efficiency in noise removal has been achieved by using this method

A.O. Boudraa and J.C.Cexus proposed a method "Denoising via Empirical Mode Decomposition[1]. The EMD method decomposes the signal into intrinsic mode function known as IMF's by sifting algorithm starting from the IMF's with high frequency to the IMF's with the low frequency. However on adding the IMF's basically they should result in whole signal. The method works well. But the general problem with the regular EMD that may arise in the Mode Mixing in which the IMF's loose there individual physical existence.

M.S. Redfern, R.E. Hughes, , D.B. Chaffin in 1993[6] proposed "High pass filtering to remove electrocardiographic interference from Torso EMG recording". In this, the technique provides a better performance than the standard high pass filter that were introduced in past. The problem that lies with EMG filtering from ECG contamination is that sometimes both signals got overlapped. The standard high pass filter however can alter the EMG frequency content while filtering, thus resulting inn distorted signal.

Organization of the thesis

Chapter 3. In this chapter a PLI noise removal technique has been studied. Ensemble empirical mode decomposition (EEMD)[11] is an noise removal technique which is basically based on emd but in repetitive sense. EEMD removes mode mixing effect induced by regular standard emd.

Chapter 4. In this chapter a PLI noise removal techniques based on IIR notch filter has been studied. This technique[12,13] finds non zero initial conditions for the notch notch filter for the purpose of transient suppression. And thus after having initial conditions the technique further focuses on removal of the PLI noise

Chapter 5. In this chapter another PLI removal technique has been studied[22]. This technique makes use of the adaptive algorithm which makes use of the LMS and an Matching pursuit algorithm for the removal of the PLI noise effectively.

Chapter 6. In this chapter an Electrocardiogram signal (ECG) removal technique has been studied[7]. In this technique an ICA technique is used for extracting the ECG independent component is used. After this an adaptive filter is there for estimating the ECG noise by making use of the Recursive least squares for the purpose of updation of filter coefficients.

Chapter 7. In this chapter an another Electrocardiogram noise removal technique has been studied[16]. In this technique Adaptive noise cancellation(ANC) concept is there which adjusts its coefficients using the RLS algorithm.

Chapter 8. In this chapter the efficiency of the various methods has been discussed.

Chapter 9. In this chapter the future scope of noise removal from EMG signals has been discussed.

CHAPTER 3

PLI NOISE REMOVAL USING EEMD

3.1 Introduction:-.

Various techniques has been proposed for proper conditioning of the signal. They proved to be efficient up to certain extent.

A technique has been proposed by Xu Zhang and Ping Zhou in[11]. The technique basically make use of the EMD[1,2,10,11] (Empirical mode decomposition). EMD has been widely used for the decomposition of signals but it suffers from the mode mixng effect. This mode mixing thus decreases the efficiency of decomposition of the signals at different scales.

Mode mixing basically arises when an IMF (Intrinsic mode function) consist of oscillations of variable scales caused by the occasanality of the driving mechanism. During mode mixing, the IMF had not have its individual physical existence, rather it may represent a different physical process. The mode mixing restricts the IMF's to have there individual physical existence. Also because of this intermittence , the EMD algorithm becomes unstable and also sometimes because of this intermittence the EMD cannot extract similar scales signals. As because of this mode mixing problem another method came into existence which helps in removal of the mode mixing effect that caused by the standard / regular EMD this new method is Ensemble empirical mode decomposition (EEMD)[2,10,11]. The signals in study of[11]have been taken from the thenar muscles of the Amyotropic lateral sclerosis patients. Also with this SEMG recordings (clean signals) were also used in[11].

3.2 THE EEMD APPROACH:-

This technique in[11] basically makes use of the empirical mode decomposition in ensemble sense. This EMD technique is used for the decomposition of the input signal into several components , called the IMF's (intrinsic mode functions). However the basic problem that lies with EMD is mode mixing has been removed by the EEMD. This method (eemd) is an noise assisted approach. The basic principle of this method is the use of the Ensemble approach. The Sifting process (as that was performed in the EMD method) worked on the group of signals (Noise + signal).

EEMD proved to be a great improvement over the standard EMD. EEMD is a truly adaptive data analysis method. EEMD is basically a noise assisted data analysis method (NADA). The EEMD method, applied on ensemble of noisy signal. Each signal is derived with a combination of original signal added with a different white noise. The white noise which is added is different in combination. The white noise considered should have a proper finite amplitude.

Let the noisy signal be y(t) , consisting of the original signal and an added noise in it. The original signal being x(t) , and with a different white noise every time say w(t) , the equation can given as :-

$$y(t) = x(t) + w(t)$$

Now in the above given equation the noisy signal can be decomposed using the standard EMD method. Actually the added white noise will usually provide a, uniform reference of scale distribution. During the addition of the signal with the white background having uniform distribution , basically the bits of the added signal themselves projected onto the proper scales of the reference which were established by this white noise (in the background).

The basic reason of addition of the different noise {white noise} is that different white noise {added everytime} cancels each other in time space of the mean of ensembles.

The purpose of this addition is to bring the final resulting intrinsic mode functions to the comparable scales which are independent of the local time-domain characteristics of signal itself. It produces the IMF's which do have there individual physical existence resulting reduced mode mixing effect as which was produced by the regular EMD.

3.3 <u>Empirical Mode Decomposition</u>

EMD stands for empirical mode decomposition. Empirical mode decomposition is an adaptive and efficient method. This method can be used for decomposing non stationary or non linear process / signal. This method is very efficient for analyzing real life signals. It is to be mentioned here that many of the signals are real / non linear signals. In that case this method proved to be of great importance.

Initially the input which is under consideration is being inputted to the emd process. The signal is then decomposed into oscillatory functions. These oscillatory functions are known as IMF's {intrinsic mode functions}.

Mainly these IMF's are basically extracted in decreasing order. Here with decreasing order it is to be mean that the extracted IMF's contains higher frequency oscillation as compared to the successor. For example if a signal has been decomposed and let say that ten IMF's have been extracted in a series {1,2,3,…10} so, the initially extracted IMF, the first IMF contains higher frequency oscillations as compared to second or third IMF. Decomposition does not mean degradation of the signal it simply means breaking of the signal into separate components {oscillatory modes}.

As with EMD the signal can be decomposed into IMF's , so if these IMF's have to be added together then they should gives up the whole signal {input signal}. Also as in case of wavelet approach [33,34] whose basis function is fixed, the EMD method derives its basis function directly from the input signal. However because of this fixed basis function they are not able to match many real signals.

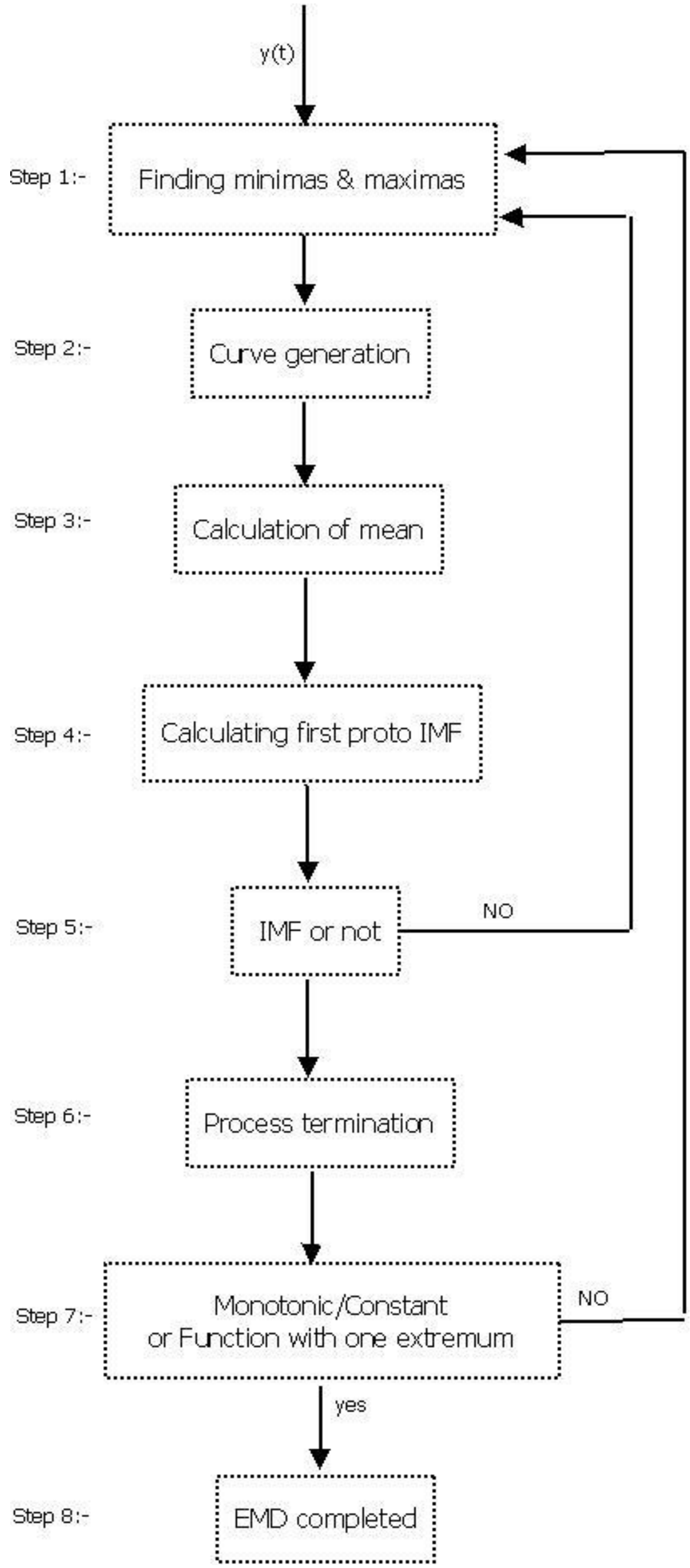

FIG 3.1-This figure shows the flow chart of EMD process[2]

As discussed above that the emd technique [1,2,10,11] decompose the signal into oscillatory components with decreasing order of frequency, but there should be proper criteria for an oscillatory component to be an IMF, that is there should be some conditions that are to be followed for the determination of the component to be an IMF.

The decomposed component should follow the following conditions:-

(i) The number of zero crossings and the number of extermas should be equal or they should be atmost differ by one.

(ii) The mean of the two envelops i.e. the lower envelope and the upper envelope should be zero.

If these conditions are provided, then an decomposed component may be said to be an IMF.

Here the upper envelope consist of the maximas which are connected through the spline curve and the lower envelope consist of the minimas which were also connected through the curve.
This process of finding IMF has been repeated again and again till the process stops . The process is known as sifting process.

Now finding the mean of the above given envelops. i.e. the upper envelope and the lower envelope, the equation of the mean can be given as:-

$$m_e(t)=\frac{f_l(t)+f_u(t)}{2}$$

Here $m_e(t)$ denotes the mean of the two envelops.

$f_u(t)$ denotes the upper envelope (of maximas)

$f_l(t)$ denotes the lower envelope (of minimas)

Now after finding this mean, this mean has to be subtracted from the input signal. i.e. from y(t).
So getting z_1 (t) is called first proto IMF. z_1 (t) i.e. the proto IMF still may contain maximas and minimas {extremas} and also the zero crossings in them. So the required sifting process is applied on it so as to find the IMF's as it may contain many IMF's.
Now the sifting continues till we get the initial / first IMF. After finding the first IMF, this has to be subtracted from the original signal. As the remaining signal may also contain the IMF's, so now the sifting process is applied on the remaining signal {Original signal-first IMF}.

As after repetitive application of the sifting process, there should some stopping criteria to terminate the process. Various techniques are available that can be used as stopping criteria; one of the widely used criteria is Sum of difference.

$$SD(\text{sum of difference}) = \sum_{t=0}^{T} \frac{\left| z_{k-1}(t) - z_k(t) \right|^2}{z^2_{k-1}}$$

For the application of the SD, a threshold value is needed, for comparing SD with that threshold value to get the IMF, now if the value of SD is smaller than the threshold , first Intrinsic Mode Function is obtained.

$$y(t) - g_1(t) = w_1(t)$$

As after subtracting the first IMF from the original signal, residue is obtained. As this residue is part of the signal itself, the remaining part of the signal may also contain information which may be useful, then again the same procedure is applied again on the residual signal.

So this can be written as:-

$$w_{i-1}(t) - g_i(t) = w_i(t) \qquad i = 1,2,3, \ldots\ldots N$$

so after getting all the required IMF's there is need to terminate the whole process. The procedure can be terminated if the residual which is left, should be either a constant or it may be a monotonic slope or it contain only one extremum.

As we have explained above that EEMD approach is being developed to improve the previous technique, that is the EMD method for its problem of mode mixing effect. Here in EEMD [5,11] method the sifting process is applied on the ensemble of noisy signals.

The purpose of addition white noise is that final IMF's come in comparable scales which in turn thus reduces the mode mixing effect which was introduces by the standard emd

Now let $g_{ij}(t)$ represents the resultant IMF's that were obtained from EMD.

$$g_i(t) = \frac{1}{M^T} \sum_{j=1}^{M^T} g_{ij}(t)$$

Where i denotes the IMF order

j denotes trial index

M^T denotes total number of trials

Here in this technique the filtering can be done by filtering every IMF. This can be done with the help of an notch filter of second order. An output $\beta_i(t)$ can be obtained[11] after applying the notch filter on every IMF.

Now to calculate how much power line noise component present in the IMF, then e_i has to be analyzed,

$$e_i = \frac{var(g_i(t) - \beta_i(t))}{var(g_i(t))}$$

The notch filter that is applied in this process is an second order having a cutoff frequency of 60Hz and its harmonics (120Hz,..). in the above expression the var operator denotes variance of the considered time series(x(t)).

After having the IMF's from the EEMD method, notch filter is applied. However the notch filter is not applied to all the IMF'S. The IMF'S are scanned for checking the percentage of power line interference presence in them. This can be done as given below:-

$$g_i(t) = \begin{cases} g_i(t) & e_i < 0.1 \\ \beta_i(t) & e_i \geq 0.1 \end{cases}$$

As given in the above expression, the notch filter is applied to selected number of IMF's.

Here $g_i(t)$ denotes the filter IMF.

Actually a criteria mentioned above is being used for checking the power line presence. If the presence of the power line component is found to be more than 10% of the total energy of the IMF, then only the considered IMF will undergo notch filtering. However if the presence of the PLI in the IMF is less than 10% then IMF is treated as it is (as original). No notch filtering is done in those cases.

EEMD method is able to decompose the signals with the problem of mode mixing as in with the EMD, thus achieving a better performance.
For analyzing the performance of EEMD method, SER values from[11] have been studied.

$$SER = 10 \lg \left\{ \frac{var(c(t))}{var\{c(t) - \bar{c}(t)} \right\}$$

Here c(t) is clean experimental SEMG signal

The SER values are at different values of SNR. The expression for SNR is:-

$$\mathrm{SNR} = 10\ \lg\left\{\frac{\mathrm{var}\{c(t)\}}{\mathrm{var}\{\alpha.\,v(t)\}}\right\}$$

From the analysis done in[11], it can be said that the Initial Intrinsic Mode Function from the EEMD process mostly consist of high frequency noise. so removing this IMF, HF noise can be removed.

By analyzing the EEMD, as stated above that EEMD makes use of EMD method for decomposing the signal into IMF's and based upon the characteristics of noise, IMF filtering can be done. However the continuos application of the EMD in EEMD makes it computationally more complex. But it gives a better efficient performance.

CHAPTER 4

PLI NOISE REMOVAL USING IIR NOTCH FILTERS

4.1 INTRODUCTION

A technique based on EEMD method has been discussed in previous chapter. Another technique based on notch filtering for the Power Line Interference removal has been applied in[12,13].

One of the commonly used method is the notch filtering. Notch filter method can be used as single notch or in multinotch (double, triple, etc) [12,13]. Here in the purpose of noise removal digital IIR notch filter with non zero initial conditions has been used [12,13]. In[15] Pie and Tseng also uses notch filtering for PLI removal in ECG signals.

4.1.1 Notch Filter

A Notch filter basically accepts the whole frequency band besides rejecting (notching) a particular frequency (mostly narrow frequency), or multiples of this particular frequency or its harmonics also. Notch filter can be formed by altering the distance of poles and zeroes from the unit circle. In case of notch filters the zeroes are placed near to the unit circle.

In [12,13] digital IIR notch filter has been used , however IIR notch filters are potentially not stable [14] also may not sometimes provides a linear phase response. But in comparison FIR filters are stable and can able to give a linear phase exactly. But with condition of meeting amplitude specifications the length of the FIR filter should be high, and as the length of the filter increases the signal delay also increases.

Its basic principle of working is that it notches (removes) the particular unwanted frequency. The notch frequency can be removed by creating a zero gain at all the unwanted frequencies and unit gain at all other required frequencies.

Although the notch filter removes the noise to some extent but during this method, whenever the noisy signal is passed through the filter, the transient response may occur and thus disturbs the output of the notch filter / considered filter.

The duration of this transient response may vary from milliseconds to several seconds. This effect is sometimes more pronounced during lower frequencies , in this case the duration of this transient response may even go upto several seconds.
So for an efficient working of the structure there is a need to remove this transient response.

However this technique is not as computationally simple as previous technique. In [12,13] technique extra computational load is there.

Normally the input given to the notch filter can be given as:-

$$u(n) = g(n) + \sum_{k=1}^{N} Q_k \sin(n\,\Phi_{Mk} + \psi_k)$$

So,

$$u(n) = g(n) + c(n)$$

Where c(n) is the sum of sinusoidal interference signals having freq. $\Phi_{Mk} \in (0,\pi)$
u(n) is corrupted signal
g(n) the desired signal to be extracted
ψ_k is the phase shift
Q_k represents the amplitude with N number of interference signals.

$$\left|H(e^{j\Phi})\right| = \begin{cases} 1 & \text{for} \quad \Phi \neq \Phi_{Mk} \\ 0 & \text{for} \quad \Phi = \Phi_{Mk}, \quad k = 1, \ldots N \end{cases}$$

As discussed above that the notch filter can be designed in a number of ways, however one of the simple and commonly used method is the pole and zero location within the circle. The type of filter under consideration sometimes depends upon the distance of poles and zeroes.

In case of notch filters the poles should be placed at notch frequencies given as $F_{pk} = re^{\pm j\Phi Mk}$ and are placed very near to the unit circle similarly with zeroes, they are placed at the same frequencies as that of poles $F_{zk} = e^{\pm j\Phi Mk}$ but on the unit circle. Here r represents pole radius.

Considering a notch filter with N notches its transfer function may be represented as:-

$$H(z) = \prod_{k=1}^{N} \frac{1 - 2\cos\Phi_{Mk} z^{-1} + z^{-2}}{1 - 2r\cos\Phi_{Mk} z^{-1} + r^2 z^{-2}}$$

$$= \frac{W(z)}{W(r^{-1}z)}$$

Here $W(z) = \sum_{k=0}^{2N} w_k z^{-k}$ represents a symmetrical polynomial.

As discussed above a simple conventional notch filters with arbitrary initial conditions may not able to provide efficient results. So a notch filter combined with non zero initial conditions perform well.

Pie and Tseng [15] proposed a technique for removal of ac interference in ECG. This technique is based on finding better initial conditions for the notch filter. In this, they basically uses vector projection technique for finding better initial conditions for the purpose of transient response reduction.

In [15] the method used for noise removal may also work well for denoising the EMG (electromyogram) signals too.

In [12,13] the proposed technique basically consist of two step operation. The first steps consist of the vector projection. The vector projection here, performs the decomposition. This decomposes first B samples of the given input signal into two components , viz

(i) The sinusoidal interference part and

(ii) The desired signal part.

After getting the desired signal part and for the reduction of transient response , this signal component (part) is used as initial values for the IIR notch filter to perform further filtering operation.

The technique [12,13,15] for the transient response suppression can be given as

4.2 Initial Conditions For IIR Notch Filters

Step 1:- In this step initially B input samples of signals as input data vector are arranged.

$$U = \{u(0)\ u(1)\ u(2)\ u(3)\ \ldots\ldots\ldots\ u(B-1)\}^T$$

$$U = G + C$$

Where G represents:-

$$G = \{g(0)g(1)g(2)g(3)\ldots\ldots\ldots g(B-1)\}^T$$
$$C = \{c(0)c(1)c(2)c(3)\ldots\ldots\ldots c(B-1)\}^T$$

Step 2:- In this step a matrix has been constructed given as:-

$$
Q = \begin{bmatrix}
1 & \cos(\Phi_{01}) & \cos(2\Phi_{01}) & \cos(3\Phi_{01}) & - & \cos\{(B-1)\Phi_{01}\} \\
0 & \sin(\Phi_{01}) & \sin(2\Phi_{01}) & \sin(3\Phi_{01}) & - & \sin\{(B-1)\Phi_{01}\} \\
1 & \cos(\Phi_{02}) & \cos(2\Phi_{02}) & \cos(3\Phi_{02}) & - & \cos\{(B-1)\Phi_{02}\} \\
0 & \sin(\Phi_{02}) & \sin(2\Phi_{02}) & \sin(3\Phi_{02}) & - & \sin\{(B-1)\Phi_{02}\} \\
1 & \cos(\Phi_{03}) & \cos(2\Phi_{03}) & \cos(3\Phi_{03}) & - & \cos\{(B-1)\Phi_{03}\} \\
0 & \sin(\Phi_{03}) & \sin(2\Phi_{03}) & \sin(3\Phi_{03}) & - & \sin\{(B-1)\Phi_{03}\} \\
- & - & - & - & - & - \\
1 & \cos(\Phi_{0N}) & \cos(2\Phi_{0N}) & \cos(3\Phi_{0N}) & - & \cos\{(B-1)\Phi_{0N}\} \\
0 & \sin(\Phi_{0N}) & \sin(2\Phi_{0N}) & \sin(3\Phi_{0N}) & - & \sin\{(B-1)\Phi_{oN}\}
\end{bmatrix}
$$

In a case of matrix like this, then for any phase the vector C should be in column space.

Step 3:- In this step a projection matrix let say Ar is being calculated

$$Ar = Q(Q^TQ)^{-1}Q^T$$

Step 4:- now calculating first B o/p samples as follows:-

$$\{y(0)y(1)y(2)y(3)\ldots\ldots\ldots\ldots\ldots y(B-1)\}^T = (I-Ar)U$$

Step 5:- now calculating B…………..M-1 o/p samples as follows:-

$$y_0(n) = w_0u(n) + w_1u(n) + \ldots\ldots + w_{2N}u(n-2N) - rw_1y_0(n-1) - \ldots\ldots\ldots - r^{2N}w_{2N}y_0(n-2N)$$

Here:- N denotes N number of notches
$y_0(n)$ denotes O/P of the filter
I denotes identity matrix
u(n) denotes input to the filter

Now in [15] Pie and Tseng does not state how to choose the B {number of samples} according to them if B is chosen in between 5 and 15 the algorithm perform well. But for the algorithm to perform more precisely the length of B should be correlated with the period of fundamental freq. of PLI and its sampling rate.

4.3 TECHNIQUE B

Another technique of reducing PLI from SEMG has been given in [13]. To make methods of transient reduction more efficient and effective a combined approach non zero initial conditions and time varying pole radius is studied [13,14,15].

Rewriting the difference equation:-

$$y_0(n) = u(n) - 2\cos\Phi_0 u(n-1) + u(n-2) + 2r(n)\cos\Phi_0 y_0(n-1) - r^2(n)y_0(n-2)$$

$$n = B\ldots\ldots\ldots M-2,\ M-1$$

Also in case of time varying pole radius it is known that for decreasing the transient response at the o/p , a temporary decrease in the pole radius should be done.

So,

$$r(n) = \bar{r}\left[1 + (v_r - 1)\exp\left(-\frac{n}{hf_{sf}}\right)\right]$$

here v_r represents variation range of r(n)

h represents exponential variation rate of r(n)

$\bar{r}$ represents final value of pole radius

$$v_r = \frac{r(o)}{\bar{r}}$$

As discussed above that with varied coefficients {time- varying} the transient response in the filter start up can be reduced as well as for small value of the pole radius too, the filter transient nature can also be minimized but one more thing to be remembered while reducing/minimizing the transient nature is the transfer characteristics of the IIR notch filter.

So in order not to effect / alter the transfer characteristics of the filter than the eq. should have to get settled to the final value i.e. $\bar{r}$ with in time duration $[0, \bar{t}_{sif}]$. This is to be with the time frame so as not to effect the transfer characteristics of the filter. Here the $\bar{t}_{sif}$ denotes the time (settling time regarding step response (time invariant filter).

Here, during $\bar{t}_{sif} = t_{sf}$ then maximum transient reduction can be achieved.

For studying the method efficiency, MSE(Mean Square Error) [12]has been given. The expression for MSE is:-

$$MSE = \frac{1}{W} \sum_{n=n_0+1}^{n_0+W} |y_0(n) - g(n)|^2$$

The MSE criteria has been considered for three cases:- in which PLI has been added artificially and individually to the clean EMG signal. Initially 60Hz PLI is added, then in second case 60Hz PLI with first harmonic 120Hz is added then 60Hz PLI with its first two harmonics i.e. 120Hz and 180Hz has been added to the clean EMG signal respectively.

Now for removing the PLI components with its harmonics, notch filter with multiple notches has been used in[12]. Initially in triple notch filter case the value of r (pole radius) is considered is 0.995, then for double and single notch filter case the value considered is 0.98.

As discussed above the number of samples to be decomposed(B) also effect the filtering results. In[12] the value taken for B is 17 which gives an optimal performance, as for optimal filtering the value should cover the filtering the PLI fundamental frequency.

CHAPTER 5

PLI REJECTION USING ADAPTIVE ALGORITHM

5.1 INTRODUCTION

This method[22] uses the widrow's least mean square adaptive technique, and an Matching pursuit algorithm [21,22,24]. Initially the reference input has been constructed mathematically by cosine function {50Hz and 150Hz}. Then the Matching pursuit algorithm has been applied on the output given by the LMS structure. In the first step the LMS adaptive structure is used for the generation of the input for reference with cosine functions {cos (w50t) {fundamental function} and cos(w150t) {harmonic function}]. The reference signal which is to be estimated, should be correlated with the interference signal.

Now after estimating the PLI i.e. PLI_{ref} , this is followed by another step involving the matching pursuit algorithm which is used for further estimating the PLI. Here in the initial step, for the purpose of the estimation of the reference PLI, the Least mean square algorithm has been used in the adaptive sense.

An adaptive structure adjusts its weights / impulse response automatically. This adjustments is done with the help of the algorithm which looks for the error signal generation during the process.

The purpose of the process is to minimize the error so as too make an good estimate for the purpose of an minimum error, the estimate should be as close as possible to the real interference (the PLI signal). However a problem with the SEMG signal is that, the signal had an low SNR.

This objective can be achieved by feeding back the O/p to the adaptive filter and thus adjusting the filter coefficients / weights through an adaptive algorithm LMS. Thus minimizing the error in the least square sense. Before applying the following algorithm the signal is preprocessed by band pass filter 20Hz-250Hz, for the case of respiratory Diaphragm electromyogram signal.

5.2 The LMS algorithm: -

The LMS algorithm [10,19,22,23] basically belong to class of adaptive filter. It basically finds the filter coefficients thus producing the least mean square of the error signal { $PLI_{actual} - PLI_{ref}$ }. This filter actually updates the weights / coefficients so as to converge to the interference (PLI). After each O/p the LMS analysis the error signal and thus updating at each step.

5.2.1 <u>The basic LMS adaptive filter structure</u>

The basic purpose of the least mean square algorithm is to adjusts the weights of the filter so as to minimize the mean square error. As the output is connected to the LMS adaptive filter, then the LMS filter analysis the error, based upon this adjusts the weights in order to minimize the error.

ALC: - ALC is the basic principle component of the adaptive system. ALC stands for Adaptive linear compiler. It basically weights and sums of the set of inputed signal.

$$S_i = \begin{Bmatrix} s_{0i} \\ s_{1i} \\ s_{2i} \\ . \\ . \\ . \\ s_{ni} \end{Bmatrix}$$

Here Si represents input signal vector

s_{oi} represents an constant during the presence of biasness in the input signal.

$$w = \begin{Bmatrix} w_0 \\ w_1 \\ w_2 \\ . \\ . \\ . \\ w_n \end{Bmatrix}$$

Here in this vector w, the w_s represent the weighting coefficients

So, $y_i = S_i^T w = W^T S_i$

here y_i is the output

Now here the purpose is to estimate the signal/response (y_i) but how much it differs from the desired one constitutes the error.

The error can be represented as follows:-

$$e_i = d_i - y_i$$

again, $e_i = d_i - S_i^T w$

here e_i represents the error.

Now we know that the purpose of the LMS algorithm is to minimize the error e_i in least mean square sense.

$$e_i^2 = d_i^2 - 2d_i S_i^T w + w^T S_i S_i^T w$$

Now, the expectations of the above relation can be expressed as:-

$$E[e_i^2] = E[d_i^2] - 2E[d_i S_i^T]w + w^T E[S_i S_i^T]w$$

$$D_v = E[d_i S_i]$$

$$D_v = E\begin{Bmatrix} d_i s_{0i} \\ d_i S_{1i} \\ d_i S_{2i} \\ . \\ . \\ . \\ d_i s_{ni} \end{Bmatrix}$$

Here Dv denoted the cross correlation vector between the input signal vector and the desired response.

Similarly input correlation matrix can be found out.
Now the expression for mean square error can be expressed as:-

$$E[e_i^2] = E[d_i^2] - 2D_v^T w + w^T K w$$

Where K denotes the input correlation matrix

Now the idea of using this algorithm is to reduce the error as much as possible i.e. to minimize the error, for this purpose, gradient method can be used.

The gradient of the mean square error(above) can be found out by the differentiating the above equation:-

$$\nabla = \begin{Bmatrix} \dfrac{\partial E[e_i^2]}{\partial w_0} \\ \dfrac{\partial E[e_i^2]}{\partial w_1} \\ . \\ . \\ . \\ \dfrac{\partial E[e_i^2]}{\partial w_n} \end{Bmatrix}$$

Here ∇ represents the gradient of the above given error.

Basically the least mean square algorithm is the implementation of the Steepest descent method.
The optimal weight vector also known as the wiener weight vector, it can be found out by, equating the above equation to zero, so the expression can be expressed as :-

$$w^*=K^{-1}Dv$$

Also, $w_{j+1} = w_j - \mu\nabla_i$

Where w_{j+1} gives the next weight vector

w_j gives the present weight vector

μ gives the rate of convergence and continuous stability.

In case of the LMS adaptive filter, the number of weights required in general should be equal to twice the total signal bandwidth to the freq. resolution of the filter.

5.2.2 <u>LMS for PLI Removal</u>

In the method given in[22] initially a reference input is needed. This reference i/p is mathematically constructed from two cosine functions i.e. from 50Hz and 150Hz functions. Now, as explained in above procedure this reference have to be correlated with the PLI signal(noise) of input in some unknown way. Here then by analyzing the error function, the reference i/p is then filtered adaptively using Least Mean Square, i.e. getting $\overline{\text{PLI}}$.

Now the estimated EMG signal can be represented as:-

$$\varepsilon_i = S_i - \overline{C}_i^T X_i^T$$

Where

S_i represents primary input (noisy signal)

$\overline{C}_i^T$ represents coefficients of the adaptive filter

X_i^T represents the adaptive signal ($\overline{\text{PLI}}$)

Now the next adaptive coefficients can be calculated as:-

$$\overline{C}_{i+1} = \overline{C}_i + 2\mu\varepsilon_i \overline{X}_i$$

$\overline{C}_{i+1}$ represents next coefficients of adaptive filter

$\overline{C}_i$ represents present coefficients of adaptive filter

μ represents rate of convergence

Now, the fir of the adaptive filter is:-

$$\overline{C}_i^T = [\overline{C}_1, \overline{C}_2, \overline{C}_3, \ldots\ldots.. \overline{C}_m]$$

The length of the LMS filter can be given as:-

$$N > \frac{f_{sf}}{f_{lp}}$$

f_{sf} denotes sampling frequency

f_{lp} denotes lowest interference frequency

So in this way the PLI can be estimated by using LMS algorithm.

Fig 5.1

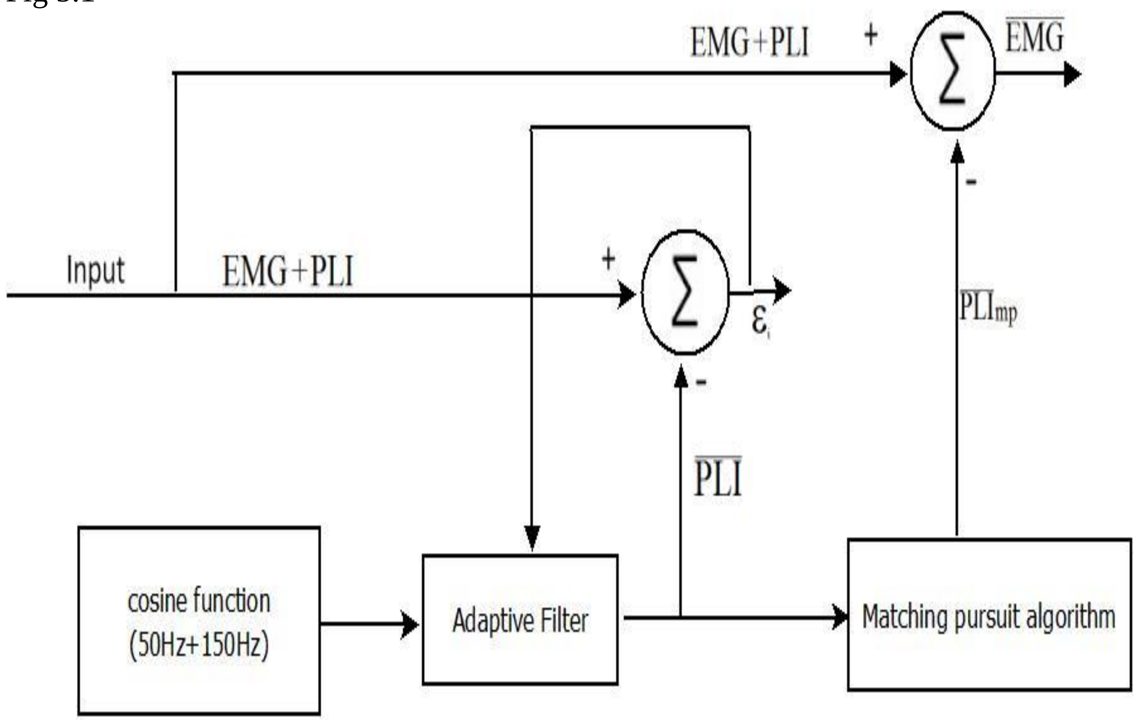

Fig 5.1-Figure shows the AMP_CO[22]

5.3 <u>**The matching pursuit algorithm**</u>: -

After applying the LMS algorithm, it estimated the PLI and than removed PLI interference from the noisy EMG signal but still some components of the EMG signal are present. So in order to denoise EMG signal without distortion, the power line interference should be estimated again, this can be done with another technique, called the matching pursuit in [21,22,24].

Matching pursuit is an algorithm which basically decomposes any considered signal into linear expansion of the waveforms, which are selected from an dictionary, which contains a large number of functions. These waveforms / atoms are chosen, depending upon how well they match the signal structures / elements. The waveforms are selected in a way as to how good / best match with the signal.

The matching algorithm is although a non-linear. But still it maintains conservation of energy, which is an efficient sign of the algorithm. The waveforms whose time and frequency properties suited / adapted to the local structures of the signal which is to be decomposed are known as time frequency atoms, and the waveforms / atoms which are the best matches to the signal structures were chosen by the MPA (Matching Pursuit Algorithm).

So with the help of matching pursuit algorithm. A signal can be decomposed into linear expansion of the selected atom. However if in a case sometimes when an structure may not correlate with any element of the dictionary then it is subdecomposed into several elements

Now, here the MP algorithm has been applied to N point's windows (using cosine packet dictionary). The cosine basis here is an product of cosine functions with the overlapping sinus shaped smooth windows (u(t)). In this[22] N=16384 samples and 70 iterations of MP have been there.

The expression for u(t) can be given as: -

$$u(t) = \begin{cases} 1 & \text{if} \quad t \geq 1 \\ \sin(\frac{\pi}{2} t) & \text{if} \quad -1 < t < 1 \\ 0 & \text{if} \quad t \leq -1 \end{cases}$$

The u(t) is chosen so as not to harm the orthogonality.

As explained above, the purpose of the matching pursuit is to decompose the signal. As suppose a function q is to decomposed in an summation of series of linear expansion function.

Let $q \in H$

Here H represents Hilbert space

A dictionary D = $\{J_\lambda\}_{\lambda\varepsilon\check{A}}$, The dictionary is basically is an family of vectors. To best match the elements of the function to be decomposed vectors has to be selected from the dictionary.

Here the purpose is to find the linear expansion of X by selecting vectors / elements from D (the dictionary), which matches the inner structures of the function as well as possible.

The decomposition of the fn. q can be given as follows: -

$$q = <q.J_{\lambda 0}>J_{\lambda 0} + V_r q$$

Here $J_{\lambda 0}$ represents element of D

This can be done by approximating successively the function of having orthogonal projections on the element / vector of D.

$V_r q$ here represents the residual vector {obtained after approximating function q in direction of $J_{\lambda 0}$}

Also the element $J_{\lambda 0}$ is orthogonal to the residual vector $V_r q$.

Then,

$$\| q \|^2 = | <q . J_{\lambda 0}>^2 | + \| V_r q \|^2 \quad \text{----------- (ii)}$$

Now, $|<q . J_{\lambda 0}>^2|$ is to be maximized, i.e. for best approximate q this has to be as large as possible. So, this can be done by minimizing $\| V_r q \|$

So,

$$|<q.J_{\lambda 0}>| \geq \beta \sup_{\lambda \varepsilon \breve{A}} |<q.J_{\lambda 0}>|$$

Here β is subproportion factor.

The value of quantity $\{|<q.J_{\lambda 0}>|\}$ given, sometimes may not be possible to reach i.e. why β is used. From the above algorithm, the above given fn. Has been decomposed, also getting the residue $V_r q$, there, now on this residue $V_r q$, the matching pursuit algorithm is again applied by orthogonally projected it on the vector of D (Dictionary).

For obtaining the above vector $J_{\lambda 0}$ from D, a choice function is needed, denoted by C here. The choice fn. Associates to any subset Λ of Ă {index} that belongs to Λ. Now suppose that several residues have been computed. Let say the nth order residue $\{ V_r^n X \}$ has been computed.

So,

$$\left|< V_r^n q, J_{\lambda n} >\right| \geq \beta \sup_{\lambda \varepsilon \breve{A}} \left|< V_r^n q, J_{\lambda} >\right|$$

$J_{\lambda n}$ is an element $\{ J_{\lambda n} \ \varepsilon \ D \}$ chosen from D by the choice function C. $J_{\lambda n}$ matches as well as possible the residue $V_r^n X$.

Now applying the same matching pursuit, the residue is then decomposed as:-

$$V_r^n q = < V_r^n q , J_{\lambda n} > J_{\lambda n} + V_r^{n+1} q$$

$$\| V_r^n q \|^2 = |< V_r^n q , J_{\lambda n} >|^2 + \| V_r^{n+1} q \|^2$$

Here $V_r^{n+1} q$ represents the residue of n+1th order.

Also,

$\{ V_r^{n+1} q$ is orthogonal to $J_{\lambda n} \}$

Now suppose the algorithm decomposes the function q into residue with order m so, function q can be written as sum of continued residues:-

$$q = \sum_{n=0}^{z-1} (V_r^n q - V_r^{z+1} q) + V_r^z q$$

Here $V_r^n q$ represents the residue but it is not an desirable components.

The initial term of the above eq. represents the estimated PLI (power line interference) i.e. $\overline{\text{PLI}}_{mp}$. The iterations here depends upon the length of signal decomposed.

So, after getting this estimated $\overline{\text{PLI}}_{mp}$, a denoised EMG can be obtained. Thus getting:-

$$\overline{\text{EMG}} = \text{noisy EMG} - \text{estimated PLI}$$
$$= \{\text{EMG} + \text{PLI}\,) - \overline{\text{PLI}}_{mp}.$$

Hence a clean PLI free signal can be obtained.

For analysis of the performance of the AMP_CO(Adaptive Matching Pursuit Algorithm with Cosine Packet Dictionary) method following are the parameters are used[22] :-

Total power is given as:-

$$T_{pw} = \sum_{i=f_1}^{f_2} (R_s(i))^2$$

The Mean Power frequency is given by:-

$$MF = \frac{\sum_{i=f_1}^{f_2} (R_S(i))^2 . f(i)}{\sum_{i=f_1}^{f_2} (R_S(i))^2}$$

Here f (i) denotes frequency bin

R_s(i) denotes spectral density amplitude per f(i)

Total power in percentage is given by

$$T_{pws}\% = \frac{T_{pws}}{T_{pwr}} .100$$

Here T_{pwr} denotes total power (of raw signals)

T_{pws} denotes total power (of clean signals)

In[22], the application of the given method has suppressed more than 90% of power line interference extraction & more than 95% of the EMG has been preserved. The AMP_CO method removes the PLI component efficiently.

CHAPTER 6

ECG ARTIFACT REMOVAL USING ICA & ADAPTED FILTER

6.1 Introduction

In previous chapters, techniques for PLI noise reduction has been discussed. However EMG sometimes got also effected from the Electrocardiogram signal while sometimes recording SEMG signal from muscles like trunk muscles, muscles near shoulders etc. Because of having weak nature, it sometimes got corrupted by various kinds of noise such as ECG. However this is basically a signal from heart which provides information of the correct working of the heart. But the signal under consideration in EMG and ECG likely let the EMG contaminate. However EMG can be taken / studied from most of part of the body but whenever the part under consideration resides near to the heart then it may be possible that it may got corrupted by ecg signal. For example the case with the trunk muscle [9].

Fig 6.1

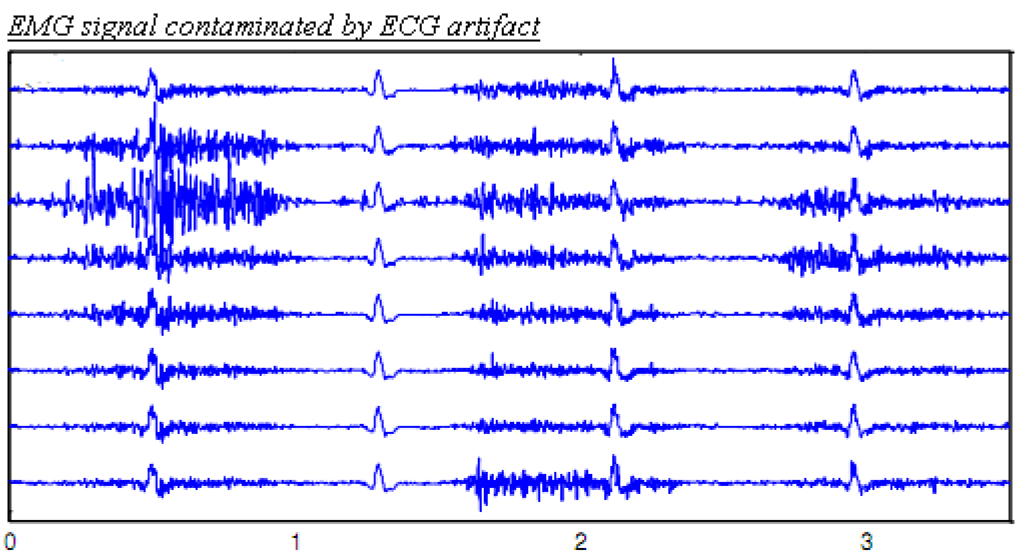

Figure shows the EMG signal contaminated by ECG signal[7]

Previously many techniques have been studied for noise removal in various signals, but here the problem lies is that both signals sometimes overlap each other {in terms of frequency and amplitude}. In this technique[7] an ICA and adapted filter is used, which proves an efficient method in ECG noise reduction. In this the reference signal is extracted through ICA and is estimated through FIR filter coefficients gets updated through RLS. In[7] this technique uses eight channel clean surface EMG has been superposed by the eight channel Electrocardiograph noise.

6.2 <u>Independent Component Analysis</u>

For proper noise removal and for better diagnostic purpose more efficient and effective techniques are needed.

In recent years various techniques had been proposed. One of the them is a special case [10] of blind source separation is ICA (Independent component analysis) and another one Adaptive filters.

Independent Component Analysis (ICA): - ICA is one of the most useful and prominent technique in signal processing. ICA, is basically a multi-variate statistical transformation which tries to recover the statistically independent component from the original signal. ICA [7,9,10,17,18] actually transforms multivariate / random vector / signal into several independent source components which are statistically independent mutually.

Considering the case of noisy EMG (Electromyographic) signal. This electromyographic signal may got effected from several kinds of noise such as PLI {power line interference} noise, also ECG (Electrocardiographic) signal.

However ECG is an signal but still it will considered as a noise in case we are attempting to consider a noisy EMG signal. ICA approach can be applied whenever the signals / processes / vectors are mutually statistically independent from each other and the process involving the ECG and EMG signal can be treated as mutually independent process. So ICA algorithm can be applied.

As if a EMG signal taken from trunk muscle [7] may also contain ecg artifact. The problem lies in filtering EMG signal i.e. removing the ECG component is that they sometimes overlap each other and during filtering loss of signal portion may occur. Considering ICA technique, the signals should be of non- Gaussian nature. ICA is basically a special case if blind source separation method.

ICA method can be applied for the separation of the mixed signals based on the following two assumptions: -

(i) Subcomponents of the mix signals should be non Gaussian.
(ii) Source signals should be independent of each other.
(iii) The mixing matrix should be linear.

The ICA being used here for separating the independent components that are present here in the mixture {EMG + ECG}. The purpose of using ICA is to get the reference signal for estimation of the ecg signal. After the reference ecg signal from ICA. This ECG artifact can be estimated using an adaptive filter (FIR digital filter), using the correction criteria. One of the widely used correction criteria is RLS (Recursive Least Squares). The RLS helps in estimating the ECG signal by updating the coefficients of the digital filter. A good estimate of the ECG artifact can be obtained by using the recursive least squares. In ICA, the observed vector $x_i(n)$ can be considered as the mixture of the ECG + EMG signals. The purpose is to separate these as Independent components.

6.2.1 <u>The ICA technique</u>: -

The basic model for the ICA technique can be represented as: -

$$x_i(n) = Bs_i(n)$$

Where $x_i(n)$ represents statistically, mutually IC's i.e. they can be written as:-
$s_i(n) = [s_1(n) , s_2(n) , s_3(n) ……………s_N(n)]^T$

$x_i(n)$ represents the observed vector i.e. $x_i(n) = [x_1(N) , x_2(n) , x_3(n) ,………. x_M(n)]^T$
B is some scalar mixing matrix [M*N]

ICA basically focusing on two things: -

(i) To maximize the non-gaussinity.
(ii) To minimize the mutual information.

Here in the above mentioned representation of ICA the problem is to find the original source signal $s_i(n)$ from the given observations $x_i(n)$.
Considering the first definition, the ICA then first finds the original source signal.

i.e. $y(n) = wx_i(n)$ ------------(3)
the function here y(n) represents the estimation of the statistically mutually IC's.

So these can be represented as:-

$y(n) = \{y_1(n),y_2(n),y_3(n),y_4(n),………,y_N(n)\}^T$

Now for decorrelation i.e. for minimizing the mutual information b/w the independent components, we have to find the matrix w in equation (3); as if we can say that for any b≠a viz. $y_a(n)$ and $y_b(n)$ are not correlated i.e. we can say that for any a≠b, the information on $y_a(n)$ must not give any information on $y_b(n)$.

As also according to another definition of ICA, non gaussianity should be maximized. Also it is to remembered that the number of observable mixture should be equal to number of independent components. Actually in the given situation the independent components and the mixing matrix are both unknown.

Here the purpose is to estimate B by using the given observable vector $x_i(n)$. In this study FastICA [7,25] fixed point algorithm has been used. Before moving further some preprocessing has to be applied.

6.2.2 <u>The Preprocessing steps</u>

Here the preprocessing consists of two steps:-

(i) Centering, and

(ii) Whitening.

(i) **Centering**:- This technique has to be done to make the signal / observable variable of zero mean. In this, the sample mean has to be subtracted from the original mixture{signal and artifact}.

$$x_i\{centered\} = x_i - E(x_i)$$

Even after this preprocessing step the mixing matrix(B) remains the same as before preprocessing.

(ii) **Whitening**:- Whitening is basically done for the purpose of removal of correlation between the data observed. As in the previous step (centering), the data has been made zero mean, now it is to be said that the independent components are zero mean random vector y and are now uncorrelated having variances equal to unity.

One of the useful and famous method of whitening is EVD. EVD basically stands for Eigen value decomposition.

So, it is to be written

$$E(x_i x_i^{\,T}) = LD_M L^T$$

Here D_M represents diagonal values of $E(x_i x_i^{\,T})$.

L represents orthogonal matrix of Eigen vectors of $E(x_i x_i^{\,T})$.

So the whitening can be done by:-

$$U = LD^{-1/2}LT$$

Where U represents the whitening matrix

The whitened data can be obtained as:-

$$W_D = Ux_i$$

So from this, a new white vector can be obtained.

Now the preprocessing on the data is done, so now applying the Fast ICA,

6.2.3 <u>THE FAST ICA ALGORITHM</u>:-

We have discussed the basic independent component analysis above (as general technique). Now the algorithm that is used below is FastICA[7,10,25].

FastICA is a very popular and widely used algorithm available for ICA. This algorithm (FastICA) is a fixed point ICA. This algorithm basically maximizes non gaussianity. FastICA (fixed point) is based on minimizing the mutual information, the expression can be given as:-

$$MI_m(s) = \int k_s(s) \log \frac{k_s(s)}{\Pi k_{si}(s_i)} d(s)$$

k_s is some non linearity function

$MI_m(s)$ is mutual information (in case of fixed point algorithm)

Considering expression for the fast ICA algorithm:-

$$MI_m(s) = Q(s) - \sum_i Q_{si} + \frac{1}{2}\log\frac{\Pi R_{ii}}{\det R_{ss}}$$

Here $MI_m(s)$ represents the mutual information.
R_{ss} represents the correlation matrix
R_{ii} represents the i_{th} diagonal element of the correlation matrix.

Now the purpose here is to maximize the Negenotropy denoted by Q. Negenotropy is basically an entropy's normalized version.

The Negentropy Q(s) can be defined as follows:-

$$Q(s) = H_D(Sgs) - H_D(s)$$

Here, $H_D(Sgs)$ represents a signal (gaussian) of same cov. Matrix {as s}.
So,

$$Q_Z(w) = \{E(Z(w^T v\} - E\{Z(v)\}^2$$

Here v represents the standard gaussian variable.
Z is one unit constant function
Q is the Negentropy
Again:-

$$w+ = E\{vg(w^T v)\} - e\{g'(w^T v)\}w$$

so:-

$$w = \frac{w^+}{\| w^+ \|}$$

From this expression the independent components can be extracted.
From this expression the independent components can be extracted. From the above mentioned procedure the reference ECG signal can be derived. Now after getting the reference signal (r(t)), so then by using the RLS (Recursive least squares) algorithm[7,10,16,20] as an correction criteria:-

The equation can be represented as:-

$$\bar{r}(t) = h(t) * r(t)$$

where the $\bar{r}(t)$ represents the estimated ECG signal. This ECG can be estimated by making use of the FIR filter whose IR(impulse response)/coefficients can be updated using RLS.

Now getting the denoised version of EMG signal,

$$\bar{s}(t) = x(t) - \bar{r}(t)$$

here x(t) represents the original noisy signal and

$\bar{s}(t)$ represents the denoised signal.

6.3 THE RLS ALGORITHM

Now the after having the reference signal, the ECG signal is then can be estimated using RLS as an correction criteria.
Using the above expression it can be written as:-

$$e(n) = x(n) - E(r(n))$$

$$\text{then, } e(n) = x(n) - \bar{r}(t)$$

where E(r(n)) presents the estimate of ECG signal.

The cost function can be represented as:-

$$C_f(n) = \sum_{i=1}^{n} \chi^{n-i} |e(i)|^2 + \gamma\chi^n \| w(n) \|^2$$

where $C_f(n)$ represents the cost function.

$w(n)=\{w_0(n),w_1(n),w_2(n),\ldots\ldots..w_{N-1}(n)\}^T$

here χ represents forgetting factor, here this is exponentially forgetting factor, basically the use of this forgetting factor is to let the data in past should be forgotten.
w(n) are the coefficients vector of finite impulse response filter.(at any time instant n)
γ represents some positive constant, for regularization purpose.

The basic principle of the recursive least algorithm is to minimize the least squares basically in turn the cost function $C_f(n)$, the cost function is actually represents the weighted least squares as shown above. The RLS algorithm keeps on updating the weights of the coefficient vector.
Taking some initial values the algorithm can be started:-
$J(0) = \gamma^{-1}I$,
$w(0) = 0$
here I represents an identity matrix.

Also we can define:-

$u(n)=\{r_r(n),r_r(n-1),r_r(n-2),\ldots\ldots\ldots\ldots.r_r(n-N+1)\}^T$

The u(n) here in general sense is the tap input vector at any time instant n
Also the gain vector can be defined as-

$$G(n)=\frac{J(n-1)\,u(n)}{\chi+u^{H}\,J(n-1)\,u(n)}$$

here G(n) can be defined as the gain vector.

$\beta(n)=x(n)-w^{H}(n-1)u(n)$

$\beta(n)$ is the a priori estimation error
also it can given as :-

$w(n)=w(n-1)+G(n)\,\beta^{*}(n)$

by this the old weights gets updated by a factor $\beta^{*}(n)$. Here H in above equations represents the hermitian transpose operator and * denotes complex conjugate of the parameter.
Now,

$$J(n)=\chi^{-1}J(n-1)-\chi^{-1}G(n)u^{H}(n)J(n-1)$$

here J(n) represents an inverse correlation matrix of dimension M*M.

so, there by using RLS updation in coefficients can be achieved and thus a better noise estimation can be achieved.

The method gives better results than its previous method, the Butterworth filter30[7].

CHAPTER 7

ECG NOISE REMOVAL USING ADAPTIVE FILTERING

7.1 Introduction

Many techniques have been proposed in recent past for the removal of ECG noise from the EMG signal. In [7] an ICA and Adapted filter technique has been used.

This technique is based on Adaptive noise cancellation (ANC) concept[7,10,16,19,20]. This adaptive noise cancellation filter makes use of the adaptive algorithm (RLS algorithm). RLS algorithm proves to be an advanced and efficient technique than its previous LMS algorithm. This algorithm (RLS) has a much higher convergence rate and proves very fruitful in increasing the SNR in real time use.

In this technique (ANC) the input signal (noisy) first goes through some preprocessing steps. In preprocessing here, notch filter and high pass filter is used for the removal of PLI and motion artifact respectively. The notch filter notches (remove) the unwanted frequency. After these, the signal goes through the ANC (with RLS Algorithm) filter.

7.2 The concept of Adaptive noise cancellation

The adaptive filter, as its name suggest basically adjusts itself according to the environment. Here the adjustment is done in the parameters according to some algorithm taking into account the considered signal (error).

Sometimes a process increases average power of the output noise, when an noise has been getting subtracted from the considered signal (Input signal).

As shown in the figure below:- the signal corrupted with noise can be given as:-

$$x(n)=s(n)+r(n)$$

Here x(n) is the desired signal and s(n) is the clean signal where r(n) represents the noise

Now:-

Let $r_i(n)$ be the reference signal coming from the noise source, so the error signal can be given as:-

$$e(n)=x(n)-y(n)$$

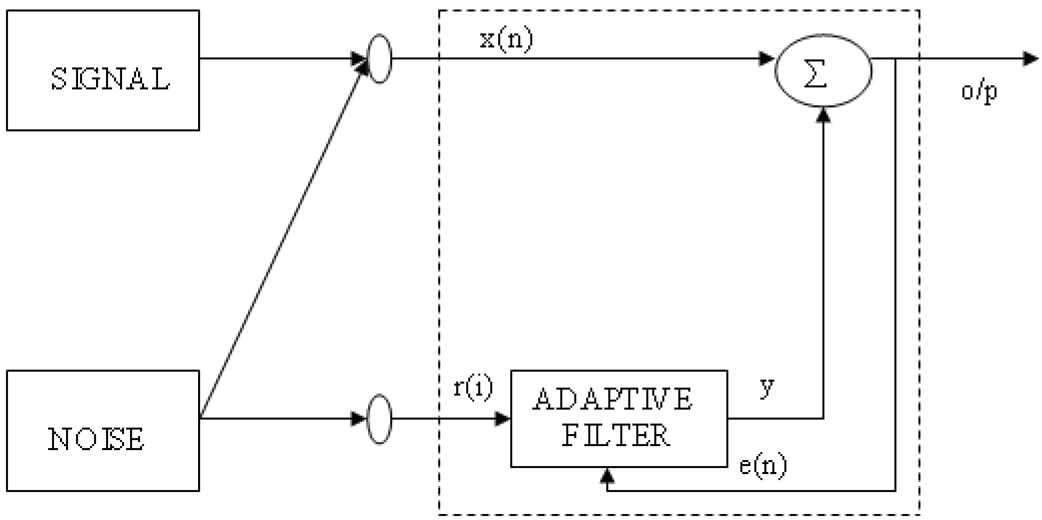

FIG 7.1-Figure shows the adaptive noise cancellation concept[19]

where y(n) represents the filter output here the purpose of the adaptive filter is to minimize the above error.
Now y(n) can be written as:-

$$y(n) = \sum_{i=0}^{N-1} w_i(n)\, r_i(n-1)$$

So, by analyzing the error, the weights of the filter can be adjusted, thus minimizing the error. However the above technique can also be performed by any other filter but here adaptive filter has been used[16].

One the purpose of using the adaptive filter is that it increases the SNR of the output, thus reducing the error. The adaptive filter basically reduces (minimizes), the mean square error value of the error (e(n)).

This follows from the above process that in the whole process the signal s(n) remains unchanged., i.e. in the adaptive noise cancellation process it basically focuses on reducing / minimizing the mean square value of o/p noise signal. The above adaptive filter adjust its parameter with the help of iterative algorithm RLS(recursive least square). This algorithm update the values of the filter coefficients/weights by analyses the error signal.

In ANC filtering using recursive least square an, EMG signal has been under consideration for the removal of ECG noise. In EMG (electromyography) signals, sometimes ECG signals got mixed/overlap whenever the EMG signal from muscles which are near consideration. As discussed above sometimes standard filtering techniques were not very efficient in the ECG removal from EMG signals.

In these cases ECG signal overlap some portions of the EMG signals, so it is sometimes difficult to remove ECG from EMG signal. But as efficient ANC technique using the RLS has been discussed in[16,20]. Which proves an efficient method for the ECG noise removal.

Now consider the signal:-

$$x(n) = s(n) + r(n)$$

here x(n) represents the EMG signal(contaminated), s(n) represents the clean EMG signal and r(n) is the ECG artifact.

The reference signal for ECG has been taken separately[16]. The ECG signal can be estimated using the FIR filter:-

$$r'(n) = \sum_{i=0}^{N-i} w_i * r_c(n-i)$$

here w_i represent the coefficients of the filter {coefficient vector}and N denotes the order of the filter.
These coefficients/weights are updated by the ANC filter using RLS by analyzing the error function.
Now the denoised EMG can be given as:-

$$s'(n) = x(n) - r'(n) = e(n)$$

here $s'(n)$ represents the denoised EMG signal (using ANC filter).

So, this can also be written it as:-

$$s'(n) = x(n) - \sum_{i=0}^{N-i} w_i * r_c(n-i)$$

the coefficient vector (consist of filter coefficients) can be given as:-

$$w(n) = \{w_0(n), w_1(n), w_2(n), w_3(n), \ldots\ldots\ldots w_{N-1}(n)\}^T$$

now the purpose of the adaptive filter is to minimize the error signal so as the estimated ECG signal can be as close as possible to the ECG signal present in the contaminated EMG signal.

The cost function can given as:-

$$C_f(n) = \sum_{i=1}^{n} \chi^{n-1} \left| e(i) \right|^2 + \gamma\chi^n \left\| w(n) \right\|^2$$

here $C_f(n)$ denotes the cost function which is summation of the weights least squares (e(i)). The purpose here is to minimize this error i.e. the cost function (C_f). here this minimization is done by the Recursive least squares algorithm. here in the above expression χ is called the forgetting factor(ff).

To start the process of error minimization by analyzing error function and thus keeps on updating the weights. The algorithm then can be started by taking some initial values:-
$J(0) = \gamma^{-1}I$,
$w(0) = 0$

here I represents an identity matrix.

Also we can define:-

$u(n)=\{r_c(n),r_c(n-1),r_c(n-2),\ldots\ldots\ldots\ldots r_c(n-N+1)\}^T$

The u(n) here in general sense is the tap input vector at any time instant n
Also the gain vector can be defined as-

$$G(n) = \frac{J(n-1)\,u(n)}{\chi + u^H\, J(n-1)\, u(n)}$$

Here G(n) can be defined as the gain vector.

$\beta(n)=x(n)-w^H(n-1)u(n)$

$\beta(n)$ is the a priori estimation error
also it can given as :-

$w(n)=w(n-1)+G(n)\ \beta^*(n)$

by this the old weights gets updated by a factor $\beta^*(n)$. Here H in above equations represents the hermitian transpose operator and * denotes complex conjugate of the parameter.
Now,

$$J(n)= \chi^{-1}J(n-1)-\chi^{-1}G(n)u^H(n)J(n-1)$$

here J(n) represents an inverse correlation matrix of dimension M*M.

so using the above relation the errors function can be minimized using Adaptive filter making use of the RLS.

For analyzing the performance SNR values[16] have been found out

So, the signal to noise ratio (SNR) of the denoised EMG have been found out for analyzing the performance.

$$\mathrm{SNR} = 10 \log 10 \frac{\mathrm{var(s)}}{\mathrm{var}(s - r')}$$

Here var represents the variance operator

In[16] another parameter used in performance checking is Coherence (in frequency domain). The expression for Coherence among denoised signal and clean signal can be given as:-

$$\mathrm{Coh}(f) = \frac{\left| P_{s(n)s'(n)}(f) \right|}{P_{s(n)}(f) P_{s'(n)}(f)}$$

Here $P_{s(n)s'(n)}(f)$ denotes the cross spectral density among s(n) and $s'(n)$.

$P_{s(n)}(f)$ and $P_{s'(n)}(f)$ denotes auto spectra

In this method *ff*(forgetting factor) can be used as a parameter for controlling the convergence rate. The value of *ff* taken in[16] is equal to 0.1. The method usually provides good performance but when the SNR decreases more, then its performance is not up to the mark.

CHAPTER 8

Discussions

Table 1:-

Noise Removal	Methods: IIR notch filter with Time varying pole radius (MSE)	Traditional notch filter with zero initial conditions (MSE)	IIR notch filter with non zero initial conditions (MSE)	Percentage improvement in MSE using IIR notch filter with non zero initial conditions in comparison to next best proposed method %
60Hz noise removal	**$2.35*10^{-4}$**	$7.4002*10^{-4}$	**$1.8289*10^{-4}$**	22.17
60Hz,120Hz noise removal	--	**$11*10^{-4}$**	**$2.3079*10^{-4}$**	79.02
60Hz,120Hz & 180Hz noise removal	--	**$36*10^{-4}$**	**$1.5472*10^{-4}$**	95.70

Table 1 shows the mean square error of three methods, IIR notch filter with non zero initial conditions with time varying pole radius, traditional notch filter with zero initial conditions and IIR notch filter with non zero initial conditions[12,13].

Table 2:-

Method / SNR (db)	IIRNC SER(db)	IIRC SER(db)	EMD SER(db)	EEMD SER(db)	Percentage improvement in SER using EEMD in comparison to next best proposed method (%)
-6	7.062	3.138	**7.305**	**9.150**	25.25
-2	10.27	4.025	**10.50**	**11.22**	6.85
2	**12.75**	4.390	12.70	**13.56**	6.35
6	**14.34**	4.515	14.18	**15.10**	5.29

Table:- 2 shows the SER of various techniques. In this IIR causal , non causal, EMD and EEMD techniques SER has been there[11]. In this a percentage improvement has been given EEMD in comparison to next best proposed method.

In table:-1 three PLI noise removal techniques [12,13] have been studied, based upon there results i.e. the Mean square error , it can be seen that for the removal of PLI & its harmonics, the infinite impulse response notch filter having non zero initial conditions, which is an improved version of the technique proposed by Pie & Tseng[15], outperforms with minimum error among the three techniques. A percentage improvement criterion is also shown in the table.

In table 2, another PLI removal techniques have been studied[11], among these techniques EEMD have better SER. A percentage improvement criterion has been shown there.

Table 3:-

SNR (db)	AF&ICA (SER)	BW30Hz (SER)	ANC (SER)	Percentage improvement in SER using AF&ICA in comparison to next best proposed method (%)
-1	**36.58**	**22.26**	21.30	64.33
0	**39.10**	22.91	**28.32**	38.06
2	**43.78**	23.93	**31.06**	40.95
5	**50.35**	24.88	**37.61**	33.87
8	**57.30**	25.40	**40.37**	41.93

Table 3 shows the SER values at various SNR levels. In this, adapted filter, Butterworth filter and adaptive noise cancellation technique has been studied[7,16]. In this table an ECG artifact removal technique have been studied[7,16], among these, technique based on adapted filter & ICA have better SER's. a percentage improvement criterion is also shown in figure.

For perfect detection and proper analyzing a clean (noise free) signal is needed. Some of the techniques provides better results but side by side sometimes, they also increase calculation time. Also in real scenarios where sometimes SNR is not always high, then in these cases where signal to noise ratio is more less, then regular techniques may not provide very efficient results. Some methods needs reference signals for estimation of noise.

However it is not always possible to provide a reference signal. In case of EEMD, although it provides a better performance[11] but it also increase the burden of calculation. Also in [7] when the Electrocardiogram noise that is present in Electromyogram signal does not have a dominating nature it provides a acceptable performance. But the technique should be so strong that it should remove the noise whether it is dominating or not.

So, such techniques should be fond which are so efficient that they can maintain there efficiency in changing scenarios such as changing SNR levels, without adding more complex computational load.

CHAPTER 9

Future scope

EMG now a day becomes a very popular technique for the detection of the muscle abnormalities and various diseases. Various methods have been proposed for the denoising of the EMG signal. Many methods have promises very efficient outcomes. But still, some methods requires updation as some of these promises efficient results but at a more computational demand. Also some methods are there whose results decreases dramatically due to sudden decrease in the SNR values. However this is not the case with all of these. But still in general a balance is required which provides higher efficiency with low computational complexity.

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