Mechanical System Design in Reliability

A

Project Report

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In

COMPUTATIONAL DESIGN

By

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CERTIFICATE

This is to certify that the thesis entitled **"Mechanical System Design in Reliability"** submitted by **Prashant Rathi (2k14/cdn/11)**, during the session 2014-2016 for the award of M.Tech degree of Delhi Technological University, Delhi is absolutely based upon his work done under my supervision and guidance and that neither this thesis nor any part of it has been submitted for any degree/diploma or any other academic award.

The assistance and help received during the course of investigation have been fully acknowledged. He is a good student and I wish him good luck in future.

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DECLARATION

I, **Prashant Rathi**, hereby certify that the work which is being presented in this thesis entitled **"Mechanical System Design in Reliability"**, is submitted, in the partial fulfillment of the requirements for degree of Master of Technology at Delhi Technological University is an authentic record of my own work carried under the supervision of **Dr. Girish Kumar**. I have not submitted the matter embodied in this seminar for the award of any other degree or diploma also it has not been directly copied from any source without giving its proper reference.

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ABSTRACT

In this thesis a multistage compressor system is designed using Markovian approach with the purpose to improve its reliability. The design analysis helped in identifying the key factors that affects the system reliability and there exists good scope to improve the system reliability by controlling the contributing factors.

Different models are developed for the system to achieve the reliability goal by adding standby redundancy to the system components. All the feasible states and, failure and repair transitions are identified to develop the system model. Keeping in mind the limitation of the Markov model the failure and repair rates are taken as constant. The sets of ordinary differential equations are obtained for the change of probability of being in respective system states with respect to time in each model. The system of rate equations is solved using Runge – Kutta method in MATLAB. The system reliability assessment is based on the sum of probabilities of all working states. Sensitivity analysis is also carried out by varying the repair rates of constituent components in the system. These results are helpful to design the highly reliable systems for thermal power plant, machineries used in medical field, aerospace and aviation industries..

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LIST OF SYMBOLS

λ_1	Failure rate of low pressure compressor
λ_{11}	Failure rate of standby low pressure compressor
λ_2	Failure rate of intercooler
λ_{22}	Failure rate of standby intercooler
λ_3	Failure rate of high pressure compressor
λ_{33}	Failure rate of standby high pressure compressor
μ_1	Repair rate of low pressure compressor
μ_{11}	Repair rate of standby low pressure compressor
μ_2	Repair rate of intercooler
μ_{22}	Repair rate of standby intercooler
μ_3	Repair rate of high pressure compressor
μ_{33}	Repair rate of standby high pressure compressor
R(t)	Reliability function
P(t)	Probability of particular state at time t
O	Operating state of a component
S	Standby state of a component
F	Failed state of a component
$\sum$	Summation

LIST OF ABBREVIATIONS

MTTF	Mean time to repair
MTTR	Mean time to failure
ETA	Event tree analysis
FMEA	Failure mode and effects analysis
FMECA	Failure mode, effects and criticality analysis
FTA	Fault tree analysis
MA	Markov analysis
PNA	Petri net analysis
TT	Truth table
RBD	Reliability block diagrams
LPC-1	low pressure compressor
LPC-11	Standby low pressure compressor
IC-2	Intercooler
IC-22	Standby intercooler
HPC-3	High pressure compressor
HPC-33	Standby high pressure compressor

CHAPTER 1

INTRODUCTION

Numerous industries such as power plant, aviation, manufacturing, refineries, gas pipeline, and space industry, etc must run continuously without any down time. In such industries failure of any mechanical subsystem can lead to stop the working of whole system. Especially in power plant, failure of any main unit can lead to stop the power generation. This type of situation is very costly and highly undesirable. Therefore to design a highly reliable mechanical system is most desired for such industries.

Reliability is a widely used concept, sometimes without a precise definition. It is simply summarized as the ability of an item to be functional. The concept of reliability has been used for technical systems for more than 50 years and is a field of research common to mathematics, operational research, graph theory, physics, etc. According to ISO (1986), reliability is defined as; *"The ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time."* Reliability is an inherent attribute of most of the system or component and it is an important consideration in the engineering design process. The analysis of reliability guides practicing engineers in selecting an appropriate design strategy and in improving performance of the system.

Reliability design is an iterative process that begins with the specification of the reliability goals consistent with system effectiveness. Once the reliability goals have been established at the system level, these goals must be translated at individual component/subcomponent levels and part specifications. This task generally requires a reliability block analysis. Once the individual component and part requirements have been determined, various design methods can be applied in order to meet the goals. These methods include the proper selection of parts and material, stress and strength analysis, derating, simplification, identification of technologies and use of redundancy. After completion of preliminary and detailed design along with initial development and prototyping, a failure analysis may be performed to determine whether specifications are being met. This is to provide a systematic approach for identifying, ranking and eliminating failure modes. Further, this require the use of reliability testing, including, perhaps a formalized reliability growth test program. Once the reliability goals have been achieved, verification that safety margins are also being met must be made. Fault tree analysis can be a useful tool in identifying critical failure mode.

Redundancy is a common approach to improve the reliability of a system. Adding redundancy increases the cost and complexity of a system design. However, if the cost of failure is high enough, redundancy may be attractive option. Redundancies can be categorized as active or passive (standby) redundancy. In case of active redundancy, all redundant components are in operation and share the load with the main component. The redundant or back up component in passive or standby system start operating only when one or more fail. The standby systems are generally much more reliable than active redundant system.

Markov chain is an useful tool to analyze the systems with standby redundancies as it is capable to handle state dependencies. The method is a state-space approach. The likelihood of any event in the chain is determined only by the immediately preceding state and is independent of any other past events.

In this thesis, a multistage compressor used in thermal power plant has been considered as a system for design in reliability. This system has three component low pressure compressor, intercooler and high pressure compressor in series combination. To achieve high reliability goal, standby redundancies have been added in system. Markov transition state models are developed for the system. Reliability is assessed by creating state space probabilistic equation with the help of Markov transition diagram. These equations are further solved by Runge-kutta method with the help of MATLAB.

CHAPTER 2

LITERATURE REVIEW

This chapter deals with literature review mainly covering historical background, reliability design and Markov process methodology.

2.1 Historical Background

Reliability was introduced in 1816, when the word reliability was first used by the poet Samuel Taylor Coleridge. An early application of reliability was in the field of the telegraph. By 1915, radios with a few vacuum tubes began to appear in the public. Automobiles came into more common use by 1920 and Mc Lenn (2011) represented mechanical applications of reliability. In the 1920, product improvement through the use of statistical quality control was promoted by Dr.Walter A Shewhart (1924) at Bell Labs.

On a parallel path with product reliability was the development of statistics in the twentieth. Statistics as a tool for making measurements would become inseparable from the development of reliability concepts. Wallodie Weibull (1996) was working in Sweden during this period and investigated the fatigue of materials. During this time, he created a distribution, which we now call Weibull distribution. By the 1940, reliability engineering still did not exist. Much of the reliability work of this period also had to do with testing new materials and material fatigue and the first published articles were about this aspect. In 1948 the Reliability Society was formed by the Institute of Electrical and Electronics Engineers (IEEE) published in Danson (1998). The military was gradually started with cost considerations at the beginning of 1950. They could not afford to have half of their essential equipment non-functional all of the time. In 1957 Robert Lusser (1950) pointed out in a report, that 60% of the failures of one Army missile system were due to components and the current methods for obtaining quality and reliability were inadequate and that something more was needed. Papers were being published at conferences showing the growth of this field. Ed Kaplan combined his nonparametric statistics paper on vacuum tube reliability with Paul Meier's (1958) biostatistics paper. the nonparametric maximum likelihood estimate (known as Kaplan-Meyer) of reliability functions from censored life data in 1958.

The 1960 saw several events, one of the most important being that a strong commitment to exploration would turn into the National Aeronautical and Space Administration (NASA), a driving force for improved reliability of components and systems. 1962 was a key year with the first issue of Military Handbook 217 by the Navy and a Failure Modes and Effect Analysis (FMEA) handbook (non-military applications) was issued in 1968 by Mclenn (2011).

During the 1970, work progressed across a variety of fronts, while 1980 and 1990 were decades of great changes. During these decades, the failure rate of many components dropped by a factor of 10. Software became important to the reliability of systems. By the end of 1980**,** programs could be purchased for performing FMEA, Fault Tree Analysis (FTA), reliability predictions, block diagrams and Weibull Analysis by Mclenn (2011). The Challenger disaster caused people to stop and re-evaluate how they estimate risk. This single event spawned a reassessment of probabilistic methods.

New technologies such as micro-electro mechanical systems (MEMS), hand-held GPS, Li-I batteries and hand-held devices that combined cell phones and computers all represent challenges to maintain reliability during the 2000. Product development time continued to over the decades and what had been done in three years was now done in 18 months or less. Consumers have become more aware of reliability failures and the cost to them by Mclenn (2011). Nowadays, reliability has become part of everyday life and consumer expectations, and the reliability tools and methods must be closely tied to the development process itself.

2.2 Design for reliability

Reliability is an inherent attribute of a system, component or a product. It is an important consideration in the engineering design process. Reliability designs an iterative process that begins with the specification of reliability goals consistent with cost and performance objectives. Method has been developed for reliability analysis and to meet the reliability goals.

The usefulness of the reliability analysis for the systems was discussed almost half century back by Morse (1958), Barlow and Hunter (1960), Sandler (1963). It has always been considered as a useful tool for design of systems, risk analysis, production availability studies. Various methods exist in literature for reliability, like Reliability Block Diagrams (RBD), Monte Carlo Simulation, Markov Modeling, Failure Mode and Effect Analysis (FMEA)), Fault Tree Analysis (FTA) and Petri Nets (PN) proposed by Singer (1990), Bradley and Dawson (1998), Modarres and Kaminsky (1999), Bing et al. (2000), Cochran (2000), Gandhi et al. (2003), Parveen et al.(2003), Adamyan and David (2004), Arthur (2004), Barbady et al. (2006), Panja and Ray (2007), Bhamare et al. (2008). In 1970, Vesely developed a computer code “KITT” to analyze the repairable systems and evaluated system reliability parameters with an assumption that the failure and repair events of considered system components must be independent. Buzacot (1970) computed reliability measures of a system based on successive reduction of complex models and determined the intervals based on parallel and series sets, which were referred as minimal cut and path sets. Exponential distribution was used to model system failure and repair rates. Kim et al. (1972) proposed a technique for computing the reliability of complex systems and suggested a three phase approach. In the first phase, all series parallel subsystems were reduced to non series parallel subsystems. In the second stage, all the possible paths were traced from source to sink and in the third phase, system reliability is calculated based on these paths. Collins (1975) investigated the possible failure modes in the case of a helicopter and recommended some useful corrective measures. Burns (1975) carried out reliability analysis in nuclear mechanical systems.

2.3 Markov Process

A Markov model needs identification of possible states of the system, their transition paths, and the rate parameters of the transitions. Each state represents the different condition of the system. The transition from one state to another state occurs, with failure and repair rate exponentially distributed. It is a widely used technique for many applications, including evaluation of reliability. The work related to application of Markov approach is reported below.

Sahner and Trivedi (1986) proposed hierarchical modeling using the Markov approach for a complex system to deal with the problem of state space explosion. The authors suggested a mechanism for decomposition and aggregation based on functional similarity. The proposed approach allows for both combinatorial and Markov models and can analyze each model to produce a distribution function. Kim and Park (1994) proposed system reliability based on Markov model for a phased mission. Pukite and Pukite (1998) in their book presented various modeling and analysis techniques for reliability, maintainability, availability, safety and supportability of complex computer systems that included sub-classes of Markovian approaches, Petri net, Monte Carlo simulation. The authors also listed advantages and limitations of each modeling technique, with special emphasis on Markov modeling. Xie *et al.* (2000) investigated the use of exponential distribution as an approximation to Weibull distribution for reliability and maintainability studies. The proposed framework addressed optimal maintenance in respect of time and spare allocation.

Ajah *et al.* (2006) introduced hierarchical Markov based reliability modeling for energy and industrial systems. The authors suggested decomposition of the reliability/availability problems in three levels (components, units and system) and aggregation based on functional and structural similarities. The proposed methodology reduced the problem of state space explosion problem for large systems. Carter and Malerich (2007) studied impact of the exponential repair assumption on reliability assessment. The authors observed that the exponential repair assumption inflated system reliability. Guo and Yang (2008) presented a methodology for the automatic creation of Markov models for reliability evaluation of safety instrumented systems. Andrews (2009) reviewed the state-of-the-art techniques, including the Markov approach for system reliability evaluation. The author also discussed the likely applications in the context of the recent advances in the assessment techniques.

Welte (2009) presented an approach, with gamma distribution transformed to a Markov Process (MP), with sequence of states having exponentially distributed sojourn times. The approximation of the gamma distribution into exponential distribution yielded good results. Some of the recent work of the researchers on reliability modeling considered features such as imperfect repairs, common cause failure, human error, etc. and used Markov approach, which included Hajeeh and Jabsheh, 2009; Hajeeh, 2011; 2011a; Jain *et al.*, 2014.

It is evident from the literature review that there are very limited attempts to address the problem of system design from reliability view point using state space models. In this work system design for a desired reliability goal is attempted using Markov model.

CHAPTER 3

RELIABILITY TOOLS AND TECHNIQUES IN DESIGN PHASE

In this chapter, various reliability tools and techniques used in design phase will be discussed. Modeling of a various mechanical systems in reliability is done by these reliability tools.

There are various methods and approaches available in the field of reliability theories. These are mainly classified as:

a. Methods for fault avoidance
    - Parts derating and selection,
    - Stress-strength analysis;
    - Part count.

b. Methods for architectural analysis and dependability assessment
    - Bottom-up method (mainly dealing with effects of single faults)
        - Event tree analysis (ETA)
        - Failure mode and effects analysis (FMEA)
        - Failure mode, effects and criticality analysis (FMECA)
    - Top-down methods (able to account for effects arising from combination of faults)
        - fault tree analysis (FTA),
        - Markov analysis (MA),
        - Petri net analysis (PNA),
        - •truth table (TT),
        - reliability block diagrams (RBD);

c. methods for estimation of measures for basic events, e.g.

- failure rate prediction,
- human reliability analysis (HRA)
- statistical reliability methods

Another distinction is whether these methods work with sequences of events or time dependent properties. If this is taken into account, the following comprehensive categorization results:

- Sequence dependent: ETA, MA, PTA, functional analysis, Dynamic FTA
- Sequence independent: FMEA, FTA, RBD

These analysis methods allow evaluation of qualitative characteristics as well as estimation of quantitative ones, in order to predict long-term operating behaviour. It should be noticed that the validity of any result is clearly dependent on the accuracy and correctness of the input data for the basic events.

3.1 The Part Count Approach

The "Part Count" is simplest, most pessimistic, inductive approach where every component failure is assumed to cause system failure. The Part Count method can be found named or described, by many standards, such as the military US standards. Under this approach, obtaining an upper bound on the probability of system failure is especially straight forward. All the components are listed along with their estimated probabilities of failure. The individual component probabilities are then added and this sum provides an upper bound on the probability of system failure. The failure probabilities can be failure rates, un-reliabilities, or un-availabilities depending on the particular application.

For a particular system, the "Part Count" technique can provide a very pessimistic estimate of the system failure probability and the degree of pessimism is generally not quantifiable. It is conservative because if critical components exist, they often appear redundantly, so that no single failure is actually catastrophic for the system. Furthermore, a component can often depart from its normal operating mode in several different ways and these failure modes will not, in general, all have an equally deleterious effect on system operation. If the relevant failure modes for the system operation are not known then it is necessary to sum the failure probabilities for all the possible failure modes.

The principal advantage is that this approach can be used in very early design phases when information is limited or missing. Another advantage of the method is its simplicity.

The analysis provides a very pessimistic estimate of the system failure probability and the degree of pessimism is generally not quantifiable.

3.2 Stress-strength analysis

Stress-Strength analysis is a method to determine the capability of a component or an item to withstand electrical, mechanical, environmental, or other stresses that might be a cause of their failure. Where reliability is the probabilistic measure of assurance of the component performance. This analysis determines the physical effect of stresses on a component, as well as the mechanical or physical ability of the component. Probability of component failure is directly proportional to the applied stresses. The specific relationship of stresses versus component strength determines component reliability.

Stress-Strength analysis is primarily used in determination of reliability or equivalent failure rate of mechanical components. It is also used in physics of failure to determine likelihood of occurrence of a specific failure mode due to a specific individual cause in a component. Evaluation of stress against strength and resultant reliability of parts depends upon evaluation of the second moments, the mean values and variances of the expected stress and strength random variables. This evaluation is often simplified to one stress variable compared to strength of the component. In general terms, the strength and stress shall be represented by the performance function or the state function, which is a representative of a multitude of design variables including capabilities and stresses. Positive value of this function represents the safe state while negative value represents the failure state.

The advantage of stress-strength analysis is that it can provide accurate representation of component reliability as a function of the expected failure mechanisms. It includes variability of design as well as variability of expected applied stresses, and their mutual correlation. In this sense, the technique provides a more realistic insight into effects of multiple stresses and is more representatives of the physics of component failure, as many factors – environmental and mechanical – can be considered, including their mutual interaction.

One disadvantage is that, in the case of multiple stresses, and especially when there is an interaction or correlation between two or more stresses present, the mathematics of problem solving can become very involved, requiring professional mathematical computer tools. Another disadvantage is possible wrong assumption concerning distribution of one or more random variables, which, in turn, can lead to erroneous conclusions.

3.3 Parts derating and selection

Derating can be defined as the practice of limiting electrical, thermal and mechanical stresses on devices to levels below their specified or proven capabilities in order to enhance reliability. If a system is expected to be reliable, one of the major contributing factors must be a conservative design approach incorporating part derating. The allowed stress levels are established as the maximum levels in circuit applications. Parts are selected, taking into account two criteria; a part's reliability and its ability to withstand the expected environmental and operational stresses when used in a system. Each component type, whether electronic (active or passive) or mechanical, must be evaluated to ensure that its temperature rating, construction, and other specific attributes like mechanical or other, are adequate for the intended environments.

Derating a part means subjecting it to reduced operational and environmental stresses, the goal being to reduce its failure probability to within the period of time required for proper product operation. When comparing the rated component strength to the expected stress, it is important to allow for a margin, which may be calculated based on the cumulative or fatigue stress and the component strength, or based on other engineering analysis criteria and methods. This margin allows the desired part reliability to be achieved regarding the particular fault modes and the respective causes.

The benefit of the part selection and derating practices is the achievement of the product's desired reliability.

The only limitation is when there is no information on part reliability in any of the available databases or from the part manufacturer. In such a case, limitation extends to the part derating, when the derating guidelines involve reliability guidelines.

3.4 Functional Analysis

Functional Analysis is a qualitative method and an important step in a system reliability analysis. In order to identify all potential failures, the analyst has to understand the various functions of the system, each functional block in the system and the performance criteria related to all those functions. The objectives of a Functional Analysis are to:

- Identify all the functions of the system
- Identify and classify the functions required in different operational modes

- Provide hierarchical decomposition of the system functions
- Describe how each function is realized
- Identify interrelationships between functions
- Identify interfaces with other systems and with the environment

Functional Trees or Functional Block Diagrams may be needed to illustrate complex systems.

Advantages: Functional Analysis provides an understanding of the systems functionality, interconnection between functions, and a base for further reliability.

Limitations: Wrong assumptions can lead to erroneous conclusions.

3.5 Failure Modes and Effects Analysis (FMEA)

Failure Mode and Effect Analysis (FMEA) was one of the first systematic techniques for failure analysis. It was developed by reliability engineers in the 1950 to study problems that may arise from malfunction of military systems. FMEA is an inductive method or a bottom-up approach. Induction involves reasoning from individual cases to a general conclusion. An FMEA is often the first step in a system reliability study. It connects given initiating causes to their end results or consequences. These consequences are often failure of a system or component. It involves reviewing all components, assemblies and sub-systems if possible, in order to identify failure modes and, causes and effects of such failures. For each component, the failure modes and their resulting effects on the rest of the system are recorded in a specific FMEA worksheet.

In the consideration of a certain system, a particular fault or initiating condition is postulated and an attempt is made to ascertain the effect of that fault or condition on system operation, an inductive system analysis is being conducted. It starts from failure initiators and basic event initiators, and then proceeds upwards to determine the resulting system effects of a given initiator. A set of possible causes are analyzed for their effects. There are several standards and procedures providing guidelines for this method, such as older military standard.

Advantages: An FMEA offers a systematic review of all components, assemblies and subsystems if possible, in order to identify failure modes and the causes and effects of such failures. It connects single failures with their effects and identifies the causes of those failures. The output of an FMEA is input to other reliability analyses such as Fault Tree, Event Tree, Reliability Block Diagram, etc.

Limitations: The analysis is limited to single failures and is time-consuming.

3.6 Reliability Block Diagram (RBD)

A Reliability Block Diagram is a success- oriented network describing the function of the system. RBD is an inductive model wherein a system is divided into blocks that represents distinct elements such as components or subsystems. These elemental blocks are then combined according to system-success pathways as shown in Figure 2.1. RBDs are generally used to represent active elements in a system, in a manner that allows an exhaustive search for and identification of all pathways for success. Dependencies among elements can be explicitly addressed.

Initially developed top-level RBDs can be successively decomposed until the desired level of detail is obtained. Alternately, series components representing system trains in detailed RBDs can be logically combined, either directly or through the use of Fault Trees, into a super-component that is then linked to other super-components to form a summary model of a system. Such a representation can sometimes result in a more transparent analysis. Separate blocks representing each system are structurally combined to represent both potential flow paths through the system.

The model is solved by enumerating the different success paths through the system and then using the rules of Boolean algebra to continue the blocks into an overall representation of system success. When an element is represented by a block it usually means that the element is functioning. Each element has also a probabilistic model of performance, such as Weibull,. If the system has more than one function, each function must be considered individual.

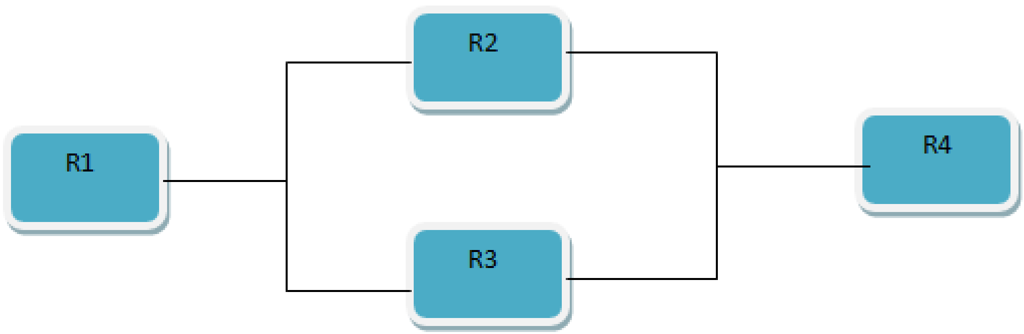


Figure 2.1: reliability block daigram

Some of the advantages of using RBD are

- Often constructed almost directly from the system functional diagram; this has the further advantage of reducing constructional errors and/or systematic depiction of functional paths relevant to system reliability.
- Deals with most types of system configuration including parallel, redundant, standby and alternative functional paths.
- Capable of complete analysis of variations and trade-offs with regard to changes in system performance parameters.
- Provides (in the two-state application) for fairly easy manipulation of functional or nonfunctional paths to give minimal logical models.
- Capable of sensitivity analysis to indicate the items dominantly contributing to overall system reliability.

- Capable of setting up models for the evaluation of overall system reliability and availability in probabilistic terms.
- Results in compact and concise diagrams for a total system.

Some of the limitations using RBD are:

- Does not, in itself, provide for a specific fault analysis, i.e. the cause-effect(s) paths or the effect-cause(s) paths are not specifically highlighted.
- Requires a probabilistic model of performance for each element in the diagram.
- It will not show spurious or unintended outputs unless the analyst takes deliberate steps to this end.
- It is primarily directed towards success analysis and does not deal effectively with complex repair and maintenance strategies or general availability analysis.
- It is in general limited to non-repairable systems.
- The analysis is limited to single failures and is time-consuming.

3.7 Event Tree Analysis (ETA)

Event Tree Analysis has been used in risk and reliability analyses of a wide range of technological systems. It is an inductive method and the most common way of analyzing an accident progression. An Event Tree is a logic tree diagram, starting from a basic initiating event and provides a systematic coverage of the time sequence of event propagation to its potential outcomes or consequences. The Initiating Event can be identified by FMECA, PHA, HAZOP, etc.

The ETA is a natural part of most risk analyses but they can be used as a design tool to demonstrate the effectiveness of protective systems in a plant. In quantitative ET this method can be used independently or, is often combine with fault tree analysis. ET and FT are known as complement to each other. ET can also be used for human reliability assessment.

The major benefit of an event tree is the possibility to evaluate consequences of an event, and thus provide for possible mitigation of a highly probable, but unfavorable consequence. The event tree analysis is thus beneficial when performed as a complement to fault tree analysis. An event tree analysis can also be used as a tool in the fault mode analysis. When starting bottom up, the analysis follows possible paths of an event to determine probable consequences of a failure.

Limitations: the analyst has to describe the different scenarios and the result will be displayed in chronological development of event chains, which needs detailed system knowledge and understanding of the system.

3.8 Fault Tree Analysis (FTA)

Fault Tree Analysis (FTA) is one of the most important logic and probabilistic techniques used in system reliability and safety assessment. FTA can be simply described as an analytical technique, whereby an undesired state of the system is specified usually a state that is critical from reliability standpoint, and the system is then analyzed in the context of its environment and operation to find all realistic ways in which the undesired event can occur.

The FT itself is a graphic model of the various parallel and sequential combinations of faults that will result in the occurrence of the predefined undesired event. A variety of elements are available for building a fault tree such as gates and events.

The faults can be events that are associated with component hardware failures, human errors, software errors, or any other pertinent events which can lead to the undesired event. A FT shows the logical interrelationships of basic events that lead to the undesired event, the top event of the FT. A fault tree is tailored to its top event that corresponds to some particular system failure mode, and the fault tree thus includes only those faults that contribute to this top event. Moreover, these faults are not exhaustive. They cover only the faults that are assessed to be realistic by the analyst.

Intrinsic to a fault tree is the concept that an outcome is a binary event i.e., either success or failure. A fault tree is composed of a complex of entities known as “gates” that serve to permit or inhibit the passage of fault logic up the tree. The gates show the relationships of events needed for the occurrence of a “higher” event. The “higher” event is the output of the gate; the “lower” events are the “inputs” to the gate. The gate symbol denotes the type of relationship of the input events required for the output event.

The qualitative evaluations basically transform the FT logic into logically equivalent forms that provide more focused information. The principal qualitative results that are obtained are the minimal cut sets (MCS) of the top event. A cut set is a combination of basic events that can cause the top event. A minimal cut set (MCS) is the smallest combination of basic events that result in the top event. The basic events are the bottom events of the fault tree. Hence, the minimal cut sets relate the top event directly to the basic event causes. The set of MCS for the top event represent all the ways that the basic events can cause the top event. A more descriptive name for a minimal cut set may be “minimal failure set.” Top event frequencies, failure or occurrence rates, and availabilities can also be calculated. These characteristics are particularly applicable if the top event is a system failure. This method is used in System Safety Analysis as well as in System Reliability Analysis. The FT can include basic events of Common Cause. The quantification of those events is made according to Common Cause Failure methods..

Some of the advantages of using FTA are:

- Can be started in early stages of a design and further developed in detail concurrently with design development. Identifies and records systematically the logical fault paths from a specific effect, back to the prime causes by using Boolean algebra.
- Allows easy conversion of logical models into corresponding probability measures.
- Assists in decision-making as a base and support tool due to variety of information obtained by a FTA.

Some of the disadvantages to using FTA are:

- FTA is not able to represent time or sequence dependency of events correctly.
- FTA has limitations with respect to reconfiguration or state-dependent behavior of systems.

These limitations can compensate for by combining FTA with Markov models, where Markov models are taken as basic events in fault trees.

3.9 Markov Chains Models

The main idea of Markov-chains based models is directly or indirectly to build a Markov chain to represent the system behavior. Markov modeling is a probabilistic method that allows the statistical dependence of the failure or repair characteristics of individual components to be adapted to the state of the system. Hence, Markov modeling can capture the effects of both order dependent component failures and changing transition rates resulting from stress or other factors. For this reason, Markov analysis is suitable for dependability evaluation of functionally complex system structures and complex repair and maintenance strategies.

The proper field of application of this technique is when the transition failure or repair rates depend on the system state or vary with load, stress level, system structure such as in stand-by, maintenance policy or other factors. In particular, the system structure and the maintenance policy induce dependencies that cannot be captured by other, less computationally intensive techniques. The size of a Markov model in terms of the number of states and transitions grows exponentially with the number of components in the system. For a system with many components, the solution of a system using a Markov model may be infeasible, even if the model is truncated. However, if the system level can be divided into independent modules, and the modules solved separately, then the separate results can be combined to achieve a complete analysis.

Some of the advantages of using Markov model are:

- It provides a flexible probabilistic model for analyzing system behavior.
- It is adaptable to complex redundant configurations, complex maintenance policies, complex fault-error handling models including intermittent faults, fault latency, reconfiguration, and the degraded modes of operation and common cause failures.
- It provides probabilistic solutions for modules to be plugged into other models such as block diagrams and fault trees.
- It allows accurate modeling of the event sequences with a specific pattern or order of occurrence.

Some of the limitations using Markov model are:

- As the number of system components increases, there is an exponential growth in the number of states resulting in laborious analysis.
- The model can be difficult for users to construct and verify, and requires specific software for the analysis.
- The numerical solution step is available only with constant transition rates.

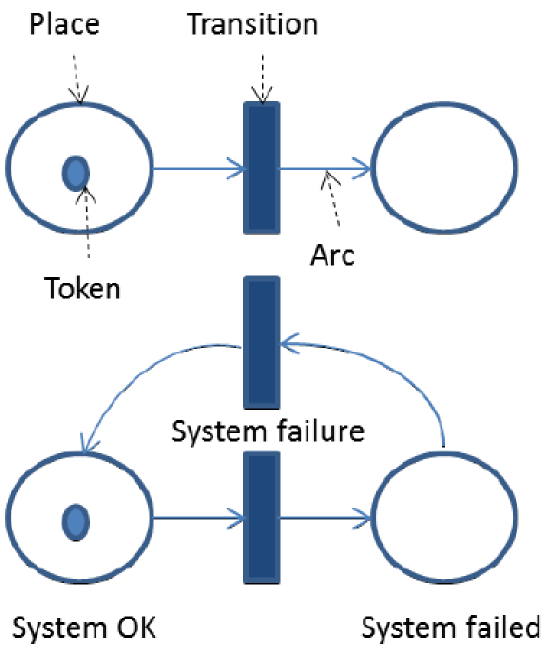
- Specific measures, such as MTTF and MTTR, are not immediately obtained from the standard solution of the Markov model, but require direct attention.

3.10 Petri Nets (PN)

Petri nets (PN) are a graphical tool for the representation and analysis of complex logical interactions between components or events in a system. Typical complex interactions that are naturally included in the Petri net language are concurrency, conflict, synchronization, mutual exclusion and resource limitation. The static structure of the modeled system is represented by a Petri net graph as exemplified in the Figure 2.2.

A condition is valid in a given situation if the corresponding place is marked, i.e. contains at least one token •. The dynamics of the system are represented by means of the movement of the tokens in the graph. A transition is enabled if its input places contain at least one token. An enabled transition may fire, and the transition firing removes one token from each input place and puts one token into each output place. The distribution of the tokens into the places is called marking. Starting from an initial marking, the application of the enabling and firing rules produces all the reachable markings called the reachability set. The reachability set provides all the states that the system can reach from an initial state.

Standard Petri nets do not carry the notion of time. However, many extensions have appeared in which timing is superimposed onto the Petri net. If a constant firing rate is assigned to each transition, the dynamics of the Petri nets can be analyzed by means of a continuous Markov time chain whose state space is isomorphic with the reachability set of the corresponding Petri net.



Petri net graph is composed of three primitive elements:

- *Places* (usually drawn as circles) that represent the conditions in which the system can be found;
- *Transitions* (usually drawn as bars) that represent the events that may change a condition in to another one;
- *Arcs* (drawn as arrows) that connect places to transitions and transition to places and represent the logical admissible connections between conditions and events.

The *tokens* are a discrete number of marks. A *transition* is enabled if its input *places* contain at least one *token*.

Figure 2.2: Example of a generic Petri net diagram

The key element of the Petri net analysis is a description of the system structure and its dynamic behavior in terms of primitive elements like places, transitions, arcs and tokens, of the Petri net language; this step requires the use of ad hoc software tools:

a) Structural qualitative analysis

b) Quantitative analysis: if constant firing rates are assigned to the Petri net transitions the quantitative analysis can be performed via the numerical solution of the corresponding Markov model, otherwise simulation is the only viable technique.

The Petri net can be utilized as a high level language to generate Markov models, and several tools in performance dependability analysis are based on this methodology. Petri nets provide also a natural environment for simulation. The use of Petri nets is recommended when complex logical interactions need to be taken into account mainly concurrency, conflict, synchronization, mutual exclusion, resource limitation. Moreover, PN are usually an easier and more natural language to describe a Markov model.

Some of the advantages of using PN are:

- Petri nets are suitable for representing complex interactions among hardware or software modules that are not easily modeled by other techniques.
- Petri Nets are a viable way of generating Markov models. In general, the description of the system by means of a Petri net requires far fewer elements than the corresponding Markov representation.
- The Markov model is generated automatically from the Petri net representation and the complexity of the analytical solution procedure is hidden to the modeler who interacts only at the Petri net level.
- In addition, the PN allow a qualitative structural analysis based only on the property of the graph. This structural analysis is, in general, less costly than the generation of the Markov model, and provides information useful to validate the consistency of the model.

Since the quantitative analysis is based on the generation and solution of the corresponding Markov model, most of the limitations are shared with the Markov analysis. The PN methodology requires the use of software's.

During the PDP of safety critical systems, other properties can be important, e.g. system safety. Some of the methods described in this chapter, e.g. FMEA, FTA, MA, ETA and FMECA are used for both reliability and safety analysis.

CHAPTER 4

SYSTEM MODELING

In this chapter, reliability model is developed for a multistage compressor system, in which three components are working as a series system. Such a system is commonly used in gas power plant. Standby redundancy and repairs are employed to achieve the higher system reliability goals.

The highly reliable systems are required in a gas power plant to provide continuous power supply. The system reliability is increased mainly by incorporating redundancy and repairs. Both these options are utilized in this work at the component level. When the reliability goals are not met in the basic series model, standby redundancy and repair options are explored. In standby redundancy, when the main component fails, its standby starts working. The selection of the component for standby redundancy is based on failure rate of components in the system and component with highest failure rate is chosen first for redundancy. At each stage of redundancy addition, the system reliability is checked and if goals are not met next redundancy is added. Therefore, four models have been developed by adding redundancy one by one to different component to achieve target reliability.

4.1 Description of multi stage compressor

Multistage compressor is a main unit of thermal/gas power plant. It is used to compress the air at high pressure and supply to combustor. This compressed air is used to ignite the fuel. It is also used for cooling of generator. Multi stage compressor comprises of three components (Refer Fig. 4.1) arranged in series and these are described as:

Low pressure compressor (LPC) – It compress the ambient air and increase the pressure and temperature.

Intercooler (IC) – compressed air delivered from LPC passes through intercooler. In intercooler cooling of air take place while pressure remains constant. Subsequently, compressed air is delivered to HPC.

High pressure compressor (HPC) – It further increase the pressure of the air up to a required pressure.

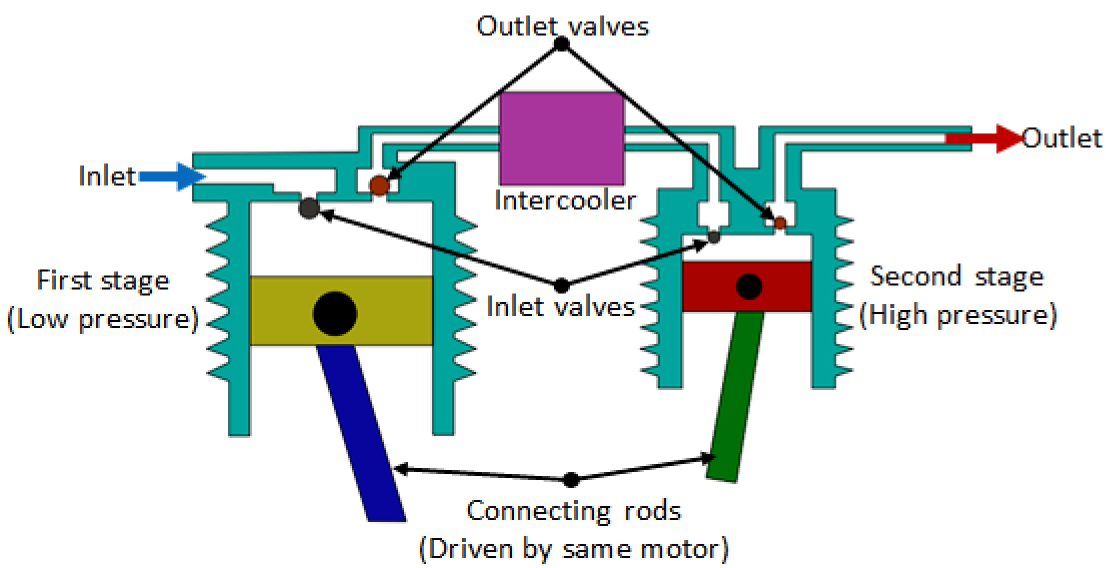


Figure 4.1: Reciprocating multistage compressor

4.2 Assumption

The assumptions used in developing the models for the multistage compressor are:

- Every component nit has multiple states, which are active, standby and failed.
- Two or more failure in system simultaneously will not be considered because the probability of occurrence of these states is negligible.
- The standby component is cold standby. The assumption is acceptable because the component in standby will not wear out and failed.

- It is assumed that switching will always be successful and considered as perfect.
- All failure and repair rates are constant over time and statistically independent. This assumption facilitates the mathematical modeling without loosing the generality.
- The repaired component has the priority to be sent back to the standby state. This assumption is based on the scenario that operating units will normally not be stopped.
- A repaired component is considered as good as new.
- Sufficient repair facilities are provided, i.e. no waiting time to start the repairs.
- Standby units are of the same nature and capacity as the active units.
- Failure and repair rates follow exponential distribution.

4.3 Deriving system state space of the system

In this project, each component is assumed to have three states; active, standby and failed. The possible states of the system are derived in terms of the states of the component. Using combinatorial theory, total state space size is calculated by multiplication of no. of states of each component. All feasible states of system are extracted and other infeasible states are rejected. For example, the state in which two or more components fail simultaneously is not feasible as the probability of occurrence of that state is negligible. Similarly, two or more component cannot be in standby states simultaneously. The state space for each model is detailed in the respective sections.

A methodology is proposed in the next section for achieving the reliability target.

4.4 Step wise methodology to achieve reliability goal

Steps to achieve more than 90 percent reliability for the multi stage compressor system at 10000 hour of operation are presented below.

Step 1: set reliability goal

Set the reliability goal for the system.

Step 2: Define basic configuration

Define the basic configuration of system in form of reliability block diagram.

Step 3: Specify reliability goal for a mission time

Specify the reliability goal for the system for a mission time.

Step 4: Asses reliability of basic system

At initial level asses the reliability of basic system.

Step 5: Reliability goal Check

If reliability goal is achieved, system is ready otherwise move to next step.

Step 6: Select component with highest failure rate

If reliability goal is not achieved than select the component has highest failure rate and add standby redundancy to that component.

Step 7: Reliability goal Check

If reliability goal achieved than system is ready otherwise add standby redundancy to next component has second highest failure rate.

Step 8: continue the process till reliability goal is not achieved.

A flow chart for the proposed methodology is given in Fig. 4.2.

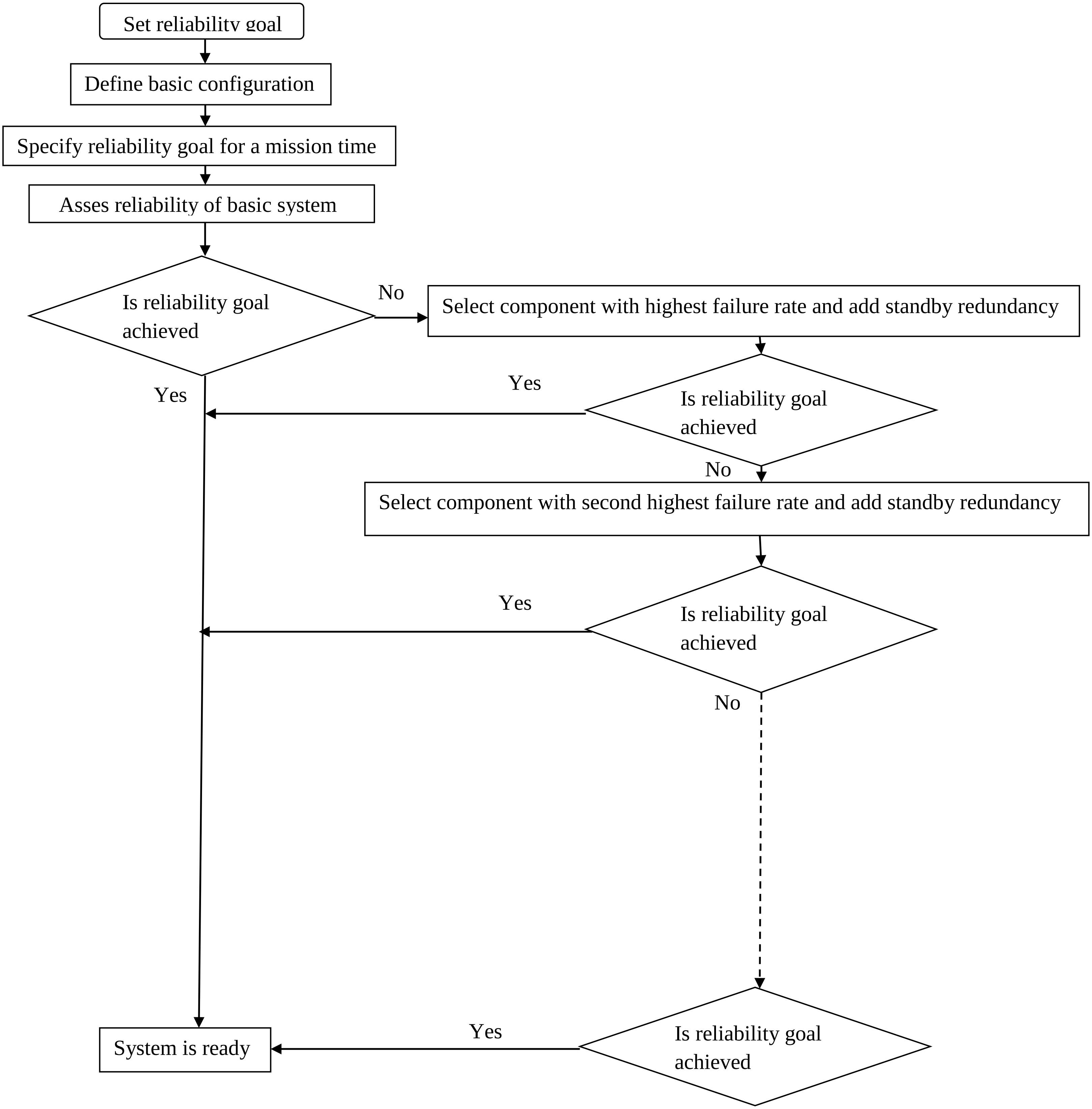


Figure 4.2: Reliability goal solution framework

4.5 System modeling

In this section four models are developed.

4.5.1 Model 1- Basic series model

In this basic model one LPC, one IC, one HPC are connected in series as shown in Fig. 4.3. Each component has 2 states, operating. 'O' and failed, 'F'. Standby state is not feasible in basic series model as there is no standby component. According to combinatorial theory, the total no. of states is 2 × 2× 2=8, in which only 4 states are feasible states for system as shown in Table 4.1. The state in which two or more components fail simultaneously is not feasible.

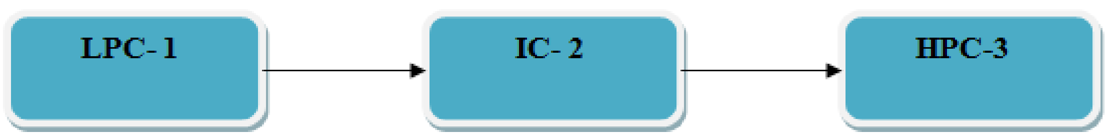


Figure 4.3: RBD of multistage compressor (for basic model)

Table 4.1: System state space for basic model (Model-1)

System state	LPC-1	IC-1	HPC-1	System status	Feasible Yes/No
1	O	O	O	working	Yes
2	O	O	F	failed	Yes
3	O	F	O	failed	Yes
4	F	O	O	failed	Yes
5	O	F	F	failed	No
6	F	O	F	failed	No
7	F	F	O	failed	No
8	F	F	F	failed	No

The Markov model of the system is developed in terms of ‘4’ feasible states shown in Table 4.1. All the possible transitions among the states are also thought of for system model. In Markov model, every transition state depends upon its just previous state and it is independent of past history. The developed model is shown in Fig. 4.4

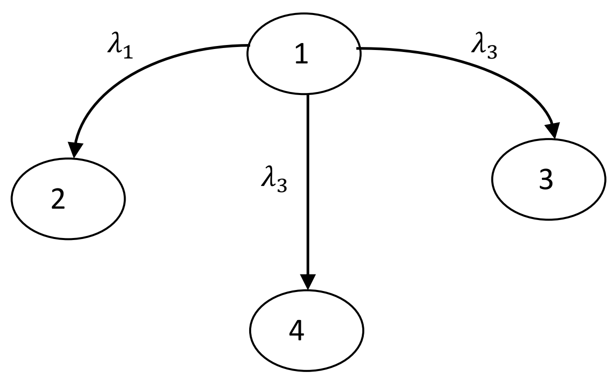


Figure 4.4: Basic series model (Model-1)

4.5.2 Model 2- One standby model

Consider the system RBD as per Fig. 4.5 for this model. In addition to the three component in series as considered in basic model, one standby LPC is added. Here, LPC-1 and LPC-11 are in standby redundancy, due to this both component have 3 states (operating, standby and failed). IC-2 and HPC-3 each have 2 states (operating and failed).Therefore, total no of possible states are 3×3×2×2=36, in which only 13 states are feasible for system and shown in the Table 4.2.

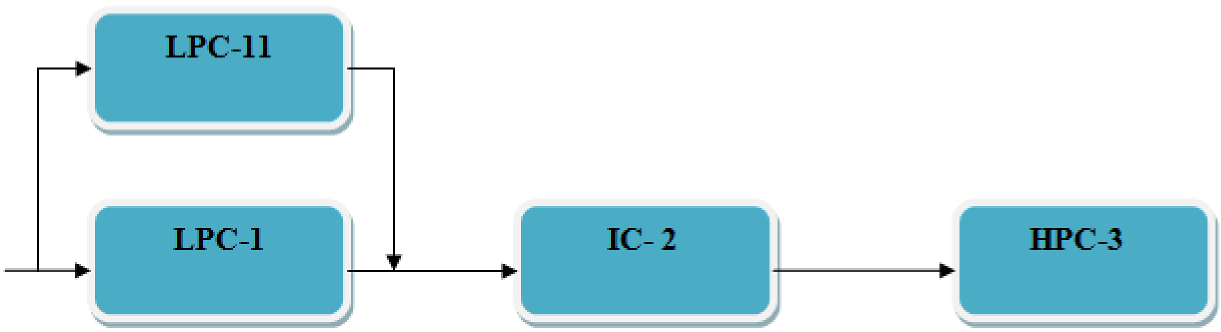


Figure 4.5: RBD of system for model-2 (One standby model)

Table 4.2: System state space for Model -2 (One standby model)

System state	LPC-1	IC-1	HPC-1	LPC-11	System status
1	O	O	O	S	working
2	O	O	O	F	working
3	O	O	S	O	working
4	O	O	F	O	working
5	O	O	F	F	failed
6	O	F	O	S	failed
7	O	F	O	F	failed
8	O	F	S	O	failed
9	O	F	F	O	failed
10	F	O	O	S	failed
11	F	O	O	F	failed
12	F	O	S	O	failed
13	F	O	F	O	failed

The Markov model of the system is developed in terms of '13' feasible states shown in Table 4.2. All the possible transitions among the states are also thought of for system model. The developed model is shown in Fig. 4.6.

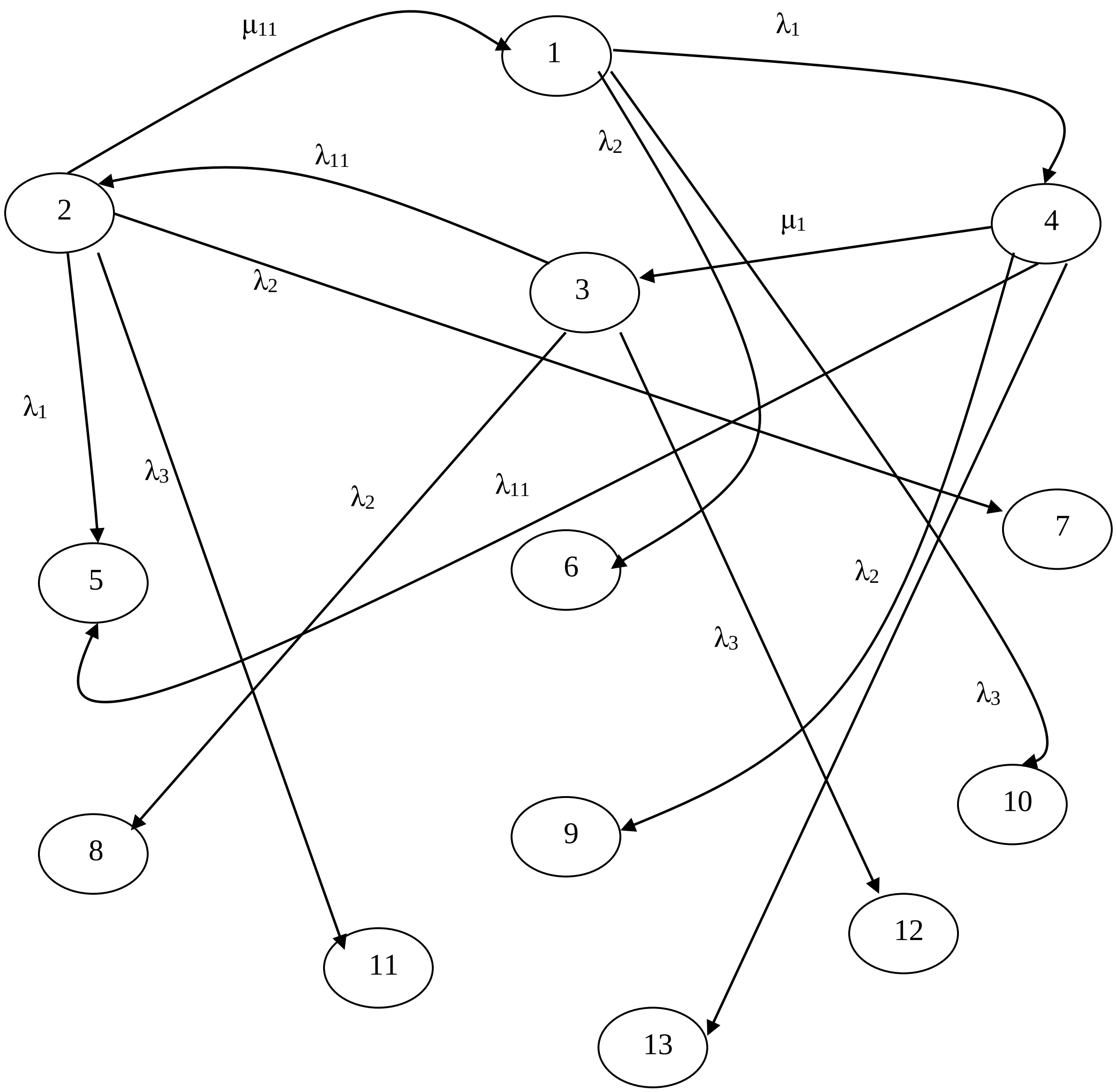


Figure 4.6: Model-2 (One standby model)

4.5.3 Model 3 – Two standby model

Consider a system with two redundant components for the model. In this model, system has total five components; two LPC, two IC and one HPC. System configuration is shown in Fig. 4.7. There are standby for LPC-1 and IC-2 and these are LPC-11 and IC-22 respectively.

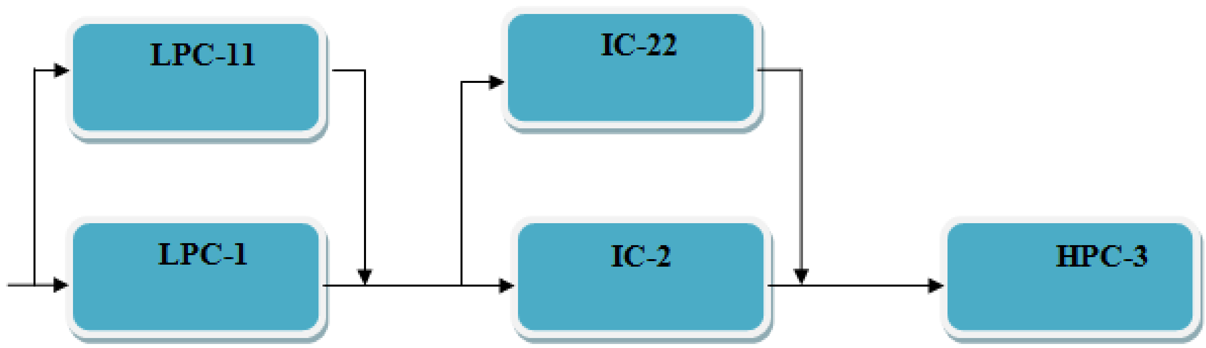


Figure 4.7: RBD of system for model-3 (Two standby model)

Here, Components LPC-1 with LPC-11 and IC-2 with IC-22 are in passive redundancy, due to which each component have 3 states (operating, standby and failed). Since there only one HPC it has component 2 states (operating and failed).Therefore, total no of states are 3×3×3×3×2=162, in which 40 states are the feasible states for the system and these are listed Table 4.3

Table 4.3: System state space for Model -3 (Two standby model)

System state	LPC-1	IC-1	HPC-1	LPC-11	IC-22	System status
1	O	O	O	S	S	working
2	O	O	O	S	F	working
3	O	O	O	F	S	working
4	O	O	O	F	F	working
5	O	O	F	S	S	failed

6	O	O	F	S	F	failed
7	O	O	F	F	S	failed
8	O	O	F	F	F	failed
9	O	S	O	S	O	working
10	O	S	O	F	O	working
11	O	S	F	S	O	failed
12	O	S	F	F	O	failed
13	O	F	O	S	O	working
14	O	F	O	S	F	failed
15	O	F	O	F	O	working
16	O	F	O	F	F	failed
17	O	F	F	S	O	failed
18	O	F	F	F	O	failed
19	S	O	O	O	S	working
20	S	O	O	O	F	working
21	S	O	F	O	S	failed
22	S	O	F	O	F	failed
23	S	S	O	O	O	working
24	S	S	F	O	O	failed
25	S	F	O	O	O	working
26	S	F	O	O	F	failed
27	S	F	F	O	O	failed
28	F	O	O	O	S	working
29	F	O	O	O	F	working

30	F	O	O	F	S	failed
31	F	O	O	F	F	failed
32	S	O	F	O	F	failed
33	F	O	F	O	F	failed
34	F	S	O	O	O	working
35	F	S	O	F	O	failed
36	F	S	F	O	O	failed
37	F	F	O	O	O	working
38	F	F	O	O	F	failed
39	F	F	O	F	O	failed
40	F	F	F	O	O	failed

The Markov model of the system is developed in terms of ‘40’ feasible states shown in Table 4.3. All the possible transitions among the states are also thought of for system model. The developed model is shown in Fig. 4.8.

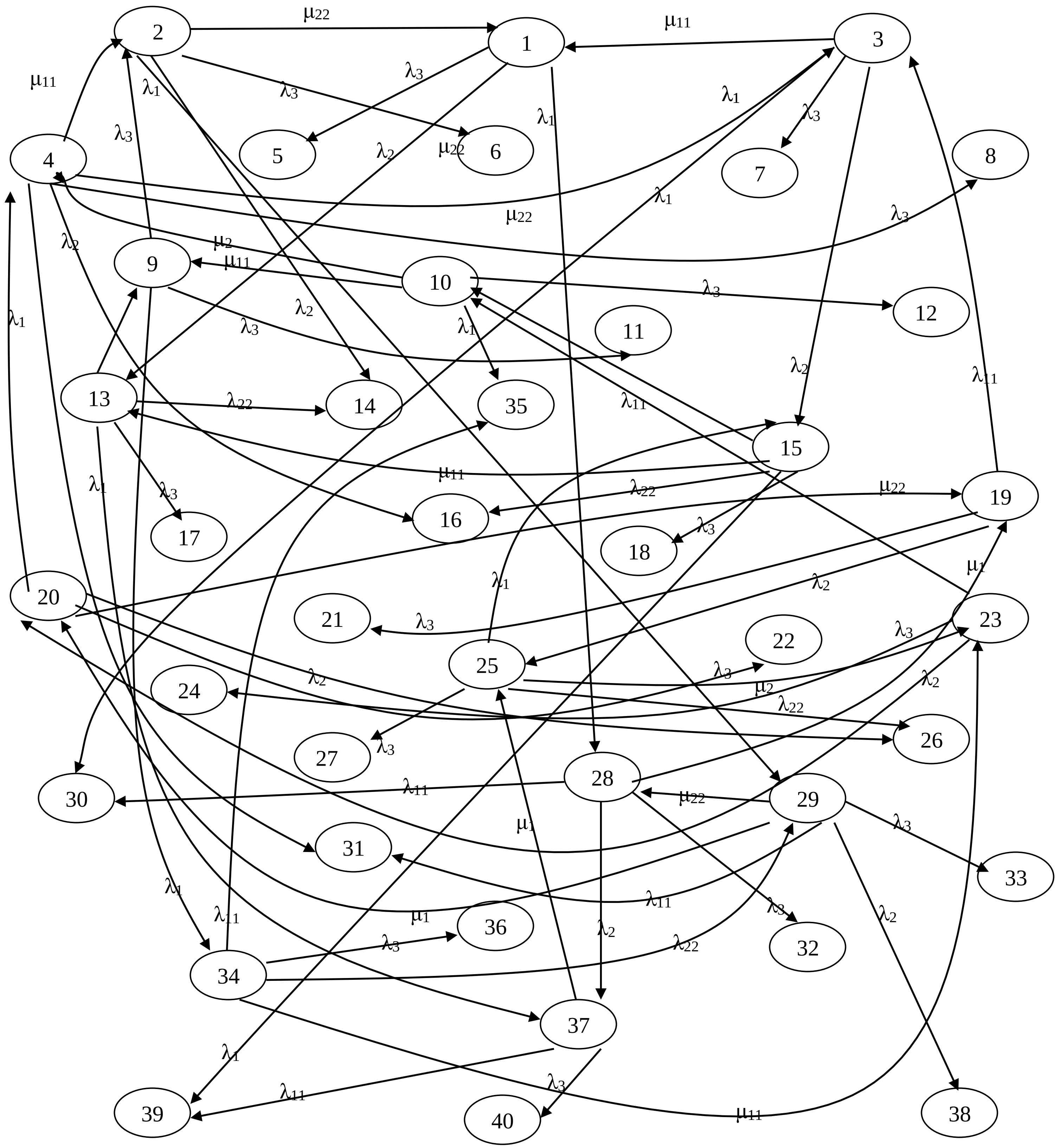


Figure 4.8: Model-3 (Two standby model)

4.5.4 Model 4 -Three standby model

Consider a system with three redundant components for the model. In this model, system has 6 components; two LPC, two IC and two HPC as shown in Fig. 4.9.

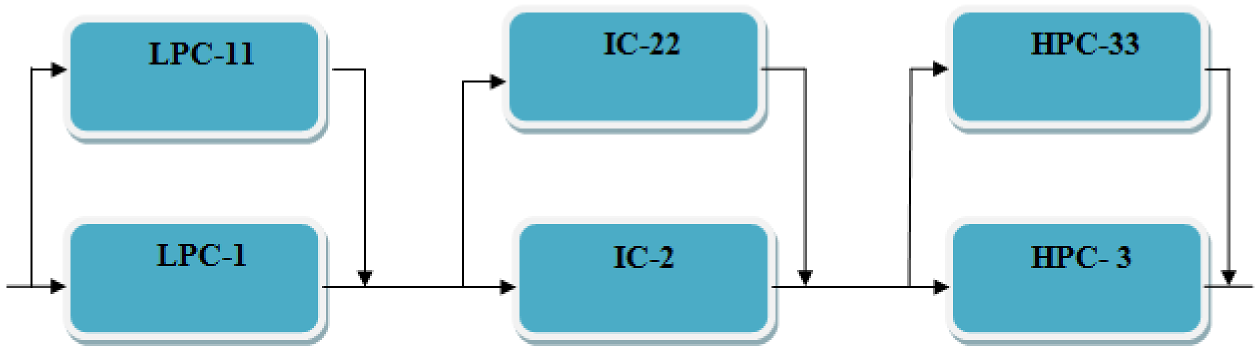


Figure 4.9: RBD of system for model-4 (Three standby model)

Here, each Component LPC-1, IC-2, HPC-3 has one standby redundancy, due to which each has 3 states (operating, standby and failed). Therefore, total no of states are 3×3×3×3×3×3=729, in which 113 states are the feasible states for system as shown in Table 4.4

Table 4.4: System state space for Model -4 (Three standby model)

System state	LPC-1	IC-1	HPC-3	LPC-11	IC-22	HPC-33	System status
1	O	O	O	S	S	S	working
2	O	O	O	S	S	F	working
3	O	O	O	S	F	S	working
4	O	O	O	S	F	F	working
5	O	O	O	F	S	S	working
6	O	O	O	F	S	F	working
7	O	O	O	F	F	S	working
8	O	O	O	F	F	F	working

9	O	O	S	S	S	O	working
10	O	O	S	S	F	O	working
11	O	O	S	F	S	O	working
12	O	O	S	F	F	O	working
13	O	O	F	S	S	O	working
14	O	O	F	S	F	O	working
15	O	O	F	F	S	O	working
16	O	O	F	F	F	O	working
17	O	S	O	S	O	S	working
18	O	S	O	S	O	F	working
19	O	S	O	F	O	S	working
20	O	S	O	F	O	F	working
21	O	S	S	S	O	O	working
22	O	S	S	F	O	O	working
23	O	S	F	S	O	O	working
24	O	S	F	F	O	O	working
25	O	F	O	S	O	S	working
26	O	F	O	S	O	F	working
27	O	F	O	F	O	S	working
28	O	F	O	F	O	F	working
29	O	F	S	S	O	O	working
30	O	F	S	F	O	O	working
31	O	F	F	S	O	O	working
32	O	F	F	F	O	O	working

33	S	O	O	O	S	S	working
34	S	O	O	O	S	F	working
35	S	O	O	O	F	S	working
36	S	O	O	O	F	F	working
37	S	O	S	O	S	O	working
38	S	O	S	O	F	O	working
39	S	O	F	O	S	O	working
40	S	O	F	O	F	O	working
41	S	S	O	O	O	S	working
42	S	S	O	O	O	F	working
43	S	S	S	O	O	O	working
44	S	S	F	O	O	O	working
45	S	F	O	O	O	S	working
46	S	F	O	O	O	F	working
47	S	F	S	O	O	O	working
48	S	F	F	O	O	O	working
49	F	O	O	O	S	S	working
50	F	O	O	O	S	F	working
51	F	O	O	O	F	S	working
52	F	O	O	O	F	F	working
53	F	O	S	O	S	O	working
54	F	O	S	O	F	O	working
55	F	O	F	O	S	O	working
56	F	O	F	O	F	O	working

57	F	S	O	O	O	S	working
58	F	S	O	O	O	F	working
59	F	S	S	O	O	O	working
60	F	S	F	O	O	O	working
61	F	F	O	O	O	S	working
62	F	F	O	O	O	F	working
63	F	F	S	O	O	O	working
64	F	F	F	O	O	O	working
65	O	O	F	S	S	F	failed
66	O	O	F	S	F	F	failed
67	O	O	F	F	S	F	failed
68	O	O	F	F	F	F	failed
69	O	S	F	S	O	F	failed
70	O	S	F	F	O	F	failed
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73	O	F	O	F	F	S	failed
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79	O	F	F	F	O	F	failed
80	O	F	F	F	F	O	failed

81	O	F	F	F	F	F	failed
82	S	O	F	O	S	F	failed
83	S	O	F	O	F	F	failed
84	S	S	F	O	O	F	failed
85	S	F	O	O	F	S	failed
86	S	F	O	O	F	F	failed
87	S	F	S	O	F	O	failed
88	S	F	F	O	O	F	failed
89	S	F	F	O	F	O	failed
90	F	O	O	F	S	S	failed
91	F	O	O	F	S	F	failed
92	F	O	O	F	F	S	failed
93	F	O	O	F	F	F	failed
94	F	O	S	F	S	O	failed
95	F	O	S	F	F	O	failed
96	F	O	F	O	S	F	failed
97	F	O	F	O	F	F	failed
98	F	O	F	F	S	O	failed
99	F	O	F	F	F	O	failed
100	F	S	O	F	O	F	failed
101	F	S	S	F	O	O	failed
102	F	S	F	O	O	F	failed
103	F	S	F	F	O	O	failed
104	F	S	F	F	O	F	failed

105	F	F	O	O	F	S	failed
106	F	F	O	O	F	F	failed
107	F	F	O	F	O	S	failed
108	F	F	O	F	O	F	failed
109	F	F	S	F	O	O	failed
110	F	F	S	F	F	O	failed
111	F	F	F	O	O	F	failed
112	F	F	F	O	F	O	failed
113	F	F	F	F	O	O	failed

The Markov model of the system is developed in terms of '113' feasible states shown in Table 4.4. All the possible transitions among the states are also thought of for system model.

In the next chapter, mathematical models are developed for all the four system models developed in this chapter.

CHAPTER 5

Markov Modeling

In this chapter, mathematical modeling of the system for the models developed in the previous chapter is presented.

5.1 Overview of Markov approach

The mathematical models developed in this work are based on the Chapman Kolmogorov differential equations which are obtained from the transition diagram of the system. These models are used for evaluating the system reliability. The differential equations are developed using Markov birth-death process. In birth process, there is one step change in the probability function in forward direction due to failures of the components. While due to repairs of the components, there is one backward change in the probability function like death process.

The reliability analysis is related with a discrete state continuous time model, called a Markov process. Markov graph consists of nodes and branches to display the Markov process pictorially for all the models. The nodes represent the states in a system and the branches represent the respective transitional probabilities. The state of the system defines the condition at any instant of time and the information is useful in analyzing the current state and in the prediction of the failure state of the system. In the Markovian approach, the state of a system with probability P_{ij}, indicates the probability of a system moving from state i to state j. This probability P_{ij} is called the transition probability. In a Markov process, the fundamental assumption made is that the transition probability from i to j depends entirely on states i and j, and is independent of all previous states except the last one i.e., state i. The other assumptions of Markov models are:

- At any given time the system is either in operating or standby or in failed state.
- The state of the system changes as time progresses.

- The transition of the system from one state to the other takes place instantaneously. It means time span between two states is assumed to be zero.
- The failure and repair rates are constant (follows exponential distribution).

Let the probability of n occurrences in time t be denoted by $P_n(t)$, i.e.,

Probability(x = n, t) = $P_n(t)$ (n = 0, 1, 2 . . .)

Then, $P_0(t)$ represent the probability of zero occurrences in time t. The probability of zero occurrences in time (t + Δt) is given by:

$$P_0(t+\Delta t) = (1-\lambda\Delta t)P_0(t) + (\mu\Delta t)P_1(t)$$

Similarly, $$P_1(t+\Delta t) = (\lambda\Delta t)P_0(t) + (1-\mu\Delta t)P1(t)$$

The above equations show the probability of one occurrence in time (t + Δt) and is composed of two parts, namely, probability of zero occurrences in time t multiplied by the probability of one occurrence in the interval Δt and the probability of one occurrence in time t multiplied by the probability of no occurrences in the interval Δt. Then simplifying and putting $\Delta t \to 0$, one gets

$$\frac{d}{dt}P_0(t) = -\lambda P_0(t) + \mu P_1(t)$$

5.2 Markov modeling of the basic series model (Model-1)

Following the methodology suggested in Section 5.1, the differential equations associated with the transition diagram (Figure 4.4) of the model- 1 are derived as:

$$\frac{d}{dt}P_1(t) = -(\lambda_1+\lambda_2+\lambda_3)P_1(t)$$

$$\frac{d}{dt}P_2(t) = \lambda_3 P_1(t)$$

$$\frac{d}{dt}P_3(t) = \lambda_2 P_1(t)$$

$$\frac{d}{dt}P_4(t) = \lambda_1 P_1(t)$$

Where, initial condition at time t = 0; $P_1(t) = 1$ and $P_1(t), P_2(t)$ ……………….. $P_4(t) = 0$

and at any time ‘t’

$$\sum_{n=1}^{n=4} Pn\ (t) = 1$$

5.3 Markov modeling of model - 2

Following the methodology suggested in Section 5.1, the differential equations associated with the transition diagram (Figure 4.6) of the model- 2 are derived as:

$$\frac{d}{dt}P_1(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_1(t) + \mu_{11}P_2(t)$$

$$\frac{d}{dt}P_2(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_2(t) + \lambda_{11}P_3(t) - \mu_{11}P_2(t)$$

$$\frac{d}{dt}P_3(t) = -(\lambda_2+\lambda_3)\,P_3(t) - \lambda_{11}P_3(t) + \mu_1 P_4(t)$$

$$\frac{d}{dt}P_4(t) = -(\lambda_2+\lambda_3)\,P_4(t) - \lambda_{11}P_4(t) + \lambda_1 P_1(t) - \mu_1 P_4(t)$$

$$\frac{d}{dt}P_5(t) = \lambda_1 P_2(t) + \lambda_{11}P_4(t)$$

$$\frac{d}{dt}P_6(t) = \lambda_2 P_1(t)$$

$$\frac{d}{dt}P_7(t) = \lambda_2 P_2(t)$$

$$\frac{d}{dt}P_8(t) = \lambda_2 P_3(t)$$

$$\frac{d}{dt}P_9(t) = \lambda_2 P_4(t)$$

$$\frac{d}{dt}P_{10}(t) = \lambda_3 P_1(t)$$

$$\frac{d}{dt}P_{11}(t) = \lambda_3 P_2(t)$$

$$\frac{d}{dt}P_{12}(t) = \lambda_3 P_3(t)$$

$$\frac{d}{dt}P_{13}(t) = \lambda_3 P_4(t)$$

With initial condition at time t = 0; $P_1(t) = 1$ and $P_1(t), P_2(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots P_{13}(t) = 0$

and at any time 't'

$$\sum_{n=1}^{n=13} Pn\,(t) = 1$$

5.4 Markov modeling of model -3

Following the methodology suggested in Section 5.1, the differential equations associated with the transition diagram (Figure 4.8) of the model- 3 are derived as:

$$\frac{d}{dt}P_1(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_1(t) + \mu_{22}\,P_2(t) + \mu_{11}\,P_3(t)$$

$$\frac{d}{dt}P_2(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_2(t) + \lambda_{22}\,P_9(t) - \mu_{22}\,P_2(t) + \mu_{11}\,P_4(t)$$

$$\frac{d}{dt}P_3(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_3(t) + \lambda_{11}\,P_{19}(t) - \mu_{11}\,P_3(t) + \mu_{22}\,P_4(t)$$

$$\frac{d}{dt}P_4(t) = -(\lambda_1+\lambda_2+\lambda_3)\,P_4(t) + \lambda_{22}\,P_{10}(t) + \lambda_{11}\,P_{20}(t) - \mu_{11}\,P_4(t) - \mu_{22}\,P_4(t)$$

$$\frac{d}{dx}P_5(t) = \lambda_3\,P_1(t)$$

$$\frac{d}{dt}P_6(t) = \lambda_3\,P_2(t)$$

$$\frac{d}{dt}P_7(t) = \lambda_3\,P_3(t)$$

$$\frac{d}{dt}P_8(t) = \lambda_3\,P_4(t)$$

$$\frac{d}{dt}P_9(t) = -(\lambda_1+\lambda_3)\,P_9(t) - \lambda_{22}\,P_9(t) + \mu_{11}\,P_{10}(t) + \mu_2\,P_{13}(t)$$

$$\frac{d}{dt}P_{10}(t) = -(\lambda_1+\lambda_3)\,P_{10}(t) - \lambda_{22}\,P_{10}(t) + \lambda_{11}\,P_{23}(t) - \mu_{11}\,P_{10}(t) + \mu_2\,P_{15}(t)$$

$$\frac{d}{dt}P_{11}(t) = \lambda_3\,P_9(t)$$

$$\frac{d}{dt}P_{12}(t) = \lambda_3\,P_{10}(t)$$

$$\frac{d}{dt}P_{13}(t) = -(\lambda_1+\lambda_3)\,P_{13}(t) - \lambda_{22}\,P_{13}(t) + \lambda_2\,P_1(t) + \mu_{11}\,P_{15}(t) - \mu_2\,P_{13}(t)$$

$$\frac{d}{dt} P_{14}(t) = \lambda_2 P_2(t)$$

$$\frac{d}{dt} P_{15}(t) = -(\lambda_1+\lambda_3) P_{15}(t) - \lambda_{22} P_{15}(t) + \lambda_{11} P_{25}(t) + \lambda_2 P_3(t) - \mu_{11} P_{15}(t) - \mu_2 P_{15}(t)$$

$$\frac{d}{dt} P_{16}(t) = \lambda_2 P_4(t) + \lambda_{22} P_{15}(t)$$

$$\frac{d}{dt} P_{17}(t) = \lambda_3 P_{13}(t)$$

$$\frac{d}{dt} P_{18}(t) = \lambda_3 P_{15}(t)$$

$$\frac{d}{dt} P_{19}(t) = -(\lambda_{11}+ \lambda_2+\lambda_3) P_{19}(t) + \mu_1 P_{28}(t) + \mu_{22} P_{20}(t)$$

$$\frac{d}{dt} P_{20}(t) = -(\lambda_{11}+ \lambda_2+\lambda_3) P_{20}(t) + \lambda_{22} P_{23}(t) + \mu_1 P_{29}(t) - \mu_{22} P_{20}(t)$$

$$\frac{d}{dt} P_{21}(t) = \lambda_3 P_{19}(t)$$

$$\frac{d}{dt} P_{22}(t) = \lambda_3 P_{20}(t)$$

$$\frac{d}{dt} P_{23}(t) = -(\lambda_{11}+ \lambda_{22}+\lambda_3) P_{23}(t) + \mu_1 P_{34}(t) + \mu_2 P_{25}(t)$$

$$\frac{d}{dt} P_{24}(t) = \lambda_3 P_{23}(t)$$

$$\frac{d}{dt} P_{25}(t) = -(\lambda_{11}+ \lambda_{22}+\lambda_3) P_{25}(t) + \lambda_2 P_{19}(t) + \mu_1 P_{37}(t) - \mu_2 P_{25}(t)$$

$$\frac{d}{dt} P_{26}(t) = \lambda_{22} P_{25}(t) + \lambda_2 P_{20}(t)$$

$$\frac{d}{dt} P_{27}(t) = \lambda_3 P_{25}(t)$$

$$\frac{d}{dt} P_{28}(t) = -(\lambda_{11}+ \lambda_{22}+\lambda_3) P_{25}(t) + \lambda_2 P_{19}(t) + \mu_1 P_{37}(t) - \mu_2 P_{25}(t)$$

$$\frac{d}{dt} P_{29}(t) = -(\lambda_{11}+ \lambda_{22}+\lambda_3) P_{25}(t) + \lambda_2 P_{19}(t) + \mu_1 P_{37}(t) - \mu_2 P_{25}(t)$$

$$\frac{d}{dt} P_{30}(t) = \lambda_{22} P_{25}(t) + \lambda_2 P_{20}(t)$$

$$\frac{d}{dt} P_{31}(t) = \lambda_{22} P_{25}(t) + \lambda_2 P_{20}(t)$$

$$\frac{d}{dt} P_{32}(t) = \lambda_3 P_{28}(t)$$

$$\frac{d}{dt} P_{33}(t) = \lambda_3 P_{29}(t)$$

$$\frac{d}{dt} P_{34}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3) P_{34}(t) + \lambda_1 P_9(t) - \mu_1 P_{34}(t) + \mu_2 P_{37}(t)$$

$$\frac{d}{dt} P_{35}(t) = \lambda_{11} P_{34}(t) + \lambda_1 P_{10}(t)$$

$$\frac{d}{dt} P_{36}(t) = \lambda_3 P_{34}(t)$$

$$\frac{d}{dt} P_{37}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3) P_{37}(t) + \lambda_2 P_{28}(t) + \lambda_1 P_{13}(t) - \mu_2 P_{37}(t) - \mu_1 P_{37}(t)$$

$$\frac{d}{dt} P_{38}(t) = \lambda_{22} P_{37}(t) + \lambda_2 P_{29}(t)$$

$$\frac{d}{dt} P_{39}(t) = \lambda_{11} P_{37}(t) + \lambda_1 P_{15}(t)$$

$$\frac{d}{dt} P_{40}(t) = \lambda_3 P_{37}(t)$$

With initial condition at time t = 0; $P_1(t) = 1$ and $P_1(t), P_2(t)$ ……………….. $P_{13}(t) = 0$

and at any time ‘t’

$$\sum_{n=1}^{n=40} Pn\,(t) = 1$$

On the similar lines mathematical models for the system model-4 is also developed and the system of ‘113’ differential equations are derived which are given in Appendix.

In the system reliability assessment the probabilities all the states where system is functional are considered. The solutions and results of the all four models are discussed in the next chapter.

CHAPTER 6

RESULTS AND DISCUSSION

Reliability is evaluated from the Markov models developed in the previous chapter. These mathematical models are solved using Matlab. Failure and repair rates of each component remain same for in all the models. Failure rates and repair rates for the components are as following:

$\lambda_1 = \lambda_{11} = 1/11000$, $\lambda_2 = \lambda_{22} = 1/14000$, $\lambda_3 = \lambda_{33} = 1/22000$,

$\mu_1 = \mu_{11} = 1/600$, $\mu_2 = \mu_{22} = 1/500$, $\mu_3 = \mu_{33} = 1/600$

6.1 Reliability of system for model -1

The system of equations for model-1 is solved using Runge-kutta method in Matlab. The system reliability values are obtained up to 50,000 hours. Since there is only one working state in the model, Reliability value at any time ‘t’ is the probability of state 1, i.e.

Reliability = P_1

Reliability-time curve for model-1 is shown in Fig. 6.1. As the failed states are absorbing states in this model, no repair can be considered for reliability assesment. From Figure 6.1, it is evident that reliability of the basic system model is decreasing with time.

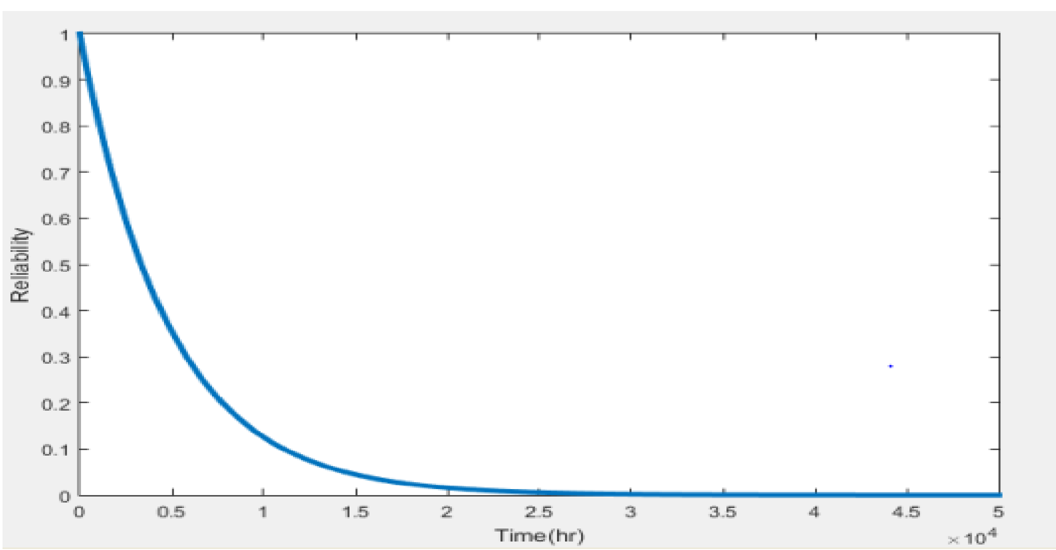

Figure 6.1: Reliability - time curve for model-1

6.2 Reliability of system for model - 2

On the similar lines, the reliability results are obtained for model-2 with one standby redundancy summing the probabilities of working states, i.e.

Reliability = $P_1 + P_2 + P_3 + P_4$

Reliability-time curve for model-2 with no repair and with repair are evaluated. Reliability–time curve with no repair is shown in Figure 6.2

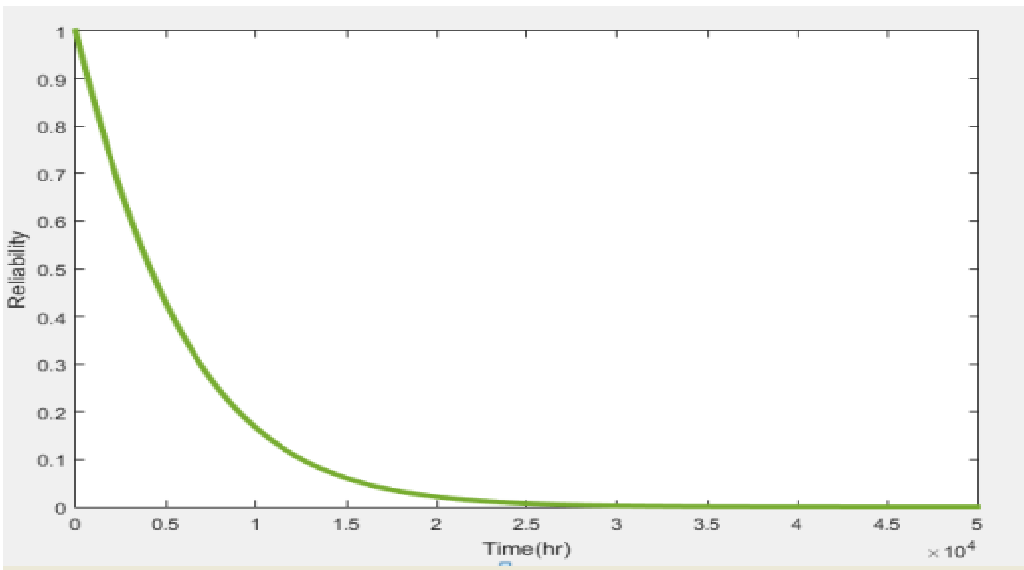

Figure 6.2: Reliability - time curve for model-2 with no repair

Reliability–time curve with repair for model-2 is shown in Figure 6.3

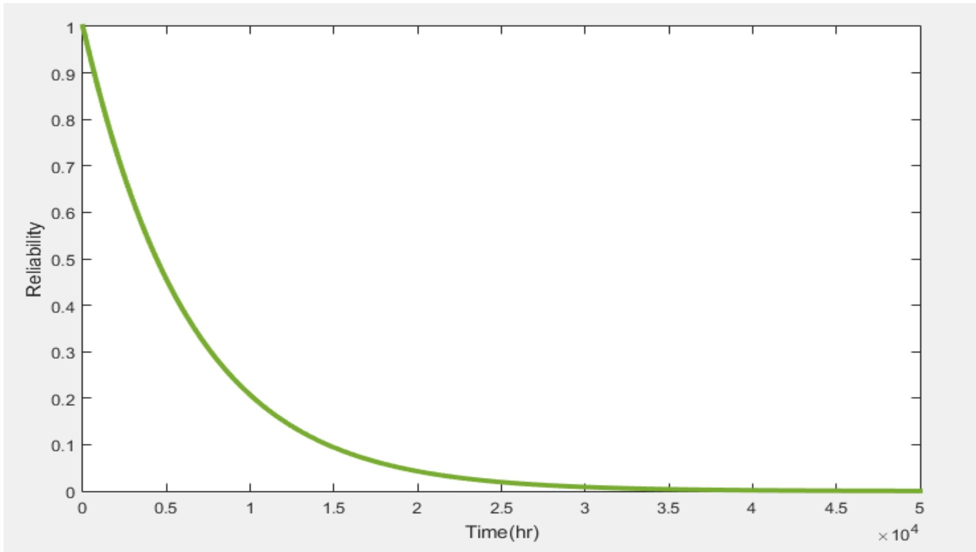

Figure 6.3: Reliability - time curve for model-2 with repair

From Fig. 6.2, it is evident that reliability of the system with one standby redundancy is increased in model-2 as compared with model-1. Also, it is observed from Fig 6.3, that the reliability has been further improved when repair is combined in the one component standby model. However, the system reliability goal is still not achieved. Therefore, one more component is chosen for standby redundancy to improve the reliability resulting into next higher model, i.e. model-3 with two component redundancy.

6.3 Reliability of system for model - 3

On the similar lines, the reliability results are obtained for model-3 with two standby redundancy summing the probabilities of working states, i.e.

Reliability = $P_1 + P_2 + P_3 + P_4 + P_9 + P_{10} + P_{13} + P_{15} + P_{19} + P_{20} + P_{23} + P_{25} + P_{28} + P_{29} + P_{34} + P_{37}$

Reliability-time curve for model-3 with no repair and with repair are evaluated. Reliability–time curve with no repair is shown in Figure 6.4. Reliability–time curve with repair for model-3 is shown in Figure 6.5

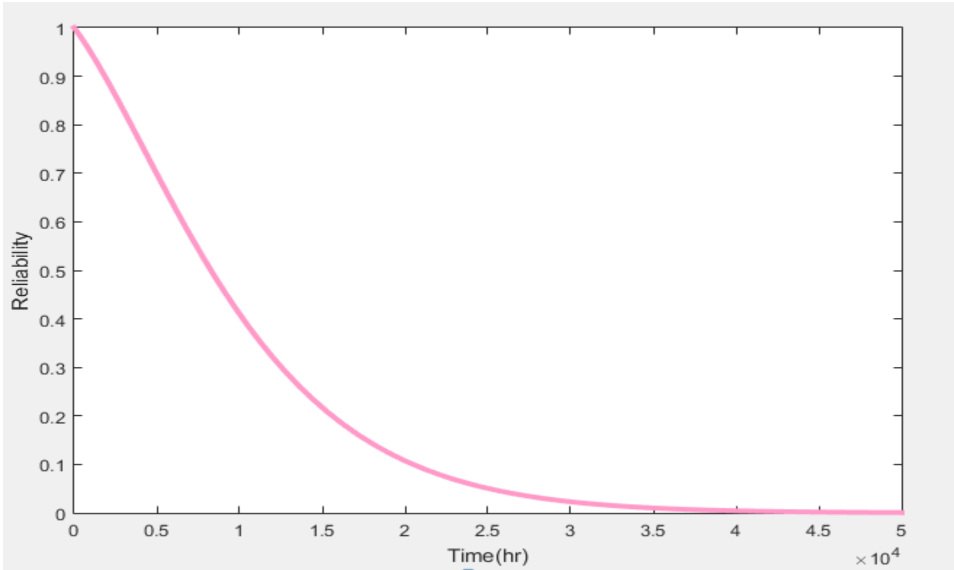

Figure 6.4: Reliability - time curve for model-3 with no repair

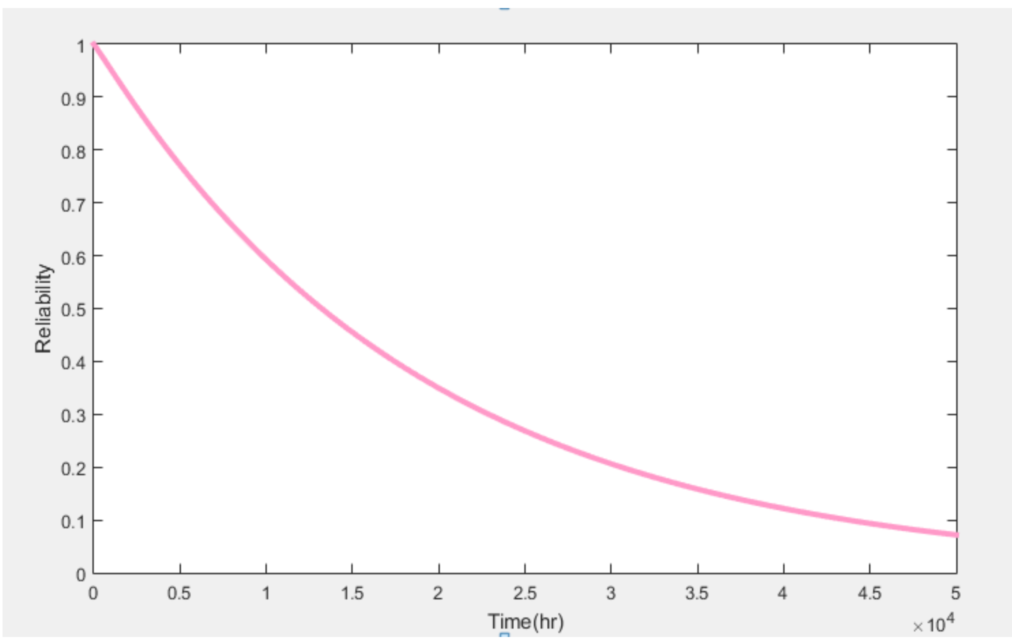

Figure 6.5: Reliability - time curve for model-3 with repair

From Fig. 6.4, it is evident that reliability of the system with two standby redundancies is increased in model-3 as compared with model-2. Also, it is observed from Fig 6.5, that the reliability has been further improved when repair is combined in the two component standby model. However, the system reliability goal is still not achieved. Therefore, one more component is chosen for standby redundancy to improve the reliability resulting into next higher model, i.e. model-4 with three component redundancy.

6.4 Reliability of system for model -4

On the similar lines, the reliability results are obtained for model-4 with thre standby redundancy summing the probabilities of working states, i.e.

Reliability = $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} + P_{12} + P_{13} + P_{14} + P_{15} + P_{16} + P_{17} + P_{18} + P_{19} + P_{20} + P_{21} + P_{22} + P_{23} + P_{24} + P_{25} + P_{26} + P_{27} + P_{28} + P_{29} + P_{30} + P_{31} + P_{32} + P_{33} + P_{34} + P_{35} + P_{36} + P_{37} + P_{38} + P_{39} + P_{40} + P_{41} + P_{42} + P_{43} + P_{44} + P_{45} + P_{46} + P_{47} + P_{48} + P_{49} + P_{50} + P_{51} + P_{52} + P_{53} + P_{54} + P_{55} + P_{56} + P_{57} + P_{58} + P_{59} + P_{60} + P_{61} + P_{62} + P_{63} + P_{64}$

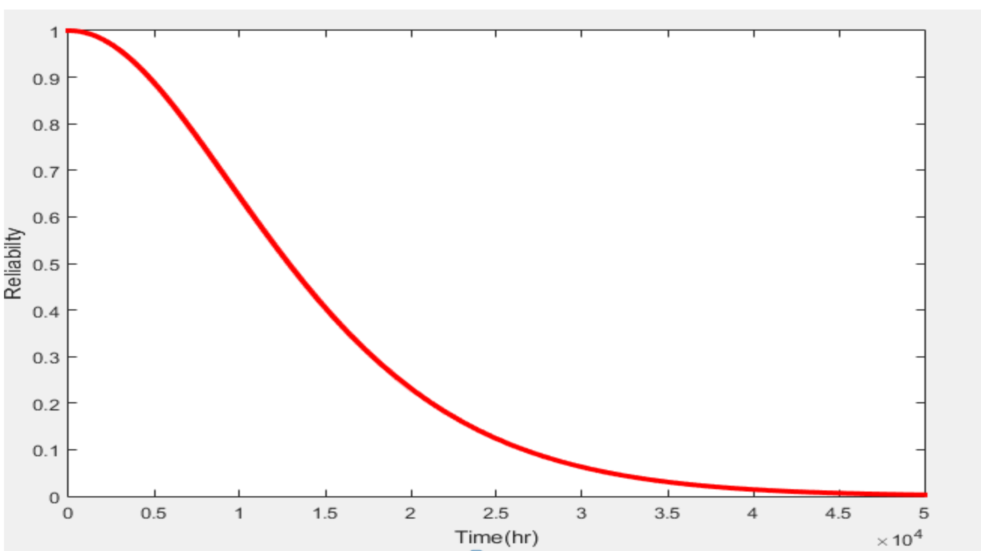

Figure 6.6: Reliability - time curve for model-4 with no repair

Reliability-time curve for model-3 with no repair and with repair are evaluated. Reliability–time curve with no repair is shown in Figure 6.4. Reliability–time curve with repair for model-3 is shown in Figure 6.5

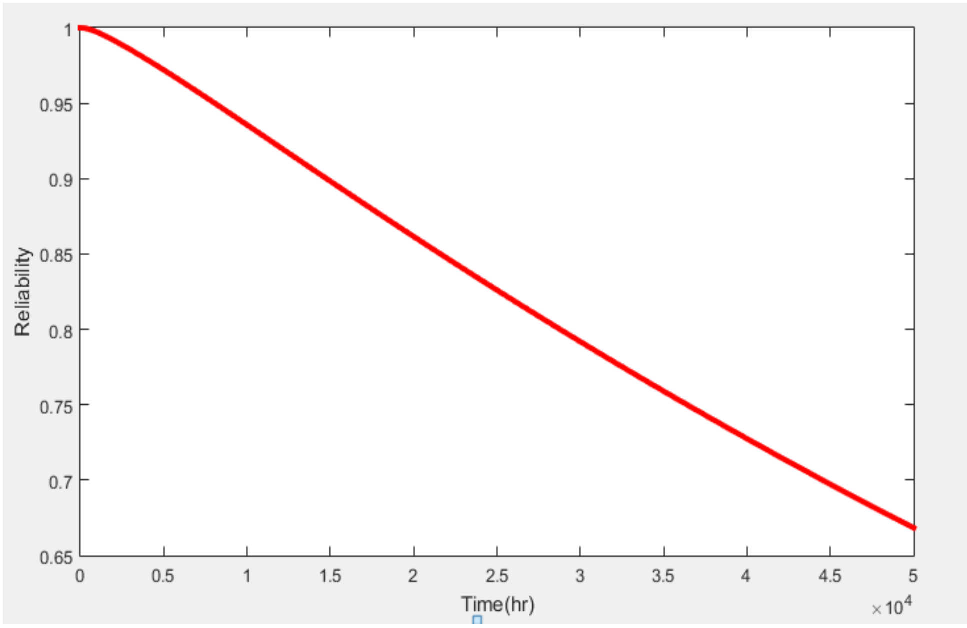

Figure 6.7: Reliability - time curve for model-4 with repair

From Fig. 6.6, it is evident that reliability of the system with two standby redundancies is increased in model-4 as compared with model-3. Also, it is observed from Fig 6.7, that the reliability has been further improved when repair is combined in the three component standby model. In this case reliability has improved substantially and the reliability goal has been achieved when repair is combined.

The comparison of the system reliability in case of no repair and with repair for all system models is shown in the Figure 6.8 and Figure 6.9.

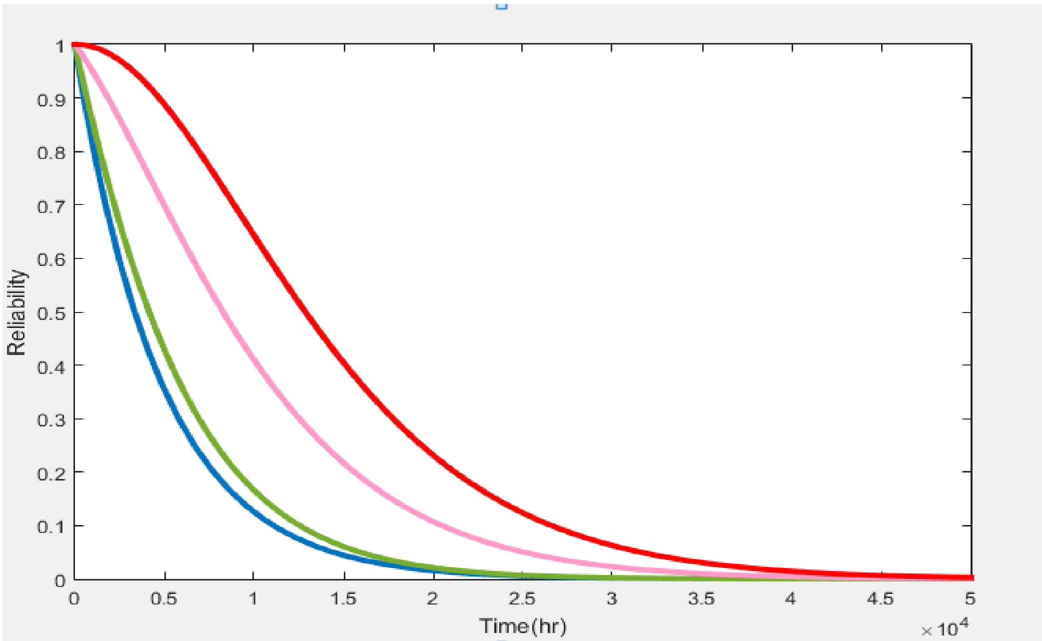

Figure 6.8: Reliability - time curves for all models with no repair

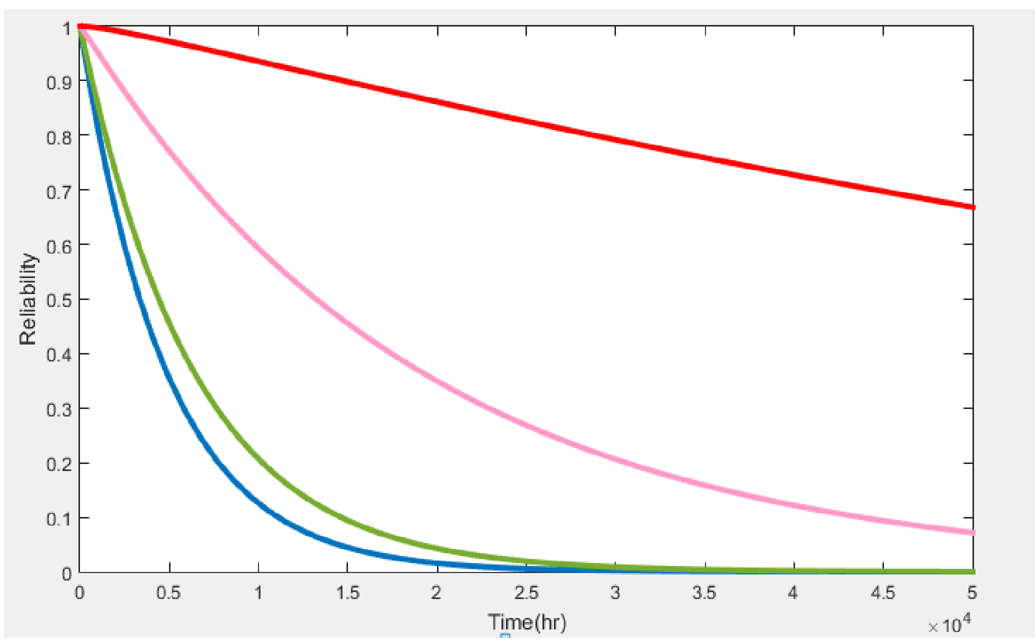

Figure 6.9: Reliability - time curves for all models with repair

The comparison of the system reliability with and without repair for each of model at 10000 hr is shown in the Fig. 6.10.

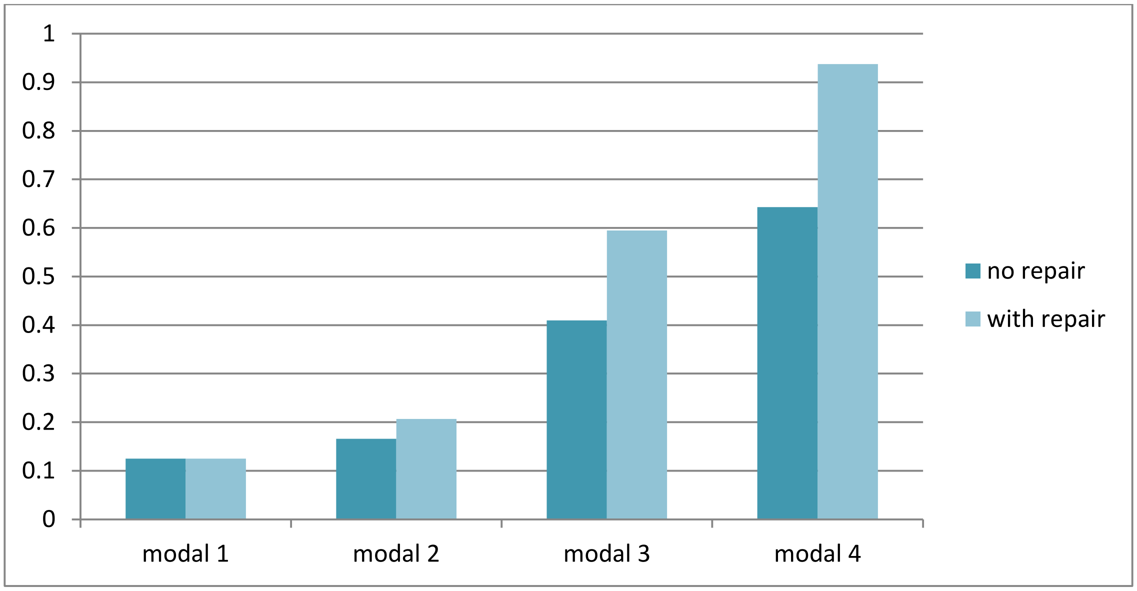

Figure 6.10: Comparison of reliability for all four models at 10000 hr

6.5 Sensitivity analysis: Sensitivity analysis is performed by varying the values of repair rates. At different values of repair rates the reliability of the system varies with time. This is carried to investigate the effect employing more repair resources in terms of multiple repair crews on system reliability.

First analysis is carried for LPC-1. Considering the input values as; $\lambda_1 = \lambda_{11} = 1/11000$, $\lambda_2 = \lambda_{22} = 1/14000$, $\lambda_3 = \lambda_{33} = 1/22000$, $\mu_{11} = 1/600$, $\mu_2 = \mu_{22} = 1/500$, $\mu_3 = \mu_{33} = 1/600$, the system reliability values obtained for three different repair rates for LPC-1 are given in Table 6.1.

Table 6.11: Variation of reliability with change of repair rate of LPC-1

μ_1	1/600	1/500	1/100

Reliability	0.9374	0.9419	0.9614

Similarly, the effect of varying the repair rate for LPC-11 on system reliability is studied. Considering input values: $\lambda_1 = \lambda_{11} = 1/11000$, $\lambda_2 = \lambda_{22} = 1/14000$, $\lambda_3 = \lambda_{33} = 1/22000$, $\mu_1 = 1/600$, $\mu_2 = \mu_{22} = 1/500$, $\mu_3 = \mu_{33} = 1/600$ the reliability values for different repair rate are given in Table 6.2.

Table 6.2: variation of reliability with change of repair rate of LPC-11

μ_{11}	1/600	1/500	1/100
Reliability	0.9374	0.9389	0.9454

On the similar lines the effect of varying repair rates is studied and the results are tabulated in Table 6.3.

Table 6.3:Variation of reliability with change of repair rate for IC-2, IC-22, HPC-3 and HPC-33.

Main Intercooler IC-2		Standby Intercooler IC-22		Main HPC HPC-3		Standby HPC HPC-33	
Repair rate	Reliability	Repair rate	Reliability	Repair rate	Reliability	Repair rate	Reliability
1/500	0.9374	1/500	0.9374	1/600	0.9374	1/600	0.9374
1/400	0.9383	1/400	0.9385	1/500	0.9388	1/500	0.9377
1/100	0.9408	1/100	0.9421	1/100	0.9446	1/100	0.9387

In the next chapter, conclusion and future scope of the work is presented.

CHAPTER 7

<u>CONCLUSION AND SCOPE FOR FUTURE WORK</u>

In this chapter, conclusion and scope for future work is presented.

7.1 Conclusion

In this thesis, Markov model of multistage compressor system is presented for design in reliability. To design a mechanical system in reliability, reliability evaluation of repairable system are performed which is based on the state space method called Markov approach. Constant failure and repair rates are considered for the different components in the system. Standby redundancy has been used to increase the reliability of system. System reliability value for basic system model, i.e. model-1 at 10000 hours is 0.1252 that has been increased by using standby redundancies in model-4 with no repair to 0.6428 and with repair to 0.9374. From sensitivity analysis part it can be seen that increase in repair rate of low pressure compressor, intercooler and high pressure compressor in the system, reliability increases. Table 6.1 show that by increasing in repair rate of LPC-1 from 0.00166 to 0.01, reliability of system increases by 2.5%. Table 6.3 shows that by increasing in repair rate of IC-2 from 0.002 to 0.01, reliability of system increases by 0.36%. Table 6.32 shows that by increasing in repair rate of HPC-3 from 0.00166 to 0.01, reliability of system increases by 0.76%.

The proposed analysis is useful for design engineer to design a repairable system to achieve the higher reliability goals and also helpful for them in taking decision for appropriate design policy.

7.2 Future scope

This section presents a brief on potential future directions.

- In this work, the costs are not considered for the reliability analysis. It will be meaningful, if it is linked with the cost incurred of redundancies and analysis is carried in terms of reliability gain Vs cost.
- An exponential distribution is assumed for failure and repair time due to the limitation of Markov approach. This assumption can be relaxed and appropriate non-exponential distribution such as Weibull for failure time and Lognormal for repair time can be considered for more realistic analysis.
- Software can be developed based on the proposed methodology to design a system with reliability goal.
- Warm standby can be considered in place of cold standby redundancy, which is more realistic in nature.
- Imperfect switching can also be considered in system modeling.

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APPENDIX A1

Markov modeling of model 4:

The following differential equations associated with the transition diagram of model 4 are formed as below:

$$\frac{d}{dt}P_1(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_1(t) + \mu_{33}\, P_2(t) + \mu_{22}\, P_3(t) + \mu_{11}\, P_5(t)$$

$$\frac{d}{dt}P_2(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_2(t) + \lambda_{33}\, P_9(t) + \mu_{11}\, P_6(t) - \mu_{22}\, P_4(t) - \mu_{33}\, P_2(t)$$

$$\frac{d}{dt}P_3(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_3(t) + \lambda_{22}\, P_{17}(t) + \mu_{33}\, P_4(t) - \mu_{22}\, P_3(t) + \mu_{11}\, P_7(t)$$

$$\frac{d}{dt}P_4(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_4(t) + \mu_{11}\, P_8(t) - \mu_{22}\, P_4(t) - \mu_{33}\, P_4(t) + \lambda_{22}\, P_{18}(t) + \lambda_{33}\, P_{10}(t)$$

$$\frac{d}{dt}P_5(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_4(t) - \mu_{11}\, P_4(t) - \mu_{22}\, P_4(t) - \mu_{33}\, P_4(t) + \lambda_{11}\, P_{10}(t) + \lambda_{22}\, P_{20}(t) + \lambda_{33}\, P_{20}(t)$$

$$\frac{d}{dt}P_6(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_4(t) - \mu_{11}\, P_6(t) + \mu_{22}\, P_8(t) - \mu_{33}\, P_6(t) + \lambda_{11}\, P_{34}(t) + \lambda_{33}\, P_{11}(t)$$

$$\frac{d}{dt}P_7(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_7(t) - \mu_{11}\, P_7(t) - \mu_{22}\, P_7(t) + \mu_{33}\, P_8(t) + \lambda_{11}\, P_{25}(t) + \lambda_{22}\, P_{19}(t)$$

$$\frac{d}{dt}P_8(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_8(t) - \mu_{11}\, P_8(t) - \mu_{22}\, P_8(t) - \mu_{33}\, P_8(t) + \lambda_{11}\, P_{36}(t) + \lambda_{22}\, P_{20}(t) + \lambda_{33}\, P_{12}(t)$$

$$\frac{d}{dt}P_9(t) = -(\lambda_1 + \lambda_2 + \lambda_{33})\, P_9(t) + \mu_{11}\, P_{11}(t) + \mu_{22}\, P_{18}(t) + \mu_3\, P_{13}(t)$$

$$\frac{d}{dt}P_{10}(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_{10}(t) + \mu_{11}\, P_{12}(t) - \mu_{22}\, P_{10}(t) + \mu_3\, P_{14}(t) + \lambda_{22}\, P_{21}(t)$$

$$\frac{d}{dt}P_{11}(t) = -(\lambda_1 + \lambda_2 + \lambda_{33})\, P_{11}(t) - \mu_{11}\, P_{11}(t) + \mu_{22}\, P_{12}(t) + \mu_3\, P_{15}(t) + \lambda_{11}\, P_{37}(t)$$

$$\frac{d}{dt}P_{12}(t) = -(\lambda_1 + \lambda_2 + \lambda_3)\, P_{12}(t) - \mu_{11}\, P_{12}(t) - \mu_{22}\, P_{12}(t) + \mu_3\, P_{16}(t) + \lambda_{11}\, P_{38}(t) + \lambda_{22}\, P_{22}(t)$$

$$\frac{d}{dt}P_{13}(t) = -(\lambda_1 + \lambda_2 + \lambda_{33})\, P_{13}(t) + \mu_{11}\, P_{15}(t) + \mu_{22}\, P_{14}(t) - \mu_3\, P_{13}(t) + \lambda_3\, P_1(t)$$

$$\frac{d}{dt}P_{14}(t) = -(\lambda_1 + \lambda_2 + \lambda_{33})\, P_{14}(t) + \mu_{11}\, P_{16}(t) - \mu_{22}\, P_{14}(t) - \mu_3\, P_{14}(t) + \lambda_{22}\, P_{23}(t) + \lambda_3\, P_3(t)$$

$$\frac{d}{dt} P_{15}(t) = -(\lambda_1 + \lambda_2 + \lambda_{33}) P_{15}(t) - \mu_{11} P_{15}(t) + \mu_{22} P_{16}(t) - \mu_3 P_{15}(t) + \lambda_{11} P_{39}(t) + \lambda_3 P_5(t)$$

$$\frac{d}{dt} P_{16}(t) = -(\lambda_1 + \lambda_2 + \lambda_{33}) P_{16}(t) - \mu_{11} P_{16}(t) - \mu_{22} P_{16}(t) - \mu_3 P_{16}(t) + \lambda_{11} P_{40}(t) + \lambda_{22} P_{24}(t) + \lambda_3 P_7(t)$$

$$\frac{d}{dt} P_{17}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{17}(t) + \mu_{11} P_{19}(t) + \mu_2 P_{25}(t) + \mu_{33} P_{18}(t)$$

$$\frac{d}{dt} P_{18}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{18}(t) + \mu_{11} P_{20}(t) + \mu_2 P_{26}(t) - \mu_{33} P_{18}(t) + \lambda_{33} P_{21}(t)$$

$$\frac{d}{dt} P_{19}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{19}(t) - \mu_{11} P_{19}(t) + \mu_2 P_{27}(t) + \mu_{33} P_{20}(t) + \lambda_{11} P_{41}(t)$$

$$\frac{d}{dt} P_{20}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{20}(t) - \mu_{11} P_{20}(t) + \mu_2 P_{28}(t) - \mu_{33} P_{20}(t) + \lambda_{11} P_{42}(t) + \lambda_{33} P_{22}(t)$$

$$\frac{d}{dt} P_{21}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{21}(t) + \mu_{11} P_{22}(t) + \mu_2 P_{29}(t) + \mu_3 P_{23}(t)$$

$$\frac{d}{dt} P_{22}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{22}(t) - \mu_{11} P_{22}(t) + \mu_2 P_{30}(t) + \mu_3 P_{24}(t) + \lambda_{11} P_{43}(t)$$

$$\frac{d}{dt} P_{23}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{23}(t) + \mu_{11} P_{24}(t) + \mu_2 P_{31}(t) - \mu_3 P_{23}(t) + \lambda_3 P_{17}(t)$$

$$\frac{d}{dt} P_{24}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{24}(t) - \mu_{11} P_{24}(t) + \mu_2 P_{32}(t) - \mu_3 P_{24}(t) + \lambda_{11} P_{44}(t) + \lambda_3 P_{19}(t)$$

$$\frac{d}{dt} P_{25}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{25}(t) + \mu_{11} P_{27}(t) - \mu_2 P_{25}(t) + \mu_{33} P_{26}(t) + \lambda_2 P_1(t)$$

$$\frac{d}{dt} P_{26}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{20}(t) + \mu_{11} P_{28}(t) - \mu_2 P_{26}(t) - \mu_{33} P_{26}(t) + \lambda_2 P_2(t) + \lambda_{33} P_{29}(t)$$

$$\frac{d}{dt} P_{27}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{20}(t) - \mu_{11} P_{27}(t) - \mu_2 P_{27}(t) + \mu_{33} P_{28}(t) + \lambda_{11} P_{45}(t) + + \lambda_2 P_6(t)$$

$$\frac{d}{dt} P_{28}(t) = -(\lambda_1 + \lambda_{22} + \lambda_3) P_{28}(t) - \mu_{11} P_{28}(t) - \mu_2 P_{28}(t) - \mu_{33} P_{28}(t) + \lambda_{11} P_{46}(t) + + \lambda_2 P_6(t) + \lambda_{33} P_{30}(t)$$

$$\frac{d}{dt} P_{29}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{29}(t) + \mu_{11} P_{30}(t) - \mu_2 P_{29}(t) + \mu_3 P_{31}(t) + + \lambda_2 P_9(t)$$

$$\frac{d}{dt} P_{30}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{30}(t) - \mu_{11} P_{30}(t) - \mu_2 P_{30}(t) + \mu_3 P_{32}(t) + \lambda_{11} P_{47}(t) + + \lambda_2 P_{11}(t)$$

$$\frac{d}{dt} P_{31}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{31}(t) + \mu_{11} P_{32}(t) - \mu_2 P_{31}(t) - \mu_3 P_{31}(t) + + \lambda_2 P_{13}(t) + \lambda_3 P_{25}(t)$$

$$\frac{d}{dt} P_{32}(t) = -(\lambda_1 + \lambda_{22} + \lambda_{33}) P_{32}(t) - \mu_{11} P_{32}(t) - \mu_2 P_{32}(t) - \mu_3 P_{32}(t) + \lambda_{11} P_{48}(t) + + \lambda_2 P_{15}(t) + \lambda_3 P_{27}(t)$$

$$\frac{d}{dt} P_{33}(t) = -(\lambda_{11} + \lambda_2 + \lambda_3) P_{33}(t) + \mu_1 P_{49}(t) + \mu_{22} P_{35}(t) + \mu_{33} P_{34}(t)$$

$$\frac{d}{dt} P_{34}(t) = -(\lambda_{11} + \lambda_2 + \lambda_3) P_{34}(t) + \mu_1 P_{50}(t) + \mu_{22} P_{36}(t) - \mu_{33} P_{34}(t) + \lambda_{33} P_{37}(t)$$

$$\frac{d}{dt} P_{35}(t) = -(\lambda_{11} + \lambda_2 + \lambda_3) P_{35}(t) + \mu_1 P_{51}(t) - \mu_{22} P_{35}(t) + \mu_{33} P_{36}(t) + \lambda_{22} P_4(t)$$

$$\frac{d}{dt} P_{36}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_3) P_{36}(t) + \mu_1 P_{52}(t) - \mu_{22} P_{36}(t) - \mu_{33} P_{36}(t) + \lambda_{22} P_{42}(t) + \lambda_{33} P_{38}(t)$$

$$\frac{d}{dt} P_{37}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_3) P_{37}(t) + \mu_1 P_{53}(t) + \mu_{22} P_{38}(t) + \mu_3 P_{39}(t)$$

$$\frac{d}{dt} P_{38}(t) = -(\lambda_{11} + \lambda_2 + \lambda_{33}) P_{38}(t) + \mu_1 P_{54}(t) - \mu_{22} P_{38}(t) + \mu_3 P_{40}(t) + \lambda_{22} P_{40}(t)$$

$$\frac{d}{dt} P_{39}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{39}(t) + \mu_1 P_{55}(t) + \mu_{22} P_{40}(t) - \mu_3 P_{39}(t) + \lambda_3 P_{33}(t)$$

$$\frac{d}{dt} P_{40}(t) = -(\lambda_{11} + \lambda_2 + \lambda_{33}) P_{40}(t) + \mu_1 P_{56}(t) - \mu_{22} P_{40}(t) - \mu_3 P_{40}(t) + \lambda_{22} P_{44}(t) + \lambda_3 P_{35}(t)$$

$$\frac{d}{dt} P_{41}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_3) P_{41}(t) + \mu_1 P_{57}(t) + \mu_2 P_{45}(t) + \mu_{33} P_{42}(t)$$

$$\frac{d}{dt} P_{42}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_3) P_{42}(t) + \mu_1 P_{58}(t) + \mu_2 P_{46}(t) - \mu_{33} P_{42}(t) + \lambda_{33} P_{43}(t)$$

$$\frac{d}{dt} P_{43}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{43}(t) + \mu_1 P_{59}(t) + \mu_2 P_{47}(t) + \mu_3 P_{44}(t)$$

$$\frac{d}{dt} P_{44}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{44}(t) + \mu_1 P_{60}(t) + \mu_2 P_{48}(t) - \mu_3 P_{44}(t) + \lambda_3 P_{41}(t)$$

$$\frac{d}{dt} P_{45}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_3) P_{45}(t) + \mu_1 P_{61}(t) - \mu_2 P_{45}(t) + \mu_{33} P_{46}(t) + \lambda_2 P_{33}(t)$$

$$\frac{d}{dt} P_{46}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{46}(t) + \mu_1 P_{62}(t) - \mu_2 P_{46}(t) - \mu_{33} P_{46}(t) + \lambda_2 P_{34}(t) + \lambda_{33} P_{47}(t)$$

$$\frac{d}{dt} P_{47}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{47}(t) + \mu_1 P_{63}(t) - \mu_2 P_{47}(t) + \mu_3 P_{48}(t) + \lambda_2 P_{37}(t)$$

$$\frac{d}{dt} P_{48}(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) P_{48}(t) + \mu_1 P_{64}(t) - \mu_2 P_{48}(t) - \mu_3 P_{48}(t) + \lambda_2 P_{39}(t) + \lambda_3 P_{45}(t)$$

$$\frac{d}{dt} P_{49}(t) = -(\lambda_{11} + \lambda_2 + \lambda_3) P_{49}(t) - \mu_1 P_{49}(t) + \mu_{22} P_{51}(t) + \mu_{33} P_{50}(t) + \lambda_1 P_1(t)$$

$$\frac{d}{dt}P_{50}(t) = -(\lambda_{11}+\lambda_2+\lambda_3)P_{50}(t) - \mu_1 P_{50}(t) + \mu_{22}P_{52}(t) - \mu_{33}P_{50}(t) + \lambda_1 P_2(t) + \lambda_{33}P_{53}(t)$$

$$\frac{d}{dt}P_{51}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3)P_{51}(t) - \mu_1 P_{51}(t) - \mu_{22}P_{51}(t) + \mu_{33}P_{52}(t) + \lambda_1 P_3(t) + \lambda_{22}P_{57}(t)$$

$$\frac{d}{dt}P_{52}(t) = -(\lambda_{11}+\lambda_2+\lambda_3)P_{52}(t) - \mu_1 P_{52}(t) - \mu_{22}P_{52}(t) - \mu_{33}P_{52}(t) + \lambda_1 P_4(t) + + \lambda_{22}P_{58}(t) + \lambda_{33}P_{54}(t)$$

$$\frac{d}{dt}P_{53}(t) = -(\lambda_{11}+\lambda_2+\lambda_{33})P_{53}(t) - \mu_1 P_{53}(t) + \mu_{22}P_{54}(t) + \mu_{33}P_{55}(t) + \lambda_1 P_9(t)$$

$$\frac{d}{dt}P_{54}(t) = -(\lambda_{11}+\lambda_2+\lambda_{33})P_{54}(t) - \mu_1 P_{54}(t) - \mu_{22}P_{54}(t) + \mu_3 P_{56}(t) + \lambda_1 P_{10}(t) + \lambda_{22}P_{59}(t)$$

$$\frac{d}{dt}P_{55}(t) = -(\lambda_{11}+\lambda_2+\lambda_{33})P_{55}(t) - \mu_1 P_{55}(t) +- \mu_{22}P_{56}(t) - \mu_3 P_{55}(t) + \lambda_1 P_{13}(t) + \lambda_3 P_{49}(t)$$

$$\frac{d}{dt}P_{56}(t) = -(\lambda_{11}+\lambda_2+\lambda_{33})P_{56}(t) - \mu_1 P_{56}(t) - \mu_{22}P_{56}(t) - \mu_3 P_{56}(t) + \lambda_1 P_{14}(t) + \lambda_{22}P_{60}(t) + \lambda_3 P_{51}(t)$$

$$\frac{d}{dt}P_{57}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3)P_{57}(t) - \mu_1 P_{57}(t) + \mu_2 P_{61}(t) + \mu_{33}P_{58}(t) + \lambda_1 P_{17}(t))$$

$$\frac{d}{dt}P_{58}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3)P_{58}(t) - \mu_1 P_{58}(t) + \mu_2 P_{62}(t) - \mu_{33}P_{58}(t) + \lambda_1 P_{18}(t) + \lambda_{33}P_{59}(t)$$

$$\frac{d}{dt}P_{59}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_{33})P_{59}(t) - \mu_1 P_{59}(t) + \mu_2 P_{63}(t) + \mu_3 P_{60}(t) + \lambda_1 P_{21}(t)$$

$$\frac{d}{dt}P_{60}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_{33})P_{60}(t) - \mu_1 P_{60}(t) + \mu_2 P_{64}(t) - \mu_3 P_{60}(t) + \lambda_1 P_{23}(t) + \lambda_3 P_{57}(t)$$

$$\frac{d}{dt}P_{61}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3)P_{61}(t) - \mu_1 P_{61}(t) - \mu_2 P_{61}(t) + \mu_{33}P_{62}(t) + \lambda_1 P_{25}(t) + \lambda_2 P_{49}(t)$$

$$\frac{d}{dt}P_{62}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_3)P_{62}(t) - \mu_1 P_{62}(t) - \mu_2 P_{62}(t) - \mu_{33}P_{62}(t) + \lambda_1 P_{26}(t) + \lambda_2 P_{50}(t) + \lambda_{33}P_{63}(t)$$

$$\frac{d}{dt}P_{63}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_{33})P_{63}(t) - \mu_1 P_{63}(t) - \mu_2 P_{63}(t) + \mu_3 P_{64}(t) + \lambda_1 P_{29}(t) + \lambda_2 P_{53}(t)$$

$$\frac{d}{dt}P_{64}(t) = -(\lambda_{11}+\lambda_{22}+\lambda_{33})P_{64}(t) - \mu_1 P_{64}(t) - \mu_2 P_{64}(t) - \mu_3 P_{64}(t) + \lambda_1 P_{31}(t) + \lambda_2 P_{55}(t) + \lambda_3 P_{61}(t)$$

$$\frac{d}{dt}P_{65}(t) = \lambda_3 P_2(t) + \lambda_{33} P_{13}(t)$$

$$\frac{d}{dt}P_{66}(t) = \lambda_3 P_4(t) + \lambda_{33} P_{14}(t)$$

$$\frac{d}{dt}P_{67}(t) = \lambda_3 P_6(t) + \lambda_{33} P_{15}(t)$$

$$\frac{d}{dt}P_{68}(t) = \lambda_3 P_8(t) + \lambda_{33} P_{16}(t)$$

$$\frac{d}{dt}P_{69}(t) = \lambda_3 P_{18}(t) + \lambda_{33} P_{23}(t)$$

$$\frac{d}{dt}P_{70}(t) = \lambda_3 P_{20}(t) + \lambda_{33} P_{24}(t)$$

$$\frac{d}{dt}P_{71}(t) = \lambda_2 P_3(t) + \lambda_{22} P_{25}(t)$$

$$\frac{d}{dt}P_{72}(t) = \lambda_2 P_4(t) + \lambda_{22} P_{26}(t)$$

$$\frac{d}{dt}P_{73}(t) = \lambda_2 P_7(t) + \lambda_{22} P_{27}(t)$$

$$\frac{d}{dt}P_{74}(t) = \lambda_2 P_8(t) + \lambda_{22} P_{28}(t)$$

$$\frac{d}{dt}P_{75}(t) = \lambda_2 P_{10}(t) + \lambda_{22} P_{29}(t)$$

$$\frac{d}{dt}P_{76}(t) = \lambda_2 P_{12}(t) + \lambda_{22} P_{30}(t)$$

$$\frac{d}{dt}P_{77}(t) = \lambda_3 P_{26}(t) + \lambda_{33} P_{31}(t)$$

$$\frac{d}{dt}P_{78}(t) = \lambda_2 P_{14}(t) + \lambda_{22} P_{31}(t)$$

$$\frac{d}{dt}P_{79}(t) = \lambda_3 P_{28}(t) + \lambda_{33} P_{32}(t)$$

$$\frac{d}{dt}P_{80}(t) = \lambda_2 P_{16}(t) + \lambda_{22} P_{32}(t)$$

$$\frac{d}{dt}P_{81}(t) = \lambda_3 P_{34}(t) + \lambda_{33} P_{39}(t)$$

$$\frac{d}{dt}P_{82}(t) = \lambda_3 P_{36}(t) + \lambda_{33} P_{40}(t)$$

$$\frac{d}{dt} P_{83}(t) = \lambda_3 P_{42}(t) + \lambda_{33} P_{44}(t)$$

$$\frac{d}{dt} P_{84}(t) = \lambda_3 P_{38}(t) + \lambda_{22} P_{47}(t)$$

$$\frac{d}{dt} P_{85}(t) = \lambda_2 P_{36}(t) + \lambda_{22} P_{46}(t)$$

$$\frac{d}{dt} P_{86}(t) = \lambda_2 P_{38}(t) + \lambda_{22} P_{47}(t)$$

$$\frac{d}{dt} P_{87}(t) = \lambda_3 P_{46}(t) + \lambda_{33} P_{44}(t)$$

$$\frac{d}{dt} P_{88}(t) = \lambda_2 P_{40}(t) + \lambda_{42} P_{48}(t)$$

$$\frac{d}{dt} P_{89}(t) = \lambda_1 P_{5}(t) + \lambda_{11} P_{49}(t)$$

$$\frac{d}{dt} P_{90}(t) = \lambda_1 P_{6}(t) + \lambda_{11} P_{50}(t)$$

$$\frac{d}{dt} P_{91}(t) = \lambda_1 P_{7}(t) + \lambda_{11} P_{51}(t)$$

$$\frac{d}{dt} P_{92}(t) = \lambda_1 P_{8}(t) + \lambda_{11} P_{52}(t)$$

$$\frac{d}{dt} P_{93}(t) = \lambda_1 P_{11}(t) + \lambda_{11} P_{53}(t)$$

$$\frac{d}{dt} P_{94}(t) = \lambda_1 P_{12}(t) + \lambda_{11} P_{54}(t)$$

$$\frac{d}{dt} P_{95}(t) = \lambda_3 P_{50}(t) + \lambda_{33} P_{55}(t)$$

$$\frac{d}{dt} P_{96}(t) = \lambda_3 P_{52}(t) + \lambda_{33} P_{56}(t)$$

$$\frac{d}{dt} P_{97}(t) = \lambda_1 P_{15}(t) + \lambda_{11} P_{55}(t)$$

$$\frac{d}{dt} P_{98}(t) = \lambda_1 P_{16}(t) + \lambda_{11} P_{56}(t)$$

$$\frac{d}{dt} P_{99}(t) = \lambda_1 P_{19}(t) + \lambda_{11} P_{57}(t)$$

$$\frac{d}{dt} P_{100}(t) = \lambda_1 P_{20}(t) + \lambda_{11} P_{58}(t)$$

$$\frac{d}{dt}P_{101}(t) = \lambda_1 P_{22}(t) + \lambda_{11} P_{59}(t)$$

$$\frac{d}{dt}P_{102}(t) = \lambda_3 P_{58}(t) + \lambda_{33} P_{60}(t)$$

$$\frac{d}{dt}P_{103}(t) = \lambda_1 P_{24}(t) + \lambda_{11} P_{60}(t)$$

$$\frac{d}{dt}P_{104}(t) = \lambda_2 P_{51}(t) + \lambda_{22} P_{61}(t)$$

$$\frac{d}{dt}P_{105}(t) = \lambda_2 P_{52}(t) + \lambda_{22} P_{62}(t)$$

$$\frac{d}{dt}P_{106}(t) = \lambda_1 P_{27}(t) + \lambda_{11} P_{61}(t)$$

$$\frac{d}{dt}P_{107}(t) = \lambda_1 P_{28}(t) + \lambda_{11} P_{62}(t)$$

$$\frac{d}{dt}P_{108}(t) = \lambda_2 P_{64}(t) + \lambda_{22} P_{63}(t)$$

$$\frac{d}{dt}P_{109}(t) = \lambda_1 P_{30}(t) + \lambda_{11} P_{63}(t)$$

$$\frac{d}{dt}P_{110}(t) = \lambda_3 P_{62}(t) + \lambda_{33} P_{64}(t)$$

$$\frac{d}{dt}P_{111}(t) = \lambda_2 P_{56}(t) + \lambda_{22} P_{64}(t)$$

$$\frac{d}{dt}P_{112}(t) = \lambda_1 P_{32}(t) + \lambda_{11} P_{64}(t)$$

With initial condition at time t = 0

$P_1(t) = 1,$

$P_1(t), P_2(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots P_{112}(t) = 0$

And at any time t

$\sum_{i=1}^{112} P_i(t) = 1$

APPENDIX A2

Program for reliability analysis of model 2:

```
function rl = oneredundent(t,y)
rl = zeros(13,1);
p1 = y(1);
p2 = y(2);
p3 = y(3);
p4 = y(4);
f1 = 1/11000;
f2 = 1/14000;
f3 = 1/12200;
f11 = 1/11000;
r1 = 1/600;
r11 = 1/600;
rl(1) = -(f1+f2+f3)*p1+r11*p2;
rl(2) = -(f1+f2+f3)*p2+f11*p3-r11*p2;
rl(3) = -(f2+f3)*p3-f11*p3+r1*p4;
rl(4) = -(f2+f3)*p4-f11*p4+f1*p1-r1*p4;
rl(5) = f1*p2+f11*p4;
rl(6) = f2*p1;
rl(7) = f2*p2;
rl(8) = f2*p3;
rl(9) = f2*p4;
rl(10) = f3*p1;
rl(11) = f3*p2;
rl(12) = f3*p3;
rl(13) = f3*p4;
```