

Project Report for Major-II

Waste heat recovery from Automobile exhaust using Thermoelectric Generator

Submitted in the partial fulfilment of the requirement for the award
of degree of

Master of Technology In Renewable Energy Technology

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DECLARATION

I, hereby declare that the project entitled “Waste heat recovery from Automobile exhaust using Thermoelectric Generator” being submitted by me, is an authentic work carried out under the supervision of Dr. Amit Pal Associate Professor, Mechanical Engineering Department, Delhi Technological University, Delhi. This is also certified that this dissertation has not been submitted to any other Institute/University for the award of any degree or diploma.

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CERTIFICATE

This is to certify that the dissertation entitled “Waste heat recovery from Automobile exhaust using Thermoelectric Generator” submitted by Mr. Vikrant Mishra (2K14/RET/17) in partial fulfilment for the award of the Degree of Master of Engineering in Renewable Energy Technology of Delhi Technological University, Delhi, is an authentic record of student’s own work carried out by him under my guidance and supervision.

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It is said that gratitude is a virtue. This part is dedicated to special thanks that I would like to deliver to the people who helped me in making the fulfilment of this thesis project possible.

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I would also like to thank Dr. Raj Kumar Singh for introducing me to this fascinating research field. He helped me a lot in developing prototype.

Last but definitely not the least, I would like thank all the staff of IC engine lab, DTU for their continuous help throughout the project.

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ABSTRACT

In recent years, an increasing concern of environmental degradation due to automobile exhaust emissions and the limitations of energy resources has resulted in extensive research for novel technologies of generating electrical power. A thermoelectric generator using the exhaust waste heat from an automobile has the potential to replace the existing alternator system in an automobile, and thus improve fuel economy and reduce emissions. The conversion efficiency of modern thermoelectric materials has increased more than three-times in the last twenty years. But there are many challenges in the thermal design of Exhaust based Thermoelectric Generator (ETEG) systems, such as increasing the efficiency of the heat exchangers (hot box and cold plate) maintaining a sufficient temperature difference across the thermoelectric module during different operating conditions and reducing thermal losses through the system as a whole.

Firstly, the whole ETEG system is mathematically analysed in this thesis. The concept of mismatching is also presented when modules are connected in series or parallel configuration. A working prototype of exhaust based thermoelectric generator with single module is demonstrated. The conversion efficiency and power produced by module are estimated on the basis of Seebeck effect, Fourier's law of heat conduction and Newton's law of heat convection. After this, methods to increase conversion efficiency of TEG system are discussed. One is the use of DC-DC boost converter along with TEG. This boost converter steps up the voltage induced by module. Single stage and multi-stage conversion network of DC-DC converter are also discussed which are used according the need of output voltage.

The other methods of increasing conversion efficiency are development of thermoelectric material with higher Figure of merit value (approx. 1) and thermal optimization of heat exchanger. Thermal optimization of heat exchanger includes the design of heat exchanger with some specific internal fins. The flow of exhaust gas become more turbulent due to this internal design of heat exchanger which cause a significant increase in hot side temperature of TEG. Ultimately, the conversion efficiency increases due to higher temperature gradient. In the end, results are discussed by graphical interpretation of conversion efficiency with respect to various related parameters in TEG device such as temperature gradient, heat transfer coefficient, engine speed etc.

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List of Abbreviations used

AETEG-	Automobile Exhaust based Thermoelectric Generator
CFD-	Computational Fluid Dynamics
ETEG-	Exhaust based Thermoelectric generator
PV-	Photovoltaic
TE-	Thermoelectric
TEC-	Thermoelectric cells
TEG-	Thermoelectric Generator
TEM-	Thermoelectric module
VCHP-	Variable conductance heat pipes
WHR-	Waste Heat Recovery

List of Symbols

- Q_1 – Heat transferred by Exhaust gases
- S - Seebeck Coefficient
- ZT- Dimensionless Figure of Merit of thermoelectric module
- T_1 - Hot Side Temperature
- T_2 - Cold side Temperature
- ΔT - Temperature Difference
- I - Current
- V - Voltage
- P - Power
- η_{th} - Theoretical Conversion Efficiency of TEG
- η_{act} - Actual Conversion Efficiency
- k - Thermal Conductivity
- R_1 - External Resistance or Load
- R_2 - Internal Load
- h - Convective heat transfer Coefficient
- μ - Viscosity
- ρ - Density
- Re - Reynold's number
- D - Duty cycle of DC-DC converter
- C_{CCM} - Critical capacitance value of a DC-DC converter.
- L_{CCM} - Critical inductance of a DC-DC converter.
- f_s - Switching frequency of a DC-DC converter.

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Chapter 1

Introduction

In recent years, Energy crisis has become a major challenge due to quickly increasing demands and consumption of Energy. The scientific and public awareness on environmental and energy issue has brought in key interest to the advancement in technologies and research particularly in highly efficient internal combustion engines. Normally, there are two basic methods to improve efficiency of internal combustion engine. One is by optimization of combustion process and the second is to recover waste heat of the engine exhaust gases. In a typical IC engine driven automobile, Vehicle mobility and accessories utilize only about 25% of the supplied fuel energy. During the combustion process in an automobile engine, 40% of the energy is lost through exhaust gases and about 30% is reflected in the form of the heat carried away by the engine coolant liquid [1]. This is represented pictorially in figure 1.1-

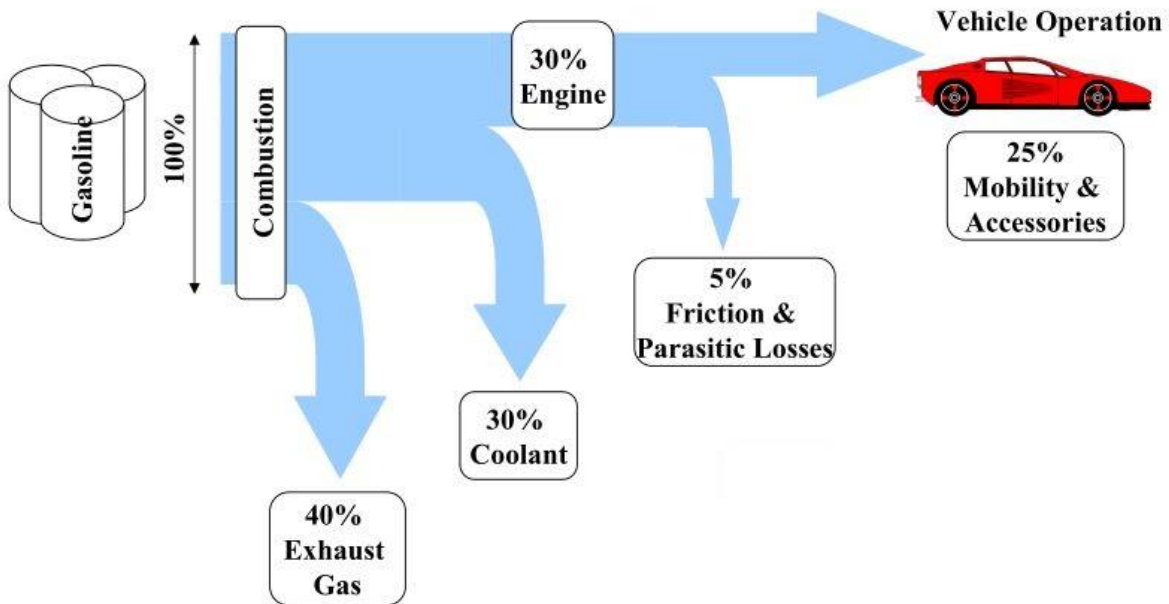


Figure 1.1- Typical energy flow in an IC engine driven automobiles [2]

So, an effort should be made to capture a significant portion of the available heat energy of exhaust gases. It can not only help in reducing engine loads and alternator size but also decrease pollutant emissions and fuel consumption [2].

There are many waste heat recovery (WHR) technologies such as Rankine bottoming cycle technique, Six-stroke internal combustion engine cycle technique, Turbocharger and Thermoelectric energy conversion technology [3]. But Thermoelectric power generation technique directly convert thermal energy into electrical energy. TEG has no moving parts and it is compact, quiet, highly reliable and environmentally friendly.

1.1 Motivation and objectives

Thermoelectric power generation offers a promising technology in the direct conversion of low grade thermal energy, such as waste-heat energy, into electrical power. In 1821, Thomas Johann Seebeck discovered the phenomenon of thermoelectricity. When a temperature gradient is established between the hot and cold junctions of two dissimilar materials (metals or semiconductors), a voltage is generated, i.e., Seebeck Voltage. Based on this Seebeck effect, thermoelectric devices can act as electrical power generators [4]. The major drawback of thermoelectric generator is their relatively low conversion efficiency (typically 5-10%). Therefore, TEGs have mostly been used in specialized electrical equipment in aerospace and military applications [5].

With technological advancement in the last two decades, The TEG has shown ever increasing potential for application that can be widely used in power generation from automobile exhaust waste heat, industrial waste heat, solar energy and other natural thermal energies. The total energy consumption in India, as per Global Energy Statistical yearbook 2015, is 872 Mtoe. The transportation sector consumes approximately 30% (261.6 Mtoe) of the total energy consumption [6]. So there is a great scope for thermoelectric power generator in automotive applications, which can reduce fuel consumption. The main objective of this thesis is to improve the conversion efficiency of thermoelectric power generation system.

1.2 Description of ATEG Projects in Industries

This section briefly discusses the endeavors of some industry-based projects which try to involve TEG as part of the solutions for the fuel economy improvement for vehicles. Although Seebeck gave the concept of thermoelectricity in 1821, thermoelectric generator was not applied to automobiles until 1963, when Neild et al. [7] build and reported the first automobile exhaust based thermoelectric generator. This experiment was not successful, but some further advancements in TEG research are given below:-

BSST's automotive waste heat recovery project

BSST is a subsidiary of Amerigon Incorporated. It was started in the year 2000 to develop advanced thermoelectric systems. The researchers were mainly focused toward the advancement in thermoelectric materials and TEG structures. BMW, Visteon and Ford supported BSST's TEG design and development in the area of vehicle system simulation and subsystem hardware development. There are many aspects on which BSST's top technological innovation focused such as thermal isolation, convection, high power density, mechanical transport and construction [8].

BSST's automotive waste heat recovery project achieved up to 12% fuel economy improvement on the basis of performance simulation for the system architecture and model of established subsystem designs. In the late 2010, a complete TE system installed into both BMW and Ford vehicles. To validate fuel savings and emissions reduction performance, the vehicles were tested over a wide range of driving condition [9].

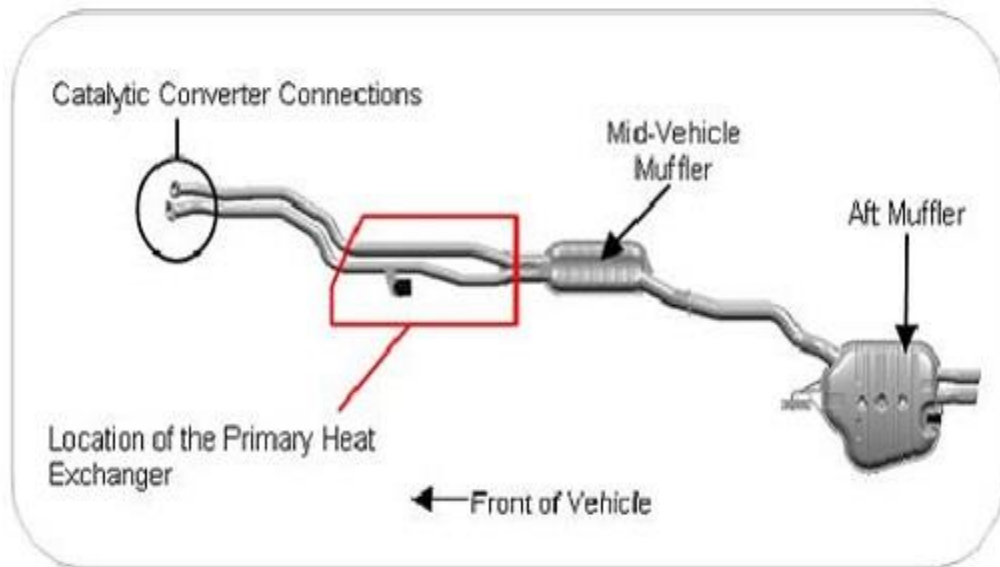


Figure 1.2- BSST automotive waste heat recovery project [9]

Hi-Z Technology Incorporation

The technological innovation of Hi-Z include the patented quantum well thermoelectric modules. Hi-Z's mission was to develop, manufacture and commercialize the next generation of

high efficiency thermoelectric modules, called Quantum wells, which guaranteed 15% to 40% conversion efficiency [10].

GMC Sierra Pick-up Truck

In 2004, The AETEG project was launched by Clarkson University and several companies such as Hi-Z technology incorporation, GM Powertrain Division and Delphi Harrison Thermal system. The purpose was to investigate the feasibility of the TEG application on a GM Pick-up truck and to develop a commercial plan for the designed AETEG system [11]. The target was to achieve power output of 330 watts, being supplied with heat from the exhaust and cooled by the conventional coolant circuit. The conversion efficiency of electronics device was in the order of 80-90%. With different types of configurations, the maximum amount of electricity created was 140 W to 225 W [11].



**330 W TEG for GM
1999 Sierra Pickup
Truck
Equipped with
power
management
system to
charge 12 V and
42 V batteries.
Suitable for
light trucks and
passenger
vehicles.**

Figure 1.3- 330 W TEG for GM Sierra Pick-up Truck ^[11]

Efficient TEG manufacturing project in KELK

KELK (former Komatsu Electronics Inc.) is a part of Japan based multinational corporation 'Komatsu Limited' which manufactures mining, construction and military equipment, Industrial machinery and thermoelectric modules [12].

It was announced in January 2009 that Komatsu would begin production and sales of world's most efficient TEGs. Its maximum output power is 24 W and maximum conversion efficiency is of the order of 7.2%. Bismuth Telluride (Bi_2Te_3) was utilized as thermoelectric material [13].

BMW Vision-Efficient-Dynamics Program

BMW, a leading industry in the world in automobile sector, launched its ATEG program Vision-efficient-dynamics in March 2009. A prototype vehicle was developed by the company in which TEG was fitted. The TE material used was Bismuth-Telluride. The vehicle used was BMW 530i. During driving speed of 130 Km/h at highway, the power production reached levels of 200 watts. The Figure of merit value (ZT value) was claimed to be around 0.4. Prediction was made on the basis of tests and observations that the fuel consumption savings are in the order of 1-8% depending on driving condition. The company also claims future progress majorly depends on how well the laboratory demonstrated materials find their way to the commercial market [14].



Figure 1.4- Prototype vehicle of BMW with TEG mounted [2]

Some previous applications of high power thermoelectric generators are shown in the table 1.1.

Table 1.1- Typical cases of the applications of high-power TEGs ^[15]

S. No.	Manufacturer	Power (W)	Efficiency (%)	Hot side Temp. °C	Material	Application	Price \$/W
1.	BMW, GM BSST, Hi-Z	800	6	500	Co-Sb	Automobile exhaust heat	13
2.	Plantec, Showa Denko, Komatsu	21.6	6.2	280	n: Ce-Co-Sb p: La-Fe-Sb	Solid waste incineration heat	-
3.	Thermonamic	500	4.8	250	Bi ₂ Te ₃	General	<2
4.	Komatsu/KELK Showa Cable Systems	200	7.2	280	Bi ₂ Te ₃	Boiler waste heat	-
5.	Komatsu	500	10~11	700	n: Co-Sb p: Mn-Si	Gas turbine afterheat	-
6.	GM, Hi-Z	1000	4.5	250	BiTe	Oil turbine afterheat	11

1.3. Problem Statement

TEGs have many potential application in Automobiles but there are certain problems which should be discussed and solved to improve the conversion efficiency of thermoelectric power generator.

- Figure of merit value of thermoelectric materials should be maximized in the temperature range of exhaust gases (300-500°C), so there is a need to deal with challenges of material development and system design in case of TEG. As soon as the thermoelectric materials with high figure of merit value will be developed, the chances of better heat to electrical conversion efficiency will be more.

- Location of TEG should be decided in such a way that it is taking maximum advantage of the heat produced by the engine exhaust gases. So there is a challenge to identify the hot spot in the exhaust pipe.
- Another problem in case of previous TEG prototypes is of low rate of heat transfer from exhaust gas to thermoelectric base plate (Hot side). Internal finning or diffuser arrangement should be used in order to minimize the temperature difference between the gas and hot side of the device.
- Generator mass is also a significant factor which should be considered while designing TEG. To keep mass of the TEG in the optimum range is still a challenge.
- Thermoelectric material stability is an important factor due to high temperature of exhaust gases. So this is an area of concern for developing advanced thermoelectric materials.
- Low conversion efficiency is also the common problem of TEG system. To overcome this DC-DC boost converter should be used with TEG. It can help in increasing Seebeck voltage produced.

1.4. Organization of the Report

In this thesis, we have presented a detailed analysis of TEG prototype demonstration, its validation and methods to make it efficient. Second chapter presents the detailed literature review of previous researches in the field of exhaust based thermoelectric generator. In the third chapter, Mathematical analysis of TEG system is presented. Mathematical expressions for Power and efficiency are derived and the concept of mismatching is also discussed. In the fourth chapter, we have described the experimental work done in this project. Experimental work shows the demonstration of TEG prototype with single thermoelectric module. In the fifth chapter, the methods of increasing conversion efficiency of the system are discussed. First one is use of DC-DC boost converter which takes the output voltage from thermoelectric module as input and then boost up it to higher voltage at output load. The other method is the thermal optimization of heat exchanger used in the waste heat recovery system. In the sixth chapter, the results are discussed based on both mathematical analysis and simulation. I-V characteristics, P-V characteristics and conversion efficiency curve with respect to different parameters such as figure-of-merit (ZT), convective heat transfer coefficient (h), temperature difference are shown in the result section.

Chapter 2

Literature Review

2.1 Automobile Exhaust based TEG as a potential energy efficiency option

In recent years, TEG technology has been considered as a potential option for automotive waste heat recovery. Many scientist and researchers are working in this field to make it popular and efficient. Ismail et al. (2009) discussed the basic concept of TE power generation and gave a review of recent patents of TE power generation with their relevant application. TE power generation is based on the principle of Seebeck Effect. When a temperature difference is created between the hot and cold ends of two dissimilar materials (metals or semiconductors), a voltage is generated. Based on this Seebeck effect, TE devices can act as electrical power generators [16].

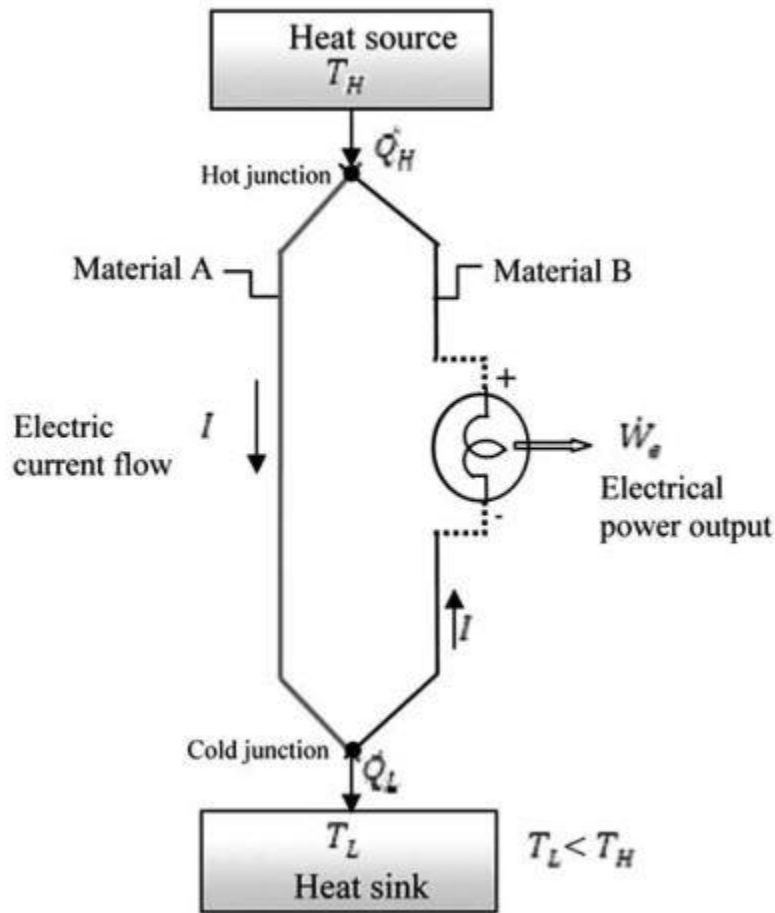


Figure 2.1- Schematic diagram of Seebeck Effect [16]

The amount of electrical power produced mainly depends on the thermoelectric Figure-of-merit ZT . It is represented dimensionless as $ZT = S^2\sigma T/k$, where S is Seebeck coefficient, σ is electrical conductivity, T is the temperature of material and k is thermal conductivity [16].

Saniya LeBlanc (2014) discussed a review of new thermoelectric materials and material performance. According to this review, the classification of TE materials is based on material structure and composition. Generally chalcogenide, skutterudite, clathrate, half-Heusler, silicide and oxide are some types of TE materials. In these types, Chalcogenide materials are widely used in demonstrated thermoelectric application with bismuth telluride (Bi_2Te_3) and lead telluride (PbTe) being the most noticeable. Bismuth Telluride and its solid solution with antimony and selenium are commercially used for thermoelectric modules in case of low temperature. Whenever there are higher temperatures (500~600°C), Lead Telluride is used for better thermoelectric properties. The properties of TE materials mainly depend on temperature which emphasize multiple challenges for application-specific materials selection [17].

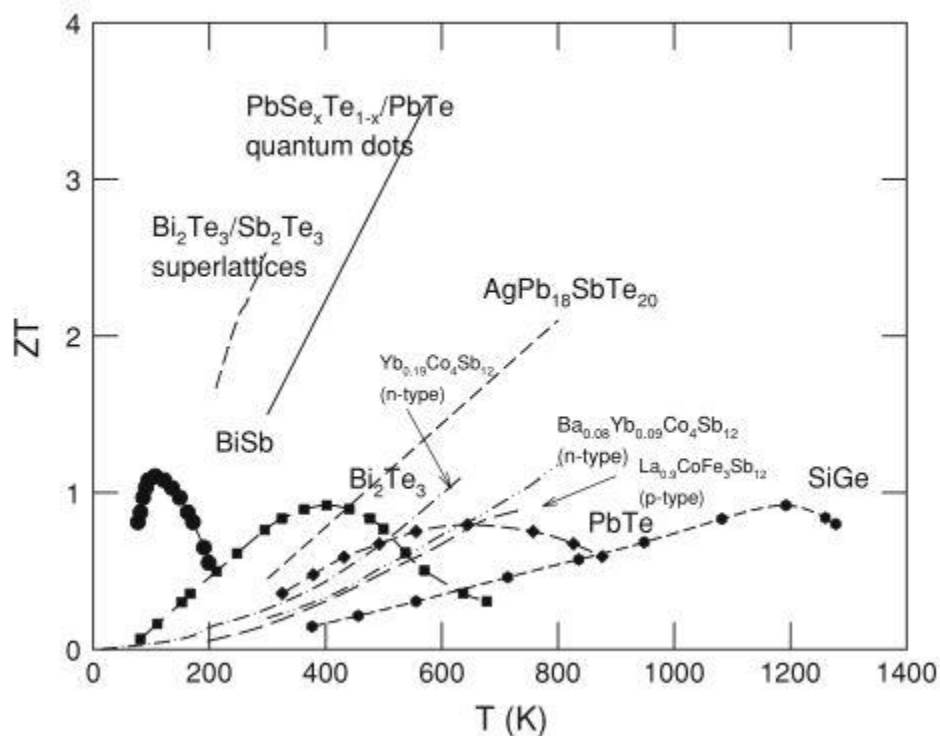


Figure 2.2- Figure of merit (ZT) versus Temperature curves for many state-of-the-practice ($ZT \leq 1$) and state-of-the-art materials ($ZT > 1$). [18]

The electricity generation potential of TEG not only depends upon TE material but also on the system design. Some components of TE devices contact directly with the TE material, affecting overall device performance. For example, a metallurgical bond exists between the TE leg and the metal interconnects. So there are design consideration which influence the selection of the solder or braze material used to form this bond. The solder/braze material should not diffuse into the TE material.

There are also challenging materials requirements for the module substrates and geometries. The substrates must be arranged in this way that they are electrically insulating and thermally conducting. The optimization of TEG power output and efficiency depends on TE material properties and dimensions as well as system level electrical and thermal resistances. Hence TEG performance is directly based on the selection of electrical and thermal contact/interface materials, substrates and heat exchangers design. It can also be concluded that major issues to resolve for TEG commercialization are [17]:

- TE material selection based on average (not peak) figure of merit,
- Thermal and chemical stability of material,
- Thermal optimization of interfaces
- Design of hot and cold side heat exchanger.

So thermoelectric generator has a potential to be used as waste heat recovery unit in automobiles and engines. Review of previous research is presented in next section. Most of the research are focused on design consideration of TEG unit and thermal optimization. In this report some new techniques such as use of DC-DC boost converter along with TEG also introduced with detailed analysis of previous studies. The main focus is on increasing the thermal to electrical conversion efficiency and making TEG a reliable and commercial technology for waste heat recovery options.

2.2 Analysis of previously developed models of TEG

Wang et al. (2013) presents a mathematical model of a Thermoelectric Generator (TEG) device using the exhaust gas of vehicles as heat source based on Fourier's law and Seebeck Effect. The model pretends the influence of various related factors on the output power and efficiency, such as vehicles exhaust mass flow rate, mass flow rate of different types of coolants, temperature of

exhaust gas, temperature difference across TEG, convection heat transfer coefficient, height of P-N couple and the ratio of external electrical resistance to internal resistance of the circuit on the output power and efficiency of the TEG system. The results showed that the output power and efficiency increase significantly by varying the convection heat transfer coefficient of the high-temperature-side than that of low-temperature-side. The results also showed that the power output achieved a peak value for optimum height of P-N couple. Besides it the peak output power value decreases when the thermal conductivity of the PN couple is decreased, and increases when the Seebeck coefficient and electric conductivity of the material are increased. Thermal conductivity is a property of material, so material with high thermal conductivity were used as hot side plate to increase rate of heat transfer and eventually to increase power output. Additionally, a maximum value of power output and efficiency of TEG device appear when external electrical resistance is greater than internal resistance. This is not usual as common circuit and with the augmentation of dimension-less figure of merit (ZT), the maximum value of output moves toward the direction of an increasing ratio of external resistance to internal resistance [19].

Shameer et al. (2013) proposed and implemented a thermoelectric waste heat energy recovery system for internal combustion engine automobiles which include gasoline vehicles and hybrid electric vehicles. In this work, An Experimental setup for waste heat recovery system using TEG, was discussed along with working procedure and design calculations. Identification of hot spot in the exhaust pipe was also analyzed using Ansys simulation [20].

Zhang et al. (2009) designed, employed and compared the parallel-connected thermoelectric power generator system. The advantage of this system is that it provide dual DC bus and it has high overall system efficiency. The low efficiency in the pure battery discharging mode can be evaded with other switching circuit. So this parallel-connected system is a superior choice for automotive application excluding the series-connected system [21].

Martins et al. (2011) assessed the potential of the use of heat pipes (HP) as a means of transferring energy from the hot exhaust gases to the TEG modules at a well-suited temperature level while diminishing the loss of efficiency due to reducing temperature. In this work, Variable Conductance Heat Pipe (VCHP) was used and its arrangement has the benefit of inducing good temperature control. Various types of heat pumps were designed, manufactured, verified and improved with the purpose of enhancing the overall heat transfer process, enabling an optimal level of electric

energy recovery using proper arrangement of TEG modules. This was comprehended by the testing of different fluids inside the heat pump and by regulating the pressure of the exhaust gas chamber. The results indicate that the use of VCHPs in conjunction with thermoelectric generators is a convincing technique for recovering waste heat energy from the automobile exhaust gases [22].

Zorbas et al. (2008) evaluated power and efficiency of TEG and also presented a commercial bismuth-telluride TE device with 31 thermocouples. The efficiency is calculated on the basis of law of thermoelectricity which are: the Seebeck effect, Peltier effect, Thomson effect and joule effect. In this experiment the module of size 2.5 cm*2.5 cm was used. The hot side of TEG is directly attached with the copper-made heater and a liquid heat exchanger was used at cold side to keep cooling temperature constant. The thermal contact resistance and thermal resistance of both top and bottom ceramics plates of TEG were taken into consideration. At hot side temperature of 220°C, the TEG device was able to produce 2.6 W of power i.e. 5.4% efficiency of TEG device. The prevailing temperatures in the exhaust pipe of a medium sized car, are sufficient to generate power with the help of TEG. This application of TEG helps in fuel savings and also reduce the pollutants extracted from the engine exhaust. In the exhaust line of vehicle the high temperature appears after the catalytic converter. So location of TEG is important and also effective design of heat exchanger is necessary for better output power and efficiency of device. Results show that it would be feasible to make a TE device even with conventional TE materials which can produce around 300 W power. In such case the fuel saving will be around 5% [23].

Kumar C.R. et al. (2011) carried out a detailed experimental work to study the performance of TEGs under various operating condition of engine. In this experiment a TEG was constructed which included a counter flow coolant cooling chamber, an exhaust gas heat exchanger (hot side heat exchanger) and 18 thermoelectric modules connected electrically in series. For better heat transfer from exhaust gas to thermoelectric module, the design of heat exchanger is very significant and also for heat flux passing through the modules, reduction of thermal resistances and thermal contact resistances is necessary. Copper and aluminum are better options for base plate which is used to make contact of TEG with exhaust pipe. It is due to the higher thermal conductivity of copper in comparison of low carbon steel and stainless steel. But when TEG is installed on low size vehicles, aluminum is better option for base plate due to its high thermal conductivity and low weight. 18 thermoelectric modules with hot and cold plates fabricated in a frame of cast iron (5

mm thickness). Thermal grease was used at the interfaces to increase the thermal conduction. The fabricated TE device neglect the need of separate cooling system for module. In this case power required to pump the coolant is the only power loss, if we compare it with the power produced by TEG. To analyze the performance of fabricated TEG, the test was carried out on a 3-cylinder 4-stroke SI engine of Maruti 800. Results showed that the addition of TEG has very little effect on exhaust emissions and exhaust back pressure. The power output of TEG system is dependent on temperature difference (ΔT), as ΔT increases, power output also increases. The presented TEG was of 0.0048 kW/kg specific power [24].

T. Wang et al. (2014) proposed a new type of open-cell metal foam-filled plate heat exchanger based thermoelectric generator system to utilize low grade waste heat. In this system TE generation works as a parasitic mode which is attached to heat exchanging process. The major portion of waste heat is captured by the process of heating water and an amount of this waste heat flux is converted by means of TEG into electricity as by product. This system mainly consists of HE-TEG unit, air supply and heating unit, cold water channel and data acquisition system [25].

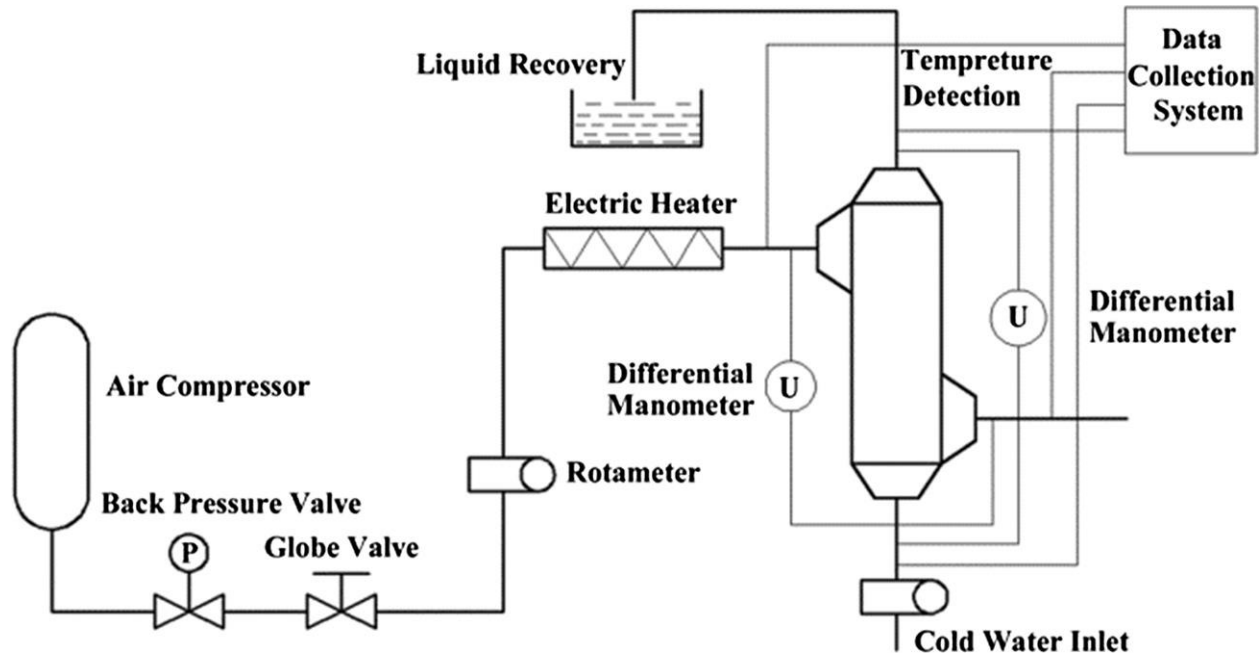


Figure 2.3 – Schematic diagram of HE-TEG waste heat recovery system ^[25]

In this work the performance of heat exchangers and thermoelectric generator units were discussed separately. The efficiency of heat exchanger between heated air and cold water was 83.56% at the experimental working condition. This was due to the use of metal foams which play a significant role in enhancing the heat transfer process. Performance of TEG is much dependent of the temperature difference. The open circuit voltage of One TE couple increased approximately linearly from 0 to 5.5 mV with the variation of temperature difference from 0 to 13.8°C. When load resistance becomes equal to internal resistance, maximum power output of one TE couple was obtained. The feasibility of increasing the number of TE couples was also demonstrated which helps in enlarging the electricity generation capacity. The maximum open circuit voltage increases up to 108.1 mV from 5.5 mV, when the number of TE couples was varied from 1 to 16 [25].

2.3 Design Considerations of Automotive Thermoelectric Generator system

For an efficient and effective automotive TEG system, the design of the complete device and its components is very significant. Design of Heat Exchangers, Location of TEG in exhaust line, Optimization of Fin distribution and Thermal performance of heat exchangers are very important parameters which should be kept in mind while designing and installing the TEG system. The overall conversion efficiency also increases when design and location of heat exchanger is optimized. In the last decade, more researchers have worked in this area which gave better result in the sense of making automotive TEG system commercial in automobile industries. Liu et al. (2014) developed an energy utilizing system which captured the heat of automobile exhaust pipe and converted the heat into electricity by means of thermoelectric power generators. In this work, heat exchanger was connected to exhaust pipe and cooling water tank. Thermoelectric modules were clamped between the heat exchanger with enough compressive force. The temperature difference between the two sides of the modules created by the flowing exhaust gas at one side and cooling water at another side. Due to temperature difference, electrical power generated and stored in batteries. In this work, simulation was done for different internal structures of heat exchanger. The plate-shaped heat exchanger was preferred for TEG application due to short distance between chassis and ground and also the height of plate-shaped heat exchanger is also short. Firstly, the simulation of the heat exchanger was done with no internal structure, in this case uneven thermal distribution occurs inside the heat exchanger due to sudden expansion of exhaust gas flowing through the exhaust pipe. The temperature found (144°C) was less and heat exchanger

could not meet the requirement. So after this, two 3-dimensional models of heat exchangers with fishbone-shaped and chaos-shaped internal structures were designed. Simulation results showed that heat exchanger with chaos-shaped internal structure have higher outlet temperature (220°C on average) than in case of fishbone-shaped (190°C). Thus, Chaos-shaped heat exchanger design is more ideally suitable in TEG application. Additionally, the thickness of the heat exchanger is also responsible for thermal performance. Heat exchanger of chaos-shaped internal structure with different thicknesses of 3 mm, 5 mm and 8 mm respectively were used for simulation comparison. The results shows that in case of 3 mm thickness heat exchanger the outlet temperature is approximately 180°C which is lesser than expected hot side temperature in automotive TEG application. So heat exchanger with 5 mm and 8 mm thicknesses were used and there was little difference in the interface temperatures. So the lighter size of TEG (5 mm thickness) is better because of reduction in weight [26].

Liu et al. (2014) also presented an analytical study to decide the location of TEG in automotive thermoelectric power generation system. The main focus was making the TE device compatible among the catalytic converter, muffler and TEG. In this work, installation position of TEG was varied and three different cases were explained on the basis of simulation result. In the first case the TEG location was at the end of exhaust system. The average interface temperature of heat exchanger in this case was just 210°C and it is known that the appropriate temperature of low-temperature TE module is in the order of 250°C to 300°C. So in the first case, temperature of heat exchanger could not meet the required temperature for efficient and effective TEG based waste heat recovery system [27].

In the second case, TEG was located between catalytic converter and muffler. The mean temperature of interface was in the order of 270°C and the interface temperature profile of heat exchanger was uniform; so it was appropriate interface temperature according to the required temperature of TE module.

In the third case, TEG was installed at upstream of catalytic converter and muffler. The average interface temperature in this case was 280°C which is in range of required TE module interface temperature, but in this case the average temperature of catalytic converter was just 190°C which is less than the ignition temperature of harmful exhaust gases (250°C). Catalytic converter does not work normally in such condition. So finally, it was concluded that case 2 is the best and TEG

should be located between catalytic converter and muffler. Infrared thermal images of case 2 was also presented which validate the best possibility of case 2 [27].

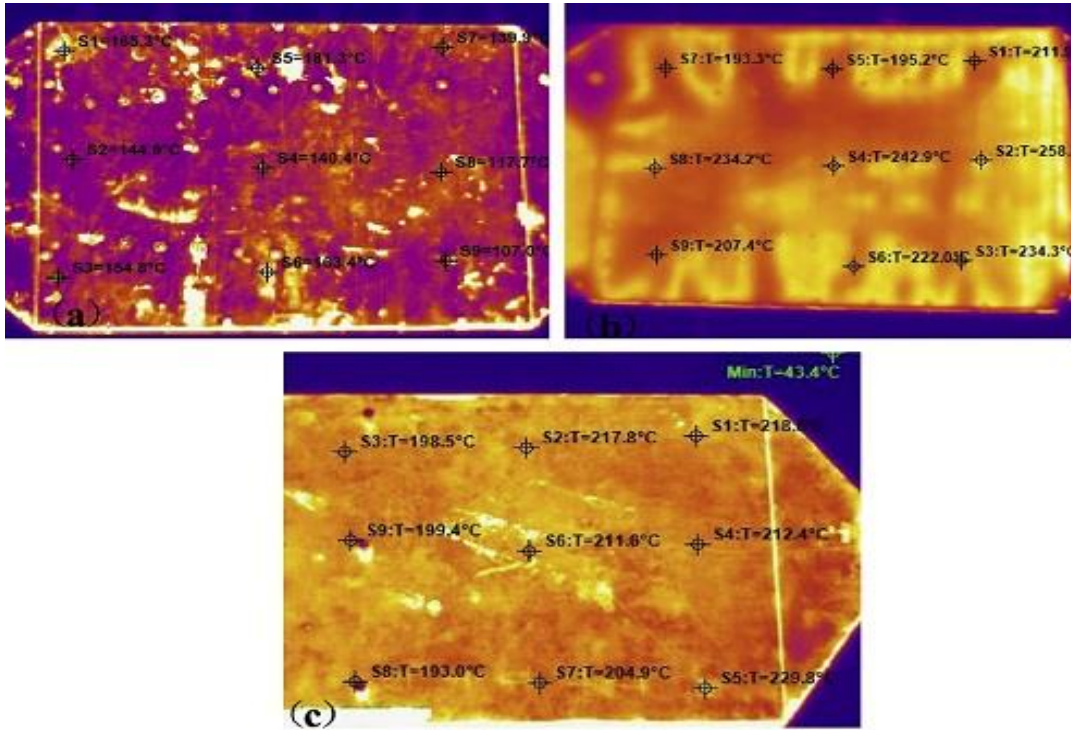


Figure 2.4- Infrared images of the heat exchanger under three cases:(a)case 1, (b)case 2, and (c)case 3 [27].

B. deok In et al. (2015) presented various cases of TEG installation in diesel engine with different thermal conditions of exhaust gas. In this work, the electricity generation characteristics of TE module were examined by means of experiments using HZ-20 to analyze the possibility of application of TEMs in actual engines. The thermoelectric modules was a bismuth-telluride module with dimensions 75mm*75mm*5mm and a constant temperature water bath was used at cold side to maintain the temperature of coolant. The flow in this experiment was constant (12 L/min). The experiment was performed on a commercial 2.2 L diesel engine. 20 thermoelectric modules were added electrically in series to fabricate the thermoelectric generator. Ten of which were HZ-20 modules and rest ten modules were HZ-14. Voltage and current generated by each module were calculated and on the basis of these voltage and current, power was measured. Then various thermal condition were explained with different kinds of heat sinks. The types of heat sinks

used were rectangular pillar-shaped heat sink, forward facing triangular pillar heat sink and backward facing triangular pillar heat sink. Results showed that highest thermal conductivity with better performance of heat transfer was achieved with the use of rectangular pillar-shaped heat sink [28].

The pressure differential was also observed higher for rectangular pillar heat sink than other two cases. Hence heat transfer capability is better for rectangular pillar heat sink. Also the generated peak voltage measured for rectangular heat sink was approximately 2.7 V which was higher than generated 2.5 V and 2.4 V in forward facing and reverse facing triangular heat sink respectively. In addition, the highest voltage level was observed when input resistance was in order of 3.3 Ω for each heat sink shape. The same trend was also observed in current measurement which was 5 A for rectangular pillar shaped heat sink (around 6.2 W) while in other two cases it is around 5.5 W. For maximum power condition, the input resistance was found approximately 0.3 Ω . Besides it, the performance of TEM increases with increase in the temperature difference between hot and cold side. So the temperature of hot side is increased until it reaches the value of durable temperature of TEM. When cold side temperature is increased, it reduces the temperature difference. So ultimately power is reduced, hence the cold side temperature should be kept low [28].

Ramade et al. (2014) presented a detailed experimental work which evaluated the performance of TEGs under different engine speeds and thermal optimization technique of TEG system was also reviewed with different kinds of heat sink. In this work, a 4-stroke 3-cylinder Maruti 800 cc SI engine was taken into consideration and the TEG device was fitted into the exhaust line of the engine. Various parameters such as voltage, current, power, heat transfer rate through hot and cold type heat exchanger, overall efficiency of fin etc. were measured on the basis of law of heat transfer and thermoelectricity. The observed efficiency was not quite, so thermal optimization of system is done to improve efficiency. So in place of single stacked type cold side heat sink, double stacked type heat sink was used which gave better temperature difference across TEG. Thermal insulation was applied on the uncovered area to neglect heat loss and counter flow type heat exchanger was arranged which increased the effective heat transfer. Results were obtained with Bismuth-Telluride thermoelectric material at hot side temperature of 250°C. Result show that efficiency of TEG device and power developed increases with increase in speed of engine. The efficiency of TEG

device was 5.0708% and power generated was 15.12 W at engine speed of 3970 rpm. Hence this generated power can be utilized to charge the car battery or to power auxiliary systems. The overall efficiency of vehicle increases [29].

Deng et al. (2013) discussed the thermal performance of the heat exchanger in automobile exhaust based thermoelectric generator. Various internal structures of heat structures were applied with different TE materials. This thermal optimization is done by the computational fluid dynamics simulation and then by infrared image capture experiment. For CFD simulation two 3-dimensional models of hexagonal-prism-shaped and plate-shaped heat exchangers were designed and taken into consideration. CFD simulation results show that the interface temperature for hexagonal prism-shaped heat exchanger is just around 120°C which is far less than the required temperature of hot side of automotive TEG system. But in case of plate-shape heat exchanger with several baffle plates, the interface temperature is around or sometimes more than 240°C. Additionally, the volume of hexagonal prism-shaped heat exchanger is very large compared to plate shaped heat exchanger which is advantageous for waste exhaust gas flowing through the heat exchanger. So all results enables plate shaped heat exchanger more suitable for waste heat recovery using TEG device [30].

Su et al. (2014) presented experimental study on thermal optimization of the heat exchanger in an automobile exhaust-based thermoelectric generator. In order to achieve temperature uniformity and higher interface temperature, three-dimensional models of different types of heat exchangers were developed and then compared with the help of CFD simulation. These types of internal structure of heat exchangers are fishbone-shaped, accordion-shaped and scatter-shaped. The CFD simulation result indicates that the accordion-shaped internal structure of heat exchanger provide a better uniform temperature distribution and also it has higher interface temperature than the other two internal structures [31].

Yiping Wang et al. (2015) discussed about improvement in the temperature uniformity of heat exchanger in TEG system. For this purpose, optimization of fin distribution was studied to maximize the electrical power generated. A CFD model of heat exchanger was constructed to describe the effect of various fin distributions on the temperature uniformity. Four Factors: length of fins, spacing between the fins, angle of fins and thickness of the fins, were considered for the optimization of fin distribution. Optimization of these four factors improved the temperature

uniformity without too great pressure loss [32]. The process flow chart of optimization of the heat exchanger is shown in the figure 2.5.

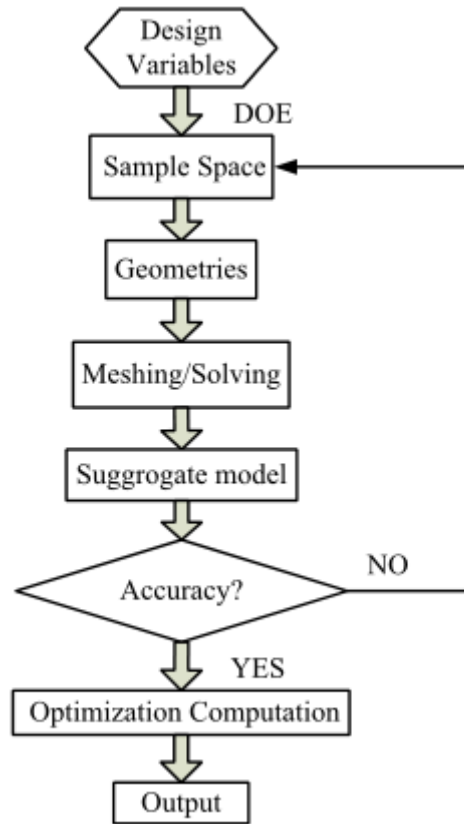


Figure 2.5 – Optimization Process flow chart for heat exchanger ^[32]

From the process flow chart of optimization process it is clear that at first, a database for different lengths of fins, angle of fins, thickness of fins and spacing between the fins, was prepared by proper DOE method. Secondly, the objective was to maintain the pressure loss and temperature uniformity, so numerical simulation was done for this purpose. Then Kriging method was used to set up a surrogate model according to the inputs and corresponding outputs. After that on the basis of the surrogate model, a multi-island genetic algorithm was used for the optimization purpose of fins distribution in heat exchanger. At last they construct a test bench to validate CFD result and also to test the temperature distribution of an empty heat exchanger with no internal fins. It was concluded that the temperature uniformity improved; the pressure loss reduced after optimization.

Hence the ultimate purpose of optimizing design of heat exchanger is to improve the fuel economy [32].

S. Bai et al. (2014) presented computational fluid dynamics (CFD) analysis of heat exchangers used in automotive TEG with six different designs. This CFD analysis was done to compare the pressure drop and heat transfer for all six different structures of heat exchanger. The vehicle used was 1.2 L petrol engine for which CFD models were developed for a typical driving cycle. The result showed that heat transfer was enhanced by 7 baffles in case of serial plate structure. The transferred heat was maximum in this case (1737 W). Serial plate structure also produced a maximum pressure drop of 9.7 kPa in a suburban cycle. The rate of heat transfer in serial plate structure was 35% greater than in the parallel plate structure and 26% greater than in the separate plate with holes. In case of heat exchanger with internal pipe structure the amount of heat released was 805 W which is 46.3% of the serial plate. Also in pipe structure a pressure drop of 6.5 kPa occurs which is 66.8% of the serial plate. For the condition of maximum power output, only the empty cavity structure and the inclined plate had pressure drops less than 80 kPa. The largest pressure drop exceeded 190 kPa. In such cases reliability and stability of engine is needed, so for this necessity the bypass mechanism with a differential pressure switch was used [33].

2.4 Possible new applications of TEG device

B. Orr et al. (2014) presented a combination of two promising technologies to recover waste heat of automobile exhaust. The useful two technologies for this purpose were thermoelectric cells (TECs) and heat pipes. In this work a bench type model was demonstrated which produced power by Thermoelectric cells using heat pipes. Exhaust gas of vehicle was used as heat source and heat pipes effectively dissipate heat from gases to hot side plate of TEG device. 8 thermoelectric cells were used and 6.03 W power produced when charging the battery. The produced power was not far off the maximum power produced in case of waste heat recovery system using thermoelectric device with variable resistor device. The heat to electrical conversion efficiency of whole system was 1.43% and the efficiency was measured as 2.31% which is approximately 1/9th of the Carnot efficiency. To increase system efficiency, thermoelectric material with higher figure of merit (ZT) can be used. At present time, availability of TE material with ZT equals to 1.2 is possible but even in this case the predicted efficiency would only raise to approximately 5%. The reason for low predicted efficiency is that each individual TE cell do not operate at its optimum voltage/current.

Moreover, the actual efficiency of system is lesser than predicted efficiency. The actual efficiency is $1/15^{\text{th}}$ of the Carnot cycle efficiency [34].

Now a days, thermoelectric device also are being utilized for conversion of solar heat into electricity. In a Solar photovoltaic cell, a considerable amount of solar energy is converted to waste heat due to thermalisation of high energy electrons and absorption of low energy photons which raise the temperature of PV cell. Although the efficiency of such type of TE device application is very low, it may be possible to use thermoelectric device with PV cells in future after further research and development in this field. Some of the previous works are described in following paragraphs.

Hashim et al. (2016) presented a model for geometry optimization of TE devices in a hybrid photovoltaic-thermoelectric system.

In this system, the operating temperature of PV cell reduces due to heat transfer into TEG and an additional amount of electricity generated due to established temperature difference across the TEG. Thus, solar energy is converted into thermal energy and then into electrical energy. 8 types of TE modules were used to analyze the dependency of maximum power output on module geometry. To simulate this, a MATLAB program was used. Simulation result showed that the power output of TEG increase up to a certain length. After that if the length of module is increased, power output of TEG decrease. So it is necessary to design a TE module with the optimal length which enables a TEG to operate at maximum power producing condition. Also larger area and large number of thermo-elements required to increase power output. Thus, power output of TE module increases at the expense of material consumption [35].

Bjork et al. (2015) discussed an analytical model for 4 different kinds of commercial PVs and a bismuth telluride TEG. The result did not show a viable option for combined PV and TEG system because the performance of PV decreases significantly with increasing temperature. The degradation of power output of PV system with temperature is much faster than the increase in power output of TEG. The low efficiency of thermoelectric modules is the main reason for this. Commercial PV cells such as Crystalline Si (c-Si), cadmium telluride (CdTe) and copper indium gallium selenide (CIGS); produced lower power for combined PV-TE system than the PV alone. Only for an amorphous Si (a-Si) cell, the performance of combined system slightly increases. So the combined PV-TE system could not be efficient and effective when it would be a sensor

application, i.e. PV cells produces power directly in daylight and TE device produces power in night due to established temperature difference [36].

Zhang et al. (2009) presented design and implementation method of a hybrid PV-TE energy source for hybrid electronic vehicle. The result showed that the proposed system can perform MPPT successfully. In this hybrid PV-TE system, an MPPT controller along with thermoelectric module, PV panel, power conditioning unit and a DSP controller was used. A 12v 24Ah battery was used as the load. The MPPT controller measures the currents and voltages of PV panel and TEG device. It also generates the switching signal to the conditioning circuit according to MPPT algorithm. Then according to Thevenin's theorem, the TEG as PV panel array can be represented by a voltage source with an internal resistor. The power conditioning unit can track the maximum power point of the energy source by turning its duty cycle of the pulse width modulating (PWM) switching signal to enable the input resistance $r_{in} = V_i/I_i$, equal to the internal resistor of energy source r_g [37].

2.5 Concluding remark from the Literature review

This literature review presented various models of exhaust based waste heat recovery system using thermoelectric generator which indicates that use of TEG in a waste heat recovery system, with proper heat transfer mechanism, can replace the traditional method of waste heat recovery and can also reduce the amount of fuel consumption in automobiles. Mostly previous researches has been focused on the making efficient TEG system. So some other technologies such as variable conductance heat pipes are open cell metal foam-filled heat exchanger, are used along with the use of TEG. The combined system of heat pipes or heat exchanger with TEG made the complete waste heat recovery unit efficient and effective.

Various designs of heat exchangers were also discussed to utilize the thermal energy of exhaust gases as much as possible. It can be concluded that heat exchangers with some internal design can enhance the heat transfer to the surface while hollow type of heat exchanger could not maximize the use of exhaust heat. The location of TEG installation is also very significance from the point of view of better efficiency. TEG should be located between catalytic converter and muffler. In some of the previous researches, design of cold side heat exchanger or heat sink was optimized by thermal analysis of different type of heat sink. Use of rectangular-pillar type heat

sink can enhance the performance of TEG device, higher thermal conductivity of heat transfer is also achieved.

Finally, some new trends of TEG application were also reviewed in which performance analysis of combined solar photovoltaic and thermoelectric generator system was demonstrated. The feasibility of combined PV and TEG system is not efficient, so geometry optimization was also done to make hybrid PV-TE system efficient and cost-effective. But the hybrid photovoltaic and thermoelectric system is still not successful as an electricity generator. A combined technology of heat pumps and TEG was also discussed which would be a better waste heat recovery option, if it is used in optimum conditions.

2.6 Gaps in the Literature Review

Though this literature review presents various methods to increase conversion efficiency, there is a few literature regarding the module mismatching problem. When thermoelectric modules are connected in series or parallel configuration, they operate with less efficiency than in case when modules operate separately. So we present mathematical analysis of module mismatching in this thesis which could help in analyzing the optimum parameters. By analyzing optimum parameters such as optimum voltage, optimum current, we can reduce the power loss due to mismatching.

In this review, DC –DC converter are just introduced but their mathematical analysis is not done. The use of DC-DC converter is very significance for better conversion efficiency. So, this thesis also presents detailed analysis of DC-DC converter which can boost up the power produced by the TEG.

Chapter 3

Mathematical Analysis of AETEG

As we have discussed in the literature review that thermoelectric generator works on the principle of Seebeck effect and there is both conductive and convective medium of heat transfer in complete assembly of AETEG system. So there is a need to analyze the whole system mathematically before the experiment procedure.

3.1 Power and Efficiency calculation

Generally, thermoelectric generators are made of several thermoelectric modules (TEMs) and TEM consists couple of assembled P-type and N-type thermoelectric materials. In this mathematical analysis we start our study by assuming a TEG made up of several TE modules. These TE modules are positioned in 'r' rows and there are 'm' thermocouples of P-type and N-type thermoelectric materials in each row. So total number of couples (n) will be equals to multiplication of number of rows and number of couples in single row ($n = r * m$). These couples are connected electrically in series and thermally in parallel.

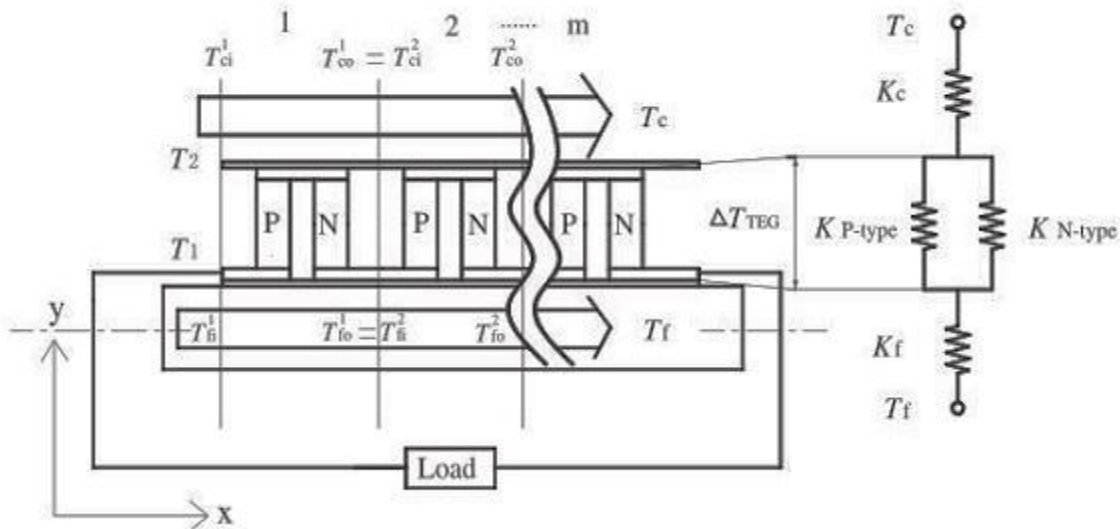


Figure 3.1- Schematic model of thermoelectric generator containing 'n' number of modules ^[38]

From figure it is clear that TEG operate between hot source and cold source, In this analysis exhaust gas is taken as hot source of TEG device and ambient air is taken as cold source. To analyze the complete TEG device, we can divide it into many small units of TE modules and study the each distinct unit of thermoelectric module first. So in such condition the outlet temperature of the former module will be considered as the inlet temperature for the next module unit. Thus calculation can be made easy.

In this analysis, there are some assumptions

- i. Thermal and flow properties of TE module should not vary with time i.e. steady state condition.
- ii. The exhaust gas which entrapped in the gaps of device should be ignored.
- iii. As the thickness of copper plate is enough small, thermal resistance of copper plate should be assumed negligible (copper plate is used as an interface between exhaust gas and module).
- iv. The thermal conductivity (k) of the thermoelectric material should not be the function of temperature, i.e. it should be constant.

Now, in this analysis we use the Seebeck effect of thermoelectricity and Fourier's law of heat conduction. Based on Fourier's law, heat is transferred from exhaust gases to the hot side of TEG device and on difference of temperature across the TEG created due to this induce voltage on the basis of Seebeck effect.

Let's consider the j^{th} unit in the theoretical model presented in figure. We can determine the equation of energy consumption for both P and N type material in j^{th} unit.

The rate of heat transfer for P material at the hot side (Q_{p1}) for j^{th} unit can be expressed as:-

$$Q_{p1} = S_{p1} \cdot I \cdot T_1 - k_p \cdot A_p \cdot \frac{(dT)}{(dy)} \quad (3.1)$$

The rate of heat transfer for N material for j^{th} unit:

$$Q_{n_1} = S_{n_1} \cdot I \cdot T_1 - k_n \cdot A_n \cdot \frac{(dT)}{(dy)} \quad (3.2)$$

Here, S_{p1} and S_{n1} are the Seebeck coefficient of P and N-type material at hot side, I is the output current, T_1 is the mean temperature of the hot side per unit length in axial direction, k_p and k_n are thermal conductivities of P and N-type materials respectively. A_p and A_n are the cross sectional area and (dT/dy) is the temperature gradient in perpendicular direction (y-direction).

Similarly we can write equations for rate of heat transfer (Q_{p2} and Q_{n2}) at the cold side of the j^{th} unit of TE module:

$$Q_{p_2} = S_{p_2} \cdot I \cdot T_2 - k_p \cdot A_p \cdot \frac{(dT)}{(dy)} \quad (3.3)$$

$$Q_{n_2} = S_{n_2} \cdot I \cdot T_2 - k_n \cdot A_n \cdot \frac{(dT)}{(dy)} \quad (3.4)$$

As we know that due to thermoelectricity concept, the waste heat energy of exhaust gases convert into equivalent electrical power if we consider our assumptions of steady state and no thermal losses. Then the correlation between temperature gradient with joule heat per unit height of P-N couple is expressed as:

$$-k_p \cdot A_p \cdot \frac{(d^2T)}{(dy^2)} = I^2 \cdot \frac{\rho_p}{A_p} \quad (3.5)$$

$$-k_n \cdot A_n \cdot \frac{(d^2T)}{(dy^2)} = I^2 \cdot \frac{\rho_n}{A_n} \quad (3.6)$$

Here ρ is termed as the electrical resistivity of the material. In this case if we use boundary conditions in equations (3.5) and (3.6), we can derive important formulae.

Boundary conditions are:

- At $y = 0$; Temperature of module (P-side temp), $T = T_1$
- At $y = H$; Temperature of module (N-side temp), $T = T_2$.

Using boundary condition and integrating equations (5) and (6), we get:

$$k_p \cdot A_p \cdot \frac{(dT)}{(dy)} = I^2 \cdot \rho_p \cdot \frac{\left(y - \frac{H}{2}\right)}{A_p} + k_p \cdot A_p \cdot \frac{(T_2 - T_1)}{H} \quad (3.7)$$

$$k_n \cdot A_n \cdot \frac{(dT)}{(dy)} = I^2 \cdot \rho_n \cdot \frac{\left(y - \frac{H}{2}\right)}{A_n} + k_n \cdot A_n \cdot \frac{(T_2 - T_1)}{H} \quad (3.8)$$

Now if we use equation (3.7) in equations (3.1) & (3.2) and equation (3.8) with equations (3.3) & (3.4), we can obtain the unit heat absorbed Q_1 (at $y=0$) and unit heat released Q_2 in j^{th} unit of TE module.

So,

$$Q_1 = (S_{p1} - S_{n1}) \cdot I \cdot T_1 + K(T_1 - T_2) - I^2 \frac{R_1}{2} \quad (3.9)$$

And

$$Q_2 = (S_{p2} - S_{n2}) \cdot I \cdot T_2 + K(T_1 - T_2) - I^2 \frac{R_1}{2} \quad (3.10)$$

Where,

$$K = k_p \cdot \frac{A_p}{H} + k_n \cdot \frac{A_n}{H}$$

$$R_1 = \rho_p \cdot \frac{H}{A_p} + \rho_n \cdot \frac{H}{A_n}$$

Hence 'K' is equivalent thermal conductance of the j^{th} unit and R_1 is the electrical resistance. The reciprocal of thermal conductance ($1/K$) gives the value of thermal resistance in the unit module.

It is also to be noted that from Newton's law of cooling for heat convection, the heat absorbed at the hot side (Q_1) of module from exhaust gas will be equal to:

$$Q_1 = h_1 \cdot A_1 \left(\frac{(t_{fi} + t_{fo})}{2} - T_1 \right) \quad (3.11)$$

Similarly the heat rejected at the cold side to the ambient air;

$$Q_2 = h_2 \cdot A_2 \left(T_2 - \frac{(t_{c_o} + t_{c_i})}{2} \right) \quad (3.12)$$

Here h_1 & h_2 are the convective heat transfer coefficient for hot side and cold side respectively and A_1 & A_2 are the heat exchanger areas of hot and cold side respectively for the j^{th} unit.

So equating the equations (9) and (11) and then equating (10) and (12) jointly, we obtain the following expressions:

$$h_1 \cdot A_1 \left(\frac{(t_{f_i} + t_{f_o})}{2} - T_1 \right) = (S_{p_1} - S_{n_1}) \cdot I \cdot T_1 + K(T_1 - T_2) - I^2 \cdot \frac{R_1}{2} \quad (3.13)$$

$$h_2 \cdot A_2 \left(T_2 - \frac{(t_{c_o} + t_{c_i})}{2} \right) = (S_{p_2} - S_{n_2}) \cdot I \cdot T_2 + K(T_1 - T_2) - I^2 \cdot \frac{R_1}{2} \quad (3.14)$$

As the temperature of exhaust gas changed when it transfer some of its heat to TEG system, in that case form heat balancing equation:

$$Q_1 = h_1 \cdot A_1 \left(\frac{(t_{f_i} + t_{f_o})}{2} - T_1 \right) = C_f \cdot M_f \cdot \frac{(t_{f_i} - t_{f_o})}{r} \quad (3.15)$$

Here t_{f_i} & t_{f_o} are temperature of exhaust gas before and after heat transfer to TEG device respectively.

Similarly for cold side,

$$Q_2 = h_2 \cdot A_2 \left(T_2 - \frac{(t_{c_o} + t_{c_i})}{2} \right) = C_c \cdot M_c \cdot \frac{(t_{c_o} - t_{c_i})}{r} \quad (3.16)$$

Here t_{c_i} and t_{c_o} are temperatures of cooling air at inlet and outlet of TEG system, 'C' indicates the specific heat capacity of the TE material and 'M' indicates mass flow rate.

In this analysis, it was assumed that all units are independent, so we divide the total external load resistance into 'n' portions and each P-N couple has the same value of external load. In this case if 'R₂' is external load resistance then,

$$\text{Load allocated to each couple} = R_2/n$$

Where n is the number of PN couple. Now we can express the electrical current in jth unit of circuit as follows:

$$I = \frac{U}{\left(\frac{R_2}{n} + R_1\right)} = \frac{\left((S_{p_1} - S_{n_1}) \cdot T_1 - (S_{p_2} - S_{n_2}) \cdot T_2\right)}{\left(\frac{R_2}{n} + R_1\right)} \quad (3.17)$$

From equation (3.17) it is observed that electrical current I is a function of hot side temperature (T₁) and cold side temperature (T₂).

Equation (3.13) to (3.17) possess some important mathematical correlations which are solved with four unknown temperature t_{fo}, T₁, T₂ & t_{co}. Two temperature t_{fi} and t_{ci} should be known as they are initial values or value of the last unit module. So to analyze the total current and power we apply iteration based calculation.

We have also assumed that electrical current is same in all PN couple is same but practically it is rarely possible, so we take summation of all the current-values found from individual modules and then we take an average. The averaged value of current can be used in calculation of power.

The output power for each unit of P-N couple can be expressed as:

$$P = (S_{p_1} - S_{n_1}) \cdot I \cdot T_1 - (S_{p_2} - S_{n_2}) \cdot I \cdot T_2 - I^2 \cdot R_1 \quad (3.18)$$

And we can also calculate the efficiency of single module as:

$$\eta = \frac{P}{Q_1} \quad (3.19)$$

For overall conversion efficiency and power output we take summation of all power values found from individual modules:

$$P_{(overall)} = r \cdot \sum_{j=1}^m P^j \quad (3.20)$$

$$\eta_{(overall)} = \frac{\sum_{i=1}^m P^j}{\sum_{j=1}^m Q_i} \quad (3.21)$$

Where P^j is the power generated from j^{th} TE module and Q_i is the total heat transferred by exhaust gases to the hot side of thermoelectric module. In the formula of overall power output 'r' is the number of rows as described earlier and m is the number of TE modules in single row.

3.2 Module mismatch

It is seen that there are discrepancies among predicted power output and actual power output when thermoelectric modules are connected electrically in series or in parallel. This phenomenon of differences between predicted and actual power output is known as module mismatch. This case generally occurs when parameters of connected thermoelectric modules differ.

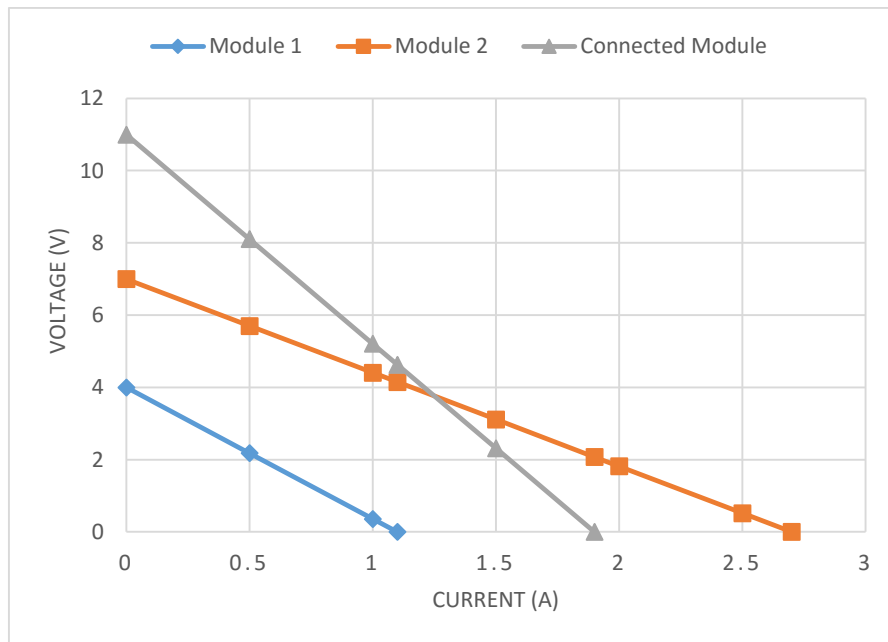


Figure 3.2- Example of a voltage-current plot for two modules

As from the graph, it is clear that voltage-current parameters are different for module 1 and module 2 because of different seebeck coefficient and different internal electrical resistance. When these module are connected, the voltage-current curve changes. The V-I curve for connected module refers to new properties of module.

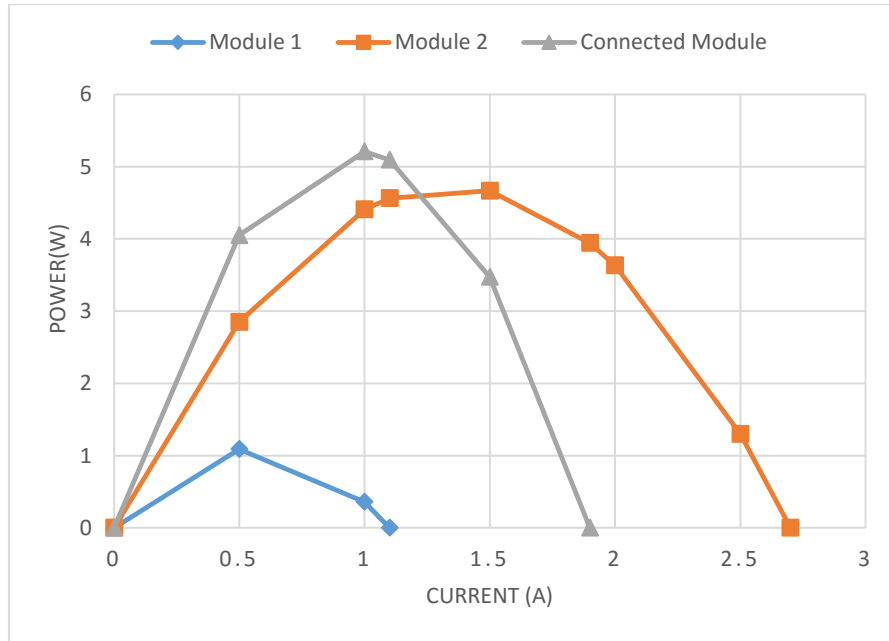


Figure 3.3- Example of a power-current curve for module 1, module 2 and the connected module

Similarly figure indicates the different power curves for two modules (Module 1 and Module 2) and it also show the power-current for connected modules. If we add the value of maximum power in each of two cases, it will be more that the peak power indicated by the power-current curve of electrically connected module.

Mathematically we can analyze the problem of module mismatch as follow-

The power generated by unit TE module from equation (3.18)

$$P = S \cdot I(T_1 - T_2) - I^2 \cdot R \quad (3.22)$$

Here, S in the Seebeck coefficient of TE couple. For maximum value of power output we use concept of maxima-minima. Hence,

$$\frac{(dP)}{(dI)} = 0 \quad (3.23)$$

On solving above eq. (3.23) we get,

$$S(T_1 - T_2) = 2I \cdot R \quad (3.24)$$

Or

$$I = S \frac{(T_1 - T_2)}{2} R \quad (3.25)$$

By putting this value of current (I) in equation (3.22), we can derive formula for maximum power produced by unit TE module,

$$\mathbf{P_{max}} = \mathbf{S^2 \frac{(T_1 - T_2)^2}{4R}} \quad (3.26)$$

If two modules are added electrically and P_1 and P_2 are the individual output power for module 1 and module 2 respectively. Then from above equation (3.26),

$$\begin{aligned} P_1 + P_2 &= (S_1)^2 \frac{(T_1 - T_2)^2}{4} R_1 + (S_2)^2 \frac{(T_1 - T_2)^2}{4} R_2 \\ &= (S_1)^2 \frac{(\Delta T)^2}{(4R_1)} \frac{((C_1)^2 + C_2)}{C_2} \end{aligned} \quad (3.27)$$

Here, C_1 and C_2 are two ratios used to simplify equations, C_1 is the ratio of seebeck coefficient of second module (S_2) to that of first module (S_1). C_2 is the ratio of electrical resistance of second module (R_2) to electrical resistance of first module (R_1)

$$C_1 = \frac{S_2}{S_1} \quad (3.28)$$

$$C_2 = \frac{R_2}{R_1} \quad (3.29)$$

Equations (3.28) and (3.29) can be used to set up mismatch ratio when modules are connected electrically.

Now with the help of following governing equations we can estimate the power ratio in case of series connection and parallel connection of thermoelectric modules. Governing equation are:

$$V = S(T_1 - T_2) - IR \quad (3.30)$$

$$I = S \frac{(T_1 - T_2)}{R} - \frac{V}{R} \quad (3.31)$$

Module mismatching for Series connection –

In series connection of modules, the current remains constant and voltage increases. Let us consider there are two modules connected in series and voltage induce through them are V_1 and V_2 respectively. Then from governing equation (3.30),

$$V_1 = S_1(T_1 - T_2) - I_s R_1 \quad (3.32)$$

$$V_2 = S_2(T_1 - T_2) - I_s R_2 \quad (3.33)$$

$$V_s = V_1 + V_2$$

$$\Rightarrow V_s = \Delta T(S_1 + S_2) - I_s(R_1 + R_2) \quad (3.34)$$

We can also calculate current passing through serially connected module (I_s) by rearranging equation (34),

$$I_s = \Delta T \frac{(S_1 + S_2)}{(R_1 + R_2)} - \frac{V_s}{(R_1 + R_2)} \quad (3.35)$$

From here we can calculate the equivalent maximum voltage (which is half of open circuit voltage) and equivalent maximum current (which is half of the short circuit current) in case of series-connection of module 1 and module 2,

$$V_{s_{\max}} = \frac{V_o}{2} = \Delta T \frac{(S_1 + S_2)}{2} \quad (3.36)$$

$$I_{s_{\max}} = \frac{I_0}{2} = \left(\frac{\Delta T}{2} \right) \cdot \frac{(S_1 + S_2)}{(R_1 + R_2)} \quad (3.37)$$

Here V_0 is the open circuit voltage and I_0 is the short circuit current. The value of maximum power will be multiplication of maximum current and maximum voltage value of series-connected module.

$$P_{s_{\max}} = V_{s_{\max}} \cdot I_{s_{\max}} = (\Delta T)^2 \frac{(S_1 + S_2)^2}{(4(R_1 + R_2))} \quad (3.38)$$

Using the values of C_1 and C_2 in above equation (3.38) we will get the value of peak power in terms of C_1 and C_2 . Hence,

$$P_{s_{\max}} = \frac{(S_1 \cdot \Delta T)^2}{(4R_1)} \cdot \frac{(1 + C_1)^2}{(1 + C_2)} \quad (3.39)$$

Now we can compare the maximum power produced by the module 1 and module 2 connected in series. This value is lower than the sum of individual peak powers produced by both module. If we divide equation (3.39) by equation (3.27) the ratio will indicate the level of mismatch.

$$\frac{(P_{s_{\max}})}{(P_1 + P_2)} = C_2 \frac{(1 + C_1)^2}{((C_2 + C_1)^2 \cdot (1 + C_2))} \quad (3.40)$$

Module mismatching for parallel connection-

Parallel connection thermoelectric modules provide that there is same voltage induced across all the TE modules but the current differs. From governing equation (31), electrical current passing through module 1 and module 2 separately are:-

$$I_1 = S_1 \frac{(T_1 - T_2)}{R_1} - \frac{V_p}{R_1} \quad (3.41)$$

$$I_2 = S_2 \frac{(T_1 - T_2)}{R_2} - \frac{V_p}{R_2} \quad (3.42)$$

The electrical current passing through full TE unit will be the sum of currents in individual module. Hence the current passing through the parallel-connected modules (I_p)-

$$I_p = I_1 + I_2$$

$$I_p = \Delta T(S_1/R_1 + S_2/R_2) - (1/R_1 + 1/R_2)V_p \quad (3.43)$$

Rearranging the equation (3.43) for the value of V_p ,

$$V_p = \Delta T(S_1/R_1 + S_2/R_2)(1/R_1 + 1/R_2)^{-1} - (1/R_1 + 1/R_2)^{-1} \cdot I_p \quad (3.44)$$

Now, to find maximum value of current and voltage induced in parallel-connected TE module, we will half the value of short circuit current (I_0) and open circuit voltage (V_o).

$$I_{p_{\max}} = \frac{I_0}{2} = \left(\frac{\Delta T}{2}\right) \left(\frac{S_1}{R_1} + \frac{S_2}{R_2}\right) \quad (3.45)$$

$$V_{p_{\max}} = \frac{V_o}{2} = \left(\frac{\Delta T}{2}\right) \left(\frac{S_1}{R_1} + \frac{S_2}{R_2}\right) \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \quad (3.46)$$

To calculate peak power output, we can multiply maximum current and maximum voltage in parallel-connected TE module.

$$\mathbf{P}_{p_{\max}} = \mathbf{I}_{p_{\max}} \cdot \mathbf{V}_{p_{\max}}$$

$$\mathbf{P}_{p_{\max}} = \left(\frac{\Delta T}{2}\right)^2 \cdot \left(\frac{S_1}{R_1} + \frac{S_2}{R_2}\right)^2 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \quad (3.47)$$

Using the values of C_1 and C_2 , we can calculate the peak power in term of C_1 and C_2 –

$$\mathbf{P}_{p_{\max}} = (\Delta T)^2 \cdot \frac{(S_1)^2}{4R_1} \cdot \frac{(C_1 + C_2)^2}{(C_2(C_2 + 1))} \quad (3.48)$$

Now we can compare the maximum power produced by parallel-connected modules with the sum of power produced by individual modules. Parallel-connected modules have lesser value of power output. If we divide equation (3.48) by equation (3.27) the ratio will indicate the level of mismatch.

$$\frac{P_{p_{\max}}}{(P_1 + P_2)} = \frac{(C_1 + C_2)^2}{(C_2 + (C_1)^2)(1 + C_2)} \quad (3.49)$$

Now we will discuss some special cases for this power ratio which are as following-

- i. When $C_1 = C_2$,

In this case, the power ratio for series connection,

$$\frac{P_{s_{\max}}}{(P_1 + P_2)} = C \frac{(1 + C)^2}{(C + C^2)(1 + C)} = 1 \quad (3.50)$$

For parallel connection,

$$\frac{P_{p_{\max}}}{(P_1 + P_2)} = \frac{(C + C)^2}{(C + C^2)(1 + C)} = 4 \frac{C}{(C + 1)^2} \quad (3.51)$$

So the condition when ratio C_1 and C_2 becomes equal, the problem of mismatch is incapacitated in series-connection of modules. But for parallel connection there is still a mismatching even the value of the parameters ratios are same.

- ii. When the seebeck coefficient of both the module are equal ($S_1 = S_2$) -

In this case the value of ratio constant C_1 will equal to 1. Putting the value $C_1=1$ in equation of power ratios (eq. 40 and eq. 49), we get

For series connection,

$$\frac{P_{s_{\max}}}{(P_1 + P_2)} = C_2 \frac{(1 + 1)^2}{(C_2 + 1^2)(1 + C_2)} = 4 \frac{C_2}{(C_2 + 1)^2} \quad (3.52)$$

For parallel connection,

$$\frac{P_{p_{\max}}}{(P_1 + P_2)} = \frac{(1 + C_2)^2}{(C_2 + 1^2)(1 + C_2)} = 1 \quad (3.53)$$

Hence for the second case, parallel connection overcome the problem of mismatch and there is similar mismatching in series-connection as in the parallel connection for the first case. From the second case it can be concluded that if the value of seebeck coefficient (S) is same for all TE module, the performance of TEG system will not be affected by mismatching in parallel configuration of connection.

iii. When the electrical resistance is same for both module-

In this case the value of C_2 will be equals to 1. This case gives same value of power ratio for both series and parallel type connection of modules.

$$\frac{P_{\max}}{(P_1 + P_2)} = \frac{(C_1 + 1)^2}{2(1 + C_1^2)} \quad (3.54)$$

From the equation (54) it is clear that when both modules have same electrical resistance, the value of power ratio will never be less than 0.5.

3.3 Analysis of Heat dissipation through fins in surroundings

In an automobile exhaust based TEG, the heat transfer from exhaust gas to the hot side of TEG is by convection. The heat transfer through the hot side plate is by conduction and then fins are used to dissipate heat of heat sink to surrounding by convection method of heat transfer.

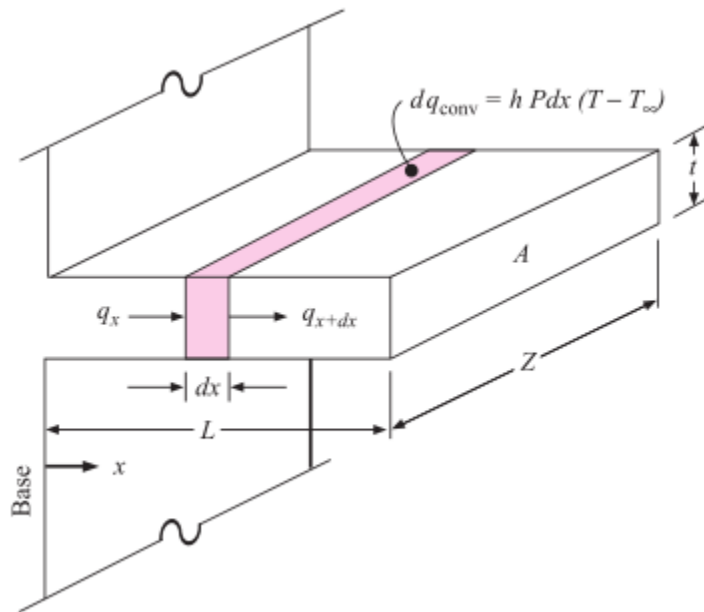


Figure 3.4- heat transfer through rectangular fin ^[39]

Let us consider a fin exposed to surrounding at temperature T_∞ . Practically, heat transfer should be analyzed in 3-D, but for ease of calculation we assume one dimensional heat transfer through fin. The base temperature of fin is T_0 .

From energy conservation law,

$$\text{Inlet energy at left face} = \text{outlet energy at right face} + \text{energy lost by convection} \quad (3.55)$$

$$\begin{aligned} \text{Energy in left face} &= q_x = -kA \frac{dT}{dx} \\ \text{Energy out right face} &= q_{x+dx} = -kA \left. \frac{dT}{dx} \right]_{x+dx} \\ &= -kA \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} dx \right) \\ \text{Energy lost by convection} &= hP dx (T - T_\infty) \end{aligned}$$

Putting all the values in equation (3.55) we get,

$$-kA \cdot \frac{dT}{dx} = -kA \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} \cdot dx \right) + h(P \cdot dx)(T - T_\infty)$$

Further simplifying above equation leads to-

$$\frac{d^2T}{dx^2} - \frac{hP}{kA} (T - T_\infty) = 0 \quad (3.56)$$

Take $T - T_\infty = \theta$; then eq. (3.56) becomes-

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA} \theta = 0 \quad (3.57)$$

Let $m^2 = hP/kA$; then solution of equation (3.57) becomes,

$$\theta = C_1 \cdot \exp(-mx) + C_2 \cdot \exp(mx) \quad (3.58)$$

Boundary conditions are:

- i. $\theta = \theta_0 = T_0 - T_\infty$; at $x=0$

- ii. Second boundary condition in this case is convection at the fin tip.

Solving equation (3.58) with boundary condition we get,

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL} \quad (3.59)$$

And we can get heat dissipated through fin by Fourier's law,

$$q = -kA \left(\frac{dT}{dx} \right)_{x=0}$$

$$q = \sqrt{hPkA} (T_0 - T_{\infty}) \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL} \quad (3.60)$$

Thus heat dissipated through fins in surrounding can be calculated from equation (3.60).

In this chapter, we have analyzed the complete AETEG system mathematically. This analysis is very significant for calculating various parameters in experiment such as heat transfer coefficient, power produced by TEG, heat to electrical conversion efficiency of the system, thermal conductance of the system etc. The module mismatching concept presented in this chapter is also significance in estimating the value of optimum voltage and current.

Chapter 4

Modelling of automobile exhaust based TEG

In this chapter we will discuss modeling of AETEG system in two sections. In the first section we present the experimental set up with single thermoelectric module prototype. In this project the complete experimental work is done with this TEG prototype of single module. While in the second section of this chapter we proposed a TEG design with 8 thermoelectric modules which are connected electrically in series and thermally in parallel.

4.1 Demonstration of Automotive waste recovery using single thermoelectric module

In this project we have experimentally demonstrated a TEG which consist of single TE module. The experimental setup and procedure are described below:-

4.1.1 Experimental Set-up

In this project, we made an automotive waste heat recovery device using a TE module of bismuth-telluride (Bi_2Te_3). We performed our experiment on a single-cylinder, four stroke engine of Hero Honda CD Dawn. The engine specification are tabulated in following table 4.1 -

Table 4.1 – Specification of vehicle/engine used in automotive waste heat recovery system^[40]

Model	Hero Honda CD dawn
Engine Type	Single cylinder, four stroke
Compression ratio	9.0:1
Maximum Power	7.80 HP (5.7 kW) @ 7500 rpm
Maximum Torque	8.04 Nm (0.8 Kgf-m) @ 4500 rpm
Bore	50 mm
Stroke	49.5 mm
Ignition	DC-CDI

The location of TEG installation was chosen between catalytic converter and muffler. Approximately 40% of fuel energy is wasted in exhaust gases. So this experiment was just to demonstrate the concept of thermoelectricity using exhaust gas of vehicle as heat source. On the

basis of this prototype, we can develop the TEG containing a large number of modules to eliminate the need of alternator and also to reduce fuel consumption in vehicles. As shown in figure 4.1, we use following parts to make this prototype with single thermoelectric module:

a. Module - In this experiment, we used a bismuth telluride (Bi_2Te_3) thermoelectric module because it performs better at comparatively low temperature difference. Other popular thermoelectric material such as lead telluride (PbTe) gives better performance at higher temperature difference through the module.

- Seebeck coefficient for bismuth telluride = $287 \mu\text{V/K}$
- Maximum value of figure of merit (Z) for bismuth telluride = 0.003 K^{-1}

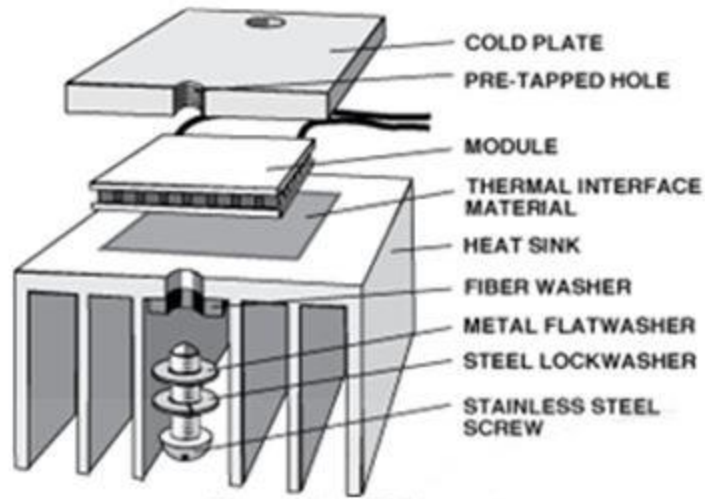


Figure 4.1- Basic structure of thermoelectric generator device [41]



Figure 4.2- Assembled TEG with single thermoelectric module

- b. Aluminum base plate** – It is used as the interface material between thermoelectric module and exhaust pipe of vehicle. It prevents module from excessive temperature of exhaust gas and allow temperature within the range for material of TE module.
- c. Thermal grease** - It is also named as thermal gel, thermal paste or heat sink paste. It is a fluid substance with viscosity which is applied on the microscopic air gaps present at thermal interfaces. Use of thermal grease on thermal interface increases thermal conductivity of the interface material and therefore, maintain the better rate of heat transfer. Thermal grease is also used to aid heat dissipation via a heat sink in many electronics components.
- d. Insulating material** – Insulating material such as asbestos is used to fill the gap between aluminum base plate and heat sink. The gap is generally created due to less cross sectional area of TEM, so to avoid heat loss to surrounding, asbestos material is used as thermal insulator.
- e. Heat Sink** - Heat sink is used to dissipate the rejected heat by means of rectangular fins. In our project we use ambient air as the cold side, so we used an open type heat sink with wire EDM machined rectangular fins. When coolant fluid is used at cold side of TEG system, the heat sink shape is like a divergent-convergent box heat exchanger with rectangular fins.
- f. Assembly element** – To locate TEG device on exhaust pipe we need some assembly elements like U-clips, nut and bolts. The material of these assembly material should be chosen in an appropriate manner that it can withstand against the very high temperature of exhaust pipe (around 500°C). In our project, we use assembly elements made of steel.



Figure 4.3 – Experimental setup for automotive waste heat recovery



Figure 4.4- TEG located on exhaust pipe of vehicle

4.1.2 Method of Experiment

As discussed in the literature review that TEG should be located between catalytic converter and muffler. This is the best compatible condition of the TEG installation. After location the thermoelectric device, we start the vehicle and accelerate it. Due to acceleration the amount of heat energy of exhaust gas is increased. This excessive heat energy heats up the silencer and exhaust pipe of the vehicle to a very high temperature in the range of 400°C to 500°C. At the cold side ambient air is used as cold fluid whose temperature is very less (~ 25-35°C) in comparison of exhaust-side temperature. So a

temperature gradient is established, the thermoelectric module placed between the hot silencer surface and heat sink, generate voltage due to this temperature difference. The voltage generated due to the Seebeck effect and it can be measured with the help of voltmeter or digital multimeter. The amount of voltage induced will be more as much as the temperature difference. We take our observation reading in the static condition (neutral condition) of vehicle. If vehicle is running, the air will circulate through the fins at higher speed. This will help in maintaining lower temperature of the heat sink side. Hence temperature difference will be more for moving vehicle, so the voltage induced will be more. We can use this induce voltage to charge low power batteries. Thus, with the help of thermoelectric module waste heat energy of exhaust gases directly convert into electricity.

4.1.3 Observations

For observations readings of hot side and cold side plate, we used temperature sensors with digital display. Voltage and current readings were taken by digital multi-meter. The experiment was performed as described in the method. The readings are shown in the following tables:-

Table 4.2 – Experimental readings of voltage induced from single TE module at different temperature gradient.

Hot Side Temp., T_h (°C)	Cold Side Temp., T_c (°C)	Temp. Difference (ΔT) (°C)	Voltage Induced from single TE module (volt)	Time (sec)
32.8	32.8	0	0	0
45.3	32.8	12.5	0.29	50
57.4	32.9	24.5	0.63	100
69.1	33.0	36.1	1.38	150
78.8	33.2	45.6	1.82	200
90.0	33.3	56.7	2.13	250
98.5	33.4	65.1	2.39	300
105.3	33.5	71.8	3.02	350
120.1	33.6	86.5	3.10	400
135.6	33.9	101.7	3.15	450
150.3	34.2	116.1	3.19	500

From the table, it is obvious that with increasing temperature difference, induced voltage also increases. But after a certain temperature difference voltage does not increase too much due to limitation of using single thermoelectric module. For higher voltage, there is need to link two or more modules. We can represent voltage-temperature gradient relationship graphically as follows:-

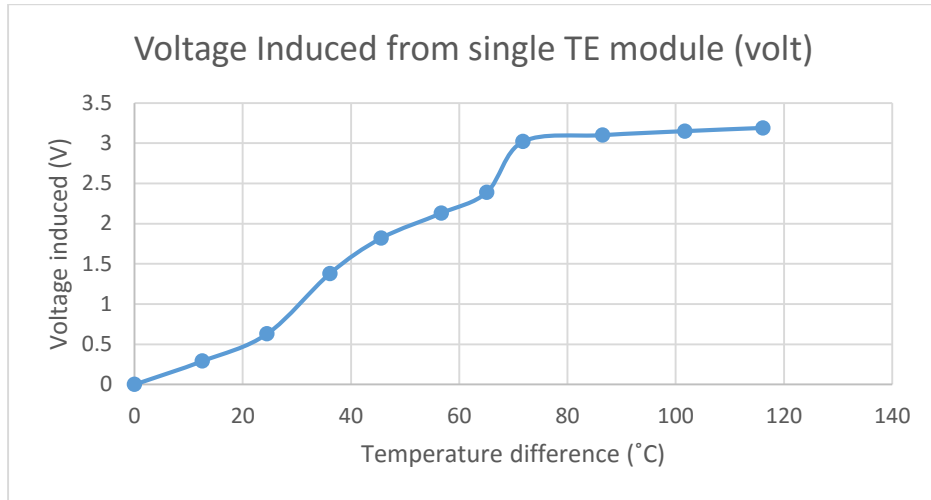


Figure 4.5 – Voltage induced from single TEG at different temperature difference

We can also calculate the power produced from the single TE module in automotive waste heat recovery system. For power calculation we need the current which is measured with the digital multimeter in milli-ampere. After measuring current we can find the power generated by single TE module from following formula:-

$$P = V \cdot I$$

Where V is in volt and I is in amperes. Power is measured in watt.

Table 4.3: Readings of current and power produced from single TE module at different temperature gradient

ΔT (°C)	Voltage induced (V)	Current (mA)	Power generated (Watt)
0	0	0	0
12.5	0.29	134	0.03886
24.5	0.63	227	0.14301
36.1	1.38	352	0.48576
45.6	1.82	439	0.79898
56.7	2.13	568	1.20984

65.1	2.39	631	1.50809
71.8	3.02	756	2.28312
86.5	3.10	820	2.542
101.7	3.15	863	2.718
116.1	3.19	923	2.944

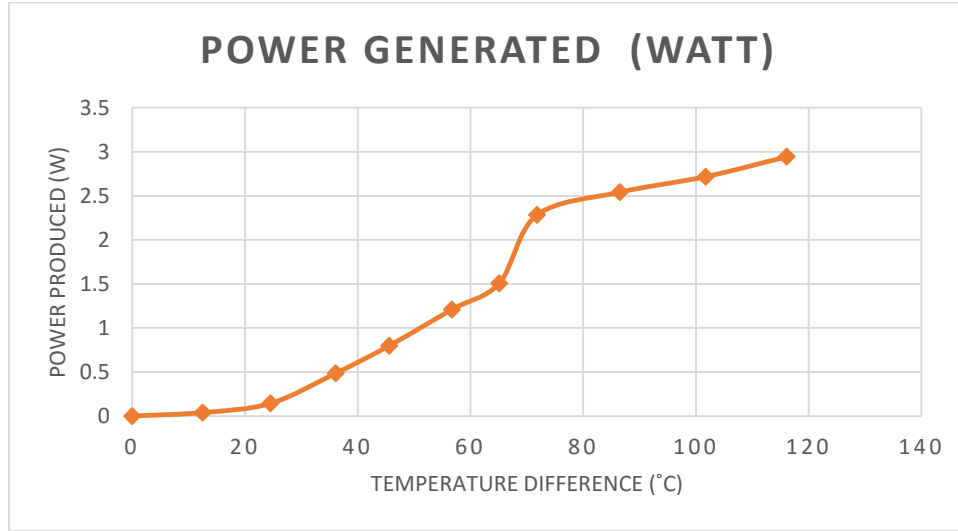


Figure 4.6 – Graphical representation of power generation by single module on different temperature gradient

4.1.4 Mathematical validation of the demonstrated prototype with single TE module

We can mathematically validate the prototype of thermoelectric generator with single module. As described before, the system was located at the exhaust pipe between catalytic converter and muffler. The temperature of exhaust gas in this region is around 350°C to 450°C. We can take exhaust temperature as 400°C in calculation.

The heat energy is transferred from exhaust gases to surface of exhaust pipe and outer side of aluminum base plate by the convection mode of heat transfer mechanism. Hence from Newton’s law of heat convection,

$$Q_1 = h \cdot A_s \cdot (T_e - T_\infty)$$

Here, h = heat transfer coefficient in $W/m^2-K = 10 W/m^2-K$

A_s = Area of surface to which heat is transferred = $\pi dl = 3.14 \cdot 5.08 \cdot 10^{-2} \cdot 0.15 = 0.0239 m^2$

T_e = Exhaust gas temperature = 400°C

T_∞ = Temperature of surface (base plate) which is being heated.

As temperature of base plate and exhaust pipe increases with time after the vehicles starts. So for mathematical validation we can take any one value of temperature from the observed temperature readings in the experiment. For example, we take $T_\infty = 105.3^\circ\text{C}$.

Then, $Q_1 = 10 * 0.0239 * (400 - 105.3) = 70.433 \text{ W}$

We can also calculate the temperature at the hot side of temperature which will be equal to the temperature at the inner face of aluminum base plate assuming no thermal contact loss.

From Fourier's law of heat conduction,

$$Q_1 = -k \cdot A \cdot \frac{dT}{dx}$$

$$Q_1 = k \cdot A \cdot \frac{(T_\infty - T_1)}{x}$$

Here k is the thermal conductivity of aluminum base plate (237 W/m-K), A is cross-sectional area of the plate (0.15*0.09 m²) and x is the thickness of the base plate (4 mm). Putting the values we get,

$$\Rightarrow 70.433 = 237 \times 0.15 \times 0.09 \frac{(105.3 - T_1)}{0.004}$$

On solving we get, $105.3 - T_1 = 0.088$

Or $T_1 = 105.22^\circ\text{C}$

We can see that the value of temperature at hot side of module does not differ too much with the value of temperature at outer face of base plate. The reason for this minute difference is the better thermal conductivity of aluminum base plate.

Calculation of theoretical efficiency

Theoretical efficiency of a thermoelectric module can be calculated from the following formula [42]:

$$\eta_{th} = \frac{\Delta T}{T_h} \cdot \frac{\sqrt{(1 + ZT_m)} - 1}{\sqrt{(1 + ZT_m)} + \frac{T_c}{T_h}} \quad (4.1)$$

Where ΔT = temperature difference between both sides of module = $105.3 - 33.5 = 71.8$ K

T_h = Hot side temperature of module (T_1) = 105.3 °C = 378.45 K

T_c = Cold side temperature of module (T_2) = 33.5 °C = 306.65 K

Z = figure of merit of thermoelectric material;

For Bismuth Telluride thermoelectric module, $Z = 3 \times 10^{-3} \text{ K}^{-1}$

$T_m = (T_h + T_c)/2 = 69.4$ °C = 342.55 K.

Putting all the values in the equation, we get,

$$\eta_{th} = 0.036 = 3.60\%$$

Calculation of actual conversion efficiency of TEG prototype

Actual efficiency is given by following formula which is already discussed in the previous chapters:-

$$\eta_{act} = \frac{P_{output}}{Q_1}$$

Putting the values, we get $\eta_{act} = 2.28312/70.433 = 0.0324 = 3.24\%$

Table 4.4 - Actual and Theoretical Efficiency Comparison

<i>For bismuth telluride module, at temperature difference of 71.8°C</i>	
Theoretical Conversion Efficiency	3.60%
Actual Conversion Efficiency	3.24%
Percentage error	10%

4.2 Proposed Design of AETEG system

As we have demonstrated the AETEG system with single module, but power produced by single module is low. The power produced can't meet the demand of power required for small electronic accessories in vehicle. So we proposed a setup for waste heat recovery of exhaust gases using a number of thermoelectric modules connected in a specific configuration (series or parallel). The setup is shown in the figure below:-

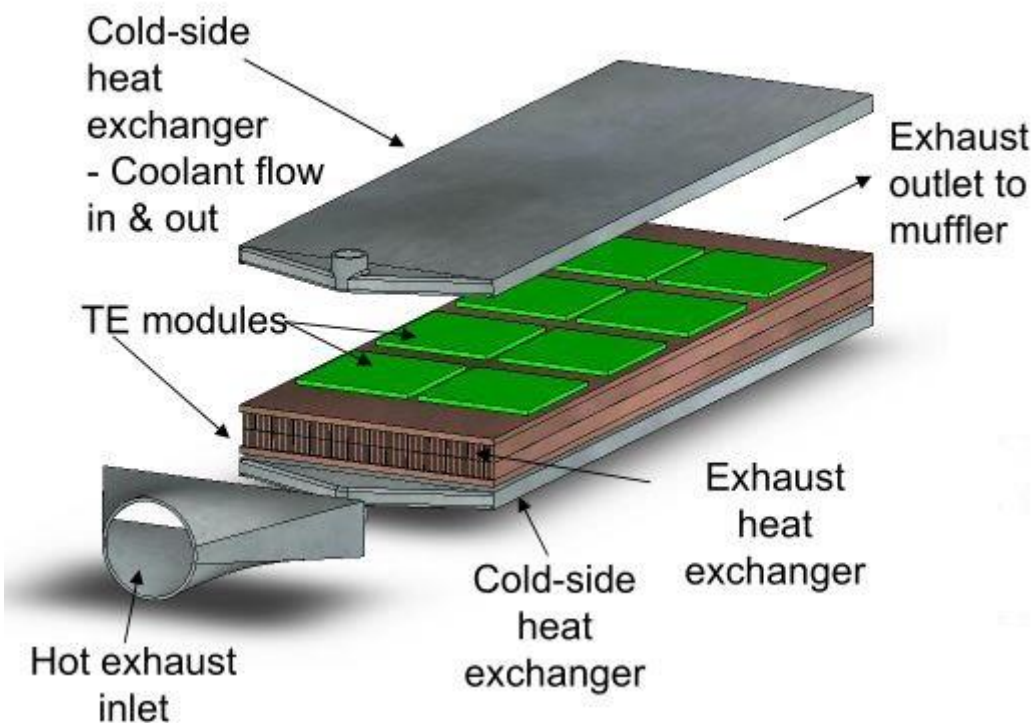


Figure 4.7– Proposed AETEG system using a number of thermoelectric modules

In this AETEG system, the hot side heat exchanger is made as integral portion of exhaust gas pipe or silencer. The exhaust gas flows into the inner part of heat exchanger and it heats up the surface of heat exchanger which comes in contact. Thermoelectric modules are placed upon the outer surface of hot side heat exchanger. The outer surface is also hot due to heat conduction from inner surface. The material of heat exchanger should be of high thermal conductivity and it should be decided by considering the thermal conductivity including cost factor of material. Generally Cast iron, Aluminum or brass are used as material for construction of hot side heat exchanger. Copper is not used due to its high cost. Thermoelectric modules, placed on the outer surface of heat

exchanger gets heated on one side while cold side heat exchanger is placed on the other side of modules. The temperature gradient is created due to different temperatures on both sides of module and the voltage induced due to Seebeck effect. The coolant used in the cold side heat exchanger could be air or water. The diagram for water cooled and air-cooled heat sink in AETEG system are shown in the following figures:-

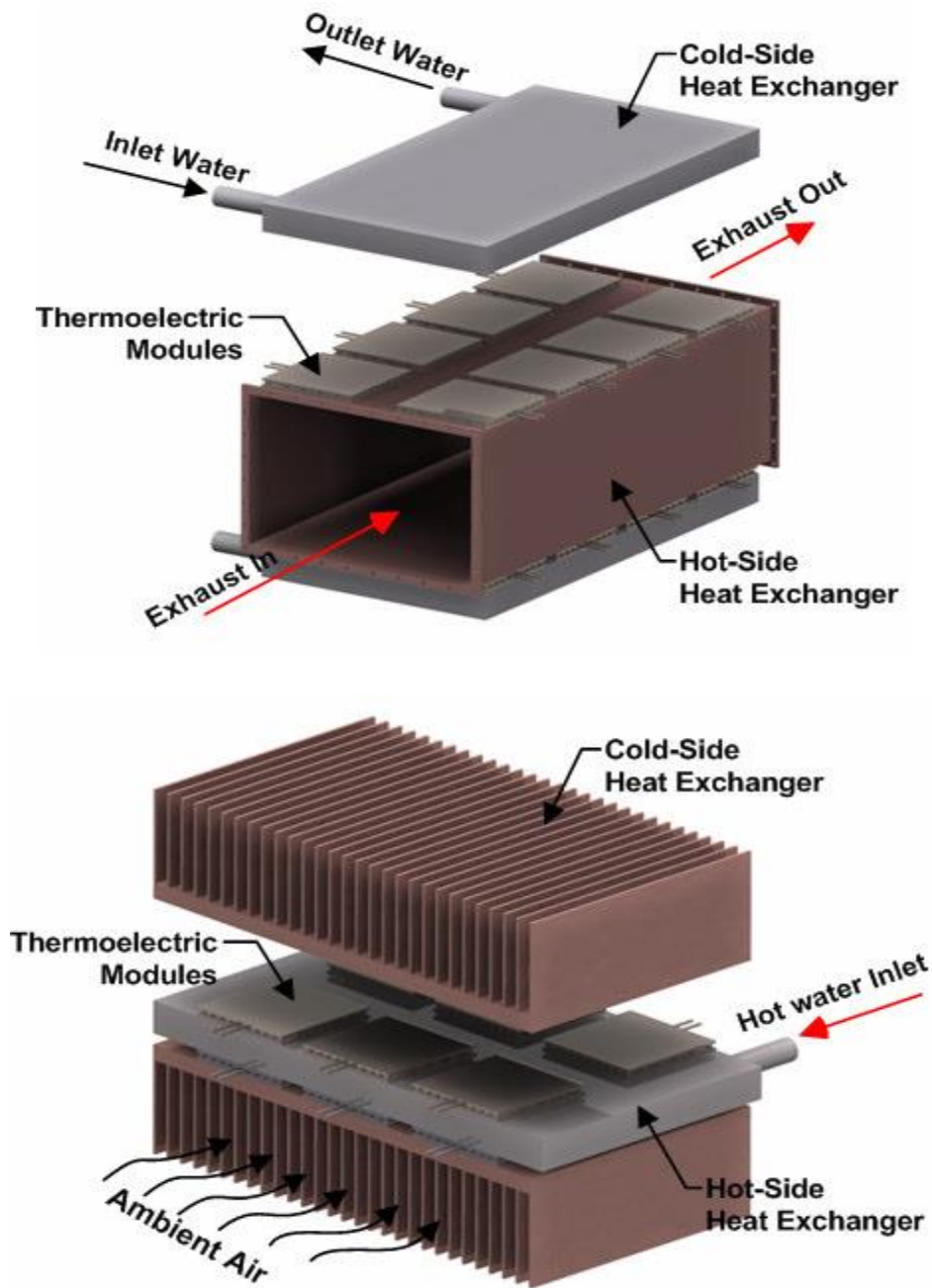


Figure 4.8 – AETEG system (a) with water cooled heat sink, (b) Air-cooled heat sink [43]

This proposed design of TEG device is more efficient and effective than TEG device with single module, the number of modules can be varied according to the need of external load. In such design exhaust gas is in direct contact with the surface of heat exchanger unlike the demonstrated prototype. As in case of demonstrated prototype, aluminum base plate is in contact with the surface of silencer, so the hot side temperature is comparatively lower than in case of the proposed design.

So the proposed system of thermoelectric generator could be successfully used in case of high power requirement, because TEG with single module is only suitable for production of low power around 2~5 watts.

Chapter 5

Methods to Increase conversion efficiency of an exhaust based TEG system

5.1 DC-DC Converter

These are generally used in power electronics circuit. Here, we are presenting some review of DC-DC converter in following sub-headings.

5.1.1 General review of DC-DC converter

In power electronics, DC-DC converters are generally used for conversion of voltage supply from the power source to the required level of voltage for the target load. In Automobile exhaust based TEG, we can also use DC-DC converter to meet the required voltage level. As in the case of automobile exhaust based TEG system, the efficiency of system is very low in the range of 3 to 5%. The voltage induced from single TE module is also very low, so there is a need to boost up the voltage induced. DC-DC Boost Converter can successfully boost up the voltage produced by Peltier module. The voltage produced by Peltier module may reach to the required level for different electronics accessories in car. The general concept of DC-DC converter is related to storage and then release of electrical energy. Electrical energy is first stored in capacitor or inductor and then it is released to load. The average voltage appeared at the converter load can be controlled by controlling the time for energy storage and release [44].

Pulse-width Modulation (PWM) signal is utilized as the switching control signal in DC-DC converter. The advantage of PWM signal is that it has linear control over the load power. If total switching period is T_s and total time in ON condition for switching device is T_{on} . With the help of these times, we can calculate duty cycle (D) of the PWM signal [44]:

$$D = \frac{T_{on}}{T_s} \quad (5.1)$$

5.1.2 Types of DC-DC converter

The classification of DC-DC converter can be done in many ways according to the need of target application. Sometimes transformers are used into the DC-DC converter to achieve the DC

isolation between the input and output. The converters which are incorporated with transformer are known as isolating converters, while others are known as non-isolating DC-DC converters are as follow:-

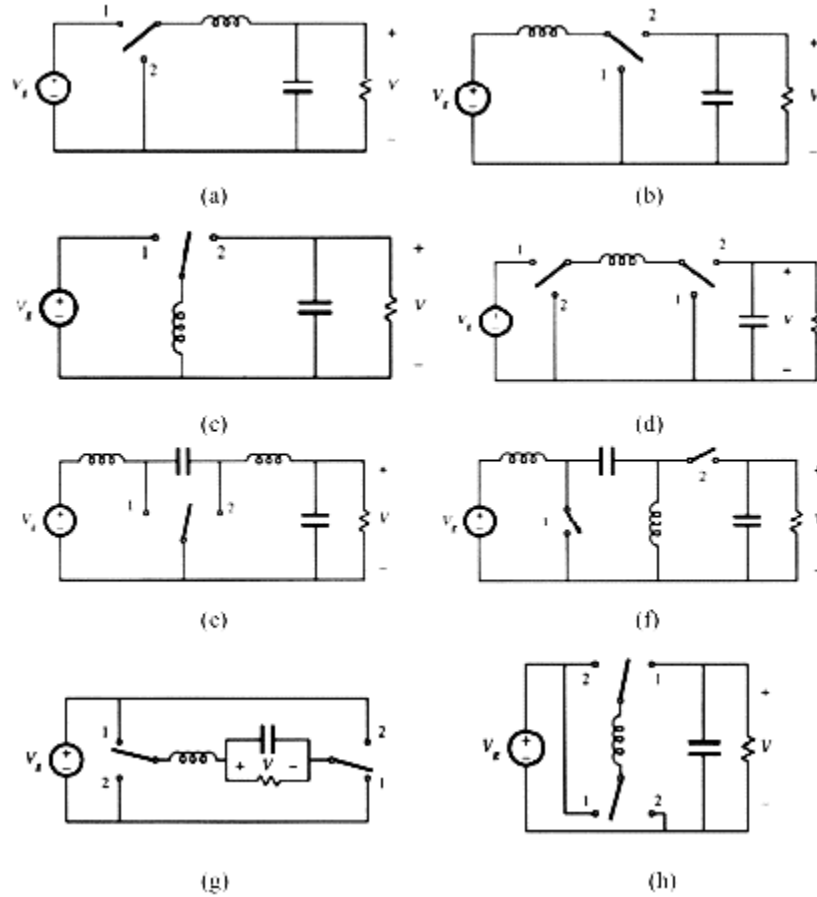


Figure 5.1- Circuit diagram of non-isolating DC-DC converters: (a) Buck (b) Boost (c) Buck-Boost (d) Non-inverting Buck-Boost (e) Cuk (f) SEPIC (g) H-Bridge (h) Watkins-Johnson [45]

Each type of converter possess conversion of voltage level to different level according to the application. Some of these converter are used to step down the voltage and other are used for step up the voltage induced. The ratio by which voltage is converted is known as transfer function. Our aim is to increase the voltage induced by TEG due to temperature gradient across it. So DC-DC boost converter will be a right choice.

The following table shows the transfer function of different DC-DC converters:-

Table 5.1 Transfer function of different DC-DC converters [45]

Converter Topology	Transfer Function
Buck	D
Boost	$1/(1-D)$
Buck-Boost	$-D/(1-D)$
Non-inverting Buck-Boost	$D/(1-D)$
Cuk	$-D/(1-D)$
SEPIC	$D/(1-D)$

5.1.3 Detailed Analysis of Boost converter

In our project, we need to increase the voltage induced from peltier module. So, use of boost converter will be a better option among all the converters described above. The first task in designing of DC-DC boost converter is the selection of switching frequency (f_s) which is decided by taking power level, cooling method and specificity of application into consideration. There are two modes of operation for DC-DC converters [45]:-

- i. Continuous Conduction Mode (CCM)
- ii. Discontinuous Conduction Mode (DCM)

Current fluctuation occurs in both types of mode, but current never goes down to zero in CCM while in case of DCM, the current is reduced to zero at or before the end of energy cycle. The Design equation of Boost converters for both types of conduction modes are discussed further in this section. Before this we can see the equivalent circuits of boost converters as follow:

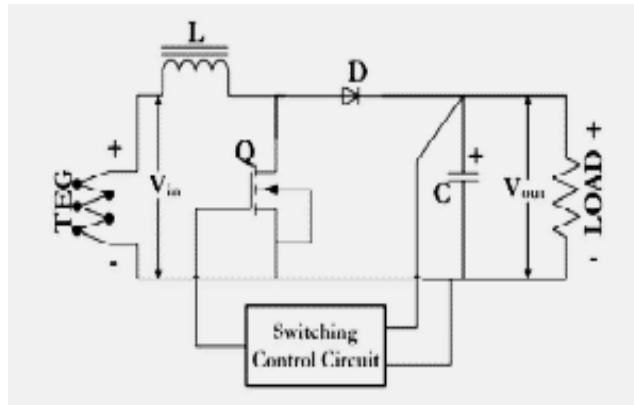


Figure 5.2 - The schematic circuit diagram of the boost converter [45]

The circuit of boost converter operates differently in switch-on and switch-off condition. The equivalent circuit are shown in figure below:-

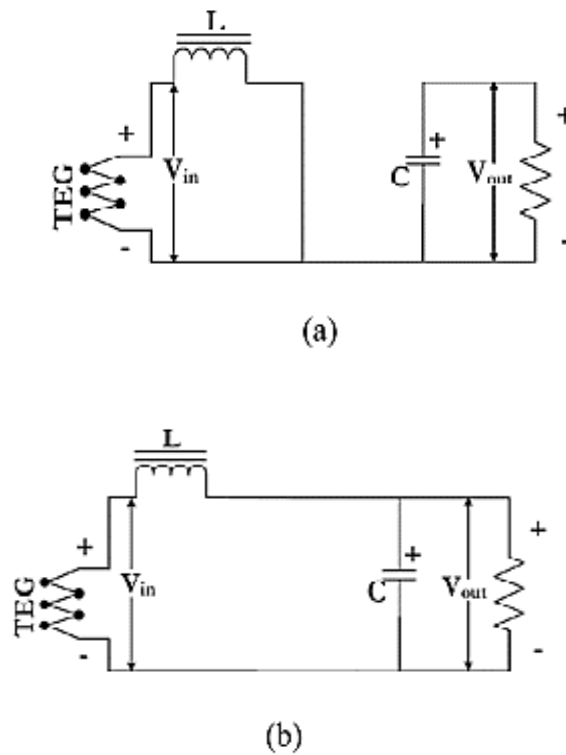


Figure 5.3- Equivalent circuits of the boost converter: (a) switch on (b) switch off [45]

Design equations for boost converter for both types of modes are as following:

CCM

Duty ratio of CCM boost converter can be expressed as:-

$$D_{CCM} = 1 - \frac{V_i}{V_o}$$

Transfer function for this mode can be calculated by rearranging the above equation:-

$$\frac{V_o}{V_i} = \frac{1}{(1 - D_{CCM})} \quad (5.2)$$

In terms of duty ratio, switching frequency and load resistance, the critical inductance of DC-DC can be represented as:-

$$L_{CCM} = D_{CCM} \cdot (1 - D_{CCM})^2 \cdot \frac{R_L}{2f_s} \quad (5.3)$$

If the value of critical inductance is smaller than actual inductance, the continuous mode of conduction prevails, otherwise, the circuit will operate under discontinuous mode.

Capacitance in CCM mode boost converter is expressed as:-

$$C_{CCM} = \frac{D_{CCM}}{R_L \cdot f_s \cdot r} \quad (5.4)$$

Where 'r' is the ripple ratio of desired output voltage for the DC-DC boost converter [44].

DCM

In this mode of boost converter operation, the duty ratio can be determined as:-

$$D_{DCM} = \left(1 - \frac{V_i}{V_o}\right) \cdot \sqrt{\lambda}$$

Where λ is the ratio of actual inductance to the minimum required inductance in continuous mode.

$$\lambda = \frac{L}{L_{CCM}}$$

Transfer function of the circuit in this mode:-

$$\frac{V_o}{V_i} = \frac{1}{1 - \frac{D_{DCM}}{\sqrt{\lambda}}} \quad (5.5)$$

Comparison of equation indicates that DCM boost converter increases magnitude of output voltage by a larger amount than the CCM boost converter.

$$C_{DCM} = \frac{(2 - \sqrt{\lambda})(\sqrt{\lambda} - D_{DCM})[2 - (1 - D)\sqrt{\lambda}]}{4R_L f_s r \sqrt{\lambda}} \quad (5.6)$$

With the help of above equations various parameters related to design equations are determined [44].

5.1.4 DC-DC conversion network in automobile exhaust based TEG

To increase the voltage level produced by automobile exhaust based TEG, DC-DC Boost Converter can be used in TEG device system by two methods. First one is the traditional method of single-stage topology and other is multi section multi-stage topology. We can describe these conversion networks as following paragraphs:-

Single stage conversion network-

Now a days, single stage conversion system of boost converter are being used in most of the TEG system. These converters are generally used to provide suitable and established voltage supply for mini electronics equipment of vehicles. In a single-stage topology of conversion network, all TE couples are connected electrically in series and a single output voltage from the connected module is used as the input voltage for the DC-DC Boost Converter. So converter acts as an external load for TEG modules. Thus, Boost converter provide an appropriate amount of voltage for electronic accessories of the car. But mostly hybrid electrical vehicle require higher output voltage due to need of automotive applications such as charging the battery, In this case the effectiveness of conversion network is not appropriate. The main reason for this less effectiveness

is very high internal resistance of TEG module due to series-connection of a number of TEG modules. However, the input resistance of DC-DC converter is usually around 1 Ohm, or even lower. The unmatched resistances tend to severely degrade the overall efficiency of the system. Secondly, the single-stage topology is in itself unreliable. If one of the TE couples in the long string fails to function, the entire device would fail, which might lead to more serious results of vehicle failure [46].

Multi stage multi section conversion network

It is the new proposed topology of DC-DC converter in which many low level DC-DC boost converter are associated with TEG modules individually. A Thermoelectric module is connected with an individual converter and so on. After this the voltage output of all the low level DC-DC converter is supplied to the input of mid-level DC-DC boost converter which further transfer the voltage to external load at comparatively higher voltage than in case of single stage conversion network [46].

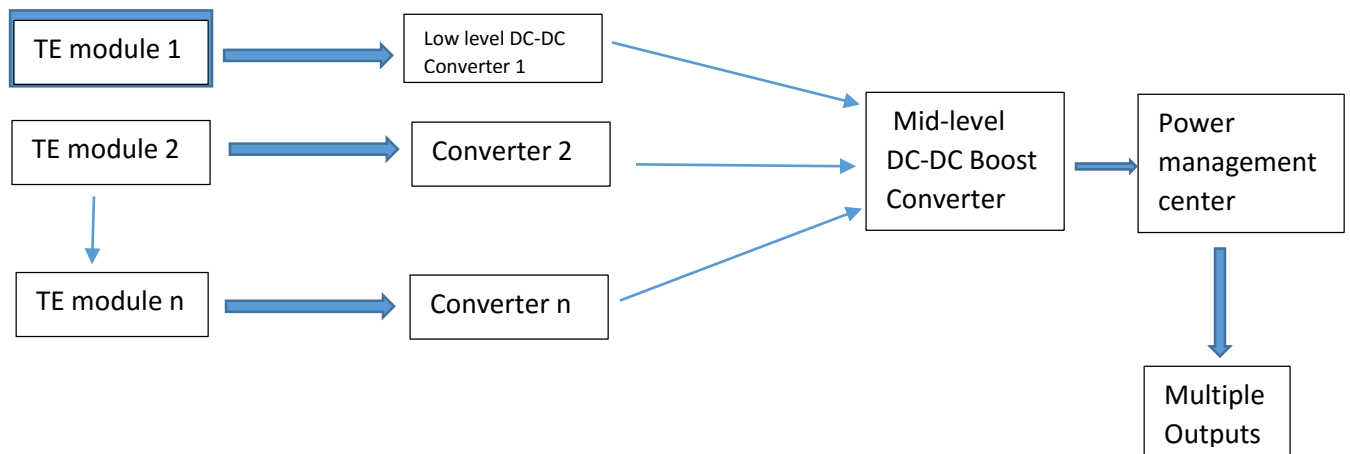


Figure 5.4- Schematic of multi-stage multi-section conversion network

Merits of multi-section multi-stage conversion network:-

- i. In this conversion network, low level boost converters first boost up the output voltage of every thermoelectric module and then a mid-level boost converter further boost up the

voltage forwarded by all low level DC-DC converters. Thus the voltage level increased by an appreciable amount and power output also increases.

- ii. In this system, a power management center is used with multiple outputs which collects output from the mid-level boost converters and generate a stable power supply for various electronics on vehicles.
- iii. This topology of network provide a high conversion efficiency in automotive waste heat recovery system.
- iv. The voltage level increased up to three times in this conversion network topology.
- v. Additionally, the parallel arrangement of thermoelectric module also enhance the high utilization of heat energy of waste exhaust gases and the complete system becomes more reliable.

5.1.5 Effect of DC-DC boost converter on demonstrated prototype

In the last chapter we have discussed about an experimental set up of TEG containing only single thermoelectric module. The voltage induced by single TE module can be increased by DC-DC boost converter to meet the voltage level requirement of target load.

The following table shows the readings of voltage induced by Peltier module with and without the use of DC-DC boost converter-

Table 5.2: Voltage produced by Single Thermoelectric module without and with the use of Boost converter

Temp. Difference (ΔT) ($^{\circ}C$)	Voltage Induced without boost converter (volt)	Voltage Induced with boost converter (volt)
0	0	0
12.5	0.29	0.55
24.5	0.63	1.20
36.1	1.38	2.52
45.6	1.82	3.35
56.7	2.13	4.02
65.1	2.39	4.48
71.8	3.02	5.56

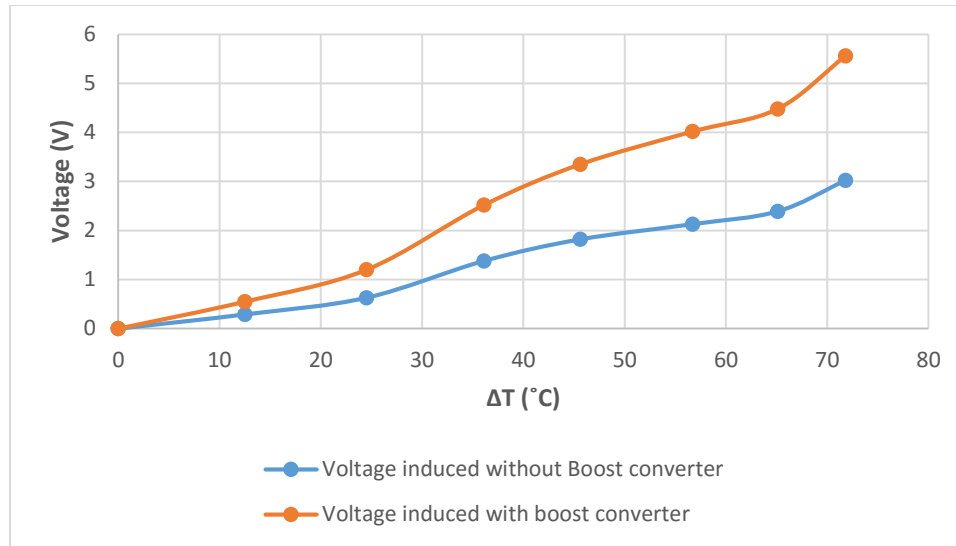


Figure 5.5- Comparison of voltage induced with and without boost converter

Boost converter is connected with TE module in series configuration, so value of current passing through the circuit will remain same, but the power output will increase due to increase in voltage.

The observation table for power output is shown below:-

Table 5.3 – Power produced by single TE module with boost converter

Temp. Difference (ΔT) (°C)	Voltage Induced with boost converter (volt)	Current (A)	Power produced (watt)
0	0	0	0
12.5	0.55	0.134	0.0737
24.5	1.20	0.227	0.2724
36.1	2.52	0.352	0.88704
45.6	3.35	0.439	1.47065
56.7	4.02	0.568	2.28336
65.1	4.48	0.631	2.82688
71.8	5.56	0.756	4.20336

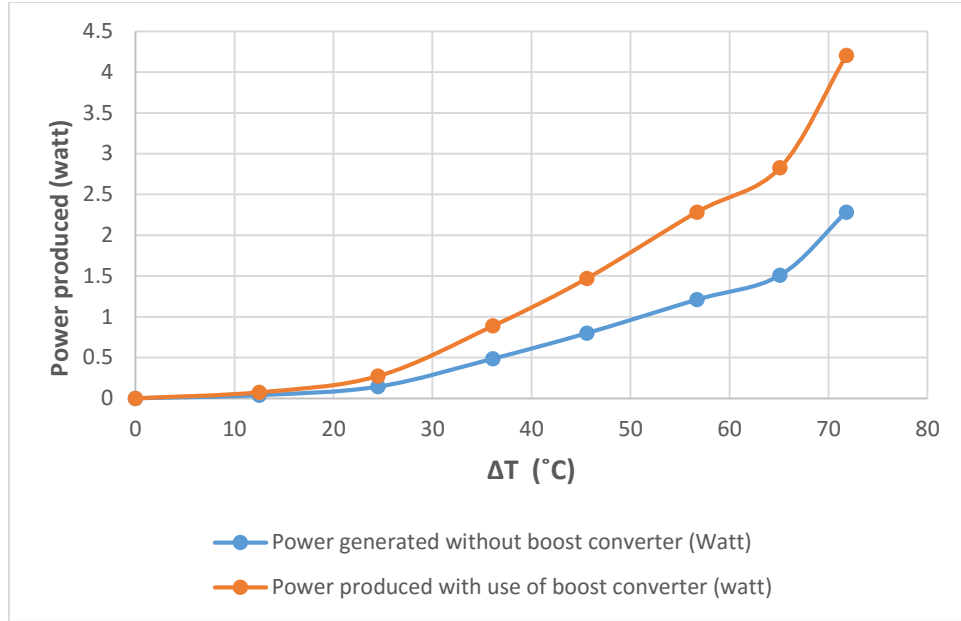


Figure 5.6- Comparison of power produced by single TE module with and without use of boost converter

Validation:-

Consider the case for the temperature difference of 71.8°C across the thermoelectric module.

- Voltage induced without the use of DC-DC boost converter = 3.02 V
- Mostly the value of duty cycle (D) for boost converter is in the range of 0.45 to 0.6. In this example we take, D = 0.5
- Transfer function for boost converter = $1/(1 - D) = 1/(1 - 0.5) = 2$
- Hence the voltage induced with the use of DC-DC boost converter = $3.02 \times 2 = 6.04$ V
- Experimental value of voltage after the use of boost converter = 5.56 V
- Percentage error in experimental value = $\frac{6.04-5.56}{6.04} \times 100\% = 7.94\%$

Hence, we can see that the experimental value of voltage induced by boost converter assisted thermoelectric module device is a little bit lesser than the theoretical value. But the amount of voltage level increased is still appreciable if we use boost converter in automotive waste heat recovery TEG device.

5.2 Thermal optimization of TEG device in automotive waste heat recovery system

Besides the use of DC-DC converter, Thermal optimization of design of heat exchanger is also a method of increasing conversion efficiency of automotive waste heat recovery system. A diverging-converging shell type heat exchanger is proposed which transfer heat of exhaust gases to the contacting side of thermoelectric module. There is a need to optimize the geometry, mounting location of heat exchanger for better heat to electrical conversion efficiency. The contact area with TE modules and other parameters related to TEG operation should also be optimized.

In this project, we optimize the design of heat structure with two different types of internal structures which enhance the heat dissipation area and more heat is transferred to one side of TE module. The design of heat exchanger is shown in the following geometries made on Solid-works software-

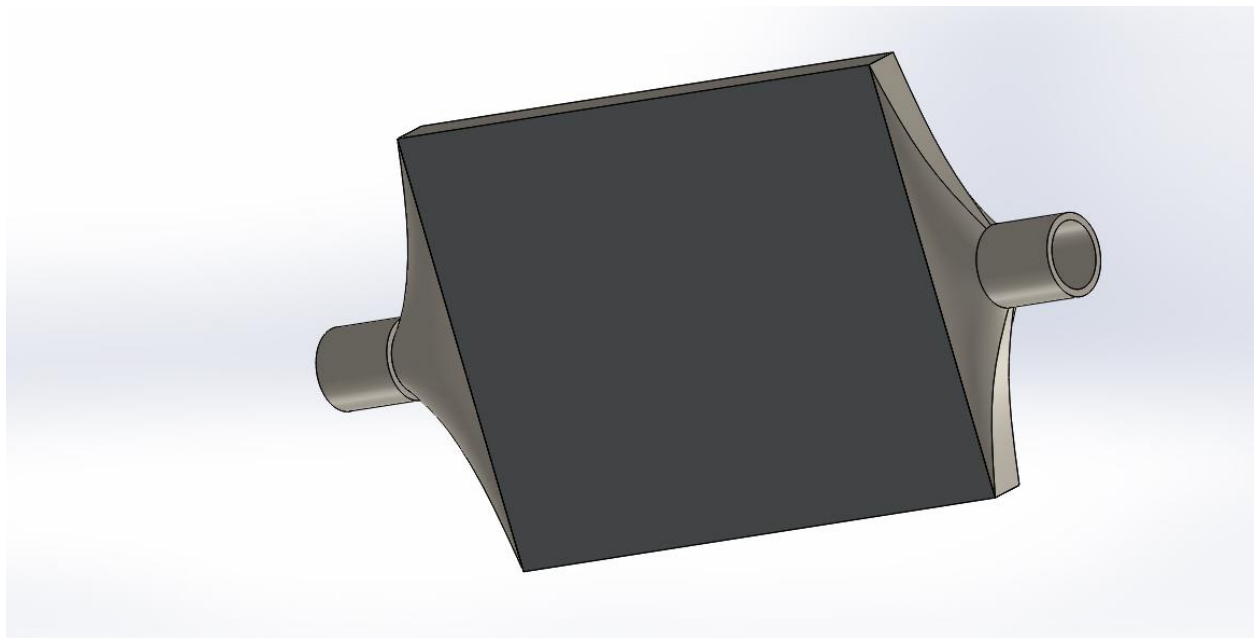


Figure 5.7 – Diverging-Converging shell type heat exchanger with no fins.

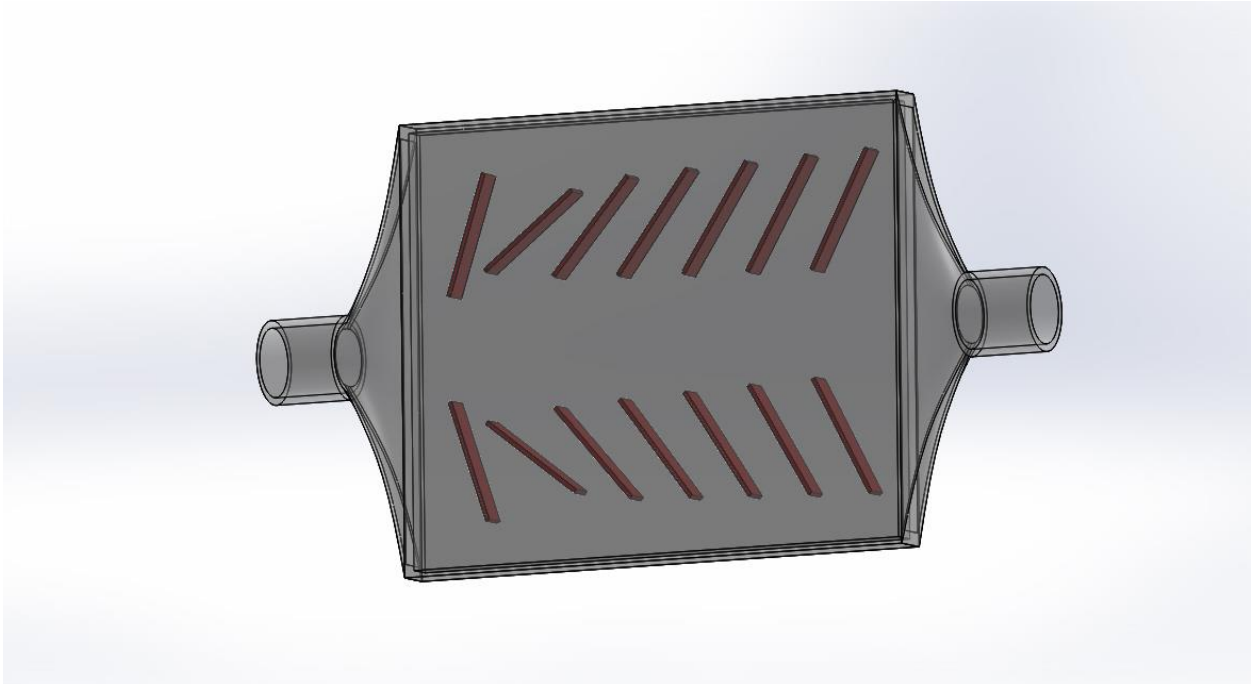


Figure 5.8 – Heat exchanger with fishbone-shaped structure

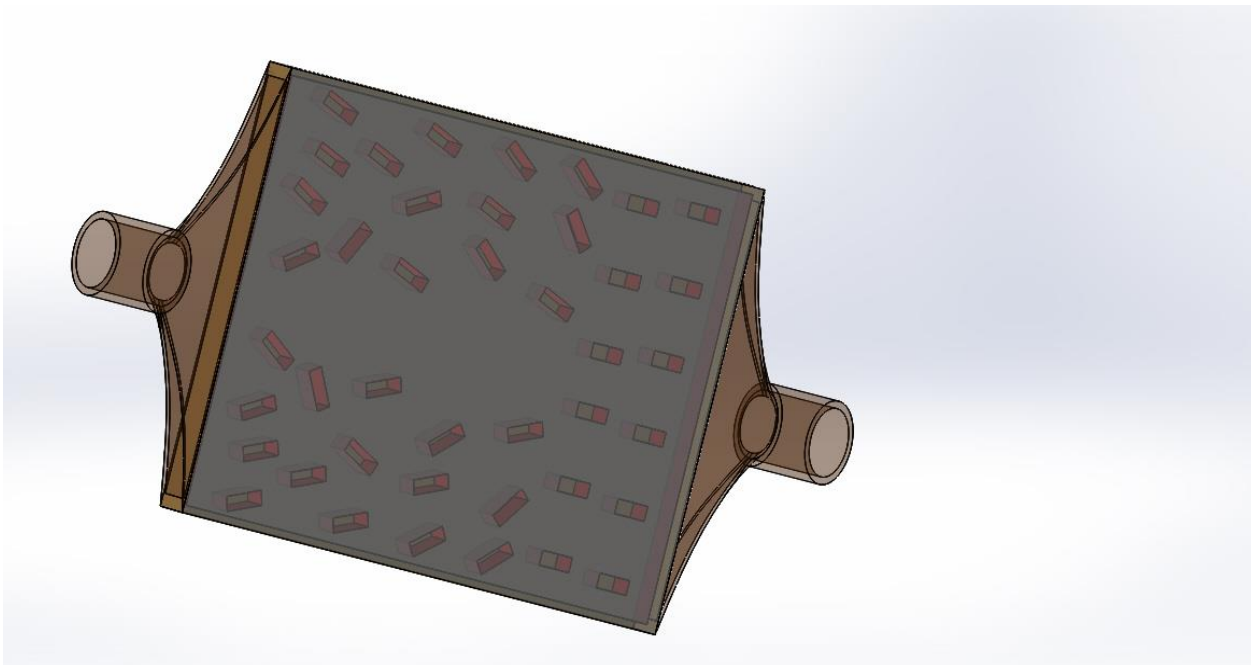


Figure 5.9- Heat exchanger with scatter-shaped

After the design of the heat exchanger on solid works, we used the computational fluid dynamics (CFD) software to simulate the flow of automobile exhaust gas within the heat exchanger. The CFD simulation in fluent also shows the temperature distribution on the wall for these three types of heat exchangers. The steps involved in the simulation are as following:-

- i. Geometry:-** Three types of heat exchanger geometries are designed in the Solid works software. First one is the heat exchanger of diverging-inlet, converging-outlet and with no fins. Second design is of fishbone-shaped internal structure. Third one is heat exchanger with scatter-shaped internal structure.

The main purposes, for adapting these type of shapes, are:

- a.** To minimize the back pressure across the ends of new design of heating chamber.
 - b.** To maximize the temperature profile across the surface of heating chamber for better utilization.
- ii. Meshing:-** All three design of heat exchangers are imported in ANSYS Fluent as .IGS format. There is a need of defining two domains for CFD of fluid-solid interfaces i.e. Solid domain and fluid domain. Meshing is started after cleaning up the solid-works design using the MERGE function across the circular ends of all designs for removing the excess joints. After this, hit and trail method is used to mesh independent results. The meshing parameters used in all heat exchanger geometries are shown in table-

Table 5.4 – Meshing parameters for Heat exchanger prototype design

S. No.	Features	Selection
1.	Advanced Size Function	Proximity
2.	Relevance Center	Fine
3.	Initial size seed	Full Assembly
4.	Smoothing	High
5.	Transition	Slow
6.	Span Angle Center	Fine

Besides the above parameters, other meshing function such as INFLATION and BODY SIZING were also used to obtain the accurate results at interface surfaces as well as in both

Solid and Fluid Domains. The range of orthogonal quality is kept from 0.2 to 0.3 and the range for skewness was kept 0.0005-0.001.

- iii. Set up:-** In this simulation the standard k-ε model is adapted and standard wall function was used as Near-Wall treatment. Different governing equations are used for fluid and solid domain. In case of fluid domain, mass conservation, momentum conservation and energy conservation equations were used. Equations are presented as following:

The Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \quad (5.7)$$

The Momentum conservation equation:

$$\frac{\partial(\rho U)}{\partial t} + \nabla \cdot (\rho U^2) = - \nabla p + \nabla \tau + S_M \quad (5.8)$$

The Energy conservation equation:

$$\frac{\partial(\rho h_{tot})}{\partial t} - \frac{\partial \rho}{\partial t} + \nabla(\rho U h_{tot}) = \nabla(\lambda \cdot \Delta T) + \nabla(U\tau) + US_M + S_E \quad (5.9)$$

In case of solid domain, only the energy conservation equation is used, which is shown below:

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot (\rho U_s h) = \nabla(\lambda \cdot \Delta T) + S_E \quad (5.10)$$

In this k-ε model of CFD simulation boundary conditions are set by checking whether the flow is turbulent or not. Reynold's number is calculated for this purpose.

Reynold's number,

$$R_e = \frac{\rho \cdot V \cdot d}{\mu}$$

Where, ρ = density of exhaust air

V = velocity

D = inlet diameter and μ = Dynamic viscosity

We calculate the value of Reynold's number by putting the all required value which is obtained more than 2000. Hence flow is called turbulent. Now boundary conditions are decide as follow:

- For inlet boundary condition, mass flow rate of 0.06 kg/s is selected with inlet temperature of exhaust gas as 673 K. Turbulence intensity and hydraulic diameter was initialized.
 - For outlet boundary condition, gauge pressure value was assumed as 0 Pa with coupled wall interface between fluid domain and solid domain.
- iv. **Solution:-** We set all governing equations for second high order with the maximum iteration of 500. After this CFD solver analyze the input conditions and prepare a result.
- v. **Result:-** In this step of CFD simulation, results are displayed by CFD Post-solver. We analyze the temperature contour only in our simulation but many-parameters such as velocity contour, pressure contour, streamline flow etc. can be shown by this CFD post solver software. Based on the Result of CFD simulation, the following results are analyzed on the basis of temperature contour obtained –

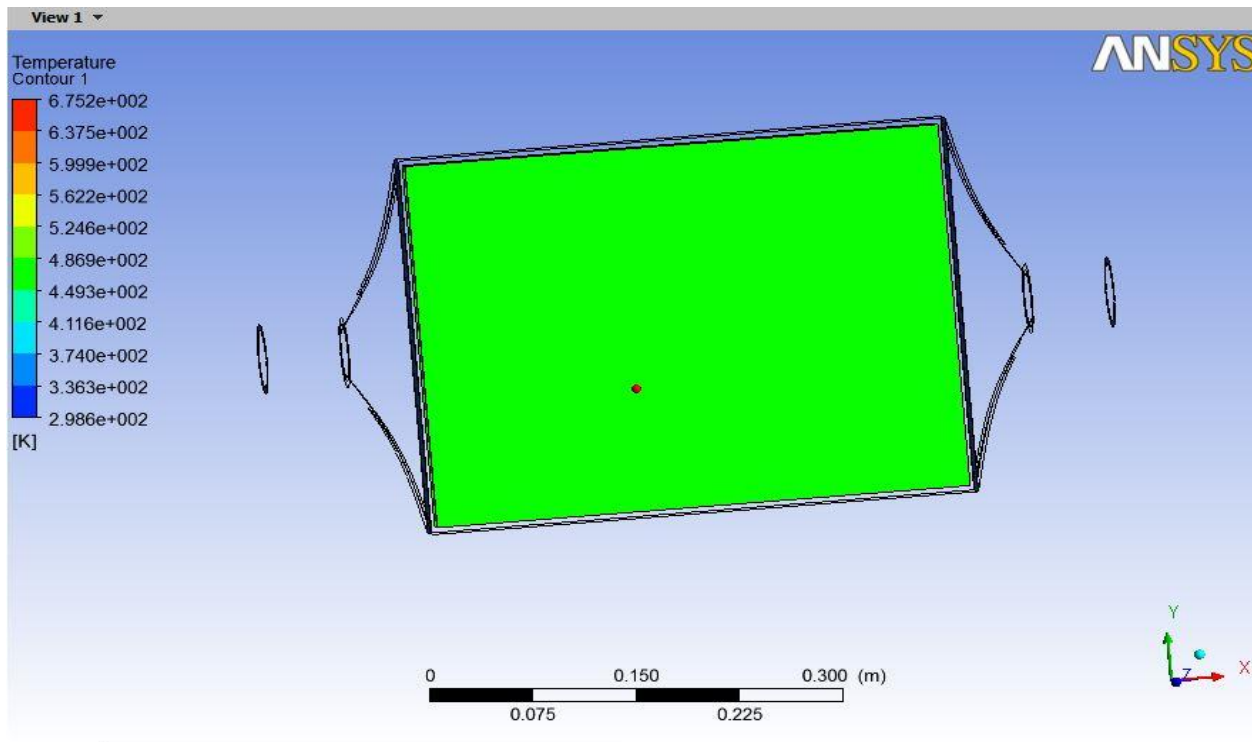


Figure 5.10 – Temperature contour of heat exchanger with no internal structure

Figure shows the temperature contour for heat exchanger with no fins inside the heat exchanger. The results indicates that outer surface is not effectively heated by exhaust gases in such case. The average value of temperature, obtained at outer surface of heat exchanger with no fins, is around 450 K (177°C). In this case conversion efficiency of TEG system is comparatively low. To enhance the conversion efficiency heat structure with some internal designs. Heat exchangers with internal designs are described next. One is heat exchanger with fishbone-shaped internal structure and other is scatter-shaped internal structure.

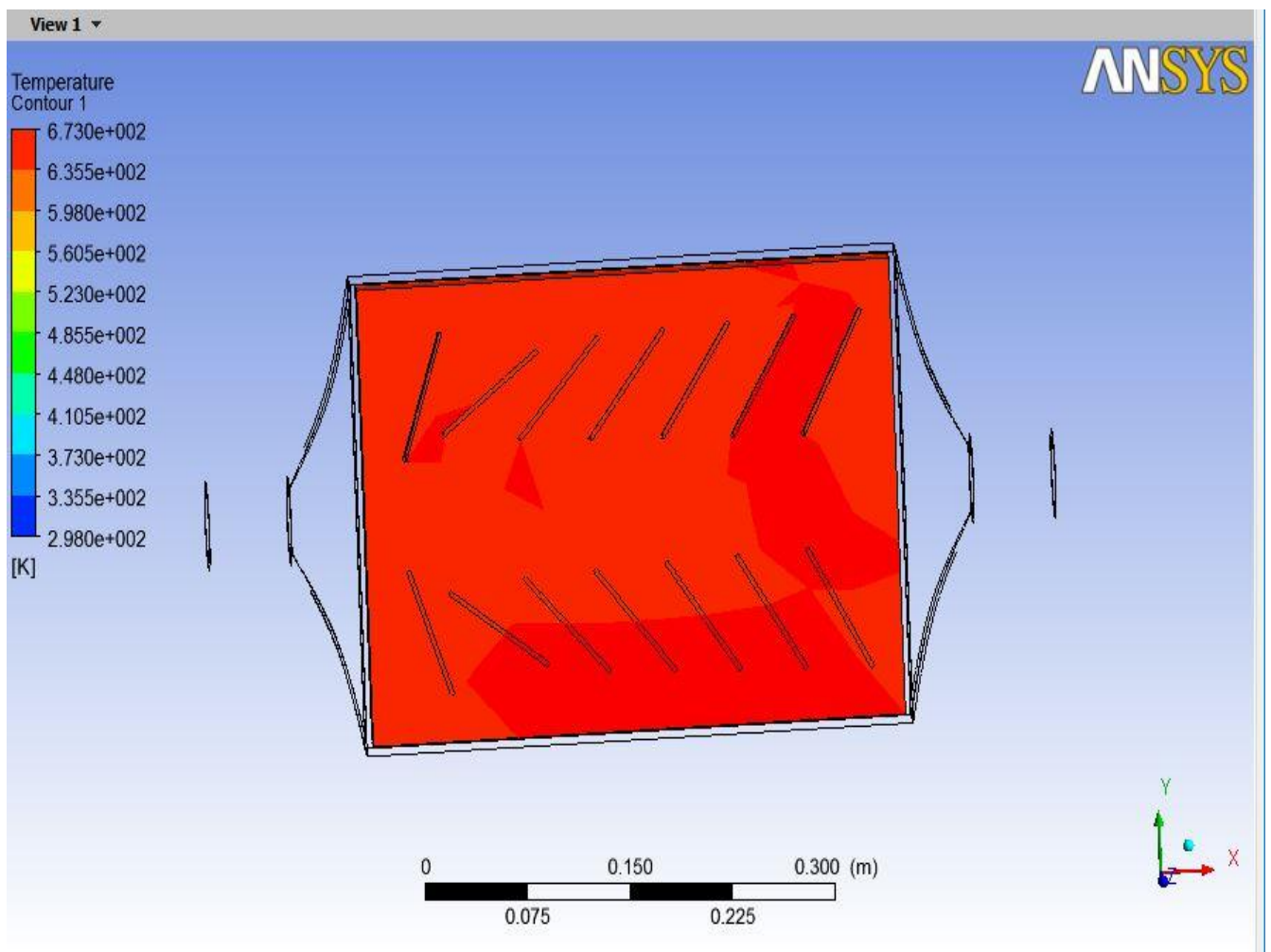


Figure 5.11 – Temperature contour of heat exchanger with fishbone-shaped internal structure

In the above figure, heat exchanger with 14 internal fins is used. The structure looks like fishbone-shaped and heat transfer area is more in this case in comparison with the heat structure with no fins. The average temperature obtained on the surface of the heat exchanger in this case is around 600 K (327°C). In this case, the maximum temperature is little bit higher than the maximum temperature in scatter-shaped internal structure. But the simulation results indicate that average temperature in heat exchanger with scatter-shaped internal structure is around 610 K (337°C) which is little bit higher than the average temperature in fishbone-shaped. If the number of baffle plates or fins is increased in heat exchanger with scatter-shaped type internal structure, the uniformity of temperature distribution would be more. So it could be a better method to increase heat to electrical conversion efficiency of AETEG system.

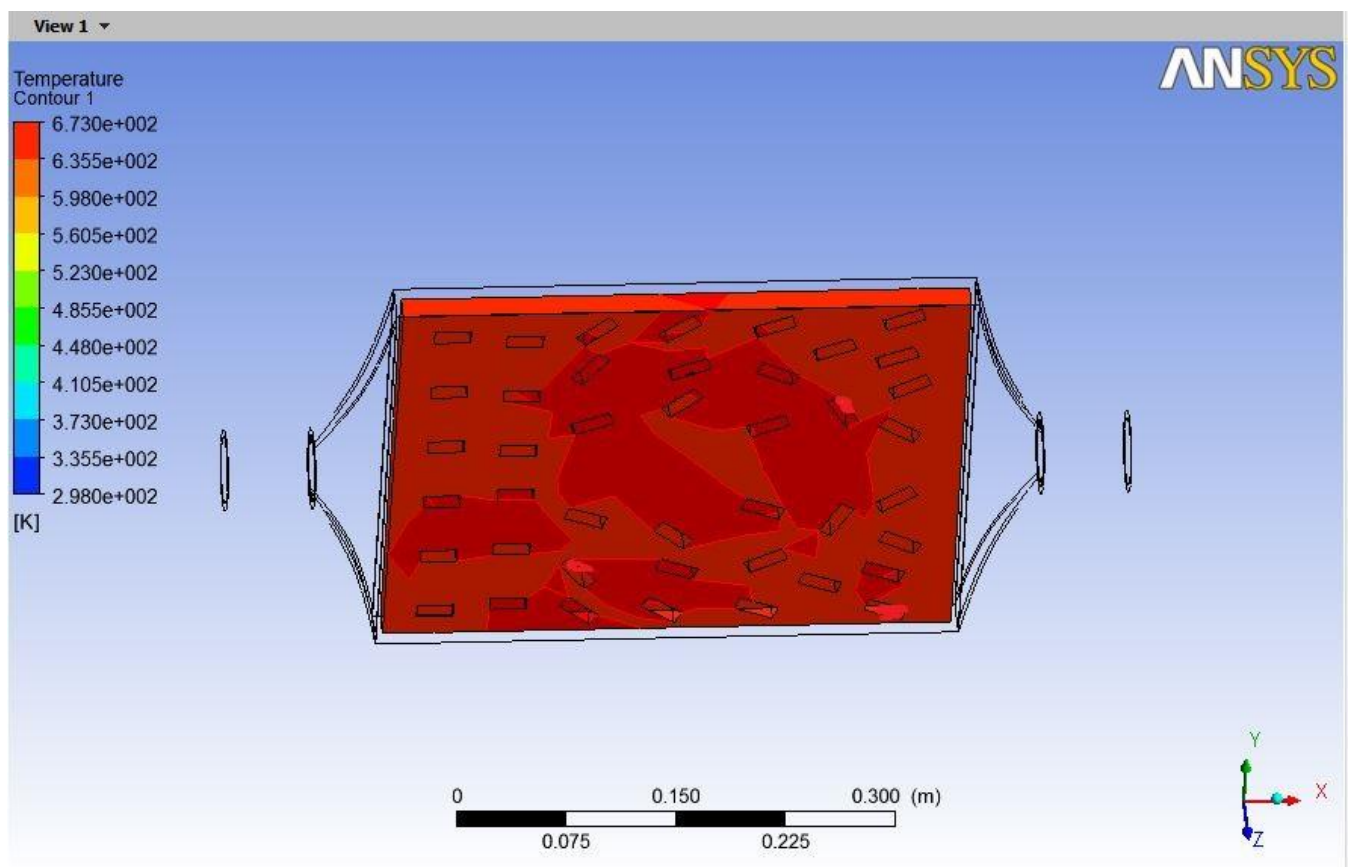


Figure 5.12 - Temperature contour of heat exchanger with scatter-shaped internal structure.

Thus, heat exchanger with internal fins structure provide better heat transfer of exhaust heat energy to surface of heat exchanger. The heat exchanger with no fins could not meet the requirement of surface temperature of heat exchanger. But if we use fins in internal portion of heat exchanger, the flow of exhaust gas become more turbulent and more heat is dissipated to heat exchanger surface. The hot side temperature of TEG is increased due to this and ultimately conversion efficiency of system increases.

5.3 Increasing conversion efficiency by developing thermoelectric material with high dimensionless figure of merit (ZT)

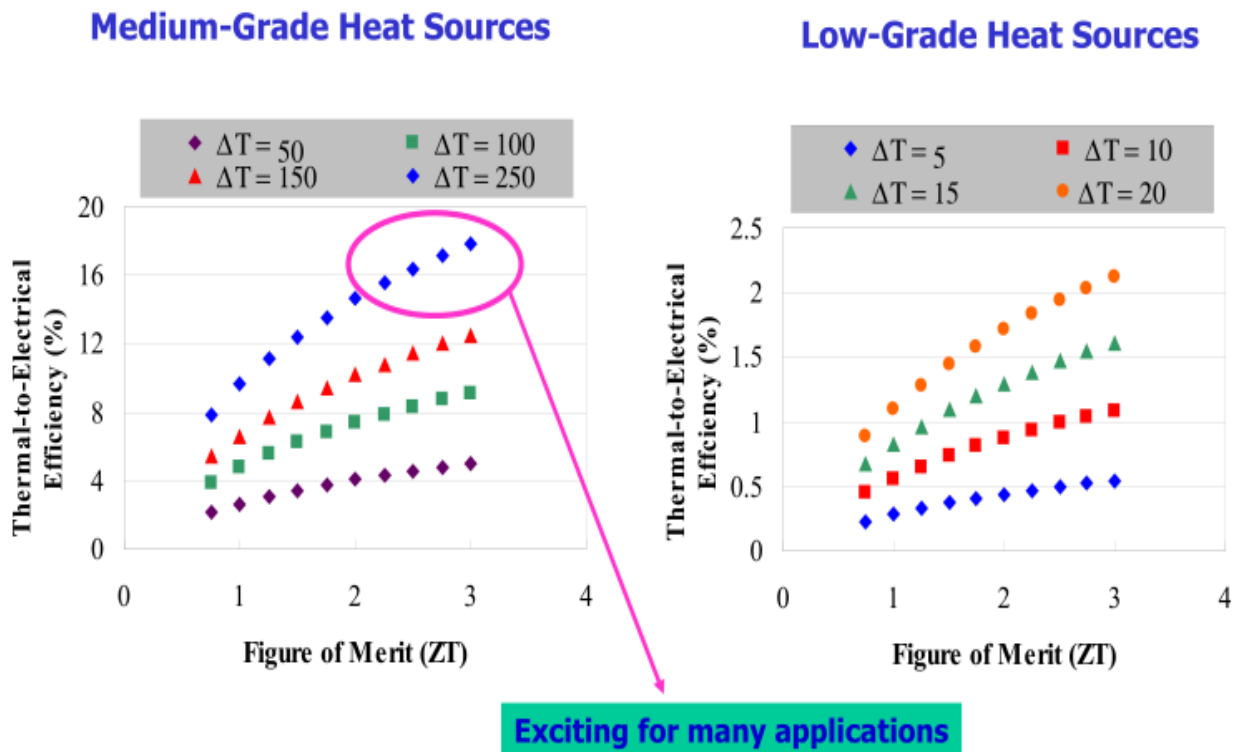


Figure 5.13- Dependency of Conversion efficiency on Figure of merit (ZT) in case of medium and low-grade heat sources [48]

The figure 5.13 illustrate that the conversion efficiency of TEG system is greatly dependent of figure of merit. In case of automobile exhaust waste heat recovery, the heat source temperature is in the range of medium-grade heat source. So there is a potential of replacing alternator in vehicles by using thermoelectric waste heat recovery system with thermoelectric material of high Figure of

merit value. At present, commercially available thermoelectric material have ZT value around 0.4~0.6. So there is need to research to develop thermoelectric material and semiconductor with higher ZT , approximately equals to one. If this would have been made possible, thermoelectric conversion efficiency raise by a considerable amount.

Chapter 6

Results and Discussions

6.1 Based on the mathematical modelling

The detailed mathematical analysis of automotive waste heat recovery system presented in the chapter 3rd, gives many correlations among the different parameters in this system. Some of which are elaborated as following:-

I-V and P-V characteristics of TEG device-

Figures 6.1 & 6.2 shows the current-voltage and power-voltage characteristics of thermoelectric module. The I-V curve shows the decreasing trend of current with the increasing voltage difference which is approximately linear and validate the relation of proportionality of voltage with current ($V=IR$). For high resistance load, current will be low and vice versa. In the P-V curve, the power increase firstly up to 6.55 watt, then decreases till zero. Thus maximum power is obtained at optimum voltage of 9.1 V. The relationship between power and voltage is non-linear because,

$$P = \frac{V^2}{R}$$

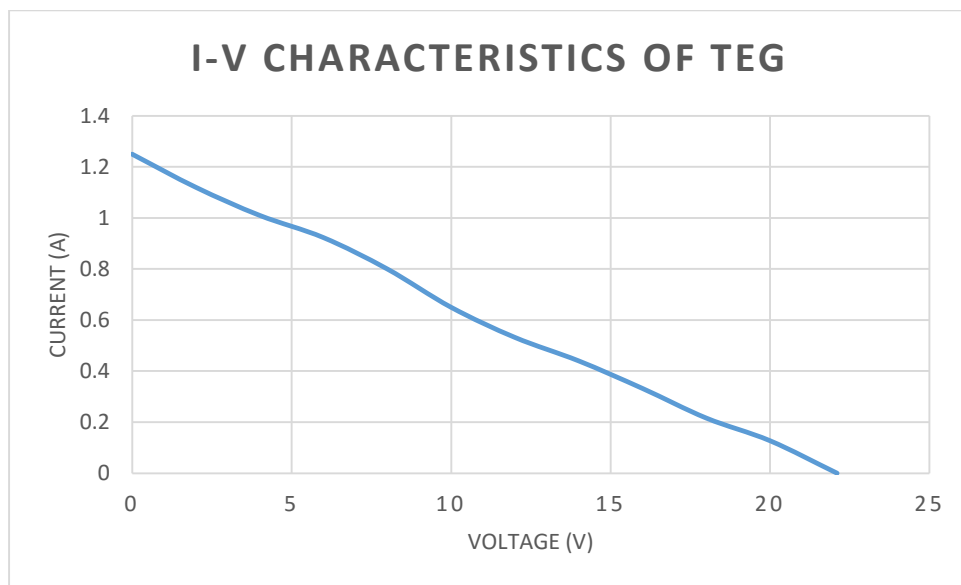


Figure 6.1 - I-V Characteristics of single thermoelectric module

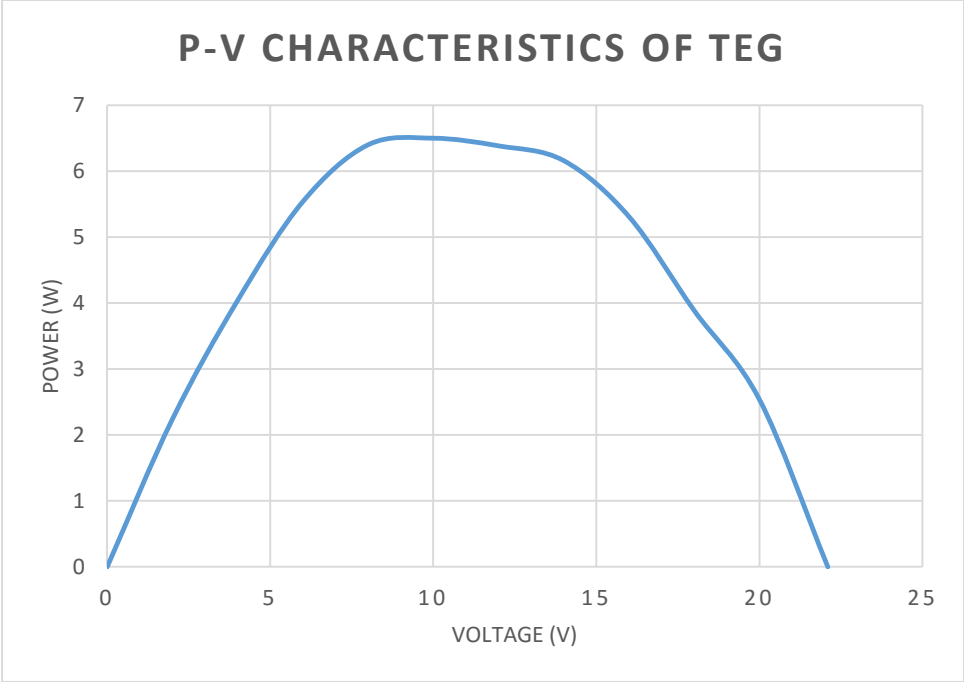


Figure 6.2- P-V characteristics of single TE module

Change in voltage and power produced by TEG with respect to time-

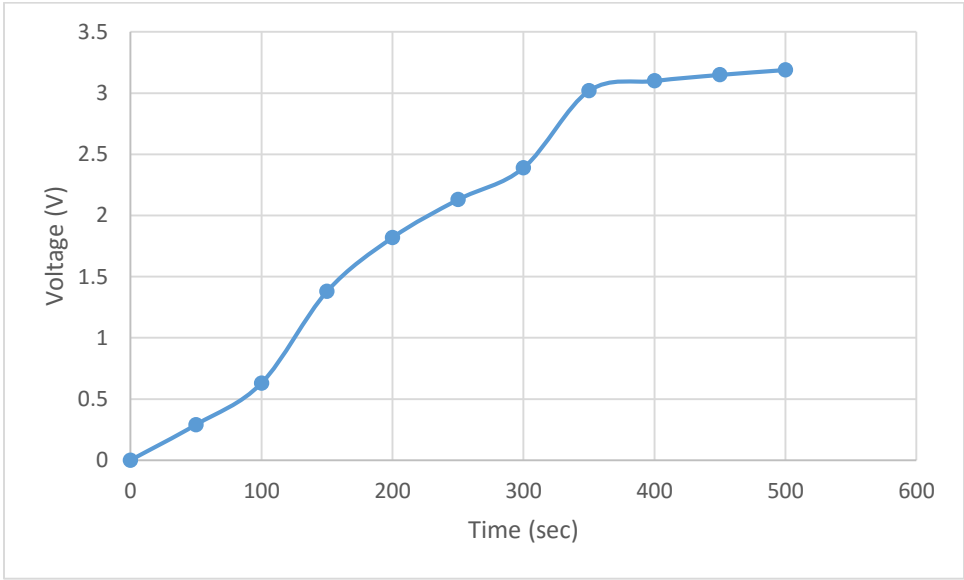


Figure 6.3- Voltage variation since the time vehicle starts

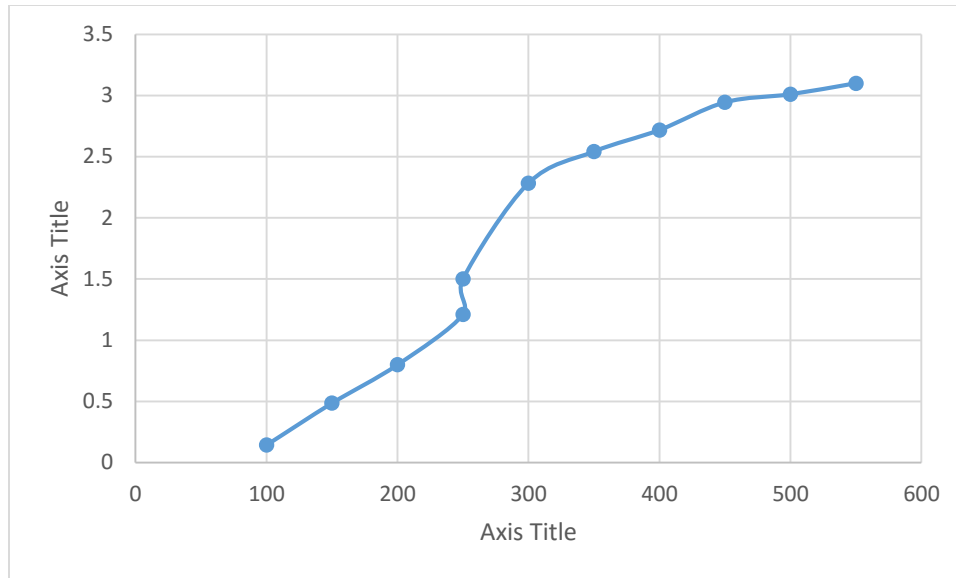


Figure 6.4- Variation in power output by single TE module with time

As vehicle is started, exhaust gas started flowing through exhaust pipe. Thermoelectric module located on exhaust pipe starts inducing voltage due to temperature difference. Voltage increase with time because temperature difference increases across the TE module. Due to increment in voltage, power produced also increases which is shown in figure 6.4.

Effect of Engine rpm on voltage induced-

Voltage increases as engine rpm are increased as indicated in figure 6.5-

Table 6.1- Voltage induced at different engine RPM

S. No.	Engine RPM	Voltage induced (V)
1.	2020	2.10
2.	2276	2.53
3.	2434	2.86
4.	2604	3.02
5.	2848	3.31
6.	3050	3.76
7.	3243	4.17

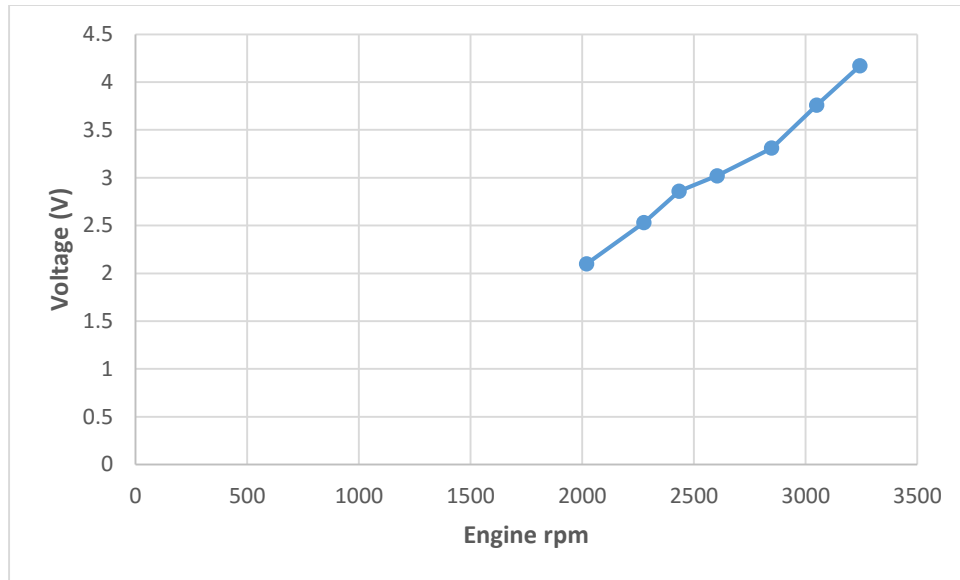


Figure 6.5- Voltage readings at different Engine RPM

6.2 Effect of various parameters on Conversion Efficiency

6.2.1 Effect of using DC-DC boost converter

Mostly research related to automotive TEG system have a main purpose to increase the heat to electrical conversion efficiency. In our work we also presented some methods to increase the conversion efficiency. First one was the use of DC-DC boost converter in parallelization of TE module.

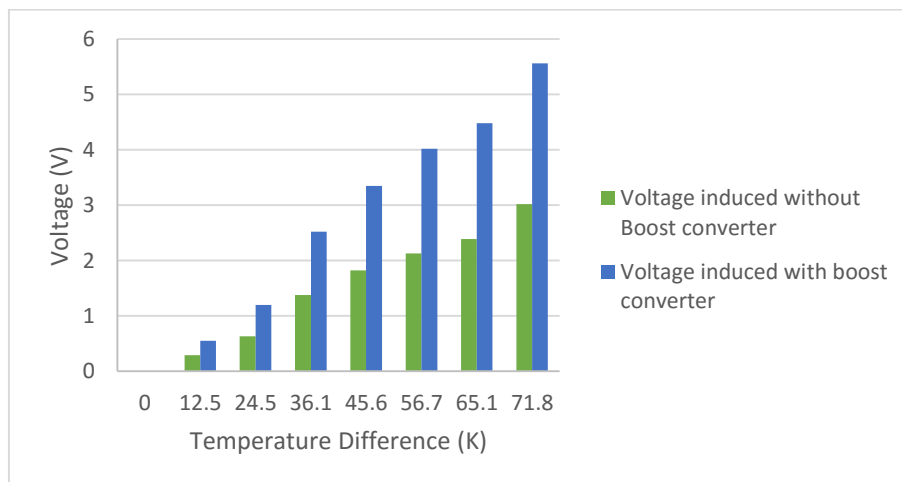


Figure 6.6- Voltage comparison for TEG circuit with or without boost converter

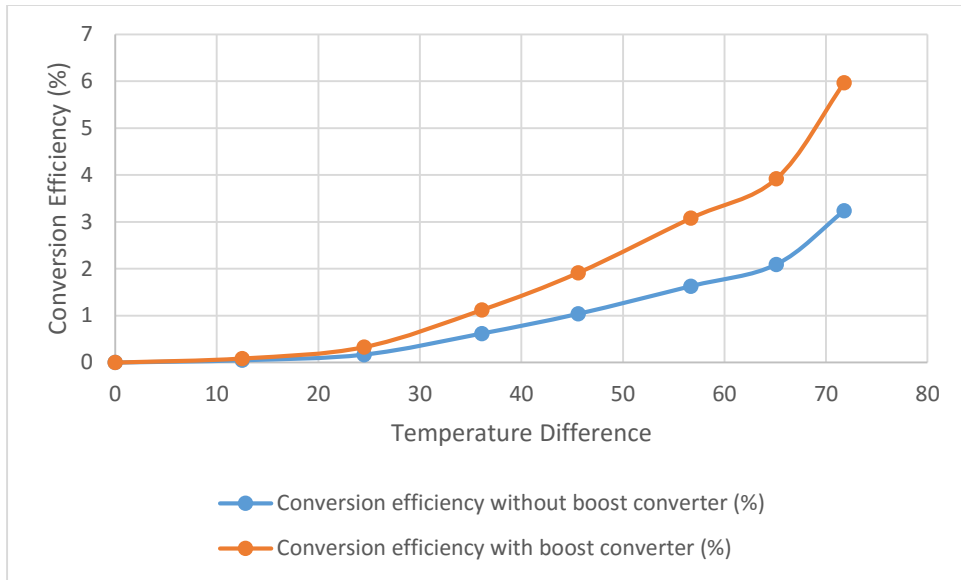


Figure 6.7- Efficiency comparison in case of system without and with boost converter

Thus, use of booster circuit increases the conversion efficiency of TEG system by enough amount.

6.2.2 Effect of figure of merit (ZT)

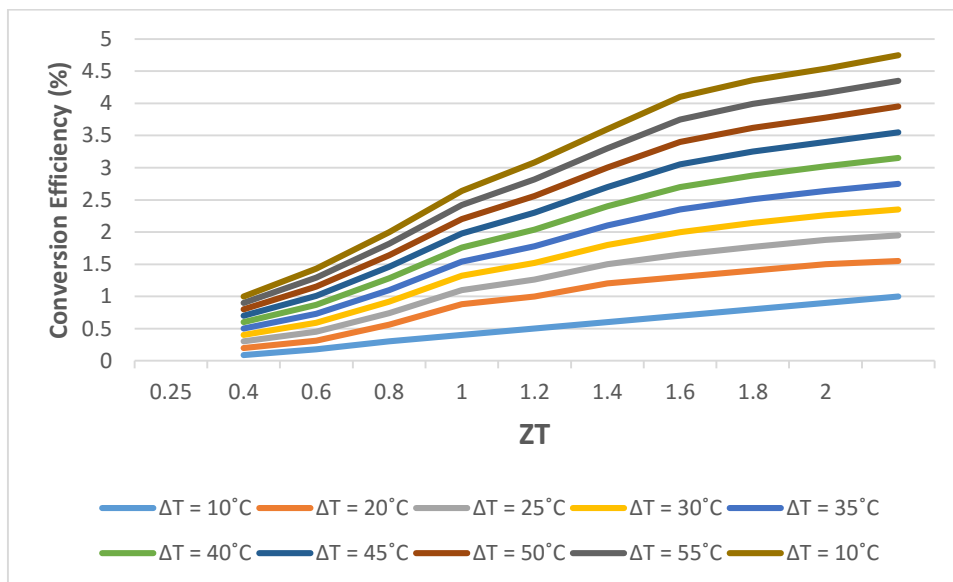


Figure 6.8- Efficiency vs ZT curve at different temperature gradient

Figure of merit is a main parameter for the performance evaluation of TEG. Conversion efficiency increases with the increase in figure of merit value. The module used in the experiment was bismuth telluride module which have ZT number in the range of 0.4-0.5. The trends show that increase rate of conversion efficiency with respect to ZT, have a higher value for higher temperature gradient across TEG.

6.2.3 Effect of convective heat transfer coefficient (h)

The convection heat transfer coefficient (h) for outside coolant (air or water) should be as high as possible because conversion efficiency increases with respect to the increasing convective heat transfer coefficient. The efficiency also increases with increase in convective heat transfer coefficient of exhaust air flowing in the inner volume of heat exchanger. The power produced by TEG also increases with the increment in the value of convective heat transfer coefficient. The trend of efficiency vs ‘h’ is shown in figure 6.9-

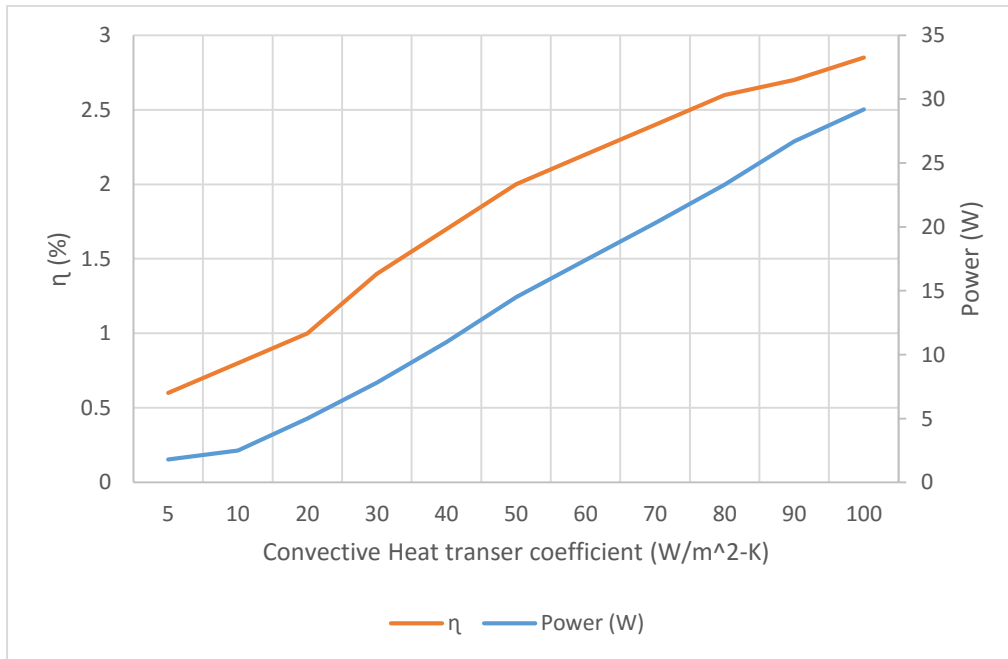


Figure 6.9- Efficiency and power produced by TEG as a function of convective heat transfer coefficient

6.2.4 Effect of hot side temperature of TE module

The hot side temperature of TEG system significantly affects conversion efficiency of the system. As high as the hot side temperature will be, the temperature gradient across the thermoelectric module will be more. This will emphasize the high voltage induction by the Peltier module. The trends for effect of hot side temperature are shown in below-

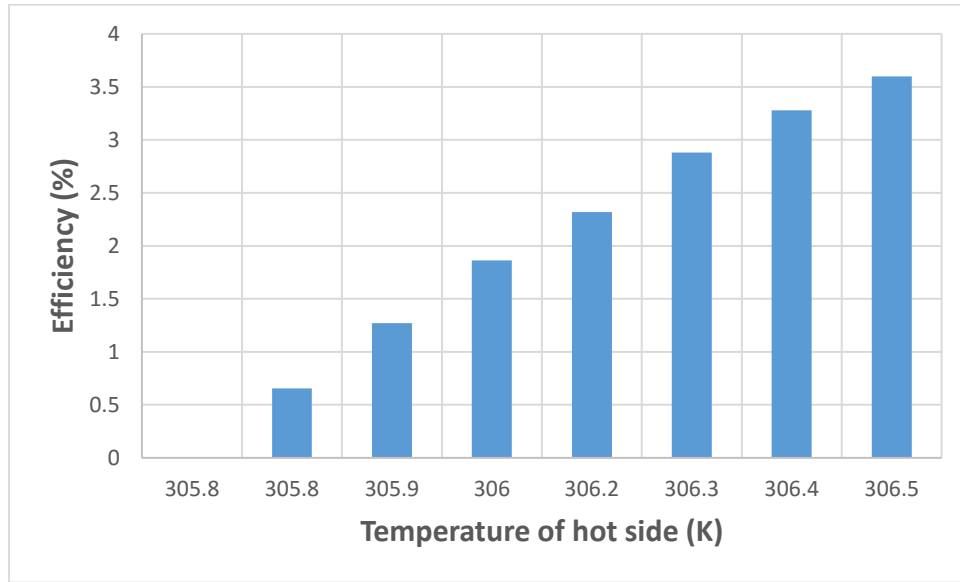


Figure 6.10- The clustered column diagram for Efficiency (η) vs hot side temp. (T_h)

6.2.5 Comparison of Actual and theoretical efficiency at different temperature gradient

Table 6.2- Theoretical efficiency readings at different temperature gradient

T_h (K)	T_c (K)	ΔT (K)	T_{mean} (K)	Theoretical Efficiency (%)
305.8	305.8	0	305.8	0
318.3	305.8	12.5	312.05	0.653
330.4	305.9	24.5	318.15	1.27
342.1	306	36.1	325.6	1.862
351.8	306.2	45.6	329	2.32
363	306.3	56.7	334.65	2.88
371.5	306.4	65.1	338.95	3.28
378.3	306.5	71.8	342.4	3.6

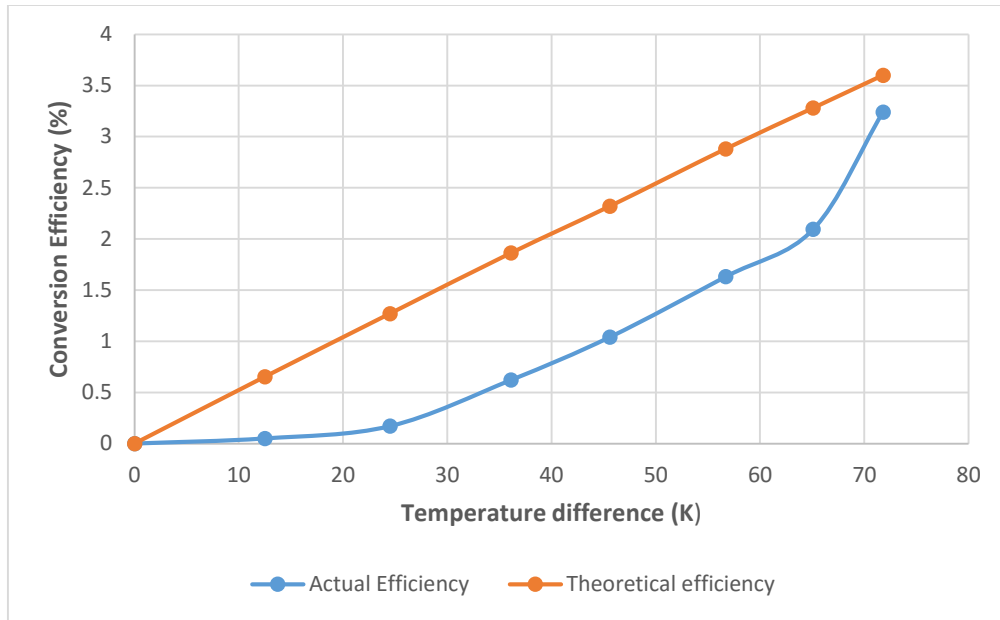


Figure 6.11- Comparison of Actual and theoretical efficiency

6.3 Results Based on the thermal optimization

Table 6.3 – Average temperature for different structure of heat exchanger used in AETEG system

Type of heat exchanger	Average Temperature (K)
Heat exchanger with no internal fins	450
Heat exchanger with fishbone-shaped internal structure	600
Heat exchanger with scatter-shaped internal structure	610

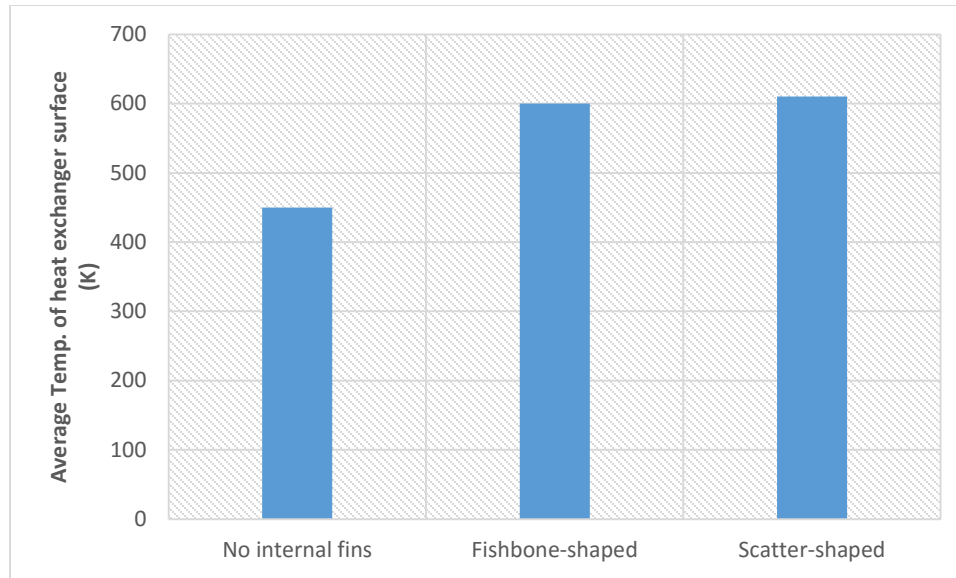


Figure 6.12- Comparison of average temperature of heat exchanger with different internal structure

Figure 6.12 clearly indicates that average temperature of heat exchanger surface increases by a considerable amount when it is constructed with some internal design instead of making it hollow. However, Scatter-shaped heat exchanger provide better temperature uniformity in case when more baffles are use inside the heat exchanger. So Heat exchanger with the Scatter-shaped internal structure is better optimized design to maximize the waste heat recovery from automobile exhaust gases.

Chapter 7.

Conclusions and Recommendations for future applications

7.1 Conclusion

The overall goal of thesis was to develop and validate a thermoelectric generator with study of methods of improving heat-to-electrical conversion efficiency of the system. Firstly we presented a detailed review of previous researches in the field of automotive waste heat recovery using thermoelectric generator. The review concludes that TEG could be more beneficial if the concerned parameters in the automotive WHR system are optimized.

In this work, we gave a description of mathematical analysis of TEG and discussed the problem of mismatching when modules are connected. A prototype TEG was demonstrated on the exhaust pipe of a single cylinder four stroke engine of hero Honda CD dawn bike. The efficiency of single thermoelectric module is quite low which could not replace alternator of vehicle entirely. To utilize the maximum heat energy from waste exhaust gases, the TEG device should consist a large number of modules which are connected in series or parallel configuration. The low conversion efficiency of thermoelectric module is still a limitation but DC-DC boost converter used in parallel configuration with module could increase the voltage and power produced to 1.5~2 times. Thermal optimization of heat exchanger can also enhance the power output of TEG due to increase in temperature difference across the TEG.

In the end, the result were discussed on the basis of mathematical analysis and demonstrated prototype. The result shows that voltage induced increases with increasing temperature gradient across the TEG. Hence the power produced and conversion efficiency also increases with increasing temperature gradient.

7.2 Recommendations for future applications

Future researches should be focused on developing methods of increasing efficiency of AETEG device along with cost effectiveness. In this field, the development of new thermoelectric material with high figure of merit value could be a better option. DC-DC converters with better performance can also be used to raise the voltage level produced by the TEG. Design of TEG device should be considered by using optimum parameters. Various parameters such as design of

heat exchanger, type of fins, length of TE module, surface area are directly related to the performance of the complete AETEG system. So these parameters should be optimized.

If all the above written methods for increasing conversion efficiency are taken into action, the waste heat recovery system using thermoelectric generator, can be commercialized and an enough amount of fuel consumption can also be reduced. Moreover, TEGs do not cause any environmental degradation, these are simple, compact and extremely reliable. TEGs also have no mechanical moving part in the system, so these are quiet in operation.

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