A Dissertation on

PONDEROMOTIVE FORCE DRIVEN SELF GENERATED MAGNETIC FIELD IN LASER PLASMA INTERACTIONS

Submitted in the partial fulfillment of the requirements of the degree of

MASTER OF TECHNOLOGY

In

NUCLEAR SCIENCE & ENGINEERING

By

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2K14/NSE/03

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DECLARATION

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

PLACE:

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Abstract

In my thesis a differential equation governing magnetic field depending on plasma frequency(ω_{pe}), laser intensity parameter (a_0), laser frequency (ω_l), plasma density which varies along transverse direction (n_0) and Electric field $\left(E(\xi)\right)$ is derived. This differential equation is second order linear and inhomogeneous. For deriving this differential equation I have done literature review on laser plasma interaction. I have studied basics of plasma and laser systems. I have also gone through the process of generating ultra-short laser pulse which is very important for my research as these ultra-short laser pulse are described by high peak intensities. These ultra-short laser pulse are amplified by Chirped Pulse Amplification as conventional amplifying methods have lot of limitations. Also these ultra-short laser pulse can introduce the effect of non linearities in the plasma which can catalyse (Pondermotive Force) the process of generation of magnetic field.

The purpose of my research is to study the characteristics and behaviour of magnetic field for respective electric field envelope. Since this magnetic field is self-induced after laser irradiation of the target, I have to study the process of self-induced plasma magnetisation such as Biermann Battery, Inverse Faraday Effect, Plasma Magnetisation Due to Fast Currents or Fountain Effect, magnetisation by Pondermotive force, Weibel Instability etc.

I had analytically solved the differential equation for different piece wise electric field pulse profile such as Sine, Triangular, Sawtooth Increasing, Sawtooth Decreasing, Rectangular Triangular and Square pulse. The response of the magnetic field for these profiles were analysed and maximum amplitude compared to all other electric field pulse profile was found for Square pulse but with heavy oscillations. The second best amplitude was with Sawtooth Decreasing and interestingly it had minor oscillation unlike Square profile. The maximum amplitudes of Sine, Triangular, Sawtooth Increasing, Rectangular triangular electric field pulse profiles were found to be nearly same with minor fluctuations.

I have also tried to see the variation of magnetic field with laser intensity parameter (a_0), plasma density (n_0), and differential length (d_l) for all electric field pulse profiles and analysed that the magnetic field increases with increase in laser intensity parameter (a_0) and plasma density (n_0) and decreases with increase in differential length (d_l) and this result matches computational and theoretical results.

My results can be applied in applications where shape of electric field and magnetic field along with their peak amplitude are considered. We can choose Square electric field Pulse profile if only matter of consideration is peak amplitude and not the oscillations in envelope. If oscillations in the envelope and peak both are important better option is Sawtooth Decreasing electric field pulse profile

CONTENTS

Units and Values

Here are the most used units and values used in this M.Tech Dissertation

Temperature T	T-*eV*	
Plasma frequency ω_{pe}	*radian/sec*	
Laser frequency ω_l	*radian/sec*	
Speed of light c	*cm/sec*	3×10^{10}
Electric Field E	*V/cm*	
Electron charge e	*stat coulomb*	4.8×10^{-10}
Electron mass m	*grams*	9.1×10^{-28}
Laser intensity parameter a_0	*no units*	*0.3 to 0.7*
Plasma density n_0	cm^{-3}	10^{19}
Differential length d_l	*cm*	50×10^{-4}
Laser wavelength λ	*cm*	10^{-4}

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Chapter 1

Introduction

Experience from every day suggests that universe is composed of either solid, liquid or gas but actually the 99% universe is composed of plasma. Plasma is fourth state of matter formed after ionising the gas either by heating or by electromagnetic irradiation etc. **Plasma is defined as Quasineutral gas of charged particles and neutral particles exhibiting collective behaviour**. All the stars including sun in the universe is expected to composed of entirely plasma.

Nuclear fusion is the source of energy in stars**. In nuclear fusion two lighter nuclei recombine to form single nucleus with release of tremendous amount of energy**. But for recombining two nuclei one must overcome the repulsive force that is involved. In sun nuclear fusion occurs at several thousands of temperature and this temperature provides thermal energy so that nuclei recombination takes place [1]. But in practice generating such a large temperature for nuclear fusion is highly difficult. Thus here the study of plasma becomes important where large temperatures can be attained.

Various ways have been proposed to attain controlled thermo nuclear fusion in the laboratory. Magnetic confinement was the first method produced where initially Deuterium (D) – Tritium (T) plasma of density $10^{14} cm^{-3}$ is confined using powerful magnetic fields for ten seconds [2]. Since magnetic fields are used to confine plasma their study becomes important. With the invention of laser, the study of self-induced magnetic fields in plasma became more prominent because laser can deposit large amount of energy for very short time as low as femto seconds. With the advent of Chirped Pulse Amplification the laser irradiation became dominant in plasma study. Chirped Pulse Amplification can deposit several hundreds of mega joules of energy in laser pulse which can be as short as of femto seconds. This invention has revolutionised completely the study of high intensity laser plasma interaction.

The topic of connexion of energy from high intensity laser to any other target is of great prominence [4,5]. The coupling of laser energy to plasma have been discussed in laser plasma interaction chapter of this thesis. When laser interacts with plasma several phenomena can occur such as generation of highly energised charged particle, generation of magnetic fields of several order. The detailed discussion of self-induced magnetic fields in laser irradiated plasma is discussed in one of the chapters of this thesis

The magnetic fields are of great importance in laser fusion tests, as they can affect the energy movement from the light absorption zone to the ablation region. The thermal conductivity will fall with an increase in electron mean free path to the Larmor radius ratio. It is also likely that the strength of these fields can become analogous to those that may occur in many cosmological bodies. Hence producing models of such cosmological systems in the workroom may open up new paths of research in astrophysics. The origin of magnetic fields in laser irradiated plasma has been of great relevance in both theory and experiments.

1.1 Scope of the Project Dissertation

This project includes basic discussion on plasma and laser systems. It also includes detailed mechanisms that are involved in the generation of seed magnetic fields since this topic is part of my project. The mechanisms includes Biermann Battery, Inverse Faraday Effect, Plasma Magnetisation Due to Fast Currents or Fountain Effect, magnetisation by Pondermotive force, Weibel Instability etc.

My results can be applied in applications where shape magnetic field follows the shape of electric field. I have considered six electric field pulse profiles like sine, triangular, sawtooth decreasing, sawtooth increasing, rectangular triangular and square. Depending on our requirement we can choose respective pulse profile. We can choose Square electric field Pulse profile if only matter of consideration is peak amplitude and not the oscillations in envelope, since square has high fluctuations which falls from peak to zero rapidly. If oscillations in the envelope and peak both are important better option is Sawtooth Decreasing electric field pulse profile. If we don't require oscillations at all and peak amplitude of magnetic fields is not of concern then we can go to triangular or sawtooth increasing electric field pulse profiles as in both pulse profiles the shape is smooth unlike sinusoidal or sawtooth increasing or square or rectangular triangular.

1.2 Plasma

Plasma is considered to be fourth state of matter other being solid, liquid and gas. Sun has plasma as one major component. It is formed after heating gas to larger temperature. Hence it can be called as superheated gas. **At this temperature atoms absorbs sufficient energy to separate electrons from itself to form positive ions. Hence plasma is made up of electrons, neutrals and positive ions and it is most abundant state of matter in the whole universe**.

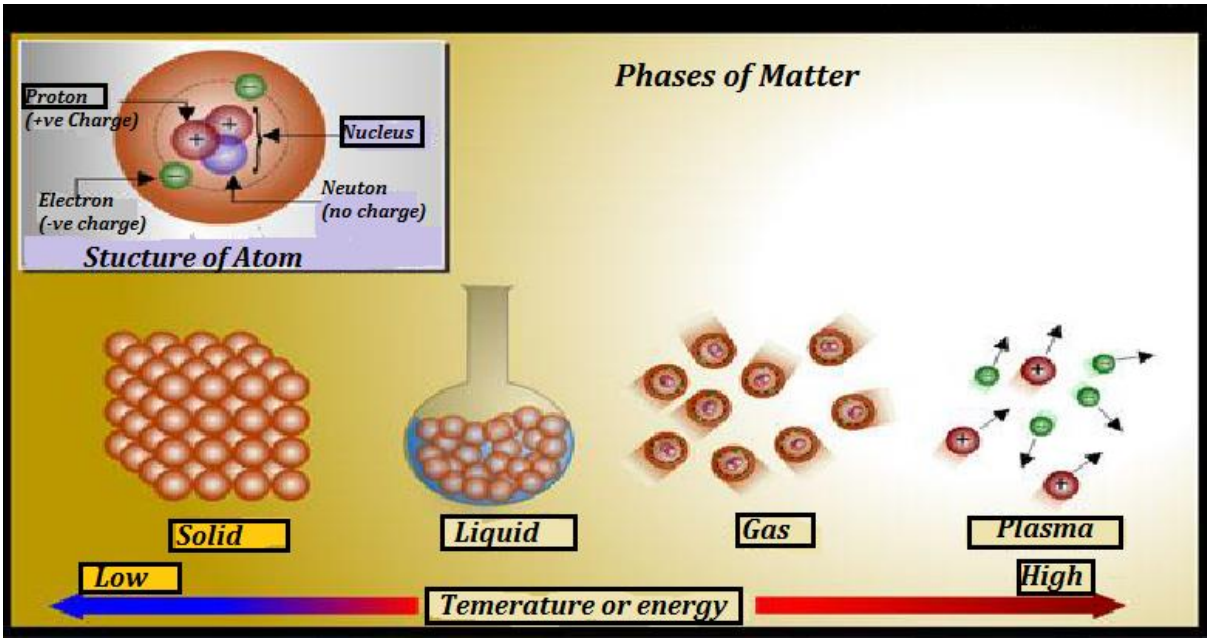


Fig 1.1 Phases of Matter

Ionization can be achieved even by different means such as by intense laser or electromagnetic field from a microwave generator or by the application of electric field so that the atoms absorbs sufficient energy to dissolve into electrons and positive ions. The definition of plasma goes as follows "**A plasma is Quasineutral gas of charged and neutral particle which exhibits collective behaviour**." Collective behaviour means the motion of charged particle is not just confined to local region but it effects even remote location in plasma. This means the motion of charged particle is dependent on the state of plasma even in the remote region. Hence collective behaviour in the definition. Quasineutral means that the plasma is neutral enough so that even a minor disturbance or external electric field will not have any effect after small distance. This distance is called as Deby length. Thus in plasma particles are shielded from external disturbance. Hence at any region in plasma we can take density of electron n_e as equal to the density of ions n_i.

$n_e \approx n_i \approx n$, $where\ n$ is commonly called as plasma density.

Because of presence of charged particle plasma is electrically conductive and instantly gives response to electric and magnetic fields. This feature quite different from solids, liquids, gases and hence it is considered to be distinctive state of matter. Like gas plasma also do not have any distinctive shape or definite volume unless enclosed in container. But unlike gas it can form various structures under the influence of external fields.

1.2.1 Plasma Conditions:

Every ionized gas cannot be called as plasma there are certain conditions that are to be met.

$$\lambda_d \ll L,$$

$$N_D \gg 1,$$

$$\omega_p \tau > 1,$$

where λ_d is the Deby length [2] which is nothing but when an electric field introduced inside a plasma the range upto which its effect can be felt. After this length it will not affect any of the charged particle. N_D is total number of particles in the Deby sphere. Deby sphere is the spherical volume enclosing the effect of external electric field with radius of Deby length. ω_p is the plasma frequency with which the charged particle of plasma oscillates. τ is the mean time between collisions with the neutral atom.

Even the partially ionized gas can have the characteristics of plasma when it can satisfy above conditions. Plasma density n and kT_e are important parameters that characterise plasma. The plasma density can vary from 10^6 to 10^{34} in order of 10^{28} and kT_e can vary from 0.1 to 10^6 eV. The importance of the range of density can be visualised if we know that the density difference between water and air is just 10^5.

1.2.2 Some Parameters of Plasma

Plasma Electron Frequency:

This is the natural electron oscillation in plasma and represented ω_{pe}. The electric field inside plasma is responsible for this electron oscillation. Its value in cgs is as given

$$\omega_{pe} = \sqrt{\frac{4\pi n_0 e^2}{m}},$$

where e, m, n_0 are charge of electron, mass of electron and density of plasma.

Plasma Density:

Plasma density is one important parameter in my project work. Generally plasma density is considered to be uniform but there can be variation or gradient in plasma density also. Plasma density is represented by n_0. In my work I have taken plasma density is varying along transverse direction as represented below.

$$n_0(y) = n_0^0\left(1 + \frac{y}{d_l}\right)$$

We can see from the above expression and can visualise it as density changing as we move upwards in plasma space. Where n_0^0 is initial plasma density and d_l is length gradient which according above ramp equation is responsible for its slope.

1.3 LASERS

Light Amplification by Stimulated Emission of Radiation (LASER)

Laser is radiation emitted by stimulated emission and enhanced by optical amplification. The main difference between normal light and laser is it emits light coherently. This coherence phenomena allows laser to be focussed at a spot sharply. The other feature of laser is collimation which is the spread of laser. Coherence and collimation allows laser to be narrow over large distance. Collimation is main property for being laser being used as Laser Pointer.

1.3.1 Operation and Design of laser:

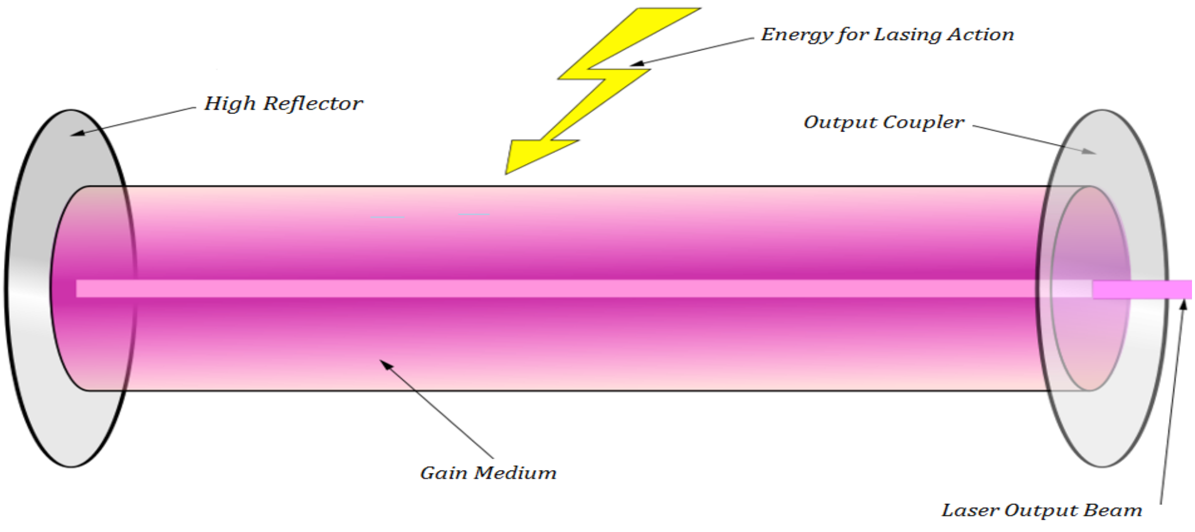
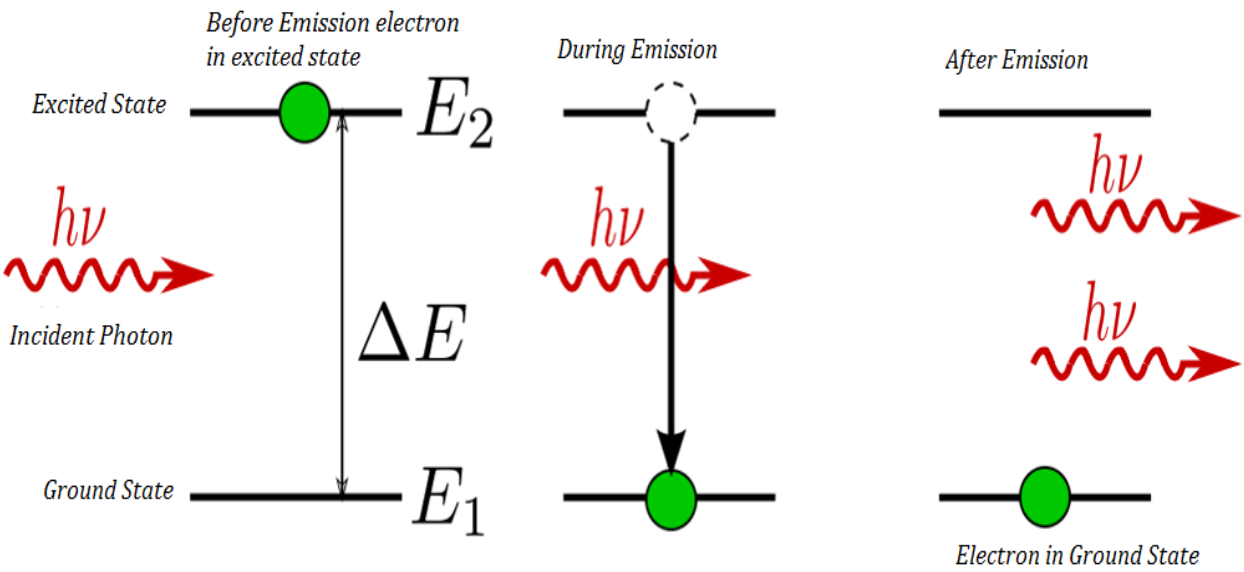


Fig 1.2 Design of Laser

Laser construction has basic component such as gain medium, energy for lasing action, High Reflector, output coupler, Laser output beam port. Gain medium is component where stimulated emission undergoes amplifications before getting emitted. High Reflector is used in the complete reflection of stimulated photon so that it multiplies. After sufficient energy supplied for stimulated emission to occur, the emission multiplies with help of high reflector. Output reflector is partial reflector which allows part of enhanced stimulated emission to reflect back and part to be transmitted as Laser output. Coming to operation of laser. It can be explained as follows. When an electron in ground state gets energy moves to excited state then after sometime the electron reaches back to ground state. This emission is called as spontaneous emission. But instead of this procedure if electron in excited state is forced by external photon to move back to lower energy state which can be ground state or some intermediate energy state before its time to return, then the resultant emission is called as stimulated emission. Then process can be clearly seen in below diagram.



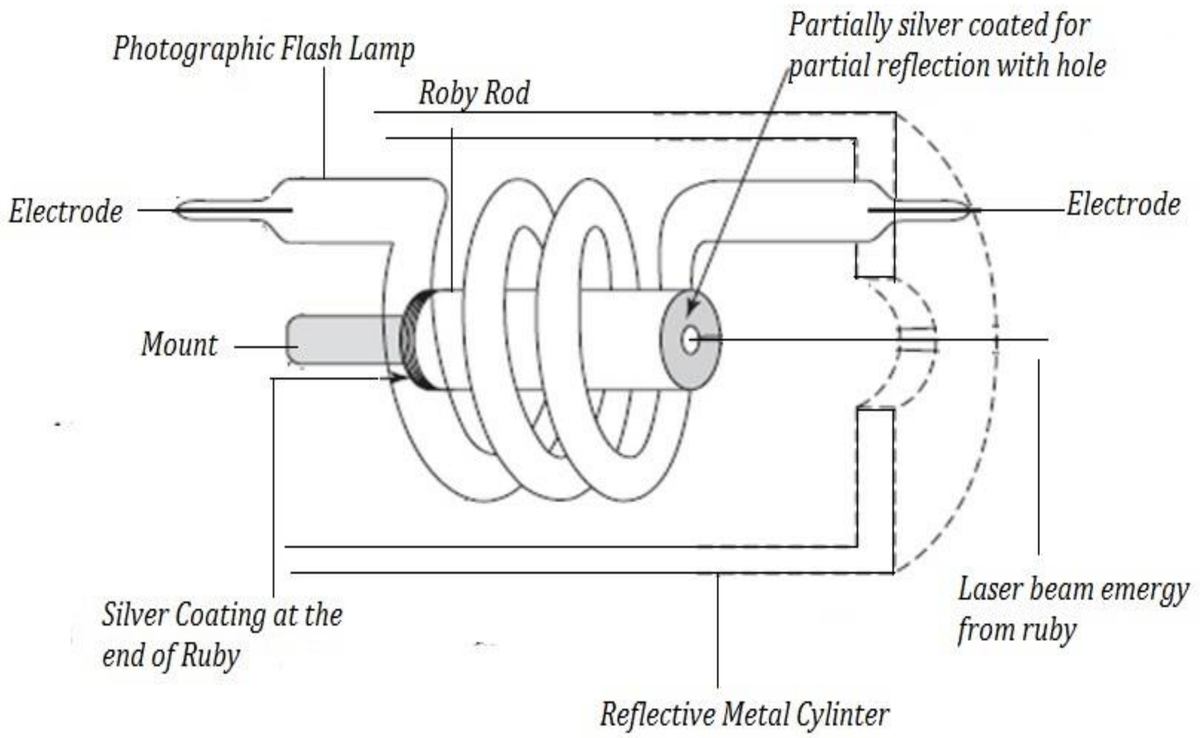
$$E_2 - E_1 = \Delta E = h\nu$$

Fig 1.3 Process of Stimulated Emission

1.3.2 Types of Lasers:

Laser medium can be solid, liquid or gas and can be chosen depending on requirement.

Solid state laser have solid crystal structure such as ruby crystal as laser medium. The crystal structure acts as active medium. The crystal structure is surrounded by neon flash lamp which acts as energy provider for lasing action to happen inside the crystal. Both the ends of crystal are properly polished so that one end acts as complete reflector and other end acts as partial reflector and partial transmitter. These reflections helps in amplifying the intensity of laser. Semiconductor lasers can also be termed as solid state laser but often treated as separate class because of difference in pumping process. Semiconductor lasers employ pumping electrically. Most common solid state laser is neodymium-doped yttrium aluminium garnet (Nd:YAG). Neodymium-doped glass (Nd:glass) and ytterbium-doped glasses or ceramic



Cross Sectional Stucture of Solid state Laser

Fig 1.4 Solid State Laser

The gas laser can have mixture of gases such as helium and neon, carbon dioxide (CO_2) laser, carbon monoxide laser (CO) inside a tube of length ranging from 0.25m to 1m and the diameter around 1cm. Here the lasing action is produced by the discharge of electric current through the gas. At both ends we have two reflecting mirror.one can reflect completely and other reflect partially. When dc discharge is provided there will be stimulated emissions which are amplified by two reflectors and laser is produced. There are also chemical lasers where discharge is produced by chemical reaction. Gas laser are much advantageous as the active medium is relatively inexpensive and also the active medium is nearly impossible to be damaged. Also heat can be relatively removed quickly from gas lasers when compared with other. He-ne lasers are used in reading bar code and making holograms.

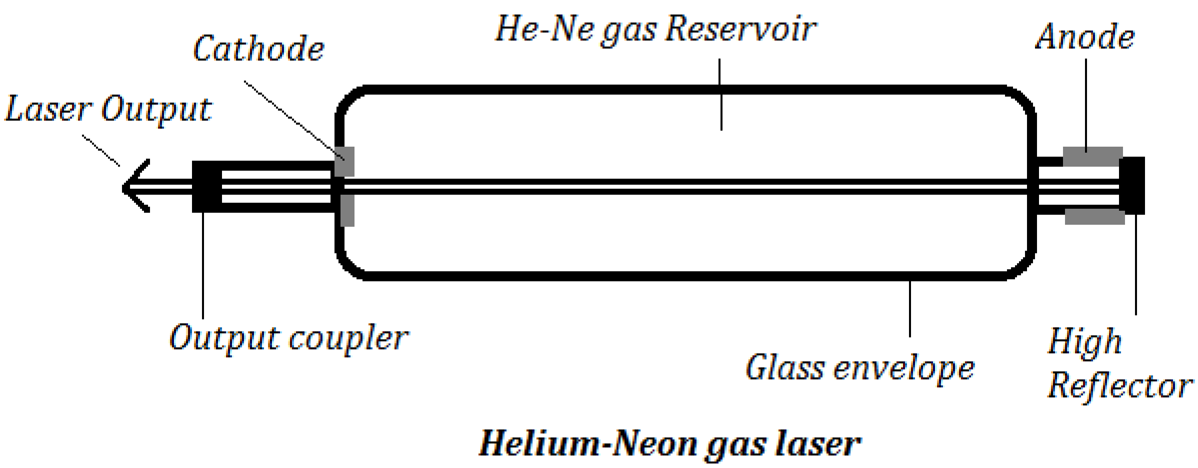


Fig 1.5 Gas Laser

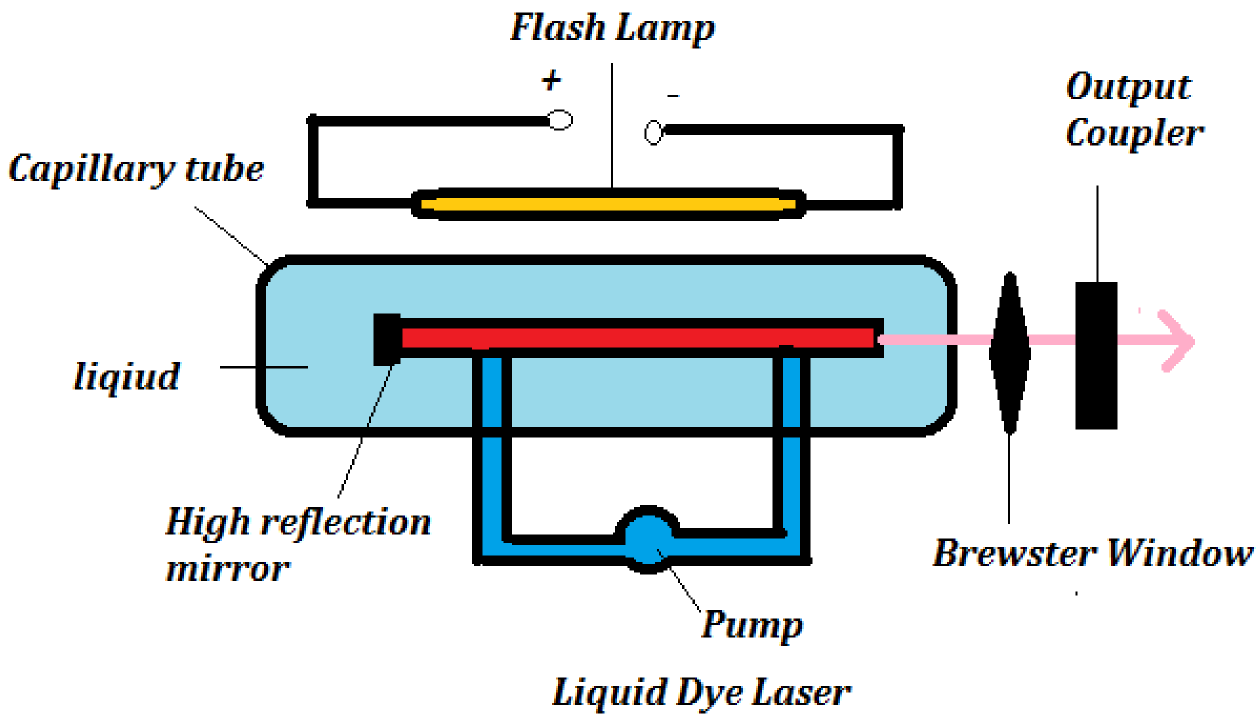
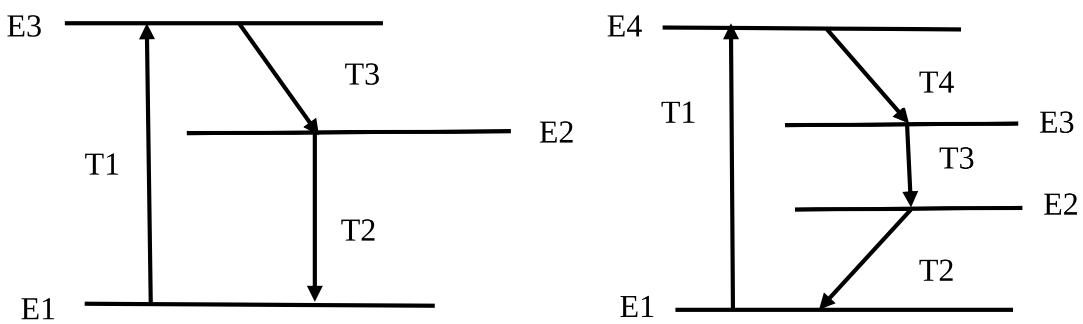


Fig 1.6 Liquid Laser

Liquid laser uses liquid as reflector and amplification medium of stimulated emission. The lasing medium or active medium is generally complex organic dye such as rhodamine 6G doped liquid hence also called dye lasers. The pump source can be flash lamp or another laser. The operation can be either continuous mode or pulsed mode. These lasers are used in bio medical sensing and display screens since they are cost effective.

1.3.3 3 and 4 level lasers:

The main mechanism to attain lasing action is population inversion. Population inversion is nothing but at a given time for continuous laser operation to occur the number of electrons in higher energy state should be more than number electrons in lower energy state most probably ground state. This condition is possible when the electron in higher energy state takes more time to de- excite. Consider an atom having only two states that is excited and ground state in this case electrons cannot stand for longer time in excited state so we say that 2 level lasers are not possible. Now consider a third energy state where electron instead of falling to ground state will fall on intermediate state then electron can stay bit longer in intermediate state and population inversion is possible. This intermediate energy state is called as Meta Stable state. So we can say that to attain population inversion we must require atleast 3 energy levels. But even with 3 level laser continuous lasing is not possible, because for only a certain period of time population inversion exists as electron from Meta stable state moves back to ground state. So if we eliminate the possibility of electron falling back to ground state from Meta stable state by placing one more energy state above ground state then continuous population inversion can be achieved and resulting in continuous lasing action.



3 Level Laser

4 Level laser

Fig 1.7 3 & 4 Level Laser Operation

In 3 level laser case T3 is fast decay and T2 is far greater than T3. Similarly in 4 level laser case T4 is fast decay and T3, T2 both are far greater than T4. Population inversion can be achieved at E3 or E2.

1.3.4 Generating short Pulse Laser:

In my work I have dealt with ultra-short laser pulse of duration in femto seconds ($\approx 10^{-15} sec$). Ultra-short laser pulses are defined by high peak intensities and they lead nonlinear interactions with medium they interact with. Since pondermotive force is nonlinear interaction in plasma we can say that ultra-short laser pulse enhances this force in laser plasma interaction. Hence it is of important consideration to know the mechanism these ultra-short laser pulse generation.

There are generally two methods with these pulses can be generated they are

1. Q-Switching
2. Mode locking

1 Q-Switching:

It's the method of generating ultra-short laser pulse using concept of varying Q factor inside laser resonator. A variable attenuator is kept inside the laser resonator and this acts as Q switch. To increase Q factor we must stop partial reflection at output coupler of laser resonator for some time and this can be done by placing variable attenuator before output coupler. So when attenuator is 'ON' the radiation does not pass towards output coupler and instead completely reflected back continuously for some time. So for this time being there will not be any laser output and instead due to increase in reflections, the intensity of stimulated emissions keep on increasing. Now when the variable attenuator is 'OFF' the operation of out coupler comes into play and we can get short pulse laser with increased intensity. Again after some time the variable attenuator gets 'ON' and the entire procedure repeats. Basically the variable attenuator acts as an 'ON-OFF' switch hence the name Q-Switching. But Q Switching produces nano second laser pulse and to get femto second laser pulse we must use mode locking

2 Mode locking:

Inside the laser resonator there are multiple modes that will be oscillating. The phase of each mode is different and is not fixed which severely effects the intensity of laser. In mode locking method the different modes that oscillate independently are made to oscillate at fixed phase. So we can say all the modes are phase locked and due to these all oscillating pulses contribute constructively in attaining a peak intensity for particular duration which can be upto femto seconds long. We can visualise this process with formation of tides in ocean. As minor ripples whose phase are similar in ocean work constructively to form huge tide which can exist for some time here also the different oscillating modes when made to oscillate at a fixed phase will work constructively to form high peak laser for short duration.

1.3.5 Chirped Pulse Amplification:

The laser pulse used in my work is of large intensity and ultra-short duration. We use chirped amplification process in amplifying the ultra-short laser pulse. Chirped Pulse amplification has been of great importance in gaining high power short laser pulse. In conventional laser the amplification is done only by chain of optical amplifiers which can be problematic if we have application of high power laser as at high power generation number of nonlinear distortions like self-focussing, self-phase modulation, filamentation come into play in optical amplifiers. So at high power laser generation chirped pulse amplification is used. In the first step of chirped pulse amplification the pulse is stretched temporally by using a pair of gratings as shown in figure.

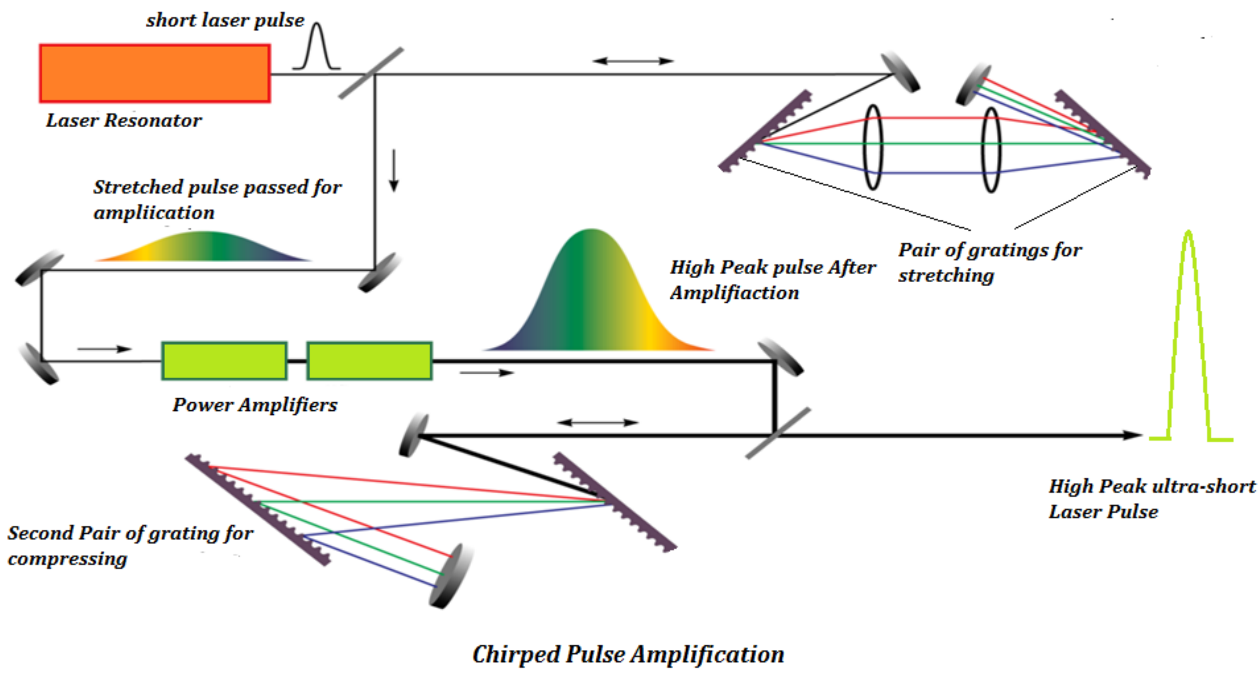


Fig 1.8 Process of Chirped Pulse Amplification

The stretching is done by relative delay of different frequency components in the low power short laser pulse. This process is called chirping. Now this stretched pulse is amplified by conventional optical amplifiers. Since the stretched pulse is of low power there will not be any distortions that I mentioned above come into play. Now the stretched amplified pulse is again compressed by suitable grating arrangement as shown in figure so that pulse duration remains same as original pulse but with large peak value. The distance between the gratings decides the stretching and compressing phenomena pulse.

1.3.6 Various Pressures inside Laser produced plasma:

There are different kinds of pressure experienced on the plasma produced by laser. They are as follows

1. Light Pressure P_L

2. Electron Pressure P_e

3. Ion Pressure P_i

4. Ablation pressure P_a

Light Pressure P_L is caused directly by laser after its irradiation on Target. Its expression is given as follows

$$P_L = \frac{I_L}{c}(1+R) \cong 3.3Mbar\left(\frac{I_L}{10^{16}\frac{W}{cm^2}}\right)(1+R),$$

where R is laser reflectivity whose value ranges from 0 to 1 and c is the speed of light. If irradiance is large then we can have light pressure as close to 1Gbar which effects significantly in magnetic field generation.

Second and third Pressure are in the corona region i.e. cold electron pressure, hot electron pressure and ion pressure. These pressures are associated with different temperature range of cold electron T_e, hot electron T_H and for ions T_i. The hot and cold electron temperatures are obtained in the corona if the electrons have different velocity distribution as seen from Fountain Effect. After knowing ideal gas equation can be used to get respective pressure.

$$P_e = n_e k_B T_e \cong 1.6Mbar\left(\frac{n_e}{10^{21}cm^{-3}}\right)\left(\frac{T_e}{Kev}\right),$$

$$P_H = n_H k_B T_H,$$

$$P_i = n_i k_B T_i.$$

Ablation is the fourth kind of pressure which is associated with flow of heated plasma from solid target. This ablation pressure drives a shock wave into the target which causes it to compress.

1.3.7 Laser Intensity Parameter a_0

Lasers having intensities of order $10^{18}\ W/cm^2$ has Electric Field Intensity of order $10^{12} V/cm$ and this can easily ionise atoms to generate plasma. This generation of Plasma was possible because the Laser induced electric field will be much greater than the columbic attraction force within atoms. Electrons that will be oscillating within laser electric field attain energy which is called as quiver energy. The intensity of laser can be I_L can be expressed in terms of normalized laser vector potential a_0 which in turn can be expressed in terms of quiver momentum as follows

$$a_{0=} \frac{P_{oscillation}\ or\ (quiver\ momentum = mv_{osc})}{mc}$$

$$a_0 = \frac{eE}{m\omega c} = \sqrt{\frac{I_L {\lambda_{um}}^2}{1.37 \times 10^{18}}},$$

where I is intensity and $\lambda_{um}$is wavelength. So we can say value of $a_0$is relativistic measure of laser intensity *I*. Depending on the value of $a_0 < 1\ or a_0\ > 0$ the intensity I_L will be above or below the relativistic limit. The relativistic factor can be calculated as follows.

$\gamma = \sqrt{(1 + {a_0}^2)}$ for circularly polarised laser.

$\gamma = \sqrt{1 + \frac{{a_0}^2}{2}}$ for linearly polarised Laser..

Chapter 2

Review of Literature

The purpose of my research is to study the characteristics and behaviour of magnetic field for respective electric field envelope. Since this magnetic field is self-induced after laser irradiation of the target, I have to study the process of self-induced plasma magnetisation such as Biermann Battery, Inverse Faraday Effect, Plasma Magnetisation Due to Fast Currents or Fountain Effect, magnetisation by Pondermotive force, Weibel Instability etc.

2.1 Laser Plasma Interaction to generate self-Induced Magnetic Fields

In this we are going to see various mechanisms for which magnetic fields can be generated in plasma when it interacts with laser. Study of generation of magnetic fields in different topics of physics such as in cosmic environment, laser produced plasma etc., has been important since many decades. Main consideration is how initially magnetic field free plasma can generate huge magnetic field in itself. It can be understood if we can know the mechanisms with which we are getting electron current density j_e along with electric field E. If we can understand these, then from Maxwell's equation $\nabla \times B = u_0 j \; and \; \nabla \times E = -\frac{\partial B}{\partial t}$, we can easily infer the generation of magnetic field [6,7,8,9,10,11,12,13]. In plasma, a separation must be created between ions and electrons so that electric field is established which in turns is responsible for the generation of current by accelerating electron against the force of attraction of ions. This work is accomplished by generation of hot electrons when intense laser interacts with plasma. The generated current or current density creates magnetic field as interpreted from Maxwell's equations. There are different method with which a magnetic field can be created in plasma after laser interactions such as Biermann Battery, Inverse Faraday Effect, Pondermotive Force developed from the interaction if intense ultra-short laser pulse with plasma, Weibel Instabilities, Thermal Instabilities etc. All these methods or mechanisms describe how ions and electrons are separated suitably to get electric field E which in turns accelerate the electrons providing current density ultimately paving path for magnetic field generation. Also these methods are cable of generating small to large magnetic fields. Here we mainly concentrated on first three methods of magnetic field generation which are discussed in detailed below.

2.1.1 The $\nabla n_e \times \nabla T_e$ Mechanism (Biermann Battery or Thermo Electric Mechanism):

Sclutter and Biermann [14], about sixty years ago demonstrated this Biermann Battery mechanism for rotating magnetized stars in non-relativistic plasma regime. They stated that the non-parallel density gradient ∇n_e [15] and temperature gradient ∇T_e will create a rotating magnetic field whose magnitude depends on the angle between the two gradients. When we write equation of motion for plasma then one of the force terms on the right hand side will be due to pressure gradient. This pressure gradient ∇P_e is responsible for this Biermann Battery.

where $P_e = K_B n_e T_e$, where K_B *is boltzmann constant*

Now neglecting linear and nonlinear inertia of electron, i.e., $m\frac{\partial v}{\partial t} = 0$ and writing equation of motion, we obtain

$$0 = -\nabla P_e - e n_e E,$$

Getting Electric field from this, we get

$$E = -\frac{\nabla P_e}{e n_e},$$

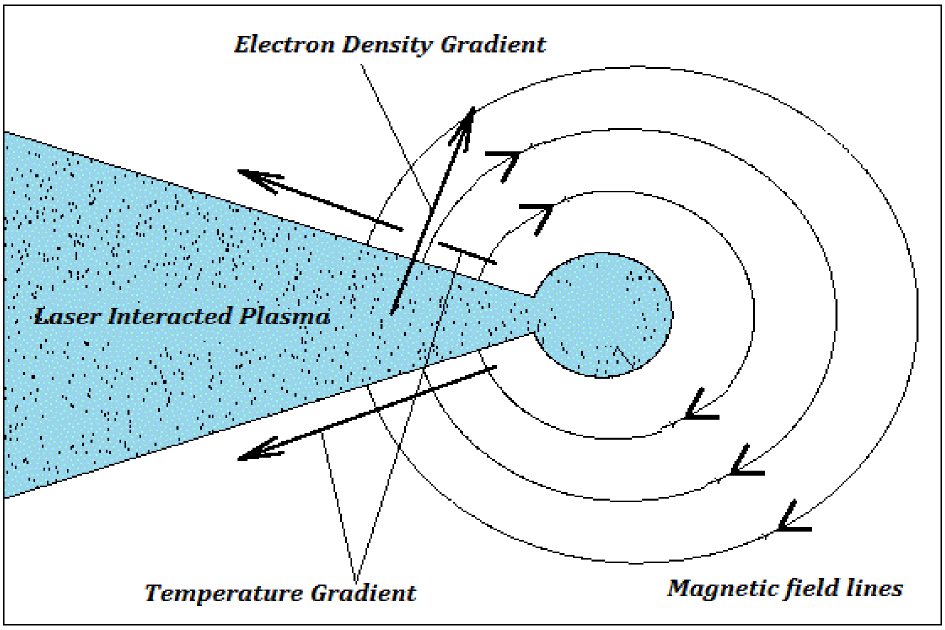


Fig 2.1 Biermann Battery

But from one of Maxwell's equation

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}.$$

Substitute E from above in Maxwell's equation and also $P_e = K_B n_e T_e$, we obtain

$$\frac{\partial B}{\partial t} = \frac{c}{en_e}\nabla \times \nabla P_e \ ,$$

$$\frac{\partial B}{\partial t} = \frac{ck_B}{en_e}\nabla \times \nabla(n_e T_e) \ .$$

Now using vector identity we can solve above equation to obtain

$$\frac{\partial B}{\partial t} = \frac{ck_B}{en_e}\nabla T_e \times \nabla n_e \ .$$

If we carefully analyse above equation we see that when temperature gradient $\nabla T_e$and density gradient ∇n_e are non-parallel then there always exists a rotating magnetic field B. it can be noted this phenomena is most dominant in generation of toroidal magnetic field [16].

$$\boxed{\frac{\partial B}{\partial t} = \frac{ck_B}{en_e}\nabla T_e \times \nabla n_e.}$$

This Biermann battery can also be visualised from above diagram.

For target irradiated with large intensity laser this Biermann mechanism can most probably occur on the outer region of laser spot [17]. The density gradient is at the edge of laser spot on surface plasma and the temperature gradient points towards the axis of laser beam. The generated magnetic field is toroidal and has scale size is comparable to the laser spot which falls to zero at the centre of laser spot. Magnetic field of approximately 340 MG [18]have also been at critical density surface of laser induced plasma.

2.1.2 The Inverse Faraday Effect:

The Inverse Faraday Effect is the phenomena where magnetic field gets created due to rotational electromagnetic waves. A circularly polarised laser can induce magnetic field using this phenomena in plasma the field gets created because electron quivers with energy attained from high intensity laser and if the laser is circularly polarised then electron follows or gain circular motion resulting in formation flowing electric current which in turns assists in generation of magnetic field. The creation of magnetic field from inverse Faraday Effect in cold plasma can be established as follows.

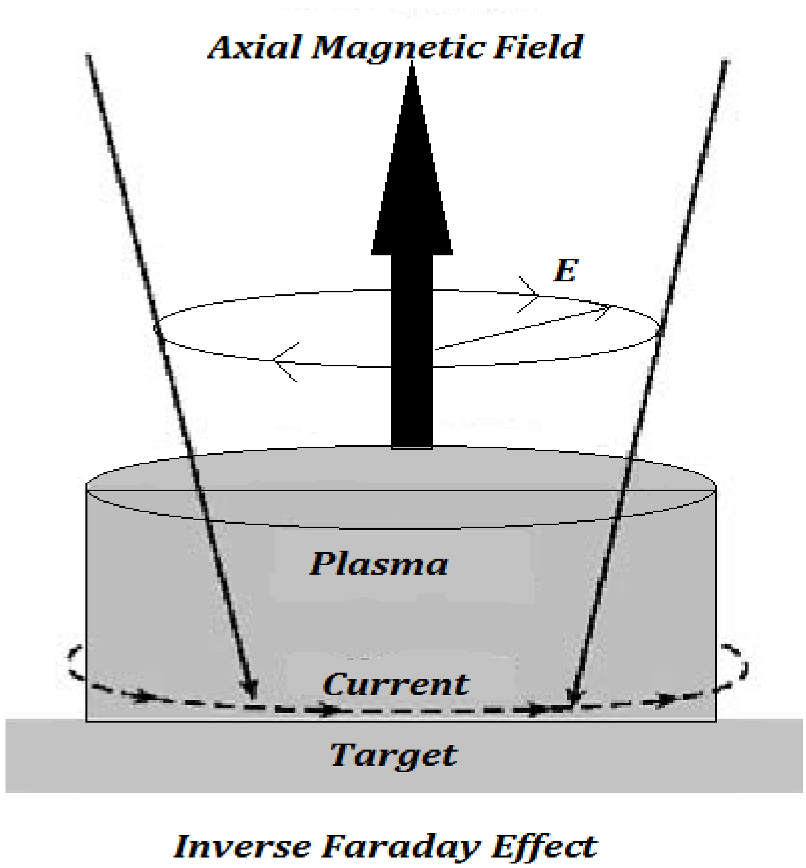


Fig 2.2 Inverse Faraday Effect

The equation of motion due to applied electric field E only can be as shown

$$m_e \frac{\partial v}{\partial t} = -eE ,$$

where v is electron velocity due to applied electric field E which came into existence due to plasma interaction with laser. Now since laser is to be circularly polarise along $\hat{z}$ directions its equation can be written as

$$E = E_0 \left(\frac{\hat{x} + i\hat{y}}{\sqrt{2}} \right) e^{-i(\omega t - kz)} ,$$

where $\hat{x}$ and $\hat{y}$ are unit vector along x and y direction ,respectively.

Substituting this circularly polarised electric field in equation shown above , we obtain

$$m_e \frac{\partial v}{\partial t} = -eE_0 \left(\frac{\hat{x} + i\hat{y}}{\sqrt{2}} \right) e^{-i(\omega t - kz)},$$

Solving above partial differential equation for v we get

$$v = v_0 \left(\frac{\hat{x} + i\hat{y}}{\sqrt{2}} \right) e^{-i(\omega t - kz)}.$$

We can clearly see from above equation for circularly polarised laser or electric field induced by laser interaction with cold plasma, will induce electron motion also in circular direction

Since electrons also satisfy continuity equation, the continuity equation can be written as follows.

$$\frac{\partial n_e}{\partial t} = -\nabla.(n_e v).$$

The electron density is assumed to be consisting of all background component n_0 and perturbed component n_1.

$$n_e = n_0 + n_1.$$

n_0 does not depend on time but n_1 is dependent on time as follows

$$n_1 \approx e^{-i\omega t}.$$

Substituting $\frac{\partial}{\partial t} = -i\omega;$

$$i\omega n_1 = v.(\nabla n_0).$$

Now the current density can be obtained from

$$J = -e < n_1 v >,$$

where $< >$ indicates median or average value to be taken.

Substituting n_1 from above in current density equation, we get

$$J = \frac{ie}{\omega} < v.(\nabla n_0)v >.$$

Substituting v from above calculations in J we finally get

$$J = \frac{e^3 {E_0}^2}{2m_e \omega^3} \nabla n_0 \times \hat{z},$$

$\hat{z}$ is unit vector along z direction. Also wave number vector k is parallel to z direction.

We can infer from above current density equation that the generated current density is due to density gradient ∇n_0 on the surface of plasma and it is toroidal due to cross product. This current density generates axial magnetic field B according to following Maxwell's equation.

$$\nabla \times B = \frac{4\pi}{c} J.$$

Solving for B we can get magnetic field from inverse Faraday Effect. Studies have shown that several order of Mega Gauss fields can be generated from inverse Faraday Effect.

2.1.3 Magnetic Field from Fountain Effect for finite geometry of Plasma:

In this process resonance absorption plays the important role. When laser hits the plasma target hot electrons created due to resonance absorption will flow in the direction of density gradient which is along the edge of the focal spot on surface of the plasma. This effect has counter effect of cold electrons flowing inwards creating Toroidal Magnetic Field due to net current density gradient or net current from inward and outward flow of electron. This is called as Fountain Effect [19] and is considered in finite geometry of plasma only as in infinite plasma the net current would be zero resulting in zero magnetic field. The direction or orientation of the magnetic field generated from this effect is same as from the Biermann or thermoelectric effect. Here the generated field can also be asymmetrical depending on the angle of incidence of laser on plasma and process of resonance absorption.

The mathematics involved can be shown as follows [20].

The electric field generated during return current interaction is given as

$$E = \eta j_b,$$

where η is resistivity and j_b is return current density.

Applying Quasineutrality we will get the net current density as

$$j = j_b + j_{fast},$$

where j is total current density and j_{fast} is current density due to fast or hot electrons.

From Maxwell's equation, we have

$$\nabla \times B = \mu_0 j_b.$$

The total electric field is given by

$$E = -\eta j_{fast} + \frac{\eta}{\mu_0} \nabla \times B.$$

Now we can get magnetic field directly as shown below.

$$\frac{\partial B}{\partial t} = \nabla \times \eta j_{fast} - \nabla \times \frac{\eta}{\mu_0} \nabla \times B.$$

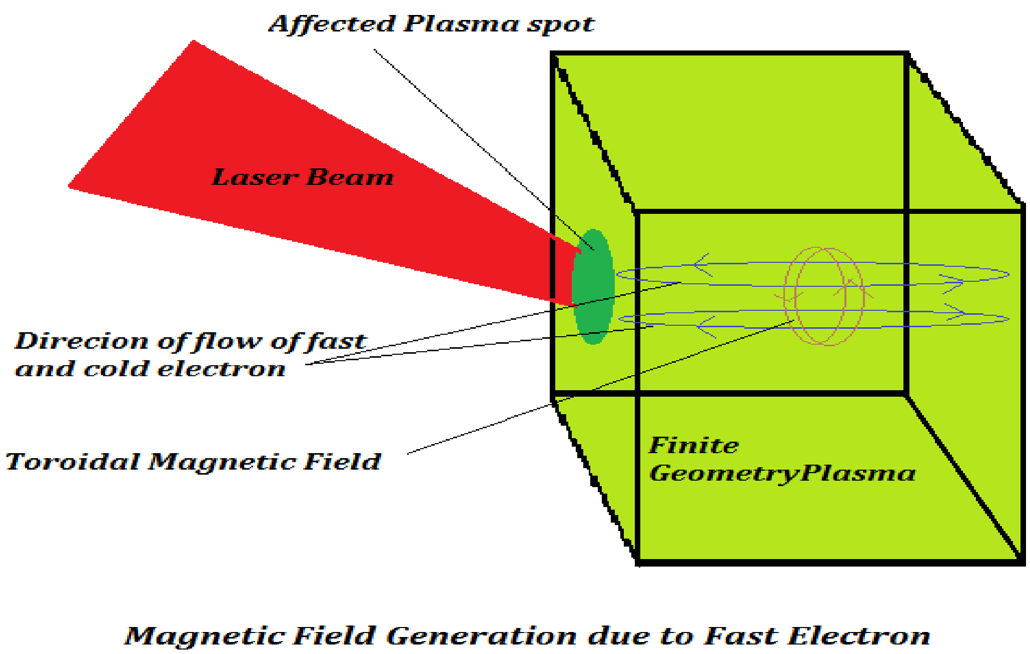


Fig 2.3 Magnetic field generation due to fast electrons

Clark *et al* have recorder the maximum magnetic field due to Fountain Effect using high energy proton is as close to as 32MGauss which shows the importance of the Fountain Effect in magnetic field generation of finite geometry plasma [21,22].

2.1.4 Plasma Magnetisation by a Pondermotive Effect:

The nonuniform nature of electromagnetic beam, in this case laser is responsible for pondermotive force. If the laser pulse is of nonuniform nature then pondermotive force have been considered as alternative mechanism in the magnetic field generation in plasma. The term pondermotive force is the radiation pressure exerted by the nonuniform intense laser or high energy nonuniform electromagnetic beam that are interacting with plasma. The nonlinear Lorentz force in the magnetic and electric fields of nonuniform electromagnetic wave is responsible for the creation of low frequency pondermotive force which mainly acts on the electron as their mass is less compared to ions. This effect on the electron effects in the generation of current density which in turn creates the magnetic field. Thus plasma is magnetised by the pondermotive force or nonuniform electromagnetic beam. The pondermotive force is proportional to the intensity of the laser beam can swiftly push the electron from the high intensity field region and pile up in the low intensity field region [22].

Consider single particle motion n the electromagnetic field.

Let us assume the monochromatic electromagnetic field due to intense laser is given by

$$E = E_0 \cos(\omega t).$$

From Maxwell's equation

$$\frac{\partial B}{\partial t} = -\nabla \times E.$$

Thus these here we have sum up some of the mechanisms which are responsible for the magnetisation of plasma especially magnetic field due Biermann Battery, Magnetic field due to Inverse Faraday Effect, Magnetic field generated from Fountain effect, Magnetic field Generated from Nonlinear nature of electromagnetic pulse. Each of the mechanism has the potential to generate stationary or non-stationary magnetic fields whose intensity depends on various plasma conditions. Such as density gradient, temperature gradient, nature of incident laser or electromagnetic beam, geometry of plasma etc. the generated magnetic are critically important in understanding the behaviour of plasma. In my work I have considered these effects and attained a second order non homogenous equation in magnetic field. We have analytically solved the differential equation for various pulse profiles which are discussed in next chapter to study the behaviour generated magnetic field [24].

Chapter 3

Report on the Present Investigation

3.1 Second Order Differential Equation in Magnetic Field

Plasma density is one important parameter in my research. Generally plasma density is considered to be uniform but there can be variation or gradient in plasma density also. Plasma density is represented by n_0. In this work we have taken plasma density is varying along transverse direction as represented below.

$$n_0(x) = n_0^0\left(1 + \frac{y}{d_l}\right).$$

We can see from above expression and can visualise it as density changing as we move upwards in plasma space. Where n_0^0 is initial plasma density and d_l is length gradient which according above ramp equation is responsible for its slope.

Now the current density can be obtained from following equation

$$J_a = -n_0(x)ev_2.$$

Equation of Motion

$$m\frac{d\vec{v}_2}{dt} + \left(\overrightarrow{v_1}.\vec{\nabla}\right)\vec{v}_1 = -eE_2 - \frac{e}{c}\left(\overrightarrow{v_1} \times \overrightarrow{B_1}\right).$$

Since E_2 is small we can neglect $-eE_2$ from above equation;

Substituting $\frac{d}{dt} = -i2\omega_l \; and \; \nabla = i2k$ in the above equation

$$m(-i2\omega_l)\overrightarrow{v_2} = -\frac{e}{c}\left(\overrightarrow{v_1} \times \overrightarrow{B_1}\right).$$

Obtaining v_2 from above expression

$$\vec{v}_2 = \frac{e}{4im\omega_l c}\left(\vec{v}_1 \times \vec{B}_1\right).$$

$B = \frac{c}{\omega}(\vec{k} \times \vec{E})$ from Maxwell’s equation

But we have $\vec{v}_1 = \frac{eE_1}{im\omega}$ also$B_1 = B$. Substitute these in above

$$\vec{v}_2 = \frac{-e^2 {E_1}^2 K}{4m^2 {\omega_l}^3}\hat{x}.$$

Substitute this $\vec{v}_2$ $in\ J_a\ above\ and\ E_1 = E$

$$J_a = J_x = n_0(1+\frac{y}{dl})\frac{e^3E^2K}{4m^2{\omega_l}^3}\hat{x}.$$

From Maxwell's equation

$$\vec{\nabla} \times B_a = \frac{4\pi}{c}J_x + \frac{1}{c}\frac{\partial E_a}{\partial t}.$$

Taking curl on both sides

$$\vec{\nabla} \times \vec{\nabla} \times B_a = \frac{4\pi}{c}\vec{\nabla} \times J_x + \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{E}_a\right).$$

Calculating $\vec{\nabla} \times J_x\ and\ from\ Maxwell\ equation\ \vec{\nabla} \times E_a = \frac{-1}{c}\frac{\partial B_a}{\partial t}$, we obtain

$$-\vec{\nabla}^2 B_a = \frac{-4\pi}{c}\frac{n_0 e^3 E^2 K}{d_l 4m^2{\omega_l}^3}\hat{z} + \frac{4\omega^2}{c^2}B_a.$$

$$\boxed{\vec{\nabla}^2 B_a + \frac{4{\omega_l}^2}{c^2}B_a = \frac{4\pi}{c}\frac{n_0 e^3 E^2 K}{d_l 4m^2{\omega_l}^3}\hat{z},}$$

where $E^2\ in\ above\ expression\ is\ equal\ to$ $\boxed{E^2 = E^2(\zeta) = {E_0}^2\ f(\xi),}$

where $f(\xi)$ describes the shape of various electric field pulse profile. The detailed description of each profile with expression is shown in next chapter.

Now adjusting above equation in terms of $\omega^2{}_{pe} = \frac{4\pi n_0 e^2}{m}\ and\ {a_0}^2 = \frac{e^2{E_0}^2}{m^2c^2{\omega_l}^2}$.

$We\ get\ second\ order\ differential\ equation\ in\ terms\ of\ magnetic\ field\ B_a(\zeta)\ and$

$electric\ field\ pulse\ envelope\ f(\zeta)\ as\ fallows$

$$\frac{\partial^2 B_a}{\partial \xi^2} + \frac{4{\omega_l}^2}{c^2} B_a = \frac{{\omega_{pe}}^2 cmK{a_0}^2}{\omega_l e d_l} f(\xi),$$

This derivation is arrived by considering cgs system, where $f(\xi)$ is envelope or our electric field pulse profile.

3.2 Electric Field Pulse Profiles

The deferential equation in the magnetic field from the previous chapter is dependent on $f(\xi)$ which is our pulse profile function. This function is part of electric field as shown below.

$$E^2(\xi) = {E_0}^2\, f(\xi).$$

The description of each of the pulse profile in detailed is as shown.

3.2.1 Sine Pulse Electric Field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2\, f(\xi),$$

where $f(\xi)$ describes the sine electric field pulse profile whose expression is as follows [24]

$$f(\xi) = \sin\left(\frac{\pi\xi}{L}\right),\ for\ 0 \le \xi \le L.$$

The shape of the pulse profile is follows.

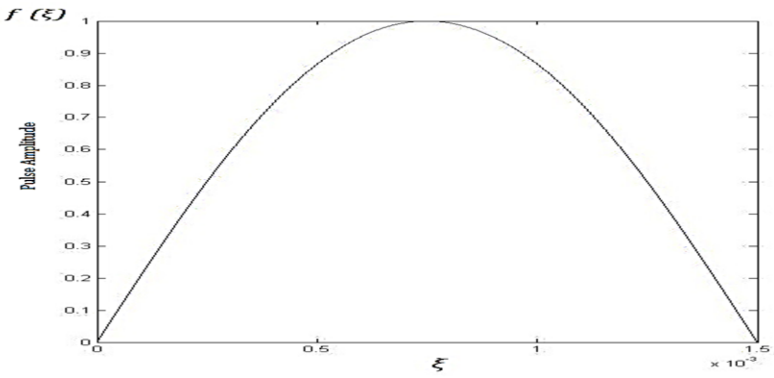


Fig 3.1 Sine Electric Field Pulse Profile.

3.2.2 Triangular Electric Field Pulse Profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2\, f(\xi),$

where $f(\xi)$ describes the Triangular electric field pulse profile whose expression is as follows. The practical triangular electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with two limits [25].

$$f(\xi) \quad = 1 - \cos\left(\frac{\pi\xi}{L}\right),\ for\ 0 \le \xi \le \frac{L}{2},$$
$$= 1 + \cos\left(\frac{\pi\xi}{L}\right) for\ \frac{L}{2} \le \xi \le L.$$

The shape of the pulse profile is as shown

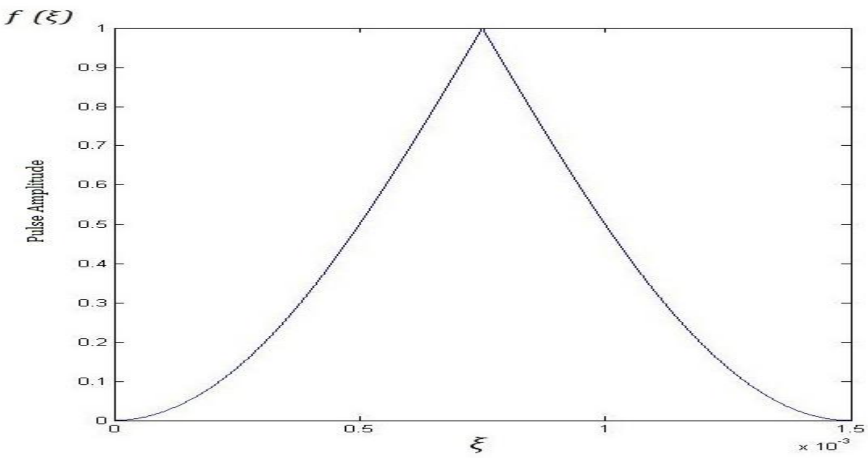


Fig 3.2 Triangular Electric Field Pulse Profile.

3.2.3 Sawtooth Decreasing Electric Field Pulse profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2\, f(\xi),$

where $f(\xi)$ describes the Sawtooth Decreasing electric field pulse profile whose expression is as follows. The practical Sawtooth Decreasing electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with two limits [25].

$$f(\xi) \quad = 1 - \cos\left(\frac{29\pi\xi}{L}\right),\ for\ 0 \le \xi \le \frac{L}{30},$$
$$= 1 + \cos\left(\frac{\pi\xi}{L}\right) for\ \frac{L}{30} \le \xi \le L.$$

The pulse shape looks as follows

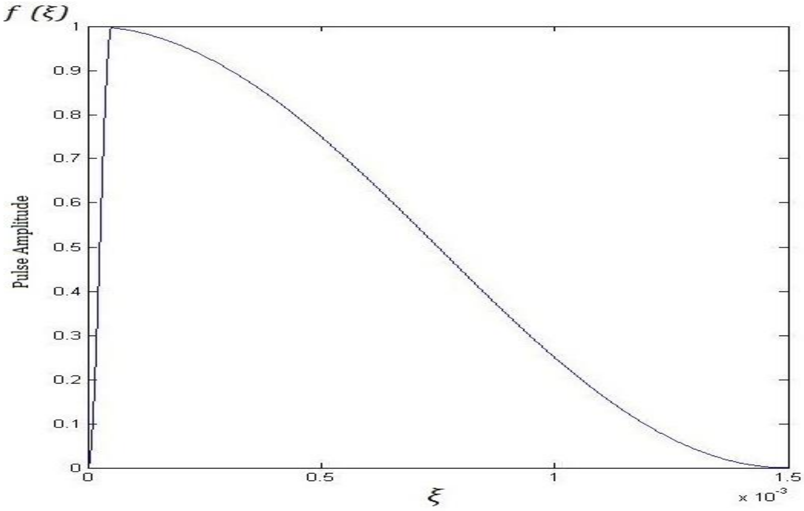


Fig 3.3 Sawtooth Decreasing Electric Field Pulse Profile.

3.2.4 Sawtooth Increasing Electric Field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2\, f(\xi),$$

where $f(\xi)$ describes the Sawtooth Increasing electric field pulse profile whose expression is as follows. The practical Sawtooth Increasing electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with two limits [25].

$$f(\xi) \quad = 1 - \cos\left(\frac{30\pi\xi}{58L}\right),\ for\ 0 \le \xi \le \frac{29L}{30},$$
$$= 1 + \cos\left(\frac{15\pi\xi}{L}\right),\ for\ \frac{29L}{30} \le \xi \le L.$$

The pulse profile looks as follows

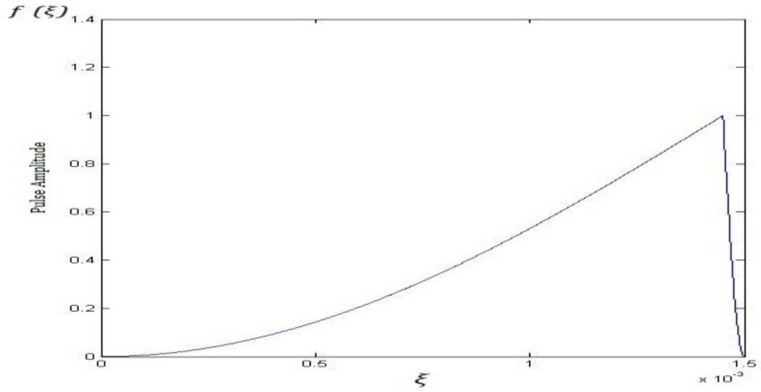


Fig 3.4 Sawtooth Increasing Electric Field Pulse Profile.

3.2.5 Rectangular Triangular Electric field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2\, f(\xi),$$

were $f(\xi)$ describes the Rectangular Triangular electric field pulse profile whose expression is as follows. The Rectangular Triangular electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with three limits [24].

$$\begin{aligned} f(\xi) &= \sqrt{(1-\cos(2\pi\xi/L))},\ for\ 0 \le \xi \le L/4, \\ &= 1, for\ \frac{L}{4} \le \xi \le 2L/3, \\ &= \sqrt{(1+co\,s\left(\frac{3\pi\xi}{L}\right))/2}, for\,\frac{2L}{3} \le \xi \le L \end{aligned}$$

The Rectangular Triangular Electric Field Pulse profile looks as follows

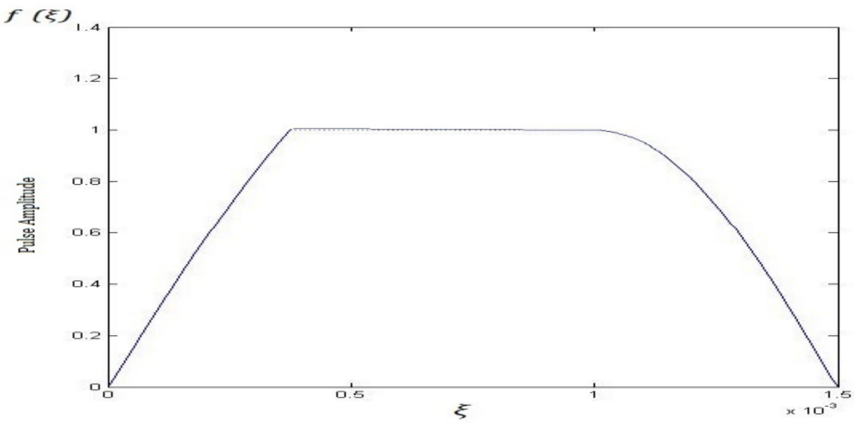


Fig 3.5 Rectangular Triangular Electric Field Pulse Profile.

3.2.6 Square Electric Field Pulse Profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2 \, f(\xi),$

where $f(\xi)$ describes the Square electric field pulse profile whose expression is as follows. The Square electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with three limits [24].

$$
\begin{aligned}
f(\xi) \quad &= 0,\ for\ 0 \le \xi \le L/6, \\
&= 1, for\ \frac{L}{6} \le \xi \le 10L/12, \\
&= 0, for\ \frac{10L}{12} \le \xi \le L.
\end{aligned}
$$

The shape of Square Electric Field Pulse is follows

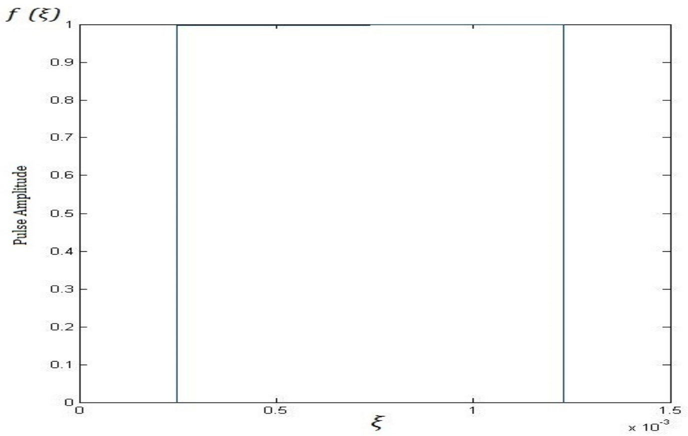


Fig 3.6 Square Electric Field Pulse Profile.

3.3 Solving the Differential Equation for Respective Pulse Profiles

The second order linear inhomogeneous differential equation which we obtained is as follows

$$\vec{\nabla}^2 B_a + \frac{4{\omega_l}^2}{c^2} B_a = \frac{4\pi}{c} \frac{n_0 e^3 E^2 K}{d_l 4m^2 {\omega_l}^3} \hat{z}.$$

Now adjusting above equation in terms of $\omega^2{}_{pe} = \frac{4\pi n_0 e^2}{m}$ $and\ {a_0}^2 = \frac{e^2 {E_0}^2}{m^2 c^2 {\omega_l}^2};$

$$\frac{\partial^2 B_a}{\partial \xi^2} + \frac{4{\omega_l}^2}{c^2} B_a = \frac{{\omega_{pe}}^2 cmK {a_0}^2}{\omega_l e d_l} f(\xi).$$

This derivation is arrived by considering cgs system, where $f(\xi)$ is envelope or our electric field pulse profile, ω_l laser frequency, ω_{pe} is plasma frequency, a_0 laser intensity parameter, e is charge of electron, d_l differential length parameter describing the slope of density ramp which varies along transverse direction, m is mass of electron, n_0 is density at origin, *c* is the speed of light, *K* is given by $\frac{\omega_l}{c}$.

Now assigning constants particular variable as follows

$$U^2 = \frac{4{\omega_l}^2}{c^2},$$

$$A^2 = \frac{{\omega_{pe}}^2 cmK{a_0}^2}{\omega_l e d_l},$$

$$D^2 = \frac{\partial^2}{\partial \xi^2} \; and \; B_a = B(\xi).$$

Therefore the above differential equation reduces to

$$(D^2 + U^2)B(\xi) = A^2 f(\xi).$$

Now employing constant coefficients method to solve the above differential equation. The solution to the homogenous part or the complementary solution is as follows and this solution is same for all the electric field pulse profiles as there is no dependence of the solution on $f(\xi)$ or $f(\xi) = 0$.

$$(D^2 + U^2)B_c(\xi) = 0.$$

The solution to this homogenous part is

$$B_c(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi),$$

where $B_c(\xi)$ is the complementary solution of magnetic field.

Now we can see this part of solution gets added to the solution of all the electric field pulse profiles irrespective of the shape of pulse profile.

Now the solution of differential equation with inhomogeneous part is purely dependent on the components with which $f(\xi)$ is built. This part of solution is called as particular solution. So we have calculated the Particular solution of differential equation individually for individual pulse profiles.

3.3.1 Sine Electric Field Pulse Profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2 f(\xi),$

where $f(\xi)$ describes the sine electric field pulse profile whose expression is as follows [24]

$f(\xi) = \sin\left(\frac{\pi\xi}{L}\right),\ for\ 0 \le \xi \le L.$

Now substituting this pulse profile in the differential equation

$$(D^2 + U^2)B(\xi) = A^2 \sin\left(\frac{\pi\xi}{L}\right),$$

$$B_p(\xi) = \frac{A^2 \sin\left(\frac{\pi\xi}{L}\right)}{(D^2 + U^2)}\ ,for\ 0 \le \xi \le L,$$

where $B_p(\xi)$ particular solution to the magnetic field

According to the constant coefficients methods for sinusoidal pulse of form sin(bx) or cos(bx), substitute $D^2 = -b^2$

Here in our problem $b = \frac{\pi}{L}$.

Substituting these we finally get

$$B_p(\xi) = \frac{A^2 \sin\left(\frac{\pi\xi}{L}\right)}{\left(-(\frac{\pi}{L})^2 + U^2\right)}\ ,for\ 0 \le \xi \le L.$$

Now the total solution to magnetic field is sum of complementary solution and particular solution

$$B(\xi) = B_c(\xi) + B_p(\xi)\ for\ 0 \le \xi \le L,$$

$$B(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2 \sin\left(\frac{\pi\xi}{L}\right)}{\left(-\left(\frac{\pi}{L}\right)^2 + U^2\right)}, for\ 0 \leq \xi \leq L.$$

This is the solution to magnetic field for sine electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$.

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B(\xi) = 18.8558 \sin\left(\frac{\pi\xi}{L}\right) + 0.3144 \sin(U\xi),\ for\ 0 \leq \xi \leq L.}$$

Thus this is the final solution to the differential equation for sinusoidal electric field pulse profile.

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

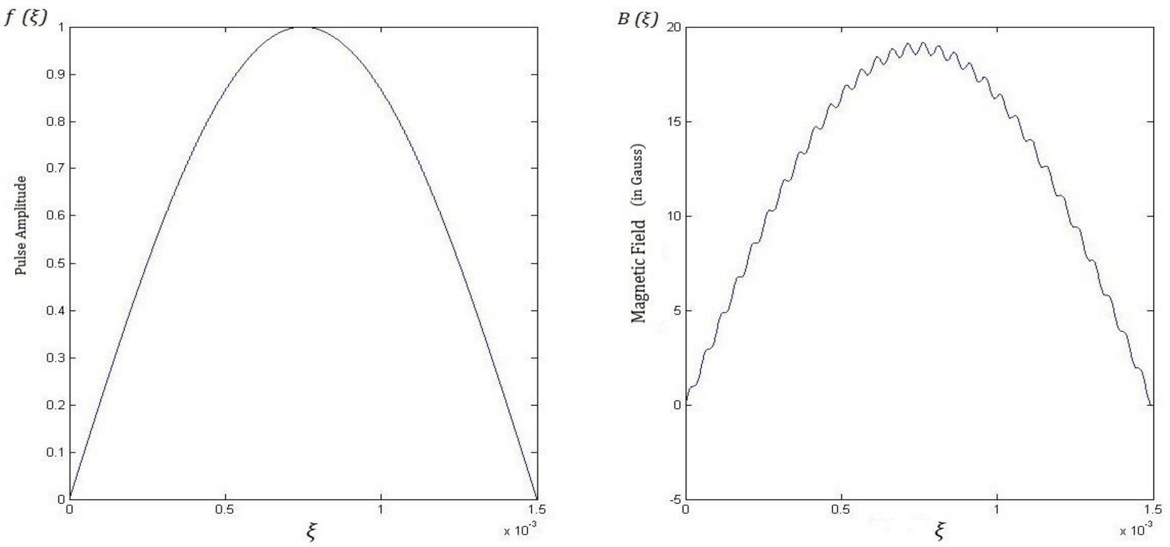
Code:

```
l=1.5*10^-3;
t=[0:0.0000001:l];
f=1/l;
y=sin(pi*f*t);
subplot(1,2,1);
plot(t,y);
x=[0:0.000001:1.5*10^-3];
L=1.5*10^-3;
g=pi/L;
wl=1.8*10^15;
```

```
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=30*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
c2=A*pi/(k3*L*k2);
ba1=c2*sin(k2*x)+(A/k3)*sin(g*x);
subplot(1,2,2);
plot(x,ba1,'b');
xlabel('pulse length=L');
ylabel('generated magnetic field');
hold on;
```

This is the program clearing involving the steps to solve the differential equation for sinusoidal electric field pulse profile.

The output to the above program is as shown



If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of electric field pulse profile but there is significant amount of oscillation or fluctuations in magnetic field whose account can be understood from the dominance or significant presence of sinusoidal terms in the complementary solution. Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value of constants c_1 and c_2 governing the complementary solution is either negligible or zero.

The maximum value of magnetic field for sinusoidal electric field pulse profile observed is 19.163 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile.

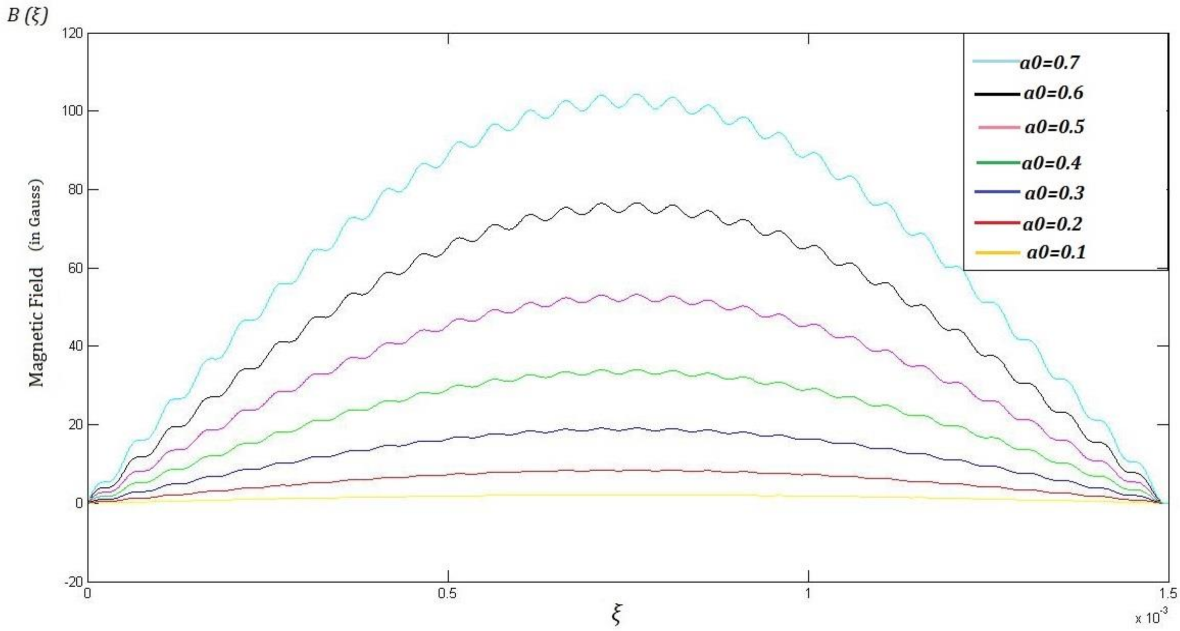
Table showing variation of magnetic field with respect to the respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile.

Sine Pulse

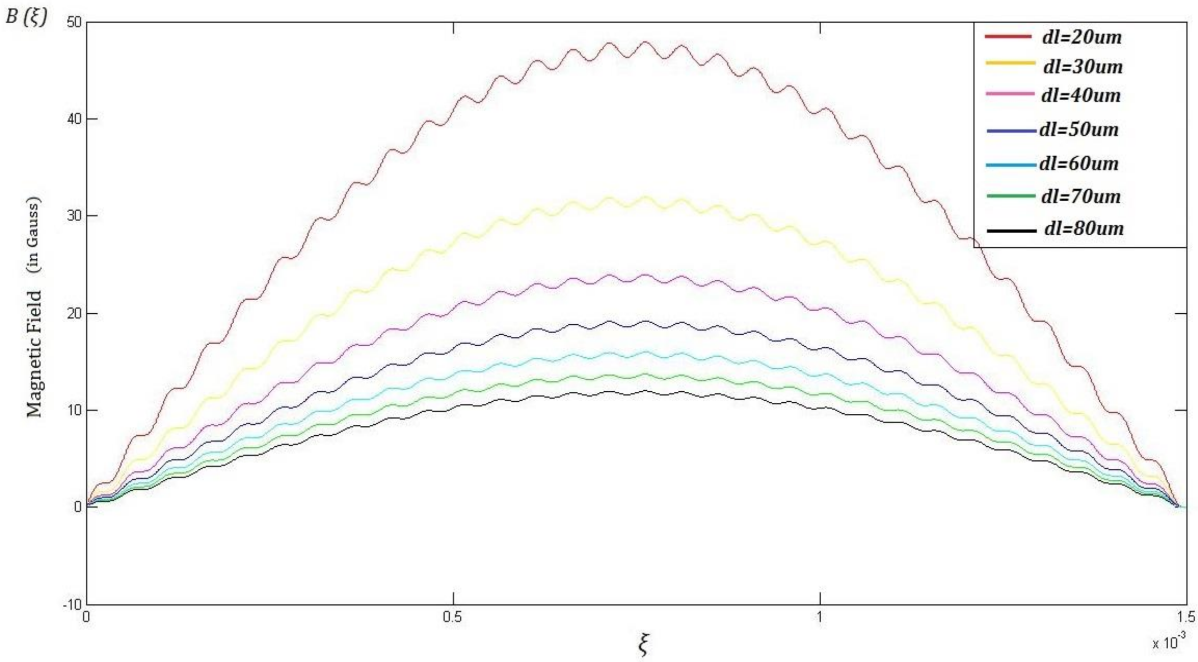
a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	n_0 in $\times 10^{19} cm^{-3}$	$\max of\ B(\xi)$ $in\ Gauss$
0.1	2.1293	20	47.9081	0.1	1.9163
0.2	8.517	30	31.9388	0.5	9.5816
0.3	19.163	40	23.954	1	19.163
0.4	34.068	50	19.163	2	38.3265
0.5	53.2313	60	15.9694	4	76.653
0.6	76.653	70	13.688	6	114.97
0.7	104.333	80	11.977	8	153.306

Table 3.1 Variation in Amplitude of Magnetic Fields for Sine Electric Field Pulse Profiles.

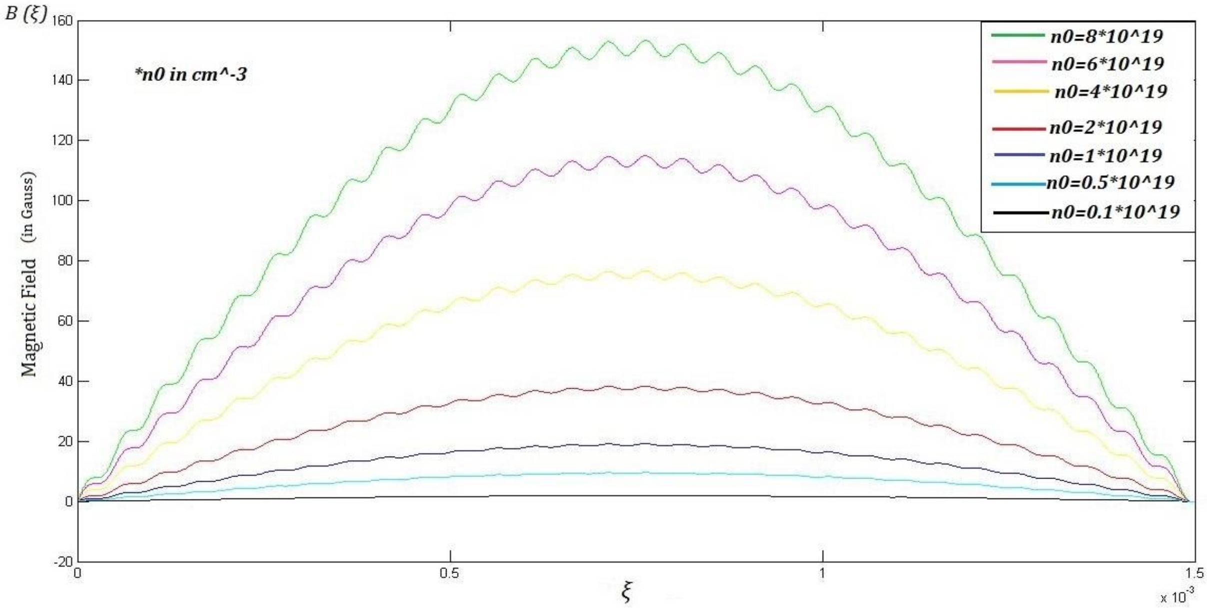
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0).



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)



From these plots we can realise that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp.

Thus this is the general solution of magnetic field for Sine Electric Field Pulse profile.

3.3.2 Triangular Electric Field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2\, f(\xi),$$

where $f(\xi)$ describes the Triangular electric field pulse profile whose expression is as follows. The practical triangular electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece function with two limits [25].

$$\begin{aligned} f(\xi) \quad &= 1 - \cos\left(\frac{\pi\xi}{L}\right),\ for\ 0 \le \xi \le \frac{L}{2}, \\ &= 1 + \cos\left(\frac{\pi\xi}{L}\right), for\ \frac{L}{2} \le \xi \le L. \end{aligned}$$

Now obtaining particular solution of nonhomogeneous part we get two particular solution since the triangular electric field pulse profile is piecewise built with two limits. The magnetic field from the first limit is represented as $B_{1p}(\xi)$ and the second limit is represented as $B_{2p}(\xi)$. Now the mathematics involved is as shown.

$$(D^2 + U^2)B_{1p}(\xi) = A^2\,(1 - \cos\left(\frac{\pi\xi}{L}\right)), for\ 0 \le \xi \le \frac{L}{2},$$

$$B_{1p}(\xi) = \frac{A^2}{U^2} - \frac{A^2\cos\left(\frac{\pi\xi}{L}\right)}{U^2 - \left(\left(\frac{\pi}{L}\right)^2\right)}, for\ 0 \le \xi \le \frac{L}{2}.$$

Thus this the particular solution of first limit. The total solution of the first limit is sum of complementary solution and particular solution which is as shown. $B_1(\xi)$ represents the total solution upto first limit of triangular electric field pulse profile.

$$B_1(\xi) = B_c(\xi) + B_{1p}(\xi), for\ 0 \le \xi \le \frac{L}{2},$$

$$B_1(\xi) = c_1\cos(U\xi) + c_2\sin(U\xi) + \frac{A^2}{U^2} - \frac{A^2\cos\left(\frac{\pi\xi}{L}\right)}{U^2 - \left(\left(\frac{\pi}{L}\right)^2\right)}, for 0 \le \xi \le \frac{L}{2}.$$

This is the solution upto first limit magnetic field for triangular electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_1(\xi) = -18.861\cos\left(\frac{\pi\xi}{L}\right) + 18.8558,\ for\ 0 \le \xi \le \frac{L}{2},}$$

Now obtaining particular solution to second limit of electric field pulse profile, we get

$$(D^2 + U^2)B_{2p}(\xi) = A^2\,(1 + \cos\left(\frac{\pi\xi}{L}\right)), for\ \frac{L}{2} \le \xi \le L.$$

$$B_{2p}(\xi) = \frac{A^2}{U^2} + \frac{A^2 \cos\left(\frac{\pi\xi}{L}\right)}{U^2 - \left((\frac{\pi}{L})^2\right)}, for\ \frac{L}{2} \leq \xi \leq L.$$

Thus this the particular solution of second limit. The total solution of the second limit is sum of complementary solution and particular solution which is as shown. $B_2(\xi)$ represents total solution upto second limit of triangular electric field pulse profile.

$$B_2(\xi) = B_c(\xi) + B_{2p}(\xi), for\ \frac{L}{2} \leq \xi \leq L,$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{U^2} + \frac{A^2 \cos\left(\frac{\pi\xi}{L}\right)}{U^2 - \left((\frac{\pi}{L})^2\right)}, for\ \frac{L}{2} \leq \xi \leq L.$$

This is the solution upto second limit magnetic field for triangular electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_2(\xi) = 18.861 \cos\left(\frac{\pi\xi}{L}\right) + 18.858, for \frac{L}{2} \leq \xi \leq L.}$$

The total solution for the magnetic field for triangular electric field pulse profile is as shown below.

$$\boxed{\begin{aligned} B(\xi)\ &= -18.861 \cos\left(\frac{\pi\xi}{L}\right) + 18.8558,\ for\ 0 \leq \xi \leq \frac{L}{2}, \\ &= 18.861 \cos\left(\frac{\pi\xi}{L}\right) + 18.858, for \frac{L}{2} \leq \xi \leq L. \end{aligned}}$$

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

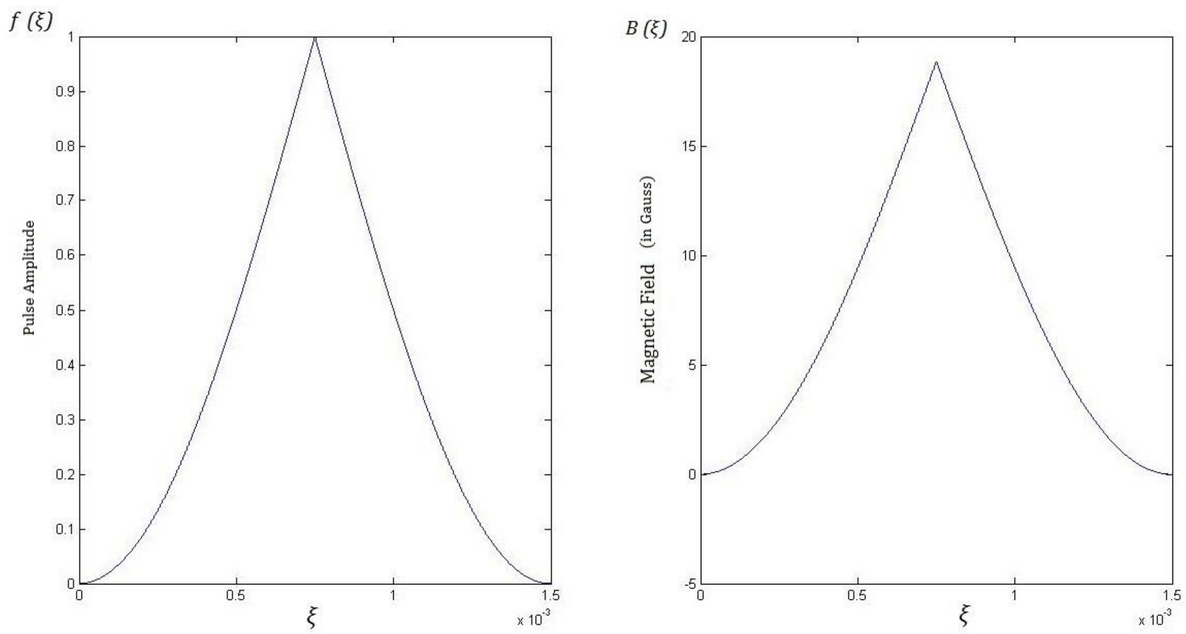
Code:

```
l=1.5*10^-3;
t1=[0:0.0000001:l/2]
e1=1-cos(pi*t1/l);
t2=[l/2:0.0000001:l];
e2=1+cos(pi*t2/l);
subplot(1,2,1);
plot(t1,e1);hold on;plot(t2,e2);
syms c2 c3
x1=[0:0.000000001:0.75*10^-3];
L=1.5*10^-3;
g=pi/L;
wl=1.8*10^15;
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=50*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
b1=A/k3-(A/(k3-(g)^2))*cos(g*x1);
bg=7.19*10^-3;
b1g=(A/(k3-(g)^2))*g;
```

```
eqn1=[0.99*c2-0.047*c3-0;-0.047*k2*c2+0.99*k2*c3-b1g+(A/(k3-
(g)^2))*g*sin(g*L/2)];
S=solve([eqn1]);
x2=[0.75*10^-3:0.000001:1.5*10^-3];
b2=subs(S.c2)*cos(k2*x2)+subs(S.c3)*sin(k2*x2)+A/k3+(A/(k3-
(g)^2))*cos(g*x2);
subplot(1,2,2);
plot(x1,b1,'b'); hold on; plot(x2,b2,'b');
hold on;
```

This is the program clearing involving the steps to solve the differential equation for triangular electric field pulse profile.

The output to the above program is as shown



If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of electric field pulse profile. Also when compared to response for sine electric field pulse profile there are no significant oscillation or fluctuations in magnetic field.

Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value constants c_1 *and* c_2 governing the complementary solution is either negligible or zero. Here in this case we can see in solution that these complementary constants are zero. Thus there are no oscillations in the envelope.

The maximum value of magnetic field for triangular electric field pulse profile observed is 18.858 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile. This is lesser than the maximum value for sinusoidal pulse profile but the difference margin is very less.

Table showing variation of magnetic field with respect to the laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile.

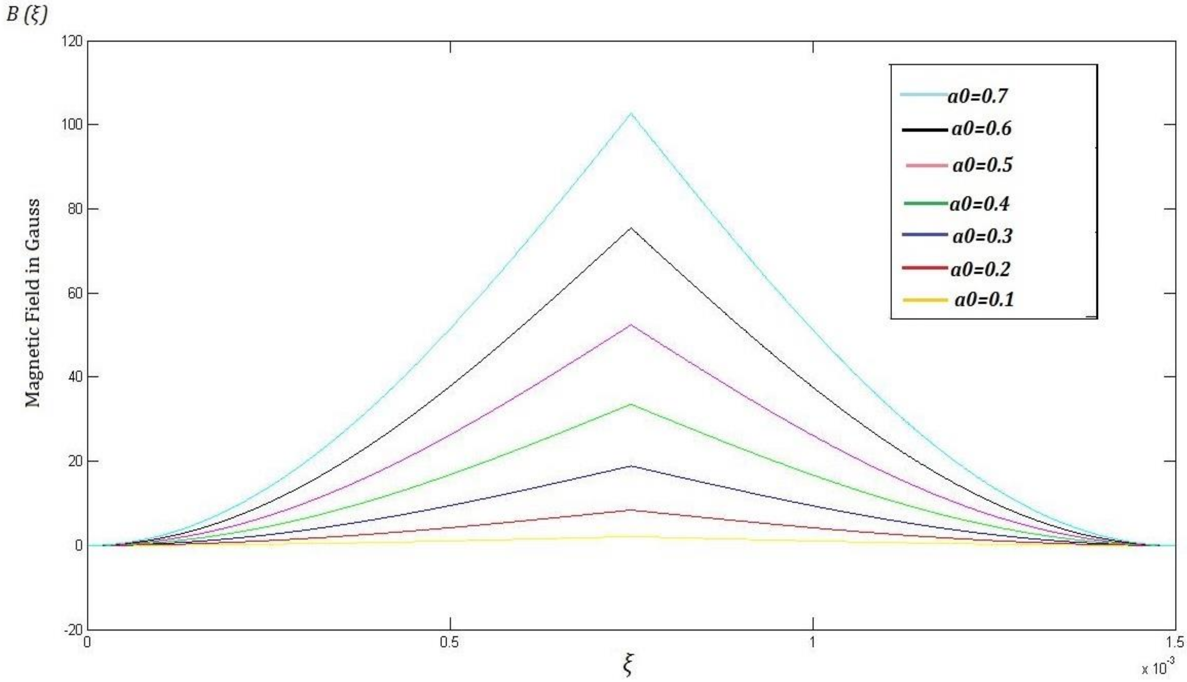
Triangular Pulse

a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	$n_0\ in$ $\times 10^{19} cm^{-3}$	$\max of\ B(\xi)$ $in\ Gauss$
0.1	2.0951	20	47.1396	0.1	1.8856
0.2	8.3804	30	31.4264	0.5	9.4279
0.3	18.858	40	23.5698	1	18.858
0.4	33.5215	50	18.858	2	37.7117
0.5	52.3774	60	15.7132	4	75.4234
0.6	75.4234	70	13.4685	6	113.1351
0.7	102.6596	80	11.7849	8	150.8468

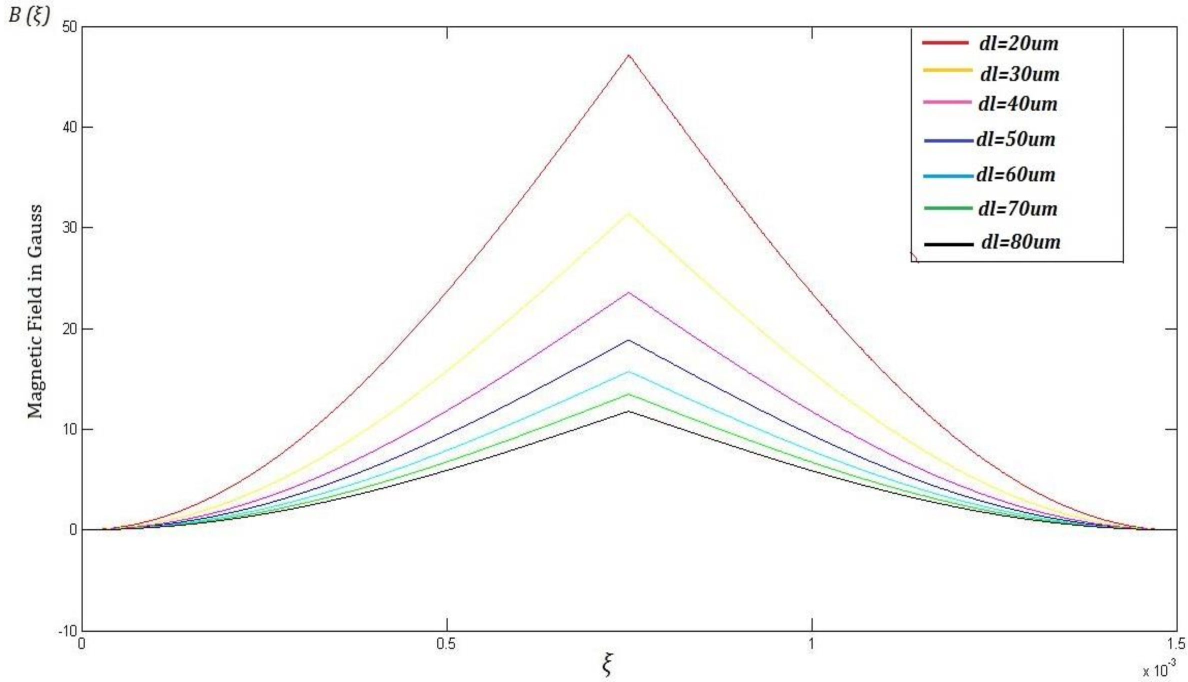
Table 3.2 Variation in Amplitude of Magnetic Fields for Triangular Electric Field Pulse Profiles.

The table clearly shows that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp. These can also be seen in the Matlab output plots as follows.

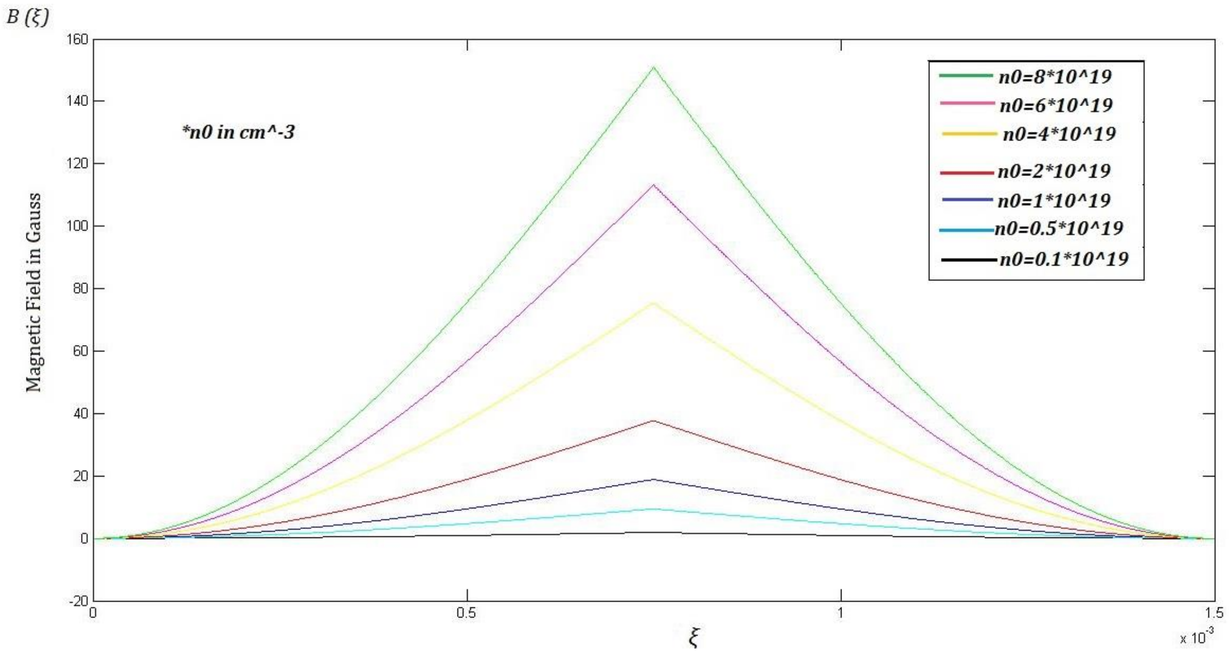
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0)



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)



3.3.3 Sawtooth Decreasing Electric Field Pulse Profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2\, f(\xi),$

where $f(\xi)$ describes the Sawtooth decreasing electric field pulse profile whose expression is as follows. The practical Sawtooth decreasing electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece wise function with two limits [25].

$$f(\xi) \quad = (1-\cos\left(\frac{29\pi\xi}{L}\right))/2\,, for\ 0 \le \xi \le \frac{L}{30},$$

$$= \frac{1+\cos\left(\frac{\pi\xi}{L}\right)}{2}, for\ \frac{L}{30} \le \xi \le L.$$

Now obtaining particular solution of nonhomogeneous part, we get two particular solutions since the sawtooth decreasing electric field pulse profile is piecewise built with two limits. The magnetic field from the first limit is represented as $B_{1p}(\xi)$ and the second limit is represented as $B_{2p}(\xi)$. Now the mathematics involved is as shown.

$$(D^2+U^2)B_{1p}(\xi) = \frac{A^2}{2}\left(1-\cos\left(\frac{29\pi\xi}{L}\right)\right),\ for\ 0 \le \xi \le \frac{L}{30},$$

$$B_{1p}(\xi) = \frac{A^2}{2U^2} - \frac{A^2 \cos\left(\frac{29\pi\xi}{L}\right)}{2\left(U^2 - \left(\left(\frac{29\pi}{L}\right)^2\right)\right)}, for\ 0 \le \xi \le \frac{L}{30}.$$

Thus this the particular solution of first limit. The total solution of the first limit is sum of complementary solution and particular solution which is as shown. $B_1(\xi)$ represents total solution upto first limit of sawtooth decreasing electric field pulse profile.

$$B_1(\xi) = B_c(\xi) + B_{1p}(\xi), for\ 0 \le \xi \le \frac{L}{30}.$$

$$B_1(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{2U^2} - \frac{A^2 \cos\left(\frac{29\pi\xi}{L}\right)}{2\left(U^2 - \left(\left(\frac{29\pi}{L}\right)^2\right)\right)}, 0 \le \xi \le \frac{L}{30}.$$

This is the solution upto first limit magnetic field for sawtooth decreasing electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 is laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which, we get is

$$\boxed{B_1(\xi) \quad = 2.876 \cos(U\xi) - 12.3056 \cos\left(\frac{29\pi\xi}{L}\right) + 9.427,\ for\ 0 \le \xi \le \frac{L}{30}.}$$

Now obtaining particular solution to second limit of electric field pulse profile ,we get

$$(D^2 + U^2)B_{2p}(\xi) = \frac{A^2}{2}\ (1 + \cos\left(\frac{\pi\xi}{L}\right)), for \frac{L}{30} \le \xi \le L,$$

$$B_{2p}(\xi) = \frac{A^2}{2U^2} + \frac{A^2 \cos\left(\frac{\pi\xi}{L}\right)}{2U^2 - 2\left((\frac{\pi}{L})^2\right)}, for\ \frac{L}{30} \le \xi \le L.$$

Thus this is the particular solution of second limit. The total solution of the second limit is sum of complementary solution and particular solution which is as shown. $B_2(\xi)$ represents total solution upto second limit of Sawtooth decreasing electric field pulse profile.

$$B_2(\xi) = B_c(\xi) + B_{2p}(\xi), for\ \frac{L}{30} \leq \xi \leq L.$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{2U^2} + \frac{A^2 \cos\left(\frac{\pi\xi}{L}\right)}{2U^2 - 2\left((\frac{\pi}{L})^2\right)}, for\ \frac{L}{30} \leq \xi \leq L.$$

This is the solution upto second limit magnetic field for Sawtooth decreasing electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$B_2(\xi) = 5.7968 \cos(U\xi) + 0.6023 \sin(U\xi) + 9.4300 \cos\left(\frac{\pi\xi}{L}\right) + 9.427,$$
$$for\ \frac{L}{30} \leq \xi \leq L.$$

The total solution for the magnetic field for triangular electric field pulse profile is as shown below.

$$\begin{aligned} B(\xi) &= 2.876 \cos(U\xi) - 12.3056 \cos\left(\frac{29\pi\xi}{L}\right) + 9.427,\ for\ 0 \leq \xi \leq \frac{L}{30}, \\ &= 5.7968 \cos(U\xi) + 0.6023 \sin(U\xi) + 9.4300 \cos\left(\frac{\pi\xi}{L}\right) + 9.427, \\ &\qquad for\ \frac{L}{30} \leq \xi \leq L. \end{aligned}$$

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

Code:

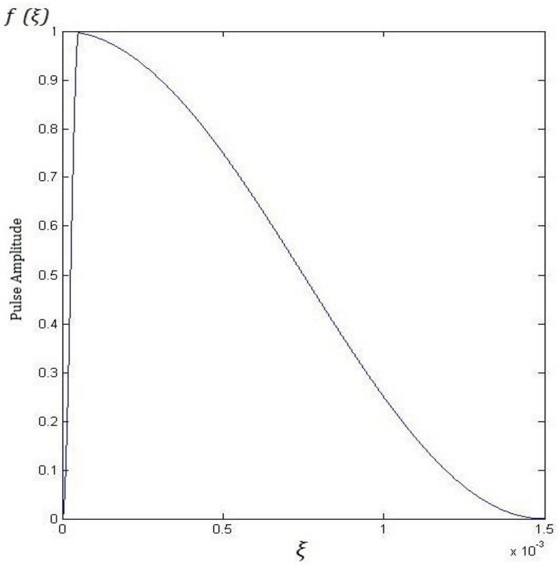
```
syms c2 c3
l=1.5*10^-3;
t1=[0:0.0000001:l/30];
e1=(1-cos(29*pi*t1/l))/2;
t2=[l/30:0.00000001:l];
e2=(1+cos(pi*t2/l))/2;
subplot(1,2,1)
plot(t1,e1);hold on;plot(t2,e2);
x1=[0:0.00001:5*10^-5];
L=1.5*10^-3;
g=pi/L;
wl=1.8*10^15;
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=50*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
c1=(-(A/k3)+(A/(k3-(29*g)^2)))/2;
b1=c1*cos(k2*x1)+0.5*A/k3-(0.5*A/(k3-(29*g)^2))*cos(29*g*x1);
bg=c1*cos(k2*L/30)+0.5*A/k3-(0.5*A/(k3-
(29*g)^2))*cos(29*g*L/30);
b1g=-k2*c1*sin(k2*L/30)+(0.5*A/(k3-
(29*g)^2))*29*g*sin(29*g*L/30);
eqn1=[0.99*c2-0.00318*c3-bg+(0.5*A/k3)+(0.5*A/(k3-
(g)^2))*cos(pi/30);0.00318*k2*c2+0.99*k2*c3-b1g+(0.5*A/(k3-
(g)^2))*g*sin(g*L/30)];
```

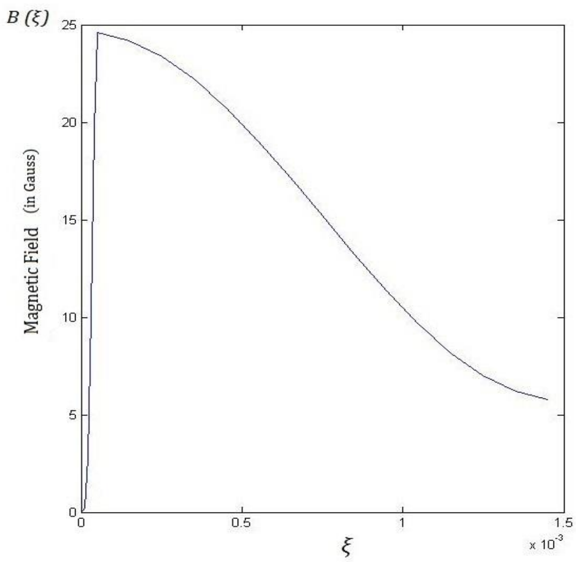
```
S=solve([eqn1]);
x2=[5*10^-5:0.0001:1.5*10^-3];
b2=subs(S.c2)*cos(k2*x2)+subs(S.c3)*sin(k2*x2)+0.5*A/k3+(0.5*A
/(k3-(g)^2))*cos(g*x2);
subplot(1,2,2)
plot(x1,b1,'b'); hold on; plot(x2,b2,'b');
hold on;
```

This is the program clearing involving the steps to solve the differential equation for Sawtooth decreasing electric field pulse profile.

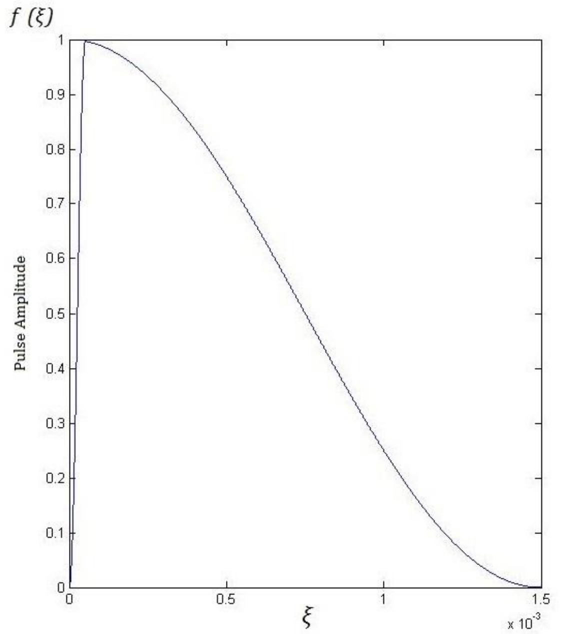
The output to the above program is as shown in the next page.

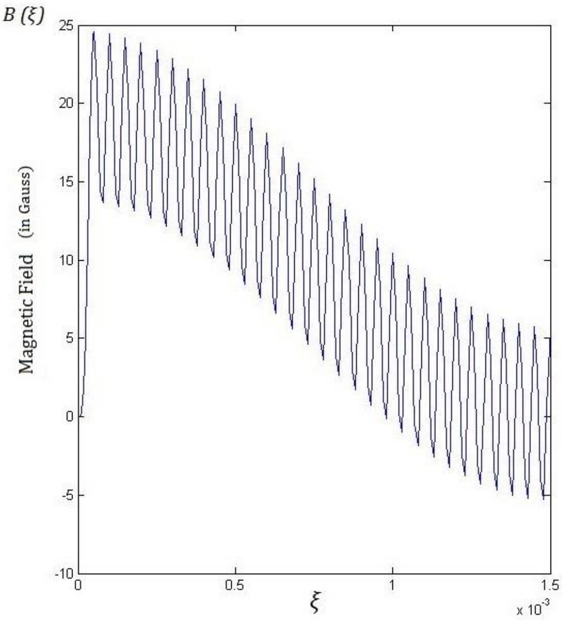
If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of sawtooth decreasing electric field pulse profile. Also when compared to response for sine electric field pulse profile there will be significant oscillation or fluctuations in magnetic field. The oscillations are not seen in the output since we have increased the step size to properly show the envelope. The actual output with oscillations can be seen when we decrease the step size. It has been shown in next page





Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value constants c_1 *and* c_2 governing the complementary solution is either negligible or zero. Here in this case we can see in solution that these complementary constants are much significant. Thus there will be oscillations in the envelope.





Sawtooth Decreasing Pulse

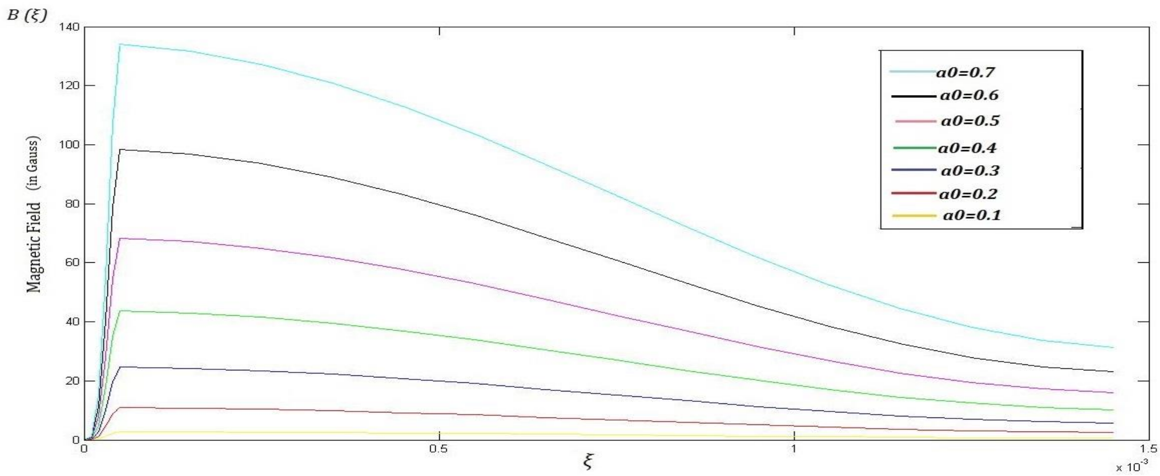
a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	$n_0\ in\ per\ \times 10^{19} cm^3$	$\max of\ B(\xi)$ $in\ Gauss$
0.1	2.7335	20	61.5041	0.1	2.4602
0.2	10.9341	30	41.0027	0.5	12.3008
0.3	24.6016	40	30.7520	1	24.6016
0.4	43.7362	50	24.6016	2	49.2032
0.5	68.3378	60	20.5014	4	98.4065
0.6	98.4065	70	17.5726	6	147.6097
0.7	133.9422	80	15.3760	8	196.8130

Table 3.3 Variation in Amplitude of Magnetic Fields for Sawtooth Decreasing Electric Field Pulse Profiles.

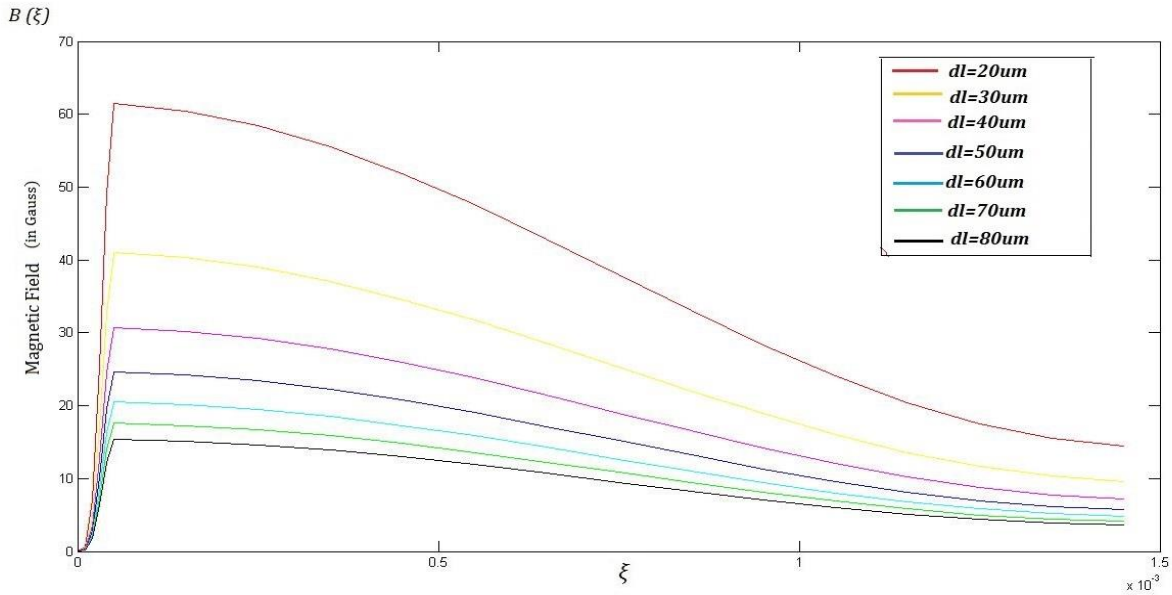
Above Table shows variation of magnetic field with respect to the laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile.

The maximum value of magnetic field for sawtooth decreasing electric field pulse profile observed is 24.606 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile. This is significantly larger than the maximum value for sinusoidal or triangular electric field pulse profile.

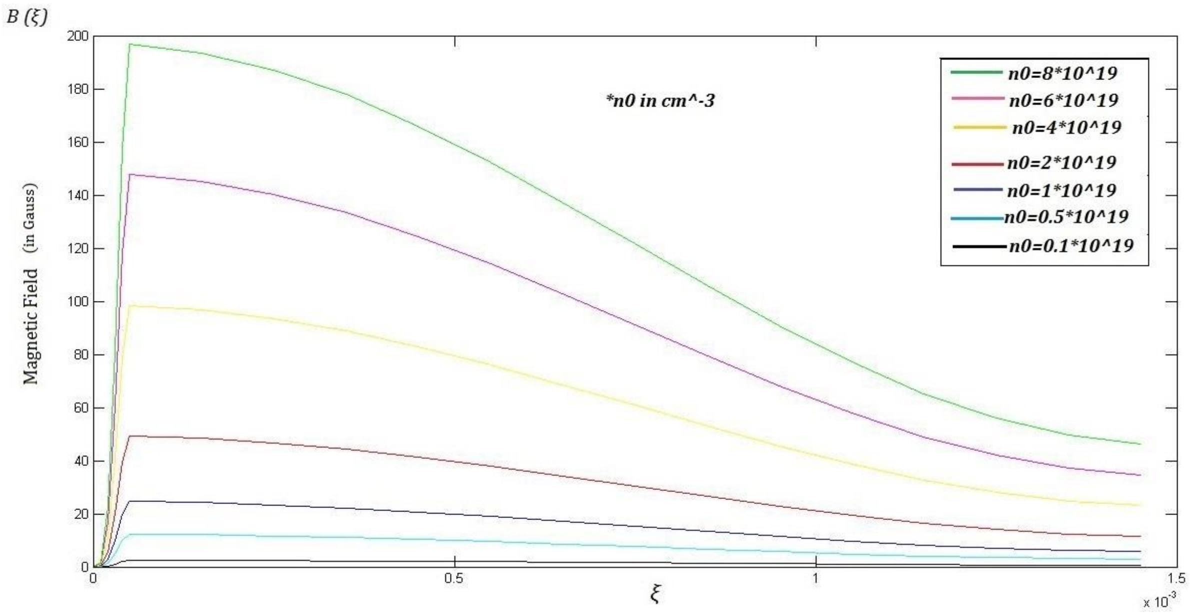
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0)



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)



These plots clearly shows that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp.

3.3.4 Sawtooth Increasing Electric Field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2\, f(\xi),$$

where $f(\xi)$ describes the Sawtooth increasing electric field pulse profile whose expression is as follows. The practical Sawtooth increasing electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece wise function with two limits [25].

$$\begin{aligned} f(\xi) \quad &= 1 - \cos\left(\frac{30\pi\xi}{58L}\right),\ for\ 0 \le \xi \le \frac{29L}{30}, \\ &= 1 + \cos\left(\frac{15\pi\xi}{L}\right),\ for\ \frac{29L}{30} \le \xi \le L. \end{aligned}$$

Now obtaining particular solution of nonhomogeneous part, we get two particular solutions since the sawtooth increasing electric field pulse profile is piecewise built with two limits. The magnetic field from the first limit is represented as $B_{1p}(\xi)$ and the second limit is represented as $B_{2p}(\xi)$. Now the mathematics involved is as shown.

$$(D^2 + U^2)B_{1p}(\xi) = A^2\left(1 - \cos\left(\frac{30\pi\xi}{58L}\right)\right),\ for\ 0 \le \xi \le \frac{29L}{30},$$

$$B_{1p}(\xi) = \frac{A^2}{U^2} - \frac{A^2\cos\left(\frac{30\pi\xi}{58L}\right)}{U^2 - \left(\left(\frac{30\pi}{58L}\right)^2\right)}, for\ 0 \le \xi \le \frac{29L}{30}.$$

Thus this the particular solution of first limit. The total solution of the first limit is sum of complementary solution and particular solution which is as shown. $B_1(\xi)$ represents total solution upto first limit of sawtooth increasing electric field pulse profile.

$$B_1(\xi) = B_c(\xi) + B_{1p}(\xi), for\ \ 0 \le \xi \le \frac{29L}{30}.$$

$$B_1(\xi) = c_1\cos(U\xi) + c_2\sin(U\xi) + \frac{A^2}{U^2} - \frac{A^2\cos\left(\frac{30\pi\xi}{58L}\right)}{U^2 - \left(\left(\frac{30\pi}{58L}\right)^2\right)}, for\ 0 \le \xi \le \frac{29L}{30}.$$

This is the solution upto first limit magnetic field for sawtooth increasing electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_1(\xi) = -18.858\cos\left(\frac{15\pi\xi}{58L}\right) + 18.858,\ for\ 0 \le \xi \le \frac{29L}{30},}$$

Now obtaining particular solution to second limit of electric field pulse profile we get

$$(D^2 + U^2)B_{2p}(\xi) = A^2\left(1 + \cos\left(\frac{15\pi\xi}{L}\right)\right), for\ \frac{29L}{30} \le \xi \le L.$$

$$B_{2p}(\xi) = \frac{A^2}{U^2} + \frac{A^2 \cos\left(\frac{15\pi\xi}{L}\right)}{U^2 - \left(\left(\frac{15\pi}{L}\right)^2\right)}, for\ \frac{29L}{30} \le \xi \le L.$$

Thus this the particular solution of second limit. The total solution of the second limit is sum of complementary solution and particular solution which is as shown. $B_2(\xi)$ represents total solution upto second limit of Sawtooth increasing electric field pulse profile.

$$B_2(\xi) = B_c(\xi) + B_{2p}(\xi), for\ \frac{29L}{30} \le \xi \le L.$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{U^2} + \frac{A^2 \cos\left(\frac{15\pi\xi}{L}\right)}{U^2 - \left(\left(\frac{15\pi}{L}\right)^2\right)}, for\ \frac{29L}{30} \le \xi \le L.$$

This is the solution upto second limit magnetic field for Sawtooth increasing electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$B_2(\xi) = 0.473 \cos(U\xi) + 5.203 \sin(U\xi) + 20.114 \cos\left(\frac{15\pi\xi}{L}\right) + 18.858,$$
$$for\ \frac{29L}{30} \le \xi \le L.$$

The magnetic field response for sawtooth increasing electric field pulse profile is as shown

$$B(\xi) = -18.858 \cos\left(\frac{15\pi\xi}{58L}\right) + 18.858,\ for\ 0 \le \xi \le \frac{29L}{30},$$
$$= 0.473 \cos(U\xi) + 5.203 \sin(U\xi) + 20.114 \cos\left(\frac{15\pi\xi}{L}\right) + 18.858,$$
$$for\ \frac{29L}{30} \le \xi \le L.$$

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

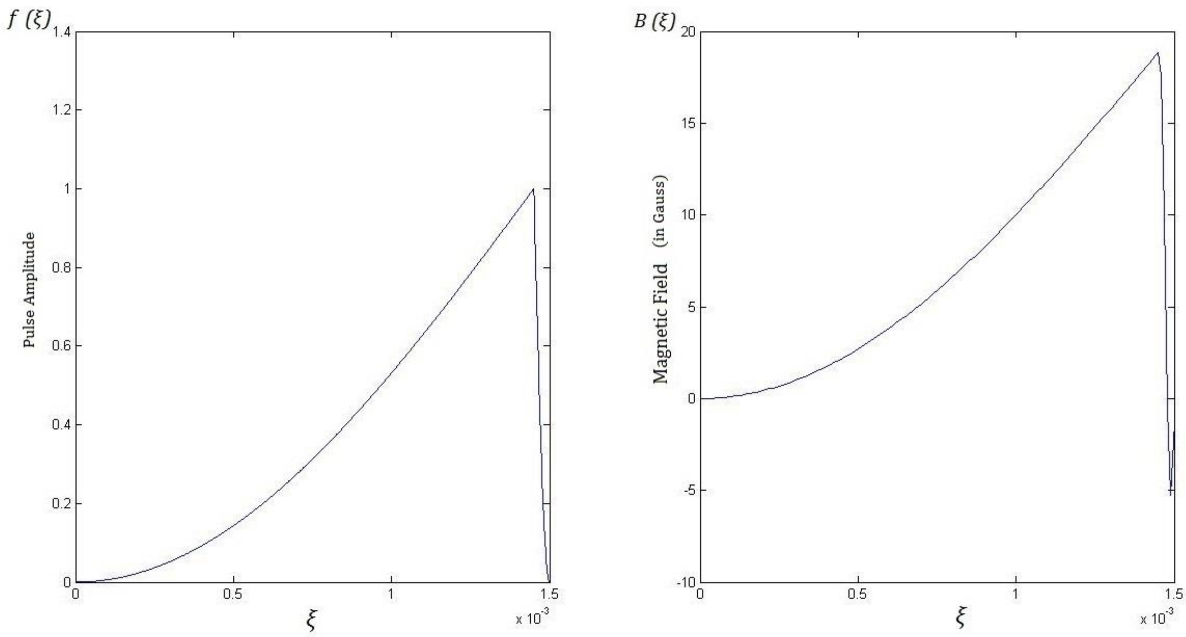
Code:

```
syms c2 c3
l=1.5*10^-3;
t1=[0:0.000001:29*l/30];
e1=1-cos((30/58)*pi*t1/l);
t2=[29*l/30:0.000001:l];
e2=1+cos(15*pi*t2/l);
subplot(1,2,1);
plot(t1,e1);hold on;plot(t2,e2);
x1=[0:0.00001:1.45*10^-3];
L=1.5*10^-3;
g=pi/L;
wl=1.8*10^15;
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=50*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
b1=A/k3-(A/(k3-(30/58*g)^2))*cos(30*g*x1/58);
bg=A/k3;
b1g=(A/(k3-(30/58*g)^2))*30*g/58;
```

```
eqn1=[0.99*c2-0.09*c3-0;0.09*k2*c2+0.99*k2*c3-b1g-(A/(k3-
(15*g)^2))*15*g];
S=solve([eqn1]);
x2=[1.45*10^-3:0.00001:1.5*10^-3];
b2=subs(S.c2)*cos(k2*x2)+subs(S.c3)*sin(k2*x2)+A/k3+(A/(k3-
(15*g)^2))*cos(15*g*x2);
subplot(1,2,2);
plot(x1,b1,'b'); hold on; plot(x2,b2,'b');
hold on;
```

This is the program clearing involving the steps to solve the differential equation for Sawtooth increasing electric field pulse profile.

The output to the above program is as shown



If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of electric field pulse profile. Also when compared to response for sine electric field pulse profile there are no significant oscillation or fluctuations in magnetic field. Even the variation in step size recorded same output

Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value constants c_1 *and* c_2 governing the complementary solution is either negligible or zero. Here in this case we can see in solution that these complementary constants are zero for $B_1(\xi)$. Thus there are no oscillations in the envelope of the magnetic field.

The maximum value of magnetic field for sawtooth increasing electric field pulse profile observed is 18.858 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile. This is lesser than the maximum value for sawtooth decreasing, sinusoidal pulse profile but exactly equal to triangular electric field pulse profile response. The difference of margin between sawtooth increasing and sine pulse response is very less

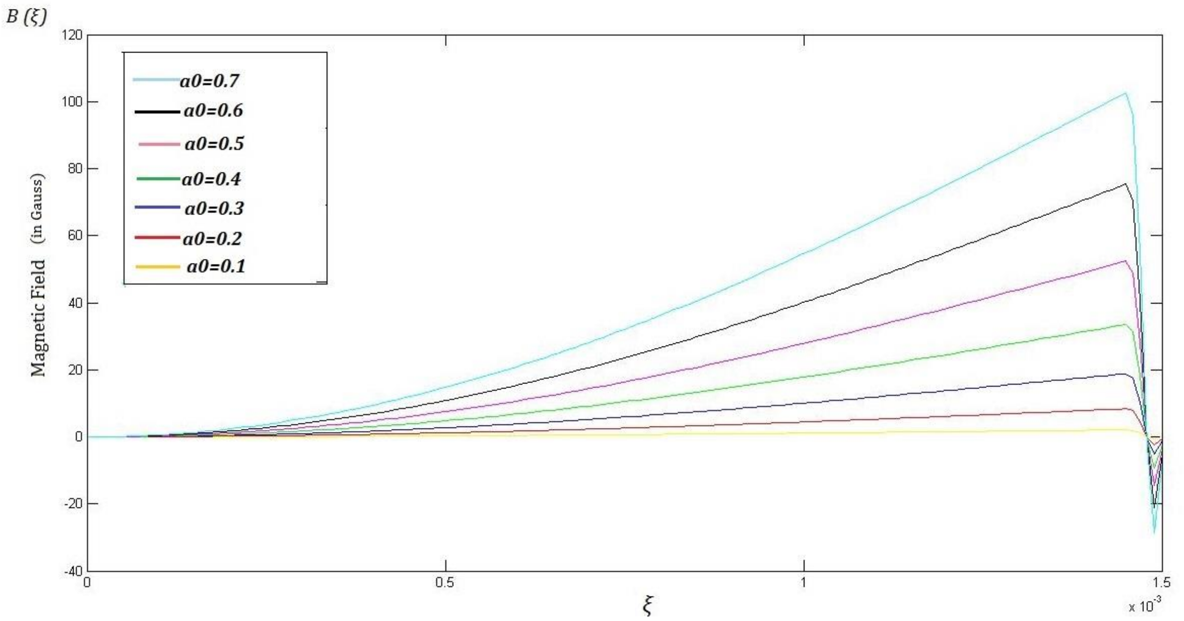
Table showing variation of magnetic field with respect to the respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile.

Sawtooth Increasing Pulse

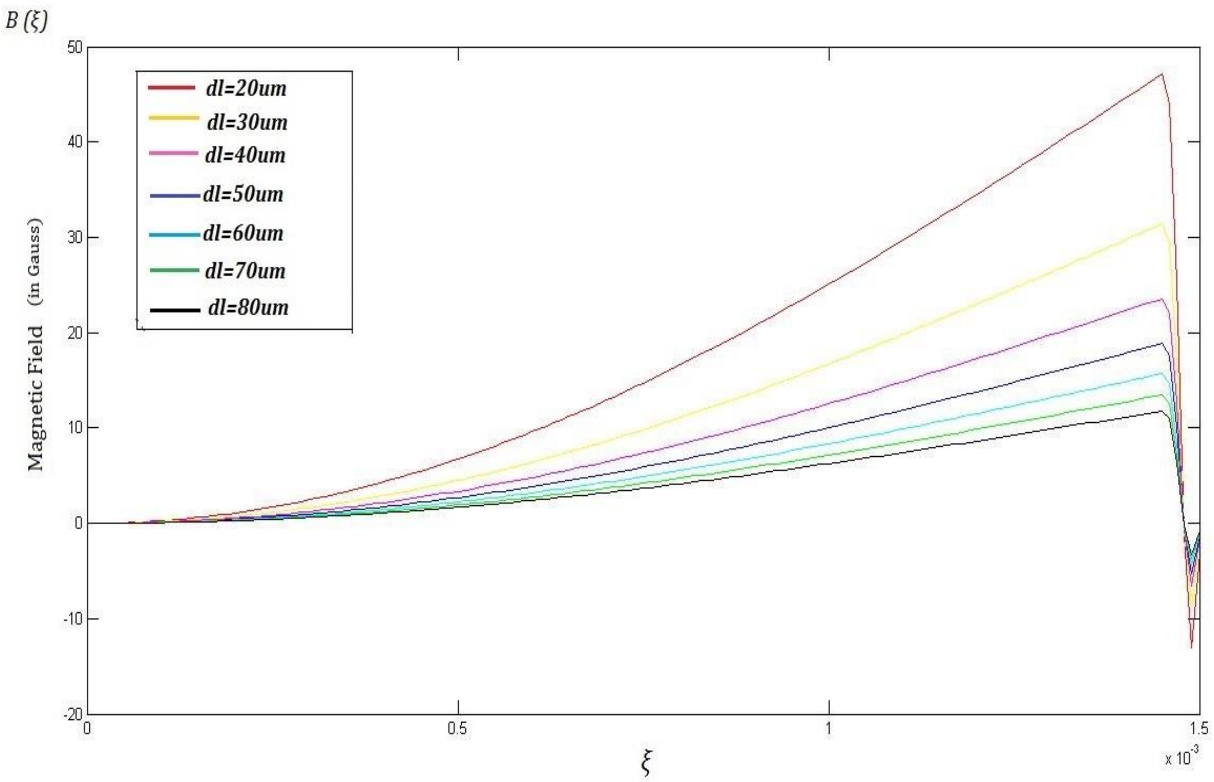
a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	$n_0\ in$ $\times 10^{19} cm^{-3}$	$\max of\ B(\xi)$ $in\ Gauss$
0.1	2.0951	20	47.1396	0.1	1.8856
0.2	8.3804	30	31.4264	0.5	9.4279
0.3	18.8558	40	23.5698	1	18.8558
0.4	33.5215	50	18.8558	2	37.7117
0.5	52.3774	60	15.7132	4	75.4234
0.6	75.4234	70	13.4685	6	113.1351
0.7	102.6596	80	11.7849	8	150.8468

Table 3.4 Variation in Amplitude of Magnetic Fields for Sawtooth Increasing Electric Field Pulse Profiles.

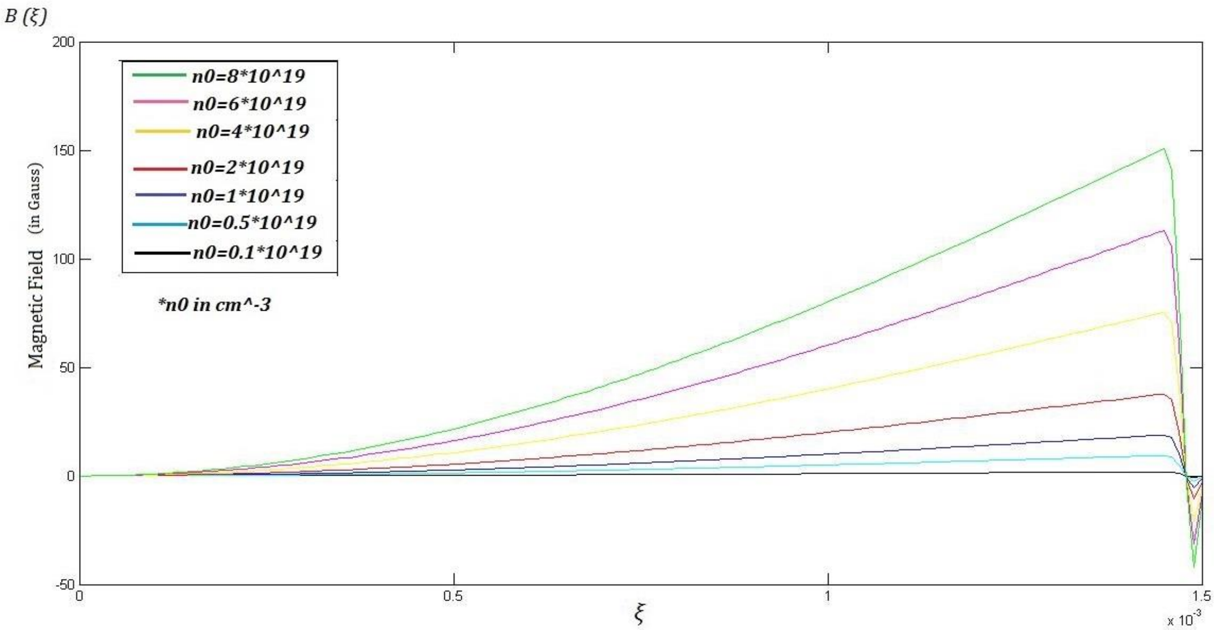
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0)



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)



These plots clearly shows that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp.

3.3.5 Rectangular Triangular Electric Field Pulse Profile:

The electric field can be represented as follows

$$E^2(\xi) = {E_0}^2 \, f(\xi),$$

where $f(\xi)$ describes the Rectangular triangular electric field pulse profile whose expression is as follows. The practical rectangular triangular electric field pulse can be constructed from sinusoidal or cosine term as follows. This pulse is piece wise function with three limits [24].

$$f(\xi) = \sqrt{(1 - \cos(2\pi\xi/L))}, \; for \; 0 \le \xi \le L/4,$$
$$= 1, for \; \frac{L}{4} \le \xi \le 2L/3,$$
$$= \sqrt{[(1 + \cos(3\pi\xi/L))]/2}, for \; \frac{2L}{3} \le \xi \le L.$$

The profile can be reduced to following form by applying laws of trigonometry

$$f(\xi) = \sqrt{2}\sin(\pi\xi/L), \; for \; 0 \le \xi \le L/4,$$
$$= 1, for \; \frac{L}{4} \le \xi \le 2L/3,$$
$$= -\cos\left(\frac{3\pi\xi}{2L}\right) for \; \frac{2L}{3} \le \xi \le L.$$

Now obtaining particular solution of nonhomogeneous part, we get three particular solutions since the rectangular triangular electric field pulse profile is piecewise built with three limits. The magnetic field from the first limit is represented as $B_{1p}(\xi)$, second limit is represented as $B_{2p}(\xi)$ and the third limit is represented as $B_{3p}(\xi)$. Now the mathematics involved is as shown.

$$(D^2 + U^2)B_{1p}(\xi) = A^2\sqrt{2}\sin(\pi\xi/L),\ for\ 0 \le \xi \le L/4,$$

$$B_{1p}(\xi) = \frac{A^2\sqrt{2}\sin(\pi\xi/L)}{U^2 - \left((\frac{\pi}{L})^2\right)}, for\ 0 \le \xi \le \frac{L}{4}.$$

Thus this the particular solution of first limit. The total solution of the first limit is sum of complementary solution and particular solution which is as shown. $B_1(\xi)$ represents total solution upto first limit of rectangular triangular electric field pulse profile.

$$B_1(\xi) = B_c(\xi) + B_{1p}(\xi), for\ 0 \le \xi \le \frac{L}{4}.$$

$$B_1(\xi) = c_1\cos(U\xi) + c_2\sin(U\xi) + \frac{A^2\sqrt{2}\sin(\pi\xi/L)}{U^2 - \left((\frac{\pi}{L})^2\right)}, for\ 0 \le \xi \le \frac{L}{4}.$$

This is the solution upto first limit magnetic field for rectangular triangular electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which, we get is

$$\boxed{B_1(\xi) = 26.6696\sin\left(\frac{\pi\xi}{L}\right) - 0.4447\sin(U\xi),\ for\ 0 \le \xi \le L/4.}$$

Now obtaining particular solution to second limit of electric field pulse profile, we obtain

$$(D^2 + U^2)B_{2p}(\xi) = A^2 for\ \frac{L}{4} \leq \xi \leq 2L/3,$$

$$B_{2p}(\xi) = \frac{A^2}{U^2} for\ \frac{L}{4} \leq \xi \leq 2L/3.$$

Thus this the particular solution of second limit. The total solution of the second limit is sum of complementary solution and particular solution which is as shown. $B_2(\xi)$ represents total solution upto second limit of rectangular triangular electric field pulse profile.

$$B_2(\xi) = B_c(\xi) + B_{2p}(\xi), for\ \frac{L}{4} \leq \xi \leq 2L/3,$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{U^2}, for \frac{L}{4} \leq \xi \leq 2L/3.$$

This is the solution upto second limit magnetic field for rectangular triangular electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is.

$$\boxed{B_2(\xi) = -0.0095 \cos(U\xi) - 0.766 \sin(U\xi) + 18.8558, for\ \frac{L}{4} \leq \xi \leq 2L/3;}$$

Now obtaining particular solution to third limit of electric field pulse profile ,we get

$$(D^2 + U^2)B_{3p}(\xi) = -A^2 \cos\left(\frac{3\pi\xi}{2L}\right) for\ \frac{2L}{3} \leq \xi \leq L,$$

$$B_{3p}(\xi) = \frac{-A^2 \cos(3\pi\xi/2L)}{U^2 - \left((\frac{3\pi}{2L})^2\right)}, for\ \frac{2L}{3} \leq \xi \leq L.$$

Thus this the particular solution of third limit. The total solution of the third limit is sum of complementary solution and particular solution which is as shown. $B_3(\xi)$ represents total solution upto third limit of rectangular triangular electric field pulse profile.

$$B_3(\xi) = B_c(\xi) + B_{3p}(\xi), for\ \frac{2L}{3} \leq \xi \leq L.$$

$$B_3(\xi) = c_1\cos(U\xi) + c_2\sin(U\xi) + \frac{-A^2\cos(3\pi\xi/2L)}{U^2 - \left((\frac{3\pi}{2L})^2\right)}, for\ \frac{2L}{3} \leq \xi \leq L.$$

This is the solution upto third limit magnetic field for rectangular triangular electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4}cm, and\ n_0 = 10^{19}cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_3(\xi) = -0.0213\cos(U\xi) - 0.772\sin(U\xi) - 18.867\cos\left(\frac{3\pi\xi}{2L}\right),\quad for\ \frac{2L}{3} \leq \xi \leq L.}$$

The magnetic field response for rectangular triangular electric field pulse profile is as shown

$$\begin{aligned} B(\xi) &= 26.6696\sin\left(\frac{\pi\xi}{L}\right) - 0.4447\sin(U\xi),\ for\ 0 \leq \xi \leq L/4, \\ &= -0.0095\cos(U\xi) - 0.766\sin(U\xi) + 18.8558, for\ \tfrac{L}{4} \leq \xi \leq 2L/3, \\ &= -0.0213\cos(U\xi) - 0.772\sin(U\xi) - 18.867\cos\left(\frac{3\pi\xi}{2L}\right), \\ & for\ \frac{2L}{3} \leq \xi \leq L. \end{aligned}$$

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

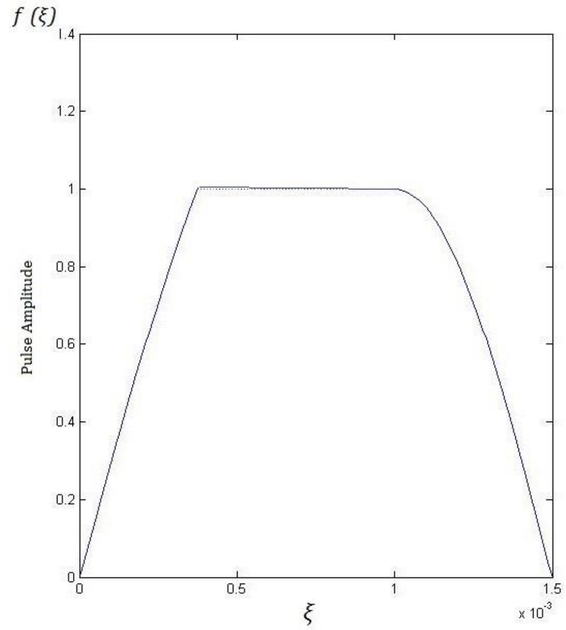
Code:

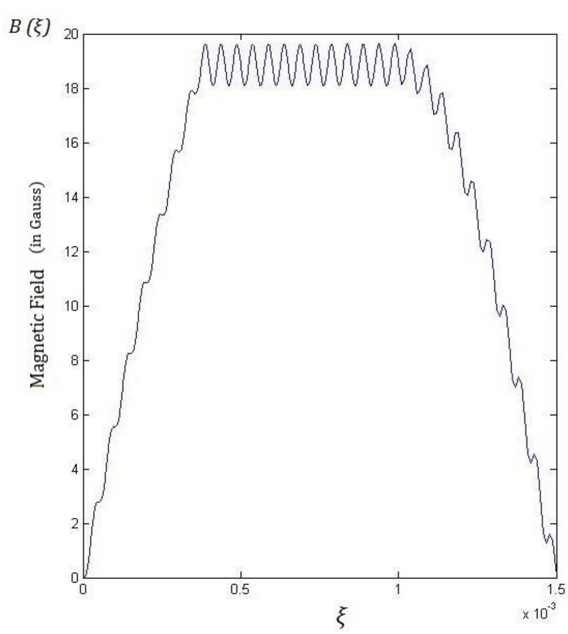
```
syms c2 c3 c4 c5
t1=[0:0.00001:0.38*10^-3];
e=1;
d=pi/(1.5*10^-3);
e1=e*sqrt(1-cos(2*d*t1));
t2=[0.375*10^-3:0.00001:1*10^-3];
t3=[1*10^-3:0.00001:1.5*10^-3];
e3=e*sqrt((1+cos(3*d*t3))/2);
subplot(1,2,1)
plot(t1,e1);hold on;plot(t2,e);hold on;plot(t3,e3);
L=1.5*10^-3;
x1=[0:0.000001:L/4];
g=pi/L;
wl=1.8*10^15;
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=50*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
c1=-(1.414*A*g/(k3-(g)^2))/k2;
b1=c1*sin(k2*x1)+(1.414*A/(k3-(g)^2))*sin(g*x1);
```

```
x2=[L/4:0.000001:10^-3];
bg=c1*sin(k2*L/4)+(1.414*A/(k3-(g)^2))*sin(g*L/4);
b1g=k2*c1*cos(k2*L/4)+(1.414*A/(k3-(g)^2))*g*cos(g*L/4);
eqn1=[-0.99*c2+0.023*c3-bg+A/k3;0.023*k2*c2-0.99*k2*c3-b1g];
S=solve([eqn1]);
b2=subs(S.c2)*cos(k2*x2)+subs(S.c3)*sin(k2*x2)+A/k3;
bg2=subs(S.c2)*cos(k2*2*L/3)+subs(S.c3)*sin(k2*2*L/3)+A/k3;
b2g=-subs(S.c2)*k2*sin(k2*2*L/3)+subs(S.c3)*k2*cos(k2*2*L/3);
eqn2=[0.99*c4-0.063*c5-bg2+(-A/(k3-
(1.5*g)^2))*cos(1.5*g*2*L/3);0.061*k2*c4+0.99*k2*c5-
b2g+(A/(k3-(1.5*g)^2))*1.5*g*sin(1.5*g*2*L/3)];
D=solve([eqn2]);
x3=[10^-3:0.00001:L];
b3=subs(D.c4)*cos(k2*x3)+subs(D.c5)*sin(k2*x3)+(-A/(k3-
(1.5*g)^2))*cos(1.5*g*x3);
subplot(1,2,2)
plot(x1,b1,'b'); hold on; plot(x2,b2,'b'); hold on;
plot(x3,b3,'b'); hold on;
```

This is the program clearing involving the steps to solve the differential equation for Rectangular triangular electric field pulse profile.

The output to the above program is shown in the next page.





If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of rectangular triangular electric field pulse profile. Also when compared to response for triangular or sawtooth increasing electric field pulse profile there will be significant oscillation or fluctuations in magnetic field.

Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value constants c_1 and c_2 governing the complementary solution is either negligible or zero. Here in this case we can see in solution that these complementary constants are much significant. Thus there will be oscillations in the envelope.

Table showing variation of magnetic field with respect to the respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile is in the next page.

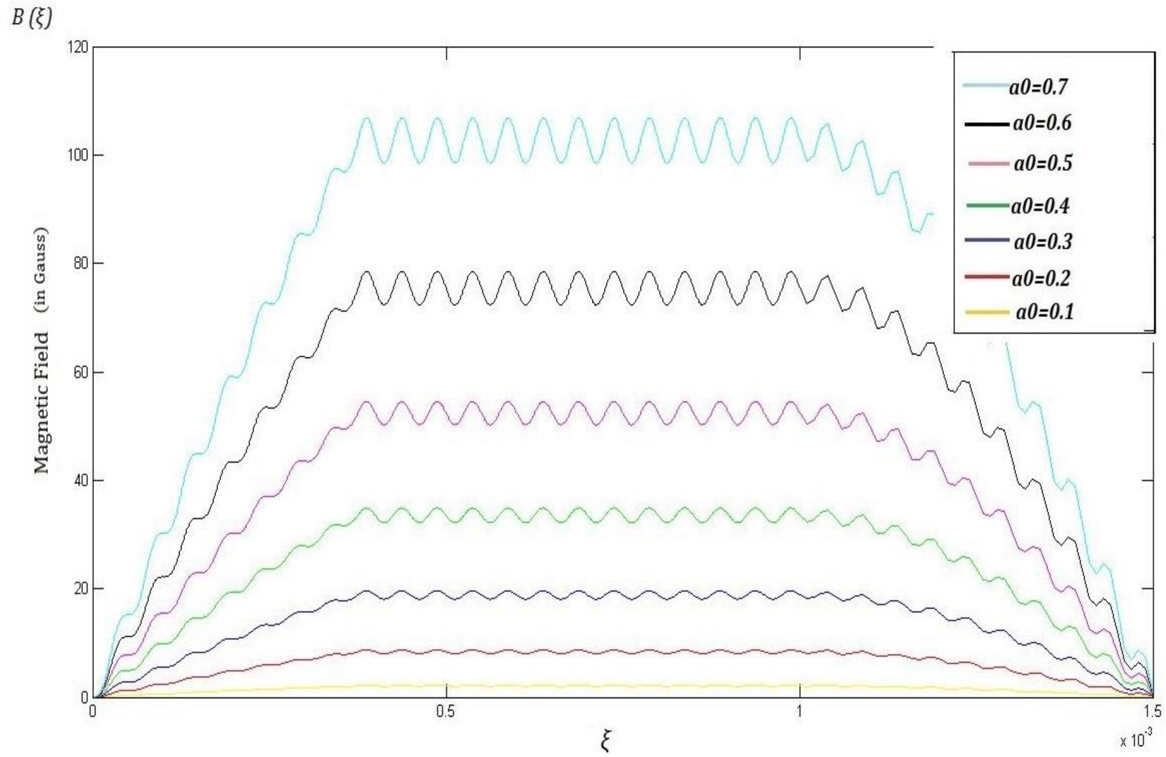
Rectangular Triangular pulse

a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	$n_0\ in$ $\times 10^{19} cm^{-3}$	max $of\ B(\xi)$ $in\ Gauss$
0.1	2.1803	20	49.0570	0.1	1.9623
0.2	8.7212	30	32.7047	0.5	9.8114
0.3	19.6228	40	24.5285	1	19.6228
0.4	34.8850	50	19.6228	2	39.2456
0.5	54.5078	60	16.3523	4	78.4912
0.6	78.4912	70	14.0163	6	117.7368
0.7	106.8352	80	12.2642	8	156.9823

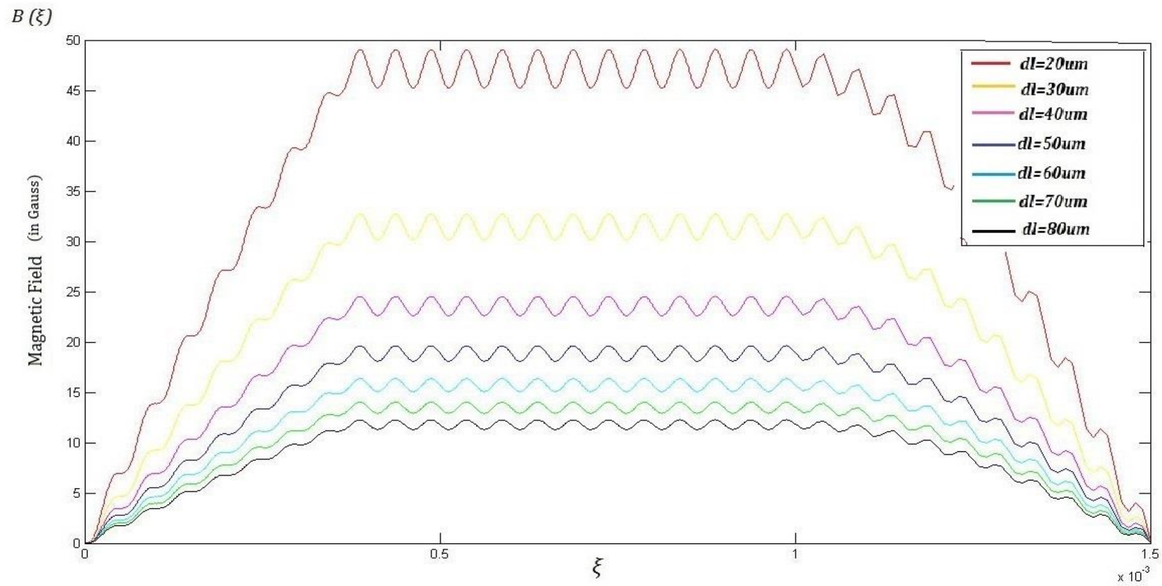
Table 3.5 Variation in Amplitude of Magnetic Fields for Rectangular Triangular Electric Field Pulse Profiles.

The maximum value of magnetic field for rectangular triangular electric field pulse profile observed is 19.6288 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile. This is lesser than the maximum value for sawtooth decreasing but it is slightly larger than triangular and sawtooth increasing. The margin of difference is much smaller. Its maximum value is almost equal to the maximum value of sinusoidal.

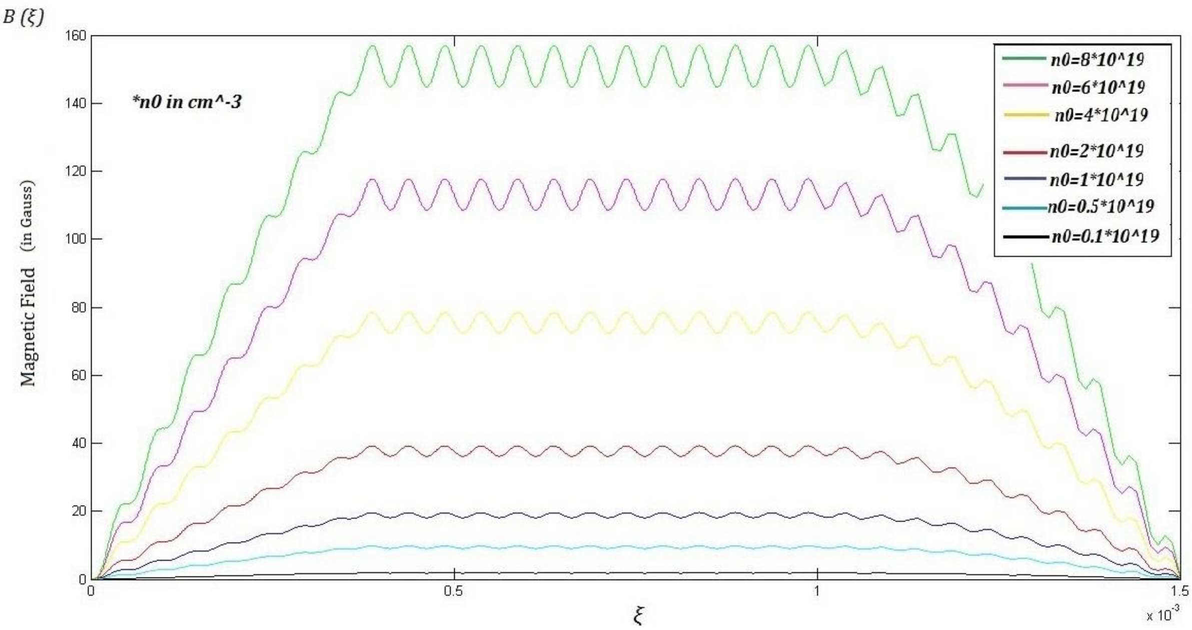
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0)



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)



These plots clearly shows that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp.

3.3.6 Square Electric Field Pulse Profile:

The electric field can be represented as follows

$E^2(\xi) = {E_0}^2 \, f(\xi),$

where $f(\xi)$ describes the Square electric field pulse profile whose expression is as follows. The practical Square electric field pulse can be constructed as follows. This pulse is piece wise function with three limits.

$$
\begin{aligned}
f(\xi) \quad &= 0,\ for\ 0 \le \xi \le L/6,\\
&= 1, for\ \tfrac{L}{6} \le \xi \le \tfrac{10L}{12},\\
&= 0, for\ \tfrac{10L}{12} \le \xi \le L.
\end{aligned}
$$

Now obtaining particular solution of nonhomogeneous part, we get three particular solutions since the square electric field pulse profile is piecewise built with three limits. The magnetic field from the first limit is represented as $B_{1p}(\xi)$, second limit is represented as $B_{2p}(\xi)$ and the third limit is represented as $B_{3p}(\xi)$. Now the mathematics involved is as shown.

$$(D^2 + U^2)B_{1p}(\xi) = 0,\ for\ 0 \le \xi \le L/6,$$

$$B_{1p}(\xi) = 0, for\ 0 \le \xi \le \frac{L}{6}.$$

Clearly the particular solution to the first limit is same as complementary solution since the first limit of square electric field pulse profile is zero. The total solution of the first limit is sum of complementary solution and particular solution which is as shown. $B_1(\xi)$ represents total solution upto first limit of rectangular triangular electric field pulse profile.

$$B_1(\xi) = B_c(\xi) + B_{1p}(\xi), for\ 0 \le \xi \le \frac{L}{4}.$$

$$B_1(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + 0, for\ 0 \le \xi \le \frac{L}{6}.$$

This is the solution upto first limit magnetic field for square electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3},$

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_1(\xi) = 0,\ \ for\ o \leq \xi \leq \frac{L}{6}.}$$

Now obtaining particular solution to second limit of electric field pulse profile, we get

$$(D^2 + U^2)B_{2p}(\xi) = A^2;\ \frac{L}{6} \leq \xi \leq 10L/1,$$

$$B_{2p}(\xi) = \frac{A^2}{U^2}, for \frac{L}{6} \leq \xi \leq 10L/12.$$

Thus this the particular solution of second limit. The total solution of the second limit is sum of complementary solution and particular solution which is as shown. $B_2(\xi)$ represents total solution upto second limit of square electric field pulse profile.

$$B_2(\xi) = B_c(\xi) + B_{2p}(\xi), for\ \frac{L}{6} \leq \xi \leq 10L/12.$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + \frac{A^2}{U^2}, for \frac{L}{6} \leq \xi \leq 10L/12.$$

This is the solution upto second limit magnetic field for square electric field pulse profile. Applying boundary condition to get $c_1\ and\ c_2$

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3},$

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$\boxed{B_2(\xi) = -19.0414 \cos\left(U\xi\right) + 0.3058 \sin\left(U\xi\right) + 18.8558\ , for\ \frac{L}{6} \leq \xi \leq \frac{L}{1.2}.}$$

Now obtaining particular solution to third limit of square electric field pulse profile we get

$$(D^2 + U^2)B_{3p}(\xi) = 0 for \frac{10L}{12} \leq \xi \leq L.$$

$$B_{3p}(\xi) = 0, for \frac{12L}{10} \leq \xi \leq L.$$

Thus this the particular solution of third limit and this is equal to the complementary solution since inhomogeneous part is zero. $B_3(\xi)$ represents total solution upto third limit of square electric field pulse profile.

$$B_3(\xi) = B_c(\xi) + B_{3p}(\xi), for \frac{12L}{10} \leq \xi \leq L.$$

$$B_2(\xi) = c_1 \cos(U\xi) + c_2 \sin(U\xi) + 0, for \frac{12L}{10} \leq \xi \leq L.$$

This is the solution upto third limit magnetic field for square electric field pulse profile. Applying boundary condition to get c_1 and c_2

Taking $a_0 = 0.3,\ d_l = 50 \times 10^{-4} cm, and\ n_0 = 10^{19} cm^{-3}$,

where a_0 laser intensity parameter, d_l differential length affecting slope of density ramp, n_0 is plasma density at beginning or origin. Taking remaining all values in cgs the final solution which we get is

$$B_3(\xi) == -0.1706 \cos(U\xi) - 1.293 \sin(U\xi),\ for\ \ L/1.2 \leq \xi \leq L.$$

The magnetic field response for square electric field pulse profile is as shown

$$\begin{aligned} B(\xi) \quad &= 0\ for\ 0 \leq \xi \leq L/6, \\ &= -19.0414 \cos(U\xi) + 0.3058 \sin(U\xi) + 18.8558,\ for\ \frac{L}{6} \leq \xi \leq \frac{L}{1.2}, \\ &= -0.1706 \cos(U\xi) - 1.293 \sin(U\xi),\ for\ \ L/1.2 \leq \xi \leq L. \end{aligned}$$

We have tried to write a program of this solution in Matlab and analyse the result. The program is also helpful to us in analysing the variation of magnetic field with respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin

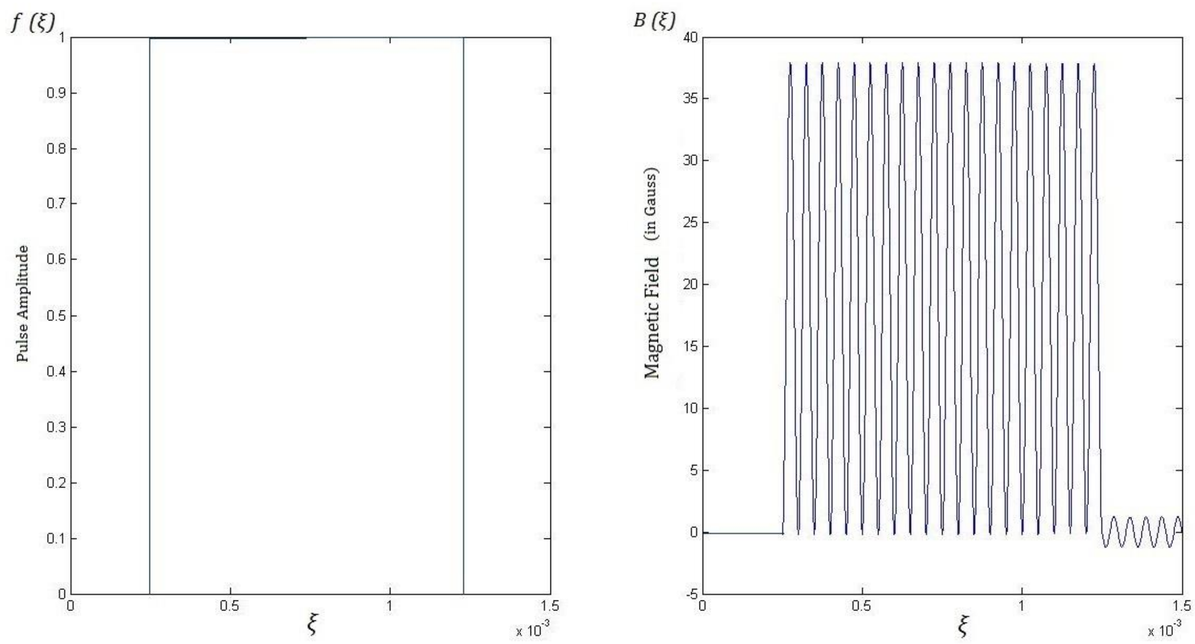
Code:

```
l=1.5*10^-3;
t1=[0:0.000001:l/6];
b1=0;
t2=[l/6:0.000001:l/1.2];
b2=1;
t3=[l/1.2:0.000001:l];
b3=0;
subplot(1,2,1);
plot(t1,b1);hold on;plot(t2,b2);hold on;plot(t3,b3);
syms c2 c3 c4 c5
L=1.5*10^-3;
g=pi/L;
wl=1.8*10^15;
c=3*10^10;
k1=wl/c;
k2=1.256*10^5;
k3=k2^2;
dl=50*10^-4;
e=4.38*10^-10;
m=9.1*10^-28;
f= 3.1816e+009;
n0=1*10^19;
wpe=sqrt(f*n0);
a0=0.3;
A=(wpe^2*m*k1*a0^2*c)/(4*wl*e*dl);
x1=[0;0.000001:L/6];
b1=0;
```

```
x2=[L/6:0.000001:L/1.2];
eqn1=[0.99*c2-0.0159*c3+A/k3;0.0159*k2*c2+0.99*k2*c3+0];
S=solve([eqn1]);
b2=subs(S.c2)*cos(k2*x2)+subs(S.c3)*sin(k2*x2)+A/k3;
bg=subs(S.c2)*cos(k2*L/1.2)+subs(S.c3)*sin(k2*L/1.2)+A/k3;
b1g=-subs(S.c2)*k2*sin(k2*L/1.2)+subs(S.c3)*k2*cos(k2*L/1.2);
x3=[L/1.2:0.000001:L];
eqn2=[0.99*c4-0.0159*c5-bg;0.0159*k2*c4+0.99*k2*c5-b1g];
D=solve([eqn2]);
b3=subs(D.c4)*cos(k2*x3)+subs(D.c5)*sin(k2*x3);
subplot(1,2,2);
plot(x1,b1,'b'); hold on; plot(x2,b2,'b'); hold on;
plot(x3,b3,'b');
hold on;
```

This is the program clearing involving the steps to solve the differential equation for Square electric field pulse profile.

The output to the above program is shown



If we carefully concentrate on the shape of magnetic field pulse we see its envelope is following the shape of rectangular triangular electric field pulse profile but with huge oscillations. The oscillations are much larger than what we have seen in the case sinusoidal, sawtooth decreasing or rectangular triangular electric field pulse profile.

Since for every pulse profile, the complementary solution is same and since the complementary solution is composed of sinusoidal terms there will be significant oscillations in the envelope of magnetic field of all the pulse profiles unless the value constants c_1 and c_2 governing the complementary solution is either negligible or zero. Here in this case we can see in solution that these complementary constants are much significant. Thus there will be oscillations in the envelope. Another reason for oscillations can also be given as follows. Since this pulse is not practically feasible and also this pulse can be constructed by sum of infinite number sinusoidal terms. As the differential equation is linear, if solution to sinusoidal will also be sinusoidal and solution to infinite sinusoidal will be sum of infinite sinusoidal. Thus output has significant oscillations since we can construct square wave with infinite sinusoidal wave.

Table showing variation of magnetic field with respect to the respect to laser intensity parameter (a_0), (d_l) differential length parameter describing the slope of density ramp and (n_0) density at origin for sinusoidal electric field pulse profile is as shown

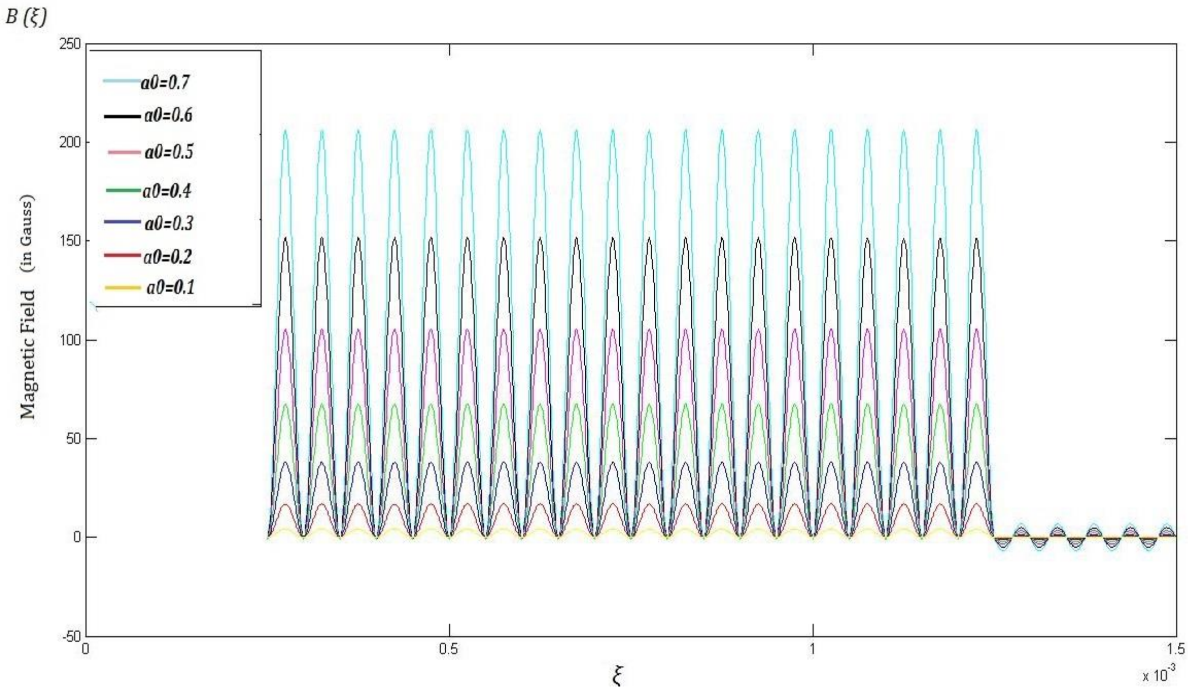
Square Pulse

a_0	$Max\ of\ B(\xi)$ $in\ Gauss$	d_l(um)	$Max\ of\ B(\xi)$ $in\ Gauss$	$n_0\ in$ $\times 10^{19} cm^{-3}$	$\max of\ B(\xi)$ $in\ Gauss$
0.1	4.2111	20	94.7492	0.1	3.7900
0.2	16.8443	30	63.1661	0.5	18.9498
0.3	37.8997	40	47.3746	1	37.8997
0.4	67.3772	50	37.8997	2	75.7994
0.5	105.2769	60	31.5831	4	151.5987
0.6	151.5987	70	27.0712	6	227.3981
0.7	206.3427	80	23.6873	8	303.1975

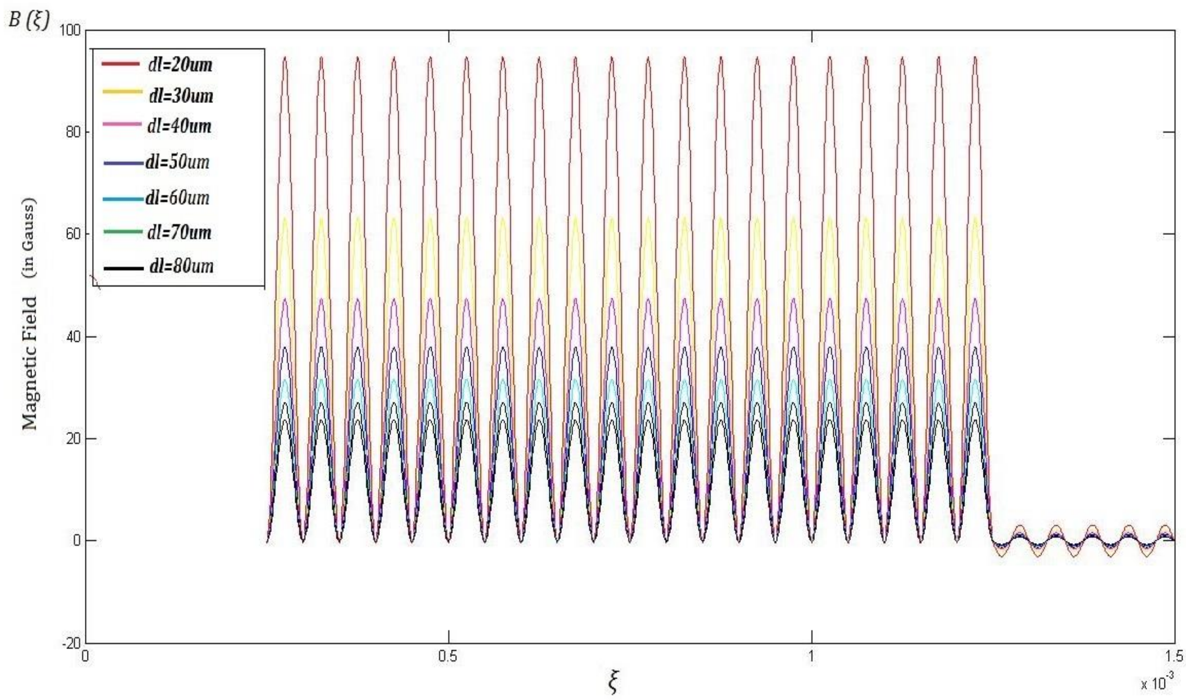
Table 3.6 Variation in Amplitude of Magnetic Fields for Square Electric Field Pulse Profiles.

The maximum value of magnetic field for sawtooth increasing electric field pulse profile observed is 37.8997 Gauss for laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile. This is much larger than the maximum value for remaining pulse profile that we have taken. But important consideration is even though its maximum value is largest it also has larger fluctuations which almost falls to zero.

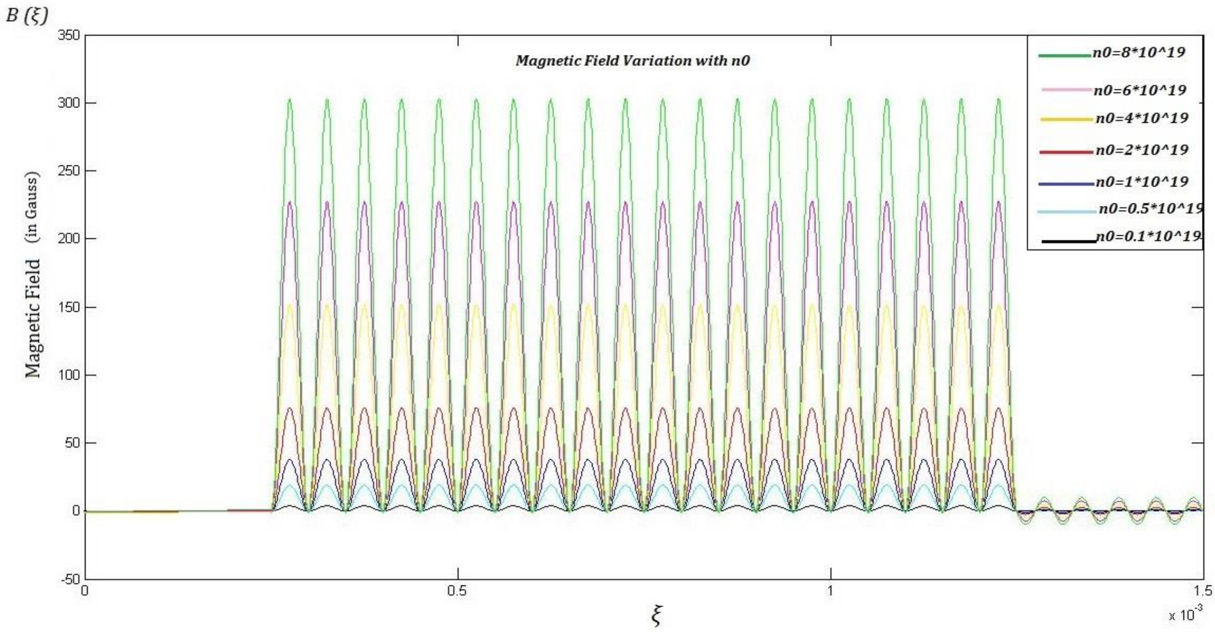
Matlab output showing variation of magnetic field with respect to laser intensity parameter (a_0) can be seen in the next page.



Matlab output showing variation of magnetic field with respect to (d_l) differential length parameter describing the slope of density ramp.



Matlab output showing variation of magnetic field with respect to plasma density (n_0)

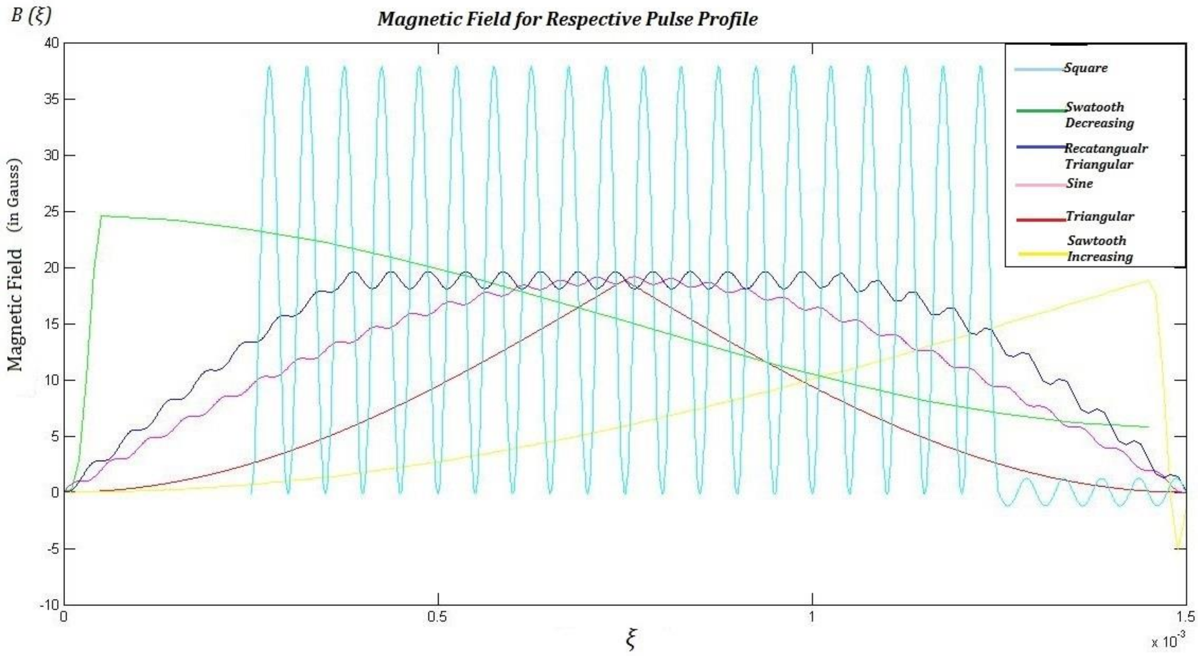


These plots clearly shows that magnetic field increases with increase in laser intensity parameter and plasma density. Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp.

Chapter 4

4.1 Results and Discussion

In the previous chapter we have solved the second order linear differential equation to obtain magnetic field response for different electric field profiles like sine, triangular, sawtooth decreasing, sawtooth increasing, rectangular triangular and square. Here we try to analyse the results that we obtained in the previous chapter. The magnetic field response of all the pulses combine is as shown below. Here we should note that the values of laser intensity parameter ($a_0 = 0.3$), ($d_l = 50 \times 10^{-4} cm$) differential length parameter describing the slope of density ramp and ($n_0 = 10^{19} cm^{-3}$) density at origin for sinusoidal electric field pulse profile.



Observing above magnetic field response of all pulses combine and trying to analyse individual pulse we get

4.1.1 Sine pulse:

The magnetic field response follows the path of sine electric field pulse profile but has oscillations in the output envelope. These oscillations are due to complementary solution of the differential equation which we have already seen in earlier chapter. The maximum magnetic field that was observed was 19.163 Gauss. For this pulse we can say that it can be used only if we are interested in getting exactly sine magnetic field response.

4.1.2 Triangular pulse:

The magnetic field response follows the shape of electric field response but the interesting thing here is that this pulse profile is oscillations free and the curve is smooth. The decreasing the step size also does not alter the shape. This means in this profile complementary solution does not have any effect resulting in no oscillations. The maximum electric field that was observed with this profile is 18.858 Gauss which is slightly less than the sine pulse but margin is very small. We can use this pulse profile when we want complete oscillations free magnetic field response with less importance to peak amplitude.

4.1.3 Sawtooth Increasing Pulse:

The magnetic field response follows the shape of electric field response but the interesting thing here is that this pulse profile is oscillations free and the curve is smooth similar to the response of triangular pulse profile.. The decreasing the step size also does not alter the shape. This means here in this profile complementary solution does not have any effect resulting in no oscillations as in the case of triangular pulse profile. The maximum electric field that was observed with this profile is 18.858 Gauss which exactly same as that of maximum value of triangular pulse and is slightly less than the sine pulse but margin is very small. We can use this pulse profile when we want complete oscillations free magnetic field response with less importance to peak amplitude as with the case of triangular pulse profile.

4.1.4 Rectangular Triangular Pulse:

This profile response can be compared to that of the sine electric field pulse profile. The magnetic field response follows the path of rectangular electric field pulse profile but has oscillations in the output envelope similar to sine pulse. These oscillations are due to complementary solution of the differential equation which we have already seen in earlier chapter. The maximum magnetic field that was observed was 19.6228 Gauss. For this pulse we can say that it can be used only if we are interested in getting exactly rectangular triangular magnetic field response.

4.1.5 Sawtooth Decreasing pulse:

This pulse profile offers the interesting result as we can see its peak value is significantly larger than remaining pulse except square pulse and the envelope has fluctuations. The fluctuations can be observed if decrease the step size and the reason for the fluctuations on the envelope is significant complementary solution. The magnetic field response follows the shape of sawtooth decreasing electric field pulse profile. The maximum amplitude that can be seen for this pulse profile is 24.0616 Gauss. We can use this pulse if we are interested in shape, high peak amplitude and low oscillations.

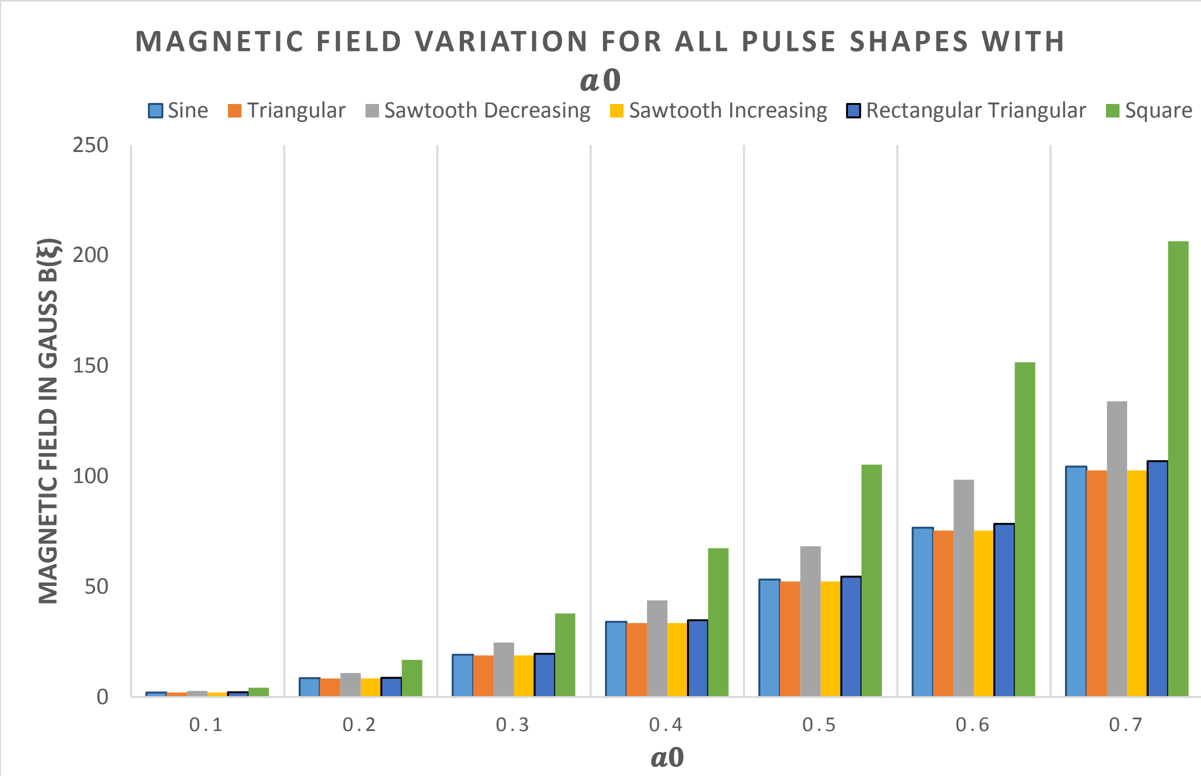
4.1.6 Square Pulse:

The maximum amplitude with this pulse profile is 37.8997 Gauss which largest of compared to all the remaining pulse profile. Even here the magnetic field envelope follows the shape of square pulse profile. But interesting consideration is that even though it has largest peak amplitude it also has heavy fluctuations on the envelope which falls to as low as zero. The complementary solution is highly significant for this pulse profile. We can opt for this pulse profile only if criteria for consideration is maximum peak amplitude and we do not care about occurrence of oscillations. Else we can opt for other pulse profile.

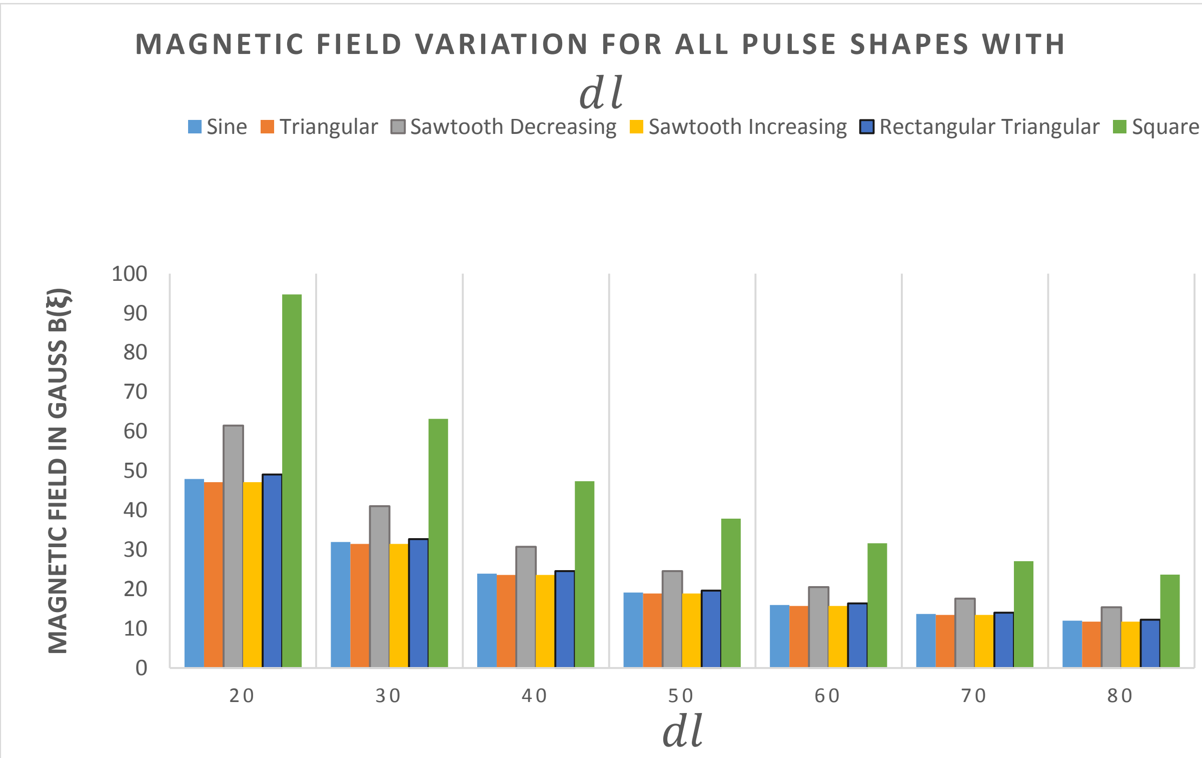
Also magnetic field increases with increase in laser intensity parameter (a_0) and plasma density (n_0). Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp. This result is followed by all the pulse profile.

4.2 Bar Graphs:

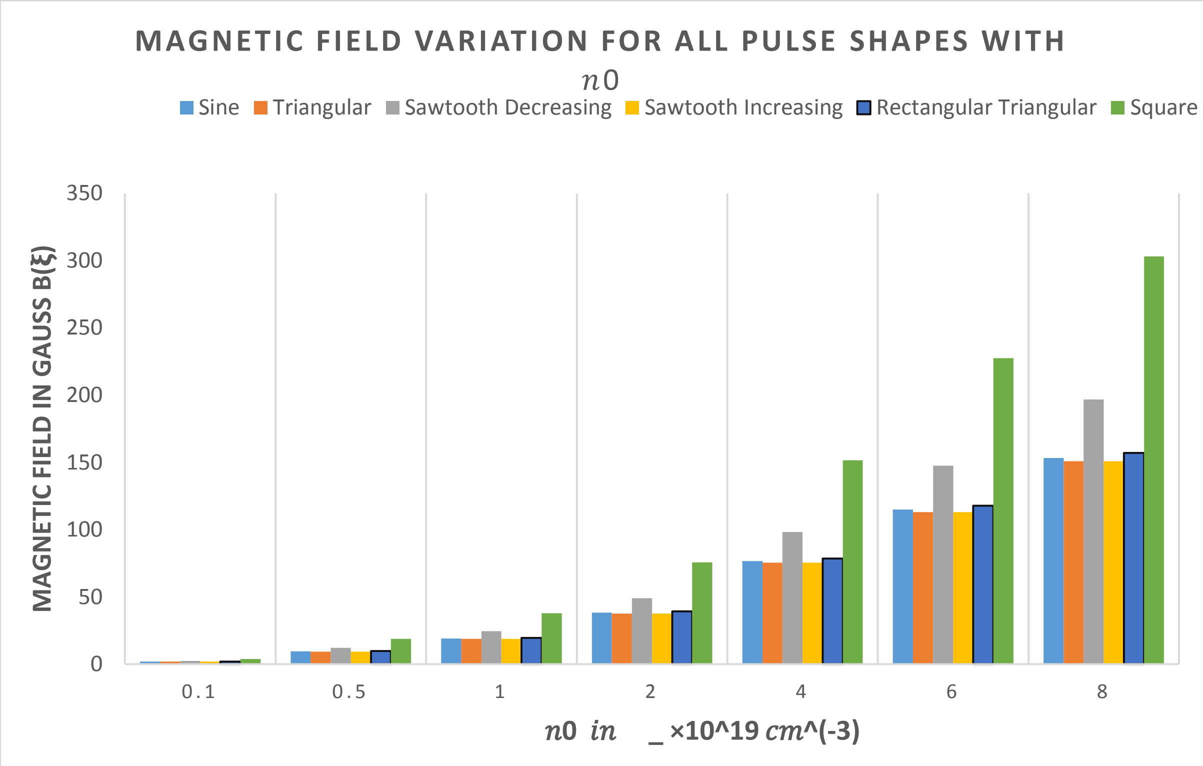
The variation of magnetic field of all the pulse profile with respect to laser intensity parameter (a_0) is as shown



The variation of magnetic field of all the pulse profile with respect to plasma density (n_0) is as shown



The variation of magnetic field of all the pulse profile with respect to (d_l) differential length parameter relating the slope of density ramp is shown.



Chapter 5

Conclusion and Summary

In this project dissertation we have established the response of magnetic field for various electric field pulse profiles such as sine, triangular, sawtooth decreasing, sawtooth increasing, rectangular triangular and square pulse. Second order linear differential equation was derived for laser irradiated plasma. This equation is solved for above mentioned electric field pulse profiles to obtain magnetic field response. The response of the magnetic field for these profiles were analysed and maximum amplitude compared to all other electric field pulse profile was found for Square pulse but with heavy oscillations. The second best amplitude was with Sawtooth Decreasing and interestingly it had minor oscillation unlike Square profile. The maximum amplitudes of Sine, Triangular, Sawtooth Increasing, Rectangular triangular electric field pulse profiles were found to be nearly same with minor fluctuations.

Also for all these electric field profiles magnetic field increases with increase in laser intensity parameter (a_0) and plasma density (n_0). Magnetic field decreases with increase in (d_l) differential length parameter describing the slope of density ramp. This result is followed by all the pulse profile.

In future we can try to improve the amplitude of magnetic field pulse profiles without effecting the shape of the pulse profile by employing different methods such as taking different plasma density profile instead ramp varying along transverse direction, introducing the effect of temperature gradient etc. This procedure can also be applied for the relativistic case of laser irradiated plasma.

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