Chapter – 1

Introduction

1.1 Motivation

Consolidation phenomenon was first described by Terzaghi in 1925 as the squeezing out of the pore water from the soil under a long term static loading without replacement of water with air. According to Terzaghi, consolidation can be expressed mathematically as,

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \qquad (1.1)$$

Where Cv is the coefficient of consolidation, u denotes the pore water pressure, z and t are displacement and time respectively.

Consolidation behaviour of soils depends on various factors such as structure of soil grains, void ratio, pore-water pressure, etc. For coarse grained soils like sands, the permeability is sufficiently high such that the consolidation takes place nearly instantaneously with the applied load. For cohesive soils, such as clays, the permeability is very small and the consolidation of a thick clay deposit may take years or even decades to complete.

Soils naturally occur in layers of clay and sand. Thus combined behaviour of such layered soil system requires a detailed study for greater accuracy in assessment of soil behaviour and soil structure interactions. Thus in the present study natural occurrence of layered soils has been related with artificially prepared layered sand-clay sample.

The objective of this study is to propose a simple numerical solution to the one-dimensional consolidation due to constant rate of loading, which can be used for estimation of the degree of consolidation, settlement and pore-water pressure distributions more efficiently. Pore pressure dissipation at regular time intervals can be used to analyse the typical soil behaviours in varying conditions.

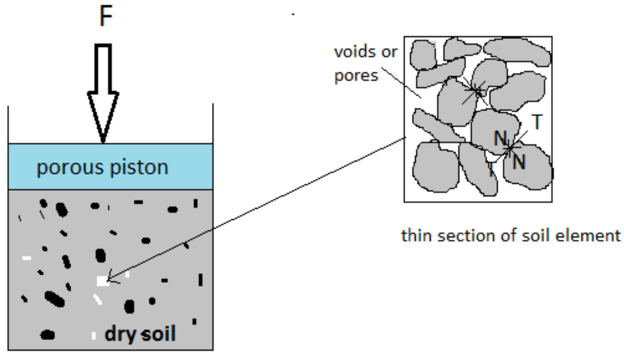
1.2 Objective of study

The main objective of this study is to evaluate the consolidation characteristics of the thin clay lamina in sand acting as multi-layered soil system. Ground is heterogeneous always constituting various kinds of soils in layers. Thus, study of these multi-layered soils is essential for accurate analysis of the settlement of the ground. Many scientists have worked in this field to evaluate the consolidation characteristics of such soils.

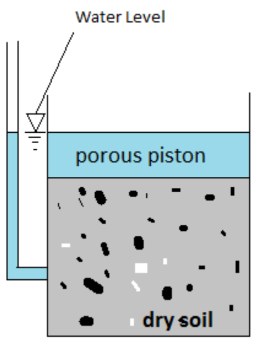
Sands or granular soils possess high permeability, thus allowing the water to flow easily in a lesser duration of time. Such type of soils experience immediate settlement on application of loads. Most of the consolidation settlement in these soils is observed in primary consolidation stage itself. On the other hand clay or fine grained stiff soils have low permeability. Thus settlement takes place after load application for long durations. Settlement due to consolidation in clays can continue for years after the construction of the buildings. This consolidation settlement is due to restructuring of soil solids and is related to secondary consolidation. The orientation of this work is towards studying the effects of thin clay lamina in granular soils in context to the consolidation. Introduction of clay layer to the sandy strata can slow down the settlement process.

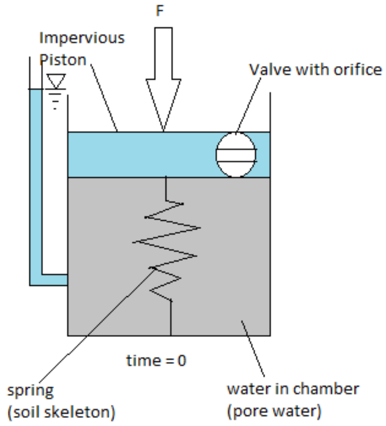
During construction of buildings or superstructure, load is transmitted to the soil through the foundation. Due to this load the underlying soil experiences settlement, which can be large or small depending upon the soil strata and the loads. This settlement needs to be estimated prior to the construction to utmost accuracy in order to prevent future damages.

The process of consolidation is generally explained with an analogy of a spring, a container with a valve, and water. In this system, the spring depicts the compressibility or the structure of soil and the water which fills the container accounts for the pore water in the soil. Figure 1.1 gives the idealization of the consolidation phenomenon.

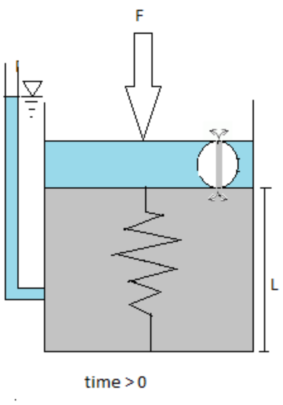


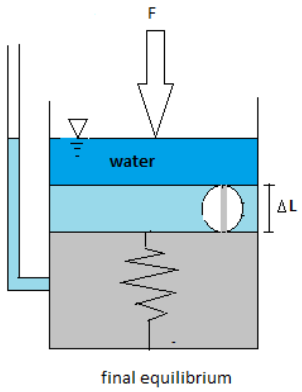
(a) Dry soil with pores under applied force





(b) Saturated soil (c) Soil-water spring analogy system





(d) At t>0, $\Delta u > \Delta\sigma$ (e) Final equilibrium, $\Delta u=0$

Fig 1.1 Idealization of Consolidation

When drainage occurs due to applied stresses the pore water pressure is greater than the normal values because it is carrying part of the applied stress as opposed to the soil particles. Thus pore water pressure dissipation study can aid in evaluating soil behaviour on application of constant rate loadings and varying loads.

The total settlement S, can be calculated as the sum of the following three constituents:

$$S = S_i + S_p + S_s \quad (1.2)$$

Where; S_i = immediate settlement

S_p = primary consolidation settlement

S_s = secondary consolidation settlement

Secondary compression or settlement is difficult to compute as no such evidences are there to differentiate the primary and secondary consolidation stages precisely. Secondary compression is significant in case of fine grained soils with lesser permeability.

Settlement occurs slowly in low permeability soils thus accounting to secondary consolidation. Due to secondary compression highly viscous water present between the points of contact in soil is forced out. In such cases when secondary compression continues, settlement can take years to complete especially for saturated clays.

Figure 1.2 shows the sand-clay layered soil strata in natural conditions under applied effective stress.

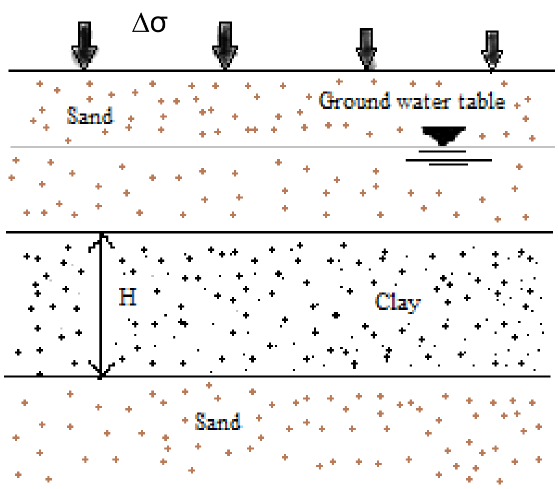


Fig 1.2 Sand-clay layers under applied effective stress

Variation in pore pressure and effective stress in the clay layer can be shown as in figure 1.3 below. Pore pressure and effective stress attain maximum values at the centre and decrease to almost zero at boundaries.

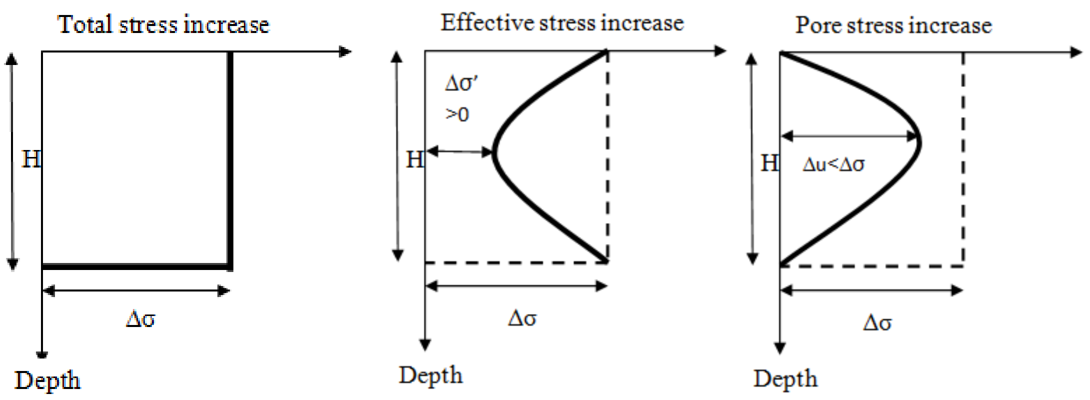


Fig 1.3 Variation of total, pore-water and effective stresses in clay layer

Consolidation stages for the soil are shown in figure 1.4. Initial compression occurs due to preloading when sample is placed in position before the load is applied. Primary consolidation occurs after load is applied and pore water starts dissipating, thus causing deformation. Secondary compression occurs after pore water pressure dissipates completely and hygroscopic fluid comes out due to rearrangement of particles. The dissipation of excess pore pressure is associated with an increase in effective stress and volumetric strain.

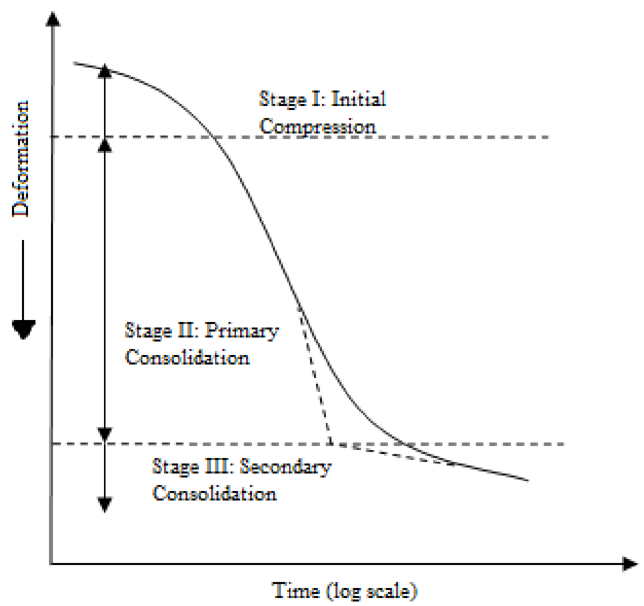


Fig 1.4 Stages of Consolidation

1.3 Major scientists and their contribution

Table 1.1 Major Scientists & their Contribution

Scientists	Contribution
Karl Von Terzaghi(1925)	Theory of 1-D consolidation
M.A. Biot (1940)	Theory of 3-D consolidation
Gray (1945)	Consolidation of double-layered soil
Ji et al. (1948)	Quasi-dynamic solution for symmetric consolidation of isotropic soil
Schiffman (1958)	Theory of 1-D consolidation for time dependent loading with varying permeability and coefficient of consolidation
Geng et al. (1962)	Solution of non-linear 1-D consolidation using Laplace transform
Mikasa (1963)	Theory of 1-D consolidation in terms of compression strain
Davis & Raymond (1965)	Relation of permeability of soil with coefficient of consolidation
Gibson & Hussey (1967)	Analysis of 'rate of consolidation' depending upon thickness of soil strata.
Schiffman and Stein (1970)	Analytical solution for 1-D consolidation of layered soil system
Oslon.et al. (1977)	Incorporated Ramp loading as mode of load application in 1-D consolidation
Desai and Saxena (1977)	Consolidation analysis of layered anisotropic foundation
Cai et al. (1981)	Semi analytical 1-D consolidation solution with variation of cyclically loaded soil compressibility.
Booker and Rowe (1983)	1-D consolidation of periodically layered soil
Abid and Pyrah (1988)	Finite element method for coupled approaches in 1-D consolidation
Lee et al. (1992)	1-D consolidation of layered systems
Xie and Pan (1995)	Analytical solution for layered system under time dependent loading

Scientists	Contribution
Sloan and Abbo (1999)	Biot consolidation analysis with automatic time stepping and error control
Xie et al. (2001)	1-D non-linear consolidation of double layered soil
Chen et al. (2004)	1-D non linear consolidation of multilayered soil using differential quadrature method
Hsu and Lu (2006)	Ramp loading in 1-D consolidation with varying coefficient of consolidation
Conte et al. (2006)	Analysis of coupled consolidation of unsaturated soil under plain strain loading.
Wang et al. (2007)	Mesh-less method for consolidation analysis of saturated soils with anisotropic damage
Ai et al. (2008)	State space solution of 3-D consolidation in layered soils
Menendez et al. (2009)	Mathematical modelling of consolidation in elastic soil with incompressible fluid by FEM
Huang and Griffith (2010)	Coupled and uncoupled solutions of consolidation in layered soils by finite element method
Rani et al. (2011)	Consolidation of mechanically isotropic however hydraulically anisotropic clay layer incorporating compression of pore fluid and solid constituents.
Kim and Mission (2011)	Numerical analysis of 1-D consolidation in layered clay using compression strain
Tewatia et al. (2012)	Settlement trends in primary and secondary consolidation
Trivedi et al. (2014)	Consolidation of clay gouge amid permeating rock mass

1.4 Physical Background : Error Function

The error function and the complementary error function are significant special functions which are noticed in solving the heat diffusion problems, mass and momentum transport, probability, the theory of errors and various branches of mathematical physics.

Error function is of sigmoid shape and is used in probability distributions and partial differential equations. It is defined as:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{1.3}$$

The complementary error function, denoted *erfc*, is defined as

$$erfc(x) = 1 - \mathrm{erf}(x) \tag{1.4}$$

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = e^{-x^2} erfcx(x) \tag{1.5}$$

In the above equation *erfcx* is the scaled complementary error function. The error and complementary error functions are applicable in solutions of the heat flow problems when the boundary conditions can be provided by the Heaviside step function.

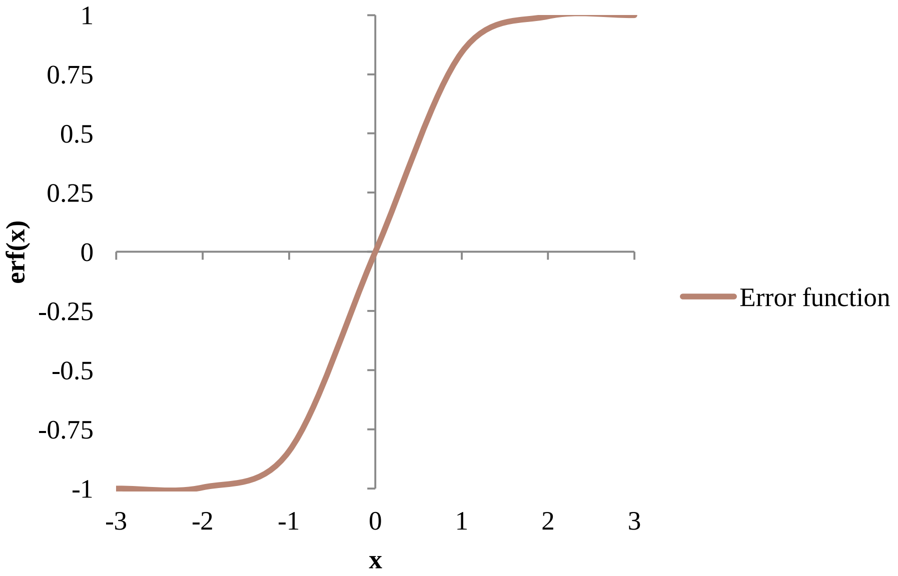


Fig. 1.5 Plot of Error function

The imaginary error function, denoted by *erfi*, is mathematically given as,

$$erfi(z) = -ierf(iz) = \frac{2}{\sqrt{\pi}} e^{x^2} D(x), \tag{1.6}$$

where *D*(*x*) is known as the Dawson function.

The error function is related to the cumulative distribution Φ by

$$\emptyset(x) = \frac{1}{2} + \frac{1}{2} \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) \tag{1.7}$$

Abramowitz and Stegun (1964) provided various approximations with different accuracy. This gives one a choice to opt for the fastest approximation suitable for a given application. In order of increasing accuracy, these estimations are:

a) $\mathrm{erf}(x) \approx 1 - \frac{1}{(1+a_1x+a_2x^2+a_3x^3+a_4x^4)^4}$ (Maximum error: 5×10^{-4})

where a_1 = 0.278393, a_2 = 0.230389, a_3 = 0.000972, a_4 = 0.078108

b) $\mathrm{erf}(x) \approx 1 - (a_1t + a_2t^2 + a_3t^3)e^{-x^2}$,

$t = \frac{1}{1+px}$ (Maximum error: 2.5×10^{-5})

where p = 0.47047, a_1 = 0.3480242, a_2 = −0.0958798, a_3 = 0.7478556

c) $\mathrm{erf}(x) \approx 1 - \frac{1}{(1+a_1x+a_2x^2+\cdots+a_6x^6)^{16}}$ (Maximum error: 3×10^{-7})

where a_1 = 0.0705230784, a_2 = 0.0422820123, a_3 = 0.0092705272, a_4 = 0.0001520143 , a_5 = 0.0002765672, a_6 = 0.0000430638

d) $\mathrm{erf}(x) \approx 1 - (a_1t + a_2t^2 + \cdots + a_5t^5)e^{-x^2}$

$t = \frac{1}{1+px}$ (Maximum error: 1.5×10^{-7})

where p = 0.3275911, a_1 = 0.254829592, a_2 = −0.284496736, a_3 = 1.421413741, a_4 = −1.453152027, a_5 = 1.061405429

All of these approximations are valid for $x \geq 0$.

For all ranges of values, there exists an approximation with a maximum error of 1.2 x 10^{-7} which can be represented as follows:

$$\mathrm{erf}(x) = \begin{cases} 1 - T, for\ x \geq 0 \\ T - 1, for\ x < 0 \end{cases}$$

With,

$$T = t.\exp(-x^2 - 1.26551223 + 1.00002368.t + 0.37409196.t^2 + 0.09678418.t^3 - 0.18628806.t^4 + 0.27886807.t^5 - 1.13520398.t^6 + 1.48851587.t^7 - 0.82215223.t^8 + 0.17087277.t^9)$$

And

$$t = \frac{1}{1 + 0.5|x|}$$

Generalized form of error function can be given as,

$$E_n(x) = \frac{n!}{\sqrt{\pi}} \int_0^x e^{-t^n} dt = \frac{n!}{\sqrt{\pi}} \sum_{p=0}^{\infty} (-1)^p \frac{x^{np+1}}{(np+1)p!}$$

Some of the values of the generalised error function are,

- $E_0(x) = \frac{x}{e\sqrt{\pi}}$
- $E_2(x)$ is the error function, *erf(x)*.
- $E_1(x) = (1 - e^{-x})/\sqrt{\pi}$

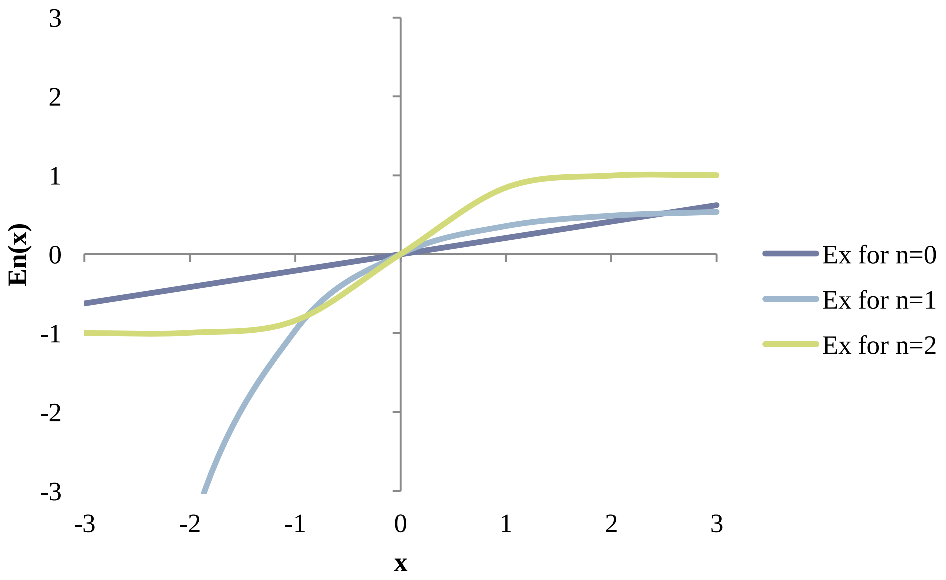


Fig. 1.6 Generalised Error function E_n (x)

1.5 Organization of report

Chapter 1 It gives the idea of consolidation process and mechanism of settlement occurring due to consolidation. It provides information about the various stages of consolidation and types of settlement. Also it introduces the error function and its application in solving the parabolic differential equations.

Chapter 2 Discusses about the various works and researches done in the field of consolidation. The literature review of the previous work is incorporated in this chapter. Various methods used for evaluation of consolidation parameters such as coefficient of consolidation, rate of consolidation, and time factor under constant and varying loads are mentioned.

Chapter 3 It deals with the numerical analysis of the 1-D consolidation. Theory of consolidation as given by conventional method of Fourier analysis has been illustrated. Estimation of one dimensional consolidation solution using the error function is also discussed in detail.

Chapter 4 Materials and Methods of experimental analysis are discussed in this chapter. Standard experiment procedures are provided as per the Indian standard codes. Determination procedures of Atterberg limits and other properties of soils are discussed.

Chapter 5 It includes the results obtained from the experimental as well as numerical analysis .Comparison of theoretical methods has also been illustrated. Validation of theoretical results with experimental results is done in this chapter.

Chapter 6 This chapter gives the conclusions inferred from the project work. Merits and Merits and limitations of the employed numerical method are discussed. Future scope of this method has been discussed.

Chapter 7 This chapter presents an account of several research publications that have been used and studied for reference in the project work.

Chapter – 2

Literature Review

2.1 Consolidation: Basic Concept

Consolidation is a process of expulsion of water from the voids in a soil on application of load or stress without replacement of water with air. As compression proceeds, soil grains rearrange themselves into a more stable and denser configuration. Due to squeezing out of pore fluid there is a decrease in the volume which results in surface settlement (**Holtz and Kovac, 1981).** Thus consolidation can be termed as the reduction in volume of fully saturated soil due to expulsion of pore-water under external loading. Consolidation is associated with reduction in volume of soil due to drainage of pore water. Excess pore water pressure is developed in the soil due to application of stresses as the pore fluid is assumed to be incompressible in nature. This pore water pressure in excess causes increase in effective stresses in the soil.

Consolidation of soil was first conducted by Karl Von Terzaghi in 1925. Terzaghi gave the theory of one dimensional consolidation of soils and derived the partial differential equation for consolidation phenomena in terms of pore water pressure (u). He coined the term constant coefficient of consolidation (Cv).

Dissipation of pore water through the soil is governed by Darcy's law (1856). Darcy's law states that the rate of fluid flow in a permeable medium is directly proportional to the drop in vertical elevation between two places in the medium and in directlyproportional to the distance between them. The distribution of pore water pressure varies depending on time, applied stress and depth. It is generally in the form of isochrones of excess pore pressure.

While applying Terzaghi's consolidation theory, the pressure at the ground level causing consolidation is assumed to be applied instantaneously. Practically, building or embankment loads are never placed on the ground instantaneously rather they are acted upon gradually over a certain time period. Limitations of Terzaghi's theory have been discussed by many researchers and have been modified from time to time.

Solution of one dimensional consolidation equation of **Terzaghi (1943)** is given using Fourier analysis. The consolidation equation is a parabolic differential equation which can be solved analytically using series given by **Fourier (1807).**

One of the assumptions in Terzaghi's theory is homogeneous nature of soil which is rarely experienced in reality. Since soils are usually heterogeneous in nature, their actual consolidation in the field is mostly the result of the interaction of a series of compressible layers of soils having different consolidation characteristics.

2.2 General Theories

Many 1-D consolidation theories developed for multi-layered soils have analytical solutions which extended from the conventional consolidation equation of **Terzaghi (1943)** and are expressed in terms of excessive pore pressure **(Schiffman and Stein 1970; Lee et al. 1992; Xie et al. 1999).** Some of the derived closed-form solutions were complex and hence limited to the case of double-layered soil **(Gray 1945; Xie et al. 2002).**

Gray (1945) first talked about the nature of consolidation for two contiguous layers of different soils with varying compressibility. The general analytical solution for the one-dimensional consolidation of a layered soil system has been developed by **Schiffman & Stein (1970). Desai & Saxena (1977)** analysed the consolidation behaviour of layered anisotropic foundations.

Abid & Pyrah (1988) presented some guidelines for using the finite-element (FE) method to predict 1D consolidation behaviour of a soil using both diffusion and coupled approaches.

Lee et al. (1992) developed more efficient analytical solution, which illustrated that the effects of permeability and coefficient of volume compressibility of soil on the consolidation of layered soil system are not same and cannot be employed into a single coefficient of consolidation.

The volume compressibility of the soil layer m_v also plays an important role in the rate of consolidation. **Xie & Pan (1995)** further developed an analytical solution for a layered system under time-dependent loading. **Pyrah (1996)** showed that the 1D consolidation behaviour of layered soils consisting of two layers with the same value of the c_v, but different k and mv were quite different. **Zhu & Yin (1999**) gave more analytical solutions for different loading cases.

Various consolidation tests have been carried out considering the pore water pressure dissipation as a major parameter governing the process of consolidation. However, certain field observations have produced the results mostly in terms of compression strain. **Mikasa (1963)** introduced for the first time, a consolidation equation in terms of compression strain for saturated clays having homogeneous consolidation characteristics throughout its depth. Considering different assumptions and limitations of the Terzaghi theory, a more accurate and generalized formulation has been presented for the case of finite strains, nonlinear consolidation parameters, and consideration of self-weight effects on consolidation by **Mikasa et al. (1998)**.

Mikasa (1963) introduced for the first time, a consolidation equation in terms of compression strain for saturated clays having homogeneous consolidation characteristics throughout its depth. When the coefficient of consolidation *Cv* is assumed to be constant and infinitely small strains are considered, the generalized 1-D consolidation equation becomes,

$$\frac{\partial \varepsilon}{\partial t} = Cv\ \frac{\partial^2 \varepsilon}{\partial z^2} \qquad (2.1)$$

Mikasa's theory was studied further for multi-layered clays by **Kim and Mission (2011)**. The equation given by Terzaghi is a kind of diffusion equation and thus it can be solved mathematically for a homogeneous soil with drained or undrained boundary conditions. For a layered soil, the solution can be estimated using finite difference method in which different properties are assigned to different layers.

Olson (1977) studied further the Terzaghi's 1-D consolidation theory for ramp loading cases, employing consolidation due to vertical flow, radial flow and combined vertical and radial flow. He discretised the ramp loading into very small loads with small increments, and applied Terzaghi's theory and the principle of superposition, to produce a mathematical expression for the excessive hydrostatic pressure. The solutions were also presented graphically in the form of U-Tv plots. In Terzaghi's theory of one-dimensional consolidation, as well as **Olson's (1977)** theory, it is assumed that c_v remains the same during consolidation. **Hsu and Lu (2006**) further extended Olson's work, allowing and computing the rate of consolidation for the variations in c_v with the applied stress.

Tewatia and Bose (2003) discussed the beginning of secondary consolidation of soils such as Sawan Bhaadon soil, and bentonite clay using the concept proposed by

Robinson (1996). Trivedi and Sud (2004) conducted extensive study on the collapsible behaviour of coal ash to examine the factors influencing the collapse settlement of compacted coal ash due to wetting. This study with proper assumptions can be used for collapsible soils as well.

Generally, the process of consolidation is formulated using numerical and analytical methods. Finite-element programs are one of the numerical methods for solving such consolidation problems. A numerical model based on dual-Lagrangian framework was presented by **Fox and Lee, (2008)** for analysis of coupled large strain consolidation and solute transport in saturated porous media.

The first version finite difference solution of consolidation was written by **Terry Howard** at the University of California, Berkeley in **1970**. The program was then modified by **Wong** in and modifications were continued during the period from 1979 through 1983. The current version of the program **(CONSOL Version 3.0**), was developed by **Tan (2003).** The program was re-written in Visual Basic Version 6, which now runs under Windows 2000 and Windows XP. There are no constraints on the number of soil types, the number of layers, the number of load options.

Recently, **Trivedi, Banik, Sukumar et al (2014)** studied the effect of clayey gouge in a permeating rock mass. They derived an expression for 1-D consolidation in 2-D drainage. They introduced a dimensionless ration constant r, which is a function of shape factor and rock mass characteristics.

2.3 Applications

2.3.1 Fourier Series

The consolidation equation is a parabolic differential equation which can be solved analytically using series given by **Fourier (1807).** Fourier introduced the series for the purpose of solving the heat equation in a metal plate. The consolidation equation given by **Terzaghi (1925)** is a partial differential equation of parabolic nature as one-dimensional heat equation.

Fourier series can give both real and imaginary part of the pore-water pressure function. Generally real part is taken as the principal part and imaginary part is neglected. Fourier series has various uses in electrical engineering. It is used for vibration analysis; signal processing, digital image processing, quantum mechanics, econometrics, and thin-walled shell theory. In engineering applications, Fourier series is usually assumed to converge everywhere except at discontinuities. Specifically, Fourier series converges absolutely and uniformly to *f*(*x*) whenever the differential of *f*(*x*) is square-integrable.

2.3.2 Error Function

The error function is obtained by integrating the normalized Gaussian distribution.

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{2.2}$$

The error function can also be expressed in terms of other functions and series.

$$\operatorname{erf} x = \frac{1}{\sqrt{\pi}} \gamma\left(\frac{1}{2}, x^2\right)$$

$$= \frac{2x}{\sqrt{\pi}} M\left(\frac{1}{2}, \frac{3}{2}, -x^2\right) = \frac{2x}{\sqrt{\pi}} e^{-x^2} M\left(1, \frac{3}{2}, x^2\right)$$

$$= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(2n+1)}$$

where (γ) is the incomplete gamma function, M is the first kind confluent hypergeometric function and the series solution is a Maclaurin series.
Transient flow in a semi-infinite solid is governed by the diffusion equation, given as

$$\frac{\partial^2 T}{\partial^2 x^2} = \frac{1}{\propto} \frac{\partial T}{\partial t}$$

where α is thermal diffusivity. The solution of the diffusion equation is a function of the erf(x) depending on the boundary condition employed. One dimensional consolidation equation is similar to the diffusion equation. Thus, it can be analysed numerically using the error function like the diffusion equation. An absolute exponential approximation for the complementary error function has been given by **Chiani, Dardari & Simon (2003)** as,

$$erfc(x) \approx \frac{1}{6} e^{-x^2} + \frac{1}{2} e^{-\frac{4}{3}x^2}, x > 0 \tag{2.3}$$

Chapter – 3

Numerical Analysis

3.1 Fourier Series

Fourier invented this series for the purpose of solving the heat flow equation in a metal plate in **1807**. Fourier's main idea was to model a complicated heat source as a superposition of simple sine and cosine waves, and to express the solution as a superposition of the corresponding Eigen solutions. This superimposed solution accounts as Fourier series.

Fourier transform was first introduced by **Fourier (1822)** in Fourier’s Analytical Theory of Heat to account for the complex variables. The Fourier transform basically is an extension of Fourier series when the period of the represented function is allowed to tend to infinity (**Taneja 2008).** There is a relation between the concept of Fourier series and Fourier transform for functions which zero outside of an interval. Fourier transforms are commonly used to solve the differential equations.

A Fourier series is an infinite trigonometric series of the form

$$f(x) = a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x) + a_3\cos(3x) + b_3\sin(3x) + \cdots \quad (3.1)$$

which can be written using summation notation as

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right) \quad (3.2)$$

The coefficients a_0, a_n, and b_n are related to the periodic function $f(x)$ by definite integrals:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \quad (3.3)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \quad (3.4)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx \quad (3.5)$$

An even Fourier series has only cosine terms and it can be represented by

$$f(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) + \cdots. \qquad (3.6)$$

An odd Fourier series has only the sine terms, and can be used to approximate an odd function, so

$$f(x) = b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) + \cdots \qquad (3.7)$$

The conditions imposed on *f (x)* to make Eq. (3.1) valid are that *f (x)* have only a finite number of finite discontinuities and only a finite number of extreme values, maxima, and minima in the interval $[0, 2\pi]$.1 Functions satisfying these conditions are called piecewise regular. The conditions themselves are called as the Dirchlet conditions**.**

A uniformly convergent series of continuous functions *(*sin *nx*, cos *nx)* always yields a continuous function. If

(a) *f (x)* is continuous, $-\pi \le x \le \pi$,

(b) $f(-\pi) = f(+\pi)$, and

(c) *f'(x)* is sectionally continuous,

Fourier series for *f (x)* will converge uniformly. These restrictions do not demand that *f (x)* be periodic, but they will be satisfied by continuous, differentiable, periodic functions

with period 2π. The computation of the general Fourier series is based on the following integral identities;

$$\int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = \pi\delta_{mn} \qquad (3.8)$$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = \pi\delta_{mn} \qquad (3.9)$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0 \qquad (3.10)$$

$$\int_{-\pi}^{\pi} \sin(mx)dx = 0 \qquad (3.11)$$

$$\int_{-\pi}^{\pi} \cos(mx)dx = 0 \qquad (3.12)$$

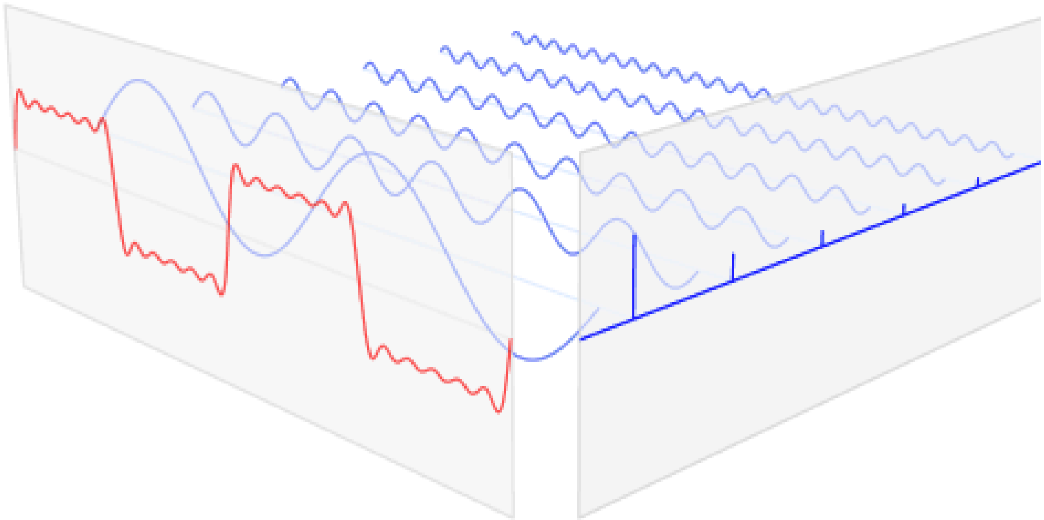


Fig. 3.1 Fourier series of sine function

(Source: http://en.wikipedia.org/wiki/Fourier_series)

3.2 Theory of Consolidation using Fourier Series

According to Terzaghi, consolidation can be expressed mathematically as,

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \tag{3.13}$$

This equation is solved using the Fourier series. The boundary conditions for the consolidation phenomena occurring in a soil sample of height H are,

(i) At $t = 0$, at any distance z, $\overline{u} = \overline{u_0} = \Delta\sigma$ (constant)

(ii) At $t = \infty$, at any distance z, $\overline{u_0} = 0$

(iii) At $t = \infty$, t $z = 0$, $\overline{u} = 0$ and at $z = H$, $\overline{u} = 0$

If $\overline{u}$ is assumed to be a product of some function of z and t, it may be represented by the following expression:

$$\overline{u} = f_1(z).f_2(t) \tag{3.14}$$

Eq. 3.13 may therefore be written as

$$f_1(z).\frac{\partial}{\partial t}[f_2(t)] = c_v\, f_2(t).\frac{\partial^2}{\partial z^2}\,[f_1(z)]$$

or

$$\frac{\frac{\partial^2}{\partial z^2}[f_1(z)]}{f_1(z)} = \frac{\frac{\partial}{\partial t}[f_2(t)]}{c_v\, f_2(t)} \tag{3.15}$$

The left hand term does not contain *t* and hence is constant if *t* is considered variable.

Similarly, the right hand term is constant when z is considered variable, hence each term must be equal to a constant (say, - A^2) and they may be represented by the following relations,

$$\frac{\partial^2}{\partial z^2}\ [f_1(z)] = \ -A^2 f_1(z) \tag{3.16}$$

and

$$\frac{\partial^2}{\partial t^2}\ [f_2(t)] = \ -A^2 c_v f_2(t) \tag{3.17}$$

Equations (3.16) and (3.17) are satisfied respectively by the following expression,

$$f_1(z) = C_1 \cos(Az) + C_2 \sin(Az) \tag{3.18}$$

And $f_2(t) = C_3 \varepsilon^{-A^2\, c_v\, t}$ (3.19)

Where C_1, C_2, C_3 = arbitrary constants

ε = base of hyperbolic or Napierian logarithm = 2.718

Substituting the above values, Eq. (3.14) becomes

$$\bar{u} = C_4 \cos(Az) + C_5 \sin(Az)\, \varepsilon^{-A^2\, c_v\, t} \tag{3.20}$$

The solution of equation (3.20) must satisfy the boundary conditions stated above,

Thus, at time *t* when z = 0, $\bar{u} = 0$: C_4 = 0

Hence Eq. (3.14) reduces to

$$\bar{u} = C_5 \sin(Az)\, \varepsilon^{-A^2\, c_v\, t}$$

Also, at time *t*, at z = H, $\bar{u} = 0 = C_5 \sin(AH)\, \varepsilon^{-A^2\, c_v\, t}$ (3.21)

Eq. (3.21) is satisfied when AH= nπ , where n is any integer.

$$\bar{u} = C_5 \left(\sin \frac{\mathrm{n\pi z}}{\mathrm{H}}\right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v\, t}$$

The above expression may be written in the following form:

$$\overline{u} = B_1 \left(\sin\frac{\pi z}{H}\right) \varepsilon^{\frac{-\pi^2}{H^2} c_v\, t} + B_2 \left(\sin\frac{2\pi z}{H}\right) \varepsilon^{\frac{-4\pi^2}{H^2} c_v\, t} + \quad \ldots \ldots \ldots B_n \left(\sin\frac{n\pi z}{H}\right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v\, t} + \ldots \ldots \ldots \tag{3.22}$$

Therefore, $\overline{u} = \sum_{n=1}^{n=\infty} B_n \left(\sin\frac{n\pi z}{H}\right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v\, t}$ (3.23)

At $t = 0$, $\varepsilon^{\frac{-n^2\pi^2}{H^2} c_v\, t} = 1$ and $\overline{u} = \overline{u_0}$

Thus, $\overline{u_0} = \sum_{n=1}^{n=\infty} B_n \left(\sin\frac{n\pi z}{H}\right)$ (3.24)

Using Fourier analysis,

$\int_0^{\pi} \sin mx \sin nx\, dx = 0$

And $\int_0^{\pi} \sin^2 nx dx = \frac{\pi}{2}$

Therefore, $\int_0^{H} \sin\frac{m\pi z}{H} \sin\frac{n\pi z}{H}\, dz = 0$

And $\int_0^{H} \sin^2 \frac{n\pi z}{H}\, dz = \frac{H}{2}$

Hence, $\int_0^{H} \left(\bar{u}_0 . sin\frac{n\pi z}{H}\right) dz = \sum_{m=1}^{m=\infty} B_m \int_0^{H} \left(sin\frac{m\pi z}{H} sin\frac{n\pi z}{H}\right) dz + B_n \int_0^{H} (sin\frac{n\pi z}{H})^2\, dz$

On multiplication by $sin\frac{n\pi z}{H}$ the R.H.S of Eqn. (3.24) splits into two parts:

Hence $\int_0^{H} \left(\bar{u}_0\, sin\frac{n\pi z}{H}\right) dz = B_n \frac{H}{2}$ (3.25)

Or $B_n = \frac{2}{H} \int_0^{H} \left(\bar{u}_0\, sin\frac{n\pi z}{H}\right) dz$ (3.26)

Substituting this in equation (3.23), we get

$\bar{u} = \sum_{n=1}^{n=\infty} \left[\frac{2}{H} \int_0^{H} \left(\bar{u}_0 . sin\frac{n\pi z}{H}\right) . dz\right] . \left(sin\frac{n\pi z}{H}\right) . e^{-\frac{n^2 . \pi^2}{H^2} . c_v . t}$ (3.27)

Or $\bar{u} = \sum_{n=1}^{n=\infty} \frac{2\Delta\sigma}{n\pi} (1 - cos\, n\pi) . \left(sin\frac{n\pi z}{H}\right) . e^{-\frac{n^2\pi^2}{H^2} . c_v . t}$ (3.28)

When n = even, (1-$cos\, n\pi$) = 0,

When n = odd

Substituting n = 2N+1, N = integer, above equation becomes:

$$ū = \frac{4}{n}.\Delta\sigma.\sum_{N=1}^{N=\infty}\frac{1}{2N+1}\left[sin\frac{(2N+1)\pi z}{H}\right].e^{-\frac{n^2.\pi^2}{H^2}.c_v.t} \quad (3.29)$$

This equation represents variation of excess hydrostatic pressure ū with depth z at any time in terms of applied consolidating pressure $\Delta\sigma$.

Now the settlement for pressure increment, $\Delta\rho = m_v.\Delta\sigma'.dz$ (3.30)

Where $\Delta\sigma'$= effective pressure increment at time t, $\Delta\sigma' = \Delta\sigma - ū$

Thus, $\Delta\rho = m_v.(\Delta\sigma - ū).dz$

Integrating between the limits 0 to H, the settlement ρ of the clay at time t is given by:

$$\rho = m_v\left[\Delta\sigma.H - \int_0^H ū.dz\right]$$

Substituting ū from equation (3.29) and integrating,

$$\rho = m_v.\Delta\sigma.H\left[1 - \frac{8}{\pi^2}.\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2}.e^{-\frac{(2N+1)^2.\pi^2}{H^2}.c_v.t}\right] \quad (3.31)$$

At t = ∞, when process of consolidation is complete, the ultimate or final settlement ρ_f is given by: $\rho_f = m_v.\Delta\sigma.H$ (3.32)

The ratio of ρ to ρ_f expressed as a ratio, is termed as “Degree of Consolidation” U;

$$U = \frac{\rho}{\rho_f} \quad (3.33)$$

$$U = \left[1 - \frac{8}{\pi^2}.\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2}.e^{-\frac{(2N+1)^2.\pi^2}{H^2}.c_v.t}\right] \quad (3.34)$$

A dimensionless parameter is being introduced known as time factor T_v defined by following equation:

$$T_v = \frac{c_v.t}{d^2} \quad (3.35)$$

Where, d=drainage path for case of double drainage,

d = H/2. Thus, Eqn. (3.34) may be written as:

$$U = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} . e^{-\frac{(2N+1)^2.\pi^2}{4}.T_v}\right] \tag{3.36}$$

Or U = f (T_v)

3.3 Error Function

Error function is also called as the Gauss error function. It was introduced by **Gauss (1809)** in terms of Gaussian integral by assuming a normal distribution of errors. It is basically the integral of normalised Gaussian distribution. It is a special function having sigmoid characteristics which occurs in probability distributions, statistics and partial differential equations. Error function is called so because of its use in error measurement of probability distributions in the history. However, it has nothing to do with errors when used in mathematical physics. It is defined as:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{3.37}$$

Over the total range of values, there is a numerical approximation for error function with a maximum error of1.2×10^{-7}, as follows:

$$\text{erf}(x) = \begin{cases} 1 - T, for\ x \geq 0 \\ T - 1, for\ x < 0 \end{cases}$$

with

$$T = t.\exp\,(-x^2 - 1.26551223 + 1.00002368.t + 0.37409196.t^2 + 0.09678418.t^3 - 0.18628806.t^4 + 0.27886807.t^5 - 1.13520398.t^6 + 1.48851587.t^7 - 0.82215223.t^8 + 0.17087277.t^9)$$

And

$$t = \frac{1}{1 + 0.5|x|}$$

Approximation of error function can be done in Power series or Taylor series for x<2 as,

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} \tag{3.38}$$

$$= \frac{2}{\sqrt{\pi}}(z - \frac{z^3}{3} + \frac{z^5}{10} - \frac{z^7}{42} + \frac{z^9}{216} - \cdots.)$$

A useful asymptotic expansion of the complementary error function for large real x *i.e.* *x>2* is,

$$erfc(x) = \frac{e^{-x^2}}{x\sqrt{\pi}}\sum_{n=0}^{N-1}(-1)^n \frac{(2n-1)!!}{(2x^2)^n} + R_N(x)$$

Where,

$$R_N(x) := \frac{(-1)^N}{\sqrt{\pi}} 2^{-2N+1} \frac{(2N)!}{N!} \int_x^{\infty} t^{-2N} e^{-t^2} dt$$

3.4 Theory of Consolidation using Error Function

Consider the one dimensional consolidation equation for z ϵ [0, +∞],

$$\frac{\partial^2 u}{\partial^2 x^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \tag{3.39}$$

With boundary conditions,

$u(x, 0) = f(x)$

And $u(0, t) = 0$

Theory of 1-D consolidation using the description as per the source (http://ewp.rpi.edu/hartfrod/~ernesto/F2010/CINVESTAV/Notes/ch06.pdf) can be given as,

$$u_p(x, t) = Ae^{-v^2 \propto t} \sin(vx) \tag{3.40}$$

where v is an arbitrary constant and A = A(v). Here v is similar to the Eigen values except that in this case it assumes continuous rather than discrete values.
Superimposing solutions of the above equation to produce a more general solution,

$$u(x, t) = \int_0^{\infty} A(v) e^{-v^2 \propto t} \sin(vx) du \tag{3.41}$$

Since the initial condition must also be satisfied this requires that

$$f(x) = \int_0^{\infty} A(v) \sin(vx) du \tag{3.42}$$

But this is precisely the Fourier sine integral representation of *f*(*x*) with Fourier coefficients given by

$$A(v) = \frac{2}{\pi}\int_0^\infty f(\varepsilon)\sin(v\varepsilon)d\varepsilon \qquad (3.43)$$

Therefore the resulting solution becomes,

$$u(x,t) = \frac{2}{\pi}\int_0^\infty \int_0^\infty e^{-v^2\propto t} f(\varepsilon)\sin(v\varepsilon)\sin(ux)d\varepsilon du \qquad (3.44)$$

However, since

$$\sin(v\varepsilon)\sin(u\varepsilon) = \frac{1}{2}[\cos\big(v(\varepsilon - x)\big) - \cos(v(\varepsilon + x))]$$

And

$$\int_0^\infty e^{-a^2x^2}\cos(bx)dx = \frac{\sqrt{\pi}}{2a}e^{-b^2/4a^2}$$

the required solution can be expressed as

$$u(x,t) = \frac{1}{2\sqrt{\propto \pi t}}\int_0^\infty f(\varepsilon)[e^{-(\varepsilon-x)^2/4\alpha t} - e^{-(\varepsilon+x)^2/4\alpha t}]d\varepsilon \qquad (3.45)$$

Specifically,

If $f(x) = const = u_0$

Then the solution becomes

$$u(x,t) = \frac{2u_0}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{\propto t}}} e^{-v^2}\,dv = u_0 \mathrm{erf}\,(\frac{x}{2\sqrt{\propto t}}) \qquad (3.46)$$

Where

$$\mathrm{erf}(z) = \frac{2}{\pi}\int_0^z e^{-v^2}\,dv$$

is the error function.

Coefficient of consolidation Cv has been written as α for ease of work.

The settlement ρ of the clay at time t is given by:

$$\rho = m_v \left[\Delta\sigma . H - \int_0^H \bar{u} . dz\right] \tag{3.47}$$

Therefore, at t = ∞, when process of consolidation is complete, the final settlement ρ_f is given by:

$$\rho_f = m_v . \Delta\sigma . H \tag{3.48}$$

The ratio of ρ to ρ_f expressed as a percentage, is called as “Degree of Consolidation” U;

$$U = \frac{\rho}{\rho_f}$$

On substituting the values for u in the above equation we get,

$$U = 1 - \left(\frac{1}{H}\int_0^H \text{erf}\left(\frac{x}{2\alpha\sqrt{t}}\right) dz \, . \, \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v \, t}\right) \tag{3.49}$$

Using the numerical approximations for error function the above equation becomes,

$$U = 1 - \left(1 - \frac{1}{(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)^4} \, . \, \varepsilon^{\frac{-\pi^2}{H^2} c_v \, t}\right) \tag{3.50}$$

Where, a_1 = 0.278393, a_2 = 0.230389, a_3 = 0.000972, a_4 = 0.078108

Chapter – 4

Materials and Methods

4.1 Material

The soils used in this consolidation study are bentonite and sand. Bentonite specimen was brought from N.K Enterprise, Industrial estate, Okhla Phase- III. Sand was taken from soil mechanics laboratory, DTU passing 425μm sieve and retaining on 300μm sieve. Laboratory tests to determine various index and engineering properties of bentonite and sand were conducted according to Indian Standard methods of testing.

4.2 Methods

The experimental study was carried out as follows:

1. Specific gravity of bentonite and sand specimen
2. Hydrometer test for grain size analysis of bentonite
3. Determination of Atterberg Limits

i) Liquid limit determination by Casagrande apparatus

ii) Plastic limit determination

4. Determination of consolidation properties of bentonite.

4.2.1 Specific Gravity [IS: 2720 (Part III)]

The specific gravity of soil is the ratio of the weight of the soil solids to the weight of equal volume of water. It is measured with the help of a volumetric flask in a very simple experimental setup where the volume of the soil is found out and its weight is divided by the weight of equal volume of water.

$$G = \frac{W2 - W1}{[(W2 - W1) - (W3 - W4)]}$$

W1 (gm) = Weight of bottle

W2 (gm) = Weight of bottle + Dry soil

W3 (gm) = Weight of bottle + Soil + Water

W4 (gm) = Weight of bottle + Water

Test procedure: Weight of the empty clean and dry Density bottle is recorded, **W1**. About 150-200 gm of a dry sample is placed in the Density bottle. Weight of the Density bottle containing the dry soil sample is then recorded, **W2**. Distilled water is added to fill about half to three-fourth of the Density bottle. The sample is soaked for 10 minutes. The mixture is stirred rigorously with a glass rod to ensure removal of all the entrapped air. Density bottle is filled with distilled (water to the mark) and the exterior surface of the Density bottle is wiped off with a clean dry cloth. Weight of the Density bottle and contents is determined, **W3**. Density bottle is emptied and cleaned. Then it is filled with distilled water only (to the mark). The weight of the Density bottle and distilled water is noted, **W4**.

4.2.2 Grain Size Analysis by Hydrometer Test [IS: 2720 (Part IV)]

The hydrometer method is based on the measurement of velocity of soil particles in a sedimentation solution and the dry mass of soil in the solution in different intervals of time. The velocity of falling particles and dry mass of soil at a specific depth are measured by a hydrometer. The results are combined with Stokes' law, which gives the relation between velocity of a spherical particle and its diameter while settling within its solution. The tests are carried out according to procedure mentioned in IS 2720 Part 4 1985.

4.2.3 Liquid Limit determination [IS: 2720 (Part V)]

Liquid limit is the minimum water content at which the soil is still in liquid state but has a small shearing strength against flowing. In other words it is the water content at which soil suspension gains an infinitesimal strength from zero strength. It is the minimum water content at which part of soil cut by groove of standard dimensions, will flow together for a distance of 12 mm (1/2 inch) under an impact of 25 blows.
Plot the flow curve on semi-log plot with water content as the ordinate and no. of blows as abscissa. The water content corresponding to 25 blows is taken as the liquid limit of the soil.

Fig 4.1 Liquid limit apparatus

4.2.4 Plastic limit determination [IS: 2720 (Part V)]:- Plastic limit is the minimum water content at which a soil just begins to crumble when rolled into a thread of 3 mm.

4.2.5 Consolidation by Odeometer [IS 2720(part 15)]

The process of compression resulting from long term static load and gradual reduction of voids by expulsion of pore water is termed as consolidation. The permeability of an undisturbed sample of clay can be determined indirectly at different void ratios by a consolidation test. Odeometer is used to determine the deformation caused by applied load on the soil sample. Consolidometer mainly consists of a consolidation ring of 20 mm in which the sample is filled. This ring is then placed in the Odeometer assembly with porous stones at top and bottom for double drainage case. Loading is the then applied in double increments for every 24 hours. Dial gauge records the deformation in the soil sample for an applied load at various time intervals. This deformation is analysed for 24 hours and then summed up as the total settlement for a particular load for primary consolidation stage. In laboratory experiments majority of the primary consolidation is assumed to occur for a time span of 24 hours only.

Sample preparation: Bentonite clay and fine sand were taken in required weight for different samples. Consolidation ring was cleansed and markings were made in the ring using marker for the required thickness of layers. Thickness of clay layer was adjusted according to the clay-mix ratio for each test. Then the clay and sand were filled in layers corresponding to the markings provided in the ring. Thicknesses of clay lamina for various samples were 0.4, 0.8, 1.2, 1.6 and 2 cm. Thickness of sad layer was adjusted in respect to the clay layer and the size of the ring.

Fig 4.2 Consolidation Ring with sample

Fig 4.3 Consolidation test apparatus

Chapter – 5

Results and Discussion

5.1 Experimental Results

5.1.1 Specific Gravity

Table 5.1 Specific gravity of soil samples

S. No	Sample	Specific Gravity
1	Bentonite	2.32
2	Sand	2.68

5.1.2 Liquid limit of Clay

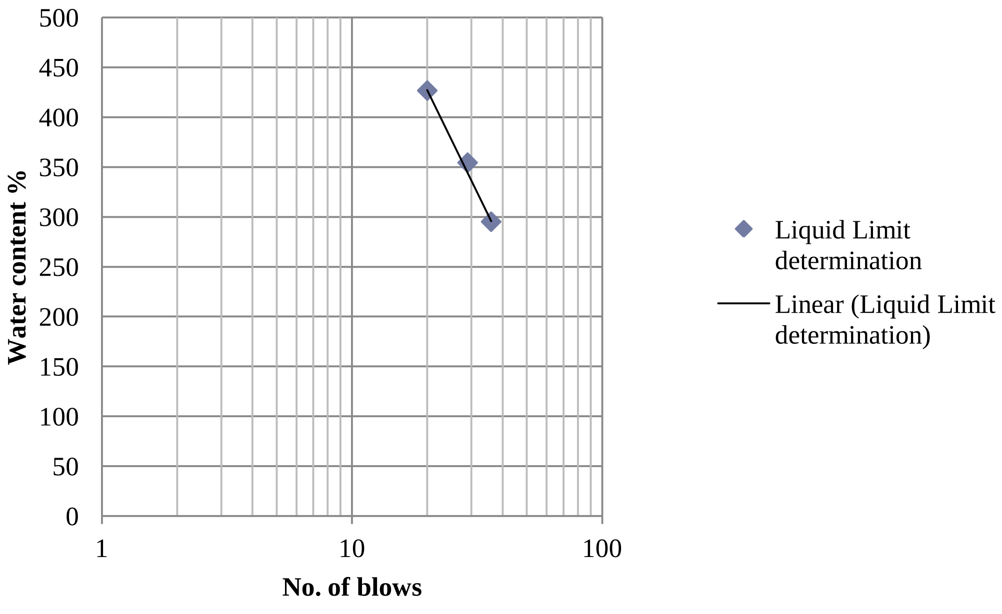


Fig. 5.1 Liquid limit determination

Liquid Limit of bentonite clay sample = 386%

5.1.3 Plastic limit of Clay

Plastic Limit of bentonite sample = 111.48%

5.1.4 Grain size analysis of Clay

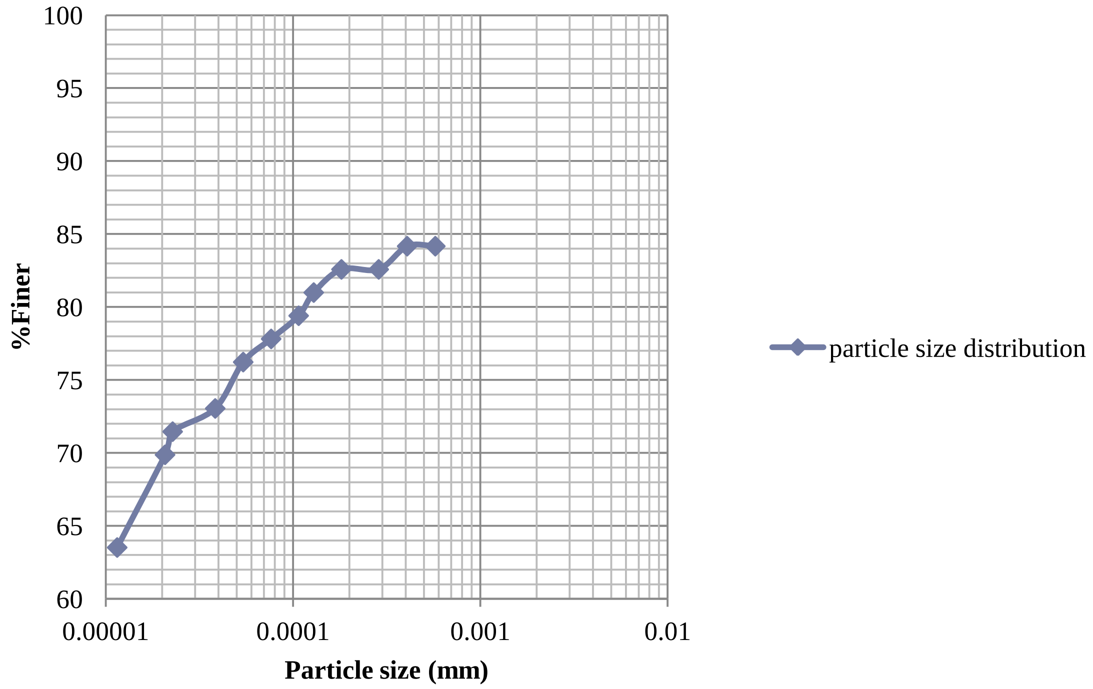


Fig 5.2 Particle size distribution of bentonite clay

5.1.5 Consolidation Properties

As the present study focuses on role of clay lamina in sand, thus ratio of clay layer to the total thickness of sample is considered. Thickness of clay layer is denoted by tc and thickness of sand layer is given by ts. Therefore, Clay can be expressed as a ratio of total thickness tc/(tc+ts). The clay thickness to total thickness ratios considered in this work are 0.2, 0.4, 0.6, 0.8 and 1.0. The consolidation characteristics determined from the tests are represented in following figures.

a) For pure clay of thickness 2cm $,\frac{tc}{(tc+ts)} = 1.0$

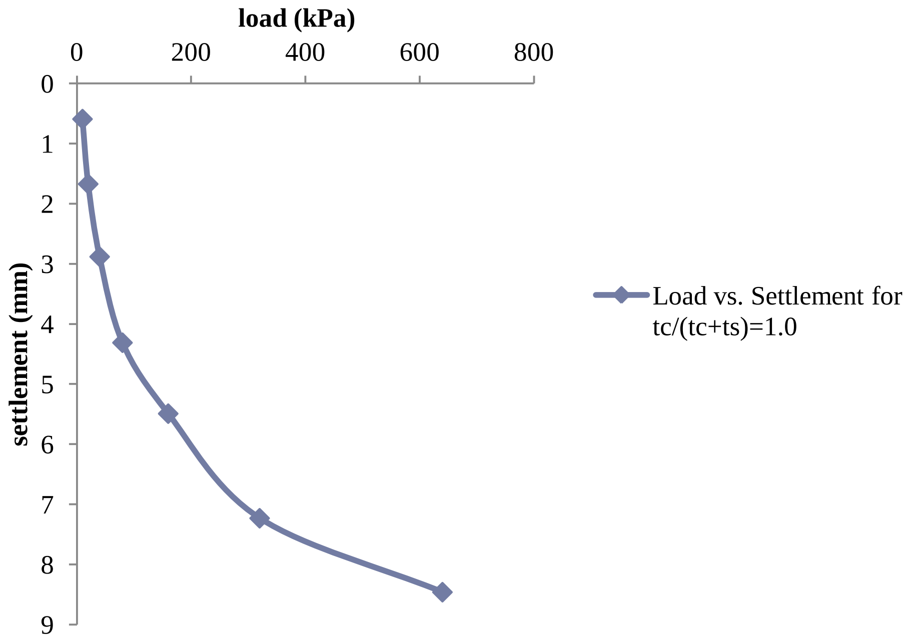


Fig 5.3 Load Settlement curve

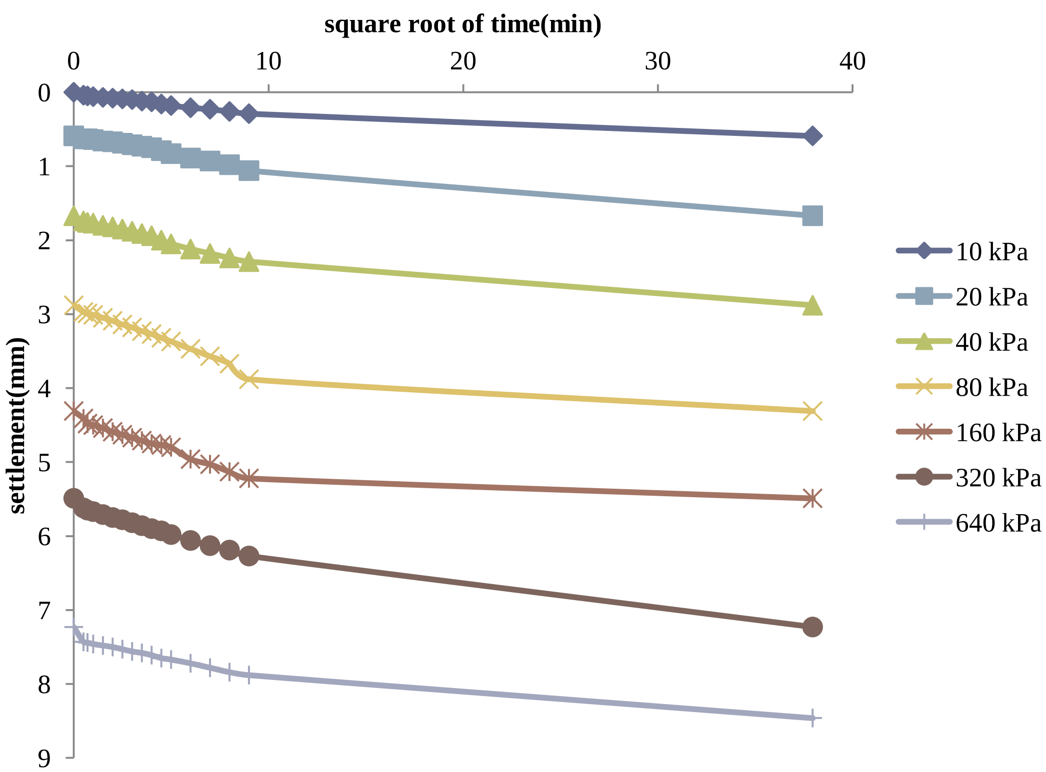


Fig 5.4 Square root of time vs. Settlement

b) For clay layer of thickness 1.6 cm, $\frac{tc}{(tc+ts)}$ = 0.8

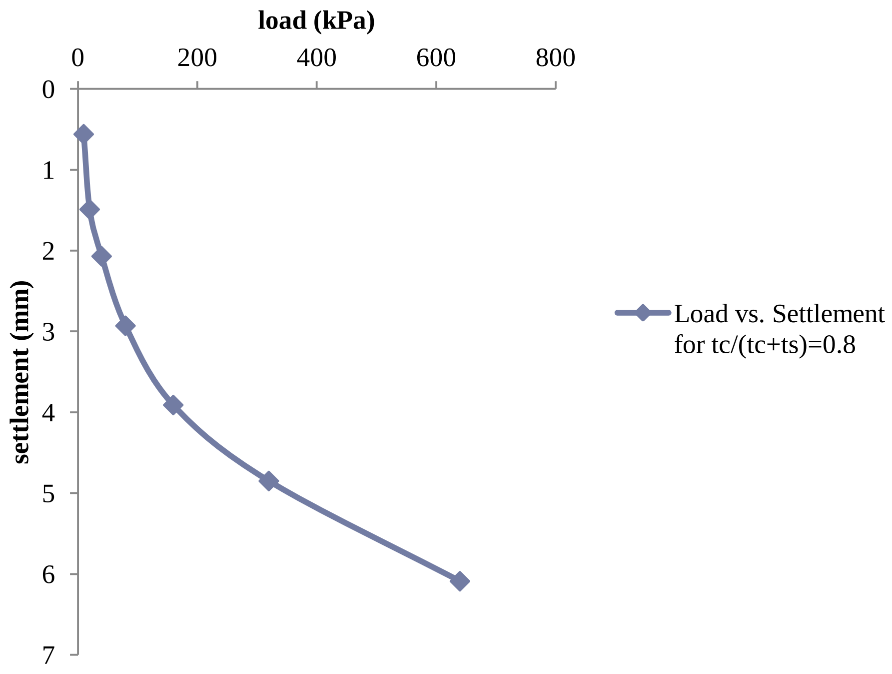


Fig 5.5 Load Settlement curve

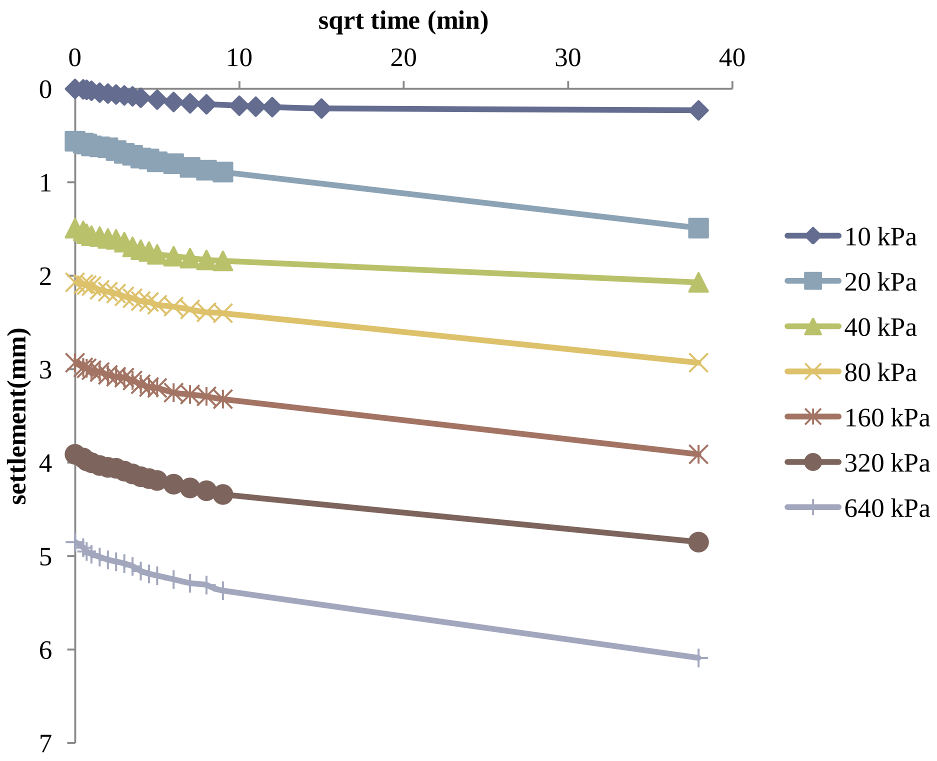


Fig 5.6 Square root of time vs. Settlement

c) For clay layer of thickness 1.2 cm, $\frac{tc}{(tc+ts)}$ = 0.6

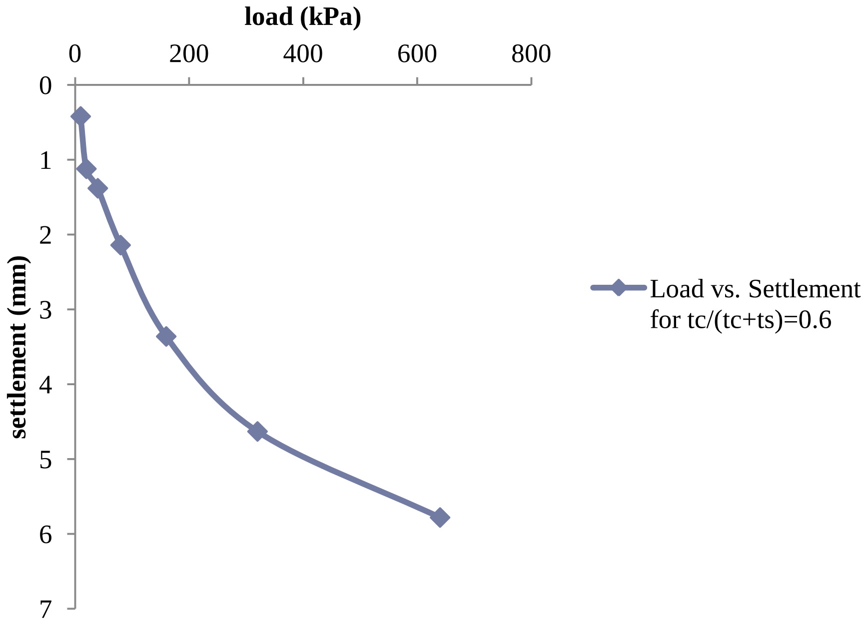


Fig 5.7 Load vs. Settlement

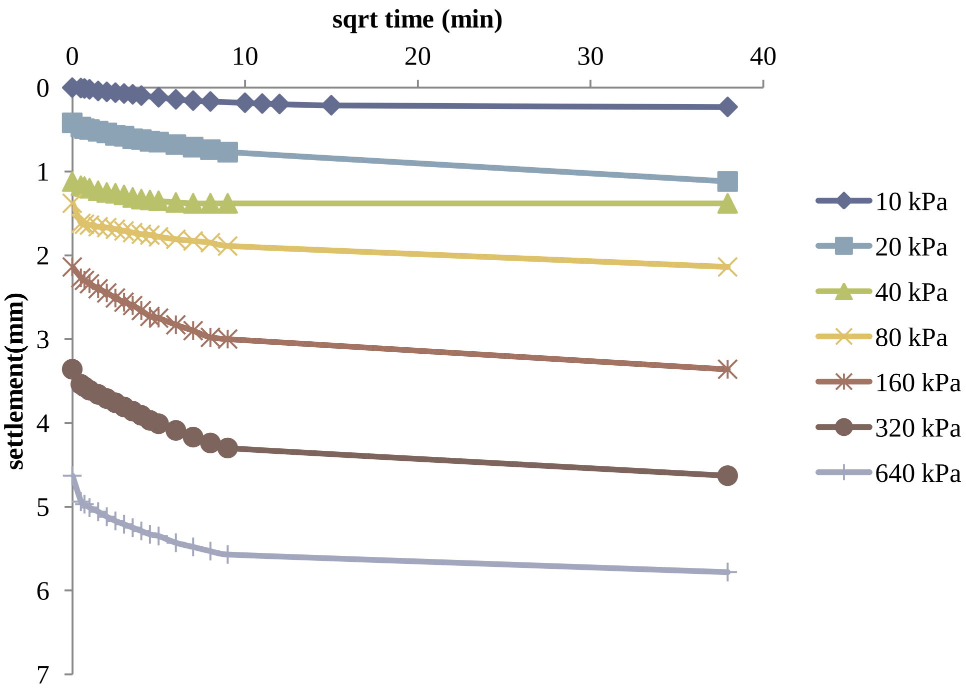


Fig 5.8 Square root of time vs. Settlement

d) For clay layer of thickness 0.8 cm, $\frac{tc}{(tc+ts)}$ = 0.4

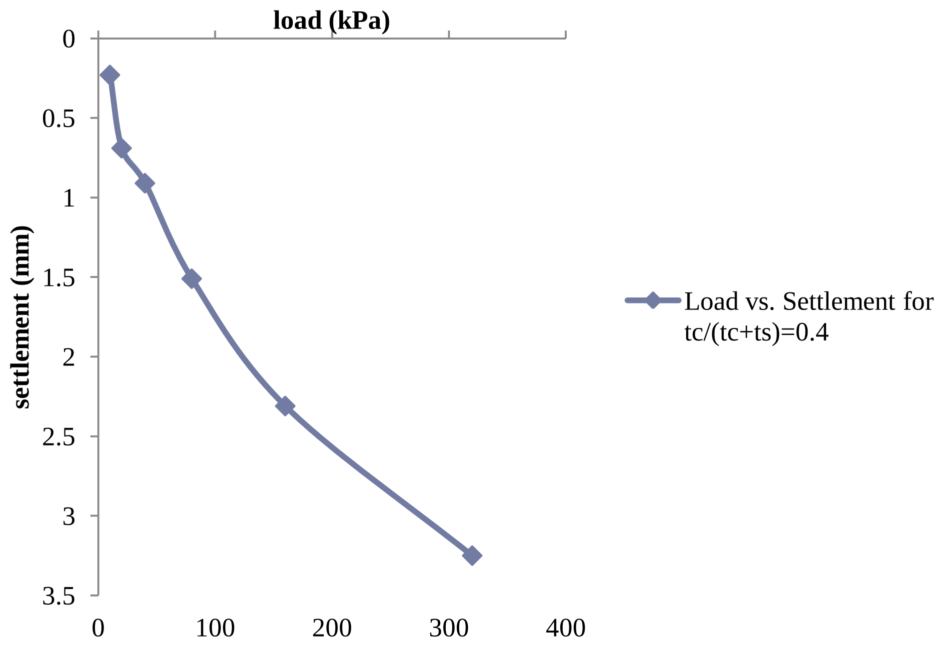


Fig 5.9 Load vs. Settlement

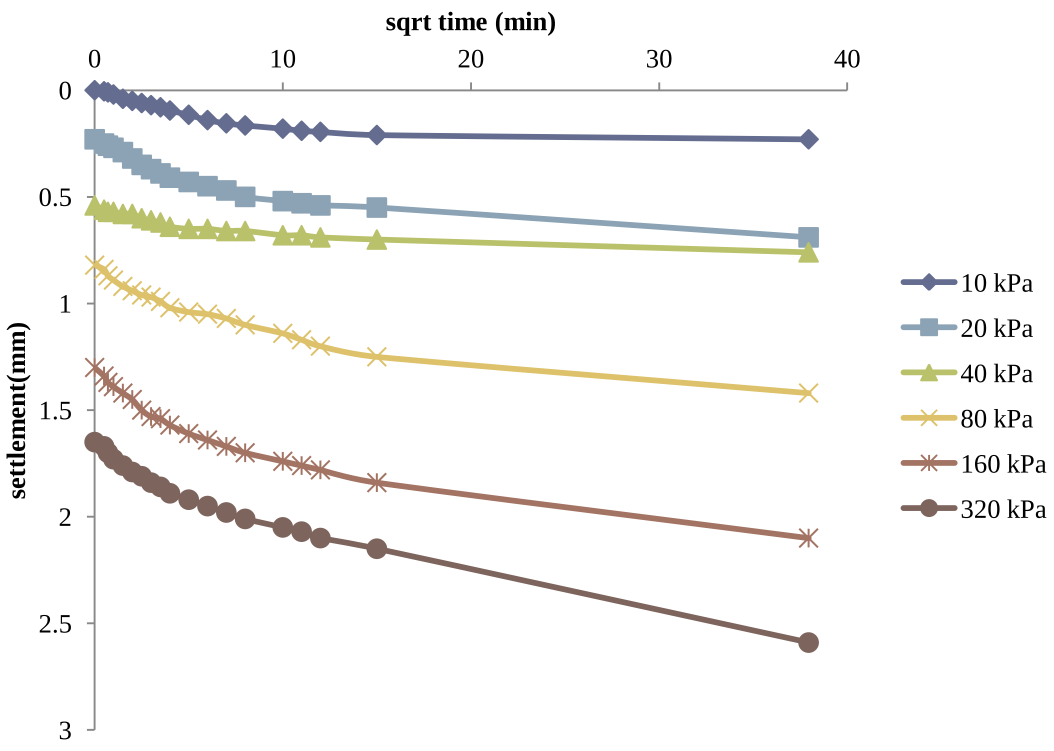


Fig 5.10 Square root of time vs. Settlement

e) For clay layer of thickness 0.4 cm, $\frac{tc}{(tc+ts)} = 0.2$

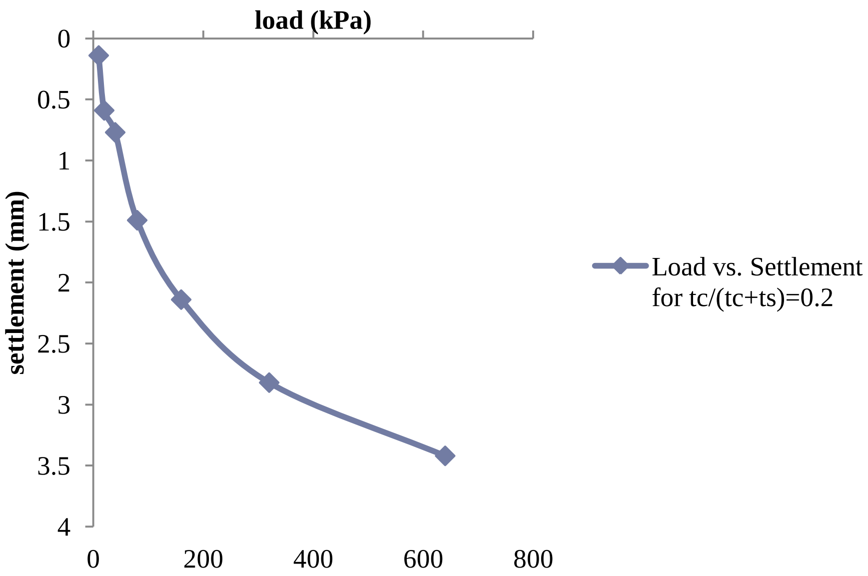


Fig 5.11 Load vs. Settlement

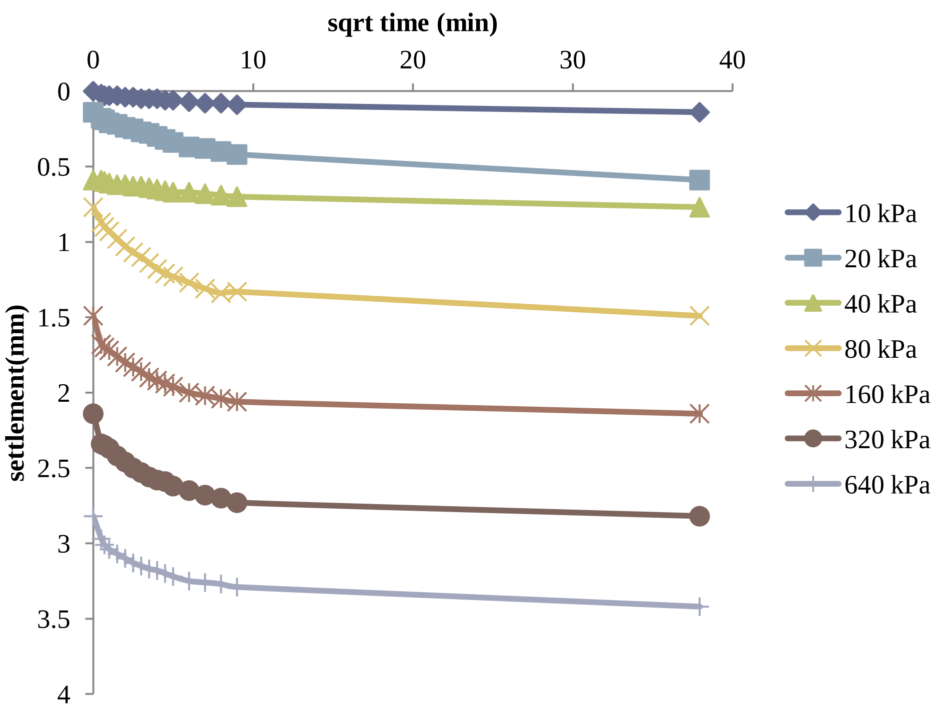


Fig 5.12 Square root of time vs. Settlement

Using Square root of time fitting method and height of solids method for calculation of consolidation parameters we get the following results for all the soil samples.

Table 5.2 Consolidation characteristics for different samples

S. No	Sample	C_v (cm^2/sec) Coefficient of consolidation	a_v (cm^2/kg) Compressibility	m_v (cm^2/kg) Volume compressibility	k (cm/sec) Permeability
1	tc/(tc+ts)=1.0	8.82 X 10^{-6} cm^2/sec	1.30 cm^2/kg	0.59 cm^2/kg	5.20 x 10^{-6} cm/sec
2	tc/(tc+ts)=0.8	1.13 X 10^{-5} cm^2/sec	1.24 cm^2/kg	0.56 cm^2/kg	6.35 x 10^{-6} cm/sec
3	tc/(tc+ts)=0.6	1.15 X 10^{-5} cm^2/sec	0.954 cm^2/kg	0.42 cm^2/kg	4.83 x 10^{-6} cm/sec
4	tc/(tc+ts)=0.4	1.32 X 10^{-5} cm^2/sec	0.49 cm^2/kg	0.23 cm^2/kg	3.05 x 10^{-6} cm/sec
5	tc/(tc+ts)=0.2	1.35 X 10^{-5} cm^2/sec	0.26 cm^2/kg	0.14 cm^2/kg	1.9 x 10^{-6} cm/sec

5.2 Numerical Results

Using the error function distribution, the pore water pressure distribution can be given at regular time intervals for different samples with varying thickness of sand and clay for incremental loads with load increment ratio of 1.

a) **For** $\frac{tc}{(tc+ts)} = 0.2$

i) **Applied pressure = 10 kPa**

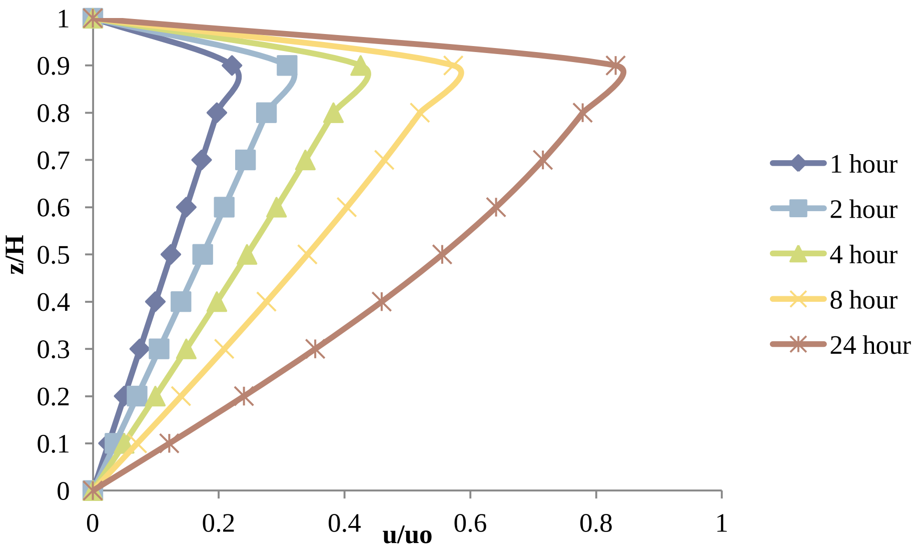


Fig 5.13 Pore pressure distribution

ii) Applied pressure = 20 kPa

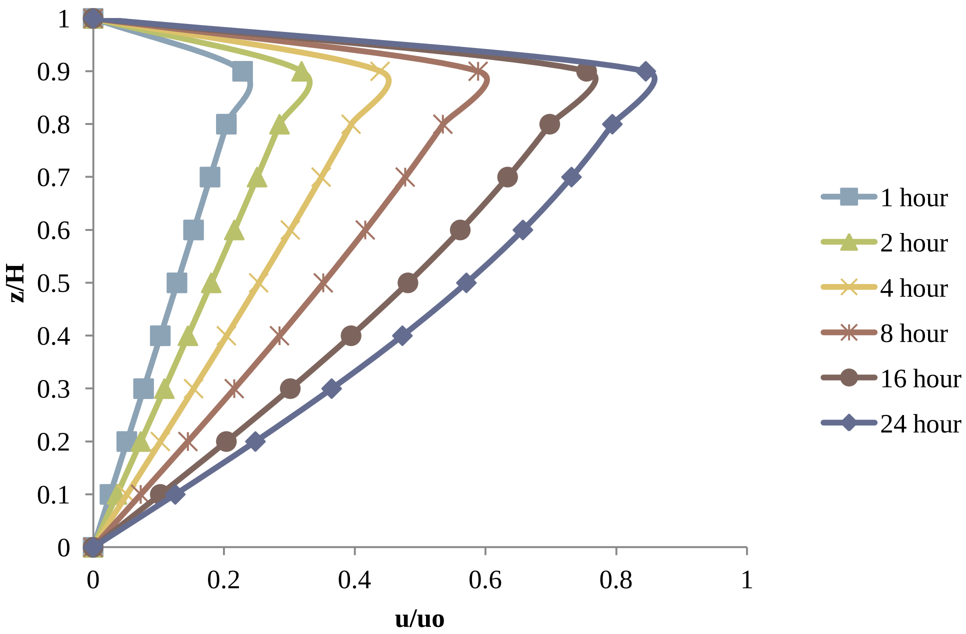


Fig 5.14 Pore pressure distribution

iii) Applied pressure = 40 kPa

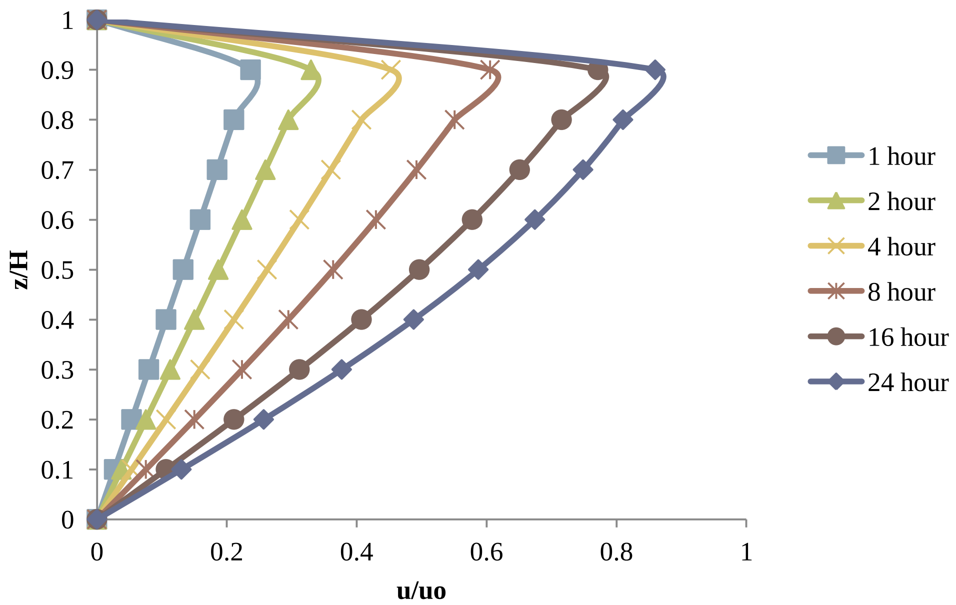


Fig 5.15 Pore pressure distribution

iv) Applied pressure = 80 kPa

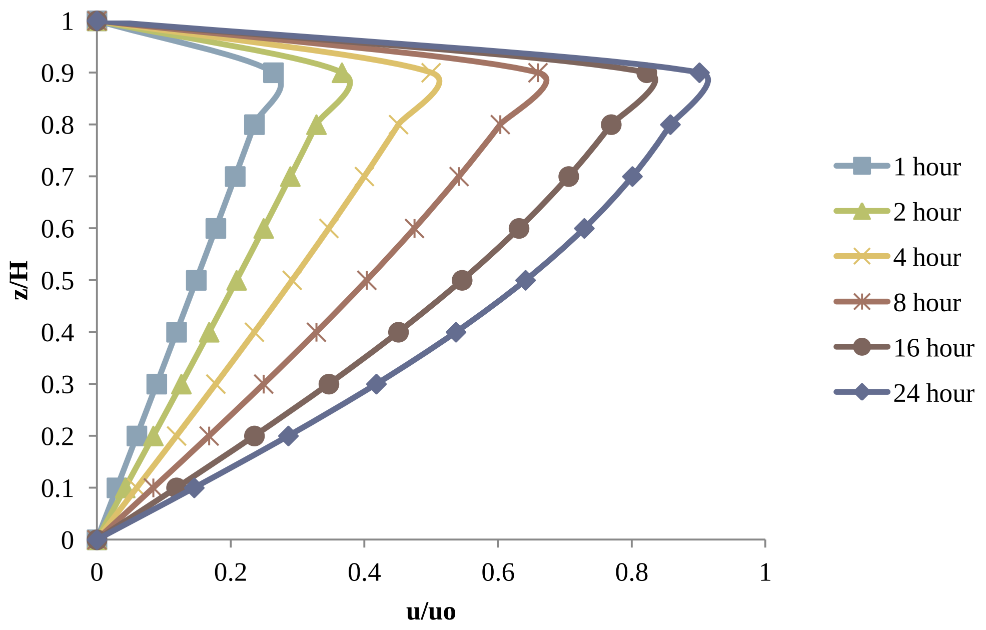


Fig 5.16 Pore pressure distribution

v) Applied pressure = 160 kPa

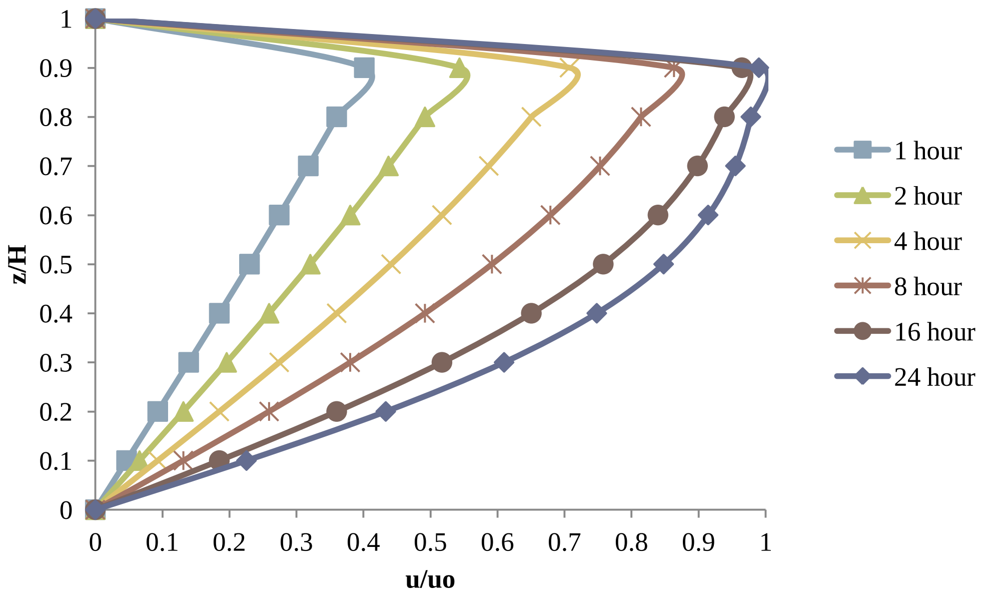


Fig 5.17 Pore pressure distribution

vi) Applied pressure = 320 kPa

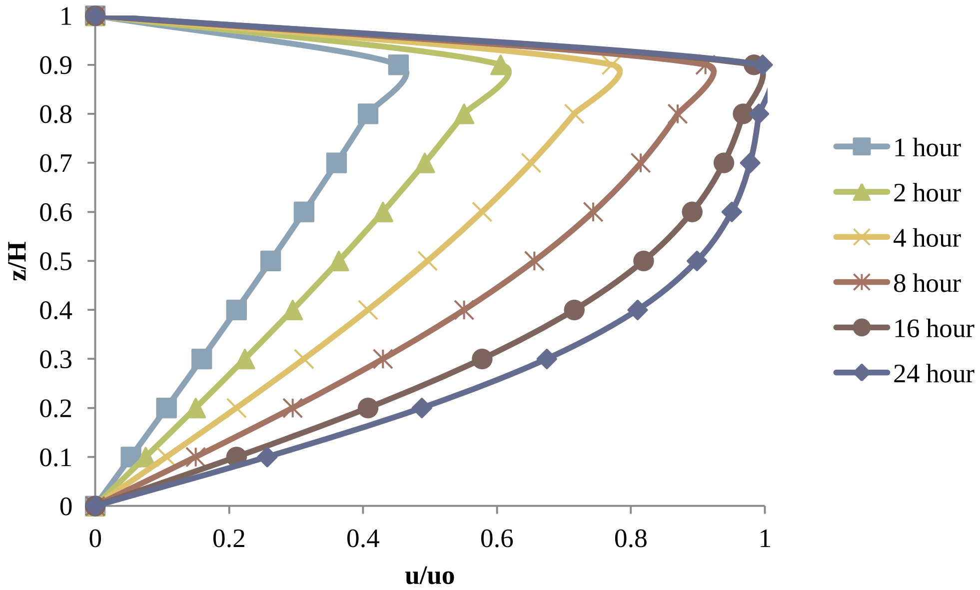


Fig 5.18 Pore pressure distribution

vii) Applied pressure = 640 kPa

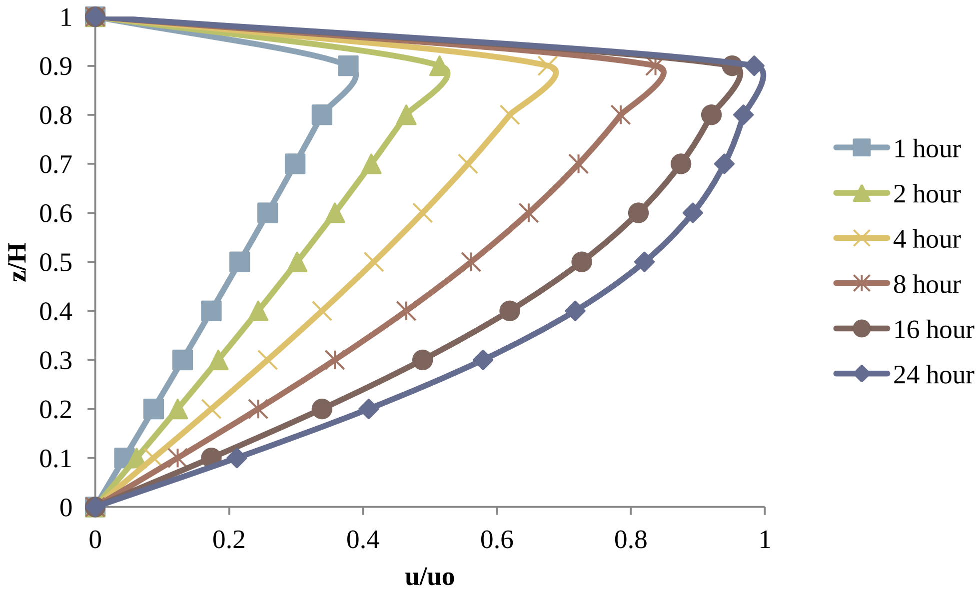


Fig 5.19 Pore pressure distribution

viii) Pore pressure distribution at 24 hours for different applied pressures for tc/(tc+ts)= 0.2

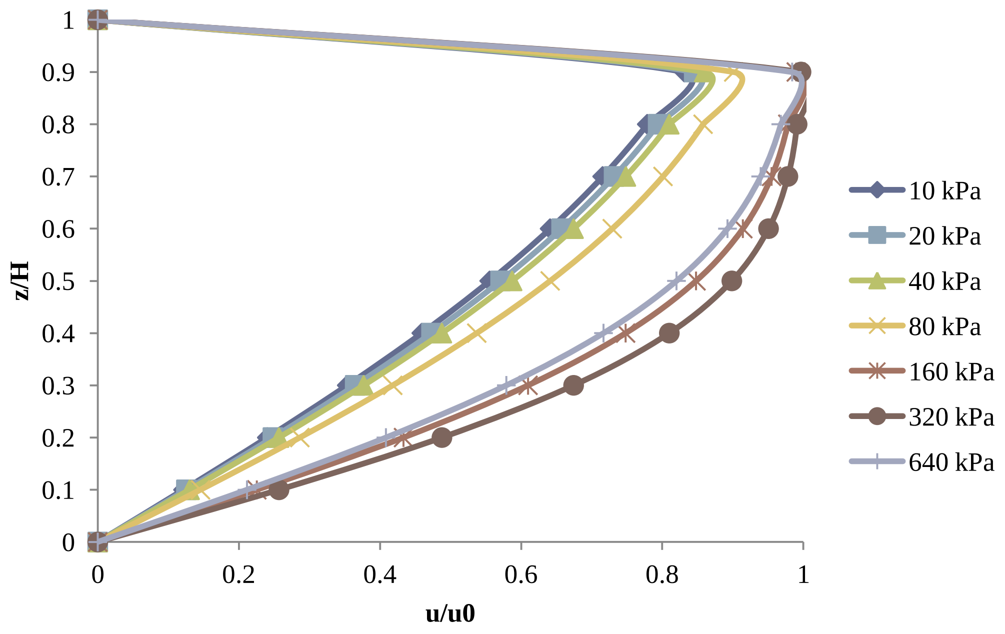


Fig 5.20 Pore pressure distribution

b) For $\frac{tc}{(tc+ts)} = 0.4$

i) Applied pressure = 10 kPa

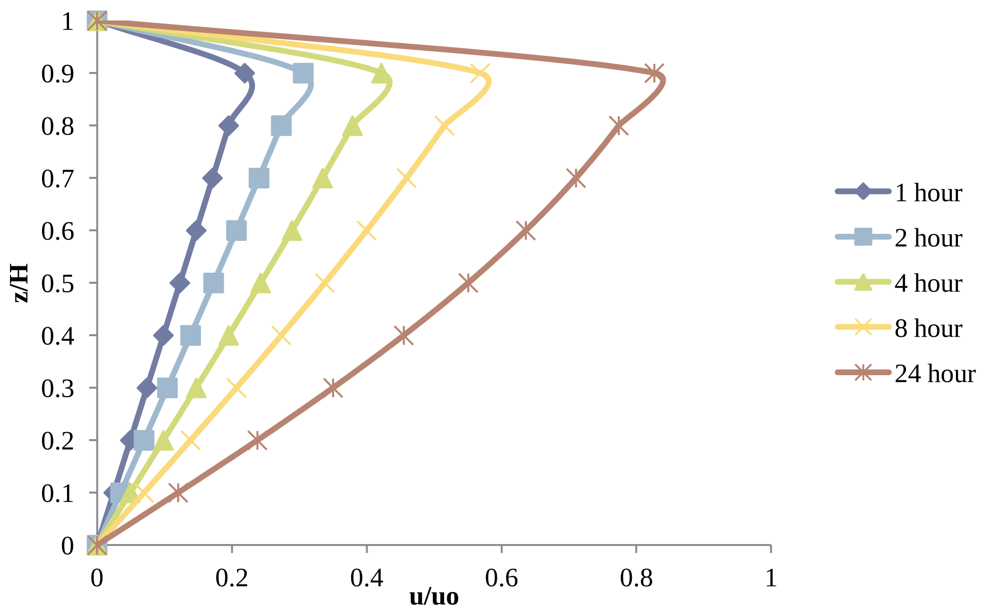


Fig 5.21 Pore pressure distribution

ii) Applied pressure = 20kPa

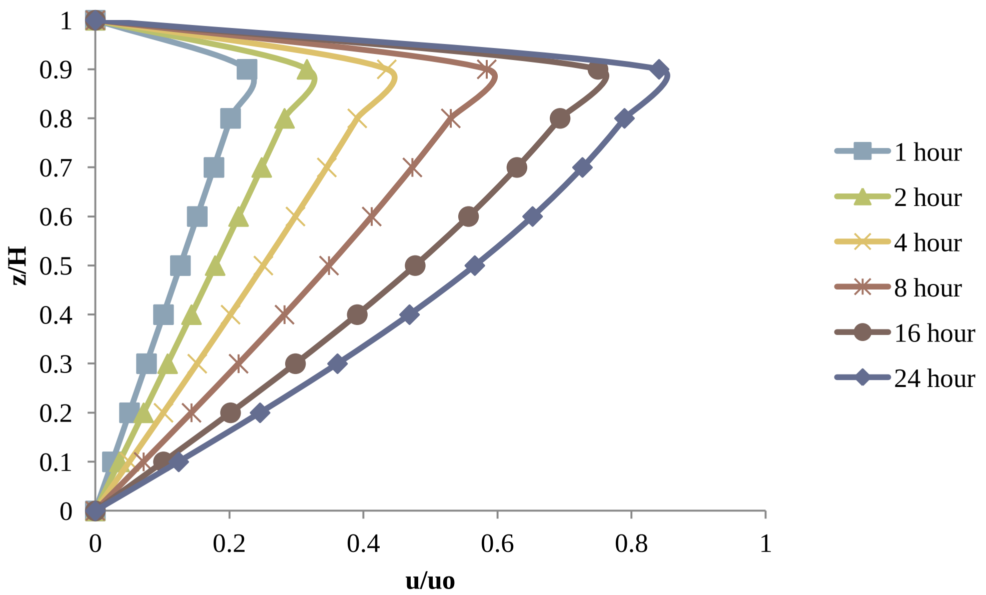


Fig 5.22 Pore pressure distribution

iii) Applied pressure = 40 kPa

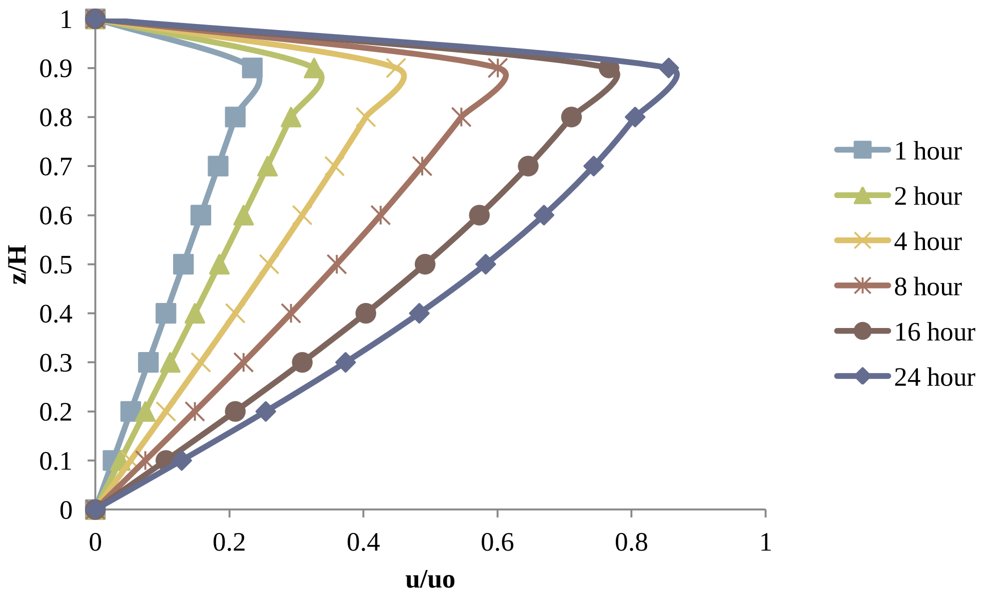


Fig 5.23 Pore pressure distribution

iv) Applied pressure =80 kPa

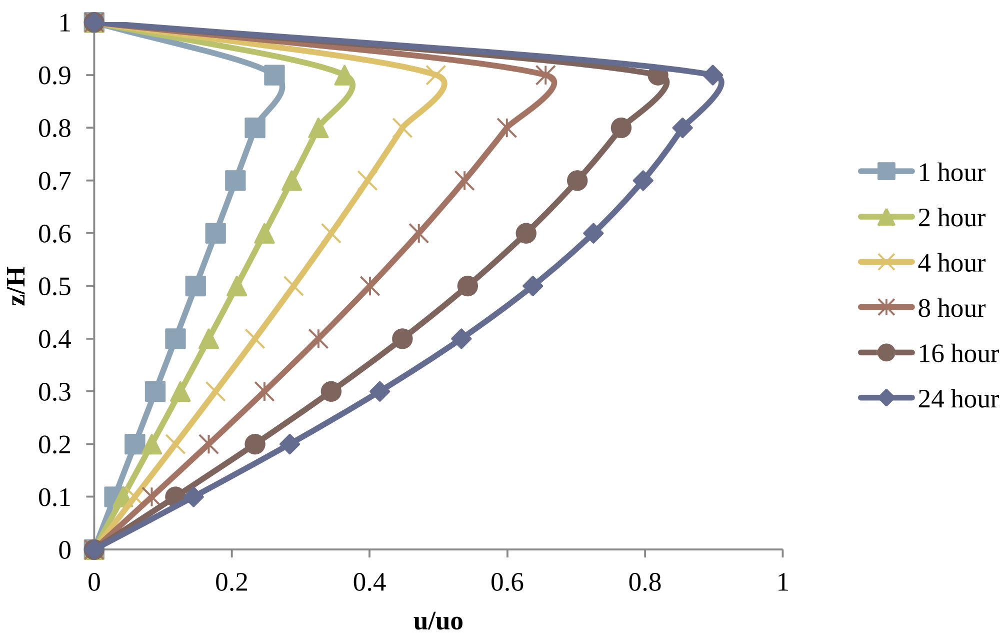


Fig 5.24 Pore pressure distribution

v) Applied pressure = 160 kPa

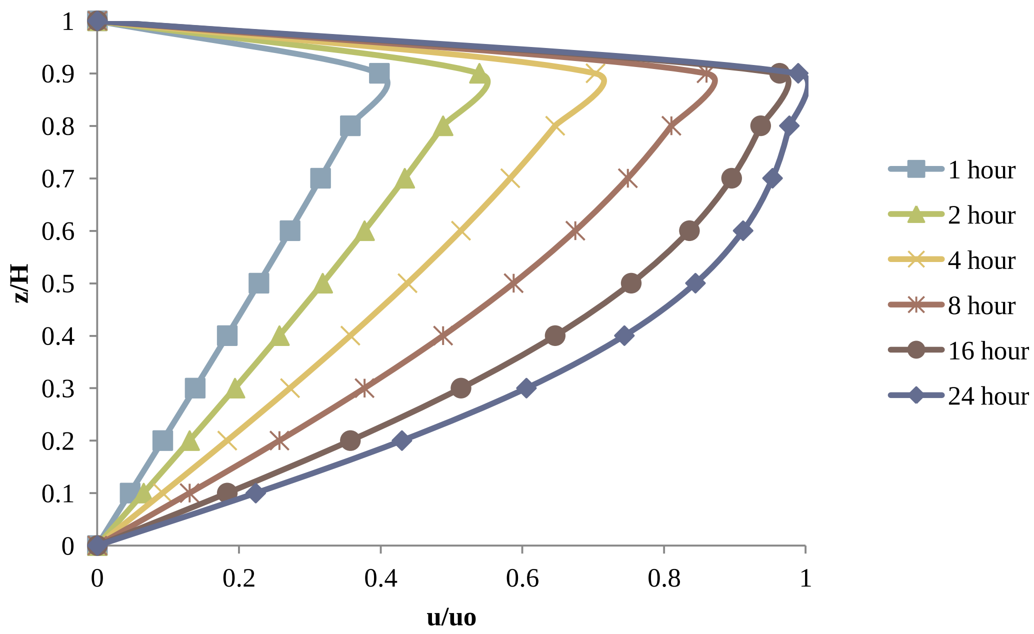


Fig 5.25 Pore pressure distribution

vi) Applied pressure = 320 kPa

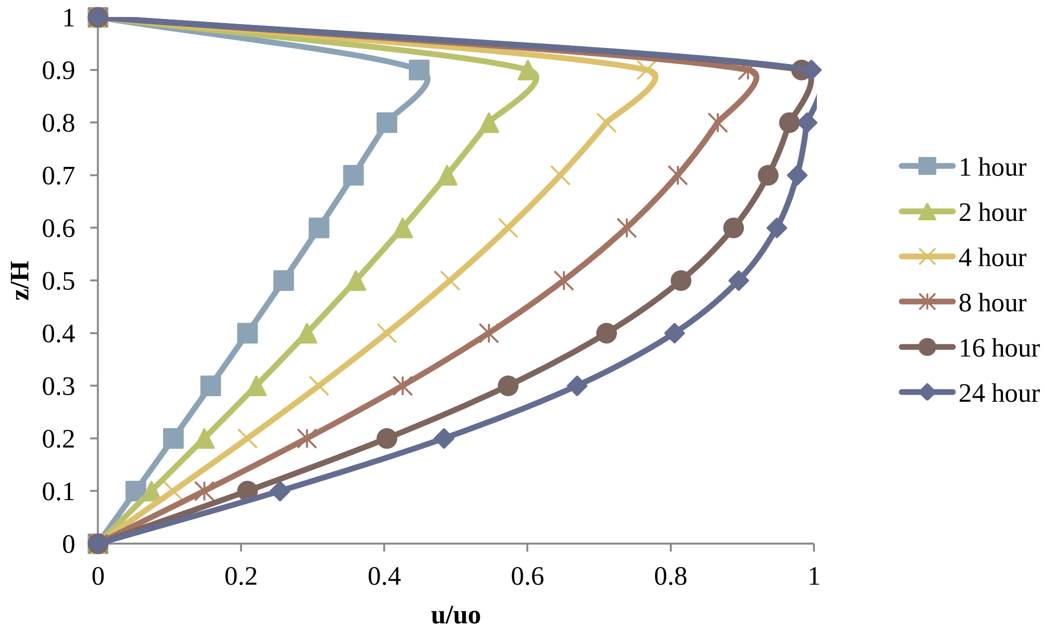


Fig 5.26 Pore pressure distribution

vii) Pore pressure distribution at 24 hours for different loadings for tc/(tc+ts) = 0.4

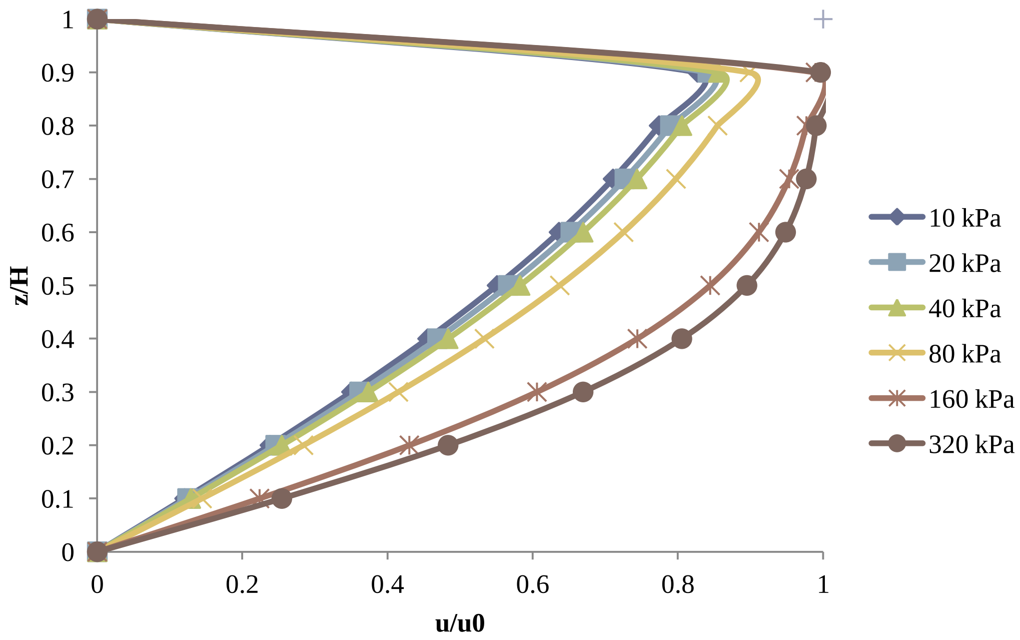


Fig 5.27 Pore pressure distribution

c) **For** $\frac{tc}{(tc+ts)} = 0.6$

i) **Applied pressure = 10 kPa**

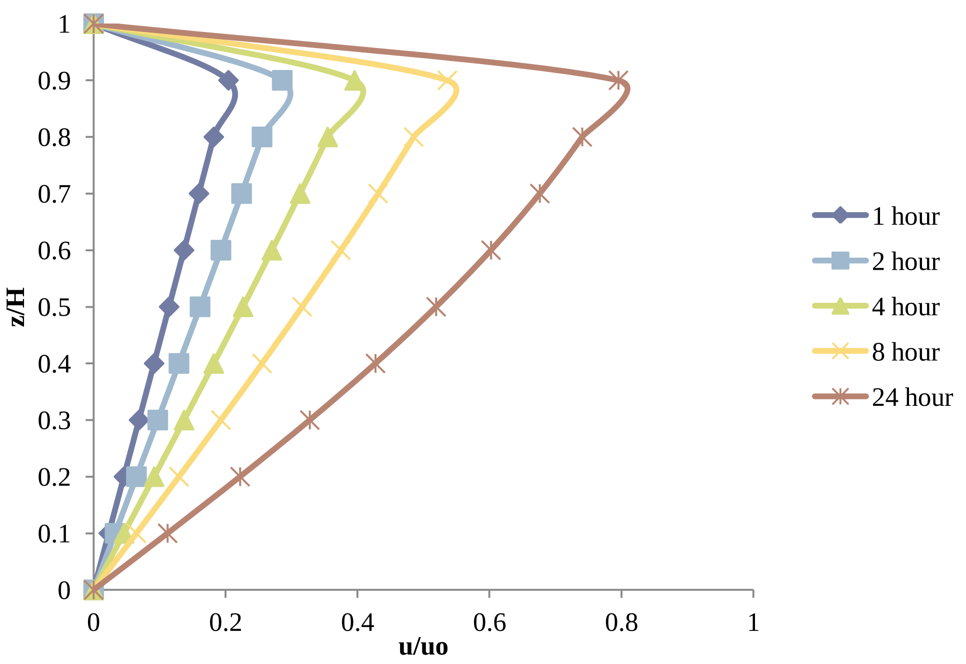


Fig 5.28 Pore pressure distribution

ii) **Applied pressure = 20 kPa**

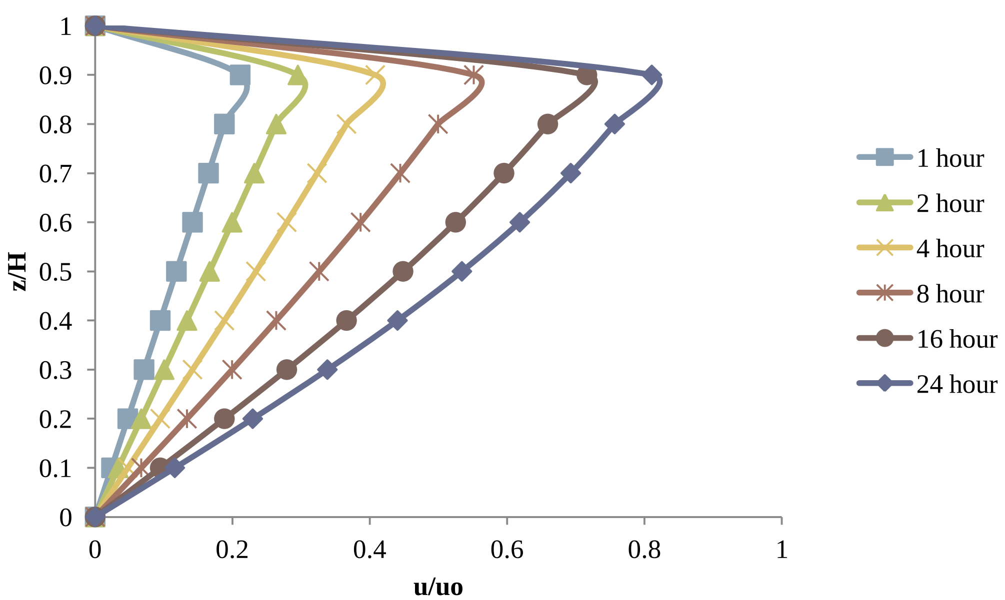


Fig 5.29 Pore pressure distribution

iii) Applied pressure = 40 kPa

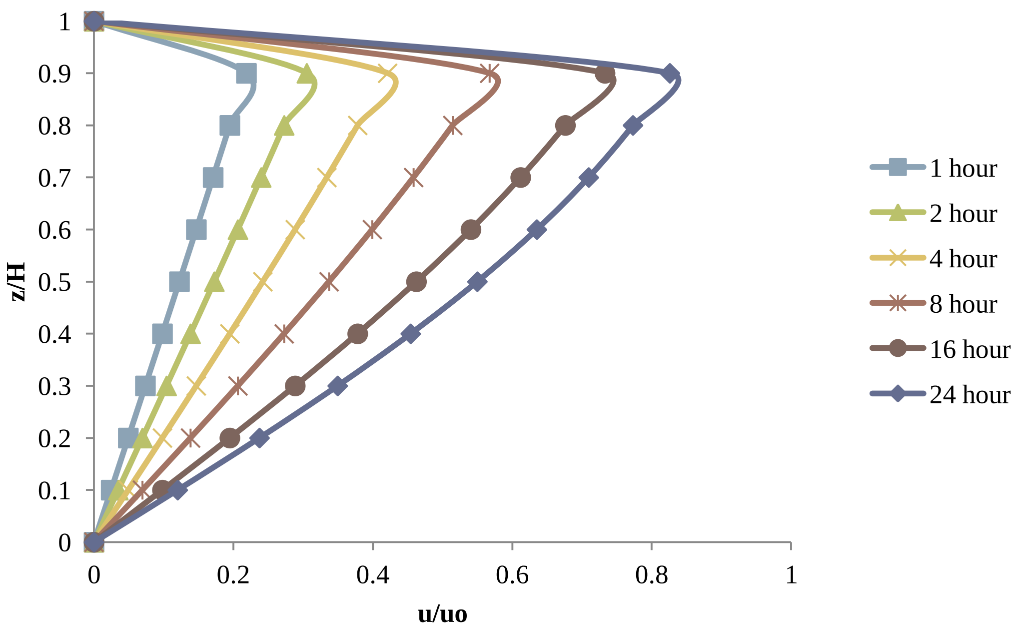


Fig 5.30 Pore pressure distribution

iv) Applied pressure = 80 kPa

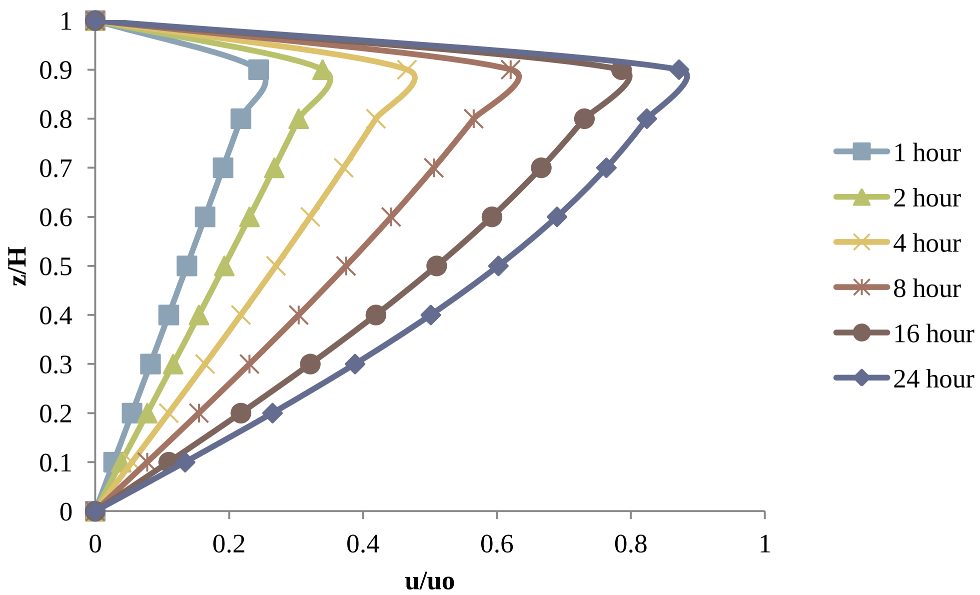


Fig 5.31 Pore pressure distribution

v) Applied pressure = 160 kPa

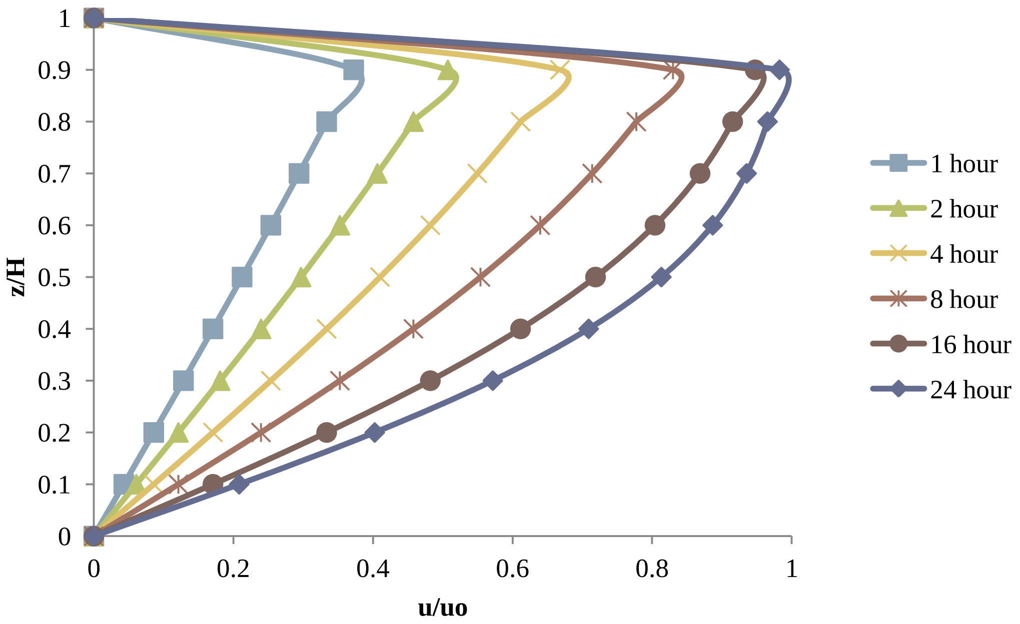


Fig 5.32 Pore pressure distribution

vi) Applied pressure = 320 kPa

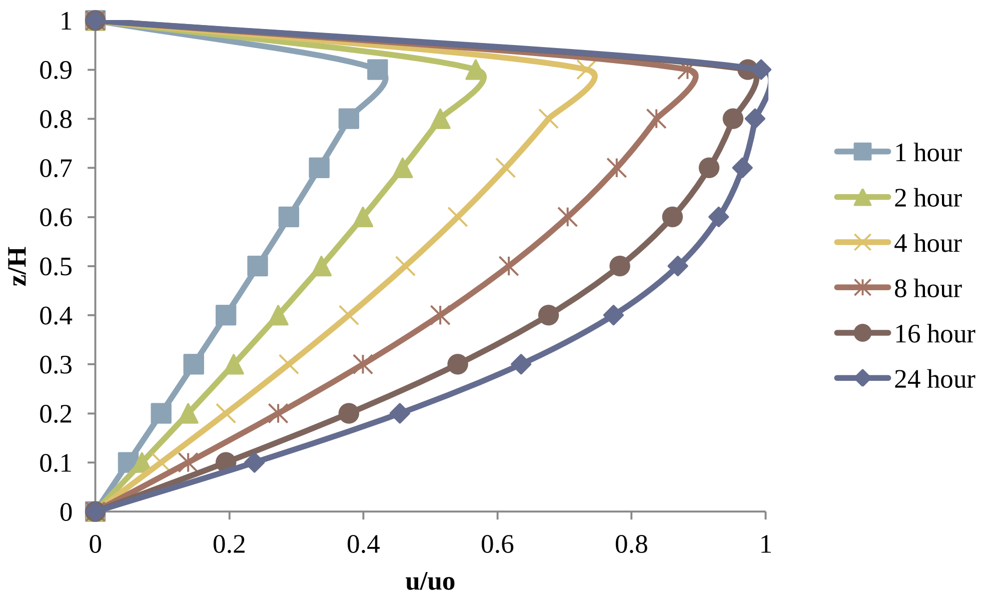


Fig 5.33 Pore pressure distribution

vii) Applied pressure = 640 kPa

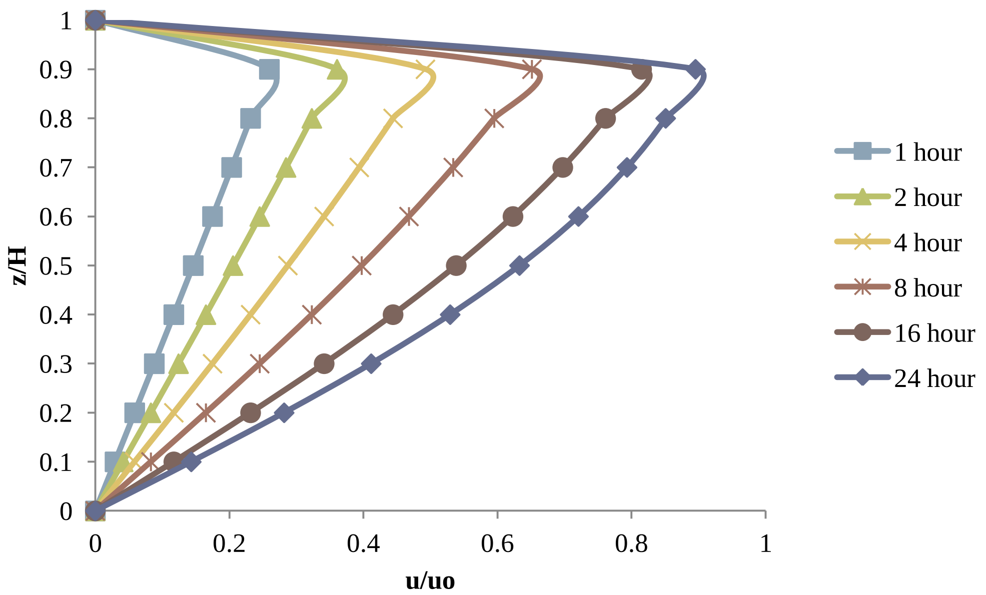


Fig 5.34 Pore pressure distribution

viii) Pore pressure distribution at 24 hours for different loadings for tc/(tc+ts)=0.6

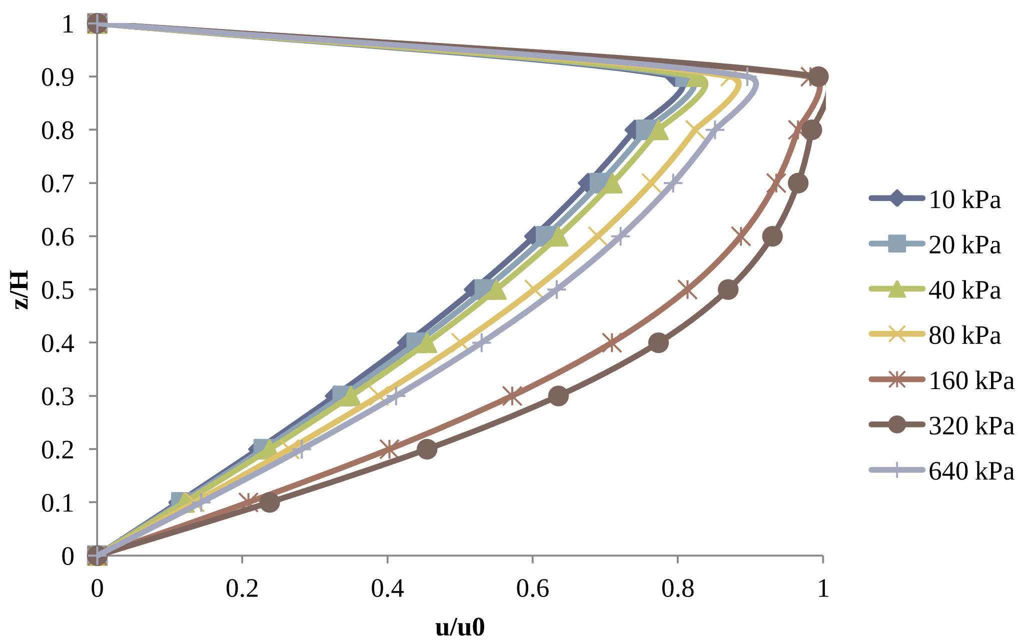


Fig 5.35 Pore pressure distribution

d) **For** $\frac{tc}{(tc+ts)} = 0.8$

i) **Applied pressure = 10 kPa**

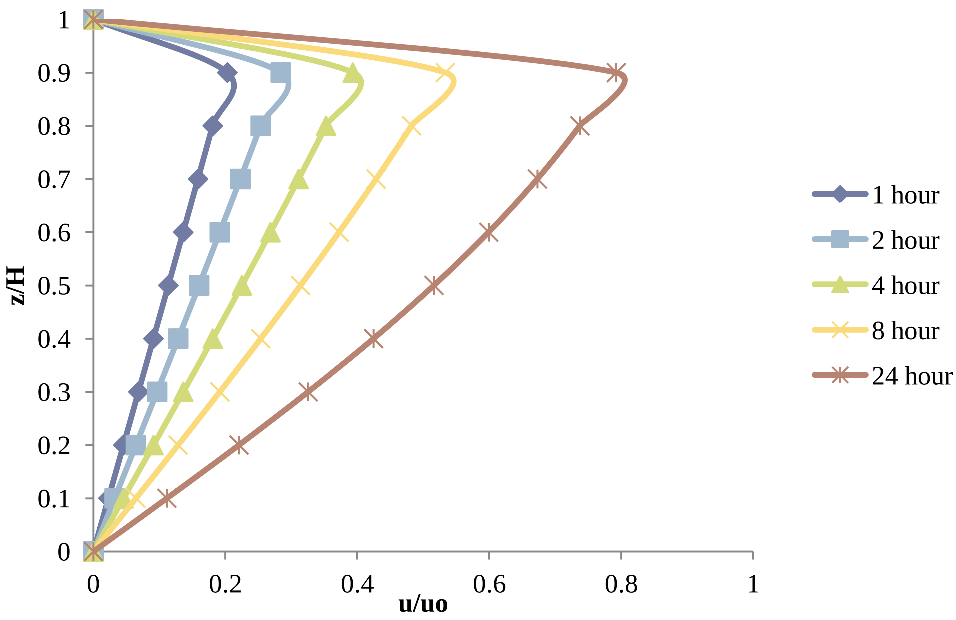


Fig 5.36 Pore pressure distribution

ii) **Applied pressure = 20 kPa**

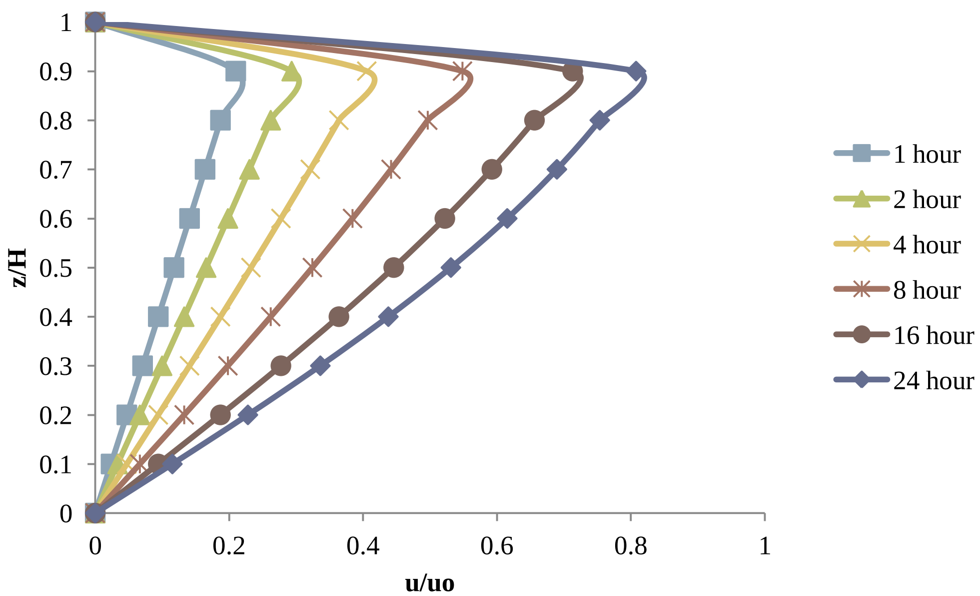


Fig 5.37 Pore pressure distribution

iii) Applied pressure = 40 kPa

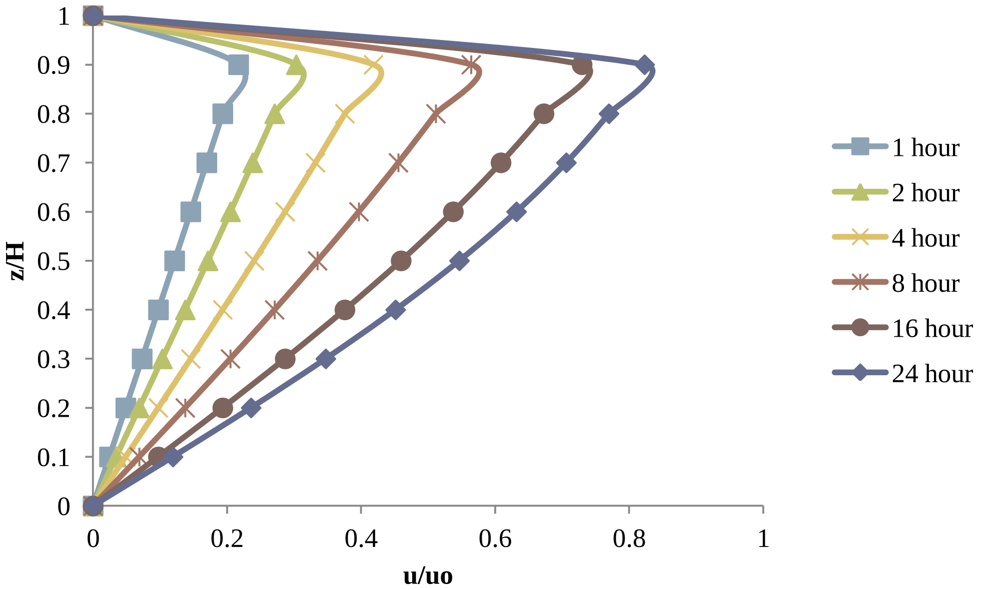


Fig 5.38 Pore pressure distribution

iv) Applied pressure = 80 kPa

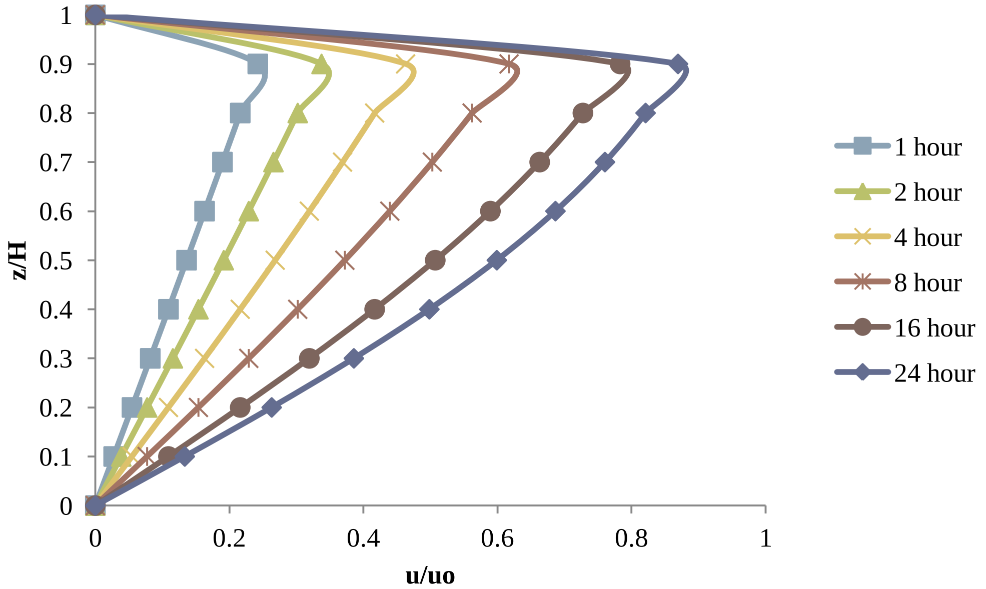


Fig 5.39 Pore pressure distribution

v) Applied pressure = 160 kPa

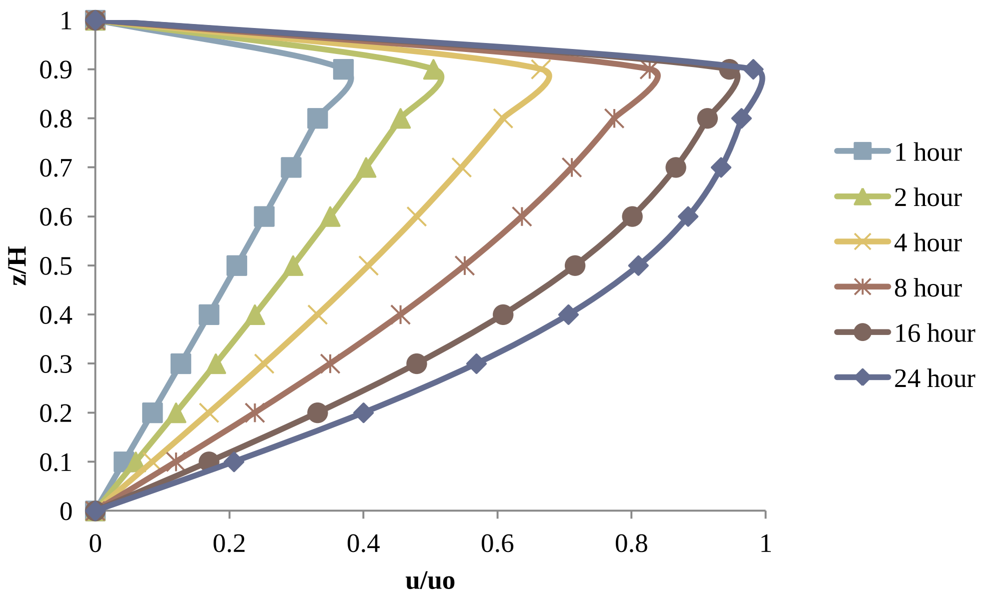


Fig 5.40 Pore pressure distribution

vi) Applied pressure = 320 kPa

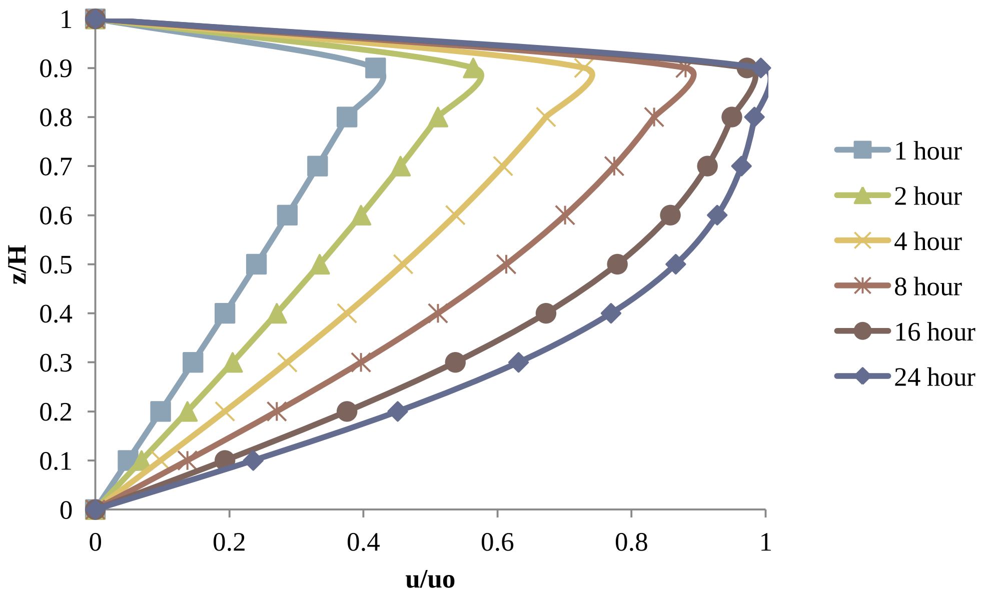


Fig 5.41 Pore pressure distribution

vii) Applied pressure = 640 kPa

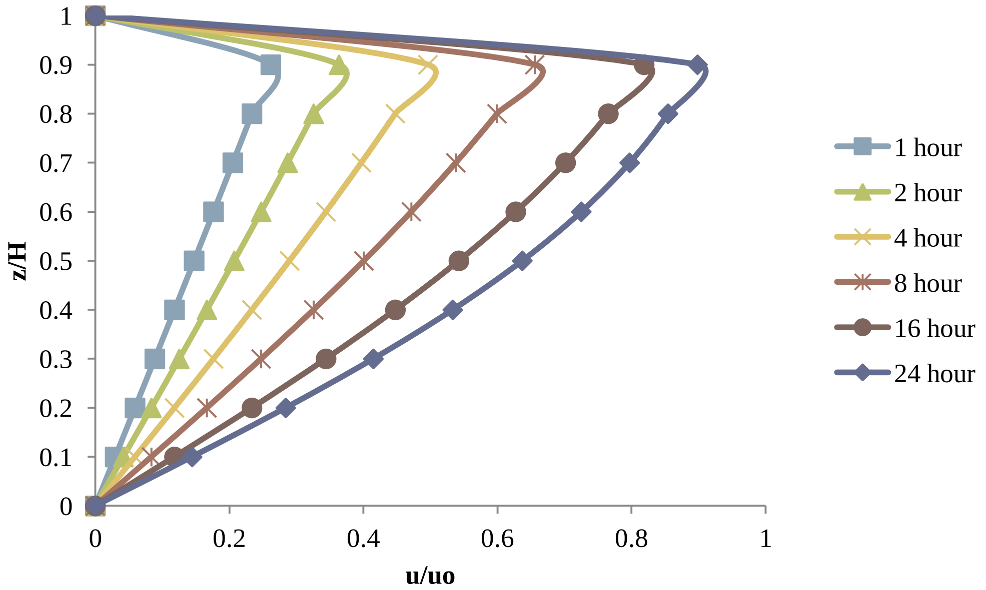


Fig 5.42 Pore pressure distribution

viii) Pore pressure distribution at 24 hours for different loadings for tc/(tc+ts)=0.8

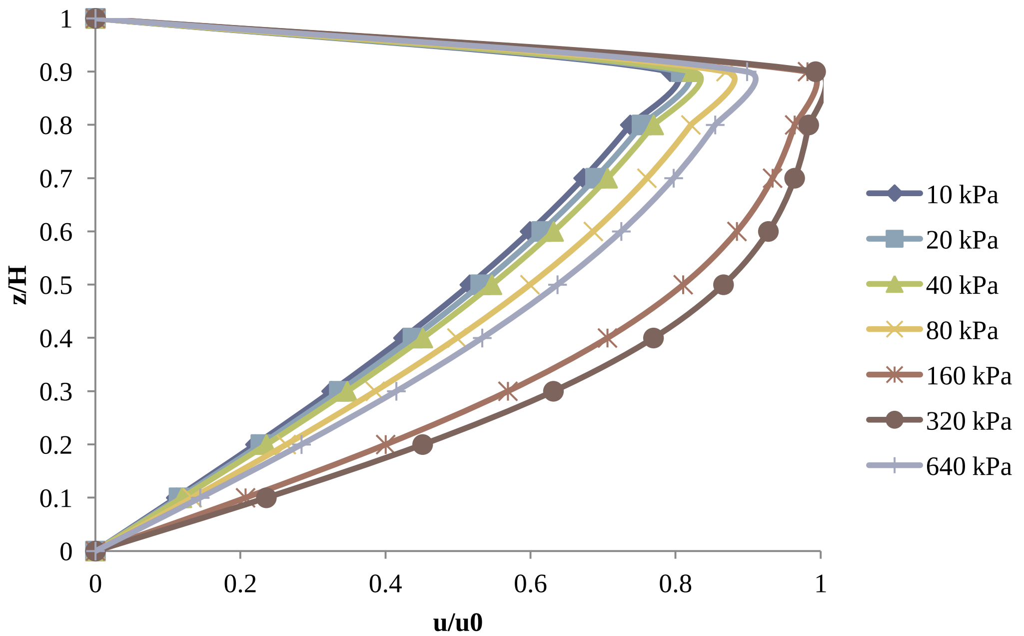


Fig 5.43 Pore pressure distribution

e) For $\frac{tc}{(tc+ts)} = 1.0$

i) Applied pressure = 10 kPa

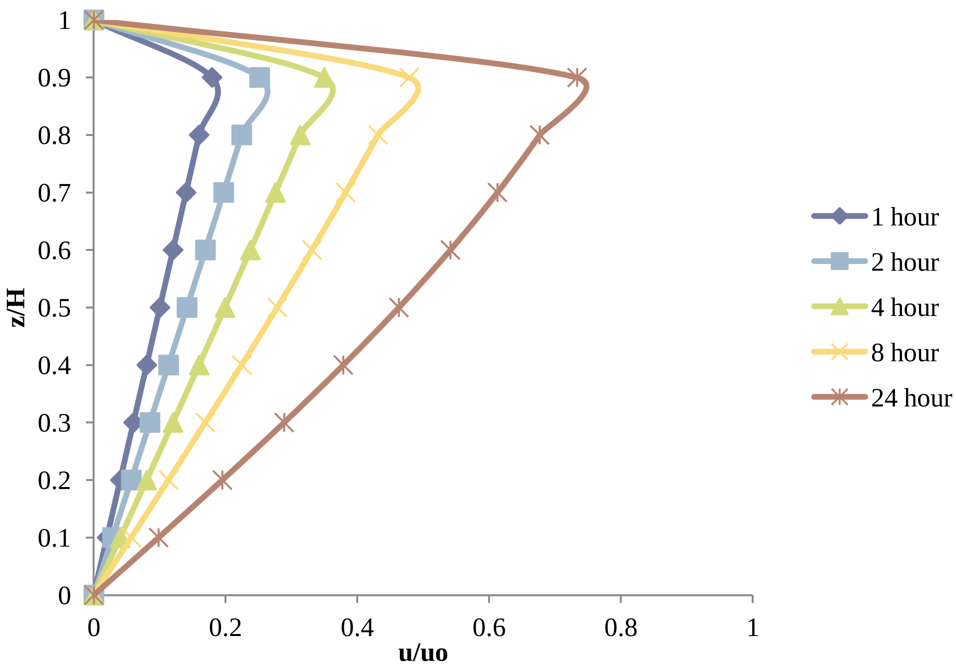


Fig 5.44 Pore pressure distribution

ii) Applied pressure = 20 kPa

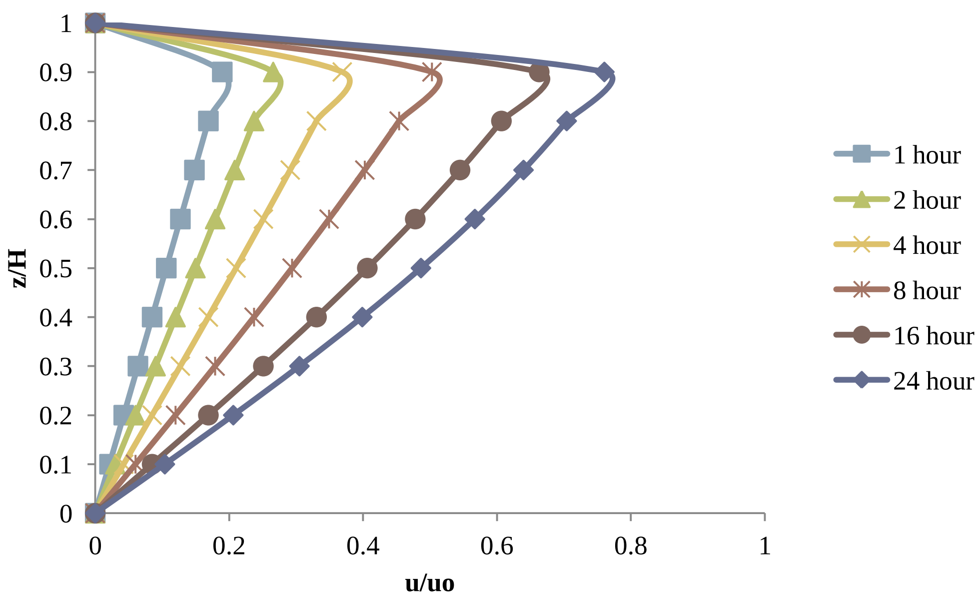


Fig 5.45 pore pressure distribution

iii) Applied pressure = 40 kPa

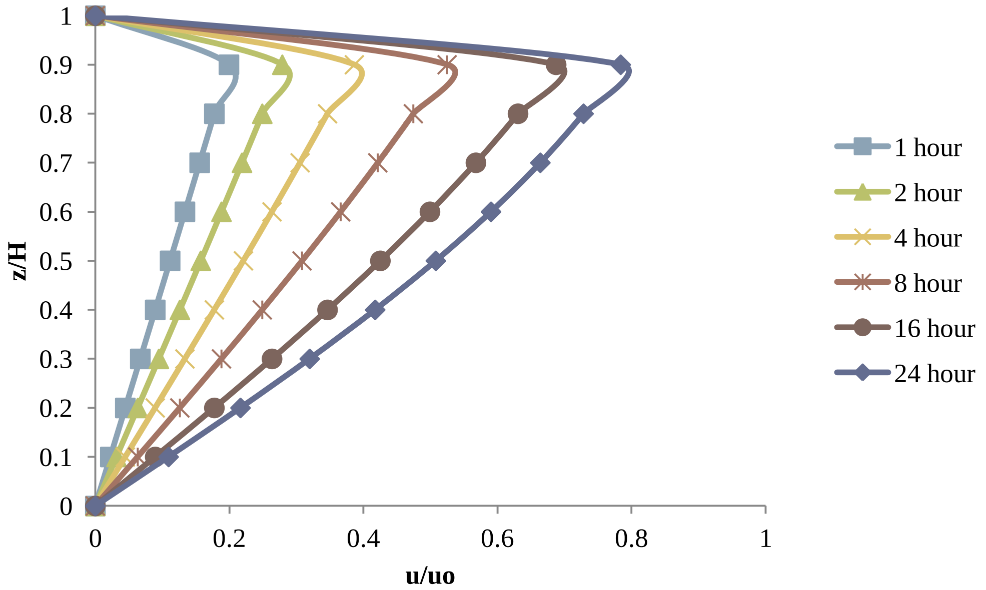


Fig 5.46 Pore pressure distribution

iv) Applied pressure = 80 kPa

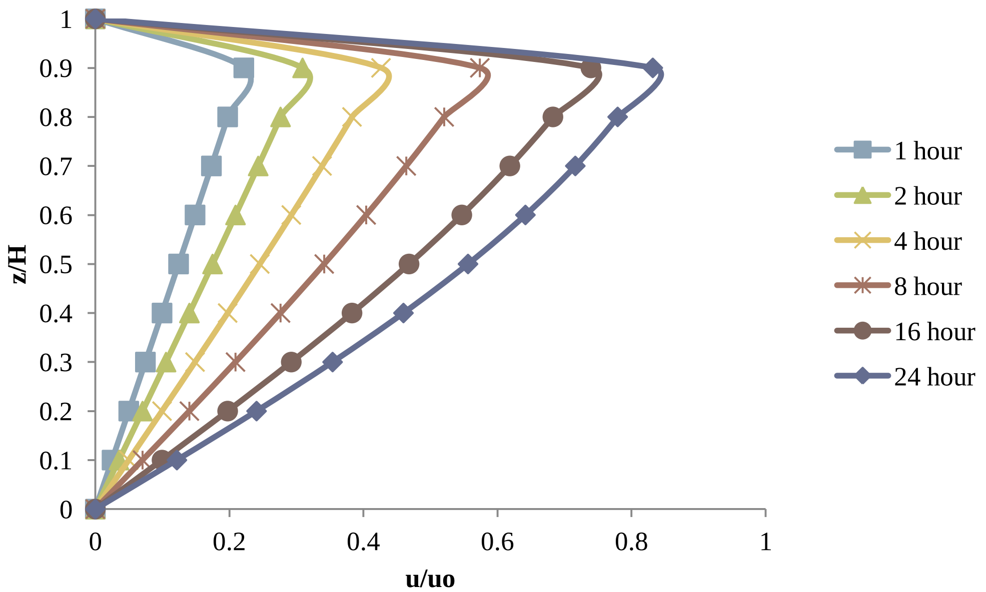


Fig 5.47 Pore pressure distribution

v) Applied pressure = 160 kPa

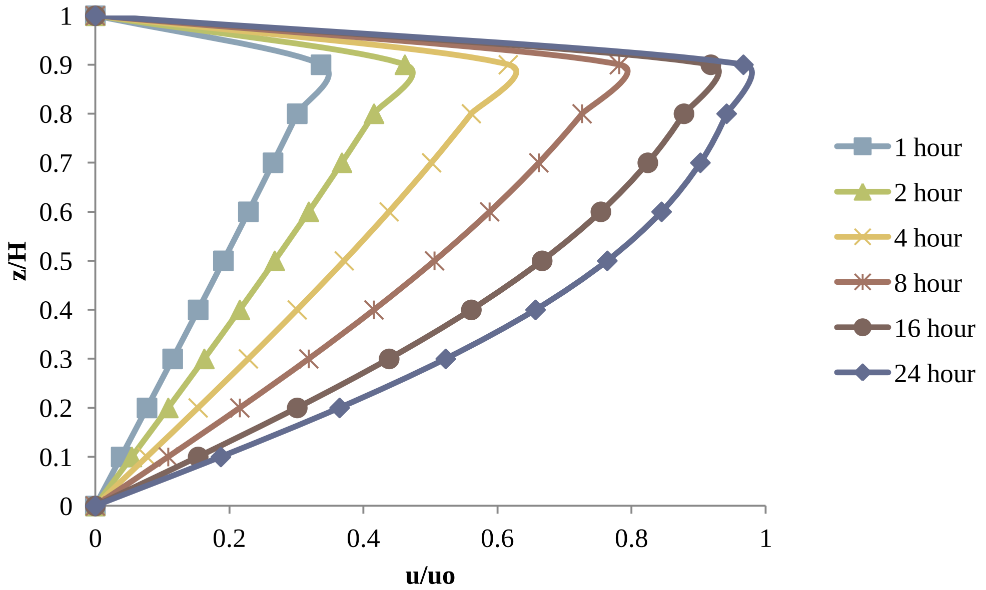


Fig 5.48 Pore pressure distribution

vi) Applied pressure = 320 kPa

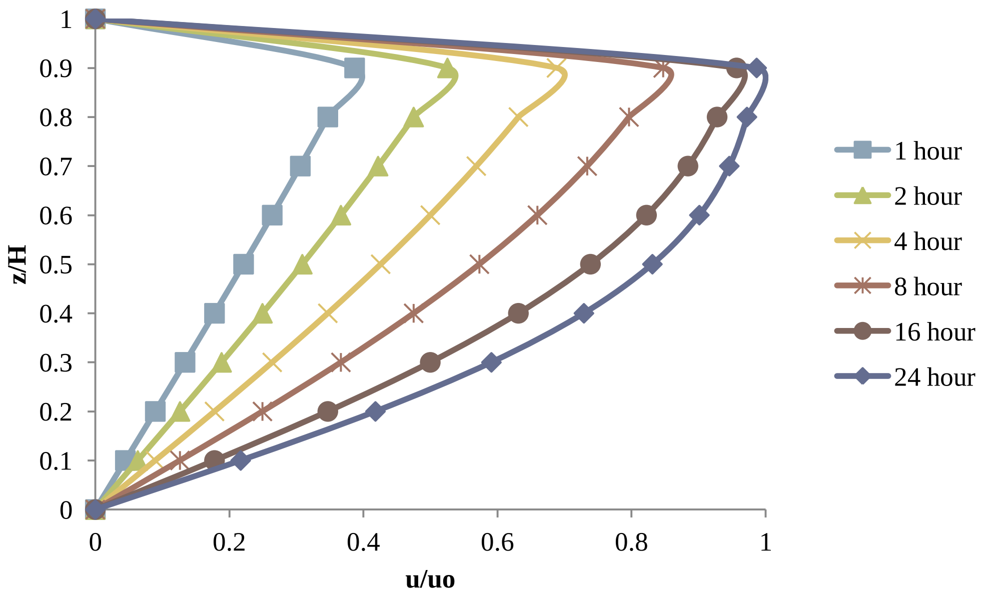


Fig 5.49 Pore pressure distribution

vii) Applied pressure = 640 kPa

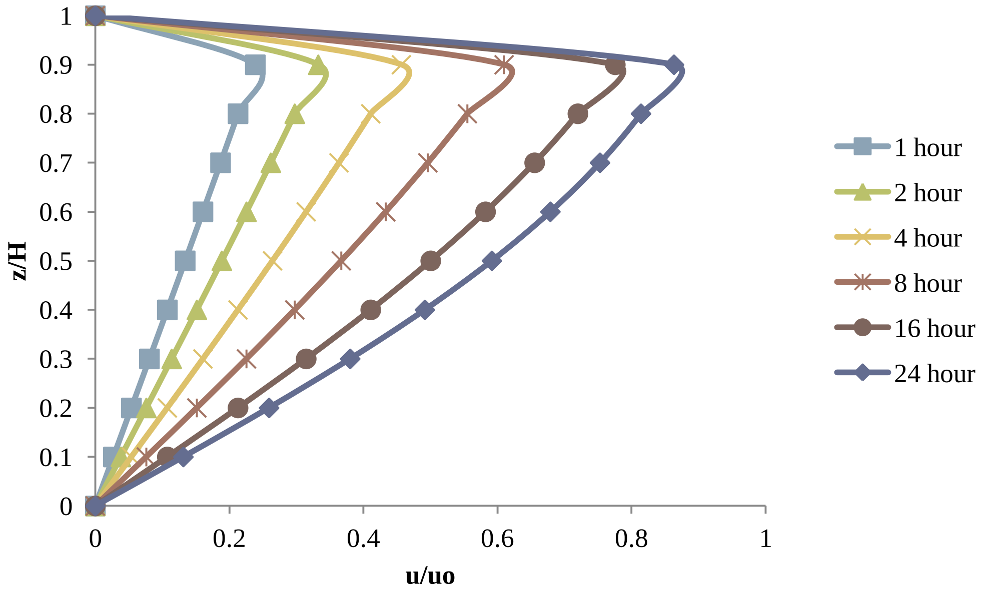


Fig 5.50 Pore pressure distribution

viii) Pore pressure distribution at 24 hours for different loadings for tc/(tc+ts)=1.0

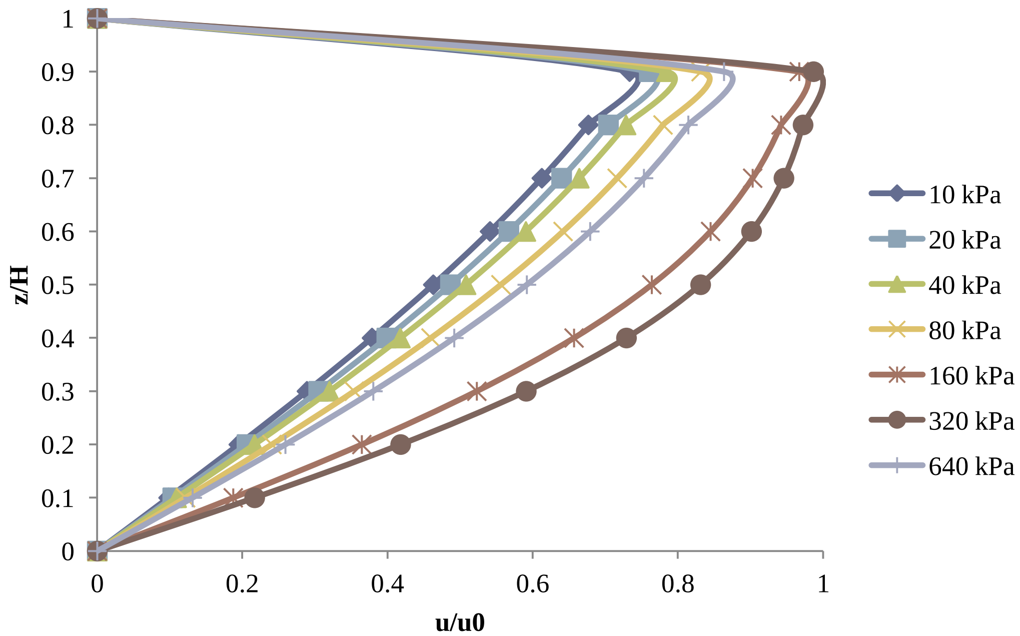


Fig 5.51 Pore pressure distribution

5.3 Theoretical Comparison of Models

Error function is a special function which is used for solving the partial differential equations. The error function is an entire function; it has no singularities (except that at infinity) and it always converges. The error function can be evaluated for arbitrary complex arguments *z* and it is referred as complex error function. Thus it is suitable for analytic analysis of complex variables.

Table 5.3: Comparison of two models

S. No	**Fourier Series**	**Error Function**
(a)	It is used for periodic functions only	It can be used for non periodic functions as well.
(b)	Series varies with in the domain of (0,2π).	Function varies with in the domain of (0, +∞).
(c)	Basic equation of Fourier series f(x) = $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n . \cos nx + \sum_{n=1}^{\infty} b_n . \sin nx$ Where $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x). \cos nx \, . dx$ $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x). \sin nx \, . dx$	Basic equation of Error function $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$
(d)	Series is discrete in nature	Function is continuous
(e)	Applications: 1-D consolidation 1-D heat flow Full-wave rectifier	Applications: Probability and Statistics Heat equation Digital communication

Comparison of theoretical Fourier series and Error function for degree of consolidation graphically is shown in figure below.

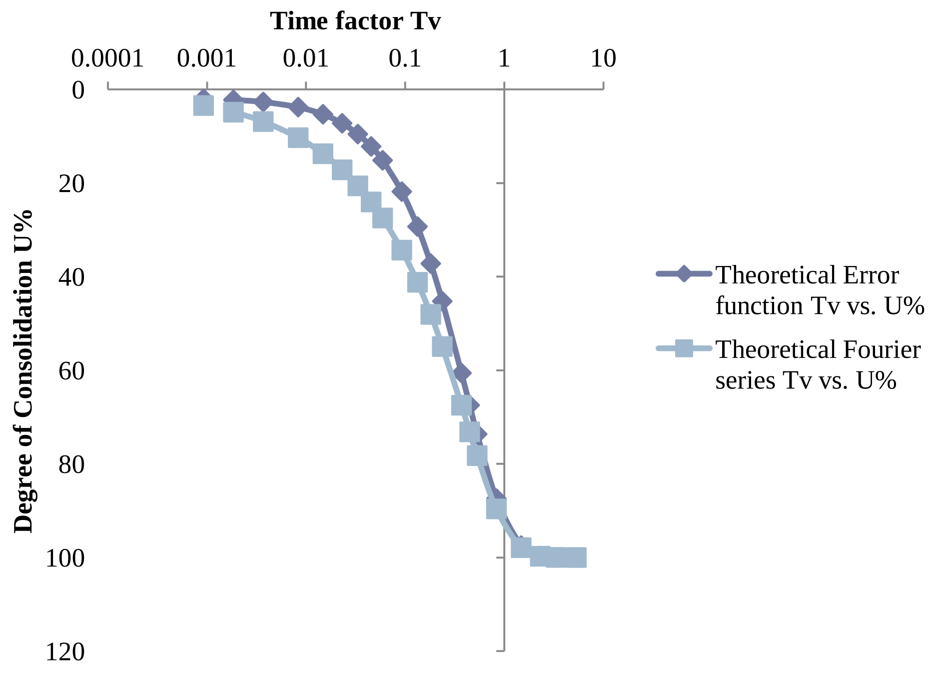


Fig 5.52 Comparison of theoretical results

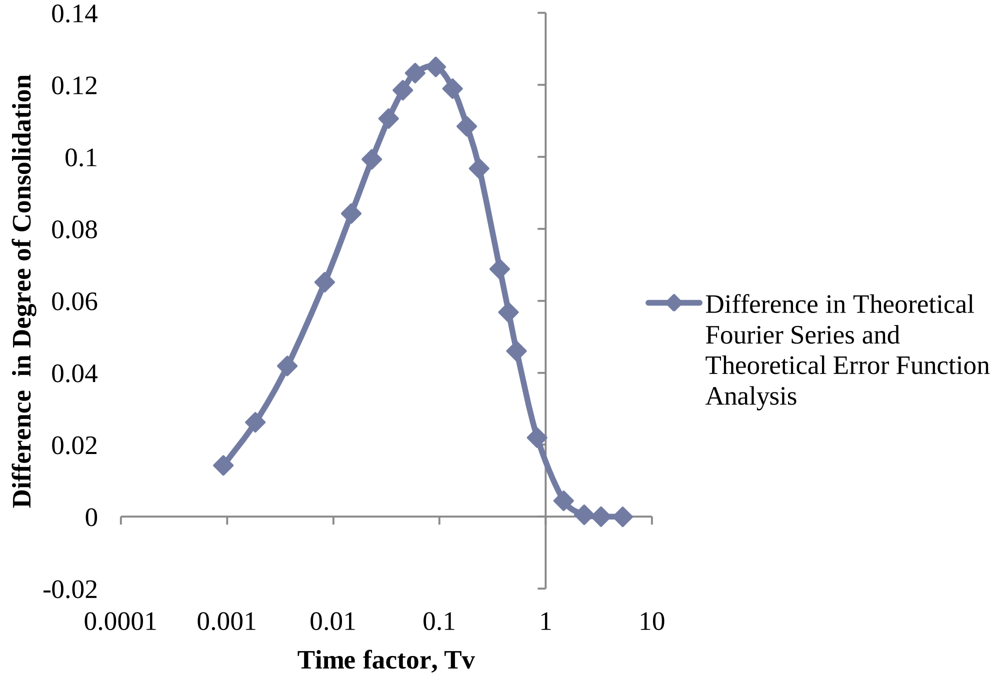


Fig 5.53 Variation in Degree of Consolidation

5.4 Validation of Numerical Model with Expermental Analysis

Several experiments were conducted for different compositions of soil sample. Experimental results using Fourier series are compared to the experimental results obtained using Error function and validation of result is done. The variations in values of degree of consolidation with time factor are calculated using the two methods and the subsequent result has been plotted. For different samples of soil the resulting degree of consolidation values with time factor are shown below.

i) For tc/(tc+ts)=0.2

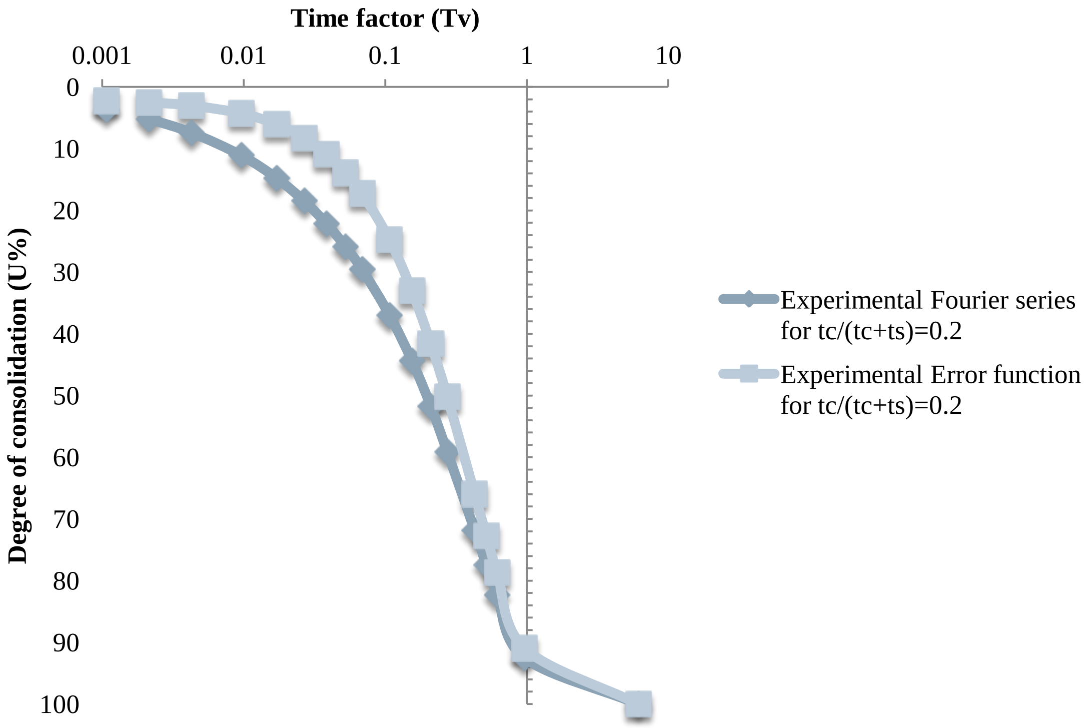


Fig 5.54 Variation of degree of consolidation with time factor

ii) For tc/(tc+ts)=0.4

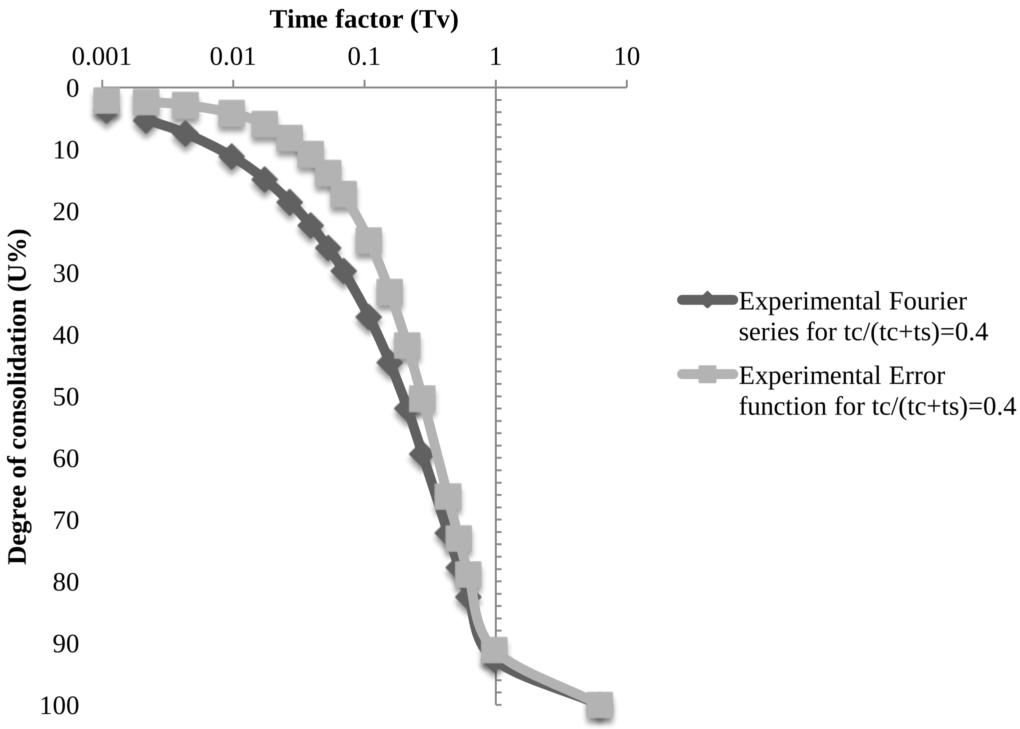


Fig 5.55 Variation of degree of consolidation with time factor

iii) For tc/(tc+ts)=0.6

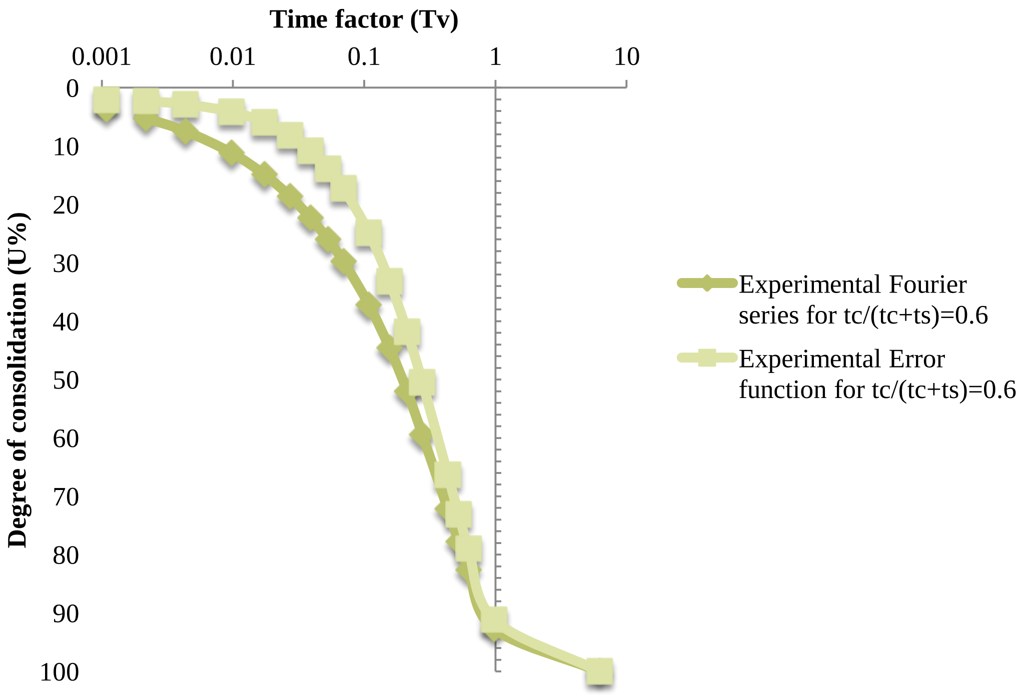


Fig 5.56 Variation of degree of consolidation with time factor

iv) For tc/(tc+ts)=0.8

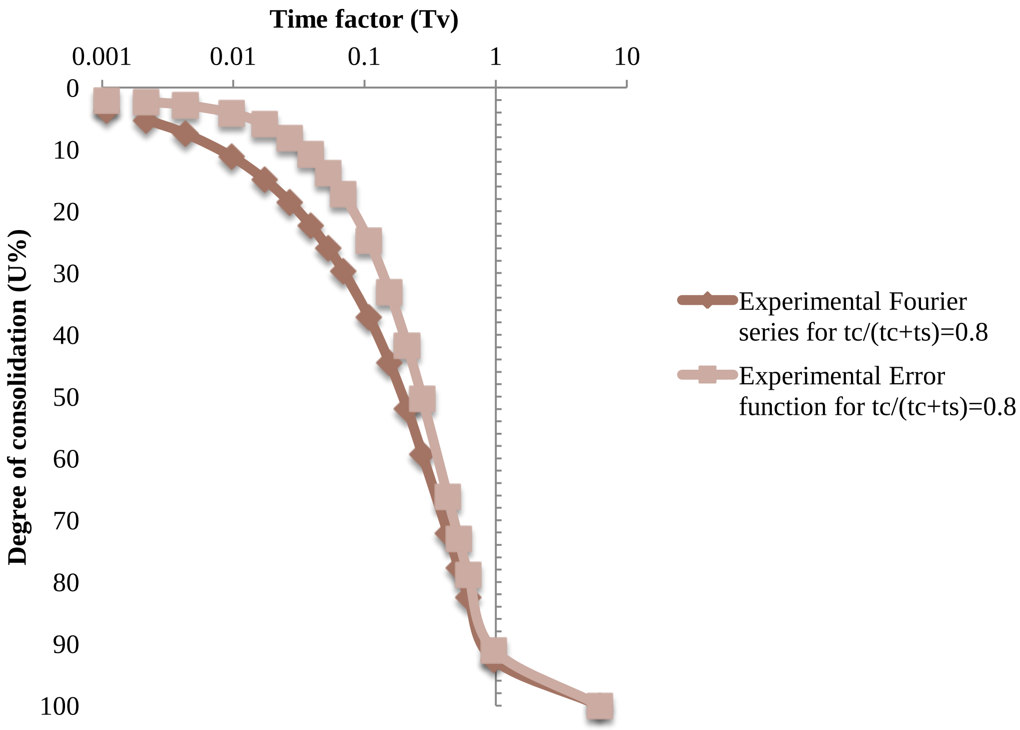


Fig 5.57 Variation of degree of consolidation with time factor

v) For tc/(tc+ts)=1.0

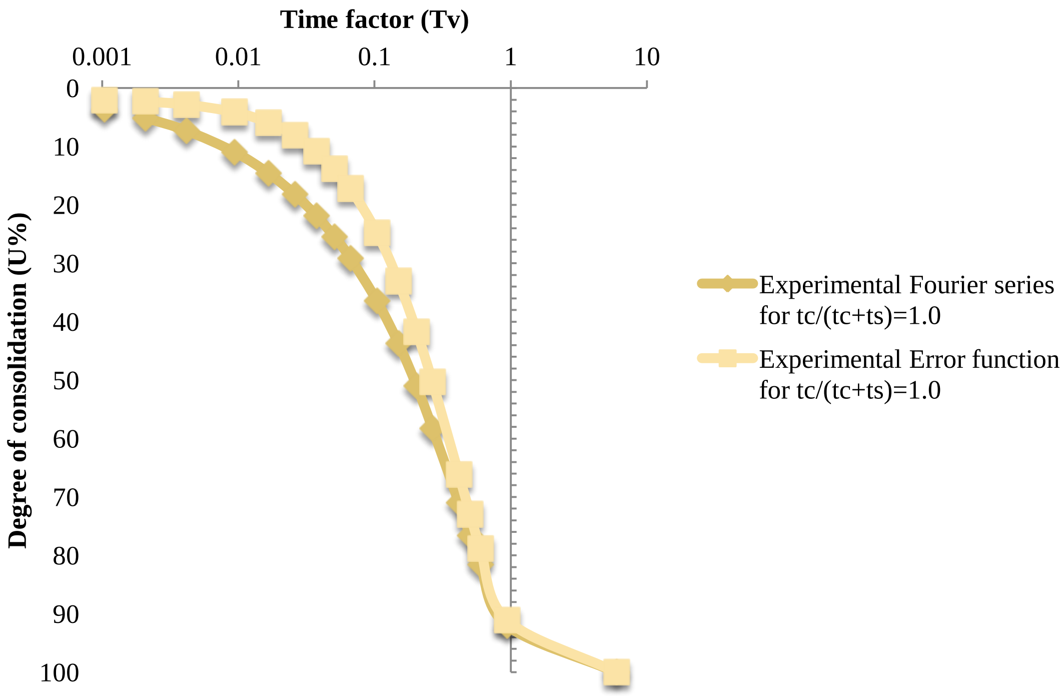


Fig 5.58 Variation of degree of consolidation with time factor

5.5 Discussion

Hence from the numerical analysis various consolidation parameters can be estmated efficiently. Liquid limit of the bentonite clay comes out to be 386% and the plastic limit comes out as 111.48%. This shows clay is highly plastic. Grain size analysis shows that maximum portion of the clay constitutes of particles with size less than 0.002 mm. Sand used is fine with particle size lesss than 0.425 mm. Specific gravity for the bentonite is 2.318. Permeability of the clay layer is very less thus affecting the consolidation of samples. Coefficient of permeability for the samples is in the range of 1×10^{-6} cm/sec to 1×10^{-5} cm/sec. If the permeability of the material is less, the deformations are retarded by the pore fluid.

Coefficient of consolidation and coefficient of volume compressibility increases with the increasing thickness of sand layer in the sample. Coefficient of permeability decreases with the increasing thickness of clay layer in the sample. Consolidation is slowed down by the clay content in the sample. Thus consolidation settlement for the samples with greater clay thicknesses is large but with slower rate of consolidation. For such soils long term consolidation behavioural study is essential.

Pore pressure distribution curves for the samples as obtained using the error function analysis show that the pore pressure is increasing with depth. Higher pore pressure values are observed at the bottom due to presence of clay layer which acts a rigid plate at the bottom.

Pore pressure distribution at 24 hours for higher loading cases like, 320 kPa and 640 kPa takes on a curvilinear form. This might be due to the secondary compression taking place in the sample.

The comparison for the degree of consolidation with respect to the time factor for both the methods shows that the error function analysis results deviates from the Fourier analysis results initially and converge towards the end of consolidation.

Chapter – 6

Conclusion

6.1 Conclusions

- In case of sandy soils or sands maximum consolidation takes place in primary stage causing immediate settlement. Addition of clay layers to the sand can slow down the consolidation process at initial stages.
- Permeability of bentonite clay is very less thus it can be used as a lining material for containment structures where the underlying soil is sandy in nature to prevent percolation.
- As the consolidation is slower in clay, thus ultimate settlement will include the secondary compression settlement as well.
- Bentonite clay thin lining can be used in sanitary landfills to prevent seepage of leachates into the ground and hence reduce groundwater contamination.
- Analytically from the curves of the two models it can be concluded that with the application of Error function, consolidation of stiff clays can be achieved at a faster rate.
- For complex analysis of the consolidation and pore pressure dissipation this method is more reliable.
- Also, this technique is suitable for consolidation of layered soil system as assigning of different properties for different layers in the analysis is easy
- Thus with the help of Error function for numerical analysis process of consolidation can be studied for any kind of soil at any time.
- Amount of dissipation of pore pressure can be determined at any instant during the process of consolidation using numerical solution easily.

6.2 Limitations

- Error function method gives good results for higher degree of consolidation at larger time intervals but for lesser time in initial stages results are fluctuating.
- This analysis done in the present study is for clayey soils but for sandy soils its use is yet to be studied.
- The Pore pressure dissipation and degree of consolidation can not be perfectly correlated with the actual and experimental results. It only gives a close idea about the consolidation.
- Error approximated for initial stages of consolidation exceeds the permissible limits sometimes.

6.3 Future Scope

- Application of Various methods of numerical and analytical analysis can be studied and error analysis can be done for various theories and methods.
- For multilayered soil systems, numerical analysis methods such as error function and approximation can be developed as they are easier to work with.
- Different numerical approximations of error function can be studied and the most accurate one can be evaluated and used.

Chapter – 7

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