

**OPTIMAL AVAILABILITY ANALYSIS OF BRAKE DRUM  
MANUFACTURING SYSTEM BY USING MARKOVIAN  
APPROACH**

**A THESIS SUBMITTED IN FULFILMENT OF  
THE REQUIREMENT FOR THE AWARD OF THE DEGREE**

**OF**

**MASTER OF TECHNOLOGY**

**IN**

**PRODUCTION ENGINEERING**

**BY**

**SMRITI MISHRA**

**(ROLL NO- 2K12/PRD/21)**

**GUIDED BY**

**Dr. R.S. WALIA**

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**Mechanical, Production, Industrial & Automobile  
Engineering Department  
Delhi Technological University**

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# CERTIFICATE

Date:- \_\_\_\_\_

This is to certify that report entitled **“Optimal Availability Analysis of Brake Drum Manufacturing System by using Markovian Approach ”** by **Ms. Smriti Mishra** is the requirement of the partial fulfilment for the award of Degree of **Master of Technology (M. Tech.)** in **Production Engineering** at **Delhi Technological University**. This work was completed under my supervision and guidance. She has completed her work with utmost sincerity and diligence. The work embodied in this project has not been submitted for the award of any other degree to the best of my knowledge.

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## CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this thesis entitled, “**Optimal Availability Analysis of Brake Drum Manufacturing System by using Markovian Approach**” in partial fulfilment of the requirements for the award of **Master of Technology Degree in Production Engineering at Delhi Technological University, Delhi** is an authentic work carried out by me under the supervision of Mr. Girish Kumar and Dr. R.S. Walia in Mechanical, Production, Industrial and Automobile Engineering department.

The matter embodied in this report has not submitted to any other university/institute for award of any degree.

**(Smriti Mishra)**

This is to certify that above statement made by the candidate is correct to best of my knowledge.

**Dr. R. S. Walia**

**Mr. Girish Kumar**

**Date:**

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## List of Symbols

$\lambda_i$ (i= 1,2,3,4.....)	Failure rate of each component
$\mu_i$ (i= 1, 2, 3, 4.....)	Repair rate of each component
R(t)	Reliability function
F(t)	Failure function
h(t)	Hazard function
H(t)	Cumulative hazard function
$R_s$	Reliability of Series System
$R_p$	Reliability of Parallel System
$A_v$	Availability
A, B, C	Working State of Furnace, PDC and Shot blasting
a, b, c	Failure State of Furnace, PDC, Shot blasting
$\Sigma$	Standard Deviation
Z	Level of Confidence
N	Number of Samples
E	Margin of Error

## **List of Abbreviations**

CBM	Condition Based Maintenance
PM	Preventive Maintenance
CM	Corrective Maintenance
RBD	Reliability Block Diagram
FTA	Fault Tree Analysis
MTTF	Mean time to failure
MTTR	Mean time to repair
MTBF	Mean time between failure
PDC	Pressure Die Casting
SB	Shot Blasting

## **Abstract**

In this thesis availability analysis is carried out for a Brake Drum manufacturing system using Markovian approach with the purpose to improve its operational availability. The analysis helped in identifying the key factors that affects the system reliability and there exists good scope to improve the system availability by controlling the contributing factors.

Separate models are developed for active and passive redundancy cases. All the feasible states and, failure and repair transitions are identified to develop the system model. Keeping in mind the limitation of the Markov model the failure and repair rates are taken as constant. The set of ordinary differential equations are obtained for the change of probability of being in respective system states with respect to time in each model. This system of rate equations is solved using Runge- Kutta method in MATLAB. The system availability assessment is based in the sum of probabilities of all working states. Sensitivity analysis is also carried out by varying the repair rates of constituent components in the system which helped in identifying the critical factors and assessing their impact on the system availability. These results are helpful to identify the more sensible elements for overall plant availability and suggest some maintenance and operational action to reduce down-time and maintenance costs.

# Chapter- 1

## Introduction

*Nowadays modern technologies are used to increase the productivity of the system. To increase the production and to reduce the production cost highly reliable technologies are required. These technologies should be highly available and maintainable also so that after any failure during manufacturing it can restore its initial stage easily. Availability of the system can be increase by reducing the load of the system. For doing this different type of redundancy is used. In this thesis standby redundancy is used to increase system availability then Markov model is used to evaluate the availability of the system by using Runge-Kutta method.*

**1.1 Introduction of Research:** While modern technologies have heightened the world to an unparalleled level of productivity, our advanced society has become more elegant and unguarded because of the enhancing dependence on advanced technological system that frequently require intricate operations and highly revolutionary management. From any respect, the system availability is a crucial measure to be considered in system operation and risk management. When designing a highly reliable and available system, there arises an important question how to obtain a balance between the availability and other resources e.g., cost, volume and weight.

As a result, addition of redundant components or increase of component with main components availability of the system leads to the increase of the system availability by reducing load of the system. From domestic to industrial applications, the interference of automated systems in our daily life has substantially increased and our dependency on that system also increased. Failures in air traffic control systems, nuclear reactors, or hospital patient monitoring systems can bring catastrophic consequences. In order to increase the dependability of computing systems, an effective evaluation of their reliability is required and its improving factor should present. The importance of reliable operation has been realized in large complex process industries such as chemical, fertilizer, sugar, paper, oil refinery, textile etc.

It is difficult to attain in any industry high productivity with less number of redundant component, storage capacity, less losses and minimum number of failure. These industries work continuously in such a dangerous habitat to reduce the chance of failure and increase the availability of the system. So the high system availability can be attaining by quantitative

analysis method using various concepts of industrial engineering and operation research. This evaluation can be done by using some accurate knowledge about some constant parameters like failure and repair rate of the systems / subsystems which are subjected to random failures because of many reasons like poor design, lack of operative skills and manufacturing techniques etc.

**1.2. Objective of the Research:** The failed systems can achieve its initial working states after doing number of repairs and / or replacement of some components in minimum possible down time by using effective maintenance planning and control This chapter presents some quantitative method of evaluating system reliability, and represent that stochastic modelling technique is the best and effective way for reliability/availability evaluation. In stochastic modelling technique combination of both Markov model and software is used. Markov model helps in generating the transition state whereas software is used to solve these transition states. Different type of redundancy is used to improve availability of the system by reducing total load of the system. It is assume that system failed independently because of different reasons such as poor design, manufacturing problem etc. Reliability plays an important role in the overall performance of a manufacturing system. Machine reliability is the most important factor in most of the performance-related studies.

Good design is very much important for the component with having high reliability. Some industrial components which bear high stress during operating condition require good product design to reduce maintenance. Reliability/availability/maintainability evaluation of production systems pertaining to process industries has assumed ever-increasing importance. Main advantage of this process is that the industry in terms of higher productivity and lower maintenance costs. Maintenance can be produced as a well organized manner to convince a acceptable level of reliability during the useful life of a physical advantage.

The earliest maintenance technique is basically breakdown maintenance (also called unplanned maintenance, or run-to-failure maintenance), which takes place only at breakdowns occur. A later maintenance technique is time-based preventive maintenance which is also called planned maintenance technique in which regular maintenance action has been taken to reduce the failure of the system. With the fast development of advanced technology, products have become more compound whereas better quality and higher reliability are essential. To increase reliability makes the cost of preventive maintenance higher. Nowadays, preventive maintenance has become a major expense of many industrial companies. So to reduce maintenance cost more

efficient maintenance approaches such as condition-based maintenance (CBM) are being proposed. CBM is a maintenance program that endorses maintenance actions based on the information collected through condition monitoring.

CBM attempts are used to eliminate useful maintenance tasks by taking maintenance actions only when there is possibility of unusual behaviours of a physical benefit. If a CBM program, is properly accepted and effectively execute then it can reduce maintenance cost by reducing the number of a useful scheduled preventive maintenance operations. A system of components working in a random environment is subjected to wear and damage over time and may fail unexpectedly. The components are replaced or repaired upon failure, and such unlikely events of failure are at the same time also considered in practice as opportunities for preventive maintenance on other components.

To keep a system in normal working condition, taking proper maintenance becomes even more important during its serviced life. According to the earlier studies maintenance was classified into two categories, corrective maintenance (CM) and preventive maintenance (PM). Normally, PM is more useful than CM because it is always to keep a system in an available condition so that the large loss caused by unpredictable fails can be avoided. Maintenance actions are generally of two types: Corrective Maintenance (CM) and Preventive Maintenance (PM). The quality of maintenance actions in both CM and PM is an emerging area of research and is also vitally important when maintenance policies are being developed in practice. PM can be either perfect or imperfect and the study of their impact on the performance of the manufacturing systems after maintenance action is very important. A perfect PM is assumed to restore the equipment to be as good as new while imperfect PM brings the system to failed state. There always exist priorities to repair different failed items when limited repair facilities are available.

When a less priority item going for repair and at the same time an item of higher preference gets fail then the less priority unit is pre-empted for repair and higher priority item is deal first for the repair. This is called pre-empted repair policy. After the completion of the repair of higher priority product the pre-empted unit is repaired from that particular point it was stopped. Various number of performance parameters are used in the process industry to describe the performance parameters of a plant regarding its reliability and maintainability. The study of repairable systems is a basic and important topic in reliability engineering.

The system reliability and the system availability are very essential in power plants, industrial systems, and manufacturing systems. According to the earlier studies, an assumption is considered that a perfect repair model was commonly studied in repairable systems by considering that the failed system would be repaired as good as new after failures. But practically, the systems after repair can be brought to one of the possible states after doing a repair. These states are considered “as good as new”, “as bad as old”, “better than old but worse than new”, “better than new”, and “worse than old”.

**1.3. Method of the Research:** System layout can be considered of two type- Series and Parallel and the availability of both the approaches can be calculated by different-different methods. There are basically three states of the practical system which can be considered and that are working state, good but not working state and last one is failed state. There are two types of failure in the system and that basically are – revealed and unrevealed. The revealed failures consist of failure that brings the components from good state or pending-to-failed state to failed state whereas the unrevealed failures bring the components from good state to pending-to-failed state. The preventive maintenance and corrective maintenance action are performed on transition to pending-to-failed and failed states of the components to achieve the initial state of the system.

Many standard techniques are used for evaluation of reliability and availability analysis such as Reliability Block Diagram (RBD), Fault tree Analysis (FTA) and Reliability Graph (RG). RBDs which help in comprehension of the functions of each subsystem, whereas the Fault tree Analysis technique determines, if failure modes occur at one level that produce critical failures at a higher level in the system. In spite of these methods are simple and precise but the main disadvantage of this method are that they are essentially static in nature. Complex systems which incorporate repair method and having non exponential probability distributions are not possible to be solved with these techniques.

In Markov approach, the probability of generating a transition from one state to another must be constant, i.e. controlled by a constant rate, and times between transitions are controlled by the negative exponential distribution; the approach must shortfall memory, i.e. the future states of the system are independent of all past states except the just previous one; and the states of the system must be recognizable. If the previous conditions are fulfilled the first stage of the Markovian model is used to generate the state transition diagram to evaluate the availability. In the markovian model each node represents one of the distinct system states, and the edges



of the diagram represent the transition probabilities or rate of occurrence between the states in the direction represent by an arrow drawn on the edge. Markov approach is improvement to such techniques as it provides the potential to insert repair in the system. Markov approach enclose mainly two concepts which are The “state” of the system and the “transitions” in the system from operating to non-operating and vice versa. One of the most important characteristics of any Markov model is that the transition probability  $p_{ij}$  depends only on states  $i$  and  $j$  and is completely independent of all past states except the last one. This property of the model is called ‘Memorylessness’. Markov models are easy to construct but very typical mathematical approach is required to solve this model. Many simplifying methods are exist which are used to model of complex systems. Although ordained with these advantages, the major drawback of Markov methods is the explosion of the number of states as the size of the system increases.

**1.3. Structure of the Thesis:** In this thesis a layout of brake drum manufacturing system is considered in which three subcomponents are present, Furnace, Pressure die casting machine and shot blasting machine. This is a hybrid type of system whose reliability block diagram is a combination of series and parallel network. Furnace, pressure die casting and shot blasting are connected in series manner. In this system three PDCs are used which are connected in parallel manner and two shot blasting machines are used which are also connected in parallel manner. With the help of this reliability block diagram a markov transition state model is developed which represent the transition of system and subsystem from full available state to failed state.

At initial level system is taken as 100% available state but during working its humiliation started and moderately it reaches from 100% available state to 0 level state. In this markov model each transition state of the system is shown. There are three category of the Markov model- Working state, Good but not working state and failed state. Availability of the system depends only on the working states. All the states are depends only its just previous state. After failure initial state of the system can be attain by normally repair of all the system and subsystem. Availability can be assess by creating state space probabilistic differential equations with the help of Markov transition diagram. These equations are further solved by Runge- Kutta method with the help of MATLAB.

## **Chapter- 2**

### **Literature Review**

#### **2.1. Introduction:**

The maintenance concept was first identified by Gitz and Gerards. It is concerned with implementing maintenance, training staff maintenance, integrating maintenance with management enterprise and spare part inventory. It is also concerned with developing repairing material and technique.

#### **2.2. Review on Reliability:**

To evaluate the reliability of any system many publishers proposed different methods. Reliability block diagram is one of the best method for evaluating reliability of series, parallel and any hybrid system.

Dhillon in [1984] continued his research by analyzing reliability of a transit system in which he proposed two mathematical model of transit system. State probability system of markov model is used to solve this transit equation.

Mahmood M et al. [1987] derived a mathematical model for a system having two parallel redundant active units, with common – cause failure and a cold standby unit. All repair time distributions were assumed to be arbitrary and different. The analysis was carried out under the assumption of having a single service facility for repair and replacement. Supplementary variable, Laplace transforms technique was used to calculate the various state probabilities. The explicit expression for the steady state probabilities and steady state availability was also derived.

Wang proposed maintenance models related to maintenance policies. He categories these policies into age replacement policy, random age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, repair limit policy and opportunistic maintenance policy. These policies are useful for availability improvement of the system.

Dhillon [1992] evaluated reliability and availability of the system with warm standby and considering common cause of failure. The standby and switching mechanism of considering two unit parallel system reliability. Laplace transform of system state probability are used for availability, MTTF and MTBF consideration.

Singh & Goyal [2000] proposed Behavior Analysis of a Biscuit Making Plant using Markov Regenerative Modeling” in which methodology to study the transient behaviour of repairable mechanical biscuit shaping System pertaining to a biscuit manufacturing plant is present. This methodology is for determining availability by markov model. Failure and repair rates are taken constant and state space probability equations are generated which are solved by Laplace method for availability evaluation.

Goyal et al. [2001] evaluated reliability of a part of rubber tube production system under preemptive resume priority repair under preemptive resume priority repair discipline is based on markov modeling. The system undergoes for preventive maintenance (PM) and corrective maintenance (CM) on its transition to pending-to-failed and failed states respectively. In this system repair is done on the basis of priority. Performance analysis of the system has been carried out which helped in identifying the critical factors and assessing their impact on the system availability.

Shakuntla et al. [2002] derived Certain stochastic processes with discrete states in continuous time can be converted into Markov process by the well-known method of including supplementary variables technique. In this paper a mathematical model of the steel industry which manufactures the stainless steel plates and also made an attempt to improve its availability was developed and Lagrange’s approach is used for solving these differential equations.

### **2.3. Review on Maintenance and Redundancy:**

Maintenance and redundant policies are used to prevent the system from failure and to increase availability of the system by doing periodic inspection and by reducing load of the system by connecting them in parallel manner.

Wu and Chan [2003] proposed importance of a component state in Multi State systems and explained the solution of this multi state system by using heuristics, met heuristics, neural networks and fuzzy techniques.

Yuo-Tern Tsai et al. [2003] published a paper on “A study of availability-centered preventive maintenance for multi-component systems” in which preventive maintenance is considered in three action mechanical service, repair and replacement. In this model of reliability the mean-up and mean-down times of each component are also investigated and the replacement intervals of components are determined based on availability maximization. At least one interval is chosen as PM interval of system in periodic maintenance policy. Then the scheduling is progressed step by step and is terminated until the system extended life reaching to its expected

Cher et al [2006] proposed a model on practical framework of predictive maintenance of scheduling of multi state system. By this scheduling system-perspective using the failure times of the overall system as estimated from its performance degradation trends can be derived.

Gurler and Kaya [2008] proposed maintenance policy with multistate system whereas Huang and Yuan[2009] proposed two state preventive maintenance policy for multi state deterioration system under periodic inspection of the system with multiple action of preventive maintenance.

### **2.3. Review on Availability:**

Availability of the system can be evaluated by different approaches such as monte carlo, fault tree method. Bayesian approach and markov method. Many publishers published their paper on following approaches.

Singh [1991] formulated mathematical models for a standby redundant complex system under preemptive repeat repair policy; standby redundant complex system having two classes with many components under preemptive resume repair policy. This repair is done on the basis of priority of the sub-system. Some researchers have worked on the problems of mission reliability of systems.

Coetzee [1999], proposed the maintenance policy for considering the procedure the work force. So, down-top-down requirement is developed. Feedback from the inner cycle to the outer cycle is not apparent in Coetzee maintenance framework.

Rakopoulos et al. [2006] published a paper on “Availability analysis of hydrogen/natural gas blends combustion in internal combustion engines” in which computational model is developed for hydrocarbon fuels so that a comparison is performed under the assumption that hydrogen combustion will be feasible in conditions that do not depart exceedingly from current engine configurations.

Sisworahardjo et al. [2007] published a paper on “Reliability and availability analysis of low power portable direct methanol fuel cells” in which a methodology of model for calculating reliability and availability of low power menthol cell is proposed. Two techniques are used for availability analysis, first reliability block diagram is constructed and the probabilistic state space markov model is used which incorporate three different states—operational, derated, and fully faulted states and then homogenous poisson process is used for solving the equation.

Yong Sun et al. [2007] published a paper on “Prediction of System Reliability for Multiple Component Repairs” in which a split system approach is presented for reliability prediction of complex system under preventive maintenance. This extended model can be used to determine the remaining life of the system and to describe the changes in reliability with PM actions. A sequential preventive maintenance can increase remaining useful life of the system.

Romulo I. Zequeira et al. (2007) proposed the study of the determination of the optimal maintenance policy for a manufacturing facility and the optimal buffer inventory to satisfy the demand during the interruption period due to a maintenance action. The possibility of imperfect production and that opportunities for the fabrication of the buffer inventory and opportunities to carry out a maintenance action to the random production facility is considered.

Guo and Yang [2008] presented a methodology for automatic creation of Markov models for reliability assessment of safety instrumented systems. Many related factors in the proposed model such as failures modes, self diagnostic, restorations, common causes and voting is included and simplification of Markov model is done by states merging.

Rakopoulos et al. [2008] publishes a paper on “Availability analysis of a syngas fuelled spark ignition engine using a multi-zone combustion model” in which zero dimensional, multizone model is developed and validate to predict engine performance. Within the framework of the multi-zone model, the various availability components constituting the total availability of each of the multiple zones of the simulation are identified and calculated separately.

Mo et al. (2008) proposed mission reliability analysis of fault tolerant multiple phased systems by deriving several efficient formulations. The proposed methodology allowed random phase durations on exponentially distributed repair activities and repair policies. Whereas Distefano and Puliafito (2009) proposed a methodology, called dynamic reliability block diagram (DRBD), for modeling and analysis of dynamic dependant behaviours of complex systems. The authors demonstrated the methodology with the help of several case studies. The authors

also compared the DRBD approach with the other existing methodologies and demonstrated its effectiveness.

Ji-wen Sun et al. [2008] published a paper named "Availability modeling and analysis of serial-parallel hybrid multioperational manufacturing system considering dimensional quality, tool degradation and system configuration" in which a machine configuration was proposed for evaluating the system-level reliability of a manufacturing system, while the absence of proper consideration of the product quality and the tool degradation on tool failure when determining the machine-level reliability is likely to result in unanticipated machine downtimes or inappropriate maintenance decisions. First the model is developed for evaluating machine level reliability then this developed model is used for system level reliability. The analytical procedure and the effectiveness of the proposed methodology are demonstrated through an example of a serial-parallel hybrid system for cylinder head gasket machining.

Savita Garg et al. [2009] published a paper on "Availability analysis of crank-case manufacturing in a two-wheeler automobile industry" This unit fails either by normal working state or by partial failed state. This system is under both corrective and preventive maintenance. Probabilistic differential equations are generated for analysing the availability. Laplace and Runge-Kutta approach are used for solving these equation. According to this analysis successful program of preventive and routine maintenance will reduce equipment failures, extend the life of the equipment, and increase the system availability upto considerable limits

Enrico Zio , Francesco Di Maio [2009] proposed a fuzzy network for reliability and availability estimation . Data from failure dynamic scenarios of the system are used to create a library of reference trajectory patterns to failure. By this failure system remaining life can be predicted by fuzzy similarity analysis. The prediction on the failure time is dynamically updated as time goes by and measurements of signals representative of the system state are collected.

Rajiv Khanduja et al. [2009] published a paper on "Performance Analysis of Screening Unit in a Paper Plant Using Genetic Algorithm" in which all subsystems are arranged in series and parallel configuration and mathematical formulation of markov death- birth model is done for availability evaluation. Now this model is solved by genetic algorithm.

Distefano and Puliafito (2009) proposed a method of dynamic reliability block diagram (DRBD), for modeling and evaluation of dynamic dependant behaviours of complex systems.

The authors also compared the DRBD approach with the other existing methodologies and demonstrated its effectiveness.

Yongjin Kwon et al. (2009) published a paper on “Remote, condition-based maintenance for web-enabled robotic system”. In this paper, mathematical modeling of system availability has been derived in order to account for other failures that might occur in the subsystems of the robot. Compared to the schedule-based maintenance strategies, the proposed approach shows great potential for improving overall production efficiency, while reducing the cost of maintenance. The current trends in industry include an integration of information and knowledge-base network with a manufacturing system, which coined a new term, e-manufacturing. From the perspective of e-manufacturing any production equipment and its control functions do not exist alone, instead becoming a part of the holistic operation system with distant monitoring, remote quality control, and fault diagnostic capabilities.

Laszlo Sikos and Jiri Klemes [2010] published a paper on “Reliability, availability and maintenance optimisation of heat exchanger networks” in which comprehensive up-to-date commercial software tools are used for reliability evaluation of heat exchanger. This idea is used to apply the combination of specific HEN optimisation and reliability software packages have several advantages over the commonly used approach.

Mahmood Reza Haghifam and Moein Manbachi [2010] studied about a combined model of heat and power system for availability analysis. He evaluated reliability and availability in his paper by using state space and continuous markov chain and present all contributing factors that can improve the availability of the system.

Roberto de Lieto Vollaro and Mauro Davoli [2010] estimated plant reliability installed in hospital by constructing RBD of all components working in the system and in this RBD MTTR, MTBF and MDT are evaluated and system reliability is estimated by using markovian chain and constructing probabilistic differential equations.

F.J.G. Carazas et al. [2010] published a paper on “Availability analysis of heat recovery steam generators used in thermal power plants” in which technology for design and construction and operation of steam generator are considered and its availability is evaluated based on the time to failure and time to repair data recorded during the steam generator operation and fault tree

analysis method is used for availability evaluation. This. Components maintenance policy can be improved through the use of reliability centered maintenance (RCM) concepts.

Sharma et al. [2011] published a paper on “Reliability analysis of complex multi-robotic system uses GA and fuzzy methodology” in which various reliability parameters of robotic system by using Real Coded Genetic Algorithms (RCGAs) and Fuzzy Lambda-Tau Methodology (FLTMs) is proposed. Optimal value of Mean time to Failure and Mean time between Failures is obtained by using genetic algorithm. Triangular fuzzy numbers are used to enhance the reliability of the system. Petrinets are considered to represent the relation between different components of the system.

A.K. Amjad et al. [2011] published a paper on “Availability analysis of n-heptane and natural gas blends combustion in HCCI Engines” in which he proposed a solution for combustion timing control is using a binary fuel blend in which two fuels with different auto-ignition characteristics are blended at various ratios on a cycle-bicycle basis. This study is focussed on different percentage of natural gases in blended fuels.

Omran Musa Abbas et al. [2011] developed a Predictive Markov-chain Condition – Based Tractor Failure Analysis Algorithm in which a repairable agricultural tractor is analysed for availability evaluation which is subjected to repeated failure. This system is kept under periodic inspection. Condition-Based Maintenance (CBM) is a methodology that strives to identify incipient faults before they become critical to enable more accurate planning of preventive actions. For availability analysis Markov chain closed system is used. Hence, six performances measures of the model process are used to find the optimal algorithm parameters that maximize the system availability. The model decision variables are working hours, time to repair and number of failure.

Gu Yingkui, Li Jing [2012] proposed multistate system reliability by considering multiple possible state of the system. This model allows both system and its component to consider more than two level of performance. multi-state reliability models provide more realistic and more precise representations of reliability.

Resham Vinayak and S. Dharmaraja [2012] proposed Semi-Markov Modelling Approach for Deteriorating Systems with Preventive Maintenance. Different preventive and corrective techniques have been applied to restore efficient service of the system. In this paper a



performance model to study the impact of load on degradation, using Markov reward models is proposed. Closed form solution of reliability, availability and maintainability is obtained.

Yuan [2013] presented a two-stage preventive maintenance (PM) policy for the multi-state deterioration system under periodic inspection and with multiple candidate actions for PM. they assumed that: (1) such actions except replacement are imperfect, (2) the inspection and action times can be ignored, (3) the system can be modelled by a multi-state discrete time Markov chain whose transition probabilities will change and be updated only at the instant after completing each PM, and (4) the risks of such imperfect actions will be updated only at the instant after completing each PM.

Olga Fink et al. [2013] published a paper on “Predicting component reliability and level of degradation with complex-valued neural networks” in which multi layer feed forward neural networks based on multi-valued neurons (MLMVN), a specific type of complex valued neural networks are used to evaluate reliability of the system. Reliability model is formulated in the form of time series to extract complex dynamic pattern to predict long term system reliability.

Wang et al. [2013] proposed Reliability and availability analysis of redundant BCHP (building cooling, heating and power) system. This analysis is helpful in deciding the redundancy of equipment failure. Space state method of probabilistic markov model is used for availability analysis. Failure rate, repair rate, MTTF is deduce and analyzed simultaneously. Hybrid layout of the system is used for analyzing reliability. Due to the more equipment in the hybrid cooling system, the reliability of the cooling sub-system decreases greatly.

Aggarwal et al. [2013] published a paper on “Performance Modelling and Availability Analysis of Skim Milk Powder Production System of a Dairy Plant” in which availability is evaluated by considering exponential failure and repair rate. Chapman-Kolmogorov differential equations are derived from the transition diagram. These differential equations are solved by using normalizing conditions to compute the availability under steady state condition.

## Chapter- 3

### Basic Reliability and Availability concepts

*Reliability and availability are very essential factor of industrial engineering. A system is reliable only when it is available and performs its task economically over specified period of time. Reliability can be expressed in terms of availability. MTTF is the main factor of reliability evaluation, if MTTF is known then it can be easily evaluated that when the system will fail and when it require repair. But MTTF of any system cannot be evaluated exactly so range of MTTF is calculated with the help of confidence interval. Confidence interval gives a range in which MTTF of the system varies.*

This chapter concerns the definitions of reliability, maintainability, and availability, applied, in a general sense, to mechanical systems. The key features of reliability are discussed, and the more important mathematical models of the probability functions are described as well as their applicability in availability simulation studies.

**3.1. Reliability:** The reliability of a mechanical system, equipment item, or component, can be defined from two different points of view: qualitative and quantitative. The following definition of Lenahan is typically qualitative: ‘A plant is reliable if it is available when required and performs its designed capability safely and economically over its lifetime’. In this case the term ‘reliable’ involves some aspects associated with the performance of the ‘plant’, such as ‘safely’ and ‘economically’, which sometimes are difficult to quantify, and even more difficult to group together as a single figure. However, this is a very useful definition in such cases when, for example, a maintenance strategy for a plant is being designed and implemented. Reliability is an inherent characteristic and a specific measure of any component device or a system which describes its ability to perform its intended function. The reliability measures used in power station indicates how well the system performs its basic function of supplying electrical energy to its customers. Reliability levels are interdependent with economics since increased investment is necessary to achieve increased reliability or even to maintain reliability at current acceptable levels. On the other hand, when the objectives around of the term ‘reliability’ are more deterministic, a quantitative definition, such as the following founded in, is more useful: ‘Mechanical Reliability is the probability that a component, unit, or system will perform its prescribed duty without failure for a given time when operated correctly in a specified environment.’

This definition brings into focus four important factors, namely,

- The reliability of a device is expressed as a probability
- The device is required to give adequate performance
- The duration of adequate performance is specified
- The environmental and operating conditions are prescribed

**3.1.1. Reliability Expressed as a Probability:** It is the ratio of the number of times we can expect an event to occur to the total number of trials undertaken. The maximum value of this fraction is 1 and minimum value is 0. For Example the reliability of the safety valve of a high pressure boiler is 0.91. This means that out of 1000 safety valve tested under identical conditions approx. 910 would be satisfactory.

**3.1.2. Adequate Performance:** This is the second element in the definition of reliability. It describe in unambiguous terms, what is expected of a device or system. For Example, we may specify that the performance of a safety valve of a high- pressure steam boiler would be adequate if the valve is released when the pressure reaches a limit of 350 kg/sq. cm. It is also possible to state that the valve operates between 350 kg/sq. cm and 351 kg/sq. cm. This means that a particular valve may not be released every time at the specified pressure of 350 kg/sq. cm.

**3.1.3. Duration of Adequate Performance:** This is one of the most important elements in the definition since it represents a measure of period for which the performance is satisfactory. Most units or a system fails when operated over long period. Deterioration of materials and parts is natural, and consequently the performance level of the unit will also go down with time.

**3.1.4. Operating Condition:** The environmental or operating conditions in which we expect a device to function adequately could be with regard to temperature, humidity, shock, vibration and so on. Air conditioner which performs satisfactorily in temperature zones may not have the same performance characteristics for hot and arid climatic conditions. It can be seen that reliability of any system decreases with time due to the failure of the components. Relationship between reliability and time is exponential.

Mathematically, the reliability function  $R(t)$  is the probability that a system will be successfully operating without failure in the interval from time 0 to time  $t$ ,

$$R(t) = P(T > t), t \geq 0 \quad (2.1)$$

Where T is a random variable representing the failure time or time-to-failure. The failure probability, or unreliability, is then

$$F(t) = 1 - R(t) = P(T \leq t)$$

This is known as the distribution function of T.

If the time-to-failure random variable T has a density function  $f(t)$ , then

$$R(t) = \int_t^{\infty} f(x) dx$$

This can be interpreted as the probability that the failure time T will occur between time t and the next interval of operation,  $t + \Delta t$ . The three functions, R(t), F(t) and f(t) are closely related to one another. If any of them is known, all the others can be determined.

### 3.2. Terms and Symbols of Reliability:

**3.2.1. Cumulative Hazard Function H(t):** The area under the hazard function from 0 to t. H(t) is not a probability.

**3.2.2. Hazard Function h(t):** The instantaneous conditional probability of failure in a small interval (t, t + dt) divided by the width of the interval.

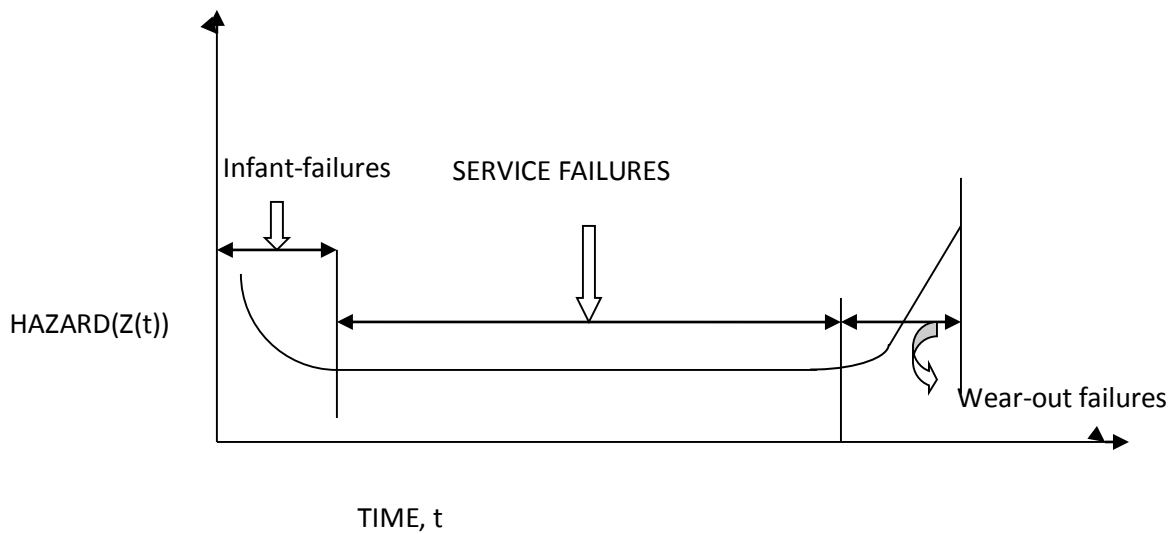
**Failure Rate Function:** A function depicting the number of failures per unit of time at a particular time. The failure rate function  $fr(t)$  is related to the hazard function in that its plot over time has the same shape. Only the Y axis values differ.

**3.2.3. Mean time to failure (MTTF):** These are the main terms which are used in reliability evaluation.

Usually we are interested in the expected time to next failure, and this is termed mean time to failure. Suppose that the reliability function for a system is given by R(t), the MTTF can be computed as:

$$MTTF = \int_0^{\infty} tf(t)dt = \int_0^{\infty} R(t)dt$$

**3.2.4. Bath Tub Curve:** A chart between failure rate or hazard, Z(t) and time, t is known as "BATH TUB CURVE"



**Fig 3.2: Bath tub Curve**

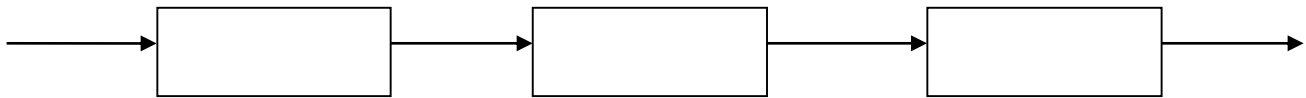
There are three clear zones in bath tub curve.

- The first is the short initial period during which the failures are called infant or early failures. This period is the break-in period when components fail due to defects in manufacture, being inherently weak because of weak parts, bad assemble, poor insulation, and so on.
- The failures in the second zone are termed service failure. During this period, the failure rate is constant. These failures are random and are fairly evenly distributed.
- The failures in the third zone are wear-out failures. The incidence of failure in this zone is high since most of the components will have exceeded their service life, and consequently would have deteriorated.

**3.3. Reliability Network:** An engineering system can form various different configurations in conducting reliability analysis. This section presents such commonly occurring configuration. These are as follows:

- Series Network
- Parallel Network

**3.3.1. Series Network:** This is the simplest reliability network and its block diagram is shown in fig.



**Fig 3.3: Series Network**

Each block diagram in the fig represents a unit/component. If any of the unit fails the series system fails, in other words, all the units must work normally for the success of the series system.

**3.3.1.1. Reliability of Series Network:** If we let  $E_i$  denotes the events that the  $i_{th}$  it is successful, and then the reliability of the series system is expressed by

$$R_s = P ( E_1 E_2 E_3 \dots E_n )$$

Where

$R_s$  is the series system reliability

$P(E_1 E_2 E_3 \dots E_n)$  is the occurrence probability of events  $E_1 E_2 E_3 \dots E_n$

Four independent units we have

$$R_s = P ( E_1 ) ( E_2 ) ( E_3 ) \dots P ( E_n )$$

If we let  $R_i = P ( E_i )$  for  $I=1, 2, 3, 4 \dots, n$  then we have

$$R_s = R_1 R_2 R_3 \dots R_n$$

Or we have in general

$$R_s = \prod_{i=1}^n R_i$$

Where  $R_i$  Is the unit reliability; for  $I = 1, 2, 3 \dots n$ .

For constant failure rate  $\lambda_i$ , of the unit 1, the reliability of the unit I is given by

$$R_i = e^{-\lambda_i t} \dots \dots \dots (d)$$

Where  $R_i (t)$  is the reliability of unit i at time t

From eq<sup>n</sup> (c) & (d) we get

$$R_s(t) = e^{-\sum_{i=1}^n \lambda_i(t)}$$

Also:

$$R_s(t) = e^{-(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n)t}$$

Thus for a series system, constant mean failure rate

$$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n$$

$$MTTF = \frac{1}{\lambda_s}$$

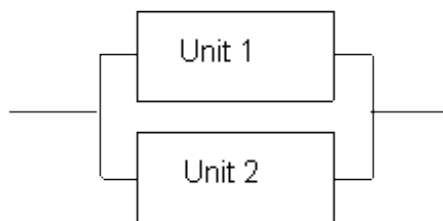
$$MTTF = \frac{1}{(\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n)}$$

Availability ( $A_s$ ) of the series system means that the probability that it is in the working state is also multiplied.

$$A_s = A_1 A_2 A_3 \dots A_n$$

**3.3.2. Parallel Network:** In this case all  $n$  units operate simultaneously and at least one such unit must work normally for the system success. The  $n$  unit parallel system block diagram is shown in figure.

**3.3.2.1. Reliability of parallel system:** A system is required to run for a given time  $t$  and its unit's exhibit mean failure rates  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ . As it is an active parallel system it survives if at least one unit survives. Hence reliability of parallel system is :



**Fig: 3.4: Parallel network**

$$R_p = 1 - \prod_{i=1}^n f_i$$

Where

F is the failure probability of unit 1: for  $i= 1,2,3,\dots,\dots,\dots,n$

$R_p$  is the reliability of parallel system

Hence

$$R_p = 1 - \{(1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})\dots\dots\dots (1 - e^{-\lambda_n t})\}$$

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{(\lambda_1 + \lambda_2)}$$

(In case system comprises of only two units)

**3.3. Availability:** Reliability is a measure of successful system operation over a period of time or during a mission. During the mission time, no failure is allowed. Availability is a measure that allows for a system to be repaired when failures occur. Availability is defined as the probability that the system is in normal operation. Availability (A) is a measure of successful operation for repairable systems. Mathematically,

$$A = \frac{\text{system up time}}{\text{system up time} + \text{system down time}}$$

because the system is “up” between failures,

$$A = \frac{MTTF}{MTTF + MTTR}$$

where MTTR stands for mean time to repair

Mean time to failure (MTTF), and mean time between failures (MTBF):

It is important to distinguish between the concepts Mean Time To Failure (MTTF) and Mean Time Between Failures (MTBF). The MTTF is the expected time to failure of a component or system. That is, the mean of the time to failure (TTF) for that component or system. The MTBF is the expected time to failure after a failure and repair of the component or system. With the MTBF, it is easily seen that some assumptions are necessary as to the state of the component after its repair. The terms, Time Between Failures and Mean Time Between Failures are usually reserved for the study of repairable systems.

Like MTTF and MTTR, MTBF is an expected value of the random variable time between failures. Mathematically,



$$MTBF = MTTF + MTTR$$

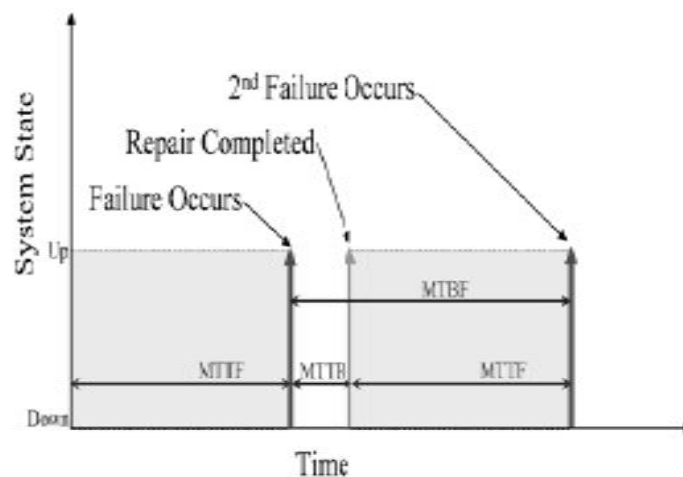
If MTTR can be reduced, availability will increase. A system in which failures are rapidly diagnosed and recovered is more desirable than a system that has a lower failure rate but the failures take a longer time to be detected, isolated, and recovered. Figure B.3 shows pictorially the relationship between MTBF, MTTR, and MTTF. From the figure it is easy to see that MTBF is the sum of MTTF and MTTR.

Mean Time to Failure= 1/ failure time(hours)

$$MTTF = \frac{1}{\lambda}$$

Mean Time to Failure= 1/ repair time(hours)

$$MTTR = \frac{1}{\mu}$$



*Fig 3.5: Curve between Time and System State*

**3.4. Reliability Improvement:** Many failures can be attributed to improper design and over stressing of the components. As such it would be justifiable to use some of the more powerful techniques of analysis only after taking suitable precautions about the simple but often effective matters.

As a first approach to improving reliability, we can use superior components and parts with lower failures rates. However we would immediately realize components of high reliability will require more time and money to development. They may also be larger in size and weight. Generally, the objective is not merely to produce a system with the highest reliability but to evolve a system which reflects an optimum total cost. The major items contributing to the total cost are researched and development, production, spares and maintenance.

In order to design and develop a highly reliable component or unit we would undoubtedly require a correspondingly high investment in research and development activities and this will be reflected in considerable measure in total cost. Similarly the production facilities must be sufficiently sophisticated to enable manufacture of precision components, which result that the production cost would also increase with the requirements of greater reliability factor. In figure the cost curve will have a point of minima corresponding to the optimum cost.

Redundancy may also be introduced to increase system reliability. Redundancy is the availability provision of more than one items (means) to perform a given task.

Redundancy involves the use of two or more identical components or assemblies. When one component or assembly fails others are taken to online to perform the task. The redundant component or assembly is used only when the primary fails. This is called cold stand by redundancy

Classification according to the structure.

- Active Redundancy.
- Passive Redundancy
- K out of n Redundancy

Classification according to level.

- Low Level Redundancy
- High Level Redundancy

Maintenance policy also affects redundancy and availability. Maintenance can be divided into following two types:

- Preventive Maintenance(P.M)
- Corrective Maintenance (C.M)

In preventive maintenance inspection is performed at schedule times on the basis of operating age and state of degradation. The actions are performed before a failure occurs or a change is noticed in performance. P.M mainly includes minor adjustment and actions such as inspection, lubrication, replacement of age-dependent parts etc .It is necessary aspect of system availability.

In corrective maintenance which is called repair also, the action is taken to restore the system to operating conditions by repairing the failed components of the system. Corrective maintenance is also an important measure of effectiveness the correctness of maintenance policy depends on the estimation of operating time, down time and maintenance cost.

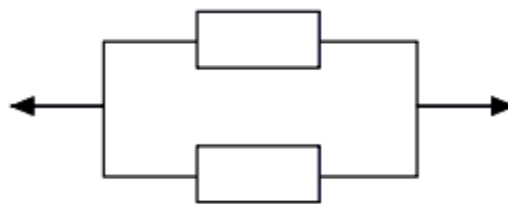
## Chapter-4

### Availability Evaluation Technique

*Availability evaluation is most essential thing to know the life of the system. There are various method of availability evaluation such as reliability block diagram method, fault tree analysis method, monte carlo simulation , markov state space model etc. In all, these model in this thesis markov state space model is used to evaluate the availability. This is the probabilistic model in which transition of the system from full available state to failed state is shown. With the help of transition model differential equations are generated which can be solved with the help of different techniques.*

There are many techniques in reliability analysis. The most widely used techniques in computing systems are reliability block diagrams, network diagrams, fault tree analysis and Monte Carlo simulation, which will be introduced in the following sections.

**4.1. Reliability block diagram:** A reliability block diagram is one of the standard and most common method of reliability and availability analysis. The main advantage of reliability block diagram is that it is very simple method of reliability evaluation of complex system. A reliability block diagram shows the system reliability structure. This block diagram is constructed with the help of different blocks and these blocks represent the working and failure state of the system. There are two types of reliability block diagram – series configuration and parallel configuration. For many complex system hybrid type of system can also be used which is the combination of series and parallel system.



*Fig 4.1: RBD of two components connected in parallel way*

These block diagrams are generally used to represent the communication networks which consist of individual links. Main goal of this reliability network is to analyse the system reliability. The main goal of the network is to implement the programs by connecting different sites that contain processing elements and resources. For simple network diagrams, analysis is not difficult and reliability block diagrams can be solved by different approaches such as

markov approach, montecarlo approach etc. Figure shown below represents the network diagrams that are connected through series or parallel links.

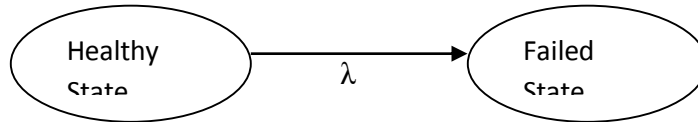


*Fig 4.2: Network Diagram*

Selection of reliability block diagram depends upon the convenience and ease of the usage. In reliability block diagram different modules are used and each module represents the different state of the system, each transition state can be easily represented by these modules. This network diagram is used in complex system where different nodes are connected with the help of the links.

**3.2. Markov Model:** Markov model is another widely used technique in reliability evaluation. It reduce most disadvantages of other techniques such as Montecarlo, fault tree method as it is more adaptable to be execute in reliability evaluation in different enumerating systems. To evaluate the dependability metrics of a system, it is necessary to study the behaviour of the system. This behaviour can be studied during the working of the system, by generating its behaviour model. Alternatively, one model of the system is also constructed at the time of designing to forecast its behaviour and its transition state from working to failure state. With the help of this transition model complete behaviour of the system can be predicted. The generated model is implemented in an habitat where it will be subject to in conceptual effects (for example, the availability of services the system depends on might display random behaviour). The transition model represents the stochastic probabilistic behaviour of the system and different-different mathematical parameters are used to solve the model and with the help of these parameters availability can be evaluated. In any case Markov model is prove useful as it can solve different probabilistic equations by using mathematical algorithm that allows using applicable figure or random distributions of that figure. A Markov approach is a mathematical transition model that is useful in the study of the availability/reliability of any complex/hybrid systems. In any given system which is solved by markov approach, different transition states that are working or failed are given and their relationship, transition path and directions are also given which are helpful for availability evaluation. In this analysis two

factors are important and that are failure and repair of the system. When markov model is constructed each state is represent buy a block of some particular geometric symbols and their transitions are represents by the line having arrows as shown in the figure below for a single component that has just two states: healthy and failed.



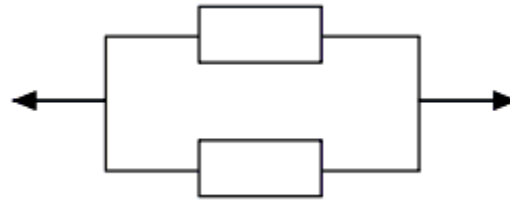
*Fig 4.3: Markov Transition State*

The symbol 1 represents that this is the transition state after failure from full working condition whereas state 0 represents the state of full working condition.  $P_j(t)$  represents the probability of the transition state 2 of the system at time t. If the system is working at any initial time  $t = 0$ , then the probabilities of the two states are  $P_0(0) = 1$  and  $P_1(0) = 0$  where 1 represents failed state and 0 represents working state. Probability of transition from State 0 to state 1 at constant rate is represent at particular increment of time t is  $\lambda dt$ . For a stochastic model Markov property is the probability of the future behaviour of the system and its process when the current state is known exactly and this state is depend upon its just previous state and totally independent of past history.

**3.2.1. A Simple Markov Model for a Two-Unit System:** By using the effect of failure and repair rate in two unit system Markov Model can be To describe the various techniques for including the effect of repairs, for explaining the markov model simple 2-unit system is considered which is shown in the figure below. This system will perform its intended function if either Unit 1 or Unit 2, or both are operational.

This system can be in one of four possible states,

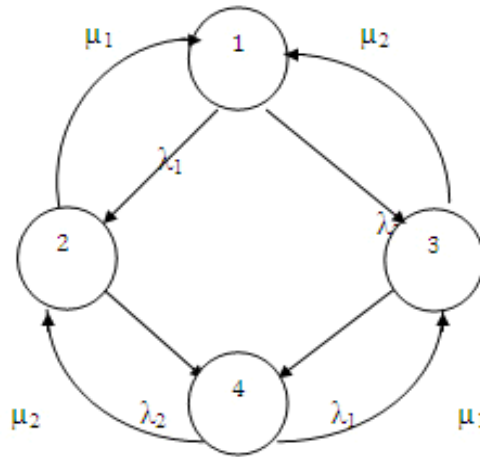
- Both units are working
- Unit 1 failed but Unit 2 is working
- Unit 2 failed but Unit 1 is working
- Both units failed



*Fig 4.4: Two*

*Parallel*

*Component Model in*



*Fig 4.5: Transition Diagram of two component system*

Equation of the following transient state model is:

$$- (\lambda_1 + \lambda_2)P_1(t) + \mu_1 P_2(t) + \mu_2 P_3(t)$$

$$- (\mu_1 + \lambda_2)P_2(t) + \lambda_1 P_4(t) + \mu_2 P_4(t)$$

$$- (\lambda_1 + \mu_2)P_3(t) + \lambda_2 P_1(t) + \mu_1 P_4(t)$$

$$- (\mu_1 + \mu_2)P_4(t) + \lambda_1 P_3(t) + \lambda_2 P_2(t)$$

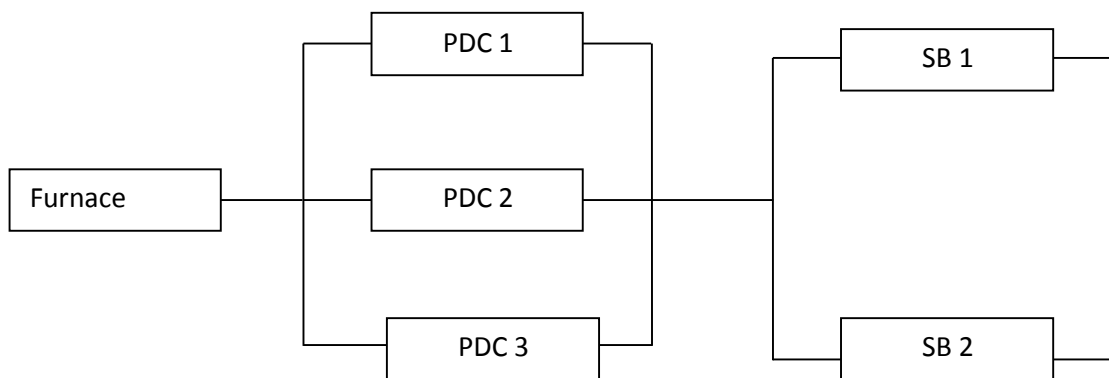
These equations can be solved by different approaches. In this thesis Runge- Kutte approach is used for solving these equations.

## Chapter-5

### System Description

*In this thesis a system of brake drum manufacturing system is taken for availability evaluation. There are many steps in brake drum manufacturing unit. Main layout is focussed for evaluation in which one furnace, three pressure die casting and two shot blasting machines are connected in hybrid manner. Furnace, PDC and SB are connected in series. With the help of this layout state space diagram is constructed for availability evaluation.*

In this thesis brake drum manufacturing system is taken in which three components are present. The process flow diagram presenting the process of the brake drum preparation multiple component system is shown in Figure.



*Fig. 5.1: Layout of the system*

One furnace having capacity of 250 kg is used to melt the raw material of brake drum. Three pressure die casting machine and two shot blasting machines are used in this hybrid system. Furnace, pressure die casting and furnace are connected in series manner whereas all three PDC's are connected in parallel way and two shot blasting machines are also connected in parallel manner.

**5.1. Pressure Die Casting:** Pressure die casting for aluminium metal and its alloy are used for very fast manufacturing of industrial and other related components having simple or complex structure. This technique has many advantages when a component is required in mass. However, for aeronautical industry, military, automotive applications and in every industry good mechanical properties and reliability are of first importance. It is, therefore, essential that



the best characteristics of design should be used and optimum casting technique with minimum cost should be adopted. Pressure die cast products are used in the form of various electrical, electronic, mechanical instruments and components used in domestic and technical field. Aluminium and its alloy are used for brake drum manufacturing Because of its high melting point, high strength and toughness. Silicon is used to increase the strength of the component in die casting machine. Molten metal for a single shot is filled into a cylindrical chamber through a pouring apparatus, a reciprocating piston is then used to forces the metal into the die chamber, the complete procedure is being completed in a few seconds so that iron contamination can be virtually eliminated. In this technique injection pressure of molten metal is in the range of 70-140MP which is feasible to enable lower metal to be employed and more sophisticacy can be achieved. The castings are less liable to entrapped air and a higher standard of soundness ensues from the smaller amount of liquid and solidification shrinkage occurring within the die. Total power requirement for brake drum manufacturing is 40 H.P.



*Fig 5.2: Pressure Die Casting Machine*

These pressure die casting machines are in active redundancy. In active redundancy mode more than one machine is connected with other in parallel way to reduce the work load of the system. These three machines work simultaneously to reduce the work load of single machine. By doing this efficiency of the system increases and failure rate decreases. Al and its alloys are used with pressure range 1500 psi to 4500 psi

**5.2. Shot blasting:** Shot blasting is used for removing the gate and riser from the casting product. It is used for cleaning action. Shot blasting systems use a high performance, airless and centrifugal wheel for propelling blast media at a high velocity, in a controlled pattern and direction. Shot blasting is method which is used to achieve good cleaning action and surface

preparation for finishing operations. In general shot blasting concentrated abrasive particles are used at high speed (upto 65-110 m/second) in a controlled manner at the material body for removing surface contaminates due to the abrasive impact on the product surface.

Shot blasting is commonly used for:

- The cleaning of iron, steel, non-cast parts, forgings and casting products.
- Mechanical cleaning of sheets, rods, coils, wire, etc.
- Shot peening to alter mechanical properties (increasing resistance to fatigue for springs, gears) etc

The criteria used for selecting the type of shot blasting system depends on the size and shape of the parts, the condition of the surface to be cleaned, final surface finish specification and overall process required.

#### **5.2.1. Shot blasting systems are comprised of 6 basic subsystems:**

- Abrasive delivery method  
By Compressed Air  
By Centrifugal turbines
- Abrasive recovery and cleaning
- Dust collection
- Blast Cabinet
- Part movement and support system
- Controls and instrumentation

These two shot blasting are also connected in standby redundancy form to increase the efficiency of the whole system and reduce failure rate. This layout is used for evaluating availability of the system of brake drum manufacturing system.

**5.3. Brake drum manufacturing process:** In brake drum manufacturing system three subsystems are mainly important which are a cupola furnace for melting the aluminium and its alloy, pressure die casting machine for casting the brake drum and last is shot blasting machine for cleaning process. First metal is being melted in the cupola furnace after that molten metal is poured in the mould cavity of pressure die casting for casting process. After the drum is cast and cooled it is moved for turning and machining processes. These turning and machining process are done in automatic or semi-automatic cnc machine.



*Fig 5.3: Brake Drum*

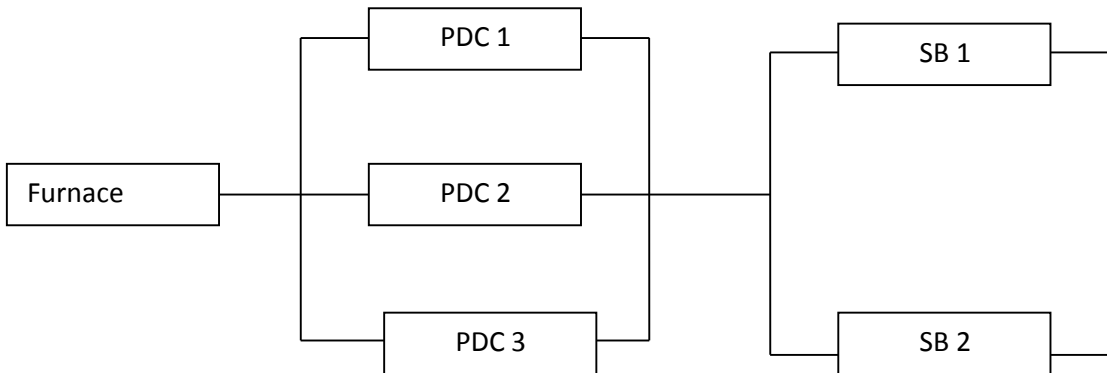
After that these brake drums are moved for the cleaning line. Cleaning process is done in shot blasting machine in which high velocity abrasives are fired on the drum through nozzle. After cleaning tensile test is done to check the strength of the brake drum. Thousand of brake drum are manufactured in a day. So after a certain number of manufacturing of brake drum, failure of the machine is started. So repair process has to be done to achieve the performance of the system. By considering this failure and repair rate availability of the system is evaluated by using markov state-space model.

## Chapter- 6

### Experimental Procedure and System Modelling

*In this chapter complete experimental procedure of reliability, availability and MTTF evaluation is presented. Availability evaluation methods are discussed in previous chapters. Reliability is evaluated with the help of reliability block diagram method and MTTF is calculated with the help of confidence interval. All differential equations of markov model is solved with the help of matlab and sensitivity analysis is done to show the effect of failure and repair rate on availability.*

In this system a markov model is constructed in which standby redundancy is used to operate the system properly. In standby redundancy more than one component is connected in parallel manner. In case of failure of one component in parallel configuration second component start working. System will fail if all components fail in particular configuration. Layout of the system is as under:



*Fig 6.1: Layout of the System*

In this system one furnace, 3 PDC and 2 Shot Blasting machine are connected in series manner whereas all three PDC are in parallel manner and two S.B. are also in parallel way.

**6.1. Reliability of the system:** This is a hybrid type of structure in which series and parallel both type of configuration are present. Reliability of the system can be evaluated by following formula. After analysis it had been seen that in furnace failure occur after 500 hours, in PDC 1 failure occur after 2166.6 hours (after 60000 components), in PDC 2 failure occur after 2666.6 hours (after 80000 components), in PDC 3 it occur after 3333.3 hours (after 1 lakh component), in SB 1 it occur after 500 hours and in SB 2 failure occur after 600 hrs.

$\lambda = 1/\text{failure time(hrs)}$

Taking  $\lambda_1 = .002$ ,  $\lambda_2 = .00461$ ,  $\lambda_3 = .00375$ ,  $\lambda_4 = .0003$ ,  $\lambda_5 = .002$ ,  $\lambda_6 = .0016$

$\mu_1 = .01$ ,  $\mu_2 = .0461$ ,  $\mu_3 = .0375$ ,  $\mu_4 = .003$ ,  $\mu_5 = .02$ ,  $\mu_6 = .016$

Time taken: 200 hours

In this layout Furnace, Pressure Die Casting and Shot Blasting machines are in series. So reliability of the system is:

$$R_s = R_1 * R_2 * R_3$$

$$R_1 = e^{-\lambda_1 t}$$

$$R_1 = e^{-.00461 * 200} = 0.67$$

$$R_2 = [1 - \{1 - e^{-\lambda_2 t}\} * \{1 - e^{-\lambda_3 t}\} * \{1 - e^{-\lambda_4 t}\}]$$

$$R_2 = 0.98$$

$$R_3 = [1 - \{1 - e^{-\lambda_5 t}\} * \{1 - e^{-\lambda_6 t}\}]$$

$$R_3 = 0.95$$

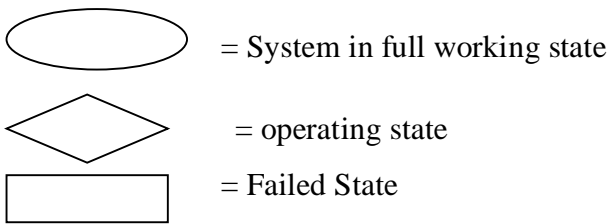
$$R_s = 0.67 * 0.98 * 0.95 = 0.6237$$

So reliability of the system without considering repair rate is 62.37 %

**6.2. Assumptions:** There are the following assumptions.

- The states of all subsystems are mutually independent and the failure and repair rates are constant over time and follows exponential distribution.
- There are sufficient repair or replacement facilities available.
- There is only one failure at a time.
- When one subsystem fails, it is instantaneously replaced by one of the standby Subsystems and the switchover devices are perfect and the repaired sub-system behave as new sub-system.
- Failure and repair rates are statistically independent.
- Repair is carried on the basis of importance of the sub-system.

### 6.3. Notation:



A= Furnace in working State

B= pressure die casting 1 in working state

B<sub>1</sub>= Standby PDC 2 in working state

B<sub>2</sub>= Standby PDC 3 in working sate

C= Shot blasting 1 in working state

C<sub>1</sub>= Shot Blasting 2 in working state

a,b,c = Furnace, PDC, SB in failure state

$\lambda$ = failure rate

$\mu$ = repair rate

$\lambda_i$ ,  $i=1, 2, 3, 4, 5, 6$  represents the failure rate of subsystem A, B, B<sub>1</sub>, B<sub>2</sub>, C, C<sub>1</sub>

$\mu_i$ ,  $i=1, 2, 3, 4, 5, 6$  represents the repair rate of subsystem A, B, B<sub>1</sub>, B<sub>2</sub>, C, C<sub>1</sub>

The Markov model of this layout is constructed as under and its working and failure state is shown in following table. This table represents the working and failed state of the system.

**Table 6.1: Working and Failure State of the system**

	1	2	3	4	5	6	System State
1	O	O	S.B.	S.B.	O	S.B	WORKING
2	F	O	S.B.	S.B.	O	S.B	FAILED
3	O	F	O	S.B.	O	S.B	WORKING
4	O	O	S.B.	S.B.	F	O	WORKING

5	F	F	O	S.B.	O	S.B	FAILED
6	O	F	F	O	O	S.B	WORKING
7	O	F	O	S.B.	F	O	WORKING
8	F	O	S.B.	S.B.	F	O	FAILED
9	O	O	S.B.	S.B.	F	F	FAILED
10	F	F	F	O	O	S.B	FAILED
11	O	F	F	F	O	S.B	FAILED
12	O	F	F	O	F	O	WORKING
13	F	F	O	S.B.	F	O	FAILED
14	O	F	O	S.B.	F	F	FAILED
15	F	F	F	O	F	O	FAILED
16	O	F	F	F	F	O	FAILED
17	O	F	F	O	F	F	FAILED

Probabilistic model of this system is developed with the help of above table in which failure and repair of all subsystems are represented. Mathematical equations are generated with the help of all these states of the subsystem. Each mathematical equation represents the transition of subsystem from full working state to failed state. Failure and repair rate are taken as constant. Every transition state depends upon it's just previous state and it is independent of past history. These equations can be solved by different mathematical approaches. In this project Runge-Kutta method is used to solve this equation by using Matlab.

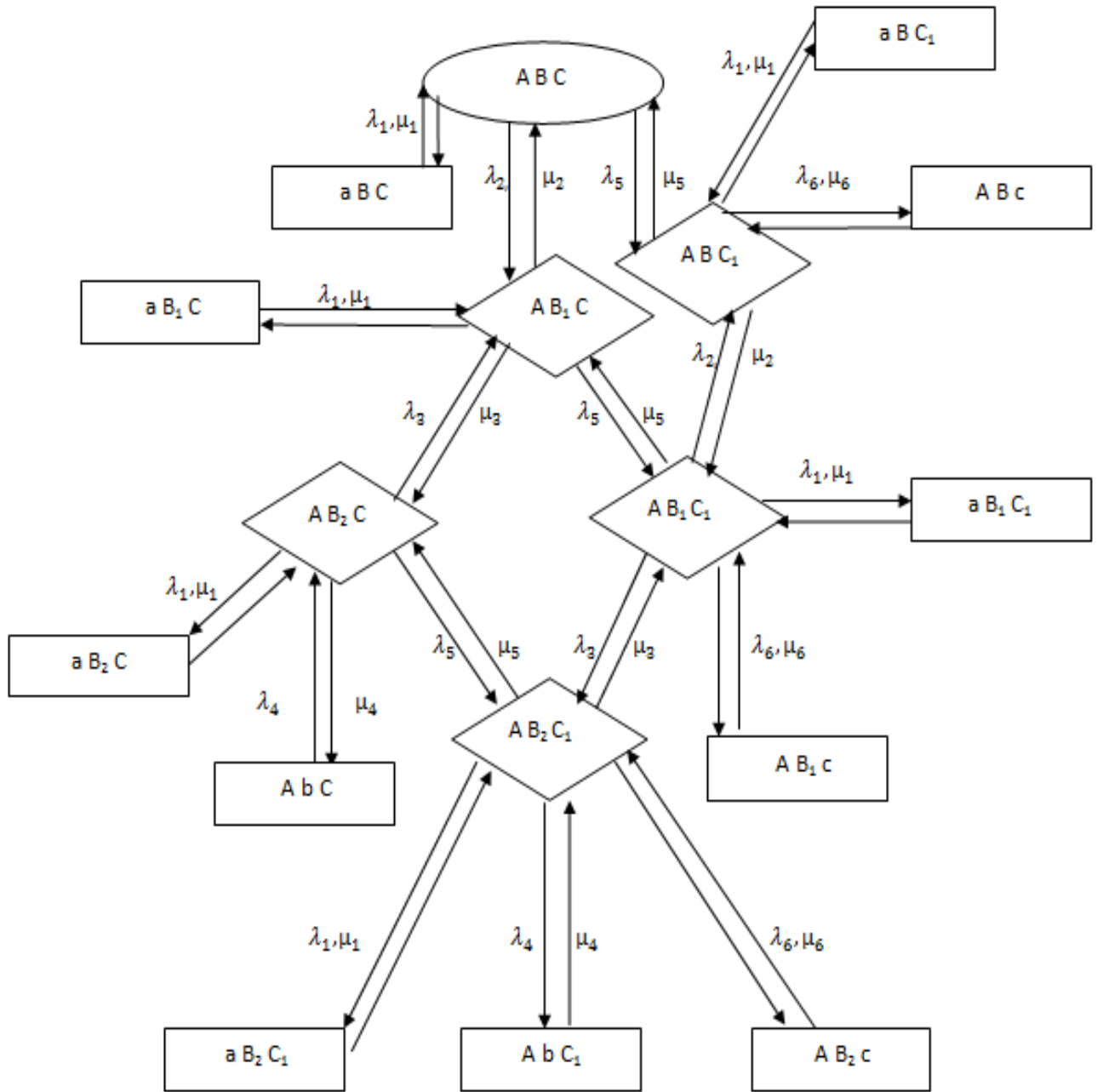


Fig 6.2: Transition State of the system



## 6.4. Mathematical Formulation of the model:

Model can be formulised in the form of differential equation as under:

$$\frac{dp_1}{dt} = -(\lambda_1 + \lambda_2 + \lambda_5)P_1(t) + \mu_1P_2(t) + \mu_2P_3(t) + \mu_5P_4(t)$$

$$\frac{dp_2}{dt} = -\mu_1P_2(t) + \lambda_1P_1(t)$$

$$\frac{dp_3}{dt} = -(\lambda_1 + \lambda_3 + \lambda_5 + \mu_2)P_3(t) + \mu_1P_5(t) + \mu_3P_6(t) + \mu_5P_7(t) + \lambda_2P_1(t)$$

$$\frac{dp_4}{dt} = -(\lambda_1 + \lambda_2 + \lambda_6 + \mu_5)P_4(t) + \mu_1P_8(t) + \mu_2P_7(t) + \mu_6P_9(t) + \lambda_5P_1(t)$$

$$\frac{dp_5}{dt} = -\mu_1P_5(t) + \lambda_1P_3(t)$$

$$\frac{dp_6}{dt} = -(\lambda_1 + \lambda_4 + \lambda_5 + \mu_3)P_6(t) + \mu_1P_{10}(t) + \mu_4P_{11}(t) + \mu_5P_{12}(t) + \lambda_3P_3(t)$$

$$\begin{aligned} \frac{dp_7}{dt} = & -(\lambda_1 + \lambda_3 + \lambda_6 + \mu_2 + \mu_5)P_7(t) + \mu_1P_{13}(t) + \mu_3P_{12}(t) + \mu_6P_{14}(t) + \lambda_2P_4(t) \\ & + \lambda_5P_3(t) \end{aligned}$$

$$\frac{dp_8}{dt} = -\mu_1P_8(t) + \lambda_1P_4(t)$$

$$\frac{dp_9}{dt} = -\mu_6P_9(t) + \lambda_6P_4(t)$$

$$\frac{dp_{10}}{dt} = -\mu_1P_{10}(t) + \lambda_1P_6(t)$$

$$\frac{dp_{11}}{dt} = -\mu_4P_{11}(t) + \lambda_4P_6(t)$$

$$\begin{aligned} \frac{dp_{12}}{dt} = & -(\lambda_1 + \lambda_4 + \lambda_6 + \mu_3 + \mu_5)P_{12}(t) + \mu_1P_{15}(t) + \mu_2P_{16}(t) + \mu_5P_{17}(t) + \lambda_3P_7(t) \\ & + \lambda_5P_6(t) \end{aligned}$$

$$\frac{dp_{13}}{dt} = -\mu_1P_{13}(t) + \lambda_1P_7(t)$$

$$\frac{dp_{14}}{dt} = -\mu_6 P_{14}(t) + \lambda_6 P_7(t)$$

$$\frac{dp_{15}}{dt} = -\mu_1 P_{15}(t) + \lambda_1 P_{12}(t)$$

$$\frac{dp_{16}}{dt} = -\mu_4 P_{16}(t) + \lambda_4 P_{12}(t)$$

$$\frac{dp_{17}}{dt} = -\mu_6 P_{17}(t) + \lambda_6 P_{12}(t)$$

These equations are solved by Runge- Kutta method. In all above equation all states are not working states. Some states are failed state due to the failure of furnace, PDC, shot blasting.

**6.5. Availability of the system:** Availability can be evaluated with the help of all available state. Failed state does not give any contribution in system availability.

$$Av = P_1 + P_2 + P_3 + P_4 + P_7 + P_{12}$$

$$Av = 84.23\%$$

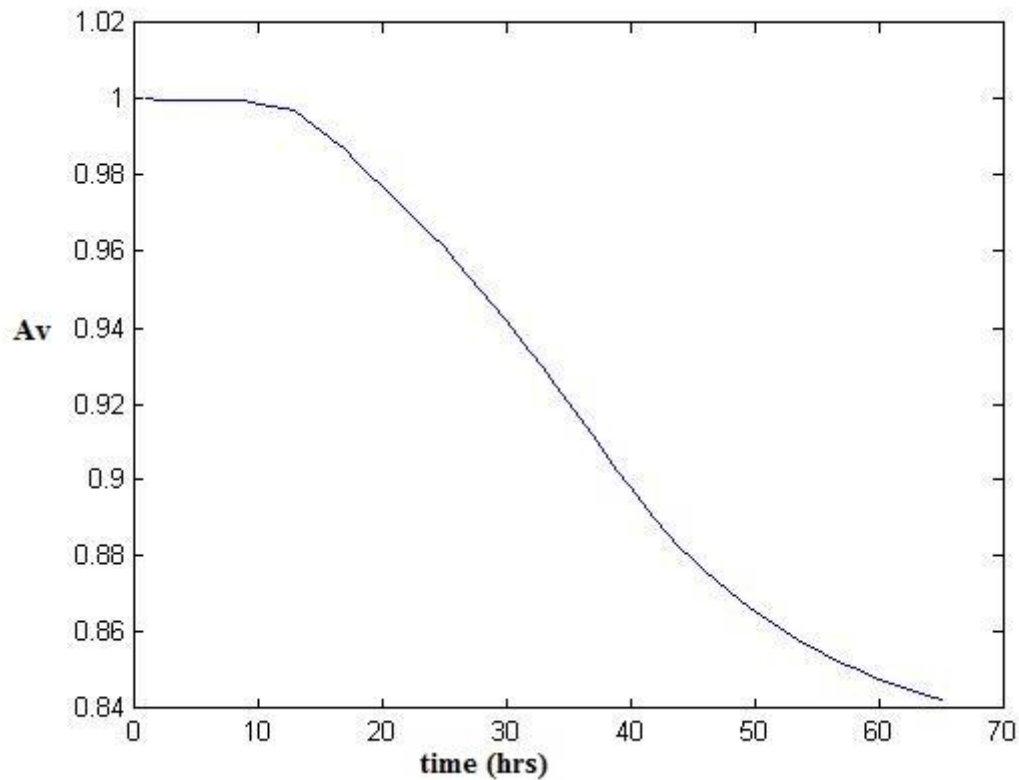


Fig 6.3. Graph Between Availability and Time

**6.6. Sensitivity Analysis:** Sensitivity analysis is done by varying the value of failure and repair rate. At different value of failure and repair rate availability value changes time at particular time interval. Sensitivity analysis is done as under:

Taking  $\lambda_1= 0.002, \lambda_2 = 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$   
 $\mu_2= 0.0461, \mu_3= 0.0375, \mu_4= 0.003, \mu_5= 0.02, \mu_6= 0.016$  all these data constant and changing the value of  $\mu_1$  :

*Table 6.2: Variation of availability with change of repair rate of furnace*

$\mu_1$	0.01	0.02	0.04	0.08
Availability	0.8423	0.9034	0.9450	0.9679

Taking  $\lambda_1= 0.002, \lambda_2 = 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$   
 $\mu_1= 0.01, \mu_3= 0.0375, \mu_4= 0.003, \mu_5= 0.02, \mu_6= 0.016$  all these data constant and changing the value of  $\mu_2$  :

*Table6. 3: Variation of availability with change of repair rate of PDC1*

$\mu_2$	0.0461	0.0922	0.1844	0.3688
Availability	0.8423	0.8424	0.8425	0.8426

Taking  $\lambda_1= 0.002, \lambda_2 = 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$   
 $\mu_1= 0.01, \mu_2= 0.0461, \mu_4= 0.003, \mu_5= 0.02, \mu_6= 0.016$  all these data constant and changing the value of  $\mu_3$ :

*Table 6.4: Variation of availability with change of repair rate of PDC2*

$\mu_3$	0.0375	0.075	0.15	0.30
Availability	0.8423	0.8424	0.8425	0.8425

Taking  $\lambda_1= 0.002, \lambda_2= 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$

$\mu_1= 0.01, \mu_2= 0.0461, \mu_3= 0.0375, \mu_5= 0.02, \mu_6= 0.016$  all these data constant and changing the value of  $\mu_4$ :

**Table 6.5: Variation of availability with change of repair rate of PDC3**

$\mu_4$	0.003	0.006	0.012	0.024
0.003	0.8423	0.8423	0.8424	0.8425

Taking  $\lambda_1= 0.002, \lambda_2= 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$

$\mu_1= 0.01, \mu_2= 0.0461, \mu_3= 0.0375, \mu_4= 0.003, \mu_6= 0.016$  all these data constant and changing the value of  $\mu_5$ :

**Table 6.6: Variation of availability with change of repair rate of SB1**

$\mu_5$	0.02	0.04	0.08	0.16
0.02	0.8423	0.84.49	0.84.65	0.84.73

Taking  $\lambda_1= 0.002, \lambda_2= 0.00461, \lambda_3= 0.00375, \lambda_4= 0.0003, \lambda_5= 0.002, \lambda_6= 0.0016$

$\mu_1= 0.01, \mu_2= 0.0461, \mu_3= 0.0375, \mu_4= 0.003, \mu_5= 0.02$  all these data constant and changing the value of  $\mu_6$ :

**Table 6.7: Variation of availability with change of repair rate of SB2**

$\mu_6$	0.016	0.032	0.064	0.128
0.016	0.8423	0.84..50	0.84.66	0.84.74

**6.7. Availability Analysis by Using Active Redundancy:** In active redundancy system load of the system is reduced by joining multiple components in parallel manner. Loads are distributed among there parallel components. Three PDC and two SB machines are connected to reduce the load of the system and improve availability of the system. Active redundancy transition diagram can be seen as under:

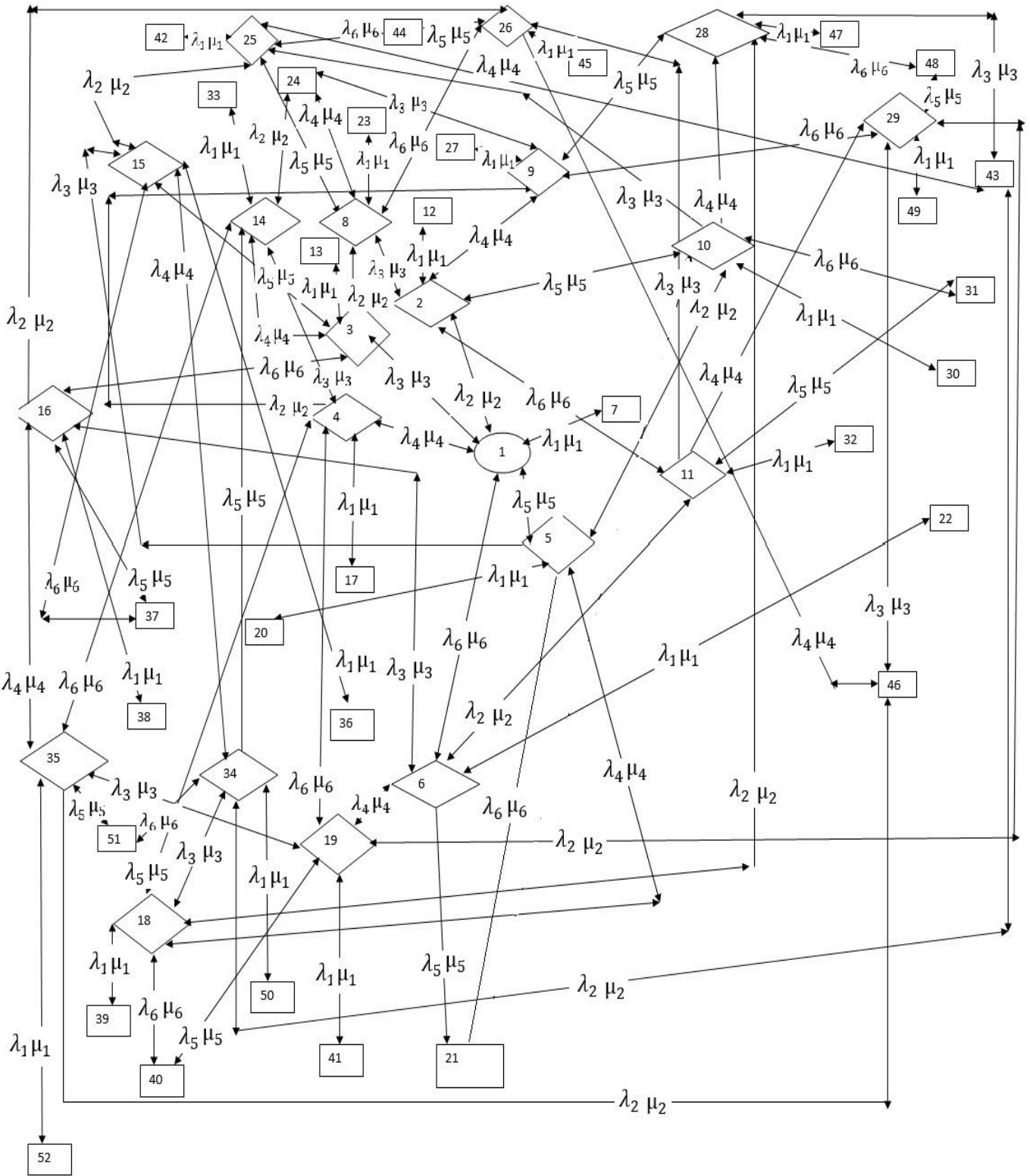

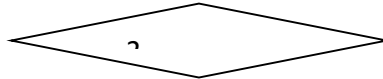
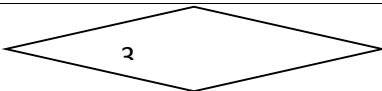
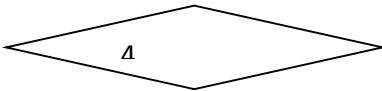
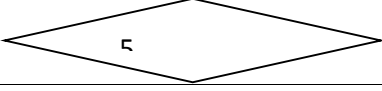

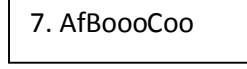
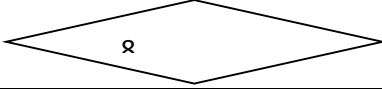


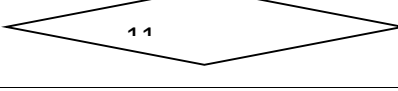
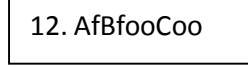
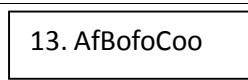
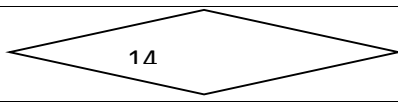

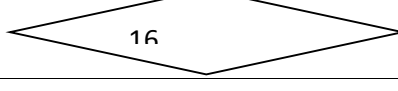
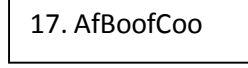
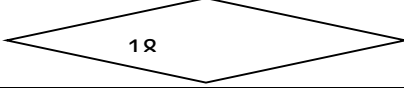
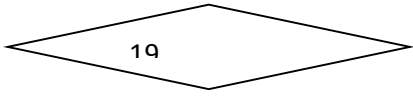
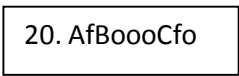
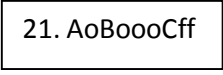

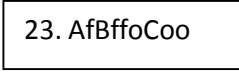
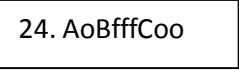
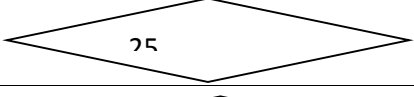
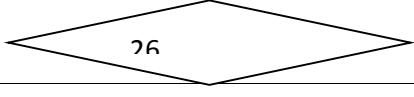

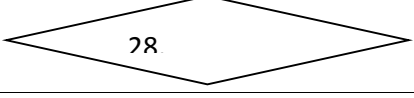
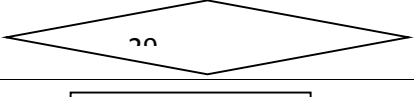
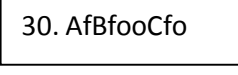
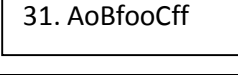
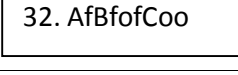
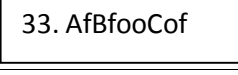
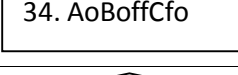
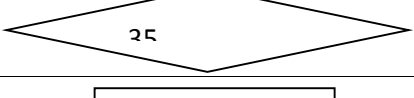
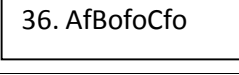
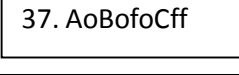


Fig 6.4: Transition Diagram in Active Redundancy

**Table 6.8: Working and Failure State of the System**

	1	2	3	4	5	6	System State Notation	System
1	O	O	O	O	O	O		WORKING
2	O	F	O	O	O	O		FAIL
3	O	O	F	O	O	O		WORKING
4	O	O	O	F	O	O		WORKING
5	O	O	O	O	F	O		WORKING
6	O	O	O	O	O	F		WORKING
7	F	O	O	O	O	O		FAIL
8	O	F	F	O	O	O		WORKING
9	O	F	O	F	O	O		WORKING
10	O	F	O	O	F	O		WORKING
11	O	F	O	O	O	F		WORKING
12	F	F	O	O	O	O		FAIL
13	F	O	F	O	O	O		FAIL
14	O	O	F	F	O	O		WORKING
15	O	O	F	O	F	O		WORKING
16	O	O	F	O	O	F		WORKING
17	F	O	O	F	O	O		FAIL
18	O	O	O	F	F	O		WORKING

19	O	O	O	F	O	F		WORKING
20	F	O	O	O	F	O		FAIL
21	O	O	O	O	F	F		FAIL
22	F	O	O	O	O	F		FAIL
23	F	F	F	O	O	O		FAIL
24	O	F	F	F	O	O		FAIL
25	O	F	F	O	F	O		WORKING
26	O	F	F	O	O	F		WORKING
27	F	F	O	F	O	O		FAIL
28	O	F	O	F	F	O		WORKING
29	O	F	O	F	O	F		WORKING
30	F	F	O	O	F	O		FAIL
31	O	F	O	O	F	F		FAIL
32	F	F	O	O	O	F		FAIL
33	F	O	F	F	O	O		FAIL
34	O	O	F	F	F	O		FAIL
35	O	O	F	F	O	F		WORKING
36	F	O	F	O	F	O		FAIL
37	O	O	F	O	F	F		FAIL

38	F	O	F	O	O	F	38. AfBofoCof	FAIL
39	F	O	O	F	F	O	39. AfBoofCfo	FAIL
40	O	O	O	F	F	F	40. AoBoofCff	FAIL
41	F	O	O	F	O	F	41. AfBoofCof	FAIL
42	F	F	F	O	F	O	42. AfBffoCfo	FAIL
43	O	F	F	F	F	O	43. AoBfffCfo	FAIL
44	O	F	F	O	F	F	44. AoBffoCff	FAIL
45	F	F	F	O	O	F	45. AfBffoCof	FAIL
46	O	F	F	F	O	F	46. AoBfffCof	FAIL
47	F	F	O	F	F	O	47. AfBfofCfo	FAIL
48	O	F	O	F	F	F	48. AoBfofCff	FAIL
49	F	F	O	F	O	F	49. AfBfofCof	FAIL
50	F	O	F	F	F	O	50. AfBoffCfo	FAIL
51	O	O	F	F	F	F	51. AoBoffCff	FAIL
52	F	O	F	F	O	F	52. AfBoffCof	FAIL

### 6.7.1. Mathematical Equation of the Transition diagram:

These are the mathematical differential equation showing transformation state of the system.

$$\frac{dp_1}{dt} = -(l_1 + l_2 + l_3 + l_4 + l_5 + l_6)P_1(t) + mu_1P_7(t) + mu_2P_2(t) + mu_3P_3(t) + mu_4P_4(t) + mu_5P_5(t) + mu_6P_6(t)$$



$$\begin{aligned}\frac{dp_2}{dt} = & -(l_1 + mu_2 + l_3 + l_4 + l_5 + l_6)P_2(t) + mu_1P_{12}(t) + l_2P_1(t) + mu_3P_8(t) \\ & + mu_4P_9(t) + mu_5P_{10}(t) + mu_6P_{11}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_3}{dt} = & -(l_1 + l_2 + mu_3 + l_4 + l_5 + l_6)P_3(t) + mu_1P_{13}(t) + mu_2P_8(t) + l_3P_1(t) \\ & + mu_4P_{14}(t) + mu_5P_{15}(t) + mu_6P_{16}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_4}{dt} = & -(l_1 + l_2 + l_3 + mu_4 + l_5 + l_6)P_4(t) + mu_1P_{17}(t) + mu_2P_9(t) + mu_3P_{14}(t) \\ & + l_4P_1(t) + mu_5P_{18}(t) + mu_6P_{19}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_5}{dt} = & -(l_1 + l_2 + l_3 + l_4 + mu_5 + l_6)P_5(t) + mu_1P_{20}(t) + mu_2P_{10}(t) + mu_3P_{15}(t) \\ & + mu_4P_{18}(t) + l_5P_1(t) + mu_6P_{21}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_6}{dt} = & -(l_1 + l_2 + l_3 + l_4 + l_5 + mu_6)P_6(t) + mu_1P_{22}(t) + mu_2P_{11}(t) + mu_3P_{16}(t) \\ & + mu_4P_{19}(t) + mu_5P_{21}(t) + l_6P_1(t)\end{aligned}$$

$$\frac{dp_7}{dt} = -(mu_1)P_7(t) + l_1P_1(t)$$

$$\begin{aligned}\frac{dp_8}{dt} = & -(l_1 + mu_2 + mu_3 + l_4 + l_5 + l_6)P_8(t) + mu_1P_{23}(t) + l_2P_3(t) + l_3P_2(t) \\ & + mu_4P_{24}(t) + mu_5P_{25}(t) + mu_6P_{26}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_9}{dt} = & -(l_1 + mu_2 + l_3 + mu_4 + l_5 + l_6)P_9(t) + mu_1P_{27}(t) + l_2P_4(t) + mu_3P_{24}(t) \\ & + l_4P_2(t) + mu_5P_{28}(t) + mu_6P_{29}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_{10}}{dt} = & -(l_1 + mu_2 + l_3 + l_4 + mu_5 + l_6)P_{10}(t) + mu_1P_{30}(t) + l_2P_5(t) + mu_3P_{25}(t) \\ & + mu_4P_{28}(t) + l_5P_2(t) + mu_6P_{31}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_{11}}{dt} = & -(l_1 + mu_2 + l_3 + l_4 + l_5 + mu_6)P_{11}(t) + mu_1P_{32}(t) + l_2P_6(t) + mu_3P_{26}(t) \\ & + mu_4P_{29}(t) + mu_5P_{31}(t) + l_6P_1(t)\end{aligned}$$

$$\frac{dp_{12}}{dt} = -(mu_1)P_{12}(t) + l_1P_2(t)$$

$$\frac{dp_{13}}{dt} = -(mu_1)P_{13}(t) + l_1P_3(t)$$

$$\begin{aligned} \frac{dp_{14}}{dt} = & -(l_1 + l_2 + mu_3 + mu_4 + l_5 + l_6)P_{14}(t) + mu_1P_{33}(t) + mu_2P_{24}(t) + l_3P_4(t) \\ & + l_4P_3(t) + mu_5P_{34}(t) + mu_6P_{35}(t) \end{aligned}$$

$$\begin{aligned} \frac{dp_{15}}{dt} = & -(l_1 + l_2 + mu_3 + l_4 + mu_5 + l_6)P_{15}(t) + mu_1P_{36}(t) + mu_2P_{25}(t) + l_3P_5(t) \\ & + mu_4P_{34}(t) + l_5P_3(t) + mu_6P_{37}(t) \end{aligned}$$

$$\begin{aligned} \frac{dp_{16}}{dt} = & -(l_1 + l_2 + mu_3 + l_4 + l_5 + mu_6)P_{16}(t) + mu_1P_{38}(t) + mu_2P_{26}(t) + l_3P_6(t) \\ & + mu_4P_{35}(t) + mu_5P_{37}(t) + l_6P_3(t) \end{aligned}$$

$$\frac{dp_{17}}{dt} = -(mu_1)P_{17}(t) + l_1P_4(t)$$

$$\begin{aligned} \frac{dp_{18}}{dt} = & -(l_1 + l_2 + l_3 + mu_4 + mu_5 + l_6)P_{18}(t) + mu_1P_{39}(t) + mu_2P_{28}(t) \\ & + mu_3P_{34}(t) + l_4P_5(t) + l_5P_4(t) + mu_6P_{40}(t) \end{aligned}$$

$$\begin{aligned} \frac{dp_{19}}{dt} = & -(l_1 + l_2 + l_3 + mu_4 + l_5 + mu_6)P_{19}(t) + mu_1P_{41}(t) + mu_2P_{29}(t) + mu_3P_{35}(t) \\ & + l_4P_6(t) + mu_5P_{40}(t) + l_6P_4(t) \end{aligned}$$

$$\frac{dp_{20}}{dt} = -(mu_1)P_{20}(t) + l_1P_5(t)$$

$$\frac{dp_{21}}{dt} = -(mu_5 + mu_6)P_{21}(t) + l_5P_6(t) + l_6P_5(t)$$

$$\frac{dp_{22}}{dt} = -(mu_1)P_{22}(t) + l_1P_6(t)$$

$$\frac{dp_{23}}{dt} = -(mu_1)P_{23}(t) + l_1P_8(t)$$

$$\frac{dp_{24}}{dt} = -(mu_2 + mu_3 + mu_4)P_{24}(t) + l_2P_{14}(t) + l_3P_9(t) + l_4P_8(t)$$

$$\begin{aligned} \frac{dp_{25}}{dt} = & -(l_1 + mu_2 + mu_3 + l_4 + mu_5 + l_6)P_{25}(t) + mu_1P_{42}(t) + l_2P_{15}(t) + l_3P_{10}(t) \\ & + mu_4P_{43}(t) + l_5P_8(t) + mu_6P_{44}(t) \end{aligned}$$

$$\begin{aligned} \frac{dp_{26}}{dt} = & -(l_1 + mu_2 + mu_3 + l_4 + l_5 + mu_6)P_{26}(t) + mu_1P_{45}(t) + l_2P_{16}(t) + l_3P_{11}(t) \\ & + mu_4P_{46}(t) + mu_5P_{44}(t) + l_6P_8(t) \end{aligned}$$

$$\frac{dp_{27}}{dt} = -(mu_1)P_{27}(t) + l_1P_9(t)$$

$$\begin{aligned}\frac{dp_{28}}{dt} = & -(l_1 + mu_2 + l_3 + mu_4 + mu_5 + l_6)P_{28}(t) + mu_1P_{47}(t) + l_2P_{18}(t) \\ & + mu_3P_{43}(t) + l_4P_{10}(t) + l_5P_9(t) + mu_6P_{48}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_{29}}{dt} = & -(l_1 + mu_2 + l_3 + mu_4 + l_5 + mu_6)P_{29}(t) + mu_1P_{49}(t) + l_2P_{19}(t) \\ & + mu_3P_{46}(t) + l_4P_{11}(t) + mu_5P_{48}(t) + l_6P_9(t)\end{aligned}$$

$$\frac{dp_{30}}{dt} = -(mu_1)P_{30}(t) + l_1P_{10}(t)$$

$$\frac{dp_{31}}{dt} = -(mu_5 + mu_6)P_{31}(t) + l_5P_{11}(t) + l_6P_{10}(t)$$

$$\frac{dp_{32}}{dt} = -(mu_1)P_{32}(t) + l_1P_{11}(t)$$

$$\frac{dp_{33}}{dt} = -(mu_1)P_{33}(t) + l_1P_{14}(t)$$

$$\begin{aligned}\frac{dp_{34}}{dt} = & -(l_1 + l_2 + mu_3 + mu_4 + mu_5 + l_6)P_{34}(t) + mu_1P_{50}(t) + mu_2P_{43}(t) \\ & + l_3P_{18}(t) + l_4P_{15}(t) + l_5P_{14}(t) + mu_6P_{51}(t)\end{aligned}$$

$$\begin{aligned}\frac{dp_{35}}{dt} = & -(l_1 + l_2 + mu_3 + mu_4 + l_5 + mu_6)P_{35}(t) + mu_1P_{52}(t) + mu_2P_{46}(t) \\ & + l_3P_{19}(t) + l_4P_{16}(t) + mu_5P_{51}(t) + l_6P_{14}(t)\end{aligned}$$

$$\frac{dp_{36}}{dt} = -(mu_1)P_{36}(t) + l_1P_{15}(t)$$

$$\frac{dp_{37}}{dt} = -(mu_5 + mu_6)P_{37}(t) + l_5P_{16}(t) + l_6P_{15}(t)$$

$$\frac{dp_{38}}{dt} = -(mu_1)P_{38}(t) + l_1P_{16}(t)$$

$$\frac{dp_{39}}{dt} = -(mu_1)P_{39}(t) + l_1P_{18}(t)$$

$$\frac{dp_{40}}{dt} = -(mu_5 + mu_6)P_{40}(t) + l_5P_{19}(t) + l_6P_{18}(t)$$

$$\frac{dp_{41}}{dt} = -(mu_1)P_{41}(t) + l_1P_{19}(t)$$

$$\frac{dp_{42}}{dt} = -(mu_1)P_{42}(t) + l_1P_{25}(t)$$

$$\frac{dp_{43}}{dt} = -(mu_2 + mu_3 + mu_4)P_{43}(t) + l_2P_{34}(t) + l_3P_{28}(t) + l_4P_{25}(t)$$

$$\frac{dp_{44}}{dt} = -(mu_5 + mu_6)P_{44}(t) + l_5P_{26}(t) + l_6P_{25}(t)$$

$$\frac{dp_{45}}{dt} = -(mu_1)P_{45}(t) + l_1P_{26}(t)$$

$$\frac{dp_{46}}{dt} = -(\mu u_2 + \mu u_3 + \mu u_4)P_{46}(t) + l_2 P_{35}(t) + l_3 P_{29}(t) + l_4 P_{26}(t)$$

$$\frac{dp_{47}}{dt} = -(\mu u_1)P_{47}(t) + l_1 P_{28}(t)$$

$$\frac{dp_{48}}{dt} = -(\mu u_5 + \mu u_6)P_{48}(t) + l_5 P_{29}(t) + l_6 P_{28}(t)$$

$$\frac{dp_{49}}{dt} = -(\mu u_1)P_{49}(t) + l_1 P_{29}(t)$$

$$\frac{dp_{50}}{dt} = -(\mu u_1)P_{50}(t) + l_1 P_{34}(t)$$

$$\frac{dp_{51}}{dt} = -(\mu u_5 + \mu u_6)P_{51}(t) + l_5 P_{35}(t) + l_6 P_{34}(t)$$

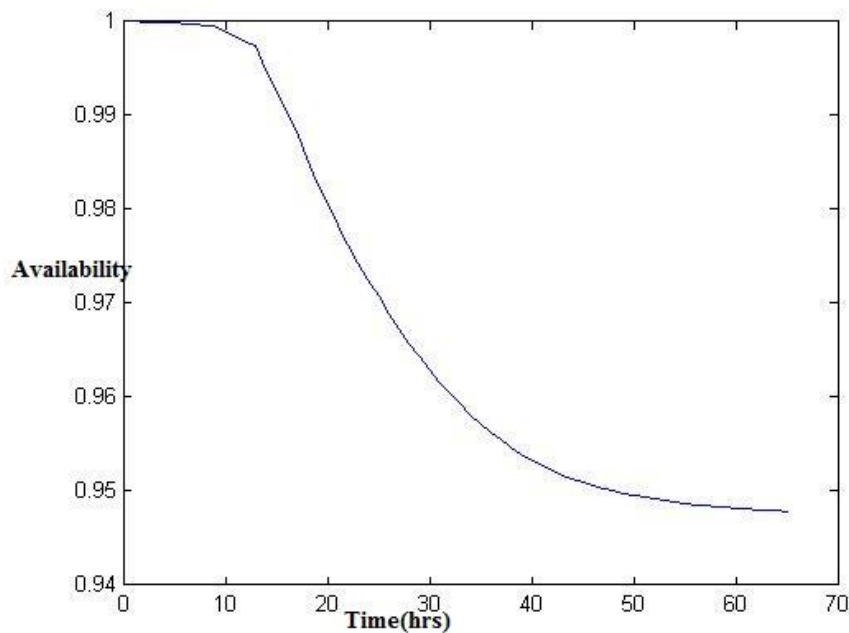
$$\frac{dp_{52}}{dt} = -(\mu u_1)P_{52}(t) + l_1 P_{35}(t)$$

These equations can be solved with the help of matlab by using Runge-Kutta method.

**6.7.2. Availability of the System:** Availability can be evaluated with the help of available state in which system is working by using failure and repair rate of each component.

$$\text{Availability} = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_8 + P_9 + P_{10} + P_{11} + P_{14} + P_{15} + P_{16} + P_{18} + P_{19} + P_{25} + P_{26} + P_{28} + P_{29} + P_{34} + P_{35}$$

$$\mathbf{Av = 93.38\%}$$



*Fig 6.5: Curve between Time and availability*

# Chapter- 7

## Conclusion and Future Scope

**7.1. Conclusion:** In this thesis Markov model of brake drum manufacturing system is presented for analysing its reliability and availability. The reliability and the availability evaluation of repairable system are performed based on the state space method by using Markov Model considering constant failure and repair rates during the period of operation. For increasing the availability of the system redundant components are used. Standby redundancy is used to increase the availability of the system. System model evaluation in 200 hours indicates that system reliability is 0.6237 and availability is 0.8423. By using active redundancy in the same system, availability increases further. Availability of the system by using active redundancy is 0.9338. Availability of the system in the active redundancy increases because load of each component decreases due to sharing of the load. Chances of failure decreases and availability increases. Down time cost of the system in case of active redundancy also reduces. Therefore, active redundancy is preferred to enhance the availability. From sensitivity analysis part it can be seen that increase in repair rate of furnace, pressure die casting, and shot blasting in the system, availability increases. Table 1 show that by increasing the repair rate of furnace from 0.01 to .08 availability of the system increases by 14.91% . Table 2 shows that increase in repair rate of pressure die casting from 0.0461 to 0.3688; there is nominal increment in availability i.e. 0.023%. The similar study can be carried out by varying the repair rates for standby model. Table 5 shows that increase in repair rate from 0.02 to .08 improves its availability by 0.59%.

The proposed analysis is useful for the plant engineers to optimize their maintenance resources and also helpful for them in taking decisions for appropriate maintenance policy.

**7.2. Future Scope:** This section presents a brief on potential future directions.

- In this work, the costs are not considered for the availability analysis. It will be meaningful, if it is linked with the cost incurred on the different maintenance actions and analysis is carried in terms of availability gains Vs cost on repair.
- Sensitivity analysis can also be carried out for active redundancy.
- The CI estimate for MTTF can be evaluated for both standby and active redundancy.

- An exponential distribution is assumed for failure and repair time due to the limitation of the Markovian approach. This assumption can be relaxed and appropriate non-exponential distributions such as Weibull for failure time and Log-normal for repair time can be considered for more realistic analysis.
- Application for decision making based on the methodology suggested in this thesis.
- Integration of availability assessment with decision making models taking into account different factors.

## REFERENCE

**Aggarwal A.K.** (2013), “Performance Modeling and Availability Analysis of Skim Milk Powder Production System of a Dairy Plant”, *International Journal of Agriculture and Food Science Technology*. ISSN 2249-3050, Volume 4, Number 9 (2013), pp. 851-858.

**Amjad A. K.** (2011), “Availability analysis of n-heptane and natural gas blends combustion in HCCI engines”, *Energy*, Volume 36, Issue 12, Pages 6900–6909.

**Atul Goyal and Parvinder Singh** (2000), “Behavior Analysis of a Biscuit Making Plant using Markov Regenerative Modeling”, *ISSN : 2319 – 3182*, Volume-2, Issue-3.

**Atul Goyal, S.K. Sharma, Pardeep Gupta** (2001), “Availability analysis of a part of rubber tube production system under preemptive resume priority repair”, *International Journal of Industrial Engineering*, 16(4), 260-269.

**C.D. Rakopoulos and E.G. Giakoumis** (2008) Availability analysis of a turbocharged diesel engine operating under transient load conditions”, *Energy* 29 1085–1104

**C.D. Rakopoulos, M.A. Scott, D.C. Kyritsis, E.G. Giakoumis** (2006), “Availability analysis of hydrogen/natural gas blends combustion in internal combustion engines”, *Energy* 33 248–255.

**Cher** (2006), “Optimal, opportunistic maintenance policy using genetic algorithms”. *Journal of Quality in Maintenance Engineering*, Volume 1, No. 3, 25-34.

**Dhillon B.S.** (1987), “Engineering Maintenance: A Modern Approach”, *CRC Press*, Boca Raton, Florida.

**Dhillon B.S.** (1992), “Reliability and Availability analysis of a system with warm standby and common cause failures”, *Micro electron Reliability*, Volume 33, No. 9, pp. 1343-1349.

**Distefano and Puliafito** (2009), “Dependability evaluation with dynamic reliability block diagrams and dynamic fault trees”, *Dependable and Secure Computing, IEEE Transactions on* Volume:6, Issue:1, ISSN :1545-5971, Page(s):4 – 17.

**Enrico Zio and Francesco Di Maio** (2009), “Processing dynamic scenarios from a reliability analysis of a nuclear power plant digital instrumentation and control system”, *Energy*

Department, Polytechnic of Milan Via Ponzio 34/3, 20133 Milano, Italy, *Annals of Nuclear Energy*, Volume 36, Issue 9, Pages 1386–1399.

**F.J.G. Carazas, C.H. Salazar, G.F.M. Souza** (2010), “Availability analysis of heat recovery steam generators used in thermal power plants”, *Energy* 36 (2011) 3855e3870.

**Fink O.** (2013), “Predicting component reliability and level of degradation with complex-valued neural networks”, *Reliability Engineering and System Safety* 121 (2014) 198–206.

**Garg Savita, Jai Singh, D.V. Singh** (2009), “Availability analysis of crank-case manufacturing in a two-wheeler automobile industry”, *Applied Mathematical Modeling* 34 (2010) 1672–1683.

**Gu Yingkui and Li Jing** (2012), “Multi-State System Reliability: A New and Systematic Review”, *Reliability Engineering*(2009) 02090 (–2000102 ) 531 – 536.

**Guo and Yang** (2008), “Automatic creation of Markov models for reliability assessment of safety instrumented systems”, *Reliability Engineering & System Safety*, Volume 93, Issue 6, June 2008, Passages 829–837.

**Gurler and Kaya** (2008), “A maintenance policy for a system with multi-state component: an approximate solution”, *Reliability Engineering & System Safety* 76(2): 117-127 .

**Huang and Yuan** (2013), “A dynamic opportunistic maintenance policy for continuously monitored systems”. *Trans. J. of Quality Management Engineering*, Volume 12, No. 3, 294 – 305.

**J.-J. Wang, Chao Fu, Kun Yang, Xu-Tao Zhang, Guo-hua Shi, John Zhai** (2013), “Reliability and availability analysis of redundant BCHP (building cooling, heating and power) system”, *Energy* 61 (2013) 531e540.

**J.W. Sun, Li-feng Xi, Shi-chang Du, Bo Ju** (2008), “Reliability modeling and analysis of serial-parallel hybrid multioperational manufacturing system considering dimensional quality, tool degradation and system configuration”, *International Journal Production Economics* 114, 149–164.

**L. Sikos, J. Klemes** (2009), “Reliability, availability and maintenance optimization of heat exchanger networks”, *Applied Thermal Engineering*.



**M. Colledani and A. Yemane** (2013), “Impact of machine reliability data uncertainty on the design and operation of manufacturing systems”, *Forty Sixth CIRP Conference on Manufacturing Systems 2013, Procedia CIRP* 7 ( 2013 ) 557 – 562.

**Mahmood M.** (1987), “Development of performance evaluation system for screening unit of paper plant”, *International Journal of Industrial Engineering*, 14(3), 220-225.

**Mahmood Reza Haghifam, Moein Manbachi** (2010), “Reliability and availability modelling of combined heat and power (CHP) systems”, *Electrical Power and Energy Systems* 33, 385–393.

**N.S. Sisworahardjo, M.S. Alam, G. Aydinli** (2007), “Reliability and Sensitivity Analysis of Low Power Portable Direct Methanol Fuel Cell”, 10.1109/EURCON.2007.4400616. *The International Conference on "Computer as a Tool*.

**Omran Musa Abbas, Hassan Ibrahim Mohammed and El Nougomi Abdelgadir Omer** (2011), “Development of Predictive Markov-chain Condition – Based Tractor Failure Analysis Algorithm”, *Research Journal of Agriculture and Biological Sciences*, 7(1): 52-67, 2011.

**Rajiv Khanduja , P. C. Tewari, R.S. Chauhan** (2009),“Performance Analysis of Screening Unit in a Paper Plant Using Genetic Algorithm”, *Journal of Industrial and Systems Engineering* Volume 3, No. 2, pp 140-151.

**Resham Vinayak and S. Dharamraja** (2012),“Semi-Markov Modeling Approach for Deteriorating Systems with Preventive Maintenance”, *International Journal of Performability Engineering* Volume 8, No. 5, September 2012, pp. 515- 526

**Roberto de Lieto Vollaro and Mauro Davoli** (2010), “Plant Reliability in Hospital Facilities”, *Energy Procedia*, Volume 45, 2014, Pages 1195–1204

**Romulo I. Zequeira, Jose E. Valdes, Christophe Berenguer** (2007), “Optimal buffer inventory and opportunistic preventive maintenance under random production capacity availability”, *International Journal of Production Economics* 02/2008; 111(2):686-696.

**Shakuntla, A.K. Lal, S.S. Bhatia** (2002), “Availability Analysis of Polytube Industry When Two Sub-System Are Simultaneous Fail”, *Bangladesh Journal of Scientific and Industrial Research* ISSN 0304-9809, Volume 46, No 4 (2002), BJSIR 2011; 46(4): 475-480.

**Sharma S. P.** (2011),“Reliability analysis of complex multi-robotic system using GA and fuzzy methodology”, *Applied Soft Computing* 12 (2012) 405–415.

**Singh** (1991), “A new algorithm for multi-area reliability evaluation—Simultaneous decomposition-simulation approach”, *Electric Power Systems Research*, Volume 21, Pages 129–136.

**Wu and Chan** (2003), “Performance utility-analysis of multi-state systems”, *Ieee Transactions on Reliability*, 2003, v. 52 n. 1, p. 14-21.

**Y.-T. Tsai, Kuo-Shong Wang, Lin-Chang Tsai** (2004),“A study of availability-centered preventive maintenance for multi-component systems”, *Reliability Engineering and System Safety* 84 (2004) 261–270.

**Yong jinkwon, Richard Chiou, Leonard Stepanski** (2009), “A review on degradation models in reliability analysis”, *Proceedings of the 4<sup>th</sup> World Congress on Engineering Asset Management, Athens, Greece*, 1-16.

**Yong Sun, Lin Ma, and Joseph Mathew** (2007), “Determination of preventive maintenance lead time using hybrid analysis”, *International journal of plant engineering and management*, 2007.10(1), p.13-18.

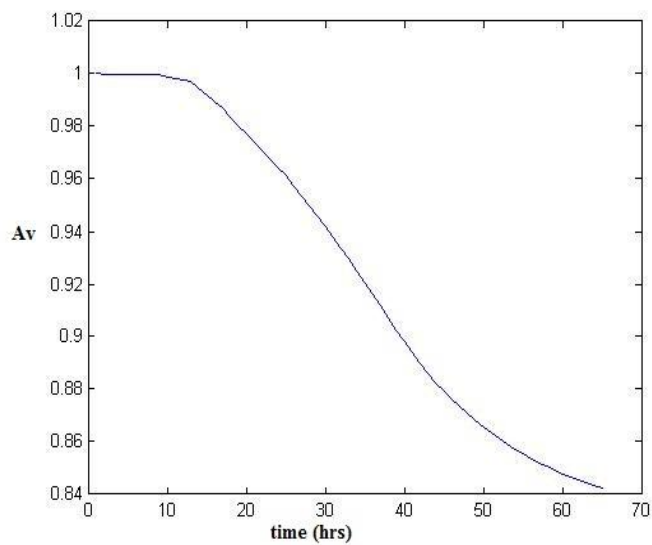
## APPENDIX A1

### Fig A1-Graphs of Sensitivity Analysis:

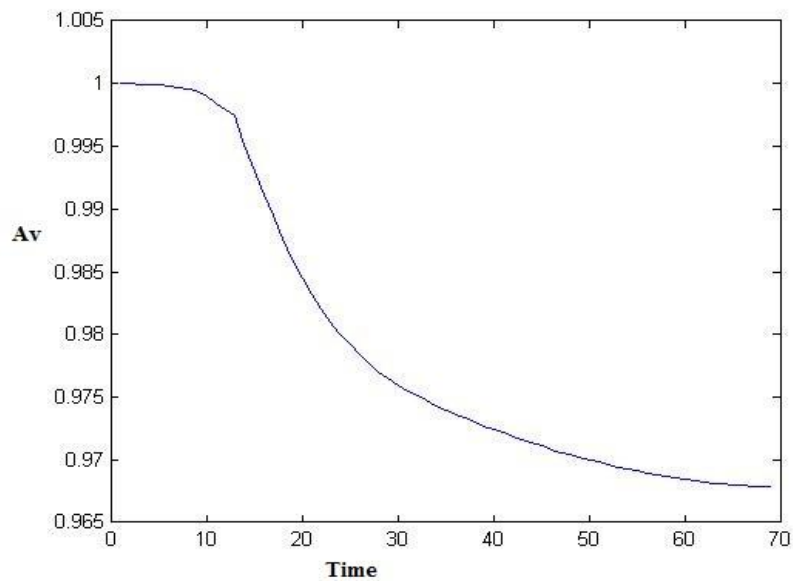
These are the graphs of sensitivity analysis between availability and time by varying the value of  $\mu_1$  and other factors remain constant.

Taking  $\lambda_2 = 0.00461$ ,  $\lambda_3 = 0.00375$ ,  $\lambda_4 = 0.0003$ ,  $\lambda_5 = 0.002$ ,  $\lambda_6 = 0.0016$   $\mu_2 = 0.0461$ ,  $\mu_3 = 0.0375$ ,  $\mu_4 = 0.003$ ,  $\mu_5 = 0.02$ ,  $\mu_6 = 0.016$

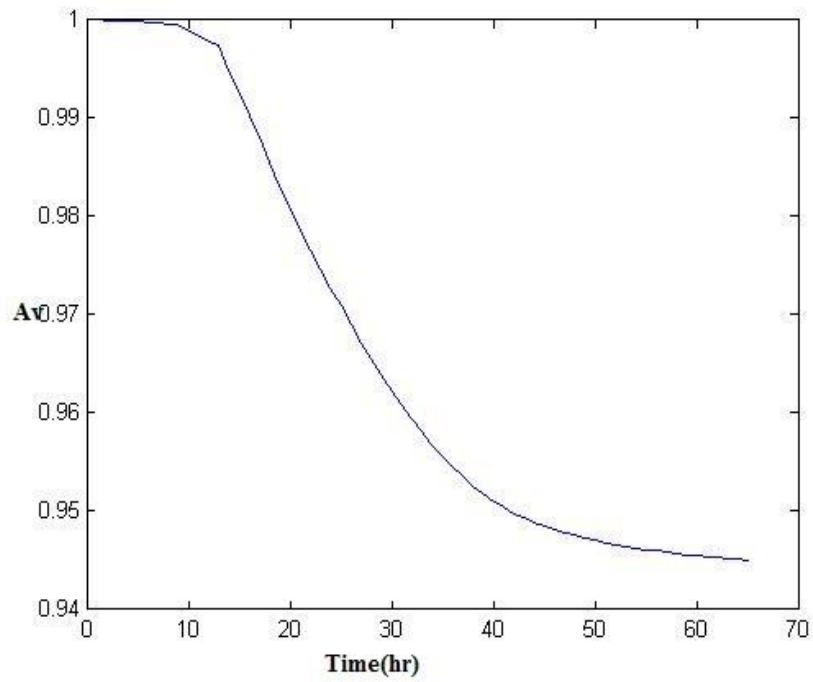
At repair rate of furnace .01 and failure rate .002



At repair rate of furnace .02 and failure rate .002



At repair rate .04 and failure rate .002



At repair rate .08 and failure .002

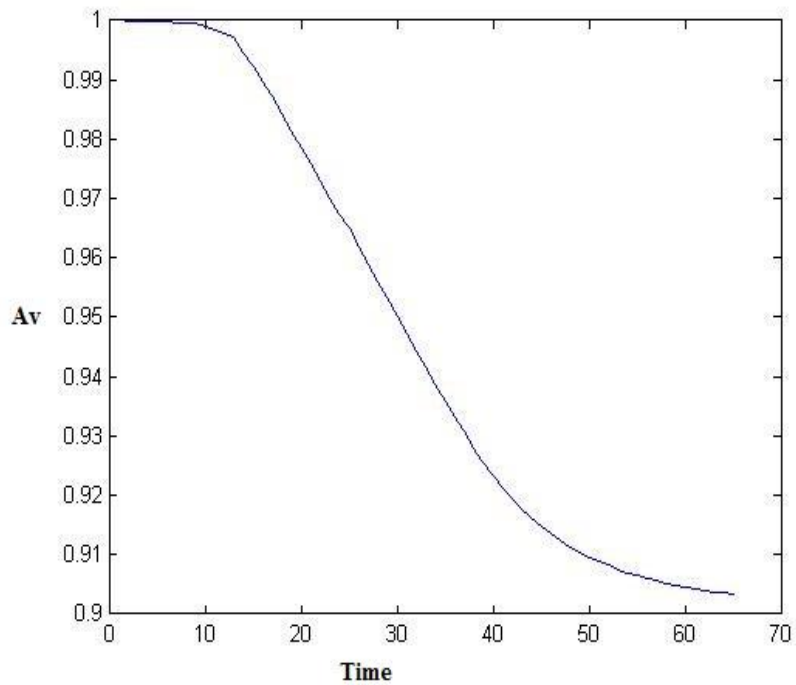
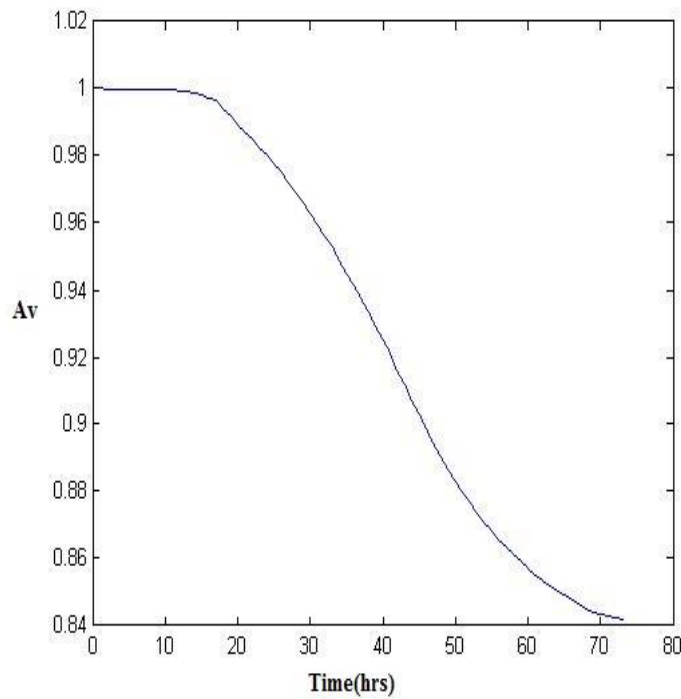


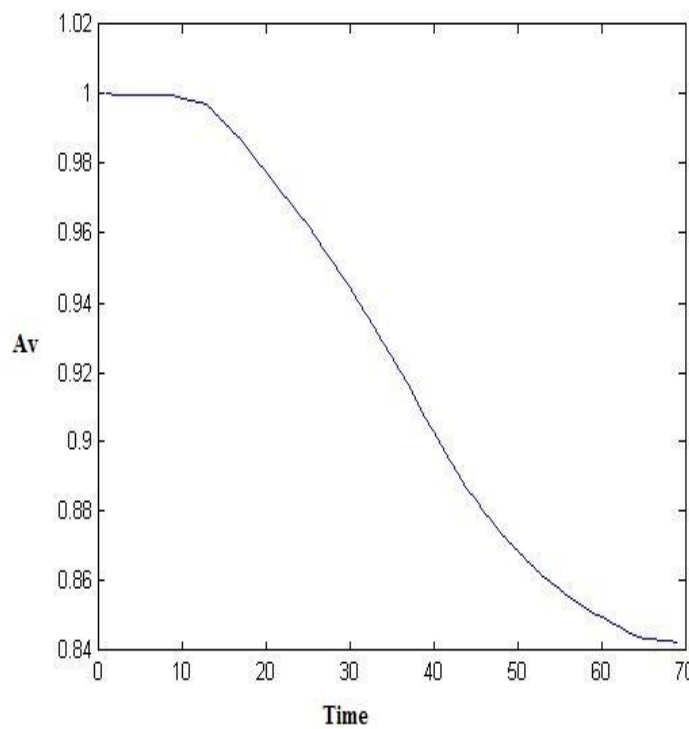
Fig A1-1 Graphs between availability and time by varying  $\mu_1$

Taking  $\lambda_1=0.002$ ,  $\lambda_2=0.00461$ ,  $\lambda_4=0.0003$ ,  $\lambda_5=0.002$ ,  $\lambda_6=0.0016$   $\mu_1=0.01$ ,  $\mu_2=0.0461$ ,  $\mu_4=0.003$ ,  $\mu_5=0.02$ ,  $\mu_6=0.016$  all these data constant and changing the value of  $\mu_3$ :

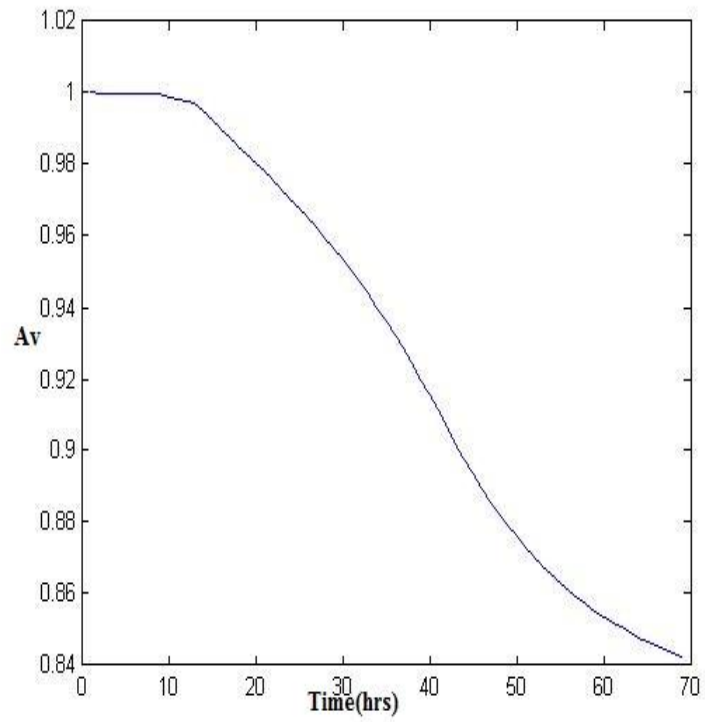
Repair rate of PDC2 at .0375 and failure rate .00375



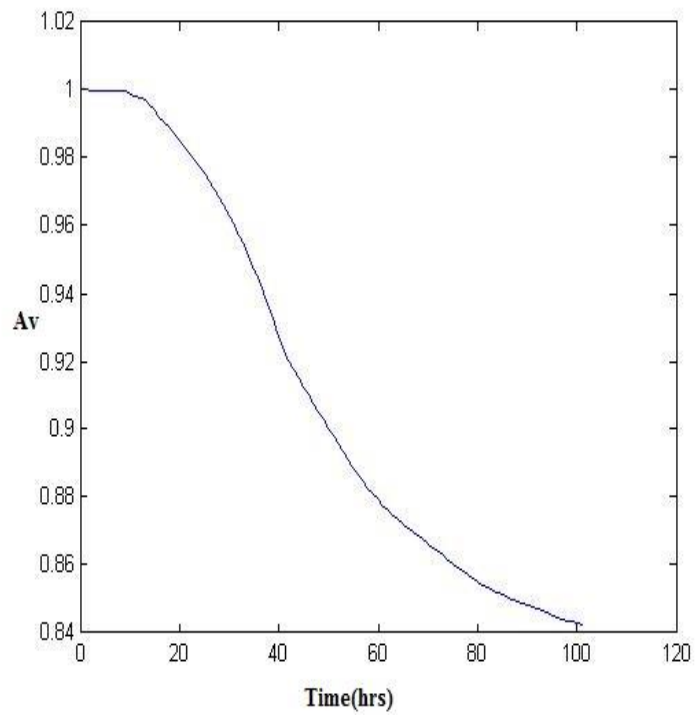
At repair rate .075 and failure rate .00375



At repair rate .15 and failure rate .00375



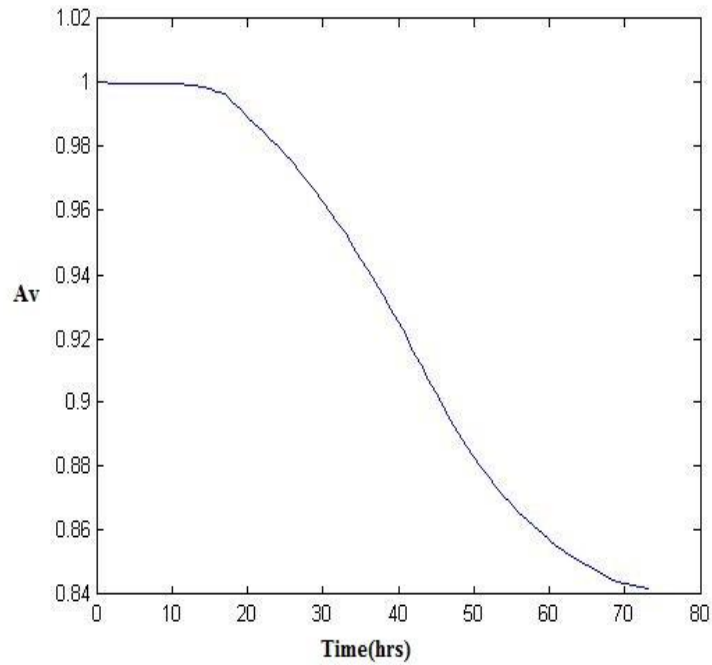
At repair rate .30 and failure rate .00375



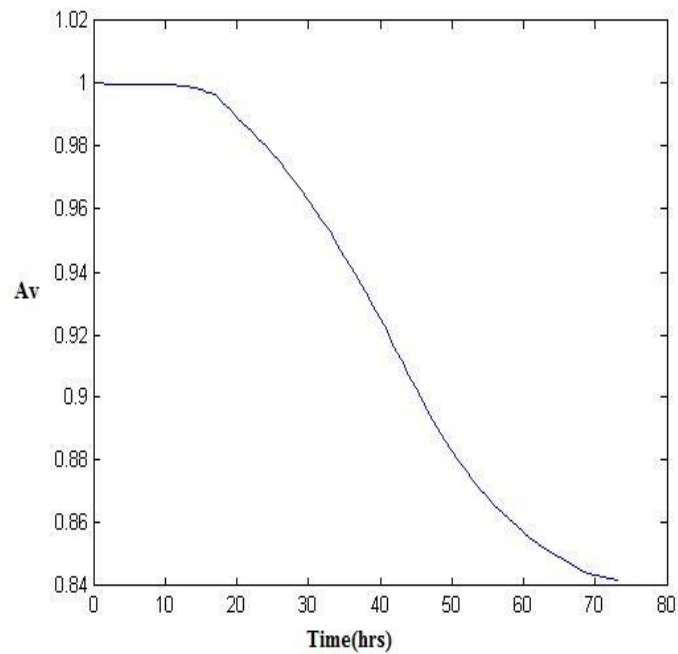
*Fig A1-3: Curve between Time and Availability by varying  $\lambda_3$  and  $\mu_3$*

Taking  $\lambda_1= 0.002$ ,  $\lambda_2= 0.00461$ ,  $\lambda_3=0.00375$ ,  $\lambda_5= 0.002$ ,  $\lambda_6= 0.0016$   $\mu_1= 0.01$ ,  $\mu_2= 0.0461$   $\mu_3= 0.0375$ ,  $\mu_5= 0.02$ ,  $\mu_6= 0.016$  all these data constant and changing the value of  $\lambda_4$ ,  $\mu_4$ :

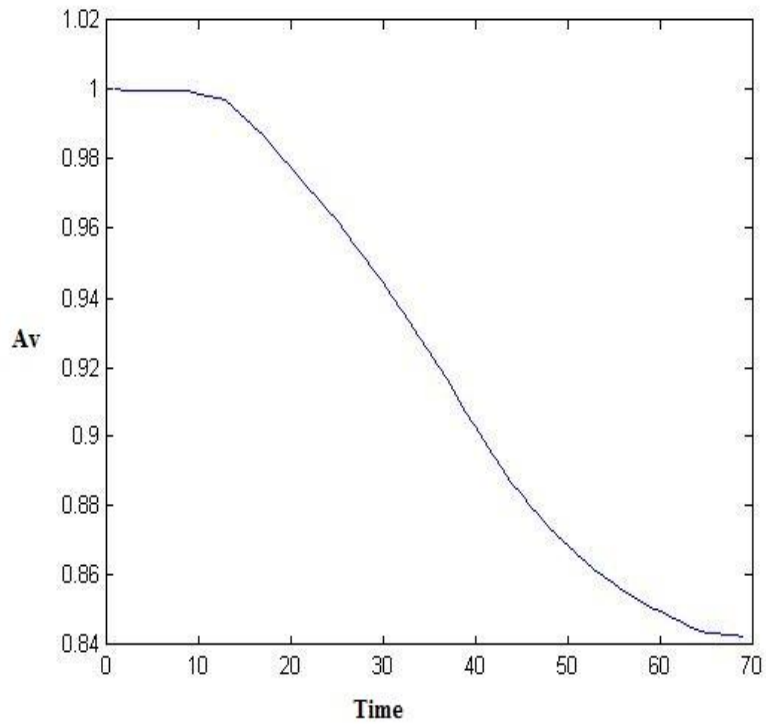
Repair rate of PDC3 at .003 and failure rate .0003



At repair rate .006 and failure rate .0003



At repair rate .012 and failure rate .0003



At repair rate .024 and failure rate .0003

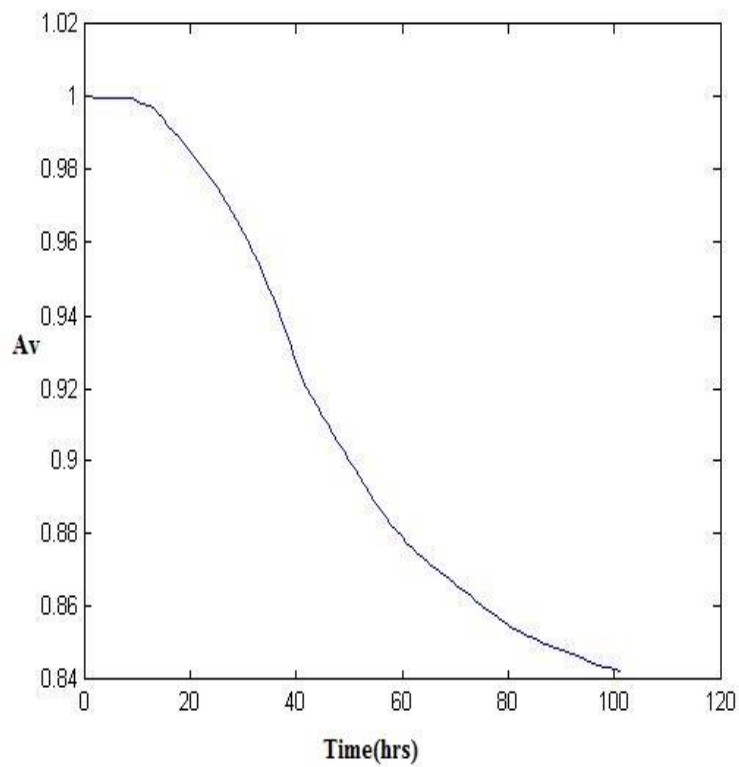
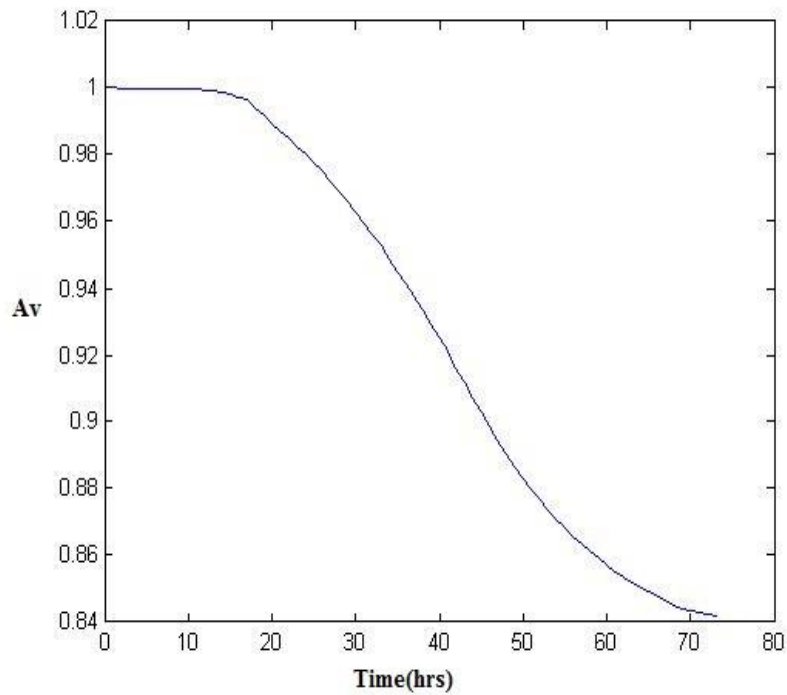


Fig A1-4: Curve between Time and Availability by varying  $\lambda$  and  $\mu$

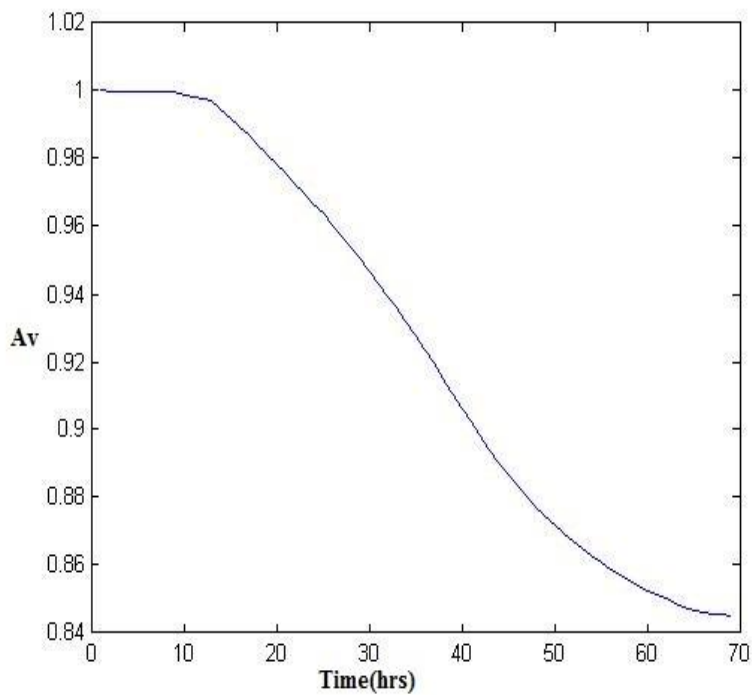


Taking  $\lambda_1= 0.002$ ,  $\lambda_2= 0.00461$ ,  $\lambda_3= 0.00375$ ,  $\lambda_4= 0.0003$ ,  $\lambda_6= 0.0016$ ,  $\mu_1= 0.01$ ,  $\mu_2= 0.0461$ ,  $\mu_3= 0.0375$ ,  $\mu_4= 0.003$ ,  $\mu_6= 0.016$  all these data constant and changing the value of  $\lambda_5$ ,  $\mu_5$

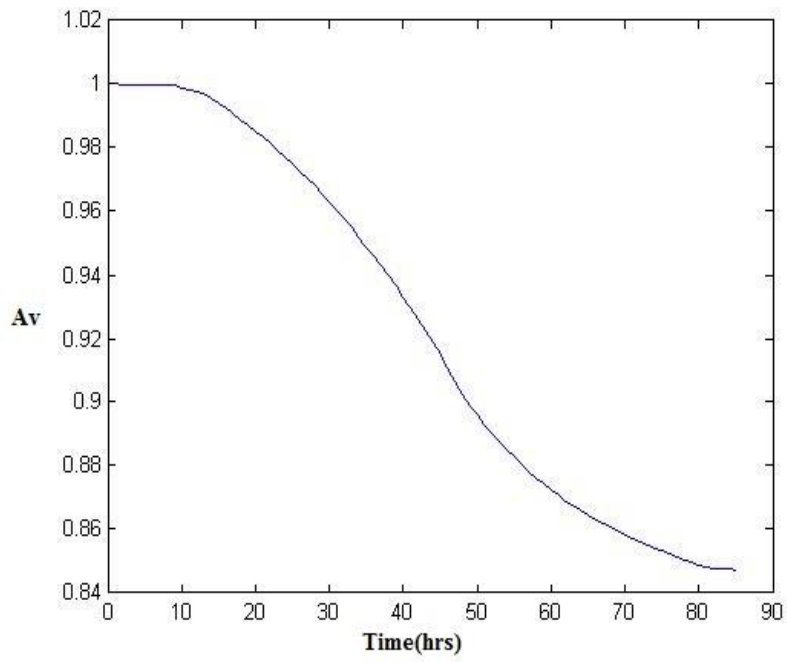
Repair rate of SB1 at .02 and failure rate .002



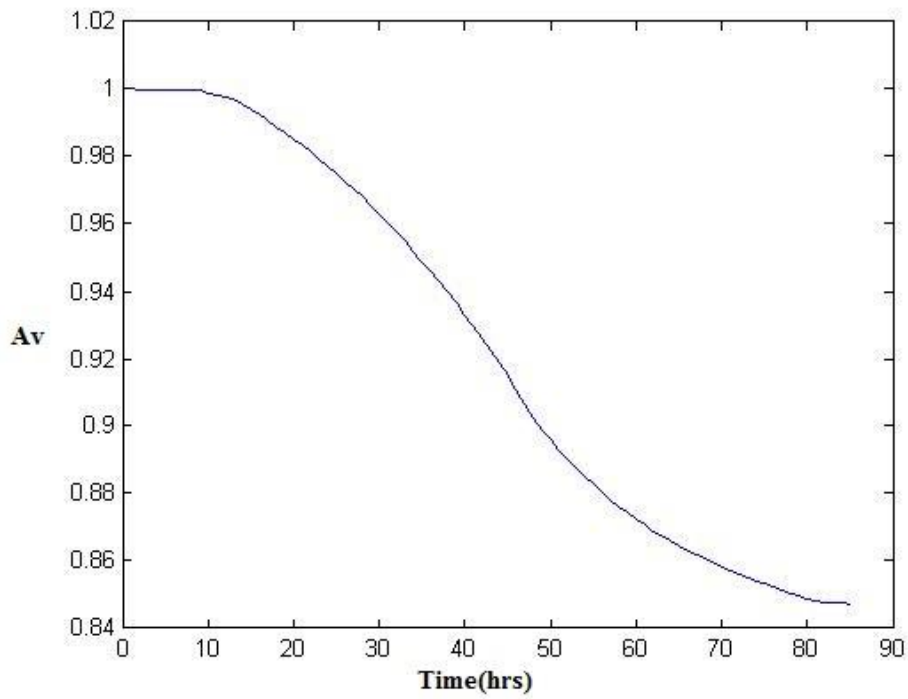
At repair rate .04 and failure rate .002



At repair rate .08 and failure rate .002



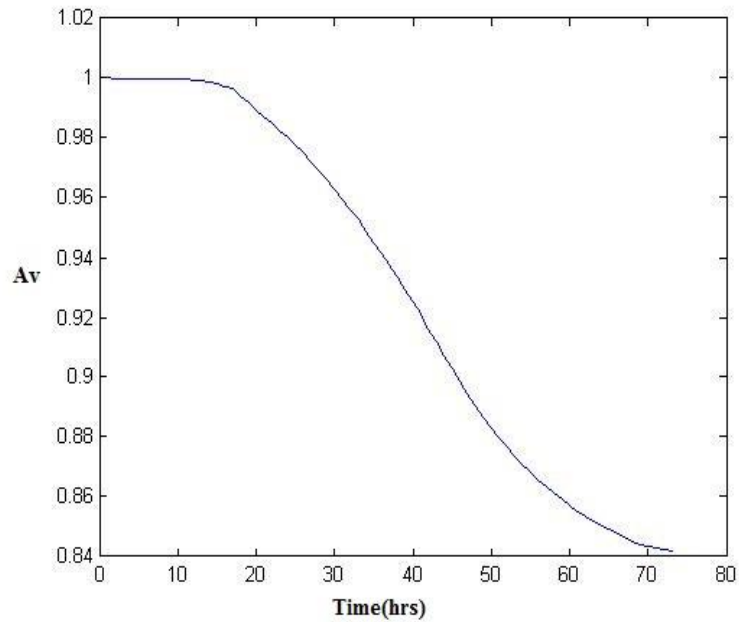
At repair rate .16 and failure rate .002



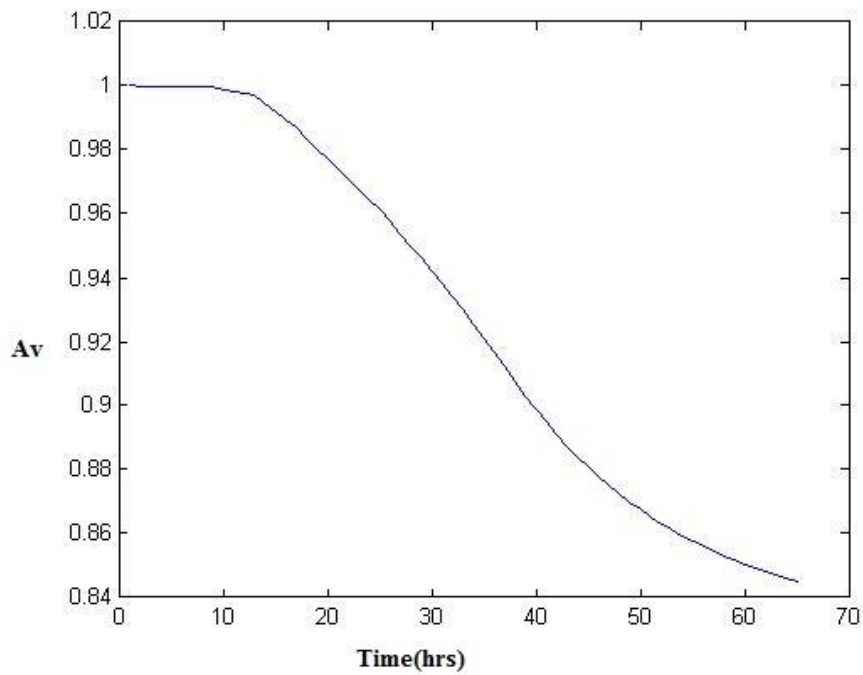
*Fig A1-5: Curve between Time and Availability by varying  $\mu$ s*

Taking  $\lambda_1=0.002$ ,  $\lambda_2=0.00461$ ,  $\lambda_3=0.00375$ ,  $\lambda_4=0.0003$ ,  $\lambda_5=0.002$ ,  $\mu_1=0.01$ ,  $\mu_2=0.0461$ ,  $\mu_3=0.0375$ ,  $\mu_4=0.003$ ,  $\mu_5=0.02$  all these data constant and changing the value of  $\lambda_6$ ,  $\mu_6$ :

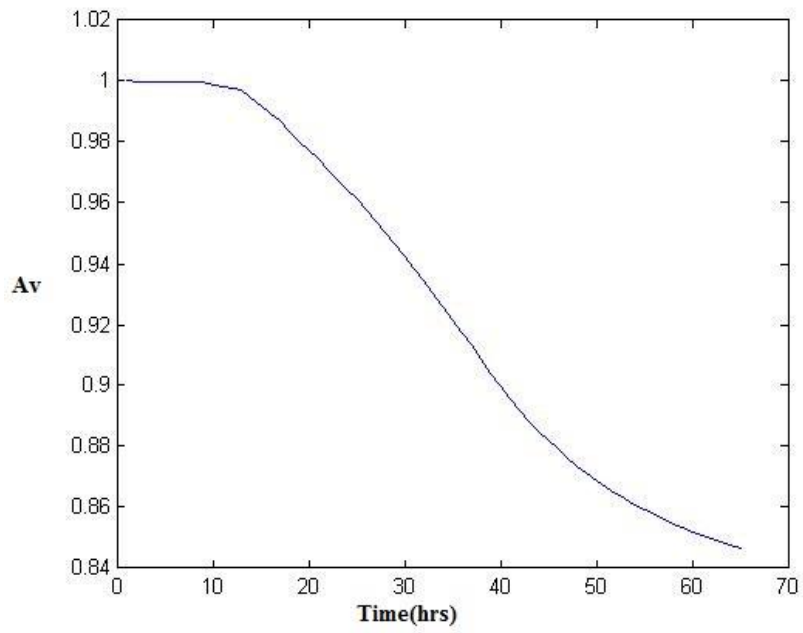
Repair rate of SB2 at .016 and failure rate .0016



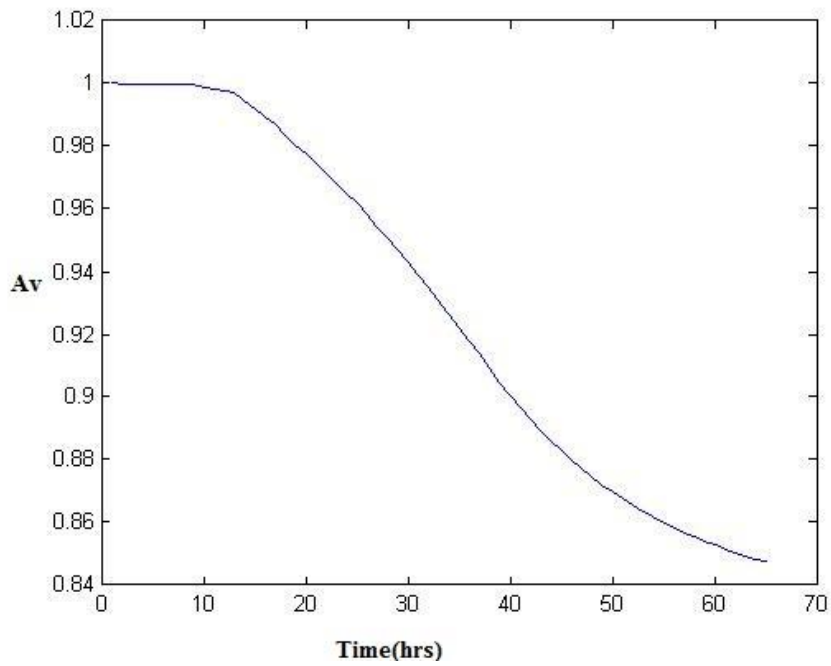
At repair rate .032 and failure rate .0016



At repair rate .064 and failure rate .0016



At repair rate .128 and failure rate .0016



*Fig A1-6: Curve between Time and Availability by varying  $\mu_6$*

## Appendix A2

### Programs for availability analysis

#### A2-1: Program of Availability Analysis in Standby System:

Functiondydt = reliability1(t,y)

mu1=0.08;  
mu2=0.0461;  
mu3=0.0375;  
mu4=0.003;  
mu5=0.02;  
mu6=0.016;

l1=0.002;  
l2=0.00461;  
l3=0.00375;  
l4=0.0003;  
l5=0.002;  
l6=0.0016;

dydt = [ -  
(l1+l2+l3+l4+l5+l6)\*y(1)+mu1\*y(7)+mu2\*y(2)+mu3\*y(3)+mu4\*y(4)+mu5\*y(5)+mu6\*y(6);  
-  
(l1+mu2+l3+l4+l5+l6)\*y(2)+mu1\*y(12)+l2\*y(1)+mu3\*y(8)+mu4\*y(9)+mu5\*y(10)+mu6\*y(11);  
-  
(l1+l2+mu3+l4+l5+l6)\*y(3)+mu1\*y(13)+mu2\*y(8)+l3\*y(1)+mu4\*y(14)+mu5\*y(15)+mu6\*y(16);  
-  
(l1+l2+l3+mu4+l5+l6)\*y(4)+mu1\*y(17)+mu2\*y(9)+mu3\*y(14)+l4\*y(1)+mu5\*y(18)+mu6\*y(19);  
-  
(l1+l2+l3+l4+mu5+l6)\*y(5)+mu1\*y(20)+mu2\*y(10)+mu3\*y(15)+mu4\*y(18)+l5\*y(1)+mu6\*y(21);  
-  
(l1+l2+l3+l4+l5+mu6)\*y(6)+mu1\*y(22)+mu2\*y(11)+mu3\*y(16)+mu4\*y(19)+mu5\*y(21)+l6\*y(1);

$$-\mu_1 y(7) + l_1 y(1);$$

$$-$$

$$(l_1 + \mu_2 + \mu_3 + l_4 + l_5 + l_6) y(8) + \mu_1 y(23) + l_2 y(3) + l_3 y(2) + \mu_4 y(24) + \mu_5 y(25) + \mu_6 y(26);$$

$$-$$

$$(l_1 + \mu_2 + l_3 + \mu_4 + l_5 + l_6) y(9) + \mu_1 y(27) + l_2 y(4) + \mu_3 y(24) + l_4 y(2) + \mu_5 y(28) + \mu_6 y(29);$$

$$-$$

$$(l_1 + \mu_2 + l_3 + l_4 + \mu_5 + l_6) y(10) + \mu_1 y(30) + l_2 y(5) + \mu_3 y(25) + \mu_4 y(28) + l_5 y(2) + \mu_6 y(31);$$

$$-$$

$$(l_1 + \mu_2 + l_3 + l_4 + l_5 + \mu_6) y(11) + \mu_1 y(32) + l_2 y(6) + \mu_3 y(26) + \mu_4 y(29) + \mu_5 y(31) + l_6 y(1);$$

$$-\mu_1 y(12) + l_1 y(2);$$

$$-\mu_1 y(13) + l_1 y(3);$$

$$-$$

$$(l_1 + l_2 + \mu_3 + \mu_4 + l_5 + l_6) y(14) + \mu_1 y(33) + \mu_2 y(24) + l_3 y(4) + l_4 y(3) + \mu_5 y(34) + \mu_6 y(35);$$

$$-$$

$$(l_1 + l_2 + \mu_3 + l_4 + \mu_5 + l_6) y(15) + \mu_1 y(36) + \mu_2 y(25) + l_3 y(5) + \mu_4 y(34) + l_5 y(3) + \mu_6 y(37);$$

$$-$$

$$(l_1 + l_2 + \mu_3 + l_4 + l_5 + \mu_6) y(16) + \mu_1 y(38) + \mu_2 y(26) + l_3 y(6) + \mu_4 y(35) + \mu_5 y(37) + l_6 y(3);$$

$$-\mu_1 y(17) + l_1 y(4);$$

$$-$$

$$(l_1 + l_2 + l_3 + \mu_4 + \mu_5 + l_6) y(18) + \mu_1 y(39) + \mu_2 y(28) + \mu_3 y(34) + l_4 y(5) + l_5 y(4) + \mu_6 y(40);$$

$$-$$

$$(l_1 + l_2 + l_3 + \mu_4 + l_5 + \mu_6) y(19) + \mu_1 y(41) + \mu_2 y(29) + \mu_3 y(35) + l_4 y(6) + \mu_5 y(40) + l_6 y(4);$$

$$-\mu_1 y(20) + l_1 y(5);$$

$$-(\mu_5 + \mu_6) y(21) + l_5 y(6) + l_6 y(5);$$

$$s - \mu_1 y(22) + l_1 y(6);$$

$$-\mu_1 y(23) + l_1 y(8);$$

$$-(\mu_2 + \mu_3 + \mu_4) y(24) + l_2 y(14) + l_3 y(9) + l_4 y(8);$$

$$-$$

$$(l_1 + \mu_2 + \mu_3 + l_4 + \mu_5 + l_6) y(25) + \mu_1 y(42) + l_2 y(15) + l_3 y(10) + \mu_4 y(43) + l_5 y(8) + \mu_6 y(44);$$

-

$$(l1+\mu2+\mu3+l4+l5+\mu6) * y(26) + \mu1 * y(45) + l2 * y(16) + l3 * y(11) + \mu4 * y(46) + \mu5 * y(44) + l6 * y(8);$$

$$-\mu1 * y(27) + l1 * y(9);$$

-

$$(l1+\mu2+l3+\mu4+\mu5+l6) * y(28) + \mu1 * y(47) + l2 * y(18) + \mu3 * y(43) + l4 * y(10) + l5 * y(9) + \mu6 * y(48);$$

-

$$(l1+\mu2+l3+\mu4+l5+\mu6) * y(29) + \mu1 * y(49) + l2 * y(19) + \mu3 * y(46) + l4 * y(11) + \mu5 * y(48) + l6 * y(9);$$

$$-\mu1 * y(30) + l1 * y(10);$$

$$-(\mu5+\mu6) * y(31) + l5 * y(11) + l6 * y(10);$$

$$-\mu1 * y(32) + l1 * y(11);$$

$$-\mu1 * y(33) + l1 * y(14);$$

-

$$(l1+l2+\mu3+\mu4+\mu5+l6) * y(34) + \mu1 * y(50) + \mu2 * y(43) + l3 * y(18) + l4 * y(15) + l5 * y(14) + \mu6 * y(51);$$

-

$$(l1+l2+\mu3+\mu4+l5+\mu6) * y(35) + \mu1 * y(52) + \mu2 * y(46) + l3 * y(19) + l4 * y(16) + \mu5 * y(51) + l6 * y(14);$$

$$-\mu1 * y(36) + l1 * y(15);$$

$$-(\mu5+\mu6) * y(37) + l5 * y(16) + l6 * y(15);$$

$$-\mu1 * y(38) + l1 * y(16);$$

$$-\mu1 * y(39) + l1 * y(18);$$

$$-(\mu5+\mu6) * y(40) + l5 * y(19) + l6 * y(18);$$

$$-\mu1 * y(41) + l1 * y(19);$$

$$-\mu1 * y(42) + l1 * y(25);$$

$$-(\mu2+\mu3+\mu4) * y(43) + l2 * y(34) + l3 * y(28) + l4 * y(25);$$

$$-(\mu5+\mu6) * y(44) + l5 * y(26) + l6 * y(25);$$

$$-\mu1 * y(45) + l1 * y(26);$$

$$-(\mu2+\mu3+\mu4) * y(46) + l2 * y(35) + l3 * y(29) + l4 * y(26);$$

$$-\mu1 * y(47) + l1 * y(28);$$

$$-(\mu5+\mu6) * y(48) + l5 * y(29) + l6 * y(28);$$

$$-\mu1 * y(49) + l1 * y(29);$$

$$-\mu1 * y(50) + l1 * y(34);$$

$$-(\mu5+\mu6) * y(51) + l5 * y(35) + l6 * y(34);$$

$$-\mu1 * y(52) + l1 * y(35)];$$

## A2-1.Program of Availability Analysis in Standby System:

```
functiondydt = reliabilitystandby(t,y)
```

```
mu1 = 0.01;  
mu2 = 0.0461;  
mu3 = 0.0375;  
mu4 = 0.003;  
mu5 = 0.02;  
mu6 = 0.016;
```

```
l1 = 0.016;  
l2 = 0.00461;  
l3 = 0.00375;  
l4 = 0.0003;  
l5 = 0.002;  
l6 = 0.0016;
```

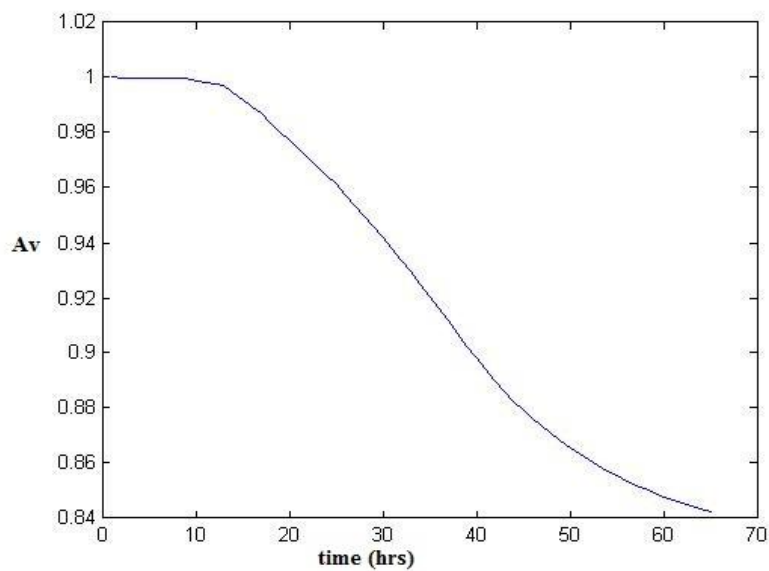
```
dydt = [-(l1+l2+l5)*y(1)+mu1*y(2)+mu2*y(3)+mu5*y(4);  
-mu1*y(2)+l1*y(1);  
-(mu2+l1+l3+l5)*y(3)+l2*y(1)+mu1*y(5)+mu3*y(6)+mu5*y(7);  
-(mu5+l1+l2+l6)*y(4)+l5*y(1)+mu1*y(8)+mu2*y(7)+mu6*y(9);  
-mu1*y(5)+l1*y(3);  
-(mu3+l1+l4+l5)*y(6)+l3*y(3)+mu1*y(10)+mu4*y(11)+mu5*y(12);  
-(mu5+mu2+l1+l3+l6)*y(7)+l5*y(3)+l3*y(4)+mu1*y(13)+mu2*y(12)+mu6*y(14);  
-mu1*y(8)+l1*y(4);  
-mu6*y(9)+l6*y(4);  
-mu1*y(10)+l1*y(6);  
-mu4*y(11)+l4*y(6);  
-  
(mu3+mu5+l1+l4+l6)*y(12)+l3*y(7)+l5*y(6)+mu1*y(15)+mu4*y(16)+mu6*y(17);  
-mu1*y(13)+l1*y(7);  
-mu6*y(14)+l6*y(7);  
-mu1*y(15)+l1*y(12);  
-mu4*y(16)+l4*y(12);  
-mu6*y(17)+l6*y(12)];
```



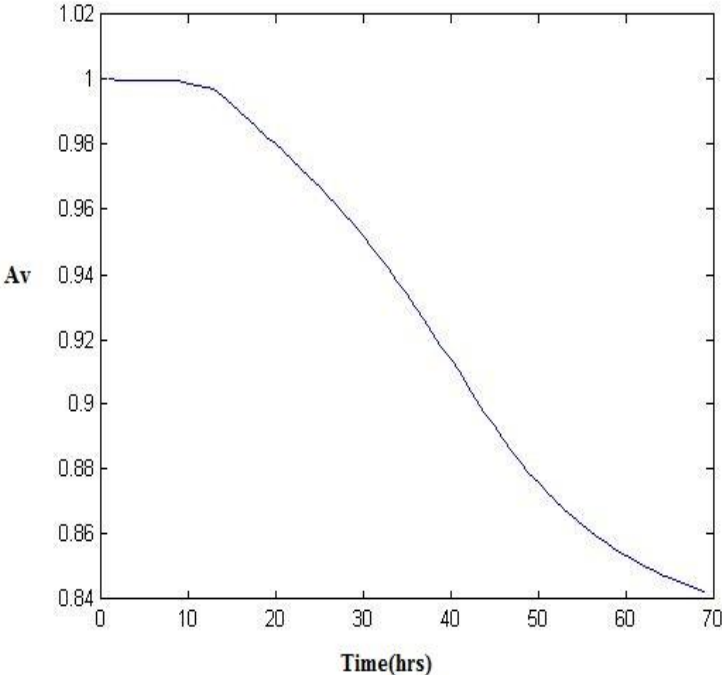
These are the graphs of sensitivity analysis between availability and time by varying the value of  $\mu_2$  and other factors remain constant.

Taking  $\lambda_1= 0.002$ ,  $\lambda_2= .00461$ ,  $\lambda_3=0.00375$ ,  $\lambda_4= 0.0003$ ,  $\lambda_5= 0.002$ ,  $\lambda_6= 0.0016$   $\mu_1= 0.01$ ,  $\mu_3= 0.0375$ ,  $\mu_4= 0.003$ ,  $\mu_5= 0.02$ ,  $\mu_6= 0.016$  all these data constant and changing the value of  $\lambda_2$ ,  $\mu_2$

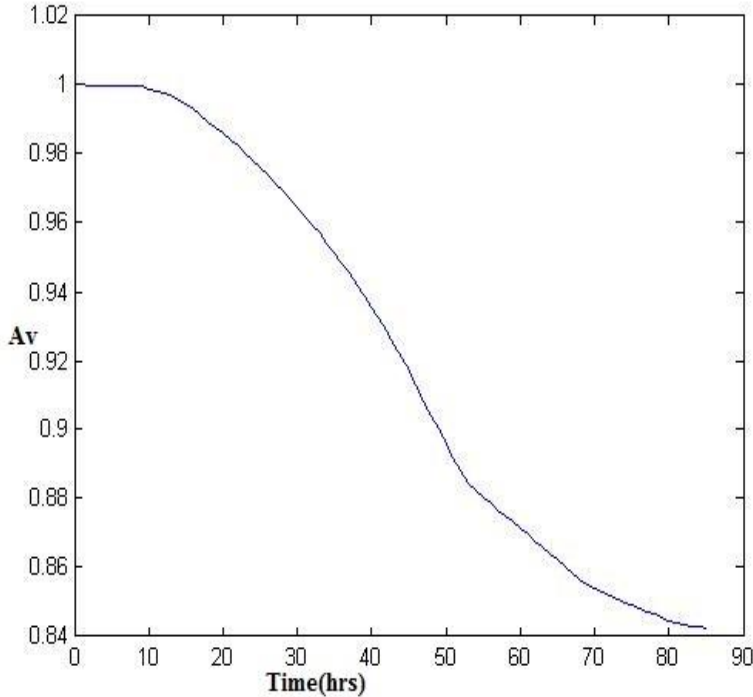
Repair rate of PDC1 at .0461 and failure rate .00461



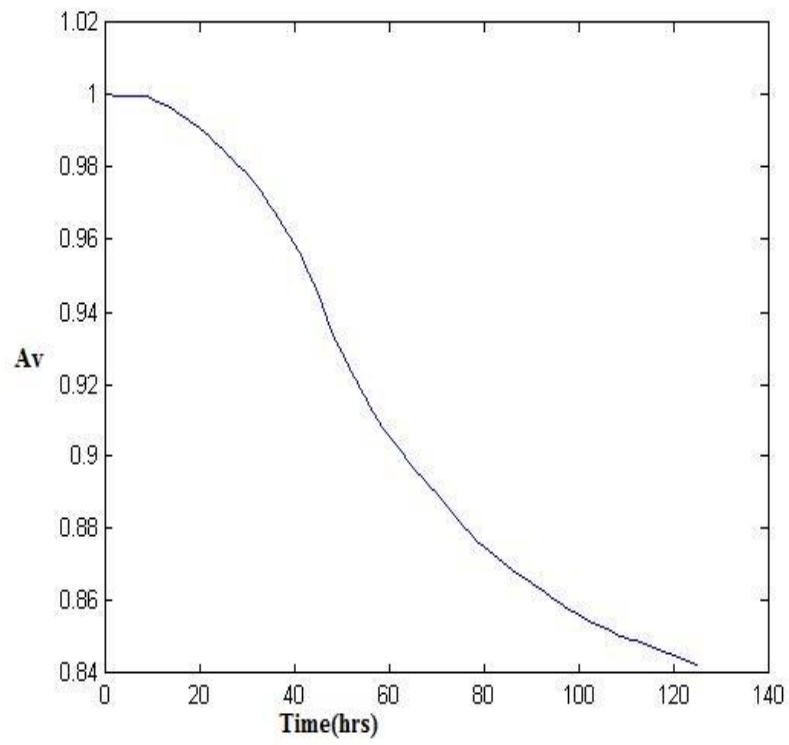
Repair rate of PDC1 at .0922 and failure rate .00461



At repair rate .1844 and failure rate .00461



At repair rate .3688 and failure rate .00461



*Fig A1-2: Curve between Time and Availability by varying  $\lambda_2$  and  $\mu_2$*