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CERTIFICATE

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This is to certify that the report entitled **"An Advanced FCM and Level Set based Image Segmentation Method for Medical Applications"** submitted by Bandaru Vasu Mani Kumar, Roll. No. *2K12/SPD/23*, in partial fulfillment for the award of degree of Master of Technology in Signal Processing & Digital Design at **Delhi Technological University, Delhi**, is a bonafide record of student's own work carried out by him under my supervision and guidance in the academic session 2013-14. The matter embodied in dissertation has not been submitted for the award of any other degree or certificate in this or any other university or institute.

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ABSTRACT

Images often suffer from noise and intensity inhomogeneity this makes segmentation challenging. Especially in medical images accurate segmentation of the voxels is necessary. They often corrupted by noise and non-uniformity. Fuzzy c means clustering is one of the popular method in medical image segmentation. But this can't dealt with noise and intensity inhomogeneity. Recently level set based active contour models are also used in medical image analysis. Precise segmentation capability of active contour models make them attractive. Chumming Li's model is proposed to deal with intensity inhomogeneity by using an energy function based on K-means clustering. In this report we propose a new energy model based on Li's model. Our addition is twofold, first we introduce a fuzzy factor into the energy function which is somewhat similar to fuzzy c-means clustering in continuous domain and secondly we utilize a special function which will be advantageous for segmenting noisy image. The proposed method can dealt with intensity inhomogeneity and noise as well. Even in the presence on noise it can result in smooth boundaries. The proposed method is verified on different MRI image which contain noise and intensity inhomogeneity and also on some natural images as well as synthetic images. The results shows that the proposed method is showing improved performance when compared with the state of the art techniques when dealing with images containing inhomogeneity and noise.

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CHAPTER 1

INTRODUCTION

1.1 IMAGE SEGMENTATION

Image segmentation is a challenging task in image analysis as the images contain complex boundaries and often affected by noise. The main types of segmentation methods are Region based, Texture based, Edge based, Clustering and Active contours. Medical images mostly contain complicated structures and their precise segmentation is necessary for clinical diagnosis [39]. Image segmentation and contour extraction are the most intuitive methods for medical image visualization [49]. A detailed review about segmentation can be found in [35-37].

Medical imaging is essential and important to biomedical research and clinical applications due to its valuable intra vital information. Among various medical image modalities, magnetic resonance imaging (MRI) provides high contrast images that has been widely used in the interpretation and visualization of various anatomical structures. Segmentation in MR images plays a fundamental role and simplifies subsequent analysis procedures by extracting certain useful anatomical structures. Intuitively, the segmentation work is carried out by experts such as doctors and physicians. However, as the amount of MR image data is exploding nowadays, manual segmentation has the following disadvantages

(1) Due to the complex anatomical structure in MR images, the slice-by-slice manual segmentation is time-consuming and tedious.

(2) The identification of target boundary is subjective and the segmentation results with user-intervention are prone to operator bias.

(3) Manual segmentation is less effective and impractical to the huge amount of MR image data.

Consequently, a wide variety of studies have been devoted to semi- or fully automatic computer-aided segmentation for achieving fast and objective segmentation with high accuracy to facilitate subsequent analyses.

In this report a level set and fuzzy c-means clustering based segmentation technique has been proposed. For the better understanding of the rest of the paper basic knowledge required about FCM, Curve evolution and Level set are being discussed in the following sections.

1.2 **Literature Survey of FCM**

Medical images mostly contain complicated structures and their precise segmentation is necessary for clinical diagnosis [39]. Image segmentation and contour extraction are the most intuitive methods for medical image visualization [49]. Image segmentation is a challenging task in image analysis as the images contain complex boundaries and often affected by noise. FCM is one of the mostly used algorithms in medical image applications because of its fuzzy nature where one pixel can belong to multiple clusters which lead to better performance than crisp methods [39]. Most of the times, MRI images, contains noise and intensity inhomogeneity. Conventional FCM couldn't dealt with these things [50], so many FCM variations are proposed in the recent years to overcome disadvantages of conventional FCM by using the spatial information or by estimating the bias field for in-homogeneity or by making changes to the cost function.

K.Xiao et.al [40] performed Gaussian smoothing on input image and proposed a method to find the weightage for each feature using bootstrapping technique when dealing with multiple features. To deal with intensity inhomogeneity Pham et.al [41] proposed Adaptive FCM, the centroids are multiplied by a unknown multiplier filed which represents the inhomogeneity .First and second order regularization terms are included in the cost function to make the multiplier field slowly varying and smooth. Jude et.al [42] used multidimensional features formed by GLCM feature generation model to include spatial information into FCM. Extra dimensions make the process time consuming to overcome this distance metric based compression is proposed which selects the representative pixels of the groups and perform clustering on them which resulted in fast and effective clustering. Enhanced FCM is proposed by Szilagyi et.al [43] to fasten the segmentation process as well as reduce the noise effect. For achieving this a new factor which includes the pixel intensity along with mean of neighbourhood pixels is calculated before the segmentation process. Computational complexity is reduced by considering the histogram count of the voxels into the cost function. Noise inhomogeneity effect is reduced in a faster way by GCFFCM proposed by Jingjing Song et.al.[44] GCFFCM algorithm uses a gain field to deal with inhomogeneity and uses the histogram of the image to reduce the computational time. Abbas Biniaz et.al [45] proposed Gaussian spatial FCM, in which to reduce the noise effect the membership function is updated twice in the algorithm. First update is similar to the conventional FCM, in the second update a spatial factor is included which consists the Gaussian average of neighbourhood membership values. Another method to deal with noise affect is proposed by Shan Shen et.al [46] called as Improved FCM. In IFCM the distance term in FCM is modified by including two terms feature attraction and distance attraction which are calculated by considering weighted average of neighbourhood intensity differences and weighted average of spatial difference respectively. The parameters that are giving weightage to these terms are optimized at every step by using a simple artificial neural network model. Aprior probability and fuzzy spatial information are used in ISFCM proposed by Zulaikha Bevi et.al [47] to overcome the noise affect in MRI images. Cluster centres are initialized using histogram based FCM in the first step to achieve faster convergence and the membership updating equation is modified in a way to use probability obtained by the ratio of number of pixels belongs to the cluster to the number of pixels in the neighbourhood and spatial information.

2.2 Literature Survey of Active Contours

Active contours are the models which will are used to extract the objects by making curves evolve by minimizing a defined energy. These are sometimes referred as snakes. The energies used in these models will be such that they will minimized when the curves evolving are at the boundaries of the required object. Usually these energies will have two main components internal energy and external energy [5]. Internal energy will make the evolving curve smooth and regularize it and external energy will guide the motion of the curve towards its optimal position. The active contour models have applications in various domains like computer vison, image processing, medical analysis etc. In the recent years they gain lead in movie industry where tracking of objects or persons is of crucial importance. They can be applied to 2D as well 3D situations. They will automatically search for their minimum energy positions but sometimes they may settle at local minimum.

There is freedom to use different types of external energies to guide the motion of the curve. The commonly used external energies are edge based [7],[20],[24] and region based [1],[2],[5], [26]. Integration of level set approach and edge based active contour models and applying them to image processing application was pioneered by Malladi *et al.* [24]. Until then Snake models are implemented based on parameterized approaches which face lot of complexities. Their proposed method includes an energy term based on edge information into level set model which normally utilizes curvature motion. A similar approach based on level sets was proposed independently by Caselles *et al.* [18]. After few years Caselles *et al.* [7] propose one more approach based on edge information called as Geodesic Active Contours. A gradient flow based model which is similar to Caselles *et al.* [7] was proposed by Satyanand *et al.* [20] in the same year. Mumford Shah *et al.* [1] proposed a region based approach to approximate an image using a piece wise smooth model of it. The energy term used here is minimized when the approximation contains smooth regions as well as sufficient number of edges that can model the given image. The implementational approach for a special case of the Mumford Shah model is given by Chan-Vese [5]. In [5] a piece wise constant approximation of an image is obtained instead of piece wise smooth. The required approximation of image is achieved is using level set model developed by *Sethian et al.* [4]. The multiphase extension of [5] is proposed in [26] by Vese *et al.* where the similar algorithm as in [5] is used but with more number of level sets. In the recent years there are many advancements were made based on the Chan-Vese [5] model. One of such is proposed by C. Li *et al.* [2] to overcome the intensity inhomogeneity affect that occur in images. An energy function based on K-means clustering model is used. The intensity is modelled using a multiplicative bias field term and a local intensity clustering property is utilized to deal efficiently with inhomogeneity. Chen *et al.*[29] also proposed a method to dealt with intensity inhomogeneity almost with similar approach as in [2] but using an additive bias field. Region based model depends less on initial level set. But edge base models commonly suffer with the problem of level set initialization. B.N. Li *et al.*[10] have used fuzzy c-means clustering’s membership function to initialize level set to overcome the before mentioned problem. They also used the membership function to select the parameters required in level set model. Cui *et al.*[16] have utilized the local intensity clustering property in forming a new FCM based clustering. L.Tang *et al.* [27] have utilized the concept of introducing fuzziness into the level set approach. They have developed an energy function by integrating FCM_S1&S2 model proposed by Chen *et al.* [57] and level set model by Samson *et al.* [33] .They showed that this model can successfully segment out images with high noise.

CHAPTER 2

BACKGROUND

2.1 FUZZY C-MEANS CLUSTERING

FCM [38, 39] finds its applications in variety of problems varying from data analysis to segmentation of images. In FCM it is possible for a data sample to belong to multiple clusters at the same time. The similarity is indicated by the membership value. In FCM a data sample is assigned with a membership value based on its similarity with the cluster centre. The membership values lie between 0 to 1, more the similarity higher the membership value [38]. Defuzzification is applied at the end of the clustering process to decide the clustering. FCM is a repetitive algorithm; the solution is achieved by repetitively updating the cluster centre and membership value. These updating equations are obtained by solving the cost function.

Let $\boldsymbol{X} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \dots \boldsymbol{x_N}\}$ denotes the data with **N** data samples; it has to be partitioned into c-clusters by minimizing the following cost function

$$J = \sum_{j=1}^{N} \sum_{i=1}^{c} u_{ij}^{m} \left\| x_j - v_i \right\|^2 \tag{2.1}$$

where, $\boldsymbol{u_{ij}}$ represents the membership of $\boldsymbol{x_j}$ with the $\boldsymbol{i^{th}}$cluster, $\boldsymbol{v_i}$ is the $\boldsymbol{i^{th}}$ cluster centre, $\|.\|$ is a norm metric and $\boldsymbol{m}$ is a constant. The parameter **m** controls the fuzziness of the resulting partition.

Taking the derivative of the equation and make it equal to zero by using Lagrange method we can get the following equations

$$v_i = \frac{\sum_{j=1}^{N} u_{ij}^{m} x_j}{\sum_{j=1}^{N} u_{ij}^{m}} \tag{2.2}$$

$$u_{ij} = \sum_{k=1}^{c} \left(\frac{\|x_j - v_i\|}{\|x_j - v_k\|} \right)^{-2} \tag{2.3}$$

and $\sum_{j=1}^{c} u_{ij} = 1 \quad \forall\, i = 1,2, \dots N$

2.1.1 Algorithm:

1. Initialize U=[u_{ij}] matrix,

2. Calculate the centres vectors V=[v_i]using U matrix

$$v_i = \frac{\sum_{j=1}^{N} u_{ij}^{m}\, x_j}{\sum_{j=1}^{N} u_{ij}^{m}}$$

3. Update U

$$u_{ij} = \sum_{k=1}^{c} \left(\frac{\|x_j - v_i\|}{\|x_j - v_k\|} \right)^{-2}$$

4. If $\max_{ij}\left\{\left\|v^{old}{}_{ij} - v^{new}{}_{ij}\right\|\right\} < \epsilon$ end the process

   Otherwise go to Step 2

2.2 FCM Methods in Detail

2.2.1 BCFCM

Ahmed *et al.* [53] proposed a bias field estimation based FCM .To deal with intensity inhomogeneity intensity of MRI voxel is modelled as the sum of observed intensity and a bias field term and the objective function of FCM is modified by including neighbourhood information it acts as regularization term it helps to reduce the effect of salt and pepper noise.

The modified objective function used in BCFCM is given by

$$J = \sum_{j=1}^{N}\sum_{i=1}^{c} u_{ij}^{m}\left\|y_j - \beta_j - v_i\right\|^2 + \frac{\alpha}{N_R}\sum_{j=1}^{N}\sum_{i=1}^{c} u_{ij}^{m}\left(\sum_{k\in N(x_j)}\|y_k - \beta_k - v_i\|^2\right) \quad (2.4)$$

Here β_j is the bias field value at the j[th] voxel estimating this helps in removing the inhomogeneity effect in segmentation. The neighbourhood effect on objective function is controlled by the parameter α .For low SNR MRI signals where noise effect is heavy giving importance to neighbourhood helps so a high α value is required, for High SNR vice versa.N_R represents the size of neighbourhood to be considered. To get the updating equations for membership term, Cluster centre and bias filed term objective function is minimized using one Lagrange multiplier similar to the method used in FCM.

BCFCM works well for high SNR as well as low SNR MRI signals and helps to reduce the effect of inhomogeneity. Selection of the parameter α heavily effects the accuracy of results for example small value of α for low SNR resulted in leakage of boundaries in the segmented image. This method cannot be applied when considering multiple features as input.

2.2.2 PFCM

Yang *et al.* [54] introduced a penalty term to the FCM objective function which is inspired by Neighbourhood EM algorithm to reduce the effect of noise in the segmentation process. The penalty is formed by using the neighbourhood information similar to NEM a few changes are made to it to satisfy the criterion of FCM.

$$J_{PFCM} = \sum_{j=1}^{N}\sum_{i=1}^{c}(u_{ij})^m d^2(x_j, v_i) + \gamma\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{i=1}^{c}(u_{ij})^m(1-u_{ik})^m w_{jk} \quad (2.5)$$

where w_{jk} represents the neighborhood region if x_k is neighbor of x_j then w_{jk} =1 else w_{jk} =0 . γ is the controlling parameter similar to α in BFCM. To get the updating equations similar procedure is followed as in conventional FCM. In PFCM cluster center updating equation hasn't affected by the changes made to objective function only membership Updation will change which will result in less computational complexity.

Like BFCM, PFCM also gives satisfactory results for low SNR data also. But it also has dependence on the controlling parameter γ . It deals only with noise it can't reduce the effect of intensity inhomogeneity .PFCM can be applied even for multidimensional input feature data.

2.2.3 SFCM

To overcome the noise effect on the segmentation phase Chuang *et al.* [50] used spatial information while updating the membership function in the repetitive FCM algorithm, because the neighbourhood pixels possess same properties as the centre pixel. In contrast to BCFCM and PFCM the objective function is not changed in the Spatial FCM (SFCM) instead the membership function is updated twice.

The first Updation is similar to the conventional FCM, in the second step a spatial function is defined as sum of the membership values in spatial domain in the entire neighbourhood around the pixel under consideration.

$$h_{ij} = \sum_{k\in N(x_j)} u_{ik} \tag{2.6}$$

where, $N(x_j)$ is the neighbourhood under consideration around x_j

The spatial function is used in updating membership function again given by the equation

$$u_{ij} = \frac{u_{ij}^{p} h_{ij}^{q}}{\sum_{k=1}^{c} u_{kj}^{p} h_{kj}^{q}} \tag{2.7}$$

When surrounding pixels belongs to the same cluster as the centre pixel the membership function will gain larger values. In smooth regions clustering remains unchanged because spatial function will just strengthens the membership function. The correction of misclassified pixels from noisy regions will happen when spatial function reduces the weight of a noisy pixel by considering its neighbourhood pixels. The p and q value can be varied as per the requirement large q value implies effect of neighbourhood information in the segmentation process is more, this is suitable when the image is affected by high density noise.

The Spatial FCM works well for high as well as low density noise. It can be applied for single and multiple feature data .Compared to other methods gives superior results without any boundary leakage even at high density noise when q value is carefully selected. Parameter (p, q) selection is crucial in SFCM also. High value of q may results in blurring of fine details.

2.2.4 FCM_s1 &FCM_s2

Chen *et al.* [57] proposed a new algorithm to reduce the computational complexity taken by BCFCM. The noise reduction is taken care by taking the neighbourhood information but in contrast to BCFCM it computes the spatial term prior to the repetitive FCM algorithm. The variations of FCM proposed here are FCM_S1, FCM_S2.

The modified cost function is given by

$$J_m = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{m} \left\| x_k - v_i \right\|^2 + \alpha \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{m} \left\| \bar{x}_k - v_i \right\|^2 \qquad (2.8)$$

where $\bar{x}_k$ is made by taking the mean or median of neighbourhood pixels respectively these algorithms named FCM_S1, FCM_S2. They have proposed kernel version of these algorithms also.

These algorithms may work well in the presence of noise also since it used spatial information also in the cost function. It can be used for multidimensional data. This algorithm is dependent on parameter (α). The computational complexity will be less as the spatial term is computed only once.

2.2.5 FLICM

The limitations faced by most of the variations of FCM techniques which are trying to use the spatial information are their dependence on noise density and type of noise for which a parameter is being used. Fuzzy Local Information C- means algorithm is proposed by Krinidis and Chatzis [55] to overcome the usage of the parameter selection when dealing with segmentation of noisy images. A fuzzy factor G is introduced into the objective function of conventional FCM.

G is defined as

$$G_{ki} = \sum_{\substack{j=N_i \\ i \neq j}} \frac{1}{d_{ij}+1}(1-u_{kj})^m \left\| x_j - v_k \right\|^2 \tag{2.9}$$

and the objective function of FLICM is given as

$$J_m = \sum_{i=1}^{N}\sum_{k=1}^{c}[u^m{}_{ki} \left\| x_i - v_k \right\|^2 + G_{ki}] \tag{2.10}$$

where d_{ij} is the spatial Euclidean distance from pixel i to j, so pixels near to the centre pixel will affect the fuzzy factor most. The objective function looks like a small change to BCFCM but it resulted in better approach, by observing the cost function it is clear that when a centre pixel is a noisy pixel it will belong to a noisy cluster but the neighbourhood forces the noisy pixel to their cluster in order to minimize the objective function.

FLICM can be used when the prior knowledge of noise is not available. It is independent of any parameter selection. It can be used to multidimensional data. It can't deal with intensity inhomogeneity.

2.2.6 MDFCM

Jamal *et al.* [56] introduced a new method to reduce the noise effect by using spatial information. The segmentation is carried by considering the multiple features like mean, standard deviation, singular value and intensity of the pixel hence called multi-dimensional FCM.

The features used in FLICM can be expressed as

$$F_1 = x_j \text{ ,Pixel intensity}$$

$$F_2 = \frac{\sum_{x_i \in l} x_i}{n} \text{ , Mean}$$

$$F_3 = \delta_{11} \text{(Largest singular value of L) ,} \qquad SVD(L) = [U]\begin{bmatrix} \delta_{11} & 0 & 0 \\ 0 & \delta_{22} & 0 \\ 0 & 0 & \delta_{33} \end{bmatrix}[V^T]$$

$$F_4 = \sqrt{\frac{\sum_{x_i \in L} (x_i - E(x))^2}{n}}$$, Standard Deviation

Where L is the local window around

Before the segmentation instead of using all the features, different combination of features can be formed. The feature set selection depends on the requirement if accuracy is the constraint considering all the features gives better results. Computational cost is the criteria then selecting few important features like Eigen value, pixel intensity is recommended.

In MDFCM by considering the spatial information in terms of different features resulted in better performance than most of the variations of FCM. Even with the images corrupted by high density noise MDFCM resulted in clear boundaries and noise free clustering. The main disadvantage of this method is computational complexity because of multidimensional data.

2.2.7 WIPFCM

A new approach to use the spatial information in FCM process was introduced by Zexuan Ji *et.al* [59]. In this method image patches are considered instead of just pixels which will add additional information to the data. Weight vectors are assigned to every image patch based on their variance with respect to their neighbouring pixels. The expression to carry out this weight assignment is given by,

$$\xi_{kr} = \exp[-(\sigma_{kr} - \frac{\sum_{r \in N_k} \sigma_{kr}}{n_k})] \tag{2.11}$$

where ξ_{kr} and σ_{kr} represent the weight and variance of r^{th} pixel in k^{th} patch respectively, later it is normalized to make the sum of the weights equal to one.

The weight vector assigned here results in removing the effect of edges and noisy pixel on clustering process by assigning less weight to them.

The objective function for WIPFCM is given by

$$J = \sum_{i=1}^{c}\sum_{k=1}^{N} u_{ik}^{m} \sum_{r \in N_k} w_{kr} \left\| I_{kr} - v_{ri} \right\|^2 \tag{2.12}$$

where I_k represents the image patch of size $q \times q$ around the pixel x_k, v_i represents the cluster centre of size $q \times q$.

In this method by utilizing the spatial information it handled noise effectively, there is no external parameter to be selected like in other methods. Increased computational burden because each pixel need to be represented by a patch and additional step of calculation of their weights.

2.2.8 KWFLICM

Maoguo Gong et.al [60] proposed an variation of FLICM Kernal weighted FCM. In this approach they have introduced trade off weights to the pixels and replaced the Euclidean distance with kernel metric based distance.

In FLICM a factor is introduced which utilizes spatial distance and Euclidian distance of neighbouring pixels from the centre pixels but it fails to analyse the neighbouring pixels when the centre pixel itself is noisy , to overcome this disadvantage in KWFLICM utilizes more local information by utilizing spatial variance instead of Euclidian distance . The weight assigning technique is similar to the one used in WIPFCM, here in addition to that it utilizes spatial distance also, and it can be explained by these equations

$$w_{ij} = w_{gc} \cdot w_{sc} \tag{2.13}$$

$$w_{gc} = \begin{cases} 2 + \beta_{ij} & \sigma_j < \bar{\sigma} \\ 2 - \beta_{ij} & \sigma_j \geq \bar{\sigma} \end{cases} \tag{2.14}$$

and

$$w_{gc} = \begin{cases} 2 + \beta_{ij} & \sigma_j < \bar{\sigma} \\ 2 - \beta_{ij} & \sigma_j \geq \bar{\sigma} \end{cases} \tag{2.15}$$

Where, β_{ij} is the weight obtained using the formula given in WIPFCM after normalization, w_{gc} is made positive by keeping these conditional statements, $\bar{\sigma}$ is the average variance in the local window.

The objective function is given by

$$J_m = \sum_{i=1}^{N}\sum_{k=1}^{c} u_{ik}^{m}(1 - K(x_i, v_k)) + G_{ki}^{'} \tag{2.16}$$

and the fuzzy factor $G_{ki}^{'}$ is given by

$$G_{ki}^{'} = \sum_{i=1}^{N}\sum_{k=1}^{c} u_{ik}^{m} \sum_{\substack{i \neq j \\ j \in N_i}} w_{ij}(1 - u_{ki})^{m}(1 - K(x_i, v_k)) \tag{2.17}$$

where, N_i represents the local window around the pixel,

$K(x_i, v_k)$ is the kernel based distance measure , the kernel used in this method is Gaussian radial basis function (GRBF). For this Gaussian kernel the band width parameter used is estimated using the variance of the zero-mean input data, it can be expressed by the following relation

$$\sigma = \left(\frac{1}{N-1}\sum_{i=1}^{N}(d_i - \bar{d})^2\right)^{\frac{1}{2}} \tag{2.18}$$

$$d_i = \left\| x_i - \bar{x} \right\| \tag{2.19}$$

The kernel distance used here is obtained by projecting the input data into a high dimensional space by using the kernel. KFLICM is independent of any external parameter selection; it utilizes local information in an effective way by using the spatial variance and spatial positioning. Kernel metric distance used here may lead to better clustering performance in the presence of image artifacts, noise and outliers.

2.2.9 STRONG FCM

The data given to FCM is projected into a high dimensional space using kernels and distance between the cluster centre and data points is calculated in that domain.it may leads to better clustering of complex noised data this technique is proposed by S.R.Kannan et.al. [58] In this they have proposed two techniques based on this approach and also a new method to initialize the cluster centres to improve the performance of FCM.

The method proposed by S.R.Kannan et.al to initialize the centres is as follows, first find the median of each data sample (in case of multidimensional data) and sort the data in descending order with respect to the magnitude of medians, Now, make the data into C (no. of clusters) groups i.e. first N/C (N-total no. of samples) data samples in the sorted data form one group and rest follows the same manner. In a group find out the distance between every pair of data sample and take the average of pair of samples with maximum distance, that average value will become the prototype for the respective cluster.

First FCM technique proposed is **Robust Fuzzy C-means based kernel function (RFCMK)** in this technique the modified cost function is given by

$$J = 2\sum_{i=1}^{n}\sum_{k=1}^{c} u_{ik}^{m}(\beta - K(x_i, v_k)) \quad (2.20)$$

The Kernel function is expressed as

$$K(x_i, v_k) = -\frac{\|x_i - v_k\|^2}{\alpha} + \beta, \quad ,\beta > 0 \quad (2.21)$$

Cost function here is obtained by projecting the data into higher dimensional space by non-linear transformation and it is minimized using the properties of kernel functions. The updating expressions for membership function and cluster centres can be obtained by differentiating the cost function and making it equal to zero using Lagrange multiplier technique as done in the conventional FCM.

Second FCM technique proposed is **Tsallis entropy based fuzzzy c-means algorithm (TEFCM)**. This is an extension to the previous technique, because above technique cannot distinguish similar intensity objects of different clusters. To overcome this problem an extra term is introduced into the cost function

$$J = 2\sum_{i=1}^{n}\sum_{k=1}^{c} u_{ik}^{m}(\beta - K(x_i, v_k)) + \frac{\alpha_i}{\gamma - 1}(\sum_{i=1}^{n}\sum_{k=1}^{c} u_{ik}^{m} - 1) \tag{2.22}$$

where, $\alpha_i = \frac{n}{\zeta}(x_i + ((n/c)/N_R)\sum_{j \in N_i} x_j)$ is a term which will include data from neighborhood and ζ is a parameter to control the effect of neighborhood term.

By including this extra term spatial information is also included in the objective function which will lead to improved performance.

The above discussed methods can be used to multidimensional data, compared to FCM which will give good clustering when all the clusters are of same size and well separated these techniques which utilized higher dimensional space may give better results even when the data is not well distributed and noisy also. TEFCM performance is parameter dependent. Time consumption may be less due to because neighbourhood term is calculated only once and it is utilized in the clustering process

2.3 CLOSED PLANAR CURVES

2.3.1 Introduction

A curve can be defined as a collection of points, near each point it looks like a line with some deformation, called as curvature. A straight is a simple curve with null curvature. The difference between a function and a curve is that a curve can possess two different values for the same values of (x, y). A function can be considered as a part of the curves.

A curve is defined as

$C(p) = \{x(p), y(p)\}$ Where, $p \in [0,1]$

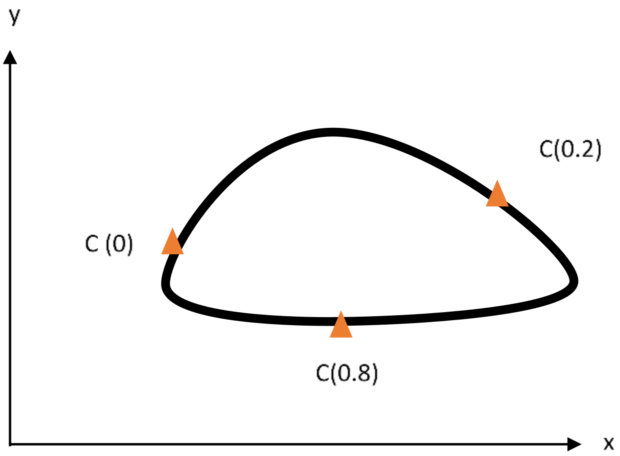


Fig. 2.1. Representation of a closed planar curve

The tangent and normal of a curve can be defined as,

By the basic theory of geometry a tangent can be obtained by taking the derivative of the curve,

So, a unit tangent can be given by,

$$\vec{t} = \frac{C_p}{|C_p|} = Cs \tag{2.23}$$

where $C_p = \dfrac{\partial C}{\partial p}$

The normal to the curve will always be in perpendicular direction to the tangent.

$|C_s| = 1$

$\langle C_s, C_s \rangle = 1$

By taking partial derivative on both sides w.r.t 's'

$\langle C_s, C_{ss} \rangle = 0$

$\therefore C_s \perp C_{ss}$

$$C_{ss} = k\vec{n} \tag{2.24}$$

where, $\vec{n}$ is normal to the curve, k represents the curvature

2.3.2 Curve evolution

The planar curves can be made to change their topology with time this is called as Curve evolution. The deformation of the curve is controlled by a velocity factor $\vec{V}$.
We can define the time changing curve as,

$$\frac{\partial C(p)}{\partial t} = \vec{V}(p,t) \tag{2.25}$$

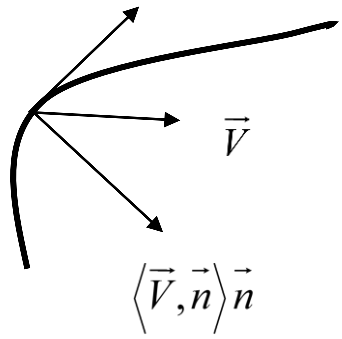


Fig. 2.2 Tangent and normal of a velocity vector of a curve

The geometry of the evolving curve is not affected by the tangential component. So, we can consider only the component in normal direction.

$$C_t = \langle \vec{V}, \vec{n} \rangle \vec{n} \tag{2.26}$$

$C_t = \vec{n}$	Constant flow
$C_t = k\vec{n}$	Curvature flow
$C_t = k^{1/3}\vec{n}$	Equi-Affine heat flow
$C_t = (g(x,y)k - \langle \nabla g(x,y), \vec{n} \rangle)\vec{n}$	Geodesic Active contours [7]

Table 1.1.Some of the basic velocity functions for curve evolution

The curve evolution can be controlled by different types of velocities. Care should be taken in order to control the motion of a curve in a homogeneous manner, otherwise it might lead to disturbances in curves. Some of the basic velocity functions that can be considered are listed in Table 1.1

The constant flow leads the curve to evolve with unit velocity in normal direction, it causes the curve to change the topology which will not happen in curvature flow as the curve changes depends on the curvature which makes it smooth. Curvature flow is rotational invariant. In this the curve vanishes as a circular point, while in equi-affine flow curve vanishes as an elliptical point. Equi-affine flow is affine invariant. When it comes to geodesic active contours they make the curvature flow depend on the edge information. In application to image processing $g(x, y)$ is defined such that it will have higher values near the edges, which makes the curve stop at the edges in the image.

2.4 LEVEL SET METHOD

Images often contain complex topologies and weak boundaries segmenting out these kind of images is problematic and most of the times results in faulty segmentation. Regular approaches like region growing fail to achieve consistent accuracy when dealing with complex boundaries.

Level set methods are used for tracking of shapes and interfaces. In the recent year's level set methods gained attention because of their successful and efficient applications. The recent developments in computer advancements made the faster phase growth of level set methods easier. It made researchers to shift from the conventional discrete domain to continuous domain and PDE's in image processing applications. The advantage of level set methods over the parameterized models is that parameterization of the contour points is not required the computations can be carried out in a fixed Cartesian grid. It shows its superiority when changing the topology of curves like merging and splitting. Achieving the same using the conventional parameterized models is quite difficult because care should be taken in case of splitting where the new points should come from and in the case of merging which points are eliminated. On the other hand level set methods can naturally change topologies.

Dervieux and Thomassett have developed the basic concepts of Level set method, but with the work of Osher and Sethian ('87) [4] only level set Method became popular for the first time. At first level set method was just used to track shapes and interfaces by Osher and Sethian but after some years the reach of the level set method has been expanded to multiple domains like image processing, computer vision, fluid dynamics etc. The Level set function is defined as a zero level set which can represent the contour of a higher dimensional space.

The planar curves discussed in the above section can be used to detect the boundaries of the objects in the image. Level set methods are the popular techniques used for numerical implementation of the curve evolution. The level set ϕ will take positive values inside the curve, outside the curve it will have negative values and zero value on the curve.

A closed planar curve or a contour can be defined as the zero level set of ϕ , Given by

$$C = \{(x, y), \phi(x, y) = 0\}$$

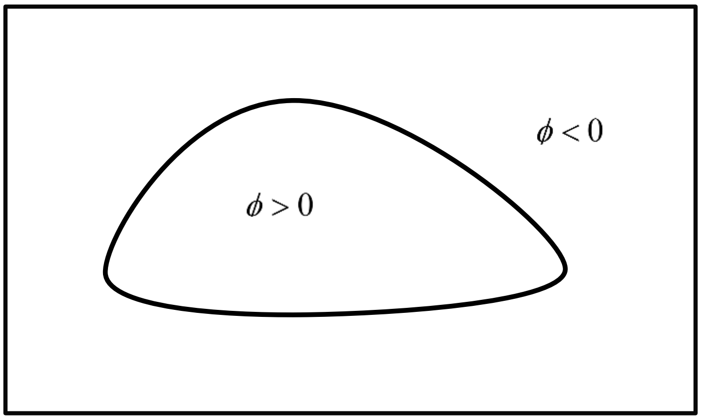


Fig. 2.3. A Curve representation using level set

To implement curvature flow now the level set will evolve so that the zero level set will change with time which will give the contour. Before that it is important to derive some basic properties of levelset which will be utilized in future.

The normal of the level set can be obtained by derivation, since zero change along the level set

2.4.1 Level set Normal:

$$\phi_s(x, y) = 0$$

$$\phi_s(x, y) = \phi_x x_s + \phi_y y_s = \langle \nabla \phi, \vec{T} \rangle$$

$$\left\langle \nabla\phi, \vec{T} \right\rangle = 0$$

$$\left\langle \frac{\nabla\phi}{|\nabla\phi|}, \vec{T} \right\rangle = 0$$ (Since the normal should have unit length, divide it by $|\nabla\phi|$)

$$\frac{\nabla\phi}{|\nabla\phi|} \perp \vec{T}$$

So,
$$\vec{N} = -\frac{\nabla\phi}{|\nabla\phi|} \tag{2.27}$$

2.4.2 Level set curvature:

The curvature can be obtained by taking the second derivative of the level set,

$$\phi_{ss}(x, y) = 0$$

$$\frac{d}{ds}\left\langle \nabla\phi, \vec{T} \right\rangle = \left\langle \frac{d}{ds}\nabla\phi, \vec{T} \right\rangle + \left\langle \nabla\phi, k\vec{N} \right\rangle = 0$$

$$\Rightarrow k\left\langle \nabla\phi, \frac{\nabla\phi}{|\nabla\phi|} \right\rangle = k\left\langle |\nabla\phi| \right\rangle = \left\langle [\phi_{xx}x_s + \phi_{xy}y_s + \phi_{yx}x_s + \phi_{yy}y_s], \vec{T} \right\rangle$$

$$\Rightarrow k\left\langle |\nabla\phi| \right\rangle = div(\nabla\phi)$$

So,
$$k = div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) \tag{2.28}$$

A new variable time can be included in time varying level set function $\phi(x, y, t)$ .The level set formulation defined by Osher and Sethian [4] is given by,

$$\frac{\partial\phi}{\partial t} = V\,|\nabla\phi| \tag{2.29}$$

The level set will evolve with time using the above equation, it can be derived as

In planar curves, $C_t = V\vec{N}$

By taking derivative w.r.to 't'

$$\frac{d}{dx}\phi(x,y,t)=0$$

$$\phi_x x_t+\phi_y y_t+\phi_t=0$$

$$\phi_t=-\langle\nabla\phi,C_t\rangle=-V\left\langle\nabla\phi,\vec{N}\right\rangle=V\left\langle\nabla\phi,\frac{\nabla\phi}{|\nabla\phi|}\right\rangle$$

$$\phi_t=V\,|\nabla\phi|$$

2.5. ACTIVE CONTOURS IN DETAIL

2.5.1 Shape Modelling with front propagation

In application to computer graphics and computer vision there is a requirement of accurately modelling the shape of an object. Malladi *et al.* [24] proposed a technique for shape modelling by extending the existed methods by overcoming their limitations. They have proposed a front propagation based level set approach which will evolve the curve based on the edge information available from the images. The edge term defined by Malladi *et al.* will be utilized to stop the curve at the evolution of the curve at the boundaries of a required object.

The energy guiding the level set will usually contain two types of energies, one is internal energy F_G which is depending on the geometry of the curve. It will regularize the curve shape from degrading and make it smooth and the second one F_A is the external energy which comes from the information of the image it will guide the curve how to evolve and where the object boundaries can occur. It can be represented as,

$$F=F_A+F_G \tag{2.30}$$

The evolution of the level set guided by the gradient descent is,

$$\frac{\partial\phi}{\partial t}=-F\,|\nabla\phi| \tag{2.31}$$

$$\frac{\partial \phi}{\partial t} = -(F_A |\nabla \phi| + F_G |\nabla \phi|) \tag{2.32}$$

If the internal energy $F_G = 0$, then the curve evolution depends on F_A, which is independent of curve's geometry , so curve evolves with a constant velocity. Malladi *et.al* defined a negative speed function F_I, that goes in addition with F_A. Which is defined as,

$$F_I = \frac{-F_A}{(M_1 - M_2)} \left(|\nabla G_\sigma * I| - M_2 \right) \tag{2.33}$$

where, $M_1 = \max(|\nabla G_\sigma * I|)$ and $M_2 = \min(|\nabla G_\sigma * I|)$. When the value of $|\nabla G_\sigma * I|$ takes maximum value (i.e., at the boundaries) we can observe from the equation that the force will become null. Which forces the curve to stop at boundaries.

But when $F_G \neq 0$ the force can't take null value, it forces the curve to flow even when the boundary is reached. To overcome this an extra term g has been proposed. It is defined as,

$$g = \frac{1}{1 + (|\nabla G_\sigma * I|)} \tag{2.34}$$

This term is added as a multiplicative term to the original force function. Because g will take values near to zero when $|\nabla G_\sigma * I|$ reaches high values. This will bring the force values also to zero, which will make the curve evolution to stop at the boundaries. There are other options for g as well, which can reach null values even faster. One of such g is given as,

$$g = e^{-|\nabla G_\sigma * I|} \tag{2.35}$$

The updated level set evolution equation with new forces is,

$$\frac{\partial \phi}{\partial t} = -(F_A + F_I)|\nabla \phi| \qquad \text{If } F_G = 0 \tag{2.36}$$

$$\frac{\partial \phi}{\partial t} = -g(F_A + F_G)|\nabla \phi| \qquad \text{If } F_G \neq 0 \tag{2.37}$$

The above discussed concepts about the curve evolution on level set function are valid only for zero level set $\phi = 0$. In a level set evolution formation when considering complete level set ϕ, there should be force values for other points as well as which are not on $\phi = 0$ as well. For this Malladi *et al.* given an algorithm to define forces for other points on level set. Let us consider a point on $\phi = c$ it can be seen as a point which is c units away from the zero level set. For every point on non-zero level set $\phi \neq 0$ find its nearest point on zero level set $\phi = 0$ and assign its force to the point on $\phi \neq 0$. This process repeats on every iteration and using these forces total level set will evolve.

2.5.2 Geodesic Active Contours

Caselles *et al.* [7] proposed edge based technique which will extend the model proposed by Malladi *et al.* [24]. This method acted as one of the state of the art model for recent advances in edge based active contour models. The main goal of this model is to take the advantage of both geometric active contour which are based on curve evolution and classical snakes which are based on energy minimization .They have proposed a Riemannian metric based term and proved that the geodesic curve with a metric derived from image information in Riemannian space serve the similar to classical snake based models. They have derived the level set evolution by minimizing the length based on the new metric.

The conventional snake model given by Kass *et al.* [34] can be represented as,

$$E = \alpha \int_0^1 \left| C_p \right|^2 dp + \beta \int_0^1 \left| C_{pp} \right|^2 dp - \lambda \int_0^1 \left| \nabla I(C) \right|^2 dp \tag{2.38}$$

where α, β and λ are positive real constants. The first two terms are to maintain smoothness of the curve that are based on curves properties and also called as internal energy. The third term is external energy obtained from image .Which will make the contour attract towards the boundaries of object in the image. In the above model the goal is to find a contour which will be at the points where $\left| \nabla I \right|$ is maximum and having good number of edges which is controlled by the internal energy. Which will be controlled by the parameters α, β and λ.

The modified model proposed by Caselles *et al.* [7] is by neglecting the second term in the snake model and by replacing the edge term with a more generalized edge indicator g . Which is strictly decreasing function.

$$E = \alpha\lambda \int_0^1 \left| C_p \right|^2 dp + \lambda \int_0^1 g(\left|\nabla I(C)\right|)^2 \, dp \tag{2.39}$$

Based on the above model, a length term in Riemannian space is derived. That is

$$L_R = \int_0^1 g(\left|\nabla I(C)\right|) \left| C_p \right| dp \tag{2.40}$$

Let us look at conventional formation of the Euclidean length term of a curve is,

$$L = \oint \left| C_p \right| dp = \oint ds \tag{2.41}$$

where ds is Euclidean arc-length and we know that $C_t = k\vec{n}$, k is Euclidean curvature

The equation (2.40) can be written as

$$L_R = \int_0^L g(\left|\nabla I(C)\right|) ds \tag{2.42}$$

By observing the above formulation we can note that, This new term is obtained weighting the conventional length with the factor $g(\left|\nabla I(C)\right|)$. This new metric contains the edge information, so when minimizing the new length term we are considering image information also in contrast to the conventional length term where only curve’s internal characteristics are considered. Which can lead to better results.

So the curve flow equations can be obtained by finding the Euler-Lagrange of the proposed energy function. The resultant curve evolution equation will be

$$C_t = g(I)k\vec{n} - (\nabla g.\vec{n})\vec{n} \tag{2.43}$$

The GAC (Geodesic Active Contour) model proposed by Caselles *et al.* [7]can be implemented using the level set evolution method proposed by Sethian *et al.*[4] . By considering the level set function ϕ where the zero level set $\phi(x) = 0$ intrinsically represents the curve to be evolved. From the previous chapter’s knowledge we can state that

$$\text{If } C_t = V\vec{n} \tag{2.44}$$

$$\text{Then } \frac{\partial \phi}{\partial t} = V\left|\nabla \phi\right| \tag{2.45}$$

So, the level set evolution equation for the proposed model will be,

$$\frac{\partial \phi}{\partial t} = g(I)k\left|\nabla \phi\right| + \nabla g.\nabla \phi \tag{2.46}$$

The proposed evolution can make the curve stop at the boundaries where the gradient value is not complete null. While in previous models the curve stops only at ideal edge ($g = 0$), which is not possible in real images. This model can also. To increase the speed of convergence an optional term $cg(I)\left|\nabla \phi\right|$ can also be added to the equation. It might also help to avoid local minimum .Where c is the Lagrange multiplier. The modified level set evolution equation is,

$$\frac{\partial \phi}{\partial t} = g(I)(c+k)\left|\nabla \phi\right| + \nabla g.\nabla \phi \tag{2.47}$$

2.5.3 Mumford-Shah Model

Mumford and Shah [1] proposed a piece-wise smooth model to segment the image, based on the following assumption a) image varies smoothly with in a region and b) varies discontinuously across the boundaries between regions. Let us assume that $I : \Omega \rightarrow \mathbb{R}$ is a gray scale image, Where Ω represents the image domain. Segmentation of the image is achieved by dividing Ω into different regions $\left\{\Omega_1, \Omega_2, ..., \Omega_N\right\}$ which are separated by a contour C, it can be represent as $\Omega = \bigcup_i \Omega_i \cup C$.

To achieve piece-wise smooth approximation of the image I the model proposed by Mumford-Shah should be minimized which is given by,

$$F^{MS}(u,C) = \int_{\Omega}\left(I-u\right)^2 dx + \mu \int_{\Omega \backslash C} \nabla u^2 dx + v\left|C\right| \tag{2.48}$$

where, μ and v are fixed parameters. u represents the piece-wise smooth approximation of the observed image I , u should be such that it should contain smoothly varying values with in a region Ω_i and varies rapidly across the boundaries. The first and second terms in the above model will make u follow the above described properties. First term will force u to close to the image I , smoothness of u is controlled by the second term.. $|C|$ gives the length the curve which will regularize the contour.

2.5.4 Two phase Chan-Vese model

The implementational approach for Mumford-Shah [1] model is proposed by Chan-Vese [5] by taking a special case of it. A Piece wise constant model is considered by Chan-Vese [5] where the piece wise smooth model u is replaced by a piece wise constant approximation of it. This can be also called as minimal partition problem.

The above mentioned Mumford-Shah model can also be represented like,

$$F^{MS}(u_1,...,u_N,\Omega_1,...,\Omega_N) = \sum_{i=1}^{N}\left(\int_{\Omega_i}\left(I-u_i\right)^2 dx + \int_{\Omega}\nabla u_i^{\,2}dx + v\left|C_i\right|\right) \tag{2.49}$$

Minimization of two phase model of Chan-Vese will achieve two separate regions Ω_1,Ω_2 in the image which are separated by a contour C. They assumed that the intensities inside these two regions are approximately piecewise constant. The fitting term used for this purpose is given by,

$$E(C) = E_1(C) + E_2(C) = \int_{\Omega_1}\left(I-c_1\right)^2 dx + \int_{\Omega_2}\left(I-c_2\right)^2 dx \tag{2.50}$$

where c_1 and c_2 represents the intensity average of image I in the regions Ω_1,Ω_2 respectively. Ω_1,Ω_2 can be assumed regions inside and outside of contour C respectively.

The above mentioned fitting function will achieve the value when the contour is exactly at the boundaries. ($E(C_0)=0$, C_0 is the boundary of the piecewise constant image). When the contour C is outside the boundary $E_1(C)>0, E_2(C)<0$ and if the contour C is outside of the boundary $E_1(C)<0, E_2(C)>0$.

The Energy function given by Chan-Vese by adding regularizing terms to the fitting energy is given by,

$$F^{CV}(c_1,c_2,C)=\mu.Length(C)+v.Area(\Omega_1)+\lambda_1\int_{\Omega_1}\left(I-c_1\right)^2 dx+\lambda_2\int_{\Omega_2}\left(I-c_2\right)^2 dx \quad (2.51)$$

where μ and v are constant parameters whose values can be positive or zero. Values of λ_1,λ_2 will affect the weightage given to the each region, most of the cases there are taken as unity. The model given above will be equivalent to the Mumford-Shah model by taking $v=0$, $\lambda_1=\lambda_2=\lambda$ and the constant approximations c_1,c_2 are replaced by piecewise smooth approximations.

2.5.4.1 Implementation of Chan-Vese model using Level sets

Chan-vese used the level set approach proposed by Osher and Sethian [4] to implement their model. The curve/contour C was represented by the zero level set $\phi_0(x)$ of the level set function $\phi(x):\Omega\rightarrow\Re$, The level set function is defined such that,

$$\phi(x)=\left\{\begin{array}{l} x\in C;\phi(x)=0; \\ x\in\Omega_1;\phi(x)>0; \\ x\in\Omega_2;\phi(x)<0; \end{array}\right\} \quad (2.52)$$

Using the level set function definition, we can change the energy function of the Chan- Vese model into the level set form by replacing the unknown C with unknown ϕ.

$$\begin{aligned} Length(C)=Length\{\phi=0\}&=\int_{\Omega}\left|\nabla H(\phi(x))\right|dx \\ &=\int_{\Omega}\delta_0\left(\phi(x)\right)\left|\nabla\phi(x)\right|dx \end{aligned} \quad (2.53)$$

$$Area(\Omega_1)=Area(\phi\geq 0)=\int_{\Omega}H(\phi(x))dx \quad (2.54)$$

where, $H(\phi(x))$ represents Heaviside function and $\delta_0\left(\phi(x)\right)$ represents the Dirac delta function. They are defined as,

$$H(z)=\begin{cases}1, if\ z\geq 0\\ 0, if\ z<0\end{cases} \text{ and } \delta_0(z)=\frac{d}{dz}H(z) \quad (2.55)$$

The Fitting energy term can also be represented using the level set function,

$$E_1(C)=\int_{\Omega_1}\left(I-c_1\right)^2dx=\int_{\phi>0}\left(I-c_1\right)^2dx=\int_{\Omega}\left(I-c_1\right)^2H(\phi(x))dx \quad (2.56)$$

$$E_2(C)=\int_{\Omega_2}\left(I-c_2\right)^2dx=\int_{\phi<0}\left(I-c_2\right)^2dx=\int_{\Omega}\left(I-c_2\right)^2(1-H(\phi(x)))dx \quad (2.57)$$

The Chan-Vese energy function in level set form is,

$$F^{CV}(c_1,c_2,\phi)=\mu\int_{\Omega}\delta_0\left(\phi(x)\right)\left|\nabla\phi(x)\right|dx+v\int_{\Omega}H(\phi(x))dx$$
$$+\lambda_1\int_{\Omega}\left(I-c_1\right)^2H(\phi(x))dx+\lambda_2\int_{\Omega}\left(I-c_2\right)^2(1-H(\phi(x)))dx \quad (2.58)$$

The optimal values of c_1 and c_2 can be achieved by minimizing the fitting energy $E(C)$ with respect to ϕ, The updating equations are given by,

$$c_1(\phi)=\frac{\int_{\Omega}I(x)H(\phi(x))dx}{\int_{\Omega}H(\phi(x))dx} \quad (2.59)$$

$$c_2(\phi)=\frac{\int_{\Omega}I(x)(1-H(\phi(x)))dx}{\int_{\Omega}(1-H(\phi(x)))dx} \quad (2.60)$$

Regularized versions of H and δ_0 are used for numerical computations and practical implementation purposes. H was approximated in the present model using the following expression and δ_0 can be obtained by taking derivative of it.

$$H_{\varepsilon}(z)=\frac{1}{2}\left(1+\frac{2}{\pi}\arctan\left(\frac{z}{\varepsilon}\right)\right) \quad (2.61)$$

To achieve the segmentation using the Chan-Vese model the level set function $\phi(x)$ has to get an extra parameter t *(time)*. As proposed by Osher *et al.* the level set function should be updated in a repetitive manner so as to achieve the optimal solution to fit the model. Time varying level set function $\phi(x,t)$ is initialized with an initial contour $\phi^0=\phi(x,0)$.

The level set update function is given by,

$$\frac{\partial\phi}{\partial t}=-\frac{\partial F}{\partial\phi} \quad (2.62)$$

Minimizing F^{CV} with respect to ϕ by keeping c_1 and c_2 using Euler – Lagrange equation results in,

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left(\lambda_2 \left(I - c_2\right)^2 - \lambda_1 \left(I - c_1\right)^2 + \mu div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - v \right) \tag{2.63}$$

2.5.4.2 Algorithm

The important steps of Chan-Vese model can be summarized as follows

- Initialize $\phi^0 = \phi(x,0)$, Iter=0
- Update $c_1(\phi^{Iter})$ and $c_2(\phi^{Iter})$ using update equations
- Update ϕ^{Iter+1}, using the above PDE
- Re-initialization (if required)
- Check if c_1 and c_2 are changing, if yes repeat the process

2.5.5 Multiphase Chan-Vese Model

Chan Vese [26] proposed a new approach ,it can be applied to multi-phase applications where the image can be segmented into multiple regions .The two phase case of the Mumford shah problem can only be used to segment an image into two regions using one level set function. It can be extended to multiphase and in [26] Chan-Vese tried to achieve this using just $\log N$ level sets for N-phase problem.

Before Chan-Vese Zhao *et.al* [51] proposed an approach deal with multiphase. In this approach they have considered N level sets to deal with N-phase problem. Each region can be defined using these level sets as $\Omega_i = \{x; \phi_i(x) > 0)$. The challenge to be faced is that N level sets defined shouldn't overlap and their union should result in complete domain Ω. To achieve this they have added a regularizing energy term $\lambda \int_\Omega \left(\sum_i H(\phi_i) - 1 \right)^2 dx$, at each time step λ is updated.

To understand the level set model proposed by Chan-Vese , let us assume $m = \log N$ level set functions $\Phi = \{\phi_1, \phi_2, ..., \phi_m\}$, and their corresponding Heaviside functions $H(\Phi)=\{H(\phi_1),H(\phi_2),...,H(\phi_m)\}$. The union of the zero level of these level set functions should result in boundaries of the image. A region can be defined using these m- Heaviside functions, this function will take the values either 0 or 1. So it can be considered as binary code, 2^m different levels can be represented using m-bits.

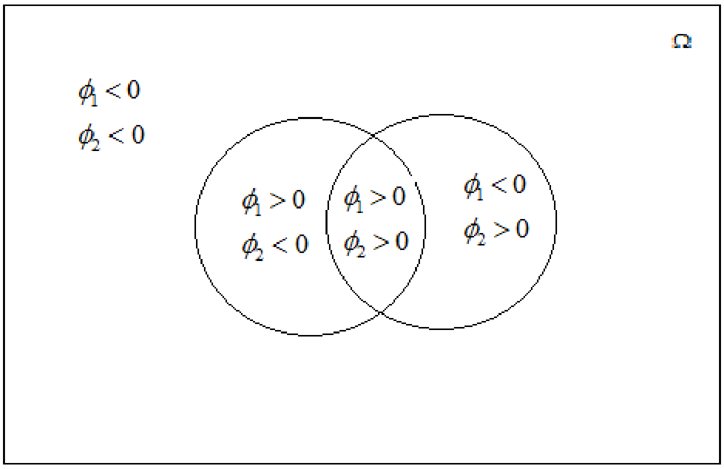


Fig. 2.1 Representation of four regions using 2 Level sets

For better understanding let us consider the 4-phase case which requires 2- level set functions which can be visualized as shown in the Fig. (2.1) and its fitting energy can be written as,

$$E(c,\Phi)=\int_\Omega (I-c_{11})^2 H(\phi_1)H(\phi_2)dx+\int_\Omega (I-c_{10})^2 H(\phi_1)(1-H(\phi_2))dx$$
$$+\int_\Omega (I-c_{01})^2 (1-H(\phi_1))H(\phi_2)dx+\int_\Omega (I-c_{00})^2 (1-H(\phi_1))(1-H(\phi_2))dx \qquad (2.64)$$

And the length term is given by,

$$Length(C)=\int_\Omega \left(|\nabla H(\phi_1(x))|+|\nabla H(\phi_2(x))|\right)dx \qquad (2.65)$$

where, $c=\{c_{11},c_{10},c_{01},c_{00}\}$ is representing the constant values for each region and $\Phi=\{\phi_1,\phi_2\}$. The parameter λ is assumed unit value here. The piece wise constant function u representing the approximation of image can be written as,

$$u=c_{11}H(\phi_1)H(\phi_2)+c_{10}H(\phi_1)(1-H(\phi_2))$$
$$+c_{01}(1-H(\phi_1))H(\phi_2)+c_{00}(1-H(\phi_1))(1-H(\phi_2)) \qquad (2.66)$$

The fitting energy term is minimized with respect to c by keeping Φ as constant , to achieve the updating equations for $c=\{c_{11},c_{10},c_{01},c_{00}\}$ which will change their values at each iteration. They are

$$c_{11}(\Phi)=\frac{\int_{\Omega} I(x)H_{\varepsilon}(\phi_1)H_{\varepsilon}(\phi_2)dx}{\int_{\Omega} H_{\varepsilon}(\phi_1)H_{\varepsilon}(\phi_2)dx} \tag{2.67}$$

,

$$c_{10}(\Phi)=\frac{\int_{\Omega} I(x)H_{\varepsilon}(\phi_1)(1-H_{\varepsilon}(\phi_2))dx}{\int_{\Omega} H_{\varepsilon}(\phi_1)(1-H_{\varepsilon}(\phi_2))dx} \tag{2.68}$$

$$c_{01}(\Phi)=\frac{\int_{\Omega} I(x)(1-H_{\varepsilon}(\phi_1))H_{\varepsilon}(\phi_2)dx}{\int_{\Omega} (1-H_{\varepsilon}(\phi_1))H_{\varepsilon}(\phi_2)dx} \tag{2.69}$$

,

$$c_{00}(\Phi)=\frac{\int_{\Omega} I(x)(1-H_{\varepsilon}(\phi_1))(1-H_{\varepsilon}(\phi_2))dx}{\int_{\Omega} (1-H_{\varepsilon}(\phi_1))(1-H_{\varepsilon}(\phi_2))dx} \tag{2.70}$$

By minimizing Euler –Lagrange equations by using the energy equation $F^{MCV}=E(c,\Phi)+Length(C)$ with respect to Φ by keeping c as constant are given by,

$$\begin{aligned}\frac{\partial\phi_1}{\partial t}&=\delta_{\varepsilon}(\phi_1)[v\left(\left(I-c_{10}\right)^2-\left(I-c_{00}\right)^2\right)\left(1-H(\phi_2)\right)\\&-\left(\left(I-c_{11}\right)^2-\left(I-c_{01}\right)^2\right)H(\phi_2)+vdiv\left(\frac{\nabla\phi_1}{|\nabla\phi_1|}\right)]\end{aligned} \tag{2.71}$$

$$\begin{aligned}\frac{\partial\phi_2}{\partial t}&=\delta_{\varepsilon}(\phi_2)[v\left(\left(I-c_{01}\right)^2-\left(I-c_{00}\right)^2\right)\left(1-H(\phi_2)\right)\\&-\left(\left(I-c_{11}\right)^2-\left(I-c_{10}\right)^2\right)H(\phi_2)+vdiv\left(\frac{\nabla\phi_2}{|\nabla\phi_2|}\right)]\end{aligned} \tag{2.72}$$

CHAPTER 3

PROPOSED METHOD

3.1 INTRODUCTION

Image segmentation has still a lot of scope to be explored. From many years research has been going on to fill this area. Still, there are many challenges to be faced, some are application specific. Complex boundaries, illumination variations, environmental noise, device noise, object specifications make the segmentation complex and a single method to deal with all these issues is not easy. There are various methods proposed based on region growing, edge detection, clustering and active contours to segment a given image.

In the recent years level set based methods are extensively been used in image segmentation applications [2]. Active contour use level set methods for mathematical implementation of their models. Level are method proposed by Osher and Sethian [4] is one of the popular approach. It acted as a base for modern active contour algorithms development. Level sets are introduced to image processing applications by Malladi *et al.* [24], Caselles *et al.*[18]. Active contour or snake models by Mumford Shah [1],Chan-Vese [5], Caselles *et al.* [7], Satyanand *et al.*[20], Chenyang *et al.*[9] and C. Li *et al.*[2] are some of the prominent models in the history.

Segmentation of medical images is still a challenging task. They often contain noise, inhomogeneity and blurred boundaries. Accurately detecting the tissue type is quite difficult. Imaging modalities like MRI images contain large of number of voxels to be segmented when considering a 3D model. MRI images contains regions like Gray matter, White matter, cerebral fluid, skin, skull, background. Most of these regions intensity properties look similar, so intensity based approaches sometimes fail to achieve good accuracies. In medical applications active contour models are very efficient in distinguishing the boundaries and extensively used in applications like detecting tumour or any deficiency in image modalities like Magnetic Resonance Image (MRI) and Position Emission Tomography (PET), function MRI (fMRI), diffusion MRI (dMRI) etc. But most of methods fail to perform better when images containing noise and inhomogeneity are given as input. There is a need to develop a method which can segment the image even in the presence of noise and intensity inhomogeneity.

In this paper the addition is twofold. (1) We add the fuzzy term to the model proposed in [2] and (2) an additional term is used to effectively deal with the noise. We propose a method by integrating active contours and fuzzy c-means clustering and a Sfcm based approach to deal with noise. An energy function is proposed by modelling the image intensity using a bias filed to nullify the intensity inhomogeneity and introducing fuzziness into it for better performance. We take help of the spatial term proposed in spatial fcm [50] method which utilizes neighbourhood membership function to reduce the effect of noise.

3.2 Proposed Model

Images are often corrupted by intensity inhomogeneity and noise it is a challenge task to segment these images. Chan-Vese[5] model is based on the assumption that intensities inside the regions are approximately piecewise constant. Which is not true in most of the cases .Chumming Li *et.al* [2,12] proposed a region based level set model to segment the images which are affected by intensity inhomogeneity. To deal with the issue they make use of bias field which will be used to model the inhomogeneity occurring in the image. Based on the properties of the bias field term they proposed a local region based energy function.

Modelling real world images as a multiplicative field added by noise. Which can be represented as,

$$I = bJ + n \tag{3.1}$$

where I is the observed image, J is the true image, b represents the bias filed term which will represent the intensity inhomogeneity and n is the additive noise.

The assumptions made in this model are

- The true image J assumed as a piecewise constant. i.e. in a given region Ω_i it will take a constant value c_i
- Bias field b is assumed to be slowly varying, which implies in a small neighbourhood b is constant.

3.2.1 Local region based energy function

In a given image I which is modelled using the Equation (3.1) and based on the assumptions made above, we can consider the value of b in a local neighbourhood region as constant. i.e. Consider a circular region $O_y \subset \{x : |x - y| \le r\}$, where r represents the radius of the circular region and $y \in \Omega$. Then,

$$b(x) \approx b(y) \text{ for } x \in O_y \tag{3.2}$$

So, intensity in the local region can be modelled as

$$I(x) = b(y)J(x) + n(x) \text{ for } x \in O_y \tag{3.3}$$

Based on the first assumption about $J(x)$, $I(x)$ can be written as

$$I(x) = b(y)c_i + n(x) \text{ for } x \in O_y \cap \Omega_i \tag{3.4}$$

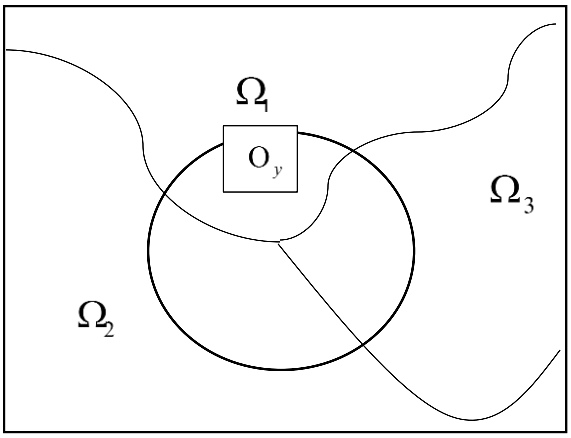


Fig 3.1 Representation of a local region O_y

The additive Gaussian noise term can be eliminated by considering, pixels in a region Ω_i are considered to be drawn from a Gaussian distribution with mean $m_i \approx b(y)c_i$.

The Local region based model mentioned above can be used to define the energy term of our model. The problem statement can be formulated as dividing the region O_y into N clusters, having centres at $m_i \approx b(y)c_i$, i varying from 1 to N. An energy by using the Fuzzy c-means [38] concept is defined to achieve this purpose. In contrast to the K-means clustering model used in [2]. This fuzzy factor will make each pixel assigned to a region based on its membership function. It will be more suitable for these type of applications on image, instead of hard assignment [16]. This membership function is also utilized to reduce the effect of noise as discussed Section 3.5. In continuous form the energy can be written as,

$$F_y = \sum_{i=1}^{N} \int_{O_y \cap \Omega_i} u_i^m(x)\left(I(x) - m_i\right)^2 dx \tag{3.5}$$

The fuzzy factor m decide the fuzziness, its value is normally taken as 2.

In a local region O_y the centre of the cluster m_i can be replaced by $b(y)c_i$ and considering a window function $W(y-x) \subset \left\{W(y-x) = 0, x \notin O_y\right\}$.

$$F_y \approx \sum_{i=1}^{N} \int_{\Omega_i} u_i^m(x) W(y-x)\left(I - b(y)c_i\right)^2 dx \tag{3.6}$$

The F_y will represent the energy after assigning the each pixel in the neighbourhood O_y to any of the N clusters. The purpose is achieved when the energy F_y is minimized. The overall energy by considering the total image domain Ω is given by,

$$F \triangleq \int F_y dy \tag{3.7}$$

i.e.,

$$F(b, c, u, \Omega_1, \Omega_2, ..., \Omega_N) \triangleq \int \left(\sum_{i=1}^{N} \int_{\Omega_i} u_i^m(x) W(y-x)\left(I - b(y)c_i\right)^2 dx \right) dy \tag{3.8}$$

where, $c = \{c_1, c_2, ..., c_N\}$ represents the constants .The selection of Kernel W selection is flexible. It should take null values outside the local region for a given x and y. i.e., $W(z) = 0$ for $|z| > r$ and inside the region it should have a sum of unity i.e., $\int W(z) = 1$. The one that is used in this model is,

$$W(z) = \begin{cases} \frac{1}{a} e^{-|u|^2 / 2\sigma^2}, for\ |u| \le r \\ 0, otherwise \end{cases} \tag{3.9}$$

The selection of the parameter a, will depend on the value of σ (standard deviation) ,such that the property $\int W(z)=1$ is satisfied. The selection of r is one of the crucial factor the assumption made on bias field will be valid only when the neighbourhood is small. In an image with more intensity inhomogeneity the bias field varies faster so the value of r should be small.

There are two purposes to be achieved using the above energy function one is segmenting the image and other is to estimate the bias field. It can be achieved by minimizing F with respect to Ω_i, c_i and b, $i=1,...,N$.

3.2.2 Two phase level set Formulation

Firstly for simplicity purpose we observe the 2 phase level set model of the proposed model. In this we divide the image into two regions Ω_1,Ω_2 using a single level set ϕ. The two regions are defined using the level set as,

In the region $\Omega_1, M_1(\phi)=H(\phi)$ and in Ω_2 $M_2(\phi)=(1-H(\phi))$

The energy function can be rewritten using these definitions as,

$$F(b,c,u,\phi) \square \int\left(\sum_{i=1}^{2}\int u_i^m(x)W(y-x)\left(I-b(y)c_i\right)^2 M_i(\phi(x))dx\right)dy \qquad (3.10)$$

The order of integration can be exchanged and the equation becomes

$$F(b,c,u,\phi) \square \int\sum_{i=1}^{2}u_i^m(x)e_i(x)M_i(\phi(x))dx \qquad (3.11)$$

where $e_i(x)$ is defined as,

$$e_i(x)=\int W(y-x)\left(I-b(y)c_i\right)^2 dy \qquad (3.12)$$

For implementational purpose $e_i(x)$ can be written in matrix form as,

$$e_i(x)=I^2 1_K-2c_i I(b*K)+c_i^2(b^2*K) \qquad (3.13)$$

where, $*$ represents the convolution and $1_K(x) = \int W(y-x)dy$

The energy used for variational level set model is given by,

$$F(b,c,u,\phi) = -F(b,c,u,\phi) + vL(\phi) + \mu R_p(\phi) \tag{3.13}$$

In this equation $F(b,c,u,\phi)$ is used as data term.

$L(\phi)$ and $R_p(\phi)$ act as regularizing terms which are defined as,

$$L(\phi) = \int \left|\nabla H(\phi)\right| dx \tag{3.14}$$

It will represents the length of the contour or zero level set of ϕ.Which will serve the purpose of keeping the curve smooth by punishing arc length of it.

and

$$R_p(\phi) = \int p\left|\nabla \phi\right| dx \tag{3.15}$$

this term is used to avoid the re-initialization in the level set evolution.

Re-initialization is one of the disadvantage of conventional level set model. During the level set evolution they develop irregularities. It may leads to numerical errors [6]. To overcome this formal approach is to stop the evolution and using a signed distance function to reshape it. It is quite difficult to predict when to apply the re-initialization some researchers also found that re-initialization as disagreement between theory and implementation. . To overcome this conflicts Li *et al.* [6] proposed a term called as distance regularized level set evolution (DRLSE). In the Level set functions signed distance will be maintained by this term.

The gradient descent form for evolution of level set function is given by,

$$\frac{\partial \phi}{\partial t} = -\frac{\partial F}{\partial \phi} \tag{3.16}$$

The minimization of the energy term $F(b,c,u,\phi)$ with respect to ϕ by keeping b,c as constants results in the following equation,

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[(u_1^m e_1 - u_2^m e_2) + v div\left(\frac{\nabla \phi}{|\nabla \phi|} \right) \right] \tag{3.17}$$
$$+\mu div(d_p(|\nabla \phi|)\nabla \phi)$$

where, $d_p(z) \triangleq \frac{p^{'}(s)}{s}$ (3.18)

To achieve the optimal solution using the energy function define , along with the level set function the values of bias field b and constants c are also should be updated in a repetitive manner. The updating equations for the bias field b , member ship function u and constants c can be obtained by minimizing $F^{Ch}(b,c,u,\phi)$ with respect to b **,** u or c by keeping the other variables constant.

<u>3.2.3 Multi-phase level set Formulation</u>

When considering segmentation as a multiphase problem the above mentioned 2-pahse model can be extended to multiphase applications. The multiphase model is utilized here to solve the segmentation problem using the proposed energy function. There will be requirement for $k \geq \log_2^N$ level sets to solve N-phase problem, by using these we can divide Ω into N regions which are represented by $M_i(\phi_1(x),...,\phi_k(x))$ and defined as,

$$M_i(\phi_1(x),...,\phi_k(x)) = \begin{cases} 1, & x \in \Omega_i \\ 0, otherwise \end{cases} \tag{3.19}$$

The energy function F for multiphase is given by,

$$F(b,c,u,\Phi) = \int \sum_{i=1}^{N} u_i^m(x) e_i(x) M_i(\Phi(x)) dy \tag{3.20}$$

where, $\Phi = \{\phi_1,...,\phi_k\}$ represents the level set vector.

The final energy function having regularizing terms is given by,

$$F(b,c,\Phi) = -F(b,c,\Phi) + v\sum_{i=1}^{k} L(\phi_i) + \mu \sum_{i=1}^{k} R_p(\phi_i) \tag{3.21}$$

Gradient descent equations can be obtained in a similar fashion as obtained in 2-phase problem, by minimizing $F(b,c,\Phi)$ with respect to Φ by keeping as b,c constants and they are obtained as,

$$\frac{\partial \phi_j}{\partial t} = \sum_{i=1}^{N} \frac{\partial M_i(\Phi)}{\partial \phi_j} u_i^m(x) e_i + \delta(\phi_j) v div\left(\frac{\nabla \phi_j}{|\nabla \phi_j|}\right) + \mu div(d_p(|\nabla \phi_j|)\nabla \phi_j) \tag{3.22}$$

where, $j = 1, 2, ..., k$

<u>3.2.4 Updating Equations</u>

3.2.4.1 Updating equation for constants

To get the update equation for c minimize $F(b,c,\Phi)$ with respect to c by keeping b,Φ,u as constants, the estimated value of the constant vector c is denoted by $\hat{c}$ and is obtained as,

If we take the derivative w.r.to c_j by taking b,Φ,u as constant, the other terms in the summation are independent of c_j, except the j^{th} term,

$$\frac{\partial}{\partial c_j} F(b,c,u,\Phi) = \int u_j^m(x) \frac{\partial e_j(x)}{\partial c_j} M_j(\Phi) dx = 0 \tag{3.23}$$

and

$$\frac{\partial e_j(x)}{\partial c_j} = -2.\int W(y-x)(I(x) - b(y)c_j)b(y) \tag{3.24}$$

By substituting $\frac{\partial e_j(x)}{\partial c_j}$ in the above equation,

$$\frac{\partial}{\partial c_j} F(b,c,u,\Phi) = \int u_j^m(x) \int W(y-x)(I(x)-b(y)c_j)b(y)M_j(\Phi)dx = 0 \tag{3.25}$$

$$\int u_j^m(x)I(x)M_j(\Phi)\left(\int W(y-x)b(y)dy\right)dx = c_j \int u_j^m(x)M_j(\Phi)\left(\int W(y-x)b^2(y)dy\right)dx \tag{3.26}$$

$$c_j = \frac{\int (b*W)IM_j(\Phi)dx}{\int (b^2*W)M_j(\Phi)dx}, \quad j = 1,2,...,N \tag{3.27}$$

3.2.4.2 Updating equation for bias field

By minimizing $F(b,c,u,\Phi)$ with respect b to by keeping c,Φ,u as constants. The estimated value of b is obtained and it is given by,

$$\frac{\partial}{\partial b(y)} F(b,c,u,\Phi) = \sum_{i=1}^{k} \int u_i^m(x) \frac{\partial e_i(x)}{\partial b} M_i(\Phi)dx = 0 \tag{3.28}$$

and

$$\frac{\partial e_i(x)}{\partial b(y)} = -2.W(y-x)(I(x)-b(y)c_i)c_i \tag{3.29}$$

By substituting $\frac{\partial e_i(x)}{\partial b(y)}$ in the above equation gives,

$$\frac{\partial}{\partial b(y)} F(b,c,u,\Phi) = \sum_{i=1}^{k} \int u_j^m(x)W(y-x)(I(x)-b(y)c_i)c_iM_i(\Phi)dx = 0 \tag{3.30}$$

$$\int W(y-x)\left(\sum_{i=1}^{k} u_i^m(x) I(x) M_i(\Phi) c_i\right) dx$$
$$= b(y)\left(\int W(y-x)\left(\sum_{i=1}^{k} u_i^m(x) M_i(\Phi) c_i^{\,2}\right) dx\right) \qquad (3.31)$$

$$\hat{b} = \frac{(IJ^{(1)}) * W}{J^{(2)} * W} \qquad (3.32)$$

where, $J^{(1)} = \sum_{i=1}^{N} c_i M_i(\Phi) u_i^m$ and $J^{(2)} = \sum_{i=1}^{N} c_i^{\,2} M_i(\Phi) u_i^{\,m}$

3.2.4.3 Updating equation for membership function

To get the updating equation for u_j, it can be done by solving the energy using Lagrange multiplier.

$$F_m = F + \lambda\left(1 - \sum_{i=1}^{k} u_i^m(x)\right) \qquad (3.33)$$

$$\because \sum_{i=1}^{k} u_i^m(x) = 1, \forall x \qquad (3.34)$$

Taking the derivative of F_m with respect to respect u_j and equating it to zero, by keeping b, c, Φ as constants. The estimated value of u_j can be obtained as,

$$\frac{\partial}{\partial u_j(x)} F_m = m u_j^{m-1}(x) e_j(x) - \lambda = 0 \qquad (3.35)$$

$$\Rightarrow u_j = \left(\frac{\lambda}{m u_j^{m-1}(x) e_j(x)}\right)^{1/(m-1)} \qquad (3.36)$$

$$\because \sum_{i=1}^{k} u_i^m(x) = 1, \forall x$$

$$\sum_{i=1}^{k}\left(\frac{\lambda}{mu_i^{m-1}(x)e_i(x)}\right)^{1/(m-1)}=1 \tag{3.37}$$

$$\lambda=\left(\sum_{i=1}^{k}\left(mu_i^{m-1}(x)e_i(x)\right)^{-1/(m-1)}\right)^{-(m-1)} \tag{3.38}$$

By, substituting value of λ, in u_j gives

$$u_j=\frac{\left(u_j^{m-1}(x)e_j(x)\right)^{-1/(m-1)}}{\sum_{i=1}^{k}\left(u_i^{m-1}(x)e_i(x)\right)^{-1/(m-1)}} \tag{3.39}$$

3.2.5 Spatial term for reducing noise effect

Using the proposed energy function we can deal with intensity inhomogeneity, In this section we are introducing a spatial term to subjugate the noise effect. A spatial term for fuzzy clustering using membership function is introduced by K.S.Chuang *et al.* [50] . We use this spatial term in our level set formulation. At every step along with bias filed, constants, membership functions, spatial term is also calculated. The spatial term is given as,

$$h_i(x)=\sum_{x_r\in N_R(x)}u_i(x_r) \tag{3.40}$$

The average of membership values of neighbouring pixels is considered in this term. It is like mean filter applied on membership function. By taking this extra spatial term into consideration, a pixels membership value is decided by the neighbouring pixels. i.e., even if the centre pixel is noisy its effect can be truncated. Using this spatial term the membership function is updated again using the following term,

$$u_i(x) = \frac{u_i^p(x)h_i^q(x)}{\sum_{j=1}^{k} u_j^p(x)h_j^q(x)} \tag{3.41}$$

where, p and q represents the weightage given to each term. If noisy is heavy by considering large values for q, more weightage is given to the spatial term. It might result in better evolution of curve.

CHAPTER 4

RESULTS

The proposed method was tested on several synthetic and medical images and the results were compared with some of the state of the art techniques.

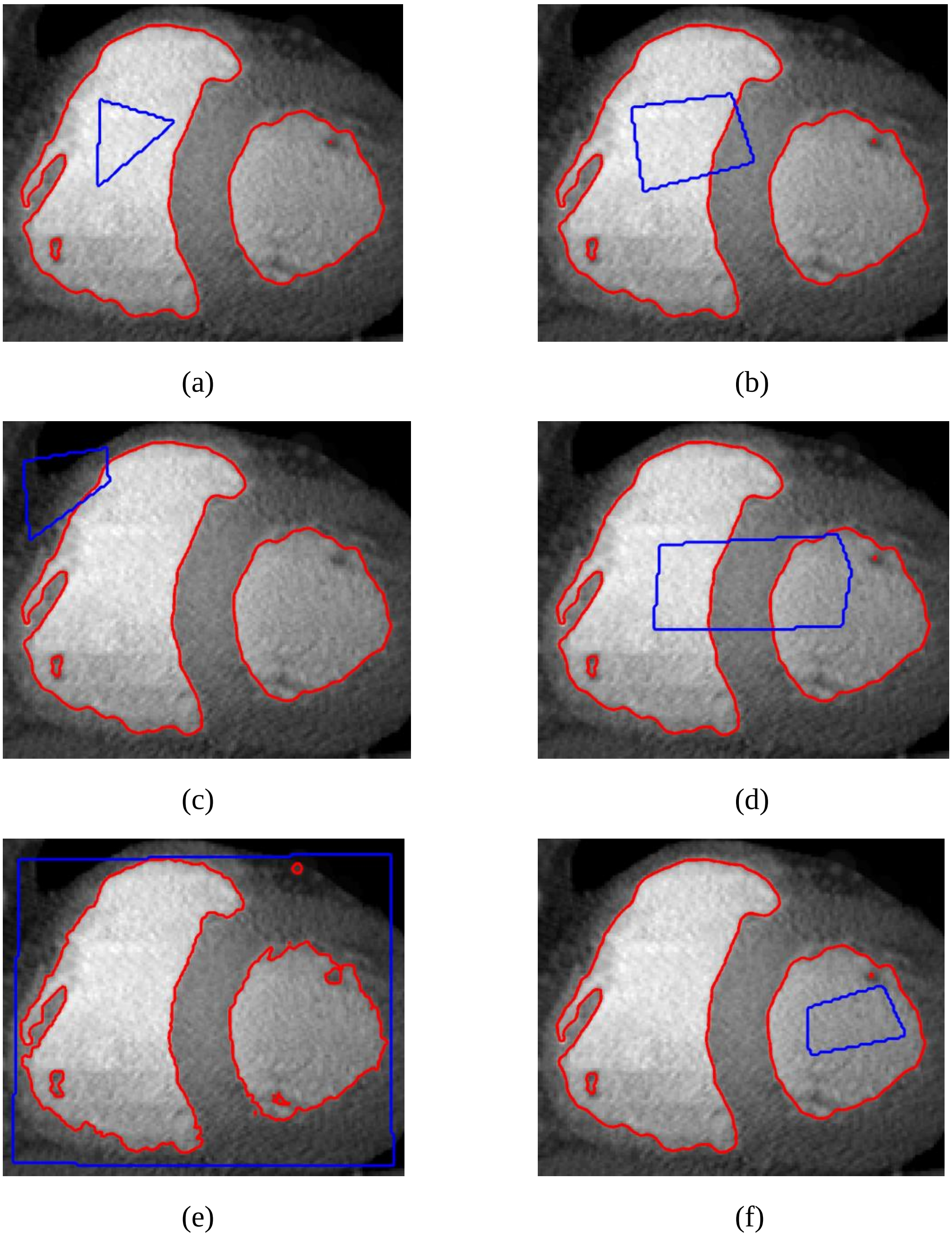
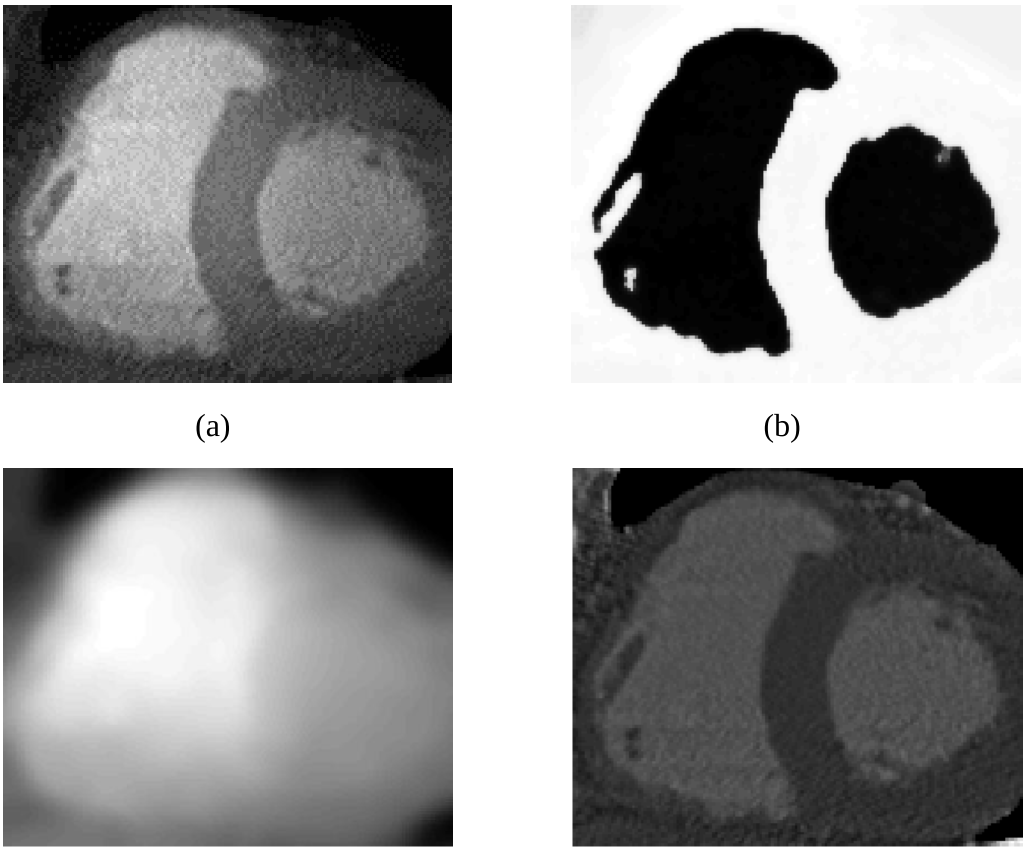


Fig.4.1 Contour evaluation using different types of Initialization

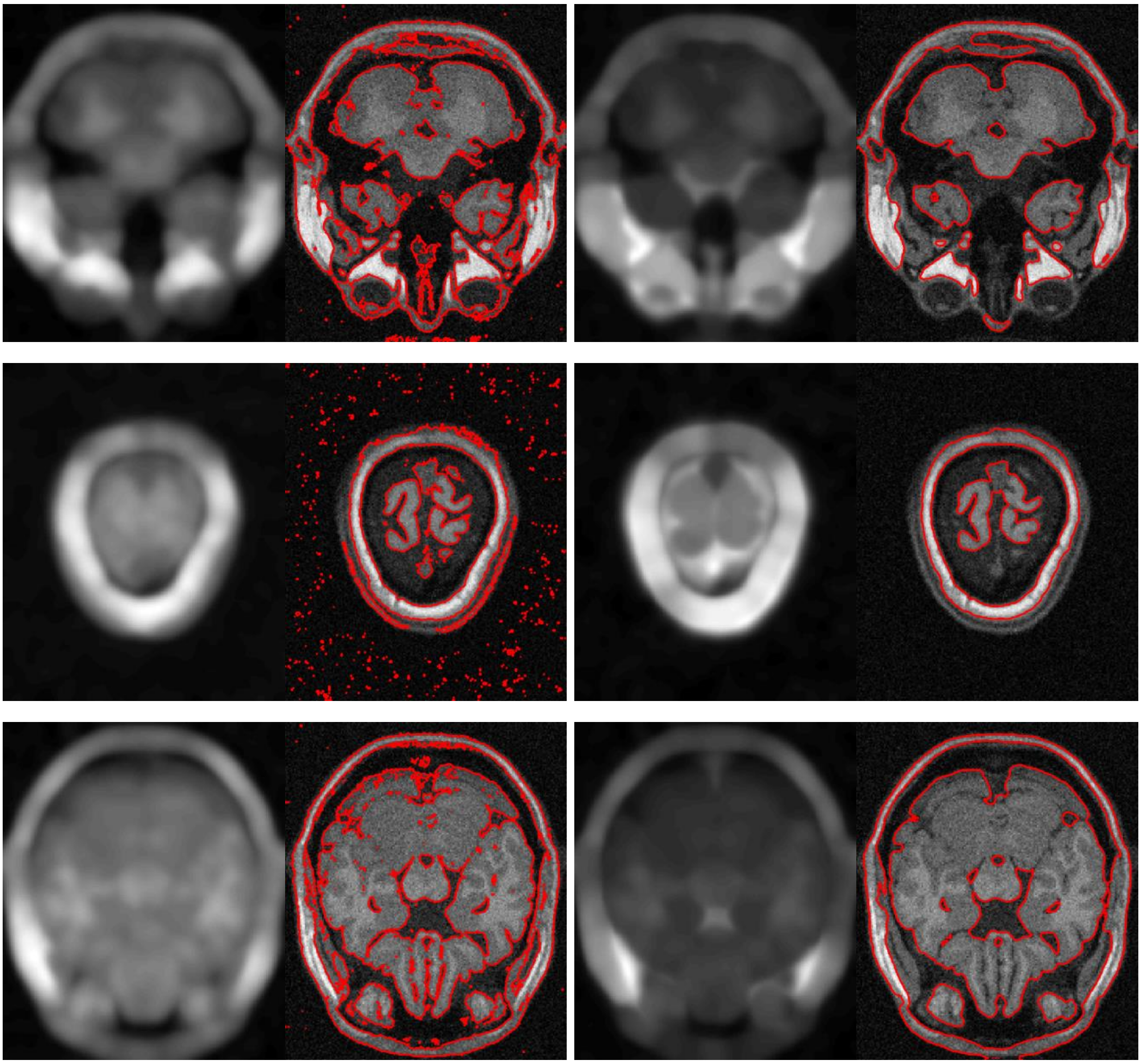
The methods were implemented in MATLAB 2013b on a 2.1 GHz Intel Core2Duo processor with 3GB RAM.

The values of parameters considered in this experimental analysis are $p=1, q=1$, $\sigma=4$, $\mu=1$, $t=0.1$ and the mask size for spatial term is considered as 3. Until and otherwise specified throughout the experiment the above specified values are only considered. Fig. 4.1 shows results of evaluation of our method on a CT scan image of heart . From the figures we can observe that independent of the initialization of initial contour. The initial contour in Fig. 4.1(a) and 4.1(f) is inside the region of interest and in Fig. 4.1(e) it is completely outside the region. In all the cases the final contour is same. Here we have used the method without the spatial term and with spatial term included the results are same, since these are non-noisy images our method works well even on clear images. The original CT scan image, the final segmented image, bias field and corrected image using the bias field are given in Fig. 4.2 (a),(b),(c),(d) respectively.



(a) (b) (c) (d)

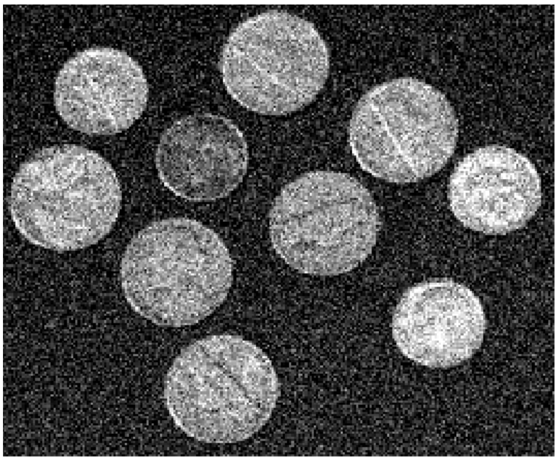
Fig. 4.2 (a) Heart CT scan image, (b) Segmented image, (c) Bias field image, (d) Bias corrected image



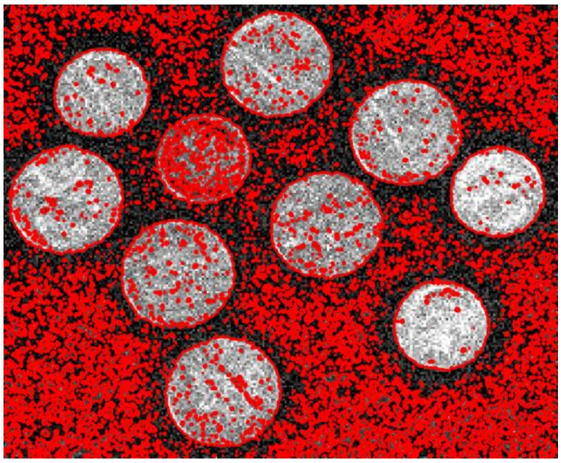
(a) (b) (c) (d)

Fig.4.3 Results of proposed method and Li's method [12] on a noisy MRI image. Column (a, b) Bias field & Final contour using Li's method [2],Column(c, d) Bias field & Final contour using proposed method.

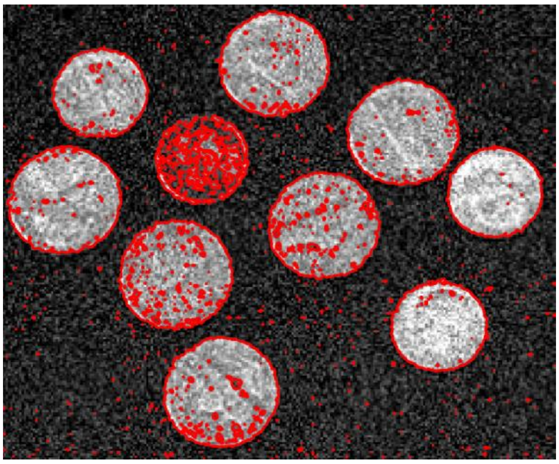
In Fig. 4.3 MRI synthetic images obtained from Brain web [61] data set are used. The MRI data used is T1 normal MRI of 1mm thickness with noise density of 9% and intensity non-uniformity of 40%. The column c&d in Fig. 4.3 are the estimated bias field and final contour obtained using the proposed method, similarly column a&b are obtained using method proposed by C.Li *et al.*[2]. From the figure we can observe that the proposed method can result in smooth contour even in the presence of noise and non-uniformity.



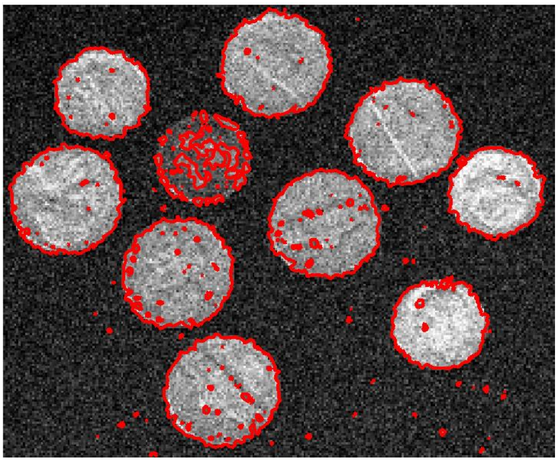
(a)



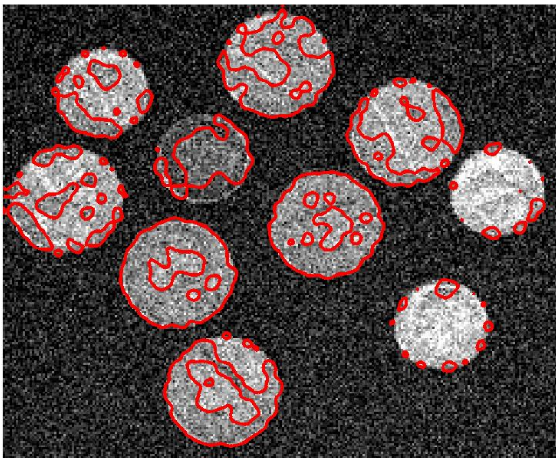
(b)



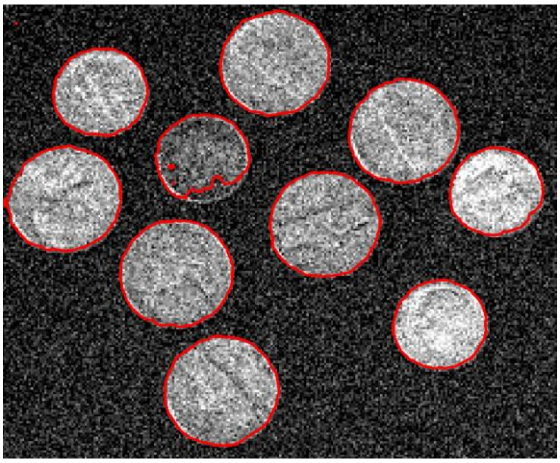
(c)



(d)



(e)



(f)

Fig. 4.4: (a) Noisy coins image with Gaussian noise density of $\sigma = 0.02$, Results using (b) C. LI *et al.*[2], (c) CV [4], (d) S.B. Arabe *et al.*[52], (e) B.N. Li *et al.*[10] ,(f) Proposed Method

To compare the proposed method with other methods, an image added with Gaussian noise of 2% is used. The results are shown in Fig. 4. Fig. 4a contains the original noisy image and Fig. 4.4(b-e) contains the results of methods in [2],[4],[52] and [10] respectively and Fig. 4.4 f contains result of the proposed method. As we can observe from the images final contour obtained by other methods is affected by some extent by the noise, by our method the effect is very less.

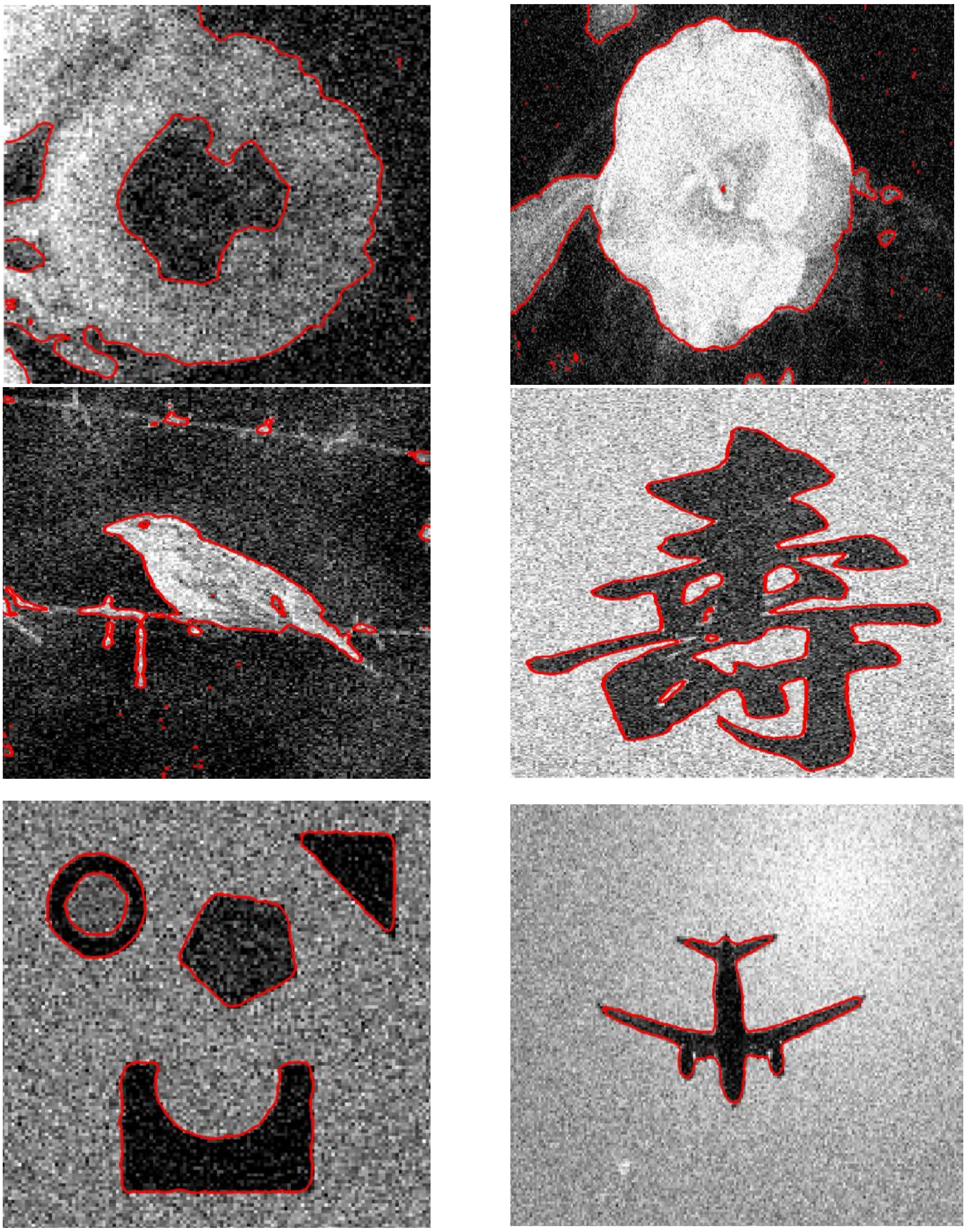


Fig. 4.5: Results of Proposed Method applied on natural and synthetic images with added Gaussian noise of $\sigma = 0.02$

The method is also verified on images other than MRI images. Some natural and synthetic images are considered in Fig. 5. They are added with a Gaussian noise of 2%. The results are satisfactory even in the presence of noise. The final contour is at the boundary of the required region by avoiding noise.

CHAPTER 5

CONCLUSION

The method proposed in this report is a region based active contour approach to segment the images corrupted by intensity inhomogeneity and noise. This was implemented using level set method. We have derived a fuzzy based energy function which will consider the region information multiplied by a weight factor. This gives it an additional information. The intensity of image is assumed to be modelled by original intensity multiplied by a bias filed. The estimation of the bias field is carried out parallel to the evolution of level set, it will be helpful to deal with inhomogeneity. In the same way an improvement to the membership updating function is done by considering the spatial information. It is done by including a second update for membership values by using a spatial term.

The proposed method can be utilized for medical applications where there is requirement for high quality and precise segmentation. This method can be computationally complex. The complexity can be reduced by careful implementation since multiple similar convolutions are used repeatedly. As a future work we suggest, to try different types of level set methods which can be used for implementation of active contours and energy functions can be developed using different FCM variations.

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