# ON THE PERFORMNACE ANALYSIS OF COMBINED (TIME SHARED) COMPOSITE MULTIPATH/SHADOWING AND UNSHADOWING FADING CHANNELS

*Thesis by*


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# DECLARATION

I hereby certify that:

- This thesis work contained original and authenticated which has been done by me under the esteemed supervision of my supervisors.
- This thesis work has not been submitted to any other Institute for any degree or diploma.
- This thesis has been written as per guidelines provided by the Institute in writing the thesis.
- Whenever I have used materials (data, theoretical analysis, and text) from other sources, I have given due credit to them by citing them in the text of the thesis and giving their details in the references.
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Rupender Singh

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# ABSTRACT

Combined (time-shared) shadowed/unshadowing fading environments are frequently encountered in different mobile realistic scenarios. These wireless channels are generally modeled as a time-shared sum of Rician multipath fading (unshadowing) and different Composite multipath/shadowing fading. In this thesis, we present the closed-form expression of composite (Weibull/Log-normal shadowed) fading using the efficient tool proposed by Holtzman. Using this result, the closed-form expression of combined (time-shared) shadowed/ Unshadowed fading is presented. We have derived closed form expressions for performance measuring parameters such as Probability Density Function (PDF) of Signal to Noise ratio (SNR), Amount of Fading (AOF), Outage Probability ($P_{out}$) and Channel Capacity(C/B). Simulated exact expressions are carried out to validate the accuracy of our derived closed form expressions.

The performance of diversity systems of MRC combiner in the presence of log normal shadowing also analyzed in this thesis. We have used Fenton-Wilkinson method to estimate the parameters for a single log-normal distribution that approximates the sum of log-normal random variables (RVs). PDF, CDF, AOF, $P_{out}$ and channel capacity are analyzed and expressed in closed form. The average bit error probabilities of MQAM and MPSK in the presence of log normal shadowing using Maximal Ratio Combining technique for L diversity branches are computed using Holtzman approximation. We proposed an analytical approach for the evaluation of important second order statistical parameters, as level crossing rate (LCR) and average outage duration (AOD) of MRC combiner in Log-Normal shadowed channels. New expressions have been derived for LCR and AOD in closed form. The results are valid for an arbitrary number of independent identically distributed diversity branches in the presence of Log -Normal shadowed. LCR and AOD have been plotted and also tabulated for different number of diversity branches. We have also proposed new statistical modeling of channel capacity for MRC combiner in the presence of log normal shadowing. This new approach to estimate higher order moments of channel capacity is simple and easy with compare to well-known results.

We also provide the performance analysis of Maximal Ratio Combiner (MRC) output for MQAM in Multiple Rician fading channels. The approximate formula of symbol error rates (SER) for MQAM over Gaussian channel have been used. Approximation is within 1 dB error for $M \geq 4$ and $0 \leq SNR \leq 30$ dB.

The proposed results in this thesis are supposed to be of great importance in assessing the performance of communication systems in composite fading channels.

**Keywords:** Composite (time shared) composite multipath/shadowing, Weibull-Lognormal shadowed (WL, Probability density function (PDF), Signal to Noise ratio (SNR), Amount of fading (AF), Outage probability ($P_{out}$), Channel Capacity(C/B), Log normal random variable, FW Method, Maximal Ratio Combining (MRC), Random Variables RVs, ABEP(Average Bit Error Probability), Level Crossing Rate (LCR), Average Outage Duration (AOD), , Independent Identically Distributed (IID), MQAM, MPSK.

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# LIST OF PUBLICATIONS

1. **Rupender Singh**, Sanjay Soni, R.S. Rao, Sandeep Kumar, "A New Closed-Form Distribution and Performance Analysis of a Composite Weibull/Log-Normal Fading Channel", (communicated) *IET Communications, 2015*.
2. **Rupender Singh**, S.K.Soni, "Performance Analysis of MRC combiner output for MQAM in Multiple Rician Fading Chanels", *International Journal of Digital Signal Processing*, vol. 7 No. 2, March 2015 (Print: ISSN 0974-9705 & Online: ISSN 0974-9594).
3. **Rupender Singh**, Sanjay Kumar Soni, "Analytical Level Crossing Rate and Average Outage Duration for MRC Combiner in Log Normal Shadowed Channels", *International Journal of Computer and Communication System Engineering*, 2015, Vol. 02, No. 02, 356-359, (*ISSN: 2312-7694*).
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10. **Rupender Singh**, S.K. Soni, "Performance Evaluation of Direct Sequence Spread Spectrum under Phase Noise Effect with SIMULINK Simulations", *International Conference (ICAREMIT-2015) 2015 at MJP Rohilkhand University, Bareilly, (TEQIP II & IEEE) Feb 21-23, 2015, pp. 40-46, Volume 07, Issue 01, Jan- June 2015, ISSN: 2321-2055 (E).*
11. **Rupender Singh**, S.K. Soni, "Performance Analysis of MRC Combiner Output Signal in Log Normal Shadowed Fading", *RAECE 2015 (IEEE) at IIT, Roorkee, Feb 13-15, 2015.*

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# LIST OF ABBREVIATIONS AND SYMBOLS

| | |
|---|---|
| ABEP | Average Bit Error Probability |
| AOD | Average Outage Duration |
| AOF | Amount of Fading |
| AWGN | Additive White Gaussian Noise |
| CCDF | Complementary Cumulative Distribution Function |
| CCI | Co-channel Interference |
| CDF | Cumulative Distribution Function |
| CF | Characteristic Function |
| EGC | Equal Gain Combining |
| IID | Independent Identically Distributed |
| LCR | Level Crossing Rate |
| LMSS | Land Mobile Satellite System |
| LOS | Line of Sight |
| MIMO | Multiple Input and Multiple Output |
| MGF | Moment Generating Function |
| MRC | Maximal Ratio Combining |
| NL | Nakagami-Lognormal Fading |
| PDF | Probability Distribution Function |
| RV | Random Variable |
| SC | Selection Combining |
| SEP | Symbol Error Probability |
| SISO | Single Input and Single Output |
| SNR | Signal to Noise Ratio |
| RL | Rayleigh-Lognormal Fading |
| WL | Weibull-Lognormal Fading |
| $T_{\gamma th}$ | Average Outage Duration |
| $\overline{\gamma}$ | Average Signal to Noise Ratio |
| $N_{\gamma th}$ | Level Crossing Rate |
| $\mu$ | Mean |

| | |
|---|---|
| $\mu_{MRC}$ | Mean of SNR at MRC Output |
| L | Number of Branches |
| $P_{out}$ | Outage Probability |
| K | Rice Factor |
| $\gamma$ | Signal to Noise Ratio |
| $\gamma_{MRC}$ | SNR at MRC Output |
| $\gamma_{th}$ | Threshold SNR |
| A | Time Share Factor |
| σ | Variance |
| $\sigma_{MRC}$ | Variance of SNR at MRC Output |

# CHAPTER 1: INTRODUCTION

## 1.1 Motivation and Background

Today wireless communication systems support not only voice communication but also high data speed and multimedia services including messaging, emails and video conferencing. These services require not only high data rates but also low delays and bit error rates (BER). These requirements are difficult to be supported due to some major constraints. First of all wireless channels faces multipath and shadow fading because of time varying nature of radio channel. These fading create severe problems in long distance wireless systems where multipath fading superimposed on shadowing fading. In addition, limited available radio spectrum and spectral efficiency for wireless applications are primary constraint in system design. Finally we need small, low power and lightweight end user terminals which restricts their capabilities. Due to these major issues and constraints, some spectral and power efficient diversity techniques are required to mitigate these problems. Diversity combining techniques have great potential to improve the performance of wireless systems in such a way that they can support high speed data as well as multimedia.

## 1.2 Thesis Objective

The objective of this thesis is four fold. First it analyzes and processes the composite multipath/shadowing fading channels. Second it represents closed form expressions for different fading parameters such as PDF, CDF, Amount of fading, Outage Probability and Channel capacity for combined unshadowing and multipath/shadowing fading. Third it develops analytical methods and tools to assess the performance of digital communication over shadowed fading channels when MRC diversity technique employed. Fourth it provides statistical characteristics of channel capacity for MRC combiner in shadowing fading channels. Emphasis is put on the development of “generic” analytical tools that can be used for a wide range of communication systems operating in various fading channels. The final goal is to provide easy to compute analytical expressions that allow the researchers or system design engineers to perform comparison and tradeoff studies among various communication type and diversity techniques combinations so as to determine the optimum choice in the face of his or her available constraints.

## 1.3 Thesis Outlined

The thesis outlines as follows. We begin in chapter 2 with review of system performance measures. In chapter 3 we review principle of characteristics and models of fading channels. The primary goal of this chapter is to introduce models, terminology and notations that will be used throughout the thesis. In chapter 4 we summarize and discuss literature already available. Then we present analysis and discussions of different fading environments with analytical results.

## 1.4 Mathematical Tools Used

1. Fenton Wilkinson Method
2. Holtzman Approximation
3. MeijerG Function
4. Gamma Function
5. Incomplete Gamma Function
6. Lower Incomplete Gamma Function
7. Bessel's Modified Function
8. Complementary Error Function

# CHAPTER 2: SYSTEM PERFORMANCE MEASURES

Wireless communications is the fastest growing communications industry. As such, it has attracted the attention of the media and our imagination. Over last decade, number of cellular system users has crossed two billions marks. At the same time, system designers have to provide better services to their large number of customers. So this chapter presents a "set of tools", those will allow system designers to evaluate the performance of communication system over fading environment.

## 2.1 Average Signal-to-Noise Ratio (SNR)

Signal to Noise ratio (SNR) is the most common performance measure characteristic of a communication system. Most often SNR is always measured at the output of the receiver. So SNR is directly related to the data detection. Of the several performance measures that exist, SNR is the simplest to evaluate and carries out an excellent indicator of the overall exactitude of the system. In fading environment, *average* SNR is the more appropriate performance measure. It is defined as statistical averaging over the probability distribution of the fading.

$$\overline{\gamma} \triangleq \int_0^\infty \gamma \, p(\gamma) d\gamma \qquad (2.1)$$

Where $p(\gamma)$ defined as a probability density function (PDF) of $\gamma$.

Average SNR also can be shown in terms of moment generating function (MGF) of $\gamma$.

$$M(s) = \int_0^\infty p(\gamma) e^{s\gamma} d\gamma \qquad (2.2)$$

On taking first order derivative of equation (2.2) with respect to s and evaluating at s=0, we can see from (2.1)

$$\overline{\gamma} = \left. \frac{dM_\gamma(s)}{ds} \right|_{s=0} \qquad (2.3)$$

So evaluation of MGF of instantaneous SNR allows evaluation of average SNR using simple mathematics.

## 2.2 Outage Probability

Outage probability $P_{out}$ is another performance parameter for diversity systems in fading environment and defined as probability that output signal to noise ratio SNR falls below a determined threshold $\gamma_{th}$.

$$P_{out} = \int_0^{\gamma_{th}} p(\gamma) d\gamma \quad (2.4)$$

$P_{out}$ is cumulative distribution function (CDF) $P(\gamma)$ of $\gamma$ at $\gamma = \gamma_{th}$. The relation between PDF and CDF is $p(\gamma) = \frac{dP(\gamma)}{d\gamma}$ and since $P(0) = 0$, then Laplace transform of PDF and CDF are related as

$$\hat{P}(s) = \frac{\hat{p}(s)}{s} \quad (2.5)$$

Where $\hat{p}(s) = M(-s)$

So $P_{out}$ can be found by taking inverse Laplace transform of equation (2.5) at $\gamma = \gamma_{th}$

$$P_{out} = \frac{1}{2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{M(-s)}{s} e^{s\gamma_{th}} ds \quad (2.6)$$

## 2.3 Average Bit Error Probability (ABEP)

ABEP is the third parameter for diversity system performance in fading environment. ABEP is most difficult to evaluate out of three parameters because conditional BEP is nonlinear function of instantaneous SNR. ABEP can be computed by taking average of conditional BEP in AWGN over PDF of SNR in fading environment. For example

$$P_b(E/\gamma) = C \exp(-a\gamma) \quad (2.7)$$

Then ABEP can be written as

$$P_b(E) \triangleq \int_0^{\infty} P_b(E/\gamma)\, p(\gamma) d\gamma = \int_0^{\infty} C \exp(-a\gamma)\, p(\gamma) d\gamma = C\, M(-a) \quad (2.8)$$

## 2.4 Amount of Fading

Generally amount of fading (AOF) is measured at combiner's output. Basically AOF is a measure of acuteness of fading channel. AOF measure describes the behavior of system with combining techniques.

So AOF can be defined as

$$AOF = \frac{var(\gamma_t)}{(E[\gamma_t])^2} = \frac{E[{\gamma_t}^2] - (E[\gamma_t])^2}{(E[\gamma_t])^2} = \frac{E[{\gamma_t}^2]}{(E[\gamma_t])^2} - 1 \quad (2.9)$$

Where $\gamma_t$ is instantaneous SNR of combiner output. In terms of MGF, AOF

$$AOF = \frac{\left.\frac{d^2M(s)}{ds^2}\right|_{s=0} - \left(\left.\frac{dM(s)}{ds}\right|_{s=0}\right)^2}{\left(\left.\frac{dM(s)}{ds}\right|_{s=0}\right)^2} \qquad (2.10)$$

## 2.5 Average Outage Duration

Average outage duration (AOD) is the duration in which on an average system remains in outage. Whenever output SNR falls below some specified threshold $\gamma_{th}$ is declared as outage state. Mathematically AOD can be shown as

$$T_{\gamma th} = \frac{P_{out}}{N_{\gamma th}} \qquad (2.11)$$

Where $P_{out}$ already have been discussed and $N_{\gamma th}$ is outage frequency or average level crossing rate (LCR) of o/p SNR $\gamma$ at $\gamma_{th}$. It can be obtained by

$$N_{\gamma th} = \int_0^{\infty} \dot{\gamma}\, f_{\gamma,\dot{\gamma}}(\gamma_{th}, \dot{\gamma})\, d\dot{\gamma} \qquad (2.12)$$

Where $f_{\gamma,\dot{\gamma}}(\gamma_{th}, \dot{\gamma})$ is the joint PDF of $\gamma$ and its time derivative.

## 2.6 Channel Capacity

The analysis of channel capacity for fading channel is a significant performance metric as it gives an estimation of the information rate that the channel can support with negligible probability of error. Mathematically it can be shown as

$$C = B \int_0^{\infty} log_2(1+\gamma)p(\gamma)d\gamma \qquad (2.13)$$

Where B is the bandwidth of channel.

# CHAPTER 3: CHARACTERIZATION AND MODELING OF FADING CHANNELS

Propagation of radio wave through wireless channels is intricate phenomenon characterized by multipath and shadowing effects. The precise mathematical expressions and descriptions of this phenomenon are either too complex or unknown for manageable communication system. However, so many authors have done considerable efforts and presented statistical modeling and characterization of different effects. Knowing the channel behavior is extremely important to design the wireless communication system optimally. The channel model helps us to design the system such that its SNR performance can be improved. The main purpose of this chapter is to review the basic characteristics and models of fading channels. These characteristics and models have been presented in detail in text books [1-5].

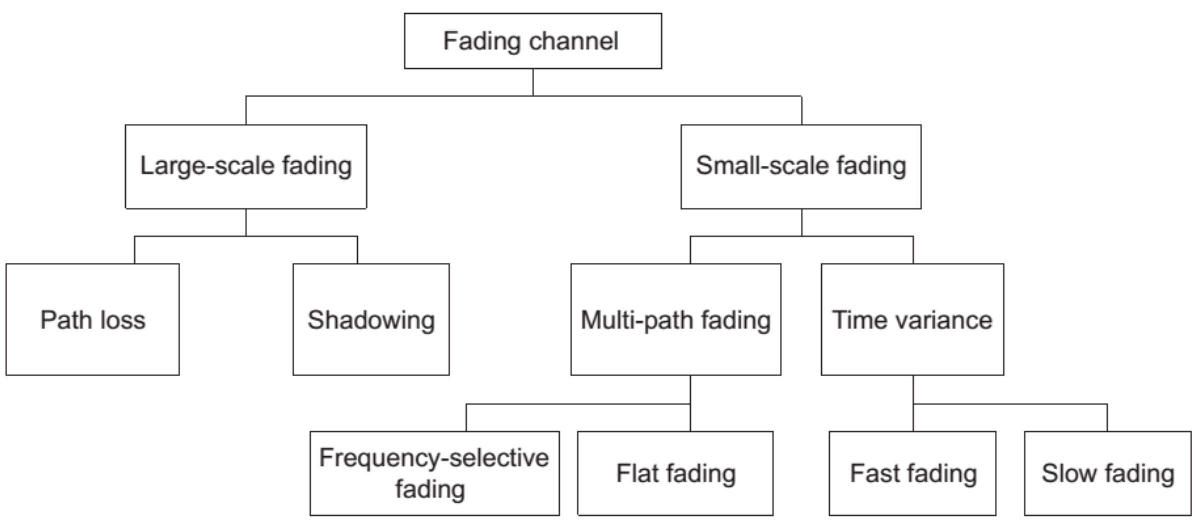


***Fig 1.1. Classification of fading in Wireless Channel***

## 3.1 Main Characteristics of Fading Channels

### 3.1.1 Slow and Fast Fading

Whenever we model fading channels, we need to distinguish between slow and fast fading. The distinction between slow and fast fading is related to coherence time which is defined as inverse of *Doppler's spread. Doppler's spread* is the measure of time duration over which fading process is correlated. The fading is said to be slow if the symbol time is smaller than the coherence time of channel, otherwise it is considered to be fast fading.

### 3.1.2 Frequency Flat and Frequency Selective Fading

The distinction between frequency flat and frequency selective fading channel is an important characteristic. If all spectral components of transmitted signal are affected by same amplitude gains and phase shifts, the channel is said to be frequency flat fading channels. This system is narrow-band system in which bandwidth of transmitted signal is smaller than coherence bandwidth of channel. This coherence bandwidth is defined as inverse of *maximum delay spread.* If all spectral components of transmitted signal are affected by different amplitude gains and phase shifts, the channel is said to be frequency selective fading channels. This system is wide-band system in which bandwidth of transmitted signal is greater than coherence bandwidth of channel.

### 3.1.3 Flat Channel Modeling

For narrow-band system, received carrier is modulated by fading amplitude $\alpha$, where $\alpha$ is a random variable with mean square $\Omega = \overline{\alpha^2}$ and PDF $p(\alpha)$. The instantaneous received power is modulated by $\alpha^2$. Thus SNR $\gamma = \alpha^2 E_s/N_0$ and average SNR $\overline{\gamma} = \Omega E_s/,$ where $E_s$ is symbol energy. Performance evaluation of communication system over fading channels will be a function of SNR $\gamma$. So PDF of SNR $\gamma$ can be obtained by change of variables in the expressions $p(\alpha)$

$$p(\gamma) = \frac{p\left(\sqrt{\frac{\Omega\gamma}{\overline{\gamma}}}\right)}{2\sqrt{\frac{\Omega\gamma}{\overline{\gamma}}}} \qquad (3.1)$$

## 3.2 Multipath Fading

Transmitted signal components are randomly delayed, reflected, scattered and diffracted. Multipath fading is the result of these effects constructively and destructively. This type of fading is fast. Therefore this fading is responsible for short term variations. Depending on behavior of radio wave propagation, there are different statistical models describing behavior of multipath fading envelope.

### 3.2.1 Rayleigh Fading

Multipath fading with no line of sight can be modeled as Rayleigh distribution. In this case, amplitude $\alpha$ of channel fading is distributed as

$$p(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right) \qquad \alpha \geq 0 \qquad (3.2)$$

So using **(3.1),** distribution of instantaneous SNR $\gamma$ is given by

$$p(\gamma) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \qquad \gamma \geq 0 \qquad (3.3)$$

k[th] moment of this PDF is given by

$$E[\gamma^k] = \Gamma(1+k)\overline{\gamma}^k \qquad (3.4)$$

Where $\Gamma(.)$ is gamma function.

AOF of Rayleigh distribution can be obtained using

$$AOF = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1 \qquad (3.5)$$

$$AOF = \frac{\Gamma(3)\overline{\gamma}^2}{(\Gamma(2)\overline{\gamma})^2} - 1 = 1 \qquad (3.6)$$

### 3.2.2 Nakagami-n (Rice) Fading

Multipath fading with strong line of sight and random weaker components can be modeled as Nakagami-n distribution. It is also known as Rician distribution. In this case, amplitude $\alpha$ of channel fading is distributed as

$$p(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} \exp\left(-\frac{2(1+n^2)\alpha^2}{\Omega}\right) I_0\left(2n\alpha\sqrt{\frac{(1+n^2)}{\Omega}}\right) \qquad \alpha \geq 0 \qquad (3.7)$$

Where n is nakagami-n parameter which value varies from 0 to ∞.

So using **(3.1),** distribution of instantaneous SNR $\gamma$ is given by

$$p(\gamma) = \frac{(1+K)e^{-K}}{\overline{\gamma}} \exp\left(-\frac{(1+K)\gamma}{\overline{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right) \qquad \gamma \geq 0 \qquad (3.8)$$

Where $K$ is Rice factor and related to $n$ by $K = n^2$.

k[th] moment of this PDF is given by

$$E(\gamma^k) = \frac{\Gamma(1+k)}{(1+K)^k} {}_1F_1(-k, 1; -K)\bar{\gamma}^k \qquad (3.9)$$

Where ${}_1F_1(.,.;.)$ is Kummer Confluent hyper geometric function.

So AOF can be shown by

$$AOF = \frac{1+2K}{(1+K)^2} \qquad K \geq 0 \qquad (3.10)$$

and varies between 0 ($K = \infty$) and 1 ($K = 0$).

### 3.2.3 Nakagami-m Fading

Nakagami-m distribution is close to central chi squared distribution can be shown by

$$p(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right) \qquad \alpha \geq 0 \qquad (3.11)$$

Where m is nakagami-m parameter which value varies from ½ to ∞.

So using **(3.1),** distribution of instantaneous SNR $\gamma$ is given by

$$p(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \qquad \gamma \geq 0 \qquad (3.12)$$

k[th] moment of this PDF is given by

$$E(\gamma^k) = \frac{\Gamma(m-k)}{\Gamma(m)m^k}\bar{\gamma}^k \qquad (3.13)$$

So AOF can be shown by

$$AOF = \frac{1}{m} \qquad m \geq \frac{1}{2} \qquad (3.14)$$

### 3.2.4 Weibull Fading

Weibull distribution is another important mathematical description of mobile system operating in the range of 800/900 MHz. In this case, amplitude $\alpha$ of channel fading is distributed as

$$p(\alpha) = c\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{\frac{c}{2}} \alpha^{c-1} exp\left[-\left(\frac{\alpha^2}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad \alpha \geq 0 \qquad (3.15)$$

So using **(3.1),** distribution of instantaneous SNR $\gamma$ is given by

$$p(\gamma) = \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\overline{\gamma}}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad \gamma \geq 0 \qquad (3.16)$$

Where c is the shape parameter. The corresponding CDF is

$$P(\gamma) = 1 - exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad \gamma \geq 0 \qquad (3.17)$$

k$^{th}$ moment of this PDF is given by

$$E[\gamma^k] = \int_0^\infty \gamma^k \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\overline{\gamma}}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] d\gamma \qquad (3.18)$$

Using $\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}} = R \; in \; (3.18) \qquad then \qquad \frac{c}{2}\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}-1} d\gamma = dR$

Then (3.18) become

$$E[\gamma^k] = \int_0^\infty (R)^{2k/c} exp[-R] \, dR \qquad (3.19)$$

$$E[\gamma^k] = \Gamma\left(1+\frac{2k}{c}\right) \qquad (3.20)$$

So AOF can be shown by

$$AOF = \frac{\Gamma\left(1+\frac{4}{c}\right)}{\Gamma^2\left(1+\frac{2}{c}\right)} - 1 \quad (3.21)$$

### 3.2.5 Log Normal Shadowing

The slow variations in mean signal level due to buildings and trees affect the quality of link in terrestrial and satellite communication system. On the basis of different indoor and outdoor experimental measurements, there is general consent that shadowing can be modeled as Log-normal distribution. So the distribution of instantaneous SNR $\gamma$ can be expressed as standard log normal expression

$$p(\gamma) = \frac{\xi}{\sigma\gamma\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma - \mu)^2}{2\sigma^2}} \quad (3.22)$$

Where $\xi$=10/ln 10=4.3429, μ(dB) is the mean of $10log_{10}\gamma$, $\sigma$(dB) is standard deviation of $10log_{10}\gamma$.

k[th] moment of this PDF is given by

$$E(\gamma^K) = \exp\left(\frac{K}{\xi}\mu + \frac{1}{2}\left(\frac{K}{\xi}\right)^2 \sigma^2\right) \quad (3.23)$$

So AOF can be shown by

$$AOF = \exp\left(\left(\frac{\sigma}{\xi}\right)^2\right) - 1 \quad (3.24)$$

### 3.2.6 Composite Multipath/Shadowing Fading

Composite multipath/shadowing fading has two components short distance multipath fading effects and long distance log normal shadowing fading. In this type of fading multipath fading superimposed over log normal shadowing fading. This scenario can be seen in urban areas with slow moving vehicles and pedestrian. This composite fading can also be seen in satellite land mobile systems due vegetative and downtown shadowing. This composite distribution can be obtained by averaging multipath fading over conditional log normal shadowing. For example, composite multipath/shadowing (weibull-lognormal) distribution can be shown by

$$p(\gamma) = \int_0^{\infty} \left\{ \frac{c}{2} \left( \frac{\Gamma\left(1+\frac{2}{c}\right)}{w} \right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[ -\left( \frac{\gamma}{w} \Gamma\left(1 + \frac{2}{c}\right)\right)^{\frac{c}{2}} \right] \right\} \left\{ \frac{\xi}{\sigma w \sqrt{2\pi}} \exp\left( -\frac{(10log_{10} w - \mu)^2}{2\sigma^2} \right) \right\} dw \qquad (3.25)$$

This type of integrations is difficult to represent in closed form expressions. There are different approximations and methods to represent in closed form expressions. Holzman approximation is one of the techniques, which have been used throughout the thesis for approximating integrations into closed form expressions.

### 3.2.7 Combined (Time Shared) Composite Multipath/Shadowing and Unshadowing fading

From his land mobile satellite characterization experiments, Lutz has shown that overall fading is a combination of unshadowing multipath fading and composite multipath/shadowing fading. Bartz and Stutzman also supported and presented the same model. This fading is based on total shadowing model presented by Lutz. The distribution of this fading can be obtained by time shared sum of unshadowing multipath fading and composite multipath/shadowing fading. Mathematically it can be shown as

$$p(\gamma) = (1-A) * PDF of\ Rician\ fading + A * PDF\ of\ Composite\ multipath/shadowed \qquad (3.26)$$

Where $A$ is time share factor, $0 < A < 1$.

When there is no direct line of sight (LOS), it is assumed to be a composite multipath/shadowing fading. When there is no shadowing, it is assumed to be a strong line of sight(LOS) multipath fading. For example, PDF of combined composite multipath/shadowing (WL) and undhadowing fading can be shown by

$$p(\gamma) = (1 - A)\frac{(1+K)e^{-K}}{\bar{y}}\exp\left(-\frac{(1+K)\gamma}{\bar{y}}\right)I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{y}}}\right)$$

$$+ A\int_0^\infty \left\{\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{w}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{w}\Gamma\left(1\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right]\right\}\left\{\frac{\xi}{\sigma w\sqrt{2\pi}}\exp\left(-\frac{(10log_{10} w - \mu)^2}{2\sigma^2}\right)\right\} dw \qquad (3.28)$$

This thesis has present statistical modeling of characteristics of combined composite multipath/shadowing (WL) and unshadowing fading channel. Holtzman approximation has been used to obtain closed form expressions.

# CHAPTER 4: LITERATURE

## 4.1 Introduction

Practical wireless channels can be modeled as composite multipath with shadowing effects. Performance analysis of these channels requires rigorous mathematics. In this chapter, we have thoroughly studied available literature which helped us to understand the research gaps.

## 4.2 Composite Multipath/Shadowing Fading

From his land mobile satellite characterization experiments, **Lutz et al [6]** has introduced the concept of combined composite multipath/shadowing and unshadowing fading channels. In this work he has recorded data between satellite and mobile terminals. He has showed that PDF of received signal power is time sharing sum of composite multipath/shadowing and unshadowing fading channel. He has also tabulated the effect of satellite elevation on different parameters including time share factor, rice factor, mean and variance. In LMSS experiment, **Barts et al [7]** shown that roadside vegetation is the main reason for severe fading due to blockage of line of sight (LOS). The model of LMSS has been presented as the time sharing sum of non-line of sight (Non LOS) and line of sight (LOS) path. Time fade simulations have been presented in this work.

**P.S. Bithas [11]** has considered composite Weibull Gamma distribution and derived closed form expressions of PDF, CDF, Characteristics function, MGF and outage using MeijerG function.

Nakagami-Gamma shadowing distribution can be modeled using generalized-K distribution model. But further derivations involved analytical and computational difficulties. **Saad Al-Ahmadi [22]** has proposed moment matching method to approximate generalized-K distribution by Gamma distribution. For small multipath fading and shadowing parameters, expressions for parameters of Gamma distribution have been expressed in adjusted form by first two positive moments matching. The adjusted Gamma PDF closely approximate generalized-K distribution in upper and lower trail regions and these approximations can be used to approximate sum of generalized-K distribution in these regions. Outage probability, ergodic capacity and outage capacity have also been presented in closed form.

The closed form of Nakagami-Lognormal Shadowing fading has been proposed using mixtures of gamma distribution by **Saman Atapattu et al [8].** They have converted PDF integration into Gaussian Hermite integration which further approximated as a power series. They have successfully derived CDF, MGF and Amount of Fading. Then they have derived PDF of SNR and shown that derived PDF of SNR is Mixtures of Gamma distributions. They compared derived results with KG distributions and established a relationship between parameters of Nakagami Lognormal and KG distribution. The same concept also discussed by **Weijun Cheng [9].** The diversity analysis (MRC and SC) for mixtures of gamma distribution has been presented by **Jaehoon Jung et al [31].** Closed form expressions have been derived for diversity and array gain.

**P. M. Shankar [10]** approximated Nakagami Lognormal distribution using Nakagami-N-Gamma distribution under multiple scattering. He has obtained PDF of Nakagami-N-Gamma distribution in terms of MeijerG function.

Nakagami-Lognormal distribution has been considered by **Amine Laourine et al [12]**. They have approximated closed form of Nakagami-Lognormal distribution using G distribution. They have compared lognormal distribution, G distribution and Gamma distribution. They have shown that G distribution is very close to lognormal distribution with compare to Gamma distribution. On the basis of closed form expression they have derived successfully channel capacity and outage probability.

**Hensen et al [13]** have presented closed form solution of CDF for Rayleigh Lognormal distribution using asymptotic expressions. They have used these expressions and shown bit error performance of mobile channel under Gaussian noise.

Point estimate mixture of Gamma distribution (PEMG) outlined to approximate composite fading distribution. **Vineet Khandelwal et al [40]** considered different approaches available to model composite fading including K, $K_G$ model in their work and their study emphasized that their PEMG based approach is simple and better approximation. Closed form expressions of few performance measures such as channel capacity and average symbol error rate and have been presented. Results validated by comparing with exact results determined using numerical quadrature method.

A new approach has been proposed to estimate the parameters of composite fast fading and shadowing using moments in logarithmic units by **Juan Reig [41].** Parameters of $\alpha - \mu$, Suzuki, Nakagami-Lognormal, K distribution and generalized K and have been estimated in their work. Parameter estimator have been derived for Nakagami-Lognormal fading. In urban environment, experimental measurement carried out in eight routes. Using K-S test, they observed that Nakagami-Lognormal distribution is best fitting in five routes, while $\alpha - \mu$ distribution is the second best fitting in two routes.

ABEP of M-QAM and M-PSK using Holtzman approximation over single lognormal and Rayleigh-lognormal fading channels have been derived by **Chaoqing Tang et al [26].** PDF, CDF, outage probability and ergodic capacity have also been derived in closed form for Rayleigh-lognormal fading channel.

**Jack M. Holtzman [28]** has proposed new method to evaluate closed form approximation of expectation of error probability using Gaussian approximation. New closed form approximation has suggested that average of any function over Gaussian distribution can be approximated as sum of the same function three times with different arguments. He has calculated error probability of direct spread spectrum multiple access system. In particular, using this approach we are able to generate unified closed form expressions throughout the thesis.

## 4.3 Sum of RVs

Minimax approximation based approach has been proposed to lognormal sum distribution by **Norman C. Beaulieu et al [17].** In their work they have compared different methods already known such as Wilkinson's, Schwartz and Yeh's, and Farley's methods. Comparative study has been given for sum of IID lognormal, sum of lognormal with different power means and sum of lognormal with different power spreads. They have compared their approach with well-known assumption that sum of IID lognormal is lognormal distributed. They concluded that this assumption is good for N=2 but poor for large number of IID lognormal distributions or difference among the dB spreads of lognormal distributions increases.

**Neelesh B. Mehta et al [18]** have given MGF based approach to sum IID lognormal distributed RVs. PDF of sum of correlated lognormal RVs and Lognormal-Rice RVs also derived in closed form. CDF and CCDF have been modeled accurately for wide range of lognormal means and variances. They have concluded that accuracy of their proposed method with compare to F-W and S-Y methods by one or more orders in magnitude.

Type IV pearson distribution approximation to sum lognormal RVs has been proposed by **Hong Nie et al [19]**. The parameters of Type IV pearson distribution has been derived by matching variance, mean, kurtosis and skewness of the two distributions. They have concluded with some advantages of their proposed approach over existing method such as closed form PDF, simple straight forward approach for parameters of PDF and high accuracy approximation. However, their approach also having some limitations.

Bounds for cumulative distribution function (CDF) of sum of two or three correlated lognormal RVs with arbitrary correlation coefficient and sum of N lognormal RVs with equal correlation coefficients have been derived as 2 fold integrals by **C. Tellambura [20].** CDF have been expressed in terms of $\emptyset, T - Function\ and\ S - Function.$ The closed form expressions have also derived for $k^{th}$ moments of SNR and AOF for MRC, SC and EGC and $P_{out}$ for SC.

Curve fitting based approach to approximate sum of two lognormal RVs has been proposed by **Rashid Abaspour et al [23].** A recursive approach has been used to estimate the mean and variances of sum of lognormal RVs. They have extended their method using surface fitting, to approximate sum of more than two lognormal RVs for non-identically distributed.

Lie-Trotter Operator Splitting Method has been proposed by **C. F. Lo [24]** to approximate sum and differences of two lognormal RVs. He has shown that sum and difference of two lognormal RVs follow a shifted stochastic lognormal process. The closed form expression with series expansion solution of PDF has also given.

Different methods are proposed to approximate PDF of sum of IID lognormal RVs. Comparative study investigated by **Norman C. Beaulieu et al [38]** for practical range of dB spread. Wilkinson's, Schwartz and Yeh's and Schleher's cumulunts Matching Method discussed in their work. CDFs for each method plotted and compared in that region, where CDFs can be approximated by a straight line. Their observations proved that CDF of Schwartz and Yeh's is less than CDF of Wilkinson's and greater than the Wilkinson's for large and small values of the argument respectively. Their discussion also led to conclusion that CDF of Wilkinson's will be poor compared to CDF of Farley's for large values argument. On the basis of outage probability, they observed that Wilkinson's approach gives more accurate results than Schwartz and Yeh's approach at lower outage probability but later is more accurate for higher outage probability.

Laplace transformation based approach to approximate sum of RVs has been proposed by **A. G. Rossberg [39].** This approach also involved FFT algorithm. Numerical examples discussed to approximate sum of two independent lognormal RVs and 111 Weibull RVs. Their approach is applicable to estimate the sum of correlated lognormal RVs. FFT of Laplace transform is one of most the expensive step of their approach for large number of mesh points.

A New quick and accurate method based on piecewise linear (PWL) curve fitting to estimate ergodic capacity and outage probability of an lognormal fading channel has been proposed by **Scott Enserink et al [25].** The proposed method is applicable to any modulation schemes such as QAM, BPSK, QPSK, 8 PSK and 16 QAM. This method can also be used to evaluate ergodic constrained capacities for other fading channels such as Nakagami. They have shown that results proposed method was well matched with Monte Carlo simulations.

A mathematical approach including hyperbolic functions, integrals by parts and partial fractions has been used by **Fabien H´eliot et al [30]** to present channel capacity of indoor UWB system over lognormal fading channel. A generic tight closed form also derived for ergodic capacity using curve fitting method for wide range of dB spread. This approach was applicable to lognormal fading channels with high SNRs and SISO/MIMO-UWB systems over IEEE 802.15.3a.

## 4.4 Diversity Combining Techniques

MGF based approach has been proposed by **Mohamed-Slim Alouini et al [15]** to evaluate symbol error rate of linearly modulated signals over generalized fading environments for single channel and MRC combining multichannel receptions. Symbol error rate of M-PSK and M-AM signals for MRC combining multichannel reception also have been evaluated.

Correlated dual diversity has been discussed for log normal fading channel by **Mohamed-Slim Alouini et al [16].** Closed form expressions of PDF, Average SNR, $k^{th}$ moment and AOF have been derived for MRC, SC and switched combining.

A unified approach for outage probabilities of MRC, SC and EGC for non-identically distributed correlated lognormal RVs have been proposed by **Dimitrios Skraparlis et al [21].** On the basis of their proposed expressions they have given comparative performance of diversity techniques as well as effect of no. of branches.

ABEP of M-QAM and M-PSK using Holtzman approximation over single lognormal and Rayleigh-lognormal fading channels have been derived by **Chaoqing Tang et al [26].** PDF, CDF, outage probability and ergodic capacity have also been derived in closed form for Rayleigh-lognormal fading channel.

**Vineet Khandelwal et al [27]** have proposed second order Taylor's series expansion to estimate average symbol error probability (ASEP) over lognormal fading channel. The symbol error probability in terms of $Q(x)$ has been approximated by using second order exponential type. This second order exponential approximation further approximated by second order Taylor's series approximation. This approach is applicable to different modulation schemes.

Meijer G function has been suggested to present Shannon capacity in closed form for different fading channels with MRC diversity by **Faissal El Bouanani et al [29].** They have validated their results using Monte Carlo simulations for all fading channels. They concluded that as increases number of branches, capacity of channel approaches the channel capacity of AWGN.

**Karmeshu et al [32]** have considered statistical analysis of characteristics of channel capacity in lognormal shadowing fading channel. PDF, first order moment, second order moment of channel capacity and outage capacity have been determined. The Holztman approximation has been used to approximate closed form expression of channel capacity in lognormal shadowing channel.

## 4.5 LCR and AOD

**Petros Karadimas et al [14]** have simulated expressions of LCR and AOD for Weibull-Lognormal shadowing fading. Under slow shadowing variations, they have approximated joint PDF of composite SNR and its derivative using Kronecker delta function. They have shown expressions for AOD as well.

The second order statistics such as LCR and AOD of weibull fading channel are studied by **Nikos C. Sagias [33].** Some mathematical manipulations led to closed form expressions of LCR and AOD. Meijer G function has been used to compute channel capacity in closed form.

**Simon L. Cotton et al [78]** considered small scale lognormal fading based on indoor propagation studies. Bessel derived autocorrelation function based lognormal fading model has been analyzed and higher order statistics such as LCR and AOD evaluated. Comparative validation was provided by comparing proposed model with Nakagami-m approximation model.

LCR and AOD in closed form were presented for diversity techniques such as MRC, SC and EGC in IID Nakagami-m fading channels by **Cyril Daniel Iskander et al [34]**. The graphical results were provided for arbitrary number of braches L for MRC and SC but for L=2 for EGC. Generalized LCR expression also derived for Rayleigh fading channel.

LCR and AOD of EGC system with CCI in Rayleigh fading were determined in closed form by **Zoran Hadzi Velkov [35].** Solutions for LCR integral expressions were derived for IID interference signals using CF method, the Beaulieu series and the Parseval's theorem. On the basis of comparison between the proposed solutions using Beaulieu series and solutions using numerical integration methods, he concluded that proposed solutions is less sensitive to the input parameters and introduce faster computation. LCR and AOD were derived in closed form in terms of gamma and beta function for dual diversity.

**Norman C. Beaulieu [36]** derived LCR and AOD of output signal of MRC combiner and EGC combiner in IID Ricean fading channels. Tight closed form results are determined for MRC combiner, while expressions for EGC combiner are derived with infinite sinusoidal series method. The results are validated for IID, isotropic scattering and perpendicular component to line of motion of mobile.

Statistical analyses in fading environment are very useful to characterize fading channels. LCR and AOD for SC combiner in Ricean-lognormal shadowing fading channel are numerically determined by **Aleksandra M. Mitic et al [37].** Traditional approach used to evaluate results for IID diversity branches.

# CHAPTER 5: PERFORMANCE ANALYSIS OF COMBINED (TIME SHARED) COMPOSITE MULTIPATH/SHADOWING (WL) AND UNSHADOWING FADING CHANNELS

## 5.1 Introduction

Wireless communication channels are weakened by inimical repercussion such as Multipath Fading and Shadowing [1]. Based on different indoor and outdoor observed measurements, a general consent has been given to model shadowing using Log-normal distribution [27, 42, 43] and multipath effect is captured by using distribution such as Nakagami, Rayleigh and Rician distribution [22].

Over last decade, the Weibull distribution becomes an important tool in various scientific fields, but its use to model fading channel become recent topic in wireless communications theory [44, 45. 46], particularly with mobile radio systems. The Weibull model exhibits an excellent fit to experimental fading channel measurements for both indoor [45] and outdoor [46] environments. Several works [33, 47] have considered weibull fading channel and analysed performance metric of communication system. In [33], second-order statistics and channel capacity are analyzed for weibull fading channel. In [47], the channel capacity in fading channels following distributions such as Nakagamm, weibull is analysed for multiple independent channels and the results are expressed in closed-form using Meijer G function.

A composite multipath/shadowing fading scenarios are often encountered in a real scenarios. Modeling of composite fading is important in analyzing the wireless communication system such as MIMO and cognitive radio network and in the modeling of interference in cellular system. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal, Nakagami-lognormal are considered in [43, 48-49]. Further, due to mathematical complexity in the closed-form expression of log-normal based composite models, various approximations to log-normal have been suggested. In [50], the gamma distribution is suggested as an alternative approach to log-normal to model shadowing effect. In [11], a composite Weibull/Gamma distrubtion is considered and performance metric such as outage probability and bit-error rate for receiver are analysed. The work in [12] analyses the composite Nakagami/Log-normal fading approximating Log-normal shadowing as an inverse Gaussian (IG) distribution, thus, ensuring the resultant expression in the closed-form. In [8], a composite multipath/shadowing fading is represented as a series of Nakagammi distribution. In [51], considering Weibull/Log-normal composite fading, second-order statistics of fading channel has been studied. However, Weibull/log-normal fading scenario has got little attention till date in terms of closed-form PDF and is not reported in literature and thus, hampering further analytical derivation of important performance metric such as Amount of Fading (AF), average channel capacity and outage probability.

In the case of mobile user there is the possibility of receiving signal from LOS and shadowed path both. This concept was discussed in [1, 15]. In early 1990s, Lutz et al. [6] and Barts and Stutzman [7] found that total fading for land–mobile satellite systems can be viewed as combination of unshadowed fading and a composite multipath/shadowed fading [1]. Statistical modeling of this realistic scenario has also received a little attention in research community till date.

In this chapter, our goal is to obtain the closed-form expression of composite Weibull/Lognormal (W-L) distribution using Holtzmanin approximation for the expectation of the function of a normal variant. The closed-form expression, thus obtained, is the weighted sum of three Weibull distribution terms with some modifications in shape and scaling parameters. This closed-form result facilitates to obtain a simple analytical approximation of the PDF of Combined (time-shared) shadowed (W-L)/unshadowed (Rician) fading. Further, the proposed closed-form PDF leads to the derivation of the closed-form solution of the performance metrics of communication system such as Amount of Fading (AF), Outage Probability ($P_{out}$) and channel capacity(C/B) using Meijer G function for both composite multipath/shadowed fading as well as combined (time-shared) shadowed/unshadowed fading channel.

The remaining of the chapter is organized as follows. In Section 5.2, the closed-form expression of composite weibull/log-normal fading is obtained and using it, the distribution of combined shadowed/unshadowed fading is expressed. Section 5.3, Amount of Fading for the composite fading channels is derived followed by combined shadowed/unshadowed fading using Kummer confluent hyper geometric function. In Section 5.4, performance measure such as outage probability of the communication system is analyzed. In Section 5.5, the derivation of average channel capacity using Meijer G function for both the composite fading as well as combined shadowed/unshadowed fading are derived and some interesting observation is made. This is followed by results and discussion in Section 5.6 which illustrates the graphical results of the closed-form expression obtained in the preceding section. This is followed by conclusion in Section 5.7.

## 5.2 The PDF of Combined Composite Multipath/shadowing and Unshadowing distribution

### *5.2.1 The Probability Density Function of the Composite Weibull/Log-normal shadowing*

In this section, we derive closed-form of weibull/log-normal composite fading where weibull represents multipath effect and log-normal model capture the effect of shadowing. PDF of SNR of composite weibull/log-normal shadowing can be obtained by averaging the conditional PDF of weibull/log-normal distribution over log-normal fading. Conditional weibull distribution is given as [1, 52]

$$p(\gamma/w) = \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{w}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{w}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad \gamma \geq 0 \qquad (5.1)$$

where c is shape parameter for weibull fading. The parameter $w$ is the average SNR at the receiver. If w is slowly varying power, the pdf in (1) becomes statistical terms and its average needs to be computed. Here, the slowly varying fading is modeled using log-normal and is expressed as

$$p(w) = \frac{1}{\sigma w\sqrt{2\pi}} \exp\left(-\frac{(log_e w - \mu)^2}{2\sigma^2}\right) \qquad (5.2)$$

where the parameters μ, σ are mean and standard deviation respectively of RV $\log_e w$. They can be expressed in dB by $\sigma_{db} = \xi\sigma$ and $\mu_{db} = \xi\mu$ where $\xi = 10/\ln(10)$ [27]. Averaging the PDF of (5.1) w.r.t. that of (5.2), we have PDF of composite WL fading

$$p(\gamma) = \int_0^\infty p(\gamma/w)\ p(w)dw \qquad (5.3)$$

Substituting the PDFs from (5.1) and (5.2), into (5.3), we have

$$p(\gamma) = \int_0^\infty \left\{\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{w}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{w}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right]\right\}\left\{\frac{1}{\sigma w\sqrt{2\pi}} \exp\left(-\frac{(log_e w-\mu)^2}{2\sigma^2}\right)\right\} dw \qquad (5.4)$$

It is difficult to calculate the results directly, in this work, we adopt the efficient tool proposed by Holtzman [28], we have, Using $log_e w = x$ in (5.2)

$$p(\gamma) = \int_0^\infty \psi(\gamma; x). \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \qquad (5.5)$$

Then finally we have PDF of WL fading

$$p(\gamma) \approx \frac{2}{3}\psi(\gamma; \mu) + \frac{1}{6}\psi(\gamma; \mu + \sqrt{3}\sigma) + \frac{1}{6}\psi(\gamma; \mu - \sqrt{3}\sigma) \tag{5.6}$$

where

$$\psi(\gamma; x) = \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(x)}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{\exp(x)}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \tag{5.7}$$

Using (5.6) and (5.7) we have CDF of WL fading

$$P(\gamma) = \frac{2}{3}\{1 - \alpha(\gamma; \mu)\} + \frac{1}{6}\{1 - \alpha(\gamma; \mu + \sqrt{3}\sigma)\} + \frac{1}{6}\{1 - \alpha(\gamma; \mu - \sqrt{3}\sigma)\} \tag{5.8}$$

where

$$\alpha(\gamma; x) = exp\left[-\left(\frac{\gamma}{\exp(x)}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right]$$

### *5.2.2 The Probability Density Function of the Combined (time-shared) composite shadowed/unshadowed fading*

Considering the case of moving user there is the possibility of receiving signal from LOS and shadowed path both. So combined (time-shared) PDF of instantaneous SNR $\gamma$ is [15]

$$p(\gamma) = (1 - A) * PDFof\ Rician\ fading + A * PDF\ of\ Composite\ multipath\ /shadowed(WL) \tag{5.9}$$

Where $A$ is shadowing time-share factor, $0 \leq A \leq 1$

Rice distribution of $\gamma$ (SNR per symbol) is given as [15]

$$p_{rice}(\gamma) = \frac{(1+K)e^{-K}}{\bar{\gamma}} \exp\left(-\frac{(1+K)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right) \tag{5.10}$$

Rice distribution is often used to model the propagation channel consisting of strong line-of-sight (LOS) and some multipath components. Using $p(\gamma)$ from (5.6) and (5.7) into (5.9) and noting the Rice distribution of SNR per symbol from (5.10), we have PDF of combined shadowed (WL)/unshadowed fading as

$$p(\gamma) = (1-A)\frac{(1+K)e^{-K}}{\overline{\gamma}}\exp\left(-\frac{(1+K)\gamma}{\overline{\gamma}}\right)I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right) + A\left(\frac{2}{3}\psi(\gamma;\mu) + \frac{1}{6}\psi(\gamma;\mu+\sqrt{3}\sigma) + \frac{1}{6}\psi(\gamma;\mu-\sqrt{3}\sigma)\right) \quad (5.11)$$

where $\psi(\gamma; x)$ is defined in (5.7).

### 5.3 Amount of Fading (AOF)

Amount of Fading is a measure of severity of fading of the channel. In this section, we compute the amount of fading of the composite weibull/log-normal fading. The amount of fading is defined as

$$AOF = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1 \quad (5.12)$$

Let us consider the approximate closed-form expression of (5.6) with (5.7). This is weighted sum of three weibull distributed terms. Considering each term separately, we have the kth moment of $\gamma$ (SNR per symbol) given as

$$E(\gamma^k) = \int_0^\infty \gamma^k \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\overline{\gamma}}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] d\gamma \quad (5.13)$$

where $\overline{\gamma} = \exp(x)$

Substituting $\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}} = R$ in (5.13) and after some straightforward calculation, we obtain

$$E[\gamma^k] = \left[\frac{\exp(x)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k \Gamma\left(1+\frac{2k}{c}\right) \quad (5.14)$$

*where $\Gamma(.)$ gamma function.*

Thus, considering all the three terms of (5.6), we have

$$E[\gamma^k] = \left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k\right] \Gamma\left(1+\frac{2k}{c}\right) \quad (5.15)$$

From (5.12), we have AOF

$$AOF = \frac{\Gamma\left(1+\frac{4}{c}\right)}{\Gamma^2\left(1+\frac{2}{c}\right)}\left[\frac{\left\{\frac{2}{3}\exp(2\mu)+\frac{1}{6}\exp(2\mu+2\sqrt{3}\sigma)+\frac{1}{6}\exp(2\mu-2\sqrt{3}\sigma)\right\}}{\left\{\frac{2}{3}\exp(\mu)+\frac{1}{6}\exp(\mu+\sqrt{3}\sigma)+\frac{1}{6}\exp(\mu-\sqrt{3}\sigma)\right\}^2}\right] - 1 \qquad (5.16)$$

k^th moment of output SNR of combined shadowing/unshadowing fading is

$$E(\gamma^k) = (1-A)\int_0^\infty \gamma^k p_{rice}(\gamma)\,d\gamma + A\int_0^\infty \gamma^k p_{comp}(\gamma)\,d\gamma \qquad (5.17)$$

The kth moment of Rice distribution is given as [1]

$$E(\gamma^k) = \frac{\Gamma(1+k)}{(1+K)^k}\,{}_1F_1(-k,1;-K)\bar{\gamma}^k \qquad (5.18)$$

where ${}_1F_1(.,.;.)$ is Kummer Confluent hyper geometric function.

Thus, the k^th moment of combined shadowed/unshadowed fading is obtained by substituting(5.16 ) and (5.18) in (5.17) and is expressed as

$$E[\gamma^k] = (1-A)\frac{\Gamma(1+k)}{(1+K)^k}\,{}_1F_1(-k,1;-K)\bar{\gamma}^k + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k\right]\Gamma\left(1+\frac{2k}{c}\right) \qquad (5.19)$$

Amount of fading can straightforward be obtained by finding first and second moment from (5.19) and substituting the results in (5.12). Complete expressions has been derived and shown in appendix.

### 5.4 Outage Probability

The outage probability is standard performance criterion of communication systems operating over fading channels and it is defined as the probability that the instantaneous error rate exceeds a specified value, or equivalently, that SNR falls below a predetermined threshold $\gamma_{th}$.

$$P_{out}(\gamma_{th}) = P[\gamma \le \gamma_{th}] \qquad (5.20)$$

Mathematically can be derived by

$$P_{out}(\gamma_{th}) = \int_0^{\gamma_{th}} p(\gamma)\,d\gamma \qquad (5.21)$$

Substitution of (5.10) into (5.21) yields the outage probability of Rice-distributed fading channel and it is expressed as

$$P_{out}(\gamma_{th})_{rice} = \int_0^{\gamma_{th}} (1 - A)\frac{(1+K)e^{-K}}{\bar{\gamma}} \exp\left(-\frac{(1+K)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right) d\gamma \qquad (5.22)$$

To solve this, replacing $I_0(z)$ in (5.22) from 8.447 tables of integrals [53] by

$$I_0(z) = \sum_{n=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2n}}{(n!)^2} \qquad (5.23)$$

And assuming $= \frac{(1+K)}{\bar{\gamma}}$, we obtain

$$P_{out}(\gamma_{th})_{rice} = \sum_{n=0}^{\infty} \frac{(1-A)a^{n+1}e^{-K}\,K^n}{(n!)^2} \int_0^{\gamma_{th}} \gamma^n \exp(-a\,\gamma)\ d\gamma \qquad (5.24)$$

Using ([54], / 0.1.03.21.0058.01),

$$P_{out}(\gamma_{th})_{rice} = \sum_{n=0}^{\infty} \frac{a^{n+1}e^{-K}\,K^n}{(n!)^2}[(-a)^{-n-1}\,(-1)^n\ \Gamma(1+n, a\gamma_{th}) - (-a)^{-n-1}\,(-1)^n\,\Gamma(1+n,0)] \qquad (5.25)$$

Considering the incomplete gamma function defined as

$$\int_0^x t^{n-1}\exp(-t)\ dt = \gamma(n,x) \quad \textit{Lower Incomplete gamma function}$$

and its relation with Kummer' confluent hypergeometric function,

$\int_0^{\gamma_{th}} \gamma^n \exp(-a\,\gamma)\ d\gamma = a^{-n-1}\,\gamma(n+1, a\gamma_{th}) = a^{-n-1}(1+n)^{-1}(a\gamma_{th})^{n+1}{}_1\mathrm{F}_1(n+1; 2+n; -a\gamma_{th})$

The expression (5.24) can also be expressed in terms of Kummer's confluent function as

$$P_{out}(\gamma_{th})_{rice} = \sum_{\boldsymbol{n=0}}^{\infty} \frac{a^{n+1}e^{-K}\,K^n}{(n!)^2} a^{-n-1}(1+n)^{-1}(a\gamma_{th})^{n+1}{}_1\mathrm{F}_1(n+1; 2+n; -a\gamma_{th}) \qquad (5.26)$$

Outage probability of composite weibull/log-normal fading can straightforward be obtained by substituting (5.6) into (5.21) which yields

$$P_{out}(\gamma_{th})_{WL} = A\,\frac{2}{3}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_1^{\frac{c}{2}}\right)\right) + A\,\frac{1}{6}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_2^{\frac{c}{2}}\right)\right) + A\,\frac{1}{6}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_3^{\frac{c}{2}}\right)\right) \quad (5.27)$$

where

$$b_1 = \frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu)}$$

$$b_2 = \frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu+\sqrt{3}\sigma)}$$

$$b_3 = \frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu-\sqrt{3}\sigma)}$$

Thus, the outage probability of combined (time-shared) weibull/log-normal and rice fading is expressed as

$$P_{out}(\gamma_{th}) = \sum_{n=0}^{\infty}\frac{(1-A)a^{n+1}e^{-K}\,K^{n}}{(n!)^2}[a^{-n-1}\,(-1)^{n}\;\Gamma(1+n,-a\gamma_{th}) - a^{-n-1}\,(-1)^{n}\,\Gamma(1+n,0)] + A\,\frac{2}{3}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_1^{\frac{c}{2}}\right)\right) + A\,\frac{1}{6}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_2^{\frac{c}{2}}\right)\right) + A\,\frac{1}{6}\left(1-\exp\left(-\gamma_{th}^{\frac{c}{2}}\,b_3^{\frac{c}{2}}\right)\right) \quad (5.28)$$

### 5.5 Average Channel Capacity

The analysis of channel capacity for fading channel is a significant performance metric as it gives an estimation of the information rate that the channel can support with negligible probability of error. In this section, we obtain the closed-form expression of the average channel capacity of the composite weibull/log-normal fading channel followed by analytical expression of channel capacity of combined (time-shared) multipath/shadowed and unshadowed (Rice) fading channel.

Channel capacity [29] is defined as

$$C = B\int_{0}^{\infty} log_2(1+\gamma)p(\gamma)d\gamma \quad (5.29)$$

Using (5.8) into (5.29) , the average channel capacity of rice fading channel is given as

$$\frac{C}{B} = \int_{0}^{\infty} log_2(1+\gamma)(1-A)\frac{(1+K)e^{-K}}{\overline{\gamma}}\exp\left(-\frac{(1+K)\gamma}{\overline{\gamma}}\right)I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right)d\gamma \quad (5.30)$$

After substituting the modified Bessel function from (5.23) into (5.30) and noting the Meijer G functions from Appendix, we obtain average channel capacity for rician fading as

$$\frac{C}{B}=\sum_{n=0}^{\infty}(1-A)\frac{a^{n+1}e^{-K}\,K^{n}}{\ln(2)\,(n!)^{2}}\int_{0}^{\infty}\gamma^{n}\,G_{2,2}^{1,2}\left[\gamma\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right]G_{0,1}^{1,0}[a\gamma|0]\,d\gamma \qquad (5.31)$$

where $a=\frac{(1+K)}{\bar{\gamma}}$, Replacing n by n-1 in (5.31) and using ([54], /07.34.21.0011.01) and [29], the average channel capacity of rician fading channel is expressed as

$$\frac{C}{B}=\sum_{n=-1}^{\infty}(1-A)\frac{a^{n}e^{-K}\,K^{n-1}}{\ln(2)\,((n-1)!)^{2}}\,G_{2,3}^{3,1}\left[a\left|\begin{matrix}-n,1-n\\0,-n,-n\end{matrix}\right.\right] \qquad 5.(32)$$

The channel capacity for the composite multipath/shadowed fading is given as

$$\frac{C}{B}=\int_{0}^{\infty}log_{2}(1+\gamma)A\left(\frac{2}{3}\psi(\gamma;\mu)+\frac{1}{6}\psi\left(\gamma;\mu+\sqrt{3}\sigma\right)+\frac{1}{6}\psi\left(\gamma;\mu-\sqrt{3}\sigma\right)\right)d\gamma \qquad (5.33)$$

Where $\psi(x)$ is given in (5.7). Substituting $\psi(x)$ in (5.33) and after taking sum assumptions, we have

$$\frac{C}{B}=\sum_{i=1}^{3}\frac{c}{2}\frac{A\,b_i^{\frac{c}{2}}d_i}{\ln(2)}\int_{0}^{\infty}\gamma^{\frac{c}{2}-1}\,ln(1+\gamma)\exp\left(-\gamma^{\frac{c}{2}}b_i^{\frac{c}{2}}\right)d\gamma \qquad (5.34)$$

Where $d_1=\frac{2}{3}$ and $d_2$ & $d_2=\frac{1}{6}$

$$b_1=\frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu)}$$

$$b_2=\frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu+\sqrt{3}\sigma)}$$

$$b_3=\frac{\Gamma\left(1+\frac{2}{c}\right)}{\exp(\mu-\sqrt{3}\sigma)}$$

Using appendix we have

$$\frac{C}{B}=\sum_{i=1}^{3}\frac{c}{2}\frac{A\,b_i^{\frac{c}{2}}d_i}{\ln(2)}\int_{0}^{\infty}\gamma^{\frac{c}{2}-1}\,G_{2,2}^{1,2}\left[\gamma\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right]G_{0,1}^{1,0}\left[\gamma^{\frac{c}{2}}b_i^{\frac{c}{2}}|0\right]\,d\gamma \qquad (5.35)$$

As per Generalization of classical Meijer's integral from two G functions ([54], /07.34.21.0013.01)

$$\int_0^\infty \gamma^{\frac{c}{2}-1}\, ln(1+\gamma)\exp\left(-b_i^{\frac{c}{2}}\gamma^{\frac{c}{2}}\right)d\gamma = \int_0^\infty \gamma^{\frac{c}{2}-1}\, G_{2,2}^{1,2}\left[\gamma \left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right] G_{0,1}^{1,0}\left[b_i^{\frac{c}{2}}\gamma^{\frac{c}{2}}|0\right]\, d\gamma =$$

$$\frac{\sqrt{k}}{l(2\pi)^{k+2l-\frac{3}{2}}}\, G_{2l,k+2l}^{k+2l,l}\left[b_i^{\frac{c}{2}}/k^k \left|\begin{matrix}I(l,-\frac{c}{2}),I(\,l,1-\frac{c}{2})\\ I(k,0),I(l,-\frac{c}{2}),I(l-\frac{c}{2})\end{matrix}\right.\right]$$

Where $I(m,p) \triangleq \frac{p}{m},\frac{p+1}{m}, \dots\dots\dots\dots\dots\dots\dots, \frac{p+m-1}{m}$ $\quad$ $and\, \frac{l}{k}=\frac{c}{2}$

And p and n are arbitrary integers.

For example, c=1.4, l=7 and k=10.

So

$$\int_0^\infty \gamma^{\frac{c}{2}-1}\, ln(1+\gamma)\exp\left(-b_i^{\frac{c}{2}}\gamma^{\frac{c}{2}}\right)d\gamma =$$

$$\frac{\sqrt{10}}{7*(2\pi)^{22.5}}\, G_{14,24}^{24,7}\left[{b_i}^{1.4}/10^{10}\left|\begin{matrix}I(7,-0.7),I(7,-0.3)\\ I(10,0),I(7,-0.7),I(7-0.7)\end{matrix}\right.\right]$$

$$I(7,-0.7) \triangleq \frac{-0.7}{7},\frac{0.3}{7},\frac{1.3}{7},\frac{2.3}{7},\frac{3.3}{7},\frac{4.3}{7},\frac{5.3}{7}$$

$$I(7,-0.3) \triangleq \frac{-0.3}{7},\frac{0.7}{7},\frac{1.7}{7},\frac{2.7}{7},\frac{3.7}{7},\frac{4.7}{7},\frac{5.7}{7}$$

$$I(10,0) \triangleq 0,\frac{1}{10},\frac{2}{10},\frac{3}{10},\frac{4}{10},\frac{5}{10},\frac{6}{10},\frac{7}{10},\frac{8}{10},\frac{9}{10}$$

**Table 5.1: Meijer G Parameters**

| ***c*** | ***l*** | ***k*** |
|---|---|---|
| 1.4 | 7 | 10 |
| 1.5 | 3 | 4 |
| 2 | 1 | 1 |
| 2.8 | 7 | 5 |
| 3 | 3 | 2 |
| 4 | 2 | 1 |
| 4.4 | 11 | 5 |

Using (Appendix), the channel capacity of composite weibull/log-normal fading channel is expressed as

$$\frac{C}{B} = \sum_{i=1}^{3} \frac{c}{2} \frac{A\, b_i^{\frac{c}{2}} d_i}{\ln(2)} \frac{\sqrt{k}}{l(2\pi)^{k+2l-\frac{3}{2}}}\, G_{2l,k+2l}^{k+2l,l} \left[ b_i^{\frac{c}{2}}/k^k \middle| \begin{matrix} I(l,-\frac{c}{2}), I(\,l,1-\frac{c}{2}) \\ I(k,0), I(l,-\frac{c}{2}), I(l-\frac{c}{2}) \end{matrix} \right] \quad (5.36)$$

Using (5.32) and (5.36), the average channel capacity of the combined (time-shared) shadowed/unshadowed fading channel is expressed as

$$\frac{C}{B} = \sum_{n=-1}^{\infty} (1-A) \frac{a^n e^{-K} K^{n-1}}{\ln(2)\,((n-1)!)^2}\, G_{2,3}^{3,1}\left[a \middle| \begin{matrix} -n, 1-n \\ 0, -n, -n \end{matrix}\right]$$
$$+ \sum_{i=1}^{3} \frac{c}{2} \frac{A\, b_i^{\frac{c}{2}} d_i}{\ln(2)} \frac{\sqrt{k}}{l(2\pi)^{k+2l-\frac{3}{2}}}\, G_{2l,k+2l}^{k+2l,l} \left[ b_i^{\frac{c}{2}} /k^k \middle| \begin{matrix} I(l,-\frac{c}{2}), I(\,l,1-\frac{c}{2}) \\ I(k,0), I(l,-\frac{c}{2}), I(l-\frac{c}{2}) \end{matrix} \right] \quad (5.37)$$

### 5.6 Results and Discussions

In this section, we present the graphical results for various performance metrics and PDF whose closed-form expressions are obtained in the preceding sections. Figure 5.1 depicts the graphical result of the closed-form of the composite weibull/log-normal pdf (Eq. 5.6) for varying SNR (dB). The plots are obtained for varying shape parameters of weibull distribution which decides the randomness of weibull RV. Also the exact result (Eq. 5.4) are also obtained. Perfect agreement confirms the accuracy of the proposed closed-from result. Increasing the shape parameter of weibull distribution indicates the increasing deterministic nature of the RV, thus, shifting the plot in upward directions.

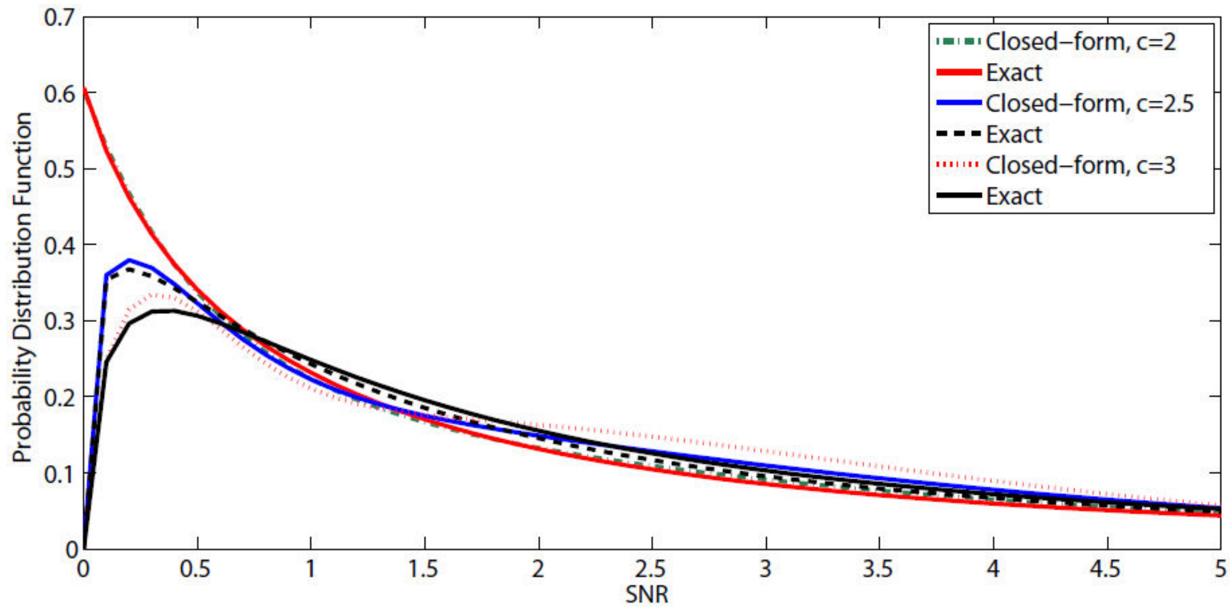


***Fig. 5.1. Comparison between exact expression (Eq. 5.4) and Closed form solution (Eq. 5.6) of composite PDF of Weibull /Log-Normal distribution (μ=1, σ=1)***

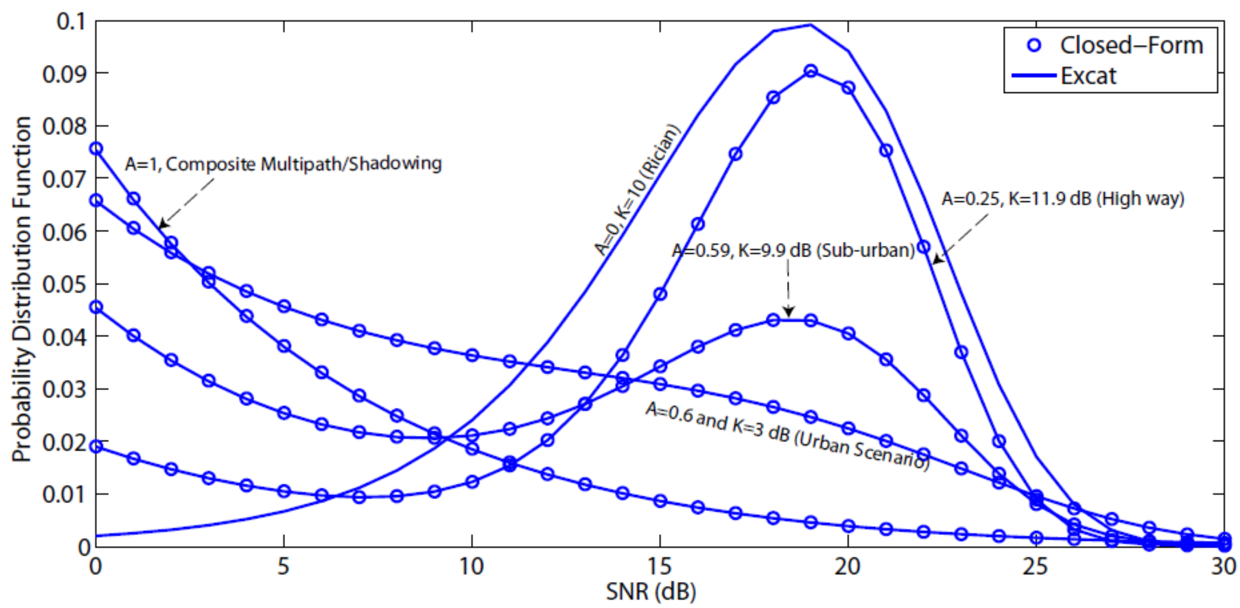


***Fig. 5.2. Probability distribution of Combined (Time-shared) multipath/shadowing and unshadowing fading for urban, sub-urban and highway [11].***

Figure 5.2 illustrates the results of PDF of combined (time-shared) shadowed/unshadowed fading (Eq. 5.11) and the exact plot uses the Eq. 5.6 in Eq. 5.11. Various shadowing cases enlisted in

[11] have been considered such as urban, sub-urban and highway scenarios. The corresponding values of A (a parameter that indicate the probability of occurrence of shadowing and unshadowing) and rice factors for these scenarios are presented in Table 5.2. It is quite clear from the PDF distribution of the combined multipath/shadowing and unshadowed fading that for urban scenario, the PDF is closer to composite multipath/shadowing fading and as the likelihood of rician scenario is more and more (moving from urban to highway), the combined pdf gets closer to rician distribution. Thus, for highway scenario, the fading is dominated by rician distribution and hence the pdf nearing the perfect rician curve.

**Table 5.2: Parameters A and K for various scenarios [6]**

| Environment | A | K(dB) |
|---|---|---|
| Urban | 0.60 | 3 |
| Suburban | 0.59 | 9.9 |
| Highway | 0.25 | 11.9 |

Amount of fading results for the combined shadowed/unshadowed fading is depicted in Fig. 5.3 (Eq. 5.12 and 5.19).

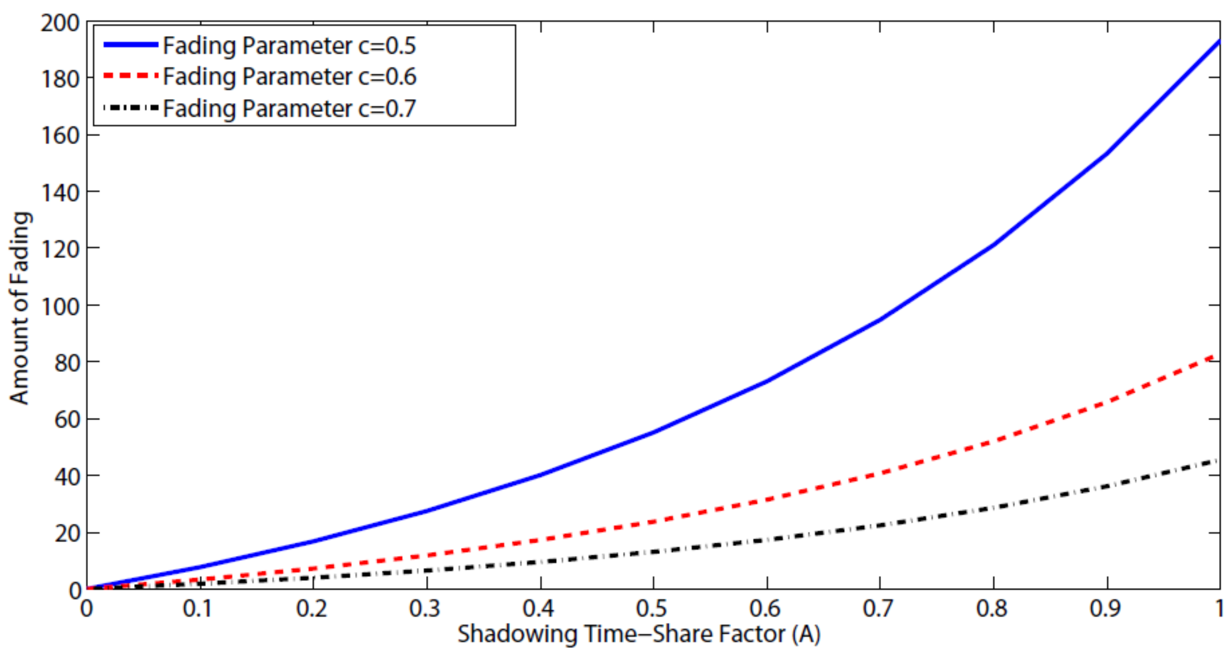


*Fig. 5.3. Amount of Fading for Combined (Time-shared) fading channel*

This figure gives an idea about the variation in the total Amount of Fading with variation in probabilistic change in the fading condition for a composite multipath/shadowing and unshadowed fading scenario. Initially, with A=0, only Rician condition dominates and hence AF remains very low. With increase in A, fading/fluctuation is dominated by multipath/shadowing condition and hence AF increases linearly with increase in fading severity. It is also worth noting that there is horizontal shift in downwards with increasing in the shape parameter of weibull distribution. This is due to dominance of shadowing effect as compared to multipath effect with increase in the shape parameter. In Figure 5.4, the outage probability results for a Rician distributed fading scenarios with various Rice factors [55] are demonstrated (Eq. 5.25).

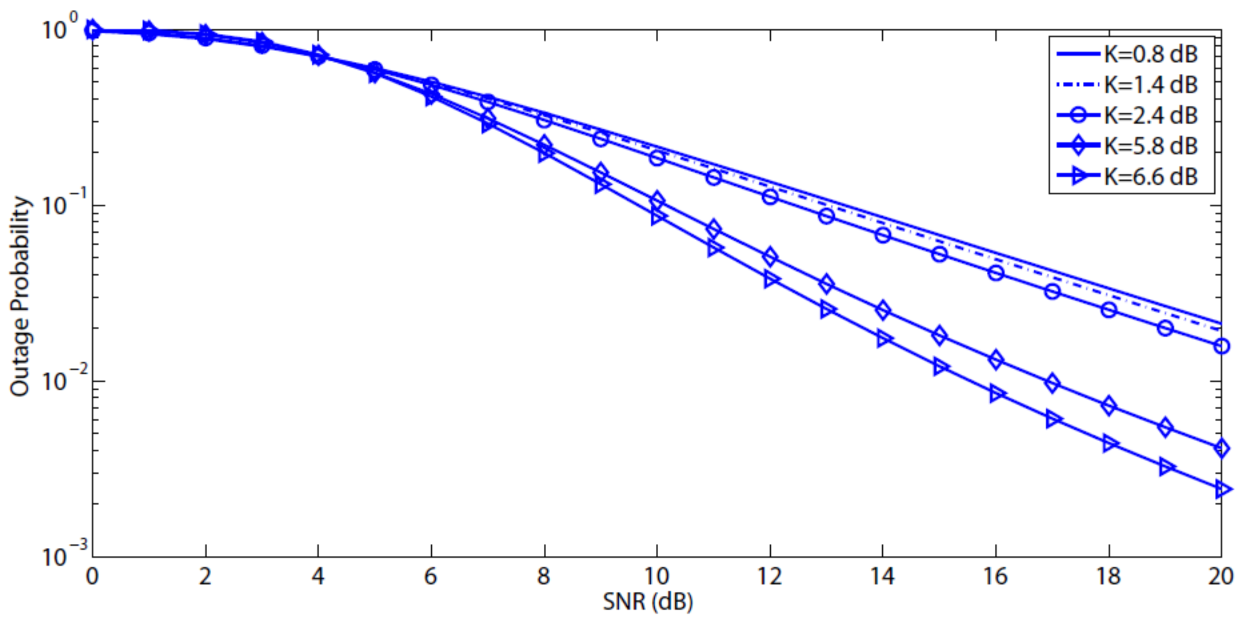


***Fig.5.4. Outage probability for different values of Rice factor K [55].***

As expected, the outage probability decreases with increase in the average SNR at the receiver. Further, the increase in the rice parameter shifts the outage probability to downward a side which is intuitively justifying.

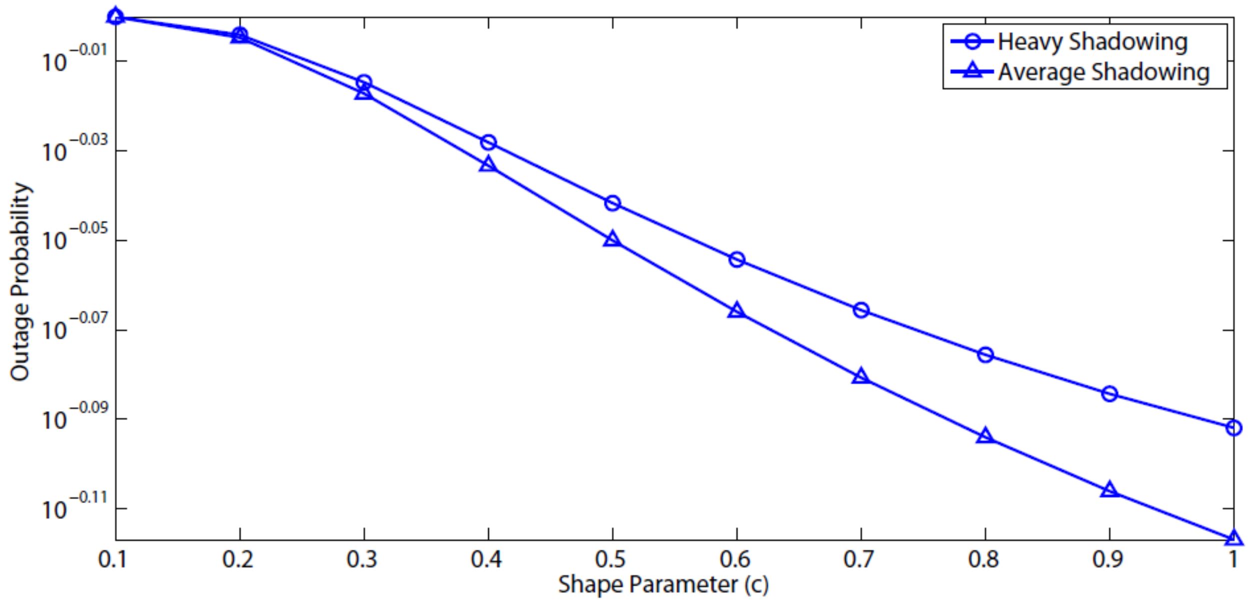


***Fig. 5.5. Outage Probability for*** $y_{th} = 5dB$ ***with two different shadowing conditions: (i) Average shadowing*** $\mu = -0.115dB, \sigma = 0.161dB$ ***(ii) Heavy shadowing*** $\mu = -3.914dB, \sigma = 0.806dB$ ***[12].***

Outage probability of composite weibull/log-normal fading is illustrated in Fig. 5.5 with varying shape parameter (Eq. 5.27). The increase in shape parameter of weibull RV indicates the less fluctuation of the multipath effect of composite fading, the outage probability is expected to decrease with increase in shape parameter and it is quite obvious from the figure. Further, the results are obtained for two different fading scenarios: heavy shadowing and average shadowing [12]. As expected, while moving from heavy shadowing to average shadowing, the results get shifted downside.

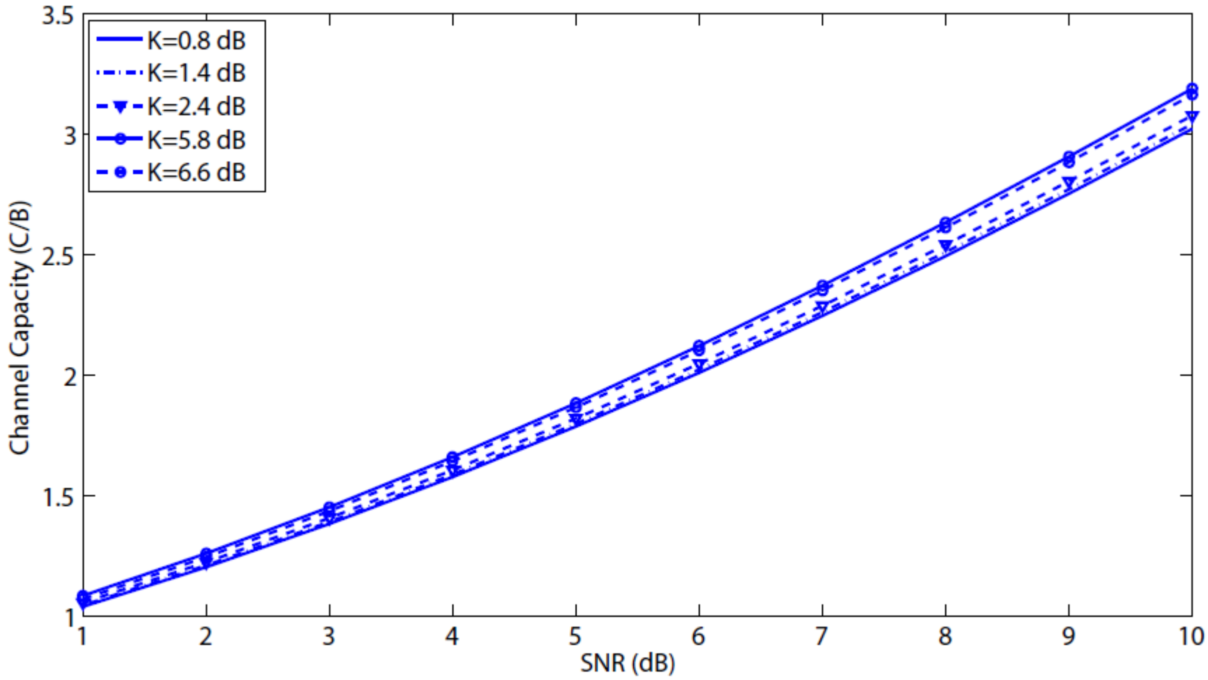


**Fig. 5.6. Average channel capacity for rice fading with different value of rice factor [55].**

Fig. 5.6 gives the graphical results for average channel capacity vs SNR (dB) for rice fading channel with different rice factors. As is obvious from the plots, that channel capacity increases with SNR and also there is a shift in upwards direction with a scenarios having strong line-of-sight condition. The Fig. 5.7 discusses about average channel capacity (Eq. 5.36), with increase in the shape parameter of multipath fading. As expected, there is an increase in channel capacity with shape parameter and also while going from average shadowing to heavy shadowing, there is shift of the plot in downside. Thus, for a given SNR, an average shadowed fading channel is supposed to have more capacity that heavy shadowed fading channel.

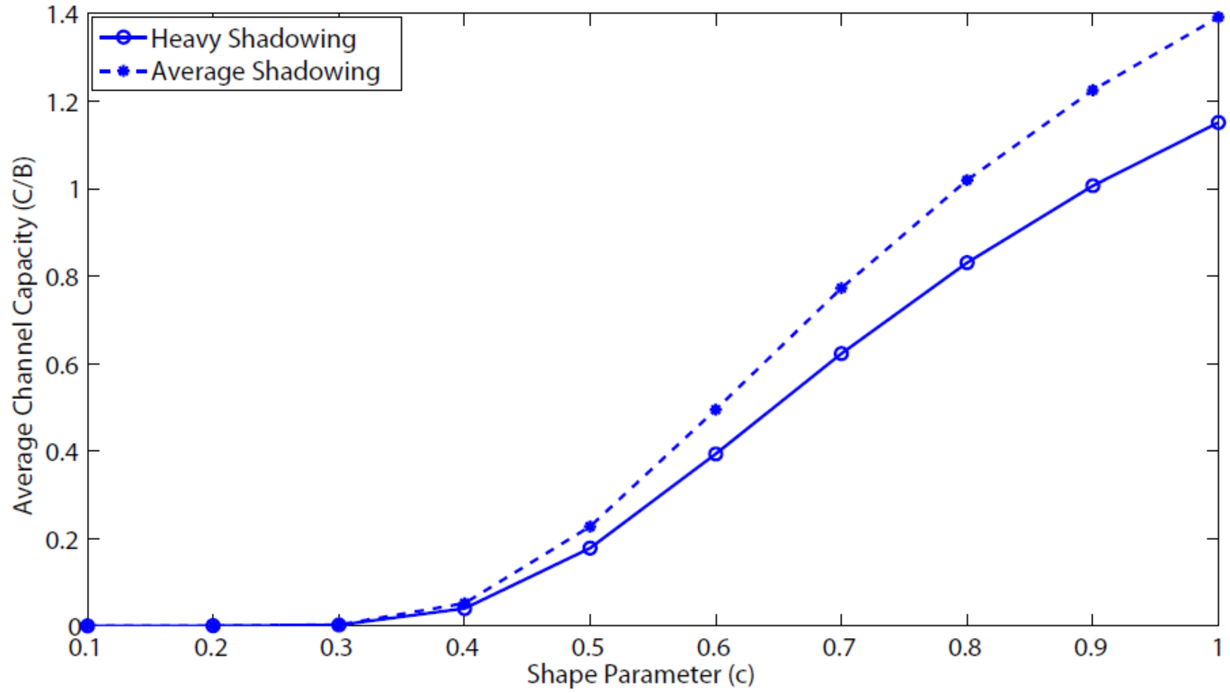


***Fig.5.7. Average Channel Capacity composite fading channel with two different shadowing conditions: (i) Average shadowing*** $\mu = -0.115dB, \sigma = 0.161dB$ ***(ii) Heavy shadowing*** $\mu = -3.914dB, \sigma = 0.806dB$ ***[12].***

### 5.7 Conclusion

In this study we have achieved closed-form expressions for PDF of instantaneous SNR, amount of fading, outage probability and channel capacity of the Composite Weibull/Log-normal fading and combined (time shared) multipath/shadowing (W-L) and unshadowing (Rician) fading. This paper has established a process for estimating the distribution of Composite/Shadowed (WL) fading when the log-normal based shadowing is considered. The procedure uses the Holtzmanian approximations to estimate the closed form of composite PDF of W-L fading. The resulting Holtzmanin approximation for Composite multipath/shadowed (WL) has the advantage of being in closed form, thereby facilitating the performance evaluation of communication links over Combined (time shared) composite multipath/shadowing (WL) and unshadowing fading channel.

# CHAPTER 6: PERFORMANCE ANALYSIS OF MRC COMBINER OUTPUT IN LOG NORMAL SHADOWED FADING

## 6.1 Introduction

Based on different indoor and outdoor observed measurements, a general consent has been given to model shadowing using Log-normal distribution [27, 42, 43]. Due to fading, signal recovery becomes difficult. When a received signal experiences fading during transmission, its envelope and phase both fluctuate over time.

One of the methods used to mitigate these degradation are diversity techniques, such as space diversity [1], [5]. Diversity combining has been considered as an efficient way to combat multipath fading and improve the received signal-to-noise ratio (SNR) because the combined SNR compared with the SNR of each diversity branch, is being increased. In this combining, two or more copies of the same information-bearing signal are combining to increase the overall SNR. The use of log-normal distribution [1], [13] to model shadowing which is random variable doesn't lead to a closed form solution for integrations involving in sum of random variables at the receiver. This distribution (PDF) can be approximated by another log normal random variable using Fenton-Wilkinson method [18].

This chapter presents Maximal-Ratio Combining procedure for communication system where the diversity combining is applied over uncorrelated branches ($\rho=0$), which are given as channels with log-normal fading.

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining technique whose SNR is the sum of the SNR's of each individual diversity branch. MRC is the optimal combining scheme, but its price and complexity are high, since MRC requires cognition of all fading parameters of the channel.

The sum of log normal random variables has been considered in [17, 18, 19, 56]. Up to now these papers has shown different techniques such as MGF, Type IV Pearson Distribution and recursive approximation. In this chapter we have approximated sum (MRC) of log normal random variables using FW method. On the basis of FW approximation, we have given amount of fading (AF), outage probability ($P_{out}$) and channel capacity (C) for MRC combiner.

### 6.2 Systems and Channel Models

Log-normal Distribution

A RV γ is log-normal, i.e. $\gamma \sim LN(\mu, \sigma^2)$, if and only if $\ln(\gamma) \sim N(\mu, \sigma^2)$. A log-normal RV has the PDF

$$p(\gamma) = \frac{\xi}{\sigma\gamma\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma-\mu)^2}{2\sigma^2}} \qquad (6.1)$$

For any $\sigma^2 > 0$. The expected value of γ is

$$E(\gamma) = e^{(\mu+0.5\,\sigma^2)}$$

And the variance of γ is

$$Var(\gamma) = e^{(\sigma^2-1)} * e^{(2\mu+\sigma^2)}$$

Where $\xi$=10/ln 10=4.3429, μ(dB) is the mean of $10log_{10}\gamma$, $\sigma$(dB) is standard deviation of $10log_{10}\gamma$.

### 6.3 Maximal Ratio Combining

The total SNR at the output of the MRC combiner is given by:

$$\gamma_{MRC}=\sum_{i=1}^{L}\gamma_i \qquad (6.2)$$

$$\gamma_{MRC}=\gamma1 + \gamma2 + \gamma3 + \gamma4 \dots\dots\dots\dots + \gamma L \qquad (6.3)$$

where *L* is number of branches.

Since the *L* lognormal RVs are independently distributed, the PDF of the lognormal sum [18]

$$p(\gamma_{MRC}) = p(\gamma_1) \otimes p(\gamma_2) \otimes p(\gamma_3) \otimes \dots\dots\dots\dots\dots\dots\dots \otimes p(\gamma_L) \qquad (6.4)$$

where $\otimes$ denotes the convolution operation.

The functional form of the log-normal PDF does not permit integration in closed-form. So above convolution can never be possible to present. Fenton (1960) estimate the PDF for a sum of log-normal RVs using another log-normal PDF with the same mean and variance. The Fenton approximation (sometimes referred to as the Fenton-Wilkinson (FW) method) is simpler to apply for a wide range of log-normal parameters.

**6.3.1 Consider the sum of *L* uncorrelated log-normal**

RVs, $\gamma$, as specified in (1) where each $\gamma$ ~LN($\mu$ , $\sigma^2$) with the expected value and variance $\mu$ and $\sigma$ respectively. The expected value and variance of $\gamma_{MRC}$ are

$$E(\gamma_{MRC}) = L\,E(\gamma)$$

And

$$Var(\gamma_{MRC}) = L\,Var(\gamma)$$

The FW approximation is a log-normal PDF with parameters $\mu_M$ and $\sigma^2{}_M$ such that

$$e^{(\mu_M + 0.5\,\sigma_M{}^2)} = L\,E(\gamma)$$

And

$$e^{(\sigma_M{}^2 - 1)} * e^{(2\mu_M + \sigma_M{}^2)} = L\,Var(\gamma)$$

Solving above equations for $\mu_M$ and $\sigma^2{}_M$ gives

$$\sigma_M{}^2 = \ln\left(\frac{e^{(\sigma^2 - 1)}}{L} + 1\right) \qquad (6.5)$$

And

$$\mu_M = \ln(L\,e^{\mu}) + 0.5(\sigma^2 - \sigma_M{}^2) \qquad (6.6)$$

So PDF of the sum of L diversity branches using F-W method is given as

$$p(\gamma_{MRC}) = \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} \qquad (6.7)$$

The different $\mu_M$ and $\sigma_M$ have been calculated for different numbers of branches L using above F-W approximation from (6.5) and (6.6) and shown in table 6.1. For calculations we have considered $\gamma$ ~LN(0.69,1.07$^2$).

**Table 6.1 $\mu_M$ and $\sigma_M$ for different number of diversity branches**

| **Number of diversity branches $L$** | $\mu_M$ | $\sigma_M$ |
|---|---|---|
| 2 | *1.59* | 0.85 |
| 4 | *2.44* | 0.65 |
| 6 | *2.90* | 0.55 |
| 8 | *3.22* | 0.48 |
| 10 | *3.46* | 0.44 |
| 15 | *3.91* | 0.36 |
| 20 | *4.21* | 0.31 |
| 25 | *4.44* | 0.28 |
| 30 | *4.63* | 0.26 |
| 50 | *5.15* | 0.20 |

In Fig 6.1, PDF of received SNR using MRC diversity techniques has presented. As we can see from the Figure that as the number of branches increases, PDF of received SNR tends towards Gaussian distribution shape. So we can conclude that FW approximation method also satisfies central limit theorem.

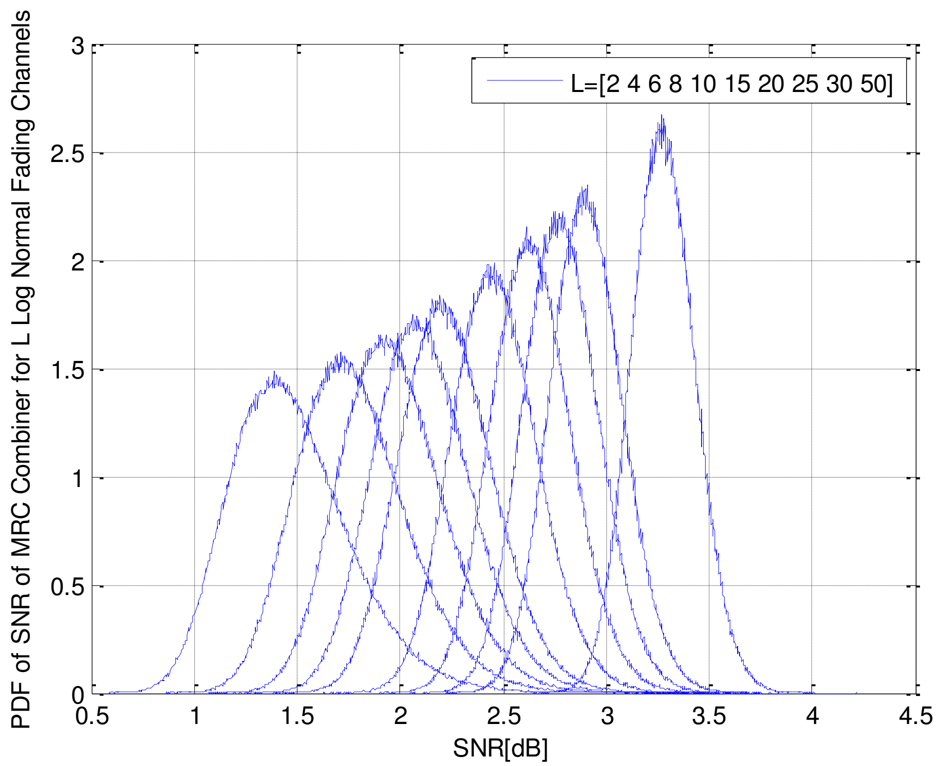


***Fig 6.1. PDF of received SNR of MRC combiner output***

**6.4 Amount of Fading (AOF) is defined as**

$$AOF = \frac{E[{\gamma_{MRC}}^2]}{(E[\gamma_{MRC}])^2} - 1 \qquad (6.8)$$

For MRC

$\gamma_{MRC} = \gamma 1 + \gamma 2 + \gamma 3 + \gamma 4 + \cdots \ldots \ldots \ldots \ldots . + \gamma L$

$E[\gamma_{MRC}] = E[\gamma 1 + \gamma 2 + \gamma 3 + \gamma 4 + \cdots . + \gamma L]$

Moment of PDF of received SNR [1]

$$E({\gamma_{MRC}}^K) = \int_0^\infty {\gamma_{MRC}}^K \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \qquad (6.9)$$

$$E({\gamma_{MRC}}^K) = \exp\left(\frac{K}{\xi}\mu_M + \frac{1}{2}\left(\frac{K}{\xi}\right)^2 \sigma_M\right) \qquad (6.10)$$

Yielding an AOF of

$$AOF = \exp\left(\left(\frac{\sigma_M}{\xi}\right)^2\right) - 1 \qquad (6.11)$$

In Table 6.2, AF is given for different numbers of diversity branches L. We can conclude that AF decreases as the number of diversity branches L increases.

**Table 6.2 AF for different number of diversity branches**

| Number of diversity branches $L$ | $\boldsymbol{\sigma_M}$ | $AOF = \exp\left(\left(\frac{\sigma_M}{\xi}\right)^2\right) - 1$ |
|---|---|---|
| 2 | 0.85 | 0.03905 |
| 4 | 0.65 | 0.022654 |
| 6 | 0.55 | 0.016168 |
| 8 | 0.48 | 0.012291 |
| 10 | 0.44 | 0.010318 |
| 15 | 0.36 | 0.006895 |
| 20 | 0.31 | 0.005108 |
| 25 | 0.28 | 0.004165 |
| 30 | 0.26 | 0.003591 |
| 50 | 0.20 | 0.002123 |

### 6.5 Outage Probability

Recall from chapter 2, section 2.2 outage probability can be defined as

$$P_{out} = \int_0^{\gamma_{th}} p(\gamma)d\gamma \qquad (6.12)$$

$$P_{out}(\gamma_{th}) = P[\gamma_{MRC} = \gamma 1 + \gamma 2 + \gamma 3 + \gamma 4 + \cdots \dots \dots \dots \dots . + \gamma \mathrm{L} \le \gamma_{th}] \qquad (6.13)$$

$$P_{out}(\gamma_{th})^{MRC} = \int_0^{\gamma_{th}} p_{\gamma_{MRC}}(\gamma_{MRC})d\gamma_{MRC} \qquad (6.14)$$

Using (6.7) into (6.14)

$$P_{out}(\gamma_{th})^{MRC} = \int_0^{\gamma_{th}} \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC}-\mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \quad (6.15)$$

$$P_{out}(\gamma_{th})^{MRC} = \frac{1}{2} erfc\left(\frac{\mu_M - 10log_{10}\gamma_{th}}{\sqrt{2}\sigma_M}\right) \quad (6.16)$$

In Fig 6.2, outage probability $P_{out}$ is shown for different numbers of diversity branches from L=1 to 50. We can conclude that as the number of L increases outage probability decreases. We can see that MRC not only improves SNR but also improves performance in sense of outage probability$P_{out}$.

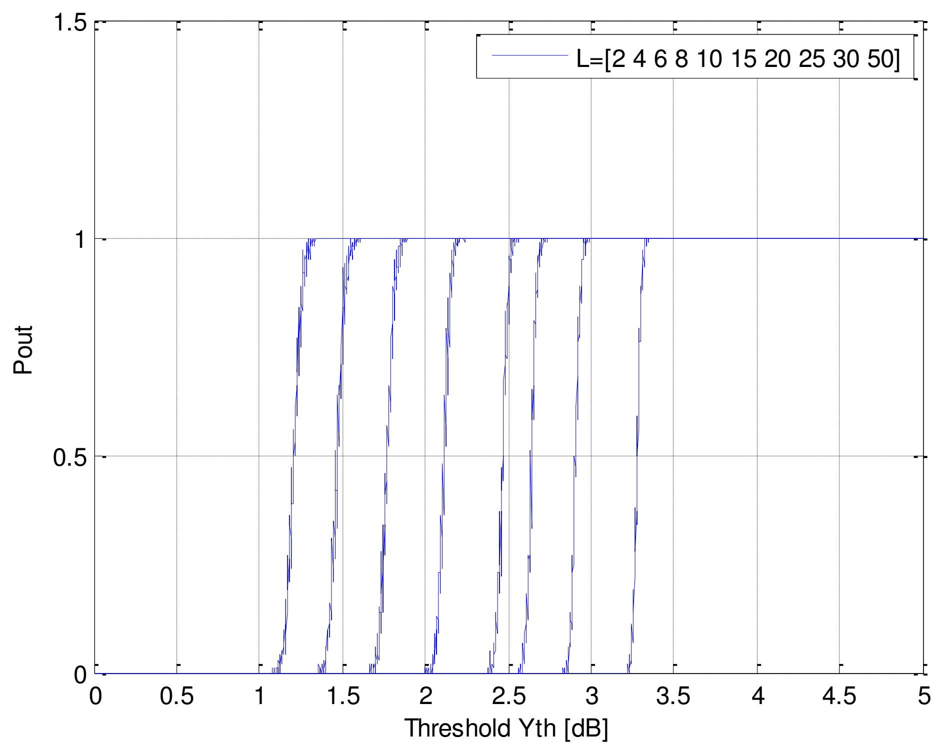


***Fig 6.2. $P_{out}$ of MRC combiner output versus threshold $\gamma_{th}$***

**6.6 Channel Capacity**

For MRC, Channel capacity [29] is defined as

$$C = B\int_0^{\infty} log_2(1+\gamma_{MRC}) p_{\gamma_{MRC}}(\gamma_{MRC}) d\gamma_{MRC} \quad (6.17)$$

Using (6.7) in (6.17) we have capacity

$$C = B\int_0^{\infty} log_2(1+\gamma_{MRC})\frac{\xi}{\sigma_M\gamma_{MRC}\sqrt{2\pi}}e^{-\frac{(10log_{10}\gamma_{MRC}-\mu_M)^2}{2\sigma^2{}_M}}d\gamma_{MRC} \tag{6.18}$$

This can be approximated using by Holtzman approximation [28] (5.6) and (5.7)

$$\frac{C}{B} \approx \frac{2}{3}\log_2\left(1+10^{\frac{\mu_M}{10}}\right)+\frac{1}{6}\log_2\left(1+10^{\frac{\mu_M+\sqrt{3}\sigma_M}{10}}\right)+\frac{1}{6}\log_2\left(1+10^{\frac{\mu_M+\sqrt{3}\sigma_M}{10}}\right) \tag{6.19}$$

Where B is the bandwidth. The values of *C/B* has been calculated and shown in Table 6.3. As we can conclude from table 6.3 that channel capacity is more in case of large number of diversity branches for MRC combiner output. The closed form in (6.19) can be validated by comparing with [13] in which channel capacity has been derived in closed form for single lognormal fading channel.

**Table 6.3 Channel capacity *C/B* for different number of diversity branches**

| **Number of diversity branches *L*** | ***C/B*** |
|---|---|
| 2 | 0.7939 |
| 4 | 1.1032 |
| 6 | 1.2860 |
| 8 | 1.4174 |
| 10 | 1.5174 |
| 15 | 1.7072 |
| 20 | 1.8350 |
| 25 | 1.9335 |
| 30 | 2.0151 |
| 50 | 2.2392 |

### 6.7 Conclusion

This study has established a process for estimating the distribution of MRC combiner output for lognormal distributed SNR (a single log-normal RV is a special case). The procedure uses the Fenton- Wilkinson approximation (Fenton, 1960) to estimate the parameters for a single log-normal PDF that approximates the sum (MRC) of log-normal RVs. Fenton Wilkinson (FW) approximation was shown to be general enough to cover the cases of sum of uncorrelated log normal RVs. We have tabulated $\mu_M$ and $\sigma_M$. We calculated amount of fading (AOF) and channel capacity (C/B). Also we represented graphically PDF of received SNR outage probability$P_{out}$. Fig (6.1) has shown that as value of L increases PDF of MRC combiner output approaches Gaussian distribution which can be proved using Central limit theorem. Table 2 depicted that fading reduces with increasing MRC branches. Table 3 has shown that channel capacity increase with increasing MRC branches.

# CHAPTER 7: ABEP OF MQAM AND MPSK FOR MRC COMBINER OUTPUT IN LOG NORMAL SHADOWED FADING CHANNEL

In this chapter, a simple accurate closed-form using Holtzmanin [28] approximation for the expectation of the function of a normal variant is employed. Then, simple analytical approximations for the ABEP of M-QAM and M-PSK modulation schemes for MRC combiner output are derived.

Using maximum likelihood coherent detection, the instantaneous BEP of M-QAM and M-PSK can be obtained for different modulation schemes employing Gray encoding at higher SNR can be written in generic form [27], [2] as

$$P_b(\beta) = \theta . Q\left(\sqrt{\lambda\beta\gamma_s}\right) \qquad (7.1)$$

Where

$$\theta = \frac{4}{log_2 M} \;\; and \; \lambda = 3.\left(\frac{1}{M-1}\right) \qquad for\; M - QAM$$

And

$$\theta = \frac{2}{log_2 M} \;\; and \; \lambda = 3.sin^2\left(\frac{\pi}{M}\right) \qquad for\; M - PSK$$

$\gamma_s$ is received SNR in additive white-gaussian noise, and $\beta$ is a non-negative random variable depends on the fading type.

### 7.1 ABEP of MQAM for MRC Combiner Output in Log Normal shadowing fading channel can be written as using (2.8) and (6.7)

$$P_S(E)_{MQAM} = \int_0^\infty P_b(\beta)_{MQAM} . \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \qquad (7.2)$$

It is difficult to calculate the results directly, in this work, we adopt the efficient tool proposed by Holtzmanin[28] to simplify Eg. (5). Taking Eg. (5-7) in [28], we have

Using $10log_{10}\gamma_{MRC} = x$ in (7.2)

$$P_S(E)_{MQAM} = \int_0^{\infty} \psi(x).\frac{1}{\sigma_M\sqrt{2\pi}} e^{-\frac{(x-\mu_M)^2}{2\sigma^2{}_M}} dx \quad (7.3)$$

Then finally we have ABEP

$$P_S(E)_{MQAM} \approx \frac{2}{3}\psi(\mu_M) + \frac{1}{6}\psi(\mu_M + \sqrt{3}\sigma_M) + \frac{1}{6}\psi(\mu_M - \sqrt{3}\sigma_M) \quad (7.4)$$

Where

$$\psi(x) = \theta.Q\left(\sqrt{\lambda 10^x \gamma_s}\right) and\ \theta = \frac{4}{log_2 M}, \lambda = 3.\left(\frac{1}{M-1}\right)$$

In Fig 7.1, 7.2 and 7.3 ABEP of MQAM has been shown for different numbers of diversity branches from L=2 to 50. The values of $\mu_M$ and $\sigma_M$ computed in table 6.1. We can conclude that as the number of L increases ABEP decreases. We can see that MRC not only improves SNR but also improves performance in sense of ABEP. Also we have concluded that with increasing M=4, 16, 64 ABEP also increases.

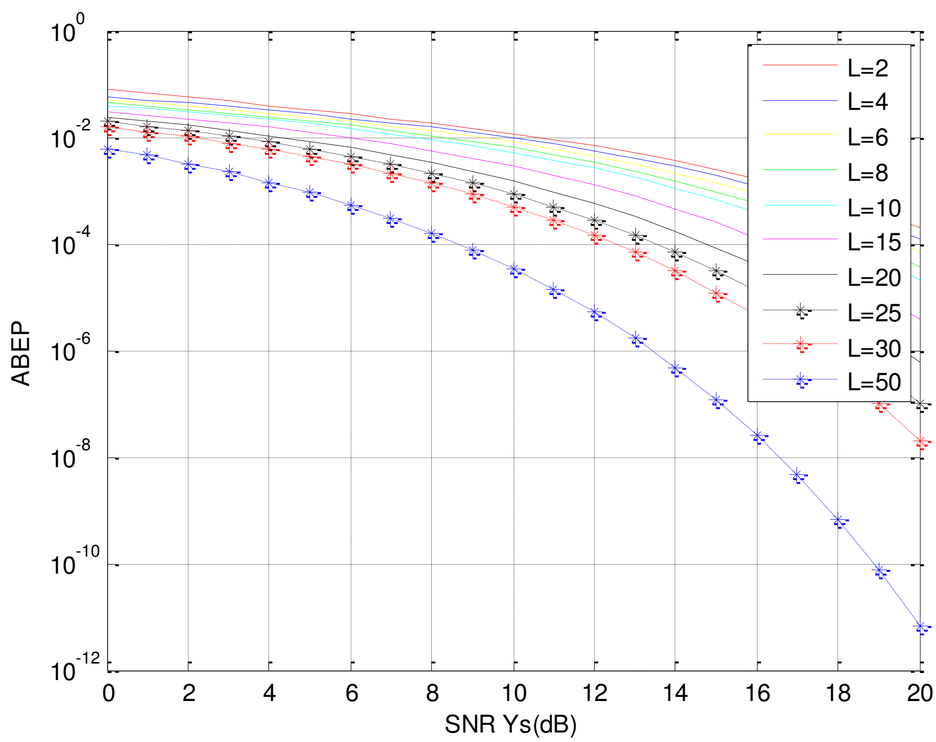


***Fig 7.1. ABEP of MQAM for MRC Combiner Output M=4***

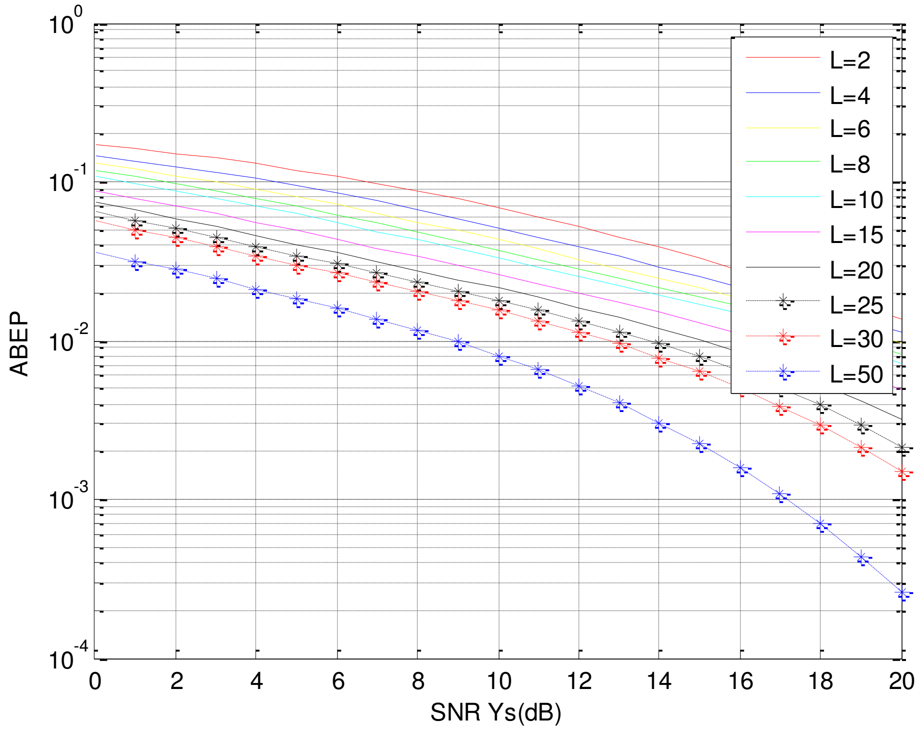


*Fig 7.2. ABEP of MQAM for MRC Combiner Output M=16*

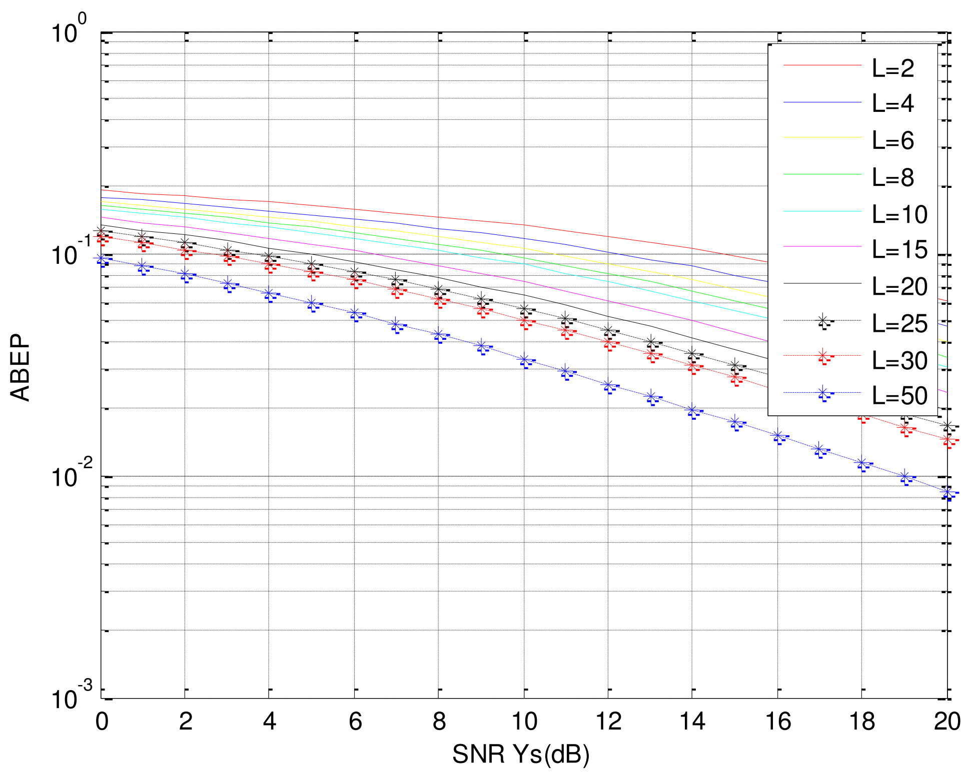


*Fig 7.3. ABEP of MQAM for MRC Combiner Output M=64*

**7.2 ABEP of MPSK for MRC Combiner Output in Log Normal shadowing fading channel can be written as using (2.8) and (6.7)**

$$P_S(E)_{MPSK} = \int_0^\infty P_b(\beta)_{MPSK} \cdot \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC}-\mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \quad (7.5)$$

It is difficult to calculate the results directly, in this work, we adopt the efficient tool proposed by Holtzmanin[28] to simplify Eg. (5). Taking Eg. (5-7) in [28], we have

Using $10log_{10}\gamma_{MRC} = x$ in (7.5)

$$P_S(E)_{MPSK} = \int_0^\infty \psi(x). \frac{1}{\sigma_M\sqrt{2\pi}} e^{-\frac{(x-\mu_M)^2}{2\sigma^2{}_M}} dx \quad (7.6)$$

Then finally we have ABEP

$$P_S(E) \approx \frac{2}{3}\psi(\mu_M) + \frac{1}{6}\psi\left(\mu_M + \sqrt{3}\sigma_M\right) + \frac{1}{6}\psi\left(\mu_M - \sqrt{3}\sigma_M\right) \quad (7.7)$$

Where

$$\psi(x) = \theta. Q\left(\sqrt{\lambda 10^{x}\gamma_s}\right) and\ \theta = \frac{2}{log_2 M}, \lambda = 3. sin^2\left(\frac{\pi}{M}\right)$$

In Fig 7.4, 7.5 and 7.5 ABEP of MPSK has been shown for different numbers of diversity branches from L=2 to 50. The values of $\mu_M$ and $\sigma_M$ computed in table 6.1. We can conclude that as the number of L increases ABEP decreases. We can see that MRC not only improves SNR but also improves performance in sense of ABEP. Also we have concluded that with increasing M=4, 16, 64 ABEP also increases.

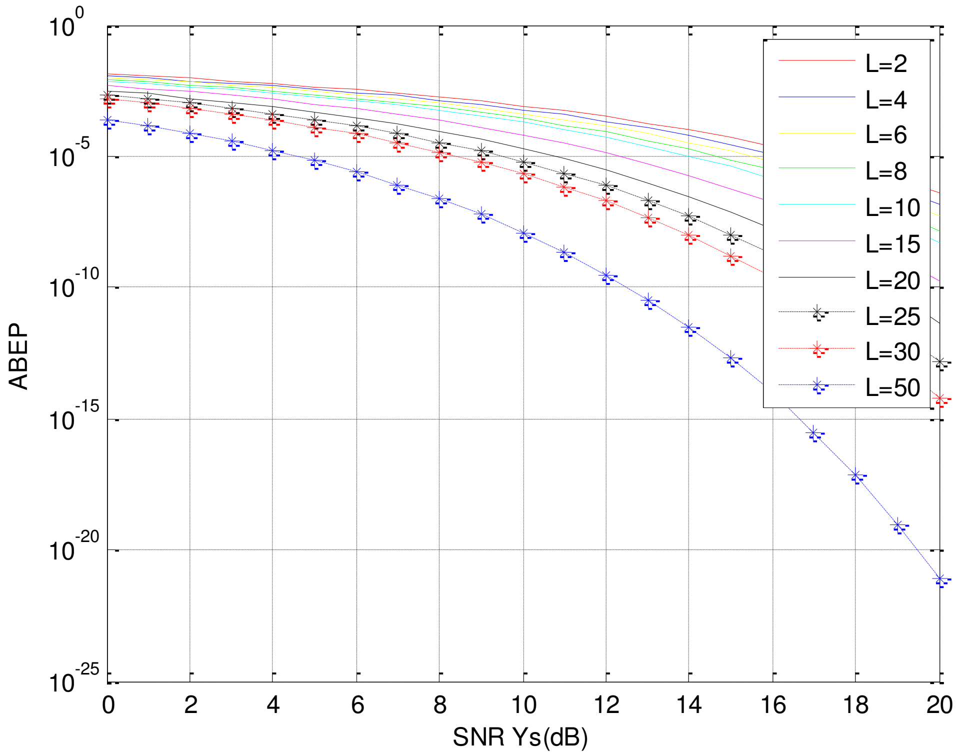


***Fig 7.4. ABEP of MPSK for MRC Combiner Output M=4***

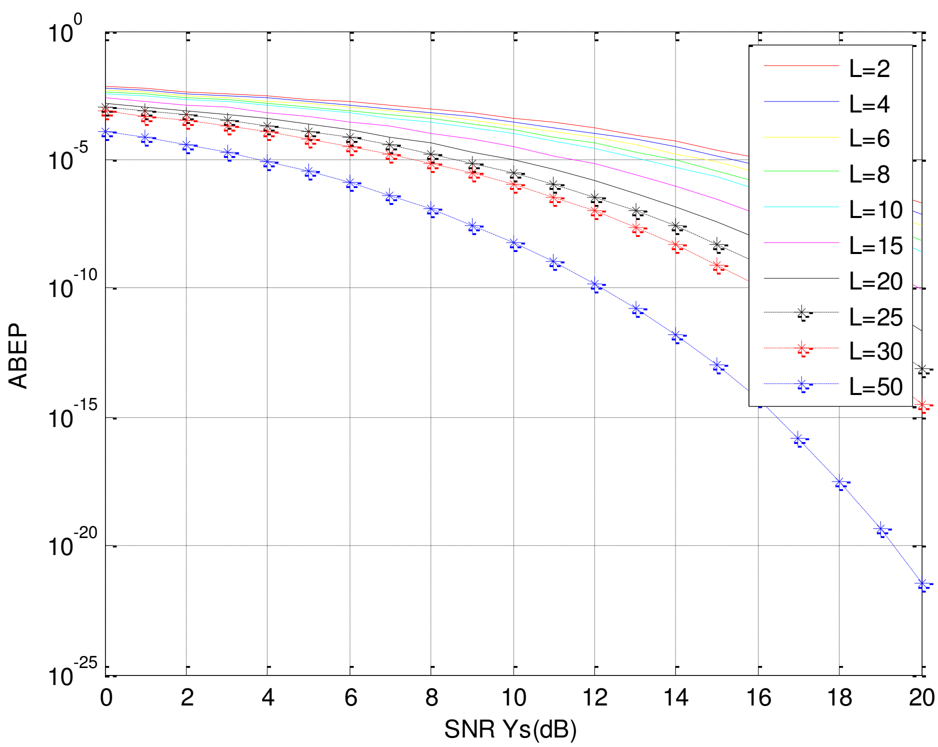


***Fig 7.5. ABEP of MPSK for MRC Combiner Output M=16***

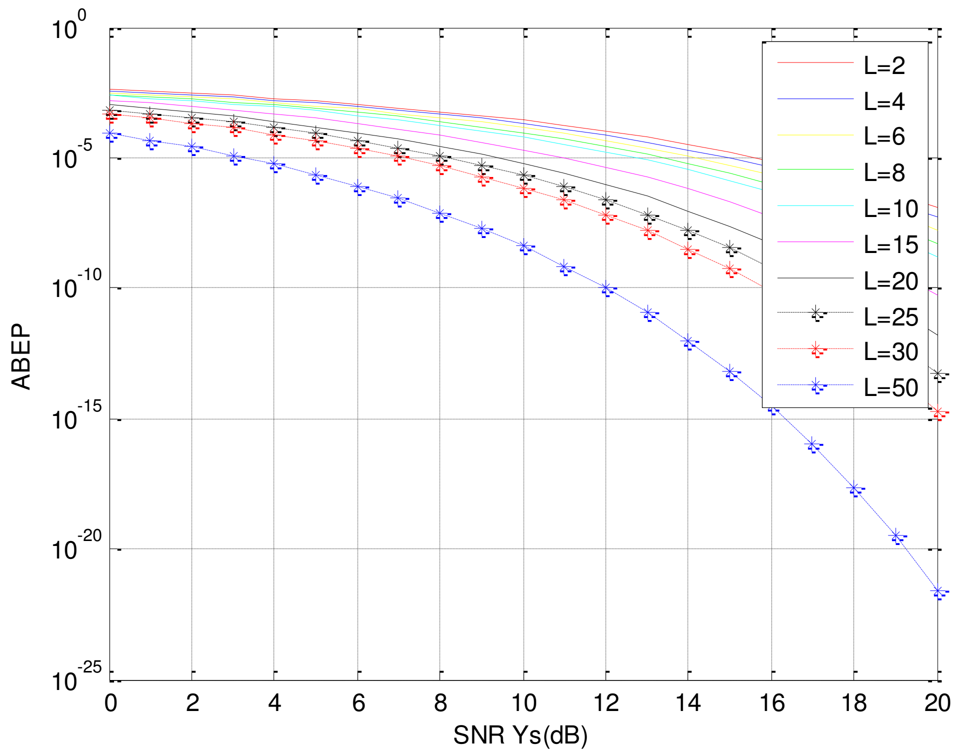


***Fig 7.6. ABEP of MPSK for MRC Combiner Output M=64***

### 7.3 Conclusion

This study has established a process for estimating ABEP of MQAM and MPSK of MRC combiner output for lognormal distributed SNR. The procedure uses the Holtzmanin approximation to approximate closed form expression of ABEP. We have used $\mu_M$ and $\sigma_M$ from table 6.1. ABEP for MQAM and MPSK for MRC combiner output in Log Normal fading channel also plotted from Fig 7.1 to 7.6 for different diversity branches L. We can conclude that MRC improves performance as well as ABEP of communication systems in fading environment.

# CHAPTER 8: ANALYTICAL LEVEL CROSSING RATE AND AVERAGE OUTAGE DURATION FOR MRC COMBINER OUTPUT IN LOG NORMAL SHADOWED FADING

## 8.1 Introduction

The log-normal distribution is often found to be the most suitable distribution to fit empirical fading channel measurements particularly for the indoor radio propagation environments [57]. The use of log-normal distribution [1], [13] to model shadowing which is random variable (SNR) can't be expressed in closed form solution for integrations involving in sum of random variables (SNRs) of MRC combiner output . This distribution (PDF) can be approximated by another log normal random variable using Fenton-Wilkinson method [18]. The level crossing rate (LCR) and average outage duration (AOD) are important metrics for evaluating the dynamic performance of diversity systems [58]. LCR and AOD have been considered in [34-36, 58, 60] of different fading channels. These papers have considered Nakagami, Rayleigh, Rician, Rician-Log normal and Rayleigh-Log normal shadowed channels. A method to derive LCR and AOD for the log normal shadowed channel has been shown in [33]. Rice has given a method to find LCR and AOD of a fading channel using the joint PDF of the fading envelope and its time derivative [61]. But traditional Rician approach to estimate LCR and AOD is more complex and not suitable to characterize discrete channels [4, 49, 62].

In this chapter, we have proposed a more reliable method for the statistical estimation of LCR and AOD. We have proposed a new and simple analytical approach to evaluate LCR and AOD of MRC combiner output in Log Normal (LN) shadowed channels. A simple joint CDF based approach for LCR and AOD has been shown. The MRC combiner output has been estimated using Fenton- Wilkinson method. Different values of Means and Variances have been tabulated for different number of diversity branches L. LCR and AOD has been plotted and tabulated for different number of diversity branches L.

## 8.2 Systems and Channel Models

Log-normal Distribution

A RV γ is log-normal, i.e. $\gamma \sim LN(\mu, \sigma^2)$, if and only if $\ln(\gamma) \sim N(\mu, \sigma^2)$. A log-normal RV has the PDF

$$p(\gamma) = \frac{\xi}{\sigma\gamma\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma-\mu)^2}{2\sigma^2}} \qquad (8.1)$$

For any $\sigma^2 > 0$. The expected value of $\gamma$ is

$$E(\gamma) = e^{(\mu+0.5\,\sigma^2)}$$

And the variance of $\gamma$ is

$$Var(\gamma) = e^{(\sigma^2-1)} * e^{(2\mu+\sigma^2)}$$

Where $\xi$=10/ln 10=4.3429, μ(dB) is the mean of $10log_{10}\gamma$, $\sigma$(dB) is standard deviation of $10log_{10}\gamma$.

CDF of log-normal RV

$$P(\gamma) = Q\left(\frac{\mu - 10log_{10}\gamma}{\sigma}\right) \qquad (8.2)$$

Where Q (.) is one dimentional standard Gaussian Q function.

## 8.3 Maximal Ratio Combining

The total SNR at the output of the MRC combiner is given by:

$$\gamma_{MRC}=\sum_{i=1}^{L}\gamma_i \qquad (8.3)$$

$$\gamma_{MRC}=\gamma 1 + \gamma 2 + \gamma 3 + \gamma 4 \ldots\ldots\ldots\ldots. + \gamma L \qquad (8.4)$$

Where *L* is number of IID branches.

Using Fenton-Wilkinson method, PDF of MRC combiner output is

$$p(\gamma_{MRC}) = \frac{\xi}{\sigma_M\gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC}-\mu_M)^2}{2\sigma^2{}_M}} \qquad (8.5)$$

And CDF of MRC combiner output is

$$P(\gamma) = Q\left(\frac{\mu_M - 10log_{10}\gamma_{MRC}}{\sigma_M}\right) \qquad (8.6)$$

## 8.4 Level Crossing Rate and Average Outage Duration

The AOD, $T_{\gamma th}$ (in seconds) is a measure of how long, on the average, the system remains in the outage state [1].

$$T_{\gamma th} = \frac{P_{out}}{N_{\gamma th}}$$

Where $P_{out}$ is outage probability.

The envelope level crossing rate $N_{\gamma th}$ is defined as the expected rate (in crossings per second) at which the signal envelope crosses the level $\gamma_{th}$ in the downward direction [2]

$$N_{\gamma th} = \int_0^{\infty} \dot{y}\, f_{\gamma,\dot{y}}(\gamma_{th}, \dot{y})\, d\dot{y}$$

Where $f_{\gamma,\dot{y}}(\gamma_{th}, \dot{y})$ is the joint PDF of $\gamma$ and its time derivative.

Above definitions are Rician approach to estimate LCR and AOD. But this approach is not suitable for discrete channels. So for discrete sampled random process, LCR can be defined as the rate at which the envelope $\gamma$ crosses a certain threshold $\gamma th$ in the positive or in the negative direction

$$N_{\gamma th} = \frac{P(\gamma 1 \leq \gamma th, \gamma 2 \geq \gamma th)}{T} \qquad (8.7)$$

Where $\gamma 1 \triangleq \gamma(\mathrm{t})$and $\gamma 2 \triangleq \gamma(\mathrm{t}+\mathrm{T})$ is second order statistics and T denotes sampling period. Here $\gamma 1$ and $\gamma 2$ are uncorrelated IID random variables so their CDF is same as shown in (8.6).

Noting here

$$P(\gamma 1 \leq \gamma th, \gamma 2 \geq \gamma th) = P(\gamma 1 \leq \gamma th) - P(\gamma 1 \leq \gamma th, \gamma th \geq \gamma 2) \qquad (8.8)$$

Or

$$P(\gamma 1 \leq u, \gamma 2 \geq u) = CDF\ of\ \gamma 1 - Joint\ CDF\ of\ \gamma 1\ and\ \gamma 2$$

CDF of $\gamma 1$ for L diversity branches

$$P(\gamma 1 \leq \gamma th) = Q\left(\frac{\mu_M - 10 log_{10} \gamma_{MRC}}{\sigma_M}\right) \qquad (8.9)$$

And Joint CDF of $\gamma 1$ and $\gamma 2$ for L diversity branches

$$P(\gamma 1 \leq \gamma th, \gamma th \geq \gamma 2) = Q\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right) Q\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right) \quad (8.10)$$

Using (8), (9) and (10) in (7), LCR of MRC combiner output in log normal shadowed channel is

$$N_{\gamma th} = \frac{Q\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right) - Q^2\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right)}{T} \quad (8.11)$$

AOD is defined as

$$T_{\gamma th} = \frac{P(\gamma \leq \gamma th)}{N_{\gamma th}} \quad (8.12)$$

Using (8.9) and (8.11) in (8.12) we have AOD of MRC combiner output in log normal shadowed channel is

$$T_{\gamma th} = \frac{T\,Q\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right)}{\left(Q\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right) - Q^2\left(\frac{\mu_M - 10log_{10}\,\gamma_{MRC}}{\sigma_M}\right)\right)} \quad (8.13)$$

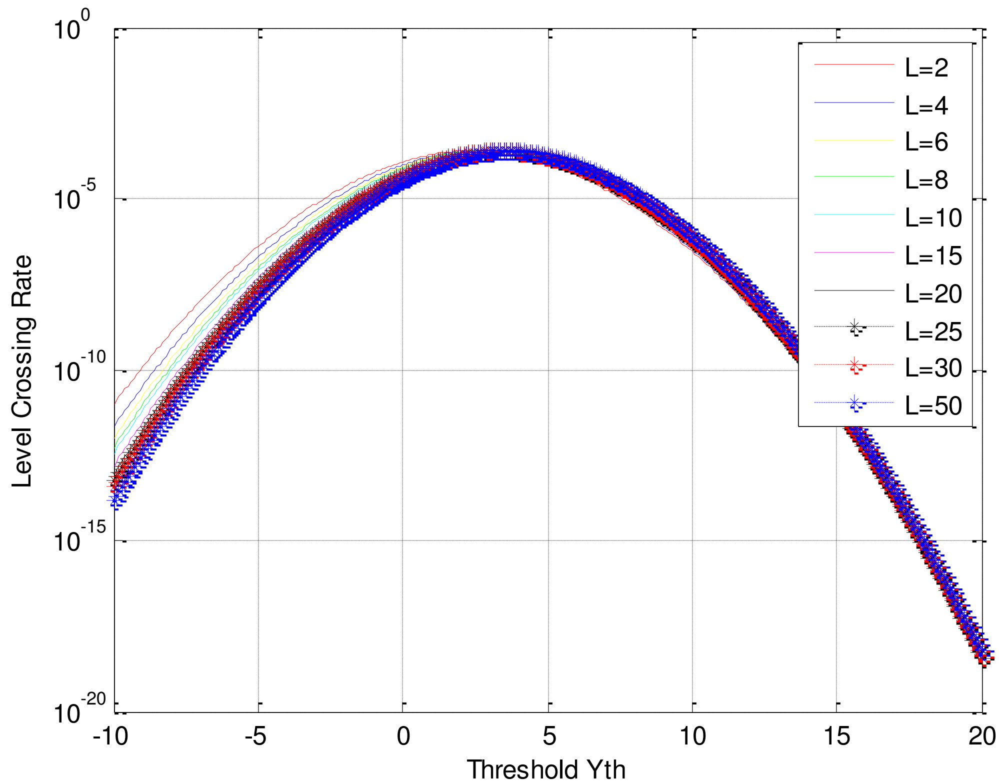


***Fig 8. 1. LCR in (8.11) of MRC combiner output in log normal fading channel for T=1000sec with $\mu_M$ and $\sigma_M$ from table 6.1.***

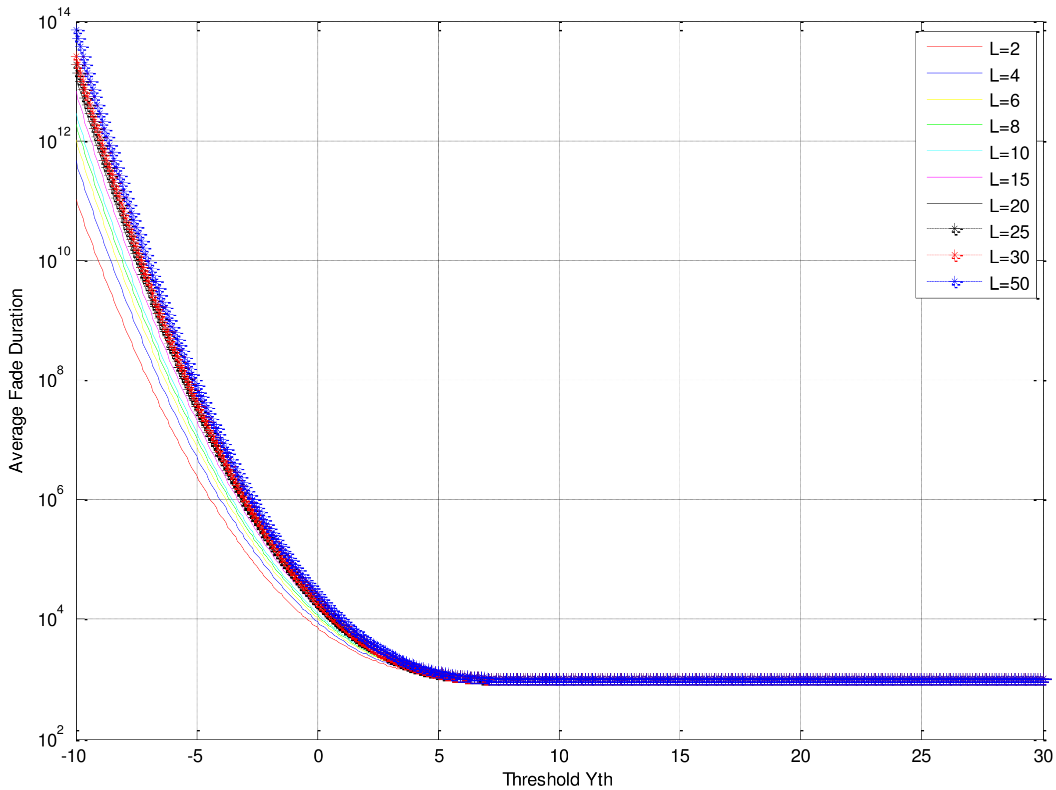


***Fig 8. 2. AOD of MRC combiner output in log normal fading channel T=1000sec with $\mu_M$ and $\sigma_M$ from table 6.1.***

**Table 8.1 Level Crossing Rate of MRC combiner output**

| $\gamma_{th} = -10dB$ | | $\gamma_{th} = -5dB$ | | $\gamma_{th} = -0dB$ | | $\gamma_{th} = 5dB$ | | $\gamma_{th} = 10dB$ | | $\gamma_{th} = 15dB$ | | $\gamma_{th} = 20dB$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $N_{\gamma th}$ |
| 2 | 6.707e-12 | 2 | 3.202e-07 | 2 | 1.099e-04 | 2 | 1.141e-04 | 2 | 3.531e-07 | 2 | 7.838e-12 | 2 | 1.179e-18 |
| 4 | 1.756e-12 | 4 | 1.679e-07 | 4 | 9.217e-05 | 4 | 1.247e-04 | 4 | 3.712e-07 | 4 | 6.24e-12 | 4 | 5.607e-19 |
| 6 | 5.328e-13 | 6 | 8.748e-07 | 6 | 7.348e-05 | 6 | 1.434e-04 | 6 | 5.058e-07 | 6 | 8.816e-12 | 6 | 7.295e-19 |
| 8 | 2.194e-13 | 8 | 5.354e-07 | 8 | 6.141e-05 | 8 | 1.574e-04 | 8 | 6.342e-07 | 8 | 1.146e-11 | 8 | 9.027e-19 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 1.071e-13 | 10 | 3.53e-08 | 10 | 5.202e-05 | 10 | 1.705e-04 | 10 | 8.01e-07 | 10 | 1.585e-11 | 10 | 1.304e-18 |
| 15 | 1.971e-14 | 15 | 1.29e-08 | 15 | 3.397e-05 | 15 | 2.006e-04 | 15 | 1.403e-06 | 15 | 3.597e-11 | 15 | 3.493e-18 |
| 20 | 4.27e-15 | 20 | 5.008e-09 | 20 | 2.2e-05 | 20 | 2.249e-04 | 20 | 2.394e-06 | 20 | 8.358e-11 | 20 | 1.045e-17 |
| 25 | 1.02e-15 | 25 | 2.004e-09 | 25 | 1.405e-05 | 25 | 2.417e-04 | 25 | 3.995e-06 | 25 | 1.97e-10 | 25 | 3.379e-17 |
| 30 | 2.526e-16 | 30 | 8.027e-10 | 30 | 8.783e-06 | 30 | 2.495e-04 | 30 | 6.505e-06 | 30 | 4.613e-10 | 30 | 1.119e-16 |
| 50 | 7.772e-19 | 50 | 1.539e-11 | 50 | 9.869e-07 | 50 | 1.912e-04 | 50 | 3.52e-05 | 50 | 1.141e-08 | 50 | 1.207e-14 |

**Table 8.2 Average Outage Duration of MRC combiner output**

| $\gamma_{th} = -10dB$ | | $\gamma_{th} = -5dB$ | | $\gamma_{th} = 0dB$ | | $\gamma_{th} = 5dB$ | | $\gamma_{th} = 10dB$ | |
|---|---|---|---|---|---|---|---|---|---|
| **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ | **Number of diversity branches *L*** | $T_{\gamma th}$ |
| 2 | 1.491e+11 | 2 | 3.122e+06 | 2 | 7.954e+03 | 2 | 1.151e+03 | 2 | 1e+03 |
| 4 | 5.695e+11 | 4 | 5.955e+06 | 4 | 9.735e+03 | 4 | 1.171e+03 | 4 | 1e+03 |
| 6 | 1.877e+12 | 6 | 1.143e+07 | 6 | 1.252e+04 | 6 | 1.210e+03 | 6 | 1.001e+03 |
| 8 | 4.557e+12 | 8 | 1.868e+07 | 8 | 1.521e+04 | 8 | 1.243e+03 | 8 | 1.001e+03 |
| 10 | 9.334e+12 | 10 | 2.833e+07 | 10 | 1.816e+04 | 10 | 1.279e+03 | 10 | 1.001e+03 |
| 15 | 5.074e+13 | 15 | 7.754e+07 | 15 | 2.84e+04 | 15 | 1.385e+03 | 15 | 1.001e+03 |
| 20 | 2.342e+14 | 20 | 1.997e+08 | 20 | 4.443e+04 | 20 | 1.1519e+03 | 20 | 1.002e+03 |
| 25 | 9.8e+14 | 25 | 4.99e+08 | 25 | 7.017e+04 | 25 | 1.692e+03 | 25 | 1.004e+03 |
| 30 | 3.959e+15 | 30 | 1.246e+09 | 30 | 1.129e+05 | 30 | 1.916e+03 | 30 | 1.007e+03 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 50 | 1.287e+18 | 50 | 6.498e+10 | 50 | 1.012e+06 | 50 | 3.882e+03 | 50 | 1.038e+03 |

### 8.5 Conclusion

This study has established a process for estimating the LCR and AOD for MRC combiner output in log normal shadowed channel. Here we have used Table 6.1. LCR and AOD has been plotted and tabulated as well for different number of diversity branches L. For low threshold level, LCR decreases with increase in number of diversity branches L, while for high threshold level (SNR), LCR increases with increase in number of diversity branches L. For low threshold level (SNR), AOD increases with increase in number of diversity branches L while for high threshold level, AOD decreases with increase in number of diversity branches L. So we can conclude that MRC combining improves system's performance in terms of LCR and AOD in log normal shadowed channel.

# CHAPTER 9: PERFORMANCE ANALYSIS OF MRC COMBINER OUTPUT FOR MQAM IN RICIAN FADING CHANNELS

## 9.1 Introduction

The emergence of transmission with high data rate in wireless communications recommence interest for linear modulation M-Ary systems, because of these systems having ability to transmit more bits per transmitted symbol. A common problem confronted in wireless systems is receiving signals from multiple paths for signal transmitting from the transmitter to the receiver due to reflection, diffraction and refraction. The total effect of multiple signals from different paths is volatility (fluctuating), i.e. evolution of fading signals at the receiver, which greatly degrades the quality of reception. Diversity technique is a best tool in which multiple faded replicas of SNR of the information signal SNR sent provided [63]. Several diversity techniques are used to mitigate the effects of multipath fading and improve the channel capacity as well as reliability of wireless mobile systems, such as: transmit/receive diversity [64], time, space and frequency diversity [78], [71], polarization diversity [79]. The best diversity technique among all is maximal ratio combining (MRC), which provides optimum results [70], [67], [75], [73]. M-ary Quadrature Amplitude Modulation (MQAM) is a well-known frequently used modulation technique in wireless communications. High spectral efficiency of MQAM makes it an attractive technique for wireless mobile communication. With the change in Rice factor K, Symbol error rate (SER) also changes. When Rice factor K is varied, the performance of the systems also varies. There are two conditions: at K=0, channel is Raileigh fading channel, and when K tends ∞, channel is AWGN channel without fading. The performance can be evaluated for different diversity techniques and message signals. In this chapter, exact analysis of bit error rate (BER) is provided for MQAM over Rician fading channels using L diversity branches. BER has been provided in closed form expressions for MQAM at the output of MRC combiner in Rician fading channels. For Rice factor and number of branches L, MQAM shows different characteristics. The goal of this chapter is to compare its characteristics for different parameters and to highlight its effect.

## 9.2 Comparing Symbol Error Rate for Exact and Approximate formula for MQAM

### 9.2.1 Exact Formula

The exact symbol error rate for the M-ary QAM under Gaussian channel is given as [3]

$$P_{AWGN}(\gamma) = 1 - \left(1 - \left[2.\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\gamma}\right)\right]\right)^2 \tag{9.1}$$

The exact probability of a symbol error for the M-QAM is shown in Figure 9.1.

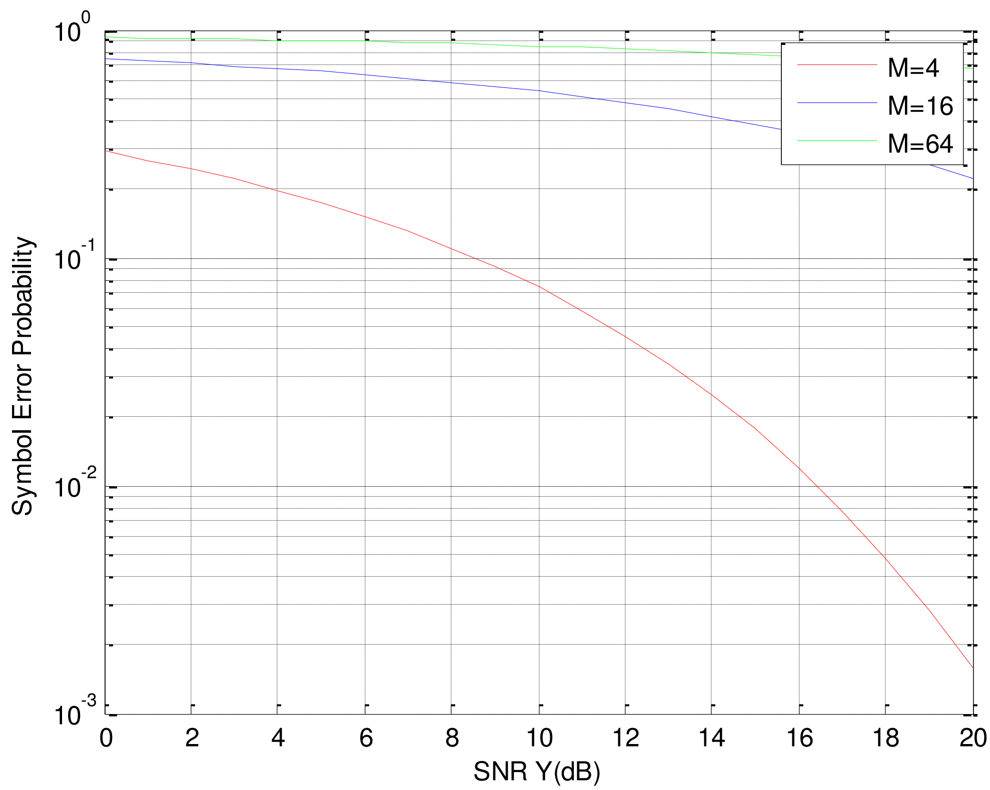


***Fig 9.1. The exact probability of a symbol error for the M-QAM***

### 9.2.2 Approximate Formula

The symbol error rate can be obtained from the approximated formula for the MQAM using quadratic square constellation over AWGN channel. The approximated formula can be written as under 1 dB error for $M \geq 4$ and $0 \leq SNR \leq 30$ dB [67]

$$P_{AWGN}(\gamma_B) = 0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}\right].log_2 M \tag{9.2}$$

The approximate probability of a symbol error for the M-QAM is shown in Figure 9.2.

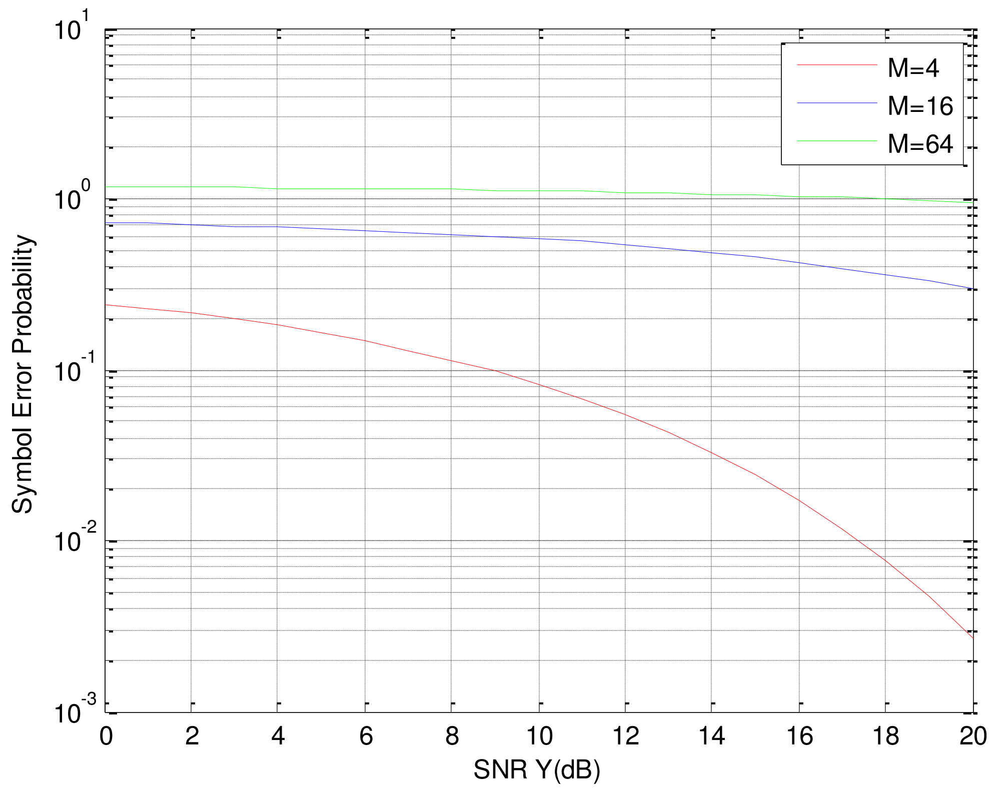


***Fig 9.2. The approximate probability of a symbol error for the M-QAM***

Also another symbol error rate can be obtained from the approximate formula for the MQAM using quadratic square constellation over AWGN channel. The approximated formula can be written as under 1 dB for M ≥ 4 and BER ≤10-3 is given as [68]

$$P_{AWGN}(\gamma_B) = 0.2.\exp\left[-\frac{1.6.\gamma}{(M-1)}\right].log_2 M \qquad (9.3)$$

### 9.3 Communication Link

Here we are considering a communications system with *M*–ary signaling employed over an AWGN and Rician fading channel (slow and flat). We have assumed that Rician fading is slow and flat in comparison to the signal duration of *T* seconds and bandwidth. The Rician fading having multiplicative distortion effects on transmitted signal by a Rice factor, *K*. The transmitted signal is also perturbed by AWGN channel with power spectral density *No/2* .The symbol error probability generated by stationary AWGN depends only on the instantaneous SNR corresponding to each symbol [74]. While for fading channel, the instantaneous SNR is a random variable. By using change of variables it can be shown that PDF of instantaneous SNR γ of over Rician fading channel [71]

$$p(\gamma) = \frac{(1+K).\exp[-K]}{\overline{\gamma}}.\exp\left[-\frac{(1+K).\gamma}{\overline{\gamma}}\right].I_0\left[2.\sqrt{K.\frac{(1+K).\gamma}{\overline{\gamma}}}\right] \quad (9.4)$$

Where $\overline{\gamma}$ is average SNR.

**9.4 Maximal Ratio Combining**

The received total SNR at the output of the MRC combiner is provided by the sum of the instantaneous SNRs on the individual branches [65]

$$\gamma_{MRC}=\sum_{i=1}^{N}\gamma_i$$

and the probability density function (*pdf*) of γ , the SNR after diversity combining, is given by

$$p(\gamma) = \left[\frac{(L+K)}{\overline{\gamma}}\right]\left[\frac{(L+K)\gamma}{K\overline{\gamma}}\right]^{\frac{L-1}{2}} \exp\left[\frac{(L+K)\gamma + K\overline{\gamma}}{\Gamma}\right]$$

$$.I_{L-1}\left[2.\sqrt{K.\frac{(L+K).\gamma}{\overline{\gamma}}}\right] \quad (9.5)$$

Where, $\overline{\gamma}$ is average SNR and L is the number of branches.

$K=\sum_{i=1}^{N} K_i$ and $I_N(.)$ is the N-order modified Bessel function of the first kind.

**9.5 Performance Analysis**

The error probability as a function of *K*, γ and L of the system can be calculated by averaging the conditional probability of error over the *pdf* of γ , i.e.

$$P_S(E) = \int_0^{\infty} P_S\left(\frac{E}{\gamma}\right).p(\gamma)d\gamma \quad (9.6)$$

Where, $P\,(E/\gamma)_S$ is the conditional probability of symbol error.

The probability of symbol error for QAM over a Gaussian channel is given as [72]

$$P_S\left(\frac{E}{\gamma}\right) = 0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}\right].log_2 M \qquad (9.7)$$

After the substitution of (9.5) and (9.7) into (9.6), we get

$$P(\gamma) = \int_0^\infty \begin{matrix} 0.2.\exp\left[-\frac{1.5.\gamma}{(M-1)}\right].log_2 M \left[\frac{(L+K)}{\overline{\gamma}}\right]\left[\frac{(L+K)\gamma}{K\overline{\gamma}}\right]^{\frac{L-1}{2}} \\ \exp\left[\frac{(L+K)\gamma + K\overline{\gamma}}{\overline{\gamma}}\right].I_{L-1}\left[2.\sqrt{K.\frac{(L+K).\gamma}{\overline{\gamma}}}\right] d\gamma \end{matrix} \qquad (9.8)$$

Using the following relation given in [53]

$$\int_0^\infty x^v e^{-\alpha x} I_{2v}\left(2\beta\sqrt{x}\right)dx = \alpha^{-(2v+1)}\beta^{2v}\exp(\frac{\beta^2}{\alpha}) \qquad (9.9)$$

On comparing (9.8) and (9.9), we have

$$v = \frac{L-1}{2}$$

$$\alpha = \frac{(L+K)(M-1) + 1.5.\overline{\gamma}}{(M-1)\overline{\gamma}}$$

And

$$\beta = \sqrt{\frac{K(L+K)}{\overline{\gamma}}}$$

Then probability of symbol error for M-QAM over IID Rician fading channels with Rician parameter $K$, diversity L, and mean symbol SNR γ , is given as follows

$$P(\gamma) = 0.2.log_2 M \left[\frac{(L+K).(M-1)}{[(L+K).(M-1) + \overline{\gamma}.1.5]}\right]^L .\exp\left[-\frac{1.5.\overline{\gamma}.K}{[(L+K).(M-1) + \overline{\gamma}.1.5]}\right] \qquad (9.10)$$

On comparing our SEP expressions with SEP of MDPSK and MPSK [74], evaluated at the output of MRC combiner using mathematical analysis for M-QAM over IID Rician fading channels (slow and flat), it can be observed that SEP expressions for MDPSK and MPSK given in [74] are in the integral expressions while our SEP expressions are simple closed form and contains only exponential functions

Moreover, it can also be observed that at *K*=0 in (9.10) yields the symbol error probability for M-QAM over L IID Rayleigh fading channel

$$P(\gamma) = 0.2.\,log_2 M \left[\frac{L.(M-1)}{[L.(M-1)+\overline{\gamma}.1.5]}\right]^L \qquad (9.11)$$

**9.6 Results Analysis and Discussion**

**Effect of Diversity and Rician parameter on the system performance**

In this section, we analyzed the effect of number of diversity branches L and Rice factor K on the system performance. The symbol error probability for M-QAM modulation over IID Rician fading channels (slow and flat) for various values of *K* and *L* are illustrated in the Figures 9.3-9.11. In figures, the symbol error probabilities of M-QAM system over Rician fading channels plotted versus the mean bit SNR for $M$ = 4, 16, and 64 and $L$ = 2, 4, and 10. For different values of $M$ and $L$, the variation in the symbol error probabilities on the mean values of the bit SNR is shown for different values of the Rice factor, i.e. $K$ = 0, 6 dB, 12 dB, and $\infty$ , which corresponds respectively to Rayleigh fading, Rician fading and AWGN.

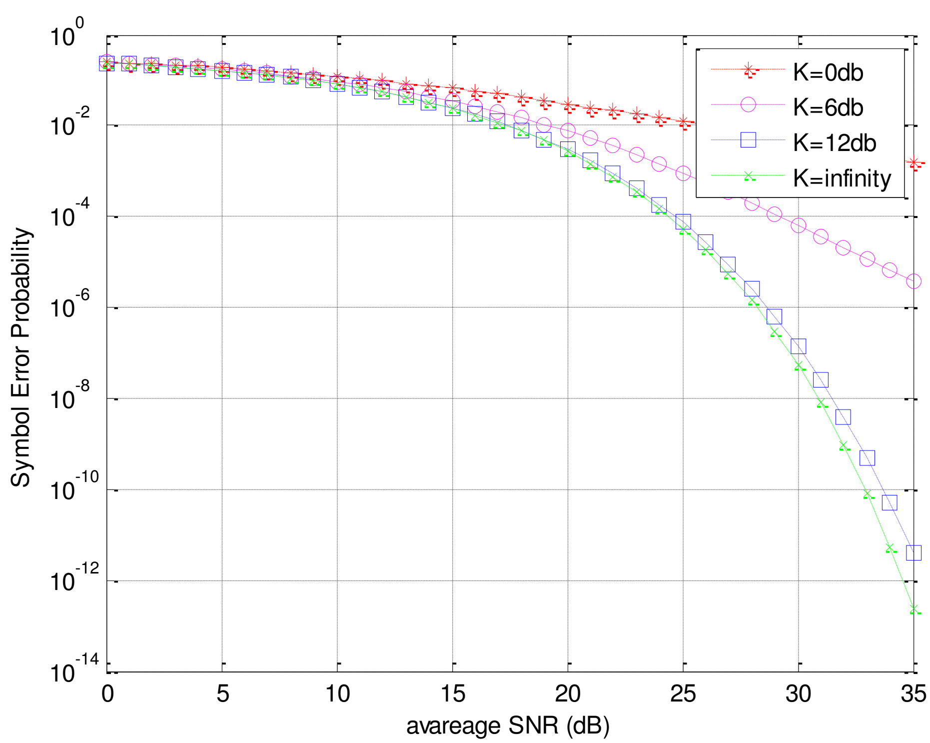


***Fig 9.3. SEP for M-QAM M=4 over Rician fading channels with L=2***

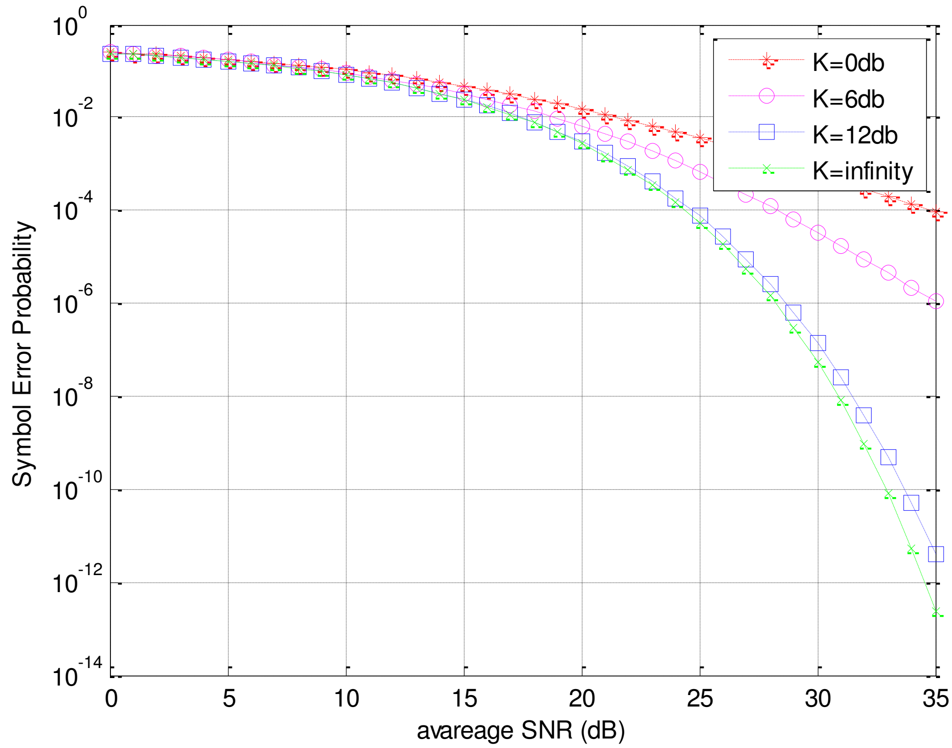


*Fig 9.4. SEP for M-QAM M=4 over Rician fading channels with L=4*

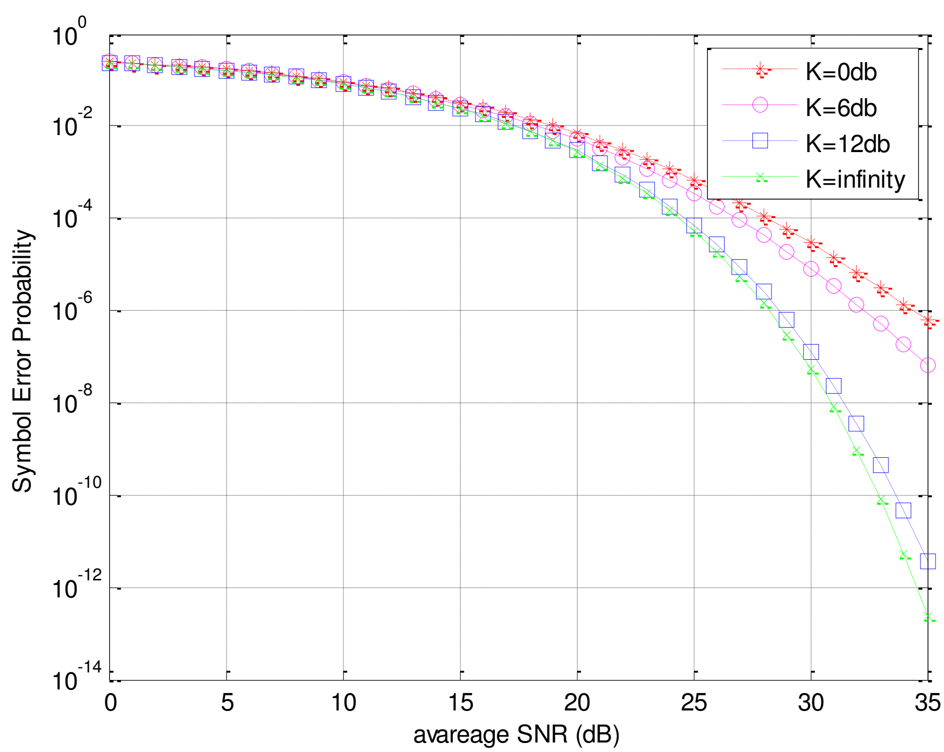


*Fig 9.5. SEP for M-QAM M=4 over Rician fading channels with L=10*

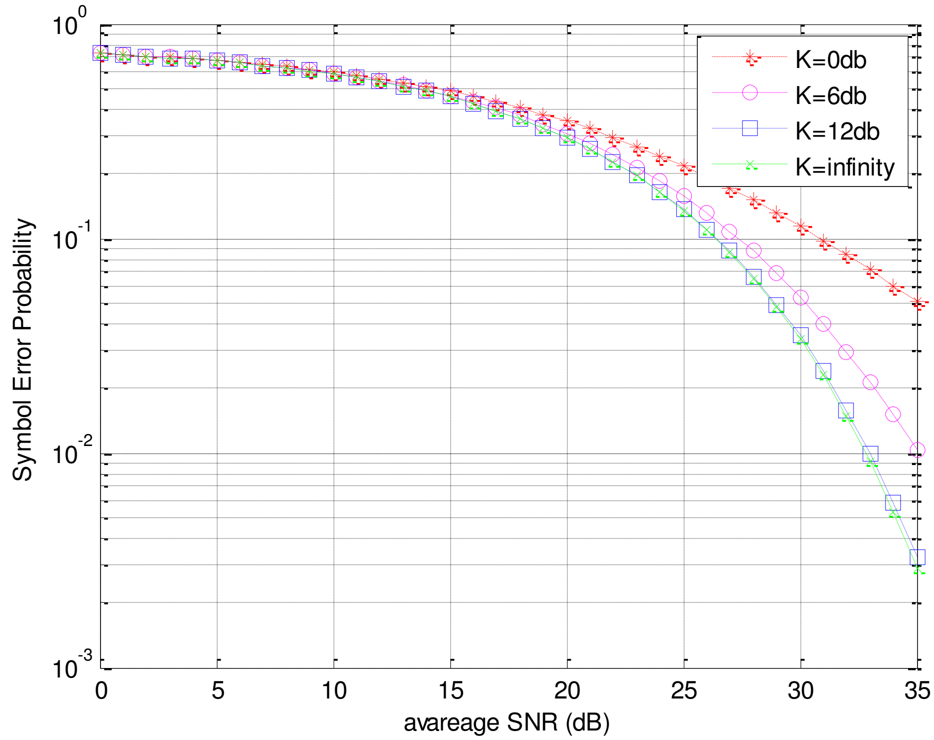


*Fig 9.6. SEP for M-QAM M=16 over Rician fading channels with L=2.*

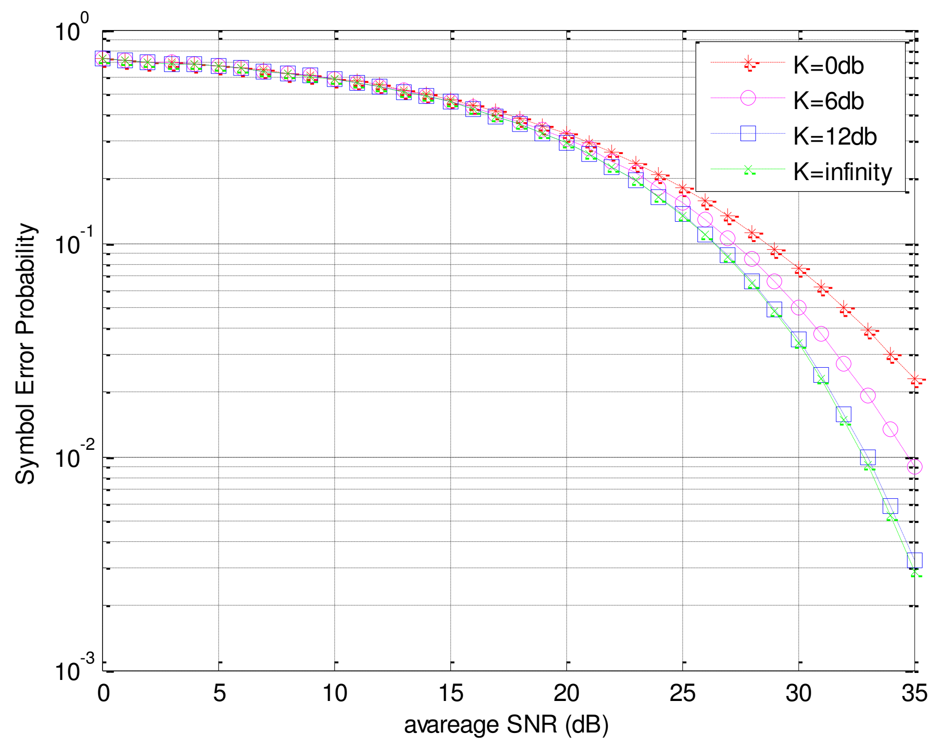


*Fig 9. 7. SEP for M-QAM M=16 over Rician fading channels with L=4.*

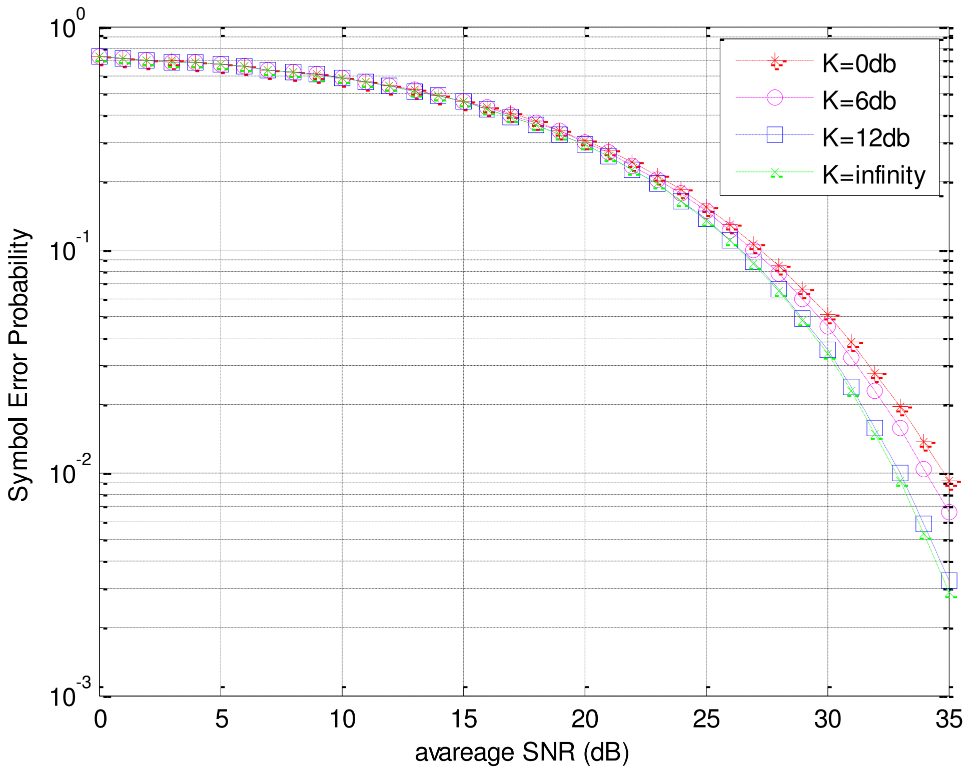


***Fig 9.8. SEP for M-QAM M=16 over Rician fading channels with L=10.***

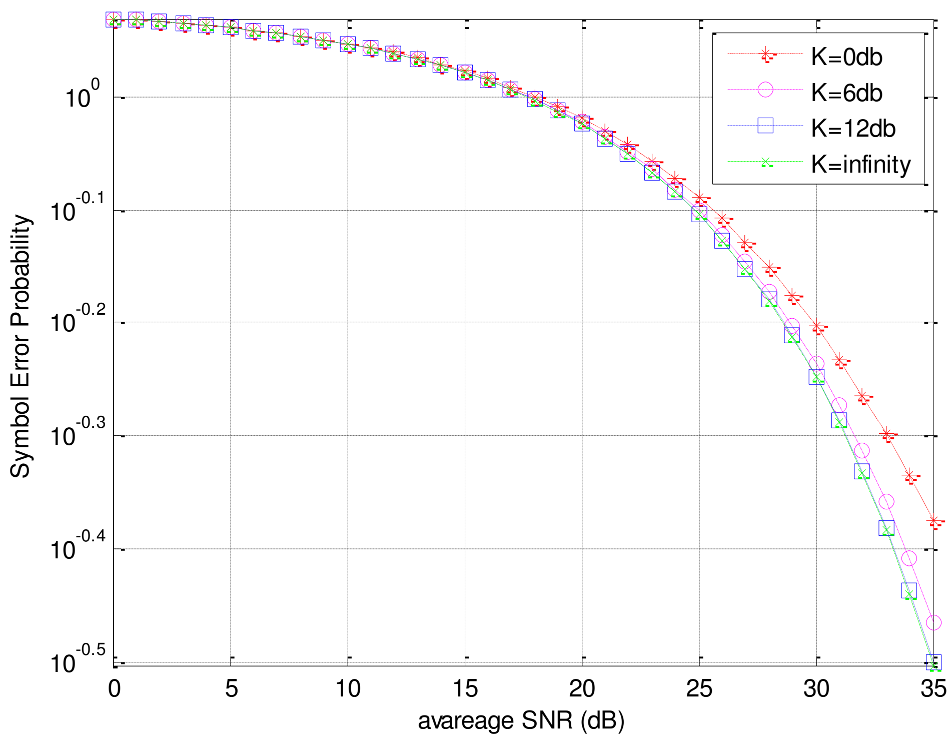


***Fig 9.9. SEP for M-QAM M=64 over Rician fading channels with L=2.***

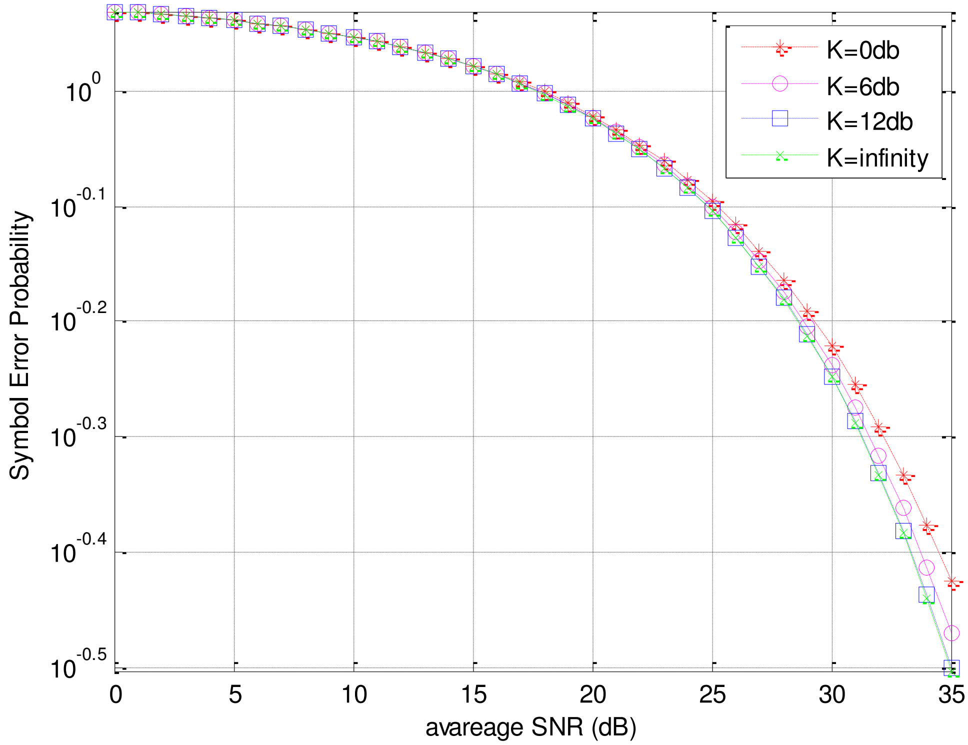


***Fig 9.10. SEP for M-QAM M=64 over Rician fading channels with L=4.***

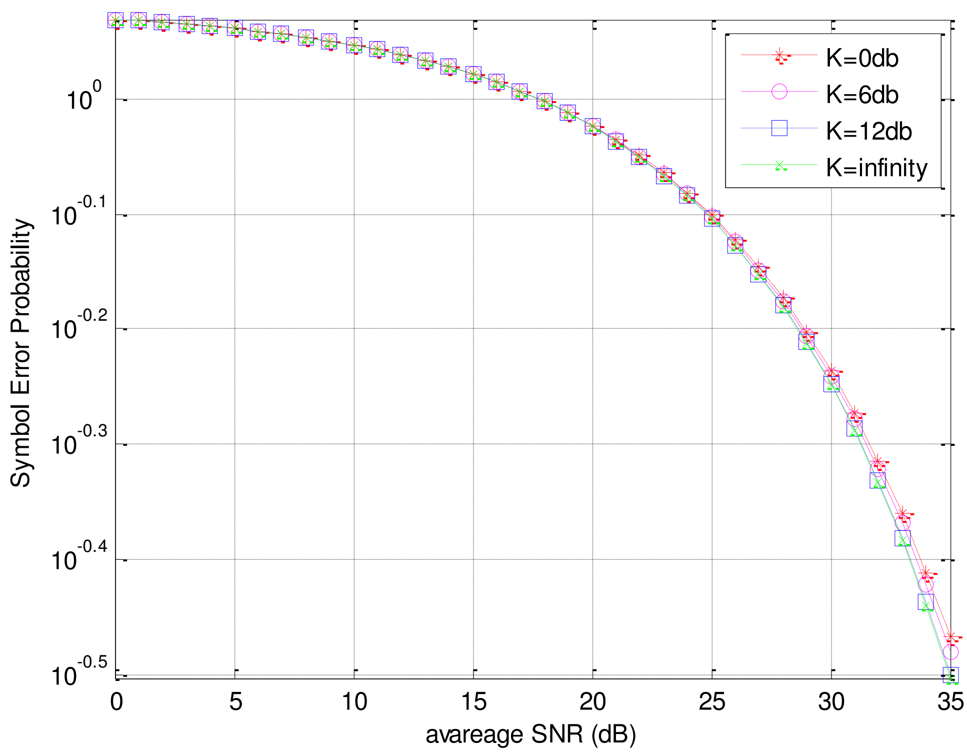


***Fig 9.11. SEP for M-QAM M=64 over Rician fading channels with L=10.***

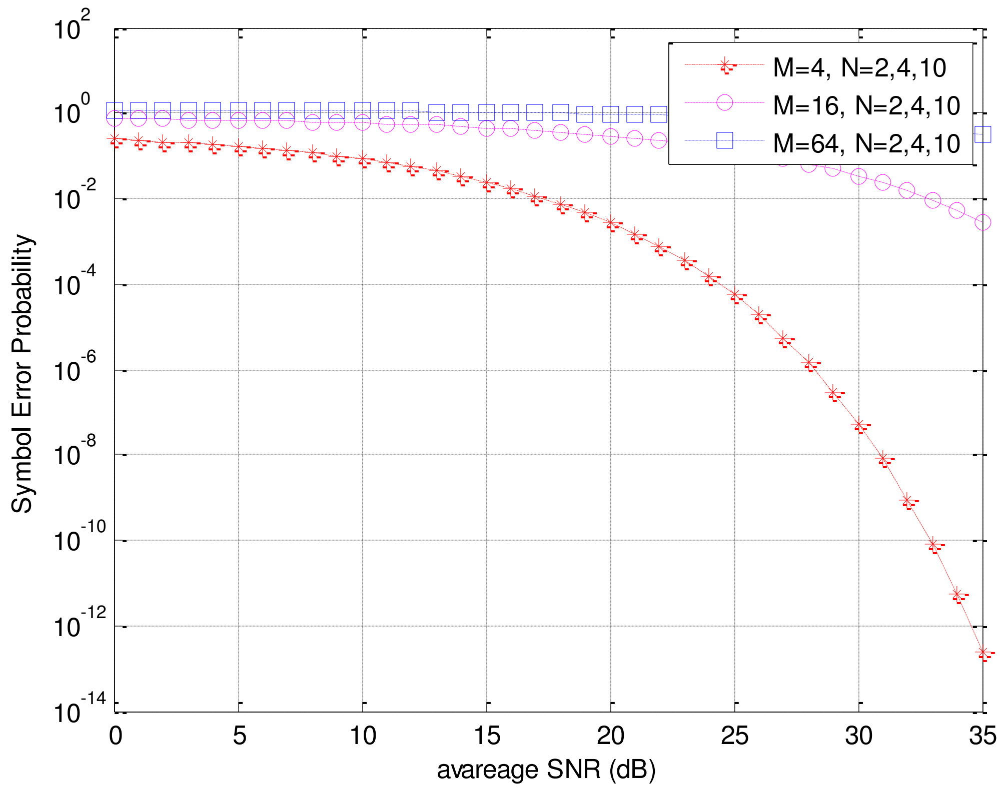


***Fig 9.12. SEP for M-QAM, M=4, 16, and 64 over Rician fading channels K=infinity with L=2, 4, 10 diversity***

### 9.7 Conclusion

From the Fig 9.3-9.11, we can conclude the following:

1. As the value of the Rician parameter *K* increases, the need for the Diversity reduces which is obvious.
2. As the number of diversity branches L increases, the reliability of communication system increases. However, the effect of diversity in improving the system performance is more at smaller values of *K*.
3. Diversity having no effect on the performance of the system when *K* approaches infinity. This is because the diversity is used to combat fading so when the channels are not fading it does not have any effect. This can be seen from Figure 9.12.

# CHAPTER 10: STATISTICAL MODEL OF CHANNEL CAPACITY FOR MRC COMBINER IN LOG NORMAL FADING CHANNELS

The maximum transmission rate of a channel can be bounded by one of the important performance measures, channel capacity. First, Shannon derived channel capacity for AWGN channel. In this chapter, a statistical modeling of channel capacity for MRC Combiner in log normal fading channels has been considered. The PDF of instantaneous channel capacity has been derived by using change of variable. We have found that PDF of channel capacity is not bell shaped for higher $\sigma_{dB}$. We have also provided first and second moments in terms of convergent series given in [82]. The approach given in [82] to estimate nth moment is quite complicated and involved rigorous mathematics. So here, we have proposed Holtzman [28] approximation based approach to compute nth moments of channel capacity.

### 10.1 Statistical Model of channel capacity for MRC combiner

Instantaneous Shannon capacity C for MRC combiner output can be written [32, 81, 82] as

$$C = log_e(1 + \gamma_{MRC}) \qquad (10.1)$$

Where $\gamma_{MRC}$ is lognormal distributed SNR of MRC combiner output, already explained in chapter 6.

From (6.7) PDF of $\gamma_{MRC}$ is

$$p(\gamma_{MRC}) = \frac{\xi}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} \qquad (10.2)$$

Using change of variable, PDF of C can be obtained as from (10.1)

$$\gamma_{MRC} = e^c - 1 \qquad (10.3)$$

So derivative of (10.3)

$$\frac{d\gamma_{MRC}}{dc} = e^c \qquad (10.4)$$

So PDF of C is

$$p(\mathrm{C}) = p(\gamma_{MRC} = e^c - 1).\frac{d\gamma_{MRC}}{dc} \qquad (10.5)$$

Using (10.4) into (10.5) we have,

$$p(\mathrm{C}) = \frac{\xi}{\sigma_M(1-e^{-c})\sqrt{2\pi}} e^{-\frac{(\xi log_e(e^c-1)-\mu_M)^2}{2\sigma^2{}_M}} \qquad c > 0 \qquad (10.6)$$

Using (10.6) CDF of channel capacity can be written as (using appendix 11.2.5)

$$P(\mathrm{C}) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{(\xi log_e(e^c-1)-\mu_M)}{\sqrt{2\pi}\sigma_M}\right)\right] \qquad c > 0 \qquad (10.7)$$

The expressions in (10.6) and (10.7) will enable us to compute the ϵ-outage capacity. In fig 10.1, (10.6) has been plotted for different number of MRC combiner branches. It can be observed from fig 10.1 that PDF of instantaneous channel capacity follows normal distribution at higher number of branches, which can be validated by central limit theorem. CDF in (10.7) also plotted in fig 10.2 for different number of MRC combiner branches L.

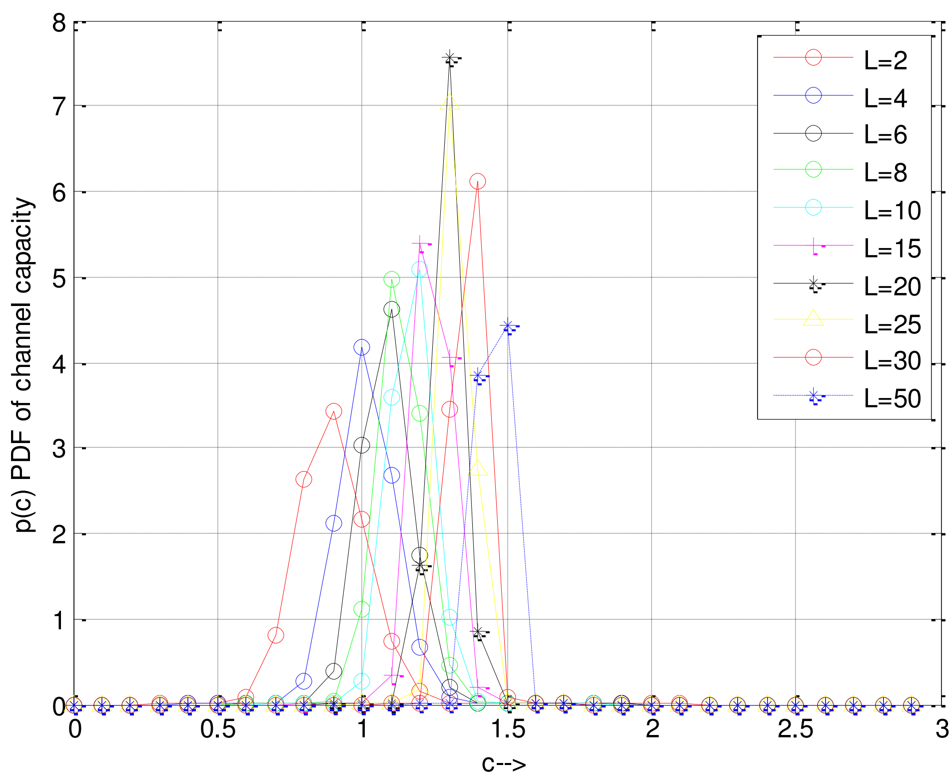


***Fig 10.1 PDF of instantaneous channel capacity for MRC combiner***

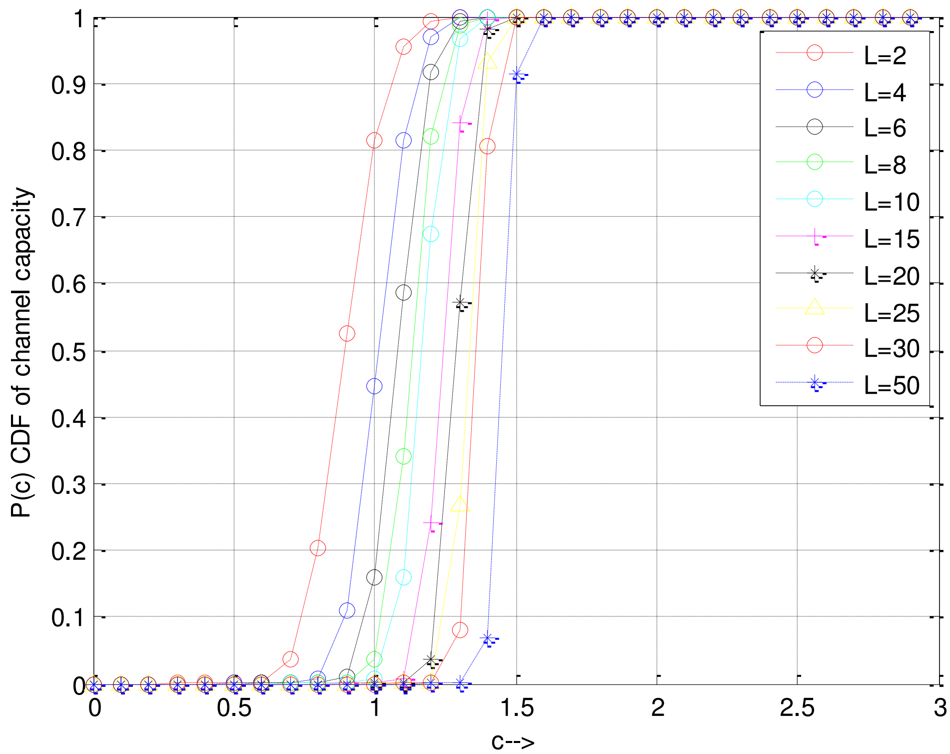


*Fig 10.2 CDF of instantaneous channel capacity for MRC combiner*

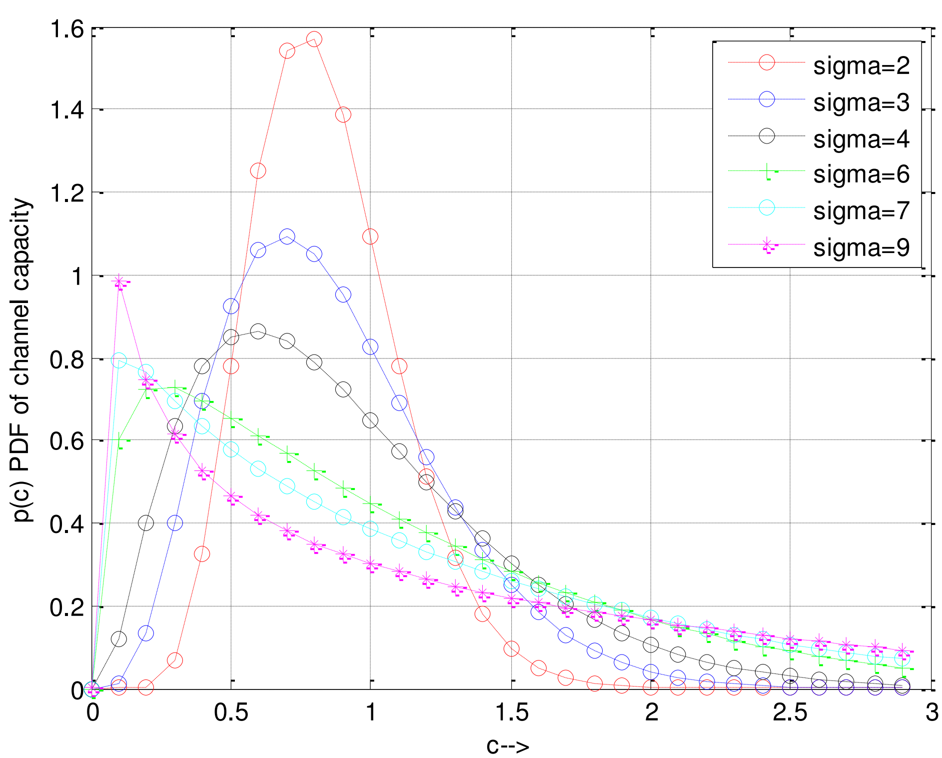


*Fig 10.3 PDF of instantaneous channel capacity for L=1 with μ=1dB [32]*

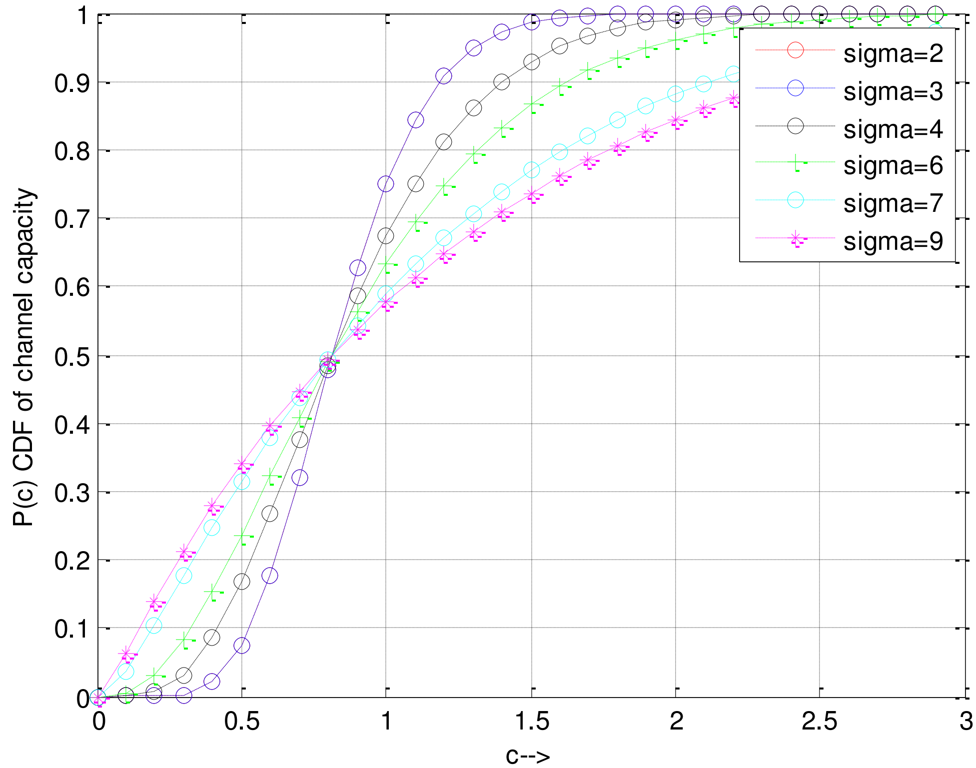


***Fig 10.4 CDF of instantaneous channel capacity for L=1 with μ=1dB [32]***

It is seen that expressions (10.6) and (10.7) are in excellent agreement for L=1 and μ=1dB with results in [32]. Fig 10.3 and fig 10.4 has been plotted for different σ(dB).

### 10.2 Statistical Characteristics of channel capacity for MRC combiner

First and second order moments is obtained by [82] in closed form expressions using infinite convergent series. In that paper, higher orders moments obtained by trimming infinite series to finite term (N=5). Here we are giving first two moments of channel capacity for MRC combiner due to [82].

First moment can be written as

$$E[C] = \int_0^\infty c \frac{\xi}{\sigma_M (1 - e^{-c})\sqrt{2\pi}} e^{-\frac{(\xi log_e (e^c - 1) - \mu_M)^2}{2\sigma^2{}_M}} dc \qquad (10.8)$$

From [82]

$$E[C] = \frac{e^{-\frac{\mu_M^2}{2\sigma^2{}_M}}}{2}\left\{\sqrt{\frac{2}{\pi}}\sigma_M + (\mu_M)erfcx\left(-\frac{\mu_M}{\sigma_M\sqrt{2}}\right) + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}erfcx\left(\frac{\sigma_M k}{\sqrt{2}} + \frac{\mu_M}{\sigma_M\sqrt{2}}\right) + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}erfcx\left(\frac{\sigma_M k}{\sqrt{2}} - \frac{\mu_M}{\sigma_M\sqrt{2}}\right)\right\} \tag{10.9}$$

Where $ercfx(x) = e^{x^2}ercf(x)$.

Same way, Second moment can be written as

$$E[C^2] = \int_0^{\infty} c^2 \frac{\xi}{\sigma_M(1-e^{-c})\sqrt{2\pi}} e^{-\frac{(\xi log_e(e^c-1)-\mu_M)^2}{2\sigma^2{}_M}} dc \tag{10.10}$$

From [82], we have

$$E[C^2] = \frac{e^{-\frac{\mu_M^2}{2\sigma^2{}_M}}}{2}\left\{\sqrt{\frac{2}{\pi}}\sigma_M\mu_M + (\mu_M^2+\sigma_M^2)erfcx\left(-\frac{\mu_M}{\sigma_M\sqrt{2}}\right) + \sum_{k=1}^{\infty}2\frac{(-1)^{k+1}}{k}\big(\psi(k)-\psi(1)\big)erfcx\left(\frac{\sigma_M k}{\sqrt{2}} + \frac{\mu_M}{\sigma_M\sqrt{2}}\right) + \sum_{k=1}^{\infty}2\frac{(-1)^{k+1}}{k}\big(\psi(k)-\psi(1)\big)erfcx\left(\frac{\sigma_M k}{\sqrt{2}} - \frac{\mu_M}{\sigma_M\sqrt{2}}\right) + \sum_{k=1}^{\infty}2\frac{(-1)^{k+1}}{k}\left(\sqrt{\frac{2}{\pi}}\sigma_M + (\mu_M - \sigma_M^2 k)erfcx\left(\frac{\sigma_M k}{\sqrt{2}} - \frac{\mu_M}{\sigma_M\sqrt{2}}\right)\right)\right\} \tag{10.11}$$

Where $\psi(.)$ is the Digamma function, which is defined by

$$\psi(x) = \frac{d\ln\big(\Gamma(x)\big)}{dx}$$

(10.9) and (10.11) are infinite convergent series expressions. For higher order moments these expressions become more complicated and lengthy. These moments are quite involved. So here we are proposing Holtzman approximation [28] based approach to compute higher orders moments of channel capacity.

### 10.3 A New approach to compute closed form of higher order moments of channel capacity for MRC combiner

From (10.1),

$$C = log_e(1+\gamma_{MRC}) \tag{10.12}$$

First moment of can be written as

$$E[C] = \int_0^\infty log_e(1+\gamma_{MRC}) \frac{1}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(log_e \gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \tag{10.13}$$

From section 5.2, using (5.5) and (5.6), (10.13) can be approximated as

$$E[C] \approx \frac{2}{3}\psi(\mu_M) + \frac{1}{6}\psi\left(\mu_M + \sqrt{3}\sigma_M\right) + \frac{1}{6}\psi\left(\mu_M - \sqrt{3}\sigma_M\right) \tag{10.14}$$

Where $\psi(x) = log_e(1+e^x)$

So from (10.13) and (10.14), we have

$$E[C] \approx \frac{2}{3}\log_e(1+e^{\mu_M}) + \frac{1}{6}\log_e\left(1+e^{\mu_M+\sqrt{3}\sigma_M}\right) + \frac{1}{6}\log_e\left(1+e^{\mu_M-\sqrt{3}\sigma_M}\right) \tag{10.15}$$

Same procedure can be used to obtain nth moment given as

$$E[C^n] = \int_0^\infty log_e(1+\gamma_{MRC})^n \frac{1}{\sigma_M \gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(log_e \gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \tag{10.16}$$

Using (10.16) we have moments as follows

$$E[C^2] \approx \frac{2}{3}\log_e(1+e^{\mu_M})^2 + \frac{1}{6}\log_e\left(1+e^{\mu_M+\sqrt{3}\sigma_M}\right)^2 + \frac{1}{6}\log_e\left(1+e^{\mu_M-\sqrt{3}\sigma_M}\right)^2 \tag{10.17}$$

$$E[C^3] \approx \frac{2}{3}\log_e(1+e^{\mu_M})^3 + \frac{1}{6}\log_e\left(1+e^{\mu_M+\sqrt{3}\sigma_M}\right)^3 + \frac{1}{6}\log_e\left(1+e^{\mu_M-\sqrt{3}\sigma_M}\right)^3 \qquad (10.18)$$

$$E[C^4] \approx \frac{2}{3}\log_e(1+e^{\mu_M})^4 + \frac{1}{6}\log_e\left(1+e^{\mu_M+\sqrt{3}\sigma_M}\right)^4 + \frac{1}{6}\log_e\left(1+e^{\mu_M-\sqrt{3}\sigma_M}\right)^4 \qquad (10.17)$$

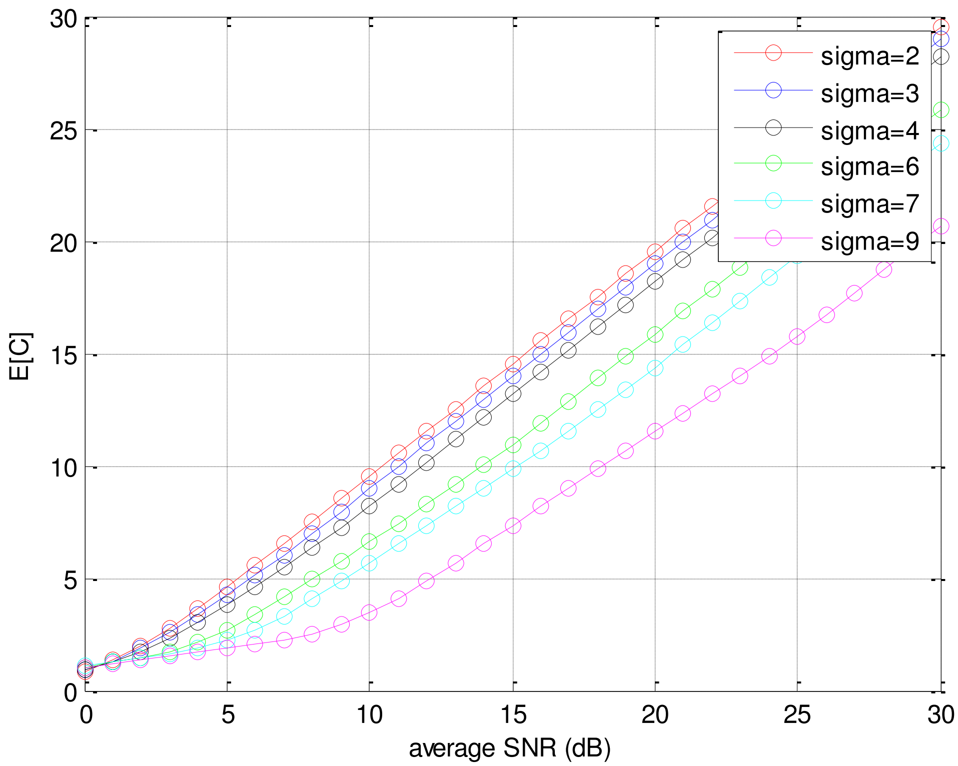


***Fig 10.5 Average channel capacity for L=1 with μ=1dB with different $\sigma_{dB}$***

Average channel capacity and second order moment of capacity are plotted in fig 10.5 and 10.6 for different values of variance. It can be observed that average capacity is low in heady shadowing. It can also be seen that as the value of variance increases, average capacity and second moment of capacity decreases. The same concept can be used at output of MRC combiner. If there are large numbers of diversity branches then there is low variance. So we can conclude that average capacity increases with increase in number of diversity branches L.

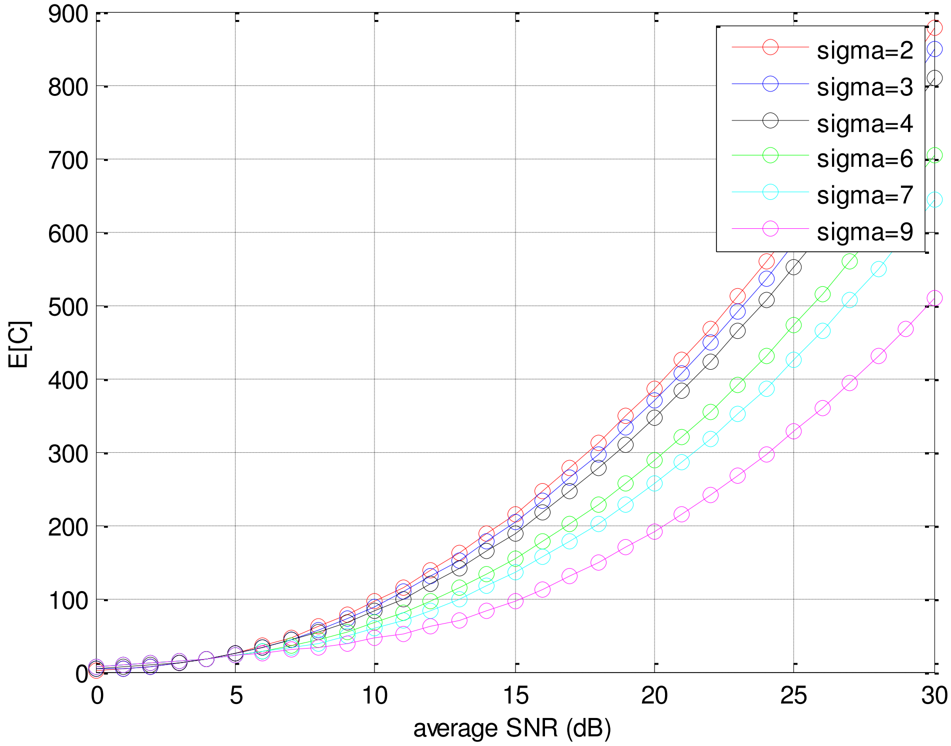


***Fig 10.6 Second order moment of instantaneous channel capacity of for L=1 with $\mu$=1dB with different $\sigma_{dB}$***

It can be observed that these closed form expressions are quite simple in comparison to expressions in (10.9) and (10.11). Further, by using these expressions higher order moments are adequately straight forward.

### 10.4 Conclusion

In this chapter, we have derived PDF and CDF of channel capacity for MRC combiner in log normal shadowing fading channel. These explicit form of PDF and CDF allows us to study the effect of number of MRC combiner branches L over channel capacity, which shown in fig 10.1 and 10.2. A new approach is proposed using Holtzman approximation to estimate higher order moments of channel capacity for MRC combiner. These new explicit expression are compared with expressions obtained in [82]. It is phenomenal that our proposed expression for higher order moments of channel capacity for MRC combiner are quite simple and easy to compute in comparison to expressions in [82].

# CHAPTER 11: APPENDIXES

## 11.1 APPENDIX I

1. $I_0(z)$ From [53] equation 8.447

$$I_0(z) = \sum_{n=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2n}}{(n!)^2}$$

2. For any z ([54], / 0.1.03.21.0058.01)

$$\int z^n \exp(-az) = a^{-n-1} \, (-1)^n \, \Gamma(1+n, -z)$$

3. In terms of Lower Incomplete Gamma Function

$$\int_0^y z^n \exp(-a\,z) \;d\,z = a^{-n-1} \, \gamma(n+1, ay)$$

Where

$$\gamma(n, x) = \int_0^x t^{n-1} \exp(-t) \;d\,t$$

4. Lower Incomplete Gamma Function in terms of Kummer Confluent Hypergeometric Function

$$\gamma(a, z) = (a)^{-1} (z)^a {}_1\mathrm{F}_1(a; 1+a; -z)$$

5. For any z ([54], /07.34.03.0228.01) in MeijerG

$$\exp(-z) = G_{0,1}^{1,0}[z|0]$$

6. For any z ([54], /07.34.03.0456.01)

$$ln(1+z) = G_{2,2}^{1,2}\left[z\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right]$$

7. Meijer's integral from two G functions ([54], /07.34.21.0011.01)

$$\int_0^\infty z^n \, G_{2,2}^{1,2}\left[z\left|\begin{matrix}1,1\\1,0\end{matrix}\right.\right] G_{0,1}^{1,0}[az|0] \, dz = \; G_{2,3}^{3,1}\left[a\left|\begin{matrix}-n, 1-n\\0, -n, -n\end{matrix}\right.\right]$$

8. Meijer's integral from two G functions ([54], /07.34.21.0013.01)

$$\int_0^\infty z^{n-1}\, G_{2,2}^{1,2}\left[z \left| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right.\right] G_{0,1}^{1,0}[(az)^n|0]\, dz$$

$$= \frac{\sqrt{k}}{l(2\pi)^{k+2l-\frac{3}{2}}}\, G_{2l,k+2l}^{k+2l,l}\left[a^n/k^k \left| \begin{matrix} I(l,-n), I(\,l,1-n) \\ I(k,0), I(l,-n), I(l-n) \end{matrix} \right.\right]$$

Where $I(m,p) \triangleq \frac{p}{m}, \frac{p+1}{m}, \dots\dots\dots\dots\dots\dots..., \frac{p+m-1}{m}$ $and \frac{l}{k} = n$ and p and n are arbitrary integers.

9. Gamma Function

$$\Gamma(t) = \int_0^\infty x^{t-1} \exp(-x)\, dx$$

$$\Gamma(n) = (n-1)!$$

10. Kummer Confluent Function of First Kind

$${}_1F_1(a; 1+a; -z) = 1 + \frac{a}{b} z + \frac{a(a+1)z^2}{b(b+1)2!} + \cdots \dots \dots . = \sum_{k=0}^{\infty} \frac{\frac{(a)_k}{(b)_k} z^k}{k!}$$

11. Complementary Error Function

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

## 11.2 APPENDIX II

### 11.2.1 AOF of Combined Composite WL and Unshadowed fading

$k^{th}$ moment from (5.19)

$E[\gamma^k] =$

$$(1-A)\frac{\Gamma(1+k)}{(1+K)^k} {}_1F_1(-k,1;-K)\overline{\gamma}^k + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^k\right] \Gamma\left(1+\frac{2k}{c}\right) \qquad (1)$$

$E[\gamma^2] =$

$$(1-A)\frac{\Gamma(3)}{(1+K)^2} {}_1F_1(-2,1;-K)\overline{\gamma}^2 + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2 + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2 + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2\right] \Gamma\left(1+\frac{4}{c}\right) \qquad (2)$$

$E[\gamma] =$

$$(1-A)\frac{\Gamma(2)}{(1+K)}{}_1F_1(-1,1;-K)\overline{\gamma} + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1 + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1 + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1\right]\ \Gamma\left(1+\frac{2k}{c}\right) \qquad (3)$$

So AOF of Combined Composite WL and Unshadowed fading from (2) and (3)

$$AOF = \frac{(1-A)\frac{\Gamma(3)}{(1+K)^2}1F1(-2,1;-K)\overline{\gamma}^2 + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2 + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2 + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^2\right]\ \Gamma\left(1+\frac{4}{c}\right)}{\left((1-A)\frac{\Gamma(2)}{(1+K)}1F1(-1,1;-K)\overline{\gamma} + A\left[\frac{2}{3}\left[\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1 + \frac{1}{6}\left[\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1 + \frac{1}{6}\left[\frac{\exp(\mu-\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right]^1\right]\ \Gamma\left(1+\frac{2k}{c}\right)\right)^2} - 1 \qquad (4)$$

### 11.2.2 K$^{th}$ Moment of Weibull Distribution

Weibull distribution

$$p(\gamma) = \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\overline{\gamma}}\right)^{\frac{c}{2}}\gamma^{\frac{c}{2}-1}exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad \gamma \geq 0 \qquad (5)$$

$$E[\gamma^k] = \int_0^\infty \gamma^k\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\overline{\gamma}}\right)^{\frac{c}{2}}\gamma^{\frac{c}{2}-1}exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right]d\gamma \qquad (6)$$

$$put\ \left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}} = R \qquad then \qquad \frac{c}{2}\left(\frac{1}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\gamma^{\frac{c}{2}-1}d\gamma = dR$$

$$E[\gamma^k] = \int_0^\infty (R)^{2k/c}exp[-R]\,dR \qquad (6)$$

$$E[\gamma^k] = \left[\frac{2}{3}\left(\frac{\exp(\mu)}{\Gamma\left(1+\frac{2}{c}\right)}\right)^k + \frac{1}{6}\left(\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right)^k + \frac{1}{6}\left(\frac{\exp(\mu+\sqrt{3}\sigma)}{\Gamma\left(1+\frac{2}{c}\right)}\right)^k\right]\ \Gamma\left(1+\frac{2k}{c}\right) \qquad (7)$$

Where $\Gamma(.) is\ gamma\ function.$

$$\Gamma(t) = \int_0^{\infty} x^{t-1} \exp(-x)\, dx$$

$$\Gamma(n) = (n-1)!$$

**11.2.3 AOF of lognormal distribution**

k[th] moment of lognormal distribution

$$E(\gamma^k) = \int_0^{\infty} \gamma^k \frac{\xi}{\sigma\gamma\sqrt{2\pi}} e^{-\frac{(10\log_{10}\gamma - \mu)^2}{2\sigma^2}} d\gamma \tag{8}$$

$on\ putting\ \frac{10\log_{10}\gamma - \mu}{\sqrt{2}\sigma} = x$ or $\frac{\xi\ln(\gamma)-\mu}{\sqrt{2}\sigma} = x$ then $\frac{\xi}{\gamma\sqrt{2}\sigma} d\gamma = dx$

$$E(\gamma^k) = \int_{-\infty}^{\infty} e^{k\frac{\sqrt{2}\sigma x + \mu}{\xi}} \frac{1}{\sqrt{\pi}} e^{-x^2} dx \tag{9}$$

We know

$$I = \int_{-\infty}^{\infty} e^{-z^2} e^{az+b}\, dz = \sqrt{\pi}\, e^{\frac{a^2}{4}+b} \tag{10}$$

From (9) using (10)

$$E(\gamma^K) = e^{\left(\frac{1}{2}\left(\frac{k}{\xi}\right)^2 \sigma^2 + \frac{k}{\xi}\mu\right)} \tag{11}$$

So AOF of lognormal distribution

$$AOF = \frac{e^{\left(\frac{1}{2}\left(\frac{2}{\xi}\right)^2 \sigma^2 + \frac{2}{\xi}\mu\right)}}{e^{\left(\left(\frac{1}{\xi}\right)^2 \sigma^2 + \frac{2}{\xi}\mu\right)}} - 1 = e^{\frac{\sigma^2}{\xi}} - 1 \tag{12}$$

**11.2.4 CDF of Weibull distribution**

CDF is defined as

$$P(\gamma) = \int_0^{\gamma_{th}} p(\gamma)\; d\gamma \tag{13}$$

From (5) into (13)

$$P(\gamma) = \int_0^{\gamma_{th}} \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\bar{\gamma}}\right)^{\frac{c}{2}} \gamma^{\frac{c}{2}-1} exp\left[-\left(\frac{\gamma}{\bar{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] d\gamma \tag{14}$$

$$put \left(\frac{\gamma_{th}}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}} = R \qquad then \quad \frac{c}{2}\left(\frac{1}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\gamma^{\frac{c}{2}-1}d\gamma = dR$$

$$P(\gamma) = \int_0^{\left(\frac{\gamma_{th}}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}} exp[-R]\ dR \qquad (15)$$

$$P(\gamma) = 1 - exp\left[-\left(\frac{\gamma}{\overline{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right] \qquad (16)$$

**11.2.5 Outage Probability of MRC combiner in Lognormal fading channel**

$$P_{out}(\gamma_{th})^{MRC} = \int_0^{\gamma_{th}} p_{\gamma_{MRC}}(\gamma_{MRC})d\gamma_{MRC} \qquad (17)$$

Using (6.7) into (17)

$$P_{out}(\gamma_{th})^{MRC} = \int_0^{\gamma_{th}} \frac{\xi}{\sigma_M\gamma_{MRC}\sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC}-\mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \qquad (18)$$

$$put\ \frac{\mu_M - 10log_{10}\gamma_{MRC}}{\sqrt{2}\sigma_M} = x \quad in\ (18) \qquad then \qquad -\frac{10}{\sqrt{2}\sigma_M\ln(10)\gamma}d\gamma = dx$$

$$P_{out}(\gamma_{th})^{MRC} = \int_{\infty}^{\frac{\mu_M-10log_{10}\gamma_{th}}{\sqrt{2}\sigma_M}} -\frac{1}{\sqrt{\pi}}e^{-x^2}dx \qquad (19)$$

$$P_{out}(\gamma_{th})^{MRC} = \frac{2}{2\sqrt{\pi}}\int_{\frac{\mu_M-10log_{10}\gamma_{th}}{\sqrt{2}\sigma_M}}^{\infty} e^{-x^2}dx \qquad (20)$$

$$P_{out}(\gamma_{th})^{MRC} = \frac{1}{2}erfc\left(\frac{\mu_M - 10log_{10}\gamma_{th}}{\sqrt{2}\sigma_M}\right) \qquad (21)$$

**11.2.6 Average SNR $\overline{\gamma}$ of MRC combiner in lognormal fading channel**

$$\overline{\gamma}_{MRC} = E[\gamma_{MRC}] = \int_0^{\infty}\gamma_{MRC}p(\gamma_{MRC})d\gamma_{MRC} \qquad (22)$$

Using (6.7) in (22)

$$\overline{\gamma} = \int_0^{\infty} \gamma_{MRC} \, \frac{\xi}{\sigma_M \gamma_{MRC} \sqrt{2\pi}} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \qquad (23)$$

$$\overline{\gamma} = \frac{\xi}{\sigma_M \sqrt{2\pi}} \int_0^{\infty} e^{-\frac{(10log_{10}\gamma_{MRC} - \mu_M)^2}{2\sigma^2{}_M}} d\gamma_{MRC} \qquad (24)$$

$$\overline{\gamma} = \exp\left(\frac{\mu_M}{\xi} + \frac{\sigma^2{}_M}{2\xi}\right) \qquad (25)$$

# CHAPTER 12: REFERENCES


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