Terahertz Radiation emission from plasma filled Free Electron Laser

A Dissertation submitted towards the partial fulfillment of
the requirement for the award of degree of

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in
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Submitted by

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CERTIFICATE

This is to certify that work which is being presented in the dissertation entitled **Terahertz Radiation emission from plasma filled Free Electron Laser** is the authentic work of **Vivek Yadav** under my guidance and supervision in the partial fulfillment of requirement towards the degree of **Master of Technology in Microwave and Optical Communication Engineering**, jointly run by Department of Electronics & Communication Engineering and Department of Applied Physics in Delhi Technological University during the year 2013-2015.

As per the candidate declaration this work has not been submitted elsewhere for the award of any other degree.

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DECLARATION

I hereby declare that all the information in this document has been obtained and presented in accordance with academic rules and ethical conduct. This report is my own, unaided work. I have fully cited and referenced all material and results that are not original to this work. It is being submitted for the degree of Master of Technology in Engineering at the Delhi Technological University. It has not been submitted before for any degree or examination in any other university.

Signature :

Name : Vivek Yadav

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ABSTRACT

Terahertz (THz, or be called T-ray) wave refers to the electromagnetic wave with the frequency between 0.1 THz (1THz=1000GHz) and 10 THz or wavelength from 30 μm to 3000 μm. In the frequency domain, this region of electromagnetic wave is located between microwave and infrared wave. Different from the great successes in the microwave and infrared light wave, the applications of THz wave are much less investigated in the past few decades. Recently, it has recently been demonstrated that THz wave exhibits several unique features in terms of applications. For example, THz wave can easily penetrate fabrics and plastics, but it will be reflected by metallic materials. Thus, THz wave has a promising application in security screening. In order to realize the applications of THz wave, the compact, efficient, and high power THz sources have become a critical element of researches in THz wave.

In this dissertation I have developed the formalism for tunable coherent terahertz radiation generation from a relativistic electron beam, modulated by two laser beams, as it passes through a helical wiggler in the presence of plasma. The lasers exert a ponderomotive force on beam electrons, and modulate their velocity. In the drift space, velocity modulation translates into density modulation. As the beam bunches pass through the wiggler, they acquire a transverse velocity, constituting a transverse current that acts as an antenna to produce coherent THz radiation.

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LIST OF SYMBOLS

$\vec{E}$	Electric Field Intensity
$\vec{B}$	Magnetic Flux Density
d	Modulator Length
L	Drift Space Length
L_w	Wiggler Length
L_b	Electron Bunch Length
v_b	Beam Velocity
c	Speed of Light
k	Wave Number
ω	Frequency
n_b	Electron Beam Density
J_1	First Order Bessel Function
m_e	Mass of Electron
ε_0	Permittivity of free space
μ_0	Permeability of free space
$\vec{F}_p$	Ponderomotive Force
r_b	Electron Bunch Radius
γ_0	Relativistic gamma factor
$\vec{P}_{av}$	Average poynting vector
n_p	Plasma density

CHAPTER 1

INTRODUCTION TO THz RADIATION

1.1 INTRODUCTION

THz radiation consists of electromagnetic waves at frequencies range from 300 GHz to the far infrared band, 10 THz corresponding wavelength of radiation in the band range from 0.03 mm to 1 mm. Terahertz radiation consists a range between microwaves and infrared light wave has active research field. It represents the region in which the electromagnetic spectrum that the frequency of electromagnetic radiation is high to measure directly counting pulses using electronic counter, and should be measured by the alternative properties of wavelength and energy. The generation of coherent electromagnetic wave in this frequency range ceases to be feasible by the conventional electronic devices used for generation of radio waves and microwaves, and requires new techniques and devices.Electron beam based sources have also proven to be effective at producing high peak and average powers, and providing wide tunability [1–3].The coherent synchrotron radiation has been observed experimentally in storage ring, where a short laser pulse interacts with a small part of the electron bunch [4–11].

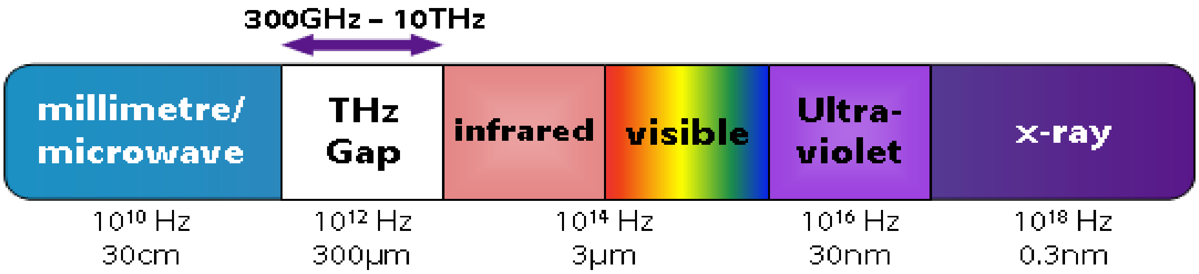

Fig.1.1 Position of THz radiation in the spectrum of electromagnetic waves

The generation of sub-picosecond pulses in the THz spectral range from 0.1 THz to 10 THz is a significant current direction in ultra-fast spectroscopy.

THz can be applied in medical imaging because of comparatively low energy of photons than microwave .It would not damage any living cells. THz radiation can sense differences in water content. THz can be applied in security screening because it able to penetrate fabrics and plastics and used to detect hidden weapons on a person remotely.

The earth atmosphere is a strong absorber of THz radiation in specific water vapor absorption band, consequently the range of THz radiation is limited to affect its efficacy in long-distance communications. The producing and detection of coherent terahertz radiation remains technically challenging, though economical commercial sources now exist in the range from 0.3 THz to 1.0 THz including backward wave oscillators, resonant-tunneling diodes and gyrotrons.

1.2 THE CHARACTERISTICS OF THz WAVE

THz wave studied extensively, not only because it is a kind of electromagnetic wave which is not known to human beings, more extensively, because it has numerous unique characteristics and broad application scenario. THz light wave ranges between the microwave and mid-infrared region of the electromagnetic spectrum. THz radiation is called millimeter wave or sub-millimeter- wave in electronics ,and it is known as far infrared ray in optics. From the observation of energy, THz energy band locates between electronic and photonic.

1.2(a) Wave-particle Duality

THz radiation is the electromagnetic wave, that's why it has all the characteristics of electromagnetic waves. THz wave has particle nature, such as interference and diffraction. So it shows Duality nature of wave and particle.

1.2(b) Penetrability

THz radiation has good penetrability on a lot of non-polar liquids and dielectric material. Therefore, THz waves are able to perspective imaging for a lot non-transparent objects and material. The penetrability of THz light wave makes it as the complement of X-ray imaging and ultrasound imaging for security checks or quality control in non-destructive testing. THz light wave imaging technologies, including the 2-D imaging, time-of-flight imaging, compound aperture imaging, as well as near-field imaging.

1.2(c)Security

THz radiation's another important characteristic is its security, compare to X-ray with kilo-eV photon energy, the energy of THz radiation is mill-eV. Its energy is lower than the energy of different types of chemical bond, that why it will not cause any harmful ionizing reaction. This is critical to the realistic applications such as the security check of human body, inspection of other biological samples, and objects etc.

1.2(d) The Resolving Power of Spectrum

Even though the THz radiation photon energy is comparatively low, the band still contains prosperity of spectral information. Many organic molecules has high absorption and dispersion characteristics in the THz band, the THz spectroscopy of material contains a prosperity of physical and chemical information, that make them the unique characteristics, such as fingerprints as a result THz spectral imaging technology able to differentiate objects morphology, as well as identify the composition of objects.

1.3 APPLICATION OF THz RADIATION

The unique characteristics of THz radiation make it has wide and significant application in the field of plasma physics and engineering, astrophysics, environmental science and engineering, biomedical engineering, materials science and engineering, information science and technology, spectroscopy and imaging technology, etc.

1.3(a)THz Technology in Biomedicine

Many biological molecules and DNA (Deoxyribo Nucleic Acid) molecular vibration and rotation locate in the THz-band level, generally the organisms has a unique response to THz wave, therefore THz radiation can be used for disease diagnosis, organisms detection and imaging.

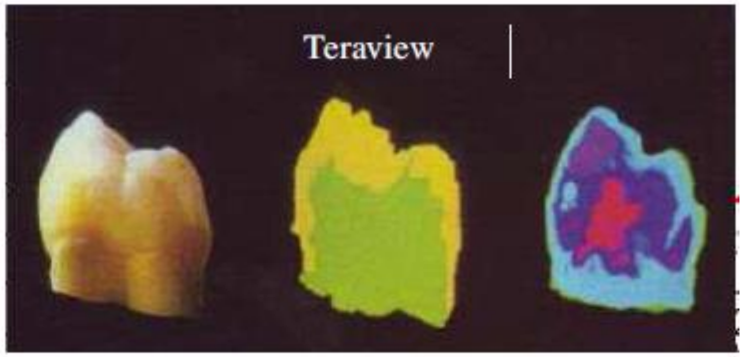

Fig. 1.2: THz tomography of human teeth[12]

Computer-aided tomography technology is the 3-D imaging technique, used in the field of X-ray. THz wave can also be functional to computer-assisted tomography. Imaging of object by X-ray tomography will reflect the distribution of absorption rate, even as the time waveform information of complete THz pulse recorded by the THz tomography. According to different requirement different physical detection may be selected, such as peak time, the electric field strength and even the spectral characteristics of that material. THz tomography can get absorption rate distribution of objects as well as the 3-D distribution of the refractive index of the objects and material.

THz-wave imaging has numerous inherent limitations

1. THz is not capable to penetrate the metal. Metal surface is almost 100% reflection index of the THz radiation that why THz light wave not able to detect metal objects container.
2. THz light wave has a strong absorption for the water.

1.3(b)Safety Monitoring and Quality Control

THz radiation can be used for detecting contaminant, chemical and biological detection; and therefore can be used to observe the procedure of food processing and food preservation. THz imaging can completely replace the CT scan, X-ray examination, and used for vital security sector and chemical and biological weapons inspection and the material non-destructive monitoring.

Fig.1.3- Detection of hidden metal weapons by THz wave.

1.3(c) Non-destructive Testing

The safety and penetrable properties of THz wave able to use for non-destructive testing of building. THz time-domain spectroscopy is used to calculate the penetrability of THz wave. Foam is the material used for Space shuttle frequently. Foam has extremely low absorption and refractive index on the THz wave that why THz waves can penetrate a few inches thick foam, and to detect the defects.

1.3(d)Short Distance Wireless Communications and Networking

THz band has wide bandwidth and more channel than the microwave, which is suitable for local area networks and broadband mobile communications. Speed of10 Gbps wireless transmission can be obtained through THz communication, that is a few hundred or few thousand times faster than current Ultra-Wide band technology, and is the opportunity for large-capacity multi-media wireless communications. Experts expect that in the next decade THz wireless network will replace the Bluetooth technology or wireless LAN.

1.3(e) Military Applications

THz technology has following applications in the military field

(i) Secure communications

(ii) THz non-destructive detection

(iii) Anti-stealth THz ultra-wide band radar.

(i) Secure Communication

The transmission of THz is lossless in outer space that why we can attain long-distance space communications with very few power. When compared with space optical communication, THz wave has wider beam width, so long-distance space communication is easier and is particularly suitable for secure communications between the satellites and stars.

(ii) Anti-stealth THz Ultra-wideband Radar

THz radar can emits thousands of frequency as well as nano-second and Pico-second pulse. It has many advantages and capabilities over ordinary radar. THz Ultra-wideband radar has a broad application in the national and military security. THz Ultra-wideband radar launch THz pulse contains prosperity of frequency, which permit stealth aircraft to lose the role of narrow-band radar. Furthermore, THz Ultra-wideband radar has strong anti-stealth facility to shape and material stealthy.

(iii) Chemical and Biological Agent Detection

THz pulse spectrum is very sensitive to constitute and molecules of their surrounding atmosphere. Therefore, in all weather conditions, as well as dust and smoke , THz technology used in the battle field of air for chemical detection and can identify of the type of thechemical and biological agents.

CHAPTER-2

Free-electron laser

2.1 Free Electron Laser

A free-electron laser is a high power microwave devices in which a relativistic electron beam (REB) propagating through a periodic magnetic field is used to amplify or generate coherent electromagnetic radiation.

A free-electron laser uses very-high speed electrons that move freely through a periodically magnetic structure [13] that's why the free electron used as a lasing medium [14]. The free-electron laser has large frequency range, as well as widely tunable [15] and ranging from microwaves to THz radiation and infrared to ultraviolet, and X-ray [16].

The word free-electron laser was given by John Madey in 1976 at Stanford University [17]. The work originate from research prepared by Hans Motz and his coworkers, who built a wiggler at Stanford in 1953 [17,18] using the wiggler magnetic arrangement. Madey used a 43-MeV electron beam[19] and 5 m long wiggler to amplify a signal.

In free-electron laser, a beam of electrons is accelerated about the speed of light. The beam passes through a wiggler, a periodic arrangement of magnets in which a side to side magnetic field produced with the poles across the beam path. The direction across the beam path is called transverse and the direction of the beam is called the longitudinal direction.

The free-electron laser (FEL) radiation intensity grows, causes extra micro-bunching of electrons, which radiate in phase with each other [20].The wavelength of the radiation can be tuned by varying the wiggler wavelength $\lambda_1 = \dfrac{\lambda_w}{2\gamma_0}$.

2.2 Ponderomotive force

A ponderomotive force is a non-linear force in which a charged particle experiences an oscillating electromagnetic field in inhomegenous medium.

The ponderomotive force $\vec{F}_p$ is expressed by

$$\vec{F}_p = \frac{-e}{2}\left(\vec{v} \times \vec{B}\right)$$

where e is the charge of the particle, B is the magnetic flux density (low enough that magnetic field exerts very little force) , v is the oscillation velocity of electron beam of frequency ω.

The above equation signifies that a charged particle in an inhomogeneous oscillating field oscillatesat the frequency of ω as well as drifts toward the weak field area.

The mechanism of the ponderomotive force is that the motion of the charge in an oscillating electric field. In homogeneous field the charge proceeds to its initial point over one cycle of oscillation while in inhomogeneous field the force experienced by the charge during the half-cycle it spend in the region with higher field amplitude towards the weak field amplitude. The force exert over the half-cycle spent in the region with a higher field amplitude is larger than the field force exert over the half-cycle spent in the region with a weak amplitude. Hence, averaged over a full cycle period there is a net force that drive the charge from high field region toward the weak field region.

2.3 Optical modulator

An optical modulator is a device used to modulate a beam of light. The beam passed over free space, or propagated through an optical fiber.

Depending on the parameter of a light beam modulators may be categorized into

(i) Amplitude modulator.

(ii) Phase modulator.

(iii) Polarization modulator.

The simplest method to obtain modulation of intensity is to modulate the current driving light source that is a laser diode. This scheme of modulation is called direct modulation. For laser diode narrow line width is required due to which a high bandwidth chirping effect when applying and removing the current to the laser so direct modulation is avoided.

2.3.1 Classification of optical modulators

According to the properties of the material which modulate the light beam, modulators are divided into two group

(1) REFRACTIVE MODULATORS-In this modulation refractive index of the material is changed. Refractive modulator use of an electro-optic effect. In Refractive modulator change of the refractive index and permittivity is due to pockels effect, kerr effect, electron-gyration.

(2) **ABSORPTIVE MODULATORS** – In this modulators absorption coefficient of the material is changed. The absorption coefficient of the material can be manipulated by the Quantum-confined excitonic absorption, Stark effect, Franz-Keldysh effect, changes of Fermi level, or changes of free carrier concentration.

2.4Helical Wiggler

A helical wiggler is an insertion device in a synchrotron. Helical wiggler is a series of magnet designed to periodically (shown in Fig 2.1) deflect a beam of charged particles inside a storage ring of a synchrotron. These deflection create a change in acceleration that will provide turn produces emission of broad synchrotron radiation tangent to the curve, but the intensity is higher due to the contribution of many magnetic dipoles in the wiggler. Furthermore, as the wavelength λ_w is decreased this means the frequency f has increased.

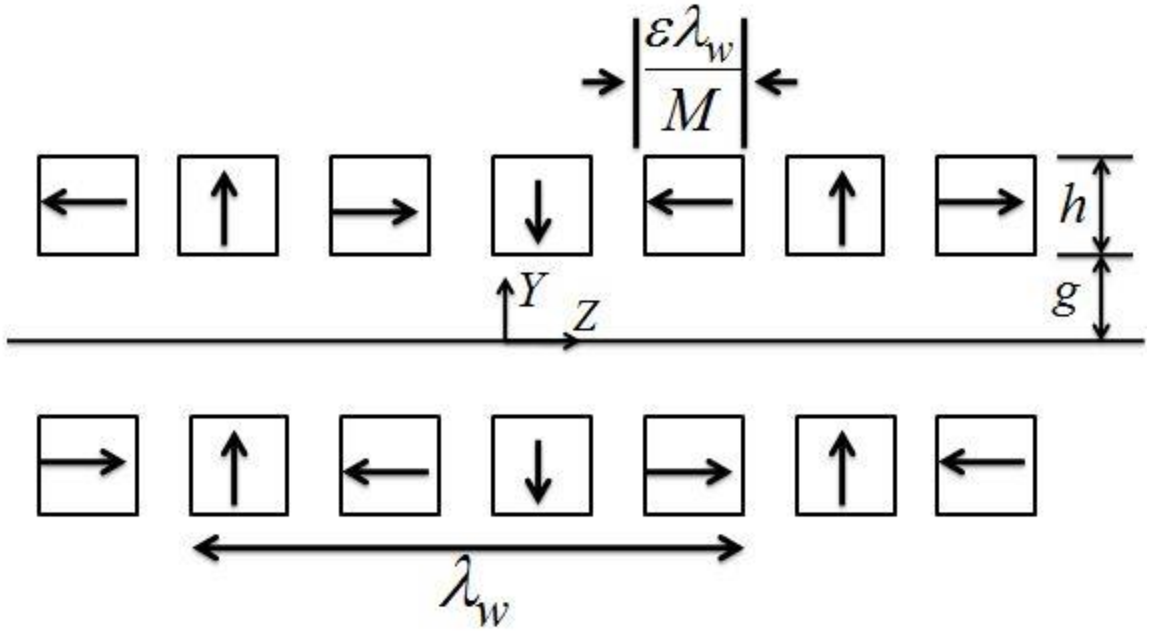

Fig:2.1- Vertical cross section view of wiggler / undulator with four blocks per period[21].

where h is the height of the blocks, g is the full gap and M' is the number of blocks per period in each half of the device. Device is effective when the width of the blocks in the X Direction is much greater than the gap width g.

2.5 Plasma

Plasma is the fourth state of matter other three states of matter are namely solid, liquid and gas. Solid is a state of matter, where the atoms are arranged in the specific positions and they cannot move freely inside the bulk of the material, though they can oscillate above their mean positions. Liquid is a state in which atoms or molecules can wander inside the bulk of the material because they have substantial kinetic energies that can overcome the potential energies due to mutual interactions. Gaseous state where the molecules have enough kinetic energies they are free to move and as a consequence you require some vessel or container to contain a gas.

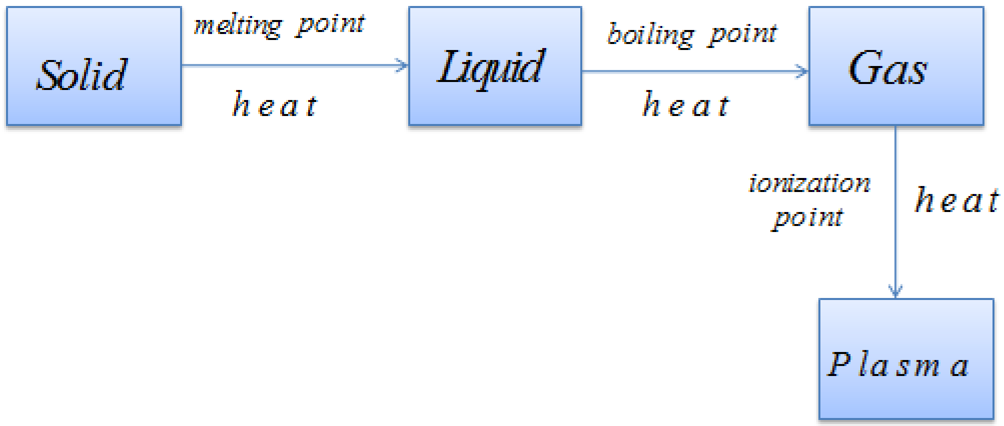

Fig-2.2: Block diagram of plasma formation

We can go from one phase of material to another phase like from solid to liquid if you heat the material. So, just by heating this up to a point a temperature called melting point, if you heat a solid then it will convert into a liquid. Similarly, if you heat a liquid to a temperature called boiling point, then the liquid will convert itself into a gas. what will happen if you heat a gas? Well if you heat a gas to high temperature of the order of 10,000 degrees Kelvin or higher than the gas can get ionized gas atoms producing ions and free electrons and they can that state of matter is called plasma as shown in figure.

A gas atom has a nucleus in the center and electrons may be in hydrogen there is one electron in helium there are two and in other gases in other atoms there are many electrons. So, electrons go around the nucleus at least one of the electrons of these orbits have to be rendered free should be removed from this atom then the atom is called ionized. In the case of hydrogen, if we want to render this electron which is rotating like this to be free you require an energy which we call as ionization potential 5 multiply by electron charge. For hydrogen this should be about 13.6 electron volts. So, we require colliding atoms to have energies of the order of 13.6 electron volt then, they will be ionized they can ionize each other or one of them will be ionized in collision. So, when an atom gets ionized, then atom gets converted into an ion plus an electron the energy required for this is of the order of ionization potential or more. So, typically the energies required for most of the atoms to be ionized are about 10 electron volts or higher.

Now, when we require 10 electron volt of energy then what temperature required. The average kinetic energy of atoms in a gas the molecules have maxwellian distribution function as a consequence if the temperature of a gas is T then

$$\text{AVERAGE KINETIC ENERGY}=\frac{3}{2}K_BT$$

Where, T is the temperature of the gas, K_B is the Boltzmann constant and if we put this usually in plasmas we talk of K_BT as temperature rather than T, the actual temperature is T, but we multiply this K_B the Boltzmann constant whose value is $K_B = 1.38\times10^{-23}$. This is joule per degree Kelvin. So, in plasmas K_B T is normally called the temperature of plasma, but if you put T is equal to 10^4 degree Kelvin K_B T is around 1 electron volt. So, we say that 1 electron volt temperature corresponds to about 10^4 degree Kelvin. So, in plasma there is a practice to call temperature in electron volt. So, typically 10^4 degrees Kelvin is equal to about 1electron volt.

A 10^5 degrees temperature will be like 10 electron volts. So it is difficult to generate plasma directly from heating the gas.

2.6 DRIFT SPACE

Drift or bunching space is a region where slow electron are speeded up or accelerated to overtake the fast moving electron and fast moving electron are deaccelerated to catch up the slow moving electron that turns into the density modulation of the beam electron

CHAPTER -3

Mathematical Analysis

3.1Overview

In this chapter, we develop the mathematical analysis for laser modulation of an electron beam and THz generation from the density bunched electron beam in a helical wiggler.

The process of THz generation is as follows: Two lasers with a frequency difference $\omega_{01} - \omega_{02}$ in the THz range apply a ponderomotive force on the beam electrons modulating their velocity. In the drift space, velocity modulation translates into density modulation. The electrons that arrive the modulator at a time when the electric field is parallel to v_b (where v_b is the initial electron beam velocity), get retarded and emerge from the modulator with reduced velocity. The electrons that arrive half wave period later when the electric field is anti-parallel to v_b get accelerated and emerge with larger velocity. In the drift space, fast moving electrons catch with slow moving electrons and cause density bunching. As the beam bunches pass through the wiggler in the presence of plasma which has the relative permittivity $|\varepsilon_r| = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$, where $\omega_p^2 = \frac{n_p e^2}{m_e \varepsilon_0}$, ω_p is plasma frequency. They acquire a transverse velocity, constituting a transverse current producing coherent THz radiation.

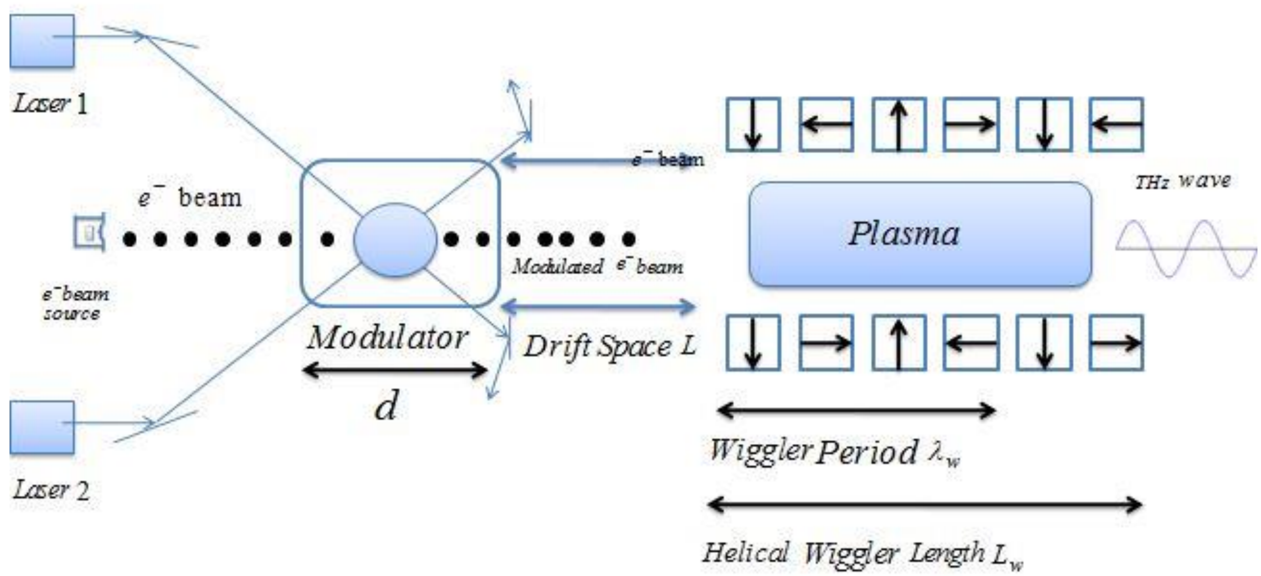

Fig.3.1. Schematic diagram of electron bunch to helical wiggler after following ref.[22].

By following the analysis of Kumar and Tripathi[22].

Consider two laser beams of frequencies ω_{01} and ω_{02} propagating in a modulator with electric field,

$$\vec{E}_{01} = A_1 e^{-i(\omega_{01}t - k_{01}z)}\hat{x} ,$$

$$\vec{E}_{02} = A_2 e^{-i(\omega_{02}t - k_{02}z)}\hat{x} ,$$

We find out the magnetic field associated with this field by using Maxwell's Equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E}_{01} = -\frac{\partial \vec{B}_{01}}{\partial t} \quad \nabla \times \vec{E}_{02} = -\frac{\partial \vec{B}_{02}}{\partial t}$$

Replace $\frac{\partial}{\partial t} \to -i\omega$ and by solving $\nabla \times \vec{E}_{01} = -\frac{\partial \vec{B}_{01}}{\partial t}$

$$\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \right) \times \left(A_1 e^{-i(\omega_{01}t - k_{01}z)}\hat{x} \right) = -\frac{\partial \vec{B}_{01}}{\partial t}$$

$$A_1 e^{-i(\omega_{01}t - k_{01}z)}\left(ik_{01}\right)\hat{y} = -i\omega_{01}\vec{B}_{01}$$

$$\vec{B}_{01} = -\frac{A_1 k_{01}}{\omega_{01}} e^{-i(\omega_{01}t - k_{01}z)}\hat{y} \tag{1}$$

Similarly, we get

$$\vec{B}_{02} = -\frac{A_2 \vec{k}_{02}}{\omega_{02}} e^{-i(\omega_{02}t - k_{02}z)}\hat{y} \tag{2}$$

A relativistic electron beam of density n_b, velocity $v_b\hat{z}$, and cross-section A_b passes through the modulator of width d. The lasers impart an oscillatory velocity to the beam electron

$$\vec{v}_1 = \frac{e\vec{E}_{01}}{m_e i\omega_{01}\gamma_0}, \tag{3}$$

$$\vec{v}_2 = \frac{e\vec{E}_{02}}{m_e i\omega_{02}\gamma_0}, \tag{4}$$

Where -e and m_e are the charge and rest mass of electrons respectively and $\gamma_0 = \left(1 - \frac{v_{0b}{}^2}{c^2}\right)^{-1/2}$ is the relativistic gamma factor. Lasers apply a non-linear ponderomotive force

$$\vec{F} = -\frac{e}{2}\left(\vec{v}_1 \times \vec{B}_{02}^{*} + \vec{v}_2^{*} \times \vec{B}_{01}\right) \tag{5}$$

First we solve –

$$\vec{v}_1 \times \vec{B}_{02}^*$$

$$\vec{v}_1 \times \vec{B}_{02}^* = \frac{eA_1 e^{-i(\omega_{01}t - k_{01}z)}\hat{x}}{m_e i\omega_{01}\gamma_0} \times \left(-\frac{A_2^* k_{02}}{\omega_{02}} e^{i(\omega_{02}t - k_{02}z)} \right)\hat{y}$$

$$= -\frac{eA_1 A_2^* k_{02}}{m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i[(\omega_{01}-\omega_{02})t - (k_{01}-k_{02})z]}\hat{z}$$

$$\vec{v}_1 \times \vec{B}_{02}^* = -\frac{eA_1 A_2^* k_{02}}{m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i(\omega t - kz)}\hat{z} \tag{6}$$

where $\omega_{01} - \omega_{02} = \omega$ and $\vec{k}_{01} - \vec{k}_{02} = \vec{k}$

Similarly we will find $\vec{v}_2^* \times \vec{B}_{01}$

$$\vec{v}_2^* \times \vec{B}_{01} = \frac{eA_1 A_2^* k_{01}}{m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i(\omega t - kz)}\hat{z} \tag{7}$$

Substituting the value of Eq.(6) and Eq. (7) in Eq. (5), we get

$$\vec{F} = -\frac{e}{2}\left(-\frac{eA_1 A_2^* k_{02}}{m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i(\omega t - kz)} + \frac{eA_1 A_2^* k_{02}}{m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i(\omega t - kz)} \right)\hat{z}$$

$$= -\frac{e^2 A_1 A_2^*}{2m_e i\omega_{01}\omega_{02}\gamma_0}(k_{01} - k_{02}) e^{-i(\omega t - kz)}\hat{z}$$

$$= -\frac{e^2 A_1 A_2^* \vec{k}}{2m_e i\omega_{01}\omega_{02}\gamma_0} e^{-i(\omega t - kz)}\hat{z}$$

$$\vec{F} = \vec{F}_0 e^{-i(\omega t - kz)} \hat{z}$$

where

$$\vec{F}_0 = -\frac{e^2 A_1 A_2^* \vec{k}}{2 m_e i \omega_{01} \omega_{02} \gamma_0} \tag{8}$$

As a practical scheme of modulation, we consider two laser beams of spot diameter $2r_0$ each, colliding with each other at an angle θ_d . Then, the length of the overlap region

$$d = \frac{2r_0}{\sin\left(\frac{\theta_d}{2}\right)} \approx \frac{2r_0}{\frac{\theta_d}{2}} = \frac{4r_0}{\theta_d}$$

In this case, ponderomotive force z component is still F_0 with k replaced by $k\cos\theta_d$. For $\theta_d \ll 1$; $k\cos\theta_d \approx k$ and above expression for ponderomotive force remains same.

Let t_0 be the time at which an electron reaches the modulator (z=0), t_1 at which it leaves the modulator at z=d, and t_2 at which it reaches the wiggler placed after a drift space of length L. Inside the modulator motion under the ponderomotive force is governed by the relativistic equation of motion

$$\frac{\partial}{\partial t}(\gamma \vec{v}) = \frac{\vec{F}_p}{m_e} \tag{9}$$

$\vec{v} = v_b \hat{z} + v_{b\omega} \hat{z}$ and $\gamma = \gamma_0 + \gamma_0^3 \frac{v_b v_{b\omega}}{c^2}$

$$\gamma \vec{v} = \left(\gamma_0 + \gamma_0^3 \frac{v_b v_{b\omega}}{c^2}\right)(v_b + v_{b\omega})\hat{z}$$

$$= \left(\gamma_0 v_b + \gamma_0 v_{b\omega} + \gamma_0^3 \frac{v_b^2 v_{b\omega}}{c^2} + \gamma_0^3 \frac{v_b v_{b\omega}^2}{c^2}\right)\hat{z}$$

In order tolinearize the equation we make the approximation, then

$$\gamma\vec{v} \cong \left(\gamma_0{}^3 v_{b\omega}\right)$$

From Eq. (9), we get

$$\frac{d}{dt}\left(\gamma_0{}^3\vec{v}\right) = \frac{\vec{F}}{m_e} \tag{10}$$

In the exponent, we take $z = v_b\left(t - t_0\right)$ and integrate Eq.(10) from t_0 to $t_1 = \left(t_0 + \frac{d}{v_b}\right)$, we get

$$\int_0^{v_{b\omega}} d\left(\gamma_0{}^3 v_{b\omega}\right) = \int_{t_0}^{t_0 + d/v_b} \frac{\vec{F}_0}{m_e} e^{-i\left[\omega t - v_b\left(t - t_0\right)k\right]} dt$$

$$\gamma_0{}^3 v_{b\omega} = \frac{\vec{F}_0}{m_e}\left[\frac{e^{-i\left[\omega t - v_b(\mathrm{t} - \mathrm{t}_0)\mathrm{k}\right]}}{-i(\omega - v_b\,\mathrm{k})}\right]_{t_0}^{t_0 + d/v_b}$$

$$\gamma_0{}^3 v_{b\omega} = \frac{\vec{F}_0}{-im_e\left(\omega - v_b k\right)}\left[e^{-i\left(\omega t_0 + \left(\frac{\omega - v_b k}{v_b}\right)d\right)} - e^{-i\left(\omega t_0\right)}\right]$$

$$\gamma_0{}^3 v_{b\omega} = \frac{\vec{F}_0}{-im\left(\omega - v_b k\right)} e^{-i\left(\omega t_0 + \left(\frac{\omega - v_b k}{2v_b}\right)d\right)}\left[e^{-i\left(\frac{\omega - v_b k}{2v_b}\right)} - e^{i\left(\frac{\omega - v_b k}{2v_b}\right)}\right]$$

$$v_{b\omega} = \frac{-\vec{F}_0}{m_e v_b \gamma_0{}^3}\frac{\sin\theta_g}{\theta_g} e^{-i(\omega t_0 + \theta_g)}\hat{z}$$

Where $\theta_g = \left(\frac{\omega - v_b k}{2v_b}\right)d$ is the modulator phase angle.

Substitutingthe value of $\vec{F}_0$ from Eq.(8) in $v_{b\omega}$

$$v_{b\omega} = \frac{-e^2 k A_1 A_2^* d}{2i\omega_{01}\omega_{02} m_e^2 v_b \gamma_0^{\ 4}} \frac{\sin\theta_g}{\theta_g} e^{-i(\omega t_0 + \theta_g)} \hat{z}$$

$$v_{b\omega} = \frac{i e^2 k A_1 A_2^* d}{2\omega_{01}\omega_{02} m_e^2 v_b \gamma_0^{\ 4}} \frac{\sin\theta_g}{\theta_g} \left[\cos(\omega t_0 + \theta_g) - i \sin(\omega t_0 + \theta_g) \right] \hat{z} \tag{11}$$

By considering real terms

$$v_{b\omega} = \frac{-e^2 k A_1 A_2^* d}{2\omega_{01}\omega_{02} m_e^2 v_b \gamma_0^{\ 4}} \frac{\sin\theta_g}{\theta_g} \sin(\omega t_0 + \theta_g) \hat{z}$$

$$v_{b\omega} = -\psi v_b \sin(\omega t_0 + \theta_g) \hat{\mathrm{z}}$$

$$\psi = \frac{k d c^2}{2 v_b^2 \gamma_0^4} \frac{e A_1}{m_e \omega_{01} c} \frac{e A_2^*}{m_e \omega_{02} c} \frac{\sin\theta_g}{\theta_g} \tag{12}$$

In the drift space, the fast moving electrons catch up with the slow moving ones that had left the modulator half wave period earlier, leading to density bunching of electrons. The time of arrival of the electron bunch at the entry point (z=d+L) to the wiggler is $t_{2=}t_0 + \left(d/v_b \right) + \left(L(1+\psi \sin(\omega\, \mathrm{t}_0 + \theta_g)) / \mathrm{v}_b \right)$. Let the beam current at the wiggler be I_2. Then, the charge passing through the cross-section in time dt_2 at the wiggler is $I_2 dt_2$. This must be equal to the charge entering the modulator between t_0 and $t_0 + dt_0$, i.e., $I_2 = I_0 \big/ \left(dt_2 / dt_0 \right)$.

I2 is a periodic function of time. We may expand it in Fourier series,

$$I_2(\mathrm{t}_2) = \mathrm{a}_0 + a_1 \cos\omega\,\mathrm{t}_2 + a_2 \cos\omega\,\mathrm{t}_2 + \ldots\ldots.. + b_1 \sin\omega\,\mathrm{t}_2 + b_2 \sin\omega\,\mathrm{t}_2 + \ldots\ldots\ldots\ldots..$$

where

$$a_1 = \frac{\omega}{\pi} \int_0^{2\pi/\omega} I_2 dt_2 \cos \omega t_2$$

$$= \frac{\omega}{\pi} \int_0^{2\pi/\omega} I_2 dt_0 \cos\left(\omega t_0 + \frac{\omega(d+L)}{v_b} + \frac{\omega L \psi}{v_b} \sin\left[\omega t_0 + \theta_g \right] \right)$$

$$b_1 = \frac{\omega}{\pi} \int_0^{2\pi/\omega} I_2 dt_2 \sin \omega t_2$$

$$= \frac{\omega}{\pi} \int_0^{2\pi/\omega} I_2 dt_0 \sin\left(\omega t_0 + \frac{\omega(d+L)}{v_b} + \frac{\omega L \psi}{v_b} \sin\left[\omega t_0 + \theta_g \right] \right)$$

If we introduce

$$\phi = \frac{\omega I_0}{\pi} \int_0^{2\pi/\omega} \exp\left(i\omega \left(t_0 + \frac{d+L}{v_b} \right) \right) \times \exp\left(\frac{i\omega L \psi \sin(\omega t_0 + \theta_g)}{v_b} \right) dt_0$$

Then $a_1 = \mathrm{Re}(\phi)$ and $b_1 = \mathrm{Im}(\phi)$. Employing the Bessel function identity,

$e^{i\alpha \sin\theta} = \sum_{n=-\infty}^{\infty} J_n(\alpha) e^{in\theta}$, we may write,

$$\phi = \frac{\omega I_0}{\pi} \sum_n \int_0^{2\pi/\omega} J_n\left(\omega L \psi / v_b \right) e^{i\left(\omega(d+L)/v_b\right)} e^{i(n+1)\omega t_0} e^{in\theta_g} dt_0$$

Using the orthonormality relation, we find that all terms in ϕ ,other than n= -1 term, vanish, giving

$$\phi = -2 I_0 J_1\left(\omega L \psi / v_b \right) e^{i\left(\omega(d+L)/v_b - \theta_g\right)}$$

Hence,

$$a_1 = -2 I_0 J_1\left(\omega L \psi / v_b \right) \cos\left((\omega(d+L)/v_b) - \theta_g \right)$$

$$b_1 = -2 I_0 J_1\left(\omega L \psi / v_b \right) \sin\left((\omega(d+L)/v_b) - \theta_g \right)$$

The ω frequency component of I_2 is

$$I_2(\omega) = \mathrm{a}_1 \cos(\omega\,\mathrm{t}_2) + \mathrm{b}_1 \sin(\omega\,\mathrm{t}_2)$$

$$= -2\,\mathrm{I}_0\, J_1\left(\omega L\psi/v_b\right)\cos(\omega\,\mathrm{t}_2 - \delta)$$

Where $\delta = \dfrac{\omega\left(d+L\right)}{v_b} - \theta_g$, $I_0 = -n_b e v_b A_b$

The beam density modulation at the wiggler can be written as

$$n_{\omega,k} = -\frac{I_2(\omega)}{e v_b A_b} = -2 n_b J_1\left(\omega L\psi/v_b\right)\cos(\omega\,\mathrm{t}_2 - \delta) \tag{13}$$

3.2 Helical wiggler magnetic field

We allow the electron bunch to enter into the helical wiggler magnetic field-

$$\vec{B} = \vec{B}_0(\hat{\rho} - \mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\mathrm{k}_0 \mathrm{z} + \varphi)}, \qquad \text{where} \quad \varphi = \omega_0 t$$

In this field, electrons acquire an oscillatory velocity which in the beam frame, moving with velocity can be written as

$$\vec{v}_0^{\,'} = \frac{e\vec{E}_0^{\,'}}{m_e i \omega_0^{'}} \tag{14}$$

where $\omega_0^{'} = \gamma_0 k_0 v_b$ and

where the prime denotes Lorentz-transformed quantities in the moving frame

$$\vec{E}_0^{\,'} = \gamma_0 v_b \hat{z} * \vec{\mathrm{B}}_0(\hat{\rho} - \mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega_0^{'} t^{'} - k_0^{'} \mathrm{z}^{'} + \varphi)}$$

$$= \gamma_0 v_b \hat{z}\, \vec{\mathrm{B}}_0(\hat{\varphi} + i\,\hat{\rho})\,\mathrm{e}^{-i(\omega_0^{'} t^{'} - k_0^{'} \mathrm{z}^{'} + \varphi)}$$

$$\vec{E}_0^{\,'} = -\frac{\gamma_0}{i} v_b\, \vec{\mathrm{B}}_0(\hat{\rho} - \mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega_0^{'} t^{'} - k_0^{'} \mathrm{z}^{'} + \varphi)} \tag{15}$$

Substitute value of eq. (15) in eq. (14), we get

$$\vec{v}_0^{\,'} = -\frac{e}{m_e i \omega_0^{'}} \frac{\gamma_0}{i} v_b\, \vec{\mathrm{B}}_0(\hat{\rho} - \mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega_0^{'} t^{'} - k_0^{'} \mathrm{z}^{'} + \varphi)}$$

$$\vec{v}_0^{'} = \frac{e}{m_e\omega_0^{'}}\gamma_0 v_b\,\vec{\mathrm{B}}_0(\hat{\rho}-\mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega_0^{'}t^{'}-k_0^{'}\mathrm{z}^{'}+\varphi)} \tag{16}$$

3.3 Non Linear current density

Non Linear current density is given by

$$\vec{J}_{NL} = -n_{\omega,k}e\vec{v}_0^{'}$$

From Eq. (13) and Eq. (16)

$$\vec{J}_{NL} = 2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^{-i(\delta-\varphi)}e^{-i\omega t_2} * e * \frac{e}{m_e\omega_0^{'}}\gamma_0 v_b\,\vec{\mathrm{B}}_0(\hat{\rho}-\mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega_0^{'}t^{'}-k_0^{'}\mathrm{z}^{'})}$$

$$\vec{J}_{NL} = 2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^{-i(\delta-\varphi)}\,\frac{e^2}{m_e\omega_0^{'}}\gamma_0 v_b\,\vec{\mathrm{B}}_0(\hat{\rho}-\mathrm{i}\,\hat{\varphi})\,\mathrm{e}^{-i(\omega\,\mathrm{t}_2+\omega_0^{'}t^{'}-k_0^{'}\mathrm{z}^{'})}$$

Put $t_2 = t^{'} - \frac{z^{'}}{v_b}$

$$\vec{J}_{NL} = \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^2 v_b\gamma_0 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{-i\left(\omega t^{'}-\frac{\omega z^{'}}{v_b}+\omega_0^{'}t^{'}-k_0^{'}z^{'}\right)}e^{i(\delta-\varphi)}$$

$$\vec{J}_{NL} = \frac{2n_b J_1\left(\frac{\omega L\eta}{v_b}\right)e^2 v_b\gamma_0 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{-i\left[\left(\omega+\omega_0^{'}\right)t-\left(\frac{\omega}{v_b}+k_0^{'}\right)z^{'}\right]}e^{i(\delta-\varphi)}$$

Let $\omega_s = \omega + \omega_0^{'}$

$$\vec{J}_{NL} = \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^2 v_b\gamma_0 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{-i\left[\omega_s t-\left(\frac{\omega}{v_b}+k_0^{'}\right)z^{'}\right]}e^{i(\delta-\varphi)} \tag{17}$$

3.4 Retarded vector potential -

$$\vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int\frac{\vec{J}\left(\vec{r},t-\frac{R}{c}\right)}{R}d^3r^{'}$$

where $R=\left|\vec{r}-\vec{r}^{'}\right|=r-z^{'}\cos\theta$

For plasma relative permittivity $\left|\varepsilon_r\right|=\left(1-\frac{\omega_p^2}{\omega^2}\right)$ where $\omega_p^2=\frac{n_p e^2}{m_e\varepsilon_0}$

Then

$$\vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int\frac{\vec{J}\left(\vec{r},t-\frac{R\left|\varepsilon_r\right|^{0.5}}{c}\right)}{r}4\pi r_b^2dz^{'}$$

$$\vec{A}(\vec{r},t)=\frac{\mu_0 r_b^2}{r}\int\vec{J}\left(\vec{r},t-\frac{R\left|\varepsilon_r\right|^{0.5}}{c}\right)dz^{'} \tag{18}$$

Substitute the value of $\vec{J}$ from Eq. (17) in Eq. (18),

$$\vec{A}(\vec{r},t)=\frac{\mu_0 r_b^2}{r}\int\frac{2n_bJ_1\left(\frac{\omega L\psi}{v_b}\right)e^2v_b\gamma_0B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{i(\delta-\varphi)}e^{-i\left(\omega_s\left(t-\frac{R\left|\varepsilon_r\right|^{0.5}}{c}\right)-\left(\frac{\omega}{v_b}+k_0^{'}\right)z^{'}\right)}dz^{'}$$

$$\vec{A}(\vec{r},t)=\frac{\mu_0 r_b^2}{r}\frac{2n_bJ_1\left(\frac{\omega L\psi}{v_b}\right)e^2v_b\gamma_0B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{i(\delta-\hat{\varphi})}\int e^{-i\left(\omega_s\left(t-\frac{(r-z^{'}\cos\theta)\left|\varepsilon_r\right|^{0.5}}{c}\right)-\left(\frac{\omega}{v_b}+k_0^{'}\right)z^{'}\right)}dz^{'}$$

$$\vec{A}(\vec{r},t)=\frac{\mu_0 r_b^2}{r}\frac{2n_bJ_1\left(\frac{\omega L\psi}{v_b}\right)e^2v_b\gamma_0B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\omega_0^{'}}e^{i(\delta-\varphi)}e^{-i\left(\omega_s\left(t-\frac{r\left|\varepsilon_r\right|^{0.5}}{c}\right)\right)}\int e^{-i\left(\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}-\frac{\omega}{v_b}-k_0^{'}\right)z^{'}}dz^{'}$$

Let $I = \int_0^{L_b} e^{-i\left(\frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} - \frac{\omega}{v_b} - k_0^{'}\right) z^{'}} dz^{'}$

where L_b is bunch length.

$$I = \frac{e^{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right) L_b} - 1}{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right)}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0 r_b^2}{r} \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right) e^2 v_b \gamma_0 B_0 \left(\hat{\rho} - i\hat{\varphi}\right)}{m_e \omega_0^{'}} e^{i(\delta-\varphi)} e^{-i\left(\omega_s\left(t - \frac{r|\varepsilon_r|^{0.5}}{c}\right)\right)} \frac{e^{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right) L_b} - 1}{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right)}$$

3.5 The Magnetic field of THz is-

$$\vec{B} = \Delta \times \vec{A} \approx \left(i \frac{\omega_s |\varepsilon_r|^{0.5}}{c}\right) \hat{\rho} \times \vec{A} \tag{19}$$

Substitute the value of $\vec{A}$ in Eq.(19), we get

$$\vec{B} = \left(i \frac{\omega_s |\varepsilon_r|^{0.5}}{c}\right) \hat{\rho} \times \left(\hat{\rho} - i\hat{\varphi}\right) \frac{\mu_0 r_b^2}{r} \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right) e^2 v_b \gamma_0 B_0}{m_e \omega_0^{'}} e^{i(\delta-\varphi)} e^{-i\left(\omega_s\left(t - \frac{r|\varepsilon_r|^{0.5}}{c}\right)\right)}$$

$$\times \frac{e^{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right) L_b} - 1}{i\left(\frac{\omega}{v_b} - \frac{\omega_s \cos\theta |\varepsilon_r|^{0.5}}{c} + k_0^{'}\right)}$$

$$\vec{B} = \left(i\frac{\omega_s\left|\varepsilon_r\right|^{0.5}}{c} \right)(-i\hat{z})\frac{\mu_0 r_b^2}{r}\frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^2 v_b\gamma_0 B_0}{m_e\omega_0^{'}}e^{i(\delta-\varphi)}e^{-i\left(\omega_s\left(t-\frac{r\left|\varepsilon_r\right|^{0.5}}{c}\right)\right)}$$

$$\times\frac{e^{i\left(\frac{\omega}{v_b}-\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}+k_0^{'}\right)L_b}-1}{i\left(\frac{\omega}{v_b}-\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}+k_0^{'}\right)}$$

Let $M = \frac{e^{i\left(\frac{\omega}{v_b}-\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}+k_0^{'}\right)L_b}-1}{i\left(\frac{\omega}{v_b}-\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}+k_0^{'}\right)}\times\frac{L_b/2}{L_b/2}$

Let $\theta_1 = \left(\frac{\omega}{v_b}-\frac{\omega_s\cos\theta\left|\varepsilon_r\right|^{0.5}}{c}+k_0^{'}\right)\frac{L_b}{2}$

Now M will become-

$$M = e^{i\theta_1}\left(\frac{e^{i\theta_1}-e^{-i\theta_1}}{i\theta_1}\right)\frac{L_b}{2}$$

$$M = \frac{e^{i\theta_1}}{\theta_1}L_b\sin\theta_1$$

Expression of magnetic field reduces to-

$$\vec{B} = \left(\frac{\omega_s\left|\varepsilon_r\right|^{0.5}}{c} \right)\frac{\mu_0 r_b^2}{r}\frac{2n_{bo} J_1\left(\frac{\omega L\psi}{v_b}\right)e^2 v_b\gamma_0 B_0}{m_e\omega_0^{'}}e^{i(\delta-\varphi)}e^{-i\left(\omega_s\left(t-\frac{r\left|\varepsilon_r\right|^{0.5}}{c}\right)\right)}\frac{e^{i\theta_1}}{\theta_1}L_b\sin\theta_1\hat{z}$$

By taking mode and squaring above expression

$$\left|\vec{B}\right|^2 = \frac{\omega_s^2 \left|\varepsilon_r\right|}{c^2} \frac{\mu_0^2 r_b^4}{r^2} \frac{4 n_b^2 J_1^2 \left(\frac{\omega L \psi}{v_b} \right) e^4 v_b^2 \gamma_0^2 B_0^2}{m_e^2 \omega_0^{'2}} L_b^2 \frac{\sin^2 \theta_1}{\theta_1^2} \tag{20}$$

3.6 Time average pointing vector

Time average pointing vector is given by

$$\vec{P}_{avg} = \hat{r} \frac{c}{2\mu_0} \left|\vec{B}\right|^2 \tag{21}$$

Substitute the value of $\left|\vec{B}\right|^2$ from Eq.(20) in Eq. (21), we get

$$\vec{P}_{avg} = \hat{r} \frac{c}{2\mu_0} \frac{\omega_s^2 \left|\varepsilon_r\right|}{c^2} \frac{\mu_0^2 r_b^4}{r^2} \frac{4 n_b^2 J_1^2 \left(\frac{\omega L \psi}{v_b} \right) e^4 v_b^2 \gamma_0^2 B_0^2}{m_e^2 \omega_0^{'2}} L_b^2 \frac{\sin^2 \theta_1}{\theta_1^2}$$

$$\vec{P}_{avg} = \hat{r} \frac{4 R_b^4}{K_0^2} \frac{\omega_s^2 \left|\varepsilon_r\right|}{\omega_{01}^2} \frac{\omega_b^4}{\omega_{01}^4} \frac{L_b^2}{r^2} \frac{\sin^2 \theta_1}{\theta_1^2} J_1^2 \left(\frac{\omega L \psi}{v_b} \right) \times c B_0^2 \big/ 2\mu_0$$

$$\vec{P}_{avg} = \hat{r} \frac{4 R_b^4}{K_0^2} \frac{\omega_s^2 \left|\varepsilon_r\right|}{\omega_{01}^2} \frac{\omega_b^4}{\omega_{01}^4} \frac{L_b^2}{r^2} \frac{\sin^2 \theta_1}{\theta_1^2} J_1^2 \left(\frac{\omega^2}{\omega_{01}^2} A \right) \times c B_0^2 \big/ 2\mu_0$$

where $A = \frac{1}{2\gamma_0^4} \frac{\omega_{01} d}{c} \frac{\omega_{01} L}{c} \frac{c^3}{v_b^3} \frac{e A_1}{m_e \omega_{01} c} \frac{e A_2^*}{m_e \omega_{01} c} \frac{\sin\left(\theta_g\right)}{\theta_g}$

$$R_b = \frac{r_b \omega_{01}}{c} \quad K_0 = \frac{c k_0}{\omega_{01}} \quad \omega_b^4 = \frac{n_{bo}^2 e^4}{m_e^2 \varepsilon_0^2} \quad \frac{\omega^2}{\omega_{01}^2} A = \frac{\omega L \psi}{v_b}$$

3.7 Amplitude of THz

Wave equation for THz wave is

$$\nabla^2 \vec{E}_\omega + \frac{|\varepsilon_r|}{c^2}\frac{\partial^2 \vec{E}_\omega}{\partial t^2} = \frac{-i\omega_s |\varepsilon_r|}{c^2 \varepsilon_0} \vec{J}_\omega^{NL} \tag{22}$$

Let $\vec{E}_\omega = \hat{x} A_\omega(z) e^{-i(\omega_s t - K_{THZ} z)}$

Where $K_{THZ} = \dfrac{\omega}{v_b} + k_0^{'}$

By Substitute value of $\vec{E}_\omega$ and $\vec{J}_\omega^{NL}$ in eq. (22), we get

$$\nabla^2 \vec{E}_\omega + \frac{|\varepsilon_r|}{c^2}\frac{\partial^2 \vec{E}_\omega}{\partial t^2} = \frac{-i\omega_s |\varepsilon_r|}{c^2 \varepsilon_0} \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right) e^2 v_b \gamma_0 B_0 (\hat{\rho} - i\hat{\varphi})}{m_e \omega_0^{'}} e^{-i\left[\omega_s t - \left(\frac{\omega}{v_b} + k_0^{'}\right) z^{'}\right]}$$

$$2iK_{THZ}\frac{\partial A_\omega}{\partial z} + 2iK_{THZ}\frac{\partial A_\omega}{\partial t} + \left(-K_{THZ}^2 + \frac{\omega^2 |\varepsilon_r|}{c^2}\right) A_\omega = \frac{-i\omega_s}{c^2 \varepsilon_0} \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right) e^2 v_b \gamma_0 B_0 (\hat{\rho} - i\hat{\varphi})}{m_e \omega_0^{'}}$$

$$\times e^{-i\left[\omega_s t - \left(\frac{\omega}{v_b} + k_0^{'}\right) z^{'}\right]}$$

At exact phase matching condition third term on left hand side becomes zero and $\dfrac{\partial A_\omega}{\partial t}$ also vanishes.(A_ω is the amplitude not varying with time).

So above eq. reduces to-

$$\frac{\partial A_\omega}{\partial z} = \frac{-i\omega_s |\varepsilon_r|}{c^2 \varepsilon_0} \frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right) e^2 v_{bo} \gamma_0 B_0 (\hat{\rho} - i\hat{\varphi})}{m_e \omega_0^{'} 2iK_{THZ}}$$

$$\frac{\partial A_\omega}{\partial z}=\frac{-i\omega_s\left|\varepsilon_r\right|}{c^2\varepsilon_0}\frac{2n_b J_1\left(\frac{\omega L\psi}{v_{bo}}\right)e^2 v_b\gamma_0 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e\gamma_0 k_0 v_b\, 2iK_{THZ}}$$

$$\frac{\partial A_\omega}{\partial z}=\frac{-i\omega_s\left|\varepsilon_r\right|}{c^2\varepsilon_0}\frac{2n_b J_1\left(\frac{\omega L\psi}{v_b}\right)e^2 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e k_0\, 2iK_{THZ}}$$

$$\frac{\partial A_\omega}{\partial z}=\frac{-\omega_s\left|\varepsilon_r\right|}{c^2\varepsilon_0}\frac{n_b J_1\left(\frac{\omega L\psi}{v_{bo}}\right)e^2 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{m_e k_0 K_{THZ}}$$

$$\frac{\partial A_\omega}{\partial z}=\frac{-\omega_s\left|\varepsilon_r\right|}{c^2}\frac{\omega_b^2 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{k_0 K_{THZ}}J_1\left(\frac{\omega L\psi}{v_b}\right)$$

$$A_\omega=\int_0^{L_w}\frac{-\omega_s\left|\varepsilon_r\right|}{c^2}\frac{\omega_b^2 B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{k_0 K_{THZ}}J_1\left(\frac{\omega L\psi}{v_b}\right)dz$$

$$A_\omega=\frac{-\omega_s\left|\varepsilon_r\right|}{c^2}\frac{\omega_b^2 L_w B_0\left(\hat{\rho}-i\hat{\varphi}\right)}{k_0 K_{THZ}}J_1\left(\frac{\omega L\psi}{v_b}\right)$$

$$\left|A_\omega\right|^2=\frac{1}{K_0^2}\frac{\omega_s^2}{\omega^2}\frac{\omega_b^4}{\omega_{01}^4}\frac{\omega_{01}^2 L_b^2}{c^2}\frac{v_b^2}{c^2}\frac{L_w^2}{L_b^2}\frac{\omega_{01}^2}{\omega^2}\left(1+\frac{K_0 v_b}{c}\frac{\omega_{01}}{\omega}\right)^{-2}J_1^2\left(\frac{\omega^2}{\omega_{01}^2}A\right)$$

$$K_0=\frac{ck_0}{\omega_{01}}\;\;\omega_b^4=\frac{n_{bo}^2 e^4}{m_e^2\varepsilon_0^2}$$

CHAPTER-4

RESULTS

4.1 Result

Laser bunched electron beam coupling to a helical wiggler appears a potential candidate for THz generation. THz power scales as square of bunch radius and beam current, linearly with bunch length, and inverse Square of wiggler wave vector. For a fixed drift space lengthL, the THz power increases with THz frequency, attains a maximum at ω =2.5 THz, for beam velocity $v_b = 0.94c$ and then falls off due to the Besselfunction character of density modulation. The frequency of the THz radiation can be tuned by the changes in the wavelengths ofthe lasers. The power conversion efficiency from the beam toTHz can be of the order of a percent. One may note that thelength of the modulator segment could befairly large as compared to the THz wavelength as the beam travels with a velocity close to the phase velocity of the ponderomotive force. However, we neglect this effect which may be as long as Lw< L. We have ignored thebeam space charge effect that may be justified as long as the growth rate exceeds the beam plasma frequency.

Parameters of electron beam and helical wiggler is given in Table 4.1.

We have carried numerical calculations of normalized THz power and amplitude for the following parameters: $\omega_1 L/c$=7$\times 10^4$, $\omega_1 d/c$=2$\times 10^3$, $\gamma_0 = 2.93, R_b = 1950, \omega_b/\omega_1$=7.98 $\times 10^{-6}, K_0 = 3.768\times10^{-3}$. We have plotted the THz power with ω/ω_1 for different values of beam velocity $v_b = 0.94c$ in Figure 4.1. Initially, THz power increases with frequency and attains a maximum value 2.7×10^{-8} at ω=2.5 THz and then falls off.

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Table 4.1 Parameters used in THz Generation

Electron beam energy	1Mev
Electron beam size	r_b =5.85 mm
Electron beam density	$n_b = 10^{15}/m^3$
Velocity of beam	v_b =0.94c
Wavelength for Laser 1	$\lambda_1 = 3\mu m$
Wavelength for Laser 2	$\lambda_2 = 3(1 + \lambda_1/\lambda_{THz})$
Length of modulator	d=6mm
Length of Drift space	L=20 cm
Length of wiggler	$L_w = 5m$
Electron bunch length	$L_b = 0.5cm$
Wiggler wavelength	$\lambda_w = 5mm$
Plasma density	$n_p = 10^{18}/m^3$

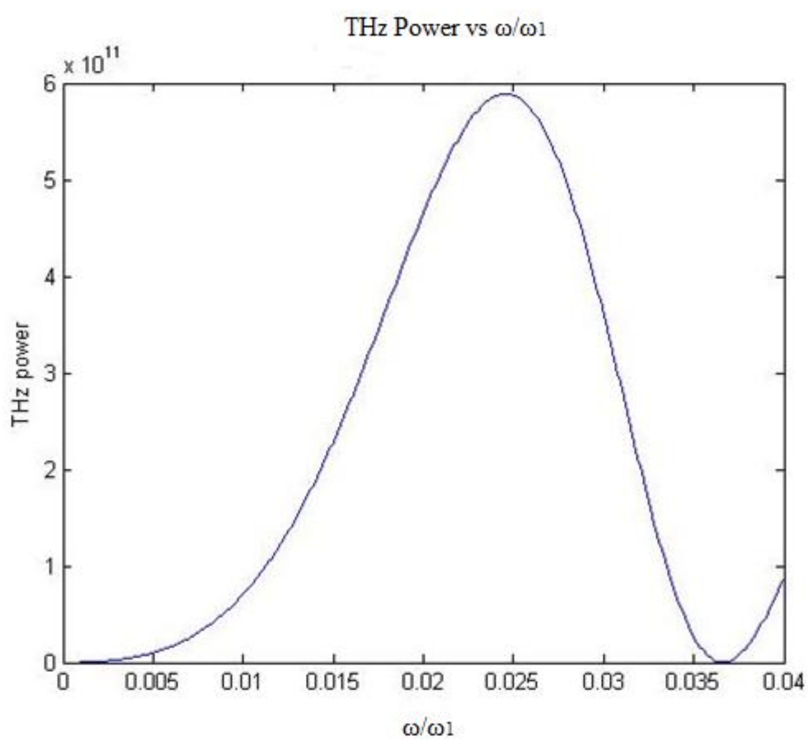

Figure 4.1 THz Power as a function of ω/ω_1 for v_b/c=0.94

Figure 4.1 shows the variation of THz power with ω/ω_1 for the beam velocity $v_b = 0.94c$. THz power attains a maximum value of 5.9×10^{11} $watt / m^2$ at 2.5 THz.

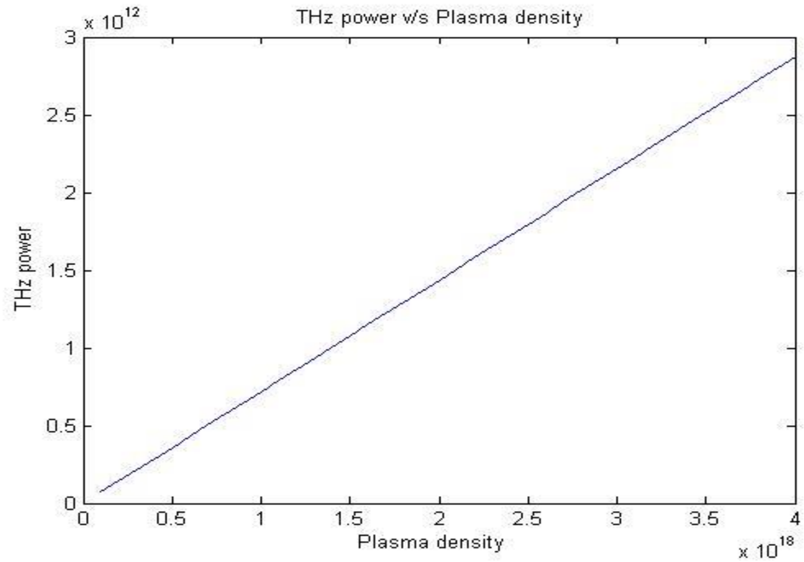

Figure 4.2 THz power variation with Plasma density n_p

Figure 4.2 shows the variation of THz power with Plasma density. It indicate that at $\omega/\omega_1 = 0.025$ and at plasma density 1×10^18 THz power is 5.9×10^11 $watt / m^2$ which satisfied the result of Fig.4.1. THz power variation is proportional to the plasma density as plasma density increase THz power increases.

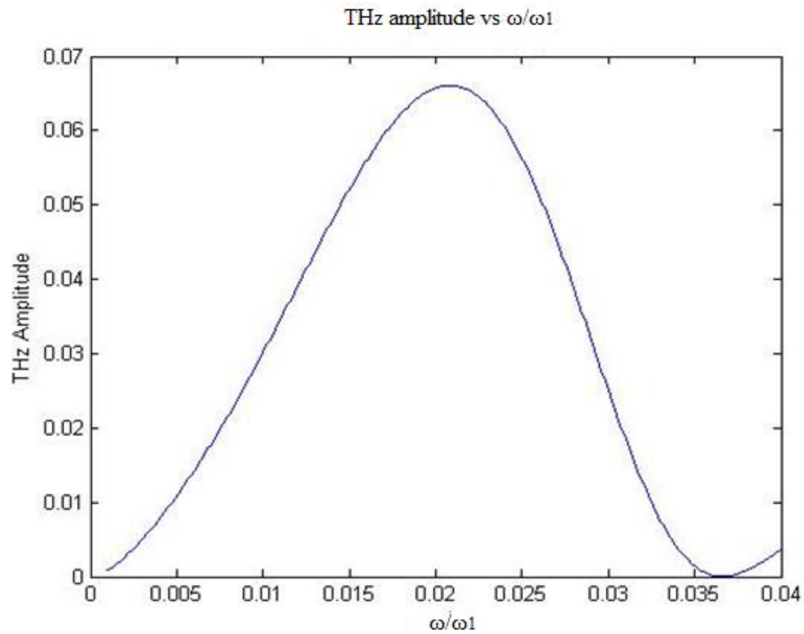

Figure 4.3 THz Amplitude as a function of ω/ω_1 for v_b/c =0.94

Figure 4.3 shows the variation power with $\omega/\omega_1$for the with $\omega/\omega_1$for the beam velocity $v_b = 0.94c$. THz amplitude increases with THz frequency to a maximum value and then falls off due to the Bessel function character of density modulation.

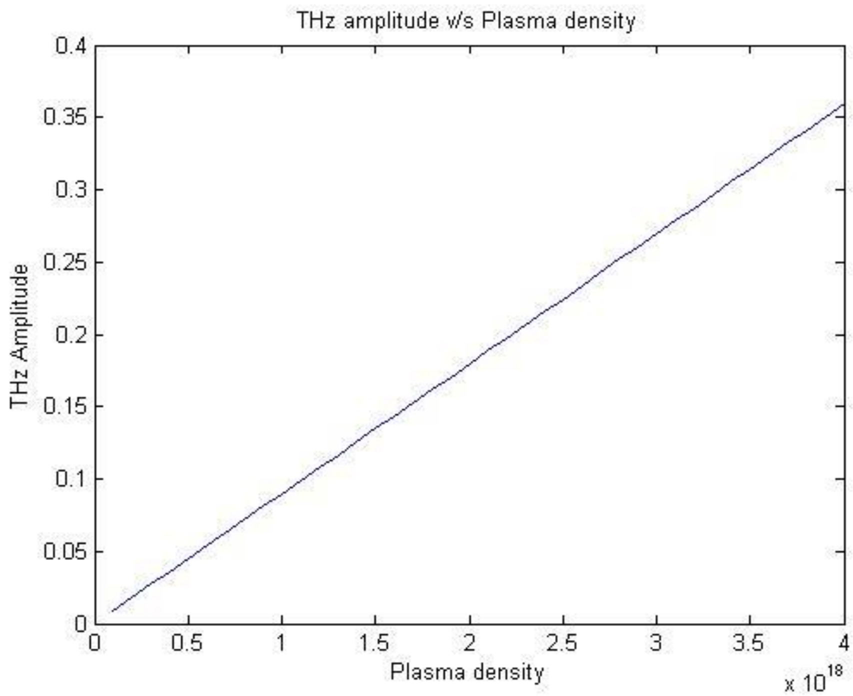

Figure 4.4 THz Amplitude variation with Plasma density n_p

Figure 4.4 shows the variation of THz amplitude with plasma density. It indicate that at $\omega/\omega_1 = 0.025$ and at plasma density 1×10^18 THz amplitude is 0.069 which satisfied the result of Fig.4.3. THz amplitude variation is proportional to the plasma density as plasma density increase THz amplitude increases.

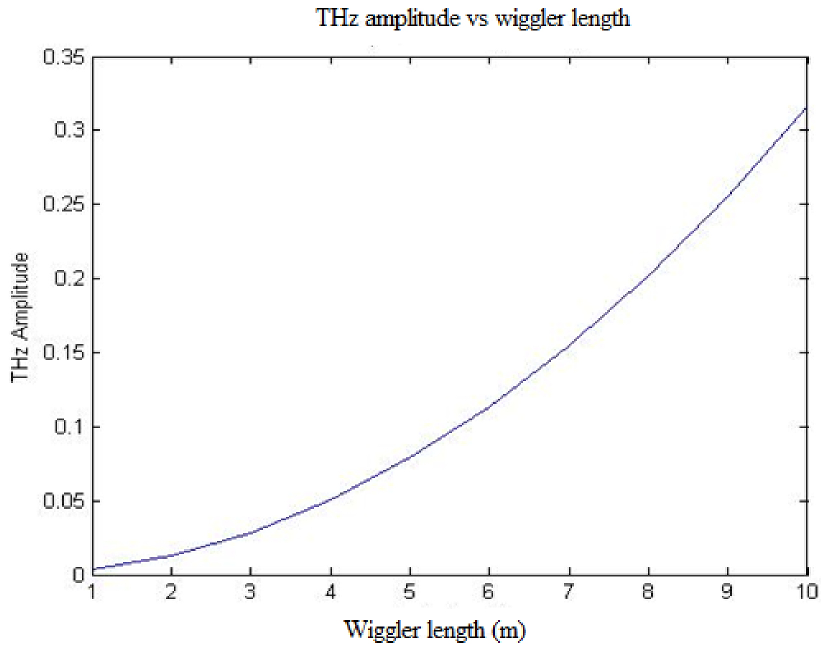

Figure 4.5 THz Amplitude variation with wiggler length

Figure 4.5 shows the graph of THz amplitude varies with length of helical wiggler at 2.1 THz frequency for the beam velocity $v_b = 0.94c$. THz amplitude is proportional to the length of helical wiggler

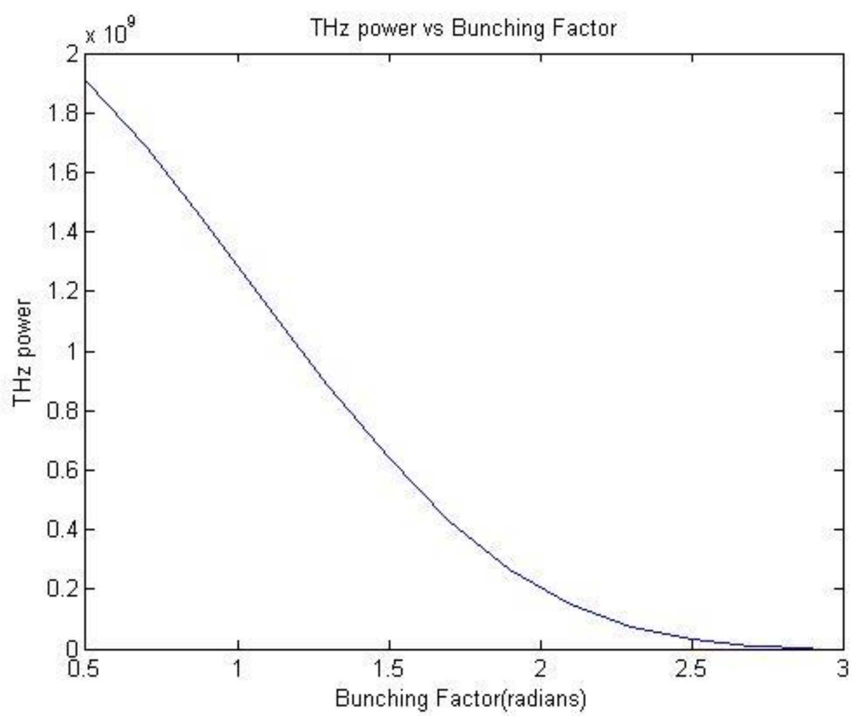

Figure 4.6THz power variation with bunching factor(radians)

The variation of THz power with bunching factor is shown in figure 4.6 which shows the characteristic of sinc function. It will attain maximum value at origin and decreases as bunching factor increases.

The mathematical expression of THz power and amplitude are derived. In the expression of THz power and amplitude various parameter are varying. The varying parameter are plasma density, frequency, wiggler length. The variation of plasma density with power and amplitude are linear and variation with frequency is like a Bessel function in which power and amplitude attain a maximum value at particular frequency and then decreases. Thz power variation with bunching factor is a sinc function in which power is maximum at the origin.

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