

A Major Project Report on

**“BUCKLING ANALYSIS OF THIN PLATES”**

Submitted in Partial Fulfillment for the Award of the Degree of

**MASTER OF TECHNOLOGY**

**IN**

**STRUCTURAL ENGINEERING**

By

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**CERTIFICATE**

This is to certify that **Mr. Vaibhav Shrivastava**, a student of final semester M.Tech (Structural Engineering), Department of Civil and Environmental Engineering, during the session 2010-2012 has successfully completed the project work on "*Buckling analysis of thin plates*" under my guidance and supervision and has submitted a satisfactory report in partial fulfillment for the award of the degree of Master of Technology.

The assistance and help received during the course of investigation have been fully acknowledged. He is a good student and we wish him good luck in future.

Dr. Sarat Kumar Panda

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# Declaration

I Certify that

- a. The work contained in this thesis is original and has been done by me under the guidance of my supervisor.
- b. The work has not been submitted to any other Institute for any degree or diploma.
- c. I have followed the guidelines provided by the University in preparing the thesis.
- d. I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.
- e. Whenever I have used materials (data, theoretical analysis, figures, and text) from other sources, I have given due credit to them by citing them in the text of the thesis and giving their details in the references.

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## *List of symbols*

The various notations and symbols used in the text or in the figures have been enlisted below for ease of reference. Symbols not contained in the list have been explained in the sections when they appear first.

$a, b$  dimension of the plate

$k$  buckling coefficient

$E$  Longitudinal Young modulus

$G$  Shear modulus

$h$  Thickness of the isotropic plates

$m, n$  Positive odd integers

$N_x$  and  $N_y$  Applied forces along  $x$  and  $y$  axes

$N_{xy}$  Buckling load

$u, v$  In-plane displacements

$w$  Out-of-plane displacement

$x, y, z$  Cartesian coordinates

## ABSTRACT

In the this project, buckling loads of rectangular plates having different sets of boundary conditions and subjected to uniform and linearly varying inplane loading are analyzed. Using the stress distribution as per the variation of inplane edge loading, the plate buckling equations are derived. Adopting Galerkin's approximation, the governing partial differential equations are converted into a set of homogeneous linear algebraic equations. The critical buckling load is obtained from the solution of the associated linear eigenvalue problem. The present buckling loads are compared with the published results wherever available.

The buckling loads obtained from the present method for plate with various boundary conditions and subjected to linearly varying inplane loading are found. Buckling mode shapes of plate for simply supported conditions with non-uniform inplane loadings are also presented. Due to the nature of the imposed constraint on the plate's lateral deflection,  $w$ , solving for the buckling load required the solution of a nonlinear partial differential equation in  $w$ . While the plates were modeled along the lines of classical plate theory, the nonlinearity arose from the fact that the plates were attached to nonlinear elastic foundations exhibiting a deformation sign dependent force-displacement relationship. This feature was introduced to model the unilateral constraint. The influence of different boundary conditions and transverse load distributions was investigated. For each case, the weak form of the governing differential equation was solved via the Galerkin's method.

# Table of Contents

<b>Title</b> .....	<b>i</b>
<b>Certificate</b> .....	<b>ii</b>
<b>Declaration</b> .....	<b>iii</b>
<b>Acknowledgements</b> .....	<b>iv</b>
<b>List of symbols</b> .....	<b>v</b>
<b>Abstract</b> .....	<b>vi</b>
<b>Table of Contents</b> .....	<b>vii</b>
<b>1. Introduction</b> .....	<b>1</b>
<b>2. Literature Review</b> .....	<b>3</b>
<b>3. Mathematical Formulation</b> .....	<b>5</b>
3.1 The Governing equation .....	5
3.2 The Von Karman Theory Of Plates .....	6
3.3 Boundary Conditions .....	16
3.3.1 Simply Supported along two opposite Edges at $x=0$ and $x=a$ .....	17
3.3.2 Clamped support along two opposite Edges at $x=0$ and $x=a$ .....	17
3.3.3 Clamped support along the Edge at $x=0$ and simply supported at $x=a$ .....	17
3.4 Galerkin’s Method .....	18
3.5 Galerkin’s method used in present problem .....	19
<b>4. Result and Discussion</b> .....	<b>21</b>
4.1 Buckling of simply supported rectangular plate with different type of inplane loading ....	22
4.1.1 Buckling Modes of the Simply Supported Rectangular Plates Uniformly Compressed in One Direction .....	26
4.2 Buckling of rectangular plate with different boundary condition applied with uniformly compression .....	29

4.2.1 CSCS Rectangular Plates.....	29
4.2.2 SCSC Rectangular Plates.....	30
4.2.3 SCSS Rectangular Plates .....	31
4.2.4 SSCC Rectangular Plates.....	32
4.2.5 SSCS Rectangular Plates .....	33
4.2.6 CCCC Rectangular Plates.....	34
4.2.7 CCCS Rectangular Plates .....	35
4.2.8 CCSC Rectangular Plates .....	36
<b>5. Conclusion .....</b>	<b>37</b>
<b>6. References.....</b>	<b>38</b>