DETERMINATION OF K-T PARAMETERS USING BUTTERWORTH FILTER

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For The Award of The Degree of

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Submitted By

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CERTIFICATE

This is to declare that A Thesis on **"Determination of K-T Parameters Using Butterworth Filter"** is a bonafide record of work done by **"Rajesh Kumar"** for partial fulfillment of requirement of award of degree in **Master of Technology (Structural Engineering)** at Delhi Technological University (Formerly known as Delhi College of Engineering).

This project has been carried out under the supervision of **Sri G. P. Awadhiya**, Associate Professor, Delhi Technological University.

I, Rajesh Kumar have not submitted the matter embodied in this thesis to any other Institute or University for the award of any other Degree or Diploma.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge

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Rajesh Kumar
Roll No: 08/STR/2K10

List of Symbols

The various notations and symbols used in the text or in the figures have been enlisted below for ease of reference. Symbols not contained in the list have been explained in the sections when they appear first.

G_0 = intensity of the ideal white noise excitation at bedrock-overburden interface
ξ_g = K-T damping coefiicient
ω_g = K-T frequency
ω = circular frequency
Δt = time step
α = normalized coefficient of transfer function in cascade realization
$H(z)$ = Transfer function in z-plane
σ_0 = RMS acceleration
Ƶ = z-Transform
Ω_c = frequency parameter in s plane
f_0 = filter cutoff frequency
ω_0 = circular filter cutoff frequency
$a(t)$ = acceleration varying with time (periodic)
$G(\omega)$ = Power spectral density function
λ_0 = first spectral moment
λ_1 = second spectral moment
λ_2 = third spectral moment
δ = shape factor
R= epicentral distance
ω_c = central frequency
N = number of samples
a_1, a_2, b_0, b_1, b_2 = coefficients of lowpass and highpass filter
M_L = magnitude of earthquake
k = rock site
s = soil site
a_{max} = maximum acceleration
E = East component
N = North component

Abstract

The spectral content and duration of some major Indian earthquake time-histories or strong-motion accelerograms have been studied with an aim of quantifying the uncertainty of ground motion. Ground motions are characterized by Kanai-Tajimi parameters based on spectral density function which has been computed using Butterworth filter. Parameters are estimated for each record based on the method of spectral moments. The statistics and dependencies of the parameters are evaluated, and in particular, correlations between the Kanai-Tajimi parameters, maximum ground acceleration, epicentral distance, and magnitude of earthquake are investigated. The estimated parameters by using correlation can be used for purposes during seismic consideration of design.

Keywords: Ground motion, Butterworth Filter, Epicentral distance, accelerograms, K-T seismic parameters etc.

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CHAPTER 1. INTRODUCTION

Earthquakes are a source of critical loading condition for structures located in the seismically active regions of the earth. A significant feature of the earthquake loading is a large number of uncertainties associated with the earthquake phenomenon. To establish the seismic loading condition for a structure, it is necessary to anticipate the number, size (intensity, duration) and location of future earthquake in the region surrounding the site during the service life of structure. Further, this assessment should be coupled with the prediction of structural response, and damage due to random vibration induced by ground motion of a given intensity. Both these steps involve uncertainty at several stages. The problem is compounded by the fact that potentially damage strong motion earthquakes occur after long intervals of time and the available data for such events is often statically insufficient.

1.1 Characteristics of Strong Ground Motion

Characteristics of strong ground motion are influenced by characteristics of the source mechanisms, travel-path geology and local site conditions.

1) Factors influencing characteristics of ground motion.

A. Characteristics of Source Mechanism :

Most earthquakes of engineering significance are of tectonic origin and are caused by slip along geological faults. While specific source mechanisms leading to a slip vary in different regions of the earth, and are not fully understood, four basic types of faulting can be identified with strong –motion earthquakes (Housner, 1977):

i. Low-angle, compressive under-thrust faulting caused by compressive forces generated due to the movement of the sea-floor crustal plate against continental plate;
ii. Com1pressive over-thrust faulting due to shear failure on an inclined fault with the upper portion of the rock moving upward under the action of compressive forces;
iii. Extensional faulting on inclined faults due to extensional strains in the earth's crust causing rocks overlaying the fault to move downwards;
iv. Strike-slip faulting which consists of a relative horizontal displacement of two sides of a fault across an essentially vertical plane.

In most earthquakes the actual slip mechanism is a combination of two or more, of the above types of faulting. Often slip occurs on an irregular surface and on more than one fault. The characteristics of the ground motion during an earthquake in the vicinity of the causative fault (near-field) are strongly dependent on the type of faulting and the time-history motion of the fault displacement.

B. Travel-Path Geology

As we move away from the fault (far-field), the nature of ground motion is primarily determined by travel-path geology. The basic characteristics of the seismic waves depend primarily on: the stress drop during the slip; total fault displacement; size of slipped area; roughness of the slipping process; fault shape; and the proximity of the slipped area to the ground surface. As the waves radiate from the fault, they undergo geometric spreading and attenuation due to loss of energy in the rocks. Since the interior of the earth consists of heterogeneous formations, the waves undergo multiple reflections refractions, dispersion and attenuation as they travel. The dissipation of seismic energy occurs due to material damping in soil. The seismic waves arriving at a site on the surface of the earth are a result of complex superposition giving rise to irregular motion, which may be modeled as a random vector varying randomly in space and time.

C. Local Site Condition or Effects

The nature of ground motion at a point on the earth surface is also influenced by the local site conditions, that is, soil properties and topography.

Local site condition or geology has been classified in three categories

I. Basin/soil

It corresponds to the two or three dimensional extent of the sedimentary basin structures. Effect of this condition is as under

a. Impedance contrast: - Seismic waves travel faster in hard rocks than in softer rocks and sediments. As the seismic waves pass from hard medium to soft medium, their speed decrease, so they must get bigger in amplitude to carry the same amount of energy. Thus, shaking becomes stronger at sites with softer soils\ layers.
b. Resonance: - tremendous increase in ground motion amplification occurs when there is resonance of signal frequency with the fundamental frequency or higher harmonics of soil layer.
c. Damping in soil: - Absorption of energy occurs and a part of energy in the seismic wave is lost instead of being transferred through the medium.
d. Basin edge:- Basin-edge induces strong surface waves near the edge
e. Trapping of waves: - Trapping of waves happen in soft sediments and due to that duration of motion is increased.

II. Topography

In this context ridge, valley and slope-variation can be discussed. Buildings located at hill tops suffer much more damage than those located at the base after destructive earthquakes in hilly areas. Some of the findings after studies are listed below:

- Ground motion amplification increases with ridge-slope.
- Maximum amplification occurs at the crest of the triangular wedge type topography relative to the base for wavelength comparable to width of the base
- De-amplification occurs in valley relative to the top of valley.
- Ridge amplification increases with elevation.

III. Lateral discontinuity effects

Due to lateral discontinuity significant increase of damage in the narrow zone happens. An amplitude amplification and local surface wave generation occurs in the softer medium in areas where a softer material lies besides a more rigid one.

An earthquake causes both translation and rotation at a point on the surface of the earth. For most problems the rotational component can be disregarded, and ground motion treated as a random vector with three orthogonal translational components-two horizontal and one vertical. Each component can be expressed either by an acceleration, velocity or displacement function of time. Although the three forms contain equivalent information and can be derived from each other by earthquake ground motion as acceleration, and derive velocity and displacement through integration, if required. Based on its characteristics, a ground acceleration time-history due to earthquakes may be classified into four broad groups (Newmark and Rosenblueth)

(i) Time-history containing essentially a single shock. Such motions occur at short distance on firm ground during moderate to shallow focus earthquakes. The records exhibit a strong unidirectional character and represent predominantly short period oscillatory motion;
(ii) Time-history containing moderately long duration and extremely irregular motion. Such motions occur on firm ground at moderate distances from the focus of moderate to large earthquakes. They contain a wide range of frequencies (0.1-3 Hz), and are generally of comparable severity in the three directions;

(iii) Time-history of long duration containing a dominant frequency of vibration. Such motions result from the filtering of the second type of ground motion through layers of soft soil and from successive wave reflections in the mantle; and

(iv) Motions consisting of large-scale, permanent deformation of ground, such as, slides or soil liquefaction.

The actual ground motion during an earthquake may contain the characteristics of two, or more, of the type of motions described above. The first type of motions can be treated as deterministic vibration problem due to its simplicity and the fourth type of motion are not amenable to formal analytical treatment while second and third type of earthquake ground motion can be modeled as random processes and, therefore, cause random vibration of ground-based systems.

The dynamic behavior of structures during an earthquake is determined primarily by the amplitude, frequency and duration of ground motion. The amplitude of strong-motion earthquake acceleration records generally exhibit:

(i) a rapid building up at the beginning of the motion;

(ii) a nearly constant value during the strong-motion shaking; and

(iii) an exponentially decaying tail.

The frequency characteristics are reflected by the Fourier amplitude spectra; power spectral density (PSD); response spectra; or response envelope spectra. The second type of earthquakes possesses broad-band characteristics and their acceleration time-history can be adequately modeled as a uniformly modulated stationary random process. The high frequency components of ground motion attenuate with distance faster than the low frequency components, and this strongly influences the spectral characteristics of ground motion as a function of distance. The important parameters defining the gross characteristics of earthquake motion at a site are: the peak values of ground acceleration (A_g), velocity (V_g), displacement (D_g); the RMS value of ground acceleration (σ_0); the response spectra (spectral acceleration (SA), spectral velocity (SV), spectral displacement (SD)); the spectrum intensity (SI); a site intensity such as NMI (I); and the duration (T).

1.2 Scope

This thesis deals with characterization of strong ground motion in terms of Kanai-Tajimi parameter. Parameters that will be finding out with relations in this thesis will be used during design of different structures with utmost safety.

By using correlations parameters are determined i.e. peak ground acceleration (PGA), natural frequency as a central frequency, Kanai-Tajimi parameters i.e. material properties (damping coefficient and natural frequency) etc.

1.3 Aim

This thesis is aimed to find out different seismic parameters by use of past earthquake data and correlate with different parameters such as epicentral distance and Kanai-Tajimi damping coefficient, epicentral distance and Kanai-Tajimi frequency, peak acceleration and Kanai-Tajimi damping coefficient etc.

In this thesis strong ground motion recorded data is using from that recorded data PGA is found out. For determining K-T parameters PSDF is plotted by use of Butterworth filter then correlation between different parameters are established to get some parameters of unknown site from study of previous earthquake record.

Kanai-Tajimi Parameters G_0, ξ_g, and ω_g

Individual power spectral density functions may have highly irregular shapes. Averaging a number of normalized power spectral density functions $G^n(\omega)$ for similar strong ground motions reveals a smooth characteristic shape. Kanai (1957) and Tajimi (1960) proposed a model for power spectral density based on a limited number of strong motion records.

$$G(\omega) = G_0 \frac{1 + \left[2\xi_g \left({}^{\omega}/_{\omega_g}\right)\right]^2}{\left[1 - \left({}^{\omega}/_{\omega_g}\right)^2\right]^2 + \left[2\xi_g \left({}^{\omega}/_{\omega_g}\right)\right]^2}$$

This thesis is of my interest because earthquake is uncertain. Some places where earthquake is frequent and some places where earthquake haven't occur from so many years but there are chances to occur so by statistically from previous data some parameters can be correlated with some parameter which is normally available and by finding out seismic parameters the seismic safety aspect up to a certain extent can be achieved by taking this seismic parameter in design and construction of all types of structures.

CHAPTER 2. LITERATURE REVIEW

In this research effort I have tried to plot the power spectral density function versus frequency relation using Digital Signal Filter techniques by processing the earthquake data. Kanai-Tajimi did this work using conventional mathematical approach. The K-T parameters evaluated from PSDF vs. Frequency curve using Vanmarcke and Lai (1980) and Shih-Sheng Paul Lai, (1982) Methodology. Brief description of research work which I have referred to produce my thesis work is as given below.

2.1 Erik H. Vanmarcke (1970) [Ref. 2]

In this paper many important statistical properties of stationary random motions depend on the spectral density function only through the spectral parameter ω_c and δ which depends on first few spectral moments has been shown. This has been used by Shih-Sheng Paul Lai, (1982).

2.2 Erick H. Vanmarcke and S. P. Lai (1980) [Ref. 3]

A simple procedure is proposed to estimate the strong-motion duration of earthquakes. The suggested strong-motion duration is approximately proportional to the quantity I_0/a_{max}^2, where a_{max} is the maximum acceleration and I_0 is the Arias intensity of the integral over time of the squared accelerations. Further after study of 140 strong ground motion duration of strong ground motion has been correlated and a relation has been suggested between strong-motion duration and peak ground acceleration

$$S_0 = 30\exp(-3.254a_{max}^{0.35})$$

Where S_0 is strong motion duration and a_{max} is peak ground acceleration in this thesis strong motion duration has not been studied but this parameter is important and can be evaluated negatively by the above relation.

2.3 Kanai, K. (1957) [Ref. 4]

In this paper Semi-empirical formula for the seismic characteristics of the ground has been given. The following formula for the seismic characteristic of the ground

$$U_s = \frac{c_1U_0}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\tau\frac{T}{T_0}\right\}^2}}$$

Where U_s = absolute amplitude of earthquake motion at free surface,

U_0 = absolute amplitude of seismic waves reaching the bottom boundary of the surface layer

T_0 = predominant period of the ground,

T = period of seismic waves

τ = apparent damping coefficient of the surface vibration

c_1 = a coefficient which depends upon the impedance ratio of the two media and is independent of T_0

2.4 Shih-Sheng Paul Lai, (1982) [Ref. 5]

The spectral content and duration of strong-motion accelerograms have been studied with an aim of quantifying the uncertainty of ground motion representation. Ground motions are characterized by the parameters of Kanai-Tajimispectral density function and by strong motion duration. Parameters are estimated for each record based on the method of spectral moments. The statistics and dependencies of the parameters are evaluated, and in particular, correlations between the Kanai-Tajimi parameters, maximum ground acceleration, epicentral distance, and magnitude of earthquake are investigated. The estimated parameters by using correlation can be used for purposes during seismic consideration of design.

2.5 Tajimi, H. (1960) [Ref. 6]

In this paper Tajimi has explained about a statistical method of determining the maximum response of the building structure during the earthquakeand expression of Kanai (1957) of semi empirical formula has been extended to get power spectral density function which has been used in this thesis to determine K-T parameters.

2.6 Tom Irvine (2000) An introduction to the filtering of digital [Ref. 7]

Paper published dated on 31 March, 2000; In this paper filtering of digital signals has been explained. Sixth order Butterworth filter design is also explained which has been used in this thesis for filtering of earthquakes. Also for phase correction refiltering has been done which has been suggested in this paper.

2.7 Tom Irvine (2000) Power spectral density units [Ref. 8]

Paper published dated 28 July, 2000; In this paper method of calculation of power spectral density function has been given. As per this paper filter the signals in a particular band width and then power in that bandwidth corresponding to band center frequency will be plotted to get Power spectral density function.Method of calculation of power spectral density has been used in this thesis and also included in this thesis.

CHAPTER 3. GROUND MOTION AND K-T PARAMETERS

3.1 Study of strong ground motion

The earth is far from quiet. It vibrates almost continuously at periods ranging from milliseconds to days and amplitudes ranging from nanometers to meters. The great majority of these vibrations are so weak that they cannot be felt or even detected without specialized measurement equipment. Such micro seismic activity is of greater importance to seismologists than engineers.

Earthquake engineers are interested primarily in strong ground motion (i.e., motion of sufficient strength to affect people and their environment). Evaluation of the effects of earthquakes at a particular site requires objective, quantitative ways of describing strong ground motion.

It is not necessary to reproduce each time history exactly to describe the ground motion adequately for engineering purposes. It is necessary, however, to be able to describe the characteristics of the ground motion that are of engineering significance and to identify a number of ground motion parameters that reflect those characteristics.

For engineering purposes, **three strong ground motion parameters of earthquake** are of primary significance:

A. Amplitude parameters

The most common way of describing a ground motion is through the time history i.e. Acceleration time history, Velocity time history and Displacement time history.

Typically, only one of these is recorded directly with the others computed form it by integration/differentiation. Note that integration produces a smoothing or filtering effect. The acceleration time history displays more high frequency content (relatively), the velocity time -history displays more intermediate frequency content (relatively), and the displacement displays more low frequency content (relatively).

A.1. Peak acceleration

Peak Horizontal Acceleration (PHA): The largest (absolute) value of the horizontal acceleration. Because of its relationship to inertial force, intensity-acceleration relationships can be used to estimate PHA when other information is not available.

Peak Vertical Acceleration (PVA): The largest (absolute) value of the vertical acceleration.

Ground motions with high peak accelerations are usually, but not always, more destructive than motions with lower peak accelerations. Damage is also related to other characteristics (e.g., frequency content and duration).

B. Frequency content parameters

B.1. Ground motion spectra

Power spectral density function: Definition

Power spectral density function (PSDF) shows the strength of the energy or power variations as a function of frequency. In other words, PSD tells us at which frequency range energy or power variations are strong and that might be quite useful for further analysis. The unit of PSD is power per frequency (bandwidth) i.e. G^2Hz^{-1} and one can obtain power within a specific frequency range by integrating PSD within that frequency range

Let a(t) is a periodic records of discrete function

The exponential form of Fourier series is

$$a(t) = \sum_{-\infty}^{\infty} c_n e^{in\omega_0 t} = c_0 + \sum_{n=1}^{\infty} \left(c_n e^{in\omega_0 t} + c_n^* e^{-in\omega_0 t} \right)$$

Because of periodic it can be represented as a real part of Fourier series and the constant term c_0 can be dealt separately

$$a(t) = Re\sum_{n=1}^{\infty} C_n e^{in\omega_0 t}$$

$$= \frac{1}{2}\sum_{n=1}^{\infty}\left(C_n e^{in\omega_0 t} + C_n^* e^{-in\omega_0 t}\right)$$

Where C_n is a complex number, and C^*_n is its complex conjugate

Its mean square value is

$$\overline{a^2} = \lim_{T\to\infty}\frac{1}{T}\int_0^T \frac{1}{4}\sum_{n=1}^{\infty}\left(C_n e^{in\omega_0 t} + C_n^* e^{-in\omega_0 t}\right)^2 dt = \sum_{n=1}^{\infty}\frac{1}{2}C_n C_n^*$$

Thus $\overline{a^2}$ is made up of discrete contributions in each frequency interval $\Delta\omega$

As the contribution to the mean square value in the frequency interval $\Delta\omega$ the power spectrum $P(\omega_n)$

$$P(\omega_n) = \frac{1}{2}C_n C_n^*$$

The mean square value then

$$\overline{a^2} = \sum_{n=1}^{\infty} P(\omega_n)$$

Now the discrete power spectral density $G(\omega_n)$ defined as the power spectrum divided by the frequency interval $\Delta\omega$

$$G(\omega_n) = \frac{P(\omega_n)}{\Delta\omega} = \frac{C_n C_n^*}{\Delta\omega}$$

The mean square value then can be written as

$$\overline{a^2} = \sum_{n=1}^{\infty} G(\omega_n) d\omega$$

For a continuous spectrum as the limiting case of $P(\omega_n)$ as $\Delta\omega \to 0$

$$\lim_{\Delta\omega\to 0} G(\omega_n) = G(\omega)$$

The mean square value is then

$$\overline{a^2} = \int_0^{\infty} G(\omega) d\omega$$

Ground Motion Spectral Density Function

The frequency content of a ground motion can also be described by a power spectra or Power Spectral Density Function. The Power Spectral Density Function can also be used to estimate the statistical properties of a ground motion and to compute stochastic response using random vibration techniques (Clough & Penzien, 1975; Vanmarcke, 1976; Yang, 1980).

A simple way to represent an earthquake ground motion is as a segment of limited duration S_0 of a stationary stochastic process the ground acceleration a(t) can be expressed as the sum of series of sinusoidal waves with frequency ω_i, random amplitude $A(\omega_i)$ and random phase angle (ϕ_i).

$$a(t) = \sum_{i=1}^{n} A(\omega_i)\sin\left(\omega_i t + \phi_i\right) \qquad 0 \le t \le S_0$$

Where S_0=duration of a stationary stochastic process

As

$$a(t) = \sum_{i=1}^{n} A(\omega_i)\sin(\omega_i t + \phi_i) \qquad 0 \le t \le S_0$$

By use of DFT $A(\omega_i)$ can be expressed as

$$A(\omega_i) = \frac{1}{S_0}\sum_{j=0}^{n-1} a(t_j)\left(e^{-in\omega t_j}\right)\Delta t$$

$t_j = j\Delta t$, $T = N\Delta t$

Where T = Total duration

and Δt = Time segment

Now,

$$[a(t)]^2 = \left[\sum_{i=1}^{n} A(\omega_i)\sin(\omega_i t + \phi_i)\right]^2$$

Using Parseval's theorem

$$\sum_{m=0}^{\infty} {a_m}^2 = |f|^2 = \int_a^b p(x) f(x)^2 dx \quad or \int_{-\infty}^{\infty} f(t)^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega$$

Hence

$$\int_0^T [a(t)]^2 \, dt = \frac{1}{\pi}\int_0^{\omega_{N/2}} [A(\omega_i)]^2 d\omega$$

Where $\omega_{N/2} = \frac{\pi}{\Delta t}$ is the Nyquist Frequency

Now for the stationary stochastic duration S_0 average intensity λ_0 can be obtained.

$$\lambda_0 = \frac{1}{S_0}\int_0^{S_0} [a(t)]^2 \, dt$$

$$= \frac{1}{\pi S_0}\int_0^{\omega_N} \left[A(\omega_j)\right]^2 d\omega$$

As average intensity is equal to mean squared acceleration

Power spectral density (PSD) is a positive real function of a frequency variable associated with a stationary stochastic process, or a deterministic function of time. Power spectral density (PSD) describes how the power of a signal or time series is distributed with frequency hence

$$\lambda_0 = \int_0^{\omega_N} G(\omega) d\omega$$

Therefore, PSD function of the ground excitation can be defined as

$$\hat{G}(\omega_i) = \frac{1}{\pi S_0} A(\omega_i)^2$$

By appropriate smoothing of $\hat{G}(\omega_i)$, One can estimate for the "true" spectral density function $G(\omega_i)$ of the underlying stationary stochastic process. Of course, the fact that the accelerogram is digitized and has finite duration imposes fundamental lower and upper limits on the range of frequency.

Determination of Statistical Parameters of PSD

For frequency domain analysis, the spectral moments of a PSD function are key statistical parameters. The j^{th} spectral moment λ_j is defined as (Vanmarcke, 1970)

$$\lambda_j = \int_0^{\infty} \omega^j G(\omega) d\omega$$

The variance of the excitation is the zero spectral moment λ_0

$$\lambda_0 = \int_0^{\infty} \omega^0 G(\omega) d\omega = \int_0^{\infty} G(\omega) d\omega = {\sigma_0}^2$$

Now, spectral moments of the Kanai-Tajimi PSD function can be expressed as using Kanai-Tajimi expression given by Pulgrano and Albowitz (1969)

$$\lambda_0 = \int_0^{\infty} \omega^0 G(\omega) d\omega$$

$$= \int_0^{\infty} \frac{\left[1 + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2\right] G_0}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right] + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2} d\omega$$

$$= G_0 \left[\int_0^{\infty} \frac{d\omega}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2} + 4\xi_g^2 \int_0^{\infty} \frac{\left(\frac{\omega}{\omega_g}\right)^2 d\omega}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\xi_g^2 \left(\frac{\omega}{\omega_g}\right)^2}\right]$$

Now, Introducing

$$J_i(\Omega^*) = \frac{4\xi_g}{\pi} \int_0^{\Omega^*} \frac{\Omega^i d\Omega}{\left(1 - \Omega^2\right)^2 + 4\xi_g^2 \Omega^2}$$

Where $\Omega^* = \frac{\omega^*}{\omega_g}$

ω^*= upper integration limit in the equation of spectral moment. Theoretical it is equal to infinity. So,

$$\lambda_0 = \frac{\pi G_0 \omega_g}{4\xi_g} J_0(\Omega^*) \left(1 + 4\xi_g^2\right)$$

Where,

$$J_0(\Omega^*) = \frac{4\xi_g}{\pi} \int_0^{\Omega^*} \frac{\Omega^0 d\Omega}{\left(1 - \Omega^2\right) + 4\xi_g^2 \Omega^2}$$

First spectral moment λ_1

$$\lambda_1 = \int_0^{\infty} \omega G(\omega) d\omega$$

$$\lambda_1 = \frac{\pi G_0 \omega_g^2}{4\xi_g} \left[J_1(\Omega^*) + 4\xi_g^2 J_3(\Omega^*) - \frac{1 + 4\xi_g^2 - 8\xi_g^4}{\pi\sqrt{1 - \xi_g^2}} \tan^{-1}\left(\frac{2\xi_g\sqrt{1 - \xi_g^2}}{1 - 2\xi_g^2}\right)\right]$$

$$J_1(\Omega^*) = \frac{4\xi_g}{\pi} \int_0^{\Omega^*} \frac{\Omega d\Omega}{\left(1 - \Omega^2\right) + 4\xi_g^2 \Omega^2}$$

$$J_3(\Omega^*) = \frac{4\xi_g}{\pi}\int_0^{\Omega^*}\frac{\Omega^3 d\Omega}{\left(1-\Omega^2\right)^2+4\xi_g^2\Omega^2}$$

Second spectral moment λ_2

$$\lambda_2 = \frac{\pi G_0\omega_g^2}{4\xi_g}\left[J_2(\Omega^*)+4\xi_g^2 J_4(\Omega^*)\right]$$

$$J_2(\Omega^*) = \frac{4\xi_g}{\pi}\int_0^{\Omega^*}\frac{\Omega^2 d\Omega}{\left(1-\Omega^2\right)^2+4\xi_g^2\Omega^2}$$

$$J_4(\Omega^*) = \frac{4\xi_g}{\pi}\int_0^{\Omega^*}\frac{\Omega^4 d\Omega}{\left(1-\Omega^2\right)^2+4\xi_g^2\Omega^2}$$

Based on Kanai's study (1957) of the frequency content for several real strong ground motions, Tajimi (1960) proposed the following functional form for the spectral density function of earthquake motion on firm ground

$$G(\omega_j) = G_0\frac{1+\left[2\xi_g\left({}^{\omega_j}/_{\omega_g}\right)\right]^2}{\left[1-\left({}^{\omega_j}/_{\omega_g}\right)^2\right]^2+\left[2\xi_g\left({}^{\omega_j}/_{\omega_g}\right)\right]^2}$$

Whereξ_g, ω_g, and G_0 are parameters to be determined from earthquake records.

Physically, the Kanai-Tajimi (K-T) PSD function may be interpreted as corresponding to an "ideal white noise" excitation at bedrock level filtered through the overlaying soil deposits. Within this context, the K-T parameters are interpreted as the soil overburden effective damping coefficient$\xi_g$and natural frequency $\omega_g. G_0$ is the intensity of the ideal white noise excitation at bedrock-overburden interface.

Due to the fact that most estimated spectra $G(\omega_j)$ are quite erratic, it is difficult to determine the parameters of the corresponding smooth K-T PSD function. The "spectral moments" method was applied to the estimation of K-T PSD parameters by several investigators, e.g., Vanmarcke (1970), Binder (1978) and Lai (1979). A brief overview of the method is presented above.

Frequency parameters Central frequency ω_c and shape factor (or bandwidth measure), δ of the random process can be directly evaluated from first few spectral moments.

$$\omega_c = \left[\frac{\lambda_2}{\lambda_0}\right]^{1/2}$$

$$\delta = \left[1-\left(\frac{{\lambda_1}^2}{\lambda_0\lambda_2}\right)\right]^{1/2}$$

The method of spectral moments computes the K-T parameters in such a way that the spectral moments, i.e., λ_0, $\lambda_1$and λ_2 of the actual ergodic power spectrum and those of the fitted K-T PSD function are the same. Since ${\sigma_0}^2$, ω_c and δ are function of the spectral moments, this suggest that the moments method will lead to a K-T PSD function with the same variance, central frequency, and shape factor as the actual power spectrum. This consistency in the pertinent statistical parameters is clearly desirable.

Because the variance of excitation, central frequency, and shape factor are functions of the spectral moments (λ_0, $\lambda_1$andλ_2), they can be expressed in terms of the K-T parameters, i.e., ξ_g, ω_g, and G_0. Hence the K-T parameters can be computed by matching the variance of excitation, the central frequency and the shape factor.

C. Duration

Degradation of stiffness and strength of certain types of structures and the buildup of pore water pressures in loose, saturated sand, are sensitive to the number of cycles of a ground motion. The duration of strong ground motion is related to the time required to release the accumulated strain energy by rupture along the fault. The strong motion duration increases with earthquake magnitude.

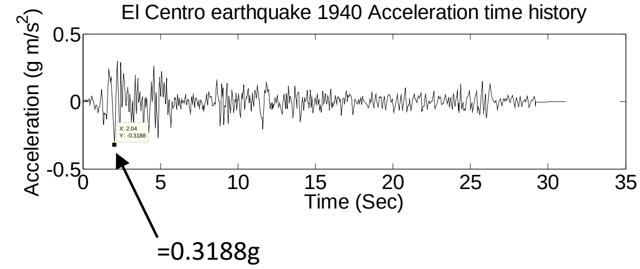
The most commonly used definition is the bracketed duration. It is defined as the time between the first and last exceedances of threshold acceleration.

3.2 Estimation of Ground Motion Parameters

A. Amplitude parameters

Peak ground acceleration

Peak ground acceleration is the absolute maximum value of the acceleration determined from the acceleration time history plot.



B. Frequency parameters

B1. Power spectral density function

Random vibration can be represented in the frequency domain by a power spectral density function. The typical units are acceleration [G^2/Hz] versus frequency [Hz]. The acceleration can also be represented by metric units, such as [$(m/sec^2)^2$ / Hz].

Note that the amplitude is actually [GRMS^2 / Hz], where RMS is root-mean-square. The RMS notation is typically omitted for brevity. The Hz value in [G^2/Hz] refers to a bandwidth rather than to the frequency in Hz along the X-axis. The RMS value of a signal is equal to the standard deviation, assuming a zero mean. The standard deviation is usually represented by sigma σ.

A pure sinusoidal function has the following relationship:

$$\text{Peak} = \sqrt{2}\ \text{RMS}$$

Random vibration, however, is very complicated. Random vibration has no simple relationship between its peak and RMS values. The peak value of a stationary random time history is typically 3 or 4 times the RMS value. A power spectral density can be calculated for any type of vibration signal, but it is particularly appropriate for random vibration.

$$PSDF = \frac{GRMS^2}{bandwidth} = G(\omega)$$

GRMS is calculated using Butterworth filter for a particular bandwidth. Details of Butterworth Filter is given in next chapter i.e. Chapter 5

B2. PSDF Parameter

Central Frequency ω_c

Power spectral density G(ω) can be used to estimate statistical properties of a ground motion, such as Central Frequency ω_c. The central Frequency is a measure of the frequency where the power spectral density is concentrated.

The n^{th} momentum of G(ω) is defined by:

$$\lambda_n = \int_0^{\omega_{N/2}} G(\omega)\omega^n d\omega$$

Central Frequency ω_c is given by (Vanmarcke 1976)

$$\omega_c = \sqrt{\frac{\lambda_2}{\lambda_0}}$$

Shape Factor δ

The shape factor δ is a measure of the dispersion of the power spectral density about central frequency and is given by (Vanmarcke 1976)

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}}$$

Shape factor is also related to damping given by Erik H. Vanmarcke (1970) as

$$\delta = \left(1 - \frac{\left[1 - \frac{1}{\pi}\tan^{-1}\left(\frac{2\xi\sqrt{1-\xi^2}}{1-\xi^2}\right)\right]}{1-\xi^2} \right)^{\frac{1}{2}}$$

The shape factor always lies between 0 and 1, with higher values corresponding to larger bandwidth.

B3. Kanai-Tajimi Parameters G_0, ξ_g, and ω_g

Individual power spectral density functions may have highly irregular shapes. Averaging a number of normalized power spectral density functions $G^n(\omega)$ for similar strong ground motions reveals a smooth characteristic shape. Kanai (1957) and Tajimi (1960) proposed a model for power spectral density based on a limited number of strong motion records.

$$G(\omega) = G_0 \frac{1 + \left[2\xi_g \left({}^{\omega}/_{\omega_g}\right)\right]^2}{\left[1 - \left({}^{\omega}/_{\omega_g}\right)^2\right]^2 + \left[2\xi_g \left({}^{\omega}/_{\omega_g}\right)\right]^2}$$

It can be seen that components with frequencies greater than $\sqrt{2}\omega_g$ is attenuated and other components is amplified.

RMS Acceleration σ_0

$$\sigma_0 = \sqrt{\frac{1}{T}\int_0^T [a(t)^2]\, dt} = \sqrt{\lambda_0}$$

Note that the integral is over the duration of strong motion 0-T. T depends on the method used to define strong motion duration.

C. Duration

The duration of strong motion increases with earthquake magnitude. Durations based on absolute acceleration levels, such as the bracketed duration, decrease with distance. Durations based on relative acceleration levels (relative to the peak) increase with distance. For engineering purposes, the bracketed duration appears to provide the most reasonable indication of the influence of duration on the potential damage.

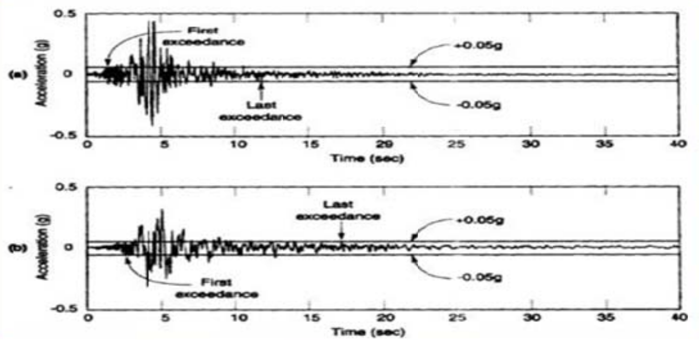


Fig. 3.1 Bracketed duration

Bracketed duration is the duration between first exceedance to last exceedance from 0.05g in acceleration time history.

CHAPTER 4. DISCRETE-TIME PROCESSING OF SIGNALS

4.1. Introduction

Discrete-Time processing is the processing of digitized discrete time sampled signals using filter.

A continuous-time signal can be represented by a function x(t) whose domain is a range of numbers (t_1, t_2), where $-\infty \leq t_1$ and $t_2 \leq \infty$. Similarly, a discrete-time signal can be represented by a function x(nT), where T is constant and n is an integer in the range (n_1,n_2) such that $-\infty \leq n_1$ and $n_2 \leq \infty$.

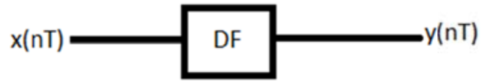
Fig. 4.1 (a) Continuous time signal (b) Discrete-Time signal

4.2 Filter

Filtering is a process by which the frequency spectrum of a signal can be modified, reshaped, or manipulated according to some desired specification. It may entail amplifying or attenuating a range of frequency components, rejecting or isolating one specific frequency component, etc.

Digital filter

The digital filter is a digital system that can be used to filter discrete-time signals. It can be represented by the block diagram. Input x(nT) and output y(nT) are related by some rule of correspondence known as transfer function.



We can indicate this fact

$Y(nT) = \mathcal{R}x(nT)$ Where $\mathcal{R}$ is an operator

Digital filters can be classified as time –invariant or time-dependent, casual or noncausal, and linear or nonlinear.

A. Time-invariance

A digital-filter is said to be time-invariant if its output to an arbitrary input does not depend on the time of application of input i.e. $\mathcal{R}x(nT-kT) = y(nT-kT)$ for all inputs x(nT) and all integers k.

A filter that does not satisfy the above criteria is said to be time dependent.

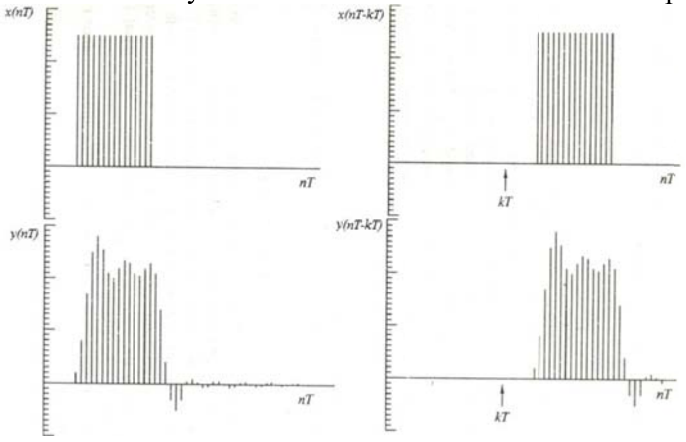


Fig. 4.2 Time Invariance: Response of x(nT) and delayed x(nT-kT)

B. Causality

A causal digital filter is one whose output value at a specific instant is independent of subsequent values of the input

i.e. if $\mathcal{R}x_1(nT) = \mathcal{R}x_2(nT)$ for $n \leq k$

for all possible distinct input $x_1(nT)$ and $x_2(nT)$ such that

$x_1(nT) = x_2(nT)$ for $n \leq k$

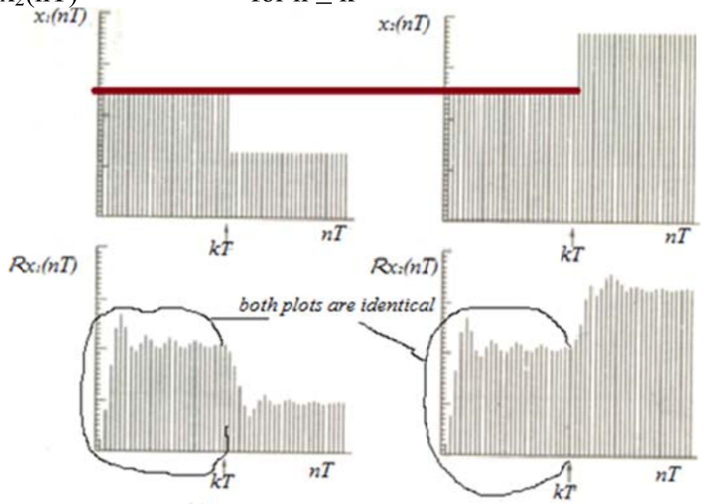


Fig. 4.3 Casuality: response of $x_1(nT)$ and $x_2(nT)$

conversely, if

$\mathcal{R}x_1(nT) \neq \mathcal{R}x_2(nT)$ for $n \leq k$

for at least one pair of distinct input $x_1(nT)$ and $x_2(nT)$ such that

$x_1(nT) = x_2(nT)$ for $n \leq k$

then the filter is noncausal.

C. Linearity

A digital filter is linear if and only if it satisfies the conditions

$\mathcal{R}\{ax(nT)\}=a\mathcal{R}x(nT)$ (Homogeneity property)

$\mathcal{R}[x_1(nT)+x_2(nT)] = \mathcal{R}x_1(nT)+ \mathcal{R}x_2(nT)$ (Additive property)

For all possible values of a (arbitrary constant) and all inputs $x_1(nT)$, $x_2(nT)$,……………

4.3 Characterization of digital filters

Digital filters are characterized in terms of difference equation. Two types of digital filters can be identified nonrecursive and recursive filters.

A. Nonrecursive filters

$y(nT) = f\{\ldots..,x(nT-T),x(nT),x(nT+T)\ldots\ldots\ldots)$

If we assume linearity and time invariance, y(nT) can be expressed as

$$y(nT) = \sum_{i=-\infty}^{\infty} a_i x(nT - iT)$$

Where a_i represents constants

Now on assuming that the filter is causal and noting that x(nT+T), x(nT+2T),………are subsequent values of the input with respect to instant nT, we must have $A_i=0$ for $i \leq -1$

And so

$$y(nT) = \sum_{i=0}^{\infty} a_i x(nT - iT)$$

If, in addition, for n<0 and $a_i=0$ for i>N

$$y(nT) = \sum_{i=0}^{n} a_i x(nT - iT) + \sum_{i=n+1}^{\infty} a_i x(nT - iT)$$

$$= \sum_{i=0}^{N} a_i x(nT - iT) + \sum_{i=N+1}^{\infty} a_i x(nT - iT) = \sum_{i=0}^{N} a_i x(nT - iT)$$

Therefore, a linear, time-invariant, causal, nonrecursive filter can be represented by an Nth-order linear difference equation. N is the order of filter.

B. Recursive filters

The output value of a recursive filter is a function of elements in the input as well as the output sequence. In the case of a linear, time-invariant, causal filter

$$y(nT) = \sum_{i=0}^{N} a_i x(nT - iT) - \sum_{i=1}^{N} b_i y(nT - iT)$$

4.4 Convolution summation

The output of a digital filter to an arbitrary input can be expressed in terms of the impulse respone of the filter.

An input x(nT) can be written as

$$x(nT) = \sum_{k=-\infty}^{\infty} x_k(nT)$$

Where $x_k(nT) = \begin{cases} x(kT) & for\ n = k \\ 0 & otherwise \end{cases}$

Alternatively

$$x_k(nT) = x(kT)\delta(nT - kT)$$

And hence

$$x(nT) = \sum_{k=-\infty}^{\infty} x(kT)\delta(nT - kT)$$

Now consider a linear time-invariant filter in which

$\mathcal{R}\delta$(nT) = h(nT) and y(nT)= $\mathcal{R}$ x(nT)

Then we have

$$y(nT) = \mathcal{R}\textstyle\sum_{k=-\infty}^{\infty} x(kT)\delta(nT - kT) = \sum_{k=-\infty}^{\infty} x(kT)\mathcal{R}\,\delta(nT - kT)$$

$$= \sum_{k=-\infty}^{\infty} x(kT)h(nT - kT) = \sum_{k=-\infty}^{\infty} h(kT)x(nT - kT)$$

Where the second from the right is deduced by a simple change of variable. This relation is of considerable importance in the characterization as well as analysis of digital filters and is known as convolution summation.

Two special cases of the convolution summation are

If the filter is causal, h(nT)=0 for n < 0, and so

$$y(nT) = \sum_{k=-\infty}^{\infty} x(kT)h(nT - kT) = \sum_{k=0}^{\infty} h(kT)x(nT - kT)$$

If, in addition, x(nT) = 0 for n<0, we have

$$y(nT) = \sum_{k=0}^{n} x(kT)h(nT - kT) = \sum_{k=0}^{n} h(kT)x(nT - kT)$$

4.5 Alternative classification of digital filters

If the impulse response of a digital filter is of finite duration such that h(nT) = 0 for n > N, then above equation gives

$$y(nT) = \sum_{k=-0}^{N} h(kT)x(nT - kT)$$

This equation is of the same as in nonrecursive section with h(0) = a_0, h(T) = a_1,…….,h(NT) = a_N. Hence if the filter is nonrecursive, the impulse response is of finite duration. If the impulse response is of infinite duration, then the filter is recursive.

Hence digital filters can be classified as **finite-duration impulse response (FIR)** and **infinite-duration impulse response (IIR)** filters.

4.6 Discrete-time transfer function (in z-plane)

The transfer function of a digital filter is defined as the ratio of the z-transform of the output to z-transform of the input.

Consider a linear, time-invariant digital filter, and let x(nT), y(nT), and h(nT) be the input, output and impulse response respectively. By using convolution summation we have

$$y(nT) = \sum_{k=-\infty}^{\infty} x(kT)h(nT-kT)$$

By using theorem of real convolution in z-Transform

$$Ƶ\sum_{k=-\infty}^{\infty} f(kT)g(nT-kT) = Ƶ\sum_{k=-\infty}^{\infty} f(nT-kT)g(kT) = F(z)G(z)$$

We get,

$$Ƶy(nT) = Ƶh(nT)\,Ƶx(nT)$$
$$Y(z) = H(z)X(z)$$
$$H(z) = \frac{Y(z)}{X(z)}$$

H(z) is the transfer function which is z transform of the impulse response of a digital filter.

<u>Derivation of H(z)</u>

The exact form of H(z) can be derived from the difference equation characterizing the filter

For causal recursive filter

$$y(nT) = \sum_{i=0}^{N} b_i x(nT-iT) - \sum_{i=1}^{N} a_i y(nT-iT)$$

And hence

$$Ƶy(nT) = \sum_{i=0}^{N} b_i z^{-i} Ƶx(nT) - \sum_{i=1}^{N} a_i z^{-i} Ƶy(nT)$$

Or

$$Y(z) = X(z)\sum_{i=0}^{N} b_i z^{-i} - Y(z)\sum_{i=1}^{N} a_i z^{-i}$$

$$Y(z)\left(1+\sum_{i=1}^{N} a_i z^{-i}\right) = X(z)\sum_{i=0}^{N} b_i z^{-i}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\sum_{i=0}^{N} b_i z^{-i}}{1+\sum_{i=1}^{N} a_i z^{-i}}$$

$$H(z) = \frac{b_0 + bz^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + \cdots \dots\dots\dots\dots\dots. + b_N z^{-N}}{1 + az^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + az^{-5} + \cdots \dots\dots\dots\dots\dots. + a_N z^{-N}}$$

4.7 Cascade Realization

When the transfer function coefficients are quantized, errors are introduced in the amplitude and phase response of the filter. This problem can overcome by realizing high-order filters as interconnections of first-and second order filter. an arbitrary transfer function can be realized by connecting a number of first and second order structure in cascade.

An arbitrary transfer function can be factored into a product of first and second order transfer function as

$$H(z) = \prod_{i=1}^{M} H_i(z)$$

Where

$$H_i(z) = \frac{b_{0i} + b_{1i}z^{-1} + b_{2i}z^{-2}}{1 + az^{-1} + a_{2i}z^{-2}}$$

With $b_{2i} = a_{2i} = 0$ for a first-order transfer function.

Hence
$$Y(z) = [H_1(z)X(z)]H_2(z) \ldots\ldots\ldots\ldots . H_M(z)$$
$$= [H_2(z)Y_1(z)]H_3(z) \ldots\ldots\ldots H_M(z)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$= H_M(z)Y_{M-1}(z)$$

Where $Y_1(z) = H_1(z)X(z)$

$Y_i(z) = H_i(z)Y_{i-1}(z)$ for i=2,3,…………M-1

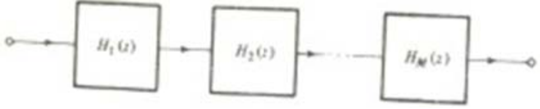
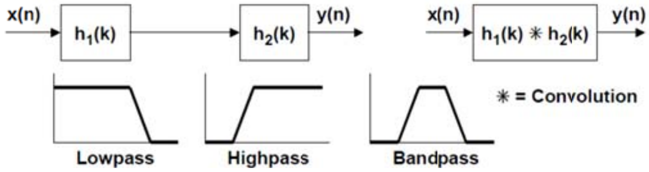


Fig. 4.4 Cascade Realization

4.8 Classification of filter on the basis of frequency selection

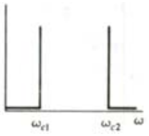
i. Bandpass filter

A Band Pass filter is a filter that passes frequencies in a desired range and attenuates frequencies below and above.



ii. Notch or Band-Reject

A filter with effectively the opposite function of the band pass is the **band-reject** or **notch** filter. Notch filters are used to remove an unwanted frequency from a signal, while affecting all other frequencies as little as possible.

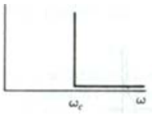


iii. Low-Pass

A third filter type is the low-pass. A low-pass filter passes low frequency signals, and rejects signals at frequencies above the filter's cutoff frequency. The cut off frequency is ω_c.

iv. High-Pass

The opposite of the low-pass is the **high-pass** filter, which rejects signals below its cutoff frequency. The cut off frequency is ω_c.



v. All-Pass or Phase-Shift

The fifth and final filter response type has no effect on the amplitude of the signal at different frequencies. Instead, its function is to change the phase of the signal without affecting its amplitude. This type of filter is called an **all-pass** or **phase-shift** filter. The effect of a shift in phase is illustrated in Figure 4.2.5.1. Two sinusoidal waveforms, one drawn in dashed lines, the other a solid line, are shown. The curves are identical except that the peaks and zero crossings of the dashed curve occur at later times than those of the solid curve. Thus, we can say that the dashed curve has undergone a **time delay** relative to the solid curve.

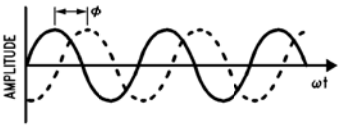


Fig: 4.5 Two sinusoidal waveforms with phase difference.

Since we are dealing here with periodic waveforms, time and phase can be interchanged—the time delay can also be interpreted as a **phase shift** of the dashed curve relative to the solid curve.

CHAPTER 5. BUTTERWORTH FILTER DESIGN

5.1 Introduction to Design of digital filter

It comprises four general steps

a) Approximation
b) Realization
c) Study of arithmetic errors
d) Implementation

(a) Approximation: The approximation step is the process of generating a transfer function that satisfies desired specification, which may concern the amplitude, phase, and possibly the time domain response of filter. The available methods for the solution of the approximation problem can be classified as direct or indirect. In direct methods, the problem is solved directly in the z-domain. In indirect methods, a continuous time transfer function is first obtained and the converted into a corresponding discrete-time transfer function.

The most frequently used approximations are

i. Butterworth
ii. Chebyshev
iii. Inverse-Chebyshev
iv. Elliptic
v. Bessel

In this thesis **Butterworth approximation** has been taken. In this approximation the **attenuation at cutoff frequencies is fixed at 3 dB** and **sixth order Butterworth filter** has been designed.

(b) Realization: The synthesis of a digital filter is the process of converting the transfer function or some other characterization of the filter into a network. This process is also referred to as realization step. The network obtained is said to be the realization of the transfer function.

The most frequently used realization methods are

i. Direct
ii. Direct canonic
iii. State-space
iv. Ladder
v. Lattice
vi. Parallel
vii. Cascade

In this thesis cascade realization has been used

(c) Arithmetic errors: During the approximation step the coefficients of transfer function are determined to a high degree of precision. When the transfer functions are quantized, errors are introduced in the amplitude and phase response of the filter. **The cascade realization is used to reduce the quantifying of errors.**

(d) Implementation: The implementation of a digital filter can assume two forms: software or hardware. In the first case, implementation involves the simulation of the filter network on a computer, workstation. In the second case, it involves the conversion of the filter network into a dedicated piece of hardware. **As in this thesis the software (mathematical calculation) has been used to filter the recorded strong motion data.**

5.2 Butterworth filter

5.2.1 Filter mathematics (in s-plane)

Butterworth filter is an analog filter. An analog filter transfer function H(s) can be represented by the equation

$$H(s) = \frac{Y(s)}{X(s)}$$

Where Y(s) and X(s) are the Laplace transform of output Y(nT) and input X(nT) in s plane where s=σ+jω

The numerator and denominator can always be written as polynomials in s. To be completely general, a transfer function can be written as given below,

$$H(s) = H_0 \frac{s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} \ldots. + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots. + a_1 s + a_0}$$

It means simply that a filter's transfer function can be mathematically described by a numerator divided by a denominator, with the numerator and denominator made up of a number of terms, each consisting of a constant multiplied by the variable "s" to some power. The a_i and b_i terms are the constants, and their subscripts correspond to the order of the "s" term each is associated with. Therefore, a_1 is multiplied by s, a_2 is multiplied by s^2, and so on. Any filter transfer function (including the 2nd-order band pass of the example) will have the general form of the above equation with the values of the coefficients a_i, and b_i depending on the particular filter.

Another way of writing a filter's transfer function is to factor the polynomials in the numerator and denominator so that they take the form:

$$H(s) = H_0 \frac{(s - z_0)(s - z_1)(s - z_2)(s - z_3) \ldots.. (s - z_n)}{(s - p_0)(s - p_1)(s - p_2)(s - p_3) \ldots.. (s - p_n)}$$

The roots of the numerator, Z_0, Z_1, z_2, ... z_n are known as **zeros,** and the roots of the denominator, p_o, p_i, ... p_n are called **poles,** z; and p; are in general complex numbers, i.e., R + jI, where R is the real part, j=$\sqrt{-1}$, and I is the imaginary part. All of the poles and zeros will be either real roots (with no imaginary part) or complex conjugate pairs. A complex conjugate pair consists of two roots, each of which has a real part and an imaginary part. The imaginary parts of the two members of a complex conjugate pair will have opposite signs and the real parts will be equal.

5.2.2 Filter Approximation for Butterworth Filter

The loss (or attenuation) of the filter in decibels is defined by

$$A(\omega) = 20log\left|\frac{X(j\omega)}{Y(j\omega)}\right| = 20log\frac{1}{|H(j\omega)|} = 10logL(\omega^2)$$

Where

$$L(\omega^2) = \frac{1}{H(j\omega)H(-j\omega)}$$

With $\omega = {}^{s}/_{j}$

$$L(-s^2) = \frac{X(s)X(-s)}{Y(s)Y(-s)}$$

The simplest lowpass approximation, the Butterworth approximation is derived by assuming that $L(\omega^2)$ is a polynomial of the form

$$L(\omega^2) = b_0 + b_1\omega^2 + \cdots + b_n\omega^{2n}$$

Such that

$$\lim_{\omega^2 \to 0} L(\omega^2) = 1$$

in a maximally flat sense.

5.2.3 Derivation of Butterworth filter

The Taylor series of $L(x+h)$, where x=ω^2 , is

$$L(x+h) = L(x) + h\frac{dL(x)}{dx} + \cdots + \frac{h^k}{k!}\frac{d^kL(x)}{dx^k}$$

The polynomial L(x) approaches unity in a maximally flat sense as x→ 0 if its first n derivatives are zero for x=0. We may therefore, assign

$$L(0) = 1$$

$$\left.\frac{d^kL(x)}{dx^k}\right|_{x=0} = 0 \quad for\ k \le n$$

Thus from equation

$$b_0 = 1\ and\ b_1 = b_2 = \cdots = b_{n-1} = 0$$

or $$L(\omega^2) = 1 + b_n\omega^{2n}$$

now for a normalized approximation in which

$$L(1) = 2$$

That is, A(ω)≈3dB at ω = 1 rad/s, $b_n = 1$ and

$$L(\omega^2) = 1 + \omega^{2n}$$

Hence Butterworth filter transfer function

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

or $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}}$ where ω_0 is cutoff frequency

or $|H(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$

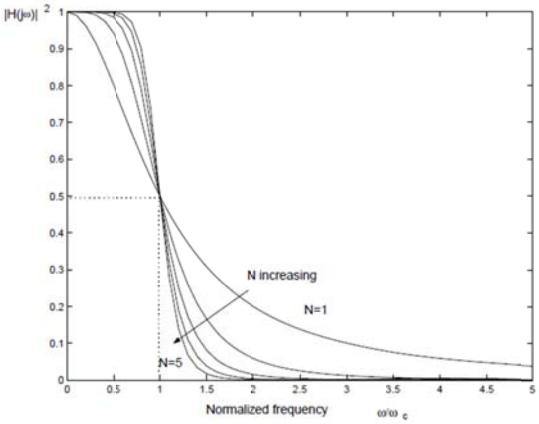


Fig. 5.1 plot of frequency response of Butterworth filter for different order

In s plane for a maximally flat sense the Butterworth filter can be written as

$$H(s) = \frac{1}{a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1s + a_0}$$

As Butterworth filter is given by

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

The poles s_k are given by

$$s_k = \exp\left(\frac{j(2k+n-1)\pi}{2n}\right),\ 1 \le k \le 2n$$

$$s_k = \cos\left(\frac{j(2k+n-1)\pi}{2n}\right) + j\sin\left(\frac{j(2k+n-1)\pi}{2n}\right),\ 1 \le k \le 2n$$

Again, only the poles in the left half s-plane are used. Effectively, only the poles for1 < k < n are used.

Note that the same pole equations are used for both lowpass and highpass filter designs. The poles are inserted into the following transfer function

$$H(s) = \frac{1}{(s-s_1)(s-s_2)\ldots\ldots(s-s_n)}$$

Butterworth filter can also be expressed using polynomials or quadratic factors.

Table 5.1(a) Butterworth Polynomials

Denominator coefficients for polynomials of the form $a_ns^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + ... + a_1s + a_0$.

n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
1	1									
2	1	1.414								
3	1	2.000	2.000							
4	1	2.613	3.414	2.613						
5	1	3.236	5.236	5.236	3.236					
6	1	3.864	7.464	9.142	7.464	3.864				
7	1	4.494	10.098	14.592	14.592	10.098	4.494			
8	1	5.126	13.137	21.846	25.688	21.846	13.137	5.126		
9	1	5.759	16.582	31.163	41.986	41.986	31.163	16.582	5.759	
10	1	6.392	20.432	42.802	64.882	74.233	64.882	42.802	20.432	6.392

Table 5.1(b) Butterworth Quadratic Factors

n	
1	$(s + 1)$
2	$(s^2 + 1.4142s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$
5	$(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$
6	$(s^2 + 0.5176s + 1)(s^2 + 1.4142s +1)(s^2 + 1.9319)$
7	$(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.2470s + 1)(s^2 + 1.8019s + 1)$
8	$(s^2 + 0.3902s + 1)(s^2 +1.1111s + 1)(s^2 + 1.6629s + 1)(s^2 + 1.9616s + 1)$
9	$(s + 1)(s^2 + 0.3473s + 1)(s^2 +1.0000s + 1)(s^2 + 1.5321s + 1)(s^2 + 1.8794s + 1)$
10	$(s^2 + 0.3129s + 1) (s^2 + 0.9080s + 1) (s^2 + 1.4142s +) (s^2 + 1.7820s+1) (s^2+1.9754s + 1)$

The conventional implementation is to apply the filter in a cascade manner rather than fully expanding the denominator in above equation. Each section of the cascade is a second order-section $H_k(s)$ given by

$$H_k(s) = \frac{1}{(s-s_k)(s-s_{n+1-k})}$$

Note that the sections are arranged to match the complex conjugate pairs of the poles.

The analog transfer function for a lowpass Butterworth filter with even order can now be written as

$$H(s) = \prod_{k=1}^{n/2} H_k(s)$$

Special Properties of Butterworth Filter

- The term Butterworth refers to a type of filter response, not a type of filter. It is sometimes called the Maximally Flat approximation, because for a response of order n, the first (2n-1) derivatives of the gain with respect to frequency are zero at frequency = 0. There is no ripple in the passband, and gain is maximally flat.
- The designer can see that there is no ripple in the passband of a Butterworth filter. The Butterworth filter, however, has a flatter response in the passband. There is a continuum of filter characteristics band in Butterworth
- Butterworth filters are one of the most commonly used digital filters in motion analysis. They are fast and simple to use. Since they are frequency-based, the effect of filtering can be easily understood and predicted. Choosing a cutoff frequency is easier than estimating the error involved in the raw data in the spline methods.

The characteristics of the normalized Butterworth filter frequency response are:

- Very close to the ideal near value
- Very smooth at all frequencies with a monotonic decrease from Largest difference between the ideal and actual responses near the transition
- Although not part of the approximation addressed, the phase curve is also very smooth.

5.3. Design of Sixth-order Butterworth filter

5.3.1. Sixth-order Lowpass Butterworth Filter

A sixth-order lowpass Butterworth filter has the poles given in Table 5.2. Only the poles on the left half of the s-plane are given.

Table 5.2 Poles of Sixth-order Lowpass Butterworth Filter

k	s_k Pole
1	$\cos(\frac{7\pi}{12}) + j\sin(\frac{7\pi}{12})$
2	$\cos(\frac{9\pi}{12}) + j\sin(\frac{9\pi}{12})$
3	$\cos(\frac{11\pi}{12}) + j\sin(\frac{11\pi}{12})$
4	$\cos(\frac{13\pi}{12}) + j\sin(\frac{13\pi}{12})$
5	$\cos(\frac{15\pi}{12}) + j\sin(\frac{15\pi}{12})$
6	$\cos(\frac{17\pi}{12}) + j\sin(\frac{17\pi}{12})$

Note the following complex conjugate pairings:

$s_4 = s_3{*}$

$s_5 = s_2{*}$

$s_6 = s_1{*}$

Apply the poles into equation as;

$$H_1(s) = \frac{1}{(s-s1)(s-s6)}$$

$$H_1(s) = \frac{1}{(s-s1)(s-s1*)}$$

$$H_1(s) = \frac{1}{\left\{s-\left[\cos\left(\frac{7\pi}{12}\right)+j\sin\left(\frac{7\pi}{12}\right)\right]\right\}\left\{s-\left[\cos\left(\frac{7\pi}{12}\right)-j\sin\left(\frac{7\pi}{12}\right)\right]\right\}}$$

$$H_1(s) = \frac{1}{\left\{s-\cos\left(\frac{7\pi}{12}\right)-j\sin\left(\frac{7\pi}{12}\right)\right\}\left\{s-\cos\left(\frac{7\pi}{12}\right)+j\sin\left(\frac{7\pi}{12}\right)\right\}}$$

$$H_1(s) = \frac{1}{\{s-cos(7\pi/12)\}^2 + \{sin(7\pi/12)\}^2}$$

$$H_1(s) = \frac{1}{s^2 - 2\cos(\frac{7\pi}{12})s + 1}$$

Similarly,

$$H_2(s) = \frac{1}{s^2 - 2\cos(\frac{9\pi}{12})s + 1}$$

$$H_3(s) = \frac{1}{s^2 - 2\cos(\frac{11\pi}{12})s + 1}$$

Normalized Frequency Parameter

Now consider a generic stage.

$$H_g(s) = \frac{1}{s^2 - \alpha s + 1}$$

Define a frequency parameter Ωc as

Consider the points at s = jΩc on the s-plane and z = exp $[j\omega_0\Delta t]$ on the z-plane.

By substitution in the equation, we get,

$$j\Omega c = \frac{\exp[j\omega_0\Delta t] - 1}{\exp[j\omega_0\Delta t] + 1}$$

$$j\Omega c = \frac{\cos[\omega_0 \Delta t] + j\sin[\omega_0 \Delta t] - 1}{\cos[\omega_0 \Delta t] + j\sin[\omega_0 \Delta t] + 1}$$

$$\Omega c = \frac{\cos[\omega_0 \Delta t] + j\sin[\omega_0 \Delta t] - 1}{j\cos[\omega_0 \Delta t] - \sin[\omega_0 \Delta t] + j}$$

$$\Omega c = \frac{\cos[\omega_0 \Delta t] + j\sin[\omega_0 \Delta t] - 1}{-\sin[\omega_0 \Delta t] + j\{\cos[\omega_0 \Delta t] + 1\}}$$

$$\Omega c = \left\{\frac{\cos[\omega_0 \Delta t] + j\sin[\omega_0 \Delta t] - 1}{-\sin[\omega_0 \Delta t] + j\{\cos[\omega_0 \Delta t] + 1\}}\right\}\left\{\frac{-j\cos[\omega_0 \Delta t] - \sin[\omega_0 \Delta t] + 1}{-\sin[\omega_0 \Delta t] - j\{\cos[\omega_0 \Delta t] + 1\}}\right\}$$

$$\Omega c = \left\{\frac{\{\cos[\omega_0 \Delta t] - 1\}\{-\sin[\omega_0 \Delta t] - j\{\cos[\omega_0 \Delta t] + 1\}\} + \{j\sin[\omega_0 \Delta t]\}\{-\sin[\omega_0 \Delta t] - j\{\cos[\omega_0 \Delta t] + 1\}\}}{\sin^2[\omega_0 \Delta t] + \{\cos[\omega_0 \Delta t] + 1\}^2}\right\}$$

$$\Omega c = \left\{\frac{\{-\cos[\omega_0 \Delta t]\sin[\omega_0 \Delta t] + \sin[\omega_0 \Delta t] - j\{\cos^2[\omega_0 \Delta t] - 1\}\} + \{\cos[\omega_0 \Delta t]\sin[\omega_0 \Delta t] + \sin[\omega_0 \Delta t] - j\sin^2[\omega_0 \Delta t]\}}{\sin^2[\omega_0 \Delta t] + \cos^2[\omega_0 \Delta t] + 2\cos[\omega_0 \Delta t] + 1}\right\}$$

$$\Omega c = \left\{\frac{2\sin[\omega_0 \Delta t]}{2\cos[\omega_0 \Delta t] + 2}\right\}$$

$$\Omega c = \left\{\frac{\sin[\omega_0 \Delta t]}{\cos[\omega_0 \Delta t] + 1}\right\}$$

$$\Omega c = \tan\left[\frac{[\omega_0 \Delta t]}{2}\right]$$

$$\Omega c = \tan[\pi f_0 \Delta t]$$

Note that Δt is the time segment duration. It is thus the inverse of the sampling rate. Furthermore, f_0 is the filter cutoff frequency.

Apply the frequency parameter to the generic transfer function.

$$\hat{H}_g(s) = H_g(s)|_{s=s/\Omega c} = H_g\left(\frac{s}{\Omega c}\right)$$

$$\hat{H}_g(s) = \frac{1}{\left(\frac{s}{\Omega c}\right)^2 - \alpha\left(\frac{s}{\Omega c}\right) + 1}$$

$$\hat{H}_g(s) = \frac{\Omega c^2}{s^2 - \alpha\Omega_c s + s^2}$$

Z-transform of Sixth-Order Lowpass Butterworth Filter

The bilinear transform is defined by

$$s = \frac{z-1}{z+1}$$

The purpose of this function is to transform an analog filter into the z-domain. The frequency transformation in equation actually follows from the bilinear transformation in equation Substitute the bilinear transform into the transfer function in equation,

$$\hat{H}_g(s) = \frac{\Omega c^2}{[(z-1)/(z+1)]^2 - \alpha\Omega c[(z-1)/(z+1)] + \Omega c^2}$$

$$\hat{H}_g(s) = \frac{\Omega c^2[z+1]^2}{[z-1]^2 - \alpha\Omega c[(z-1)][z+1] + \Omega c^2[z+1]^2}$$

$$\hat{H}_g(s) = \frac{\Omega c^2[z^2+2z+1]}{[(z^2-2z+1)] - \alpha\Omega c[(z^2-1)] + \Omega c^2[z^2+2z+1]}$$

$$\hat{H}_g(s) = \frac{[\Omega c^2 z^2 + \Omega c^2 2z + \Omega c^2]}{z^2 - 2z + 1 - \alpha\Omega c z^2 + \alpha\Omega c + \Omega c^2 z^2 + 2\Omega c^2 z + \Omega c^2}$$

$$\hat{H}_g(s) = \frac{\Omega c^2 z^2 + \Omega c^2 2z + \Omega c^2}{z^2[-\alpha\Omega c + \Omega c^2 + 1] + z[2\Omega c^2 - 2] + [\Omega c^2 + \alpha\Omega c + + 1]}$$

$$\hat{H}_g(s) = \frac{\Omega c^2 z^2 + \Omega c^2 2z + \Omega c^2}{z^2[-\alpha\Omega c + \Omega c^2 + 1] + 2z[\Omega c^2 - 1] + [\Omega c^2 + \alpha\Omega c + 1]}$$

Dividing this equation by $[\Omega c^2 - \alpha\Omega c + 1]$ and recalling equation above,

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}}$$

Set n=2.

$$\mathrm{H(z)} = \frac{b_0+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

Multiply through by z^2,

$$\mathrm{H(z)} = \frac{b_0z^2+b_1z+b_2}{z^2+a_1z+a_2}$$

Comparing it with equation above,

$$b_0 = \frac{\Omega\mathrm{c}^2}{[\Omega\mathrm{c}^2-\alpha\Omega c+1]} \qquad a_1 = \frac{2[\Omega\mathrm{c}^2-1]}{[\Omega\mathrm{c}^2-\alpha\Omega c+1]}$$

$$b_1 = \frac{2\Omega\mathrm{c}^2}{[\Omega\mathrm{c}^2-\alpha\Omega c+1]} \qquad a_2 = \frac{[\Omega\mathrm{c}^2+\alpha\Omega c+1]}{[\Omega\mathrm{c}^2-\alpha\Omega c+1]}$$

$$b_2 = b_0$$

The coefficients can be inserted into the equation given below. The resulting recursive equation for a filter section is

$$y_k = [b_0x_k+b_1x_{k-1}+b_2x_{k-2}]-[a_1y_{k-1}+a_2y_{k-2}]$$

Equation represents one of three cascade stages for a sixth-order filter. Note that there is a unique set of coefficients for each of these stages.

Equation is applied six times if refiltering is used for phase correction, again assuming a sixth-order filter.

5.3.2. Sixth-order Highpass Butterworth Filter

Recall the generic transfer function for a lowpass filter stage

$$\mathrm{H_g(s)} = \frac{1}{\mathrm{s}^2 - \alpha\mathrm{s} + 1}$$

The lowpass filter H can be transformed into a highpass filter J by changing s to 1/s.

$$\mathrm{J_g(s)} = \frac{1}{\left(\frac{1}{\mathrm{s}}\right)^2 - \left(\frac{\alpha}{\mathrm{s}}\right) + 1}$$

$$\mathrm{J_g(s)} = \frac{s^2}{1 - \alpha\mathrm{s} + \mathrm{s}^2}$$

$$\mathrm{J_g(s)} = \frac{\left(\frac{s}{\Omega\mathrm{c}}\right)^2}{1 - \alpha\left(\frac{s}{\Omega\mathrm{c}}\right) + \left(\frac{s}{\Omega\mathrm{c}}\right)^2}$$

$$\mathrm{J_g(s)} = \frac{s^2}{\Omega\mathrm{c}^2 - \alpha\Omega\mathrm{cs} + \mathrm{s}^2}$$

Recall the bilinear transform $\mathrm{s} = \frac{z-1}{z+1}$

By substitution, $$\hat{\mathrm{J}}_\mathrm{g}\mathrm{(s)} = \frac{[(z-1)/(z+1)]^2}{\Omega\mathrm{c}^2 - \alpha\Omega c[(z-1)/(z+1)] + [(z-1)/(z+1)]^2}$$

$$\hat{\mathrm{J}}_\mathrm{g}\mathrm{(s)} = \frac{[\mathrm{z}-1]^2}{\Omega\mathrm{c}^2[\mathrm{z}+1]^2 - \alpha\Omega c[(z-1)][z+1] + [\mathrm{z}-1]^2}$$

$$\hat{\mathrm{J}}_\mathrm{g}\mathrm{(s)} = \frac{[\mathrm{z}^2-2\mathrm{z}+1]}{\Omega\mathrm{c}^2[(z^2+2z+1)] - \alpha\Omega c[(z^2-1)] + \Omega\mathrm{c}^2[z^2-2z+1]}$$

$$\hat{\mathrm{J}}_\mathrm{g}\mathrm{(s)} = \frac{[z^2-2z+1]}{z^2[-\alpha\Omega c+\Omega\mathrm{c}^2+1]+z[2\Omega\mathrm{c}^2-2] + [\Omega\mathrm{c}^2+\alpha\Omega c++1]}$$

$$\hat{\mathrm{J}}_\mathrm{g}\mathrm{(s)} = \frac{[z^2-2z+1]}{z^2[-\alpha\Omega c+\Omega\mathrm{c}^2+1]+2z[2\Omega\mathrm{c}^2-1] + [\Omega\mathrm{c}^2+\alpha\Omega c++1]}$$

Dividing this equation by $[\Omega\mathrm{c}^2 - \alpha\Omega c + 1]$ and recalling equation,

$$\mathrm{H(z)} = \frac{b_0+b_1z^{-1}+\cdots+b_nz^{-n}}{1+a_1z^{-1}+\cdots+a_nz^{-n}}$$

Set n=2.

$$\mathrm{H(z)} = \frac{b_0+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$$

Multiply through by z^2,

$$\mathrm{H(z)} = \frac{b_0z^2+b_1z+b_n}{z^2+a_11z+a_2}$$

Comparing it with equation

$$b_0 = \frac{1}{[\Omega c^2 - \alpha\Omega c + 1]} \qquad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha\Omega c + 1]}$$

$$b_1 = \frac{-2\Omega c^2}{[\Omega c^2 - \alpha\Omega c + 1]} \qquad a_2 = \frac{[\Omega c^2 + \alpha\Omega c + 1]}{[\Omega c^2 - \alpha\Omega c + 1]}$$

$$b_2 = b_0$$

The coefficients can be inserted into equation. The resulting recursive equation for a filter section is

$$y_k = [b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2}] - [a_1 y_{k-1} + a_2 y_{k-2}]$$

Equation represents one of three cascade stages for a sixth-order filter. Note that there is a unique set of coefficients for each of these stages.

5.4 Refiltering For Phase Correction

The refiltering method is an effective means for approximately achieving linear phase response. This is a technique which can be applied to digital data. It is impractical for analog data since it requires time reversals.

A linear phase response is required for accurate time domain calculations. The shock response spectrum is an example of a time domain calculation which needs a linear phase input.

On the other hand, the refiltering method requires increased computational time. Another characteristic of refiltering is additional amplitude attenuation near the cutoff frequency. This could either be an advantage or a drawback depending on the original motivation for filtering the data.

The Butterworth filter, characterized by its -3 dB cutoff frequency, is known to introduce a phase shift.

Procedure

The refiltering method is applied as follows:

1. Reverse the time history.
2. Apply the filter to the time history.
3. Reverse the time history again.
4. Apply the filter to the time history.

This process is shown in Figure 3.4.1 for an input X and an output Y.

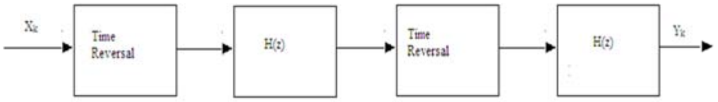


Figure 5.2 Refiltering Diagram

Note that H(z) represents the filter transfer function in terms of a Z-transform. The equivalent transfer function from X to Y is $[H(z)\ H(z^{-1})]$. Note that $z = \exp[j\omega_0 \Delta t]$, where ωis the frequency and Δt is the time step

CHAPTER 6. DETERMINATION OF K-T PARAMETERS FOR THE LISTED EARTHQUAKES

6.1 El Centro Earthquake 1940

Date: 18 May 1940

Time: 21:35 (Local time i.e. Pacific standard time)

Location of earthquake: El Centro a city in Imperial valley of California region

Component: N-S Component

Location of epicenter: Latitude 32.7601 Longitude -115.4162

Magnitude of Earthquake (M_W): 7.1

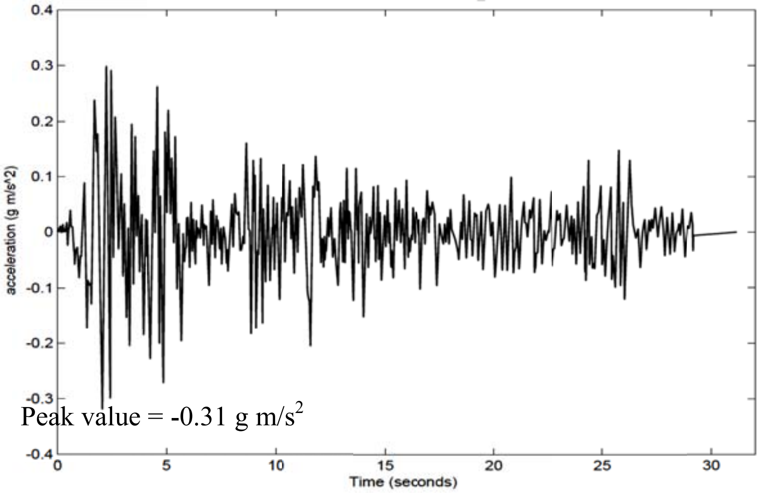
Location of station: El Centro, CA-Array Sta 9 Imperial valley region district

Sampling Rate: 50 (0.02 sec interval)

Epicentral Distance of recorded station: 11.4 km

Impact and other details: 9 people were killed due to destruction caused by earthquake. At Imperial, 80 percent of the buildings were damaged to some degree. In the business district of Brawley, all structures were damaged, and about 50 percent had to be condemned. The shock caused 40 miles of surface faulting on the Imperial Fault, part of the San Andreas system in southern California. Total damage has been estimated at about $6 million. Geology of site of **alluvium** type.

Plot of Acceleration time history of recorded ground motion data at equal time interval of 0.02 sec of El Centro Earthquake



The raw ground motion data is filtered using Butterworth filter. For filtering data in particular bandwidth, the lowpass filter is set for lower cutoff frequency while the high pass filter is applied for higher cutoff frequency. The equation used for filtering raw data is:

$$y_k=[b_0x_k+b_1x_{k-1}+b_2x_{k-2}]-[a_1y_{k-1}+a_2y_{k-2}]$$

The coefficients a_1, a_2, b_0, b_1, b_2 used in various frequency bandwidths for low pass and high pass filter are given in right side of plot of filter

The calculation of coefficients of low pass and high pass are of the form

For low pass coefficients

$$H_1:\quad b_0=\frac{\Omega c^2}{[\Omega c^2-\alpha_1\Omega c+1]};\ a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_1\Omega c+1]};\ b_1=\frac{2\Omega c^2}{[\Omega c^2-\alpha_1\Omega c+1]};\quad a_2=\frac{[\Omega c^2+\alpha_1\Omega c+1]}{[\Omega c^2-\alpha_1\Omega c+1]};\ b_2=b_0$$

$$H_2:\quad b_0=\frac{\Omega c^2}{[\Omega c^2-\alpha_2\Omega c+1]};\ a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_2\Omega c+1]};\ b_1=\frac{2\Omega c^2}{[\Omega c^2-\alpha_2\Omega c+1]};\quad a_2=\frac{[\Omega c^2+\alpha_2\Omega c+1]}{[\Omega c^2-\alpha_2\Omega c+1]};\ b_2=b_0$$

$$H_3:\quad b_0=\frac{\Omega c^2}{[\Omega c^2-\alpha_3\Omega c+1]};\ a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_3\Omega c+1]};\ b_1=\frac{2\Omega c^2}{[\Omega c^2-\alpha_3\Omega c+1]};\quad a_2=\frac{[\Omega c^2+\alpha_3\Omega c+1]}{[\Omega c^2-\alpha_3\Omega c+1]};\ b_2=b_0$$

For high pass coefficients

$$H_1:\quad b_0=\frac{1}{[\Omega c^2-\alpha_1\Omega c+1]};\quad a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_1\Omega c+1]};\quad b_1=\frac{-2\Omega c^2}{[\Omega c^2-\alpha_1\Omega c+1]};\ a_2=\frac{[\Omega c^2+\alpha_1\Omega c+1]}{[\Omega c^2-\alpha_1\Omega c+1]};\quad b_2=b_0$$

$$H_2:\quad b_0=\frac{1}{[\Omega c^2-\alpha_2\Omega c+1]};\quad a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_2\Omega c+1]};\quad b_1=\frac{-2\Omega c^2}{[\Omega c^2-\alpha_2\Omega c+1]};\quad a_2=\frac{[\Omega c^2+\alpha_2\Omega c+1]}{[\Omega c^2-\alpha_2\Omega c+1]};\quad b_2=b_0$$

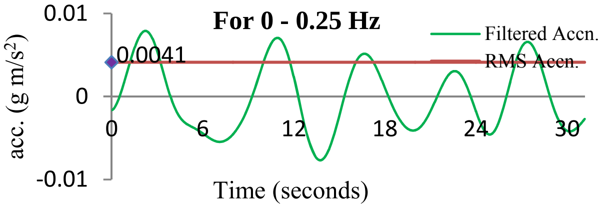
$$H_3:\quad b_0=\frac{1}{[\Omega c^2-\alpha_3\Omega c+1]};\quad a_1=\frac{2[\Omega c^2-1]}{[\Omega c^2-\alpha_3\Omega c+1]};\quad b_1=\frac{-2\Omega c^2}{[\Omega c^2-\alpha_3\Omega c+1]};\quad a_2=\frac{[\Omega c^2+\alpha_3\Omega c+1]}{[\Omega c^2-\alpha_3\Omega c+1]};\quad b_2=b_0$$

Where values of α_1, α_2, α_3 are constants which is given in next page which comes during cascading and Ω_c a frequency parameter which is dependent on filter cutoff frequency
$\Omega_c = \tan(\pi f_0\,\Delta t)$. Where f_0 is filter cutoff frequency and Δt is time step of recorded data

Plot of filtered data in particular **bandwidth which has been kept 0.25 Hz** are shown and also plot of PSDF vs Frequency in Bar graph and Line Graph is shown next to plot of filtered data.

Plot of Filtered data and corresponding filter coefficients

Δt = 0.02 sec	α_1	-0.51763809
	α_2	-1.41421356
	α_3	-1.93185165



Output RMS value = 0.0041

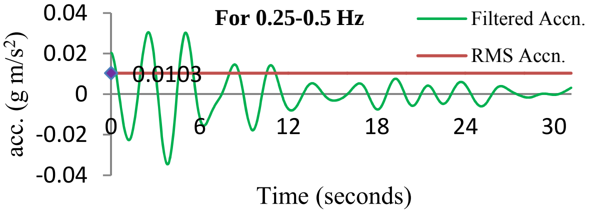
f_0	Low pass		f_0	High pass	
0.25	Ω_c	0.015709	0.01	Ω_c	0.000628

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.98289	0.983872	0.00025	0.00049	0.000247
H_2	-1.95558	0.956544	0.00025	0.00049	0.000247
H_3	-1.94015	0.941106	0.00025	0.00049	0.000246

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.99935	0.99935	0.99967	-1.99935	0.999674
H_2	-1.99822	0.998224	0.99911	-1.99822	0.999112
H_3	-1.99757	0.997575	0.99879	-1.99757	0.998787



Output RMS value = 0.0103

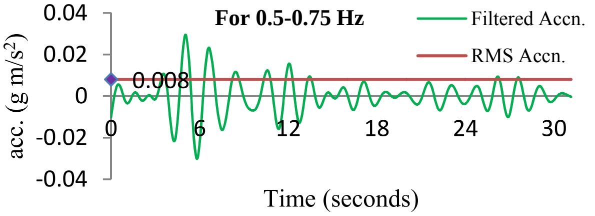
f_0	Low pass		f_0	High pass	
0.5	Ω_c	0.031426	0.25	Ω_c	0.015709

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.96413	0.968017	0.000979	0.001957	0.000979
H_2	-1.9112	0.914976	0.000965	0.00193	0.000965
H_3	-1.88191	0.885634	0.000958	0.001915	0.000958

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.98289	0.983872	0.991691	-1.98338	0.991691
H_2	-1.95558	0.956544	0.97803	-1.95606	0.97803
H_3	-1.94015	0.941106	0.970314	-1.94063	0.970314



Output RMS value = 0.008

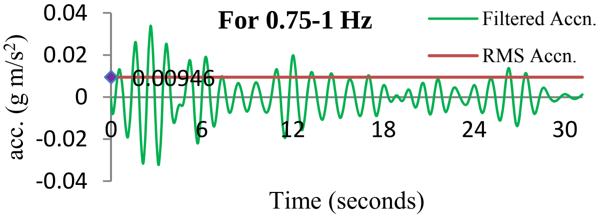
f_0	Low pass		f_0	High pass	
0.75	Ω_c	0.047159	0.5	Ω_c	0.031426

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.94378	0.952444	0.002184	0.004367	0.002184
H_2	-1.86689	0.875215	0.002125	0.00425	0.002125
H_3	-1.82521	0.833346	0.002092	0.004185	0.002092

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.96413	0.968017	0.983038	-1.96608	0.983038
H_2	-1.9112	0.914976	0.956543	-1.91309	0.956543
H_3	-1.88191	0.885634	0.941887	-1.88377	0.941887



Output RMS value = 0.00946

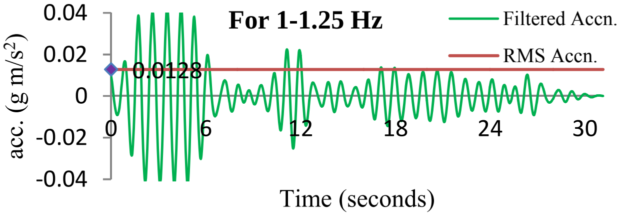
f_0	Low pass		f_0	High pass	
1	Ω_c	0.062915	0.75	Ω_c	0.047159

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.92189	0.937161	0.003849	0.007698	0.003849
H_2	-1.82269	0.837182	0.003697	0.007394	0.003697
H_3	-1.76995	0.784022	0.003615	0.007229	0.003615

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.94378	0.952444	0.974056	-1.94811	0.974056
H_2	-1.86689	0.875215	0.935527	-1.87105	0.935527
H_3	-1.82521	0.833346	0.914639	-1.82928	0.914639



Output RMS value = 0.0128

f_0	Low pass		f_0	High pass	
1.25	Ω_c	0.078702	1	Ω_c	0.062915

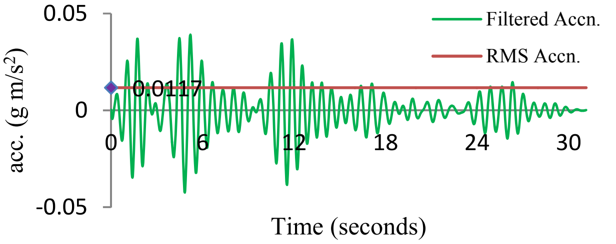
low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.89851	0.922175	0.005963	0.011926	0.005963
H_2	-1.77863	0.800803	0.005656	0.011311	0.005656
H_3	-1.71607	0.737462	0.005492	0.010985	0.005492

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.92189	0.937161	0.964762	-1.92952	0.964762
H_2	-1.82269	0.837182	0.914969	-1.82994	0.914969
H_3	-1.76995	0.784022	0.888494	-1.77699	0.888494

For 1.25-1.5 Hz



Output RMS value = 0.0117

f_0	Low pass		f_0	High pass	
1.5	Ω_c	0.094528	1.25	Ω_c	0.078702

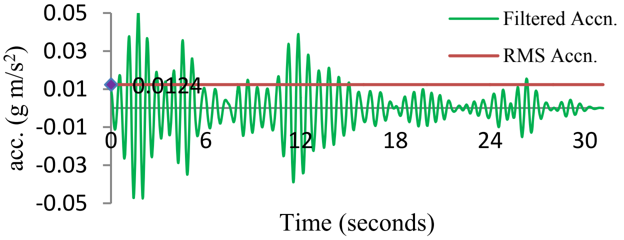
low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.8737	0.907491	0.008513	0.017025	0.008513
H_2	-1.73473	0.766007	0.007976	0.015953	0.007976
H_3	-1.66349	0.693485	0.007697	0.015393	0.007697

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.89851	0.922175	0.955171	-1.91034	0.955171
H_2	-1.77863	0.800803	0.894859	-1.78972	0.894859
H_3	-1.71607	0.737462	0.863383	-1.72677	0.863383

For 1.5-1.75 Hz



Output RMS value = 0.0124

f_0	Low pass		f_0	High pass	
1.75	Ω_c	0.110401	1.5	Ω_c	0.094528

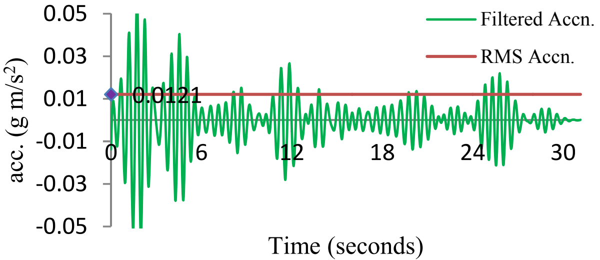
low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.84752	0.893115	0.011486	0.022973	0.011486
H_2	-1.691	0.732726	0.010637	0.021274	0.010637
H_3	-1.61214	0.651923	0.010201	0.020402	0.010201

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.8737	0.907491	0.945299	-1.8906	0.945299
H_2	-1.73473	0.766007	0.875183	-1.75037	0.875183
H_3	-1.66349	0.693485	0.839244	-1.67849	0.839244

For 1.75-2 Hz



Output RMS value = 0.0121

f_0	Low pass		f_0	High pass	
2	Ω_c	0.126329	1.75	Ω_c	0.110401

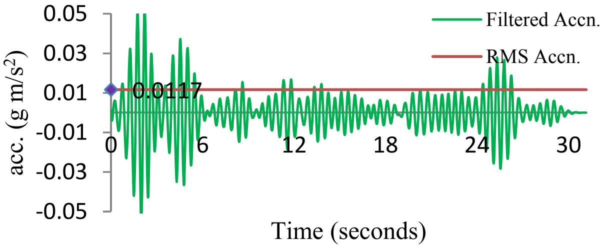
low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.82002	0.879053	0.014872	0.029744	0.014872
H_2	-1.64746	0.700897	0.013616	0.027232	0.013616
H_3	-1.56196	0.612622	0.012983	0.025966	0.012983

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.84752	0.893115	0.93516	-1.87032	0.93516
H_2	-1.691	0.732726	0.855931	-1.71186	0.855931
H_3	-1.61214	0.651923	0.816016	-1.63203	0.816016

For 2-2.25 Hz



Output RMS value = 0.0117

f_0	Low pass		f_0	High pass	
2.25	Ω_c	0.142321	2	Ω_c	0.126329

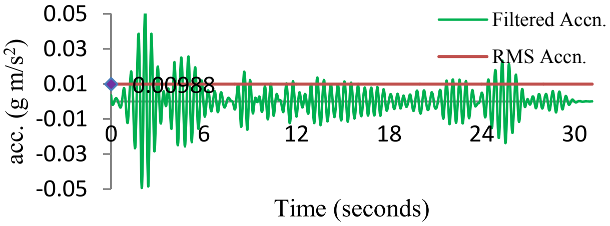
low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.79124	0.865309	0.018657	0.037315	0.018657
H_2	-1.60413	0.670458	0.016895	0.033789	0.016895
H_3	-1.51289	0.575442	0.016021	0.032042	0.016021

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.82002	0.879053	0.924768	-1.84954	0.924768
H_2	-1.64746	0.700897	0.837089	-1.67418	0.837089
H_3	-1.56196	0.612622	0.793645	-1.58729	0.793645

For 2.25-2.5 Hz



Output RMS value = 0.00988

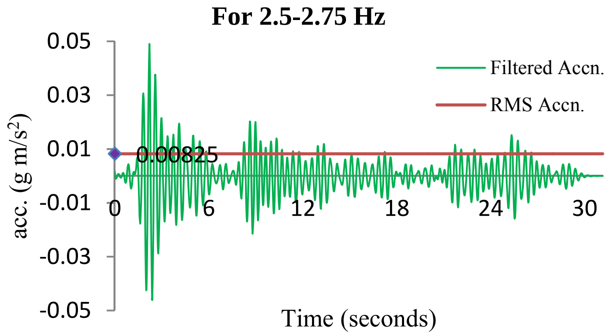
f_0	Low pass		f_0	High pass	
2.5	Ω_c	0.158384	2.25	Ω_c	0.142321

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.76125	0.851887	0.022831	0.045662	0.022831
H_2	-1.56102	0.641352	0.020455	0.040911	0.020455
H_3	-1.46487	0.540254	0.019296	0.038592	0.019296

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.79124	0.865309	0.914139	-1.82828	0.914139
H_2	-1.60413	0.670458	0.818647	-1.63729	0.818647
H_3	-1.51289	0.575442	0.772082	-1.54416	0.772082



Output RMS value = 0.00825

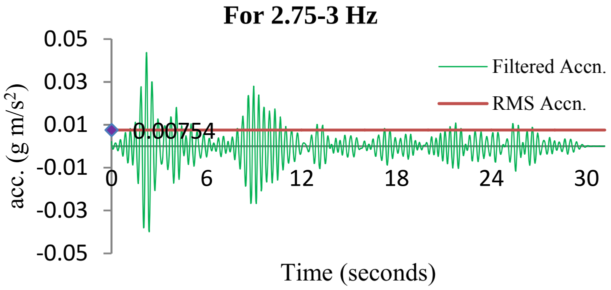
f_0	Low pass		f_0	High pass	
2.75	Ω_c	0.174528	2.5	Ω_c	0.158384

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.73008	0.83879	0.027381	0.054762	0.027381
H_2	-1.51813	0.613523	0.024282	0.048563	0.024282
H_3	-1.41785	0.506937	0.022792	0.045584	0.022792

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.76125	0.851887	0.903284	-1.80657	0.903284
H_2	-1.56102	0.641352	0.800592	-1.60118	0.800592
H_3	-1.46487	0.540254	0.75128	-1.50256	0.75128



Output RMS value = 0.00754

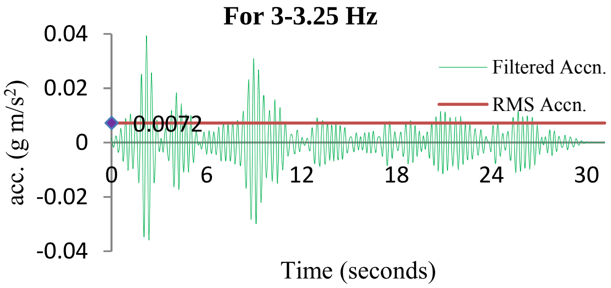
f_0	Low pass		f_0	High pass	
3	Ω_c	0.19076	2.75	Ω_c	0.174528

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.69779	0.826021	0.032296	0.064593	0.032296
H_2	-1.47548	0.58692	0.028358	0.056716	0.028358
H_3	-1.37178	0.475382	0.026493	0.052986	0.026493

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.73008	0.83879	0.892218	-1.78444	0.892218
H_2	-1.51813	0.613523	0.782914	-1.56583	0.782914
H_3	-1.41785	0.506937	0.731196	-1.46239	0.731196



Output RMS value = 0.0072

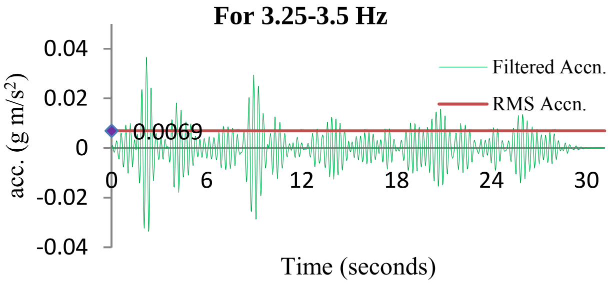
f_0	Low pass		f_0	High pass	
3.25	Ω_c	0.20709	3	Ω_c	0.19076

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.66442	0.813583	0.037566	0.075132	0.037566
H_2	-1.43307	0.561492	0.032671	0.065342	0.032671
H_3	-1.3266	0.445488	0.030385	0.060771	0.030385

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.69779	0.826021	0.880953	-1.76191	0.880953
H_2	-1.47548	0.58692	0.7656	-1.5312	0.7656
H_3	-1.37178	0.475382	0.711789	-1.42358	0.711789



Output RMS value = 0.0069

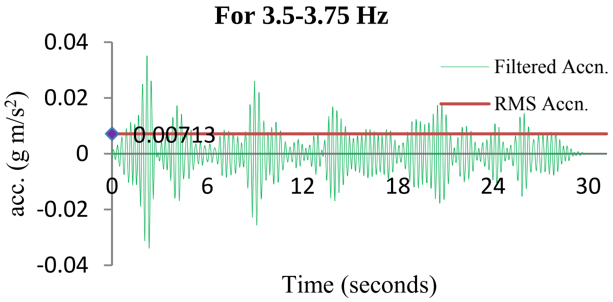
f_0	Low pass		f_0	High pass	
3.5	Ω_c	0.223526	3.25	Ω_c	0.20709

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	- .63003	0.801478	0.043178	0.086356	0.043178
H_2	-1.3909	0.537195	0.037208	0.074416	0.037208
H_3	-1.28229	0.417162	0.034457	0.068914	0.034457

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.66442	0.813583	0.869502	-1.739	0.869502
H_2	-1.43307	0.561492	0.74864	-1.49728	0.74864
H_3	-1.3266	0.445488	0.693023	-1.38605	0.693023



Output RMS value = 0.00713

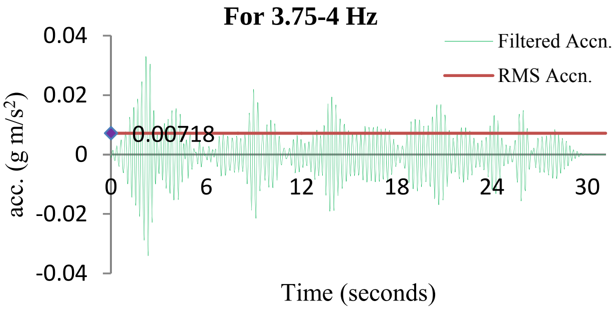
f_0	Low pass		f_0	High pass	
3.75	Ω_c	0.240079	3.5	Ω_c	0.223526

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.59464	0.789707	0.049123	0.098245	0.049123
H_2	-1.34897	0.513982	0.041956	0.083913	0.041956
H_3	-1.23878	0.390317	0.038697	0.077394	0.038697

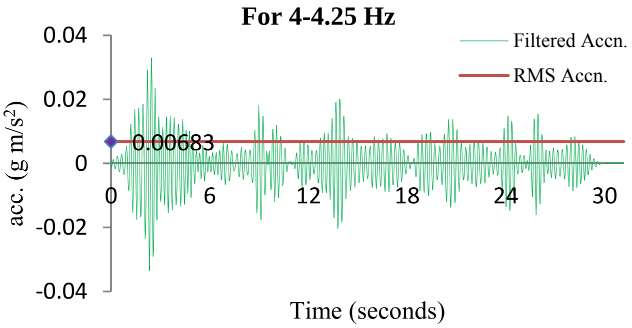
high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.63003	0.801478	0.857876	-1.71575	0.857876
H_2	-1.3909	0.537195	0.732022	-1.46404	0.732022
H_3	-1.28229	0.417162	0.674862	-1.34972	0.674862



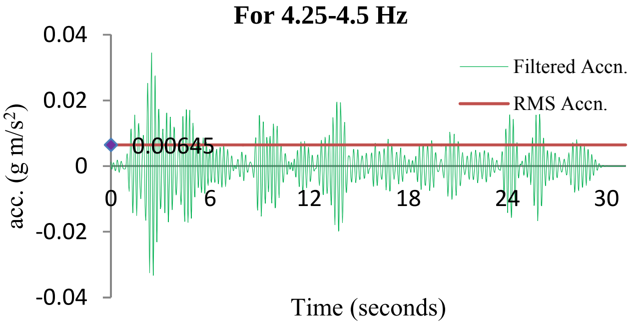
Output RMS value = 0.00718

f_0	Low pass		f_0	High pass	
4	Ω_c	0.256756	3.75	Ω_c	0.240079
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.55831	0.778272	0.055389	0.110778	0.055389
H_2	-1.30729	0.491812	0.046906	0.093812	0.046906
H_3	-1.19605	0.364873	0.043095	0.086191	0.043095
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.59464	0.789707	0.846087	-1.69217	0.846087
H_2	-1.34897	0.513982	0.715737	-1.43147	0.715737
H_3	-1.23878	0.390317	0.657274	-1.31455	0.657274



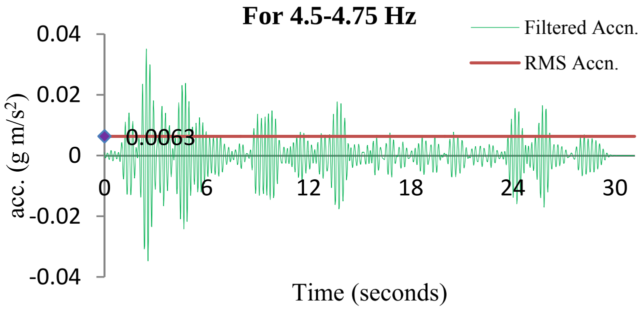
Output RMS value = 0.00683

f_0	Low pass		f_0	High pass	
4.25	Ω_c	0.273569	4	Ω_c	0.256756
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.52108	0.767175	0.061967	0.123933	0.061967
H_2	-1.26585	0.470646	0.052046	0.104093	0.052046
H_3	-1.15404	0.340756	0.047643	0.095286	0.047643
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.55831	0.778272	0.834146	-1.66829	0.834146
H_2	-1.30729	0.491812	0.699774	-1.39955	0.699774
H_3	-1.19605	0.364873	0.64023	-1.28046	0.64023



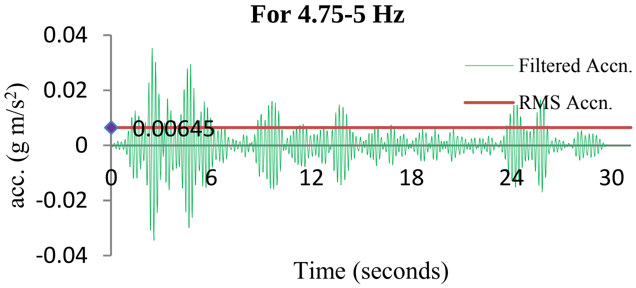
Output RMS value = 0.00645

f_0	Low pass		f_0	High pass	
4.5	Ω_c	0.290527	4.25	Ω_c	0.273569
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.48299	0.756416	0.068846	0.137691	0.068846
H_2	-1.22465	0.450445	0.057369	0.114737	0.057369
H_3	-1.11274	0.317897	0.052332	0.104663	0.052332
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.52108	0.767175	0.822064	-1.64413	0.822064
H_2	-1.26585	0.470646	0.684123	-1.36825	0.684123
H_3	-1.15404	0.340756	0.6237	-1.2474	0.6237



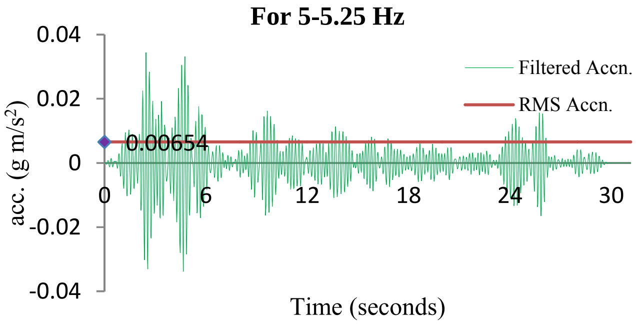
Output RMS value = 0.0063

f_0	Low pass		f_0	High pass	
4.75	Ω_c	0.30764	4.5	Ω_c	0.290527
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.44408	0.745996	0.076016	0.152032	0.076016
H_2	-1.1837	0.431175	0.062864	0.125728	0.062864
H_3	-1.07209	0.296234	0.057155	0.11431	0.057155
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.48299	0.756416	0.809852	-1.6197	0.809852
H_2	-1.22465	0.450445	0.668774	-1.33755	0.668774
H_3	-1.11274	0.317897	0.607659	-1.21532	0.607659



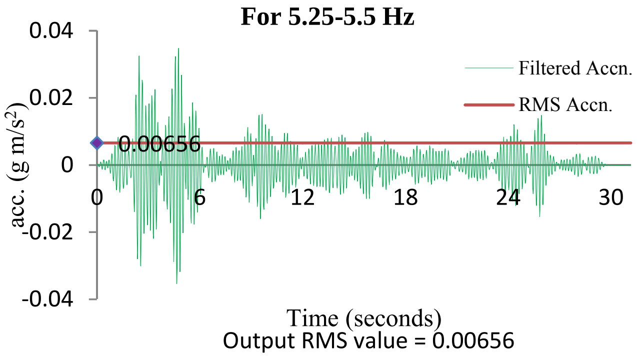
Output RMS value = 0.00645

f_0	Low pass		f_0	High pass	
5	Ω_c	0.32492	4.75	Ω_c	0.30764
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.40438	0.735915	0.083469	0.166937	0.083469
H_2	-1.14298	0.412802	0.068525	0.13705	0.068525
H_3	-1.03207	0.275708	0.062106	0.124211	0.062106
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.44408	0.745996	0.797519	-1.59504	0.797519
H_2	-1.1837	0.431175	0.653718	-1.30744	0.653718
H_3	-1.07209	0.296234	0.592081	-1.18416	0.592081


For 5-5.25 Hz

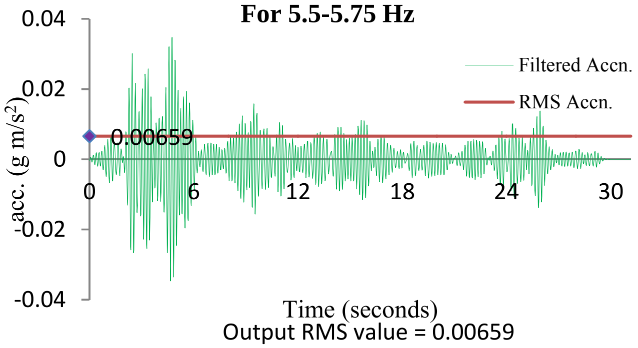
Output RMS value = 0.00654

f_0	Low pass		f_0	High pass	
5.25	Ω_c	0.342377	5	Ω_c	0.32492
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.36394	0.726174	0.091194	0.182388	0.091194
H_2	-1.1025	0.395293	0.074345	0.14869	0.074345
H_3	-0.99264	0.256263	0.067179	0.134358	0.067179
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.40438	0.735915	0.785075	-1.57015	0.785075
H_2	-1.14298	0.412802	0.638946	-1.27789	0.638946
H_3	-1.03207	0.275708	0.576944	-1.15389	0.576944


For 5.25-5.5 Hz

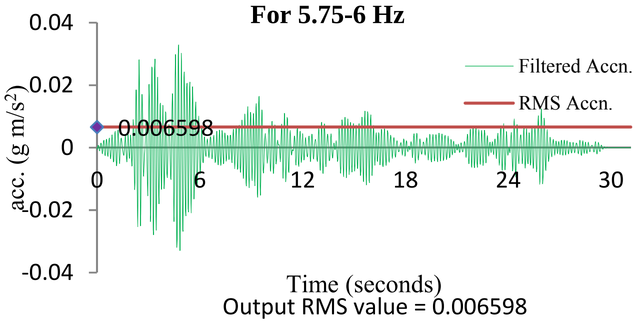
Output RMS value = 0.00656

f_0	Low pass		f_0	High pass	
5.5	Ω_c	0.360022	5.25	Ω_c	0.342377
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.3228	0.716771	0.099182	0.198365	0.099182
H_2	-1.06224	0.378619	0.080317	0.160634	0.080317
H_3	-0.95378	0.23785	0.072369	0.144738	0.072369
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.36394	0.726174	0.77253	-1.54506	0.77253
H_2	-1.1025	0.395293	0.624448	-1.2489	0.624448
H_3	-0.99264	0.256263	0.562227	-1.12445	0.562227


For 5.5-5.75 Hz

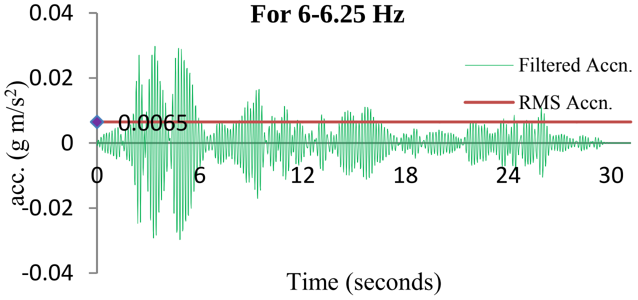
Output RMS value = 0.00659

f_0	Low pass		f_0	High pass	
5.75	Ω_c	0.377869	5.5	Ω_c	0.360022
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.28097	0.707708	0.107426	0.214852	0.107426
H_2	-1.02222	0.362752	0.086435	0.17287	0.086435
H_3	-0.91545	0.220421	0.077672	0.155345	0.077672
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.3228	0.716771	0.759892	-1.51978	0.759892
H_2	-1.06224	0.378619	0.610216	-1.22043	0.610216
H_3	-0.95378	0.23785	0.547908	-1.09582	0.547908


For 5.75-6 Hz

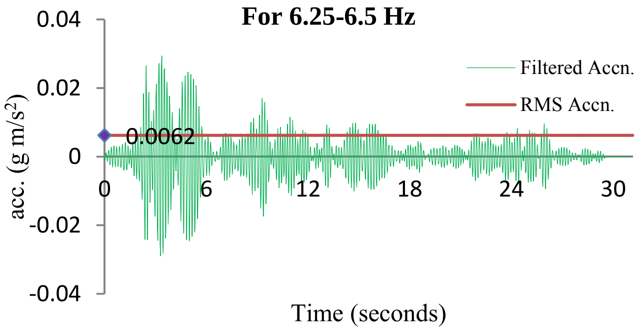
Output RMS value = 0.006598

f_0	Low pass		f_0	High pass	
6	Ω_c	0.395928	5.75	Ω_c	0.377869
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.23851	0.698984	0.115915	0.231831	0.115915
H_2	-0.98241	0.347665	0.092694	0.185388	0.092694
H_3	-0.87763	0.203933	0.083084	0.166169	0.083084
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.28097	0.707708	0.74717	-1.49434	0.74717
H_2	-1.02222	0.362752	0.596242	-1.19248	0.596242
H_3	-0.91545	0.220421	0.533968	-1.06794	0.533968


For 6-6.25 Hz

Output RMS value = 0.0065

f_0	Low pass		f_0	High pass	
6.25	Ω_c	0.414214	6	Ω_c	0.395928
low pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.19543	0.690599	0.124642	0.249285	0.124642
H_2	-0.94281	0.333333	0.099089	0.198178	0.099089
H_3	-0.84029	0.188345	0.088602	0.177204	0.088602
high pass coefficients					
	a_1	a_2	b_0	b_1	b_2
H_1	-1.23851	0.698984	0.734373	-1.46875	0.734373
H_2	-0.98241	0.347665	0.582518	-1.16504	0.582518
H_3	-0.87763	0.203933	0.520391	-1.04078	0.520391



Output RMS value = 0.0062

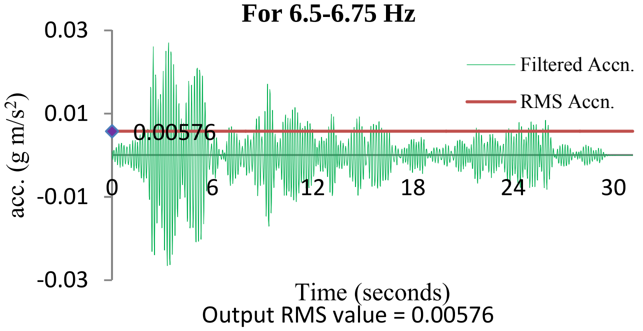
f_0	Low pass		f_0	High pass	
6.5	Ω_c	0.432739	6.25	Ω_c	0.414214

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.15179	0.682551	0.133599	0.267198	0.133599
H_2	-0.90342	0.319732	0.105616	0.211232	0.105616
H_3	-0.8034	0.17362	0.094222	0.188444	0.094222

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.19543	0.690599	0.721508	-1.44302	0.721508
H_2	-0.94281	0.333333	0.569036	-1.13807	0.569036
H_3	-0.84029	0.188345	0.507158	-1.01432	0.507158



Output RMS value = 0.00576

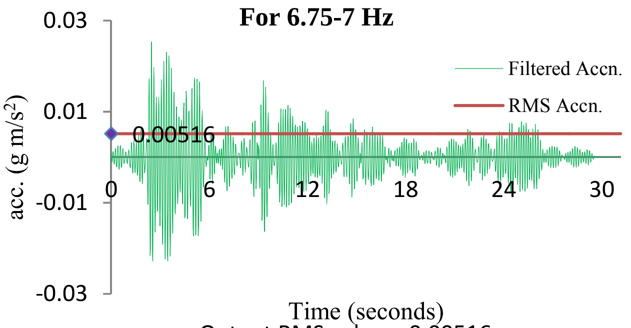
f_0	Low pass		f_0	High pass	
6.75	Ω_c	0.451517	6.5	Ω_c	0.432739

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.10759	0.674841	0.142778	0.285555	0.142778
H_2	-0.86423	0.306841	0.112271	0.224542	0.112271
H_3	-0.76694	0.159722	0.099942	0.199885	0.099942

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.15179	0.682551	0.708584	-1.41717	0.708584
H_2	-0.90342	0.319732	0.555788	-1.11158	0.555788
H_3	-0.8034	0.17362	0.494254	-0.98851	0.494254



Output RMS value = 0.00516

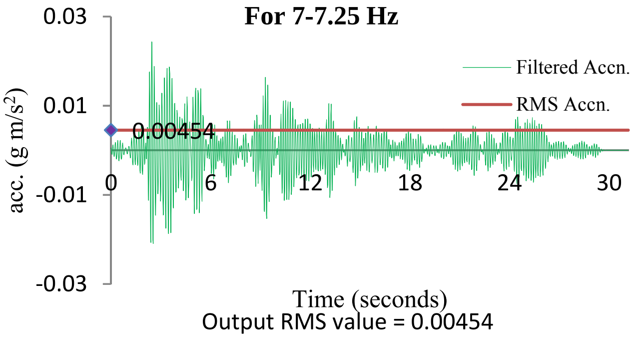
f_0	Low pass		f_0	High pass	
7	Ω_c	0.470564	6.75	Ω_c	0.451517

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.06288	0.667468	0.15217	0.30434	0.15217
H_2	-0.82523	0.294637	0.119051	0.238101	0.119051
H_3	-0.73088	0.146619	0.105761	0.211521	0.105761

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.10759	0.674841	0.695608	-1.39122	0.695608
H_2	-0.86423	0.306841	0.542767	-1.08553	0.542767
H_3	-0.76694	0.159722	0.481665	-0.96333	0.481665



Output RMS value = 0.00454

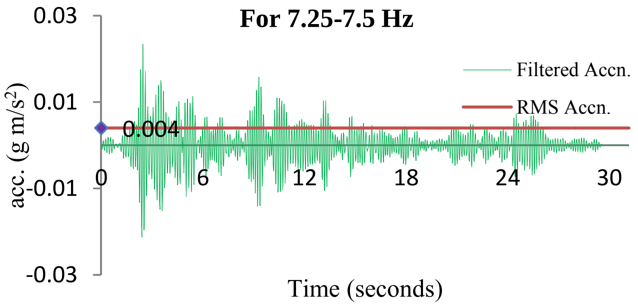
f_0	Low pass		f_0	High pass	
7.25	Ω_c	0.489895	7	Ω_c	0.470564

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.01769	0.66043	0.161769	0.323538	0.161769
H_2	-0.78642	0.283101	0.125951	0.251903	0.125951
H_3	-0.69521	0.134281	0.111675	0.223351	0.111675

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.06288	0.667468	0.682588	-1.36518	0.682588
H_2	-0.82523	0.294637	0.529967	-1.05993	0.529967
H_3	-0.73088	0.146619	0.469375	-0.93875	0.469375



Output RMS value = 0.004

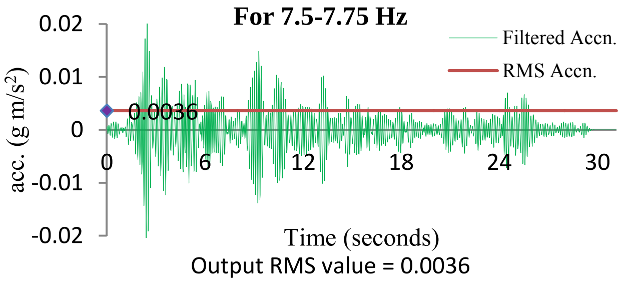
f_0	Low pass		f_0	High pass	
7.5	Ω_c	0.509525	7.25	Ω_c	0.489895

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.97204	0.653728	0.171567	0.343134	0.171567
H_2	-0.74779	0.272215	0.132971	0.265941	0.132971
H_3	-0.6599	0.122681	0.117685	0.235371	0.117685

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.01769	0.66043	0.66953	-1.33906	0.66953
H_2	-0.78642	0.283101	0.517381	-1.03476	0.517381
H_3	-0.69521	0.134281	0.457373	-0.91475	0.457373



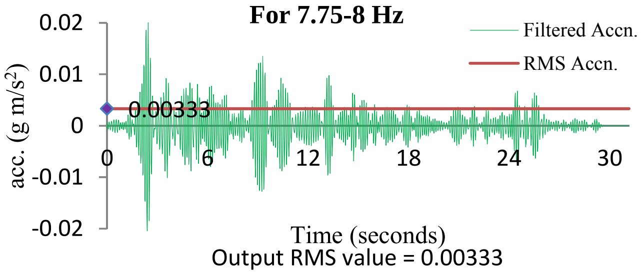
f_0	Low pass		f_0	High pass	
7.75	Ω_c	0.529473	7.5	Ω_c	0.509525

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.92595	0.647359	0.181558	0.363115	0.181558
H_2	-0.70933	0.261962	0.140106	0.280213	0.140106
H_3	-0.62492	0.111791	0.123789	0.247578	0.123789

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.97204	0.653728	0.656441	-1.31288	0.656441
H_2	-0.74779	0.272215	0.505001	-1.01	0.505001
H_3	-0.6599	0.122681	0.445644	-0.89129	0.445644



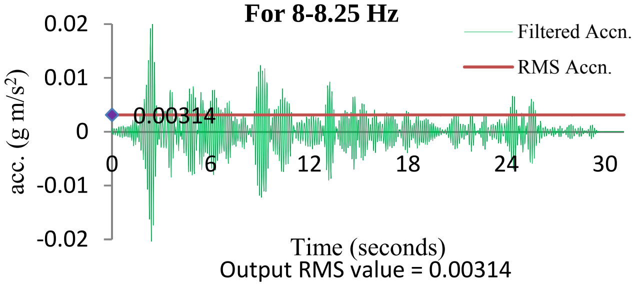
f_0	Low pass		f_0	High pass	
8	Ω_c	0.549755	7.75	Ω_c	0.529473

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.87947	0.641324	0.191733	0.383467	0.191733
H_2	-0.67103	0.252325	0.147356	0.294712	0.147356
H_3	-0.59026	0.10159	0.129986	0.259973	0.129986

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.92595	0.647359	0.643328	-1.28666	0.643328
H_2	-0.70933	0.261962	0.492822	-0.98564	0.492822
H_3	-0.62492	0.111791	0.434178	-0.86836	0.434178



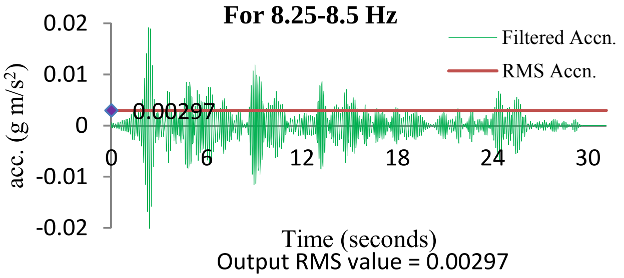
f_0	Low pass		f_0	High pass	
8.25	Ω_c	0.57039	8	Ω_c	0.549755

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.8326	0.635622	0.202088	0.404175	0.202088
H_2	-0.63289	0.243289	0.154719	0.309437	0.154719
H_3	-0.5559	0.092053	0.136276	0.272553	0.136276

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.87947	0.641324	0.630198	-1.2604	0.630198
H_2	-0.67103	0.252325	0.480838	-0.96168	0.480838
H_3	-0.59026	0.10159	0.422963	-0.84593	0.422963



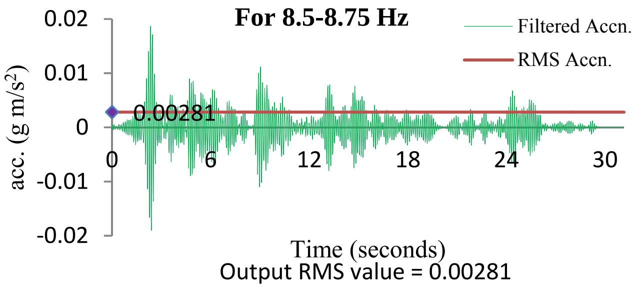
f_0	Low pass		f_0	High pass	
8.5	Ω_c	0.591398	8.25	Ω_c	0.57039

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.78538	0.630251	0.212614	0.425228	0.212614
H_2	-0.59489	0.23484	0.162192	0.324384	0.162192
H_3	-0.52182	0.083161	0.142659	0.285318	0.142659

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.8326	0.635622	0.617055	-1.23411	0.617055
H_2	-0.63289	0.243289	0.469044	-0.93809	0.469044
H_3	-0.5559	0.092053	0.411988	-0.82398	0.411988



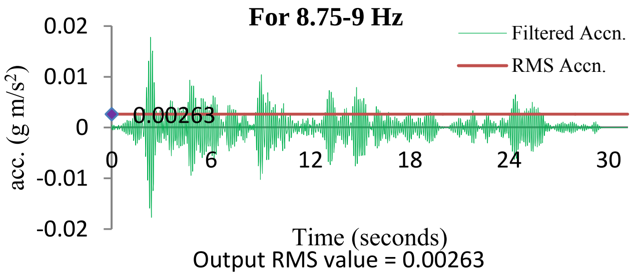
f_0	Low pass		f_0	High pass	
8.75	Ω_c	0.612801	8.5	Ω_c	0.591398

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.73783	0.625211	0.223307	0.446613	0.223307
H_2	-0.55703	0.226966	0.169776	0.339551	0.169776
H_3	-0.48799	0.074895	0.149135	0.29827	0.149135

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.78538	0.630251	0.603908	-1.20782	0.603908
H_2	-0.59489	0.23484	0.457432	-0.91486	0.457432
H_3	-0.52182	0.083161	0.401245	-0.80249	0.401245



f_0	Low pass		f_0	High pass	
9	Ω_c	0.634619	8.75	Ω_c	0.612801

low pass coefficients

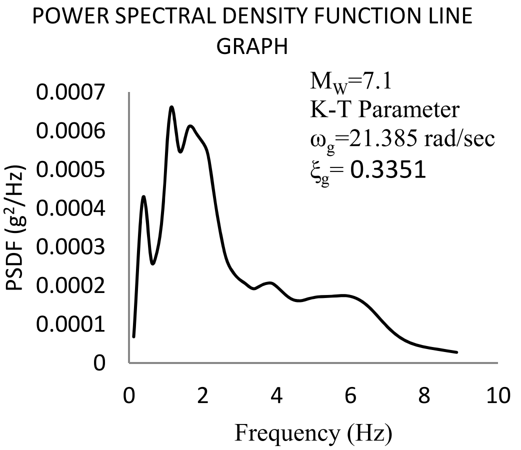
	a_1	a_2	b_0	b_1	b_2
H_1	-0.68998	0.620501	0.234159	0.468318	0.234159
H_2	-0.5193	0.219654	0.177468	0.354937	0.177468
H_3	-0.45441	0.067238	0.155704	0.311408	0.155704

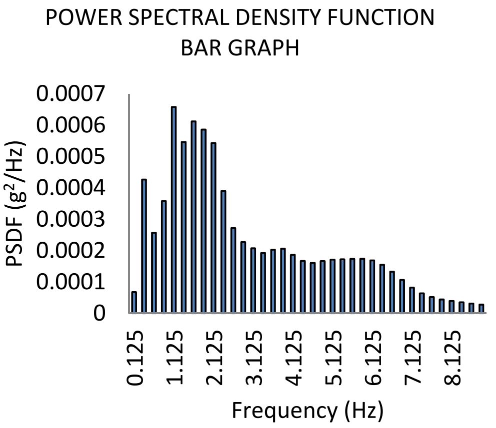
high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.73783	0.625211	0.59076	-1.18152	0.59076
H_2	-0.55703	0.226966	0.445999	-0.892	0.445999
H_3	-0.48799	0.074895	0.390722	-0.78144	0.390722

Calculation and plot (line and bar graph) of PSDF of El Centro earthquake

Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	$GRMS^2$	Bandwidth (Hz)	PSDF = $\frac{\mathrm{GRMS}^2}{\mathrm{Bandwidth}}$
0.01-0.25	0.125	0.004120738	1.7E-05	0.25	6.79219E-05
0.25-0.5	0.375	0.010330983	0.000107	0.25	0.000426917
0.5-0.75	0.625	0.008018163	6.43E-05	0.25	0.000257164
0.75-1	0.875	0.00945652	8.94E-05	0.25	0.000357703
1-1.25	1.125	0.012821477	0.000164	0.25	0.000657561
1.25-1.5	1.375	0.011685376	0.000137	0.25	0.000546192
1.5-1.75	1.625	0.012365829	0.000153	0.25	0.000611655
1.75-2	1.875	0.012099939	0.000146	0.25	0.000585634
2-2.25	2.125	0.011652497	0.000136	0.25	0.000543123
2.25-2.5	2.375	0.009878476	9.76E-05	0.25	0.000390337
2.5-2.75	2.625	0.00824861	6.8E-05	0.25	0.000272158
2.75-3	2.875	0.007541259	5.69E-05	0.25	0.000227482
3-3.25	3.125	0.00719883	5.18E-05	0.25	0.000207293
3.25-3.5	3.375	0.006932189	4.81E-05	0.25	0.000192221
3.5-3.75	3.625	0.007125979	5.08E-05	0.25	0.000203118
3.75-4	3.875	0.007177905	5.15E-05	0.25	0.000206089
4-4.25	4.125	0.006832028	4.67E-05	0.25	0.000186706
4.25-4.5	4.375	0.006456422	4.17E-05	0.25	0.000166742
4.5-4.75	4.625	0.006343397	4.02E-05	0.25	0.000160955
4.75-5	4.875	0.006453536	4.16E-05	0.25	0.000166593
5-5.25	5.125	0.00654399	4.28E-05	0.25	0.000171295
5.25-5.5	5.375	0.006560454	4.3E-05	0.25	0.000172158
5.5-5.75	5.625	0.006588891	4.34E-05	0.25	0.000173654
5.75-6	5.875	0.006598319	4.35E-05	0.25	0.000174151
6-6.25	6.125	0.006493762	4.22E-05	0.25	0.000168676
6.25-6.5	6.375	0.006222936	3.87E-05	0.25	0.0001549
6.5-6.75	6.625	0.005763523	3.32E-05	0.25	0.000132873
6.75-7	6.875	0.005160782	2.66E-05	0.25	0.000106535
7-7.25	7.125	0.004535617	2.06E-05	0.25	8.22873E-05
7.25-7.5	7.375	0.004003009	1.6E-05	0.25	6.40963E-05
7.5-7.75	7.625	0.003607466	1.3E-05	0.25	5.20552E-05
7.75-8	7.875	0.003332696	1.11E-05	0.25	4.44274E-05
8-8.25	8.125	0.003136752	9.84E-06	0.25	3.93569E-05
8.25-8.5	8.375	0.002972861	8.84E-06	0.25	3.53516E-05
8.5-8.75	8.625	0.002807518	7.88E-06	0.25	3.15286E-05
8.75-9	8.875	0.002634262	6.94E-06	0.25	2.77573E-05

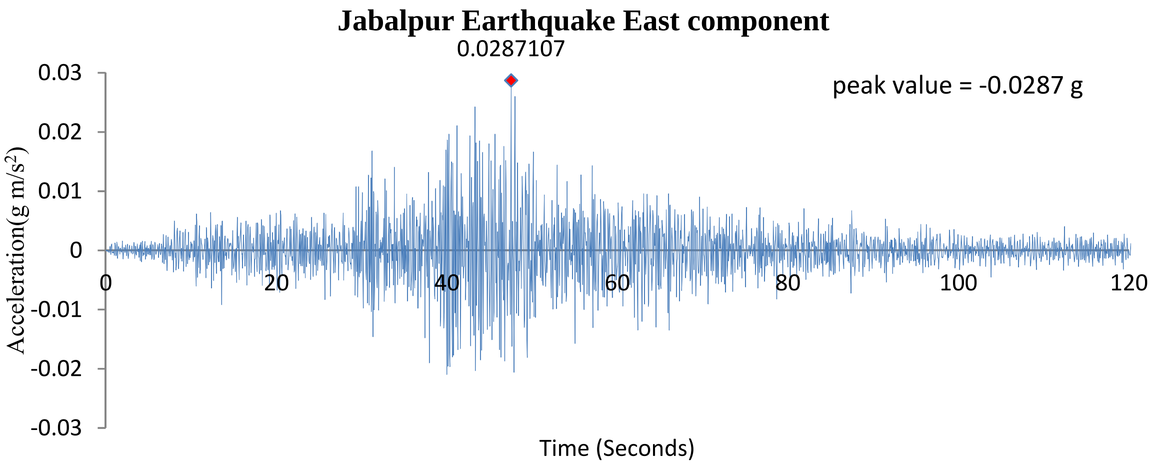


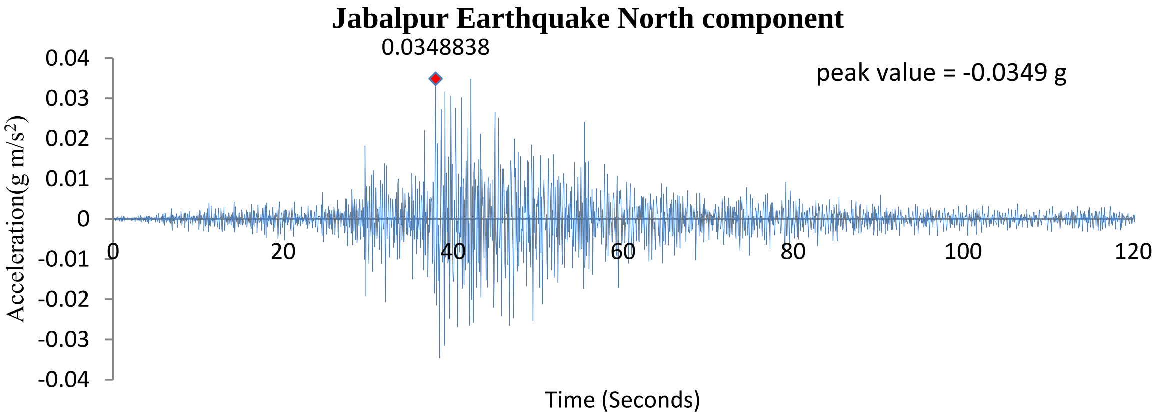


6.2 Jabalpur Earthquake

Date: 22 May 1997
Time: 04:22 AM (Local time)
Location of earthquake: 8 km southeast of the city of jabalpur
Component: N-S and E-W Component
Location of epicenter: Latitude 23.18^0 N Longitude 80.02^0E
Magnitude of Earthquake (M_W): 6.0
Location of station: Bhopal (Station code: BHPL) Latitude $23^0$14.46'N; Longitude $77^0$25.47'E
Sampling Rate: 20 (0.05 sec time interval)
Epicentral Distance of recorded station: 275 km

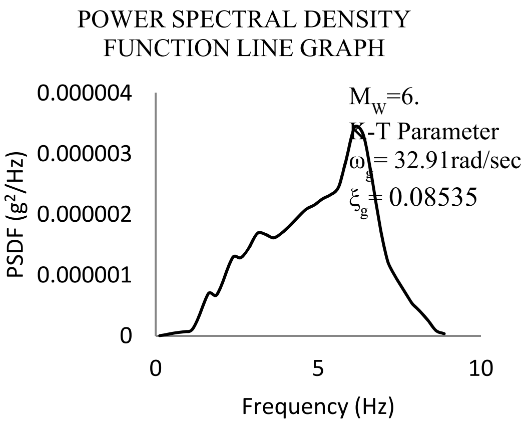
Impact and other details: The maximum damage was in the districts of Jabalpur and Mandla. About 8546 houses collapsed and about 52,690 houses were badly damaged. In all, 887 villages (or equivalent) were affected. More than 90% of houses collapsed or were badly damaged in at least two of these villages with a population of about 500. During this earthquake, about 38 persons died and about 350 were injured.. The soil in the area is known as **"black cotton soil"**; it is black or dark gray and contains a high percentage of montmorillonite. This soil has very high compressibility and shrinkage, and very high swelling characteristics.

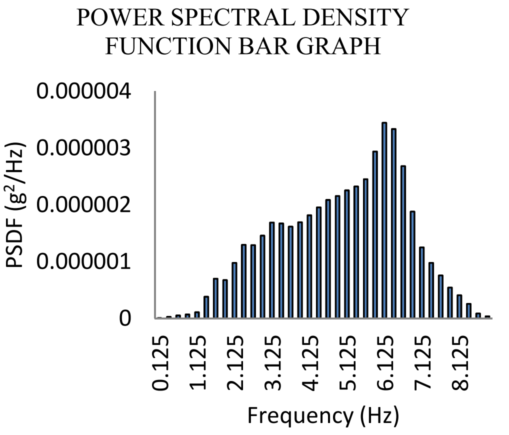




Calculation and plot (line and bar graph) of PSDF of Jabalpur earthquake 1997 (East Component)

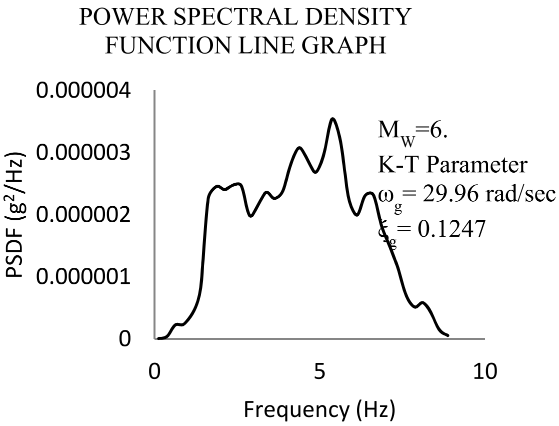
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF $=\frac{\text{GRMS}^2}{\text{Bandwidth}}$
0.01-0.25	0.125	2.6105E-05	6.815E-10	0.25	2.72587E-09
0.25-0.5	0.375	7.9355E-05	6.297E-09	0.25	2.51889E-08
0.5-0.75	0.625	0.0001126	1.268E-08	0.25	5.07148E-08
0.75-1	0.875	0.00012983	1.686E-08	0.25	6.7428E-08
1-1.25	1.125	0.00016261	2.644E-08	0.25	1.05769E-07
1.25-1.5	1.375	0.00030861	9.524E-08	0.25	3.80959E-07
1.5-1.75	1.625	0.00041791	1.747E-07	0.25	6.98611E-07
1.75-2	1.875	0.00041038	1.684E-07	0.25	6.73634E-07
2-2.25	2.125	0.00049443	2.445E-07	0.25	9.77828E-07
2.25-2.5	2.375	0.00056912	3.239E-07	0.25	1.29557E-06
2.5-2.75	2.625	0.00056782	3.224E-07	0.25	1.28969E-06
2.75-3	2.875	0.00060345	3.642E-07	0.25	1.4566E-06
3-3.25	3.125	0.00064961	4.22E-07	0.25	1.68798E-06
3.25-3.5	3.375	0.00064655	4.18E-07	0.25	1.67212E-06
3.5-3.75	3.625	0.00063541	4.037E-07	0.25	1.61499E-06
3.75-4	3.875	0.00065064	4.233E-07	0.25	1.69332E-06
4-4.25	4.125	0.00067347	4.536E-07	0.25	1.81422E-06
4.25-4.5	4.375	0.00069895	4.885E-07	0.25	1.95415E-06
4.5-4.75	4.625	0.00072185	5.211E-07	0.25	2.08426E-06
4.75-5	4.875	0.00073384	5.385E-07	0.25	2.15411E-06
5-5.25	5.125	0.00075068	5.635E-07	0.25	2.2541E-06
5.25-5.5	5.375	0.00076215	5.809E-07	0.25	2.32347E-06
5.5-5.75	5.625	0.00078269	6.126E-07	0.25	2.45042E-06
5.75-6	5.875	0.00085688	7.342E-07	0.25	2.93694E-06
6-6.25	6.125	0.00092783	8.609E-07	0.25	3.44345E-06
6.25-6.5	6.375	0.00091267	8.33E-07	0.25	3.33189E-06
6.5-6.75	6.625	0.00081883	6.705E-07	0.25	2.68196E-06
6.75-7	6.875	0.00068583	4.704E-07	0.25	1.88143E-06
7-7.25	7.125	0.00055919	3.127E-07	0.25	1.25076E-06
7.25-7.5	7.375	0.00049402	2.441E-07	0.25	9.76237E-07
7.5-7.75	7.625	0.00043526	1.895E-07	0.25	7.57817E-07
7.75-8	7.875	0.00036803	1.354E-07	0.25	5.41785E-07
8-8.25	8.125	0.00031906	1.018E-07	0.25	4.072E-07
8.25-8.5	8.375	0.00025267	6.384E-08	0.25	2.5536E-07
8.5-8.75	8.625	0.00014638	2.143E-08	0.25	8.57121E-08
8.75-9	8.875	9.3167E-05	8.68E-09	0.25	3.47203E-08

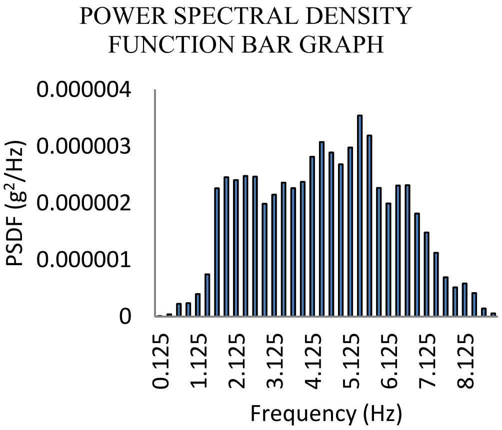


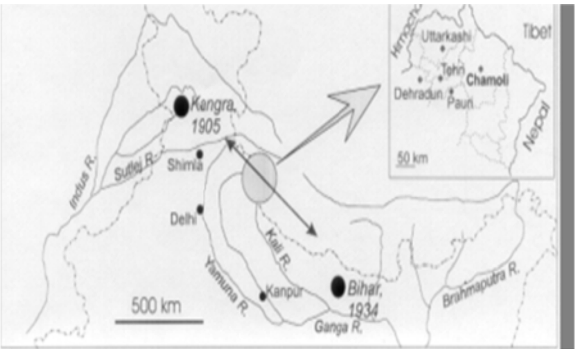


Calculation and plot (line and bar graph) of PSDF of Jabalpur earthquake 1997 (North Component)

Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF =$\frac{\text{GRMS}^2}{\text{Bandwidth}}$
0.01-0.25	0.125	4.01314E-05	1.61E-09	0.25	6.44213E-09
0.25-0.5	0.375	9.77509E-05	9.56E-09	0.25	3.82209E-08
0.5-0.75	0.625	0.000236587	5.6E-08	0.25	2.23893E-07
0.75-1	0.875	0.000241913	5.85E-08	0.25	2.34087E-07
1-1.25	1.125	0.000315025	9.92E-08	0.25	3.96964E-07
1.25-1.5	1.375	0.000431419	1.86E-07	0.25	7.44491E-07
1.5-1.75	1.625	0.000751697	5.65E-07	0.25	2.26019E-06
1.75-2	1.875	0.000783379	6.14E-07	0.25	2.45473E-06
2-2.25	2.125	0.000775316	6.01E-07	0.25	2.40446E-06
2.25-2.5	2.375	0.000786773	6.19E-07	0.25	2.47605E-06
2.5-2.75	2.625	0.000785199	6.17E-07	0.25	2.46615E-06
2.75-3	2.875	0.000704918	4.97E-07	0.25	1.98764E-06
3-3.25	3.125	0.000732833	5.37E-07	0.25	2.14817E-06
3.25-3.5	3.375	0.000768031	5.9E-07	0.25	2.35949E-06
3.5-3.75	3.625	0.000752127	5.66E-07	0.25	2.26278E-06
3.75-4	3.875	0.000770376	5.93E-07	0.25	2.37392E-06
4-4.25	4.125	0.000838892	7.04E-07	0.25	2.81496E-06
4.25-4.5	4.375	0.00087709	7.69E-07	0.25	3.07715E-06
4.5-4.75	4.625	0.000850048	7.23E-07	0.25	2.89032E-06
4.75-5	4.875	0.000819151	6.71E-07	0.25	2.68403E-06
5-5.25	5.125	0.000862755	7.44E-07	0.25	2.97738E-06
5.25-5.5	5.375	0.00094097	8.85E-07	0.25	3.5417E-06
5.5-5.75	5.625	0.000893122	7.98E-07	0.25	3.19067E-06
5.75-6	5.875	0.00075282	5.67E-07	0.25	2.26695E-06
6-6.25	6.125	0.00070587	4.98E-07	0.25	1.99301E-06
6.25-6.5	6.375	0.00075968	5.77E-07	0.25	2.30846E-06
6.5-6.75	6.625	0.00076097	5.79E-07	0.25	2.3163E-06
6.75-7	6.875	0.000673512	4.54E-07	0.25	1.81447E-06
7-7.25	7.125	0.000608463	3.7E-07	0.25	1.48091E-06
7.25-7.5	7.375	0.000529976	2.81E-07	0.25	1.1235E-06
7.5-7.75	7.625	0.000415084	1.72E-07	0.25	6.89178E-07
7.75-8	7.875	0.000358157	1.28E-07	0.25	5.13106E-07
8-8.25	8.125	0.000381579	1.46E-07	0.25	5.82409E-07
8.25-8.5	8.375	0.000321052	1.03E-07	0.25	4.12298E-07
8.5-8.75	8.625	0.000188057	3.54E-08	0.25	1.41462E-07
8.75-9	8.875	0.000117122	1.37E-08	0.25	5.48702E-08



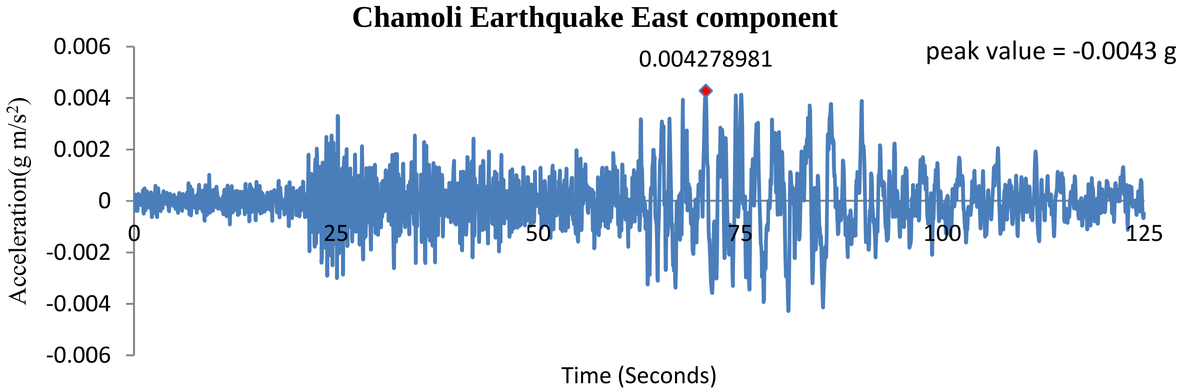


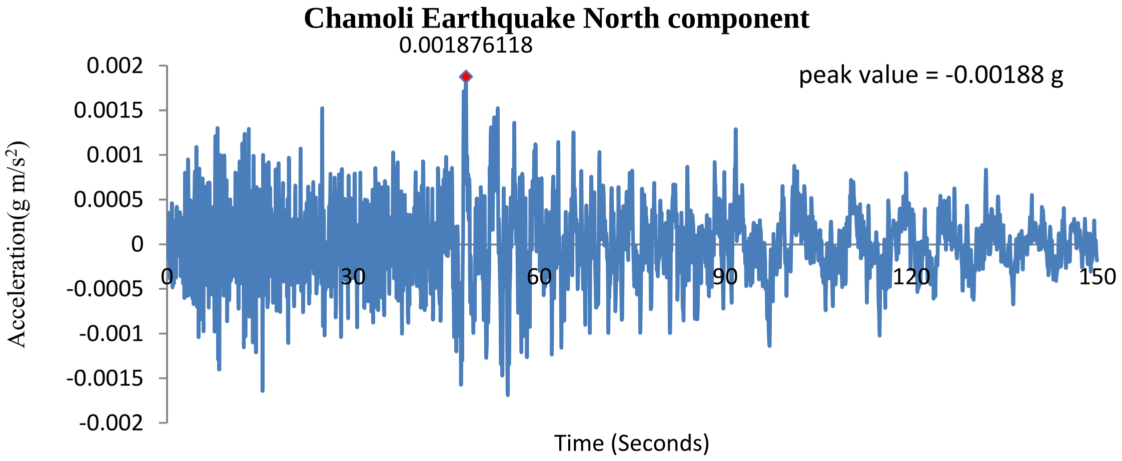


6.3 Chamoli Earthquake

Date: 29 March 1999
Time: 00:35:13.59 hours (local time)
Location of earthquake: Chamoli (Uttarakhand)
Component: N-S and E-W Component
Location of epicenter: Latitude 30.4^0 N Longitude 79.3^0 E
Magnitude of Earthquake (M_W): 6.8
Location of station: Bhopal
Sampling Rate: 20 (0.05 sec interval)
Epicentral Distance of recorded station: 798 km

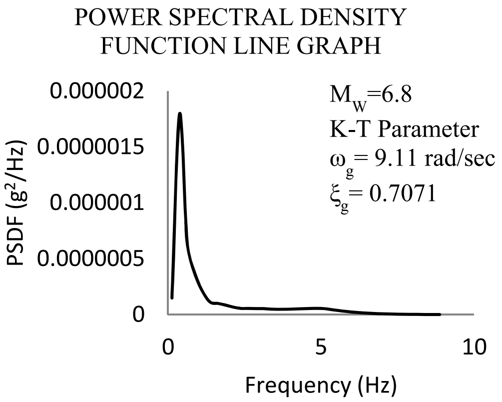
Impact and other details: Severe ground deformations resulted from the earthquake. Formation of ground fissures were reported from many areas. Landslides and changes in the groundwater flow were also reported. Well-developed ground cracks were seen in Gopeshwar, Chamoli and Bairagna. Cracks were observed in asphalt roads at several locations. The death toll was 103. Several hundred people injured and approximately 50,000 houses were damaged. Over 2,000 villages were affected by the earthquake. The site of earthquake was rock type.

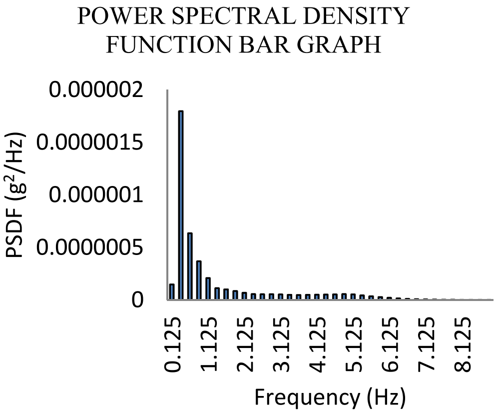




Calculation and plot (line and bar graph) of PSDF of Chamoli earthquake 1999(East Component)

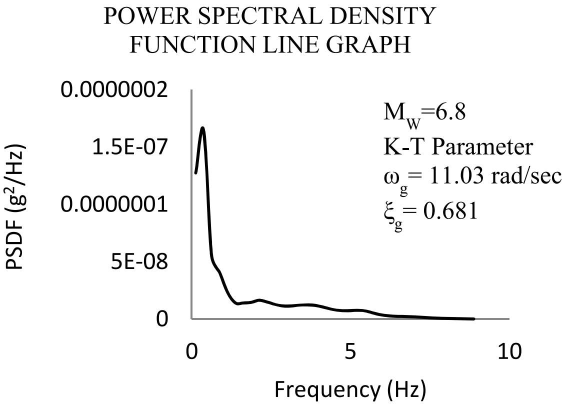
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF $=\frac{\text{GRMS}^2}{\text{Bandwidth}}$
0.01-0.25	0.125	0.000192265	3.6966E-08	0.25	1.47864E-07
0.25-0.5	0.375	0.000669958	4.4884E-07	0.25	1.79537E-06
0.5-0.75	0.625	0.000398373	1.587E-07	0.25	6.34805E-07
0.75-1	0.875	0.000302981	9.1798E-08	0.25	3.67191E-07
1-1.25	1.125	0.00022742	5.172E-08	0.25	2.0688E-07
1.25-1.5	1.375	0.000166794	2.782E-08	0.25	1.11281E-07
1.5-1.75	1.625	0.000157937	2.4944E-08	0.25	9.97764E-08
1.75-2	1.875	0.000145764	2.1247E-08	0.25	8.49882E-08
2-2.25	2.125	0.000129474	1.6764E-08	0.25	6.70545E-08
2.25-2.5	2.375	0.000117544	1.3817E-08	0.25	5.52668E-08
2.5-2.75	2.625	0.000116667	1.3611E-08	0.25	5.44443E-08
2.75-3	2.875	0.000115783	1.3406E-08	0.25	5.36227E-08
3-3.25	3.125	0.000114997	1.3224E-08	0.25	5.28975E-08
3.25-3.5	3.375	0.000109924	1.2083E-08	0.25	4.83331E-08
3.5-3.75	3.625	0.000108067	1.1678E-08	0.25	4.6714E-08
3.75-4	3.875	0.00010994	1.2087E-08	0.25	4.83475E-08
4-4.25	4.125	0.000111151	1.2354E-08	0.25	4.9418E-08
4.25-4.5	4.375	0.000113463	1.2874E-08	0.25	5.14952E-08
4.5-4.75	4.625	0.000115664	1.3378E-08	0.25	5.35131E-08
4.75-5	4.875	0.000118339	1.4004E-08	0.25	5.60166E-08
5-5.25	5.125	0.00011457	1.3126E-08	0.25	5.25056E-08
5.25-5.5	5.375	0.000104137	1.0844E-08	0.25	4.3378E-08
5.5-5.75	5.625	9.32249E-05	8.6909E-09	0.25	3.47635E-08
5.75-6	5.875	8.17412E-05	6.6816E-09	0.25	2.67265E-08
6-6.25	6.125	6.99163E-05	4.8883E-09	0.25	1.95531E-08
6.25-6.5	6.375	6.02171E-05	3.6261E-09	0.25	1.45044E-08
6.5-6.75	6.625	5.06827E-05	2.5687E-09	0.25	1.0275E-08
6.75-7	6.875	4.27201E-05	1.825E-09	0.25	7.30004E-09
7-7.25	7.125	3.7791E-05	1.4282E-09	0.25	5.71264E-09
7.25-7.5	7.375	3.27799E-05	1.0745E-09	0.25	4.29808E-09
7.5-7.75	7.625	2.83039E-05	8.0111E-10	0.25	3.20444E-09
7.75-8	7.875	2.49497E-05	6.2249E-10	0.25	2.48995E-09
8-8.25	8.125	2.15397E-05	4.6396E-10	0.25	1.85584E-09
8.25-8.5	8.375	1.55944E-05	2.4318E-10	0.25	9.72736E-10
8.5-8.75	8.625	9.96966E-06	9.9394E-11	0.25	3.97577E-10
8.75-9	8.875	5.1482E-06	2.6504E-11	0.25	1.06016E-10

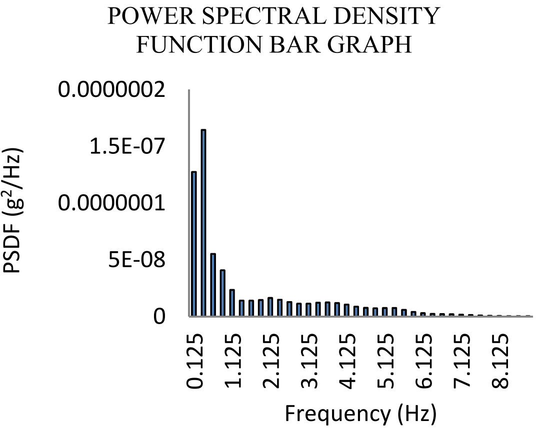




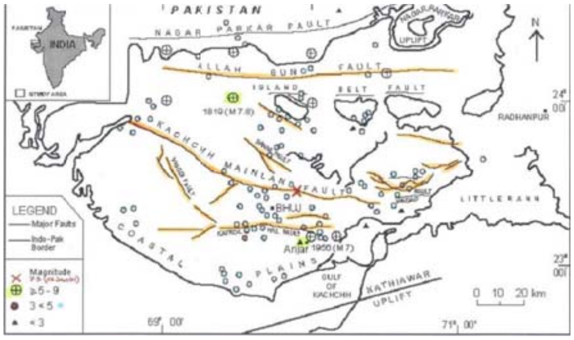
Calculation and plot (line and bar graph) of PSDF of Chamoli earthquake 1999 (North Component)

Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF $=\frac{\mathrm{GRMS}^2}{\mathrm{Bandwidth}}$
0.01-0.25	0.125	0.000178478	3.19E-08	0.25	1.27417E-07
0.25-0.5	0.375	0.0002027	4.11E-08	0.25	1.64349E-07
0.5-0.75	0.625	0.000117395	1.38E-08	0.25	5.51264E-08
0.75-1	0.875	0.000100848	1.02E-08	0.25	4.06815E-08
1-1.25	1.125	7.64373E-05	5.84E-09	0.25	2.33706E-08
1.25-1.5	1.375	5.91001E-05	3.49E-09	0.25	1.39713E-08
1.5-1.75	1.625	5.91527E-05	3.5E-09	0.25	1.39962E-08
1.75-2	1.875	6.03077E-05	3.64E-09	0.25	1.45481E-08
2-2.25	2.125	6.39809E-05	4.09E-09	0.25	1.63742E-08
2.25-2.5	2.375	6.0738E-05	3.69E-09	0.25	1.47564E-08
2.5-2.75	2.625	5.64542E-05	3.19E-09	0.25	1.27483E-08
2.75-3	2.875	5.32294E-05	2.83E-09	0.25	1.13335E-08
3-3.25	3.125	5.30992E-05	2.82E-09	0.25	1.12781E-08
3.25-3.5	3.375	5.50924E-05	3.04E-09	0.25	1.21407E-08
3.5-3.75	3.625	5.54826E-05	3.08E-09	0.25	1.23133E-08
3.75-4	3.875	5.44282E-05	2.96E-09	0.25	1.18497E-08
4-4.25	4.125	5.11868E-05	2.62E-09	0.25	1.04803E-08
4.25-4.5	4.375	4.66564E-05	2.18E-09	0.25	8.70727E-09
4.5-4.75	4.625	4.35838E-05	1.9E-09	0.25	7.59818E-09
4.75-5	4.875	4.23023E-05	1.79E-09	0.25	7.15793E-09
5-5.25	5.125	4.32697E-05	1.87E-09	0.25	7.48908E-09
5.25-5.5	5.375	4.32515E-05	1.87E-09	0.25	7.48277E-09
5.5-5.75	5.625	3.84207E-05	1.48E-09	0.25	5.90459E-09
5.75-6	5.875	3.18954E-05	1.02E-09	0.25	4.06926E-09
6-6.25	6.125	2.73274E-05	7.47E-10	0.25	2.98715E-09
6.25-6.5	6.375	2.43436E-05	5.93E-10	0.25	2.37044E-09
6.5-6.75	6.625	2.35089E-05	5.53E-10	0.25	2.21068E-09
6.75-7	6.875	2.25131E-05	5.07E-10	0.25	2.02736E-09
7-7.25	7.125	2.04822E-05	4.2E-10	0.25	1.67809E-09
7.25-7.5	7.375	1.76667E-05	3.12E-10	0.25	1.24845E-09
7.5-7.75	7.625	1.42249E-05	2.02E-10	0.25	8.0939E-10
7.75-8	7.875	1.17933E-05	1.39E-10	0.25	5.56325E-10
8-8.25	8.125	9.95915E-06	9.92E-11	0.25	3.96739E-10
8.25-8.5	8.375	7.86267E-06	6.18E-11	0.25	2.47286E-10
8.5-8.75	8.625	5.71573E-06	3.27E-11	0.25	1.30678E-10
8.75-9	8.875	3.62588E-06	1.31E-11	0.25	5.25879E-11





6.4 Bhuj Earthquake

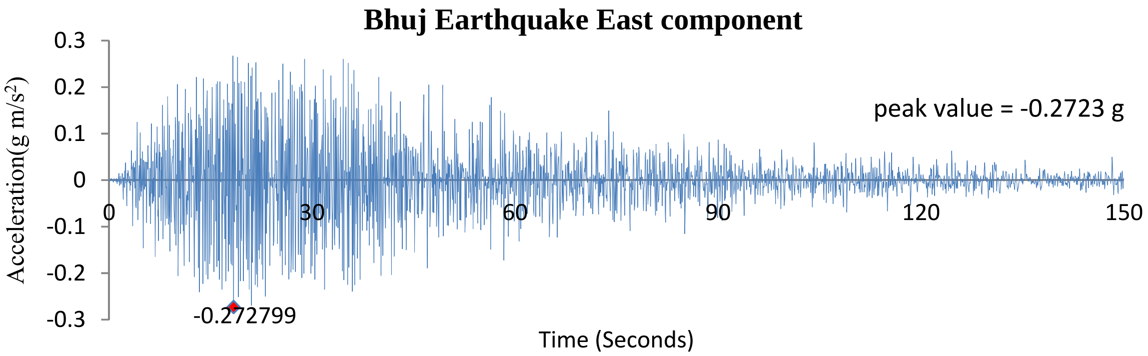


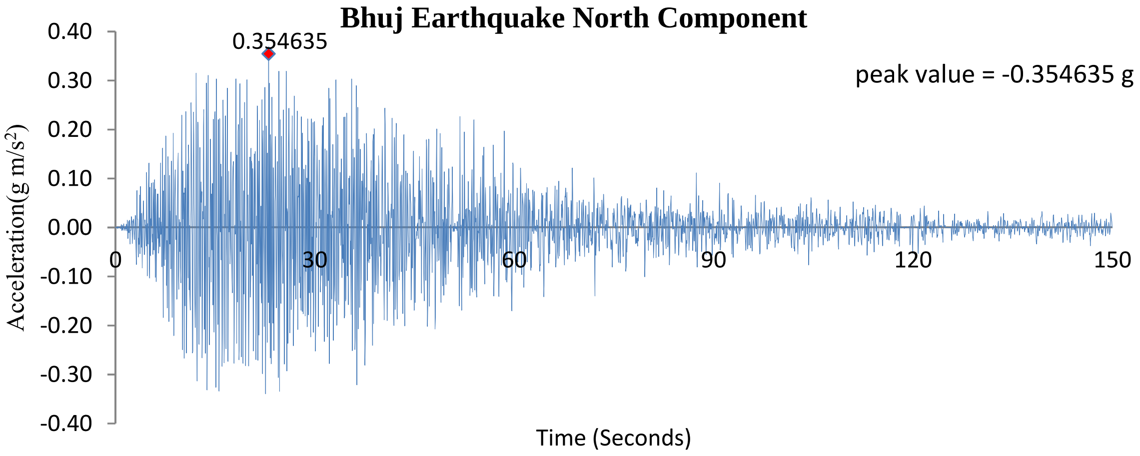
Date: 26 January 2001
Time: 8:46 AM (local time)
Location of earthquake: Kutch area (Gujarat)
Component: N-S and E-W Component
Location of epicenter: Latitude 23.4^0 N Longitude 70.3^0 E
Magnitude of Earthquake (M_W): 7.7
Location of station: Bhuj
Sampling Rate: 20 (0.05 sec interval)
Epicentral Distance of recorded station: 99 km

Impact and other details: Over 18,600 persons are reported to be dead and over 167,000 injured; the number of deaths is expected to rise with more information coming in. The estimated economic loss due to this quake is placed at around Rs.22,000 Crores. The earthquake was felt in most parts of the country and a large area sustained damages. About 20 districts in the state of Gujarat sustained damage.

The geology of Kutch is very complex.

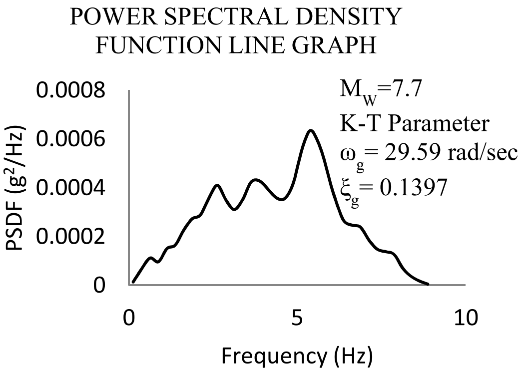
- Most of Kutch outside the Great and Little Rann and the Gulf of Kutch is on high ground, generally overlain by hard volcanic rock (Basalt) or deep layers of weathered Sandstones, Shales, marls and limestone
- Other areas, for instance around the towns of Anjar are additionally underlain by clays
- Coastal and lowland areas are on water logged loose sand

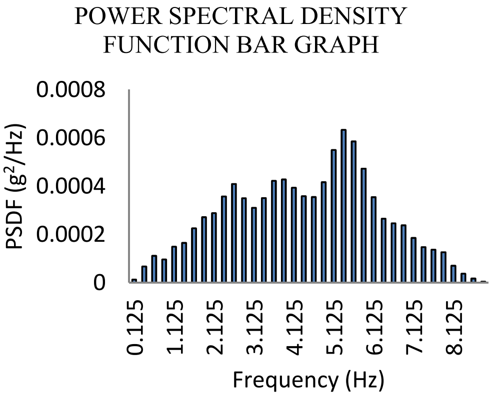




Calculation and plot (line and bar graph) of PSDF of Bhuj earthquake 2001 (East Component)

Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF = $\frac{\text{GRMS}^2}{\text{Bandwidth}}$
0.01-0.25	0.125	0.00182865	3.34E-06	0.25	1.33758E-05
0.25-0.5	0.375	0.00410224	1.68E-05	0.25	6.73136E-05
0.5-0.75	0.625	0.00527949	2.79E-05	0.25	0.000111492
0.75-1	0.875	0.00491274	2.41E-05	0.25	9.65402E-05
1-1.25	1.125	0.00612314	3.75E-05	0.25	0.000149971
1.25-1.5	1.375	0.00642523	4.13E-05	0.25	0.000165135
1.5-1.75	1.625	0.00751308	5.64E-05	0.25	0.000225786
1.75-2	1.875	0.00824642	6.8E-05	0.25	0.000272014
2-2.25	2.125	0.00849914	7.22E-05	0.25	0.000288941
2.25-2.5	2.375	0.00945769	8.94E-05	0.25	0.000357791
2.5-2.75	2.625	0.01012006	0.000102	0.25	0.000409663
2.75-3	2.875	0.00936333	8.77E-05	0.25	0.000350688
3-3.25	3.125	0.0088105	7.76E-05	0.25	0.000310499
3.25-3.5	3.375	0.00936645	8.77E-05	0.25	0.000350922
3.5-3.75	3.625	0.01028218	0.000106	0.25	0.000422892
3.75-4	3.875	0.01034433	0.000107	0.25	0.000428021
4-4.25	4.125	0.00992746	9.86E-05	0.25	0.000394218
4.25-4.5	4.375	0.00947319	8.97E-05	0.25	0.000358965
4.5-4.75	4.625	0.009427	8.89E-05	0.25	0.000355473
4.75-5	4.875	0.01020762	0.000104	0.25	0.000416782
5-5.25	5.125	0.01173243	0.000138	0.25	0.000550599
5.25-5.5	5.375	0.01258706	0.000158	0.25	0.000633736
5.5-5.75	5.625	0.01210208	0.000146	0.25	0.000585842
5.75-6	5.875	0.01087898	0.000118	0.25	0.000473409
6-6.25	6.125	0.00941639	8.87E-05	0.25	0.000354674
6.25-6.5	6.375	0.00815218	6.65E-05	0.25	0.000265832
6.5-6.75	6.625	0.00784697	6.16E-05	0.25	0.0002463
6.75-7	6.875	0.00771559	5.95E-05	0.25	0.000238122
7-7.25	7.125	0.00681563	4.65E-05	0.25	0.000185811
7.25-7.5	7.375	0.0060768	3.69E-05	0.25	0.00014771
7.5-7.75	7.625	0.00586442	3.44E-05	0.25	0.000137566
7.75-8	7.875	0.00562624	3.17E-05	0.25	0.000126618
8-8.25	8.125	0.00420345	1.77E-05	0.25	7.06759E-05
8.25-8.5	8.375	0.00305835	9.35E-06	0.25	3.7414E-05
8.5-8.75	8.625	0.00210145	4.42E-06	0.25	1.76644E-05
8.75-9	8.875	0.00108104	1.17E-06	0.25	4.67459E-06

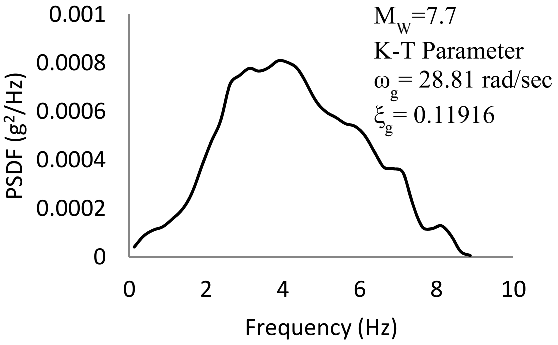




Calculation and plot (line and bar graph) of PSDF of Bhuj earthquake 2001 (North Component)

Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	$GRMS^2$	Bandwidth (Hz)	PSDF $=\frac{GRMS^2}{Bandwidth}$
0.01-0.25	0.125	0.003194038	1.02E-05	0.25	4.08075E-05
0.25-0.5	0.375	0.004653582	2.17E-05	0.25	8.66233E-05
0.5-0.75	0.625	0.005263511	2.77E-05	0.25	0.000110818
0.75-1	0.875	0.005585647	3.12E-05	0.25	0.000124798
1-1.25	1.125	0.006273953	3.94E-05	0.25	0.00015745
1.25-1.5	1.375	0.006991539	4.89E-05	0.25	0.000195526
1.5-1.75	1.625	0.008091188	6.55E-05	0.25	0.000261869
1.75-2	1.875	0.009612635	9.24E-05	0.25	0.000369611
2-2.25	2.125	0.010867966	0.000118	0.25	0.000472451
2.25-2.5	2.375	0.011847514	0.00014	0.25	0.000561454
2.5-2.75	2.625	0.013354654	0.000178	0.25	0.000713387
2.75-3	2.875	0.013683888	0.000187	0.25	0.000748995
3-3.25	3.125	0.01394263	0.000194	0.25	0.000777588
3.25-3.5	3.375	0.013840751	0.000192	0.25	0.000766266
3.5-3.75	3.625	0.013968805	0.000195	0.25	0.00078051
3.75-4	3.875	0.014219116	0.000202	0.25	0.000808733
4-4.25	4.125	0.014158628	0.0002	0.25	0.000801867
4.25-4.5	4.375	0.013931351	0.000194	0.25	0.00077633
4.5-4.75	4.625	0.013331686	0.000178	0.25	0.000710935
4.75-5	4.875	0.012637664	0.00016	0.25	0.000638842
5-5.25	5.125	0.012225477	0.000149	0.25	0.000597849
5.25-5.5	5.375	0.011993395	0.000144	0.25	0.000575366
5.5-5.75	5.625	0.011723698	0.000137	0.25	0.00054978
5.75-6	5.875	0.011603631	0.000135	0.25	0.000538577
6-6.25	6.125	0.011203873	0.000126	0.25	0.000502107
6.25-6.5	6.375	0.010428345	0.000109	0.25	0.000435002
6.5-6.75	6.625	0.009622484	9.26E-05	0.25	0.000370369
6.75-7	6.875	0.00953658	9.09E-05	0.25	0.000363785
7-7.25	7.125	0.009329816	8.7E-05	0.25	0.000348182
7.25-7.5	7.375	0.007418681	5.5E-05	0.25	0.000220147
7.5-7.75	7.625	0.005570668	3.1E-05	0.25	0.000124129
7.75-8	7.875	0.005392697	2.91E-05	0.25	0.000116325
8-8.25	8.125	0.005663939	3.21E-05	0.25	0.000128321
8.25-8.5	8.375	0.004698462	2.21E-05	0.25	8.83022E-05
8.5-8.75	8.625	0.002386587	5.7E-06	0.25	2.27832E-05
8.75-9	8.875	0.001211271	1.47E-06	0.25	5.86871E-06

POWER SPECTRAL DENSITY FUNCTION LINE GRAPH



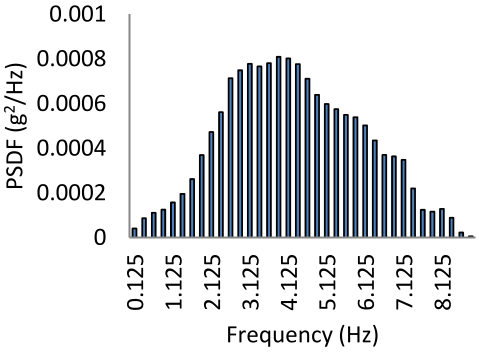
M_W=7.7

K-T Parameter

ω_g = 28.81 rad/sec

ξ_g = 0.11916

POWER SPECTRAL DENSITY FUNCTION BAR GRAPH

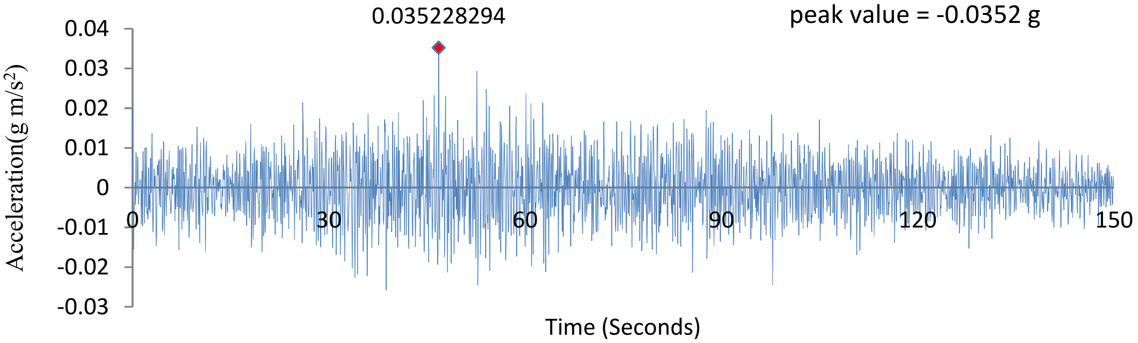


6.5 Off west coast of Sumatra (Indonesia) Earthquake

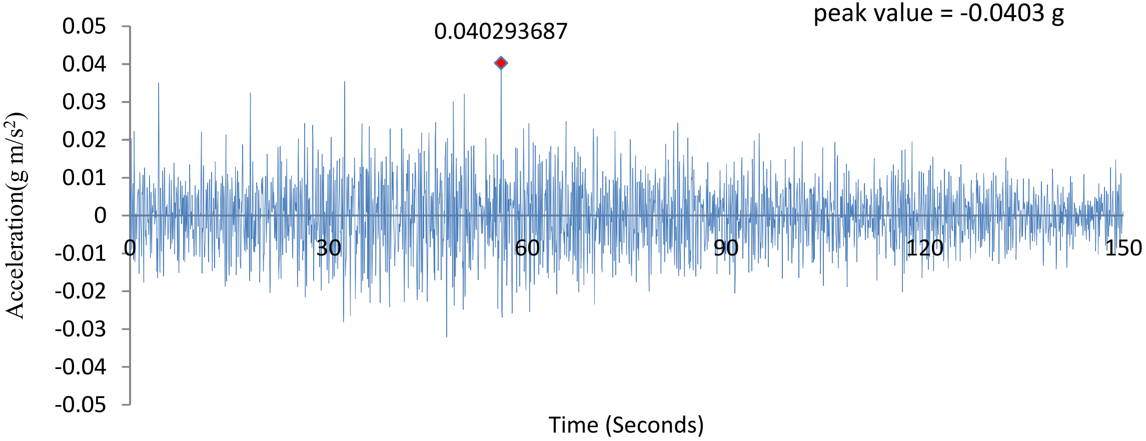
Date: 26 December 2004
Time: 6:28:53 AM (IST)
Location of earthquake: off west coast of Sumatra
Component: N-S and E-W Component
Location of epicenter: Latitude 3.3^0 N Longitude 96.1^0 E
Magnitude of Earthquake (M_W): 9.3
Location of station: Kodaikanal (Tamilnadu)
Sampling Rate: 20 (0.05 sec interval)
Epicentral Distance of recorded station: 2500 km

Impact and other details: The quake itself is known by the scientific community as the Sumatra–Andaman earthquake. The resulting tsunami is given various names, including the 2004 Indian Ocean tsunami, South Asian tsunami, Indonesian tsunami. The earthquake was caused by subduction and triggered a series of devastating tsunamis along the coasts of most landmasses bordering the Indian Ocean, killing over 230,000 people in fourteen countries, and inundating coastal communities with waves up to 30 meters (98 ft) high. The earthquake had the longest duration of faulting ever observed, between 8.3 and 10 minutes.

Off west coast of Sumatra Earthquake East component

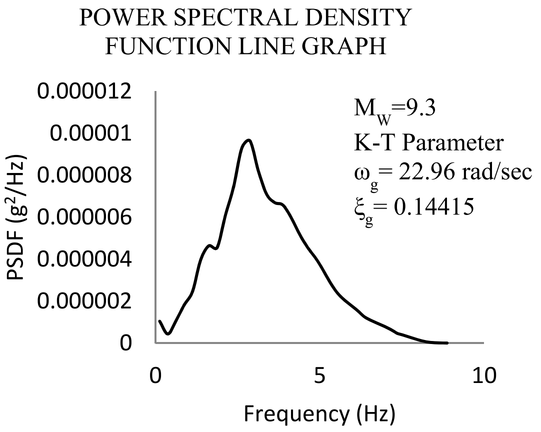


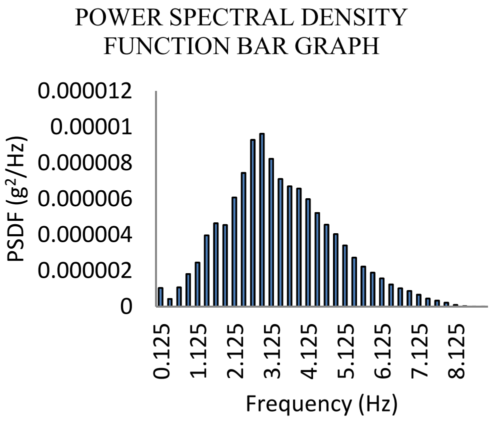
Off west coast of Sumatra Earthquake North component



Calculation and plot (line and bar graph) of PSDF of off west coast of Sumatra earthquake 2004 (East Component)

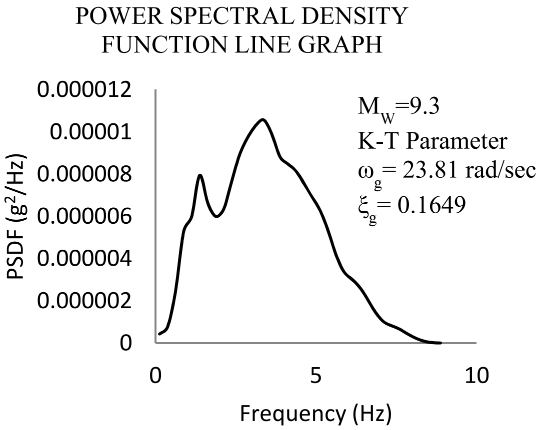
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF $=\frac{\mathrm{GRMS}^2}{\mathrm{Bandwidth}}$
0.01-0.25	0.125	0.000509194	2.593E-07	0.25	1.03711E-06
0.25-0.5	0.375	0.000328934	1.082E-07	0.25	4.32791E-07
0.5-0.75	0.625	0.000517369	2.677E-07	0.25	1.07068E-06
0.75-1	0.875	0.000674879	4.555E-07	0.25	1.82185E-06
1-1.25	1.125	0.000783634	6.141E-07	0.25	2.45633E-06
1.25-1.5	1.375	0.000996385	9.928E-07	0.25	3.97113E-06
1.5-1.75	1.625	0.001076624	1.159E-06	0.25	4.63648E-06
1.75-2	1.875	0.001066425	1.137E-06	0.25	4.54905E-06
2-2.25	2.125	0.001232495	1.519E-06	0.25	6.07618E-06
2.25-2.5	2.375	0.001363742	1.86E-06	0.25	7.43916E-06
2.5-2.75	2.625	0.001523383	2.321E-06	0.25	9.28278E-06
2.75-3	2.875	0.001551653	2.408E-06	0.25	9.63051E-06
3-3.25	3.125	0.001434559	2.058E-06	0.25	8.23184E-06
3.25-3.5	3.375	0.001333075	1.777E-06	0.25	7.10836E-06
3.5-3.75	3.625	0.001293644	1.674E-06	0.25	6.69406E-06
3.75-4	3.875	0.001282147	1.644E-06	0.25	6.57561E-06
4-4.25	4.125	0.001223349	1.497E-06	0.25	5.98633E-06
4.25-4.5	4.375	0.001141637	1.303E-06	0.25	5.21334E-06
4.5-4.75	4.625	0.001068106	1.141E-06	0.25	4.56341E-06
4.75-5	4.875	0.001004933	1.01E-06	0.25	4.03956E-06
5-5.25	5.125	0.000923709	8.532E-07	0.25	3.41295E-06
5.25-5.5	5.375	0.000826314	6.828E-07	0.25	2.73118E-06
5.5-5.75	5.625	0.000746682	5.575E-07	0.25	2.23013E-06
5.75-6	5.875	0.000687755	4.73E-07	0.25	1.89203E-06
6-6.25	6.125	0.000627797	3.941E-07	0.25	1.57652E-06
6.25-6.5	6.375	0.000555384	3.085E-07	0.25	1.2338E-06
6.5-6.75	6.625	0.00050605	2.561E-07	0.25	1.02435E-06
6.75-7	6.875	0.000466605	2.177E-07	0.25	8.70881E-07
7-7.25	7.125	0.000407123	1.657E-07	0.25	6.62996E-07
7.25-7.5	7.375	0.000336036	1.129E-07	0.25	4.51682E-07
7.5-7.75	7.625	0.000289796	8.398E-08	0.25	3.35927E-07
7.75-8	7.875	0.000233334	5.444E-08	0.25	2.17779E-07
8-8.25	8.125	0.000153561	2.358E-08	0.25	9.43241E-08
8.25-8.5	8.375	8.55131E-05	7.312E-09	0.25	2.92499E-08
8.5-8.75	8.625	4.23232E-05	1.791E-09	0.25	7.165E-09
8.75-9	8.875	1.63578E-05	2.676E-10	0.25	1.07031E-09

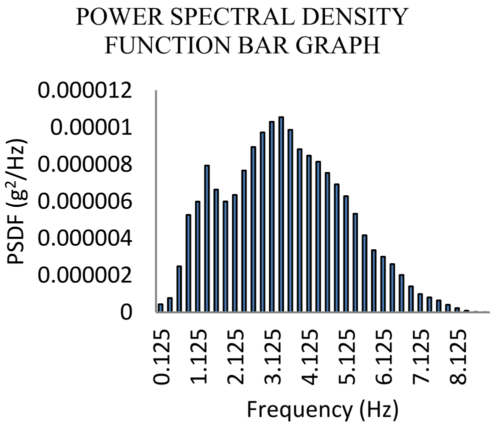




Calculation and plot (line and bar graph) of PSDF of Off west coast of Sumatra earthquake 2004 (North Component)

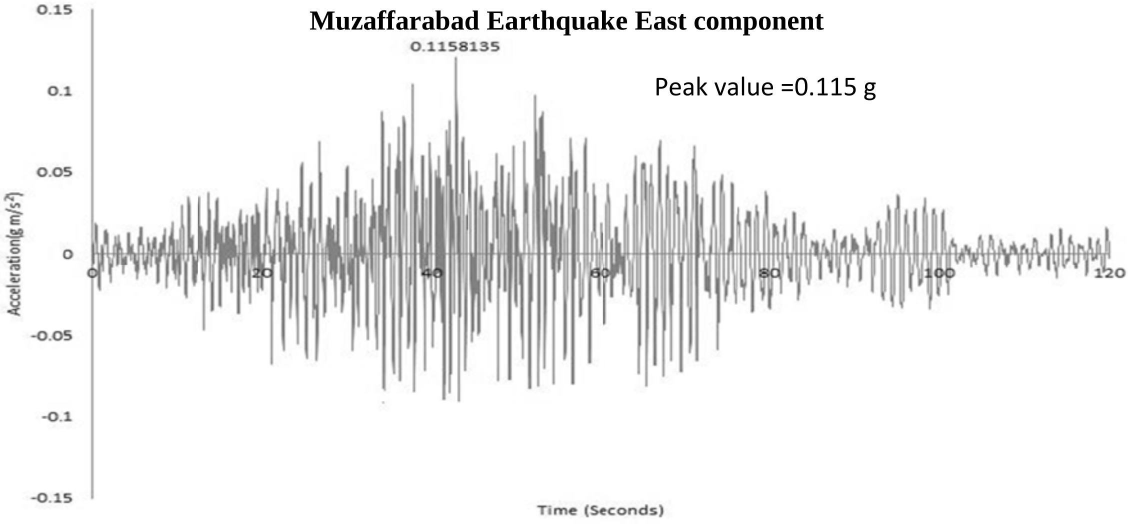
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	GRMS^2	Bandwidth (Hz)	PSDF = $\frac{\text{GRMS}^2}{\text{Bandwidth}}$
0.01-0.25	0.125	0.000327775	1.0744E-07	0.25	4.29746E-07
0.25-0.5	0.375	0.000437616	1.9151E-07	0.25	7.66031E-07
0.5-0.75	0.625	0.000788028	6.2099E-07	0.25	2.48395E-06
0.75-1	0.875	0.001147948	1.3178E-06	0.25	5.27114E-06
1-1.25	1.125	0.001223479	1.4969E-06	0.25	5.9876E-06
1.25-1.5	1.375	0.001408798	1.9847E-06	0.25	7.93885E-06
1.5-1.75	1.625	0.001288513	1.6603E-06	0.25	6.64106E-06
1.75-2	1.875	0.00122427	1.4988E-06	0.25	5.99535E-06
2-2.25	2.125	0.001259978	1.5875E-06	0.25	6.35018E-06
2.25-2.5	2.375	0.001384985	1.9182E-06	0.25	7.67273E-06
2.5-2.75	2.625	0.001495279	2.2359E-06	0.25	8.94344E-06
2.75-3	2.875	0.001560054	2.4338E-06	0.25	9.73507E-06
3-3.25	3.125	0.001605403	2.5773E-06	0.25	1.03093E-05
3.25-3.5	3.375	0.00162452	2.6391E-06	0.25	1.05563E-05
3.5-3.75	3.625	0.001571279	2.4689E-06	0.25	9.87567E-06
3.75-4	3.875	0.00148519	2.2058E-06	0.25	8.82315E-06
4-4.25	4.125	0.001455589	2.1187E-06	0.25	8.47495E-06
4.25-4.5	4.375	0.001426921	2.0361E-06	0.25	8.14442E-06
4.5-4.75	4.625	0.001373485	1.8865E-06	0.25	7.54585E-06
4.75-5	4.875	0.00131591	1.7316E-06	0.25	6.92647E-06
5-5.25	5.125	0.001253015	1.57E-06	0.25	6.28018E-06
5.25-5.5	5.375	0.001154221	1.3322E-06	0.25	5.3289E-06
5.5-5.75	5.625	0.001020768	1.042E-06	0.25	4.16787E-06
5.75-6	5.875	0.000916888	8.4068E-07	0.25	3.36273E-06
6-6.25	6.125	0.000867443	7.5246E-07	0.25	3.00983E-06
6.25-6.5	6.375	0.000808135	6.5308E-07	0.25	2.61233E-06
6.5-6.75	6.625	0.000710217	5.0441E-07	0.25	2.01763E-06
6.75-7	6.875	0.000592563	3.5113E-07	0.25	1.40452E-06
7-7.25	7.125	0.000496578	2.4659E-07	0.25	9.8636E-07
7.25-7.5	7.375	0.00044928	2.0185E-07	0.25	8.07408E-07
7.5-7.75	7.625	0.000400205	1.6016E-07	0.25	6.40656E-07
7.75-8	7.875	0.000319105	1.0183E-07	0.25	4.07311E-07
8-8.25	8.125	0.000234501	5.4991E-08	0.25	2.19964E-07
8.25-8.5	8.375	0.000142468	2.0297E-08	0.25	8.11883E-08
8.5-8.75	8.625	6.61592E-05	4.377E-09	0.25	1.75082E-08
8.75-9	8.875	2.24813E-05	5.0541E-10	0.25	2.02164E-09

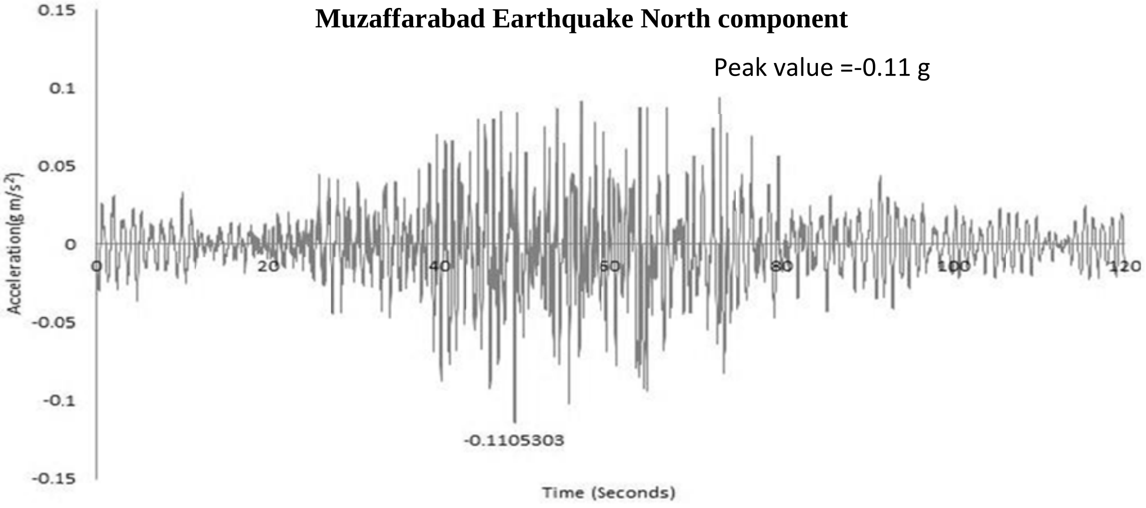




6.6 Muzaffarabad (Pakistan) Earthquake

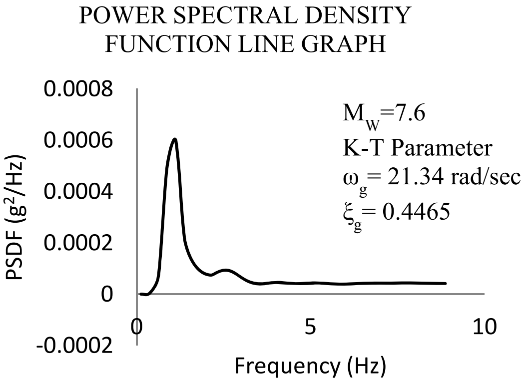
Date: 8 October 2005
Time: 9:22:37 AM (IST)
Location of earthquake: 170 km west-northwest of Srinagar, Jammu & Kashmir
Component: N-S and E-W Component
Location of epicenter: Latitude 34.5^0 N Longitude 73.1^0 E
Magnitude of Earthquake (M_W): 7.6
Location of station: Kalpal (Himachal Pradesh)
Sampling Rate: 50 (0.02 sec interval)
Epicentral Distance of recorded station: 529 km
Impact and other details: Most of the casualties resulting from the earthquake were in Pakistan, where the official confirmed death toll was 74,698.

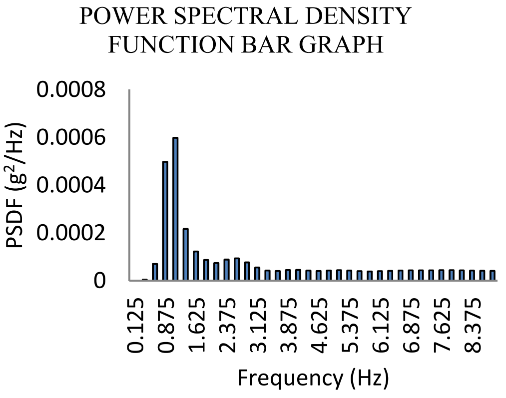




Calculation and plot (line and bar graph) of PSDF of Muzaffarabad earthquake 2005 (East Component)

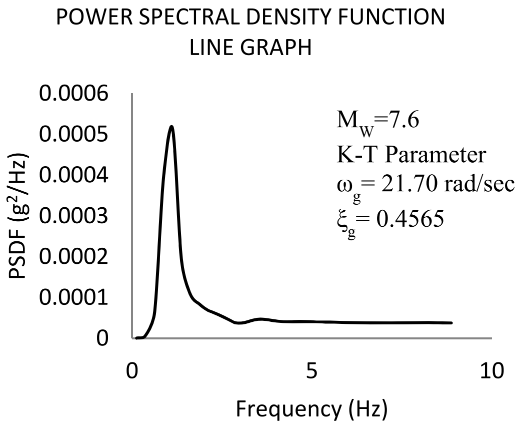
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	$GRMS^2$	Bandwidth (Hz)	PSDF $=\frac{GRMS^2}{Bandwidth}$
0.01-0.25	0.125	0.000175649	3.09E-08	0.25	1.23E-07
0.25-0.5	0.375	0.001022497	1.05E-06	0.25	4.18E-06
0.5-0.75	0.625	0.004186261	1.75E-05	0.25	7.01E-05
0.75-1	0.875	0.011156128	0.000124	0.25	0.000498
1-1.25	1.125	0.012230119	0.00015	0.25	0.000598
1.25-1.5	1.375	0.007368745	5.43E-05	0.25	0.000217
1.5-1.75	1.625	0.005543656	3.07E-05	0.25	0.000123
1.75-2	1.875	0.004664342	2.18E-05	0.25	8.7E-05
2-2.25	2.125	0.004307828	1.86E-05	0.25	7.42E-05
2.25-2.5	2.375	0.004701974	2.21E-05	0.25	8.84E-05
2.5-2.75	2.625	0.00483637	2.34E-05	0.25	9.36E-05
2.75-3	2.875	0.004383812	1.92E-05	0.25	7.69E-05
3-3.25	3.125	0.003723199	1.39E-05	0.25	5.54E-05
3.25-3.5	3.375	0.0032705	1.07E-05	0.25	4.28E-05
3.5-3.75	3.625	0.003196904	1.02E-05	0.25	4.09E-05
3.75-4	3.875	0.003330165	1.11E-05	0.25	4.44E-05
4-4.25	4.125	0.003371976	1.14E-05	0.25	4.55E-05
4.25-4.5	4.375	0.003262547	1.06E-05	0.25	4.26E-05
4.5-4.75	4.625	0.003198816	1.02E-05	0.25	4.09E-05
4.75-5	4.875	0.003256429	1.06E-05	0.25	4.24E-05
5-5.25	5.125	0.003309329	1.1E-05	0.25	4.38E-05
5.25-5.5	5.375	0.003269093	1.07E-05	0.25	4.27E-05
5.5-5.75	5.625	0.003175461	1.01E-05	0.25	4.03E-05
5.75-6	5.875	0.003125899	9.77E-06	0.25	3.91E-05
6-6.25	6.125	0.003158902	9.98E-06	0.25	3.99E-05
6.25-6.5	6.375	0.003226188	1.04E-05	0.25	4.16E-05
6.5-6.75	6.625	0.003270664	1.07E-05	0.25	4.28E-05
6.75-7	6.875	0.003283755	1.08E-05	0.25	4.31E-05
7-7.25	7.125	0.003286518	1.08E-05	0.25	4.32E-05
7.25-7.5	7.375	0.003294867	1.09E-05	0.25	4.34E-05
7.5-7.75	7.625	0.003307396	1.09E-05	0.25	4.38E-05
7.75-8	7.875	0.003310664	1.1E-05	0.25	4.38E-05
8-8.25	8.125	0.00329506	1.09E-05	0.25	4.34E-05
8.25-8.5	8.375	0.00326659	1.07E-05	0.25	4.27E-05
8.5-8.75	8.625	0.003241217	1.05E-05	0.25	4.2E-05
8.75-9	8.875	0.003226716	1.04E-05	0.25	4.16E-05

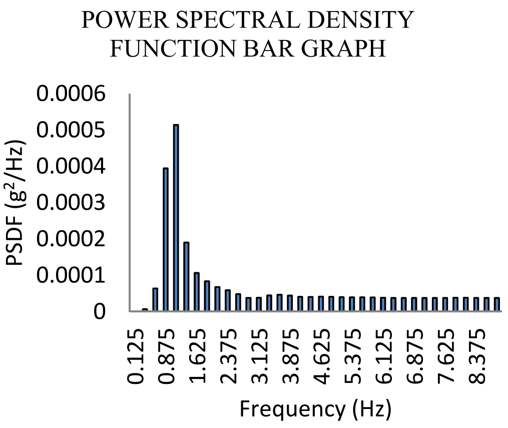




Calculation and plot (line and bar graph) of PSDF of Muzaffarabad earthquake 2005 (North Component)

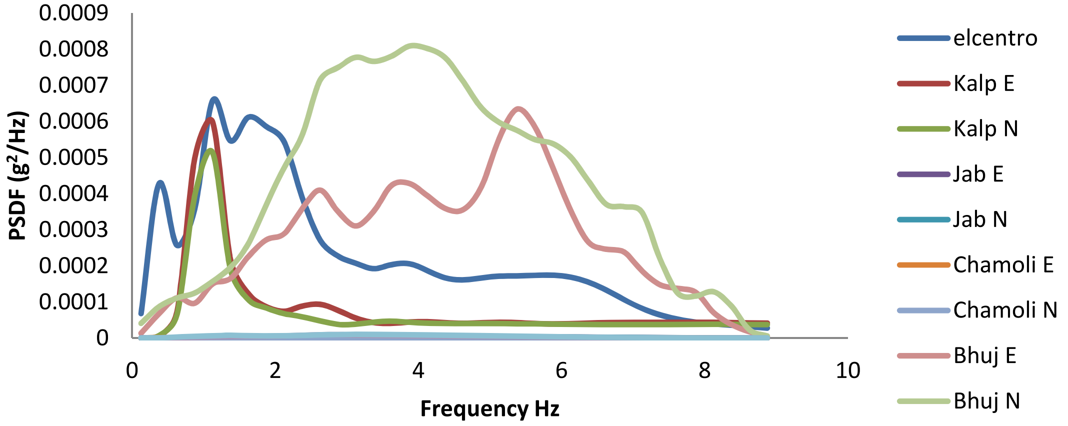
Band Pass filter (Hz)	Band Centre frequency	Overall GRMS value	$GRMS^2$	Bandwidth (Hz)	PSDF $=\frac{GRMS^2}{Bandwidth}$
0.01-0.25	0.125	0.0001674	2.802E-08	0.25	1.12087E-07
0.25-0.5	0.375	0.00130315	1.698E-06	0.25	6.79277E-06
0.5-0.75	0.625	0.00398356	1.587E-05	0.25	6.3475E-05
0.75-1	0.875	0.00992506	9.851E-05	0.25	0.000394027
1-1.25	1.125	0.0113289	0.0001283	0.25	0.000513376
1.25-1.5	1.375	0.00688746	4.744E-05	0.25	0.000189748
1.5-1.75	1.625	0.00515494	2.657E-05	0.25	0.000106294
1.75-2	1.875	0.00456224	2.081E-05	0.25	8.32562E-05
2-2.25	2.125	0.00410856	1.688E-05	0.25	6.75212E-05
2.25-2.5	2.375	0.0038391	1.474E-05	0.25	5.89548E-05
2.5-2.75	2.625	0.00347212	1.206E-05	0.25	4.82224E-05
2.75-3	2.875	0.00307088	9.43E-06	0.25	3.77211E-05
3-3.25	3.125	0.00308712	9.53E-06	0.25	3.81213E-05
3.25-3.5	3.375	0.003334	1.112E-05	0.25	4.44622E-05
3.5-3.75	3.625	0.00340884	1.162E-05	0.25	4.64807E-05
3.75-4	3.875	0.00331018	1.096E-05	0.25	4.38291E-05
4-4.25	4.125	0.003191	1.018E-05	0.25	4.073E-05
4.25-4.5	4.375	0.00317	1.005E-05	0.25	4.01957E-05
4.5-4.75	4.625	0.00319778	1.023E-05	0.25	4.09031E-05
4.75-5	4.875	0.00318566	1.015E-05	0.25	4.05938E-05
5-5.25	5.125	0.00314765	9.908E-06	0.25	3.96307E-05
5.25-5.5	5.375	0.00313697	9.841E-06	0.25	3.93622E-05
5.5-5.75	5.625	0.00314203	9.872E-06	0.25	3.94895E-05
5.75-6	5.875	0.00312338	9.756E-06	0.25	3.9022E-05
6-6.25	6.125	0.00308442	9.514E-06	0.25	3.80547E-05
6.25-6.5	6.375	0.00305666	9.343E-06	0.25	3.73727E-05
6.5-6.75	6.625	0.003053	9.321E-06	0.25	3.72832E-05
6.75-7	6.875	0.00305543	9.336E-06	0.25	3.73427E-05
7-7.25	7.125	0.00305116	9.31E-06	0.25	3.72384E-05
7.25-7.5	7.375	0.00305203	9.315E-06	0.25	3.72596E-05
7.5-7.75	7.625	0.0030695	9.422E-06	0.25	3.76873E-05
7.75-8	7.875	0.00309093	9.554E-06	0.25	3.82154E-05
8-8.25	8.125	0.00309432	9.575E-06	0.25	3.82994E-05
8.25-8.5	8.375	0.00307723	9.469E-06	0.25	3.78773E-05
8.5-8.75	8.625	0.00305836	9.354E-06	0.25	3.74142E-05
8.75-9	8.875	0.00305184	9.314E-06	0.25	3.72548E-05



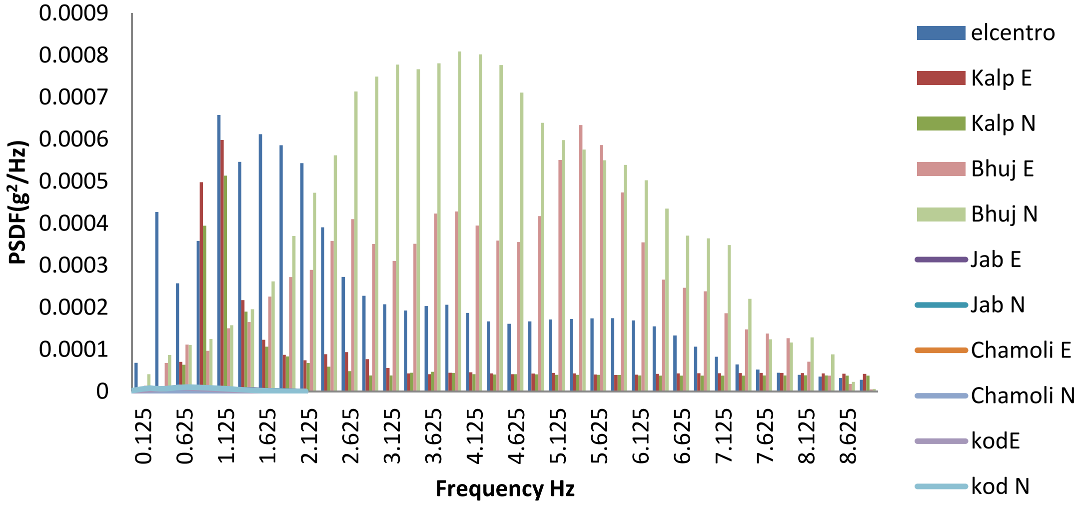


6.7 Superposition of PSDF of Studied Earthquakes

PSDF of Earthquakes Line graph



PSDF of Earthquakes Bar graph



6.8 Summary of Ground Motion Parameters of Studied Earthquakes

Earthquake	Station	Component	M_W	R (km)	Site	a_{max} (g m/s^2)	σ_0 (g m/s^2)
El Centro 1940	Imperial valley	E	7.1	11.4	s	0.3	0.04583
Jabalpur 22 May 1997	Jabalpur	E	6	275	s	0.028	0.00311
	Jabalpur	N	6	275		0.035	0.0036
Chamoli 29 March 1999	Jabalpur	E	6.8	798	k	0.0043	0.00075
	Jabalpur	N	6.8	798		0.002	0.00029
Bhuj 26 Jan 2011	Bhuj	E	7.7	99	s	0.267	0.04472
	Bhuj	N	7.7	99		0.365	0.05657
Off West Coast of Sumatra 26 Dec 2004	Kodaikanal	E	9.3	2500	s	0.03522	0.00503
	Kodaikanal	N	9.3	2500		0.04029	0.00594
Muzaffrabad Pakistan 08 Oct. 2005	Kalp (H. P.)	E	7.6	529	k	0.121	0.03317
	Kalp (H. P.)	N	7.6	529		0.0904	0.02916

Earthquake	Component	ω_c (rad/sec)	δ	ω_g (rad/sec)	ζ_g	G_0 (m^2/s^3)
El Centro 1940	E	28.132691	0.71225	19.89582	0.707	0.16968
Jabalpur 22 May 1997	E	32.308744	0.31557	32.189045	0.086	0.00014
	N	30.159265	0.34723	29.989352	0.106	0.00019
Chamoli 29 March 1999	E	16.275617	0.69982	11.918364	0.681	8.5E-06
	N	18.253821	0.64039	15.755226	0.505	1.3E-06
Bhuj 26 Jan 2011	E	30.165834	0.34853	29.992652	0.107	0.03015
	N	28.42664	0.37904	28.189124	0.129	0.04867
Off West Coast of Sumatra 26 Dec 2004	E	23.529536	0.35515	23.383344	0.1113	0.00019
	N	24.849661	0.36894	24.666168	0.1213	0.00034
Muzaffrabad Pakistan 08 Oct. 2005	E	35.49254	0.68958	27.080763	0.6464	0.16318
	N	38.602293	0.68632	29.82074	0.635	0.15945

CHAPTER 7. CORRELATION STUDIES OF SEISMIC PARAMETERS

To assessing relationship between seismic parameters statistics are presented by plotting graph between Kanai-Tajimi frequency and damping as a function of maximum acceleration, epicentral distance as well as magnitude of earthquake from the computed records of ground motion parameters.

7.1 Correlation Between Central Frequency (ω_c) And Kanai-Tajimi Frequency (ω_g)

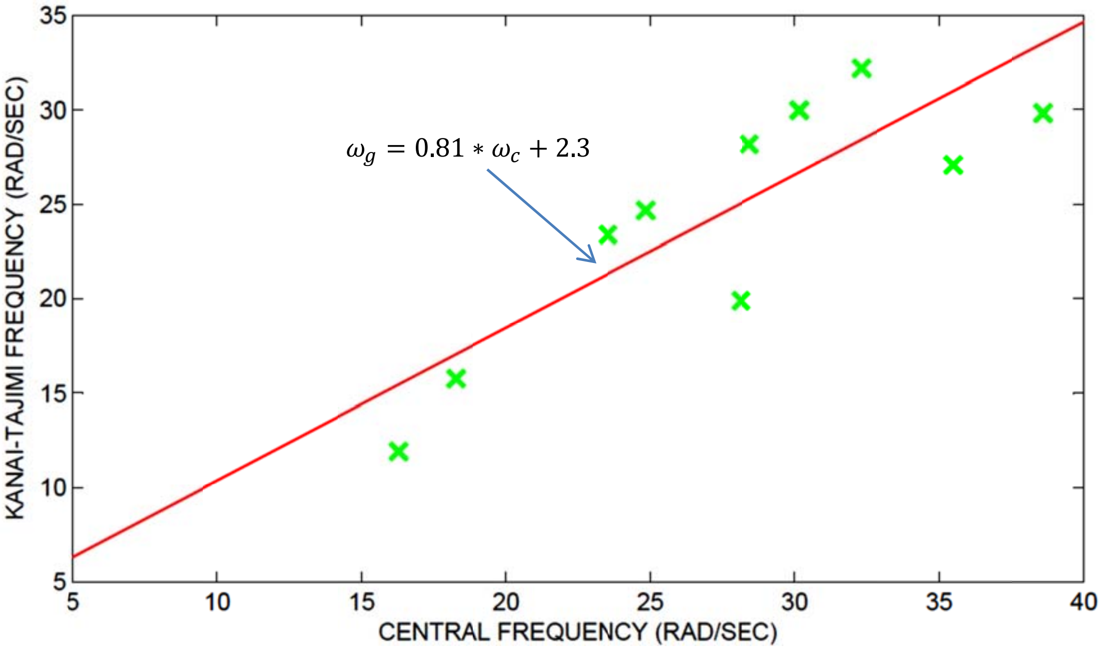


Fig 7.1 Scattergram of central frequency versus Kanai-Tajimi frequency

The central frequency and Kanai-Tazimi frequency computed from the strong motion data of different earthquake and the scattergram of central frequency versus Kanai-Tazimi frequency is presented in figure 6.1. Based on linear regression or fitting data linearly the relationship between K-T frequency and central frequency can be expressed as follows

$$\omega_g = 0.81 * \omega_c + 2.3 \qquad (7.1)$$

From equation 6.1. K-T frequency can be easily predicted for a given strong motion record.

7.2 Correlation studies between maximum ground acceleration and K-T frequency as well as K-T damping coefficient

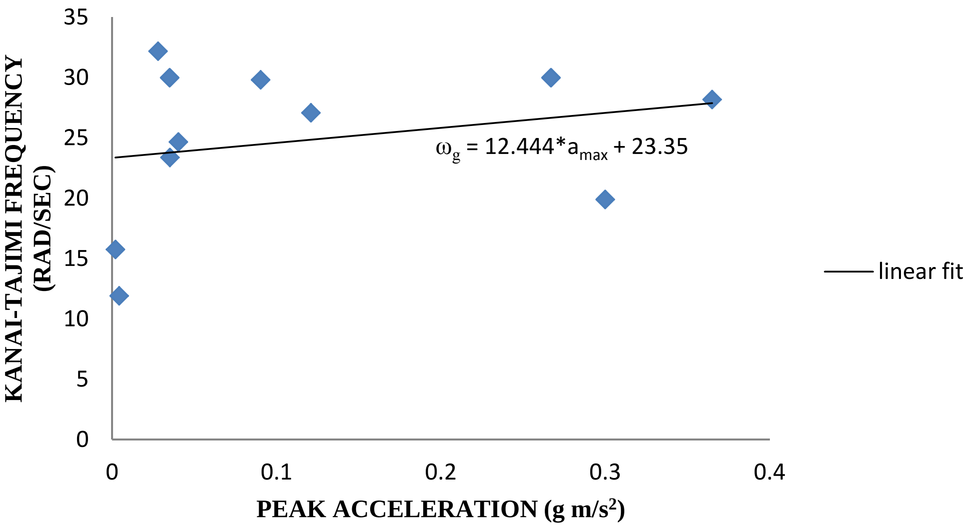


Fig 6.2 Plot of statistics of K-T frequency versus Peak acceleration

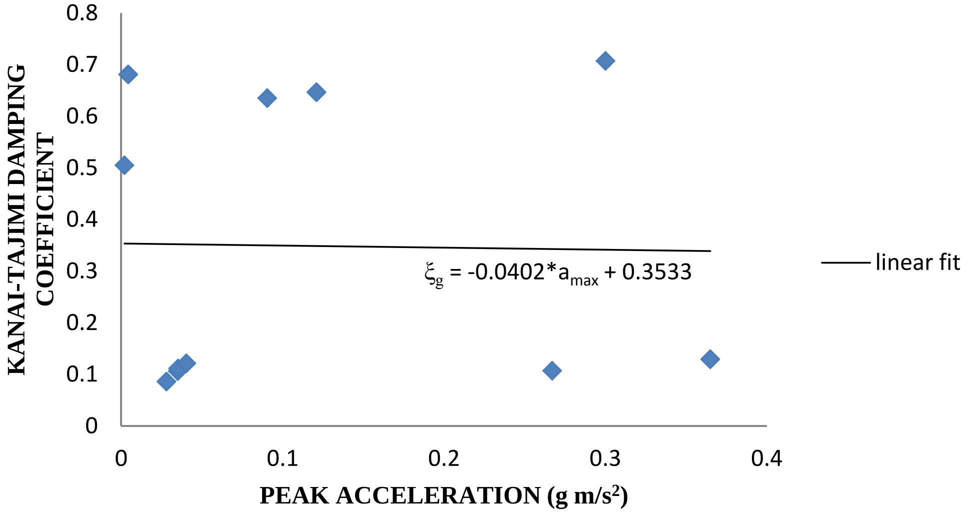


Fig 7.3 Plot of statistics of K-T damping coefficient versus Peak acceleration

Fig. 7.2 and 7.3 are plots of K-T frequency and damping coefficient with peak acceleration. The empirical equation that can be suggested by linear curve fitting as

$$\omega_g = 12.444*a_{max} + 23.35 \qquad (7.2)$$

$$\xi_g = -0.0402*a_{max} + 0.3533 \qquad (7.3)$$

where ω_g is Kanai –Tajimi frequency in rad/sec and ξ_g is Kanai-Tajimi damping coefficient

a_{max} is peak ground acceleration

7.3 Correlation studies between Epicentral distance and K-T frequency as well as K-T damping coefficient

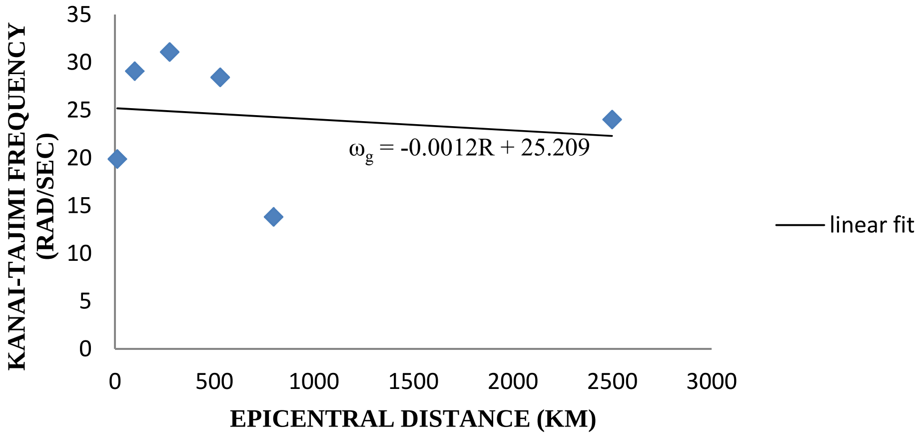


Fig 7.4 Plot of statistics of K-T frequency versus Epicentral distance

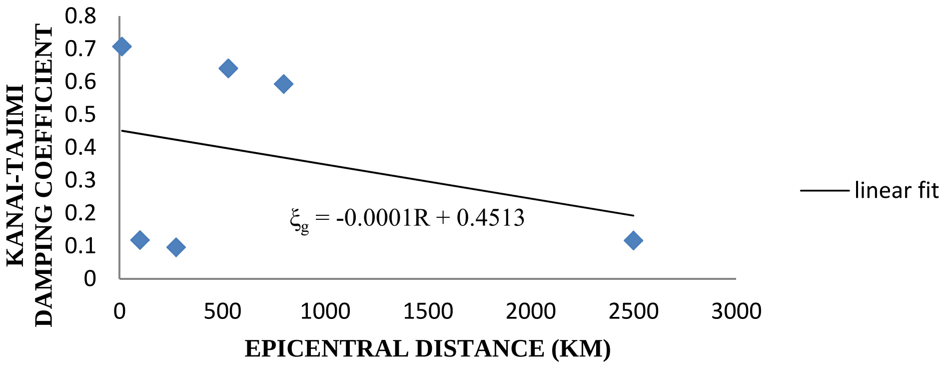


Fig 7.5 Plot of statistics of K-T damping coefficient versus Epicentral distance

Fig 7.4 and 7.5 are the plots of K-T frequency and K-T damping coefficient versus Epicentral distance R for the strong ground motions. As the mean K-T frequency decreases with increasing epicentral distance because the higher frequency contents of the travelling seismic would be filtered out through distance. Based on the results the following empirical equation is suggested by linear curve fit as

$$\omega_g = -0.0012*R + 25.209 \qquad (7.4)$$

$$\xi_g = -0.0001*R + 0.4513 \qquad (7.5)$$

Where ω_g is in rad/sec and R is in kilometers.

Equation (6.3) is quite similar given by ‘Shih-Sheng Paul Lai’ in his paper

$$\omega_g = 27 - 0.09\ R \qquad (7.6)$$

.

7.4 Correlation studies between Richter Magnitude and K-T frequency as well as K-T damping coefficient

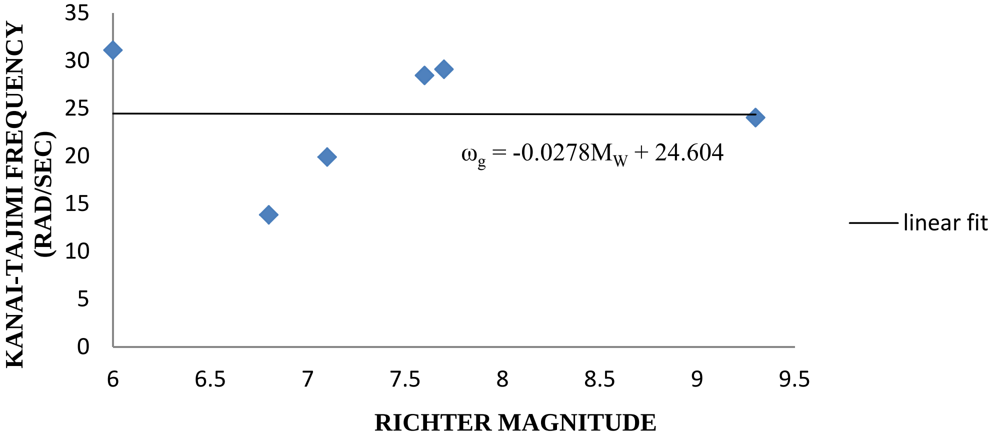


Fig 7.6 Plot of statistics of K-T frequency versus Richter Magnitude

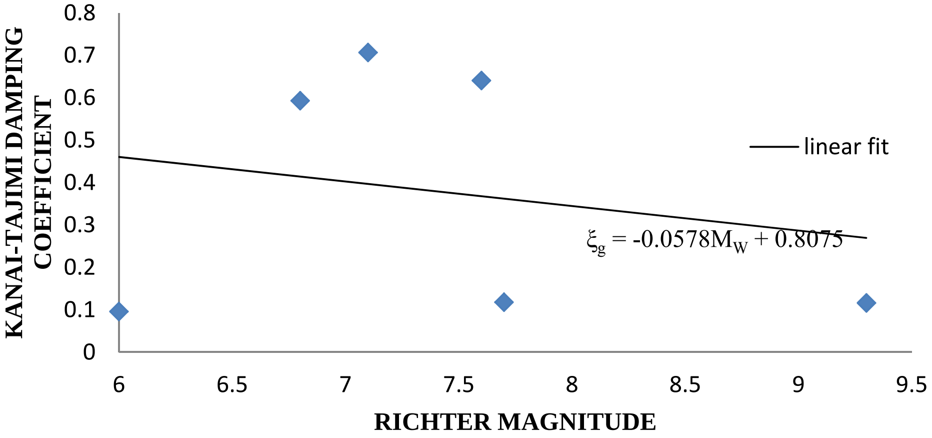


Fig 7.7 Plot of statistics of K-T damping coefficient versus Richter Magnitude

Fig 7.6 and 7.7 are the plots of K-T frequency and K-T damping coefficient versus Richter Magnitude (M_W) for the strong ground motions. Here K-T frequency generally decreases with increasing magnitude of earthquake except when magnitude of earthquake is in between 6.6 to 7.7. From above plot the following relationship can be proposed by linear curve fit as

$$\omega_g = -0.0278M_W + 24.604 \tag{7.7}$$

$$\xi_g = -0.0578M_W + 0.8075 \tag{7.8}$$

where ω_g is Kanai –Tajimi frequency in rad/sec and ξ_g is Kanai-Tajimi damping coefficient

M_W is Richter Magnitude

CHAPTER 8. CONCLUSIONS AND FUTURE SCOPE OF WORK

8.1 Conclusions

In order to assess the overall seismic safety of constructed facilities, it is necessary to consider three major sources of uncertainty, i.e., the representation of earthquake environment, the dynamic structural properties, and the method of dynamic analysis. In this paper only uncertainty of ground motion representation has been examined. Specifically, the characterization of strong ground motion in terms of the K-T PSD function has been investigated.

As illustrated four seismic parameters are needed in the statistical description of a ground motion: (i) peak ground acceleration; (ii) strong-motion duration; (iii) K-T frequency; and (iv)K-T damping. The first two seismic parameters can be assumed negatively correlated. Hence, the frequency content of a design ground motion can be defined by only three independent seismic parameters.

Following regression equation has been suggested by Vanmarcke and Lai (1980) for the relation between strong-motion duration and peak ground acceleration

$$S_0 = 30\ \exp(-3.254 a_{max}^{0.35})$$

Where S_0 is strong motion duration and a_{max} is peak ground acceleration

K-T damping coefficient and K-T ground frequency can be estimated by using correlation equation as given in **Chapter 7** for given magnitude of earthquake, peak ground acceleration etc.

8.2 Future Scope of Work

More strong motion data collected and seismic parameters can be found out of additional earthquakes. From those seismic parameters by regression we can get better correlation between seismic parameters mainly K-T frequency and damping coefficient.

REFERENCES

Journals

1. Arias, A. (1970) A measure of earthquake intensity, in seismic design of Nuclear Power Plants R. Hansen, Editor, M.I.T. Press, Cambridge
2. Erik H. Vanmarcke, (1970). Parameters of the spectral density function: their significance in the time and frequency domains, M.I.T., Dept. of Civil Engineering, Research Report
3. Erik H. Vanmarcke and S. P. Lai (1980). Strong –motion duration of earthquakes, M.I.T., Dept. of Civil Engineering, Research Report
4. Kanai, K. (1957). Semi-empirical formula for the seismic characteristics of the ground, Bull. Earthquake Res. Inst., Univ. Tokyo, 35
5. Shih-Sheng Paul Lai, (1982). Statistical Characterization of Strong Ground Motions Using Power Spectral Density Function, Bull. Of the seismological society of America, Vol. 72, No. 1, pp. 259-274, Februuary 1982
6. Tajimi, H. (1960). A statistical method of determining the maximum response of the building structure during the earthquake
7. Tom Irvine (2000) An introduction to the filtering of digital signals Revision A
8. Tom Irvine (2000) Power spectral density units

Books

9. Application of Random Vibrations by N. C. Nigam and S. Narayanan Chapter 7 Response of structures to earthquakes “Narosa Publishing House”
10. Geotechnical Earthquake Engineering by Steven L. Kramer ‘Chapter 3’ “Pearson Education, Inc.”
11. Digital Filters by Andreas Antoniou ‘ Chapter 1 Article 1.3, 1.4, 1.7; Chapter 3 Article 3.2, 3.7; Chapter 4 Article 4.1, 4.7; Chapter 5 Article 5.1, 5.3;’ “Tata McGraw-Hill Publishing Company Limited” Sixth Reprint 2003
12. Discrete-Time Signal Processing by Alan V. Oppenheim, Ronald W. Schafer, John R. Buck ‘Chapter 2 Article 2.1 to 2.5 & Chapter 7 Article 7.1’ “Pearson Education,Inc.” Seventh Impression : 2011
13. Dynamics of structures by Anil K. Chopra ‘Chapter 6’ “Pearson Education, Inc.”
14. Dynamics of structures by J. L. Humar ‘Chapter 9’ “publisher: Taylor & Francis Group”
15. Signals & systems by Alan V. Oppenheim,Alan S. Willsky and S. Hamid Nawab ‘Page No. 446-47, 703-706’ PHI learning private Ltd.
16. Stochastic Processes and Random Vibrations Theory and Practices by Julius Solnnes ‘Chapter 1’ “John Willey and Sons Publication”
17. Structural Dynamics by Mario Paz and William Leigh ‘Chapter 19 & 21’ “Springer”

Websites

http://www.cosmos-eq.org
http://www.imd.gov.in/section/seismo/static/welcome.htm
http://www.nicee.org
http://nisee2.berkeley.edu

APPENDIX - A

FILTER COEFFICIENT FOR TIME STEP 0.05 Sec (Used to filter data of Jabalpur, Chamoli, Bhuj, Off west coast of Sumatra earthquake)

f_0	Low pass		f_0	High pass	
0.25	Ω_c	0.03929	0.01	Ω_c	0.001571

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.95415	0.960195	0.00154	0.00308	0.00154
H_2	-1.88903	0.894874	0.001538	0.003076	0.001538
H_3	-1.85338	0.859106	0.001537	0.003073	0.001537

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.99837	0.998375	0.999185	-1.99837	0.999185
H_2	-1.99556	0.995567	0.997781	-1.99556	0.997781
H_3	-1.99394	0.993949	0.996972	-1.99394	0.996972

f_0	Low pass		f_0	High pass	
0.5	Ω_c	0.078702	0.25	Ω_c	0.03929

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.89851	0.922175	0.006034	0.012068	0.006034
H_2	-1.77863	0.800803	0.005834	0.011667	0.005834
H_3	-1.71607	0.737462	0.005724	0.011448	0.005724

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.95415	0.960195	0.978587	-1.95717	0.978587
H_2	-1.88903	0.894874	0.945977	-1.89195	0.945977
H_3	-1.85338	0.859106	0.92812	-1.85624	0.92812

f_0	Low pass		f_0	High pass	
0.75	Ω_c	0.118358	0.5	Ω_c	0.078702

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.83393	0.886045	0.013281	0.026563	0.013281
H_2	-1.6692	0.716634	0.012449	0.024897	0.012449
H_3	-1.58691	0.631999	0.012014	0.024027	0.012014

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.89851	0.922175	0.955171	-1.91034	0.955171
H_2	-1.77863	0.800803	0.894859	-1.78972	0.894859
H_3	-1.71607	0.737462	0.863383	-1.72677	0.863383

f_0	Low pass		f_0	High pass	
1	Ω_c	0.158384	0.75	Ω_c	0.118358

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.76125	0.851887	0.023092	0.046183	0.023092
H_2	-1.56102	0.641352	0.021037	0.042073	0.021037
H_3	-1.46487	0.540254	0.020009	0.040017	0.020009

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.83393	0.886045	0.929995	-1.85999	0.929995
H_2	-1.6692	0.716634	0.846459	-1.69292	0.846459
H_3	-1.58691	0.631999	0.804726	-1.60945	0.804726

f_0	Low pass		f_0	High pass	
1.25	Ω_c	0.198912	1	Ω_c	0.158384

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.68124	0.81976	0.035278	0.070556	0.035278
H_2	-1.45424	0.574062	0.031313	0.062627	0.031313
H_3	-1.34908	0.460234	0.029405	0.058811	0.029405

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.76125	0.851887	0.903284	-1.80657	0.903284
H_2	-1.56102	0.641352	0.800592	-1.60118	0.800592
H_3	-1.46487	0.540254	0.75128	-1.50256	0.75128

f_0	Low pass		f_0	High pass	
1.5	Ω_c	0.240079	1.25	Ω_c	0.198912

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.59464	0.789707	0.049662	0.099324	0.049662
H_2	-1.34897	0.513982	0.043047	0.086095	0.043047
H_3	-1.23878	0.390317	0.039973	0.079947	0.039973

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.68124	0.81976	0.87525	-1.7505	0.87525
H_2	-1.45424	0.574062	0.757076	-1.51415	0.757076
H_3	-1.34908	0.460234	0.702328	-1.40466	0.702328

f_0	Low pass		f_0	High pass	
1.75	Ω_c	0.282029	1.5	Ω_c	0.240079

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.50214	0.761754	0.066074	0.132147	0.066074
H_2	-1.24522	0.460427	0.056051	0.112103	0.056051
H_3	-1.13331	0.329173	0.051538	0.103076	0.051538

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.59464	0.789707	0.846087	-1.69217	0.846087
H_2	-1.34897	0.513982	0.715737	-1.43147	0.715737
H_3	-1.23878	0.390317	0.657274	-1.31455	0.657274

f_0	Low pass		f_0	High pass	
2	Ω_c	0.32492	1.75	Ω_c	0.282029

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.40438	0.735915	0.084353	0.168706	0.084353
H_2	-1.14298	0.412802	0.070175	0.14035	0.070175
H_3	-1.03207	0.275708	0.063968	0.127935	0.063968

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.50214	0.761754	0.815974	-1.63195	0.815974
H_2	-1.24522	0.460427	0.676412	-1.35282	0.676412
H_3	-1.13331	0.329173	0.61562	-1.23124	0.61562

f_0	Low pass		f_0	High pass	
2.25	Ω_c	0.368919	2	Ω_c	0.32492

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.30197	0.712198	0.104349	0.208698	0.104349
H_2	-1.0422	0.370587	0.085298	0.170595	0.085298
H_3	-0.93455	0.229016	0.077164	0.154328	0.077164

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.40438	0.735915	0.785075	-1.57015	0.785075
H_2	-1.14298	0.412802	0.638946	-1.27789	0.638946
H_3	-1.03207	0.275708	0.576944	-1.15389	0.576944

f_0	Low pass		f_0	High pass	
2.5	Ω_c	0.414214	2.25	Ω_c	0.368919

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.19543	0.690599	0.125921	0.251843	0.125921
H_2	-0.94281	0.333333	0.101324	0.202649	0.101324
H_3	-0.84029	0.188345	0.091055	0.182111	0.091055

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.30197	0.712198	0.753541	-1.50708	0.753541
H_2	-1.0422	0.370587	0.603197	-1.20639	0.603197
H_3	-0.93455	0.229016	0.540892	-1.08178	0.540892

f_0	Low pass		f_0	High pass	
2.75	Ω_c	0.461006	2.5	Ω_c	0.414214

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.0853	0.671112	0.148939	0.297878	0.148939
H_2	-0.84471	0.300654	0.118181	0.236362	0.118181
H_3	-0.74886	0.153073	0.105592	0.211183	0.105592

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.19543	0.690599	0.721508	-1.44302	0.721508
H_2	-0.94281	0.333333	0.569036	-1.13807	0.569036
H_3	-0.84029	0.188345	0.507158	-1.01432	0.507158

f_0	Low pass		f_0	High pass	
3	Ω_c	0.509525	2.75	Ω_c	0.461006

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.97204	0.653728	0.17328	0.346559	0.17328
H_2	-0.74779	0.272215	0.135813	0.271625	0.135813
H_3	-0.6599	0.122681	0.12074	0.24148	0.12074

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.0853	0.671112	0.689103	-1.37821	0.689103
H_2	-0.84471	0.300654	0.53634	-1.07268	0.53634
H_3	-0.74886	0.153073	0.475483	-0.95097	0.475483

f_0	Low pass		f_0	High pass	
3.25	Ω_c	0.560027	3	Ω_c	0.509525

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.85608	0.638432	0.19883	0.39766	0.19883
H_2	-0.65194	0.247733	0.154178	0.308356	0.154178
H_3	-0.57304	0.096739	0.136482	0.272964	0.136482

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.97204	0.653728	0.656441	-1.31288	0.656441
H_2	-0.74779	0.272215	0.505001	-1.01	0.505001
H_3	-0.6599	0.122681	0.445644	-0.89129	0.445644

f_0	Low pass		f_0	High pass	
3.5	Ω_c	0.612801	3.25	Ω_c	0.560027

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.73783	0.625211	0.225484	0.450968	0.225484
H_2	-0.55703	0.226966	0.173251	0.346501	0.173251
H_3	-0.48799	0.074895	0.152813	0.305626	0.152813

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.85608	0.638432	0.623627	-1.24725	0.623627
H_2	-0.65194	0.247733	0.474918	-0.94984	0.474918
H_3	-0.57304	0.096739	0.417446	-0.83489	0.417446

f_0	Low pass		f_0	High pass	
3.75	Ω_c	0.668179	3.5	Ω_c	0.612801

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.61767	0.614052	0.253144	0.506288	0.253144
H_2	-0.46294	0.209715	0.193015	0.386031	0.193015
H_3	-0.40444	0.05686	0.169738	0.339476	0.169738

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.73783	0.625211	0.59076	-1.18152	0.59076
H_2	-0.55703	0.226966	0.445999	-0.892	0.445999
H_3	-0.48799	0.074895	0.390722	-0.78144	0.390722

f_0	Low pass		f_0	High pass	
4	Ω_c	0.726543	3.75	Ω_c	0.668179

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.49595	0.604941	0.281717	0.563434	0.281717
H_2	-0.36953	0.195816	0.213467	0.426934	0.213467
H_3	-0.32212	0.0424	0.187273	0.374546	0.187273

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.61767	0.614052	0.557931	-1.11586	0.557931
H_2	-0.46294	0.209715	0.418163	-0.83633	0.418163
H_3	-0.40444	0.05686	0.365326	-0.73065	0.365326

f_0	Low pass		f_0	High pass	
4.25	Ω_c	0.788336	4	Ω_c	0.726543

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.37301	0.597868	0.311117	0.622233	0.311117
H_2	-0.27666	0.185136	0.234611	0.469221	0.234611
H_3	-0.24076	0.031333	0.205443	0.410886	0.205443

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.49595	0.604941	0.525224	-1.05045	0.525224
H_2	-0.36953	0.195816	0.391336	-0.78267	0.391336
H_3	-0.32212	0.0424	0.34113	-0.68226	0.34113

f_0	Low pass		f_0	High pass	
4.5	Ω_c	0.854081	4.25	Ω_c	0.788336

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.24917	0.592823	0.341261	0.682521	0.341261
H_2	-0.18421	0.177578	0.256459	0.512918	0.256459
H_3	-0.16011	0.023524	0.224282	0.448563	0.224282

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.37301	0.597868	0.492721	-0.98544	0.492721
H_2	-0.27666	0.185136	0.36545	-0.7309	0.36545
H_3	-0.24076	0.031333	0.318023	-0.63605	0.318023

f_0	Low pass		f_0	High pass	
4.75	Ω_c	0.92439	4.5	Ω_c	0.854081

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.12473	0.589798	0.37207	0.744141	0.37207
H_2	-0.09204	0.173071	0.279033	0.558067	0.279033
H_3	-0.07994	0.018876	0.243832	0.487664	0.243832

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.24917	0.592823	0.460499	-0.921	0.460499
H_2	-0.18421	0.177578	0.340448	-0.6809	0.340448
H_3	-0.16011	0.023524	0.29591	-0.59182	0.29591

f_0	Low pass		f_0	High pass	
5	Ω_c	1	4.75	Ω_c	0.92439

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.8E-16	.588791	0.40347	0.80694	0.40347
H_2	-1.3E-16	0.171573	0.302363	0.604726	0.302363
H_3	-1.1E-16	0.017332	0.264146	0.528292	0.264146

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-0.12473	0.589798	0.428633	-0.85727	0.428633
H_2	-0.09204	0.173071	0.316277	-0.63255	0.316277
H_3	-0.07994	0.018876	0.274704	-0.54941	0.274704

f_0	Low pass		f_0	High pass	
5.25	Ω_c	1.081794	5	Ω_c	1

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.124734	0.589798	0.435385	0.87077	0.435385
H_2	0.092038	0.173071	0.326484	0.652967	0.326484
H_3	0.07994	0.018876	0.285285	0.570571	0.285285

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	-1.8E-16	0.588791	0.397198	-0.7944	0.397198
H_2	-1.3E-16	0.171573	0.292893	-0.58579	0.292893
H_3	-1.1E-16	0.017332	0.254333	-0.50867	0.254333

f_0	Low pass		f_0	High pass	
5.5	Ω_c	1.17085	5.25	Ω_c	1.081794

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.249172	0.592823	0.467742	0.935484	0.467742
H_2	0.184214	0.177578	0.35144	0.70288	0.35144
H_3	0.160114	0.023524	0.307322	0.614644	0.307322

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.124734	0.589798	0.366266	-0.73253	0.366266
H_2	0.092038	0.173071	0.270258	-0.54052	0.270258
H_3	0.07994	0.018876	0.234734	-0.46947	0.234734

f_0	Low pass		f_0	High pass	
5.75	Ω_c	1.268494	5.5	Ω_c	1.17085

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.373015	0.597868	0.500467	1.000933	0.500467
H_2	0.276665	0.185136	0.377283	0.754566	0.377283
H_3	0.24076	0.031333	0.330339	0.660678	0.330339

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.249172	0.592823	0.335913	-0.67183	0.335913
H_2	0.184214	0.177578	0.248341	-0.49668	0.248341
H_3	0.160114	0.023524	0.215852	-0.4317	0.215852

f_0	Low pass		f_0	High pass	
6	Ω_c	1.376382	5.75	Ω_c	1.268494

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.495954	0.604941	0.533484	1.066968	0.533484
H_2	0.369527	0.195816	0.404071	0.808143	0.404071
H_3	0.322119	0.0424	0.354432	0.708864	0.354432

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.373015	0.597868	0.306213	-0.61243	0.306213
H_2	0.276665	0.185136	0.227118	-0.45424	0.227118
H_3	0.24076	0.031333	0.197643	-0.39529	0.197643

f_0	Low pass		f_0	High pass	
6.25	Ω_c	1.496606	6	Ω_c	1.376382

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.617671	0.614052	0.566716	1.133432	0.566716
H_2	0.462938	0.209715	0.431872	0.863744	0.431872
H_3	0.404443	0.05686	0.37971	0.759419	0.37971

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.495954	0.604941	0.277247	-0.55449	0.277247
H_2	0.369527	0.195816	0.206572	-0.41314	0.206572
H_3	0.322119	0.0424	0.18007	-0.36014	0.18007

f_0	Low pass		f_0	High pass	
6.5	Ω_c	1.631852	6.25	Ω_c	1.496606

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.73783	0.625211	0.60008	1.20016	0.60008
H_2	0.557031	0.226966	0.460759	0.921519	0.460759
H_3	0.487992	0.074895	0.406298	0.812595	0.406298

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.617671	0.614052	0.249095	-0.49819	0.249095
H_2	0.462938	0.209715	0.186694	-0.37339	0.186694
H_3	0.404443	0.05686	0.163104	-0.32621	0.163104

f_0	Low pass		f_0	High pass	
6.75	Ω_c	1.785628	6.5	Ω_c	1.631852

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.856078	0.638432	0.63349	1.266981	0.63349
H_2	0.651938	0.247733	0.490816	0.981633	0.490816
H_3	0.573045	0.096739	0.434339	0.868679	0.434339

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.73783	0.625211	0.221845	-0.44369	0.221845
H_2	0.557031	0.226966	0.167484	-0.33497	0.167484
H_3	0.487992	0.074895	0.146726	-0.29345	0.146726

f_0	Low pass		f_0	High pass	
7	Ω_c	1.962611	6.75	Ω_c	1.785628

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.972037	0.653728	0.666853	1.333705	0.666853
H_2	0.747789	0.272215	0.522135	1.044269	0.522135
H_3	0.659895	0.122681	0.463998	0.927997	0.463998

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.856078	0.638432	0.195588	-0.39118	0.195588
H_2	0.651938	0.247733	0.148949	-0.2979	0.148949
H_3	0.573045	0.096739	0.130924	-0.26185	0.130924

f_0	Low pass		f_0	High pass	
7.25	Ω_c	2.169168	7	Ω_c	1.962611

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.085301	0.671112	0.700066	1.400131	0.700066
H_2	0.844707	0.300654	0.554814	1.109628	0.554814
H_3	0.748861	0.153073	0.495462	0.990925	0.495462

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	0.972037	0.653728	0.170423	-0.34085	0.170423
H_2	0.747789	0.272215	0.131106	-0.26221	0.131106
H_3	0.659895	0.122681	0.115696	-0.23139	0.115696

f_0	Low pass		f_0	High pass	
7.5	Ω_c	2.414214	7.25	Ω_c	2.169168

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.195434	0.690599	0.733018	1.466037	0.733018
H_2	0.942809	0.333333	0.588962	1.177925	0.588962
H_3	0.840287	0.188345	0.528946	1.057893	0.528946

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.085301	0.671112	0.146453	-0.29291	0.146453
H_2	0.844707	0.300654	0.113987	-0.22797	0.113987
H_3	0.748861	0.153073	0.101053	-0.20211	0.101053

f_0	Low pass		f_0	High pass	
7.75	Ω_c	2.710619	7.5	Ω_c	2.414214

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.301965	0.712198	0.765588	1.531175	0.765588
H_2	1.042202	0.370587	0.624695	1.24939	0.624695
H_3	0.934551	0.229016	0.564695	1.129391	0.564695

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.195434	0.690599	0.123791	-0.24758	0.123791
H_2	0.942809	0.333333	0.097631	-0.19526	0.097631
H_3	0.840287	0.188345	0.087015	-0.17403	0.087015

f_0	Low pass		f_0	High pass	
8	Ω_c	3.077684	7.75	Ω_c	2.710619

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.404385	0.735915	0.797636	1.595273	0.797636
H_2	1.142981	0.412802	0.662131	1.324262	0.662131
H_3	1.032069	0.275708	0.602989	1.205977	0.602989

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.301965	0.712198	0.102558	-0.20512	0.102558
H_2	1.042202	0.370587	0.082096	-0.16419	0.082096
H_3	0.934551	0.229016	0.073616	-0.14723	0.073616

f_0	Low pass		f_0	High pass	
8.25	Ω_c	3.545733	8	Ω_c	3.077684

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.502142	0.761754	0.82901	1.658019	0.82901
H_2	1.245219	0.460427	0.70139	1.40278	0.70139
H_3	1.133306	0.329173	0.64414	1.288279	0.64414

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.404385	0.735915	0.082883	-0.16577	0.082883
H_2	1.142981	0.412802	0.067455	-0.13491	0.067455
H_3	1.032069	0.275708	0.06091	-0.12182	0.06091

f_0	Low pass		f_0	High pass	
8.5	Ω_c	4.1653	8.25	Ω_c	3.545733

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.594641	0.789707	0.85953	1.71906	0.85953
H_2	1.348968	0.513982	0.742579	1.485158	0.742579
H_3	1.238781	0.390317	0.688493	1.376987	0.688493

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.502142	0.761754	0.064903	-0.12981	0.064903
H_2	1.245219	0.460427	0.053802	-0.1076	0.053802
H_3	1.133306	0.329173	0.048967	-0.09793	0.048967

f_0	Low pass		f_0	High pass	
8.75	Ω_c	5.027339	8.5	Ω_c	4.1653

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.681239	0.81976	0.888987	1.777975	0.888987
H_2	1.454244	0.574062	0.785771	1.571542	0.785771
H_3	1.34908	0.460234	0.736407	1.472814	0.736407

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.594641	0.789707	0.048767	-0.09753	0.048767
H_2	1.348968	0.513982	0.041254	-0.08251	0.041254
H_3	1.238781	0.390317	0.037884	-0.07577	0.037884

f_0	Low pass		f_0	High pass	
8.75	Ω_c	5.027339	8.5	Ω_c	4.1653

low pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.681239	0.81976	0.888987	1.777975	0.888987
H_2	1.454244	0.574062	0.785771	1.571542	0.785771
H_3	1.34908	0.460234	0.736407	1.472814	0.736407

high pass coefficients

	a_1	a_2	b_0	b_1	b_2
H_1	1.594641	0.789707	0.048767	-0.09753	0.048767
H_2	1.348968	0.513982	0.041254	-0.08251	0.041254
H_3	1.238781	0.390317	0.037884	-0.07577	0.037884

APPENDIX – B

STEPS INVOLVED FOR OUTPUT IN THIS THESIS AND DETAILS

Step 1: Calculate coefficient of Butterworth filter

The coefficients a_1, a_2, b_0, b_1, b_2 used in various frequency bandwidths for low pass and high pass filter

The calculation of coefficients of low pass and high pass are of the form

For low pass coefficients

$$H_1: \quad b_0 = \frac{\Omega c^2}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad b_1 = \frac{2\Omega c^2}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_1 \Omega c + 1]}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad b_2 = b_0$$

$$H_2: \quad b_0 = \frac{\Omega c^2}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad b_1 = \frac{2\Omega c^2}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_2 \Omega c + 1]}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad b_2 = b_0$$

$$H_3: \quad b_0 = \frac{\Omega c^2}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad b_1 = \frac{2\Omega c^2}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_3 \Omega c + 1]}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad b_2 = b_0$$

For high pass coefficients

$$H_1: \quad b_0 = \frac{1}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad b_1 = \frac{-2\Omega c^2}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_1 \Omega c + 1]}{[\Omega c^2 - \alpha_1 \Omega c + 1]}; \quad b_2 = b_0$$

$$H_2: \quad b_0 = \frac{1}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad b_1 = \frac{-2\Omega c^2}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_2 \Omega c + 1]}{[\Omega c^2 - \alpha_2 \Omega c + 1]}; \quad b_2 = b_0$$

$$H_3: \quad b_0 = \frac{1}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad a_1 = \frac{2[\Omega c^2 - 1]}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad b_1 = \frac{-2\Omega c^2}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad a_2 = \frac{[\Omega c^2 + \alpha_3 \Omega c + 1]}{[\Omega c^2 - \alpha_3 \Omega c + 1]}; \quad b_2 = b_0$$

Where values of α_1, α_2, α_3 are constants which is given in next page which comes during cascading and Ω_c a frequency parameter which is dependent on filter cutoff frequency

$\Omega_c = \tan(\pi f_0 \Delta t)$. Where f_0 is filter cutoff frequency and Δt is time step of recorded data

The coefficients a_1, a_2, b_0, b_1, b_2 used in various frequency bandwidths for low pass and high pass filter

Step 2: filtering of data in bandwidth (for El Centro earthquake 1559 points of discrete data)

<u>For low pass filter 1st stage in cascading (For H_1)</u>

In MS Excel cells are written as A1, A2 ……..,B1,B2,….etc.

Raw data has been feed into cell A, hence 1st data is A1, A2, …………etc. and output will be in consecutive cells of column B, C, D,………..

For filtering difference equation is used

$$y_k=[b_0x_k+b_1x_{k-1}+b_2x_{k-2}]-[a_1y_{k-1}+a_2y_{k-2}]$$

Operations involved in MS Excel

Reverse time history (for this the last data will be treated as first data)

B1559=b_0*A1559

B1558=b_0*A1558+b_1*A1559-a_1*B1559

B1557=(b_0*A1557+b_1*A1558+b_2*A1559)-(a_1*B1558+a_2*B1559)

……………………………………………………………..

………………………………………………………………..

B1=(b_0*A1+b_1*A2+b_2*A3)-(a_1*B2+a_2*B3)

Reverse time history (the output data as 1st row data will be treated as first data and so on)

Output will be in column C

C1=b_0*B1

C2=b_0*B2+b_1*B1-a_1*C1

C3=(b_0*B3+b_1*B2+b_2*B1)-(a_1*C2+a_2*C1)

……………………………………………………………..

……………………………………………………………………..
C1559=(b_0*B1559+b_1*B1558+b_2*B1557)-(a_1*C1558+a_2*C1557)

For low pass filter 2nd stage in cascading (For H_2)
Reverse time history (for this the last data will be treated as first data)
Output will be in column D
D1559=b_0*C1559
D1558=b_0*C1558+b_1*C1559-a_1*D1559
D1557=(b_0*C1557+b_1*C1558+b_2*C1559)-(a_1*D1558+a_2*D1559)
…………………………………………………………..
……………………………………………………………..
D1=(b_0*C1+b_1*C2+b_2*C3)-(a_1*D2+a_2*D3)
Reverse time history (the output data as 1st row data will be treated as first data and so on)
Output will be in column E
E1=b_0*D1
E2=b_0*D2+b_1*D1-a_1*E1
E3=(b_0*D3+b_1*D2+b_2*D1)-(a_1*E2+a_2*E1)
…………………………………………………………..
……………………………………………………………..
E1559=(b_0*D1559+b_1*D1558+b_2*D1557)-(a_1*E1558+a_2*E1557)
For low pass filter 3rd stage in cascading (For H_3)
Reverse time history (for this the last data will be treated as first data)
Output will be in column F
F1559=b_0*E1559
F1558=b_0*E1558+b_1*E1559-a_1*F1559
F1557=(b_0*E1557+b_1*E1558+b_2*E1559)-(a_1*F1558+a_2*F1559)
…………………………………………………………..
……………………………………………………………..
F1=(b_0*E1+b_1*E2+b_2*E3)-(a_1*F2+a_2*F3)
Reverse time history (the output data as 1st row data will be treated as first data and so on)
Output will be in column G
G1=b_0*F1
G2=b_0*F2+b_1*F1-a_1*G1
G3=(b_0*F3+b_1*F2+b_2*F1)-(a_1*G2+a_2*G1)
…………………………………………………………..
……………………………………………………………..
G1559=(b_0*F1559+b_1*F1558+b_2*F1557)-(a_1*G1558+a_2*G1557)
For high pass filter 1st stage in cascading (For H_1)
Reverse time history (for this the last data will be treated as first data)
Output will be in column H
H1559=b_0*G1559
H1558=b_0*G1558+b_1*G1559-a_1*H1559
H1557=(b_0*G1557+b_1*G1558+b_2*G1559)-(a_1*H1558+a_2*H1559)
…………………………………………………………..
……………………………………………………………..
H1=(b_0*G1+b_1*G2+b_2*G3)-(a_1*H2+a_2*H3)

Reverse time history (the output data as 1st row data will be treated as first data and so on)
Output will be in column I
I1=b_0*H1
I2=b_0*H2+b_1*H1-a_1*I1
I3=(b_0*H3+b_1*H2+b_2*H1)-(a_1*I2+a_2*I1)
…………………………………………………………..
……………………………………………………………..
I1559=(b_0*H1559+b_1*H1558+b_2*H1557)-(a_1*I1558+a_2*I1557)

For high pass filter 2nd stage in cascading (For H_2)

Reverse time history (for this the last data will be treated as first data)
Output will be in column J
J1559=b_0*I1559
J1558=b_0*I1558+b_1*I1559-a_1*J1559
J1557=(b_0*I1557+b_1*I1558+b_2*I1559)-(a_1*J1558+a_2*J1559)
…………………………………………………………..
……………………………………………………………..
H1=(b_0*I1+b_1*I2+b_2*I3)-(a_1*J2+a_2*J3)
Reverse time history (the output data as 1st row data will be treated as first data and so on)
Output will be in column K
K1=b_0*J1
K2=b_0*J2+b_1*J1-a_1*K1
K3=(b_0*J3+b_1*J2+b_2*J1)-(a_1*K2+a_2*K1)
…………………………………………………………..
……………………………………………………………..
K1559=(b_0*J1559+b_1*J1558+b_2*J1557)-(a_1*K1558+a_2*K1557)

For high pass filter 3rd stage in cascading (For H_3)

Reverse time history (for this the last data will be treated as first data)
Output will be in column L
L1559=b_0*K1559
L1558=b_0*K1558+b_1*K1559-a_1*L1559
L1557=(b_0*K1557+b_1*K1558+b_2*K1559)-(a_1*L1558+a_2*L1559)
…………………………………………………………..
……………………………………………………………..
L1=(b_0*K1+b_1*K2+b_2*K3)-(a_1*L2+a_2*L3)
Reverse time history (the output data as 1st row data will be treated as first data and so on)
Output will be in column M
M1=b_0*L1
M2=b_0*L2+b_1*L1-a_1*K1
M3=(b_0*L3+b_1*L2+b_2*L1)-(a_1*M2+a_2*M1)
…………………………………………………………..
……………………………………………………………..
M1559=(b_0*L1559+b_1*L1558+b_2*L1557)-(a_1*M1558+a_2*M1557)

Final output of filtered data for a particular bandwidth is in column M

Step 3: calculate root mean square value

Output will be in column N
Operation will be
N1=power(M1,2)
N2=power(M2,2)
…………………
…………………….
N1559=power(M1559,2)

Average of value of column N is in cell O4
O4=AVERAGE(N1:N1559)
RMS of the average value is in P4
P4=SQRT(O4)

Final output of rms value of the filtered data in a particular bandwidth is in cell O4.

Step 5: calculation of Kanai –Tajimi parameters in MS Excel by using formulas given in Chapter 3

APPENDIX - C

RAW DATA OF ELCENTRO EARTHQUAKE (1559 POINTS)

0	0.06987	-0.29941	-0.00772	-0.12753	0.00419	-0.02496	-0.03095	0.04977	0.01571	0.06208	-0.01204	0.01605
0.0063	0.0887	-0.00421	0.01064	-0.17561	-0.01819	-0.04574	-0.03192	0.08446	-0.01402	0.03378	0.03031	0.02239
0.00364	0.04524	0.29099	0.029	-0.22369	-0.04057	-0.02071	-0.02588	0.05023	-0.04374	0.00549	0.07265	0.04215
0.00099	0.00179	0.2238	0.04737	-0.27177	-0.06294	0.00432	-0.01984	0.016	-0.07347	-0.02281	0.11499	0.06191
0.00428	-0.04167	0.15662	0.06573	-0.15851	-0.02417	0.02935	-0.01379	-0.01823	-0.0399	-0.05444	0.07237	0.08167
0.00758	-0.08513	0.08943	0.02021	-0.04525	0.0146	0.01526	-0.00775	-0.05246	-0.00633	-0.0403	0.02975	0.03477
0.01087	-0.12858	0.02224	-0.0253	0.06802	0.05337	0.01806	-0.01449	-0.08669	0.02724	-0.02615	-0.01288	-0.01212
0.00682	-0.17204	-0.04495	-0.07081	0.18128	0.02428	0.02086	-0.02123	-0.06769	0.0608	-0.01201	0.01212	-0.01309
0.00277	-0.12908	0.01834	-0.04107	0.14464	-0.0048	0.00793	0.01523	-0.0487	0.03669	-0.02028	0.03711	-0.01407
-0.00128	-0.08613	0.08163	-0.01133	0.108	-0.03389	-0.00501	0.0517	-0.0297	0.01258	-0.02855	0.03517	-0.05274
0.00368	-0.08902	0.14491	0.00288	0.07137	-0.00557	-0.01795	0.08816	-0.01071	-0.01153	-0.06243	0.03323	-0.02544
0.00864	-0.09192	0.2082	0.01709	0.03473	0.02274	-0.03089	0.12463	0.00829	-0.03564	-0.03524	0.01853	0.00186
0.0136	-0.09482	0.18973	0.03131	0.09666	0.00679	-0.01841	0.16109	-0.00314	-0.00677	-0.00805	0.00383	0.02916
0.00727	-0.09324	0.17125	-0.02278	0.1586	-0.00915	-0.00593	0.12987	0.02966	0.0221	-0.04948	0.00342	0.05646
0.00094	-0.09166	0.13759	-0.07686	0.22053	-0.02509	0.00655	0.09864	0.06246	0.05098	-0.03643	-0.02181	0.08376
0.0042	-0.09478	0.10393	-0.13095	0.18296	-0.04103	-0.02519	0.06741	-0.00234	0.07985	-0.02337	-0.04704	0.01754
0.00221	-0.09789	0.07027	-0.18504	0.14538	-0.05698	-0.05693	0.03618	-0.06714	0.06915	-0.03368	-0.07227	-0.04869
0.00021	-0.12902	0.03661	-0.14347	0.1078	-0.01826	-0.04045	0.00495	-0.04051	0.05845	-0.01879	-0.0975	-0.02704
0.00444	-0.07652	0.00295	-0.1019	0.07023	0.02046	-0.02398	0.0042	-0.01388	0.04775	-0.00389	-0.12273	0.00722
0.00867	-0.02401	-0.03071	-0.06034	0.03265	0.00454	-0.0075	0.00345	0.01274	0.03706	0.011	-0.08317	0.03517
0.0129	0.02849	-0.00561	-0.01877	0.06649	-0.01138	0.00897	0.00269	0.00805	0.02636	0.02589	-0.04362	-0.00528
0.01713	0.08099	0.01948	0.0228	0.10033	-0.00215	0.00384	-0.05922	0.03024	0.05822	0.01446	-0.00407	-0.04572
-0.00343	0.1335	0.04458	-0.00996	0.13417	0.00708	-0.00129	-0.12112	0.05243	0.09009	0.00303	0.03549	-0.08617
-0.024	0.186	0.06468	-0.04272	0.10337	0.00496	-0.00642	-0.18303	0.02351	0.12196	-0.0084	0.07504	-0.0696
-0.00992	0.2385	0.08478	-0.02147	0.07257	0.00285	-0.01156	-0.12043	-0.00541	0.10069	0.00463	0.1146	-0.05303
0.00416	0.21993	0.10487	-0.00021	0.04177	0.00074	-0.02619	-0.05782	-0.03432	0.07943	0.01766	0.07769	-0.03646
0.00528	0.20135	0.05895	0.02104	0.01097	-0.00534	-0.04082	0.00479	-0.06324	0.05816	0.03069	0.04078	-0.01989
0.01653	0.18277	0.01303	-0.01459	-0.01983	-0.01141	-0.05545	0.0674	-0.09215	0.03689	0.04372	0.00387	-0.00332
0.02779	0.1642	-0.03289	-0.05022	0.04438	0.00361	-0.04366	0.13001	-0.12107	0.01563	0.02165	0.00284	0.01325
0.03904	0.14562	-0.07882	-0.08585	0.1086	0.01863	-0.03188	0.08373	-0.0845	-0.00564	-0.00042	0.00182	0.02982
0.02449	0.16143	-0.03556	-0.12148	0.17281	0.03365	-0.06964	0.03745	-0.04794	-0.0269	-0.02249	-0.05513	0.01101
0.00995	0.17725	0.00771	-0.15711	0.10416	0.04867	-0.05634	0.06979	-0.01137	-0.04817	-0.04456	0.04732	-0.00781
0.00961	0.13215	0.05097	-0.19274	0.03551	0.0304	-0.04303	0.10213	0.0252	-0.06944	-0.03638	0.05223	-0.02662
0.00926	0.08705	0.01013	-0.22837	-0.03315	0.01213	-0.02972	-0.03517	0.06177	-0.0907	-0.02819	0.05715	-0.00563
0.00892	0.04196	-0.03071	-0.18145	-0.1018	-0.00614	-0.01642	-0.17247	0.04028	-0.11197	-0.02001	0.06206	0.01536
-0.00486	-0.00314	-0.07156	-0.13453	-0.07262	-0.02441	-0.00311	-0.13763	0.0188	-0.11521	-0.01182	0.06698	0.03635
-0.01864	-0.04824	-0.1124	-0.08761	-0.04344	0.01375	0.0102	-0.10278	0.04456	-0.11846	-0.02445	0.07189	0.05734
-0.03242	-0.09334	-0.15324	-0.04069	-0.01426	0.01099	0.0235	-0.06794	0.07032	-0.1217	-0.03707	0.02705	0.03159
-0.03365	-0.13843	-0.11314	0.00623	0.01492	0.00823	0.03681	-0.0331	0.09608	-0.12494	-0.04969	-0.01779	0.00584
-0.05723	-0.18353	-0.07304	0.05316	-0.02025	0.00547	0.05011	-0.03647	0.12184	-0.165	-0.05882	-0.06263	-0.01992
-0.04534	-0.22863	-0.03294	0.10008	-0.05543	0.00812	0.02436	-0.03984	0.0635	-0.20505	-0.06795	-0.10747	-0.00201
-0.03346	-0.27372	0.00715	0.147	-0.0906	0.01077	-0.00139	-0.00517	0.00517	-0.15713	-0.07707	-0.15232	0.01589
-0.03201	-0.31882	-0.0635	0.09754	-0.12578	-0.00692	-0.02714	0.0295	-0.05317	-0.10921	-0.0862	-0.12591	-0.01024
-0.03056	-0.25024	-0.13415	0.04808	-0.16095	-0.02461	-0.00309	0.06417	-0.03124	-0.06129	-0.09533	-0.0995	-0.03636
-0.02911	-0.18166	-0.2048	-0.00138	-0.19613	-0.0423	0.02096	0.09883	-0.0093	-0.01337	-0.06276	-0.07309	-0.06249
-0.02766	-0.11309	-0.12482	0.05141	-0.14784	-0.05999	0.04501	0.1335	0.01263	0.03455	-0.03018	-0.04668	-0.0478
-0.04116	-0.04451	-0.04485	0.1042	-0.09955	-0.07768	0.06906	0.05924	0.03457	0.08247	0.00239	-0.02027	-0.03311
-0.05466	0.02407	0.03513	0.15699	-0.05127	-0.09538	0.05773	-0.01503	0.03283	0.07576	0.03496	0.00614	-0.04941
-0.06816	0.09265	0.1151	0.20979	-0.00298	-0.06209	0.0464	-0.08929	0.03109	0.06906	0.04399	0.03255	-0.0657
-0.08166	0.16123	0.19508	0.26258	-0.01952	-0.0288	0.03507	-0.16355	0.02935	0.06236	0.05301	0.00859	-0.082
-0.06846	0.22981	0.12301	0.16996	-0.03605	0.00448	0.03357	-0.06096	0.04511	0.08735	0.03176	-0.01537	-0.0498
-0.05527	0.29839	0.05094	0.07734	-0.05259	0.03777	0.03207	0.04164	0.06087	0.11235	0.01051	-0.03932	-0.0176
-0.04208	0.23197	-0.02113	-0.01527	-0.04182	0.01773	0.03057	0.01551	0.07663	0.13734	-0.01073	-0.06328	0.0146
-0.04259	0.16554	-0.0932	-0.10789	-0.03106	-0.00231	0.0325	-0.01061	0.09239	0.12175	-0.03198	-0.03322	0.0468
-0.04311	0.09912	-0.02663	-0.20051	-0.02903	-0.02235	0.03444	-0.03674	0.05742	0.10616	-0.05323	-0.00315	0.079
-0.02428	0.0327	0.03995	-0.06786	-0.02699	0.01791	0.03637	-0.06287	0.02245	0.09057	0.00186	0.02691	0.0475
-0.00545	-0.03372	0.10653	0.06479	0.02515	0.05816	0.01348	-0.08899	-0.01252	0.07498	0.05696	0.01196	0.016
0.01338	-0.10014	0.17311	0.01671	0.0177	0.03738	-0.00942	-0.0543	0.0068	0.08011	0.01985	-0.003	-0.0155
0.03221	-0.16656	0.11283	-0.03137	0.02213	0.0166	-0.03231	-0.01961	0.02611	0.08524	-0.01726	0.00335	-0.00102
0.05104	-0.23299	0.05255	-0.07945	0.02656	-0.00418	-0.02997	0.01508	0.04543	0.09037	-0.05438	0.0097	0.01347

0.02795	0.02214	-0.00121	0.02917	-0.0679	0.0203	-0.03071	-0.03778	-0.03203	0.02107	0.01417	-0.03612	-0.00374
0.04244	0.02483	0.01953	0.03015	-0.04862	0.03009	-0.03007	-0.02281	-0.01652	0.01158	0.02039	-0.02828	-0.00368
0.05692	0.01809	0.04027	0.03113	-0.02934	0.03989	-0.01863	-0.00784	-0.00102	0.0078	0.02661	-0.02044	-0.00361
0.03781	-0.00202	0.02826	0.00388	-0.01006	0.03478	-0.00719	0.00713	0.00922	0.00402	0.03283	-0.0126	-0.00355
0.0187	-0.02213	0.01625	-0.02337	0.00922	0.02967	0.00425	0.0221	0.01946	0.00024	0.03905	-0.00476	-0.00349
-0.00041	-0.00278	0.00424	-0.05062	0.02851	0.02457	0.0157	0.03707	0.0297	-0.00354	0.04527	0.00307	-0.00342
-0.01952	0.01656	0.00196	-0.0382	0.04779	0.03075	0.02714	0.05204	0.03993	-0.00732	0.03636	0.01091	-0.00336
-0.00427	0.0359	-0.00031	-0.02579	0.02456	0.03694	0.03858	0.06701	0.05017	-0.0111	0.0275	0.00984	-0.0033
0.01098	0.05525	-0.00258	-0.01337	0.00133	0.04313	0.02975	0.08198	0.06041	-0.0078	0.01862	0.00876	-0.00323
0.02623	0.07459	-0.00486	-0.00095	-0.0219	0.04931	0.02092	0.03085	0.07065	-0.0045	0.00974	0.00768	-0.00317
0.04148	0.06203	-0.00713	0.01146	-0.04513	0.0555	0.02334	-0.02027	0.08089	-0.0012	0.00086	0.00661	-0.00311
0.01821	0.04948	-0.00941	0.02388	-0.06836	0.06168	0.02576	-0.0714	-0.00192	0.0021	-0.01333	0.01234	-0.00304
-0.00506	0.03692	-0.01168	0.03629	-0.04978	-0.00526	0.02819	-0.01225	-0.08473	0.0054	-0.02752	0.01807	-0.00298
-0.00874	-0.00145	-0.01396	0.01047	-0.0312	-0.0722	0.03061	-0.08644	-0.07032	-0.00831	-0.04171	0.0238	-0.00292
-0.03726	0.04599	-0.0175	-0.01535	-0.01262	-0.06336	0.03304	-0.05035	-0.0559	-0.02203	-0.02812	0.02953	-0.00285
-0.06579	0.04079	-0.02104	-0.04117	0.00596	-0.06451	0.01371	-0.01426	-0.04148	-0.03575	-0.01453	0.03526	-0.00279
-0.026	0.03558	-0.02458	-0.06699	0.02453	-0.04566	-0.00561	0.02183	-0.05296	-0.04947	-0.00094	0.02784	-0.00273
0.0138	0.03037	-0.02813	-0.05207	0.04311	-0.03681	-0.02494	0.05792	-0.06443	-0.06319	0.01264	0.02042	-0.00266
0.05359	0.03626	-0.03167	-0.03715	0.06169	-0.03678	-0.02208	0.094	-0.0759	-0.05046	0.02623	0.013	-0.0026
0.09338	0.04215	-0.03521	-0.02222	0.08027	-0.03675	-0.01923	0.13009	-0.08738	-0.03773	0.0169	-0.03415	-0.00254
0.05883	0.04803	-0.04205	-0.0073	0.09885	-0.03672	-0.01638	0.03611	-0.09885	-0.025	0.00756	-0.00628	-0.00247
0.02429	0.05392	-0.04889	0.00762	0.06452	-0.01765	-0.01353	-0.05787	-0.06798	-0.01227	-0.00177	-0.00621	-0.00241
-0.01026	0.04947	-0.03559	0.02254	0.03019	0.00143	-0.01261	-0.04802	-0.0371	0.00046	-0.01111	-0.00615	-0.00235
-0.0448	0.04502	-0.02229	0.03747	-0.00414	0.02051	-0.0117	-0.03817	-0.00623	0.00482	-0.02044	-0.00609	-0.00228
-0.01083	0.04056	-0.00899	0.04001	-0.03848	0.03958	-0.00169	-0.02832	0.02465	0.00919	-0.02977	-0.00602	-0.00222
-0.01869	0.03611	0.00431	0.04256	-0.07281	0.05866	0.00833	-0.01846	0.05553	0.01355	-0.03911	-0.00596	-0.00216
-0.02655	0.03166	0.01762	0.04507	-0.05999	0.03556	0.01834	-0.00861	0.0864	0.01791	-0.02442	-0.0059	-0.00209
-0.03441	0.00614	0.00714	0.04759	-0.04717	0.01245	0.02835	-0.03652	0.11728	0.02228	-0.00973	-0.00583	-0.00203
-0.02503	-0.01937	-0.00334	0.0501	-0.03435	-0.01066	0.03836	-0.06444	0.14815	0.00883	0.00496	-0.00577	-0.00197
-0.01564	-0.04489	-0.01383	0.04545	-0.03231	-0.03376	0.04838	-0.06169	0.08715	-0.00462	0.01965	-0.00571	-0.0019
-0.00626	-0.0704	0.01314	0.0408	-0.03028	-0.05687	0.03749	-0.05894	0.02615	-0.01807	0.03434	-0.00564	-0.00184
-0.01009	-0.09592	0.04011	0.02876	-0.02824	-0.04502	0.0266	-0.05618	-0.03485	-0.03152	0.02054	-0.00558	-0.00178
-0.01392	-0.07745	0.06708	0.01671	-0.00396	-0.03317	0.01571	-0.06073	-0.09584	-0.02276	0.00674	-0.00552	-0.00171
0.0149	-0.05899	0.0482	0.00467	0.02032	-0.02131	0.00482	-0.06528	-0.071	-0.01401	-0.00706	-0.00545	-0.00165
0.04372	-0.04052	0.02932	-0.00738	0.00313	-0.00946	-0.00607	-0.04628	-0.04616	-0.00526	-0.02086	-0.00539	-0.00158
0.03463	-0.02206	0.01043	-0.00116	-0.01406	0.00239	-0.01696	-0.02728	-0.02132	0.0035	-0.03466	-0.00532	-0.00152
0.02098	-0.00359	-0.00845	0.00506	-0.03124	-0.00208	-0.0078	-0.00829	0.00353	0.01225	-0.02663	-0.00526	-0.00146
0.00733	0.01487	-0.02733	0.01128	-0.04843	-0.00654	0.00136	0.01071	0.02837	0.02101	-0.0186	-0.0052	-0.00139
-0.00632	0.01005	-0.04621	0.0175	-0.06562	-0.01101	0.01052	0.0297	0.05321	0.01437	-0.01057	-0.00513	-0.00133
-0.01997	0.00523	-0.03155	-0.00211	-0.05132	-0.01548	0.01968	0.03138	-0.00469	0.00773	-0.00254	-0.00507	-0.00127
0.00767	0.00041	-0.01688	-0.02173	-0.03702	-0.012	0.02884	0.03306	-0.06258	0.0011	-0.00063	-0.00501	-0.0012
0.03532	-0.00441	-0.00222	-0.04135	-0.02272	-0.00851	-0.00504	0.03474	-0.12048	0.00823	0.00128	-0.00494	-0.00114
0.03409	-0.00923	0.01244	-0.06096	-0.00843	-0.00503	-0.03893	0.03642	-0.0996	0.01537	0.00319	-0.00488	-0.00108
0.03287	-0.01189	0.02683	-0.08058	0.00587	-0.00154	-0.02342	0.04574	-0.07872	0.02251	0.0051	-0.00482	-0.00101
0.03164	-0.01523	0.04121	-0.06995	0.02017	0.00195	-0.00791	0.05506	-0.05784	0.01713	0.00999	-0.00475	-0.00095
0.02403	-0.01856	0.0559	-0.05931	0.02698	0.00051	0.00759	0.06439	-0.03696	0.01175	0.01488	-0.00469	-0.00089
0.01642	-0.0219	0.03253	-0.04868	0.03379	-0.00092	0.0231	0.07371	-0.01608	0.00637	0.00791	-0.00463	-0.00082
0.00982	-0.00983	0.00946	-0.03805	0.04061	0.01135	0.00707	0.08303	0.0048	0.01376	0.00093	-0.00456	-0.00076
0.00322	0.00224	-0.0136	-0.02557	0.04742	0.02363	-0.00895	0.03605	0.02568	0.02114	-0.00605	-0.0045	-0.0007
-0.00339	0.01431	-0.01432	-0.0131	0.05423	0.0359	-0.02498	-0.01092	0.04656	0.02852	0.00342	-0.00444	-0.00063
0.02202	0.00335	-0.01504	-0.00063	0.03535	0.04818	-0.041	-0.0579	0.06744	0.03591	0.01288	-0.00437	-0.00057
-0.01941	-0.0076	-0.01576	0.01185	0.01647	0.06045	-0.05703	-0.04696	0.08832	0.04329	0.02235	-0.00431	-0.00051
-0.06085	-0.01856	-0.04209	0.02432	0.01622	0.07273	-0.0292	-0.03602	0.1092	0.03458	0.03181	-0.00425	-0.00044
-0.10228	-0.00737	-0.02685	0.0368	0.01598	0.02847	-0.00137	-0.02508	0.13008	0.02587	0.04128	-0.00418	-0.00038
-0.07847	0.00383	-0.01161	0.04927	0.01574	-0.01579	0.02645	-0.01414	0.10995	0.01715	0.02707	-0.00412	-0.00032
-0.05466	0.01502	0.00363	0.02974	0.00747	-0.06004	0.05428	-0.03561	0.08982	0.00844	0.01287	-0.00406	-0.00025
-0.03084	0.02622	0.01887	0.01021	-0.0008	-0.05069	0.03587	-0.05708	0.06969	-0.00027	-0.00134	-0.00399	-0.00019
-0.00703	0.01016	0.03411	-0.00932	-0.00907	-0.04134	0.01746	-0.07855	0.04955	-0.00898	-0.01554	-0.00393	-0.00013
0.01678	-0.0059	0.03115	-0.02884	0.00072	-0.03199	-0.00096	-0.06304	0.04006	-0.00126	-0.02975	-0.00387	-6.00E-05
0.01946	-0.02196	0.02819	-0.04837	0.01051	-0.03135	-0.01937	-0.04753	0.03056	0.00645	-0.04395	-0.0038	

Sample Output value i.e. filtered data in frequency bandwidth 0-0.25 Hz

-0.0016595	0.004035	0.007777	0.00165738	-0.00311	-0.00444	-0.00546	-0.003541	0.00185	0.006994	0.00235932	-0.00642	-0.0060041
-0.0016511	0.004158	0.007745	0.00153061	-0.00314	-0.00447	-0.00546	-0.00348	0.001964	0.007013	0.00220282	-0.00651	-0.0059062
-0.0016368	0.00428	0.007711	0.00140477	-0.00317	-0.00449	-0.00545	-0.003418	0.002077	0.007029	0.00204501	-0.0066	-0.0058061
-0.001617	0.004401	0.007672	0.00127994	-0.00319	-0.00452	-0.00545	-0.003355	0.002191	0.007041	0.00188599	-0.00668	-0.0057038
-0.0015918	0.004522	0.00763	0.00115617	-0.00322	-0.00455	-0.00544	-0.003292	0.002305	0.00705	0.00172584	-0.00676	-0.0055994
-0.0015615	0.004642	0.007585	0.00103354	-0.00325	-0.00457	-0.00543	-0.003227	0.002419	0.007056	0.00156465	-0.00684	-0.0054928
-0.0015263	0.00476	0.007537	0.00091211	-0.00327	-0.0046	-0.00542	-0.003161	0.002533	0.007059	0.0014025	-0.00691	-0.0053843
-0.0014863	0.004878	0.007485	0.00079194	-0.0033	-0.00462	-0.00541	-0.003094	0.002647	0.007058	0.00123947	-0.00698	-0.0052739
-0.0014418	0.004994	0.007429	0.00067309	-0.00332	-0.00465	-0.0054	-0.003027	0.002761	0.007053	0.00107567	-0.00705	-0.0051615
-0.0013929	0.005109	0.007371	0.00055561	-0.00335	-0.00468	-0.00539	-0.002958	0.002875	0.007046	0.00091118	-0.00711	-0.0050473
-0.0013398	0.005223	0.007309	0.00043957	-0.00337	-0.0047	-0.00538	-0.002888	0.002988	0.007034	0.00074607	-0.00717	-0.0049313
-0.0012828	0.005336	0.007244	0.000325	-0.00339	-0.00473	-0.00536	-0.002817	0.003101	0.007019	0.00058046	-0.00723	-0.0048136
-0.0012218	0.005446	0.007176	0.00021196	-0.00342	-0.00476	-0.00535	-0.002746	0.003214	0.007001	0.00041441	-0.00728	-0.0046942
-0.0011572	0.005556	0.007105	0.0001005	-0.00344	-0.00478	-0.00533	-0.002673	0.003327	0.006979	0.00024803	-0.00733	-0.0045733
-0.0010891	0.005664	0.007031	-9.34E-06	-0.00346	-0.00481	-0.00532	-0.002599	0.003439	0.006954	8.14E-05	-0.00738	-0.0044508
-0.0010175	0.00577	0.006954	-0.0001175	-0.00348	-0.00483	-0.0053	-0.002524	0.003551	0.006925	-8.54E-05	-0.00742	-0.0043268
-0.0009427	0.005874	0.006874	-0.000224	-0.0035	-0.00486	-0.00528	-0.002448	0.003662	0.006893	-0.0002523	-0.00747	-0.0042015
-0.0008647	0.005976	0.006791	-0.0003288	-0.00352	-0.00488	-0.00526	-0.002371	0.003772	0.006857	-0.0004191	-0.0075	-0.0040748
-0.0007838	0.006077	0.006705	-0.0004317	-0.00354	-0.00491	-0.00524	-0.002293	0.003882	0.006817	-0.0005859	-0.00754	-0.0039468
-0.0007	0.006175	0.006616	-0.0005329	-0.00356	-0.00493	-0.00522	-0.002214	0.003991	0.006774	-0.0007524	-0.00757	-0.0038176
-0.0006134	0.006272	0.006525	-0.0006323	-0.00358	-0.00496	-0.0052	-0.002133	0.004099	0.006728	-0.0009187	-0.00759	-0.0036873
-0.0005243	0.006366	0.006432	-0.0007298	-0.0036	-0.00498	-0.00518	-0.002052	0.004206	0.006678	-0.0010846	-0.00762	-0.0035558
-0.0004325	0.006458	0.006335	-0.0008254	-0.00362	-0.00501	-0.00515	-0.00197	0.004312	0.006625	-0.00125	-0.00764	-0.0034234
-0.0003384	0.006548	0.006237	-0.0009192	-0.00364	-0.00503	-0.00513	-0.001886	0.004418	0.006568	-0.0014149	-0.00765	-0.00329
-0.000242	0.006635	0.006136	-0.001011	-0.00366	-0.00505	-0.0051	-0.001802	0.004522	0.006507	-0.0015792	-0.00767	-0.0031557
-0.0001434	0.00672	0.006032	-0.001101	-0.00368	-0.00507	-0.00507	-0.001717	0.004625	0.006443	-0.0017427	-0.00768	-0.0030207
-4.26E-05	0.006803	0.005927	-0.0011889	-0.00369	-0.0051	-0.00505	-0.00163	0.004727	0.006376	-0.0019055	-0.00768	-0.0028848
6.01E-05	0.006883	0.005819	-0.001275	-0.00371	-0.00512	-0.00502	-0.001543	0.004827	0.006306	-0.0020673	-0.00768	-0.0027483
0.00016475	0.00696	0.00571	-0.0013591	-0.00373	-0.00514	-0.00499	-0.001454	0.004927	0.006232	-0.0022282	-0.00768	-0.0026112
0.00027122	0.007035	0.005598	-0.0014412	-0.00375	-0.00516	-0.00496	-0.001364	0.005024	0.006154	-0.002388	-0.00768	-0.0024735
0.00037942	0.007107	0.005484	-0.0015214	-0.00377	-0.00518	-0.00492	-0.001274	0.005121	0.006074	-0.0025467	-0.00767	-0.0023353
0.00048927	0.007176	0.005369	-0.0015997	-0.00379	-0.0052	-0.00489	-0.001182	0.005215	0.00599	-0.0027042	-0.00766	-0.0021968
0.00060067	0.007242	0.005252	-0.001676	-0.00381	-0.00522	-0.00486	-0.001089	0.005308	0.005903	-0.0028604	-0.00765	-0.0020578
0.00071354	0.007306	0.005133	-0.0017503	-0.00383	-0.00524	-0.00482	-0.000996	0.0054	0.005812	-0.0030152	-0.00763	-0.0019187
0.00082781	0.007366	0.005013	-0.0018228	-0.00385	-0.00526	-0.00479	-0.000901	0.005489	0.005719	-0.0031686	-0.00761	-0.0017792
0.00094338	0.007424	0.004892	-0.0018933	-0.00387	-0.00527	-0.00475	-0.000805	0.005577	0.005622	-0.0033204	-0.00759	-0.0016397
0.00106017	0.007478	0.004769	-0.0019619	-0.00389	-0.00529	-0.00471	-0.000709	0.005663	0.005523	-0.0034707	-0.00756	-0.0015001
0.00117812	0.00753	0.004645	-0.0020286	-0.00391	-0.00531	-0.00468	-0.000611	0.005747	0.00542	-0.0036192	-0.00753	-0.0013604
0.00129714	0.007578	0.004519	-0.0020935	-0.00393	-0.00532	-0.00464	-0.000513	0.005829	0.005314	-0.003766	-0.00749	-0.0012209
0.00141715	0.007623	0.004393	-0.0021565	-0.00395	-0.00534	-0.0046	-0.000413	0.005909	0.005206	-0.003911	-0.00746	-0.0010814
0.00153808	0.007665	0.004266	-0.0022176	-0.00397	-0.00535	-0.00456	-0.000313	0.005986	0.005094	-0.0040541	-0.00742	-0.0009422
0.00165985	0.007703	0.004137	-0.002277	-0.00399	-0.00537	-0.00451	-0.000212	0.006062	0.00498	-0.0041952	-0.00737	-0.0008032
0.00178238	0.007738	0.004008	-0.0023345	-0.00401	-0.00538	-0.00447	-0.000109	0.006135	0.004863	-0.0043342	-0.00733	-0.0006645
0.00190561	0.00777	0.003879	-0.0023904	-0.00403	-0.00539	-0.00443	-6.31E-06	0.006205	0.004743	-0.0044712	-0.00728	-0.0005262
0.00202945	0.007798	0.003748	-0.0024445	-0.00406	-0.0054	-0.00438	9.76E-05	0.006274	0.004621	-0.004606	-0.00723	-0.0003884
0.00215383	0.007823	0.003618	-0.0024969	-0.00408	-0.00541	-0.00434	0.0002022	0.00634	0.004496	-0.0047386	-0.00717	-0.0002511
0.00227869	0.007845	0.003487	-0.0025476	-0.0041	-0.00542	-0.00429	0.0003077	0.006403	0.004368	-0.0048688	-0.00711	-0.0001143
0.00240394	0.007863	0.003355	-0.0025968	-0.00412	-0.00543	-0.00424	0.0004139	0.006464	0.004239	-0.0049967	-0.00705	2.18E-05
0.0025295	0.007878	0.003223	-0.0026443	-0.00415	-0.00544	-0.00419	0.0005209	0.006522	0.004106	-0.0051222	-0.00699	0.0001572
0.00265532	0.007889	0.003092	-0.0026903	-0.00417	-0.00544	-0.00415	0.0006286	0.006577	0.003972	-0.0052453	-0.00692	0.0002918
0.00278131	0.007896	0.00296	-0.0027347	-0.00419	-0.00545	-0.0041	0.0007369	0.00663	0.003835	-0.0053658	-0.00685	0.0004255
0.0029074	0.0079	0.002828	-0.0027777	-0.00422	-0.00545	-0.00404	0.0008459	0.00668	0.003696	-0.0054837	-0.00678	0.0005584
0.00303352	0.007901	0.002696	-0.0028193	-0.00424	-0.00546	-0.00399	0.0009555	0.006727	0.003555	-0.005599	-0.0067	0.0006903
0.00315959	0.007898	0.002565	-0.0028595	-0.00427	-0.00546	-0.00394	0.0010657	0.006771	0.003411	-0.0057115	-0.00662	0.0008213
0.00328554	0.007891	0.002433	-0.0028983	-0.00429	-0.00546	-0.00388	0.0011765	0.006812	0.003266	-0.0058214	-0.00654	0.0009511
0.00341129	0.007881	0.002303	-0.0029358	-0.00432	-0.00547	-0.00383	0.0012877	0.00685	0.003119	-0.0059284	-0.00646	0.0010798
0.00353677	0.007867	0.002172	-0.002972	-0.00434	-0.00547	-0.00377	0.0013994	0.006885	0.002971	-0.0060327	-0.00637	0.0012073
0.00366191	0.00785	0.002043	-0.003007	-0.00437	-0.00547	-0.00372	0.0015116	0.006917	0.00282	-0.006134	-0.00628	0.0013336
0.00378663	0.007829	0.001913	-0.0030409	-0.00439	-0.00547	-0.00366	0.0016241	0.006946	0.002668	-0.0062324	-0.00619	0.0014585
0.00391086	0.007805	0.001785	-0.0030736	-0.00442	-0.00546	-0.0036	0.001737	0.006971	0.002514	-0.0063279	-0.0061	0.0015821

0.001704	0.005077	0.00104667	-0.00336	-0.0035634	0.000619	0.00294573	-0.0018	-0.0042191	0.002739	0.006396	0.00045223	-0.00414
0.001825	0.005058	0.00095563	-0.00341	-0.0035229	0.000699	0.00292085	-0.0019	-0.0041678	0.002874	0.006359	0.00033007	-0.00415
0.001944	0.005036	0.00086467	-0.00345	-0.0034809	0.000778	0.00289352	-0.002	-0.0041133	0.003008	0.006319	0.00020861	-0.00417
0.002062	0.005013	0.00077382	-0.00349	-0.0034376	0.000857	0.00286375	-0.0021	-0.0040555	0.00314	0.006276	8.79E-05	-0.00418
0.002178	0.004986	0.00068311	-0.00353	-0.0033929	0.000936	0.00283155	-0.0022	-0.0039945	0.003271	0.00623	-3.20E-05	-0.00419
0.002292	0.004958	0.00059258	-0.00357	-0.0033469	0.001013	0.00279693	-0.0023	-0.0039303	0.0034	0.006181	-0.00015096	-0.0042
0.002405	0.004927	0.00050227	-0.00361	-0.0032995	0.00109	0.00275989	-0.0024	-0.003863	0.003528	0.006129	-0.00026901	-0.0042
0.002516	0.004894	0.0004122	-0.00364	-0.0032507	0.001166	0.00272047	-0.00249	-0.0037925	0.003654	0.006074	-0.00038605	-0.00421
0.002625	0.004859	0.00032241	-0.00368	-0.0032007	0.001242	0.00267866	-0.00259	-0.0037189	0.003778	0.006016	-0.00050203	-0.00421
0.002732	0.004822	0.00023293	-0.00371	-0.0031493	0.001316	0.00263449	-0.00268	-0.0036423	0.0039	0.005954	-0.0006169	-0.00421
0.002837	0.004783	0.00014379	-0.00374	-0.0030966	0.001389	0.00258797	-0.00278	-0.0035626	0.004021	0.005891	-0.00073059	-0.00421
0.00294	0.004741	5.50E-05	-0.00377	-0.0030426	0.001462	0.00253913	-0.00287	-0.00348	0.004139	0.005824	-0.00084306	-0.00421
0.003042	0.004698	-3.33E-05	-0.0038	-0.0029874	0.001534	0.00248799	-0.00295	-0.0033945	0.004256	0.005754	-0.00095426	-0.0042
0.003141	0.004652	-0.0001213	-0.00383	-0.0029309	0.001604	0.00243457	-0.00304	-0.0033062	0.00437	0.005682	-0.00106412	-0.0042
0.003238	0.004605	-0.0002087	-0.00386	-0.0028732	0.001673	0.0023789	-0.00313	-0.003215	0.004482	0.005607	-0.00117261	-0.00419
0.003333	0.004556	-0.0002957	-0.00388	-0.0028142	0.001741	0.00232102	-0.00321	-0.0031211	0.004592	0.00553	-0.00127968	-0.00418
0.003426	0.004505	-0.0003822	-0.00391	-0.0027541	0.001808	0.00226093	-0.00329	-0.0030245	0.004699	0.00545	-0.00138527	-0.00417
0.003517	0.004452	-0.0004681	-0.00393	-0.0026928	0.001874	0.00219869	-0.00337	-0.0029252	0.004804	0.005367	-0.00148935	-0.00415
0.003605	0.004397	-0.0005535	-0.00395	-0.0026304	0.001938	0.00213432	-0.00345	-0.0028233	0.004907	0.005282	-0.00159187	-0.00414
0.003692	0.00434	-0.0006382	-0.00397	-0.0025669	0.002001	0.00206786	-0.00353	-0.002719	0.005007	0.005195	-0.00169278	-0.00412
0.003776	0.004282	-0.0007224	-0.00399	-0.0025022	0.002063	0.00199935	-0.0036	-0.0026122	0.005104	0.005106	-0.00179205	-0.00411
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0.004014	0.004097	-0.0009708	-0.00404	-0.0023019	0.002239	0.00178189	-0.00381	-0.0022777	0.005379	0.004824	-0.00207963	-0.00405
0.004089	0.004033	-0.0010523	-0.00405	-0.0022331	0.002295	0.00170557	-0.00388	-0.0021618	0.005466	0.004726	-0.00217195	-0.00402
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0.004485	0.003615	-0.0015247	-0.0041	-0.001801	0.002592	0.00121073	-0.00421	-0.001425	0.00592	0.004098	-0.00268654	-0.00386
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0.004972	0.002739	-0.0023086	-0.00406	-0.0009408	0.002958	0.00017042	-0.00455	6.30E-05	0.006452	0.002816	-0.00344149	-0.00346
0.005001	0.002654	-0.0023739	-0.00405	-0.0008595	0.002979	6.96E-05	-0.00457	0.00020388	0.00648	0.002694	-0.00349745	-0.00342
0.005027	0.002568	-0.0024383	-0.00403	-0.000778	0.002997	-3.20E-05	-0.00458	0.00034531	0.006504	0.002571	-0.00355125	-0.00338
0.005051	0.002482	-0.0025016	-0.00402	-0.0006961	0.003013	-0.0001343	-0.00458	0.00048718	0.006525	0.002447	-0.0036029	-0.00334
0.005071	0.002395	-0.0025638	-0.004	-0.0006139	0.003027	-0.0002372	-0.00458	0.00062939	0.006543	0.002323	-0.00365239	-0.0033
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0.005105	0.00222	-0.0026851	-0.00396	-0.000449	0.003047	-0.0004445	-0.00458	0.00091447	0.006568	0.002074	-0.0037449	-0.00321
0.005118	0.002131	-0.0027441	-0.00394	-0.0003664	0.003054	-0.0005487	-0.00457	0.00105714	0.006575	0.001949	-0.00378792	-0.00317
0.005129	0.002042	-0.0028019	-0.00391	-0.0002836	0.003058	-0.0006533	-0.00456	0.00119978	0.006579	0.001823	-0.00382879	-0.00313
0.005137	0.001953	-0.0028587	-0.00389	-0.0002009	0.00306	-0.000758	-0.00455	0.00134229	0.006579	0.001698	-0.00386752	-0.00308
0.005142	0.001863	-0.0029143	-0.00386	-0.0001181	0.003059	-0.0008629	-0.00453	0.00148457	0.006576	0.001572	-0.00390411	-0.00304
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0.005143	0.001592	-0.0030742	-0.00378	0.00012967	0.003044	-0.0011776	-0.00446	0.00190908	0.006546	0.001196	-0.00400112	-0.0029
0.005138	0.001502	-0.0031251	-0.00374	0.00021193	0.003033	-0.0012823	-0.00443	0.0020495	0.006529	0.001071	-0.00402924	-0.00286
0.005131	0.001411	-0.0031749	-0.00371	0.00029394	0.003021	-0.0013867	-0.00439	0.00218922	0.006509	0.000947	-0.00405527	-0.00281
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