# Chapter 1

# Introduction

Microwave communication systems require different types of passive and active components. The important passive components are antennas, filters, couplers, multiplexers etc. In passive component design, miniaturization (size) and performance (bandwidth) are the main features to be achieved while keeping the cost of the design as low as possible. These desired features can be achieved by choosing suitable technology and innovative technical designs. Optimal component design is possible by choosing the design techniques that take advantage of the physics of a component. The cost of the microwave component and, subsequently, the cost of the microwave system is decreased by reducing the design cycle of the component.

The coupler in microwave systems is also used as a passive component to split or combine a signal, and is available for various purposes. In particular, a directional coupler is applied to a linear amplifier to improve inter-modulation properties between stations of recent mobile communication systems. The performance of directional couplers can be evaluated by indices that include coupling, directivity, and reflection loss. On the bases of the evaluation results, a directional coupler with wide bandwidth and high directivity can be used as the main module of communication systems or testing instruments.

The directional couplers are circuits that are essential in many optical communications systems and microwave bands. A directional coupler is a device capable of extracting a part of a signal that travels over a transmission line or waveguide, leaving the rest of power not coupled direct route at the exit. Between its main applications we can mention the measurement of the power, the measurement of the stationary wave, sampling of the control signal, the combination of the microwave signals. They can even be used for performing signal processing tasks: part of balanced amplifiers, mixers, phase shifters, modulators, and demodulators.

## 1.1 Thesis Outline

The objective of the project is to study the design of a coupled microstrip line directional coupler and its properties and then to calculate the directional coupler's length, width and spacing between the lines from the known specifications. The known specifications are coupling, port impedances and the operational frequency.

After calculating physical dimensions of the directional coupler and simulating the coupler designed, the structure of the coupler is modified in order to obtain high isolation and thus high directivity.

The thesis is organized in six chapters. Chapter 1 and chapter 6 describe the introduction and conclusion respectively. The outline of the remaining chapters is given below.

**Chapter 2** gives the basic review of microstrip lines. The two port parameters generally used in microwave components are explained. The mode propagation, various parameters and losses in microstrip lines are explained in this chapter.

**Chapter 3** explains the basic study behind the directional coupler. The general analysis of parallel coupled lines is done describing the coupled mode theory. The even- and odd-mode analysis of coupled microstrip lines is done in this chapter.

**Chapter 4** presents the design of directional coupler and discusses the various methods present in the literature to improve the directivity of the directional coupler.

**Chapter 5** shows the conventional coupler design and the modified coupler design and their simulation results.

# Chapter 2

# Review of Microstrip Lines

## 2.1 Planar Transmission Structures

One of the principal requirements for a transmission structure to be used as a circuit element in microwave integrated circuits (MICs) is that the structure should be planar in configuration. A planar configuration implies that the characteristics of the element can be determined by the dimensions in a single plane. For example, width of the transmission line on a dielectric substrate can be adjusted to change its impedance. When the impedance can be varied by dimensions in a single plane, the circuit fabrication can be conveniently carried out by techniques of photolithography and photoetching of thin films. Use of these techniques has led to the development of hybrid and monolithic MICs [1].

There are several transmission structures that satisfy the requirement of being planar. The most common of these are: (i) microstrip, (ii) coplanar waveguide, (iii) slotline, and (iv) coupled microstrip lines.

***Microstrip Line***

A microstrip line is the most popular transmission structure. Microstrip has a very simple geometric structure but the electromagnetic fields involved are actually complex [2]. Microstrip cannot support a pure TEM, or any other simple electromagnetic field mode and this leads to the propagation of hybrid modes called as quasi-TEM mode. In microstrip, the simple transitions to coaxial circuits are feasible.

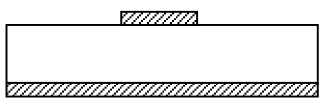

**Figure 2.1 Microstrip Line**

***Slotline***

This transmission line structure consists of a dielectric substrate metalized on one side only. The metallization has a completely separating narrow slot etched into it to form a slot line. Slotline suffers from following disadvantages:

(a) Characteristics impedances below about 60 ohms are difficult to realize.
(b) Circuit structures often involve difficult registration problems (especially with metallization on the opposite side of the slot).
(c) The Q-factor is significantly lower than the other structure considered here.

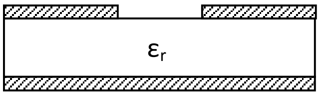


**Figure 2.2 Slotline**

***Coplanar Waveguide***

The structure supports quasi-TEM mode of transmission. The metallization is formed on the one side of the substrate alone. Each side-plane conductor is grounded and the centre strip carries the signal; thus much less field enters the substrate when compared with microstrip. The structure has been used in special applications such as mixers. The open nature of the structure causes radiation at higher frequencies.

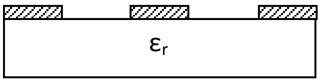


**Figure 2.3 Coplanar Waveguide**

***Coupled Microstrip Lines***

Coupled lines are lines which are laid alongside each other in order to permit coupling between the two lines. In such a configuration there is a continuous coupling between the electromagnetic fields of the two lines. Coupled lines are used extensively as a basic element of the directional couplers, filters and variety of other useful circuits.

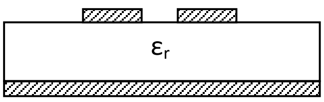


**Figure 2.4 Coupled Microstrip Line**

The planar structures discussed above are studied with the help of S-parameters and ABCD parameters which are discussed in the next section.

## 2.2 Two Port Parameters

### 2.2.1 The Scattering Matrix

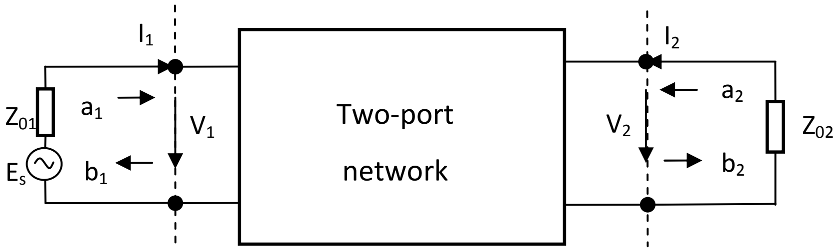


**Figure 2.5 Incident and Reflected voltages shown in two-port network**

At high frequency, two port networks are best characterized in terms of scattering parameters, rather than in terms of the admittance or hybrid parameters. Scattering parameters are defined in terms of travelling waves, which are the natural variables to be used in a transmission line environment [3].

The scattering or *S* parameters of a two-port network are defined in terms of the wave variables as:

$$S_{11} = \left.\frac{b_1}{a_1}\right| \, for \; a_2 = 0 \tag{2.1}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right| \, for \; a_1 = 0 \tag{2.2}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right| \, for \; a_2 = 0 \tag{2.3}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right| \, for \; a_1 = 0 \tag{2.3}$$

where $a_n = 0$ implies a perfect impedance match (no reflection from terminal impedance) at port *n*.

These definitions may be written as:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.5)$$

where the matrix containing the $S$ parameters is referred to as the scattering matrix or $S$ matrix, which may simply be denoted by [$S$].

The parameters $S_{11}$ and $S_{22}$ are also called the reflection coefficients, whereas $S_{12}$ and $S_{21}$ the transmission coefficients. These are the parameters directly measurable at microwave frequencies. The $S$ parameters are in general complex, and it is convenient to express them in terms of amplitudes and phases. Often their amplitudes are given in decibels (dB), which are defined as:

$$20 \log|S_{mn}|\, dB \qquad m, n = 1,2 \quad (2.6)$$

where the logarithm operation is base 10. For filter characterization, we may define two parameters:

$$L_A = -20 \log|S_{mn}|\, dB \qquad m, n = 1,2 (m \neq n) \quad (2.7)$$

$$L_R = 20 \log|S_{nn}|\, dB \qquad n = 1,2 \quad (2.8)$$

In network analysis or synthesis, it may be desirable to express the reflection parameter $S_{11}$ in terms of the terminal impedance $Z_{01}$ and the so-called input impedance $Z_{in1} = V_1/I_1$, which is the impedance looking into port 1 of the network.

$$S_{11} = \frac{V_1/\sqrt{Z_{01}} - \sqrt{Z_{01}} I_1}{V_1/\sqrt{Z_{01}} + \sqrt{Z_{01}} I_1} \quad (2.9)$$

Replacing $V_1$ by $Z_{in1}I_1$ results in the desired expression:

$$S_{11} = \frac{Z_{in1} - Z_{01}}{Z_{in1} + Z_{01}} \quad (2.10)$$

Similarly, we can have

$$S_{22} = \frac{Z_{in2} - Z_{02}}{Z_{in2} + Z_{02}} \quad (2.11)$$

where $Z_{in2} = V_2/I_2$ is the input impedance looking into port 2 of the network.

The $S$ parameters have several properties that are useful for network analysis. For a reciprocal network $S_{12} = S_{21}$. If the network is symmetrical, an additional property, $S_{11} = S_{22}$, holds. Hence, the symmetrical network is also reciprocal. For a lossless passive network the transmitting power and the reflected power must equal to the total incident power. The mathematical statements of this power conservation condition are

$$S_{21}S_{21}^* + S_{11}S_{11}^* = 1 \; or \; |S_{21}|^2 + |S_{11}|^2 = 1 \qquad (2.12)$$

$$S_{12}S_{12}^* + S_{22}S_{22}^* = 1 \; or \; |S_{12}|^2 + |S_{22}|^2 = 1 \qquad (2.13)$$

Any circuit consisting of resistors, capacitors, inductors, transformers and length of transmission line will be reciprocal. Any circuit that happens to be symmetrical, whether reciprocal or not, will of course have a symmetrical matrix, but the converse does not hold.

A two port network for which the reverse transmission coefficient for the input matched is zero is known as a unilateral two port network. Thus a unilateral two port network is a one that works in only one direction, while a reciprocal network is a one that acts equally well in the both directions.

Although scattering parameters are undoubtedly the most useful and the most commonly used parameters for characterizing a two port network constructed with microstrip lines, they do not always present the simplest way of dealing with certain problems. Many microwave networks consist of a cascade connection of two or more two-port networks in this case it is convenient to define a 2 x 2 transmission line, or ABCD matrix, for each two port network. This is discussed in the next section.

### 2.2.2 The Transmission (ABCD) Parameters

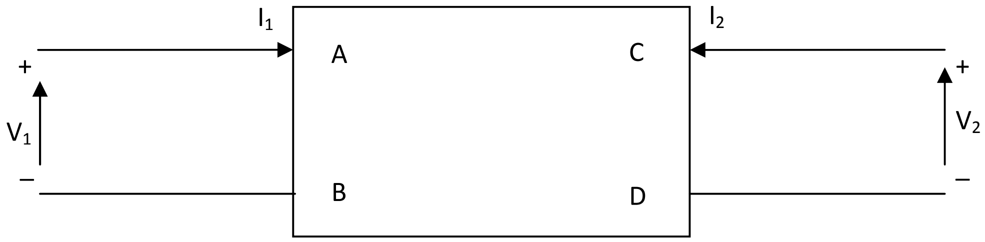


**Figure 2.6 ABCD Parameters of Two port Network**

The *ABCD* matrix of a two-port is defined using voltages and currents. The *ABCD* matrix is defined by:

$$V_1 = AV_2 + B(-I_2) \tag{2.14}$$

$$I_1 = CV_2 + D(-I_2) \tag{2.15}$$

In matrix form this provides the relation,

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \tag{2.16}$$

Depending on the properties of the *ABCD* matrix, the structures can be classified into the following categories:

*Reciprocal*: In this case, the determinant of the ABCD matrix is equal to unity:

$$AD - BC = 1 \tag{2.17}$$

*Symmetrical*: In this case, the parameters *A* and *D* are equal:

$$A = D \tag{2.18}$$

## 2.3 Microstrip Lines

Microstrip line is one of the most popular planar transmission line, primarily because it can be fabricated by photolithographic processes and is easily integrated with other passive and active microwave devices [4].

The general geometry of microstrip is shown in the figure 2.7. A conductor of width $W$ is printed on a thin, grounded dielectric substrate of thickness $h$ and relative permittivity $\varepsilon_r$; a sketch of electric field lines is shown in figure 2.7(b).

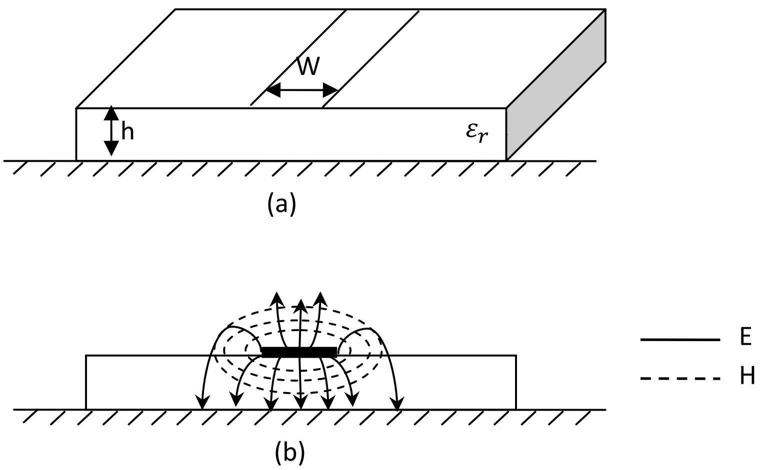


**Figure 2.7 Microstrip transmission line. (a) Geometry. (b) Electric and Magnetic field lines**

### 2.3.1 The Quasi-TEM Mode of Propagation

The microstrip involves an abrupt dielectric interface between the substrate and the air above it. Any transmission line which is filled with a uniform dielectric can support a single, well defined mode of propagation, at least over a specified range of frequencies. Transmission lines which do not have such a uniform dielectric filling cannot support a single mode of propagation; microstrip is within this category. The phase velocity of TEM fields in the dielectric region would be $c/\sqrt{\varepsilon_r}$, but the phase velocity of TEM fields in the air region would be $c$. Thus, a phase match at dielectric-air interface would be impossible to attain for a TEM type wave.

In actuality, the exact fields of a microstrip line constitute a hybrid TM-TE wave. So, the fields are referred to as quasi-TEM.

The static or quasi-static solutions of these modes can lead to the good approximation of Static-TEM parameters discussed in the next section.

### 2.3.2 Static-TEM Parameters

The microstrip synthesis problem consists of finding the values of width w and length $l$ corresponding to the characteristic impedance $Z_0$ and electrical length $\theta$ defined at the network design stage. Initially a suitable substrate of thickness h and relative permittivity $\varepsilon_r$ will have to be chosen. The choice also depends upon certain limitations of frequency. The synthesis actually yields the normalized width-to-height ratio *w/h* initially, as well as a quantity called the effective microstrip permittivity $\varepsilon_{eff}$ . This quantity is unique to mixed-dielectric transmission line systems and it provides a useful link between various wavelengths, impedances, and propagation velocities [2].

**The Characteristic Impedance $Z_O$**

For any TEM-type transmission line the characteristic impedance at high frequencies may be expressed in any one of three alternate forms

$$Z_O = \sqrt{\frac{L}{C}} \tag{2.19}$$

$$Z_O = v_p L \tag{2.20}$$

$$Z_O = \frac{1}{v_p C} \tag{2.21}$$

Equation 2.20 and 2.21 involve the phase velocity $v_p$ of the wave travelling along the line. The phase velocity is given by

$$v_p = \frac{1}{\sqrt{LC}} \tag{2.22}$$

When the substrate of the microstrip line is removed we have an air-filled line along which the wave will travel at c, the velocity of light in free space is $3*10^8$m/s. the characteristic impedance of this air-filled 'microstrip', $Z_{01}$,

$$Z_{01} = \sqrt{\frac{L}{C_1}} \tag{2.23}$$

$$Z_{01} = cL \tag{2.24}$$

$$Z_{01} = \frac{1}{cC_1} \tag{2.25}$$

where L remains unaltered and $C_1$ is the capacitance per unit length for this structure.

Combining equations (2.19), (2.24), and (2.25) yields the result

$$Z_O = \frac{1}{c\sqrt{CC_1}} \tag{2.26}$$

This means we have the required characteristic impedance only if we can evaluate the capacitances per unit length of the structure, with and without the presence of the dielectric substrate.

**The effective microstrip permittivity $\varepsilon_{eff}$**

For the air-spaced microstrip line the propagation velocity is given by

$$c = \frac{1}{\sqrt{LC_1}} \tag{2.27}$$

and dividing equation (2.27) by equation (2.22), we obtain

$$\frac{C}{C_1} = \left(\frac{c}{v_p}\right)^2 \tag{2.28}$$

The capacitance ratio $C/C_1$ is termed the effective microstrip permittivity,$\varepsilon_{eff}$, an important microstrip parameter.

$$\varepsilon_{eff} = \left(\frac{c}{v_p}\right)^2 \tag{2.29}$$

The relationship between $Z_0$, $Z_{01}$, and $\varepsilon_{eff}$ is given by

$$Z_O = \frac{Z_{01}}{\sqrt{\varepsilon_{eff}}} \tag{2.30}$$

that is,

$$Z_{01} = Z_0\sqrt{\varepsilon_{eff}} \tag{2.31}$$

For very wide lines nearly all the electric field is confined to substrate dielectric, the substrate resembles a parallel plate capacitor, and therefore, at this extreme,

$$\varepsilon_{eff} \rightarrow \varepsilon_r$$

In the case of very narrow lines the field is almost equally shared between air ($\varepsilon_r = 1$) and substrate so that, at this extreme

$$\varepsilon_{eff} \approx \frac{1}{2}(\varepsilon_r + 1) \tag{2.32}$$

The range of $\varepsilon_{eff}$ is therefore

$$\frac{1}{2}(\varepsilon_r + 1) \leq \varepsilon_{eff} \leq \varepsilon_r \tag{2.33}$$

It can be convenient to express effective microstrip permittivity as follows

$$\varepsilon_{eff} = 1 + q(\varepsilon_r - 1) \tag{2.34}$$

where the new quantity, filling factor, has the bounds

$$\frac{1}{2} \leq q \leq 1 \tag{2.35}$$

**Synthesis: the width-to-height ratio *w/h***

The width to height ratio (w/h) is a strong function of $Z_0$ and of the substrate permittivity $\varepsilon_r$.

*Synthesis formulae ($Z_O$ and $\varepsilon_r$ given)*

For narrow strips (i.e. when $\mathrm{Z_0} > \{44 - 2\varepsilon_r\}$ ohms):

$$\frac{w}{h} = \left(\frac{\exp H'}{8} - \frac{1}{4\exp H'}\right)^{-1} \tag{2.36}$$

where

$$H' = \frac{Z_O\sqrt{2(\varepsilon_r + 1)}}{119.9} + \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right) \tag{2.37}$$

For wide strips (i.e. when $\mathrm{Z_0} < \{44 - 2\varepsilon_r\}$ohms):

$$\frac{w}{h} = \frac{2}{\pi}\{(d-1) - \ln(2d-1)\} + \frac{\varepsilon_r - 1}{\pi\varepsilon_r}\left\{\ln(d-1) + 0.293 - \frac{0.517}{\varepsilon_r}\right\} \tag{2.38}$$

where

$$d = \frac{59.95\pi^2}{Z_O\sqrt{\varepsilon_r}} \tag{2.39}$$

*Analysis formulae (w/h and $\varepsilon_r$ given)*

For 'narrow strips' (w/h<3.3):

$$Z_O = \frac{119.9}{\sqrt{2(\varepsilon_r + 1)}}\left[\ln\left\{4\frac{h}{w} + \sqrt{16\left(\frac{h}{w}\right)^2 + 2}\right\} - \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right)\right] \tag{2.40}$$

For 'wide strips' (w/h>3.3):

$$Z_O = \frac{119.9\pi}{2\sqrt{\varepsilon_r}}\left[\frac{w}{2h} + \frac{\ln 4}{\pi} + \frac{\ln(e\pi^2/16)}{2\pi}\left(\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right) + \frac{\varepsilon_r + 1}{2\pi\varepsilon_r}\left\{\ln\frac{\pi e}{2} + \ln\left(\frac{w}{2h} + 0.94\right)\right\}\right]^{-1}$$

where e is the exponential base: e= 2.7182818

### 2.3.3 Effect of Finite Microstrip Thickness on Characteristic Impedance

Practical microstrip circuits have a finite thickness, *t* which must influence the field distribution. For most single microstrip lines the effect of this thickness on the design parameters is very small and may often be neglected. Even in the case of microstrip circuits using thick film technology, there is usually no need to allow for thickness when calculating impedance or effective microstrip permittivity because such films invariably taper towards the strip edges [2].

With some etched microstrip circuits, on plastic substrates, and circuits where the microstrips are designed to carry at least moderate power, the thickness may be significant. An indication of change in electric field distribution is provided in figure 2.8. We can see that the increase in thickness leads to radiation of electric field in air. Thus there is power loss.

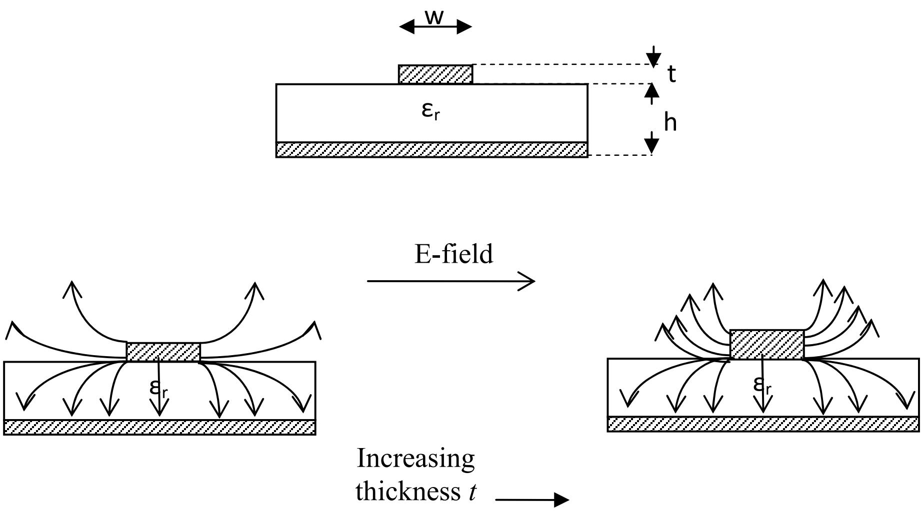


**Figure 2.8 Changes in the distribution of electric field (transverse cross-section) as a thickness of microstrip is altered**

Some accurate expressions have been reported which are now reported here.

For $w/h \leq 1$:

$$Z_O = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(8\frac{h}{w_e} + 0.25\frac{w_e}{h}\right) \quad (2.42)$$

For $w/h \geq 1$:

$$Z_O = \frac{120\pi}{\sqrt{\varepsilon_{eff}}}\left\{\frac{w_e}{h} + 1.393 + 0.667\left(\frac{w_e}{h} + 1.444\right)\right\}^{-1} \quad (2.43)$$

where with $w/h \leq 1/2\pi$

$$\frac{w_e}{h} = \frac{w}{h} + \frac{1.25t}{\pi h}\left(1 + \ln\frac{4\pi w}{t}\right) \quad (2.44)$$

with $w/h \geq 1/2\pi$

$$\frac{w_e}{h} = \frac{w}{h} + \frac{1.25t}{\pi h}\left(1 + \ln\frac{2h}{t}\right) \quad (2.45)$$

where $w_e$ is the effective microstrip width when thickness of the microstrip is finite and cannot be ignored and is denoted by $t$.

Finally, the effective microstrip permittivity $\varepsilon_{eff}$ should be evaluated using expressions given previously and the following term subtracted:

$$\Delta\varepsilon_{eff}(t) = \frac{(\varepsilon_r - 1)t/h}{4.6\sqrt{w/h}} \quad (2.46)$$

So that the final value is given by

$$\varepsilon_{eff}(t) = \varepsilon_{eff} - \Delta\varepsilon_{eff}(t) \quad (2.47)$$

For microstrip having $t/h \leq 0.005$, $2 < \varepsilon_r < 10$, and $w/h \geq 0.1$, the effects of this thickness are negligible. At smaller values of $w/h$ or greater values of $t/h$, the significance increases.

There are various losses involved in microstrip lines. These losses are discussed in the next section.

### 2.3.4 Losses in Microstrip Lines

#### *Dielectric losses*

The dielectric substrate has a complex relative permittivity given by $\varepsilon = \varepsilon_r \varepsilon_0 (1 - j \tan \delta)$ where $\tan \delta = (G/\omega C)$ is the loss tangent for the substrate. The loss tangent represents the ratio of conduction to displacement currents that flow in the dielectric region [3]. For a TEM wave propagating along a low-loss uniformly filled transmission line, the attenuation due to the losses in the dielectric material is

$$\alpha_d = \frac{GZ_0}{2} = \frac{\omega C Z_0}{2} \tan \delta \qquad neper.m^{-1} \tag{2.48}$$

A dielectric-loss effective filling factor, $q_d$, is introduced now to allow for the fact that the lossy dielectric material does not completely fill the whole microstrip cross-section. It is clear that $q_d$ will differ from a similar effective filling factor for capacitance calculations since, on the one hand, for evaluating $q_d$ there is no conductance component associated with the air region above the substrate, even though there is energy transfer through this region, while on the other hand, for $\varepsilon_{eff}$ calculations there is a capacitive component for the air region. In the above equation, C is the total capacitance per unit length of line and,

$$CZ_0 = \frac{\sqrt{\varepsilon_{eff}}}{c} \tag{2.49}$$

It follows that

$$\alpha_d = \frac{\pi f \sqrt{\varepsilon_{eff}}}{c} q_d \tan \delta \tag{2.50}$$

The effective filling factor for the dielectric loss tangent

$$q_d = \frac{electric\ energy\ stored\ per\ meter\ in\ the\ substrate}{electric\ energy\ stored\ per\ meter\ in\ the\ complete\ line} = \frac{W_S}{W}$$

Furthermore, it is shown that

$$\frac{\partial W}{\partial \varepsilon_r} = \frac{W_S}{\varepsilon_r} \; and \; \frac{\partial W}{\partial \varepsilon_{eff}} = \frac{W}{\varepsilon_{eff}} \tag{2.51}$$

giving

$$\frac{\partial \varepsilon_{eff}}{\partial \varepsilon_r} = \frac{\varepsilon_{eff}}{\varepsilon_r}\frac{W_S}{W} = \frac{\varepsilon_{eff}}{\varepsilon_r} q_d \tag{2.52}$$

Thus

$$q_d = \frac{\varepsilon_r}{\varepsilon_{eff}}\frac{\partial \varepsilon_{eff}}{\partial \varepsilon_r} \tag{2.53}$$

The variation of $\varepsilon_{eff}$ with $\varepsilon_r$ for a given line geometry is found by using $\varepsilon_{eff} = 1 + q(\varepsilon_r - 1)$, from which

$$q_d = \frac{\varepsilon_r}{\varepsilon_{eff}}\left\{q + (\varepsilon_r - 1)\frac{\partial q}{\partial \varepsilon_r}\right\} \tag{2.54}$$

It is observed that for all substrate parameter permittivity, $0.5 < q < 1.0$, and for $w/h$ =1, for which slope is about the largest

$$\frac{\partial q}{\partial \varepsilon_r} = -\frac{0.03}{\varepsilon_r^2} \tag{2.55}$$

Thus the expression

$$q_d = \frac{\varepsilon_r}{\varepsilon_{eff}} q \tag{2.56}$$

will slightly overestimate the dielectric loss, but not more than about 1% if $\varepsilon_r \geq 2.5$. Hence, equation 2.50 becomes

$$\alpha_d = \frac{\pi f\sqrt{\varepsilon_{eff}}}{c}\frac{\varepsilon_r}{\varepsilon_{eff}}\left\{\frac{\varepsilon_{eff} - 1}{\varepsilon_r - 1}\right\}\tan\delta \tag{2.57}$$

or on rearranging

$$\alpha_d = \frac{\pi\sqrt{\varepsilon_{eff}}}{\lambda_0}\left\{\frac{1 - \left(\varepsilon_{eff}\right)^{-1}}{1 - (\varepsilon_r)^{-1}}\right\}\tan\delta \quad neper.m^{-1} \tag{2.58}$$

where $\lambda_0$ is the free space wavelength.

### *Conductor losses*

At microwave frequencies, the current flows through a thin layer on the outside surface of the microstrip conductors. With the transverse dimensions much greater than the measure of this layer thickness, called the skin depth, the skin effects of an actual conductor surface may be analyzed in terms of a plane wave propagating along the normal into the conductor. For a plane wave propagating in a good conductor with no magnetic losses and with the conductivity $\sigma \gg \omega\varepsilon$, the propagation coefficient

$$\gamma = \sqrt{j\omega LG} \;\; \Rightarrow \;\; \alpha = \beta = \left(\frac{\omega LG}{2}\right)^{\frac{1}{2}}$$

$$\Rightarrow \; \alpha = \beta = \left(\frac{\omega\mu\sigma}{2}\right)^{\frac{1}{2}} \qquad (2.59)$$

Where $\mu$ is the permeability of the conductor that, for non-magnetic materials, is taken as $\mu_0$, the permeability of free space. Thus the fields and currents decay exponentially into the conductor and, at one skin depth,$\delta$ have decayed to $e^{-1}$ of their surface values, i.e. $\delta = \alpha^{-1}$. Hence the skin depth

$$\delta = \left\{\frac{2}{\omega\mu\sigma}\right\}^{\frac{1}{2}} m \qquad (2.60)$$

It may be assumed that if there are at least three skin depths of conductor thickness from each surface then the assumption of the uniform current distribution will produce errors that are negligible in comparison with other possible sources of error, such as the effects of surface roughness [3].

# Chapter 3

# Parallel Coupled Microstrip Lines and Directional Coupler

## 3.1 Introduction

A "coupled line" consists of two transmission lines placed parallel to each other and in close proximity as shown in the figure 3.1. In such configuration there is a continuous coupling between the electromagnetic fields of the two lines. Coupled lines are utilized extensively as basic elements for directional couplers, filters, and a variety of other useful circuits [1].

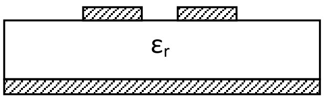


**Figure 3.1 A pair of parallel, edge-coupled microstrip line**

Because of the coupling of electromagnetic fields, a pair of coupled lines can support two modes of propagation. These modes have different characteristic impedances. The velocities of propagation of these two modes are equal when the lines are embedded in a homogeneous dielectric medium. This is a desirable property for the design of circuits such as directional couplers and filters. However, for transmission lines such as coupled microstrip lines, the dielectric medium is not homogeneous. A part of the field extends into the air above the substrate. This fraction is different for the two modes of coupled lines. Consequently, the effective dielectric constants (and their phase velocities) are not equal for the two modes. The non-synchronous feature deteriorates the performance of circuits using these type of coupled lines [1].

When the two conductors of a coupled line pair are identical we have a symmetrical configuration. This symmetry is very useful for simplifying the analysis and design of such coupled lines. If the two lines do not have the same characteristic impedance, the configuration is called asymmetric.

For the lines operating in the TEM mode or when the analysis can be based on quasi-static approximation, the properties of the coupled lines can be determined from the self and the mutual inductances and the capacitance for the lines. In the case of lines operating in the non-TEM mode, a full-wave analysis is needed for the two modes of propagation.

## 3.2 General Analysis of Coupled lines

### 3.2.1 Methods of Analysis

The four different methods that are generally employed for determining the propagation characteristics are the even- and odd-mode method, the coupled mode formulation, the graph transformation technique, and the congruent transformation technique.

The even- and odd-mode method is the most convenient way of describing the behavior of symmetrical coupled lines. In this method wave propagation along a coupled pair of lines is expressed in terms of two-modes corresponding to an even and an odd symmetry about a plane that can therefore be replaced by a magnetic or electric wall for the purpose of analysis [1].

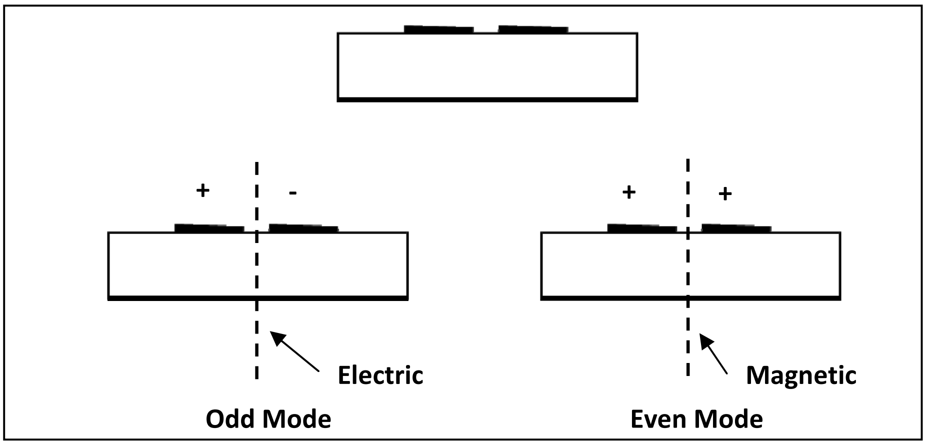


**Figure3.2 Even and Odd Mode Analysis of Coupled Microstrip Lines**

In the coupled mode approach, the wave propagation is expressed in terms of the mode of propagation on individual uncoupled lines modified by the coupling because of mutual capacitances and inductances. This approach, therefore, provides an insight into the mechanism of coupling. The method is quite general and is applicable to asymmetric coupled lines also. This approach finds application in all types of coupled systems used in various disciplines.

Coupled microstrip lines are characterized by the phase velocity and characteristic impedances of the two modes. For the purpose of synthesis, the charts and graphs can be prepared for a number of coupled line parameters and thus using tables the desired information can be obtained. This procedure is time-consuming. So, the design equations can be used to save time.

With our present work we will be discussing the first two methods to study the equivalent behavior of the coupled lines. The coupled line theory is discussed next which relates the coupled lines with the equivalent capacitance per unit length.

### 3.2.2 Coupled Line Theory

The coupled lines can be represented by the structure as shown in figure 3.3. If we assume a TEM type of propagation, then the electrical characteristics of the coupled lines can be completely determined from the effective capacitances between the lines and the velocity of propagation on the line [4].

As depicted in the figure 3.3, $C_{12}$ represents capacitance between two strip conductors in the absence of ground conductor, while $C_{11}$ and $C_{22}$ represent capacitance between the two strip conductors in the absence of another strip conductor. If the strip conductors are identical in size and location relative to the ground conductor, then $C_{11} = C_{22}$.

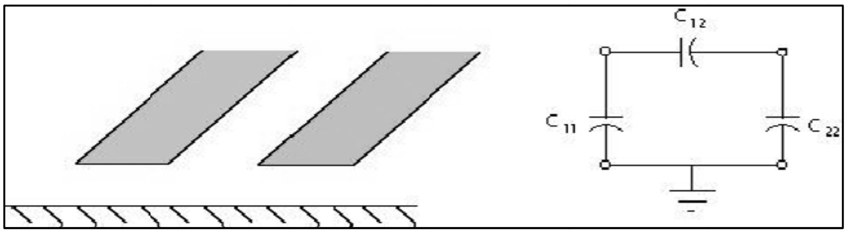


**Figure3.3 Equivalent Diagram of Coupled Microstrip Lines [4]**

Now consider two special cases of excitations for the coupled line: the even mode, where the currents in the strip conductors are equal in amplitude and in the same direction, and the odd mode, where the currents in the strip conductors are equal in amplitude but opposite in directions.

For the even mode, the electric field has even symmetry about the center line, and no current flow between the strip conductors. This leads to the equivalent circuit shown in the figure 3.4, where $C_{12}$ is effectively open circuited. Then the resulting capacitance of either line to ground for the even mode is

$$C_e = C_{11} = C_{22} \tag{3.1}$$

assuming that the two strip conductors are identical in size and location.

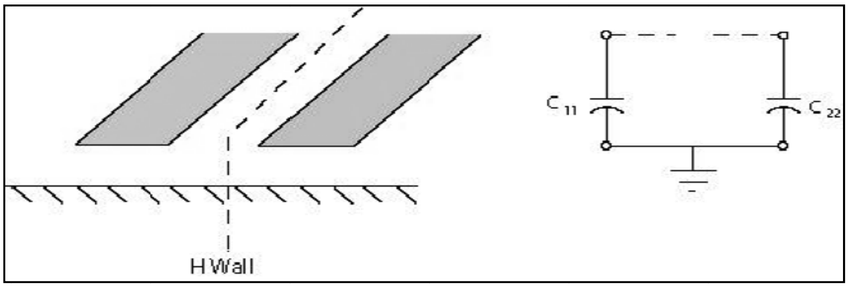


**Figure3.4 Even Mode Excitation [4]**

Then the characteristic impedance for the even mode is

$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{vC_e} \tag{3.2}$$

where $v$ is the velocity of propagation on the line.

For the odd mode, electric field lines have odd symmetry about the center line, and a voltage null exists between the two strip conductors. We can imagine this as a ground plane through the middle of $C_{12}$, which leads to the equivalent circuit as shown in figure 3.5.

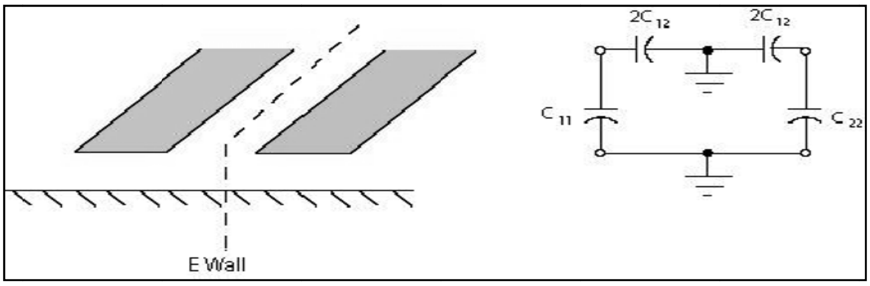


**Figure3.5 Odd Mode Excitation [4]**

In this case, the effective capacitance between either of the strip conductors and ground is

$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12} \tag{3.3}$$

and the characteristic impedance for the odd mode is

$$Z_{0o} = \frac{1}{vC_o} \tag{3.4}$$

In words, $Z_{0e}(Z_{0o})$ is the characteristic impedance of one of the strip conductors relative to ground when the coupled line is operated in the even (odd) mode. An arbitrary excitation of coupled line can always be treated as a superposition of appropriate amplitudes of even and odd modes [4].

### 3.2.3 Coupled mode Approach

In this approach coupled lines are characterized by the characteristic impedances and phase velocities for the two modes. These two modes are obtained by considering the effect of the self- and the mutual inductances and capacitances of the modes in the individual uncoupled lines [1].

For coupled mode analysis, the voltage on one line is written in terms of the currents on both lines and the self- and mutual impedances. Similarly the current in each mode is written in terms of voltages and admittances. Eliminating currents or voltages yields the coupled equations. The solution of these coupled equations determines the propagation constants for the two modes.

The couple mode analysis for coupled lines with unequal impedances is described next.

## *Analysis*

The behavior of two lossless coupled transmission lines is described in general by the following set of differential equations:

$$-\frac{dv_1}{dz} = Z_1 i_1 + Z_{in} i_2 \qquad (3.5(a))$$

$$-\frac{dv_2}{dz} = Z_{in} i_1 + Z_2 i_2 \qquad (3.5(b))$$

$$-\frac{di_1}{dz} = Y_1 v_1 + Y_m v_2 \qquad (3.5(c))$$

$$-\frac{di_2}{dz} = Y_m v_1 + Y_2 v_2 \qquad (3.5(d))$$

where $Z_j, Y_j$ (*j*=1, 2) are self-impedances and self admittances per unit length of lines 1 and 2 and $Z_m$ and $Y_m$ are mutual impedance and mutual admittance per unit length, respectively. Voltages and currents, which are function of *z*, are represented by $v_k$ and $i_k$ ($k = 1{,}2$), respectively. A time variation $e^{j\omega t}$ is assumed [1].

Eliminating $i_1$ and $i_2$ gives the following set of coupled equations for the voltages $v_1$ and $v_2$:

$$\frac{d^2 v_1}{dz^2} - a_1 v_1 - b_1 v_2 = 0 \qquad (3.6(a))$$

$$\frac{d^2 v_2}{dz^2} - a_2 v_2 - b_2 v_1 = 0 \qquad (3.6(b))$$

where

$$a_1 = Y_1 Z_1 + Y_m Z_m \quad (3.7(a))$$
$$b_1 = Z_1 Y_m + Y_2 Z_m \quad (3.7(b))$$
$$a_2 = Y_2 Z_2 + Y_m Z_m \quad (3.7(c))$$
$$b_2 = Z_2 Y_m + Y_1 Z_m \quad (3.7(d))$$

The coefficients $a_1, a_2, b_1$ and $b_2$ are constants.

Assuming a variation of the type $v(z) = v_0 e^{-\gamma z}$ for the voltages $v_1$ and $v_2$, the coupled equations reduce to the eigen-value equation

$$[\gamma^4 - \gamma^2(a_1 + a_2) + a_1 a_2 - b_1 b_2] v_0 = 0 \quad (3.8)$$

The solution leads to the following four roots of $\gamma$:

$$\gamma_{1,2} = \pm\gamma_c \text{ and } \gamma_{3,4} = \pm\gamma_\pi \quad (3.9)$$

where

$$\gamma_{c,\pi}^2 = \frac{a_1 + a_2}{2} \pm \frac{1}{2}[(a_1 - a_2)^2 + 4b_1 b_2]^{1/2} \quad (3.10)$$

The subscripts "*c*" and "$\pi$" refer to the *c* and $\pi$ modes for asymmetric coupled lines. The propagation constants for these modes, $\gamma_c$and $\gamma_\pi$, correspond to in-phase and anti-phase waves, which reduce to even and odd-mode waves, for symmetrical lines. The roots with plus or minus sign represent waves travelling in the +z or –z directions, respectively.

The ratio of voltages $v_2$ and $v_1$ on the two lines for the *c* and $\pi$ modes is obtained from the equations discussed above and is given as

$$\frac{v_2}{v_1} = \frac{\gamma^2 - a_1}{b_1} = \frac{b_2}{\gamma^2 - a_2} \quad (3.11)$$

If the corresponding ratios for the *c* and $\pi$ modes are represented by $R_c$ and $R_\pi$, then

$$R_c \left(= \frac{v_2}{v_1} \; for \; \gamma = \pm\gamma_c\right) = \frac{1}{2b_1}\left[(a_2 - a_1) + \{(a_2 - a_1)^2 + 4b_1 b_2\}^{1/2}\right] \quad (3.12)$$

$$R_\pi \left(= \frac{v_2}{v_1} \; for \; \gamma = \pm\gamma_\pi\right) = \frac{1}{2b_1}\left[(a_2 - a_1) + \{(a_2 - a_1)^2 + 4b_1 b_2\}^{1/2}\right] \quad (3.13)$$

It may be observed that $R_c$ is positive real and $R_\pi$ is negative real, thus representing in-phase and anti-phase waves.

In terms of the four waves, with propagation constants $\pm\gamma_c$ and $\pm\gamma_\pi$, the general solution for the voltages on the two lines may be written as

$$v_1 = A_1 e^{-\gamma_c x} + A_2 e^{\gamma_c x} + A_3 e^{-\gamma_\pi x} + A_4 e^{\gamma_\pi x} \tag{3.14}$$

$$v_2 = R_c(A_1 e^{-\gamma_c x} + A_2 e^{\gamma_c x}) + R_\pi(A_3 e^{-\gamma_\pi x} + A_4 e^{\gamma_\pi x}) \tag{3.15}$$

The currents $i_1$ and $i_2$ are obtained by substituting the corresponding voltages $v_1$ and $v_2$and may be written as

$$i_1 = Y_{c1}(A_1 e^{-\gamma_c z} - A_2 e^{\gamma_c z}) + Y_{\pi 1}(A_3 e^{-\gamma_\pi z} - A_4 e^{\gamma_\pi z}) \tag{3.16}$$

$$i_2 = Y_{c2} R_c(A_1 e^{-\gamma_c z} - A_2 e^{\gamma_c z}) + Y_{\pi 2} R_\pi(A_3 e^{-\gamma_\pi z} - A_4 e^{\gamma_\pi z}) \tag{3.17}$$

where $Y_{c1}$, $Y_{c2}$, $Y_{\pi 1}$, $Y_{\pi 2}$ are the characteristic admittances of lines 1 and 2 for the two modes. These are given by

$$Y_{c1} = \gamma_c \frac{Z_2 - Z_m R_c}{Z_1 Z_2 - Z_m^2} = \frac{1}{Z_{c1}} \tag{3.18}$$

$$Y_{c2} = \frac{\gamma_c}{R_c} \frac{Z_1 R_c - Z_m}{Z_1 Z_2 - Z_m^2} = \frac{1}{Z_{c2}} \tag{3.19}$$

Similar relations hold for the $\pi$ mode. Substituting the values of $R_c$and $R_\pi$ gives

$$\frac{Y_{c1}}{Y_{c2}} = -R_c R_\pi = \frac{Y_{\pi 1}}{Y_{\pi 2}} \tag{3.20}$$

The above analysis has been carried out in terms of the two independent modes of propagation termed "$c$" and "$\pi$" modes with propagation constants $\gamma_c$and $\gamma_\pi$. The voltages $v_1$and $v_2$on the two lines are related through $v_2/v_1 = R_c$ and $R_\pi$. The corresponding ratios for the currents are given by $i_2/i_1 = -1/R_\pi$and $-1/R_c$, respectively.

*Symmetric lines*

For the case of symmetric lines, $a_2 = a_1$ and $b_2 = b_1$. Therefore $v_2/v_1$

$$v_2/v_1 = R_c = +1 \qquad \text{for } c \text{ mode (even mode)} \tag{3.21}$$

$$v_2/v_1 = R_\pi = -1 \qquad \text{for } \pi \text{ mode (odd mode)} \tag{3.22}$$

## *Even and Odd mode Analysis*

The two coupled lines make a four port device. The total voltages that result when port 1 is excited by an input signal are shown in figure 3.6(a). If the two ports at one end of the structure are driven with same phase and magnitude voltages, the even-mode configuration results with four voltages and four currents at the ports. These variables are distinguished by the superscript "(e)". For the odd mode, port 3 is driven anti-phase to port 1 and the odd-mode voltages and currents are denoted by the superscript "(o)".

The total voltages at each port are given by superposition of the even- and odd-mode voltages [3]. Thus

$$V_1 = V_i^{(e)} + V_i^{(o)} \qquad V_2 = V_O^{(e)} + V_O^{(o)}$$
$$V_3 = V_i^{(e)} - V_i^{(o)} \qquad V_4 = V_O^{(e)} - V_O^{(o)} \tag{3.23}$$

The transverse electric field patterns for the even and odd modes for a microstrip line are illustrated in the figure 3.7. As was the case for the single microstrip line, rather than evaluating the inductance per unit length for each mode, each of the even- and odd-mode impedances, $Z_{0e}$ and $Z_{0o}$ , is found from both air-filled and dielectric filled line capacitances for the respective mode. The mode impedances represent the propagating wave voltage/current ratio on each line when the pair of coupled lines has been appropriately excited.

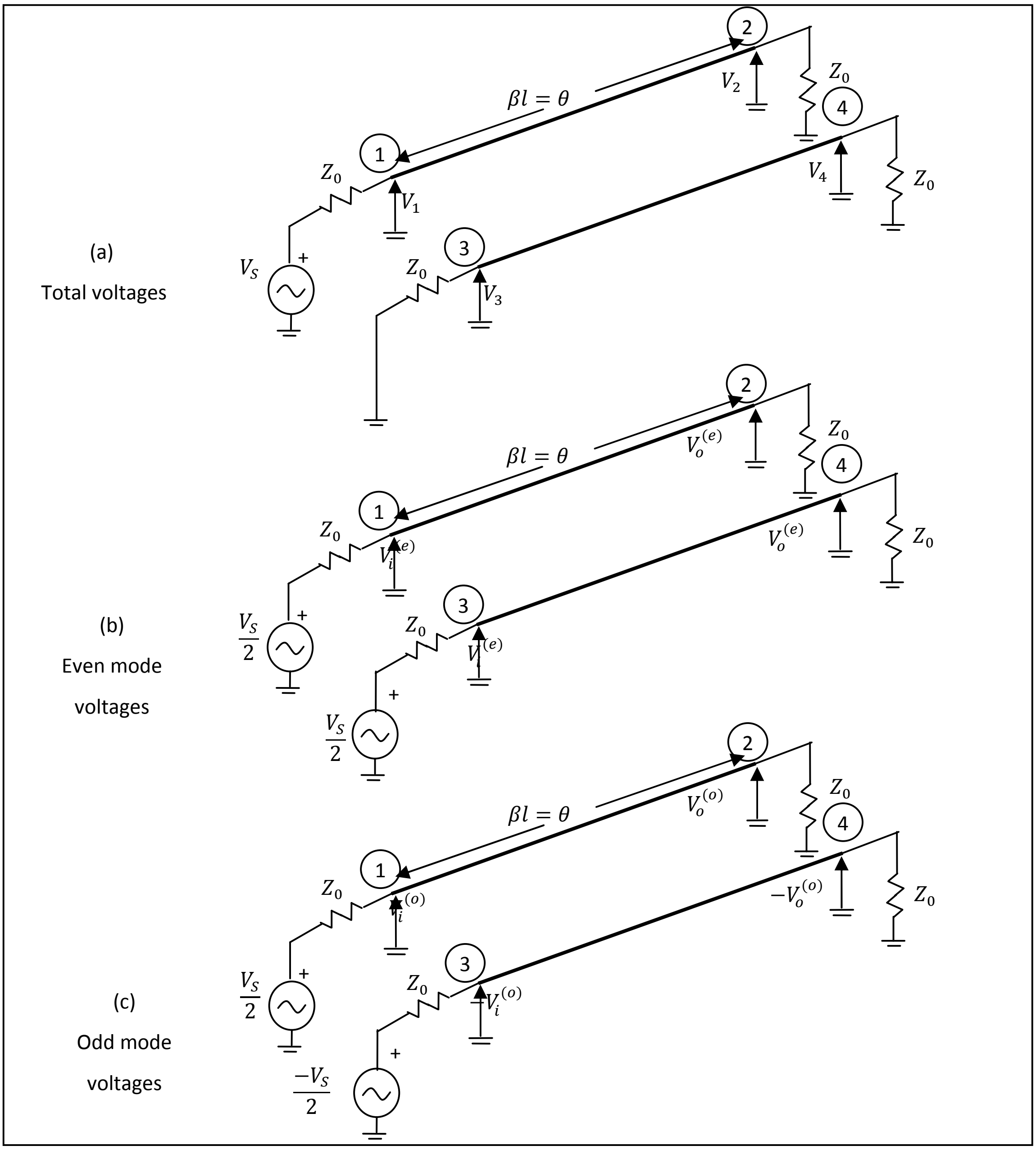


**Figure3.6 Parameters of coupled transmission lines. (a) The total voltages that result when port 1 is excited by an input signal. The voltages in (a) are decomposed into two sets, (b) the even mode voltages, and (c) the odd mode voltages [3]**

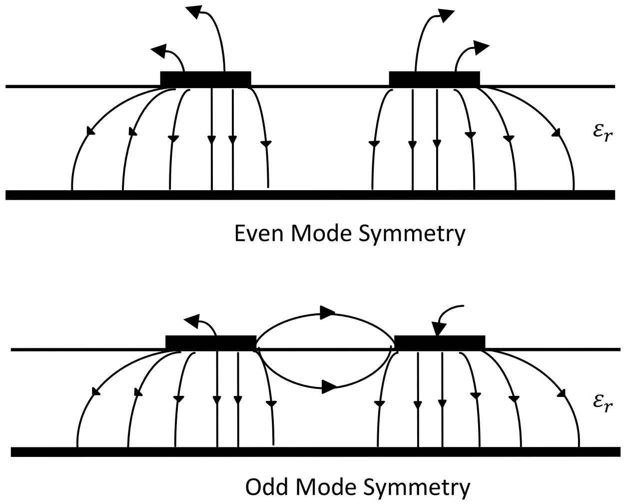


**Figure3.7 Electric field lines shown in the Even mode and Odd Mode for parallel-coupled microstrip transmission lines [3]**

Consider the line with characteristic impedance $Z_0$, as illustrated in figure 3.8. The load and source impedances give voltage reflection coefficients, $\Gamma_L$ and $\Gamma_S$ respectively, at the ends of transmission line. The voltages at each end of the line is given as

$$V_i = \frac{V_S}{2}\left[1 - \frac{\Gamma_S - \Gamma_L e^{-j2\theta}}{1 - \Gamma_S \Gamma_L e^{-j2\theta}}\right] \tag{3.24}$$

$$V_O = \frac{V_S}{2}\left[\frac{(1 - \Gamma_S)(1 + \Gamma_L)e^{-j\theta}}{1 - \Gamma_S \Gamma_L e^{-j2\theta}}\right] \tag{3.25}$$

When $\Gamma_S = \Gamma_L = \Gamma$, the above equations become

$$V_i = \frac{V_S}{2}\left[1 - \frac{\Gamma\left(1 - e^{-j2\theta}\right)}{1 - \Gamma^2 e^{-j2\theta}}\right] \tag{3.26}$$

$$V_O = \frac{V_S}{2}\left[\frac{(1 - \Gamma^2)e^{-j\theta}}{1 - \Gamma^2 e^{-j2\theta}}\right] \tag{3.27}$$

This formulation of the line voltages is also applicable to coupled lines in terms of the even- and odd-modes on the lines. For the even mode, the load and source impedances are $Z_0$ and the characteristic impedance $Z_{0e}$ so that

$$\Gamma \rightarrow \Gamma_e = \frac{Z_0 - Z_{0e}}{Z_0 + Z_{0e}}$$

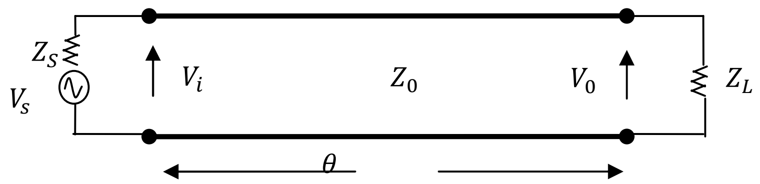


**Figure3.8 Parameters for terminated transmission line [3]**

Further, the $V_S$becomes $V_S/2$ in the figure, giving

$$V_i^{(e)} = \frac{V_S}{4}\left[1 - \frac{\Gamma_e\left(1 - e^{-j2\theta}\right)}{1 - \Gamma_e^2 e^{-j2\theta}}\right] \tag{3.28}$$

$$V_O^{(e)} = \frac{V_S}{4}\left[\frac{(1 - \Gamma_e^2)e^{-j\theta}}{1 - \Gamma_e^2 e^{-j2\theta}}\right] \tag{3.29}$$

Likewise, for the odd mode in the figure,

$$V_i^{(o)} = \frac{V_S}{4}\left[1 - \frac{\Gamma_o\left(1 - e^{-j2\theta}\right)}{1 - \Gamma_o^2 e^{-j2\theta}}\right] \tag{3.30}$$

$$V_O^{(o)} = \frac{V_S}{4}\left[\frac{(1 - \Gamma_o^2)e^{-j\theta}}{1 - \Gamma_o^2 e^{-j2\theta}}\right] \tag{3.31}$$

Note that $\theta$ in the even and odd mode equation is identical, by virtue of the assumption that the even and odd mode velocities are equal [3].

***Input match condition***

Apply an input signal to port 1 only. The input voltage $V_1 = V_i^{(e)} + V_i^{(o)}$ , so that

$$V_1 = \frac{V_S}{4}\left\{1 + 1 - \left(1 - e^{-j2\theta}\right)\left(\frac{\Gamma_e}{1 - \Gamma_e^2 e^{-j2\theta}} + \frac{\Gamma_o}{1 - \Gamma_o^2 e^{-j2\theta}}\right)\right\} \tag{3.32}$$

The condition for a matched input at port 1 in figure is $V_1 = V_S/2$, given when

$$\left(\frac{\Gamma_e}{1 - \Gamma_e^2 e^{-j2\theta}} + \frac{\Gamma_o}{1 - \Gamma_o^2 e^{-j2\theta}}\right) = 0 \tag{3.33}$$

To satisfy this condition in a frequency independent manner, i.e. for all $\theta, \Gamma_e = -\Gamma_o$ is sufficient. Now,

$$\Gamma_e = -\Gamma_o \Rightarrow \frac{Z_0}{Z_{0e}} = \frac{Z_{0o}}{Z_0} \tag{3.34}$$

i.e. $$Z_{0e}Z_{0o} = Z_0^2$$

***Isolation***

The voltage at port 4, where good isolation is required, is given by $V_o^{(e)} - V_o^{(o)}$ yielding

$$V_4 = \frac{V_S}{4}\left\{\frac{(1 - \Gamma_e^2)e^{-j\theta}}{1 - \Gamma_e^2 e^{-j2\theta}} - \frac{(1 - \Gamma_o^2)e^{-j\theta}}{1 - \Gamma_o^2 e^{-j2\theta}}\right\} \tag{3.35}$$

Again, $\Gamma_e = -\Gamma_o$ is clearly sufficient, and may likewise be shown necessary, to make $V_4 = 0$.

***The coupled port***

At port 3, the coupled port

$$V_3 = \frac{V_S}{4}\left(1 - \frac{\Gamma_e\left(1 - e^{-j2\theta}\right)}{1 - \Gamma_e^2 e^{-j2\theta}} - 1 + \frac{\Gamma_o\left(1 - e^{-j2\theta}\right)}{1 - \Gamma_o^2 e^{-j2\theta}}\right) \tag{3.36}$$

With $\Gamma_e = -\Gamma_o$

$$V_3 = -\frac{V_S}{2}\left\{\frac{\Gamma_e\left(e^{j\theta} - e^{-j\theta}\right)}{e^{j\theta} - \Gamma_e^2 e^{-j\theta}}\right\} \tag{3.37}$$

Giving,

$$V_3 = -\frac{V_S}{2}\left\{\frac{2j\Gamma_e \sin\theta}{(1-\Gamma_e^2)\cos\theta + j\sin\theta(1+\Gamma_e^2)}\right\}$$

$$= -\frac{V_S}{2}\left(\frac{2\Gamma_e}{1+\Gamma_e^2}\right)\frac{j\sin\theta}{\left(\frac{1-\Gamma_e^2}{1+\Gamma_e^2}\right)\cos\theta + j\sin\theta} \tag{3.38}$$

Now, it can be readily shown that

$$-\frac{2\Gamma_e}{1+\Gamma_e^2} = \frac{Z_{0e} - Z_{0o}}{Z_{oe} + Z_{0o}} = \boldsymbol{c} \tag{3.39}$$

where c will appear as a coupling coefficient. It is simple to show that,

$$1 - c^2 = \left(\frac{1-\Gamma_e^2}{1+\Gamma_e^2}\right)^2 \tag{3.40}$$

Thus

$$V_3 = \frac{V_S}{2}\left(\frac{j\boldsymbol{c}\sin\theta}{\sqrt{1-\boldsymbol{c}^2}\cos\theta + j\sin\theta}\right) \tag{3.41}$$

***Transmission***

With the same condition $\Gamma_e = -\Gamma_o$, the transmitted signal at port 2 may be similarly found as

$$V_2 = \frac{V_S}{2}\left(\frac{\sqrt{1-\boldsymbol{c}^2}}{\sqrt{1-\boldsymbol{c}^2}\cos\theta + \sin\theta}\right) \tag{3.42}$$

***Summary and discussion***

When $V_S$ is set as 2 V for each of the two modes, a unit input voltage at port 1 results and the following relationships are found,

$$V_1 = 1 \tag{3.43}$$

$$V_2 = \frac{\sqrt{1-\boldsymbol{c}^2}}{\sqrt{1-\boldsymbol{c}^2}\cos\theta + \sin\theta} \tag{3.44}$$

$$V_3 = \frac{j\boldsymbol{c}\sin\theta}{\sqrt{1-\boldsymbol{c}^2}\cos\theta + j\sin\theta} \tag{3.45}$$

$$V_4 = 0 \tag{3.46}$$

The maximum coupling from port 1 to port 3 occurs when the coupling length is one-quarter wavelength, i.e. $\theta = \pi/2$. Under these conditions

$$V_1 = 1$$

$$V_2 = -j\sqrt{1-\boldsymbol{c}^2}$$

$$V_3 = \boldsymbol{c}$$

$$V_4 = 0$$

To summarize, with the assumptions made earlier, the following points have been noted down

a) Port 4 always has zero output, irrespective of the electrical length of the coupling region. In practical circuits, a major cause of the poor isolation may be unequal phase velocities of the even- and odd-mode.
b) The input at each port is matched to the feed line characteristic impedance, $Z_0$, again irrespective of the electrical length of the coupling region.
c) The total output power equals the input power.

d) The maximum coupling to port 2 occurs at the frequency that gives a quarter-wave coupling length. This will be the mid-band frequency. Because of this property, these couplers are also known as quarter-wave couplers.

e) At this maximum coupling frequency, the through line voltage $V_2$ is $90^o$ out of phase with the coupled line voltage $V_3$, i.e. this coupler may be defined as a quadrature coupler. The coupled voltage $V_3$ is in phase with $V_1$ and thus $V_2$ lags $V_1$ by $90^o$, the latter phase difference being identical to the electrical length of the coupling region.

f) At frequencies other than the maximum coupling frequency, the ideal frequency response is found by evaluating the terms $|V_2(\theta)|$ and $|V_3(\theta)|$, remembering that $\theta$ is a function of frequency.

The coupling of the directional coupler is generally expressed in dB, i.e.

$$\text{coupling}, C = -20\log(c)$$

Thus it follows that

$$Z_{0e} = Z_0\left(\frac{1+c}{1-c}\right)^{\frac{1}{2}} = Z_0\left(\frac{1+10^{-C/20}}{1-10^{-C/20}}\right)^{\frac{1}{2}} \quad (3.47)$$

$$Z_{0o} = Z_0^2 \div Z_{0e} = Z_0\left(\frac{1-10^{-C/20}}{1+10^{-C/20}}\right)^{\frac{1}{2}} \quad (3.48)$$

# Chapter 4

# Design of Directional Coupler and Its Directivity Improvement

## 4.1 Design of Directional Coupler

Various design charts and methods are being developed and implemented in order to calculate the physical parameters of the directional coupler.

Earlier the methods given by Bryant and Weiss [5] calculated the capacitance, characteristic impedance and velocity of propagation of even- and odd normal modes using the physical parameters of the directional coupler. Kirschning and Jansen did a frequency dependant analysis for the even and odd mode characteristics of parallel coupled microstrip lines [6]. The design charts were plotted but they gave only the physical parameters of directional coupler versus even- and odd-mode impedance. As a result, these were not practically implemented on real applications. These design charts used backward calculations in order to determine the physical parameters of the directional coupler. The process is cumbersome and tedious. Also, the reverse process was needed where the designer required knowing about the physical geometry from impedances.

Akhtarzad *et al.* gave a design method so that a directional coupler can be implemented using synthesis technique [7]. The strip width for single microstrip line was calculated corresponding to the even and odd-mode impedances of the coupled microstrip lines. These were further corrected by Hinton [8] by correcting w/h ratios using Wheeler's formula [9].

Eroglu in his paper [10] discussed the three-step design procedure with accurate formulations to have a complete design of symmetrical two-line microstrip directional coupler including physical length at the desired operational frequency. The design procedure requires the knowledge of the port termination impedances, coupling and the operational frequency. The even and odd mode impedances are calculated in the first step. The second step calculates the physical dimensions *s/h* and *w/h*. The third step calculates the length of the coupler.

The three steps to design the directional coupler are

**a) Calculation of even and odd-mode impedances**

The even- and odd-mode impedance of the microstrip coupler is given by

$$Z_{0e} = Z_0\left(\frac{1+c}{1-c}\right)^{\frac{1}{2}} = Z_0\left(\frac{1+10^{-C/20}}{1-10^{-C/20}}\right)^{\frac{1}{2}}$$

$$Z_{0o} = Z_0^2 \div Z_{0e} = Z_0\left(\frac{1-10^{-C/20}}{1+10^{-C/20}}\right)^{\frac{1}{2}}$$

where C is the coupling required and is given in decibels.

**b) Find physical dimensions *s/h* and *w/h***

The physical dimensions are calculated by different researchers using different formulae. Akhtarzad *et al*. calculated the physical dimensions using synthesis technique [7]. The corrections were made on the calculations and are given in [8]. Eroglu [10] in his paper employed the corrections and calculated the dimensions.

The spacing ratio *s/h* of the coupler is given by

$$s/h = \frac{2}{\pi}\cosh^{-1}\left[\frac{\cosh\left[\frac{\pi}{2}\left(\frac{w}{h}\right)'_{so}\right] + \cosh\left[\frac{\pi}{2}\left(\frac{w}{h}\right)_{so}\right] - 2}{\cosh\left[\frac{\pi}{2}\left(\frac{w}{h}\right)'_{so}\right] - \cosh\left[\frac{\pi}{2}\left(\frac{w}{h}\right)_{se}\right]}\right] \tag{4.1}$$

$(w/h)_{se}$ and $(w/h)_{so}$ are the shape ratios for the equivalent single case even and odd-mode geometry.

*(w/h)* is the corrected shape ratio for the single microstrip line,

$$\frac{w}{h} = \frac{8\sqrt{\left[exp\left(\frac{R}{42.4}\sqrt{\varepsilon_r + 1}\right) - 1\right]\frac{7 + (4/\varepsilon_r)}{11} + \frac{1 + 1/\varepsilon_r}{0.81}}}{\left[exp\left(\frac{R}{42.4}\sqrt{\varepsilon_r + 1}\right) - 1\right]} \tag{4.2}$$

where $R = Z_{0e}/2$ or $R = Z_{0o}/2$.

$Z_{ose}$ and $Z_{oso}$ are the characteristic impedances that correspond to single microstrip shape ratios $(w/h)_{se}$ and $(w/h)_{so}$, respectively. They are given as

$$Z_{ose} = \frac{Z_{oe}}{2} \tag{4.3}$$

$$Z_{oso} = \frac{Z_{oo}}{2} \tag{4.4}$$

$$(w/h)_{se} = (w/h)|_{R=Z_{ose}} \tag{4.5}$$

$$(w/h)_{so} = (w/h)|_{R=Z_{oso}} \tag{4.6}$$

The corrected term $(w/h)'_{so}$ is given as

$$\left(\frac{w}{h}\right)'_{so} = 0.78\left(\frac{w}{h}\right)_{so} + 0.1\left(\frac{w}{h}\right)_{se} \tag{4.7}$$

Using the above formulae the spacing ratio, *s/h* can be calculated. After the spacing ratio, one has to find the shape ratio *w/h*. The shape ratio for the coupled lines is given as

$$\left(\frac{w}{h}\right) = \frac{1}{\pi}\cosh^{-1}(d) - \frac{1}{2}\left(\frac{s}{h}\right) \tag{4.8}$$

where

$$d = \frac{\cosh\left[\frac{\pi}{2}\left(\frac{w}{h}\right)_{se}\right](g + 1) + g - 1}{2} \tag{4.9}$$

$$g = \cosh\left[\frac{\pi}{2}\left(\frac{s}{h}\right)\right] \tag{4.10}$$

**c) Calculation of Physical Length of the Coupler**

The physical length of the coupler is obtained using

$$L = \frac{\lambda}{4} = \frac{v_p}{4f} = \frac{c}{4f\sqrt{\varepsilon_{eff}}} \tag{4.11}$$

where c=3*10$^8$ m/s, and $f$ is operational frequency in hertz. Hence the length of the directional coupler can be found if the effective permittivity constant $\varepsilon_{eff}$ of the coupled structure is known. $\varepsilon_{eff}$ can be found using

$$\sqrt{\varepsilon_{eff}} = \frac{\sqrt{\varepsilon_{effe}} + \sqrt{\varepsilon_{effo}}}{2} \tag{4.12}$$

The $\varepsilon_{effe}$ and $\varepsilon_{effo}$ are the effective permittivity constants of the coupled structure for odd and even modes, respectively. $\varepsilon_{effe}$ and $\varepsilon_{effo}$ depend on even- and odd-mode capacitances $C_e$ and $C_o$ as

$$\varepsilon_{effe} = \frac{C_e}{C_{e1}} \tag{4.13}$$

$$\varepsilon_{effo} = \frac{C_o}{C_{o1}} \tag{4.14}$$

All the capacitances are given as capacitances per unit length. $C_{e1,o1}$ is the capacitance with air as dielectric.

1) *Even-Mode Capacitance Calculation*: The even-mode capacitance $C_e$ is

$$C_e = C_p + C_f + C_f' \tag{4.15}$$

$C_p$ is the parallel plate capacitance and is defined as

$$C_p = \varepsilon_o \varepsilon_r \frac{w}{h} \tag{4.16}$$

where *w/h* is found in earlier section. $C_f$ is the fringing capacitance due to the microstrips being taken alone as if they were a single strip, which is equal to

$$C_f = \frac{\sqrt{\varepsilon_{seff}}}{2cZ_o} - \frac{C_p}{2} \tag{4.17}$$

Here, $\varepsilon_{seff}$ is the effective permittivity constant of a single strip microstrip, which can be expressed as

$$\varepsilon_{seff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} F\left(\frac{w}{h}\right) \tag{4.18}$$

where

$$F\left(\frac{w}{h}\right) = \begin{cases} \left(1 + 12\frac{h}{w}\right)^{-1/2} + 0.041\left(1 - \frac{w}{h}\right)^2, & w/h \leq 1 \\ \left(1 + 12\frac{h}{w}\right)^{-1/2}, & w/h \geq 1 \end{cases} \tag{4.19}$$

$C_f'$ is given by the following equation:

$$C_f' = \frac{C_f}{1 + A\left(h/s\right)\tanh\left(8s/h\right)}\sqrt{\frac{\varepsilon_r}{\varepsilon_{seff}}} \tag{4.20}$$

where $$A = exp\left(-0.1exp\left(2.333 - 2.53\,w/h\right)\right) \tag{4.21}$$

2) *Odd-Mode Capacitance Calculation*: The odd-mode capacitance $C_o$ is

$$C_o = C_p + C_f + C_{ga} + C_{gd} \tag{4.22}$$

$C_{ga}$ is the capacitance term in odd mode for the fringing field across the gap in the air region. It can be written as

$$C_{ga} = \varepsilon_o \frac{K(k')}{K(k)} \tag{4.23}$$

where

$$\frac{K(k')}{K(k)} = \begin{cases} \frac{1}{\pi} ln\left(2\frac{1 + \sqrt{k'}}{1 - \sqrt{k'}}\right), & 0 < k^2 < 0.5 \\ \frac{\pi}{ln\left(2\frac{1 + \sqrt{k'}}{1 - \sqrt{k'}}\right)}, & 0.5 < k^2 < 1 \end{cases} \tag{4.24}$$

$$k = \frac{s/h}{s/h + 2\,w/h} \tag{4.25}$$

$$k' = \sqrt{1 - k^2} \tag{4.26}$$

$C_{gd}$ represents the capacitance in odd mode for the fringing field across the gap in the dielectric region. It can be found using

$$C_{gd} = \frac{\varepsilon_o \varepsilon_r}{\pi} ln\left(coth\left(\frac{\pi}{4}\frac{s}{h}\right)\right) + 0.65C_f\left(\frac{0.02}{s/h}\sqrt{\varepsilon_r} + 1 - \frac{1}{\varepsilon_r^2}\right) \quad (4.27)$$

Since

$$Z_{oe} = \frac{1}{c\sqrt{C_e C_{e1}}} \quad (4.28)$$

$$Z_{oo} = \frac{1}{c\sqrt{C_o C_{o1}}} \quad (4.29)$$

Then we can write

$$C_{e1} = \frac{1}{c^2 C_e Z_{oe}^2} \quad (4.30)$$

$$C_{o1} = \frac{1}{c^2 C_o Z_{oo}^2} \quad (4.31)$$

Substituting these equations in the previous equations gives the even- and odd-mode effective permittivity $\varepsilon_{effe}$ and $\varepsilon_{effo}$. These equations help us to find the effective permittivity constant $\varepsilon_{eff}$ of the coupled structure. Thus, the length of the directional coupler can be designed.

The performance of directional couplers is characterized by the following three values.

## 4.2 Parameters of Directional Coupler

### 4.2.1 Coupling

Coupling is defined as the ratio between power coming through $P_1$ and $P_4$, for a given frequency. The expression defines the value of the coupling

$$C(dB) = 10\log\frac{P_1}{P_4} = -20\log|S_{41}| \quad (4.32)$$

### 4.2.2 Isolation

Isolation is defined as the power transferred from port 1 to port 3 of the directional coupler. It is expressed as

$$I(dB) = 10 \log \frac{P_1}{P_3} = -20 \log|S_{31}| \tag{4.33}$$

### 4.2.3 Directivity

The directivity is the ability to transfer power from the input port to the coupled port and to reject the power that can come from the through port, due to reflections on this.

It is a parameter that defines the technical and technological quality of directional coupler. The higher value of this parameter is the greater the technical quality of the component will be.

$$D(dB) = 10 \log \frac{P_4}{P_3} = -20 \log \frac{S_{31}}{S_{34}} \tag{4.34}$$

## 4.3 Improvement in Directivity of Directional Coupler

Directivity is one of the most important parameter of the directional coupler. Directivity is defined as the ratio of leakage of incident signal to that of the desired reflected signal. Lower the leakage signal, higher the directivity and better the accuracy of reflection measurement. The directivity is a measure of the coupler's ability to separate forward and reverse wave components.

$$D = I - C \ \ dB \tag{4.32}$$

The ideal coupler would have infinite directivity and isolation. But, practically this is not possible. The main reason behind the poor directivity is due to the unequal even and odd-mode phase velocities. Microstrip directional couplers suffer from poor directivity because of inhomogeneous dielectric, i.e., partly dielectric substrate, partly air. For this reason, odd and even modes excited in the coupled region exhibit different velocities and consequently different wavelengths. Typically loosely coupled backward wave microstrip couplers have directivities which decrease with increasing frequency. High directivity becomes more difficult to obtain as the coupling is loosened. The problem appears to be that the propagating velocities of the odd and even modes are not equal. The directivity becomes worse when the operating frequency is increased or the dielectric constant decreases.

Various methods have been proposed in the literature to improve the directivity and isolation of the microstrip parallel coupled-line coupler [11, 12]. Dielectric overlay is proposed to decrease the odd-mode phase velocity of the microstrip coupled-line to compensate the difference in phase velocities. Dielectric overlay [13, 14] equalizes the modal phase velocities by increasing the odd-mode effective dielectric constant and slightly lowering the even-mode value. The presence of the overlay causes a reduction in the width and separation between conductors, if the even- and odd-mode impedances are to be maintained constant, which results in increased conductor and dielectric loss compared to a conventional device.

In another approach [15], wiggly line has been proposed to compensate the phase velocity difference and improve performance of the microstrip coupled-line coupler. Wiggling the adjacent edges of the lines in the manner pictured in figure 4.1 slows the odd mode wave without much affecting the even mode. If the outsides of the lines were wiggled in unison with the insides then the even mode would be slowed down, and the wiggling would have to be more severe before velocity equalization would occur.

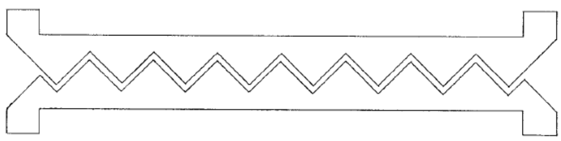

**Figure 4.1A typical wiggly line coupler[15]**

In another technique, lumped capacitances are added at the ends of the microstrip coupled-line coupler to increase the electrical length of the odd-mode at the design frequency [16, 17]. However, the capacitors values are restricted by the modal phase velocity difference.

Another attempt tries to compensate the difference in phase velocities by introducing ground-plane aperture to the ground plane of the coupler [18].

Lee presented in his paper the accurate method to design the directional coupler with high directivity with shunt loaded inductors [19].

The wiggling and slotting techniques are marked by broadband phase-velocity-compensation which is achieved by the modification of structure of the coupled-line section. The idea is based on the fact that the wiggly or slotting structure raises the odd mode inductance more strongly than that of the even mode due to the different current distributions between both modes [20].

Lei Han proposed a method with multi-element capacitive compensation to improve the directivity of the microstrip directional coupler. The directivity of the couplers with multi-compensated capacitors greatly improved compared with the conventional structure, the couplers with three- and four-compensated capacitors obtain the wider frequency response compared with the coupler with two-compensated capacitors [21].

Thus, the various methods and structures have been proposed to improve the directivity and isolation of the directional coupler. Here, in the present work discussed in this thesis, the conventional directional coupler will be designed using the synthesis method discussed before. The spacing ratio $s/h$ and the shape ratio $w/h$ will be calculated first and then the length of the directional coupler will be calculated. Then the coupling edges of the conventional directional coupler designed are modified in order to obtain high isolation and thus high directivity.

# Chapter 5

# Design and Simulation of Microstrip Directional Coupler

We have studied the various methods employed to design the microstrip directional coupler. The parameters which should be known to us are the port impedances, the required coupling level and the operational frequency at the initial stage of the design.

## 5.1 Design Specifications

Based on the known parameters, the directional coupler has been designed with the following specifications.

$$Coupling, C = -15\ dB$$

$$Port\ Impedances, Z_o = 50\ \Omega$$

$$Operational\ frequency, f = 1.5\ GHz$$

The substrate that is being used in the design is Rogers-R03210 which is having a dielectric constant, $\varepsilon_r$ of 10.2 of height, $h$ of 0.5mm.

## 5.2 Calculations and Schematic Diagram

The width, *w* of each microstrip and the spacing, *s* between the two microstrips of directional coupler is calculated using the *Agilent Advanced Design System Linecalc.*

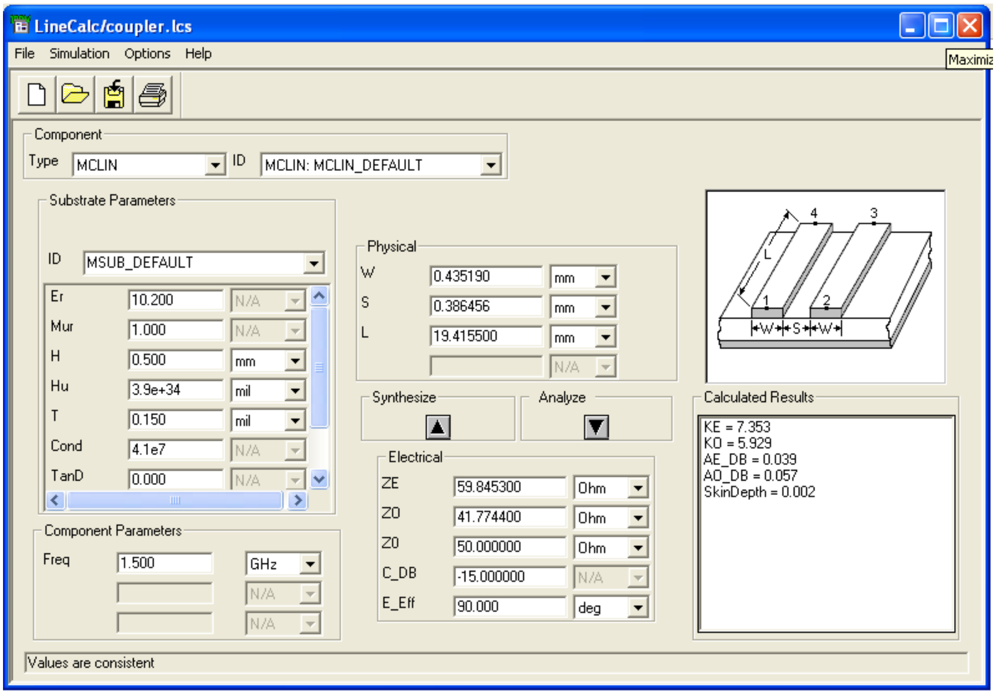


**Figure5. 1 Physical Dimensions calculated using Agilent ADS Linecalc**

The design specifications given are entered in the *Agilent ADS Linecalc* and the dimensions of the coupled microstrip lines are computed.

The width and spacing of the microstrip are calculated as

$$Width, w = 0.43mm$$

$$Space\ between\ microstrip\ lines, s = 0.39mm.$$

The length of the directional coupler is calculated here as 19.4mm. But this length is modified and calculated using the synthesis technique discussed by Eroglu [10]. The calculation is done using a MATLAB program which is scripted in the Appendix. The length of the directional coupler is found to be

$$Length, l = 18.6\ mm.$$

The schematic for the microstrip coupled line directional coupler is laid down for the calculated physical parameters and is simulated using S-parameters.

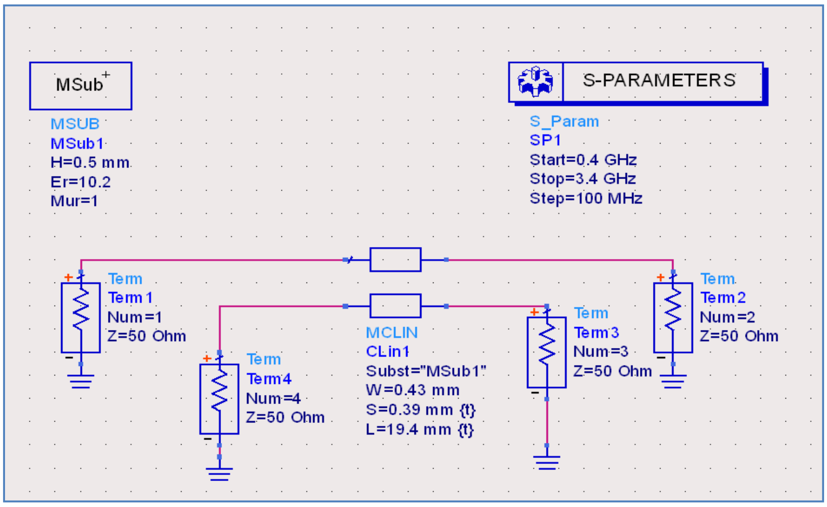


**Figure5. 2 Schematic Diagram for the Physical Parameters calculated**

The simulation results are as shown in the figure. The coupling, $S_{41}$ of the coupler simulated is -15.197 dB at 1.5GHz. The isolation, $S_{31}$ provided by the coupler is -22.094 dB. Thus the directivity of the directional coupler is,

$$D = I - C$$

$$= -22.094 + 15.197$$

$$= -6.897\ dB$$

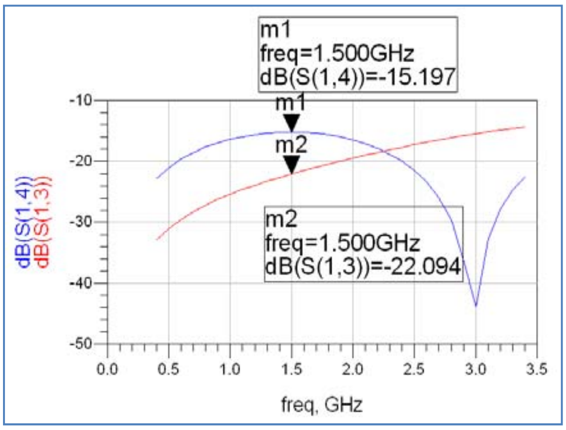


**Figure5. 3 Simulation results of designed directional coupler**

The spacing between the coupled lines and the length of directional coupler are further tuned to improve the isolation and thus the directivity.

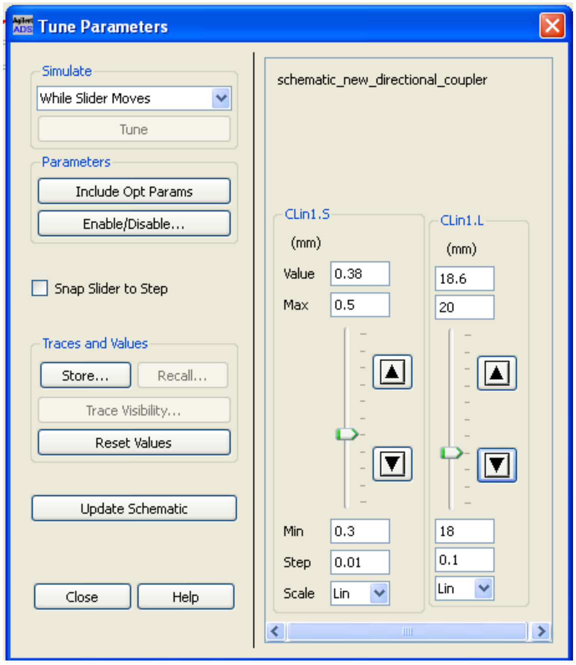


Figure 5.4 (a) Tuning of parameters

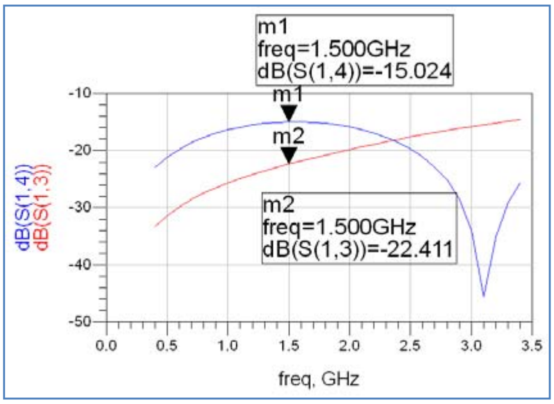


Figure 5.4 (b) Simulation results of tuned coupler

The results are seen at the length of 18.6mm and the spacing is reduced to 0.38mm. the obtained results are

$$Coupling, C = -15.024\ dB$$

$$Isolation, I = -22.411\ dB$$

$$Directivity, D = 7.387\ dB$$

The updated schematic is shown in figure below:

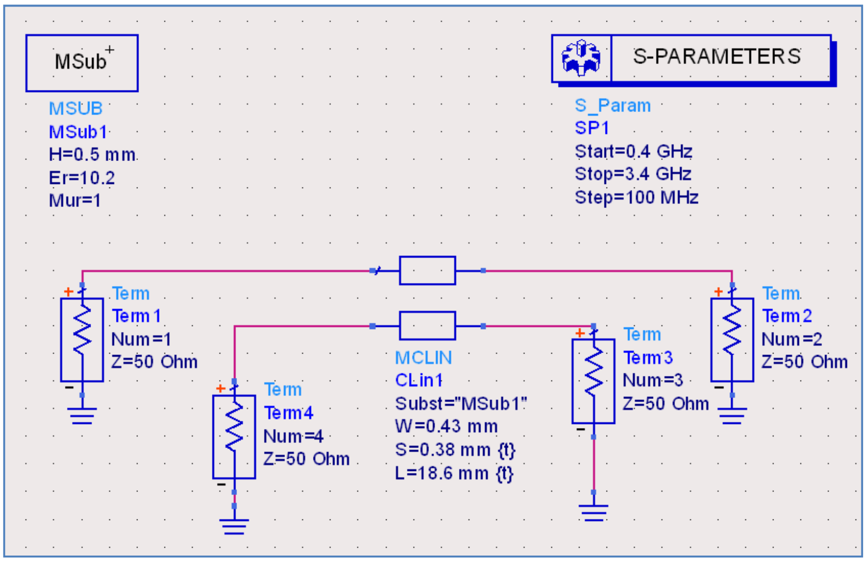


**Figure 5.5 Updated Shematic**

The final physical dimensions that are obtained from the simulation results are

$$width, w = 0.43mm$$

$$spacing\ between\ lines, s = 0.38mm$$

$$length\ of\ the\ directional\ coupler, l = 18.6mm$$

The parameters that are finally obtained are laid down in layout and thus simulated. It is discussed further.

## 5.3 Layout of Directional Coupler

The parameters obtained in the earlier section are being laid down in the layout window of *Agilent Advanced Design System*.

The layout of the conventional directional coupler is shown in the figure 5.6:

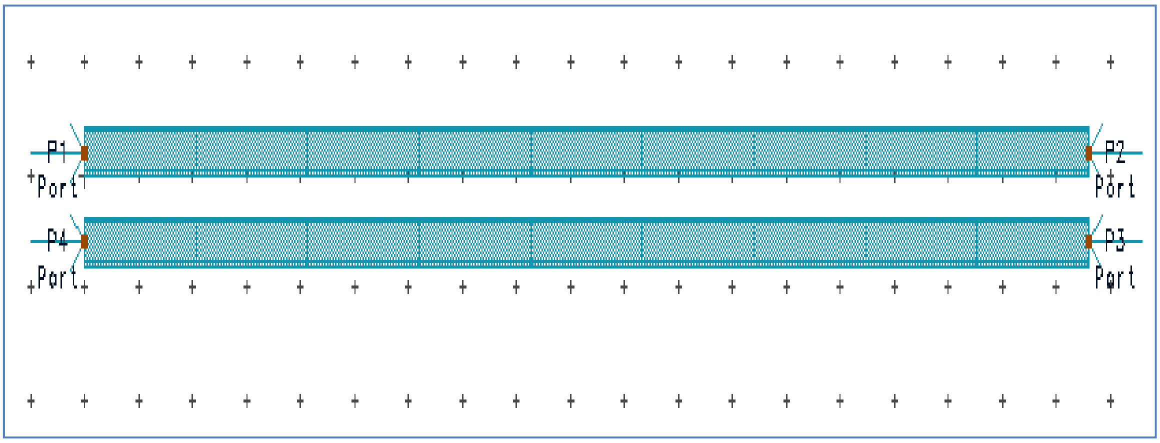


**Figure 5.6 Layout of directional coupler with specified Physical Parameters**

The substrate parameters are defined as follows:

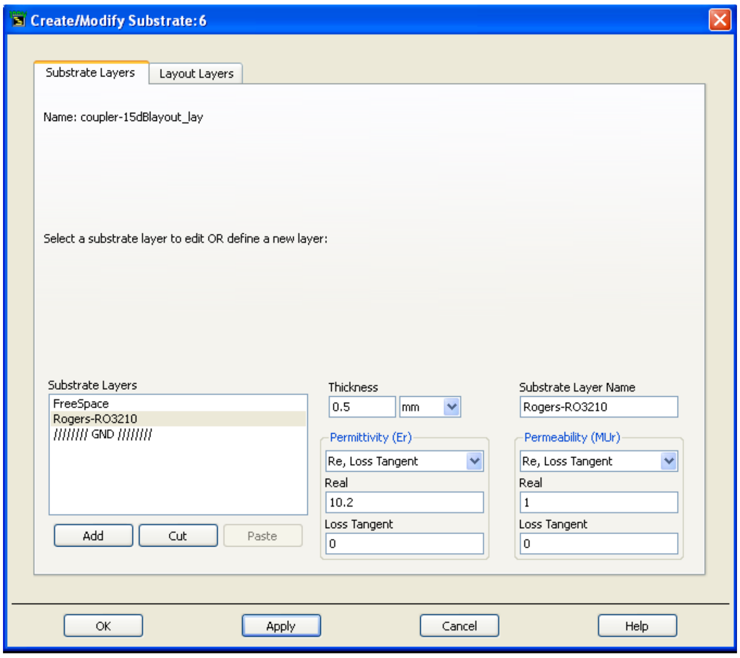


**Figure 5.7 Defining Substrate Parameters**

The S-parameters of the plotted layout are simulated using simulation controller window. The frequency range for which the layout is simulated is 0.4GHz and 2.6GHz. The simulation control window is shown in the figure 5.8.

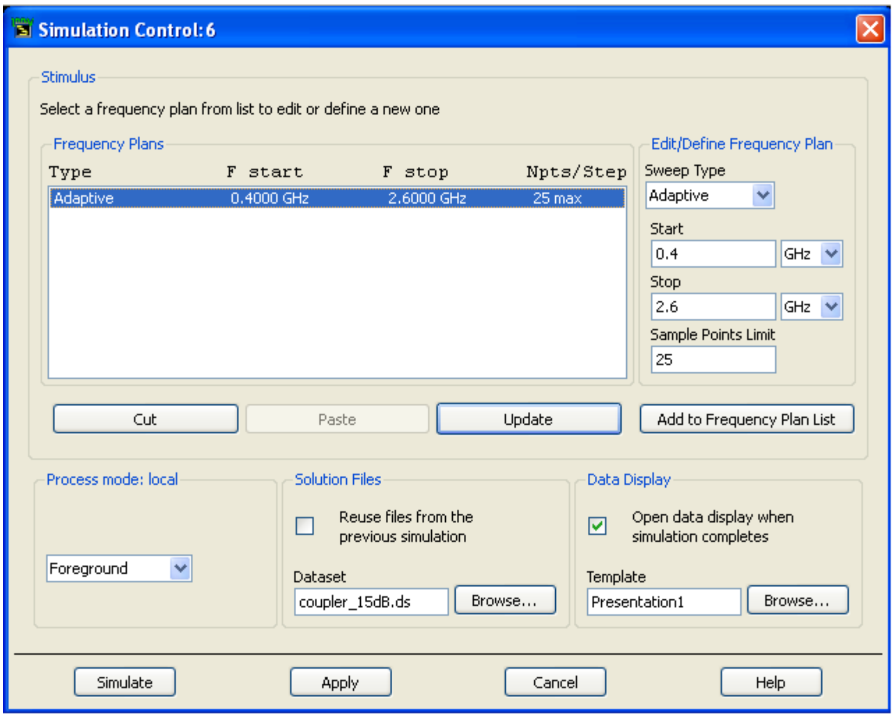


**Figure 5.8 Simulation Control Window**

The results obtained from the given layout are:

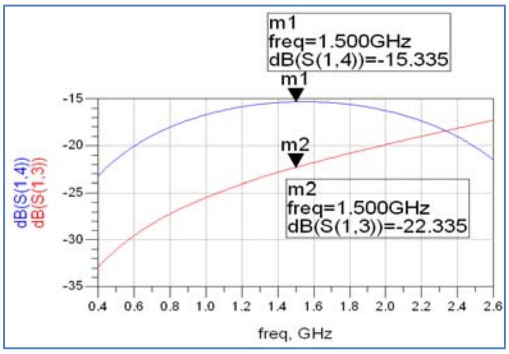


**Figure 5.9 S-parameters of the coupler designed**

The coupling obtained at 1.5GHz is -15.335dB and the isolation obtained is -22.335dB. The directivity calculated is 7dB.

## 5.4 Design of Modified Directional Coupler

It has been discussed earlier that the directivity of the microstrip coupled directional coupler can be improved by wiggling and slotting techniques. Wiggling the adjacent edges of the lines slows down the odd mode wave without much affecting the even mode. If the outsides of the lines were wiggled in unison with the insides then the even mode would be slowed down, and the wiggling would have to be more severe before velocity equalization would occur [15]. Slotting structure raises the odd mode inductance more strongly than that of the even mode due to the different current distributions between both the modes [20].

The conventional coupler that was laid down earlier is modified and the adjacent edges of the coupled microstrip lines are slotted down in the form of square wiggles.

The dimensions of the modified directional coupler are shown in the figure:

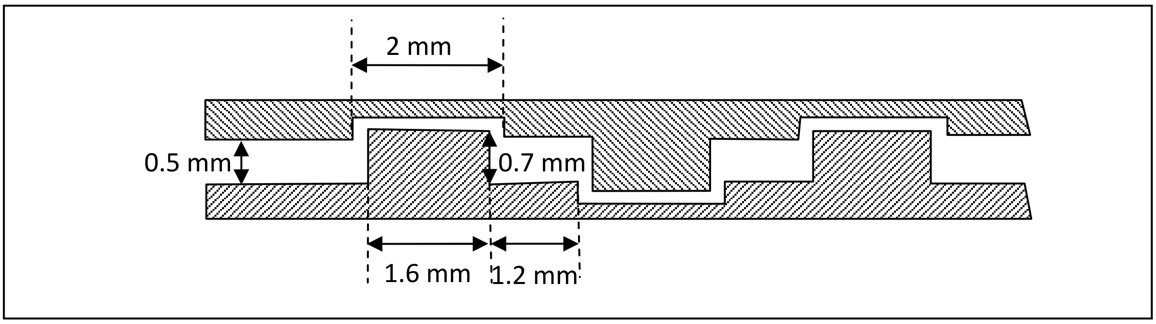


**Figure 5.10 Physical Dimensions shown of the modified Directional Coupler**

The length and width of the directional coupler remain same.

$$length, l = 18.6mm$$

$$width, w = 0.43mm$$

$$spacing\ between\ microstrip\ lines, s = 0.5mm$$

The layout of the modified directional coupler is shown in the figure:

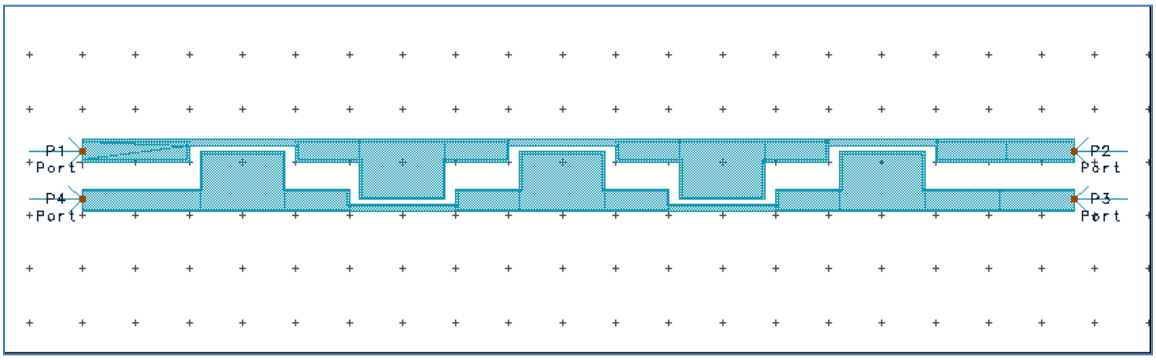


**Figure 5.11 Layout of Modified Directional Coupler**

The substrate parameters are same as earlier and the range of frequencies for which the directional coupler is simulated is also same.

S-parameters simulation results are:

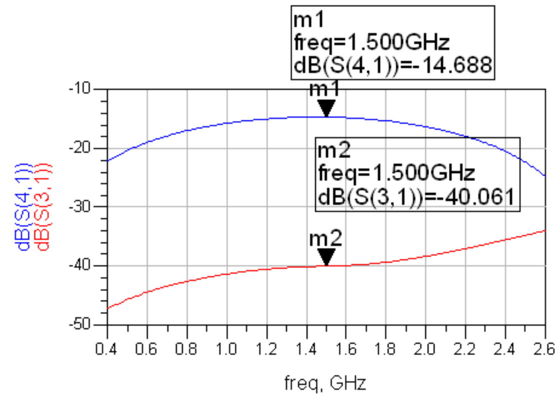


**Figure 5.12 Coupling and Isolation of Modified Directional Coupler**

The graph reads at 1.5 GHz as follows:

$$Coupling(S_{41}), C = -14.688\ dB$$

$$Isolation(S_{31}), I = -40.061\ dB$$

The forward transmission parameter $S_{21}$ simulated is as shown in the figure 5.13.

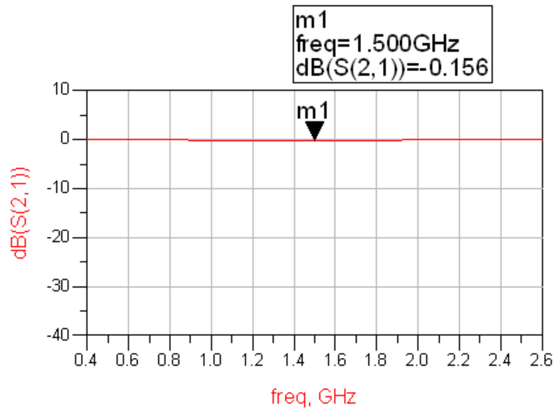


**Figure 5.13 Forward Transmission parameter $S_{21}$of directional coupler**

The result in figure shows that there is very less attenuation of the wave signal in the forward path, i.e. in travelling from port 1 to port 2. The attenuation provided by the coupler at 1.5GHz is -0.156dB from port 1 to port 2.

The modified directional coupler is symmetrical. This can be proved by plotting and comparing $S_{31}$ and $S_{42}$ parameters of the modified coupler designed.

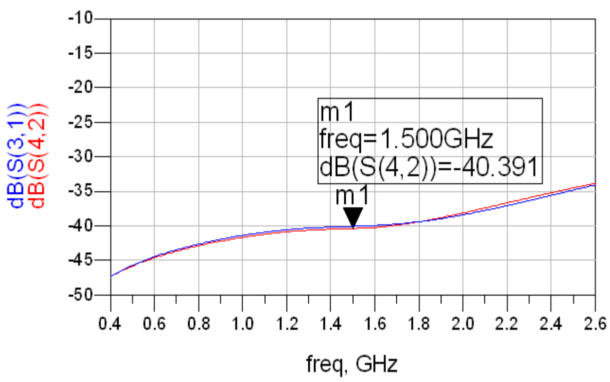


**Figure 5.14 $S_{31}$ and $S_{42}$ parameter of modified directional coupler**

The above plot explains the symmetrical behavior of the modified directional coupler. Whether the excitation is given in port 1 or it is given in port 4, the received signal at port 3 or port 2 respectively is almost the same.

# Chapter 6

# Conclusion & Future Scope

The conventional coupler was designed and simulated for the operational frequency 1.5GHz on Rogers R03210 substrate having dielectric constant,$\varepsilon_r$ 10.2 with the height,$h$ of 0.5mm. The conventional coupler was simulated using S-parameters and the results showed the coupling,$S_{41}$ of -15.335dB and the isolation,$S_{31}$ of -22.335dB and thus directivity came out to be -7dB. The coupler was modified by slotting down the coupling edges and then it was simulated. The results obtained showed the coupling to be equal to -14.688dB and isolation was seen to be equal to -40.061dB. Isolation improved with a large amount. And thus the directivity came out to be equal to -25.373dB. We can see that the directivity improved from -7dB to -25.373dB, an improvement of about 18.373dB.

The side effects of the modified coupler were that for the same spacing between the coupled lines, the coupling between the two lines increased. Thus to bring the coupling level down to the desired specification, the space between the two coupled lines was increased from 0.38mm to 0.5mm.

We can conclude that slotting and wiggling the coupling edges of the directional coupler improves the isolation and thus the directivity increases. If, in the modified directional coupler, the edges are slotted down densely, a further improvement in directivity can be seen.

Further modifications in the physical structure of microstrip directional coupler can be done in order to improve directivity of the coupler. The coupler designed can be used as power divider and filter.

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# Appendix

## MATLAB coding for calculation of length of the Directional Coupler

```
clc;
Zo= input ('enter the value of port impedance\n');
C= input ('enter the value of coupling level in decibels\n');
f= input('enter the operating frequency in hertz\n');
er=input('enter the value of dielectric constant\n');
c=3*10^8;
e0=8.854*10^-12;
%Calculation of even and odd impedance
x=1+10^(C/20);
y=1-10^(C/20);
Zoe=Zo*sqrt(x/y);
Zoo=Zo*sqrt(y/x);

% Find physical dimensions s/h and w/h
Zose=Zoe/2;
Zoso=Zoo/2;
Ae=((Zose/60)*sqrt((1+er)/2))+((er-1)/(er+1))*(0.23+0.11/er);
WbyHse=8*exp(Ae)/(exp(2*Ae)-2);
Ao=((Zoso/60)*sqrt((1+er)/2))+((er-1)/(er+1))*(0.23+0.11/er);
WbyHso=8*exp(Ao)/(exp(2*Ao)-2);
SbyH=(2/pi)*acosh((cosh((pi/2)*WbyHse)+cosh((pi/2)*WbyHso)-
2)/(cosh((pi/2)*WbyHso)-cosh((pi/2)*WbyHse)));
g=cosh((pi/2)*(SbyH));
d=0.5*(cosh((pi/2)*WbyHse)*(g+1)+g-1);
WbyH=(1/pi)*(acosh(d))-0.5*(SbyH);

% Find the physical length of directional coupler%
% Even mode Calculation
Cp=e0*er*WbyH;

xxx= 1/sqrt(1+12/WbyH);
if WbyH <= 1
    func=xxx+(0.041*(1-WbyH)^2);
else
    func=xxx;
end
eseff=((er+1)/2)+((er-1)/2)*func;
Cf=(sqrt(eseff)/(2*c*Zo))-Cp/2;

A=exp(-0.1*exp(2.33-2.53*WbyH));
yyy=1+(A/SbyH)*tanh(8*SbyH);
Cfd=(Cf/yyy)*sqrt(er/eseff);

Ce=Cp+Cf+Cfd;
```

```
% Odd mode Calculation
k=SbyH/(SbyH+2*WbyH);
kd=sqrt(1-k^2);
w=k^2;
if w<=0.5
    fracK=(1/pi)*log(2*(1+sqrt(kd))/(1-sqrt(kd)));
else
    fracK=pi/log(2*(1+sqrt(kd))/(1-sqrt(kd)));
end
Cga=e0*fracK;

Cgd=e0*er/pi*log(coth((pi/4)*SbyH))+0.65*Cf*((0.02/SbyH)*sqrt(er)+(1-
(1/er^2)));

Co=Cp+Cf+Cga+Cgd;

eeff=((c*Ce*Zoe+c*Co*Zoo)/2)^2;

%length is given by
length=c/(4*f*sqrt(eeff));
display (length)
```