High performance detection method using memo technology and v-blast architecture

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CERTIFICATE

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This is certified that the major project report entitled "**High Perfomance Dettection Method Using Memo Technology and V-Blast Architecture"** is a work of **Ratandeep Meena** (College Roll No 12/E&C/2009 & University Roll No- 13833) is a student of Delhi College of Engineering. This work is completed under my direct supervision and guidance and forms a part of master of engineering (Electronics &
Communication Engineering) course and curriculum. He has completed his work with utmost sincerity and diligence.
The work embodied in this major project has not been submitted for the award of any other degree to the best of my knowledge.

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ABSTRACT

In this paper, we investigate the QR-OSIC receiver design for the transmitter side power allocated MIMO system. Based on the properties of the function and ordering results, we develop the efficient ordering algorithms in combination with the PA scheme. From the convexity of the function, we derive the ordering strategy that makes the channel gains converge to their geometric mean. Based on this approach, the fixed ordering algorithm if first designed, for which the geometric mean is used for constant threshold. To further improve the performance, the modified scheme employing adaptive thresholds is developed using the correlation among ordering results. Theoretical analysis and simulation results show that proposed ordering schemes using QR-decomposition not only require a reduced computational complexity compared to the conventional scheme, but result in improved error performance.

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1.1 Introduction

The utilization of multiple-input multiple-output (MIMO) systems has been an active area of research as well as practical transceiver implementations for their great potential of enhancing the system's performance. The V-BLAST architecture proposed in also referred to as the BLAST ordered successive interference cancellation (B-OSIC) detector, is regarded as an attractive solution that exploits this potential. In a B-OSIC receiver, the data stream with the strongest signal-to-interference-noise ratio SINR is selected first and subtracted from the received signal, and the procedure is successively performed for the remaining multiple data streams. For equal power allocation (PA) across the transmit antenna array, it is optimal in terms of bit error rate (BER) or equivalently minimum-mean-square-error (MMSE). The knowledge of the channel is available at the transmitter; a further performance improvement can be achieved using appropriate PA schemes.

Based on the notion that the data stream with the smallest SINR degrades the overall error performance, PA schemes for the B-OSIC have been suggested in which reduces the computational complexity and the feedback overhead by adopting a diagonal pre-coding matrix for the PA. Most of the PA schemes for the closed-loop systems mainly focus on the transmitter-side processing strategies, while attempts for the joint optimization for the PA at the transmitter and the detection ordering scheme at the receiver have not been fully investigated. In the past few years, theoretical investigations have revealed that the multipath wireless channel is capable of enormous capacities, provided that the multipath scattering is sufficiently rich and is properly exploited through the use of an appropriate processing architecture. The diagonally-layered space-time architecture proposed by Foschini, now known as diagonal BLAST (Bell Laboratories Layered Space-Time) or D-BLAST is one such approach. D-BLAST utilizes multi-element antenna arrays at both transmitter and receiver and an elegant diagonally layered coding structure in which code blocks are dispersed across diagonals in space-time.

In an independent Rayleigh scattering environment, the processing structure leads to theoretical rates which grow linearly with the number of antennas (assuming equal numbers of transmit and receive antennas) with these rates approaching 90% of Shannon capacity. However, the diagonal approach suffers from certain implementation complexities which make it inappropriate for initial implementation. In this paper, we describe a simplified version of BLAST known as vertical BLAST or V-BLAST, which has been implemented in real time in the laboratory.

Using our laboratory prototype, we have demonstrated spectral efficiencies of 20 - 40 bps/Hz at average SNR ranging from 24 to 34 dB. Although these results were obtained in a relatively benign indoor environment, we believe that spectral efficiencies of this magnitude are unprecedented, regardless of propagation environment or SNR, and are simply unattainable using traditional techniques.

A single data stream is de-multiplexed into M sub streams, and each sub stream is then encoded into symbols and fed to its respective transmitter. (The encoding process is discussed in more detail below). Transmitters $1 - M$ operate co channel at symbol rate $1/T$ symbols/sec, with synchronized symbol timing. Each transmitter is itself an ordinary QAM transmitter. The collection of transmitters comprises, in effect, a vector-valued transmitter, where components of each transmitted M-vector are symbols drawn from a QAM constellation. We assume that the same constellation is used for each sub stream, and that transmissions are organized into bursts of L symbols. The power launched by each transmitter is proportional to $1/M$ so that the total radiated power is constant and independent of M.

In this project, it is to derive new detection ordering strategy and schemes from joint transceiver design, which is distinct from previous studies. To obtain a closed-form solution, a QR-factorization based approach will be employed. First the BER is provided, minimization condition, derived from the convexity of the function in the PA scheme. It is demonstrated that the ordering strategy, which makes the channel gains converge to their geometric average, achieves the improved error performance. Based on this observation, we develop the two ordering algorithms, which are identical except for the threshold adaptation.

The basic algorithm determines the detection-order using the geometric mean as a constant threshold, whereas the modified ordering scheme for robust convergence adaptively updates the threshold by taking into account the previous ordering results. The comparison of the cumulative distribution is conducted to confirm the superiority of the adaptive design. It is also shown that proposed ordering schemes using QR-decomposition obtain not only lower implementation complexity but also better BER performance compared to the conventional B-OSIC algorithm.

1.2 Problem Definition

MIMO technology through the use of multiple antennas at the transmitter and receiver sides has been an area of intense research for its promise of increased spectral efficiency and reliability. The MIMO system has been an active area of research as well as practical transceiver implementations for their great potential of enhancing the system's performance. There are three basic link performance parameters that completely describe the quality and usefulness of any wireless link, they are speed (or spectrum), range (or coverage), and reliability (or security). The use of multiple waveforms transmission in parallel constitutes a new type of radio communication. The communication using the multi-dimensional signals is to improve all the three basic link performance parameters using multiple antenna system. In MIMO, a multi-antenna system answers the question of how to achieve the higher data rates, wider coverage, and increased reliability, all without using additional frequency spectrum. The combination of multi-antenna system with multicarrier system gives an excellent performance.

The transmission in wireless communication is typically organized in packets, with a training sequence at the beginning of the packet, to allow for the channel estimation and coherent detection at the receiver. When the transmitter is unaware of the channel and the receiver does not give the feedback details Phase and Magnitude information, we speak of 'open-loop' transmission. It is good match for the wireless MIMO channel that is time varying and the rate of feeding back channel information might be low.

In this system the input data stream is de-multiplexed into sub streams and each sub stream is then encoded into symbols. With the help of multiple antennas we have build up a virtual communication system using BPSK (Binary Phase Shift Keying) modulation technique, the symbols are transmitted into the channel, which is wireless medium generally the atmosphere and this channel behavior is estimated and the noise content is to be added and is received at the receiver end antennas.

At the receiver end, the received signal is processed and the signal is estimated using QR decomposition algorithm. This algorithm estimates the BER and based on this observation we provided the power at individual transmitting antennas to control the BER and thus improves the system performance.

1.3 Objective of the Project

The main aim of the project is to derive new detection ordering strategy and schemes from joint transceiver design, which is distinct from previous studies. To obtain a closed-form solution, a QR-factorization based approach will be employed in our study. First, we provide the BER minimization condition, derived from the convexity of the Q-function in the PA scheme. It is demonstrated that the ordering strategy, which makes the channel gains converge to their geometric average, achieves the improved error performance. Based on this observation, we develop the two ordering algorithms, which are identical except for the threshold adaptation. The basic algorithm determines the detection-order using the geometric mean as a constant threshold, whereas the modified ordering scheme for robust convergence adaptively updates the threshold by taking into account the previous ordering results.

2.1 Introduction

During the past decades, wireless communication has benefitted from substantial advances and it is considered as the key enabling technique of innovative future consumer products. For the sake of satisfying the requirements of various applications, significant technological achievements are required to ensure that wireless devices have appropriate architectures suitable for supporting a wide range of services delivered to the users. In the foreseeable future, the large-scale deployment of wireless devices and the requirements of high bandwidth and high data rate applications are expected to lead to tremendous new challenges in terms of the efficient exploitation of the achievable spectral resources and constitute a substantial research challenge in the context of the emerging WLAN's and other indoor multimedia networks. Due to the physical limits imposed by the mobile radio channel which cause performance degradation and make it very difficult to achieve Journal of Theoretical and Applied Information Technology high bit rates at low error rates over the time dispersive wireless channels. Other detrimental characteristics are Co-Channel Interference (CCI), Doppler Effect, Intentional Jamming in Military Communications and Inter Symbol Interference (ISI) induced by multipath fading, however, there is an irreducible error floor that imposes a limit on the maximum attainable transmission rate. Specifically, the employment of multiple antennas at both the transmitter and the receiver, which is widely referred to as the MIMO technique, constitutes a cost-effective approach to high through put wireless communications and remote sensing.

The concept MIMO for both wired and wireless systems was first introduced by Jack Winters in 1987 for two basic communication systems. The first was for the communication between multiple mobiles and a base station with multiple antennas and second for the communication between two mobiles each with multiple antennas. Where, he introduced a technique of transmitting data from multiple users over the same frequency/time channel using multiple antennas at both the transmitter and receiver ends. Sparked off by Winters pioneering work, Salz investigated joint transmitter/receiver optimization using the minimum mean square error (MMSE) criterion. Since then, Winters and others have made further significant advances in the field of MIMO. In 1996, Raleigh and Cioffi and Foschini proposed new approaches for improving the efficiency of MIMO systems, which inspired numerous further contributions for two suitable architectures for its realization known as Vertical Bell-Labs Layered Space-Time (V-BLAST), and Diagonal Bell-Labs Layered Space-Time BLAST (D-BLAST) algorithm has been proposed by Foschini, which is capable of achieving a substantial part of the MIMO capacity.

It is capable of achieving high spectral efficiency while being relatively simple to implement. This structure offers highly better error performance than other existence detection method and still has low complexity. The basic motive was to increase the data rate in a constrained spectrum. The promises of information theoretic MIMO analysis for the channel capacity were the main trigger for this enthusiasm and also ignited the study of related areas such as MIMO Channel Modeling, Space-Time Signal Processing, Space-Time Coding, etc. The objective of such multi-channel diagonalization is to partition or distribute multi-user signals into disjoint space and resultant channel gains are maximized to optimize the overall system capacity under the constraint of a fixed transmit power. It also improves the quality (BER) or potential of achieving extraordinary data rates by transferring the signals in time domain and space domain separately, without consuming more frequency resources, frequency diversity due to delay spread, higher spectral efficiency and without increasing the total transmission power or bandwidth of the communication system by means of the deployment of multiple spatial ports, improved link reliability, beam forming, and adequate signal processing techniques at both ends of the system by using interference cancellation techniques for the communication as well as remote sensing.

In the use of multiple antennas both the transmitter and receiver improves the communication performance. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data through put and link range without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency (more bits per second per hertz of bandwidth) or to achieve a diversity gain that improves the link reliability (reduced fading).

2.2 Why is MIMO Beneficial?

Motivated by these promising improvements one question remains: Why and how are these gains in rate and reliability possible? Basically, it turns out that there are two gains that can be realized by MIMO systems. They are termed as diversity gain and spatial multiplexing gain. First, to investigate the diversity gain in an introductory form, we take a look at the single input single output (SISO) system.

In the context of wireless transmissions, it is common knowledge that depending on the surrounding environment, a transmitted radio signal usually propagates through several different paths before it reaches the receiver, which is often referred to as multipath propagation. The radio signal received by the receiver antenna consists of the superposition of the various multi paths. If there is no line-of-sight (LOS) between the transmitter and the receiver, the attenuation coefficients corresponding to different paths are often assumed to be Independent and Identically Distributed (IID). In this case the central limit theorem applies and the resulting path gain can be modelled as a complex Gaussian variable (which has a uniformly distributed phase and a Rayleigh distributed magnitude).

Due to this statistical behavior, the channel gain can sometimes become very small so that a reliable transmission is not always possible. To deal with this problem, communication engineers have thought of many possibilities to increase the so-called diversity. The higher the diversity is the lower is the probability of a small channel gain.

Some common diversity techniques are time diversity and frequency diversity, where the same information is transmitted at different time instants or in different frequency bands, as well as spatial diversity, where one relies on the assumption that fading is at least partly independent between different points in space.

The concept of spatial diversity leads directly to an expansion of the SISO system. This enhancement is denoted as single-input multiple-output (SIMO) system. In such a system, we equip the receiver with multiple antennas. Doing so usually can be used to achieve a considerable performance gain i.e. better link budget but also co-channel interference can be better combated. At the receiver, the signals are combined (i.e. if the phases of the transmission are known, in a coherent way) and the resulting advantage in performance is referred to as the diversity gain obtained from independent fading of the signal paths corresponding to the different antennas. This idea is well known and is used in many established communication systems, for example in the Global System for Mobile communications (GSM). It is clear that in the above described way, a base station can improve the uplink reliability and signal strength without adding any cost, size or power consumption to the mobile device.

As far as the ability to achieve performance in terms of diversity is concerned, system improvements are not only limited to the receiver side. If the transmitter side is also equipped with multiple antennas, we can either be in the multiple-input single-output (MISO) or multiple-input multiple-output (MIMO) case. A lot of research has been performed in recent years to exploit the possible performance gain of transmit diversity. The ways to achieve the predicted performance gain due to transmit diversity are various most of them are loosely speaking, summarized under the concept of space-time coding (STC).

Besides the advantages of spatial diversity in MIMO systems, they can also order a remarkably gain in terms of information rate or capacity. This improvement is linked with the fore mentioned multiplexing gain. In fact, the advantages of MIMO are far more fundamental as it may have appeared to the reader so far. The underlying mathematical nature of MIMO systems, where data is transmitted over a matrix rather than a vector channel, creates new and enormous opportunities beyond the just described diversity effects. Where the author points out how one may, under certain conditions, transmit a number of independent data streams simultaneously over the eign modes of a matrix channel, created by several transmit and receive antennas.

The gains achievable by a MIMO system in comparison to a SISO one can be described rigorously by information theory. A lot of research in the area of MIMO systems and STC is based on this mathematical framework introduced by Shannon. The fundamental result of error free communication below a specific rate (depending on the actual signal-to-noise ratio (SNR)) in the limit of infinite length codes is also in the MIMO case an upper bound to all communication schemes. It can be used as a design criterion for transmission schemes as well as for comparison of different MIMO communication systems.

Overall, the potential increase in data rates and performance of wireless links offered by MIMO technology has proven to be so promising that we can accept MIMO systems to be the cornerstone of many future wireless communication systems.

2.3 MIMO Channel Model

MIMO systems are an extension of smart antennas systems. Traditional smart antenna systems employ multiple antennas at the receiver, whereas in a general MIMO system multiple antennas are employed both at the transmitter and the receiver. The addition of multiple antennas at the transmitter combined with advanced signal processing algorithms at the transmitter and the receiver yields significant advantage over traditional smart antenna systems - both in terms of capacity and diversity advantage. A MIMO channel is a wireless link between M transmits and N receive antennas. It consists of MN elements that represent the MIMO channel coefficients. The multiple transmit and receive antennas could belong to a single user modem or it could be distributed among different users. The later configuration is called distributed MIMO and cooperative communications. Statistical MIMO channel models offer flexibility in selecting the channel parameters temporal and spatial correlations.

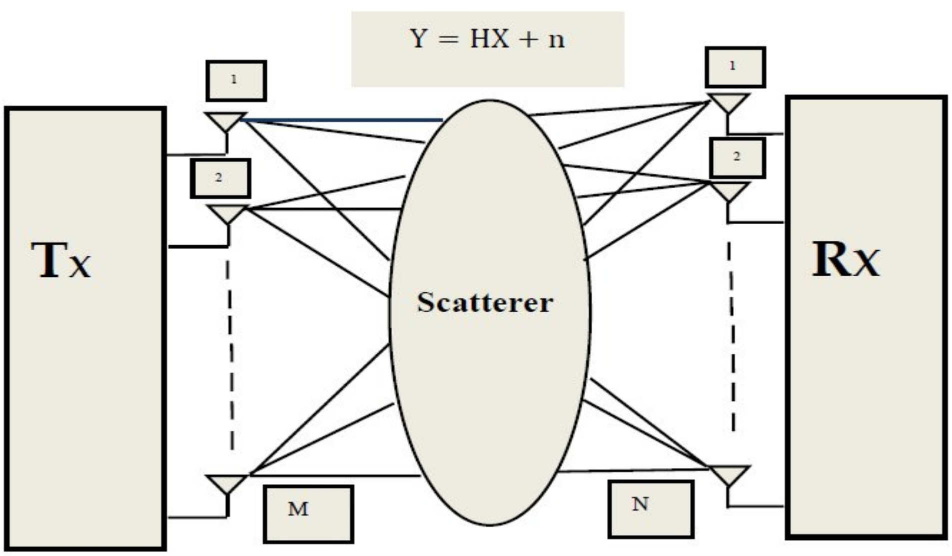


Figure: - A MIMO wireless channel

We focus on a single-user communication model and consider a point-to-point link where the transmitter is equipped with n_T antennas and the receiver employs n_R antennas. Next to the single user assumption in the depiction as point-to-point link, we suppose that no inter symbol interference (ISI) occurs. This implies that the bandwidth of the transmitted signal is very small and can be assumed frequency -° at (narrowband assumption), so that each signal path can be represented by a complex-valued gain factor. For practical purposes, it is common to model the channel as frequency -° at whenever the bandwidth of the system is smaller than the inverse of the delay spread of the channel, hence a wideband system operating where the delay spread is fairly small (for instance indoor scenes) may sometimes be considered as frequency.

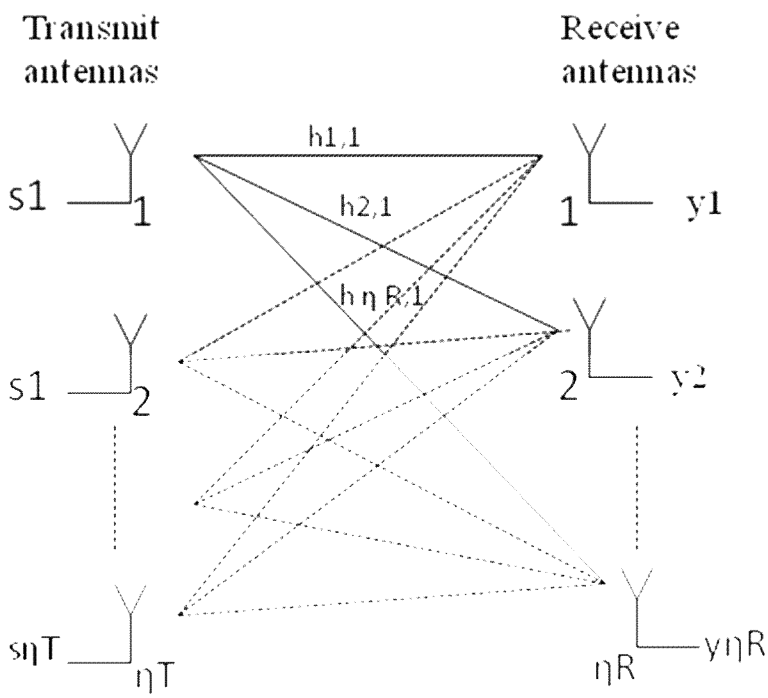


Figure: - A MIMO channel with n_T transmit and n_R receive antennas

If the channel is frequency selective, one could use an OFDM (orthogonal frequency-division multiplexing) system, to turn the MIMO channel into a set of parallel frequencies at MIMO channels, of which each obeys our stated assumptions.

In addition to these restrictions, we will further assume that we are operating in a time-invariant setup. These assumptions allow us to use the standard complex-valued baseband representation of narrowband signals that can be written in a discrete form (omitting the dependency on time).

Now let $h_{i;j}$ be the complex valued path gain from transmit antenna *j* to receive antenna *i* (the fading coefficient). If at a certain time instant the complex valued signals *s*1*; : : : ; snT (fs*1*;: : : ; snTg)* are transmitted via the *nT* antennas, respectively, the received signal at antenna *i* can be expressed as

$$y_i = \sum_{j=1}^{n_T} h_{i,j} s_j + n_i \qquad (1)$$

where *ni* represents additive noise, which will be treated later in this chapter. This linear relation can be easily written in a matrix framework. Thus, let be a vector of size n_T containing the transmitted values, and y be a vector of size n_R containing the received values, respectively. Certainly, we have s *2* C_{nT} and y *2* $C_{nR.}$ Moreover, if we define the channel transfer matrix H as

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,n_T} \\ . & . & .. & . \\ h_{n_R,1} & h_{n_R,2} & \cdots & h_{n_R,n_T} \end{bmatrix} \qquad (2)$$

$$y = Hs + n \qquad (3)$$

We obtain

This is the same matrix notation as it is used in the majority of the publications in this field, e.g. [2]. This relation, denoting a transmission only over one symbol interval, is easily adapted to the case that several consecutive vectors *f*s1*;* s2*; : : : ;* s*Lg* are transmitted (here, *L* denotes the total number of symbol intervals used for transmission) over the channel. Therefore, we arrange the transmitted, the received and the noise vectors in the matrices.

$$S = [s_1, s_2, \ldots s_L], \quad Y = [y_1, y_2, \ldots y_L], \quad N = [n_1, n_2, \ldots, n_L] \qquad (4)$$

2.4 MIMO System Channel Capacity

Multipath propagation has long been regarded as "impairment" because it causes signal fading. To mitigate this problem, diversity techniques were developed. Antenna diversity is a widespread form of diversity. Information theory has shown that with multipath propagation, multiple antennas at both transmitter and receiver can establish essentially multiple parallel channels that operate simultaneously, on the same frequency band at the same total radiated power. Antenna correlation varies drastically as a function of the scattering environment, the distance between transmitter and receiver, the antenna configurations, and the Doppler spread. Recent research has shown that multipath propagation can in fact "contribute" to capacity. Channel capacity is the maximum information rate that can be transmitted and received with arbitrarily low probability of error at the receiver. A common representation of the channel capacity is within a unit bandwidth of the channel and can be expressed in bps/Hz. This representation is also known as spectral (bandwidth) efficiency. MIMO channel capacity depends heavily on the statistical properties and antenna element correlations of the channel. Representing the input and output of a memory less channel with the random variables X and Y respectively, the channel capacity is defined as the maximum of the mutual information between X and Y

$$C = \max_{p(x)} I(X;Y) \qquad (5)$$

A channel is said to memory less if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs or outputs**.** P(x) is the probability distribution function of the input symbols X.

2.4.1 Capacity of Single-Input-Single-Output (SISO) System

According to Shannon capacity of wireless channels, given a single channel corrupted by an additive white Gaussian noise at a level of SNR, the capacity is

$$C_{SHANNON} = B.\log_2 [1 + SNR] \text{ (BPS/Hz)} \qquad (6)$$

Where: C is the Shannon limits on channel capacity, SNR is signal-to-noise ratio, B is bandwidth of channel. In the practical case of time varying and randomly fading wireless channel, the capacity can be written as:

$$C_{SHANNON} = B.\log_2 [1 + SNR\,|H|^2] \text{ (BPS/Hz)} \qquad (7)$$

Where: H is the 1x1 unit-power complex matrix Gaussian amplitude of the channel. Moreover, it has been noticed that the capacity is very small due to fading events.

Figure: Shannon capacity for SISO system

From the above expression it is clear that theoretically capacity increase as the bandwidth is increased which shown in above figure. C increases 1 bits/sec/Hz for every 3dB of SNR.

2.4.2 Capacity of Single-Input-Multiple-Output (SIMO) System

For the SIMO system, we have N antennas at receiver and only one at transmitter. If the signals received on these antennas have on average the same amplitude, then they can be added coherently to produce an N^2 increase in the signal power. On the other hand, there are N sets of noise that are added incoherently and result in an N fold increase in the noise power. Hence, there is an overall increase in the SNR.

$$SNR \approx \frac{N^2 .\text{signal power}}{N(\text{noise})} = N.SNR \qquad (8)$$

So the capacity of SIMO channel is:

$$C_{SIMO} = B.\log_2 [1+N.SNR] \text{ (BPS/Hz)} \qquad (9)$$

The capacity of SIMO system in the practical case of time-varying and randomly fading wireless channel is:

$$C_{SIMO} = B.\log_2 [1+SNR.HH^*] \text{ (BPS/Hz)} \qquad (10)$$

Where H is $1xN^R$ channel vector and ( )* is the transpose conjugate.

2.4.3 Capacity of Multiple-Input-Single-Output (MISO) System

For the SIMO system, we have M antennas at transmitter and only one at receiver. As same as the case of the SIMO system, we have capacity of MISO system.

$$C_{MISO} = B.\log_2 [1+M.SNR] \text{ (BPS/Hz)} \quad (11)$$

In the practical case of time-varying and randomly fading wireless channel, it is shown that the capacity of M x 1 MIMO system is:

$$C_{MISO} = B\log_2 [1+SNR.HH^*] \text{ (BPS/Hz)} \quad (12)$$

Compared with SISO system, the capacity of SIMO and MISO system shows improvement. The increase in capacity is due to the spatial diversity which reduces fading and SNR improvement. However, the SNR improvement is limited, since the SNR is increasing inside the log function.

2.4.4 The capacity of the MIMO Channel is analyzed in two cases

For the MIMO system, we have M antennas at transmitter and N antennas at receiver.

- **Case 1. Same signal transmitted by each antenna**

In this case, the MIMO system can be view in effect as a combination of the SIMO and MISO channels.

$$SNR \approx \frac{N^2.M^2.\text{signal power}}{N.M.(\text{noise})} = M.N.SNR \quad (13)$$

So the capacity of MIMO channels in this case is:

$$C_{MIMO} = B.\log_2 [1+M.N.SNR] \text{ (BPS/Hz)} \quad (14)$$

Thus, we can see that the channel capacity for the MIMO systems higher than that of SIMO and MIMO system. But in this case, the capacity is increasing inside the log function. This means that trying to increase the data rate by simply transmitting more power is extremely costly.

- **Case 2. Different signal transmitted by each antenna**

The big idea in MIMO is that we can send different signals using the same bandwidth and still be able to decode correctly at the receiver. Thus, it is like we are creating a channel for each one of the transmitters. The capacity of each one of these channels is roughly equal to

$$C_{MIMO} = B.\log_2\left[1 + \frac{N}{M}.SNR\right](BPS/Hz) \qquad (15)$$

But we have M_t of these channels, so the total capacity of the system is:

$$C_{MIMO} = M.B.\log_2\left[1 + \frac{N}{M}.SNR\right](BPS/Hz) \qquad (16)$$

With N≥ M, the capacity of MIMO channels is equal to:

$$C_{MIMO} = M.B.\log_2[1 + SNR]\ (BPS/Hz) \qquad (17)$$

Thus, we can get linear increase in capacity of the MIMO channels with respect to the number of transmitting antennas.

2.4.5 Capacity of Deterministic MIMO Channels

We now study the capacity of a MIMO channel in the case that the channel matrix H is deterministic. Furthermore, we assume that the channel has a bandwidth of 1 Hz and fulfills all constraints. Thus, we are investigating the vector transmission model.

$$y = \sqrt{\frac{\rho}{n_T}}Hs + n \qquad (18)$$

In the following, we assume that the channel H is known to the receiver. This is a very common assumption, although in practice hard to realize. Channel knowledge at the receiver may be maintained via training and tracking, but time-varying environments can make it difficult to estimate the channel sufficiently exact.

The capacity of the MIMO channel is defined similar to definition as

$$C = \max_{p(s)} I(s; y). \qquad (19)$$

We start by using Equation written as

$$I(s;y) = H(y) - H(y|s), \tag{20}$$

where H (¢) denotes the entropy. Because y is specified through our linear MIMO transmission model, we can use the identity $H(y|s) = H(n|s)$. Since according to our premises, the noise n and the transmit vector s are statistically independent, we can further write $H(y|s) = H(n)$.

Therefore, Equation simplifies to

$$I(s;y) = H(y) - H(n). \tag{21}$$

By our assumptions about the noise term n, the entropy H(n) can be evaluated as

$$H(n) = \ln\det(\pi e C_n) = \ln\det(\pi e I) \tag{22}$$

Thus, the maximization of the mutual information I(s; y) reduces to a maximization of H(y). To derive an expression for the entropy of y, we first investigate its covariance matrix.

The covariance matrix of y, C_y satisfies

$$C_y = E\left\{yy^H\right\} = E\left\{\left(\sqrt{\frac{\rho}{n_T}}Hs + n\right)\left(\sqrt{\frac{\rho}{n_T}}Hs + n\right)^H\right\} = \frac{\rho}{n_T}E\left\{Hss^H H^H\right\} + E\left\{nn^H\right\}, \tag{23}$$

Which can be further simplified to?

$$C_y = \frac{\rho}{n_T}HE\left\{ss^h\right\}H^H + E\left\{nn^H\right\} = \frac{\rho}{n_T}HC_sH^H + C_n \tag{24}$$

Where Cs is the covariance matrix of s. To evaluate the maximization of H(y), we need the following theorem.

Theorem 1: (Entropy-maximizing property of a Gaussian random variable). Suppose the complex random vector X_2 Cn is zero-mean and satisfies $E\{xx^H\} = C_{x.}$ then the entropy of x is maximized if and only if x is a circularly symmetric complex Gaussian random variable with $E\{xx^H\} = C_{x.}$.

Proof: Let $f_x(\xi)$ be any density function satisfying $\int_{C^n} f_x(\xi)\xi_i\xi_j^* d\xi = (C_x)_{i,j,} 1 \le i, j \le n.$

furthermore, let

$$f_{x,G}(\xi) = \frac{1}{\pi \det C_x}\exp\left[-\xi^H C_x^{-1}\xi\right] \tag{25}$$

denote a joint complex Gaussian distribution with zero-mean. Now, we can observe that $\int_{C^n} f_x(\xi)\xi_i\xi_j^* d\xi = (C_x)_{i,j,}$ and that log $f_{x,G}(\xi)$ is a linear combination of the terms $\xi_i\xi_j^*$

This means that by the construction of $f_{x,G}(\xi)$ the integral $\int_{C^n} f_{x,G}(\xi)\log f_{x,G}(\xi)d\xi$ can be split up in integrals $\int_{C^n} f_{x,G}(\xi)\xi_i\xi_j^* d\xi$ of which each yields the same as $\int_{C^n} f_x(\xi)\xi_i\xi_j^* d\xi$

Therefore, by construction, we have the identity

$$\int_{C^n} f_{x,G}(\xi)\log f_{x,G}(\xi)d\xi = \int_{C^n} f_x(\xi)\log f_{x,G}(\xi)d\xi .$$

Thus,

$$\begin{aligned} H\left(f_x(\xi)\right) - H\left(f_{x,G}(\xi)\right) &= -\int_{C^n} f_x(\xi)\log f_{x,}(\xi)d\xi + \int_{C^n} f_{x,G}(\xi)\log f_{x,G}(\xi)d\xi \\ &= -\int_{C^n} f_x(\xi)\log f_{x,}(\xi)d\xi + \int_{C^n} f_x(\xi)\log f_{x,G}(\xi)d\xi \qquad (26) \\ &= \int_{C^n} f_x(\xi)\log\frac{f_{x,G}(\xi)}{f_x(\xi)} \le 0. \end{aligned}$$

With equality if and only if

$$f_x(\xi) = f_{x,G}(\xi)$$

Thus $H\left(f_x(\xi)\right) \le H\left(f_{x,G}(\xi)\right)$ this concludes the proof.

Accordingly, the differential entropy *H*(y) is maximized when y is zero-mean circularly symmetric complex Gaussian (ZMCSCG). This, in turn implies that s must be a ZMCSCG vector, with distribution that is completely characterized by Cs. The differential entropy *H* (y)

$$H(y) = \log\det(\pi c C_y). \qquad (27)$$

Therefore, the mutual information *I*(s; y), in case of a deterministic channel H, reduces to

$$I(s;y) = \log\det\left(I + \frac{\rho}{n_T}HC_sH^H\right)\ [bps/Hz] \qquad (28)$$

This is the famous "log-det" formula, firstly derived by Telatar. In principle, we could denote the derived mutual information as a capacity since we maximized over all possible input distributions. Nevertheless, the above derivation does not tell us how to choose the covariance matrix of s to get the maximum mutual information. Therefore we keep the above notation. Thus in the following equation we write the capacity of the MIMO channel (within our power constraint) as

$$C(H) = \max_{trC_x = n_T} \log\det\left(I + \frac{\rho}{n_T} HC_s H^H \right) \left[bps/Hz \right] \quad (29)$$

2.4.6 Capacity of Random MIMO Channels

For a fading channel, the channel matrix H is a random quantity and hence the associated channel capacity *C* (H) is also a random variable. To deal with these circumstances, we define the ergodic channel capacity as the average of over the distribution of H. Definition (Ergodic MIMO channel capacity). The ergodic channel capacity of the MIMO transmission model is given by

$$C_E = E\left\{ \max_{trC_x = n_T} \log\det\left(I + \frac{\rho}{n_T} HC_s H^H \right) \right\} \quad (30)$$

According to our information theoretic basics, this capacity cannot be achieved unless coding is employed across an infinite number of independently fading blocks. After having identified the channel capacity in a fading MIMO environment, it remains to evaluate the optimal input power distribution, or covariance matrix Cs that maximizes equation. The maximization depends on an important condition we have not taken into account yet. Before being able to compute the maximization, we have to clarify if the transmitter, the receiver, or both have perfect knowledge of the channel state information (CSI). This is equivalent to the constraint that the channel matrix H is perfectly known to any or both sides of the communication system.

If the channel H is known to the transmitter, the transmit correlation matrix Cs can be chosen to maximize the channel capacity for a given realization of the channel. The main tool for performing this maximization is a technique, which is commonly referred to as water filling "or water-pouring algorithm", which we will not restate here. Besides the performance gain achievable, this method implicates a complex system, because the CSI has to be fed back to the transmitter.

Therefore, we chose to focus on the case of perfect CSI on the receiver side and on CSI at the transmitter. Of course, this implies that the maximization of Equation (31) is now more restricted than in the previous case. Nevertheless, Telatar, among others showed that the optimal signal covariance matrix has to be chosen according to

$$C_s = I$$

This means that the antennas should transmit uncorrelated streams with the same average power. With this result, the ergodic MIMO channel capacity reduces to

$$C_E = E\left\{\log\det\left(I + \frac{\rho}{n_T} H C_s H^H\right)\right\} \tag{31}$$

Clearly, this is not the Shannon capacity in a true sense, since as mentioned before, a genie with channel knowledge can choose a signal covariance matrix that outperforms $C_s = I$. Nevertheless, we shall refer to the expression in Equation (29) as the ergodic channel capacity with CSI at the receiver and no CSI at the transmitter.

Now that we have specified our MIMO transmission system in a consistent way, and having identified the corresponding ergodic MIMO channel capacity, we would like to derive another notation of the capacity formula. Therefore, we take a closer look at the term HH^H in Equation.

The term HH^H is an n_R x n_R positive semi-definite Hermitian matrix. Let the eigen decomposition of HH^H be Q^Q^H, where Q is a n_R x n_R matrix satisfying $QQ^H = Q^HQ = I$ and $A = diag\{\lambda_1, \lambda_2, ...\lambda_{n_R}\}$ with , $\lambda_t \geq 0$ denoting the ordered eigen values $(\lambda_t \geq \lambda_{t+1})$ of HH^H. Then the channel capacity can be expressed as

$$C_E = E\left\{\log\det\left(I + \frac{\rho}{n_T} Q \wedge Q^H\right)\right\} \tag{32}$$

Using the identity det (I+AB) = det (I+BA) for matrices A of size (*mxn*) and B of size (*nxm*), together with the relation $Q^HQ = I$, the above equation simplifies to

$$C_E = E\left\{\log\det\left(I + \frac{\rho}{n_T} \wedge\right)\right\} = E\left\{\sum_{t=1}^{r} \log\left(1 + \frac{\rho}{n_T} \wedge \lambda_t\right)\right\} \tag{33}$$

where r is the rank of the channel H. This expresses the capacity of the MIMO channel as the sum of the capacities of r SISO channels, each having a gain of ¸λ_i; $i = 1,\ldots\ldots\ldots.. r$.

Hence, the use of multiple antennas at the transmitter and receiver in a wireless link opens multiple scalar spatial pipes (also known as modes) between the transmitter and the receiver. This indicates the already mentioned multiplexing gain. To underline these insights, we did some numerical simulations, in which according to our IID, MIMO transmission model, we chose H to be formed by independent and Gaussian elements with unit variance shows the ergodic MIMO channel capacity with no CSI at transmitter for various numbers of transmit and receive antennas. From this, we can see that the gain in capacity is obtained by employing an extra receive antenna is around 3dB relative to the SISO system. This gain can be viewed as a consequence of the fact that the extra receive antenna effectively doubles the received power. The gain of a system with $n_T = 2$; $n_R = 1$ relative to the SISO system is small.

As far as the ergodic channel capacity is concerned there is practically no benefit in adding an extra transmit antenna to the SISO system. Note also that the SIMO channel has a higher ergodic channel capacity than the MISO channel. Finally, the capacity of a system with $n_T = 2$; $n_R = 2$ is higher and faster growing with SNR than that of the SISO system.

The growth of the ergodic channel capacity as a function of the number of antennas can be shown to obey a simple law. If we assume the channel H to be full rank, Equation indicates that when the number of transmit and receive antennas are the same, the ergodic MIMO channel capacity increases linearly by the number of antennas.

In general, the capacity increases by the minimum of the number of transmit and receive antennas. One can show that at high SNR, the ergodic channel capacity in terms of the received SNR can be described as

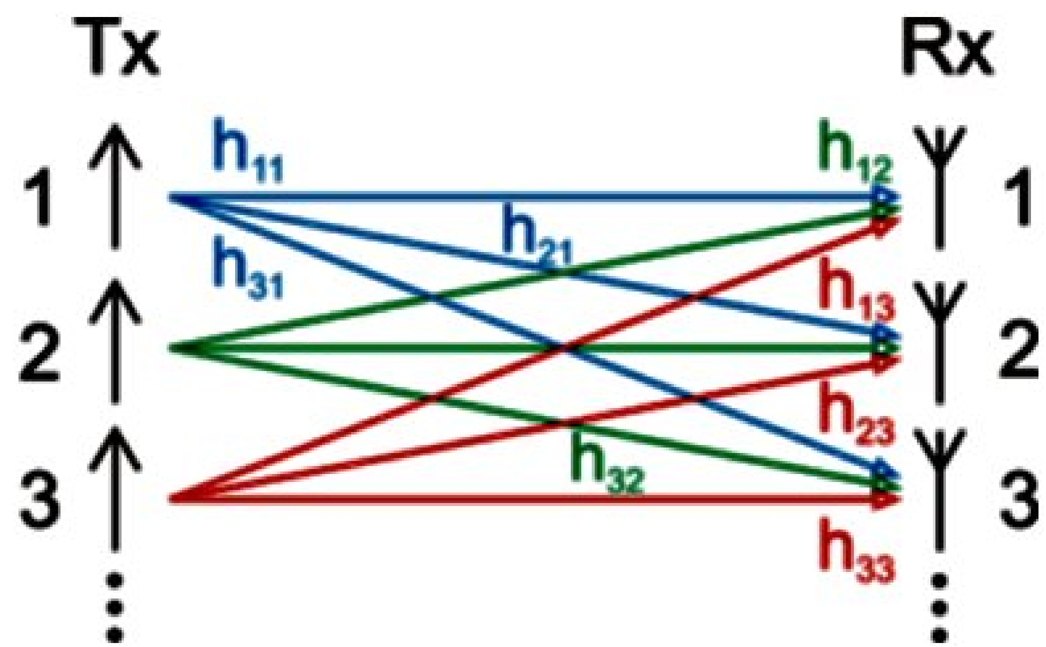
$$C_E \approx \min\{n_T, n_R\} \log\left(\frac{\rho}{n_T}\right) + \sum_{k=|n_T - n_R|+1}^{\min\{n_T, n_R\}} \log(x_k), \tag{34}$$

where X^k is a chi squared random variable with 2^k degrees of freedom. Therefore, a 3dB increase in SNR results in min $\{n_T,n_R\}$ extra bits of capacity at high SNR.

To further clarify our observation that the adding of transmit antennas to a system with a fixed number of receive antennas has a limited impact on the ergodic channel capacity, we investigate the ergodic capacity behavior for a large number of transmit antennas. In the mentioned case, using the law of large numbers, one can show that $H^H H / n_T \rightarrow I$ almost surely. As a result, the ergodic channel capacity is $n_R \log(1+\rho)$ for large n_T. This bound is rapidly reached, thus explaining the limited gain of adding extra transmit antennas.

Similar investigations can be performed for a fixed number of transmit antennas, where the capacity gain for adding one additional receive antenna also gets smaller if the number of receive antennas gets large. Now, it just remains to point out that a correlation of the entries of the channel matrix H, as it might be induced by not well separated antennas at either the transmit or receiver side. It can of course influence the shape of the presented curves massively. In general, correlation of H reduces the gains obtained in MIMO channels, as long as we are investigating a MIMO system with perfect CSI on the receiver side. Recent research shows that if only partial CSI at the receiver is available, correlation may be used to improve capacity gains.

2.5 Mathematical Description of MIMO System



In MIMO systems, a transmitter sends multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of all $N_t N_r$ paths between the N_t transmit antennas at the transmitter and N_r receive antennas at the receiver. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information. A narrowband flat fading MIMO system is modeled as

$$y = Mx + n \qquad (35)$$

where $\mathbf{y}$ and $\mathbf{x}$ are the receive and transmit vectors, respectively, and $\mathbf{H}$ and $\mathbf{n}$ are the channel matrix and the noise vector, respectively.

Referring to information theory, the ergodic channel capacity of MIMO systems where both the transmitter and the receiver have perfect instantaneous channel state information is

$$C_{Perfect-CSI} = E\left[\max_{Q;tr(Q)\leq 1} \log_2 \det\left(I + \rho HQH^H\right)\right]$$

$$= E\left[\log_2 \det(I + \rho DSD)\right] \qquad (36)$$

where $()^H$ denotes Hermitian transpose and ρ is the ratio between transmit power and noise power (i.e., transmit SNR). The optimal signal covariance $Q=VSV^H$ is achieved through singular value decomposition of the channel matrix $\mathbf{UDV}^H = \mathbf{H}$ and an optimal diagonal power allocation matrix. $S= diag(s_1,\ldots\ldots,s_{min}(N_t,N_r),0\ldots0)$ The optimal power allocation is achieved through water filling, that is

$$s_i = \left(\mu - \frac{1}{\rho d_i^2}\right)^+, \text{for } i = 1, \ldots\ldots, \min(N_t, N_r) \qquad (37)$$

where $d_1,\ldots\ldots d_{min(Nt,Nr)}$ are the diagonal elements of D, $(.)^+$ is zero if its argument is negative, and μ is selected such that

$$s_1 + \ldots\ldots\ldots + s_{min}(N_t, N_r) = N_t \qquad (38)$$

If the transmitter has only statistical channel state information, then the ergodic channel capacity will decrease as the signal covariance $\mathbf{Q}$ can only be optimized in terms of the average mutual information as

$$C_{statistical\text{-}CSI} = \max_Q E\,[\log 2 \det (I+\rho HQH^H)] \quad (39)$$

the spatial correlations of the channel has a strong impact on the ergodic channel capacity with statistical information.

If the transmitter has no channel state information it can select the signal covariance $\mathbf{Q}$ to maximize channel capacity under worst-case statistics, which means $\mathbf{Q}=1/N_t\mathbf{I}$ and accordingly

$$C_{no-CSI} = E\left[\log_2 \det\left(I + \frac{\rho}{N_t} HH^H\right)\right] \quad (40)$$

depending on the statistical properties of the channel, the ergodic capacity is no greater than $\min(N_t, N_r)$ times larger than that of a SISO system.

2.6 Noise

After stating the general linear input-output relation of the MIMO channel under more or less general assumptions, we will now go a little bit into detail on the noise term of the transmission model.

In this thesis, the noise vectors $\{n_l\}$ will be assumed to be spatially white circular Gaussian random variables with zero-mean and variance σ_N^2 per real and imaginary component. Thus,

$$n_l \sim N_c(0, 2\sigma_N^2 I), \quad (41)$$

where N_C stands for a complex-valued multivariate Gaussian probability density function. Because we will need an exact definition of the complex-valued multivariate Gaussian probability density function, we will restate it here.

- **Definition**: (Complex-valued Gaussian distribution). Let $X \in C^M$, then the probability density function $f_x(\xi)$ of x is given by

$$f_x = \frac{1}{\det(\Pi C_n)} \exp[-(\xi - \mu_x)^H c_x^{-1} (\xi - \mu_x)], \quad (42)$$

Where $C_X \underline{\underline{\Delta}} E\{(\xi - \mu_X)(\xi - \mu_{Xk})^H\}$ denotes the covariance matrix of x, $\mu_X = E\{\xi\}$ denotes the mean vector of x and $(.)^H$ stands for the complex conjugate (Hermitian transpose).Compactly, we write $X \sim \mathrm{N}_C(\mu_X, C_X)$.

There are at least two strong reasons for making the Gaussian assumption of the noise. First, Gaussian distributions tend to yield mathematical expressions that are relatively easy to deal with. Second, a Gaussian distribution of a disturbance term can often be motivated via the central limit theorem.

Throughout this thesis, we will also model the noise as temporally white. Although such an assumption is customary as well, it is clearly an approximation. In particular, N may contain interference consisting of modulated signals that are not perfectly white.

To conclude our examination of the noise term in our channel model, we summarize the statistical properties of the set of complex Gaussian vectors $\{n_l\}, l = 1, \ldots\ldots, L$:

$$
\begin{aligned}
&E\{n_l n_l^H\} = 2\sigma_N^2 I, \\
&E\{n_l n_k^H\} = 0, for l \neq k
\end{aligned}
\tag{43}
$$

The elements of the matrix H correspond to the complex-valued channel gains between each transmit and receive antenna. For the purpose of assessing and predicting the performance of a communication system, it is necessary to postulate a statistical distribution of these elements. This is also true to some degree for the design of well performing receivers, in the sense that knowledge of the statistical behavior of H could potentially be used to improve the performance of receivers.

Throughout this thesis, we will assume that the elements of the channel matrix H are zero-mean complex-valued Gaussian random variables with unit variance. This assumption is made to model the fading effects induced by local scattering in the absence of line-of-sight components. Consequently, the magnitudes of the channel gains $h_{i,j}$ have a Rayleigh distribution, or equivalently, $|h_{i,j}|^2$ are exponentially distributed. The presence of line of-sight components can be modelled by letting $h_{i,j}$ have a Gaussian distribution with a non-zero mean (This is also called Ricean fading).

After having identified the possibilities, model the complex-valued channel path gains, it re-mains to check a possible correlation between these entries. In this work, we make a commonly made assumption on H, i.e. that the elements of H are statistically independent. Although this assumption again tends to yield mathematical expressions that are easy to deal with, and allows the identification of fundamental performance limits, it is usually a rough approximation. In practice, the complex path gains $\{h_{i,j}\}$ are correlated by an amount that depends on the propagation environment as well as the polarization of the antenna elements and the spacing between them.

The channel correlation has a strong impact on the achievable system performance. Nevertheless, we will think of a rich scattering environment with enough antenna separation at the receiver and the transmitter, so that the entries of H can be assumed to be independent zero-mean complex Gaussian random variables with unit variance.

This model is often popularly referred to as the IID (identically and independently distributed) Rayleigh fading MIMO channel model. The fading itself will be modeled as block-fading, which means that the elements of H stay constant during the transmission of *L* data vectors s (or equivalently during the whole transmission duration of S) and change independently to another realization for the next block of *L* symbol periods. In practice, the duration *L* has to be shorter than the coherence time of the channel, although in reality the channel path gains will change gradually. Never the less, we will use the block fading model for its simplicity.

2.7 Fading

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The fading itself will be modeled as block-fading, which means that the elements of H stay constant during the transmission of L data vectors s (or equivalently during the whole transmission duration of S) and change independently to another realization for the next block of L symbol periods. In practice, the duration L has to be shorter than the coherence time of the channel, although in reality the channel path gains will change gradually. Nevertheless, we will use the block fading model for its simplicity.

2.8 Power Constraints, SNR Definition

The stated MIMO transmission model is now nearly ready to be investigated. What is still missing are declarations about the transmit power. Furthermore, we would like to derive expressions as a function of the signal-to-noise ratio (SNR) at the receiver, so we have to define it in terms of the already introduced quantities.

In the theoretical literature of MIMO systems, it is common to specify the power constraint on the input power in terms of an average power over the *nT* transmit antennas. This may be written as

$$\frac{1}{n_T}\sum_{i=1}^{n_T} E\left\{\left|s_{i,l}\right|^2\right\} = E_s, \quad for \;\; l = 1,......L \tag{44}$$

so that on average, we spend *Es* in power at each transmit antenna. Here *Es* denotes the mean symbol energy, as defined for example $E_s = E\left\{\left|s^{(i)}\right|^2\right\}$ (here, *i* denotes the time index of the sent symbol), where the expectation is carried out over the symbol sequence (i.e. over *i*), which in case of a white symbol sequence reduces to an averaging over the symbol alphabet.

Although this power constraint is a very common one, there is a variety of similar constraints that lead to the same basic information theoretic conclusions on MIMO transmission systems [15]. Since we will need other power constraints within this thesis, we will briefly restate them now. The power constraints can be written as

1. $E\{\left|s_{i,l}\right|^2\} = E_s$ for $i = 1; : : : n_T$; and $l = 1; : : : ;L$, where no averaging over the transmit antennas is performed.
2. $\frac{1}{L}\sum_{l=1}^{L} E\{\left|s_{i,l}\right|^2\} = E_s$,for $i = 1; : : : n_T$, what is quite similar to the power constraint, but here averaging is performed over time instead of space.
3. $\frac{1}{n_T L}\sum_{l=1}^{L}\sum_{I=1}^{nT} E\{\left|s_{i,l}\right|^2\} = E_s$ Where we average over time and space. This can equivalently be expressed as $\frac{1}{n_T.L} E\{t_r SS^H\} = E_s$

Since in most of our investigations, we want to derive expressions or curves depending on the SNR at a receive antenna, we will use a slightly adapted MIMO transmission model, in which we are using a redefinition of the power constraint.

To motivate this, we would like to express the average signal-to-noise ratio at an arbitrary receive antenna. Because we transmit a total power of $n_T E_s$ over a channel with an average path gain of magnitude one1 and a total noise power of $2\sigma_N^2$ at each receive antenna, we could state the SNR at a receive antenna as $\rho = n_T E_s /(2\sigma_N^2)$.

This would have the negative aspect, that our total transmitted power (and thus the receive SNR) is dependent on the number of transmit antennas. So, if we normalize the transmitted power by the number of transmit antennas n_T, we remove this small inconsistency.

This also motivations a slightly description of our MIMO transmission model:

$$Y = \sqrt{\frac{\rho}{n_T}} HS + N \tag{45}$$

In this context, we have following constraints on our elements of the MIMO transmission Model:

1. Average magnitude of the channel path gains $E\{t_r HH^H\} = n_R n_T$,
2. Average transmit power $E\{t_r SS^H\} = n_T L$ and
3. Average noise variance $E\{t_r NN^H\} = n_R L$

If these constraints are fulfilled, the factor $\sqrt{\frac{\rho}{n_T}}$ ensures that ρ is the average SNR at a receive antenna, independent of the number of transmit antennas (see for example also [16]).

2.9 Diversity

So far, we studied how multiple antennas can enhance channel capacity. Now we discuss how antennas can also offer diversity. Diversity provides the receiver with multiple (ideally independent) observations of the same transmitted signal. Each observation constitutes a diversity branch. With an increase in the number of independent diversity branches, the probability that all branches fade at the same time reduces. Thus, diversity techniques stabilize the wireless link leading to an improvement in link reliability or error rate.

To clarify matters, we will have a closer look at a very simple example, Assume that we transmit a data symbol *s* drawn from a scalar constellation with unit average energy. This symbol is now transmitted in a way that we can provide M identically independently Rayleigh faded versions of this symbol to the receiver. If the fading is frequency flat, the receiver sees

$$y_i = \sqrt{\frac{\rho}{M}} h_i s + n_i, i = 1,2,........,M, \tag{46}$$

Where ρ is the average SNR for each of the M diversity branches and y_i is the received signal corresponding to the i^{th} diversity branch? Furthermore, h_i denotes the channel path gain and n_i is additive ZMCSCG noise with variance 1 in the i^{th} diversity branch, whereas the noise from different branches is assumed to be statistically independent.

If we provide a receiver with multiple versions of the transmitted symbol *s*, it can be shown that the post-processing SNR can be maximized by a technique called maximum ratio combining (MRC). With perfect CSI at receiver, the M signals are combined according to $z = \sum_{i=1}^{M} h_i^* y_i$, and thus the post-processing SNR η is given by $\eta = 1/M \sum_{i=1}^{M} |h_i|^2 \rho$ With ML detection, the corresponding probability of symbol error is given by,

$$P_E \approx \overline{\mathrm{N}} Q\left(\sqrt{\frac{\eta d_{\min}^2}{2}}\right) \tag{47}$$

Where, $\overline{\mathrm{N}}$ denotes the number of nearest neighbors, $d_{\min}$ labels the minimum distance in the underlying scalar symbol constellation and $Q(.)$ is the Q-function. The error probability can be further bounded applying the Chernoff bound $Q(x) \le \exp(-x^2/2)$:

$$P_e \leq \overline{\mathrm{N}} \exp\left[-\frac{\rho d_{\min}^2}{4M} \sum_{i=1}^{M} |h_i|^2 \right] \tag{48}$$

By averaging this instant error probability with respect to the fading gains h_i ; $i = 1;:::; M$, the upper bound is obtained.

$$E\{P_e\} \leq \overline{\mathrm{N}} \prod_{i=1}^{M} \frac{1}{1+\rho d_{\min}^2 /(4M)} \tag{49}$$

In the high SNR regime, the preceding equation may be further simplified to

$$E\{P_e\} \leq \overline{\mathrm{N}} \left(\frac{\rho d_{\min}^2}{4M} \right)^{-M}, \tag{50}$$

which makes it absolutely clear that diversity effects the slope of the symbol error rate (SER) curve. The slope of the SER curve on a log-log scale, compared to the slope of a SISO system terms the mentioned diversity gain. Clearly, multiple antennas on the transmitter and/or receiver side can lead to this kind of performance gain. The answer to the question is how we can achieve the maximum diversity gain $n_T n_R$ in a MIMO system.

2.10 Functions of MIMO

MIMO can be sub-divided into three main categories, pre-coding, spatial multiplexing or SM, and diversity coding.

- **Pre-coding**: It is multi-stream beam forming, in the narrowest definition. In more general terms, it is considered to be all spatial processing that occurs at the transmitter. In (single-layer) beam forming, the same signal is emitted from each of the transmit antennas with appropriate phase (and sometimes gain) weighting such that the signal power is maximized at the receiver input. The benefits of beam forming are to increase the received signal gain, by making signals emitted from different antennas add up constructively, and to reduce the multipath fading effect. In the absence of scattering, beam forming results in a well defined directional pattern, but in typical cellular conventional beams are not a good analogy. When the receiver has multiple antennas, the transmit beam forming cannot simultaneously maximize the signal level at all of the receive antennas, and pre-coding with multiple streams is used. Note that pre-coding requires knowledge of channel state information (CSI) at the transmitter.

- **Spatial multiplexing**: It requires MIMO antenna configuration. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams into (almost) parallel channels. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher signal-to-noise ratios (SNR). The maximum number of spatial streams is limited by the lesser of the number of antennas at the transmitter or receiver. Spatial multiplexing can be used with or without transmit channel knowledge. Spatial multiplexing can also be used for simultaneous transmission to multiple receivers, known as space division multiple access. The scheduling of receivers with different spatial signatures allows good reparability.

- **Diversity Coding**: This technique is used when there is no channel knowledge at the transmitter. In diversity methods, a single stream (unlike multiple streams in spatial multiplexing) is transmitted, but the signal is coded using techniques called space-time coding. The signal is emitted from each of the transmit antennas with full or near orthogonal coding. Diversity coding exploits the independent fading in the multiple antenna links to enhance signal diversity. Because there is no channel knowledge, there is no beam forming or array gain from diversity coding.

Spatial multiplexing can also be combined with pre-coding when the channel is known at the transmitter or combined with diversity coding when decoding reliability is in trade-off.

2.11 Applications of MIMO

Spatial multiplexing techniques make the receivers very complex, and therefore they are typically combined with Orthogonal frequency division multiplexing (OFDM) or with Orthogonal Frequency Division Multiple Access (OFDMA) modulation, where the problems created by a multi-path channel are handled efficiently. The IEEE 802.16e standard incorporates

MIMO-OFDMA. The IEEE 802.11n standard, released in October 2009, recommends MIMO-OFDM.

MIMO is also planned to be used in Mobile radio telephone standards such as recent 3GPP and 3GPP2. In 3GPP, High-Speed Packet Access plus (HSPA+) and Long Term Evolution (LTE) standards take MIMO into account. Moreover, to fully support cellular environments, MIMO research consortia including IST-MASCOT propose to develop advanced MIMO techniques, e.g. multi-user MIMO (MU-MIMO).

MIMO technology can be used in non-wireless communications systems. One example is the home networking standard ITU-T G.9963, which defines a power line communications system that uses MIMO techniques to transmit multiple signals over multiple AC wires (phase, neutral and ground)

3.1 Introduction

One of the earliest communication systems that were proposed to take advantage of the promising capacity of MIMO channels is the BLAST architecture. It achieves high spectral efficiencies by spatially multiplexing coded or uncoded symbols over the MIMO fading channel. Symbols are transmitted through M antennas. Each receiver antenna receives a superposition of faded symbols. The ML decoder would select the set of symbols that are closest in Euclidean distance to the received N signals. However, it is hard to implement due to its exponential complexity. More practical decoding architectures were proposed in the literature.

3.2 V-BLAST Technique

The transmission is described as follows. A data stream is de-multiplexed into M sub-streams termed layers. For D-BLAST at each transmission time, the layers circularly shift across the M transmit antennas resulting in a diagonal structure across space and time. On the other hand, the layers are arranged horizontally across space and time for V-BLAST and the cycling operation is removed before transmission is shown in at the receiver, as mentioned previously, the received signals at each receive antenna is a superposition of M faded symbols plus additive white Gaussian noise (AWGN). Although the layers are arranged differently for the two BLAST systems across space and time, the detection process for both systems is performed vertically for each received vector. Without loss of generality, assume that the first symbol is to be detected.

<u>Main Steps for V-BLAST detection :</u>

1. Ordering: choosing the best channel.
2. Nulling: using ZF, MMSE, ML.
3. Slicing: making a symbol decision
4. Cancelling: subtracting the detected symbol
5. Iteration: going to the first step to detect the next symbol

The detection process consists of two main operations:

I) Interference suppression (nulling):

The suppression operation nulls out interference by projecting the received vector onto the null subspace (perpendicular subspace) of the subspace spanned by the interfering signals. After that, normal detection of the first symbol is performed.

II) Interference cancellation (subtraction):

The contribution of the detected symbol is subtracted from the received vector.

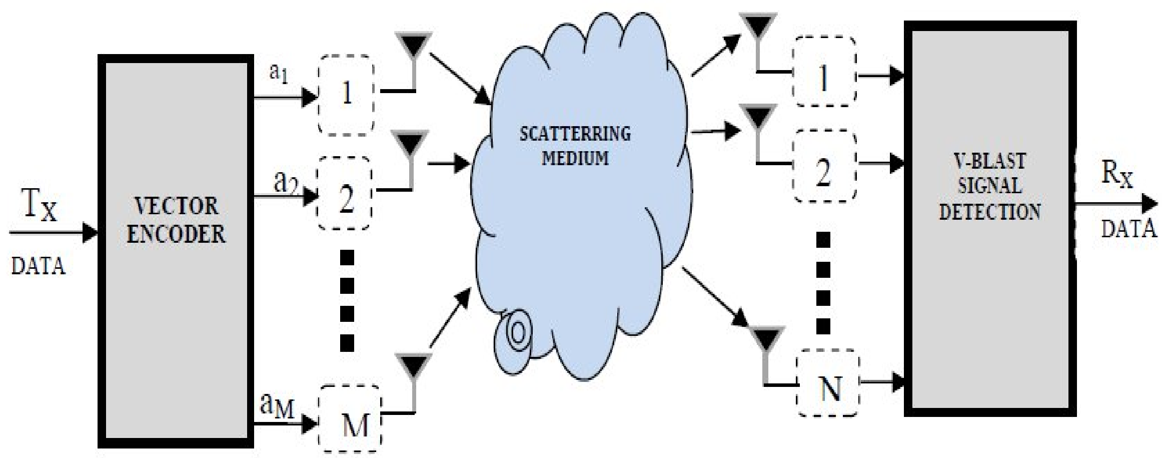


Figure: - Block diagram of V-BLAST Architecture.

BLAST detection algorithm combines linear (interference suppression) and nonlinear (serial cancellation) algorithms. This is similar to the de-correlating decision feedback multiuser detection algorithm. A drawback of BLAST algorithms is the propagation of decision errors. Also, the interference nulling operation requires that the number of receive antennas be greater than or equal to the number of transmit antennas. Furthermore, due to the interference suppression, early detected symbols benefit from lower receives diversity than later ones. Thus, the algorithm results in unequal diversity advantage for each symbol.

3.3 Difference between V-BLAST and D-BLAST

The layers of the V-BLAST can be coded or un-coded. The D-BLAST is intended to be used only with coded layers. This is the reason behind cycling which provides more spatial diversity for each layer particularly over slowly fading channels. Further, due to the diagonal structure of D-BLAST, each layer benefits from the same diversity advantage while V-BLAST layers have unequal diversity advantages. However, D-BLAST requires advanced inter-stream coding techniques to optimize the performance of the code across space and time. Finally, some space-time is wasted at the start and the end of the burst for D-BLAST.

V-BLAST takes a single data stream and de-multiplexes it into M sub-streams with M is the number of transmitter antennas. Each sub-stream is encoded into symbols and fed to a separate transmitter. The modulation method in these systems usually is M Quadrature Amplitude Modulation (QAM). QAM combines phase modulation with amplitude modulation, making it an efficient method for transmitting data over a limited bandwidth channel. BLAST's receivers operate co-channel, each receiving the signals emanating from all M of the transmitting antennas. For the sake of simplicity, it is also assumed that the channel-time variation is negligible over the L symbol periods in a burst.

3.4 V-BLAST Technique for Different Linear Detectors in a slow Fading Channel

3.4.1 Maximum Likelihood (ML):

The ML receiver performs optimum vector decoding and is optimal in the sense of minimizing the error probability. ML receiver is a method that compares the received signals with all possible transmitted signal vector which is modified by channel matrix H and estimates transmit symbol vector x according to the Maximum Likelihood principle , which is shown as:

$$\hat{x} = \arg_{x_k \in \{x_1 x_2 \ldots\ldots x_N\}} \min \|r - Hx_k\|^2 \qquad (1)$$

Where the minimization is performed over all possible transmit estimated vector symbols x. Although ML detection offers optimal error performance, it suffers from complexity issues. It has exponential complexity in the sense that the receiver has to consider |A|M possible symbols for an M transmitter antenna system with A is the modulation constellation.

3.4.2 V-BLAST Zero Forcing (ZF) characteristic:

We can reduce the decoding complexity of the ML receiver significantly by employing linear receiver front-ends to separate the transmitted data streams, and then independently decode each of the streams. Simple linear receiver with low computational complexity and suffers from noise enhancement. It works best with high SNR. The solution of the ZF is given by:

$$\hat{x} = (H^{*}H)^{-1}Hx = H^{+x} \quad (2)$$

Where, ( ) + represents the pseudo-inverse. The ZF receiver converts the joint decoding problem into M single stream decoding problems thereby significantly reducing receiver complexity. This complexity reduction comes, however, at the expense of noise enhancement which in general results in a significant performance degradation (compared to the ML decoder). The diversity order achieved by each of the individual data streams equals
N - M + 1.

3.4.3 .V-BLAST with Minimum Mean Square Error (MMSE):

The MMSE receiver suppresses both the interference and noise components, whereas the ZF receiver removes only the interference components. This implies that the mean square error between the transmitted symbols and the estimate of the receiver is minimized. Hence, MMSE is superior to ZF in the presence of noise. Some of the important characteristics of MMSE detector are simple linear receiver, superior performance to ZF and at Low SNR, MMSE becomes matched filter. Also at high SNR, MMSE becomes Zero-Forcing. MMSE receiver gives a solution of:

$$\hat{x} = D.x = \left(\frac{1}{SNR} I_{N_R} + H^{H}H\right)^{-1} . H^{H}x \quad (3)$$

At low SNR, MMSE becomes ZF:

$$\left(\frac{1}{SNR} I_{M_R} + H^H H\right)^{-1} H^H \approx \frac{1}{SNR} H^H \quad (4)$$

At high SNR, MMSE becomes ZF:

$$\hat{x} = D.x = \left(\frac{1}{SNR} I_{M_T} + H^H H\right)^{-1} H^H \approx (H^H H)^{-1} H^H \quad (5)$$

i.e., the MMSE receiver approaches the ZF receiver and therefore realizes (N-M + 1) th order diversity for each data stream.

3.4.4 V-BLAST with Maximal Ratio Combining (MRC):

MRC combines the information from all the received branches in order to maximize the ratio of signal to noise power, which gives it its name. MRC works by weighting each branch with a complex factor and then adding up the branches, MRC is intuitively appealing; the total SNR is achieved by simply adding up the branch SNRs when the appropriate weighting coefficients are used. BER for MRC in Rayleigh fading channel (1x2) with BPSK modulation

$$P_e \, MRC = P_{MRC}^2 [1 + 2\,(1 - P_{MRC})]$$

$$P_{MRC} = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b/N_O}\right)^{-1/2} \quad (6)$$

3.4.5 STBC (Space-time block codes)

STBC is a class of linear coding for MIMO systems that aims to maximize the system diversity gain rather than the data rate. A very popular STBC for a two transmit antennas setup was developed by Alamouti, which is illustrated in Fig.7. It is designed for 2x2 MIMO systems and its simplicity and high frequency have led to its wide adoption in MIMO systems. In this scheme orthogonal signals are transmitted from each antenna, which greatly simplifies receiver design. This particular scheme is restricted to using M = 2 antennas at the transmitter but can any number of receive antennas N. Two QAM symbols S1 and S2 for transmission by the Alamouti scheme are encoded in both the space and time domain at the two transmitter antennas over the consecutive symbol periods as shown in equation (20). The information bits are first modulated using a modulation scheme (for example QPSK). The encoder then takes a block of two modulated symbols s1 and s2 in each encoding operation and gives to the transmit antennas according to the code matrix.

$$S = [s_1 \quad s_2] = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \tag{7}$$

The code matrix has the following property

$$\mathrm{S.S^H} = \begin{bmatrix} |\mathrm{x}_1|^2 + |\mathrm{x}_0|^2 & 0 \\ 0 & |\mathrm{x}_1|^2 + |\mathrm{x}_0|^2 \end{bmatrix} = (|\mathrm{x}_1|^2 + |\mathrm{x}_0|^2)\mathrm{I}_2 \tag{8}$$

Where I2 is the 2x2 identity matrix.

In the above matrix the first column represents the first transmission periods and the second column, the second transmission period. The first row corresponds to the symbols transmitted from the first antenna and second row corresponds to the symbols transmitted from the second antenna. It means that during the symbol period, the first antenna transmits s1 and second antenna s2. During the second symbol period, the first antenna transmits –s2 * and the second antenna transmits s1 * being the complex conjugate of s1. This implies that we are transmitting both in space (across two antennas) and time (two transmission intervals).

$$Hence, \quad S_1 = [s_1 \quad -s_2^*] \quad and \quad S_2 = [s_2 \quad s_1^*]$$

Moreover a close look reveals that sequences are orthogonal over a frame interval, since the inner product of the sequences **S1** and **S2** is zero, i.e.

$$\mathrm{S_1.S_2} = \mathrm{s_1 s_2^* - s_2^* s_1} = 0 \tag{9}$$

$$P_e STBC = P_{STBC}^2 [1 + 2(1 - p_{STBC})]$$

$$p_{STBC} = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{2}{E_b/N_o}\right)^{-1/2} \qquad (10)$$

In a fast fading channel, the BER is of primary interest since the channel varies every symbol time; while in a slow fading situation, the FER (Frequency error rate) is more important because channel stays the same for a frame.

4.1 Introduction

MULTIPLE-INPUT multiple-output (MIMO) communication offers key advantages over single-input single-output (SISO) communication, such as diversity gain and spatial multiplexing gain. Diversity gain improves link reliability, while spatial multiplexing gain increases the transmission rate. Our goal with this paper is to investigate transmit optimization for MIMO spatial multiplexing, which is receiver-dependent. Signal reception for MIMO spatial multiplexing can employ criteria such as linear zero-forcing (ZF), minimum mean-squared error (MMSE), maximum likelihood (ML) and successive interference cancellation (SIC), or ordered SIC (OSIC), as for example, in the case of the Vertical Bell Laboratories layered space–time (V-BLAST) architecture. In order to achieve high MIMO diversity and/or spatial multiplexing gains, appropriate transceiver designs are necessary. Efforts to optimize MIMO transceivers structures have involved joint transmit–receive optimization and linear pre-coding for specific receivers.

Joint pre-coding/decoding optimization under MMSE criterion is investigated. A unified framework for joint transmit–receive design using convex optimization is proposed. Minimum bit-error rate (MBER) pre-coding for ZF equalization of block transmission and block transceivers with MMSE decision-feedback equalization (DFE) is readily applicable to MIMO systems. Pre-coding for multicarrier MIMO using an ML receiver and pair wise error probability as criterion is proposed in. These designs generally require high-complexity processing at the transmitter and the receiver, as well as high feedback overhead. Pre-coded MIMO transmission with reduced feedback has been recently proposed based on quantized channel state information (CSI) feedback and limited feedback signal design.

However, existing pre-coding schemes with reduced feedback generally also require high processing complexity. Consider simultaneous reduction of transmitter complexity and feedback overhead by constraining pre-coding to transmit power allocation, i.e., I optimize only the transmitted power of signal streams, but apply a more suitable criterion. Power allocation for multicarrier MIMO systems was considered. Where MIMO was operated in a diversity mode and the transmit power was allocated across the frequency dimension (subcarriers). As opposed to MMSE pre-coding/decoding, I consider MBER as the optimization criterion. The block transceiver design for MMSE-DFE provides a closed-form solution to approximate MBER (AMBER) pre-coding for SIC receivers. Compared with MMSE-DFE and ZF-MBER pre-coding, we provide a unified solution to MBER power allocation for ZF, SIC, and OSIC receivers.

General power allocation (with diagonal pre-coder) by minimizing error rate does *not* have a closed-form solution, and has high computational complexity. An approximate solution can be found instead, which was originally given in to allocate power across channel eigen modes. In this paper, it is applied to transmit power allocation for ZF, SIC, and OSIC receivers. Recently, it has come to our attention that a similar AMBER power allocation for V-BLAST was proposed independently. Transmitter-side power allocation ideally requires CSI or allocated power to be available at the transmitter. In some cases, CSI can be made available at the transmitter, e.g. in time-division duplex (TDD) systems, due to the reciprocity of the uplink and downlink channels. In this case, existing limited feedback schemes do not possess any advantages, since feedback overhead is not a concern.

However, power allocation is still attractive, due to the significant reduction in transmitter complexity. On the other hand, in channels that lack reciprocity in uplink and downlink, e.g. frequency-division duplex (FDD), complete CSI is not available at the transmitter, and CSI or power information has to be fed back. Regardless of availability, CSI or power feedback is imperfect, in practice, due to channel estimation, quantization, feedback delay, and/or errors introduced by feedback channel. This motivates performance analysis of power allocation under uncertain feedback.

While a general analysis is difficult, we analyze the special cases of noisy CSI and power feedback. Based on this analysis, we propose an AMBER power-allocation algorithm that takes statistical knowledge of noisy feedback into account. Furthermore, as a by-product, a modified algorithm for perfect CSI which takes into account error-propagation effects in SIC and OSIC receivers is devised.

Multi-input-multi-output (MIMO) digital communication systems are receiving an increasing attention due to their potential of increasing the overall system throughput. In such systems, MIMO decision feedback equalization (DFE) is often used to mitigate inter-symbol-interference (ISI), which results from channel multi-path propagation. In many of such systems, the transmitted symbol consists of a known training sequence followed by unknown data. An efficient equalization technique in this scenario is to rest estimate the channel impulse responses between each transmitter and each receiver using the training sequence, and then use this estimate to compute the optimal decision feedback equalizer tap coefficients corresponding to the estimated channel. The computed tap coefficients are then uploaded to the equalizer taps. For time-varying channels, the detection ordering and nulling vectors need to be updated for each time, plus the channel parameters should be tracked.

These update and tracking operations in the time domain require excessive computations. To overcome this drawback, a simplified policy for updating and tracking is proposed, where the V-BLAST (Vertical Bell Laboratories Layered Space-Time coding) detection is updated block-wise, and the channel tracking is interpolation-based, thereby creating a trade-off between complexity and performance. As an alternative approach to detecting MIMO systems in time-varying channels, the adaptive techniques may be employed. By successively detecting the transmitted symbols at each time, the adaptive decor- relating detector can suppress the co-channel interference caused by spatial multiplexing, but it requires channel estimation to determine the order of detection. The adaptive decision feedback equalizer is useful for reducing inter symbol interference in MIMO systems over frequency-selective channels. However, they are not suitable for reducing the co-channel interference; in the DFE, the transmitted symbols at each time are simultaneously detected without considering the order of detection.

The adaptive method is a blind technique, whereas the receivers are data aided. A data-aided ordered based on the recursive least squares ordered decision feed-back equalizer (RLS-O-DFE) architecture is proposed. For each time, the tap weight vectors are updated using an RLS based-time and order-update algorithm and detection ordering determined according to a least squares error (LSE) criterion. But the proposed algorithm doesn't cancel interference from detected symbols successively and the detection performance deteriorated. A variable step size blind equalization algorithm based on log-normal error function is proposed.

The algorithm has faster convergent rate and smaller the mean square error (MSE) than the constant modulus algorithm (CMA). But most cost is on the computing of error function and the complexity is increased. In this letter, an improved log-normal error function based on CMA algorithm ordered successive interference cancellation decision feedback equalization (ILNCMA- OSIC-DFE) is proposed. The algorithm improved the variable step size blind equalization algorithm based on log-normal error function restyle and added decision conditions. So the convergent rate is accelerated and the computational complexity is reduced. Then it cancels the detected symbols as interference from received symbols and overcomes the drawback of the RLS-O-DFE algorithm, such as instability and high bit error ratio (BER), while the computational complexities are increased a little. Performance analysis and simulation results show the effectiveness of the proposed algorithm.

4.2 System model

Let us consider a MIMO system with N_t transmit antennas and N_r receive antennas. The flat-fading MIMO channel is expressed by the $N_r \times N_t$ matrix H with the element h_{ji} representing the channel gain from i^{th} transmit antenna to j^{th} receive antenna. The $N_r \times 1$ received signal vector $y = [y_1, ..., x_N]^T$ is written as

$$y = \sqrt{\frac{E_s}{N_t}}\, HPX + n \qquad (1)$$

where

$X = \left[x_1, ...x_{N_t}\right]^T$ Denotes the N_r X$_1$ transmitted signal vector, and $n = \left[n_1, ...n_{N_r}\right]^T$ is the N_r dimensional noise vector with elements following complex zero σ_n^2 mean Gaussian distribution with variance of σ_n^2. E_s is the total transmitted signal energy on N_t transmit antennas and $P = \sqrt{N_t}\ [P_1, ..., P_{N_t}]$ denotes the diagonal PA pre-coding matrix.

To express the signal model for the MMSE-QR detector, an $(N_r + N_t)$ X N_t augmented channel matrix $\overline{H}$ an $(N_r + N_t)$ X$_1$ extended receive vector $\overline{y}$ and an N_t X$_1$ zero matrix $0_{Nt,1}$ can be written as

$$\overline{H} = \begin{bmatrix} H \\ \sigma_n I_{N_t} \end{bmatrix} \rightarrow ordering\ \overline{Q}\overline{R}\ and\ \ \overline{y} = \begin{bmatrix} y \\ 0_{N_t,1} \end{bmatrix} \qquad (2)$$

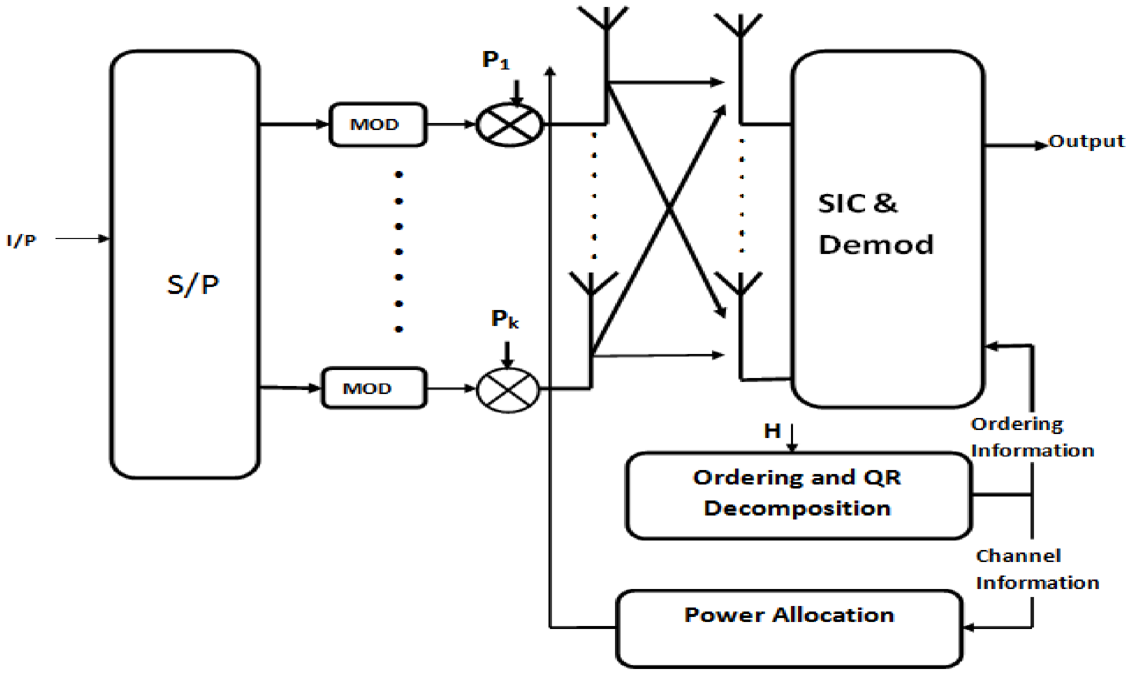


Figure. MIMO transmission model with QR-OSIC detector.

The upper triangular matrix, which is differently defined by the detection-order, determines the SINR and the post detection SINR ρ_k of the k_{th} data stream is given as

$$\rho_k = \frac{E_s}{\sigma_n^2} p_k^2 \overline{R}_{k,k}^2 - 1, \qquad k = 1, \ldots\ldots\ldots. N_t \tag{3}$$

The QR-decomposition based OSIC detection for BER-minimized PA transmission can be performed using the architecture shown in Fig.4.1. Based on the feedback information of the diagonal elements $\overline{R}_{k,k}$, transmission power P_k is assigned to each data stream. The independently encoded symbols are processed through a diagonal PA matrix and then transmitted from N_t data streams. The QR-OSIC receiver detects the transmit symbols sequentially in accordance with the designated detection-order.

4.3 MMSE Detector

The MMSE detector minimizes the mean squared error (MSE) between the actually transmitted symbols and the output of the linear detector and leads to the filter matrix.

$$G_{MMSE} = (H^H H + \sigma_n^2 I_{N_t})^{-1} H^H \quad (4)$$

The resulting filter output is given by

$$\tilde{S}_{MMSE} = G_{MMSE} y = \left(H^H H - \sigma_n^2 I_{N_t}\right)^{-1} H^H y \quad (5)$$

The estimation errors of the different layers correspond to the main diagonal elements of the error covariance matrix.

$$\phi_{MMSE} = E\left\{\left(\tilde{s}_{MMSE} - s\right)\left(\tilde{s}_{MMSE} - s\right)^H\right\}$$

$$= \sigma_n^2 (H^H H + \sigma_n^2 I_{N_t})^{-1} \quad (6)$$

With the definition of (N_r +N_t) XN_t augmented channel matrix, an (N_r + N_t) X_1 extended receive vector $\overline{y}$ and an N_t X_1 zero matrix $0_{N_t,1}$ can be written as

$$\overline{H} = \begin{bmatrix} H \\ \sigma_n I_{N_t} \end{bmatrix} \rightarrow ordering\ \overline{QR}\ and\ \overline{y} = \begin{bmatrix} y \\ 0_{Nt,1} \end{bmatrix}$$

the output of the MMSE filter now can be rewritten as

$$\tilde{S}_{MMSE} = (H^H H)^{-1} \overline{H}^H \overline{y} = \overline{H}^+ \overline{y}$$

Furthermore, the error covariance matrix becomes

$$\phi_{MMSE} = \sigma_n^2 (\overline{H}^H \overline{H})^{-1} = \sigma_n^2 \overline{H}^+ \overline{H}^{+H} \quad (7)$$

Comparing last two equations to the corresponding expression for linear zero-forcing detector in previous topic. The only difference is that the channel matrix H has been replaced by $\overline{H}$. This observation is extremely important for incorporating the MMSE criterion into the SQRD based detection algorithm.

4.4 Proposed detection algorithms (QR OSIC Algorithm)

4.4.1 MMSE QR Detection

In order to extend the QR based detection with respect to the MMSE criterion, we can apply the similarity of ZF and MMSE detection noted in previous Section. We introduce the QR decomposition of the extended channel matrix

$$\bar{H} = \begin{bmatrix} H \\ \sigma_n I_{N_t} \end{bmatrix} = \bar{Q}\bar{R} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \bar{R} = \begin{bmatrix} Q_1 \bar{R} \\ Q_2 \bar{R} \end{bmatrix}$$

where the ($N_r + N_t$) XN_t matrix $\bar{Q}$ with orthogonal columns was partitioned into the an N_r XN_t matrix Q_1 and the N_t XN_t matrix . Q_2 Obviously,

$$\bar{Q}^H \bar{H} = Q_1^H + \sigma_n^2 Q_2^H = \bar{R} \tag{8}$$

holds and from the relation $\sigma_n I_{N_t} = Q_2 \bar{R}$ it follows that

$$\bar{R}^{-1} = \frac{1}{\sigma_n} Q_2$$

i.e. the inverse $\bar{R}^{-1}$ is a by-product of the QR decomposition and Q_2 is an upper triangular matrix. This relation will be useful for the post-sorting algorithm using above equations. The filtered receive vector becomes

$$\tilde{S} = \bar{Q}^H \bar{y} = Q_1^H y = \bar{R}S - \sigma_n Q_2^H S + Q_1^H \tag{9}$$

The second term on the right hand side of the above equation including the lower triangular matrix. Q_2^H constitutes the remaining interference that cannot be removed by the successive interference cancellation procedure. This point outs the trade-off between noise amplification and interference suppression.

The optimum detection sequence now maximizes the signal-to-interference-and-noise ratio (SINR) for each layer, leading to minimal estimation error for the corresponding detection step. The estimation errors of the different layers in the first detection step correspond to the diagonal elements of the error covariance matrix.

$$\phi = \sigma_n^2 (\bar{H}^H \bar{H})^{-1} = \sigma_n^2 \bar{R}^{-1} \bar{R}^{-H} \tag{10}$$

The estimation error after perfect interference cancellation is given by $\sigma_n^2 / |\bar{r}_{k,k}|^2$. Thus, it is again optimal to choose the permutation that maximizes $|\bar{r}_{k,k}|$ in each detection step. The algorithm proposed in the next section determines an optimized detection sequence within a single sorted

QR decomposition and thereby significantly reduces the computational complexity in comparison to standard MMSE-BLAST algorithms.

4.4.2 Description of the BER Performance

The PA scheme for the average BER minimization under the assumption of the QR-decomposition of the channel matrix and no error propagation in successive cancellation of the data streams has been proposed in the PA scheme for BPSK modulation can be expressed as

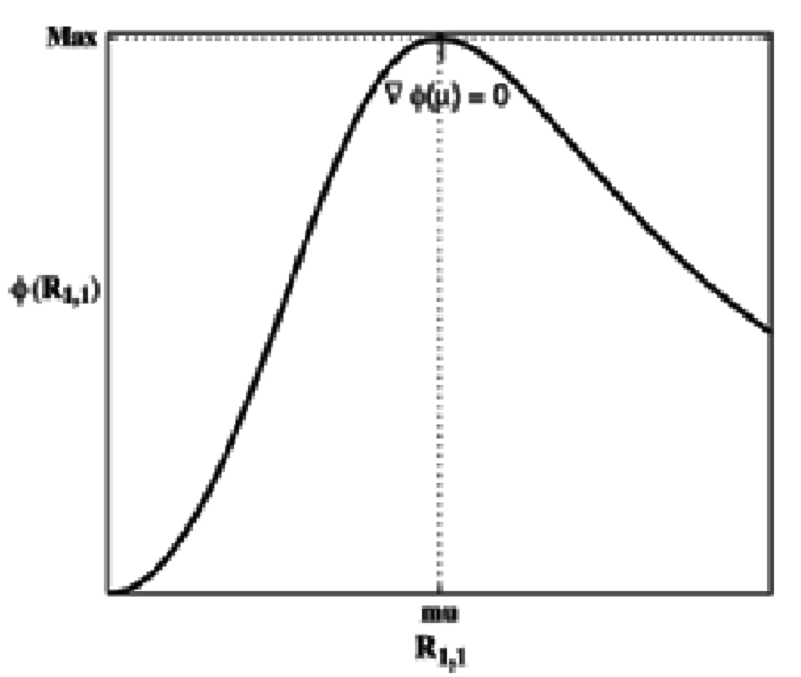
$$\text{Minimize} \quad \frac{1}{N_t}\sum_{k=1}^{N_t} Q(\sqrt{2}\,\gamma_s P_k \bar{R}_{k,k}) \approx \frac{1}{N_t}\sum_{k=1}^{N_t} Q(\sqrt{2\rho_k}) \tag{11}$$

$$\text{s.t} \quad \sum_{k=1}^{N_t} P_k^{\,2} = 1, \qquad 0 < P_k < 1,$$

$$\bar{R}_{k,k} \geq 0 \ \ k \in \{1, 2 \ldots Nt\}$$

$$\text{Where} \quad Q(x) = \sqrt{\tfrac{1}{2\pi}} \int_x^{\infty} e^{-(\frac{t^2}{2})} dt \quad \text{and} \quad \gamma_s = \sqrt{\frac{E_s}{\sigma_n^2}}$$

We assume $\bar{R}_{k,k} \geq 0$ because it is defined as the norm of the k_{th} column of the augmented channel matrix. For general constellations, the average BER of the PA can be approximated with a constellation-specific constant.



Graph of $\phi(\overline{R}_{1,1})$

It can be observed in above equations, the average BER as well as the post-detection SINR is determined by the allocated power and the channel gain. Because of the convexity property of the function the resulting BER is minimized by

(i) The detection ordering of the QR-OSIC receiver such that all diagonal elements of the matrix are equal to their geometrical average $\mu = \sqrt[N_t]{\det(\overline{R})} = \sqrt[N_t]{\prod_{k=1}^{N_t} \overline{R}_{k,k}}$,and alternatively

(ii) The PA scheme at the transmitter which makes the product of two variables P_k and $\overline{R}_{k,k}$ identical for all data streams.

As the real MIMO channel it is characterized by several spatial temporal properties, the condition isType equation here.

(i) It is not practical in spite of its optimality. On the other hand in

(ii) Different detection order leads to different $\overline{R}_{k,k}$, and hence P_k should be also differently assigned. This indicates that an appropriate detection ordering strategy incorporates with the PA scheme can achieve the improved BER performance.

Since the Q-function has convex and decreasing properties, the average BER minimization problem can be simplified to maximize the product of two variables P_k and $\bar{R}_{k,k}$

$$\text{Maximize} \quad P_1\bar{R}_{1,1}$$

$$\text{s. t.} \quad P_1\bar{R}_{1,1} = P_2\bar{R}_{2,2} = \ldots\ldots\ldots = P_{N_t}\bar{R}_{N_t,N_t} \tag{12}$$

$$\sum_{k=1}^{N_t} P_k^{\,2}, \qquad 0 < P_k < 1$$

Using the following properties of

$$P_1\bar{R}_{1,1} = P_2\bar{R}_{2,2} = \sqrt{1-P_1^2}\,(\det(\bar{R}) / \bar{R}_{1,1})$$

$$P_1^2 = (\det{}^2(\bar{R}) / (\bar{R}_{1,1}^4 + \det(\bar{R}))\ \text{And}\ \max P_1\bar{R}_{1,1} \cong \max P_1^{\,2}\bar{R}_{1,1}^2$$

The problem for two transmit antennas can be written as

$$\text{Maximize} \ \frac{\bar{R}_{1,1}^2 \det{}^2(\bar{R})}{\bar{R}_{1,1}^4 + \det{}^2(\bar{R})} = \phi(\bar{R}_{1,1}) \tag{13}$$

$$\text{s. t} \quad P_1\bar{R}_{1,1} = P_2\bar{R}_{2,2}, \ P_1^2 + P_2^2 = 1$$

To find the direction of increasing, a plot of the objective function $\phi(\bar{R}_{1,1})$ versus $\bar{R}_{1,1}$ is given. It is observed that $\phi(\bar{R}_{1,1})$ increases as $\bar{R}_{1,1}$ tends to μ. When differential calculus is applied to $\phi(\bar{R}_{1,1})$, we also obtain

$$2\bar{R}_{1,1} = \left(\bar{R}_{1,1}^4 + \det{}^2(\bar{R})\right) = 0 \tag{14}$$

$$\bar{R}_{1,1} = \sqrt{\det(\bar{R})} = \mu$$

Note that, $\rho_k \alpha P_k^{\,2}\bar{R}_{k,k}^2$ and therefore the above considerations imply that ρ_k is gradually increasing as $\bar{R}_{k,k}$ approaches to μ. In other words, the ordering strategy that makes $\bar{R}_{k,k}$ converge to μ achieves higher post-detection SINR, which also further improves the overall BER performance. It can be extended to the system with transmit antennas. To satisfy the derived strategy, we establish the fixed ordering algorithm, the architecture of which arranges the channel gains to minimize $\left|\bar{R}_{k,k} - \mu\right|$ for all k.

$$k_i = \arg\min \left| \overline{R}_{w,w} - \mu \right|$$

$$\text{s .t} \quad w\,\varepsilon\,\{k_1, \ldots\ldots, k_{i-1}\}$$

$$\mu = \sqrt[N_t]{\det(\overline{R})}$$

Where the list of N_t elements {1, 2… are rearranged with the parenthesized subscript implying the reverse order in which the elements are to be detected and the ordered set $k = \{k_1, \ldots\ldots, k_{N_t}\}$ is a permuted sequence

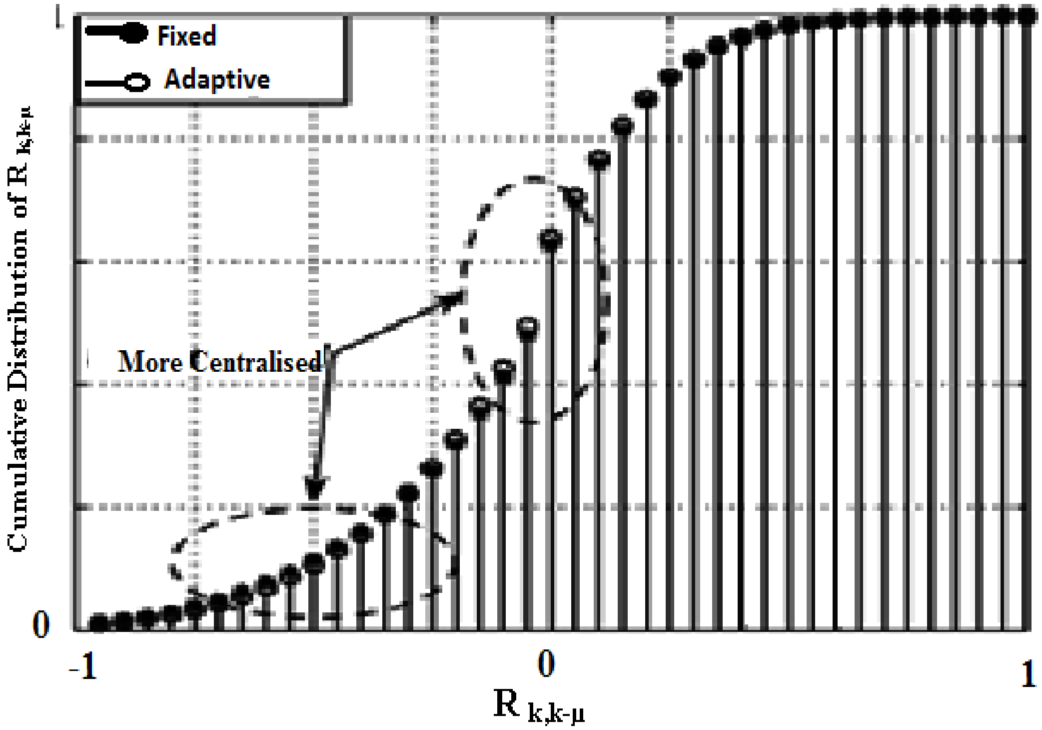


Figure. Comparison of cumulative distribution of $\overline{R}_{k,k} - \mu$

Using the correlation among ordering results, the modified ordering algorithm employing adaptive criteria can be developed for robust convergence. For instance, in N_t=3 system, selecting an element 1 as k_1, in general, result in a different $\overline{R}_{1,1}$ than if element 2 or 3 was selected. It also affects the remaining sets which decide k_2, k_3.

Moreover, channel gains are constrained via $\mu = \sqrt[N_t]{\prod_{k=1}^{N_t} \overline{R}_{k_1,k_1}}$. Motivated by the above properties, we propose the adaptive ordering design which continually renews the thresholds by controlling the weights with reference to previously determined channel gains. Substituting the variable thresholds into the fixed method, we get

$$k_i = \arg\min \left| \overline{R}_{w,w} - \mu \right| \tag{15}$$

$$\text{s . t} \quad \mu_1 = \mu; \qquad \mu_{i+1} = \frac{N_{t-i+1}}{N_t - 1} \sqrt{\mu_i / \overline{R}_{i,i}^{N_{t-i+1}}}$$

Where μ_i denotes the threshold for k_i. The adaptive ordering algorithm can be considered as the reduced sized fixed ordering process extracting the already decided gains, thus it plays a large part in balancing among ordering results. If the sign of $\overline{R}_{k,k} - \mu$ is distributed to one side serially, the adaptive ordering algorithm enables the following channel gain to be on the opposite side by adjusting μ_{i+1}. This allows more channel gains to converge to μ. To identify it, the cumulative distributions of $\overline{R}_{k,k} - \mu$ with four transmit/receive antennas are drawn in Fig.4.3. The small gap between two similar schemes is noticeable because the adaptive algorithm is equivalent to the fixed one for slight differences in $\left| \overline{R}_{k,k} - \mu \right|$.

The complexity comparison between the B-OSIC and the QR-OSIC receiver is, in a B-OSIC detector with $N_t = N_r$, the total numbers of multiplications and additions are $(43/12)N_t^4 + (20/3)N_t^3 + \vartheta(N_t^2)$, respectively. On the other hand, the OSIC receiver using QR-factorization requires $(2/3)N_t^3 + 7N_t^2 N_r + 2N_r^2 N_t + \vartheta(N_t^2)$ multiplications and additions. Because of the multiple calculations of pseudo-inverse for nulling and ordering, the B-OSIC requires higher computational cost. When $N_t = N_r$ the number of multiplications and additions are given with the complex floating point operations (flops).

$$(43/6)N_t^4 + 14N_t^3 + \vartheta(N_t^2) \qquad \text{For B-OSIC;}$$

$$(29/3)N_t^3 + \vartheta(N_t^2) \qquad \text{For QR-OSIC;}$$

4.5 Q-function

In statistics, the Q-function is the tail probability of the distribution. In other words, $Q(x)$ is the probability that a standard normal random variable will obtain a value larger than x. Other definitions of the Q-function, all of which are simple transformations of the normal cumulative distribution function, are also used occasionally.

- Definition and basic properties

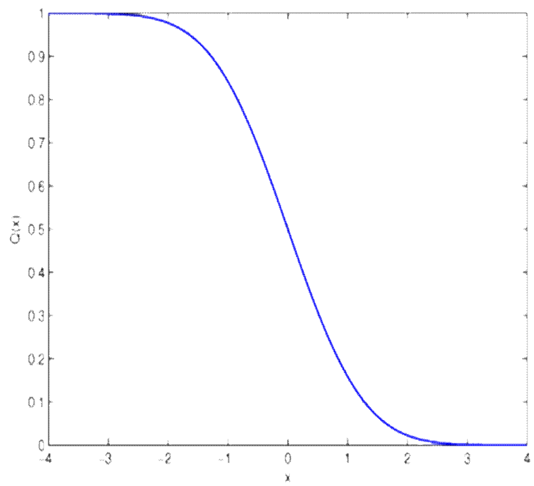


Figure. A plot of the Q function

Formally, the Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) du \qquad (16)$$

Thus,

$$Q(x) = 1 - Q(-x) = 1 - \Phi(x),$$

where $\Phi(x)$ is the cumulative distribution function of the normal Gaussian distribution.

The Q-function can be expressed in terms of the error function as

$$Q(x)=\frac{1}{2}-\frac{1}{2}erf\left(\frac{x}{\sqrt{2}}\right)=\frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right) \tag{17}$$

- **Bounds**

The Q-function is not an elementary function. However, the bounds

$$\frac{x}{1+x^2}\cdot\frac{1}{\sqrt{2\pi}}e^{-x^2/2}<Q(x)<\frac{1}{x}\cdot\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \qquad x>0,$$

Using the substitution $v = u^2 / 2$ and defining

$$\begin{aligned} a_1 &= \langle e_1, a_1\rangle e_1 \\ a_2 &= \langle e_1, a_2\rangle e_1 + \langle e_2, a_2\rangle e_2 \\ a_3 &= \langle e_1, a_3\rangle e_1 + \langle e_2, a_3\rangle e_2 + \langle e_3, a_3\rangle e_3 \\ a_k &= \sum_{j=1}^{k} \langle e_j, a_k\rangle e_j \end{aligned}$$

the upper bound is derived as follows:

$$\begin{aligned} Q(x) &= \int_x^\infty \varphi(u)\,du \\ &< \int_x^\infty \frac{u}{x}\varphi(u)\,du = \int_{x^2/2}^\infty \frac{e^{-v}}{x\sqrt{2\pi}}dv = -\frac{e^{-v}}{x\sqrt{2\pi}}\Big|_{x^2/2}^\infty = \frac{\varphi(x)}{x}. \end{aligned}$$

Similarly, using $\varphi'(u)=-u\varphi(u)$ and the quotient rule,

$$\begin{aligned} 1+\frac{1}{1+x^2}Q(x) &= \int_x^\infty \left(1+\frac{1}{x^2}\right)\varphi(u)\,du \\ &> \int_x^\infty \left(1+\frac{1}{u^2}\right)\varphi(u)\,du = -\frac{\varphi(u)}{u}\bigg|_x^\infty = \frac{\varphi(x)}{x}. \end{aligned} \tag{18}$$

Solving for $Q(x)$ provides the lower bound.

- **Chernoff bound of Q-function is**

$$Q(x) \le \frac{1}{2} e^{-\frac{x^2}{2}}, \qquad x > 0$$

4.6 QR-Decomposition

In linear algebra, a QR decomposition (also called a QR factorization) of a matrix is a decomposition of a matrix A into a product A=QR orthogonal matrix Q and an upper triangular matrix R. QR decomposition is often used to solve the linear least squares problem, and is the basis for a particular eigen value algorithm, the QR algorithm.

If A has linearly independent columns (say n columns), then the first n columns of Q form an ortho normal basis for the column space of A. More specifically, the first k columns of Q form an ortho normal basis for the span of the first k columns of A for any $1 \le k \le n$. The fact that any column k of A only depends on the first k columns of Q is responsible for the triangular form of R.

❖ **Square matrix**

Any real square matrix *A* may be decomposed as

$$A = QR,$$

where Q is an orthogonal matrix (its columns are orthogonal unit vectors meaning $Q^T Q = I$) and R is an upper triangular matrix (also called right triangular matrix). This generalizes to a complex square matrix A and an unitary matrix Q. If A is invertible, then the factorization is unique if we require the diagonal elements of R are positive.

❖ **Rectangular matrix**

More generally, we can factor a complex m×n matrix A, with $m \ge n$, as the product of an m×m unitary matrix Q and m×n upper triangular matrix R. As the bottom (m−n) rows of an m×n upper triangular matrix consist entirely of zeroes, it is often useful to partition R or both R and Q.

$$A = QR = Q\begin{bmatrix} R_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}\begin{bmatrix} R_1 \\ 0 \end{bmatrix} = Q_1 R_1$$

where R_1 is n×n upper triangular matrix, Q_1 is m×n, Q_2 is m×(m−n), and Q_1 and Q_2 both have orthogonal columns.

❖ **Computing the QR decomposition**

There are several methods for actually computing the QR decomposition, such as by means of the Gram–Schmidt process, Householder transformations, or Givens rotations. Each has a number of advantages and disadvantages.

❖ **Using the Gram–Schmidt process**

Consider the Gram–Schmidt process applied to the columns of the full column rank matrix $A = [a_1, \ldots\ldots, a_n]$, with inner product $\langle v, w \rangle = v^T w$ (or) $\langle v, w \rangle = v^* w$ for the complex case.

Define the projection:

$$\mathrm{proj}_e a = \frac{\langle e, a \rangle}{\langle e, e \rangle} e$$

Then:

$$u_1 = a_1, \qquad e_1 = \frac{u_1}{\|u_1\|}$$

$$u_2 = a_2 - \mathrm{proj}_{e_1} a_2, \qquad e_2 = \frac{u_2}{\|u_2\|}$$

$$u_k = a_k - \sum_{j=1}^{k-1} \mathrm{proj}_{e_j} a_k, \qquad e_k = \frac{u_k}{\|u_k\|}$$

We then rearrange the equations above so that the a_s are on the left, using the fact that the $\mathbf{e}_i$ are unit vectors.

$$a_1 = \langle e_1, a_1 \rangle e_1$$
$$a_2 = \langle e_1, a_2 \rangle e_1 + \langle e_2, a_2 \rangle e_2$$
$$a_3 = \langle e_1, a_3 \rangle e_1 + \langle e_2, a_3 \rangle e_2 + \langle e_3, a_3 \rangle e_3$$
$$a_k = \sum_{j=1}^{k} \langle e_j, a_k \rangle e_j$$

Where

$$\langle e_i, a_i \rangle = \|u_i\|.$$

This can be written in matrix form:

$$A = QR$$

Where

$$Q = [e_1, \ldots\ldots, e_n] \quad and \quad R = \begin{pmatrix} \langle e_1, a_1 \rangle & \langle e_1, a_2 \rangle & \langle e_1, a_3 \rangle & \cdots \\ 0 & \langle e_2, a_2 \rangle & \langle e_2, a_3 \rangle & \cdots \\ 0 & 0 & \langle e_3, a_3 \rangle & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Example: Consider the decomposition of

$$A = \begin{pmatrix} 12 & -51 & 4 \\ 6 & 167 & 68 \\ -4 & 24 & -41 \end{pmatrix}.$$

Recall that an orthogonal matrix Q has the property

$$Q^T Q = I.$$

Then, we can calculate Q by means of Gram–Schmidt as follows:

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} = \begin{pmatrix} 12 & -69 & -58/5 \\ 6 & 158 & 6/5 \\ -4 & 30 & -33 \end{pmatrix}.$$

$$Q = \begin{pmatrix} \frac{u_1}{\|u_1\|} & \frac{u_2}{\|u_2\|} & \frac{u_3}{\|u_3\|} \end{pmatrix} = \begin{pmatrix} 6/7 & -69/175 & -58/175 \\ 3/7 & 158/175 & 6/175 \\ -2/7 & 6/35 & -33/35 \end{pmatrix}.$$

Thus, we have

$$Q^T A = Q^T QR = R;$$

$$R = Q^T A = \begin{pmatrix} 14 & 21 & -14 \\ 0 & 175 & -70 \\ 0 & 0 & 35 \end{pmatrix}. \tag{48}$$

- **Relation to RQ decomposition**

The RQ decomposition transforms a matrix *A* into the product of an upper triangular matrix *R* (also known as right-triangular) and orthogonal matrix *Q*. The only difference from QR decomposition is the order of these matrices. QR decomposition is Gram–Schmidt orthogonalization of columns A, started from the first column. RQ decomposition is Gram–Schmidt orthogonalization of rows A, started from the last row.

5.1 Introduction

What is MATLAB?

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

- **Typical Uses of MATLAB**

  - Math and computation
  - Algorithm development
  - Data acquisition
  - Modeling, simulation and prototyping
  - Data analysis, exploration and visualization
  - Scientific and engineering graphics

- **Main features of MATLAB**

  - Advance algorithm for high performance numerical computation especially in the field matrix algebra.
  - A large collection of predefined mathematical functions and the ability to define one's own functions.
  - Two and three dimensional graphics for plotting and displaying data
  - A complete online help system
  - Powerful matrix or vector oriented high level programming language for individual applications.
  - Tool boxes available for solving advanced problems in several application areas

5.2 Algorithm

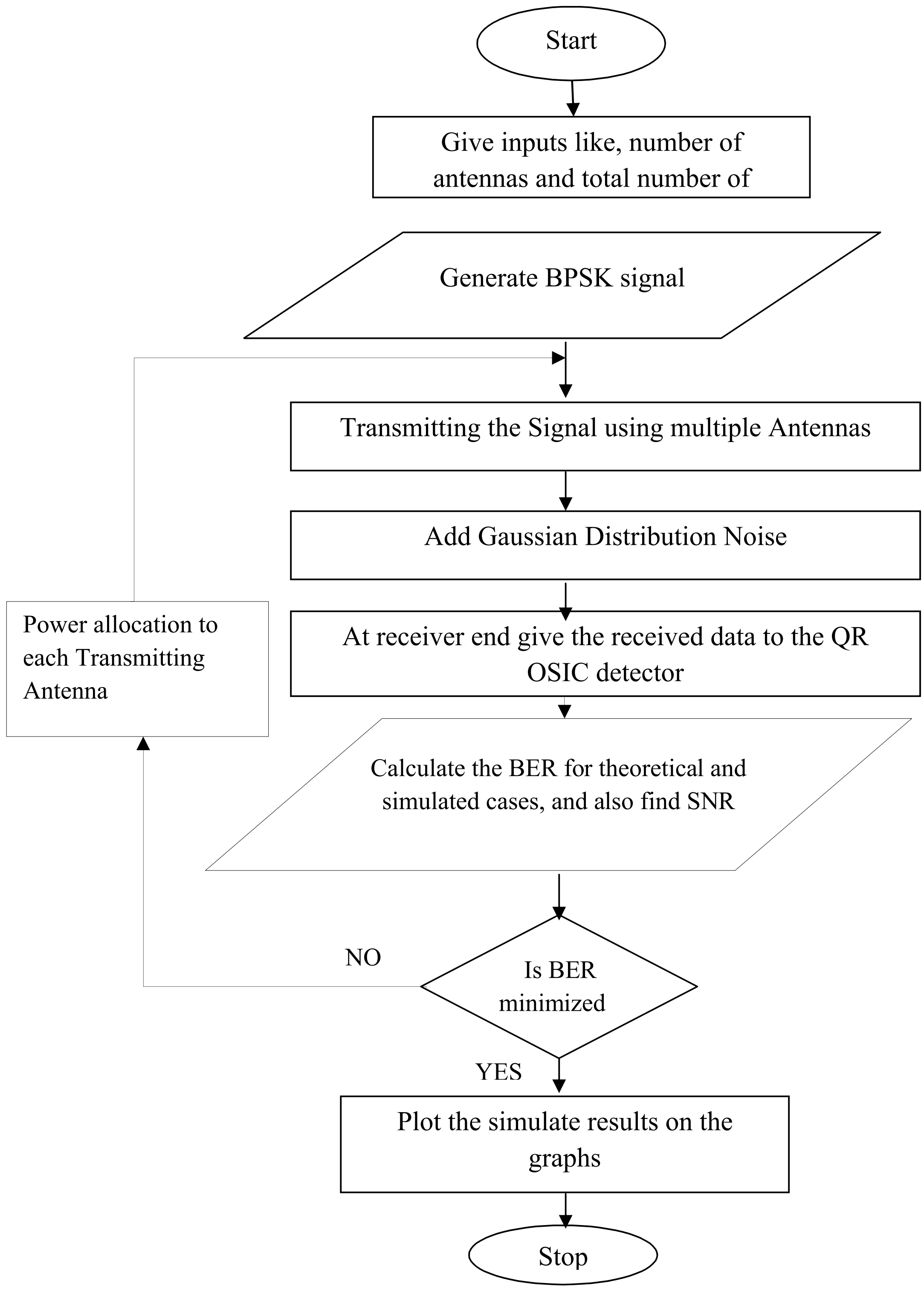
1. Initializing number of bits or symbols.
2. Also initializing number of transmitting and receiving antennas, here we are using 3×3 and 4x4 antenna systems.
3. From transmitter section we generate zero's (0) and one's (1) randomly with equal probability.
4. Modulating the generated random sequence under BPSK modulation scheme.
5. Processing the symbols and grouping them into a matrix to create a channel for data flow.
6. Transmitting the signal using multiple antennas.
7. Generating Gaussian distribution noise with 0 mean.
8. Adding the generated noise with channel properties.
9. At the receiver end using QR- OSIC detector estimating the received data.
10. Now count the total number of errors by subtracting error data stream from main data stream.
11. Calculate the simulated BER by dividing the number of errors with total number of bits or symbols.
12. If BER is not minimized, then allocate the power at each transmitting antennas.
13. Repeat the above procedure for different SNR (Eb /_No_dB) values.
14. Plot all the above generated BER in Semi-log graphs.
15. End of the program.

5.3 Functions used in MATLAB

Functions used in MAT Lab Code	**DESCRIPTON**
Clear	Erases variables and functions from memory
Clear x	erases the matrix 'x' from your workspace
Close	by itself, closes the current figure window
Figure	creates an empty figure window
hold on	holds the current plot and all axis properties so that subsequent graphing commands add to the existing graph.
hold off	sets the next plot property of the current axes to "replace"
Find	find indices of nonzero elements
STEM	Discrete sequence or "stem" plot
SORT	Sort in ascending or descending order
LEGEND	Display legend
Save	saves all the matrices defined in the current session into the file, matlab.mat, located in the current working directory
ERROR	Display message and abort function
xlabel (‘'text' ’)	writes 'text' beneath the x-axis of a plot
ylabel (‘'text' ‘)	writes 'text' beneath the y-axis of a plot
title(‘text‘)	places a title at top of graphics plot
subplot()	Allows you to create multiple plots in the same window

plot(x, y)	creates a Cartesian plot of the vectors x & y
CONV plot(y)	Convolution and polynomial multiplication creates a plot of y vs. the numerical values of the elements in the y-vector
ERFC	Complementary error function.
SQRT	Square Root.
log log(x, y)	plots log(x) vs log(y)
Grid	creates a grid on the graphics plot
RANDN	Normally distributed random numbers.

FLOW CHART


Start
Give inputs like, number of antennas and total number of
Generate BPSK signal
Transmitting the Signal using multiple Antennas
Add Gaussian Distribution Noise
At receiver end give the received data to the QR OSIC detector
Power allocation to each Transmitting Antenna
Calculate the BER for theoretical and simulated cases, and also find SNR
NO
Is BER minimized
YES
Plot the simulate results on the graphs
Stop

RESULT

We consider an uncoded MIMO system with 3*3, 4*4 transmit/receive antenna configurations and BPSK modulation. The effects of error propagation are not ignored and simulations are used to obtain the actual performance. For each of the MIMO systems and for a specific value of SNR, a quasi-static channel is assumed for the performance evaluation, for which the channel gain is constant over a frame and changed independently from frame to frame. To concentrate our point on comparing ordering algorithms, we postulate the perfect channel estimation at the receiver and error free PA information at the transmitter.

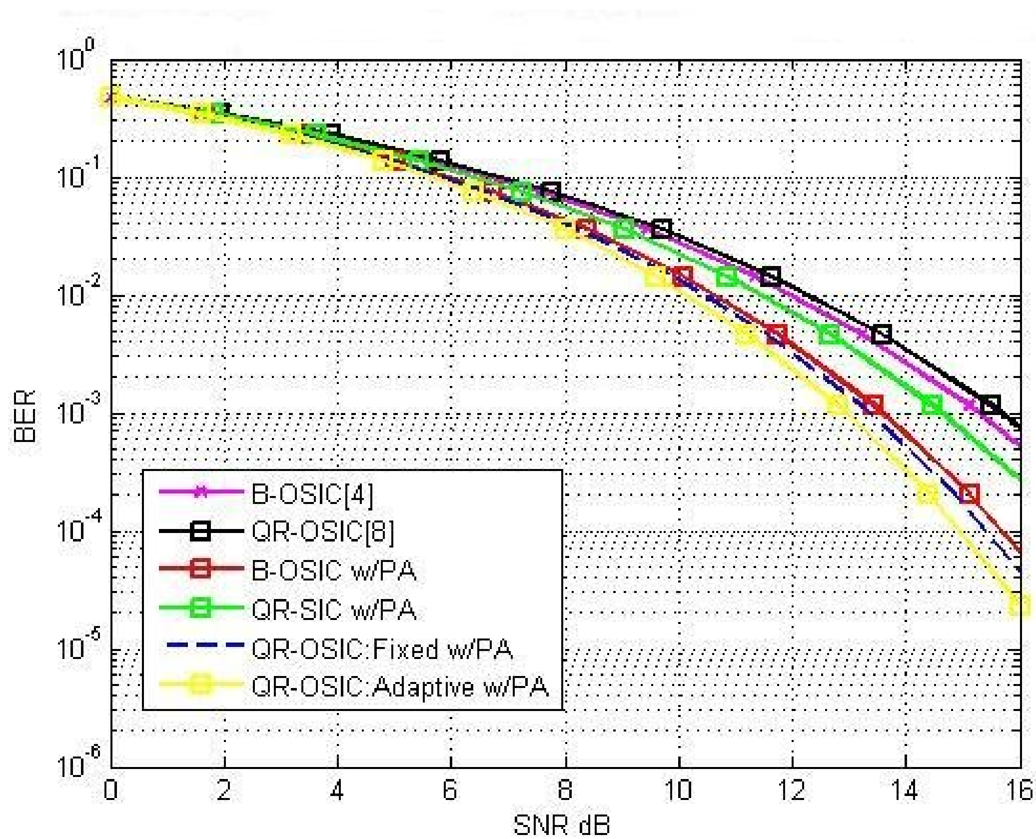


Figure 5.3 Average BER performances of MIMO system with three transmit/receive antennas.

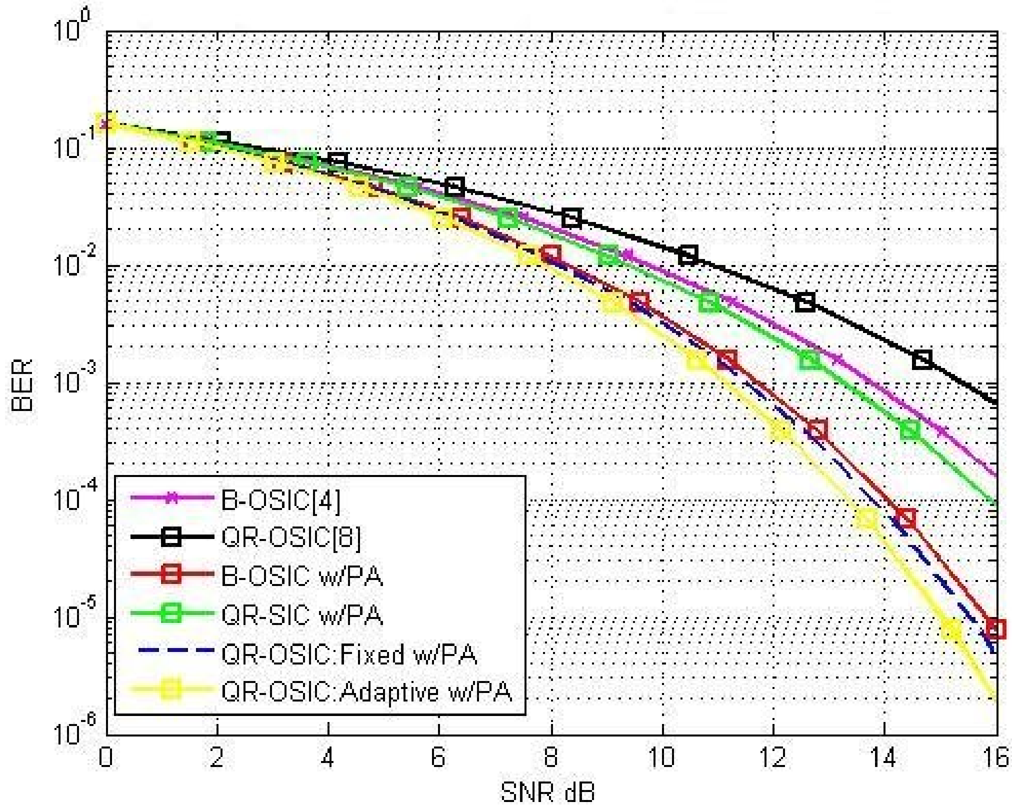


Figure 5.4 Average BER performance of MIMO systems with four transmit/receive antennas

Fig 5.3 shows the average BER performance comparison for MIMO systems with three transmit/receive antennas and the simulation results of four transmit/receive antennas are depicted in Fig 5.4. Here, the dashed line indicates a system with the BER-minimized PA scheme, whereas the solid line represents a system without the PA. The QR receiver with the PA but no ordering, denoted as QR-SIC w/ PA, has similar performance to the open-loop OSIC systems without the PA.

This demonstrates the importance of the detection order for successive detection. As expected without the PA, the B-OSIC out performs the QR-OSIC receiver. Despite the reduced complexity, however power controlled MIMO systems employing the proposed ordering strategy achieve the improved error performance compared to those with the B-OSIC algorithm. It is sufficient to confirm the superiority of the proposed design because the ordering algorithms of previous studies comply with the strategy of the B-OSIC. A further performance improvement in the high SNR region can be explained in terms of error propagation, since the PA scheme as well as the proposed QR-OSIC receiver is designed under the assumption of the error free decision in previous detection stages.

CONCLUSION

Future wireless communication systems have to be designed to integrate features such as high data rates, high quality of service and multimedia in the existing communication framework. In recent years wireless communication has taken peak state. This increased demand has led to the demand for higher network capacity and performance. Higher bandwidth, optimized modulation offer practically limited potential to increase the spectral efficiency. Hence MIMO systems utilizes space multiplex by using array of antenna's for enhancing the efficiency at particular utilized bandwidth. MIMO uses multiple inputs multiple outputs from single channel. These systems defined by spectral diversity and spatial multiplexing. MIMO describes the ways to send data from multiple users on the same frequency time channel using multiple antennas at the transmitter and receiver. A transmitter/receiver system uses multiple antennas not only transmitting data between corresponding antennas but also between adjacent antennas. The data is received in the form of MIMO Channel Matrix. MIMO system is used in many applications like Wi-Max, Wi-Fi, WLAN's, and many more signal processing applications.

In this study, we investigate the QR-OSIC receiver design for the transmitter side power allocated MIMO system. Based on the properties of the Q -function and ordering results, we develop the efficient ordering algorithms in combination with the PA scheme. Inspite of less computational effort, the proposed ordering schemes decrease the overall BER in comparison with the previously derived B-OSIC scheme. Because of the post-detection SINR increment, the coded systems with the derived approach can also be expected to achieve the performance improvement.

FUTURE SCOPE

In this project we have used QR OSIC algorithm. The future scope of the work is to improve the design further for the noise to be included in the channel and use any improved matrix inversion technique for improving the design frequency of operation. This can be done using MMSE Sorted QR Decomposition (SQRD) algorithm.

- MIMO performance can be improved by using OFDM. By incorporating OFDM the performance of the overall system can be improved.
- In the design the channel is considered to be Gaussian distribution noise. In the future work the noise is to be assumed in more complex form and the estimation of channel using different channel models is to be carried. Different channel estimation is to be simulated in MATLAB and then taken to the complete VLSI flow. The frequency of the design is to be optimized. The complete backend flow has to be completed till the tape-out of the design.
- In this project we have used BPSK modulation technique. It is however; only able to modulate at 1 bit/symbol and so it is unsuitable for high data-rate applications when bandwidth is limited. The scope of the project is to improve the performance using another modulation techniques such as QPSK and QAM.

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ABBREVIATIONS

MIMO	Multiple-input Multiple-output
MISO	Multiple-input Single-output
SISO	Single-input Single-output
SIMO	Single-input Multiple-output
BLAST	Bell Laboratories Layered Space Time
D-BLAST	Diagonal Bell Laboratories Layered Space Time
V-BLAST	Vertical Bell Laboratories Layered Space Time
B-OSIC	Blast Ordered successive interference cancellation
PA	Power allocation
BER	Bit error rate
MMSE	Minimum-mean-square error
BPSK	Binary Phase Shift Keying
CCI	Co-Channel Interference
ISI	Inter Symbol Interference
GSM	Global System for Mobile
LOS	Line-of-sight
STC	Space-time coding
OFDM	Orthogonal frequency-division multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
SNR	Signal to noise ratio
CSI	Channel state information
HSPA	High-Speed Packet Access
LTE	Long Term Evolution
MRC	Maximum ratio combining
SER	Symbol error rate
QAM	Quadrature Amplitude Modulation
ML	Maximum Likelihood
ZF	Zero Forcing

MMSE	Minimum Mean Square Error
SINR	Signal to interference noise ratio
IID	Independent and Identically Distributed
AWGN	Additive white Gaussian noise
STBC	Space-time block codes
FER	Frequency error rate
MBER	Minimum bit-error rate
DFE	Decision-feedback equalization
IID	Identically and independently distributed
TDD	Time-division duplex
FDD	Frequency-division duplex
LSE	Least squares error
CMA	Constant modulus algorithm