A Dissertation work on

FULL WAVE ANALYSIS OF DIELECTRIC RECTANGULAR WAVE GUIDE WITH HARMONICS

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DEDICATION

I would like to dedicate my work to my Father, Late Sh. Satyavir Singh and my family members. I believe that I have made it possible merely due to the prayers and great moral support from my parents throughout my studies. Specially, my Father has been a source of great inspiration throughout this course work. My parents and family members have been always around to cheer me up and were always willing to partake in my sporadic study breaks.

As it is well said 'In every conceivable manner, the family is link to our past, bridge to our future.'

CERTIFICATE

This is to certify that the work contained in this dissertation entitled "**Full Wave Analysis of Dielectric Rectangular Wave Guide With Harmonics"** submitted by Deepender Dabas (08/E&C/09) of Delhi College of Engineering in partial fulfilment of the requirement for the degree of Master of Engineering in Electronics & Communication is a bonafide work carried out under my guidance and supervision in the academic year 2009-11.

The work embodied in this dissertation has not been submitted for the award of any other degree to the best of my knowledge.

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ABSTRACT

The concept of guiding electromagnetic waves either along a single conducting wire with finite surface impedance or along a dielectric rod/slab has been known for a long time. There has been a lot of study on microwave waveguides and various methods have been evolved to find propagation constant and other parameters of a wave guide. Most of studies are being done by assuming cosine and sinusoidal fields as incidence field but very less people has assumed square field or harmonics field as incidence. Propagation modes of rectangular dielectric waveguide based on the expansion of electromagnetic field in terms of a series of circular harmonics (Bessel and modified Bessel multiplied by trigonometric function) has been done by Goell [10].

In this thesis, a non-sinusoidal signal has been considered as incidence wave to a Rectangular Dielectric waveguide. The wave function has been represented by a square wave which is distributed in two dimensions. These square waves have been represented by harmonics of sine and cosine function and their 3-D plots had been drawn using Matlab 7.0. The 3-D graphical representations of E^y_{mn} mode wave function for various harmonics (square wave) are similar as that obtained for simple cosine and sine functions [1]. These 3-D graph plots, give better visualization and understanding of field distribution in x and y directions for even and odd functions.

Modes consisting of linear combinations of the established TE, TM, or HE modes can be constructed in a wave guide. E^y_{mn} and E^x_{mn} as well as hybrid modes are supported by the waveguide. The wave guidance takes place by the total internal refection at the side walls. The field components for the E^y_{nm} modes are *Ex*, *Ey*, *Ez*, *Hx*, and *Hz*, with *Hy* = 0 and the independent set of fields for the E^x_{nm} modes are *Ex*, *Ez*, *Hx*, *Hy*, and *Hz*, with *Ey* = 0. The complete set of fields is the sum of the E^y_{nm} and E^x_{nm} modal fields.

The solutions to the rectangular dielectric guide problem have been derived by assuming guided mode propagation along the dielectric, and exponential decay of fields transverse to the dielectric surface. Thus, in the region of confinement, (inside the guide) due to reflections there is standing wave patterns and when the field goes out of the boundary of the guide, in the absence of reflection, the field moves away from the guide exponentially i.e. there is an exponential decay of fields, transverse to the dielectric surface. The fields are

assumed to be approximately square wave distributed inside the waveguide and decaying exponentially outside.

Wave function based on harmonics for even and odd functions have been derived for E^{y}_{mn} mode. Using Marcatili's, approximation method, the approximation of fields has been applied for inside and outside fields. Field at the extreme outside corner of the waveguide has been neglected as field strength is very weak at corners. Applying mode matching technique, the transverse plane of the waveguide has been divided into different regions, such that in each region canonical Eigen functions represent the electromagnetic fields. The Eigen value problem has been constructed, by enforcing the boundary conditions at the interface of each region. Assuming air dielectric interface and square wave form distribution of field inside the waveguide characteristic equations has been derived.

Solution of characteristics equations for E^{y}_{11} modes assuming three harmonics of even and odd function is calculated graphically by MathCad Tool. Calculation of transverse propagation constants for inside (u and u1) and outside (v and v1) in x and y directions of waveguide has been done by taking a particular value to the ratio c1 and c2 (u/u1=c1 and v/v1=c2). The value of c1 is optimized to F/60 and c1 is taken equal to c2. Where, F is the operating frequency. Relative dielectric constant inside the waveguide is taken as three. Comparison of results of normalized propagation constant kz/k0 using this graphical method to that of Marcatili's and Goell's methods, for a silicon dielectric waveguide with a=0.5mm and b=1mm cross section, Ey_{11} mode has been done.

This method works quite well for frequencies at the lower and middle range, when the wave is well guided, the results agree very well with the Marcatili's and Goell's method. At higher frequencies above cut-off, because of presence of harmonics, the normalized propagation constant differ from experimental or direct methods. Accurate calculations are more complicated at higher values of harmonics so it is done, only up to three harmonics. The result of three harmonic functions is very much consistent at lower frequencies and differs at higher frequencies.

ABBREVIATIONS AND SYMBOLS

Symbol	Description
μ	Reflective magnetic permeability
ε	Relative dielectric permittivity
Ψ	Wave function
c	Velocity of light
δ/ δx	Partial differentiation wrt x
ω	Angular velocity
λ	Wave length
J^2	-1
π	22/7
σ	conductivity
3-D	Three dimensional
u	Transverse propagation constant inside the waveguide in x direction
u_1	Transverse propagation constant inside the waveguide in y direction
Kz	Propagation constant inside the waveguide in z direction
v	Attenuation constant outside the waveguide in x direction
v_1	Attenuation constant outside the waveguide in y direction
e	Exponential
TE	Transverse electric
TM	Transverse magnetic
HM	Hybrid mode

Chapter 1
INTRODUCTION

1.1 General overview

With the increase in capabilities of digital computation the way of solving electromagnetic problems has changed. It is no longer necessary that analytical solutions be obtained. Many practical problems with complicated geometries for which there are no closed form analytic solutions can now be solved numerically or graphically. Nevertheless, understanding the fundamental behaviour of the solutions must still be gained from analytic solutions of canonical problems. Review of the wave guiding structures over the whole electromagnetic spectrums shows that, for frequencies below 30 GHz, mostly metal-based structures are used, and for frequencies above 30 GHz, increasing skin-depth losses in metal requires that low-loss structures be made without the use of any metallic material. Fig. 1.1 show the spectral regions in which certain guiding structures are useful. It is seen that the useful spectrum for dielectric waveguides can span from 10^9 to 10^{16} Hz [4].

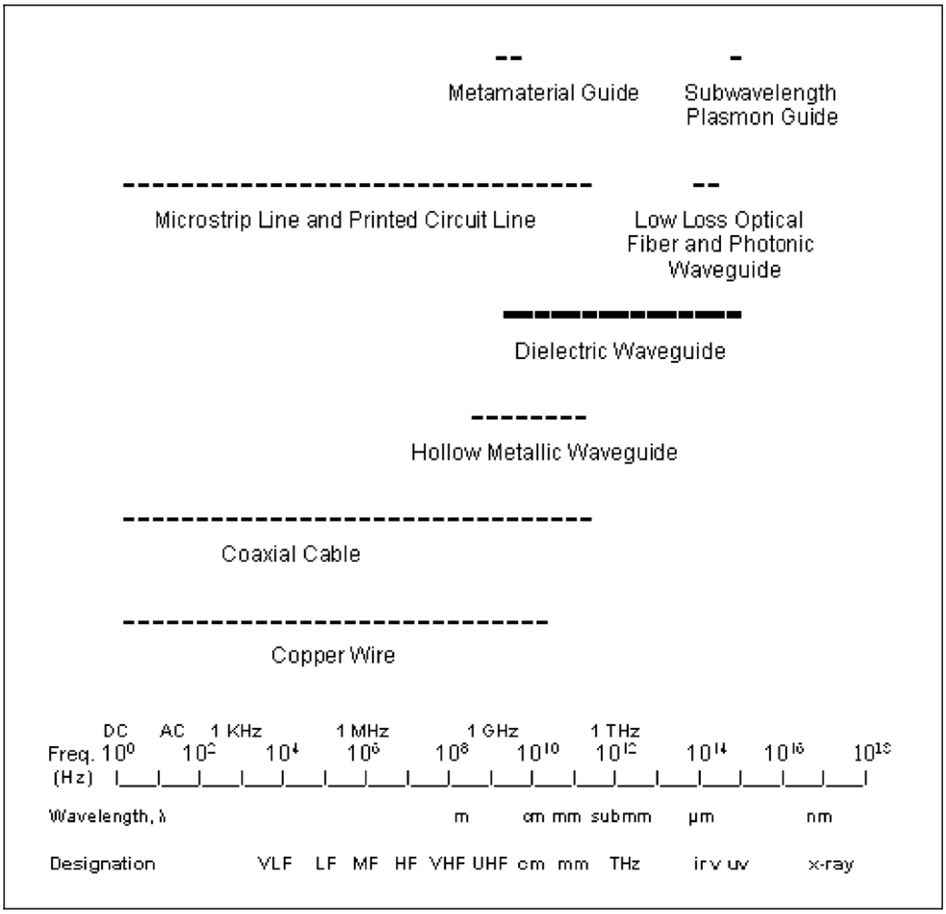

Figure 1.1. Spectral regions for different waveguides [4]

1.2 Background

The concept of guiding electromagnetic waves either along a single conducting wire with finite surface impedance or along a dielectric rod/slab has been known for a long time. As early as 1899, Sommerfeld [2] conceived the idea of guiding a circularly symmetric TM wave along a conducting wire with small surface resistivity. However, because of the large field extent outside the wire, this “open wire” line remained a novelty with limited practical applications. In 1909, Sommerfeld treated the problem of an oscillating dipole above a finitely conducting plane. He found theoretically that there existed not only a radiated wave due to the oscillating dipole, but also a surface wave that travelled along the lossy surface. In 1910, Hondros and Debye [4] demonstrated analytically that it was possible to propagate a TM wave along a lossless dielectric cylinder. Zahn in 1915 and his two students, Ruter and Schriever, confirmed the existence of such a TM wave experimentally. Not until 1936 were the propagation properties of asymmetric waves on a round dielectric rod obtained by Carson, [2] who provided a complete mathematical analysis of this problem. It was noted in their paper that in order to satisfy the boundary conditions for the general case, a hybrid wave (i.e., the coexistence of longitudinal electric and magnetic fields) must be assumed. In other words, asymmetric TE and TM modes were inextricably coupled to each other along a circular dielectric rod. They also showed that pure TE and TM waves could only exist in the circularly symmetric case and there existed one and only one mode, namely the lowest order hybrid wave called the HE_{11} mode, which possessed no cut off frequency and could propagate at all frequencies. All other circularly symmetric or asymmetric modes had cut off frequencies. Later, in 1936, Southworth [3] described more detailed experimental measurements on the phase velocity and attenuation of the circularly symmetric TM wave on a circular dielectric guide. Soon afterwards, in 1938, Schelkunoff wrote a paper on the coupled transmission line representation of the waves and the impedance concept, which became the foundation for the development of microwave circuits.

In 1943, Mallach [4] published his results on the use of the dielectric rod as a directive radiator. He showed experimentally that the radiation pattern obtained by the use of the asymmetric HE_{11} mode produced only one lobe in the principal direction of radiation. Soon after Mallach’s paper, Wegener [5] presented a dissertation in which the asymmetric HE_{11} mode, together with the lowest order circularly symmetric TE and TM modes, were analysed in detail. Elsasser [6] in 1949, published his computation on the attenuation properties of these three lowest-order modes. In a companion paper, Chandler verified experimentally Elsasser’s results on the dominant HE_{11} mode.

In the mid-1940s, Brillouin [7] summarized his research on wave propagation in periodic structures in a book (1946). In 1954, Pierce provided the results on the interaction of electron beam with slow waves guided by a periodic structure. The fundamental theory on wave propagation in periodic media is well established by these publications. At about the same time, the increasing demand for higher bandwidth low-loss transmission lines for transcontinental television and long-distance phone transmission provided the incentive to find new ways to transmit microwaves efficiently. King and Schlesinger [8] studied the dielectric image line (1954), while Goubau experimented with a conducting wire coated with a thin sheath of dielectric material, a modified version of the Sommerfeld line (1950). High loss or instability of the guided field due to the large field extent hampered further development of these approaches. These investigations together with Sommerfeld's research provided the basic understanding of the problem of wave excitation on a dielectric structure. Another notable effort was the concentrated research undertaken by the Bell Laboratory investigators on the transmission of millimetre wave in a oversized circular conducting tube supporting the low-loss circularly symmetric TM wave.

Observation of waveguide modes in optical fibers was first reported by Snitzer and Hicks [9] in 1959, then later in 1961 by Snitzer and Osterberg. In 1961, Snitzer restudied the problem of wave guidance along an optical fiber (a circular dielectric cylinder). He provided detailed numerical computations on several lower-order modes and obtained field configurations for these modes. In 1962, Yeh [10] solved the unique-canonical problem of surface wave propagation on an elliptical dielectric waveguide. Unlike the circular cylinder case where each mode can be described by a single order of Bessel function, each surface wave mode for an elliptical dielectric cylinder would require infinite sums of all orders of Mathieu functions. In 1965, Bloembergen [11] wrote a book summarizing his research on wave propagation in nonlinear dielectrics. His work on nonlinear dielectric became the backbone of the later discovery of solutions in optical fibers. Kao and Hockham in 1964 recognized that if the impurities in optical fiber can be eliminated, the fiber may become a very low-loss transmission waveguide for optical signals; and Kapron in 1970 minimized these impurities in fused silica, resulting in the successful making of optical fiber with optical transmission losses of approximately 20 dB km^{-1}.

The revolution in digital processing started in the 1950s finally took flight in the 1960s due to the rapid advances in the use of integrated circuits in digital computers. Many heretofore unsolvable engineering or scientific problems could now be solved using a relatively straight forward numerical computational approach. Yee in 1966 [12] developed the FDTD (Finite Difference Time Domain) algorithm to solve the Maxwell equations numerically; Mur in 1981 [13] developed an effective absorbing boundary condition for FDTD; Yeh and Wang in 1972 made use of the two-point boundary value numerical approach to solve the graded-index fiber problem. Yeh in 1974 became the first group who successfully adopted the finite element technique (FEM) to solve a large variety of single-mode optical waveguides; and Mariki and Yeh [14] in 1985 perfected the 3D TLM (Transmission-Line Matrix) technique based on the Schelkunoff's impedance concept to solve the arbitrarily shaped dielectric waveguide problem. Several numerical approaches (e.g., FDTD, Finite Element Method, Beam Propagation Method) have already been developed into commercial software packages where a given problem is viewed as a "blackbox" having input data and output data, that provide numerical results.

1.3 Thesis Objective

As seen, there has been a lot of study on optical and microwave waveguides and various methods have been evolved to find propagation constant and other parameters of a wave guide. Most of studies are being done by assuming co-sinusoidal field as incidence field but very less by assuming square field or harmonics field as incidence. Some has been done by Goell [15] to find propagation modes of rectangular dielectric waveguide based on the expansion of electromagnetic field in terms of a series of circular harmonics (Bessel and modified Bessel multiplied by trigonometric function).

In the present analysis the variation of longitudinal electric and magnetic fields of the modes are represented by square field inside the core as against sine (cosine) wave and by exponential decaying field outside the waveguide core. 3-D graphs of the fields are plotted assuming square field to be made of harmonics of sine and cosine functions. Field distribution for dominant mode and other modes are shown by using 3-D graphical tool of Matlab. Square wave incidence is proposed of one, two and four harmonics of sine and cosine functions. Wave function based on harmonics for even and odd functions are derived. Using Marcatili's, approximation method the approximation of field are applied for inside and outside fields. Then applying mode matching technique, the transverse plane of the waveguide has be divided into different regions, such that in each region canonical Eigen functions has be used to represent the electromagnetic fields. The Eigen value problem is then constructed by enforcing the boundary conditions at the interface of each region. Assuming air dielectric interface and square wave form distribution of field inside the waveguide characteristic equations can be derived. The characteristic equations are solved graphically by MathCad Tool. Then propagation constant and other parameters of dielectric waveguide are calculated. The results are compared to those obtained by Marcatili and Goell.

1.4 Thesis Organisation

The thesis begins with basics of Maxwell equation in chapter 2. Section 2.2 deals with the basic concepts of even and odd function distribution. Boundary conditions are explained in section 2.3.

In chapter 3 (section 3.1), a brief description of Marcatili approximation method is given and wave equations for square wave incidence are derived using two harmonics of square function in terms of sine and cosine function.

In chapter 4, inside field distribution of E^y_{mn} modes are plotted for dominant mode and for m=1 and n=3, mode also. Even and odd field distribution is shown in the graphs and outside exponential field is also shown. The field graphs are plotted using 3-D graph tool of Matlab.

In chapter 5, Applying mode matching technique, the characteristic equations are solved graphically by MathCad Tool. Then propagation constant and other parameters of dielectric waveguide are solved.

In chapter 6, The graphical results are analysed and discussed. They are compared with result obtained by Marcatili and Goell.

Chapter 2

FIELD EQUATIONS IN WAVEGUIDE

2.1 Basic Concepts

All large-scale electromagnetic wave phenomena are governed by the Maxwell equations and the appropriate boundary conditions. These equations were supported by a vast weath of experimental evidence and by previous observations such as Coulomb's law of forces between two charges Ampere's law on the interaction of current elements, the observation of Faraday on variable fields. These equations were experimentally verified by Hertz and Albert Einstein. It is customary to write Maxwell equation in either differential form or in integral form.

2.2 Maxwell Equations

On the basis of the established experimental laws, Maxwell postulated that the electromagnetic field vectors are subject to the following equations in differential form:

$$\nabla\times \mathbf{E}(\mathbf{r}, t) = -\partial \mathbf{B}(\mathbf{r}, t)/\partial t, \qquad (2.1)$$

$$\nabla\times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \partial \mathbf{D}(\mathbf{r}, t)/\partial t, \qquad (2.2)$$

where

$\mathbf{E}(\mathbf{r}, t)$ = Electric field intensity (V/m)

$\mathbf{H}(\mathbf{r}, t)$ = Magnetic field intensity (A/m)

$\mathbf{D}(\mathbf{r}, t)$ = Electric displacement vector (C/m)

$\mathbf{B}(\mathbf{r}, t)$ = Magnetic induction vector (Wb/m^2)

$\mathbf{J}(\mathbf{r}, t)$ = Electric current density (A/m)

These vectors are functions of space, **r** (in meters), and time, *t* (in seconds) The MKS system is being followed throughout. The conservation of charge law can be expressed as follows:

$$\nabla\cdot \mathbf{J}(\mathbf{r}, t) + \partial \rho/\partial t = 0, \qquad (2.3)$$

here

$\rho(\mathbf{r}, t)$ = Electric charge density (C/m^3).

This is the equation of continuity. Faraday's Law, Ampere's Law, Gauss' Law, and Coulomb's Law are included or can be derived from the Maxwell equations and the equation of continuity. For example, (2.1) is a statement of Faraday's Law, while (2.2), without the displacement current term, $\partial \mathbf{D}(\mathbf{r}, t)/\partial t$, is a statement of Ampere's Law. Maxwell postulated the existence of the displacement current term in (2.2) to express the wave nature of the electromagnetic fields.

Since the divergence of the curl of any vector vanishes identically, taking the divergence of (2.1) yields the Gauss' law

$$\nabla\cdot \partial \mathbf{B}(\mathbf{r},t)/\partial t = \partial/\partial t\,(\nabla\cdot \mathbf{B}(\mathbf{r},t)) = 0 \qquad (2.4)$$

Or $$\nabla\cdot \mathbf{B}(\mathbf{r},t) = 0. \qquad (2.5)$$

The divergence of (2.2) gives

$$\nabla\cdot \mathbf{J}(\mathbf{r},t) + \nabla\cdot \partial \mathbf{D}(\mathbf{r},t)/\partial t = 0 \qquad (2.6)$$

and from (2.3) we get

$$\partial/\partial t[\nabla\cdot (\mathbf{D}(\mathbf{r},t) - \rho(\mathbf{r},t)] = 0 \qquad (2.7)$$

Or $$\nabla\cdot \mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t). \qquad (2.8)$$

This is Coulomb's Law. We can say that divergence equations (2.5) and (2.8) are not independent relations of Maxwell equations (2.1) and (2.2) and the equation of continuity, (2.3). Limiting our investigation to linear phenomena, fields of arbitrary time variation can be constructed from harmonic solutions through the Fourier Transform Method, and there is no loss of generality with the assumption that the time-dependent variation of the fields may be factored out as follows:

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})\, e^{j\omega t}], \qquad (2.9)$$

where ω is the harmonic frequency of the wave, Re means the real part and $\mathbf{E}(\mathbf{r})$, the electric field vector, is a spatially dependent, complex function. Similar time variations are assumed for the other field and source quantities, such as

$$\mathbf{D}(\mathbf{r},t) = \mathrm{Re}[\mathbf{D}(\mathbf{r})\, e^{j\omega t}],$$
$$\mathbf{H}(\mathbf{r},t) = \mathrm{Re}[\mathbf{H}(\mathbf{r})\, e^{j\omega t}], \ldots, \text{etc.}$$

The time-harmonic Maxwell equations and the continuity equation now take the forms

$$\nabla\times \mathbf{E}(\mathbf{r}) = -j\omega \mathbf{B}(\mathbf{r}), \qquad (2.10)$$
$$\nabla\times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r}) + j\omega \mathbf{D}(\mathbf{r}), \qquad (2.11)$$
$$\nabla\cdot \mathbf{J}(\mathbf{r}) = -j\omega \rho(\mathbf{r}). \qquad (2.12)$$

The associated divergence equations are

$$\nabla\cdot \mathbf{B(r) = 0}, \tag{2.13}$$

$$\nabla\cdot \mathbf{D(r)} = \boldsymbol{\rho}\mathbf{(r)}. \tag{2.14}$$

The field vectors **E**(**r**), **D**(**r**), **H**(**r**), and **B**(**r**) are now spatially dependent complex functions. It is seen from (2.10) and (2.11) that given the source function **J**(**r**), there are four unknown quantities, **E**, **B**, **D**, and **H**, and two independent equations (2.10) and (2.11). Two additional independent equations relating the field quantities, **E**, **B**, **D**, and **H** are needed in order that deterministic solutions for these quantities may be found. The needed equations are obtained from the constitutive relations.

2.2.1 The Constitutive Relations

The constitutive relations are derived from the description of the macroscopic properties of the medium in the immediate neighbourhood of the specified field point. we assume that, at any given point in a given medium, the vector **D** and **H** may be represented as a function of **E** and **B**.

$$\mathbf{D = F_1(E,B)}, \tag{2.15}$$

$$\mathbf{H = F_2(E,B)}. \tag{2.16}$$

The functional dependencies of these functions are obtained from the macroscopic physical properties of the medium [4]. The behaviour of a material medium in an electromagnetic field can be described in terms of distributions of electric and magnetic dipoles. The medium can be characterized by two polarization density functions: **P**, the electric dipole moment per unit volume, and **M**, the magnetic dipole moment per unit volume. The polarization may be induced under the action of the field from other sources, or it may be virtually permanent and independent of external fields. The permanent polarizations will be designated by $\mathbf{P}_0$ and$\mathbf{M}_0$. Some examples of medium characteristics are as given in following section.

2.2.2 Simple Medium

A simple medium (Linear and Isotropic) is taken to be

(a) Linear, where **D** is a linear function of **E** and **H** is a linear function of **B**, and

(b) Isotropic, where **D** is parallel to **E** and **H** is parallel to **B**.

In simple medium,

$$\mathbf{D} = \mathcal{E}\mathbf{E},$$
$$\mathbf{H} = (1/\mu)\mathbf{B} \quad (2.17)$$

The parameters $\mathcal{E}$ and $\boldsymbol{\mu}$, which represent the electromagnetic properties, are, respectively, the permittivity and permeability of the medium. For isotropic inhomogeneous media, $\mathcal{E}$ and μ may be functions of positions and for frees pace,

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}_0},\ \boldsymbol{\mu} = \boldsymbol{\mu_0}, \quad (2.18)$$

where $\mathcal{E}_0$ = 8.854 × 10−12 (F/m) and μ_0 = 4π × 10−7 (H/m1) are, respectively, the free-space permittivity and free-space permeability. The relationships between the field vectors and the polarization vectors are defined as follows:

$$\mathbf{P} + \mathbf{P}_0 = \mathbf{D} - \mathcal{E}_0\mathbf{E} = (\mathcal{E} - \mathcal{E}_0)\mathbf{E} = \chi_e\mathcal{E}_0\mathbf{E}, \quad (2.19)$$
$$\mathbf{M}+\mathbf{M}_0 = (1/\mu_0)\mathbf{B} - \mathbf{H} = \{(\mu/\mu_0) - 1\}\mathbf{H} = \chi_m\mathbf{H}, \quad (2.20)$$

Where χ_e and χ_m are called the electric and magnetic susceptibilities. The electric and magnetic polarization vectors are zero in free-space. The relations (2.19) and (2.20) are definable only for time-periodic phenomena, since in general $\mathcal{E}$ and μ are functions of the frequency. The frequency dependence of the constitutive parameters is known as the *dispersive property* of the medium. Hence, these relations are applicable to other than time-periodic, time-varying fields only when, over the significant part of the frequency spectrum covered by the Fourier components of the time dependence, the constitutive parameters $\mathcal{E}$ and μ are sensibly independent of frequency.

2.2.3 Anisotropic Medium

In an anisotropic material medium, the electromagnetic properties are functions of the field directions about a point. So it can be written in matrix form as,

$$\mathbf{D} = \boldsymbol{\mathcal{E}} \cdot \mathbf{E}$$

$$\text{Where } \mathcal{E} = \begin{bmatrix} \mathcal{E}_{11} & \mathcal{E}_{12} & \mathcal{E}_{13} \\ \mathcal{E}_{21} & \mathcal{E}_{22} & \mathcal{E}_{23} \\ \mathcal{E}_{31} & \mathcal{E}_{32} & \mathcal{E}_{33} \end{bmatrix} \quad (2.21)$$

$$\mathbf{B} = \boldsymbol{\mu} \cdot \mathbf{H}$$

$$\text{Where } \mu = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix} \quad (2.22)$$

$\mathcal{E}_{ij}$ and μ_{ij} are elements of the permittivity matrix and the permeability matrix describing the anisotropic characteristics of the medium. For inhomogeneous and anisotropic medium, $\mathcal{E}_{ij}$ and μ_{ij} are functions of positions. For anisotropic and dispersive medium, $\mathcal{E}_{ij}$ and μ_{ij} are functions of the frequency.

2.2.4 Left-Handed Medium

A class of artificial media can be characterized as follows:

$$\mathbf{D} = -\boldsymbol{\varepsilon}\,\mathbf{E},$$

$$\mathbf{H} = -(1/\boldsymbol{\mu})\mathbf{B}. \qquad (2.23)$$

A left-handed material also known as Metamateria, is one with negative permittivity and negative permeability in the frequency range of interest. The index of refraction in such a metamaterial medium is also negative. The permittivity and the permeability of the medium are usually frequency dependent and lossy. In other words, they are complex quantities.

2.2.5 Conducting Medium

As per Ohm's law

$$\mathbf{J} = \sigma\mathbf{E}, \qquad (2.24)$$

where σ, a scalar constant for most conducting materials, is called the conductivity, in units of mhos per meter. This is an added relation for the source-free Maxwell equations (2.10) and (2.11). In a conducting medium ($\sigma \neq 0$), there can be no permanent distribution of free charges, because any free charge will diffuse out of the conducting medium and reside on the surface.

2.2.6 Dielectric Medium with Loss

In general, a wave propagating in a dielectric undergoes losses, and the permittivity of the material can no longer be represented by a real value. These losses can be attributed to a number of causes, including conduction, relaxation phenomena, both in the dielectric as well as in impurities, molecular resonances, and molecular structure. Because of losses, the dielectric must be represented by a complex value. The complex dielectric permittivity of that material can be written as follows

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}' - j\,\boldsymbol{\varepsilon}'' \qquad (2.25)$$

where $\boldsymbol{\varepsilon}'$ and $\boldsymbol{\varepsilon}''$ are the real and imaginary parts of the permittivity or dielectric constant.

In a dielectric, the ratio **Ɛ''**/ **Ɛ'**(= $\sigma/\omega\varepsilon''$) is a direct measure of the ratio of the conduction current to the displacement current.

2.2.7 Nonlinear Medium

The constitutive relations for a nonlinear medium have the form

D= **Ɛ**(**E**)**E** (2.26)

B=μ(**H**)**H** (2.27)

where **Ɛ**(**E**) and $\boldsymbol{\mu}$(**H**) are functions of the field strengths. Substituting these constitutive relations into (2.1) and (2.2) gives a set of equations that are nonlinear. It is important to understand the limitations imposed by describing a given medium by the macroscopic electromagnetic parameters (**Ɛ**, μ, σ). Any material medium is composed of a large number of atoms and/or molecules whose electromagnetic properties can only be accurately described by the principles of quantum electrodynamics. It is assumed that the ensemble electromagnetic properties of the material medium can be characterized by permittivity, **Ɛ** , permeability, μ, and conductivity, σ. This means that the wavelength of the electromagnetic wave phenomenon of interest must be much larger than the size of individual atoms or molecules and their separations. Characterizing the electromagnetic properties of a given medium by its permittivity (electric polarizability), permeability (magnetic polarizability), and conductivity is the cornerstone of macroscopic electrodynamics. The macroscopic behaviour of the electromagnetic field in a given medium is therefore governed by Maxwell equations (2.10) and (2.11), the continuity equation (2.12), and the constitutive relations (2.15) and (2.16) and if the medium is conductive by (2.28). The Maxwell equations can then be reduced to two independent equations with two independent variables, E and H.

2.3 Various condition and constrains on surfaces

The solution of a given electromagnetic problem is uniquely determined if it satisfies the Maxwell equations, the associated constitutive relations, the appropriate boundary conditions, and/or the radiation condition, and/or the edge condition wherever applicable are applied.

2.3.1 Boundary Conditions

Electromagnetic fields across a given boundary between two distinct media must satisfy a set of boundary conditions. Let *P* be a smooth surface separating two media, 1 and

2; let the unit vector normal to the boundary be n, pointing from medium 1 into medium 2. Consider the following three cases: (a) medium 1 and 2 are dielectrics; (b) medium 1 is a perfect conductor and medium 2 is a dielectric; and (c) medium 1 is an imperfect conductor and medium 2 is a dielectric.

Case a. Media 1 and 2 are dielectrics having constitutive parameters ($\varepsilon 1$, $\mu 1$, $\sigma 1$) and ($\varepsilon 2$, $\mu 2$, $\sigma 2$), respectively

The necessary and sufficient boundary condition on the static electric and magnetic fields across two distinct dielectric media are that (a) for the electrostatic fields, the tangential component of **E** must be continuous and the normal component of **D** must be discontinuous by the surface charge density at the boundary and (b) for the magneto static fields, the tangential component of **H** must be discontinuous by the surface current density and the normal component of **B** must be continuous at the boundary.

The necessary and sufficient boundary conditions on the time varying electromagnetic fields across two distinct dielectric media are that the tangential electric fields must be continuous across the boundary and the tangential magnetic fields must be discontinuous by the surface current density.

Case b. Medium 1 is a dielectric and medium 2 is a perfect conductor

As no electromagnetic field can exist within a perfect conductor (medium 2) and the potential on a perfect conductor is a constant. Therefore, no tangential electric field can exist on the surface of that perfect conductor while, by Ampere's law, the tangential magnetic field must be equal to the surface current density and must be in a direction normal to that surface current. All electric field lines must terminate normally on the perfectly conducting surface. Hence the boundary conditions are

$$\mathbf{n} \times \mathbf{E}_1 = 0,$$

$$\mathbf{n} \times \mathbf{H}_1 = \mathbf{J}s,$$

$$\mathbf{n} \cdot \mathbf{B}_1 = 0,$$

$$\mathbf{n} \cdot \mathbf{D}1 = \rho s, \qquad (2.28)$$

where **J**s is the total electric surface current density, which consists of the induced surface current density due to an incident electromagnetic field in medium 1 and any other impressed currents, and ρs is the surface charge density.

Case c. Medium 1 is a dielectric and medium 2 has surface impedance Zs and no field can penetrate into medium 2,

On the surface of medium 2, the surface impedance is defined as follows

$$-\mathbf{n} \times \mathbf{E}1 = Z_s\mathbf{H}1. \qquad (2.29)$$

Equation (2.29) becomes the boundary condition for the fields in medium 1. If medium 2 is a good conductor, in which the conductivity σ is large but finite, then $Zs = (1+j)(\omega\mu/2\sigma)^{1/2}$. The surface impedance boundary condition is valid only for time-harmonic fields.

2.3.2 Radiation Condition

The field associated with a finite distribution of sources or the field scattered from obstacles must satisfy conditions at infinity, which pertain to the finiteness of the energy radiated by the sources or scattered by obstacles as well as the assurance that the field at infinity represents an outgoing wave. In other words, the radiation condition not only requires that (a) for a finite three-dimensional (3D) source, the field intensities must vanish at infinity such that

lim ($r^2\mathbf{E}$) and lim ($r^2\mathbf{H}$) are bounded as $r \to \infty$, where r is the 3D radial distance from the origin,

(b) for a finite two-dimensional (2D) source, the field intensities must vanish at infinity such that lim ($r\mathbf{E}$) and lim ($r\mathbf{H}$) are bounded as $r \to \infty$, where r is 2D radial distance from the origin, but also (c) the electromagnetic wave represented by **E** and **H** must behave as an outgoing divergent traveling wave at great distances from the source. The finiteness of the energy radiated by the source is assured by condition (a) for a 3D source or by condition (b) for a 2D source.

2.3.3 Edge Condition

At sharp edges the field vectors may become infinite. But the order of this singularity is restricted by the Bouwkamp–Meixner edge condition. The energy density must be integrable over any finite domain even if this domain happens to include field singularities, that is, the energy in any finite region of space must be finite. For example, when applied to a perfectly conducting sharp edge, this condition states that the singular components of the electric and magnetic vectors are of order $\xi^{-1/2},$ where ξ is the distance from the edge, whereas the parallel components are always finite.

2.4 Classification of Fields

The propagation characteristics of guided waves are governed by the source-free Maxwell equations. Consider the source-free Maxwell equations in a simple medium in which

$$\mathbf{D} = \varepsilon \mathbf{E}, \qquad (2.30)$$

$$\mathbf{B} = \mu \mathbf{H},$$

The source-free time-harmonic Maxwell equations (2.10) and (2.11) become

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu\mathbf{H}(\mathbf{r}), \qquad (2.31)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\varepsilon\,\mathbf{E}(\mathbf{r}). \qquad (2.32)$$

Taking the curl of (2.31) and substituting (2.32) into the resultant equation yields

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - \omega^2\mu\,\varepsilon\,\mathbf{E}(\mathbf{r}) = 0. \qquad (2.33)$$

A similar equation for **H** can also be derived,

$$\nabla \times \nabla \times \mathbf{H}(\mathbf{r}) - \omega^2\mu\,\varepsilon\,\mathbf{H}(\mathbf{r}) = 0. \qquad (2.34)$$

Equation (2.33) or (2.34) is called the vector wave equation.

The solutions of this scalar wave equation had been known prior to the introduction of the vector wave equation describing the electromagnetic wave phenomenon. The scalar wave equation describing the behaviour of sound waves is

$$\nabla^2 u(\mathbf{r}) + k^2 u(\mathbf{r}) = 0, \qquad (2.35)$$

where $u(\mathbf{r})$ represent the pressure wave and k is the sound wave number. Therefore, the reduction of the vector wave equation to the well-studied scalar wave equation would be very desirable. In a simple medium, the electromagnetic fields, governed by a linear vector wave equation, are linear. This means that the superposition theorem can be applied to the fields. The complete electromagnetic fields can be obtained by superposing partial fields, which are derived from a set of new potentials, called the Debye potentials. Using these potentials, it is possible to reduce the vector wave equation to a scalar wave equation in certain special but very useful situations.

2.4.1 The Debye Potentials

Debye, had given two new potentials, $\Psi(\mathbf{r})$ and $\Phi(\mathbf{r})$, as follows:

$$\mathbf{E}(\mathbf{r}) = \nabla \times (\mathbf{a}\Psi(\mathbf{r})) - (j/\omega\,\varepsilon)\nabla \times \nabla \times (\mathbf{a}\Phi(\mathbf{r})), \qquad (2.36)$$

$$\mathbf{H}(\mathbf{r}) = \nabla \times (\mathbf{a}\Phi(\mathbf{r})) + (j/\omega\mu)\nabla \times \nabla \times (\mathbf{a}\Psi(\mathbf{r})), \qquad (2.37)$$

where **a** is a unit vector or the position vector **r**. Since the divergence of a curl is identically zero, taking the divergence of (2.36) and (2.37) gives $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{H} = 0$. The two scalar functions $\Psi(\mathbf{r})$ and $\Phi(\mathbf{r})$ satisfies a pair of second-order partial differential equations. These partial differential equations are obtained by substituting (2.36) and (2.37) into (2.33) and (2.34) (the wave equations), respectively. One can show that in a homogeneous, isotropic medium, $\Psi(\mathbf{r})$ and $\Phi(\mathbf{r})$ satisfy the scalar Helmholtz equation,

$$(\nabla^2 + k^2)\Psi(\mathbf{r}) = 0, \qquad (2.38\ a)$$

$$(\nabla^2 + k^2)\Phi(\mathbf{r}) = 0, \qquad (2.38\ b)$$

with $k^2 = \omega^2\mu\boldsymbol{\varepsilon}$. This means that by using the Debye potentials as introduced in (2.36) and (2.37), one may reduce the vector wave equations (2.33) and (2.34) to a set of scalar wave equations, (2.38 93). Since many of the solutions for the scalar Helmholtz equations are known, a number of solutions for the vector wave equations can be obtained by using (2.36) and (2.37). Since the vector wave equations for the homogeneous isotropic simple medium are linear, it follows that the superposition theorem can be used to obtain the total and complete wave fields from (2.36) and (2.37). In other words, (2.36) and (2.37) represent the complete electromagnetic field solutions in a simple medium using the Debye potentials, $\Psi(\mathbf{r})$ and $\Phi(\mathbf{r})$.

2.42 Basic Wave Types

Knowing that the linear property of electromagnetic fields in a simple medium allows us to use the superposition theorem to obtain the complete field components from partial fields (i.e., different wave types), we can consider each wave type separately. Defining the following basic wave types:

1. Transverse Electromagnetic Waves (TEM Waves). These waves contain neither an electric nor a magnetic field component in the direction of propagation. There is no E_z or H_z if $\mathbf{e}_z$ is the direction of propagation.

2. Transverse Magnetic Waves (TM or E waves). These waves contain an electric field component but not a magnetic field component in the direction of propagation. There is no H_z if $\mathbf{e}_z$ is the direction of propagation, that is, $\Psi(\mathbf{r}) = 0$.

3. Transverse Electric Waves (TE or H waves). These waves contain a magnetic field component but not an electric field component in the direction of propagation. There is no *Ez* if $\mathbf{e}_z$ is the direction of propagation, that is, $\Phi(\mathbf{r}) = 0$.

4. Hybrid waves (HE or EH waves). These waves contain all components of electric and magnetic fields. These hybrid waves are obtained by linear superposition of TE and TM waves, that is, $\Psi(\mathbf{r}) /= 0$ and $\Phi(\mathbf{r}) /= 0$.

When the guided modes propagate along a perfectly straight line path, we assume that every component of the electromagnetic wave may be represented in the form $f(u, v)e^{-j\beta z+j\omega t}$, in which z is chosen as the propagation direction and u, v are generalized orthogonal coordinates in the transverse plane. The symbol β is the propagation constant. The time-harmonic Maxwell equations in a simple homogeneous, isotropic medium (ε, μ) can now be written as

$$\frac{1}{h_2}\left(\frac{\delta\cdot E_z}{\delta\cdot v} + j\cdot\beta\cdot h_2\cdot E_v\right) + j\cdot\omega\cdot\mu\cdot H_u = \cdot 0$$

$$\frac{1}{h_1}\left(-j\cdot\beta\cdot h_1\cdot E_u - \frac{\delta\cdot E_z}{\delta\cdot u}\right) + j\cdot\omega\cdot\mu\cdot H_v = \cdot 0$$

$$\frac{1}{h_1\cdot h_2}\left[\frac{\delta(h_2\cdot E_v)}{\delta\cdot u} - \frac{\delta\cdot(h_1\cdot E_u)}{\delta\cdot v}\right] + j\cdot\omega\cdot\mu\cdot H_z = \cdot 0$$

$$\frac{1}{h_2}\left(\frac{\delta\cdot H_z}{\delta\cdot v} + j\cdot\beta\cdot h_2\cdot H_v\right)\cdot -j\cdot\omega\cdot\varepsilon\cdot E_u = \cdot 0$$

$$\frac{1}{h_1}\left(-j\cdot\beta\cdot h_1\cdot H_u - \frac{\delta\cdot H_z}{\delta\cdot u}\right)\cdot -j\cdot\omega\cdot\varepsilon\cdot E_v = \cdot 0$$

$$\frac{1}{h_1\cdot h_2}\left[\frac{\delta(h_2\cdot H_v)}{\delta\cdot u} - \frac{\delta\cdot(h_1\cdot H_u)}{\delta\cdot v}\right]\cdot -j\cdot\omega\cdot\varepsilon\cdot E_z = \cdot 0 \tag{2.39}$$

Here, $Eu,v,z = Eu,v,z(u, v)e^{-j\beta z}$ and $Hu,v,z = Hu,v,z(u, v)e^{-j\beta z}$ and ($h1$,$h2$) are the transverse coefficients of any orthogonal system of curvilinear coordinates with a longitudinal *z*-coordinate.

2.5 Propagation characteristics of guided waves along a dielectric guide

Any guided wave structure on which the guided field can extend to infinity is considered to be an open surface wave structure. There are basically two types of surface waveguides. One type is the field penetrable kind, such as the various dielectric structures that include solid dielectrics, plasmon, and photonic–periodic structures. The guided field is allowed to penetrate significantly into the core material and extend well beyond the exterior of the core surface. The other type is the field-impenetrable kind, such as the various surface impedance conducting structures, including the Sommerfeld–Goubau line. The guided field does not penetrate much beyond the surface of the guiding structure. The guided wave simply adheres to the guiding surface. Figure 2.1 shows a number of unconventional surface waveguides having structures made of metamaterial or moving material or anisotropic material. The structures shown in Fig. 2.1 are capable of supporting a finite number of guided modes as well as a continuous spectrum of unguided radiated waves. Hence, it is impossible to excite the desired guided modes along a surface waveguide without simultaneously exciting the unwanted radiated waves. Furthermore, deviations from perfect waveguide geometry or from a straight line path not only convert power among the guided modes but also scatter power into the continuous spectrum of radiated waves. The radiated wave carries power away from the guiding structure, and therefore the radiated power is considered totally lost. As far as terminology is concerned, surface waveguides encompass dielectric waveguides, optical fibers, integrated optical circuits (IOC), plasmon guides, Sommerfeld–Goubou wires, photonic waveguides, metamaterial waveguides, or any guiding structure whose guided field can extend to infinity.

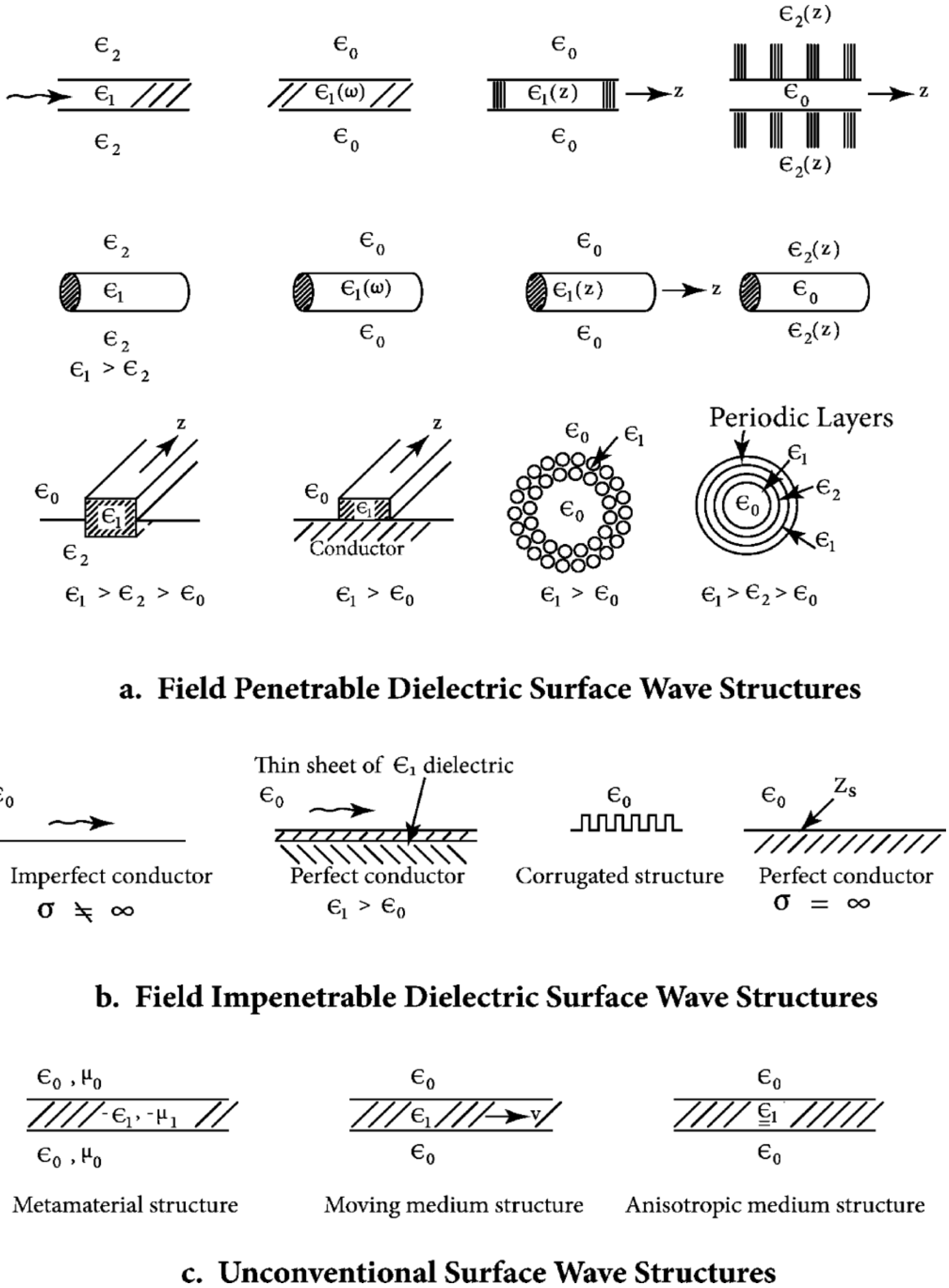

Figure 2.1. Examples of surface wave structures

2.6 Approach to the Surface Waveguide Problem

To obtain the analytical solution to the problem of wave guidance along a straight line dielectric (or surface wave) structure, the following items must be taken into consideration:

1. For guided modes propagating along a perfectly straight line path, we assume that all components of the electromagnetic wave can be represented in the form

$$f(u, v)\, e^{-j\beta z e j\omega t}, \qquad (2.40)$$

in which z is chosen as the propagation direction, (u, v) are generalized orthogonal coordinates in the transverse plane, β is the propagation constant, and ω is the angular frequency of the wave.

2. Solutions for the longitudinal fields (E_z, H_z) must satisfy the scalar wave equation

$$\nabla^2_t + (\omega^2\mu \varepsilon - \beta^2)E_z + \nabla^2_t + (\omega^2\mu \varepsilon - \beta^2) H_z = 0, \quad (2.41)$$

where ∇_t is the transverse del operator, (μ, ε) are the material parameters for the medium in which the fields reside.

3. Field solutions for a given guiding structure must satisfy the appropriate boundary conditions given below: Satisfying the boundary conditions will yield a transcendental equation for the guided modes. For a given frequency, the transcendental equation may yield a finite number of real roots. These discrete roots represent the eigenvalues of the surface wave modes. Corresponding to these eigenvalues are the eigen functions of the guided modes. The radiation solution must be included to form the complete solution. Furthermore, for a given frequency ω, there may be a case where no real root exists. In that case, a guided mode simply does not exist for that situation; only a radiated wave can be present.

4. In general, the transverse electric fields for the electromagnetic wave on an open structure satisfy certain orthogonality relations and can be expressed as follows

$$\mathbf{E}_t(x, y, z) = CE_t^{radiated}(x, y, z) + \Sigma A_p \mathbf{E}_{tp}^{guided}(x, y)\, e^{-j\beta_p z}, \quad (2.42)$$

where the subscript t signifies the transverse fields (transverse to the direction of propagation of the guided wave), the index p represents the summation index for the number of eigen surface wave modes that can exist for a given frequency, C is the amplitude coefficient for the radiated wave, A_p are the amplitude coefficients for the surface wave modes, and β_p are the propagation constants for the surface wave modes.

Chapter 3

EQUATIONS WITH WAVE INCIDENCE

3.1 Introduction

A waveguide is a hollow structure in which waves can propagate without any distortion or attenuation along the invariance direction of the guide. Such waves are generally dispersive and their dispersion relation can be obtained by solving self-adjoint eigen value problems. A Wave guide is said to be closed if it is transversally bounded, if the cross section is unbounded, one has an open waveguide for the open waveguide the presence of a continuum of radiating modes which are not guided waves makes all the questions more difficult.

3.2 Planar Dielectric Waveguides wave

The very first canonical solution for a given physical problem is usually obtained analytically for a planar structure. Analysis of a canonical slab (planar) dielectric waveguide is done leading to the detailed theoretical solutions, and also provide detailed analyses on other planar structures like a leaky slab dielectric waveguide, a multi layered (or inhomogeneous) dielectric waveguide and coupled planar dielectric waveguides.

3.2.1 Fundamental Equations with sine wave incidence

Using rectangular coordinates (*x, y, z*), the equations of E_x, E_y, H_x and H_z are written as

$$E_x := \frac{1}{p^2}\cdot\left(-j\cdot\beta\cdot\frac{\delta E_z}{\delta x} - j\cdot\omega\cdot\mu\cdot\frac{\delta H_z}{\delta y}\right)$$

$$E_y := \frac{1}{p^2}\cdot\left(-j\cdot\beta\cdot\frac{\delta E_z}{\delta y} + j\cdot\omega\cdot\mu\cdot\frac{\delta H_z}{\delta x}\right)$$

$$H_x := \frac{1}{p^2}\cdot\left(j\cdot\omega\cdot\varepsilon\cdot\frac{\delta E_z}{\delta y}\cdot -j\cdot\beta\cdot\frac{\delta H_z}{\delta x}\right)$$

$$H_y := \frac{1}{p^2}\cdot\left(-j\cdot\omega\cdot\varepsilon\cdot\frac{\delta E_z}{\delta x}\cdot -j\cdot\beta\cdot\frac{\delta H_z}{\delta y}\right) \qquad (3.1)$$

with

$$\left(\frac{\delta^2}{\delta\cdot x^2}+\frac{\delta^2}{\delta\cdot y^2}+p^2\right)\cdot\frac{E_z}{H_z}=0$$

where $p^2 = \omega^2\mu\boldsymbol{\varepsilon} - \beta^2$ (3.2)

Here β is the propagation constant of the fields and $\boldsymbol{\varepsilon}$ and μ are the constitutive parameters of the medium in which fields reside. The factor $e^{-j\beta z+j\omega t}$ is attached to all field components. For the waves guided by any planar structures, the propagation constant β must be identical in all regions of the guiding structure. Other conditions, such as the boundary conditions, the radiation condition, and/or the edge conditions, must also be satisfied by the modal fields. The modal fields may be classified into four types:

TEM [(*Ex*,*Hy*) *or* (*Ey*,*Hx*)] with *Ez* and *Hz* = 0,

TE [(*Hz*,*Ex*,*Ey*,*Hx*,*Hy*)] with *Ez* = 0,

TM [(*Ez*,*Ex*,*Ey*,*Hx*,*Hy*)] with *Hz* = 0,

HE [(*Ex*,*Ey*,*Ez*,*Hx*,*Hy*,*Hz*)] . (3.3)

3.2.2 Bound modes of slab waveguides

Considering an even TE wave in the slab, Due to symmetry, the structure is same as the lower half replaced by a magnetic wall. The tangential component of E, E_y is continuous across the interface x.

and the normal component of H, H_x. For symmetric modes the distribution of E_y is given as

$E_y(x) = \cos q_s x$: $d \geq x \geq 0$

$= \cos q_s d e^{-\gamma_x(x-d)}$: $x > d$

For anti- symmetric modes the distribution of E_y is given as

$E_y(x) = \sin q_s x$: $d \leq x \leq 0$

$= \sin q_s d e^{-\gamma_x(x-d)}$: $x > d$

Similarly the component of H, H_y (x) for even TM and odd TM mode in terms of cosine and sine is given.

3.3 Rectangular Dielectric Waveguides

Since dielectric waveguides of rectangular cross section have no closed-form solution, an exact analytic solution does not exist for the case of wave propagation along a dielectric waveguide of rectangular shape. Eigen-modes of the waveguide have to be found either numerically or using approximate techniques. Theoretical studies on geometrically simple optical and microwave dielectric waveguides have been presented in the past using approximate or numerical methods. Two most common approximate techniques are the Marcatili approach and by Schlosser and Unger approximate techniques, using rectangular harmonics,

The approximate methods are represented by an analytical approximation introduced by Marcatili and by the effective index method. The numerical techniques such as the variational methods, finite element methods and integral equation methods have been extensively used. These methods have been exclusively applied to two-dimensional problems with most of the existing techniques performing a fine discretization of the cross section. Such discretization introduces many unknowns and strong numerical instabilities. Consequently, an extension of these methods to three-dimensional problems faces many practical limitations and requires special care. The main disadvantages of them are the time and computing resources limitations and impossibility for an analytical analysis of the solution. Of all the methods for analysing dielectric waveguides which have been well addressed in the literature, the most commonly used is the mode-matching technique.

With this technique, the transverse plane of the waveguide is divided into different regions such that in each region canonical eigen functions can be used to represent the electromagnetic fields. The eigen value problem is constructed by enforcing the boundary conditions at the interface of each region.

In this work, mode matching has been done at all the air dielectric interfaces and thus the characteristic equations have been derived. With the introduction of two ratios the characteristic equations have then been solved graphically to obtain the values of the propagation constant.

3.3.1 Mode configuration in Rectangular dielectric waveguide

In wave guide generally the direction of propagation is z-axis and simplifying the field expression by transverse dependence from the longitudinal one. The common choice is to drive the field from two potentials, E and H directed along the z-axis as

$$F_e = Z_o F_e(r)$$

$$F_h = Z_o F_h(r) \qquad (3.4)$$

Where F_e and F_h are Electric and magnetic field because of field F(r) in z direction. As F_e generate magnetic field entirely contained in transverse plane therefore it is designated as TM wave while F_h generate electric filed totally in transverse plane thus designated as TE wave, so for TM mode $F_h(z)=0$ and for TE mode $F_e(z)=0$.

Rectangular dielectric waveguide modes are neither pure TM nor pure TE. E^y_{nm} and E^x_{nm} as well as hybrid modes (all six components E_x, E_y, E_z, H_x, H_y, H_z) can be supported by rectangular dielectric waveguide. The scheme adopted is based on the fact that in the limit for large aspect ratio, short wavelength, and small refractive index difference, the transverse electric field is primarily parallel to one of the transverse axes. In this, modes are designated as E^y_{mn}, if in the limit the electric field is parallel to the y-axis, and as E^x_{mn} if in the limit their electric field is parallel to x-axis. The m and n subscript are used to designate the number of maxima in x and y direction.

3.3.2 Various methods for calculating solution of rectangular wave guide

As analytic solution does not exist for rectangular dielectric waveguide, they have been solved by approximate techniques. Two approximate techniques that have been used mainly are :

(a) the Marcatili approach [9] and (b) the circular harmonic point matching technique [10]. Other notable approximate techniques by Schlosser and Unger [11], using rectangular harmonics, by Eyges, et al., using the extended boundary condition method, and by Shaw et al. [13], using a variational approach have been used and are very computationally complex.

3.4 Approximate Methods

For dielectric waveguides, the only canonical shapes with closed surfaces in the transverse plane that possess exact analytic solutions to guided wave problems are the circular cylinder and the elliptical cylinder. An exact analytic solution does not exist for the case of wave propagation along a dielectric waveguide of rectangular shape. So approximate methods are used to get the solution of rectangular wave guide.

3.4.1 Marcatili Approximate Method

Marcatili approach is that for a well guided mode in a dielectric waveguide, most of its guided power is contained within the core region of the guide. Very little power of that mode resides in the cladding region of the guide. So, if the boundary conditions are satisfied by most of the assumed fields that carry most of the guided power, then these assumed fields may be used to approximate the solution for the problem. For a rectangular waveguide shown in Fig. 3.1, very little guided power is contained in the corner regions of the guide, that is, the shaded regions. Marcatili formulated an approximate solution to this problem by ignoring the matching of fields along the edges of the shaded areas.

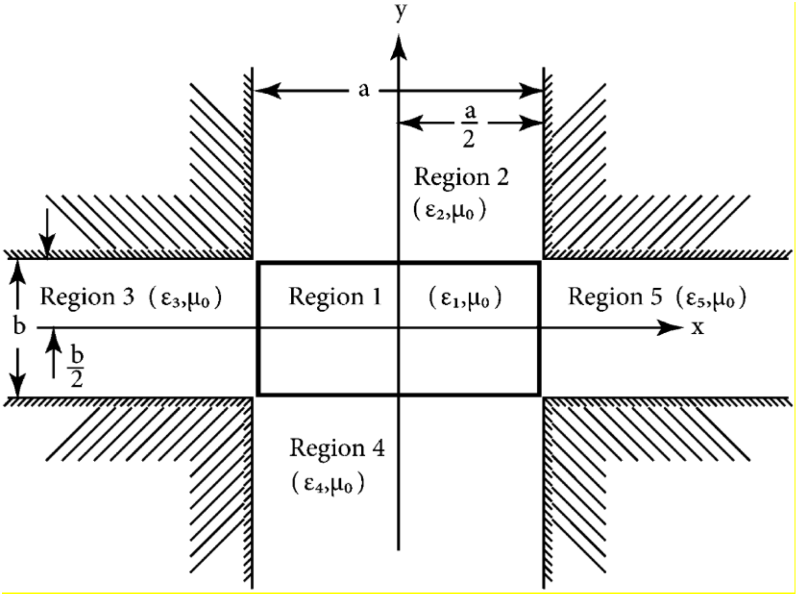

Figure 3.1. Cross-sectional rectangular geometry for Marcatili approximate approach

By ignoring the matching of the fields along the shaded boundaries, shown in Fig. 3.1, we may obtain the expressions for the fields that must satisfy the Maxwell equations and the boundary conditions along the sides of the core region, defined as region 1. It appears that there exist two independent families of modes on this rectangular dielectric waveguide: one family has most of its electric field polarized in the *y*-direction, designated as E^{y}_{nm} modes, and the other has most of its electric field in the *x*-direction, designated as E^{x}_{nm} modes. The subscripts *n* and *m* represent, respectively, the number of extrema that the field components for this mode have along the *x* and *y* directions.

The dominant modes correspond to m=n=1.These designations are independent of the coordinate systems used to describe the waveguides. It is recognized, however, that since the fields are linear, the superposition theorem applies. This means that combinations of any of the above mentioned modal fields are allowed. So new types of modes consisting of linear combinations of the established TE, TM, or HE modes may be constructed depending on the suitability of the situation. The field components for the E^y_{nm} modes are *Ex, Ey, Ez, Hx,* and *Hz,* with *Hy* = 0. The companion independent set of fields for the E^x_{nm} modes are *Ex, Ez, Hx, Hy,* and *Hz,* with *Ey* = 0. The complete set of fields is the sum of the E^y_{nm} and E^x_{nm} modal fields. If E_y is the dominant electric field for E^y_{nm} modes, then the other dominant fields must be *Ez, Hx,* and *Hz,* while *Ex* and *Hy* may be neglected. Similarly, if *Ex* is the dominant electric field for E^x_{nm} modes, then the other dominant fields must be *Ez, Hy,* and *Hz,* while *Ey* and *Hx* may be neglected.

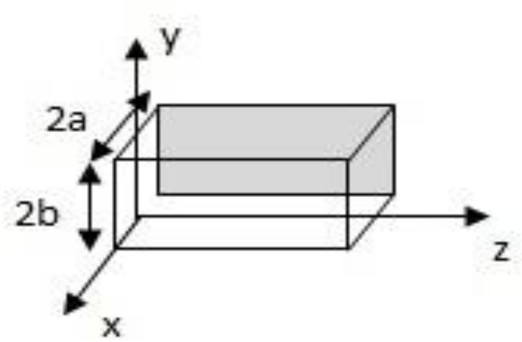

Fig 3.2 Wave Propagation along z- axis

Assuming propagation along z-axis as shown in fig 3.2 and in the view of above discussion the wave function can be written with parameters u and u1 as the transverse propagation constants inside the dielectric guide and v and v1 are the attenuation constants outside the guide. By applying the mode matching approach at dielectric interfaces in the transverse directions and applying appropriate boundary conditions. The Characteristic Equations for E^y_{mn} Modes and E^x_{mn} Modes are calculated. Based upon these equations value of u1 and v1 are calculated and the propagation constant along z direction will be calculated. E^y_{mn} Modes E^x_{mn} Modes as well as hybrid modes can be supported by the guide. The wave guidance takes place by the total internal reflection at the side walls. The solutions to the rectangular dielectric guide problem can be derived by assuming guided mode propagation along the dielectric, and exponential decay of field transverse to the dielectric surface. Thus in the region of confinement, (inside the guide) due to reflections there will be standing wave patterns and when the field goes out of the boundary of the guide, then in the absence of reflection, the field moves away from the guide exponentially i.e. there is an exponential decay of fields transverse to the dielectric surface. The fields are assumed to be approximately cosine and sinusoidally distributed inside the waveguide and decaying exponentially outside. That is, we can express the field components, as follows:

Assuming propagation along *z*-axis and in the view of above discussion the wave function can be written as either

$\Psi even = A \, Cos \, ux \, Cos \, u1y \, e^{-jkzz}$ $|x| \le a, \; |y| \le b$ (inside the guide)

$= B \, e^{-vx} \, Cos \; u1y \, e^{-jkzz}$ $|x| \ge a, \; |y| \le b$ (outside the guide)

$= C \, Cos \, ux \, e^{-v1y} \, e^{-jkzz}$ $|x| \le a, \; |y| \ge b$ (outside the guide) (3.5)

The parameters *u* and *u*1 are the transverse propagation constants inside the dielectric guide and *v* and *v*1 are the attenuation constants outside the guide.

or,

$\Psi odd = A \, Sin \, ux \, Sin \, u1y \, e^{-jkzz}$ $|x| \le a, \; |y| \le b$ (inside the guide)

$= B \, e^{-vx} \, SIn \, u1y \, e^{-jkzz}$ $|x| \ge a, \; |y| \le b$ (outside the guide)

$= C \, Sin \, ux \, e^{-v1y} \, e^{-jkzz}$ $|x| \le a, \; |y| \ge b$ (outside the guide) (3.6)

The regions $x > a$ and $y > b$ have been neglected as fields are very very weak at the corners. (approximate methods).

Applying wave equation to Equation (1) the separation parameter equations in each region become

$$u^2 + u_1^2 + k_z = k_d = \omega^2 \, \varepsilon_d \, \mu_d \qquad \text{(inside the guide) (3.7)}$$

$$-v^2 - v_1^2 + k_z^2 = 2k^20 - k^2d = 2\omega^2 \, \varepsilon_o \, \mu_o - \omega^2 \, \varepsilon_d \, \mu_d \qquad \text{(outside the guide) (3.8)}$$

Taking the case of Ψeven, i.e., Ψ an even function of *x* & *y* explicitly and then similarly for Ψodd we proceed in analyzing the rectangular dielectric waveguide. The approach is the mode matching at dielectric interfaces in the transverse directions using appropriate boundary conditions.

3.4.1.1 The E^y_{nm} Modes

As, *Hy* = 0 for all E^y_{nm} modes. Substituting this into the time-harmonic Maxwell equations in a simple medium (ε, μ) and with the assumption that the factor $e^{j\omega t - j\beta z}$ is attached to all field components and suppressed, we have, in the rectangular coordinates,

$$E_x = \frac{1}{j \cdot \omega \cdot \varepsilon} \frac{\delta H_z}{\delta \cdot y}$$

$$E_y = \frac{1}{j \cdot \omega \cdot \varepsilon} \frac{\delta H_z}{\delta \cdot x} - \frac{\beta}{\omega \cdot \varepsilon} \cdot H_x$$

$$E_z = -\left(\frac{1}{j\cdot\omega\cdot\varepsilon}\right)\frac{\delta H_x}{\delta\cdot y}$$

$$H_x = -\frac{1}{j\cdot\omega\cdot\mu}\left(\frac{\delta E_z}{\delta\cdot y} + j\cdot\beta\cdot E_y\right)$$

$$H_z = -\frac{1}{j\cdot\omega\cdot\mu}\left(-\frac{\delta E_y}{\delta\cdot x} - \frac{\delta\cdot E_x}{\delta\cdot y}\right) \tag{3.5}$$

With

$$\nabla^2 E_{x,y,z} + k^2 E_{x,y,z} = 0,$$
$$\nabla^2 H_{x,y,z} + k^2 H_{x,y,z} = 0, \tag{3.6}$$
$$k^2 = \omega^2 \mu \varepsilon.$$

Now from these equations we get the characteristics equation in $\boldsymbol{E}^y_{nm}$ modes as

(3.7)

Solving the equation, we get the equations for even and odd modes of E^y_{mn} for inside and outside fields. By applying the continuity equations we can get the characteristics equations. The characteristics equations are solved graphically and value of transverse propagation constant are derived for dominant mode. There by the value of propagation constant in z axis is calculated. The normalized propagation constant comes out to be nearly two and is lesser at lower frequencies and increases with increase in frequency.

3.4.1.2 The E^x_{mn} Modes

In a similar way when the E^y_{mn} mode is changed to E^y_{mn}, the field components are given by The $\boldsymbol{E}^x_{nm}$ Modes, and we can get the characteristics equations for E^x_{nm} Modes as

(3.8)

3.5 Field distribution in E^y_{nm} and E^x_{nm} modes

Inspection of the field expression for E^y_{nm} and E^x_{nm} modes the variations of these fields for several lower order modes is shown in fig 3.2

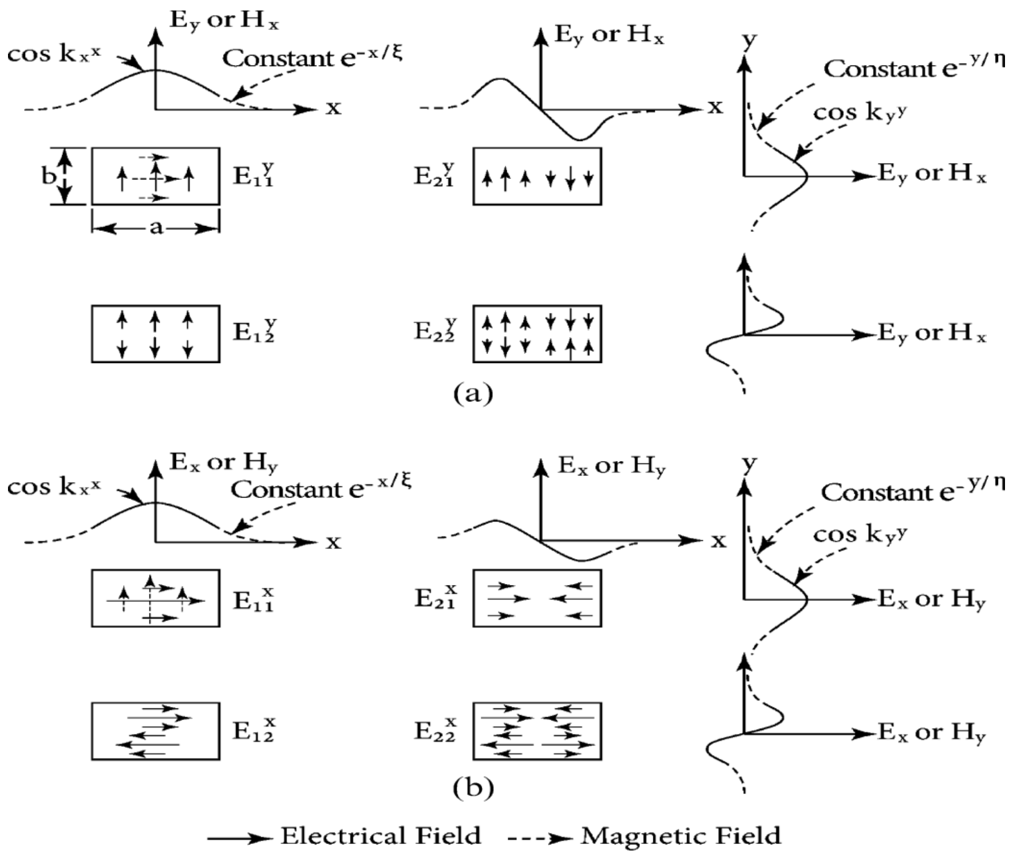

Fig 3.3 field variations of E^y_{nm} and E^x_{nm} modes for several lower order modes

3.6 Wave equation with Square wave incidence

Now, the fields are assumed to be approximately square wave distributed inside the waveguide and decaying exponentially outside. Then we express the field components, as follows. Assuming propagation along z-axis, the wave function can be written as either

$\Psi even = A\ Rect^{e}(ux)\ Rect^{e}(u1y)\ e^{-jk_z z}$ $\quad |x| \le a,\ |y| \le b$ (inside guide)

$= B\ e^{-vx}\ Rect^{e}(u1y)\ e^{-jk_z z}$ $\quad |x| \ge a,\ |y| \le b$ (outside guide)

$= C\ Rect^{e}(ux)\ e^{-v1y}\ e^{-jk_z z}$ $\quad |x| \le a,\ |y| \ge b$ (outside guide) (3.9)

Here the $Rect^{e}$ () function is even rectangular field and it can be broken in harmonics of cosine function for fields. Where u, u1, K_z are propagation constant inside the dielectric in x, y and z direction and v, v1 are attenuation constant in x and y directions outside the dielectric region. Assuming propagation in z-direction according to $e^{-jk_z z}$. The regions $x > a$ and $y > b$ have been neglected as fields are very weak at the corners.

Or, the wave function for odd distribution has been written as

$\Psi odd = A\ Rect^{o}(ux)\ Rect^{o}(u1y)\ e^{-jk_z z}$ $\quad |x| \le a,\ |y| \le b$ (inside guide)

$= B\ e^{-vx}\ Rect^{o}(u1y)\ e^{-jk_z z}$ $\quad |x| \ge a,\ |y| \le b$ (outside guide)

$= C\ Rect^{o}(ux)\ e^{-v1y}\ e^{-jk_z z}$ $\quad |x| \le a,\ |y| \ge b$ (outside guide) (3.10)

Here the $Rect^{o}$ () function is odd rectangular field and it can be broken in harmonics of sine function. The propagation constant of the dielectric can be calculated by the equations.

$u^2 + u_1^2 + k_z = kd = \omega^2 \varepsilon d\ \mu d$ (inside the guide) (3.11)

$-v^2 - v_1^2 + k_z^2 = 2k^2 0 - k^2 d = 2\omega^2 \varepsilon o\ \mu o - \omega^2 \varepsilon d\ \mu d$ (outside the guide) (3.12)

3.6.1 Fourier series expansion of rectangular wave

First we need to find the Fourier components of rectangular field and resolving that in terms of cosine and sine functions for even and odd field distribution.

(a) Assuming rectangular field as even function in distribution:-

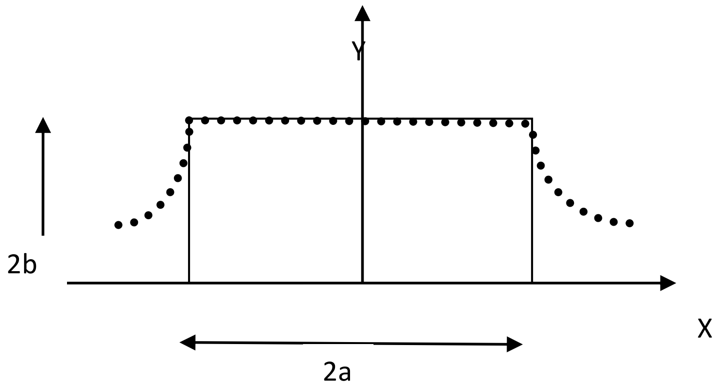

Fig 3.4 Even rectangular Field distribution inside the dielectric wave guide

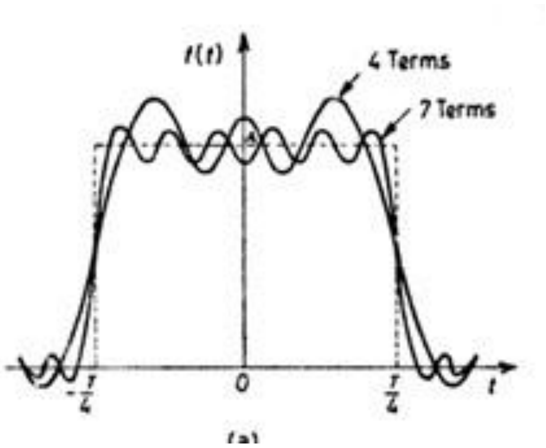

Fig 3.5 Representation of rectangular wave shape by harmonics

Assuming the rectangular co-ordinate of the wave guide with length 2a along x -axis and width 2b along y- axis. the Fourier series expansion of even Rect^e () function as harmonics of cosine function comes out to be, in terms of 2, 3 and 4 harmonics as.

$$F(x) := \frac{4b}{\pi}\cos\left(\frac{\pi x}{a}\right) - \frac{4b}{3\pi}\cdot\cos\left(\frac{3\pi x}{a}\right)$$

$$F(x) := \frac{4b}{\pi}\cos\left(\frac{\pi x}{a}\right) - \frac{4b}{3\pi}\cdot\cos\left(\frac{3\pi x}{a}\right) + \frac{4b}{5\pi}\cos\left(\frac{5\pi x}{a}\right)$$

$$F(x) := \frac{4b}{\pi}\cos\left(\frac{\pi x}{a}\right) - \frac{4b}{3\pi}\cdot\cos\left(\frac{3\pi x}{a}\right) + \frac{4b}{5\pi}\cos\left(\frac{5\pi x}{a}\right) - \frac{4b}{7\pi}\cos\left(\frac{7\pi x}{a}\right) \quad (3.13)$$

Replacing even square function as harmonics of three cosine functions the wave equation 3.9 becomes

$$\varphi_{even} := A\cdot\left(\frac{4b}{\pi}\cos\left(\frac{\pi x}{a}\right) - \frac{4b}{3\pi}\cdot\cos\left(\frac{3\pi\, x}{a}\right)\right)\cdot\left(\frac{4a}{\pi}\cos\left(\frac{\pi\, y}{b}\right) - \frac{4a}{3\pi}\cdot\cos\left(\frac{3\pi\, y}{b}\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := B\cdot(e)^{-vx}\cdot\left(\frac{4a}{\pi}\cos\left(\frac{\pi\, y}{b}\right) - \frac{4a}{3\pi}\cdot\cos\left(\frac{3\pi\, y}{b}\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := C\cdot\left(\frac{4b}{\pi}\cos\left(\frac{\pi x}{a}\right) - \frac{4b}{3\pi}\cdot\cos\left(\frac{3\pi\, x}{a}\right)\right)\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{3.14}$$

Taking m=1 and n=1, assume π /a=u and π /b=u1 and normalizing above equations we get the wave function as

$$\varphi_{even} := A\cdot\left(\cos(ux) - \frac{1}{3}\cdot\cos(3u\,x)\right)\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\, y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := B\cdot(e)^{-ux}\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\, y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := C\cdot\left[\cos((u\cdot x)) - \frac{1}{3}\cdot\cos(3u\,x)\right]\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{3.15}$$

(b) Assuming rectangular field as odd function in distribution :-

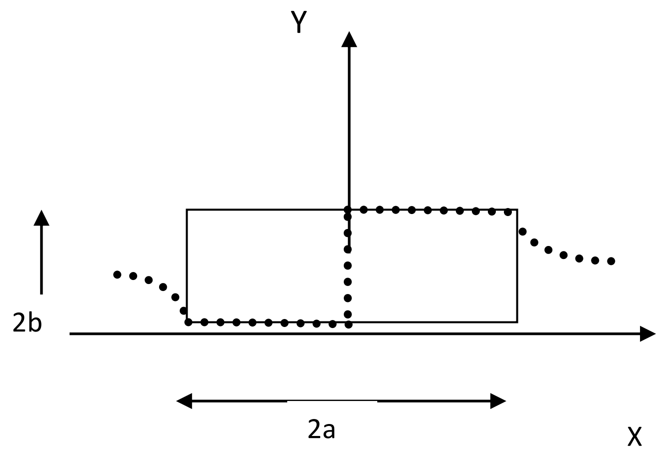

Fig 3.6 Rectangular Field distribution inside the dielectric wave guide

In the similar manner, assuming the rectangular co-ordinate of the wave guide with length 2a along x -axis and width 2b along y- axis. The Fourier series expansion of odd Rect^{o} () function as harmonics of sine function comes out to be, in terms of 2, 3 and 4 harmonics as.

$$F(x) := \frac{4b}{\pi}\sin\left(\frac{\pi x}{a}\right) + \frac{4b}{3\pi}\cdot\sin\left(\frac{3\pi x}{a}\right)$$

$$F(x) := \frac{4b}{\pi}\sin\left(\frac{\pi x}{a}\right) + \frac{4b}{3\pi}\cdot\sin\left(\frac{3\pi x}{a}\right) + \frac{4b}{5\pi}\sin\left(\frac{5\pi x}{a}\right)$$

$$F(x) := \frac{4b}{\pi}\sin\left(\frac{\pi x}{a}\right) + \frac{4b}{3\pi}\cdot\sin\left(\frac{3\pi x}{a}\right) + \frac{4b}{5\pi}\sin\left(\frac{5\pi x}{a}\right) + \frac{4b}{7\pi}\sin\left(\frac{7\pi x}{a}\right) \tag{3.16}$$

Representing harmonic function by rect^{o} function the wave function can be written in terms of **odd function** as

$$\varphi_{odd} := A\cdot\left(\frac{4b}{\pi}\sin\left(\frac{\pi x}{a}\right) + \frac{4b}{3\pi}\cdot\sin\left(\frac{3\pi x}{a}\right)\right)\cdot\left(\frac{4a}{\pi}\sin\left(\frac{\pi y}{b}\right) + \frac{4a}{3\pi}\cdot\sin\left(\frac{3\pi y}{b}\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := B\cdot(e)^{-vx}\cdot\left(\frac{4a}{\pi}\sin\left(\frac{\pi y}{b}\right) + \frac{4a}{3\pi}\cdot\sin\left(\frac{3\pi y}{b}\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := C\cdot\left(\frac{4b}{\pi}\sin\left(\frac{\pi x}{a}\right) + \frac{4b}{3\pi}\cdot\sin\left(\frac{3\pi x}{a}\right)\right)\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{3.17}$$

Assume π /a=u and π /b=u1 and normalizing above equations we get

$$\varphi_{odd} := A\cdot\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3u\,x)\right)\cdot\left(\sin\left(u_1\cdot y\right) + \frac{1}{3}\cdot\sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := B\cdot(e)^{-vx}\cdot\left(\sin\left(u_1\cdot y\right) + \frac{1}{3}\cdot\sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := C\cdot\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3u\,x)\right)\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{3.18}$$

Chapter 4
GRAPHICAL PRESENTATION OF WAVE FUNCTION

4.1 Graphical presentation

From equations the graphical presentation of fields has be plotted by using 3-D graphical tool of Matlab. The field plotted are for dominant mode of rectangular dielectric waveguide for $\mathbf{E^{y}_{mn}}$ **mode**, and $\mathbf{E^{x}_{mn}}$ **mode** with m=1 and n=1. These fields are for waveguide with dimensions 2a=1mm, 2b=0.5mm and u=(m*pi)/(2*a) and u1=(n*pi)/(2*b).[2] Graphs are plotted for 1, 2 and 4 harmonics. Outside exponential decay graph are plotted for 2 and 4 harmonics and exponential decay is similar in both x and y direction. So only one graph of one direction is shown.

4.2 Graphical presentation of even wave function

Taking propagation along z-axis, the wave function for even distribution can be written as.

$$\Psi even = A\ Rect^{e}(ux)\ Rect^{e}(u1y)\ e^{-jk_z z} \qquad |x| \le a,\ |y| \le b \text{ (inside guide)}$$

$$= B\ e^{-vx}\ Rect^{e}(u1y)\ e^{-jk_z z} \qquad |x| \ge a,\ |y| \le b \text{ (outside guide)}$$

$$= C\ Rect^{e}(ux)\ e^{-v1y}\ e^{-jk_z z} \qquad |x| \le a,\ |y| \ge b \text{ (outside guide)}$$

The 3-D graph plotted are as showed in figures with the field varying as even function around origin in x and y direction inside the dielectric and decaying exponentially outside the boundary of the dielectric. The amplitudes of field plotted are normalized so that the graphical representation becomes same in both the directions.

Firstly the field plot of dominant mode is shown and then the field plots of other modes are shown. Field gives a better understanding of even and odd function with harmonics.

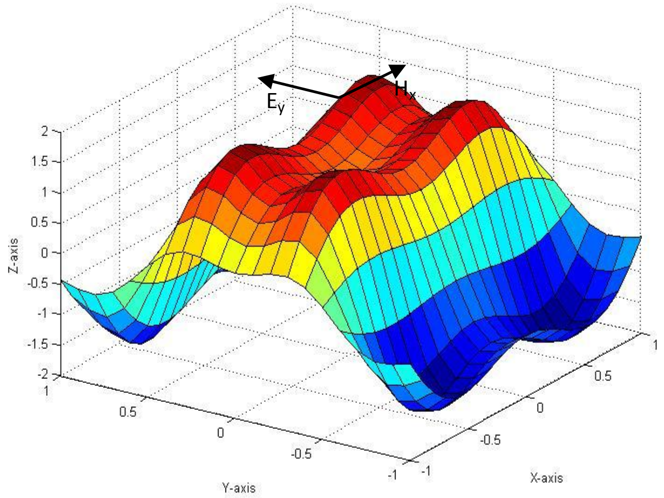

Fig. 4.1. Even field for E^y_{11} mode inside waveguide for one harmonic

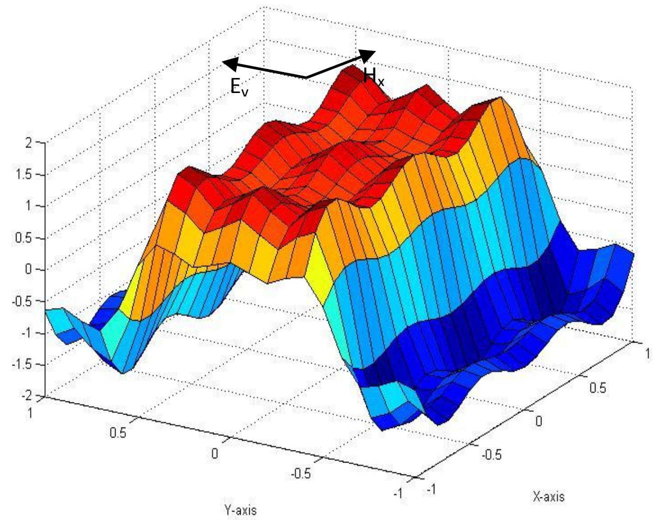

Fig. 4.2. Even field for E^y_{11} mode inside waveguide for two harmonics

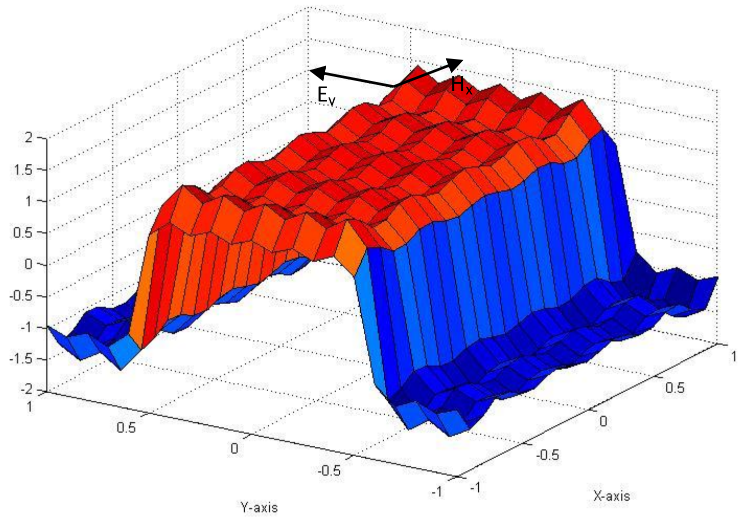

Fig. 4.3. Even field for E^{y}_{11} mode inside waveguide for four harmonics

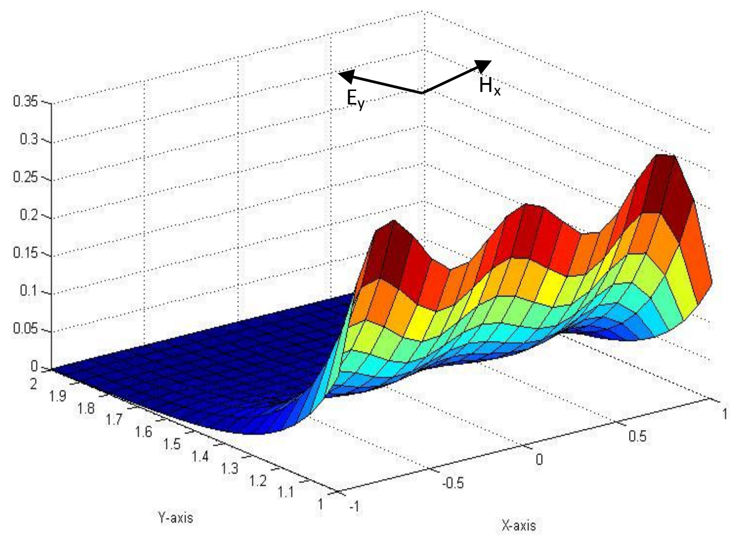

Fig. 4.4. Even field exponential decay for E^{y}_{11} mode outside the waveguide for two harmonics

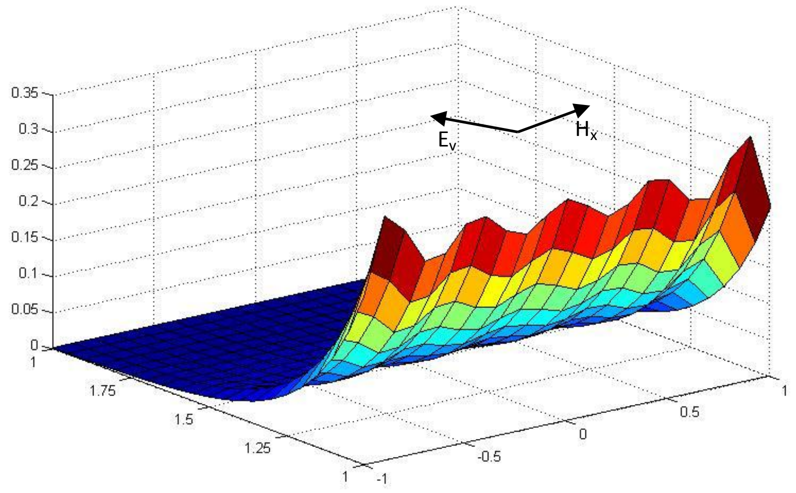

Fig. 4.5. Even field exponential decay for E^y_{11} mode outside the waveguide for four harmonics

4.3. Graphical presentation of even wave function for other modes

Field graph is plotted for m=1, n=3 for even distribution of field and similarly other mode can also be plotted for different values of m and n.

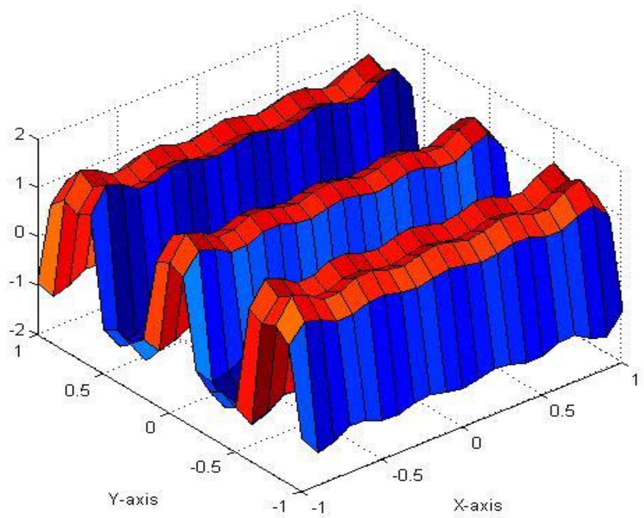

Fig. 4.6. Even field distribution for E^y_{13} mode inside waveguide for four harmonics

4.4. Graphical presentation of odd wave function

Similarly, 3-D graph has been plotted for field varying as odd function around origin in x and y direction inside the dielectric and decaying exponentially outside the boundary of the dielectric.

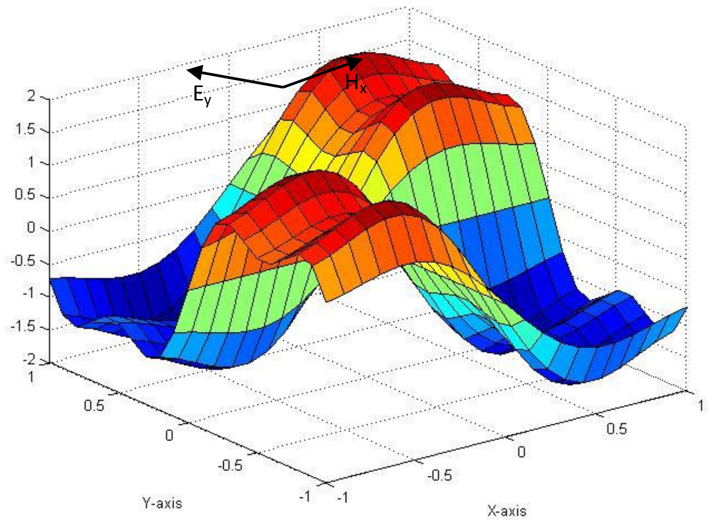

Fig. 4.7. Odd field for E^{y}_{11} mode inside waveguide for one harmonic

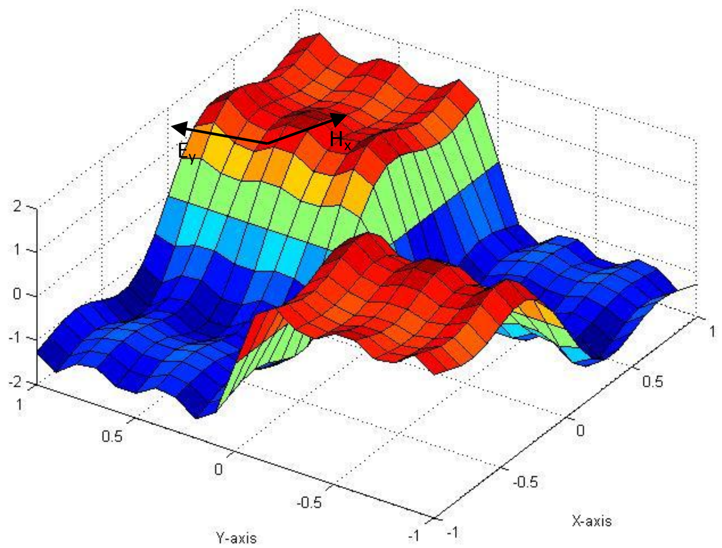

Fig. 4.8. Odd field for E^{y}_{11} mode inside waveguide for two harmonics

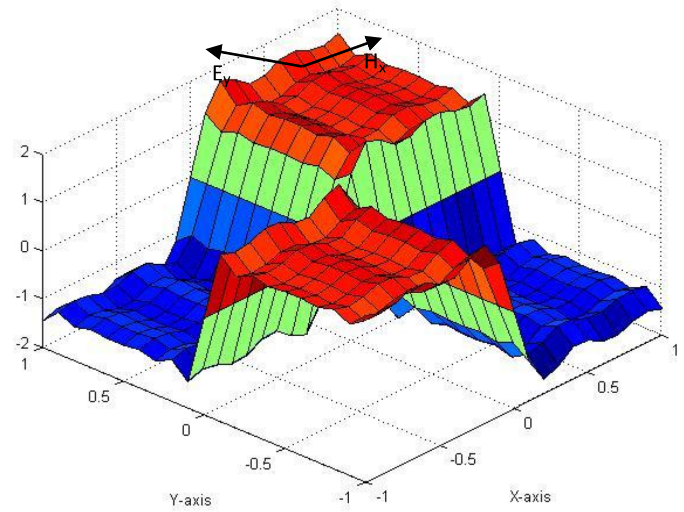

Fig. 4.9. Odd field for E^y_{11} mode inside waveguide for four harmonics

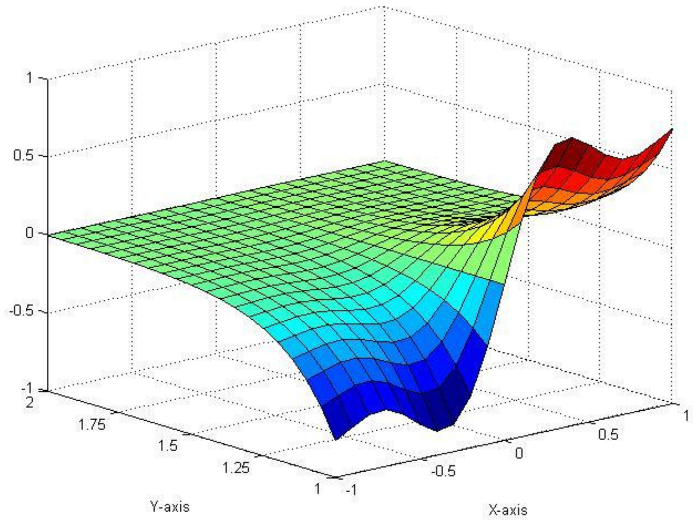

Fig. 4.10. Odd field exponential decay for E^y_{11} mode outside the waveguide for two harmonics

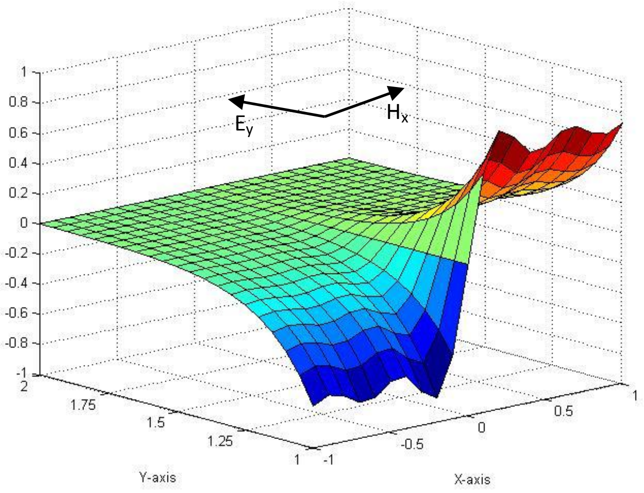

Fig. 4.11 Odd field exponential decay for E^{y}_{11} mode outside the waveguide for four harmonics.

4.5 Graphical presentation of odd wave function for other modes

Field graph is plotted for m=1, n=3 for odd distribution of field and similarly other modes can also be plotted for different values of m and n.

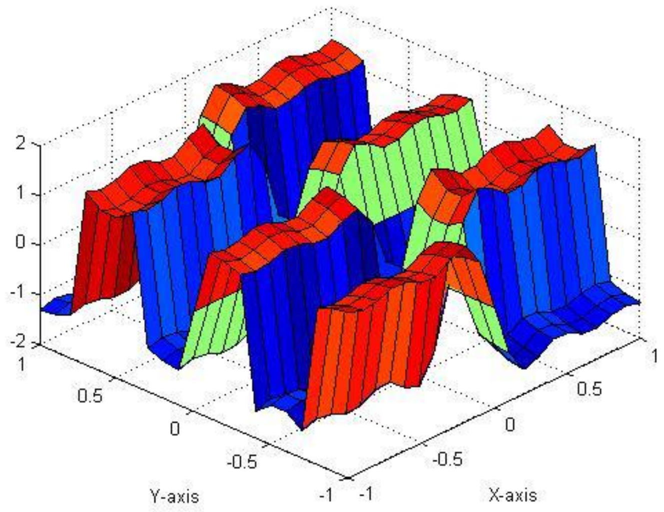

Fig. 4.12. Odd field distribution for E^{y}_{13} mode inside waveguide for four harmonics

Chapter 5

SOLUTION OF EQUATIONS WITH SQUARE WAVE INCIDENCE

5.1 Introduction

Mode matching techniques can be applied to find the characteristic equation of waveguide with square wave incidence from the equations calculated earlier by. We compare the field inside and outside the dielectric wave guide and derive the characteristics equation. Solution of characteristics equation is calculated by graphical method. Giving us value of transverse propagation constant and normalized propagation constant is calculated. It is compared with the propagation constant calculated by Marcatili and Goell methods.

5.2 Field equations for E^y_{mn}

The wave function of even symmetric square wave incidence for inside and our side field is given by

$$\varphi_{even} := A\cdot\left(\cos(ux) - \frac{1}{3}\cdot\cos(3ux)\right)\cdot\left(\cos(u_1\cdot y) - \frac{1}{3}\cdot\cos(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := B\cdot(e)^{-ux}\cdot\left(\cos(u_1\cdot y) - \frac{1}{3}\cdot\cos(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{even} := C\cdot\left[\cos((u\cdot x)) - \frac{1}{3}\cdot\cos(3ux)\right]\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{5.1}$$

And odd symmetric wave function for square wave incidence is as

$$\varphi_{odd} := A\cdot\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3ux)\right)\cdot\left(\sin(u_1\cdot y) + \frac{1}{3}\cdot\sin(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := B\cdot(e)^{-vx}\cdot\left(\sin(u_1\cdot y) + \frac{1}{3}\cdot\sin(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$\varphi_{odd} := C\cdot\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3ux)\right)\cdot(e)^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{5.2}$$

The field components of E^y_{mn} modes in terms of fields can be calculated by the equations

given as

$$E_x := \frac{1}{y}\cdot\left[\frac{d}{dx}\left(\frac{d}{dy}\varphi\right)\right] \qquad E_y := \frac{1}{y}\cdot\left[\frac{d^2}{dy^2}(\varphi) + k^2\varphi\right] \qquad E_z := \frac{1}{y}\cdot\left[\frac{d}{dy}\left(\frac{d}{dz}\varphi\right)\right]$$

$$H_x := -\left(\frac{d}{dz}\varphi\right) \qquad H_y := 0 \qquad H_z := -\left(\frac{d}{dx}\varphi\right) \tag{5.3}$$

Solving the individual component of even symmetric field inside the guide from equation 5.1

$$E_x := \frac{A\cdot u\cdot u_1}{y_d\lambda}\cdot(\sin(ux) - \sin(3u\,x))\cdot\left(\sin\left(u_1\cdot y\right) - \sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$E_y := \frac{A}{y_d\lambda}\cdot\left(\cos(ux) - \frac{1}{3}\cdot\cos(3u\,x)\right)\cdot(e)^{-jk_z\cdot Z}\cdot\left[k_d^2\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right) - u_1^2\cdot\left(\cos\left(u_1\cdot y\right) - 3\cdot\cos\left(3u_1\,y\right)\right)\right]$$

$$E_z := \frac{A\cdot u_1\cdot j\cdot k_z}{y_d\lambda}\cdot\left(\cos(ux) - \frac{1}{3}\cdot\cos(3u\,x)\right)\cdot\left(\sin\left(u_1\cdot y\right) - \sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$H_x := \left(A\cdot j\cdot k_z\right)\cdot\left(\cos(ux) - \frac{1}{3}\cdot\cos(3u\,x)\right)\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$H_y := 0 \qquad H_z := (-A\cdot u)\cdot(\sin(ux) - \sin(3u\,x))\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z} \tag{5.4 a}$$

$|x| \ge a, |y| \le t$

$$E_x := \frac{B\cdot v\cdot u_1}{y_o\lambda}\cdot e^{-vx}\cdot\left(\sin\left(u_1\cdot y\right) - \sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$E_y := \frac{B}{y_o\lambda}\cdot e^{-vx}\cdot(e)^{-jk_z\cdot Z}\cdot\left[k_o^2\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right) - u_1^2\cdot\left(\cos\left(u_1\cdot y\right) - 3\cdot\cos\left(3u_1\,y\right)\right)\right]$$

$$E_z := \frac{B\cdot u_1\cdot j\cdot k_z}{y_o\lambda}\cdot e^{-vx}\cdot\left(\sin\left(u_1\cdot y\right) - \sin\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$H_x := \left(B\cdot j\cdot k_z\right)\cdot e^{-vx}\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$H_y := 0 \qquad H_z := -B\cdot v\cdot e^{-vx}\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1\,y\right)\right)\cdot(e)^{-jk_z\cdot Z} \tag{5.4b}$$

For $|x| \leq a, |y| \geq b$

$$E_x := \frac{C \cdot u \cdot v_1}{y_o^{\lambda}} \cdot (\sin(u \cdot x) - \sin(3u\, x)) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z}$$

$$E_y := \frac{C}{y_o^{\lambda}} \cdot \left(\cos(ux) - \frac{1}{3} \cdot \cos(3u\,x)\right) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z} \cdot \left(k_o^2 + v_1^2\right)$$

$$E_z := \frac{C \cdot v_1 \cdot j \cdot k_z}{y_o^{\lambda}} \cdot \left(\cos(ux) - \frac{1}{3} \cdot \cos(3u\,x)\right) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z}$$

$$H_x := (C \cdot j \cdot k_z) \cdot \left(\cos(ux) - \frac{1}{3} \cdot \cos(3u\,x)\right) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z}$$

$$H_y := 0$$

$$H_z := -C \cdot u \cdot (\sin(u \cdot x) - \sin(3u\,x)) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z}$$

(5.4c)

5.3 Applying Boundary condition and comparing fields at interfaces

5.3.1 at x=a

At the interface x=a in y-z plane the tangential components of E & H should be continuous. Thus E_y and H_z are assumed to be continuous. So writing the equations and applying mode matching techniques eqn 5.4 gives

$$E_{y.inside} := \frac{A}{y_d^{\lambda}} \cdot \left(\cos(ux) - \frac{1}{3} \cdot \cos(3ux)\right) \cdot (e)^{-jk_z \cdot Z} \cdot \left[k_d^2 \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y)\right) - u_1^2 \cdot \left(\cos(u_1 \cdot y) - 3 \cdot \cos(3u_1 y)\right)\right]$$

$$E_{y.outside} := \frac{B}{y_o^{\lambda}} \cdot e^{-vx} \cdot (e)^{-jk_z \cdot Z} \cdot \left[k_o^2 \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y)\right) - u_1^2 \cdot \left(\cos(u_1 \cdot y) - 3 \cdot \cos(3u_1 y)\right)\right]$$

$$H_{z.inside} := (-A \cdot u) \cdot (\mathrm{sn}(ux) - \sin(3u\,x)) \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y)\right) \cdot (e)^{-jk_z \cdot Z}$$

$$H_{z.outside} := -B \cdot v \cdot e^{-vx} \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y)\right) \cdot (e)^{-jk_z \cdot Z}$$

(5.5)

Comparing field of equation 5.5 the characteristic equation for E^y_{mn} mode becomes for Ey

$$\frac{A}{y_d^{\lambda}}\cdot\left(\cos(ux)-\frac{1}{3}\cdot\cos(3ux)\right)\cdot\left[k_d^2\cdot\left(\cos(u_1\cdot y)-\frac{1}{3}\cdot\cos(3u_1 y)\right)-u_1^2\cdot\left(\cos(u_1\cdot y)-3\cdot\cos(3u_1 y)\right)\right]= \blacksquare$$

$$\frac{B}{y_o^{\lambda}}\cdot e^{-vx}\cdot\left[k_o^2\cdot\left(\cos(u_1\cdot y)-\frac{1}{3}\cdot\cos(3u_1 y)\right)-u_1^2\cdot\left(\cos(u_1\cdot y)-3\cdot\cos(3u_1 y)\right)\right] \tag{5.6a}$$

Solving for Hy

$$A\cdot u\cdot(\mathrm{sn}(ux)-\sin(3ux)) = \blacksquare\cdot B\cdot v\cdot e^{-vx} \tag{5.6b}$$

Dividing Ey by Hy equations 5.5 becomes

$$\frac{1}{y_d^{\lambda}}\cdot\frac{\left(\cos(ux)-\frac{1}{3}\cdot\cos(3ux)\right)\cdot\left[k_d^2\cdot\left(\cos(u_1\cdot y)-\frac{1}{3}\cdot\cos(3u_1 y)\right)-u_1^2\cdot\left(\cos(u_1\cdot y)-3\cdot\cos(3u_1 y)\right)\right]}{u\cdot(\sin(ux)-\sin(3ux))}= \blacksquare$$

$$\frac{1}{(v\cdot y_o)^{\lambda}}\cdot\left[k_o^2\cdot\left(\cos(u_1\cdot y)-\frac{1}{3}\cdot\cos(3u_1 y)\right)-u_1^2\cdot\left(\cos(u_1\cdot y)-3\cdot\cos(3u_1 y)\right)\right] \tag{5.7}$$

5.3.2 At x=b

Similarly applying continuity equation for at y=b for Ez and Hx

$$E_{z.inside} := \frac{A\cdot u_1\cdot j\cdot k_z}{y_d^{\lambda}}\cdot\left(\cos(ux)-\frac{1}{3}\cdot\cos(3ux)\right)\cdot\left(\sin(u_1\cdot y)-\sin(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$E_{z.outside} := \frac{C\cdot v_1\cdot j\cdot k_z}{y_o^{\lambda}}\cdot\left(\cos(ux)-\frac{1}{3}\cdot\cos(3ux)\right)\cdot e^{-v_1\cdot y}\cdot(e)^{-jk_z\cdot Z} \tag{5.8}$$

Comparing Equation (5.8) gives

$$\frac{A\cdot u_1}{y_d^{\lambda}}\cdot\left(\sin(u_1\cdot b)-\sin(3u_1 b)\right) = \blacksquare\cdot\frac{C\cdot v_1}{y_o^{\lambda}}\cdot e^{-v_1\cdot b} \tag{5.9}$$

For Hx equations are

$$H_{x.inside} := (A\cdot j\cdot k_z)\cdot\left(\cos(ux)-\frac{1}{3}\cdot\cos(3ux)\right)\cdot\left(\cos(u_1\cdot y)-\frac{1}{3}\cdot\cos(3u_1 y)\right)\cdot(e)^{-jk_z\cdot Z}$$

$$H_{x.outside} := (C \cdot j \cdot k_z) \cdot \left(\cos(ux) - \frac{1}{3} \cdot \cos(3ux) \right) \cdot e^{-v_1 \cdot y} \cdot (e)^{-jk_z \cdot Z} \quad (5.10)$$

Comparing for Hx the equation 5.10 becomes

$$A \cdot \left(\cos(u_1 \cdot b) - \frac{1}{3} \cdot \cos(3u_1 b) \right) = \blacksquare \cdot C \cdot e^{-v_1 \cdot t} \quad (5.11)$$

Dividing Ez with Hx eqautions 5.9 and 5.11 becomes

$$\frac{u_1}{y_d^{\lambda}} \cdot \frac{(\sin(u_1 \cdot b) - \sin(3u_1 b))}{\cos(u_1 \cdot b) - \frac{1}{3} \cdot \cos(3u_1 b)} = \blacksquare \cdot \frac{v_1}{y_o^{\lambda}} \quad (5.12)$$

5.4 Characteristics Equations

Equation 5.7 and 5.12 together gives the value of u1 and v1 which can be solved by taking a ration proportion of u/u1=c1 and v/v1=c2. Graphical solution of u and v are done for different values of c1 and c2 the equation after substitution becomes

$$v_1 := \frac{\varepsilon_d \cdot u_1 \cdot c_1 \cdot \left[k_o^2 \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y) \right) - u_1^2 \cdot (\cos(u_1 \cdot y) - 3 \cdot \cos(3u_1 y)) \right] \cdot (\sin(u_1 \cdot c_1 \cdot a) - \sin(3u_1 \cdot c_1 \cdot a))}{c_2 \cdot \varepsilon_o \cdot \left[k_d^2 \cdot \left(\cos(u_1 \cdot y) - \frac{1}{3} \cdot \cos(3u_1 y) \right) - u_1^2 \cdot (\cos(u_1 \cdot y) - 3 \cdot \cos(3u_1 y)) \right] \cdot \left(\cos(u_1 \cdot c_1 \cdot a) - \frac{1}{3} \cdot \cos(3u_1 \cdot c_1 \cdot a) \right)} \quad (5.13)$$

$$v_1 := \frac{\varepsilon_o \cdot u_1}{\varepsilon_d} \cdot \frac{(\sin(u_1 \cdot b) - \sin(3u_1 b))}{\cos(u_1 \cdot b) - \frac{1}{3} \cdot \cos(3u_1 b)} \quad (5.14)$$

Also the propagation equation in terms of outside and inside dielectric are given as

Other $u^2 + u_1^2 + k_z^2 = \blacksquare \cdot k_d^2 = \blacksquare \cdot \omega^2 \cdot \varepsilon_d \cdot \mu_d$ (inside waveguide)

$-v^2 - v_1^2 + k_z^2 = \blacksquare \cdot 2k_o - k_d^2 = \blacksquare \cdot 2\omega^2 \cdot \varepsilon_o \cdot \mu_o - \omega^2 \cdot \varepsilon_d \cdot \mu_d$ (outside waveguide)

From these teo equations value of v1 comes out to be

$$v_1 := \sqrt{\frac{2 \cdot k_d^2 - 2 \cdot k_o^2 - u_1^2\left(1 + c_1^2\right)}{\left(1 + c_2^2\right)}} \tag{5.15}$$

Equation 5.13, 5.14 and 5.15 gives the solution of transverse propagation constant (u and v) and from these values propagation constant in z direction can be calculated as

$$k_z \cdot \blacksquare - \sqrt{K_d^2 - u_1^2 - u^2} \tag{5.16}$$

5.5 Graphical calculation

Taking the values of a=.5mm, b=1 mm, c1=c2, c1=f/60, $\varepsilon_d = 3\varepsilon_o$ and other constant parameter values of u, u1, v and v1 are calculated. There by value of normalized propagation constant is calculated.

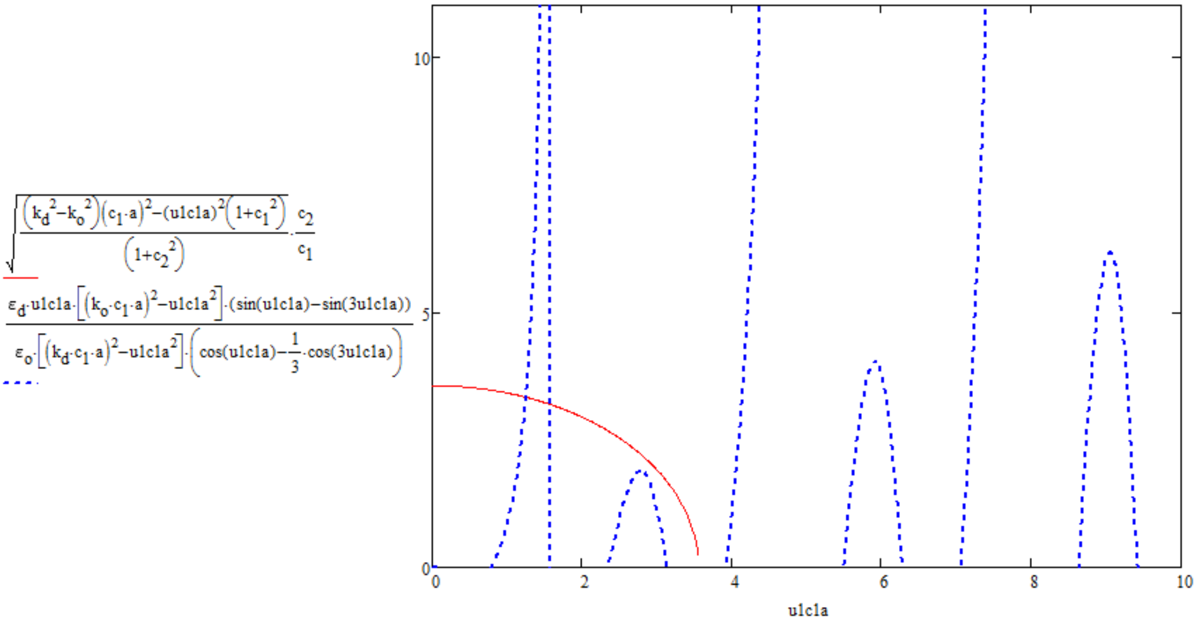

Fig 5.1 Graphical solution of u1 and v1

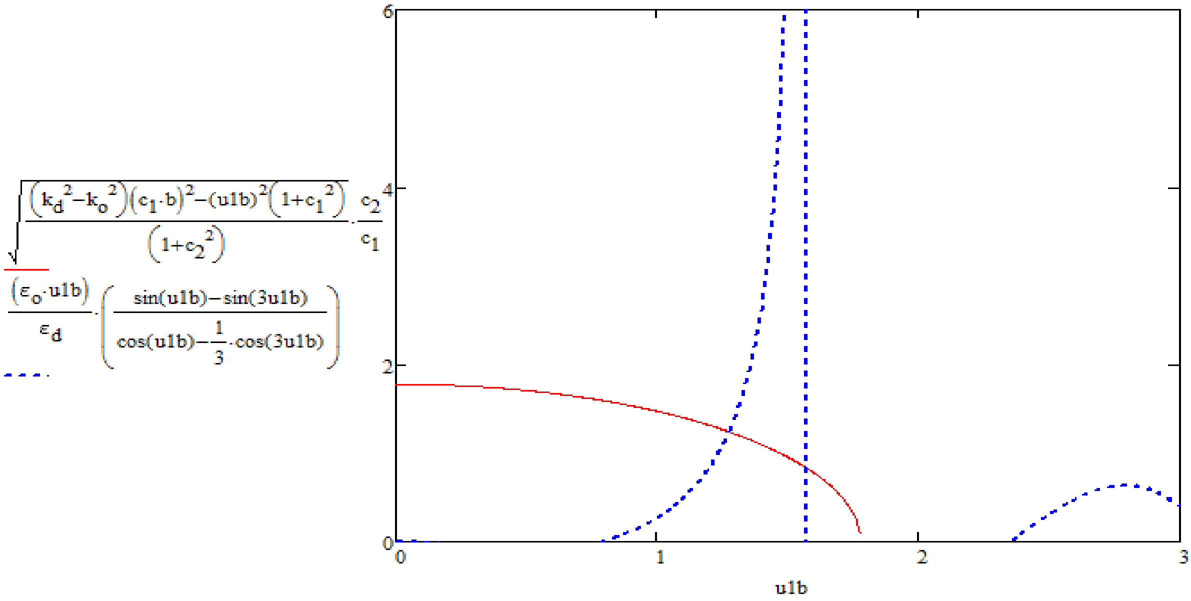

Fig5.2 graphical solution of u1 and v1

u1c1a :=

F	c_1	c_2	u_{1c1a}	v_{1c1a}	K_o	K_d	$(K_d^2 - K_c^2)$
60	1×10^9	$1 \cdot 10^9$	1.11	1.3894	$1.257 \cdot 10^3$	$2.178 \cdot 10^3$	$3.163 \cdot 10^6$
70	1.167×10^9	0	1.14	1.7335	$1.467 \cdot 10^3$	$2.541 \cdot 10^3$	$4.305 \cdot 10^6$
80	1.33×10^9	0	1.16	2.068	$1.677 \cdot 10^3$	$2.904 \cdot 10^3$	$5.622 \cdot 10^6$
90	1.5×10^9	0	1.19	2.3695	$1.886 \cdot 10^3$	$3.267 \cdot 10^3$	$7.116 \cdot 10^6$
100	1.667×10^9	0	1.23	2.6967	$2.096 \cdot 10^3$	$3.63 \cdot 10^3$	$8.785 \cdot 10^6$
110	1.833×10^9	0	1.24	3.0153	$2.305 \cdot 10^3$	$3.993 \cdot 10^3$	$1.063 \cdot 10^7$
120	2×10^9	0	1.28	3.3184	$2.515 \cdot 10^3$	$4.356 \cdot 10^3$	$1.265 \cdot 10^7$

Table 5.1 Values of c1, u1, v1, K_{o}, K_d, and K_d^2-k_o^2 with different values of Frequency

plot :=

60	1.491
70	1.548
80	1.587
90	1.613
100	1.629
110	1.647
120	1.656

Table 5.2 Values of k_d /k_o with different values of Frequency

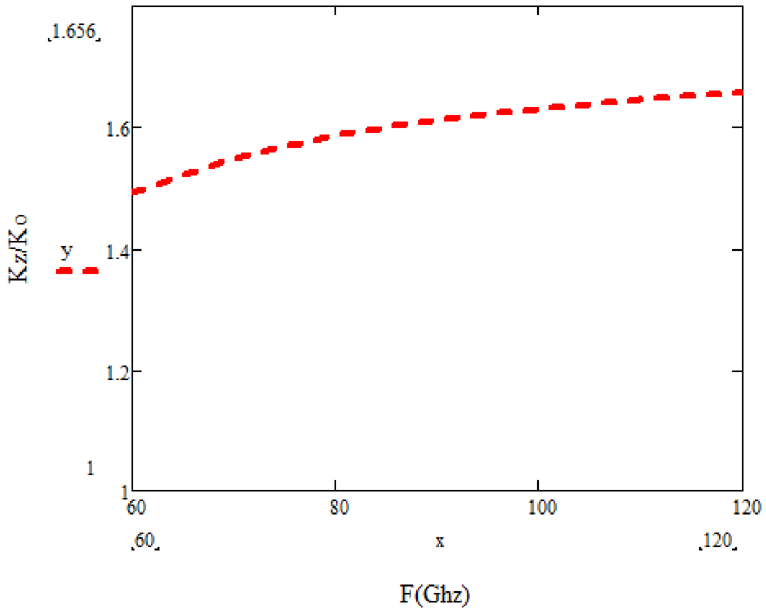

Fig 5.3 Graphical representation of k_d / k_o with Frequency

Chapter 6

RESULTS AND CONCLUSION

6.1 Results

Rectangular Dielectric waveguide solution is provided assuming square wave incidence. Here the wave function is represented in terms of square wave distribution in two dimensions. These square waves are represented by harmonics of sine and cosine function and their 3-D plots are drawn. The 3-D graphical representation of E^y_{11} mode wave function for various harmonics (square wave) are similar as that obtained for simple cosine and sine functions. Here 3-D graph plots give better visualization and understanding of field distribution in x and y directions.

Using this same field distribution the bound mode analysis is done for rectangular dielectric waveguide. A graphical solution of the characteristic equations obtained for a specific mode family (E^x_{mn} or E^y_{mn}) have been obtained by complete mode matching at the interfaces of the guide without using any approximations. The field, is assumed to be square distributed inside the waveguide in the transverse plane. In particular, solution of characteristics equations for E^y_{11} modes assuming three harmonics of even and odd function is calculated graphically. Calculation of transverse propagation constants (u, u1,v and v1) has been done taking a particular ratio of transverse propagation constants in x any y directions.

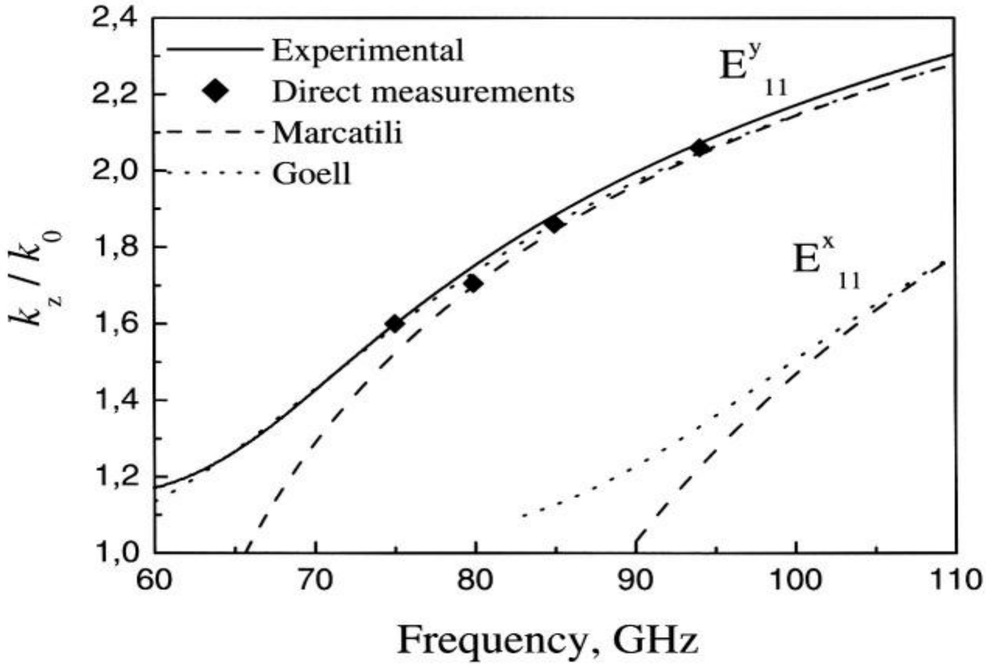

Fig 6.1 Comparison of normalized propagation constant Kz/Ko calculated by Marcatili and Goell.[4]

Here the ratio of transverse propagation constant (u/u1) is taken as one and value of c1 is optimized to a value of f/60. The relative dielectric constant inside the waveguide is taken as three. The comparison of results of the calculations of the propagation factor kz/k0 using the graphical method to Marcatili's and Goell's methods for a silicon dielectric waveguide with a=0.5mm and b=1mm cross section, Ey_{11} mode has been done with Figure 6.1

It can be seen from Figure 5.3 and 6.1 that the graphical method works quite well for frequencies at the lower and middle of the range, when the wave is well guided the results agrees very well with the Marcatili's and Goell's method. At higher frequencies above cut-off, because of presence of harmonics the normalized propagation constant differ from experimental or direct measurement. Accurate calculations are more complicated at higher values of harmonics so it is done only up to three harmonics. The result of three harmonic are very much consistent at lower frequencies and differ at higher frequencies.

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Code for Exponential decay:

```
a=1.1;
b=a/2;
u=(m*pi)/(2*a);
u1=(n*pi)/(2*b);
[X,Y]=meshgrid(-2:.2:2);
Exe1=(sin(u*X)-sin(3*u*X))*(sin(u1*Y)-sin(3*u1*Y));
surf(X,Y,Exe1);
daspect([5 5 1]);
view(-50,30);
label x;
label y;
camlight left;
```

Code for 3 harmonic even field

```
m=1;
n=1;
a=1.1;
b=a/2;
u=(m*pi)/(2*a);
u1=(n*pi)/(2*b);
[X,Y]=meshgrid(-1:.1:1);
Exe1=cos(u*X+u1*Y)+cos(u*X-u1*Y)-(1/3)*cos(3*u*X+u1*Y)-(1/3)*cos(3*u*X-
u1*Y)-(1/3)*cos(u*X+3*u1*Y)-(1/3)*cos(u*X-
3*u1*Y)+(1/9)*cos(3*u*X+3*u1*Y)+(1/9)*cos(3*u*X-3*u1*Y);
surf(X,Y,Exe1);
daspect([5 5 1]);
view(-50,30);
label x;
label y;
camlight left
```

Code for 5 harmonic even field

```
m=1;
n=1;
a=1.1;
b=a/2;
u=(m*pi)/(2*a);
u1=(n*pi)/(2*b);
[X,Y]=meshgrid(-1:.1:1);
Exe1=cos(u*X+u1*Y)+cos(u*X-u1*Y)-(1/3)*cos(3*u*X+u1*Y)-(1/3)*cos(3*u*X-
u1*Y)+(1/5)*cos(5*u*X+u1*Y)+(1/5)*cos(5*u*X-u1*Y)-(1/7)*cos(7*u*X+u1*Y)-
(1/7)*cos(7*u*X-u1*Y)+(1/9)*cos(9*u*X+u1*Y)+(1/9)*cos(9*u*X-u1*Y)-
(1/3)*cos(u*X+3*u1*Y)-(1/3)*cos(u*X-
3*u1*Y)+(1/9)*cos(3*u*X+3*u1*Y)+(1/9)*cos(3*u*X-3*u1*Y)-
(1/15)*cos(5*u*X+3*u1*Y)-(1/15)*cos(5*u*X-
3*u1*Y)+(1/21)*cos(7*u*X+3*u1*Y)+(1/21)*cos(7*u*X-3*u1*Y)-
(1/35)*cos(7*u*X+5*u1*Y)-(1/35)*cos(7*u*X-
5*u1*Y)+(1/45)*cos(9*u*X+5*u1*Y)+(1/45)*cos(9*u*X-5*u1*Y)-
(1/7)*cos(u*X+7*u1*Y)-(1/7)*cos(u*X-
7*u1*Y)+(1/21)*cos(3*u*X+7*u1*Y)+(1/21)*cos(3*u*X-
7*u1*Y1/27)*cos(3*u*X+9*u1*Y)-(1/27)*cos(3*u*X-
9*u1*Y)+(1/45)*cos(5*u*X+9*u1*Y)+(1/45)*cos(5*u*X-9*u1*Y)-
surf(X,Y,Exe1);
daspect([5 5 1]);
view(-50,30);
```

```
label x;
label y;
camlight left
```

Sine wave exponential decay

```
m=1;
n=1;
a=1.1;
b=a/2;
u=(m*pi)/(2*a);
u1=(n*pi)/(2*b);
[X,Y]=meshgrid(-1:.1:1);
Exe1=exp(-u1*Y);
Exe2=sin(u*X);
Exe3=Exe1*Exe2;
surf(X,Y,Exe3);
label x;
label y;
camlight left
```

Study of Rectangular Dielectric Wave Guide with Square Wave Incidence

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Abstract - *Rectangular dielectric waveguide modes have been studied assuming square wave incidence. Incident square wave function is based on the expansion of harmonics of sine and cosine functions. Wave guidance at above cut-of frequency are considering as surface wave having field confinement inside the waveguide and decaying exponentially outside in transverse plane (as Marcatili assumptions). Modes of propagation are defined as* E^y_{mn} *and* E^x_{mn} *with most of the electric field confinement in y and x direction respectively. Wave equations for square wave incidence are written for harmonics of even and odd functions. Graphical 3-D field distribution is plotted for even and odd symmetric fields of* E^y_{mn} *with two and four harmonics.*

Key words – Rectangular dielectric waveguide, Field distribution in E^y_{mn} ***and*** E^x_{mn} **mode, square wave incidence, harmonics of sine (cosine) function for square wave, even and odd field equations in Marcatili approx. method, 3-D graphical representation of wave function.**

I. INTRODUCTION

Dielectric waveguides are the basic building blocks of integrated optical circuits. They confine and direct the optical signal for transmission and have various applications in many optical circuits. It is required to have complete knowledge of its modes and their field distribution. The modes of dielectric waveguides are more difficult to analyze than those of the metallic rectangular waveguide because of nature of boundary [1]. Metallic rectangular waveguide finds it applications for frequencies below 30 GHz, but for frequencies above 30 GHz, dielectric waveguide has advantage. Metallic waveguide has higher losses because of increasing skin-depth with increase in frequency. Dielectric waveguide has lower losses because of absence of any metallic material. The dielectric waveguides can be used for a frequency range of 10^9 to 10^{16} Hz.

Rectangular dielectric waveguide has no closed form solution. Its Eigen modes can be found either numerically or using approximate techniques. There has been lot of studies on dielectric waveguide using approximate and numerical methods. Marcatili [2], using the approximation method, assumed that most of the field is confined inside the core of the waveguide. The properties of a rectangular dielectric waveguide field were found with sinusoidal variation in core and exponential decaying field outside the dielectric medium. Solution of Eigen equations were found by matching the inside and outside field equations. Schlosser and Unger [3] also used the approximation technique. In their work, the modes' properties were obtained by dividing the transverse plane of rectangular dielectric waveguide in to various regions and rectangular coordinates solution is assumed for each of the region. The longitudinal propagation constant was changed to match the field at specific points along the boundary. This method results, are theoretically more valid but require large computation.

In the work done by Goell propagation modes of rectangular dielectric waveguide were studied based on the expansion of electromagnetic field in terms of a series of circular harmonics (Bessel and modified Bessel multiplied by trigonometric function) [1]. The circular harmonic analysis works very well for smaller value of normalized propagation constant.

There has been lot of study by assuming co-sinusoidal field as incidence field but very less by assuming square field or harmonics field as incidence. In the present analysis the variation of longitudinal electric and magnetic fields of the modes are represented by square field inside the core as against sine (cosine) wave and by exponential decaying field outside the waveguide core. 3-D graphs of the fields are plotted assuming square field to be made of harmonics of sine and cosine functions.

II. MODES IN RECTANGULAR DIELECTRIC WAVEGUIDE

The dielectric rectangular wave guide material is assumed to have relative dielectric constant $\varepsilon_r > 1$ and surrounding medium is air extending up to infinity. Both medium are assumed to be isotropic and have permeability of free space as μ_o. Wave propagation is assumed to be in +z direction as shown in fig 1.

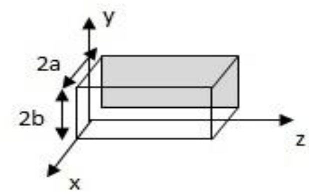

Fig. 1. The Wave propagation in rectangular dielectric wave guide in z- axis

Rectangular dielectric waveguide modes are neither pure TM nor pure TE. E^y_{nm} and E^x_{nm} as well as hybrid modes (all six components E_x, E_y, E_z, H_x, H_y, H_z) can be supported by rectangular dielectric waveguide. The scheme adopted to designate E^y_{nm} and E^x_{nm} mode is based that in the limit for large aspect ratio, short wavelength, and small refractive index difference, the transverse electric field is primarily parallel to one of the transverse axes. In this, modes are designated as E^y_{mn}, if in the limit the electric field is parallel to the y-axis, and as E^x_{mn} if in the limit their electric field is parallel to x-axis. The m and n subscript are used to designate the number of maxima in x and y direction [4].

The rectangular dielectric waveguide can have two independent families of modes. One E^y_{nm} modes having components E_y, E_z, H_x, H_y, and H_z, with $E_x = 0$. Which can be visualized as TE_x modes and other E^x_{nm} modes having components E_x, E_y, E_z, H_x, and H_z, with $H_x = 0$ which can be visualized as the TM_x modes [5] [6]. The dominant modes correspond to m=n=1 [7].

III. EQUATIONS WITH SQUARE WAVE INCIDENCE

With square wave incidence on rectangular dielectric waveguide the guided mode propagation is assumed to be well above cut off frequency. Field are assumed to be approximately square distributed inside the waveguide and decaying exponentially outside, the wave function can be written in either of the two forms. One, in even symmetric and other in odd symmetric (asymmetric) field [4].

A. For even field

Taking propagation along +z-axis, the wave function for even distribution can be written as [7].

$$\Psi even = A\, Rect^e\,(ux)\, Rect^e\,(u1y)\, e^{-jk_z z} \qquad |x| \le a,\ |y| \le b \text{ (inside guide)}$$
$$= B\, e^{-vx}\, Rect^e\,(u1y)\, e^{-jk_z z} \qquad |x| \ge a,\ |y| \le b \text{ (outside guide)}$$
$$= C\, Rect^e\,(ux)\, e^{-v1y}\, e^{-jk_z z} \qquad |x| \le a,\ |y| \ge b \text{ (outside guide)} \qquad (1)$$

Here the $Rect^e$ () function is even rectangular field and it can be broken in harmonics of cosine function for fields. Where u, u1, K_z are propagation constant inside the dielectric in x, y and z direction and v, v1 are attenuation constant in x and y directions outside the dielectric region. Assuming propagation in z-direction according to $e^{-jk_z z}$. The regions $x > a$ and $y > b$ have been neglected as fields are very weak at the corners. These field equations are written as per the approximation analysis methods and same has been used by Marcatili [2] and Goell in their work [1].

Taking Fourier series expansion of even $Rect^e$ () function as harmonics of cosine function. Writing $Rect^e$ () in terms of two harmonics, the wave function from eqn. (1) can be written as

$$\varphi_{even} := A\left(\cos(u \cdot x) - \frac{1}{3}\cdot\cos(3ux) + \frac{1}{5}\cdot\cos(5ux)\right)\left(\cos\left(u_1 \cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1 y\right) + \frac{1}{5}\cdot\cos\left(5u_1 y\right)\right)\cdot(e)^{-j\cdot k_z \cdot Z}$$

$$:= B\cdot(e)^{-v\cdot x}\cdot\left(\cos\left(u_1\cdot y\right) - \frac{1}{3}\cdot\cos\left(3u_1 y\right) + \frac{1}{5}\cdot\cos\left(5u_1 y\right)\right)\cdot(e)^{-j\cdot k_z\cdot Z} \qquad (2)$$

B. For o

Similarly taking propagation along z-axis, the wave function for odd distribution can be written as [7]

$$\Psi odd = A\, Rect^o\,(ux)\, Rect^o\,(u1y)\, e^{-jk_z z} \qquad |x| \le a,\ |y| \le b \text{ (inside guide)}$$
$$= B\, e^{-vx}\, Rect^o\,(u1y)\, e^{-jk_z z} \qquad |x| \ge a,\ |y| \le b \text{ (outside guide)}$$
$$= C\, Rect^o\,(ux)\, e^{-v1y}\, e^{-jk_z z} \qquad |x| \le a,\ |y| \ge b \text{ (outside guide)} \qquad (3)$$

In a similar way, taking Fourier series expansion of odd $Rect^o$ () function as harmonics of sine function. Writing $Rect^o$ () in terms of two harmonics the wave function from eqn.(3) can be obtained as:

$$\varphi_{odd} := A\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3ux) + \frac{1}{5}\cdot\sin(5ux)\right)\left(\sin\left(u_1\cdot y\right) + \frac{1}{3}\cdot\sin\left(3u_1 y\right) + \frac{1}{5}\cdot\sin\left(5u_1 y\right)\right)\cdot(e)^{-j\cdot k_z\cdot Z}$$
$$:= B\cdot(e)^{-V\cdot X}\left(\sin\left(u_1\cdot y\right) + \frac{1}{3}\cdot\sin\left(3u_1 y\right) + \frac{1}{5}\cdot\sin\left(5u_1 y\right)\right)\cdot(e)^{-j\cdot k_z\cdot Z}$$
$$:= C\cdot\left(\sin(u\cdot x) + \frac{1}{3}\cdot\sin(3ux) + \frac{1}{5}\cdot\sin(5ux)\right)\cdot(e)^{-v_1\cdot y}\cdot(e)^{-j\cdot k_z\cdot Z} \qquad (4)$$

IV. :TION

From equations (2) and (4) the graphical representation of fields can be plotted by using 3-D graphical tool of Matlab. The field plotted are for mainly dominant mode of rectangular dielectric waveguide for $\mathbf{E^y_{mn}}$ **mode**, with m=1 and n=1 and for $\mathbf{E^y_{13}}$ **mode** also. Graphs are plotted for 2 and 4 harmonics for even and odd functions. Outside exponential decay graph are plotted for only 4 harmonics. Only graph of one direction is plotted for exponential and it is similar in both x and y direction.

A. Graphical presentation of even wave function for dominant mode

The 3-D graph plotted are as showed in figures 2 and 3 for two and four harmonics with the field varying as even function around origin in x and y direction inside the dielectric. The even exponentially decaying field outside the boundary of the dielectric for four harmonic is shown in fig 4.

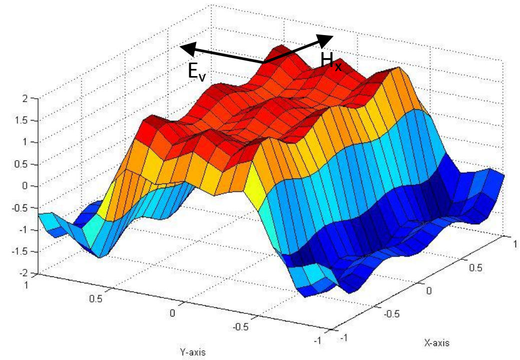

Fig. 2. Even field distribution for E^y_{11} mode inside waveguide for two harmonics

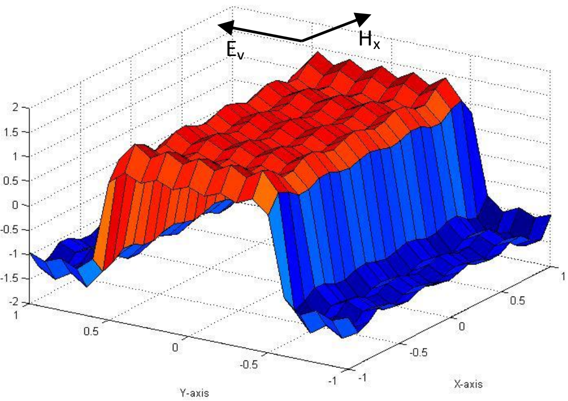

Fig. 3. Even field distribution for E^y_{11} mode inside waveguide for four harmonics

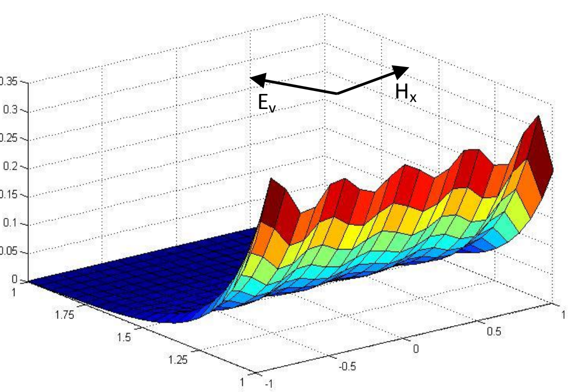

Fig. 4. Even field exponential decay for E^y_{11} mode outside the waveguide for four harmonics

B. Graphical presentation of even wave function for $\mathbf{E^y_{13}}$ *modes*

The 3-D graph of E^y_{13} mode for four harmonics with the field varying as even function inside the dielectric is shown in fig 5.

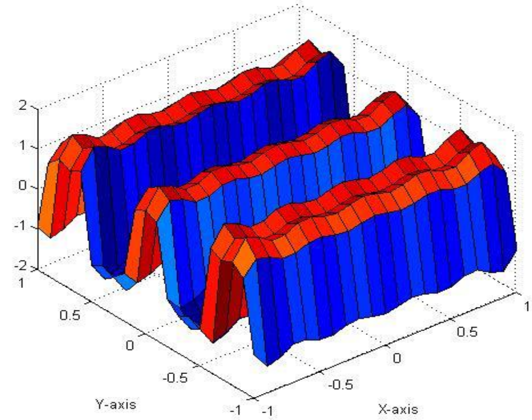

Fig. 5. Even field distribution for E^y_{13} mode inside waveguide for four harmonics

C. Graphical presentation of odd wave function for dominant mode

Similarly, 3-D graph are plotted for field varying as odd function around origin in x and y direction inside the dielectric and decaying exponentially outside the boundary of the dielectric. The graph for two harmonics is shown in fig 6 and for four harmonics is in fig 7. The odd exponentially decaying field outside the boundary of the dielectric for four harmonic is shown in fig 8.

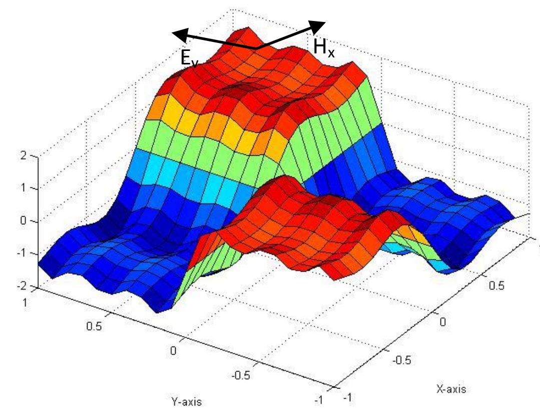

Fig. 6. Odd field distribution for E^y_{11} mode inside waveguide for two harmonics

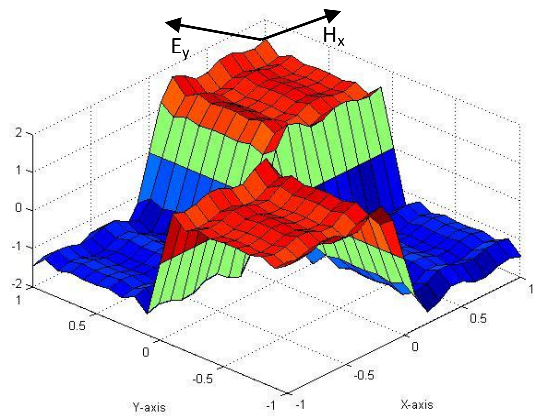

Fig. 7. Odd field distribution for E^y_{11} mode inside waveguide for four harmonics

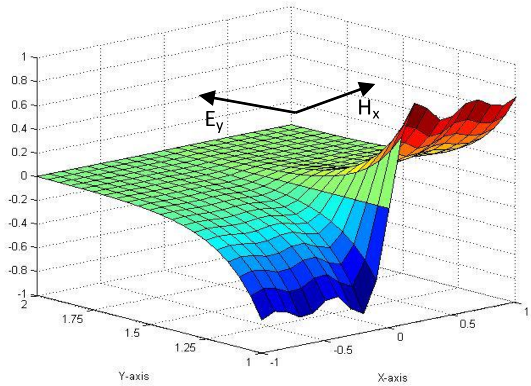

Fig. 8. Odd field exponential decay for E^y_{11} mode outside the waveguide for four harmonics.

D. Graphical presentation of odd wave function for E^y_{13} *modes*

3-D graph of E^y_{13} mode for four harmonics with the field varying as odd function inside the dielectric is shown in fig 9.

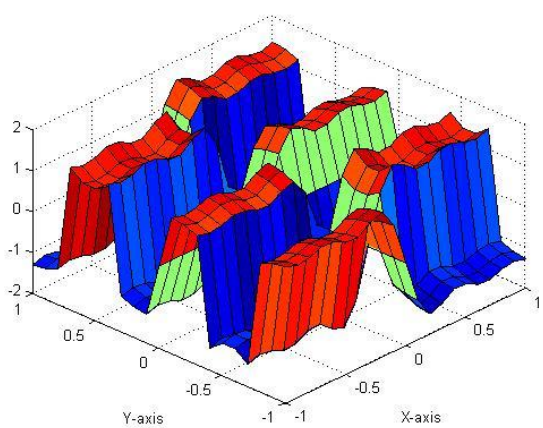

Fig. 9. Odd field distribution for E^y_{13} mode inside waveguide for four harmonics

VI. RESULTS AND DISCUSSIONS

Here the wave function is represented in terms of square wave distribution in three dimensions. These square waves incidence are represented by harmonics of sine and cosine functions for even and odd functions and their 3-D plots are drawn for E^y_{nm} mode. In a similar way field pattern for E^x_{mn} mode can be visualized by changing the E and H fields. The 3-D graphical representation of E^y_{11} and E^y_{13} mode wave function for various harmonics (for square wave) has similar pattern as that obtained for simple sine and cosine functions [7]. Here 3-D graph plots give better visualization and understanding of field distribution in x and y direction. It also gives better understanding of even and odd function distribution with square wave incidence for dominant and E^y_{13} modes. Using this same field distribution further work can be done to solve characteristics equations for waveguide by applying different techniques for various modes.

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