

# **OPTICAL EQUALIZERS**

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## **CERTIFICATE**

This is to certify that the work contained in this thesis entitled “Optical Equalizers” by Hema Saini in the requirement for the partial fulfillment for the award of the degree of Master of Technology in Microwave and Optical Communication Engineering, Delhi Technological University, New Delhi is an account of her work carried out under my guidance in the academic year 2010-2011.

This work embodied in this dissertation has not been submitted for the award of any other degree to the best of my knowledge.

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## **ABSTRACT**

Fiber optics is one of the highest bandwidth communication channel types in the current communication industry. The needs for increasing speeds and capacities in modern data communication systems have led to increasing hopes for the introduction of optical communication in a wide range of communication networks. In communications, a critical manifestation of distortion is inter-symbol interference (ISI), whereby symbols transmitted before and after a given symbol corrupt the detection of that symbol. All physical channels (at high data rates) tend to exhibit ISI. With the potential of an optical link's overwhelming advantages, it will be gainful to delve into techniques that can mitigate the effects of ISI and thus improving transmission speed and saves bandwidth.

The equalizer attempts to extract the transmitted symbol sequence by counteracting the effects of ISI, thereby improving the probability of correct symbol detection. Its purpose is to reverse the effects that the channel has on the transmitted signal, with the aim of reproducing the original signal at the receiver end. The Decision Feedback Equalizer (DFE) and Blind Equalizer, which are non-linear structures, is known to show superior performance when employed in channels that exhibit amplitude distortion. As such, it would be an ideal choice for a typical optical channel, which often suffers from amplitude distortion.

The aim of my thesis project is to analyze a typical optical channel, study dispersion effect and perform channel equalization using an adaptive DFE algorithm and Blind equalization. Evaluation can be made on the employment of the DFE algorithm and with enhancements, like Fractionally-Spaced equalization and Activity Detection Guidance, to improve its stability. Brief introduction of Blind Equalization is also given, which is the latest technique in equalization. The FSE technique, when combined with the DFE, would offer improved effectiveness to amplitude distortion.

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## **LIST OF ABBREVIATIONS**

LAN	:	Local Area Networks
ISI	:	Intersymbol Interference
BW	:	Bandwidth
PCM	:	Pulse Code Modulation
LMS	:	Least Mean Square
DFE	:	Decision Feedback Equalizer
SMF	:	Single Mode Fiber
MMF	:	Multi Mode Fiber
LED	:	Light Emitting Diode
APD	:	Avalanche Photodiode
PIN	:	Positive-Intrinsic Negative
mA	:	Milli-Ampere
WDM	:	Wavelength Division Multiplexing
RI	:	Refractive index
GVD	:	Group Velocity Division
PMD	:	Polarization Mode Dispersion
RLS	:	Recursive Least Square
NLMS	:	Normalized Least Mean Square
MMSE	:	Maximum Mean Square Error
FSE	:	Fractionally Spaced Equalizer
FIR	:	Finite Impulse Response
FBF	:	Feed Back Filter
FFF	:	Feed Forward Filter
EPI	:	Error Propagation Instant

### 1.1 Overview

Now we are in the twenty first century, the era of ‘Information technology’. The information technology has had an exponential growth through the modern telecommunication systems. Particularly, optical fiber communication plays a vital role in the development of high quality and high-speed telecommunication systems. Today, optical fibers are not only used in telecommunication links but also used in the Internet and local area networks (LAN) to achieve high signaling rates.

To guide light in a waveguide, initially metallic and non-metallic wave guides were fabricated. But they have enormous losses. So they were not suitable for telecommunication. Tyndall discovered that through optical fibers, light could be transmitted by the phenomenon of total internal reflection. During 1950s, the optical fibers with large diameters of about 1 or 2 millimetre were used in endoscopes to see the inner parts of the human body. Optical fibers can provide a much more reliable and versatile optical channel than the atmosphere, Kao and Hockham published a paper about the optical fiber communication system in 1966. But the fibers produced an enormous loss of 1000 dB/km. But in the atmosphere, there is a loss of few dB/km. Immediately Kao and his fellow workers realized that these high losses were a result of impurities in the fiber material. Using a pure silica fiber these losses were reduced to 20 dB/km in 1970 by Kapron, Keck and Maurer. At this attenuation loss, repeater spacing for optical fiber links become comparable to those of copper cable systems. Thus the optical fiber communication system became an engineering reality.[1]

It is well known that communication speed is increasing rapidly over the years. Fiber optics has become one of the highest bandwidth communication channel types available in the current communication industry today. Transmission rate at hundreds of megabytes per second over thousands of kilometres are increasingly common and recent experiments have even shown that the high-bandwidth, ultra long-distance transmission

of 2.56 terabits (trillion bits) of information per second over a distance of 4000 kilometers (2500 miles) is achievable.[1]

With the fiber optics technology's immense potential bandwidth and the fine qualities of a lighter weight cable, resistant to lightning strikes, and carries more information faster and over longer distances, the latter has found its way to many applications for the past 30 years. [2] Fiber optic telephone systems, fiber optic video transmission system and the Ethernet are just some of the many examples.

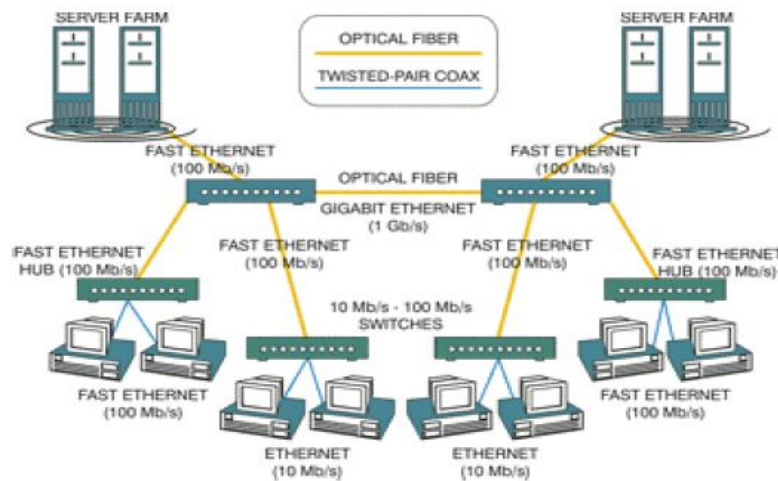


Figure 1.1: Optical Fiber Use in The Ethernet [2]

In figure 1.1, Ethernet network is shown which uses the optical fiber as the transmission medium. As the demand for data bandwidth increases, aided by the phenomenal growth of the Internet, the move to optical networking is the focus of new technologies. Nearly half a billion people have Internet access and use it regularly. [2] This inevitably created the need to expand the optical fiber's transmission capabilities. The primary goal of any data transmission is for the receiver to receive the data without any errors.

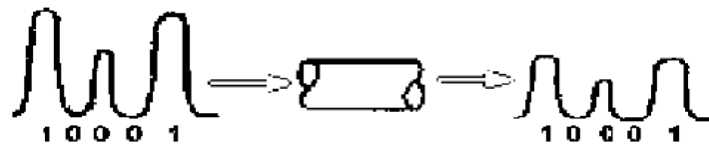
Long haul transmission in optical communications usually consists of several keys area to ensure the data transfer is error-free. These key areas include the modulation method, type of amplifier used, and the error correction scheme and dispersion compensation.

In optical communications, high-speed data transfer is often limited by signal distortion,

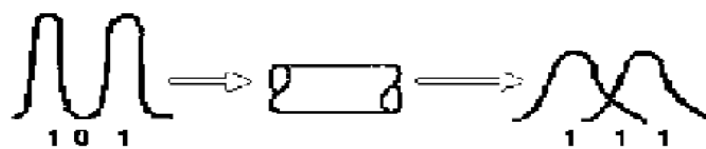
which is mainly caused by the broadening of pulses that result in Intersymbol Interference (ISI). With the potential of an optical link's overwhelming advantages, it will be gainful to delve into techniques that can mitigate the effects of ISI and thus improving transmission rate and saving bandwidth.[1]

## 1.2 Problem: Definition

In optical communications, the main source of distortion comes from pulse dispersion. Pulse dispersion can be separated into 2 main groups, namely, Intermodal Dispersion and Intramodal Dispersion. Intramodal, or chromatic, dispersion occurs in all types of fibers. Intermodal, or modal, dispersion occurs only in multimode fibers. Each type of dispersion mechanism leads to pulse spreading. As a pulse spreads, energy is overlapped. This condition is shown in Figure 1.2. The spreading of the optical pulse as it travels along the fiber limits the information capacity of the fiber.[14]



Reduction in optical fiber



Dispersion: Spreads the data

Figure 1.2: The Effects of Distortion In An Optical Fiber

The dispersion also limits the transmission distance. Regenerative repeaters, which reconstruct and boost the signal shape and power, are needed more frequently if the dispersion is severe. Such repeaters are expensive to build and install, especially when the optical link runs under the seabed in cross-ocean communications links. This gives much motivation for engineers to investigate the uses of dispersion combating techniques

to increase the transmission distance of an optical link.

## Intersymbol interference (ISI)

In telecommunication, intersymbol interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbols. [1] This is an unwanted phenomenon as the previous symbols have similar effect as noise, thus making the communication less reliable. ISI is usually caused by multipath propagation or the inherent non-linear frequency response of a channel causing successive symbols to "blur" together. The presence of ISI in the system introduces errors in the decision device at the receiver output. Therefore, in the design of the transmitting and receiving filters, the objective is to minimize the effects of ISI, and thereby deliver the digital data to its destination with the smallest error rate possible. Ways to fight intersymbol interference include adaptive equalization and error correcting codes.

The presence of ISI is readily observable in the sampled impulse response of a channel; an impulse response corresponding to a lack of ISI contains a single spike of width less than the time between symbols. As shown in figure 1.3.

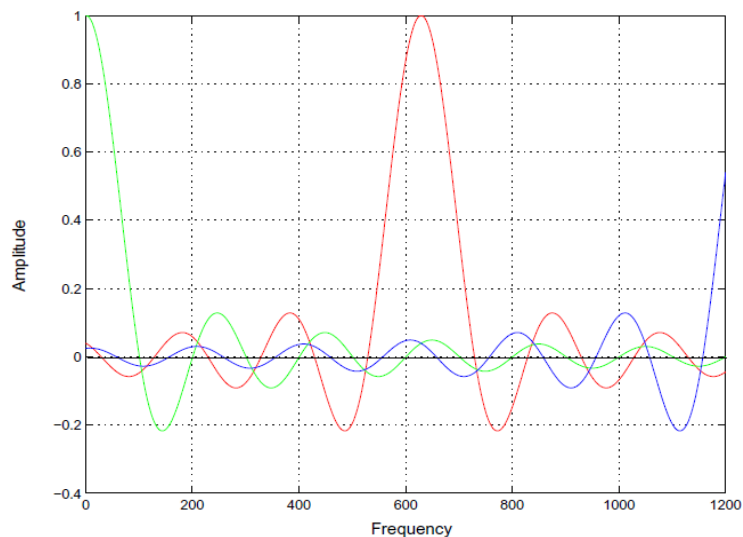


Figure 1.3: Inter Symbol Interference

The noise margin - the amount of noise required to cause the receiver to get an error - is

given by the distance between the signal and the zero amplitude point at the sampling time; in other words, the further from zero at the sampling time the signal is the better. For the signal to be correctly interpreted, it must be sampled somewhere between the two points where the zero-to-one and one-to-zero transitions cross. Again, the further apart these points are the better, as this means the signal will be less sensitive to errors in the timing of the samples at the receiver.

## Countering ISI

There are several techniques in telecommunication and data storage that try to work around the problem of intersymbol interference.

- Design systems such that the impulse response is short enough that very little energy from one symbol smears into the next symbol. As shown in figure 1. 4.

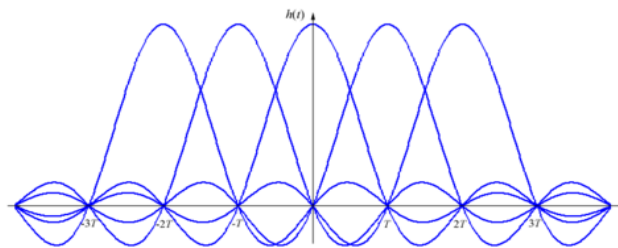


Figure 1.4: Consecutive raised-cosine impulses, demonstrating zero-ISI property

- Separate symbols in time with guard periods.
- Apply an equalizer at the receiver that, broadly speaking, attempts to undo the effect of the channel by applying an inverse filter.
- Apply a sequence detector at the receiver, that attempts to estimate the sequence of transmitted symbols using the Viterbi algorithm.

## 1.3 Aims and Objectives

The aim of this thesis is first to evaluate to the causes of dispersion in a typical optical fiber and its limiting effects to the possible communication capacity. The numerous



techniques in adaptive equalization and equalizer structures are then researched upon, to come up with a suitable and effective equalization approach for a typical optical channel.

Finally, the researches in these two main areas are combined to design an adaptive equalizer to mitigate the pulse dispersion effects of an optical link.

## **1.4 Approach**

A range of adaptive algorithms and equalizer structures were studied and evaluated. The Least Mean Square (LMS) algorithm was chosen for its robustness and computational simplicity. A series of research reveal that linear structures were inferior in channels that display amplitude distortion, due to the exorbitant noise enhancement. This can be avoided with a non-linear structure. Therefore, the Decision Feedback Equalizer (DFE) was chosen as the most suitable equalizer structure for an optical channel that suffers from amplitude distortion. Blind Equalizer is recent technology which is more accurate as compared to DFE.

An accurate way of analyzing the behavior of an adaptive DFE algorithm and the impulse response of an optical channel is by using the simulation package Matlab. Blind Equalizer is also studied by the simulation package Matlab. It allows the user to replicate specific and realistic impulse responses of a particular channel and the performance of a certain adaptive algorithm. Evaluation of the algorithm can be achieved in terms of channel estimation, convergence rate and error performance.

### 2.1 Overview on the Optical Communication Channel [3]

An optical fiber is a dielectric waveguide, normally in cylindrical form, which operates at optical frequencies. It confines electromagnetic energy in the form of light to within its surfaces and guides the light in a direction parallel to its axis. [4] The transmission capabilities are usually dictated by the structure of the optical fiber. The cylinder, in which the light propagates, is known as the core of the fiber. It is usually protected by a solid dielectric cladding, which has a refractive index that is lesser than the core. The cladding serves to reduce scattering loss that results from dielectric discontinuities at the core surface and adds mechanical strength to the fiber.

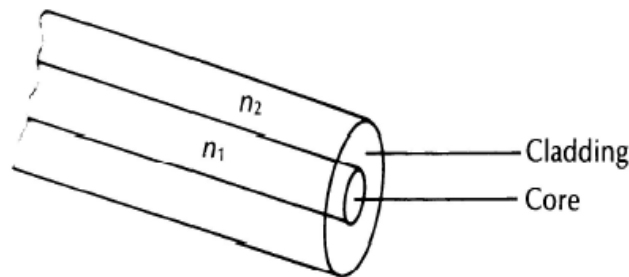


Figure 2.1: Core and Cladding of an Optical Fiber

### 2.2 Different types of fibers [1]

We know that the light or the optical signals are guided through the silica glass fibers by total internal reflection. A typical glass fiber consists of a central core glass (50  $\mu\text{m}$ ) surrounded by a cladding made of a glass of slightly lower refractive index than the core's refractive index. The overall diameter of the fiber is about 125 to 200  $\mu\text{m}$ . Cladding is necessary to provide proper light guidance i.e. to retain the light energy within the core as well as to provide high mechanical strength and safety to the core from scratches. Based on the refractive index profile we have two types of fibers (a) Step index fiber (b) Graded index fiber.

(a) Step index fiber: In the step index fiber, the refractive index of the core is uniform throughout and undergoes an abrupt or step change at the core cladding boundary. The light rays propagating through the fiber are in the form of meridional rays which will cross the fiber axis during every reflection at the core cladding boundary and are propagating in a zig-zag manner as shown in figure 2.2.

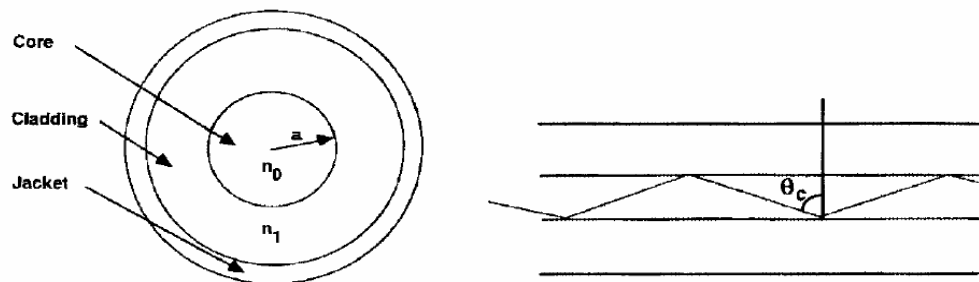


Figure 2.2: Generic optical fiber design, path of a ray propagating in step index fiber

(b) Graded index fiber: In the graded index fiber, the refractive index of the core is made to vary in the parabolic manner such that the maximum value of refractive index is at the centre of the core. The light rays propagating through it are in the form of skew rays or helical rays which will not cross the fiber axis at any time and are propagating around the fiber axis in a helical (or) spiral manner as shown in figure 2.3.

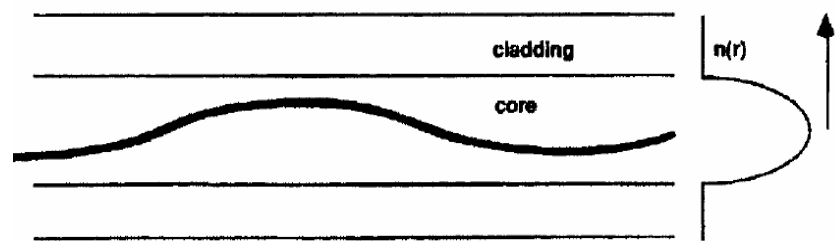


Figure 2.3: Ray path in a gradient-index fiber

Variations in material composition of the core give rise to two widely used fiber types, namely, step-index fiber and graded-index fiber. The refractive index of the core of a step index fiber is uniform throughout and undergoes an abrupt change at the cladding

boundary. Whereas in graded-index fiber, the core refractive index is made to vary as a function of the radial distance from the center of the fiber. [4]

The step and graded-index fibers can further be classified into single-mode fiber (SMF) and multi-mode fiber (MMF). The single mode fiber, though supports only one mode of propagation, is widely employed in optical communications due the lack of intermodal dispersion, which causes significant distortion to the transmission capabilities of an optical fiber. As shown in figure 2.4.

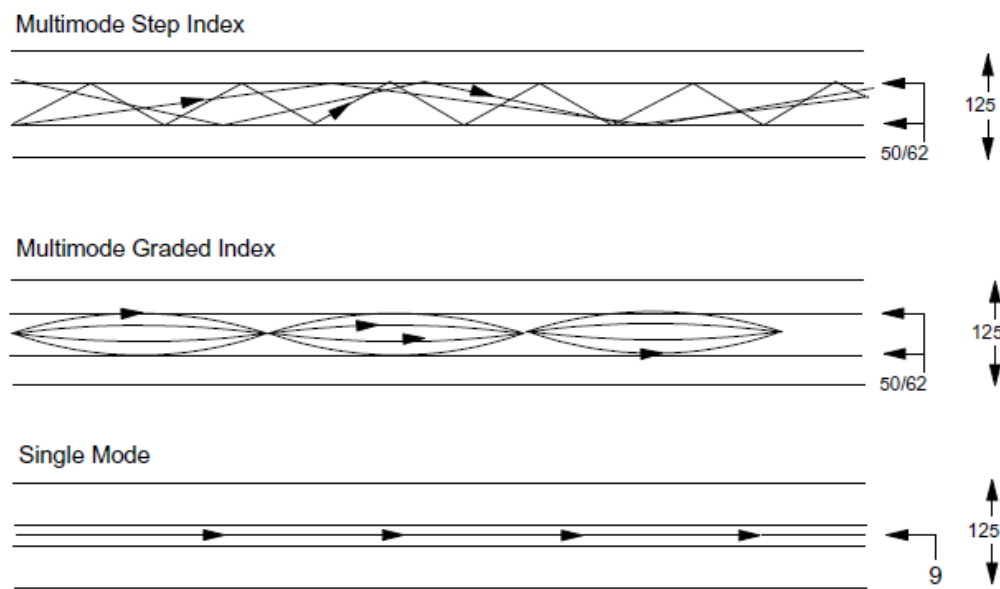


Figure2.4: illustrates the three different kinds of optical fiber

The multimode fiber offers its own range of advantages, such as larger data capacity, due to more modes being allowed in the fiber, and larger core radii, which allows easier launch of optical power into the fiber.

However, the large number of modes gives rise to significant intermodal dispersion, which is unique and detrimental to the operation of multimode fibers. Deployment of multimode fibers is then usually limited to few hundred meters, which severely limits the useful application of multimode fibers in optical communications. Equalization is usually employed in the receiver section of an optical communication system to mitigate the

dispersion effect.

### 2.3 Basic optical fiber communication system [4]

Figure 2.5 shows the basic components in the optical fiber communication system. The input electrical signal modulates the intensity of light from the optical source. The optical carrier can be modulated internally or externally using an electro-optic modulator (or) acousto-optic modulator. Nowadays electro-optic modulators (KDP, LiNbO<sub>3</sub> or beta barium borate) are widely used as external modulators which modulate the light by changing its refractive index through the given input electrical signal.

In the digital optical fiber communication system, the input electrical signal is in the form of coded digital pulses from the encoder and these electric pulses modulate the intensity of the light from the laser diode or LED and convert them into optical pulses. In the receiver stage, the photo detector like avalanche photodiode (APD) or positive-intrinsic negative (PIN) diode converts the optical pulses into electrical pulses. A decoder converts the electrical pulses into the original electric signal.

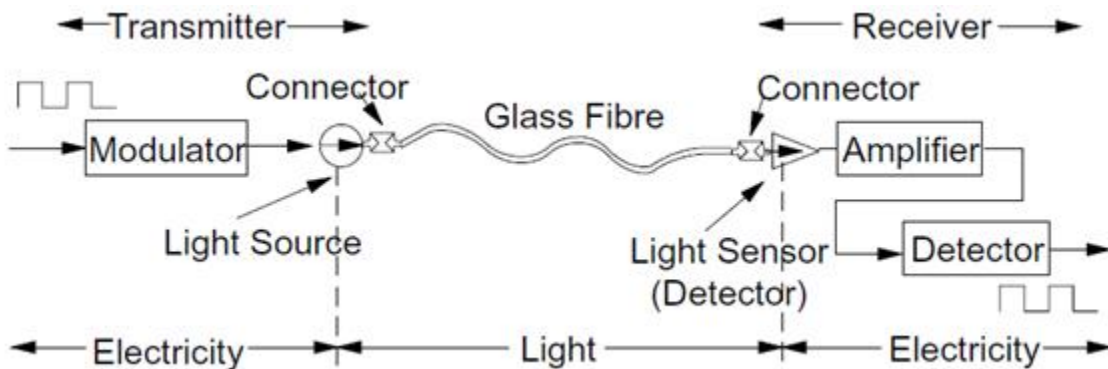


Figure 2.5 Optical communications systems

#### 2.3.1 Optical sources

Heterojunction LEDs and lasers are mostly used as the optical sources in optical fiber communication. Heterojunction means that a *p-n* junction is formed by a single crystal

such that the material on one side of the junction differs from that on the other side of the junction. In the modern GaAs diode lasers, a hetero junction is formed between GaAs and GaAlAs. This type of p-n junction diode laser or LED is used at 0.8 mm wavelength. At longer wavelengths, InP-InGaAsP heterojunction laser diodes are used.

Heterojunction lasers or LEDs are superior to conventional homojunction lasers or LEDs. Generally heterojunction lasers and LEDs have minimum threshold current density ( $10 \text{ A/mm}^2$ ), high output power (10 mW) even with low operating current ( $<500 \text{ mA}$ ), high coherence and high monochromaticity, high stability and longer life. For example in the case of a double hetero structure stripe laser, the active junction region is few microns. So the threshold current density is drastically reduced. The stripe geometry provides stability with longer lifetime for the diode. Thus it gives high power output, continuous wave operation, high efficiency, high coherence and high directionality. By means of the heterojunction formed by two different materials, both the carriers and the optical field are confined in the central active layer. The bandgap differences of adjacent layers confine the charge carriers while the step change in the indices of refraction of adjoining layers confines the optical field to the central active layer and provides an efficient waveguide structure. This dual confinement leads to both high efficiency and high power output.

### 2.3.2 Optical detectors

Semiconductor based photodiodes are used as optical detectors in the optical fiber communication systems. They have small size, high sensitivity and fast response. There are two types of photodiodes:

- (i) p-i-n photodiodes
- (ii) Avalanche photodiodes (APD)

(i) p-i-n photodiodes: A positive-intrinsic-negative (p-i-n) photodiode consists of  $p$  and  $n$  regions separated by a very lightly  $n$  doped intrinsic region. Silicon p-i-n photodiodes are used at 0.8 mm wavelength and InGaAs p-i-n photodiodes are used at 1.3 mm and 1.55 mm wavelengths. In normal operation, the p-i-n photodiode is under high reverse bias

voltage. So the intrinsic region of the diode is fully depleted of carriers. When an incident photon has energy greater than or equal to the bandgap energy of the photodiode material, the electron-hole pair is created due to the absorption of photon. Such photon generated carriers in the depleted intrinsic region where most of the incident light photons are absorbed, are separated by the high electric field present in the depletion region and collected across the reverse biased junction. This gives rise to a photocurrent flow in the external circuit.

(ii) Avalanche photodiodes (APDs): It consists of four regions  $p^+ - i - p - n^+$  in order to develop a very high electric field in the intrinsic region as well as to impart more energy to photoelectrons to produce new electron-hole pairs by impact ionization. This impact ionization leads to avalanche breakdown in the reverse biased diode. So the APDs have high sensitivity and high responsivity over  $p - i - n$  diodes due to the avalanche multiplication.

### 2.3.3 Optical amplifiers

In the long distance optical fiber communication systems, the repeaters are situated at an equal distance of 100 km. These are used to receive and amplify the transmitted signal to its original intensity and then it is passed on to the main fiber. Previously it was done by conversion of optical energy into electrical energy and amplification by electrical amplifiers and then reconversion of electrical energy into optical energy. Such methods not only increase the cost and complexity of the optical communication system but also reduce the operational bandwidth of the system. But today it is done by erbium doped optical fiber amplifiers in an elegant manner by inserting a length of 10 m fiber amplifier for every 100 km length of main fiber. By this, the signal to noise ratio is greatly improved due to optical domain operation only.

### 2.3.4 Fiber couplers

A coupler is a device which distributes light from a main fiber into one or more branch fibers. There are core interaction type couplers and surface interaction type couplers. In

core interaction type couplers, the light energy transfer takes place through the core cross-section by butt jointing the fibers or by using some form of imaging optics between the fibers (i.e. using lensing schemes such as rounded end fiber, a spherical lens used to image the core of one fiber on to the core area of the other fiber and a taper-ended fiber). In the surface interaction type the light energy transfer takes place through the fiber surface and normal to the axis of the fiber by converting the guided core modes to cladding and refracted modes.

### 2.3.5 Fiber connectors

Before connecting one fiber with the other fiber in the fiber optic communication link, one must decide whether the joint should be permanent or demountable. Based on this, we have two types of joints. A permanent joint is done by splice and a demountable joint is done by connector. Requirements of a good connector:

1. At connector joint, it should offer low coupling losses.
2. Connectors of the same type must be compatible from one manufacturer to another.
3. In the fiber link, the connector design should be simple so that it can be easily installed.
4. Connector joint should not be affected by temperature, dust and moisture. That is, it should have low environmental sensitivity.
5. It should be available at a lower cost and have a precision suitable to the application.

### 2.3.6 Different generations of optical fiber communication

Generation	Wavelength of optical Source( $\mu$ m)	Bit Rate (Mb/s)	Repeater Spacing (Km)	Loss (dB/Km)	Existed upto
1	0.8	4.5	10	1	1980
2	1.3	$1.7 \times 10^2$	50	<1	1987
3	1.55	$1.0 \times 10^4$	70	<0.2	1990
4	1.55	$1.0 \times 10^5$	100	<0.002	2000
5	1.55	$>1.0 \times 10^9$	>100	<0.002	-

Table 1: Different generations of optical fiber communication systems



Table 1 shows the different generations of optical fiber communication. In generation I, mostly GaAs based LEDs and laser diodes having emission wavelength 0.8mm were used. From 1974 to 1978, graded index multimode fibers were used. From 1978 onwards, only single mode fibers are used for long distance communication. During the second generation the operating wavelength is shifted to 1.3 mm to overcome loss and dispersion. Further InGaAsP hetero-junction laser diodes are used as optical sources. In the third generation the operating wavelength is further shifted to 1.55mm and the dispersion-shifted fibers are used. Further single mode direct detection is adopted. In the fourth generation erbium doped optical (fiber) amplifiers are fabricated and the whole transmission and reception are performed only in the optical domain. Wavelength Division Multiplexing (WDM) is introduced to increase the bit rate. In the proposed next generation (V generation), soliton based lossless and dispersion less optical fiber communication will become a reality. At that time, the data rate may increase beyond 1000 Tb/s.

### **Advantages of optical fiber communication [1]**

1. Wider bandwidth: The information carrying capacity of a transmission system is directly proportional to the carrier frequency of the transmitted signals. The optical carrier frequency is in the range  $10^{13}$  to  $10^{15}$  Hz while the radio wave frequency is about  $10^6$  Hz and the microwave frequency is about  $10^{10}$  Hz. Thus the optical fiber yields greater transmission bandwidth than the conventional communication systems and the data rate or number of bits per second is increased to a greater extent in the optical fiber communication system.

2. Low transmission loss: Due to the usage of the ultra low loss fibers and the erbium doped silica fibers as optical amplifiers, one can achieve almost lossless transmission. In the modern optical fiber telecommunication systems, the fibers having a transmission loss of 0.002 dB/km are used. Further, using erbium doped silica fibers over a short length in the transmission path at selective points; appropriate optical amplification can be achieved. Thus the repeater spacing is more than 100 km. Since the amplification is done

in the optical domain itself, the distortion produced during the strengthening of the signal is almost negligible.

3. Dielectric waveguide: Optical fibers are made from silica which is an electrical insulator. Therefore they do not pickup any electromagnetic wave or any high current lightning. It is also suitable in explosive environments. Further the optical fibers are not affected by any interference originating from power cables, railway power lines and radio waves. There is no cross talk between the fibers even though there are so many fibers in a cable because of the absence of optical interference between the fibers.

4. Signal security: The transmitted signal through the fibers does not radiate. Further the signal cannot be tapped from a fiber in an easy manner. Therefore optical fiber communication provides hundred per cent signal security.

5. Small size and weight: Fiber optic cables are developed with small radii, and they are flexible, compact and lightweight. The fiber cables can be bent or twisted without damage. Further, the optical fiber cables are superior to the copper cables in terms of storage, handling, installation and transportation, maintaining comparable strength and durability.

### 2.3.7 Fiber Transmission Windows (Bands)[4]

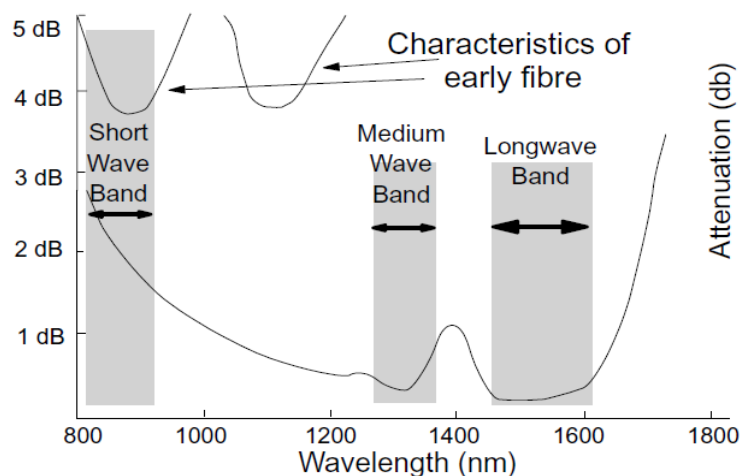


Figure 2.6: Transmission Windows.

The upper curve shows the absorption characteristics of fiber in the 1970s. The lower one is for modern fiber. In the early days of optical fiber communication, fiber attenuation was best represented by the upper curve in Figure 2.6 (a large difference from today). Partly for historic reasons, there are considered to be three “windows” or bands in the transmission spectrum of optical fiber. The wavelength band used by a system is an extremely important defining characteristic of that optical system.

#### (1) Short Wavelength Band (First Window)

This is the band around 800-900 nm. This was the first band used for optical fiber communication in the 1970s and early 1980s. It was attractive because of a local dip in the attenuation profile (of fiber at the time) but also (mainly) because you can use low cost optical sources and detectors in this band.

#### (2) Medium Wavelength Band (Second Window)

This is the band around 1310 nm which came into use in the mid 1980s. This band is attractive today because there is zero fiber dispersion here (on single-mode fiber). While sources and detectors for this band are more costly than for the short wave band the fiber attenuation is only about 0.4 dB/km. This is the band in which the majority of long distance communications systems operate today.

#### (3) Long Wavelength Band (Third Window)

The band between about 1510 nm and 1600 nm has the lowest attenuation available on current optical fiber (about 0.26 dB/km). In addition optical amplifiers are available which operate in this band. However, it is difficult (expensive) to make optical sources and detectors that operate here. Also, standard fiber disperses signal in this band. In the late 1990s this band is where almost all new communications systems operate.

### 2.3.8 Transmission Capacity

The potential transmission capacity of optical fiber is enormous. The medium wavelength band (second window) is about 100 nm wide and ranges from 1250 nm to 1350 nm (loss of about .4 dB per km). The long wavelength band (third window) is around 150 nm wide and ranges from 1450 nm to 1600 nm (loss of about .2 dB per km). The loss peaks at 1250 and 1400 nm are due to traces of water in the glass. The useful (low loss) range is therefore around 250 nm. Expressed in terms of analogue bandwidth, a 1 nm wide waveband at 1500 nm has a bandwidth of about 133 GHz. A 1 nm wide waveband at 1300 nm has a bandwidth of 177 GHz. In total, this gives a usable range of about 30 Tera Hertz ( $3 \times 10^{13}$  Hz).

Capacity depends on the modulation technique used. In the electronic world we are used to getting a digital bandwidth of up to 8 bits per Hz of analog bandwidth. In the optical world, that objective is a long way off (and a trifle unnecessary). But assuming that a modulation technique resulting in one bit per Hz of analog bandwidth is available, and then we can expect a digital bandwidth of  $3 \times 10^{13}$  bits per second.

Current technology limits electronic systems to a rate of about 10 Gbps, although higher speeds are being experimented with in research.

Current practical fiber systems are also limited to this speed because of the speed of the electronics needed for transmission and reception. The above suggests that, even if fiber quality is not improved, we could get 10,000 times greater throughput from a single fiber than the current practical limit.

### 2.3.9 G.652 “Standard” Fiber

The characteristics of single-mode fiber were specified by the International Telecommunications Union (ITU) in the 1980's. The key specifications are as follows:

Cladding diameter = 125 microns.

Mode field diameter = 9-10 microns at 1300 nm wavelength.

Cutoff wavelength = 1100-1280 nm.

Bend loss (at 1550 nm) must be less than 1 dB for travel through 100 turns of fiber wound on a spool of 7.5 cm diameter.

Dispersion in the 1300 nm band (1285-1330 nm) must be less than 3.5 ps/nm/km. At wavelengths around 1550 nm dispersion should be less than 20 ps/nm/km. The rate of change of dispersion with wavelength must be less than .095 ps/nm<sup>2</sup>/km. This is called the dispersion slope.

## 2.4 Effect of Dispersion [1]

Dispersion is the spreading out of a light pulse in time as it propagates down the fiber. Dispersion in optical fiber includes modal dispersion, material dispersion and waveguide dispersion. Each type is discussed in detail below.

### 2.4.1 Dispersion in Single-Mode Fiber

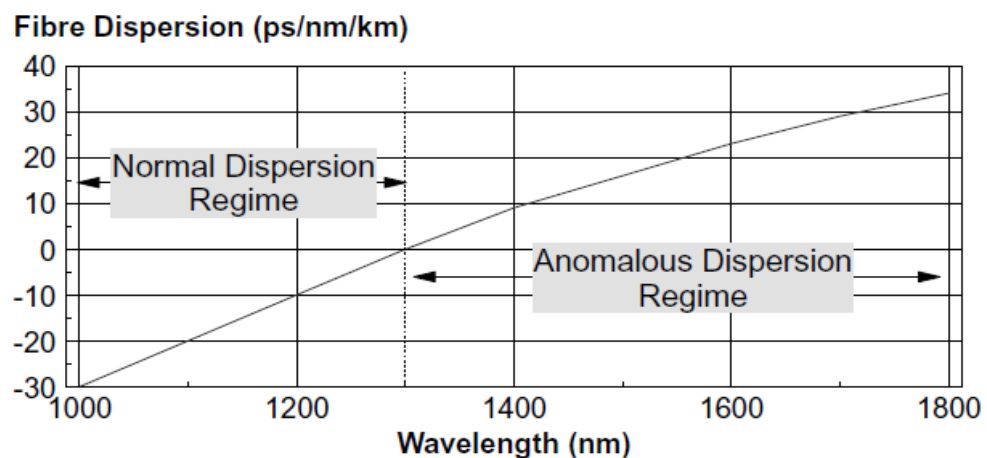


Figure 2.7: Dispersion of “Standard” Single-Mode Fiber

Since modal dispersion cannot occur in single-mode fiber, the major sources of dispersion are material (or chromatic) dispersion and waveguide dispersion.

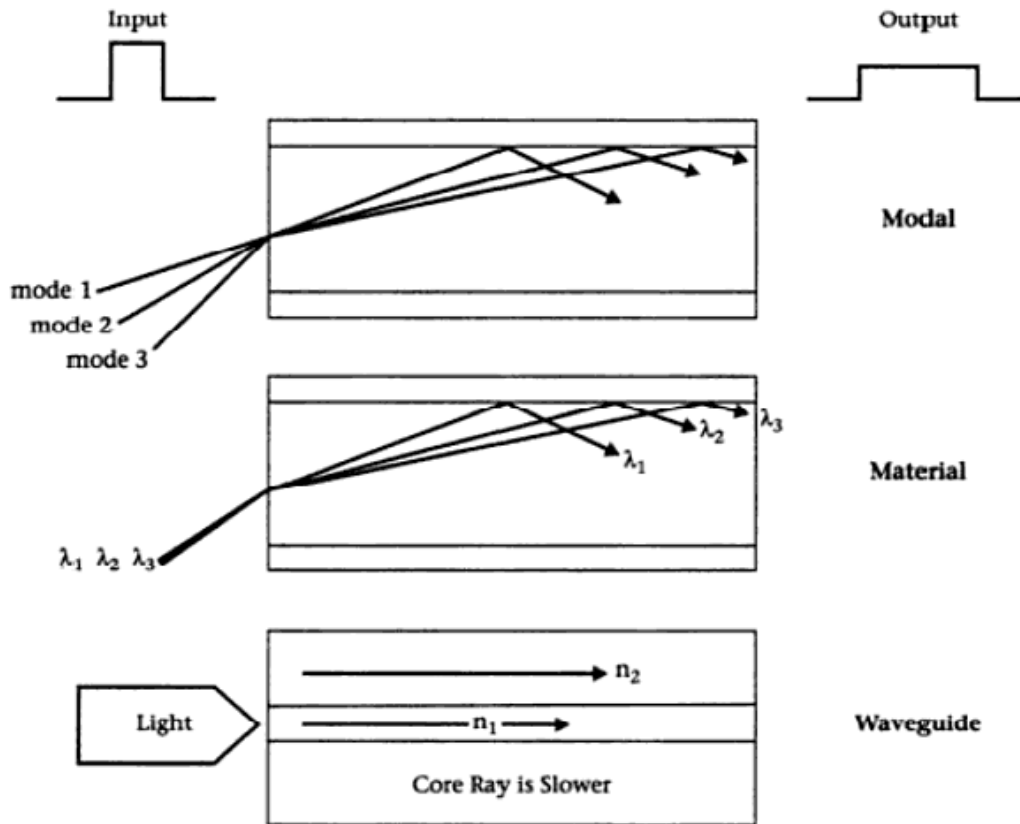


Figure 2.8: Dispersion in optical fiber

#### 2.4.2 Modal Dispersion in Multimode Fibers

Multimode fibers can guide many different light modes since they have much larger core size. This is shown as the 1st illustration in the figure 2.8. Each mode enters the fiber at a different angle and thus travels at different paths in the fiber.

Since each mode ray travels a different distance as it propagates, the ray arrives at different times at the fiber output. So the light pulse spreads out in time which can cause signal overlapping so seriously that you cannot distinguish them anymore. Modal dispersion is not a problem in single mode fibers since there is only one mode that can travel in the fiber.

### 2.4.3 Material (Chromatic) Dispersion

This is caused by the fact that the refractive index of the glass we are using varies (slightly) with the wavelength. Some wavelengths therefore have higher group velocities and so travel faster than others. Since every pulse consists of a range of wavelengths it will spread out to some degree during its travel. All optical signals consist of a range of wavelengths. This range may be only a fraction of a nanometer wide but there is always a range involved. Typically optical pulses used in communications systems range from about 0.2 nm wide to 5 nm wide for systems using single-mode fiber (with lasers).

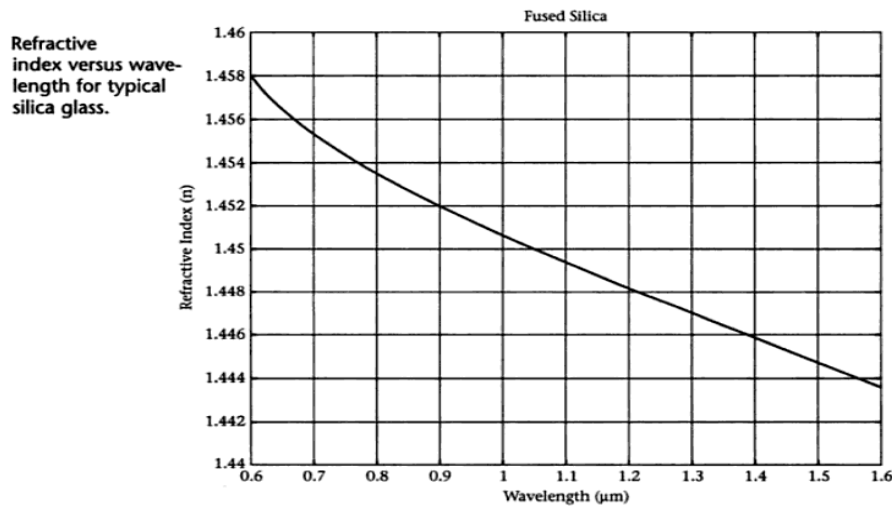


Figure 2.9 : The refractive index versus wavelength curve

The figure 2.9 shows the refractive index versus wavelength for a typical fused silica glass.

### 2.4.4 Waveguide Dispersion

Waveguide dispersion is only important in single mode fibers. It is caused by the fact that some light travels in the fiber cladding compared to most light travels in the fiber core. It is shown in the figure 2.10.

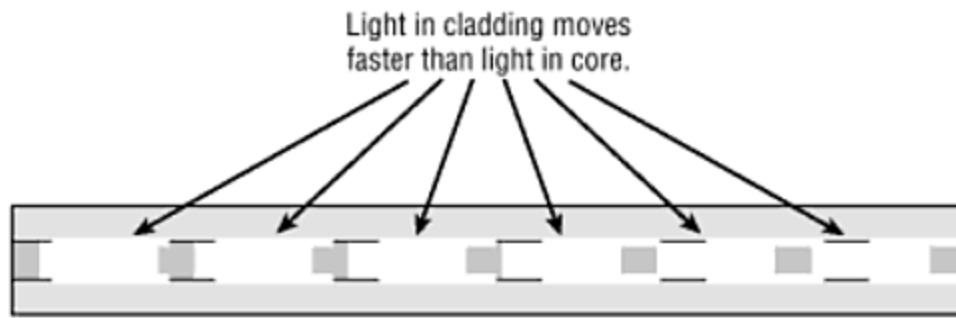


Figure 2.10: Showing the Waveguide Dispersion

Since fiber cladding has lower refractive index than fiber core, light ray that travels in the cladding travels faster than that in the core. Waveguide dispersion is also a type of chromatic dispersion. It is a function of fiber core size, V-number, wavelength and light source linewidth.

While the difference in refractive indices of single mode fiber core and cladding are minuscule, they can still become a factor over greater distances. It can also combine with material dispersion to create a nightmare in single mode chromatic dispersion.

Various tweaks in the design of single mode fiber can be used to overcome waveguide dispersion, and manufacturers are constantly refining their processes to reduce its effects. The shape (profile) of the fiber has a very significant effect on the group velocity. This is because the electric and magnetic fields that constitute the pulse of light extend outside of the core into the cladding. The amount that the fields overlap between core and cladding depends strongly on the wavelength. The longer the wavelength the further the the electromagnetic wave extends into the cladding.

The RI experienced by the wave is an average of the RI of core and cladding depending on the relative proportion of the wave that travels there. Thus since a greater proportion of the wave at shorter wavelengths is confined within the core, the shorter wavelengths “see” a higher RI than do longer wavelengths. (Because the RI of the core is higher than that of the cladding.) Therefore shorter wavelengths tend to travel more slowly than



longer ones. Thus signals are dispersed (because every signal consists of a range of wavelengths).

Notice that the two forms of dispersion cancel one another at a wavelength of 1310 nm. Thus if the signal is sent at 1310 nm dispersion will be minimized.

Dispersion (from all causes) is often grouped under the name “Group Velocity Dispersion” (GVD). On standard single-mode fiber we consider two GVD regimes - the “normal dispersion regime” and the “anomalous dispersion regime”.

#### 2.4.5 Polarization Mode Dispersion (PMD)

In single-mode fiber we really have not one but two modes (travelling on physically the same path). This is due to the fact that light can exist in two orthogonal polarizations. So we can send two possible signals without interference from one another on single-mode fiber if their polarizations are orthogonal. In normal single-mode fiber a signal consists of both polarizations. However, polarization states are not maintained in standard SM fiber. During its journey light couples from one polarization to the other randomly.

Birefringence is the name given to the characteristic found in some materials and in some geometry where the ray path exhibits a different refractive index to the different polarizations. This happens in normal single-mode fiber in that there is usually a very slight difference in RI for each polarization. It can be a source of dispersion but this is usually less than 0.5 ps/nm/km (for most applications trivial).

The effect is to cause a circular or elliptical polarization to form as the signal travels along the fiber. Dispersion resulting from the birefringent properties of fiber is called “Polarization Mode Dispersion” (PMD). More important; it is a source of “Birefringent Noise”. This is a form of modal noise. (It is also called “Polarization Modal Noise” in some publications). The mechanism involved is quite similar to the mechanism responsible for modal noise. As the light propagates along the fiber it constantly changes

in polarization in response to variations in the fiber's composition and geometry. Power is not lost but the axes of polarization and the orientation of magnetic and electric fields in relation to them constantly change. If there is a polarization sensitive device in the circuit that has significantly higher losses in one polarization mode than the other, then the effect of meeting with a signal that is constantly changing in polarization is to produce changes in total signal power. These changes constitute birefringent noise.

#### 2.4.6 Dispersion Shifted Fiber

So-called "standard" single-mode optical fiber has its dispersion null point (where waveguide dispersion and material dispersion cancel each other out) at a wavelength of 1310 nm. This was one of the reasons that almost all current long distance fiber networks operate at that wavelength. But there are a lot of very good reasons that we would like to operate in the 1550 nm band. These reasons are principally:

Fiber attenuation is a lot lower in the 1550 nm band. Erbium doped fiber amplifiers (EDFAs) operate in this band. It is true that praseodymium (Pr) doped amplifiers are available which operate in the 1300 band but their quality is not anywhere near as good as the erbium ones. WDM systems require a large amplified bandwidth and this means that we want to use the 1550 window and EDFAs.

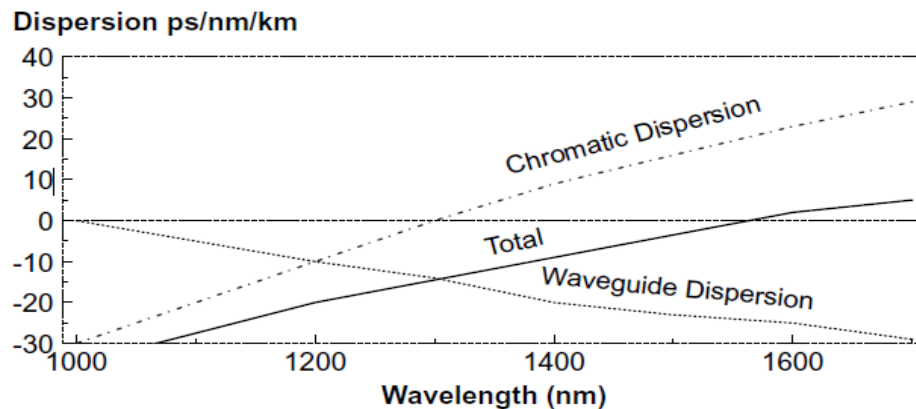


Figure 2.11: Dispersion Shifted Fiber

Dispersion of standard SM fiber at 1550 nm is very large (Around 17 ps/nm/km). We can mitigate the effects of this quite a lot by using lasers with very narrow linewidths but nevertheless it is a very significant problem.

Type of Fiber	Intermodal Dispersion	Intramodal Dispersion-Material	Intramodal Dispersion-Waveguide	Intersymbol Interference	Amplitude Distortion
Single-Mode Step Index	X	Y	Y	Y	Y
Multi Mode Step index	Y	Y	X	Y	Y
Multi Mode Graded Index	Y	Y	X	Y	Y

Table 2: Summary of dispersion properties of different optical fibers

### 3.1 Equalization Theory

For a typical optical link, the received pulse shape,  $h_r(t)$  is usually determined by the fiber impulse response,  $h_f(t)$ , and the transmitted pulse shape,  $h_t(t)$ .

$$h_r(t) = h_t(t) * h_f(t) \quad 3.1$$

where  $*$  denotes convolution. Normally, the transmitted pulse shape is known, leaving the engineer to estimate the impulse response of the fiber, which is often difficult to characterize. However, studies have shown that for a fiber that exhibits mode coupling, its impulse response is close to a cosine-squared shape in both the time and frequency domain.

$$h_f(t) = \exp[-(t^2/(2(\alpha T)^2))] / [\text{sqrt}(2\pi)](\alpha T) \quad 3.2$$

As such, it is highly probable that when a series of pulses is transmitted, overlapping will occur due to pulse broadening caused by dispersion as discussed in the previous section. Therefore, to reduce the pulse broadening that causes the resulting ISI, a suitable equalizer with a frequency response of  $H_{eq}(w)$  may be implemented.

$$H_{eq}(w) = H_{out}(w) / H_A(w) \quad 3.3$$

where  $H_{out}(w) = F[h_{out}(t)]$ , is the desired output pulse shape, and  $H_A(w) = F[h_A(t)]$ , is the total dispersive response of the system and  $F$  denotes Fourier transform.

As equalization makes up one of the main parts of an optical receiver together with the detector and amplifier, the equalizer is often designed to include the effects of the channel as well as the degradations caused by the amplifier.

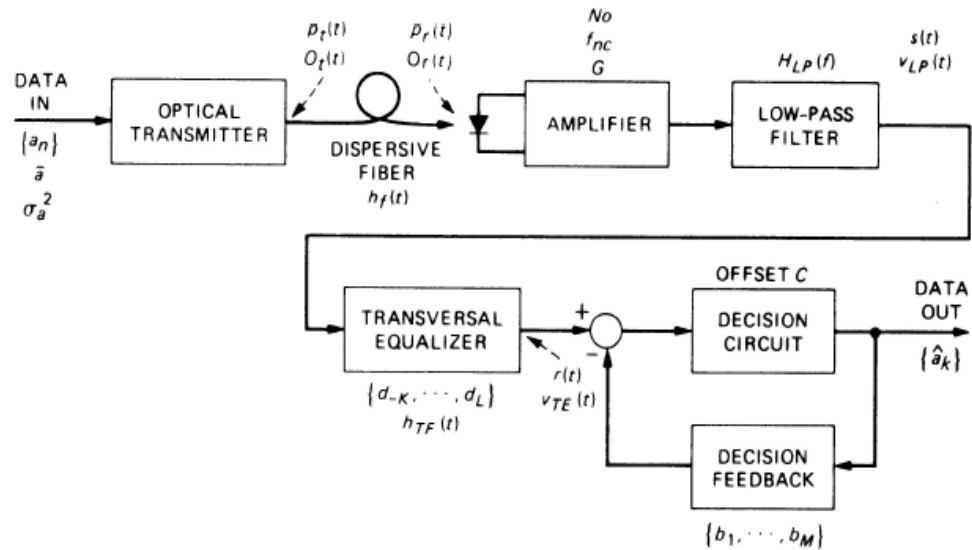


Figure 3.1 Application of a Decision Feedback Equalizer in an optical transmission system

### 3.2 Adaptive Filter Theory [8]

Adaptive equalization is increasing popular in optical fiber communications. However, with the ever-increasing demand for larger bandwidth and faster transmission speed, the increase in distortion in an optical channel is likely to be significant. Therefore, it is gainful to delve into adaptive equalization techniques to suppress distortion in an optical communication channel. Minimizing distortion will in turn, allow for longer haul communication before requiring a repeater, which will save infrastructure and equipment cost for a communication link. Adaptive filters are systems that can adjust themselves to different environments. It involves a process of filtering some input signal to match a desired response.

The filter parameters are updated by making a set of measurements of the underlying signals and applying that set to the adaptive filtering algorithm such that the difference between the filter output and the desired response is minimized in either a statistical or deterministic sense.

A simple yet effective illustration of the principle behind adaptive filtering is given in Fig (3.2).

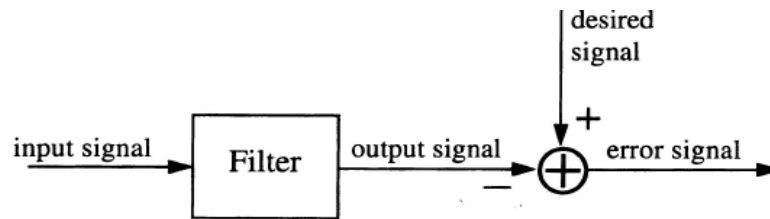


Figure 3.2: A simple diagram on the principle of adaptive filtering.

The general operation of an adaptive equalizer in an optical communication channel is to track the variations of the laser and fiber response over time. This is achieved by sending a known training sequence through the channel and obtaining the difference when the output is subtracted from the known sequence. The difference is known as the prediction error. The computed error is then used to adjust the tap coefficients so that the channel response can be estimated. This is done recursively using an adaptive algorithm like the least mean square (LMS) algorithm. Once the equalizer has converged, the actual data can then be sent and received accurately as the channel response is known and compensated for by the equalizer.

### 3.3 Comparison of Adaptive Algorithms

There exist a great number of adaptive algorithms, each with its own unique properties and applications. Before looking into the types of adaptive algorithms available, it is useful to look into factors that determine the performance of such algorithms.

**Convergence Rate** – This is defined as the number of iterations required for the algorithm to converge close enough to the optimum solution. A fast convergence rate enables the algorithm to track statistical variations when operating in a non stationary environment and requires a shorter training sequence.

**Computational Complexity** – This is defined as the number of operations required for the algorithm to make one complete iteration.

Misadjustment – this parameter provides a quantitative measure of the amount by which the final value of the mean square error, averaged over an ensemble of adaptive filters, deviates from the optimal minimum mean square error.

Numerical properties – when an algorithm is implemented numerically, inaccuracies are produced due to round-off noise and representation errors in the computer. This influences the stability of the algorithm.

### 3.4 Types of Equalizers suitable for an Optical Channel

In digital communications, an equalizer is a device that attempts to recover a signal transmitted through an ISI channel. It may be a simple linear filter or a complex algorithm.

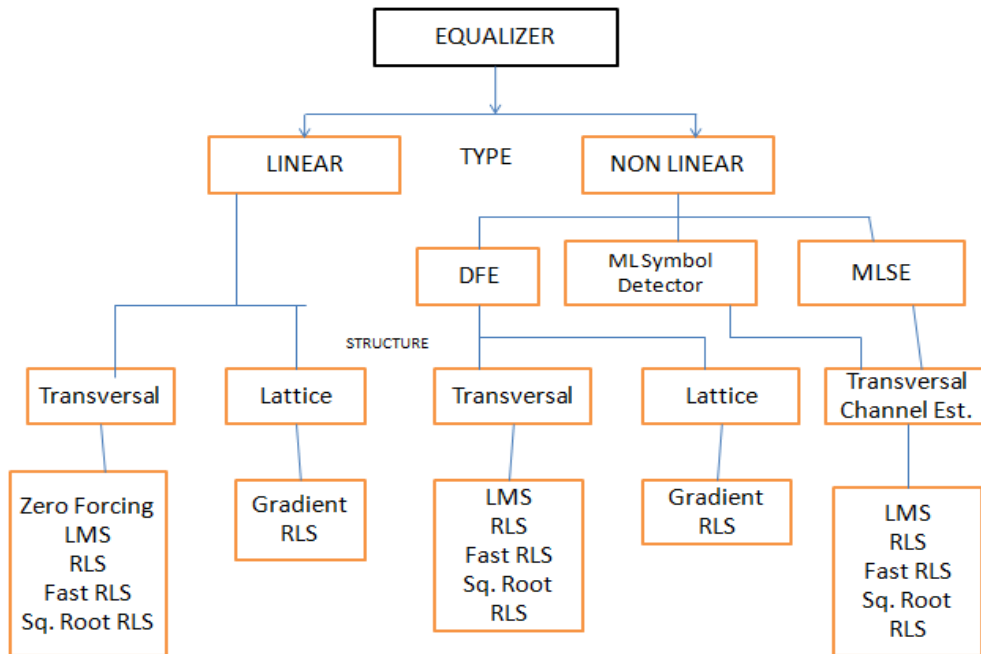


Figure 3.3: Classification of Equalizers

Several equalizer types are listed below:

- Linear Equalizer: processes the incoming signal with a linear filter

- MMSE equalizer: designs the filter to minimize  $E[|e|^2]$ , where  $e$  is the error signal, which is the filter output minus the transmitted signal.
- Zero Forcing Equalizer: approximates the inverse of the channel with a linear filter.
- Decision Feedback Equalizer: augments a linear equalizer by adding a filtered version of previous symbol estimates to the original filter output.
- Blind Equalizer: estimates the transmitted signal without knowledge of the channel statistics, using only knowledge of the transmitted signal's statistics.
- Adaptive Equalizer: is typically a linear equalizer or a DFE. It updates the equalizer parameters (such as the filter coefficients) as it processes the data. Typically, it uses the MSE cost function; it assumes that it makes the correct symbol decisions, and uses its estimate of the symbols to compute  $\hat{e}$ , which is defined above.
- Viterbi Equalizer: Finds the maximum likelihood (ML) optimal solution to the equalization problem. Its goal is to minimize the probability of making an error over the entire sequence.

The inherent property behind linear equalizers is that they are optimum with respect to the criterion of minimum probability of symbol error when the channel does not suffer from amplitude distortion. However, in a practical optical communication channel, amplitude distortion is one of the major detrimental effects.

Therefore, the investigation and research into the Decision Feedback Equalizer, which is a non-linear equalizer and capable of superior performance in amplitude distorted channels, would be very beneficial and relevant to the application in optical communications. When implementing the DFE structure, enhancements like the Fractionally Spaced Equalization, which makes the equalizer more robust to amplitude distortions, can also be considered.[7]

### **3.5 Fractionally-Spaced Equalizer (FSE)**



Conventional equalizers have tap spacing's that are spaced with respect to the symbol rate. It is known that an optimum receiver corrupted by Gaussian noise would have a matched filter sampled periodically at the symbol rate of the message and that the matched filter must be matched to the channel and corrupted signal prior to the equalizer.[16]

Since the channel response is usually unknown, the optimum matched filter must be adaptively estimated. A suboptimal result would cause degradation in performance and sensitivity to timing error from the sampling of the output.

A solution to the above is the implementation of the Fractionally Spaced Equalizer (FSE), which is based on sampling the incoming signal at least as fast as the Nyquist rate.

Fractionally-spaced equalizers have taps that are spaced closer than conventional adaptive equalizers, and with a sufficient number of taps, it is almost independent of the channel delay distortion. It means that the equalizer can negate the channel distortion without enhancing the noise. Although FSE requires increased complexity to implement, its ability to effectively compensate for an extremely wide range of delay distortion is a major feature that surpasses the complexity drawback.

Given the above properties, the FSE technique is a highly desirable application since it minimizes noise enhancement. With appropriately chosen tap spacing's; the FSE can be configured as the excellent feedforward filter.

Tap-Spacing Selection: The tap-spacing of the FSE must be close enough to sample the incoming signal as fast as the Nyquist rate. If the transmitted signal consists of pulses having a raised cosine spectrum with a roll-off factor  $\beta$ , then the signal spectrum extends upto  $(1 + \beta)/T_s$ .

And so, the sampling rate must be set to at least:

$$F_s = (1 + \beta)/2T_s$$

For example if  $\beta = 1$ , we will have a  $T_s/2$ -spaced equalizer. For  $\beta = 0.5$ , we would have a  $2T_s/3$ -spaced equalizer. In most cases,  $T_s/2$ -spaced FSEs are commonly used

The Multichannel Equalizer Structure.

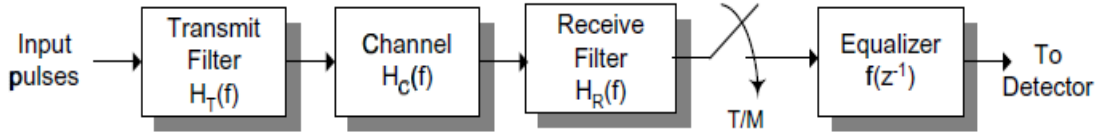


Figure 3.4: A Communication Channel with a FSE

The most general structure of a FSE is shown in fig (3.4). A tap-spacing of  $T_s/M$  is assumed, where  $M$  is an integer (usually  $M=2$ ).

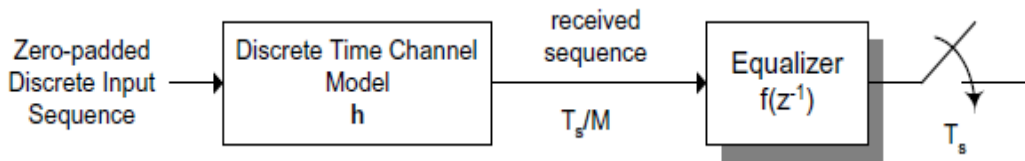


Figure 3.5: A Discrete time version of the channel with a FSE

The structure of the FSE in the discrete domain is shown in fig (3.5). Assuming the channel has an impulse-response of length  $(L_h - 1) T_s$ , the discrete-time representation of the channel will have a length  $(L_h - 1)M$ .

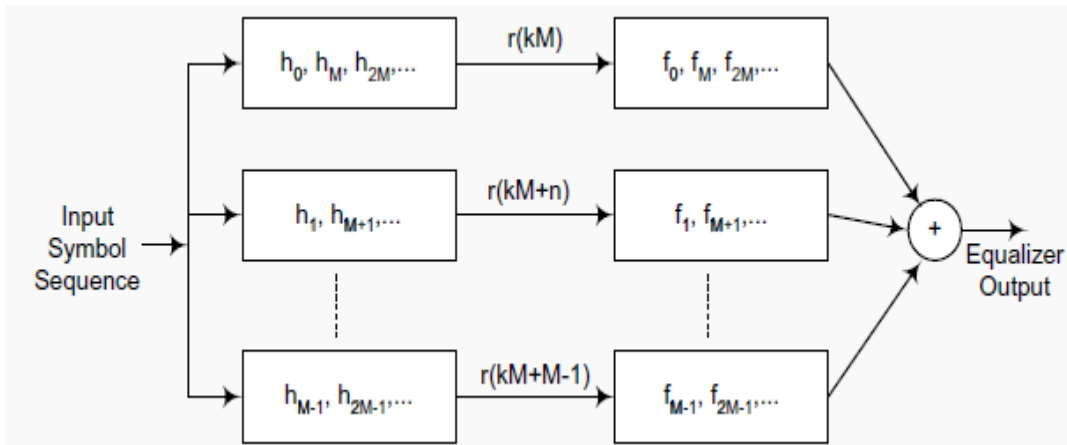


Figure 3.6: A Multichannel representation of the FSE

Let  $h$  be the  $(L_h - 1) M$ -vector representing the discrete time channel. The input pulse

sequence  $u(n)$  at a rate  $T_s$  is zero-padded with  $M$  zeros in-between samples to set the sampling rate to  $M/T_s$  and the output signal from the FSE is sampled at a rate  $T_s$  (i.e., decimated). Let  $f$  be the vector representing the fractionally-spaced equalizer tap coefficients. Because of the presence of these  $M$  zeros in-between samples, the convolution  $u * h * f$  can be easily represented as a multichannel, single-rate (i.e.,  $1/T_s$ ) structure, as shown in figure 3.6.

The channel coefficients for the first sub-channel are given by:  $h_0, h_M, h_{2M}, \dots, h_{(Lh-1)M}$ . Similarly, the coefficients for the  $m^{\text{th}}$  sub-channel is given by:  $h_m, \dots, h_{(Lh-1)M+m}$ . In a similar fashion the equalizer coefficients  $f$  are also decomposed into  $M$  sub-channels. The total response of the channel and the equalizer combination is given by:

$$h_t(n) = \sum_{m=1}^M \{h_n\}_m * \{f_n\}_m \quad 3.4$$

Where  $\{h_n\}_m$  and  $\{f_n\}_m$  represent the channel and the equalizer impulse response respectively in the  $m^{\text{th}}$  sub-channel.

### 3.5.1 Implementation of FSE with DFE

When we combine the DFE with FSE technique, we can expect an equalizer with the following qualities:

1. Minimize noise enhancement
2. Excellent amplitude distortion performance.

## 3.6 Decision Feedback Equalizer

Decision-feedback equalization was first introduced by Austin. Such equalizers are usually used in conjunction with a linear forward equalizer to remove ISI. One drawback of the linear equalization used in both partial-response maximum-likelihood (PRML) and DFE detectors is that it typically introduces high frequency boost, which causes noise enhancement.[13]

In a Decision Feedback Equalizer, both the feedforward and feedback filters are

essentially linear filters. It is a non-linear structure because of the non-linear operation in the feedback loop (decision threshold); its current output is based on the output of previous symbols.

The reason for choosing DFE over linear equalizer is that the latter's performance in channel that exhibit nulls is not effective. Noise enhancement in these regions and long impulse response are a problem. The basic reason for this problem is that in linear filtering, the desired signal and noise are processed together, causing noise enhancement problem. This is particularly detrimental to an optical channel, which suffers from amplitude distortion and attenuation.

In contrast, the decision feedback equalizer has zero noise enhancements in the feedback loop. As such, its performance is superior to the linear equalizer. The principle operation of the DFE is that a weighted sum of past decisions is subtracted from the incoming signal, with the feedback taps weights being exactly equal to the amplitude of corresponding pulse postcursors at each sampling instant. Therefore, if the decisions (output of the decision threshold) are correct, the ISI from the already-detected pulses is completely removed with no enhancement of the noise.

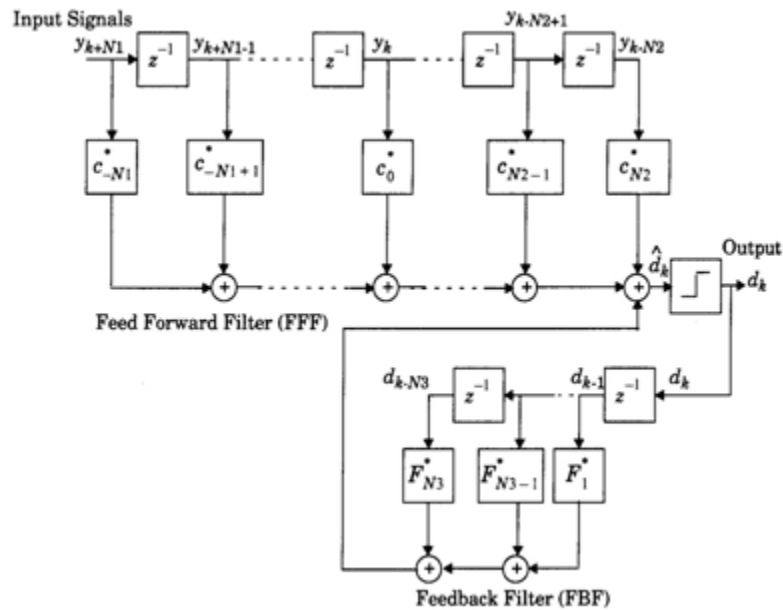


Figure 3.7: Structure of a Decision Feedback Equalizer

Figure 3.7 shows the structure of a Decision Feedback Equalizer. The input to the feed back filter comes from the output of the decision threshold device, and the coefficients are adjusted. To cancel the ISI on the current symbol as discussed earlier. The above DFE structure has  $N_1+N_2+1$  feedforward taps and  $N_3$  feedback taps.

The output of the equalizer is given by:

$$\hat{d}_k = \sum_{n=-N_1}^{N_2} C_n y_{k-n} + \sum_{i=1}^{N_3} F_i^* d_{k-i} \quad 3.5$$

where  $C_n^*$  is the tap gain and  $y_n$  is the input for the forward filter,  $F_i^*$  is the tap gain for the feedback filter and  $d_i(i < k)$  is the previous decision made on the detected signal. The DFE decodes channel inputs on a symbol-by-symbol basis and uses past decisions to remove trailing ISI. The minimum mean square error decision feedback equalizer optimizes the feedforward and feedback filter to minimize the mean-square error. Given its many advantages, there are two main problems surrounding the application of the DFE, namely, error propagation and slow convergence rate.

The feedback equalizer in the DFE is a FIR filter and, thus, its output is a linear combination of past decisions that cancel the postcursor ISI remaining after forward equalization. Usually, these equalizers are adaptive to compensate for channel variations. Typically, a FIR forward equalizer is used to remove the precursor ISI. The FIR forward equalizer is often constrained to generate its output from samples of the input waveform after and before the cursor. Such an equalizer has a high-pass transfer function that boosts the input signal at high frequency. This equalizer also boosts the input noise at high frequency causing noise enhancement, which degrades the SNR at the slicer input. The SNR is defined as the ratio of the signal power to the noise power (which consists of channel noise filtered by the FE and uncanceled ISI).

The optimal DFE filters generally have infinite length; designs in practical applications adopt finite length filters for the sake of stability and simplicity. There are two forms of Direct Form (DF) filters based on different structures: transversal (also called tapped delay line structure) and transposed DF. FFF adopts the former one because of its

insensitivity to summation delay. The ghost estimation of the FBF should be confined to one symbol cycle to handle adjacent post-echo.

Hence the traditional DFE adopts the transposed form to implement the FBF [9]. However, the transposed filter has its defects such as more resource consumption than the transversal filter. In particular, during fast-fading channel the stability of the transposed structure will be inferior to the transversal one.

### 3.6.1 Error Propagation [11]

When implementing a filter such as the DFE, where the adjustment term is based partly on past decision of symbols, there exists a probability that the decision made by the decision threshold device may be wrong. This point is often overlooked, with the common assumption of correct past decisions. This point is addressed in this section with appropriate solutions proposed.

The occurrence of one error in the DFE will cause a burst of new errors. This is due to the erroneous bit being fed back into the feedback filter of the DFE and causing a set of errors varying with the length of the feedback filter. The errors will only stop after  $M$  consecutive correct decisions ( $M$ =order of feedback filter).

To eliminate this problem, the possibility of decision errors has to be taken into account in the design process. Error propagation is the unique problem in DFE, however, its error probability can be kept under 'useable' condition with some conventional considerations. Making suitable system choices, Careful consideration of algorithm implementation, Keeping length of feedback filter short enough, Use of Tomlinson-Harashima precoding, Detection of error correction scheme.

A point to note, though, is that the error probability of the DFE cannot be improved by error correcting coding (for example, channel coding). This is because an immediate decision is required at the output of the decision threshold device. The Tomlinson-Harashima precoding and Error correction scheme would be discussed in brief detail in

the following sections.

### 3.6.2 Tomlinson-Harashima precoding [10]

With Tomlinson-Harashima (TH) precoding, the feedback filter (FBF) of the decision feedback equalizer is implemented at the transmitter to avoid the effects of error propagation in which a detection error advances through the FBF. Based on the roles of the feedforward filter (FFF) and FBF of the DFE, Tomlinson-Harashima precoding can be thought of as dealing with postcursor Intersymbol interference in the transmitter and precursor ISI in the receiver.

TH precoding has found widespread use in many data communication applications to remove ISI from phase distortions and micro reflections, and ISI from multipath. [11] TH precoding is typically implemented without adaptive compensation during the data transfer mode by keeping the transmitter precoder coefficients and receiver FFF coefficients fixed. These coefficients can be determined prior to data transmission during a training period in which the minimum mean-square error (MMSE) adjusted FFF and FBF coefficients of a DFE are calculated with adaptive techniques. After training, the FBF coefficients are relayed to the transmitter, where the FBF is implemented in conjunction with a modulo device which limits the peak transmit power. The FFF remains in the receiver and its output is passed to another modulo device, which produces the soft symbol estimates.

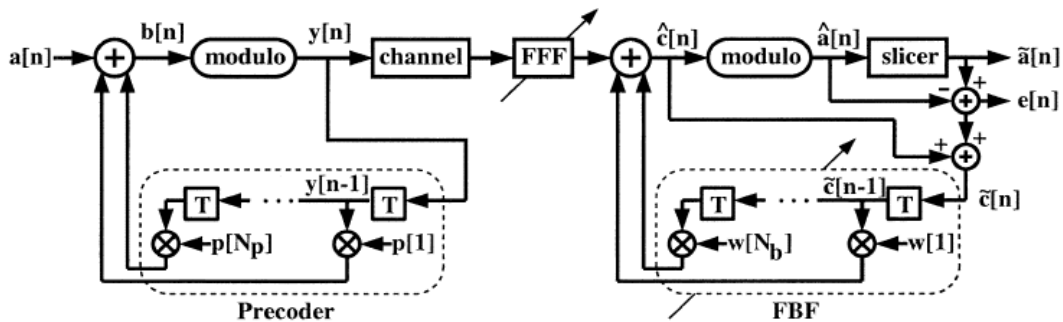


Figure 3.8: TH-precoded systems

At the end of each frame of data, the auxiliary FBF coefficients are relayed to the transmitter using a reverse channel to update the precoder periodically. The auxiliary

FBF adaptively compensates for misadjustments of the fixed transmitter precoder with respect to the actual channel at a given point in time. Thus, the magnitude of the auxiliary FBF coefficients will reflect the difference between the channel at a given point in time and the channel upon which the transmitter precoder is based. In this respect, the FBF coefficients (and hence, the effects of error propagation) can be kept small by updating the TH precoder often enough.

### **3.7 Bidirectional Zero-Based Decision Feedback Equalizers**

Over the last three decades, the use of Decision feedback equalizer (DFE) has been reported in digital communication applications to mitigate ISI. DFE provides a good compromise between performance and complexity, delivering a much better performance than a linear equalizer at a much lower complexity than that of the optimum detector the maximum likelihood sequence estimation (MLSE)[15]. Time-reversal DFE technique has drawn research interest, for the reason that the time reversal operation results in equivalent channel zeros seen from equalizer as the reciprocals of the actual channel zeros (the equivalent channel impulse response is the time reverse of the actual channel impulse response ). BiDFE has two branches, one DFE branch is used for normal time sequence equalization, and the other DFE branch is used for time-reversal sequence equalization. As channel diversity is exploited, BiDFE has better performance over traditional DFE. [15]

Here we consider the DFE where both feedforward filter (FFF) and feedback filter (FBF) are implemented using FIR filters based on channel zeros, as FIR filters are more robust in filter implementation.

#### **3.7.1 System Model and Time-Reversal Operation**

For single-user transmissions, the discrete-time baseband model is a time-invariant single-input single-output (SISO) system described by



$$y(n) = y_s(n) + w(n) \quad 3.7.1$$

$$y_s(n) = \sum_{k=0}^L x(n-k)h(k) \quad 3.7.2$$

Note that the input transmitted communication signal  $\{x(n)\}$  is independent of the additive white Gaussian noise  $\{w(n)\}$ . The channel transfer function can be expressed in a cascade form as the multiplication of first order polynomials given by,

$$H(z^{-1}) = \sum_{k=0}^L h(k)z^{-k} = \prod_{k=1}^L (1 - b(k)z^{-1}) \quad 3.7.3$$

where  $\{h(k)\}$  represent the channel impulse response and  $\{b(k)\}$  are channel zeros. Before we proceed with further discussion, the following assumptions on the channel are noted:

- 1) Channel zeros can be estimated accurately by using exiting techniques or obtained from the channel's zero distribution information as in wireless communication channels. Furthermore, no zeroes exist on the unit circle.
- 2) The channel order  $L$  is known. By reversing the observed signal  $\{y(n)\}$  before equalization, the equivalent channel impulse response changes to the time-reversal of the actual channel impulse response. The time-reversed channel transfer function can be expressed as,

$$\bar{H}(z^{-1}) = h(0) + h(1)z^{-1} + \dots + h(L)z^{-L} \quad 3.7.4$$

$$\prod_{i=1}^L (1 - b(i)z) = c \prod_{i=1}^L z(1 - b(z)^{-1}z^{-1}) \quad 3.7.5$$

Here constant  $c = (-1)^L \prod_{i=1}^L b(i)$  can be directly derived  $i=1$  from the channel zeros. We can see that the zeros of the time-reversal channel in (3.7.4) are the reciprocal of the original zeros of the channel in (3.7.5) operating in the traditional mode. In order to recover the transmitted signal, which is corrupted by multi-path effect and additive noise, a DFE can be used to equalize the time-reversal signals. In parallel, a traditional DFE can also be adopted for equalization with the normal sequence (i.e. non time reversed).

### 3.8 Detection of error correction scheme

Another method in combating error propagation in DFE is to implement an error correction scheme. In order to detect the start of feedback errors,  $T_+$  and  $T_-$  thresholds can be constructed outside of which error propagation is considered to have been initiated (designated Error Propagation Instant).

It is a simple method intended to detect feedback errors, which are constructive with respect to the desired symbol. For signals levels within the thresholds, it is deemed that error propagation has not been initiated. The objective is to minimize both the probability of incorrect estimates of EPI (false alarms) and the probability of incorrect estimates of EPI (misses) by adjusting the distance between  $T_+$  and  $T_-$ . Using the threshold scheme, upon the detection of EPI, bit reversal is applied. If EPI is estimated, then there is a high probability that the previous symbol estimate was in error and so the previous output symbol is corrected.

Another approach in using the threshold scheme is to let the feedback error detection device attempt to detect both constructive and destructive post cursor errors. If the output of the feedforward filter, falls outside these threshold pairs, EPI is deemed to exist necessitating bit reversal of the feedback estimate. The bit reversal operation can be realized by a feedback shift register, the contents of the register would be polarity reversed in order to correct the feedback error.

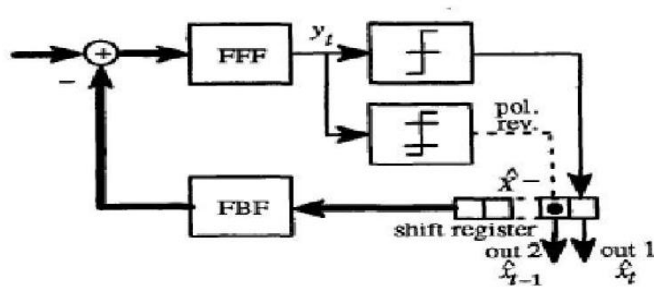


Figure 3.9: DFE model with feedback correction

### 3.9 Convergence rate of DFE

The LMS algorithm implementation with the DFE, though with its supreme advantages, does have the disadvantage of slow convergence. This is addressed with an analysis of the many techniques that have been proposed to increase the convergence rate of the DFE with the implementation of one in this thesis, the Activity Detection Guidance.[7]

### 3.9.1 Activity Detection Guidance

During high bit rate transmission in optical systems, the channel tends to have longer impulse responses. This will augment the periods of zero activity in the channel response. As the name suggests, the activity detection guidance technique is a method of detecting active taps in a communication channel.

By implementing a technique capable of detecting active taps in the channel, non-active taps can be excluded in the estimation of the channel response. This relieves the computational burden of the LMS algorithm, as well as to give a better convergence rate and asymptotic performance. The detection of the ‘active’ taps of a time-invariant channel is governed by the equation:

$$C_{k,n} = \frac{\left(\frac{1}{N} \sum_{i=1}^N i u_i y_{i-k+1}\right)^2}{\frac{1}{N} \sum_{i=1}^N (y_{i-k+1})^2} \quad 3.9.1$$

where  $i$ =time index,  $k$ =tap index, and  $N$  is the number of input samples.

$C_k$  is known as the activity measure. In order to determine a tap to be active, the value of the activity measure,  $C_k$ , must be above a certain threshold. This activity threshold is given by:

$$C_{k,n} > \sum_{i=1}^N \frac{(iu)^2 \log i}{i}$$

The accuracy improves with increasing  $N$ . [7] when a tap is detected as inactive, it will be excluded in the estimation of the active taps. Therefore, the computational burden of the adaptive algorithm is reduced.

However, the criterion discussed above is based on a few assumptions:

- The input signal,  $iu$ , bounded and wide sense stationary process
- The noise disturbance,  $nn$ , is a wide sense stationary white process, which is uncorrelated with the input signal

The activity measure given by Equation 3.9.1 requires the input signal to be uncorrelated over time. The active tap detection would not succeed if the input were colored because the correlation within the input signal would cause coupling amongst the output of the unknown channel taps.

### 3.9.2 Activity detection guidance with Tap Decoupling

Modifications to the activity measure have to be made to reduce the tap coupling effect.

$$CC_K = \sum_{i=1}^N \frac{[(iu_i - h_i u_i + \hat{H}_k y_{i-k+1}) \cdot (y_{i-k+1})]^2}{\sum_{i=1}^N (y_{i-k+1})^2} \quad 3.9.2$$

This modification would reduce the contribution of any adjacent active tap to the perceived activity of the actual tap being considered. Simulation results would prove later, in Chapter 4, that the addition of the Tap Decoupling method would give a better convergence rate and asymptotic performance.

The activity detection technique is employed in this thesis and the simulation results are shown in Chapter 4. The following sections review the techniques that have been proposed in improving the convergence rate of the DFE and serves as a comparison with the Activity Detection scheme.[7]

## 3.10 Non recursive adaptive equalization

The application of the non-recursive adaptive equalization can be analyzed into 2 stages: first estimate the channel, then compute the non-recursive DFE coefficients based on the channel estimates. The MMSE-DFE settings are computed using only the discrete Fourier transform (DFT) and the inverse discrete Fourier transform (IDFT), so that the whole computation can be carried out with  $O(M \log_2 M)$  iterations, when the fast Fourier transform (FFT) is used. [16] Direct matrix inversion can be avoided by approximating the correlation matrix by a circulant matrix.

The algorithm parallels a similar “cyclic equalization” technique applied to the linear equalizer, which uses aperiodic pseudo-random training sequence. The fast algorithm can be implemented very efficiently in practice and exhibits superiority in computational efficiency over the conventional recursive adaptive method. The main innovation of the fast start-up equalization arises from the approximation of a Toeplitz correlation matrix by a circulant matrix. A circulant matrix is asymptotically equivalent to a Toeplitz matrix. By making this approximation, most computations can be implemented with the discrete Fourier transform and inverse discrete Fourier transform very efficiently.

#### Computation Algorithm – Structured Matrices

Another approach to derive fast algorithms for the computation of the optimum feedforward and feedback filter settings of the MMSE-DFE is based on the theory of structured matrices. The feedback filter was shown to be one of the columns of the lower triangular factor of a structured correlation matrix. This allowed for an efficient computation of its settings using “generating function” algorithms for Cholesky factorization. Once the feedback filter is computed, the feedforward filter is efficiently calculated by solving a triangular system of equations by back substitution. This approach allows lower computational cost as compared to conventional techniques.

### **3.11 Modified Decision Feedback Equalizer (MDFE)**

In a conventional DFE, the feedback filter follows the feedforward filter, whereas the

FBF is performed prior to the FFF in the modified DFE. The principal behind MDFE this is that since the coefficient of the FBF can be initialized using an estimate of the channel impulse response, it is only required to initialize and train the FFF for the start-up. As a result, the MDFE can reach steady-state faster than conventional DFE.

### **3.12 Blind Equalization**

Classical equalization techniques employ a time-slot (recurring periodically for time-varying situations) during which a training signal, known in advance by the receiver, is transmitted. The receiver adapts the equalizer so that its output closely matches the known reference training signal. The more recent emergence of digital multipoint and broadcast systems has produced communication scenarios where training is infeasible or prohibited, since the inclusion of such signals sacrifices valuable channel capacity.[17]

Blind adaptive equalizers are those that do not need training to achieve convergence from an acceptable equalizer setting to a desired one. Blind equalization is desirable in multipoint and broadcast systems and necessary in noninvasive test and intercept scenarios. Even in point-to-point communication systems, blind equalization has been adopted for various reasons, including capacity gain and procedural convenience.

There are basically two different approaches to the problem of blind equalization. The stochastic gradient descent (SGD) approach which iteratively minimizes a chosen cost function over all possible choices of equalizer coefficients, while the statistical approach uses sufficient stationary statistics collected over a block of received data for a channel identification or equalization. The latter approach often exploits higher order cyclostationary statistical information directly. The intended work is focused on blind equalization method using stochastic gradient approach.

#### **Linear Blind Equalization System**

The Least Mean Square (LMS) adaptive equalizer employing a training sequence is given

by:

$$w(k + 1) = w(k) + \mu e(k)x(k) \quad 3.12.1$$

where  $\mu$  is a small step size controlling the convergence of the algorithm, and  $e(k)$  the difference between the output of the equalizer and the transmitted symbol. Naturally this algorithm requires that the channel input  $a(k - v)$  be available, the equalizer iteratively minimizes the  $E = |e(k)|^2$  mean square error(MSE) cost function in which the error is defined as :

$$e(k) = y(k) - a(k - v) \quad 3.12.2$$

If the MSE is small such that after training the equalizer output  $y(k)$  is a close estimate of the true channel input, then the decision device output can replace  $a(k - v)$  in a decision directed algorithm that continues to track the modest time variations in the channel dynamics. In blind equalization the channel input  $a(k)$  is unavailable, and thus different minimization criteria are explored. The crudest blind equalization scheme is the decision-directed scheme that updates the adaptive equalizer coefficients according to:

$$w(k + 1) = w(k) + \mu(y_k - Q[y(k)])x(k) \quad 3.12.3$$

where  $Q[y(k)] = \hat{a}(k - v)$  The ability of the equalizer to achieve desired convergence results when it is initialized with sufficiently small inter symbol interference (ISI) accounts for the key role that decision-directed algorithm plays in channel equalization. Without direct training, a blind equalization algorithm is therefore used to provide a good initialization scheme for the decision-directed equalizer because of the decision-directed equalizer's poor convergence behavior under high ISI.

Thus a better adaptive algorithm is needed for the blind equalization of linear channels when the initial coefficients are far from ideal. The general structure of the blind adaptive algorithm is shown in the Figure 3.10. Blind Adaptive equalization algorithms are often designed by minimizing special non-MSE cost functions that do not directly involve the

input  $a(k)$  while still reflect the current level of ISI in the equalizer output.

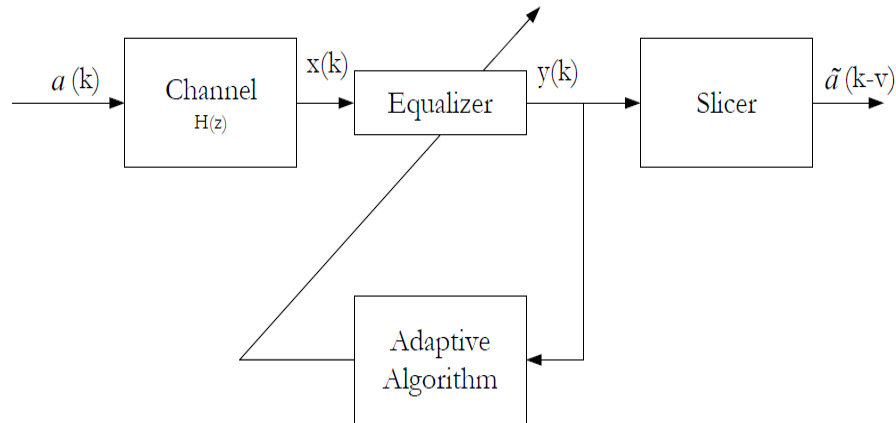


Figure 3.10: Linear Blind Equalization Systems

Let the mean cost function be defined as:

$$J_w = E\{\Psi(y(k))\}$$

where  $\Psi(\cdot)$  is a scalar function of the equalizer output. The mean cost function  $J(w)$  should be specified such that its minimum, the corresponding  $w$  results in a minimum ISI or MSE equalizer. Because of the symmetric distribution of  $\{a(k)\}$  over alphabet  $\hat{A}$  the blind equalizer is unable to distinguish between  $\pm a(k - v)$ . Thus the function  $\Psi(\cdot)$  should be even. In other words, both  $y(k) = a(k - v)$  and  $y(k) = -a(k - v)$  are acceptable objectives as global minima of the mean cost function. Using equation above, the stochastic gradient descent minimization algorithm is easily derived and is given by :

$$\begin{aligned} w(k+1) &= w(k) + \mu \frac{\partial \Psi(y(k))}{\partial w} \\ &= w(k) + \mu \Psi'(w^H x(k)) \end{aligned} \quad 3.12.4$$

Where  $\Psi' = \frac{\partial \Psi}{\partial x}$

The resulting blind equalization algorithm can be written as:

$w(k+1) = w(k) + \mu \Psi'(w^H(k)x(k))$  Hence, a blind equalizer can be defined by its cost function or its derivative. The derivative of the cost function is also called as the error function as it replaces the prediction error of the LMS algorithm. The Blind equalization when compared with the existing similar schemes gives better convergence rates at the cost of complexity.



## CHAPTER 4 - SIMULATED RESULTS

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### Simulation Results and Discussions

This chapter will present the results of the simulations performed using Matlab, based on the literature review and findings described in earlier chapters. The results were analyzed and presented using the parameters set to evaluate the performance of an adaptive equalizer, namely:

- Channel adaptation
- Asymptotic performance
- Convergence rate
- Squared difference of input to Channel and output of Equalizer

These analyzed results will be accompanied by discussions of the observations made compared to the theoretical findings in previous chapters.

The results shown are the average of ten simulations performed. The channel has a total of 2 active taps and a total tap length of 7. The noise interference applied was a zero mean white Gaussian signal, with a variance of 0.1. The adaptation step size was set at 0.005. Variations to these parameters (noise variance and step size) and their effects to the performance of the Equalizer were carried out and results are discussed.

The simulations involved the following:

- Fractionally-spaced Decision Feedback Equalizer using standard LMS
- Fractionally-spaced Decision Feedback Equalizer using standard LMS with Activity Detection Guidance
- Fractionally-spaced Decision Feedback Equalizer using standard LMS with Activity Detection Guidance and Tap Decoupling

## 4.1 Fractionally spaced Decision Feedback Equalizer

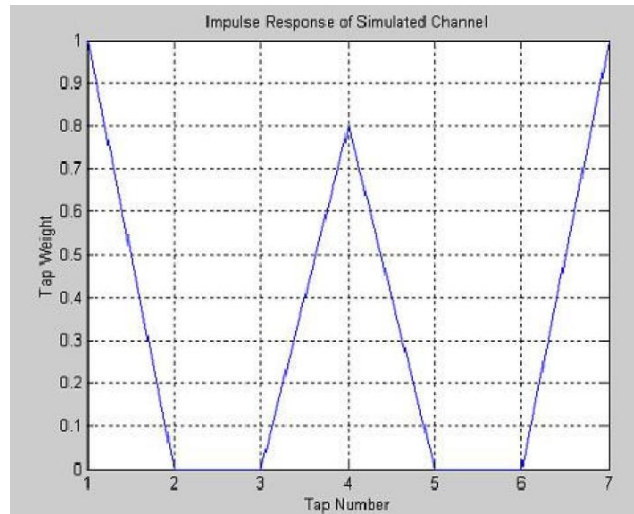


Figure 4.1.1: Simulated Channel Impulse Response

To ensure that the simulation done in this thesis is as realistic and effective as possible in Matlab, the channel impulse response of a typical optical channel is to be simulated. A typical optical communication channel has an impulse response resembling a cosine-square shape.

Figure 4.1.1 shows the simulated channel having a total of 7 taps. The feedforward section has a total 6 taps, the 1st tap has a magnitude of 1 and the 4th tap has a magnitude of 0.8. The rest of the taps are inactive or zero except for tap 7, which is the feedback tap with a magnitude of 1.

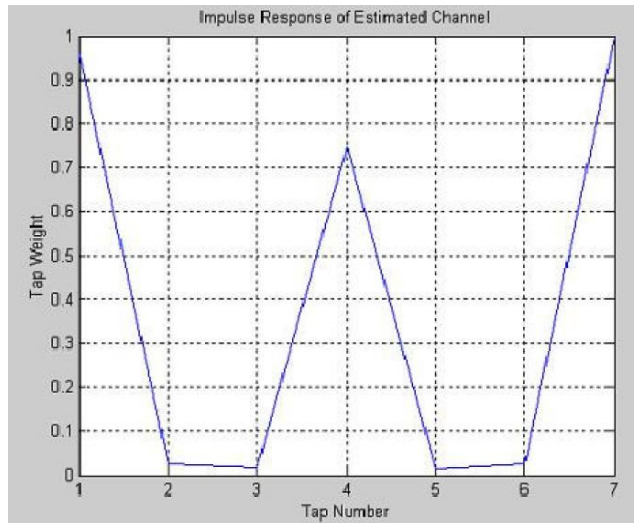


Figure 4.1.2: Estimated Channel Impulse Response

Figure 4.1.2 shows the impulse response of the estimated channel. The adaptation of the FS-DFE has been run for 30000 sample inputs to adapt the equalizer to achieve an impulse response in close proximity to the actual simulated channel. We can observe that the channel bears a fair resemblance of the simulated channel. However, the inactive taps, which were supposed to be zero, were not flat. This is due to the noisy estimates of the inactive taps in the channel.

In Figure 4.1.3, it can be observed that the asymptotic performance of the equalizer achieved is approximately  $10^{-2}$ . This is a relatively good result and shows that the equalizer filters have adapted closely to the simulated channel response.

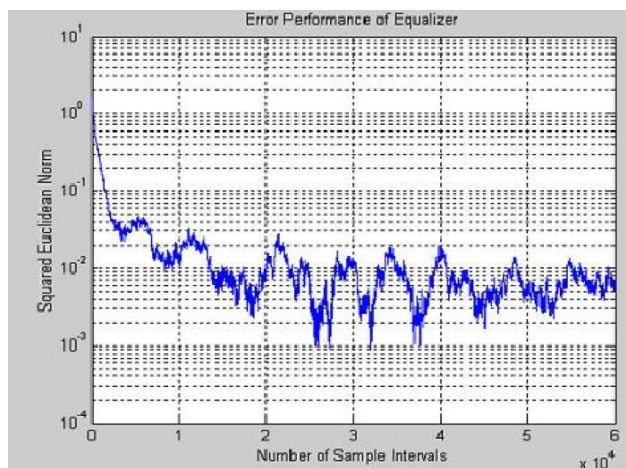


Figure 4.1.3: Asymptotic performance

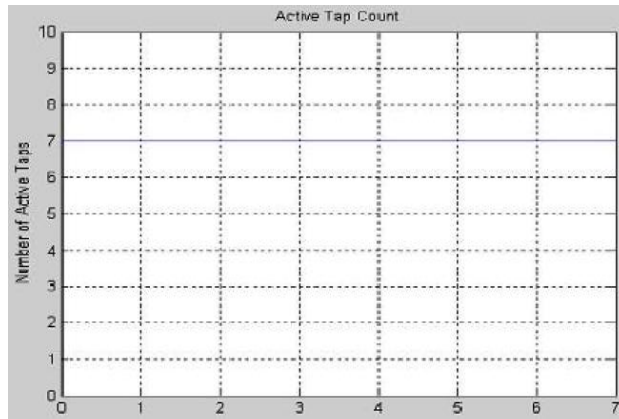


Figure 4.1.4: Active Tap Count

Figure 4.1.4 shows that the number of active taps detected were 7. This is clearly not the case as the simulated channel only has 2 active taps. By intuition, a better asymptotic performance and convergence rate can be achieved if the taps were estimated accurately and subsequently disregarding the inactive taps.

The next parameter in ascertaining the performance of the equalizer is its output. Figure 4.1.5 shows a plot of the squared difference of the input to the unknown channel and the output of the equalizer. There is a burst of errors in the initial sample inputs as the equalizer has not fully converged. The subsequent random errors indicate that the equalizer is removing the distortion in the channel but not completely.

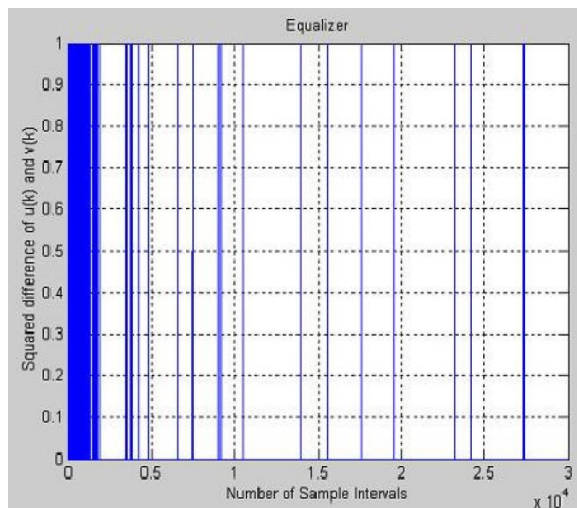


Figure 4.1.5: Squared difference of input to channel and output of equalizer

The graphs shown so far have been in 2-dimension, which is sufficient to understand and analyze the results that depicts the performance of the equalizer. However, to better illustrate the filter taps and adaptation, a 3-dimension view of the channel impulse response and adaptation is shown in Figure 4.1.6 and Figure 4.1.7. We can observe and appreciate the adaptation of the equalizer to the required impulse response of the simulated channel.

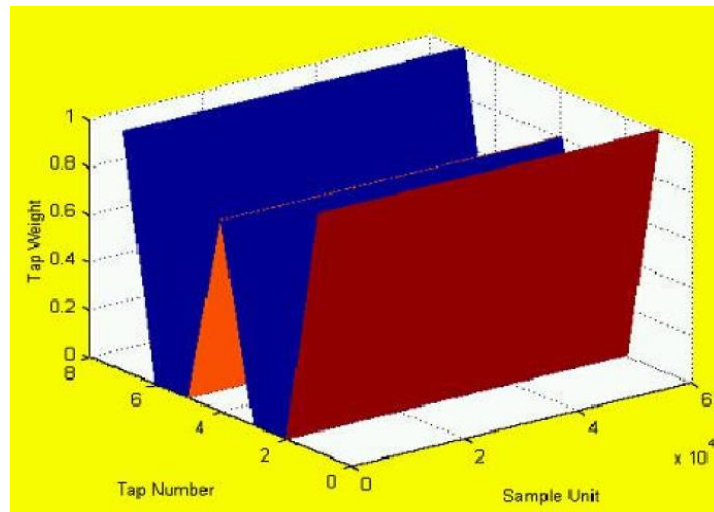


Figure 4.1.6: 3 D view of Simulated Channel

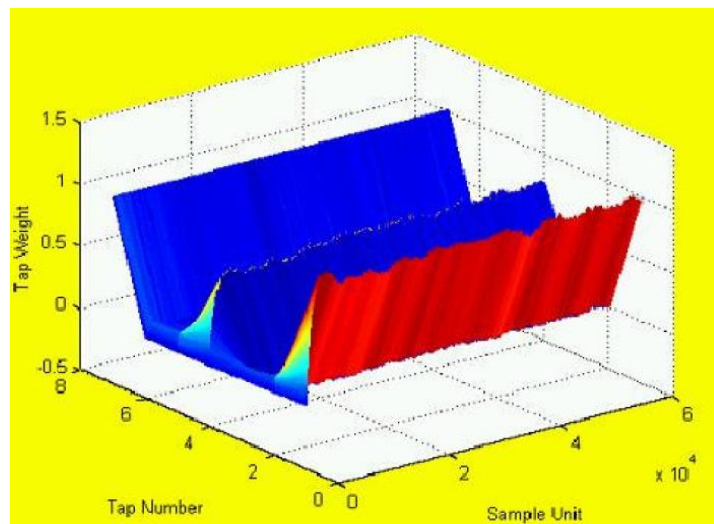


Figure 4.1.7: 3 D view of Estimated Channel

## 4.2 Fractionally spaced Decision Feedback Equalizer with Activity Detection Guidance

In Section 4.1, it was observed that the inactive taps in the estimated channel were not estimated at zero. To correct this problem of inactive taps being non-zero, an Activity Detection Guidance scheme was incorporated into the FS-DFE. Its purpose is to detect the non-active taps, disregard them in the calculation of the tap weight adaptation and set them to zero. By disregarding the inactive taps, convergence rate is most certainly improved and as shall be revealed later, so does the asymptotic performance.

To compare the channel estimation of the FS-DFE and the FS-DFE with ADG, both channel estimation graphs are presented respectively.

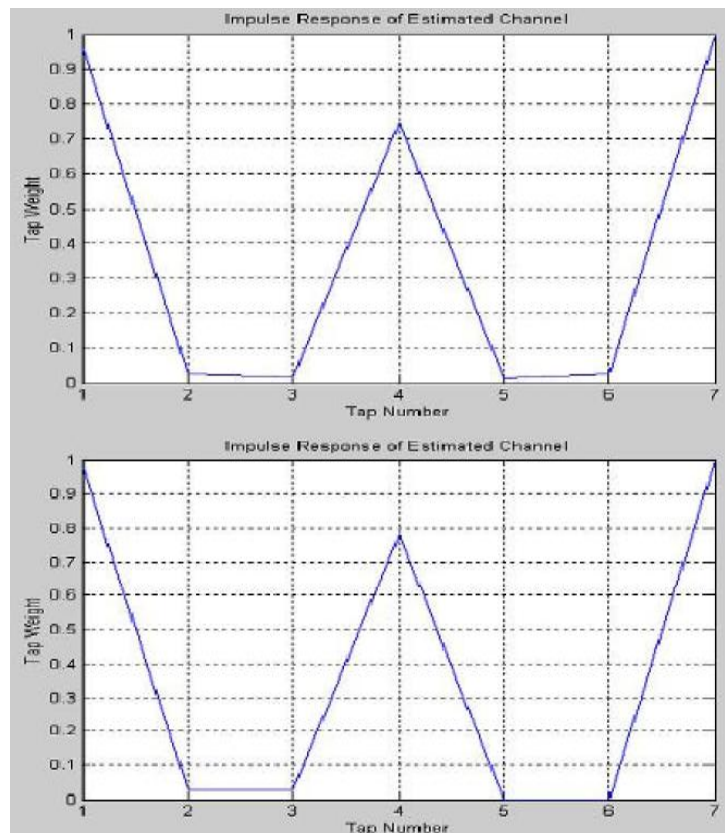


Figure 4.2.1: Comparison of Estimated Channel Response

It can be observed that the FS-DFE with ADG was capable of detecting the magnitudes and positions of the active taps in the channel and was able to ignore the estimation of the inactive taps partially. As expected, the convergence rate and asymptotic performance improved significantly.

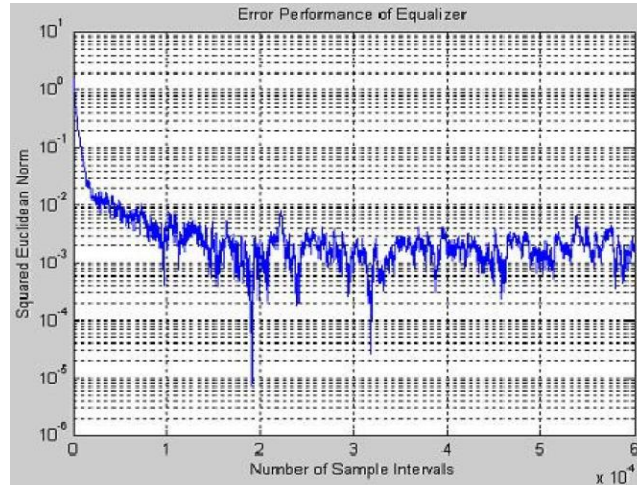


Figure 4.2.2: Asymptotic performance

The convergence rate was faster and the asymptotic performance improved from  $10^{-2}$  to  $10^{-3}$ . This is a significant improvement even though the non-active taps were not ignored in the tap weight estimation. As Figure below shows, the no. of active taps detected were still 7.

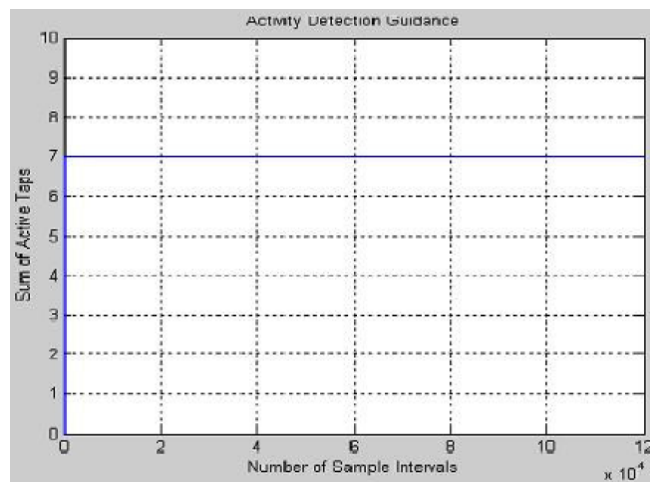


Figure 4.2.3: Active Tap Count

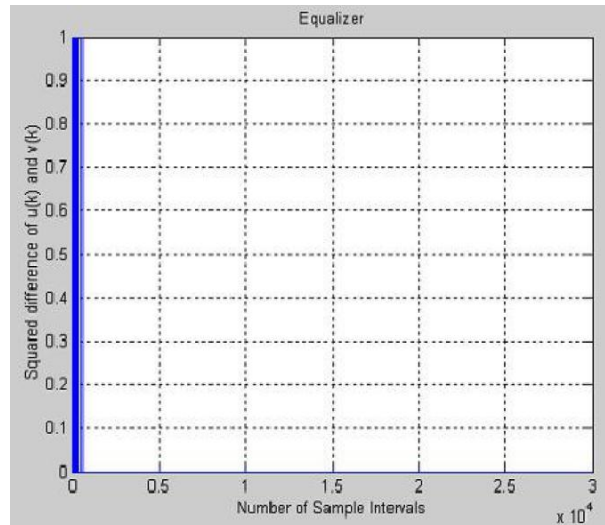


Figure 4.2.4: Output of equalizer

The output of the equalizer has improved significantly. The burst of errors in the beginning is still existing but considerably reduced and the number of errors encountered was notably lesser.

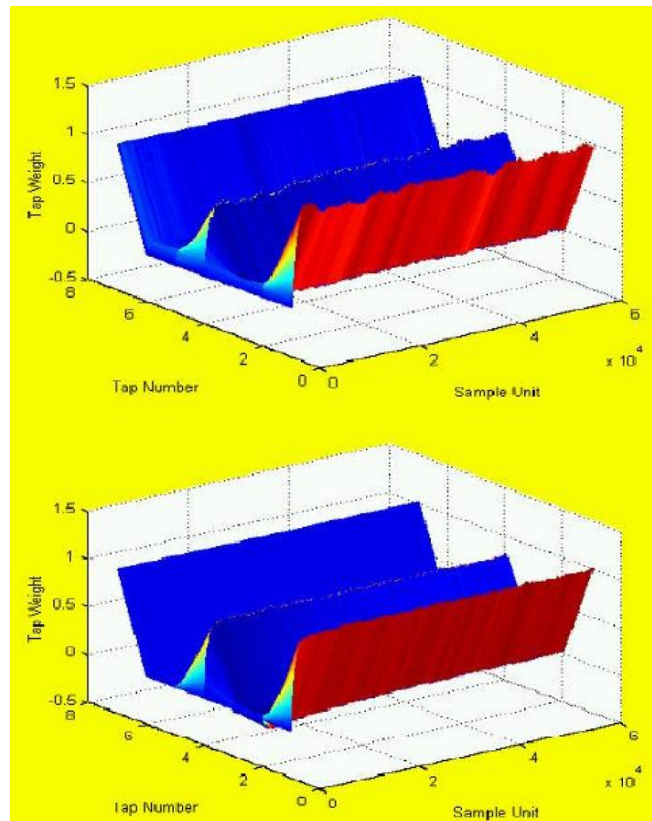


Figure 4.2.5: 3 D view of Estimated Channel



Figure 4.2.5 shows that the adaptation of the equalizer was improved. It can be observed that the adaptation was smoother with less fluctuations and resembling the simulated channel more closely.

### 4.3 Fractionally spaced Decision Feedback Equalizer with Activity Detection Guidance and Tap Decoupling

It was noted in the previous sections that the FS-DFE with ADG showed considerable improvement in convergence rate and asymptotic performance over the FS-DFE alone. However, the true number of active taps is not detected accurately and the inactive taps still suffer from the noisy estimates that are common in any communication channel.

The inability to detect the true number of active taps accurately is due to the correlation within the input signal that is causing the coupling amongst the output of the unknown channel taps. This coupling leads to the inactive taps to appear as active and the active taps to be less active. Using the principle behind the Activity Detection Guidance scheme, the tap coupling effect can be overcome with some modifications to the activity measure. The final results can be seen below.

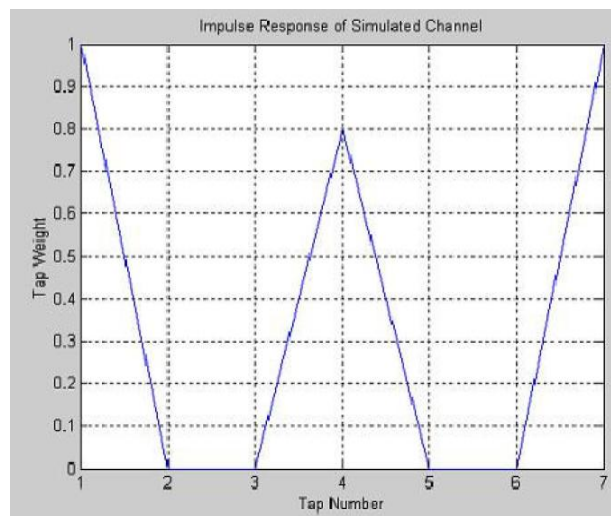


Figure 4.3.1: Simulated Channel Impulse Response

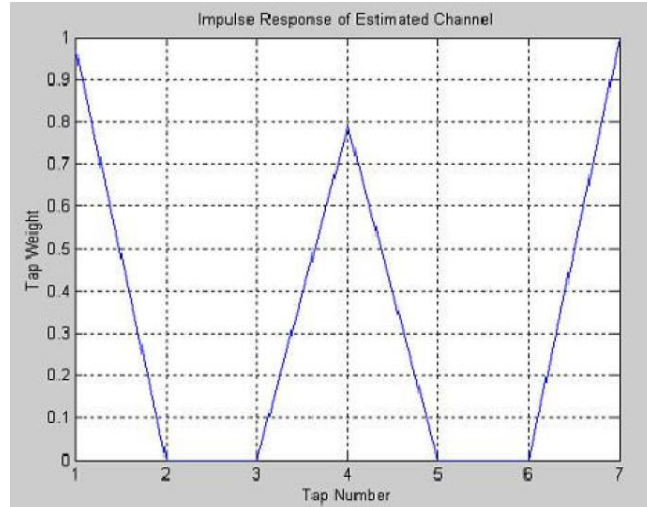


Figure 4.3.2: Estimated Channel Impulse Response

Figure 4.3.2 shows the impulse response of the estimated channel is almost an exact replica of the simulated channel. The magnitude and positions of the active taps were detected and adapted closely and the inactive taps were bound to zero.

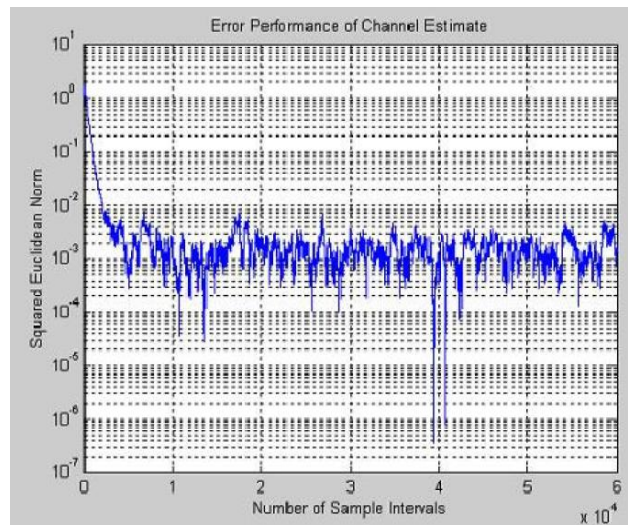


Figure 4.3.3: Asymptotic performance

Figure 4.3.3 shows the asymptotic performance of the equalizer to be  $10^{-3}$ . This is almost the same as the FS-DFE with ADG, however, the convergence rate of the FS-DFE with ADG and TD improved by 2 fold as compared to the FS-DFE with ADG. This certainly proves the theory behind the activity measure, by ignoring the inactive taps in the tap weight estimation; the convergence rate will almost certainly improve, which is proven in this case.

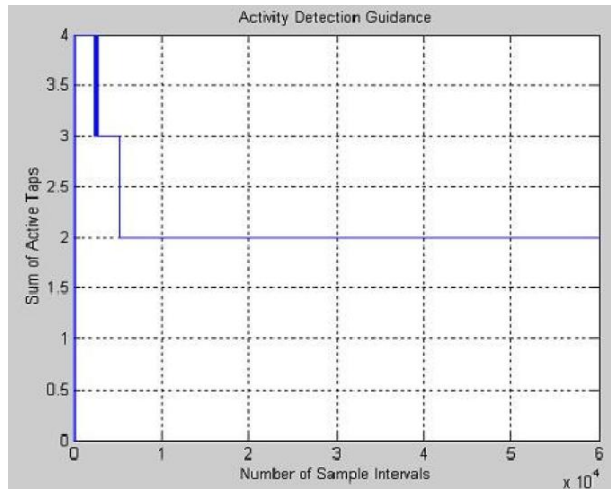


Figure 4.3.4: Active Tap Count

Figure 4.3.4 shows the true number of active taps being detected. The slight ‘jitters’ at the initial stage is most certainly due to noise as the adaptive filter converges to measure the correct number of active taps. This proves that the FS-DFE with ADG and TD has successfully been implemented and working effectively.

Figure 4.3.5 shows the accuracy of the output of the equalizer. Besides the burst of errors in the initial stage, there were zero errors for the rest of the data. This shows that the equalizer has adapted effectively and is mitigating the distortion effects of the channel successfully.

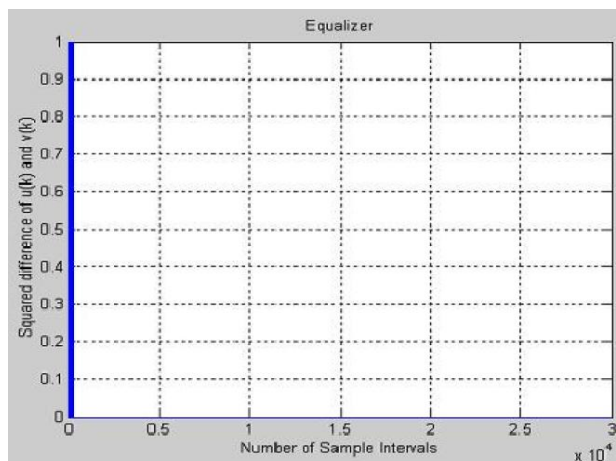


Figure 4.3.5: Equalizer Output

Figure 4.3.6 and Figure 4.3.7 shows the actual impulse response of the simulated channel and the adaptation of the equalizer over the training period in a 3 dimensional view. The result shows that the equalizer has adapted successfully close to the actual impulse response of the simulated channel.

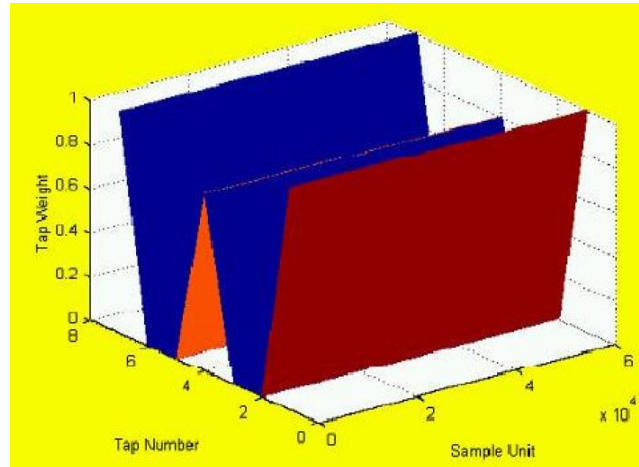


Figure 4.3.6: 3 D view of Simulated Channel

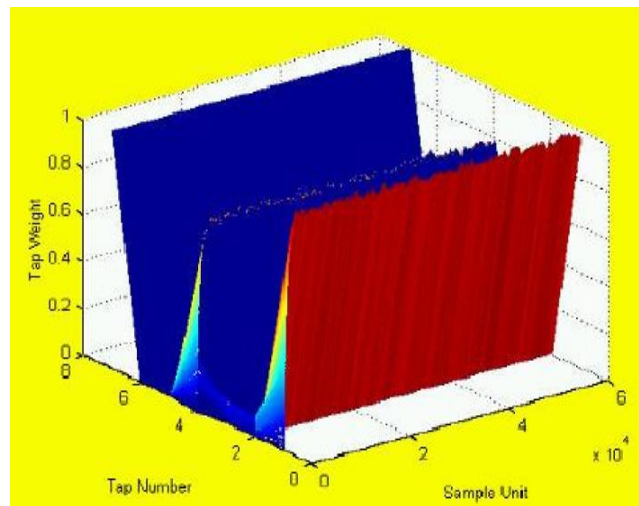


Figure 4.3.7: 3 D view of Estimated Channel

#### **4.4 Comparison between FS-DFE, FS-DFE with ADG, and FSDFE with ADG and TD**

A summary of results is presented below, comparing the critical components in assessing the performance of an adaptive equalizer.

	Channel Adaptation	Inactive Tap Detection	Steady-State error performance	Convergence Rate	Equalizer output
FS-DFE	Y	X	*	*	*
FS-DFE with ADG	Y	*	**	**	**
FS –DFE with ADG And TD	Y	**	***	***	***

Table 3: Summary of Results

Although the FS-DFE with activity detection could not detect the inactive taps completely, the asymptotic performance, convergence rate and equalizer output were superior to the FS-DFE only.

The FS-DFE with the activity detection and tap decoupling enhancements has the best performance throughout the parameters set to assess an adaptive equalizer. The convergence rate and asymptotic performance can be appreciated more clearly from Figure 4.4.1 below.

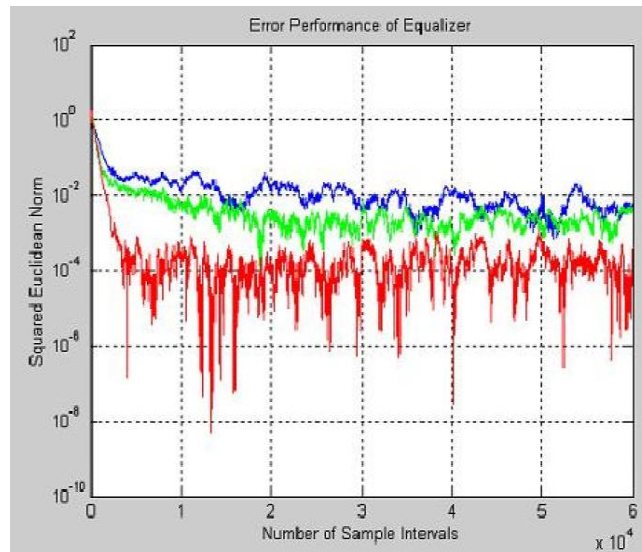


Figure 4.4.1: Comparison of Asymptotic performance

Blue – FS\_DFE, Green – FS\_DFE with ADG, Red – FS\_DFE with ADG and TD

## 4.5 Simulation Results for Blind Equalization

Total number of data samples  $n=3000$

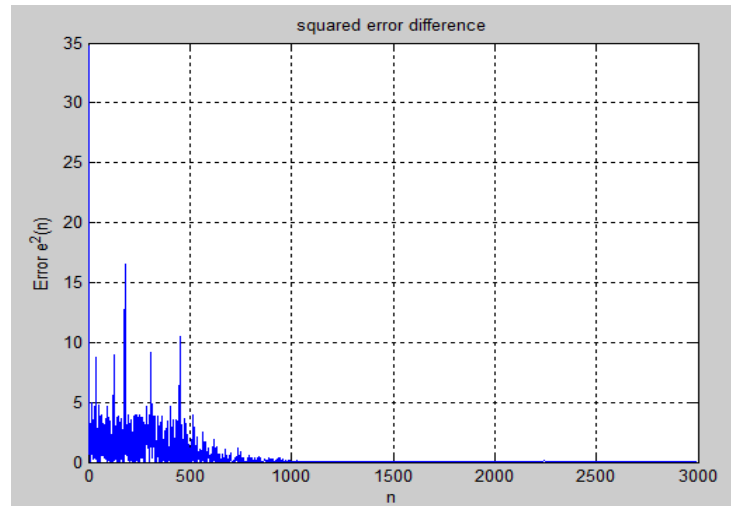


Figure 4.5.1: Plot of Squared Error versus number of samples

Figure 4.5.1 shows the asymptotic performance of the equalizer to which is '0' after few initial samples. Initially there is more error as compared to the above discussed schemes but after some samples it gives better performance. It achieves the almost idle condition for an equalizer. Although this technique is complex as compared to DFE-FSE technique but there is no need of any training sequence as well as no need of prior knowledge of channel is required. So it is superior to the adaptive DFE and its Enhancements. The above graph shows the performance of a linear Blind Equalization, now a day's research is going on the more enhanced versions of this algorithm. Blind equalization is the new research field and far superior than existing equalizers.

### 5.1 Conclusion

The objective of this thesis is to develop a suitable adaptive equalization technique to mitigate the effects of ISI and dispersion in a typical optical communication channel. With the successful development of the adaptive Decision Feedback Equalizer and blind Equalization, it offers an excellent alternative to the existing equalization techniques available in the optical communication industry. This thesis also gives brief introduction of the Blind Equalizer which is a very recent equalizer technique.

The adaptive equalizer was implemented using the Least Mean Square (LMS) technique, using stochastic gradient adaptation, for the direct equalization of the unknown channel. A number of adaptive algorithms had been analyzed and discussed. The LMS algorithm was chosen because of its simplicity in program development, robustness and stability. The FS-DFE was enhanced with the implementation of an activity detection scheme, which was devised to detect the zero activity periods and exclude them in the adaptation of the equalizer. This led to a faster convergence rate, as only the active taps need to be adapted and better asymptotic performance. Blind equalization is better technique to mitigate the effect of ISI but it is more complex as compared to existing techniques. Error performance of Blind equalizers is better than DFE equalizers.

In the nutshell, this thesis has demonstrated the successful implementation of an adaptive Decision Feedback Equalizer in a typical optical communication channel as well as new Blind equalization technique. The theoretical research and findings were successfully implemented and proven. This design approach is a promising alternative for equalization in optical communications.

## 5.2 Future Developments

Communication speed is fast increasing and optical links are the current and future answer to the high bit rate transmission of data. As data transmission speed increases, the dispersion and attenuation caused by the fiber also increases. Suggestions for some future expansion in this area of research are:

Develop an adaptive DFE using the recursive least squares algorithm on a lattice structure to provide faster convergence.

Develop a Blind Equalizer Technique because it gives better performance as compared to DFE and its enhancements.

Design an error detection scheme, with one of the techniques suggested in above Sections, and incorporate into the FS-DFE to counter the effects of error propagation. Develop the techniques suggested in above Section, to improve the convergence rate of the FS-DFE.

## 5.3 Scope

This thesis commences by giving an overview of the increasing demands in the transmission capacity of an optical communication and its distortion limitations. This is followed by a comprehensive literature review of the equalization techniques available to counter the effects of distortion in optical channels that limits the transmission capabilities. With the understanding of the background and theory, a suitable and effective adaptive equalization technique can be developed. Applying the necessary theoretical knowledge and programming acquaintance, simulation results can then be achieved and discussed in detail. Finally, the thesis will conclude with a summary of findings and recommendations for future developments in this area of research.



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## Appendix:

Adaptive decision feedback equalizers (DFE) with enhancements like Fractionally Spaced equalization and Activity Detection Guidance.

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**Abstract:** This paper introduces adaptive decision feedback equalizers (DFE). Evaluation can be made on the employment of the DFE algorithm and with enhancements, like Fractionally-Spaced equalization and Activity Detection Guidance, to improve its stability, steady-state error performance and convergence rate. The successful implementation of the DFE technique offers an excellent alternative to linear equalization. The FSE technique, when combined with the DFE, would offer improved effectiveness to amplitude distortion. As the impulse response of a typical optical link would have regions that are essentially zero, the employment of the activity detection scheme would further enhance the steady-state error performance and convergence rate.

**Keywords:** DFE, FSE, Activity detection guide, MLSE.

### 1. INTRODUCTION

Decision feedback equalization is considered to be a powerful technique to cope with channel distortions in high speed data transmissions [1]. Over the last three decades, the use of Decision feedback equalizer (DFE) has been reported in digital communication applications to mitigate ISI [2, 3]. DFE provides a good compromise between performance and complexity, delivering a much better performance than a linear equalizer at a much lower complexity than that of the optimum detector the maximum likelihood sequence estimation (MLSE).

The inherent property behind linear equalizers is that they are optimum with respect to the criterion of minimum probability of symbol error when the channel does not suffer from amplitude distortion. However, in a practical optical communication channel, amplitude distortion is one of the major detrimental effects.

Therefore, the investigation and research into the Decision Feedback Equalizer, which is a non-linear equalizer and capable of superior performance in amplitude distorted channels, would be very beneficial and relevant to the application in optical communications. When implementing the DFE structure, enhancements like the Fractionally Spaced Equalization, which makes the equalizer more robust to amplitude distortions, can also be considered.

The rest of the paper is organized as follows. Section II briefly discusses Decision Feedback Equalizer DFE's and Fractionally-Spaced Equalizer (FSE). Section III offers the simulations involved the following: Fractionally-spaced Decision Feedback Equalizer using standard LMS, Fractionally-spaced Decision Feedback Equalizer using standard LMS with Activity Detection Guidance and Fractionally-spaced Decision Feedback Equalizer using standard LMS with Activity Detection Guidance and Tap Decoupling. Finally, conclusions are generated in Section IV.

## II. DECISION FEEDBACK EQUALIZER

In a Decision Feedback Equalizer, both the feedforward and feedback filters are essentially linear filters. It is a non-linear structure because of the non-linear operation in the feedback loop (decision threshold); its current output is based on the output of previous symbols. The decision feedback equalizer has zero noise enhancements in the feedback loop. As such, its performance is superior to the linear equalizer. The principle operation of the DFE is that a weighted sum of past decisions is subtracted from the incoming signal, with the feedback taps weights being exactly equal to the amplitude of corresponding pulse postcursors at each sampling instant. [4] Therefore, if the decisions (output of the decision threshold) are correct, the ISI from the already-detected pulses is completely removed with no enhancement of the noise.

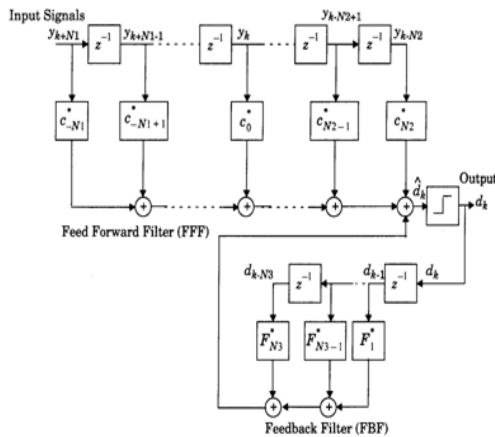


Figure 1 Structure of a Decision Feedback Equalizer. [5]

Figure 1 shows the structure of a Decision Feedback Equalizer. The input to the feedback filter comes from the output of the decision threshold device, and the coefficients are adjusted to cancel the ISI

on the current symbol as discussed earlier. The above DFE structure has  $N_1 + N_2 + 1$  feedforward taps and  $N_3$  feedback taps.[5]

The output of the equalizer is given by:

$$\hat{d}_k = \sum_{n=-N_1}^{N_2} c_n^* y_{k-n} + \sum_{i=1}^{N_3} F_i d_{k-i} \quad (1)$$

where  $C_n^*$  is the tap gain and  $y_n$  is the input for the forward filter,  $F_i$  is the tap gain for the feedback filter and  $d_i (i < k)$  is the previous decision made on the detected signal.

The DFE decodes channel inputs on a symbol-by-symbol basis and uses past decision to remove trailing ISI. The minimum mean square error decision feedback equalizer optimizes the feed forward and feedback filter to minimize the mean-square error. Given its many advantages, there are two main problems surrounding the application of the DFE, namely, error propagation and slow convergence rate.

## FRACTIONALLY-SPACED EQUALIZER (FSE)

Conventional equalizers have tap spacing that are spaced with respect to the symbol rate. It is known that an optimum receiver corrupted by Gaussian noise would have a matched filter sampled periodically at the symbol rate of the message and that the matched filter must be matched to the channel and corrupted signal prior to the equalizer.

Since the channel response is usually unknown, the optimum matched filter must be adaptively estimated. A suboptimal result would cause degradation

in performance and sensitivity to timing error from the sampling of the output. A solution to the above is the implementation of the Fractionally Spaced Equalizer (FSE), which is based on sampling the incoming signal at least as fast as the Nyquist rate.

Fractionally-spaced equalizers have taps that are spaced closer than conventional adaptive equalizers, and with a sufficient number of taps, it is almost independent of the channel delay distortion. It means that the equalizer can negate the channel distortion without enhancing the noise. Although FSE requires increased complexity to implement, its ability to effectively compensate for an extremely wide range of delay distortion is a major feature that surpasses the complexity drawback. Given the above properties, the FSE technique is a highly desirable application since it minimizes noise enhancement. With appropriately chosen tap spacing's; the FSE can be configured as the excellent feed forward filter.

Implementation of FSE with DFE : When we combine the DFE with FSE technique, we can expect an equalizer with the following qualities:

1. Minimize noise enhancement
2. Excellent amplitude distortion performance

#### Convergence rate of DFE

The LMS algorithm implementation with the DFE, though with its supreme advantages, does have the disadvantage of slow convergence. This is addressed with an analysis of the many techniques that have been proposed to increase the convergence rate of the DFE with the implementation of one in this paper, the Activity Detection Guidance.

#### Activity Detection Guidance

During high bit rate transmission in optical systems, the channel tends to have longer

impulse responses. This will augment the periods of zero activity in the channel response. As the name suggests, the activity detection guidance technique is a method of detecting active taps in a communication channel.

By implementing a technique capable of detecting active taps in the channel, non-active taps can be excluded in the estimation of the channel response. This relieves the computational burden of the LMS algorithm, as well as to give a better convergence rate and asymptotic performance.

The detection of the 'active' taps of a time-invariant channel is governed by the equation [6]:

$$C_{k,N} = \frac{(\frac{1}{N} \sum_{i=1}^N iu \cdot y_{i-k+1})^2}{\frac{1}{N} \sum_{i=1}^N (y_{i-k+1})^2} \quad (2)$$

where i=time index, k=tap index, and N is the number of input samples.

C<sub>k</sub> is known as the activity measure. In order to determine a tap to be active, the value of the activity measure, C<sub>k</sub>, must be above a certain threshold. This activity threshold is given by:

$$C_{k,N} > \sum_{i=1}^N \frac{(iu)^2 \cdot \log(i)}{i} \quad \dots(3)$$

The accuracy improves with increasing N. [6] when a tap is detected as inactive, it will be excluded in the estimation of the active taps. Therefore, the computational burden of the adaptive algorithm is reduced.

However, the criterion discussed above is based on a few assumptions:

- The input signal, iu, bounded and wide sense stationary process
- The noise disturbance, is a wide sense stationary white process,

which is uncorrelated with the input signal.

The active tap detection would not succeed if the input were colored because the correlation within the input signal would cause coupling amongst the output of the unknown channel taps.

Activity detection guidance with Tap Decoupling

Modifications to the activity measure have to be made to reduce the tap coupling effect. [6]

$$CC_k = \frac{\sum_{i=1}^N [(iu_i - h i u_i + \hat{H}_k y_{i-k+1}) \cdot (y_{i-k+1})]^2}{\sum_{i=1}^N (y_{i-k+1})^2} \dots\dots(4)$$

This modification would reduce the contribution of any adjacent active tap to the perceived activity of the actual tap being considered. Simulation results would prove that the addition of the Tap Decoupling method would give a better convergence rate and asymptotic performance.

### III. SIMULATION RESULTS AND DISCUSSIONS

This section will present the results of the simulations performed using Matlab, based on the above section.

The results shown are the average of ten simulations performed. The channel has a total of 2 active taps and a total tap length of 7. The noise interference applied was a zero mean white Gaussian signal, with a variance of 0.1. The adaptation step size was set at 0.005.

The simulations involved the following:

- Fractionally-spaced Decision Feedback Equalizer using standard LMS
- Fractionally-spaced Decision Feedback Equalizer using standard

LMS with Activity Detection Guidance

- Fractionally-spaced Decision Feedback Equalizer using standard LMS with Activity Detection Guidance and Tap Decoupling

#### 3.1 Fractionally spaced Decision Feedback Equalizer

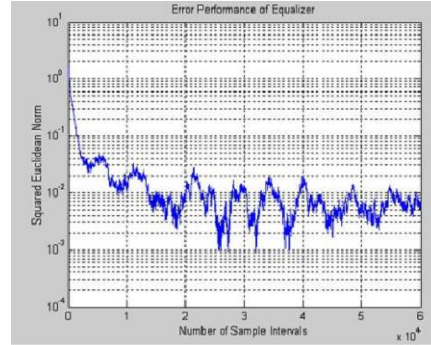


Figure 2 Asymptotic performance

In Figure 2, it can be observed that the asymptotic performance of the equalizer achieved is approximately  $10^{-2}$ . This is a relatively good result and shows that the equalizer filters have adapted closely to the simulated channel response.

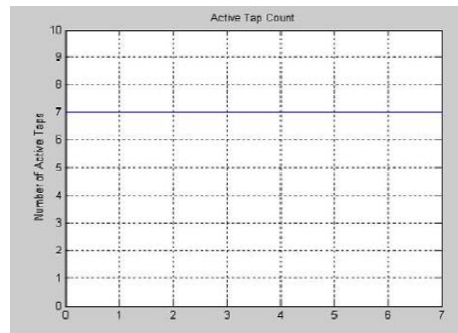


Figure 3 Active Tap Count

Figure 3 shows that the number of active taps detected were 7. This is clearly not the case as the simulated channel only has 2 active taps. By intuition, a better asymptotic performance and convergence rate can be achieved if the taps were estimated accurately and subsequently

disregarding the inactive taps.

### 3.2 Fractionally spaced Decision Feedback Equalizer with Activity Detection Guidance

It can be observed that the FS-DFE with ADG was capable of detecting the magnitudes and positions of the active taps in the channel and was able to ignore the estimation of the inactive taps partially. As expected, the convergence rate and asymptotic performance improved significantly.

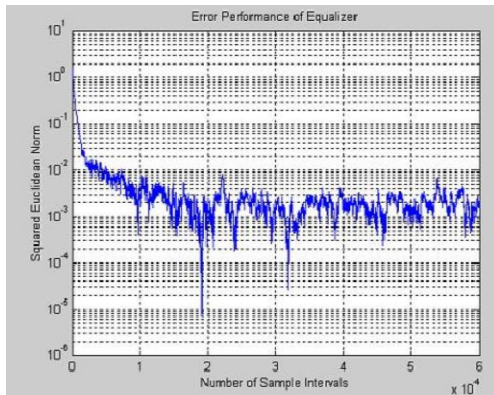


Figure 4 Asymptotic performance

The convergence rate was faster and the asymptotic performance improved from  $10^{-2}$  to  $10^{-3}$ . This is a significant improvement even though the non-active taps were not ignored in the tap weight estimation.

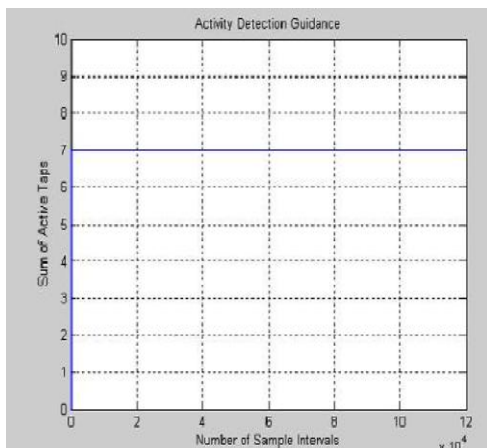


Figure 5 the no. of active taps

The no. of active taps detected is still 7.

### 3.3 Fractionally spaced Decision Feedback Equalizer with Activity Detection Guidance and Tap Decoupling.

It was noted in the previous sections that the FS-DFE with ADG showed considerable improvement in convergence rate and asymptotic performance over the FS-DFE alone. However, the true number of active taps is not detected accurately and the inactive taps still suffer from the noisy estimates that are common in any communication channel.

The inability to detect the true number of active taps accurately is due to the correlation within the input signal that is causing the coupling amongst the output of the unknown channel taps. This coupling leads to the inactive taps to appear as active and the active taps to be less active. Using the principle behind the Activity Detection Guidance scheme, the tap coupling effect can be overcome with some modifications to the activity measure.

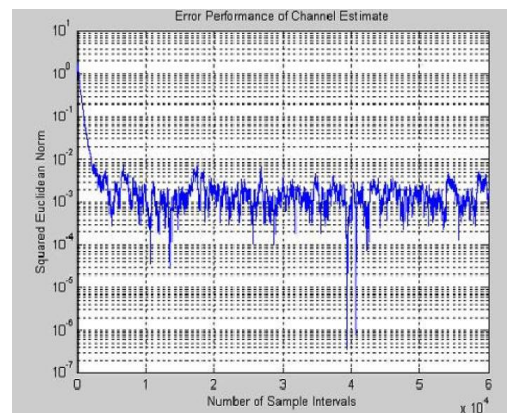


Figure 6 Asymptotic performance

Figure 6 shows the asymptotic

performance of the equalizer to be  $10^{-3}$ . This is almost the same as the FS-DFE with ADG, however, the convergence rate of the FS-DFE with ADG and TD improved by 2 fold as compared to the FS-DFE with ADG. This certainly proves the theory behind the activity measure, by ignoring the inactive taps in the tap weight estimation; the convergence rate will almost certainly improve, which is proven in this case.

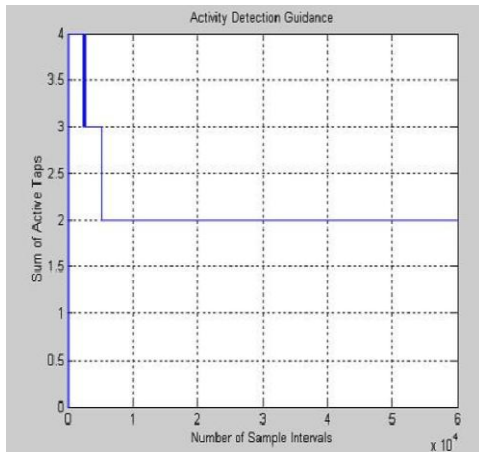


Figure 7 Active Tap Count

Figure 7 shows the true number of active taps being detected. The slight ‘jitters’ at the initial stage is most certainly due to noise as the adaptive filter converges to measure the correct number of active taps. This proves that the FS-DFE with ADG and TD has successfully been implemented and working effectively.

Although the FS-DFE with activity detection could not detect the inactive taps completely, the asymptotic performance, convergence rate and equalizer output were superior to the FS-DFE only. The FS-DFE with the activity detection and tap decoupling enhancements has the best performance throughout the parameters set to assess an adaptive equalizer.

Blue – FS\_DFE, Green – FS\_DFE with ADG, Red – FS\_DFE with ADG and TD.

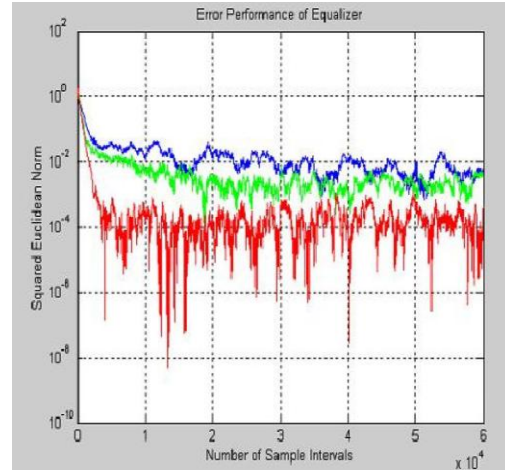


Figure 8 Asymptotic performances of all three together.

The convergence rate and asymptotic performance can be appreciated more clearly from Figure 8.

#### IV. CONCLUSION

The objective of this paper is to develop a suitable adaptive equalization technique to mitigate the effects of ISI and dispersion in a typical optical communication channel. With the successful development of the adaptive Decision Feedback Equalizer, it offers an excellent alternative to the existing equalization techniques available in the optical communication industry.

The adaptive equalizer was implemented using the Least Mean Square (LMS) technique, using stochastic gradient adaptation, for the direct equalization of the unknown channel. A number of adaptive algorithms had been analyzed and discussed. The LMS algorithm was chosen because of its simplicity in program development, robustness and stability.

The FS-DFE was enhanced with the



implementation of an activity detection scheme, which was devised to detect the zero activity periods and exclude them in the adaptation of the equalizer. This led to a faster convergence rate, as only the active taps need to be adapted and better asymptotic performance.

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