MULTIPLEXING EFFICIENCY OF MIMO ANTENNA

A dissertation submitted in partial fulfillment of the requirement towards the degree of

Master of Technology in Microwave and Optical Communication Engineering

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June, 2011

CERTIFICATE

This is to certify that the dissertation titled "**Multiplexing Efficiency of MIMO Antenna**" is the bonafide work of Raj Arun Chourasia (2K09/MOC/11) under our guidance and supervision in partial fulfillment of requirement towards the degree of Master of Technology in Microwave and Optical Communication Engineering from Delhi Technological University, New Delhi.

ACKNOWLEDGEMENT

I would like to take the opportunity to acknowledge my indebtedness towards all the people who have helped me in all my tasks and works.

My sincere gratitude is directed to my supervisor; Dr. Rajiv Kapoor. He actively involved himself in the project and offered useful support at every stage of my project, reviewed my schematics, answered my all questions and provided me with books and reading material.

I would like to acknowledge my friends and teachers at *Delhi Technological University*, *Delhi* who were kind enough to help me during my course work and teaching me with the best of their knowledge.

To the class of 2009-11, thank you for the very interesting and enjoyable two years!

I would also like to thank my dear friend Tushar, Pankaj, Chhagan and Ajay for their encouragement and support that they gave to me throughout my study and my research.

I owe my loving thanks to my Parents, my brother, my sister and friends. They have supported me very much during my study and research. Without their encouragement and understanding it would have been impossible for me to finish this work.

Thank You all !!

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ABSTRACT

A simple and intuitive metric of multiplexing efficiency is proposed for evaluating the performance of multiple input multiple output (MIMO) antennas in the spatial multiplexing mode of operation. Apart from gaining valuable insights into the impact of antenna efficiency, efficiency imbalance, and correlation on multiplexing performance, the metric is particularly useful for antenna engineers whose goal is to achieve the optimum antenna system design. Experimental results involving prototype mobile terminals highlight the effectiveness of our proposal.

Multiplexing efficiency is proposed as a simple and intuitive metric for evaluating the effectiveness of MIMO antenna terminals operating in the SM mode. Instead of comparing the ergodic capacity, the metric quantifies the performance in terms of absolute efficiency.

This thesis introduces multiplexing efficiency as a power-related metric for the SM mode of operation in MIMO systems and derives its approximate closed-form expression. The unique features of the expression are both its simplicity and the valuable insights it offers with respect to the performance impact of non ideal behaviors of multiple antennas. An example application of the metric is demonstrated for two realistic mobile terminal prototypes.

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Chapter 1

INTRODUCTION

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1.1 BACKGROUND

Despite intense academic research in multiple input multiple output (MIMO) technology for over two decades and its recent adoption in major wireless standards, performance characterization of multiple-antenna terminals is a subject of current interest. Depending on the signal-to-noise ratios (SNRs) of the received signals, different MIMO modes are required to optimize the system performance. For the low-SNR regime, diversity techniques are applied to mitigate fading, and the performance gain is typically expressed as diversity gain (in decibels, dB). Such a measure is convenient for antenna engineers since performance improvement is translated into a tangible power gain or, equivalently, an increase in coverage area. On the other hand, higher SNRs facilitate the use of spatial multiplexing (SM), i.e., the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice. In a metric based on difference in capacity is proposed for performance comparison of multiple-antenna terminals in a reverberation chamber (RC).

1.2 MOTIVATION

In the light of the above description of the current wireless networks, one can conclude that despite significant improvement on the provision of wireless services, there is an underlying strong demand for higher date rate wireless services, mainly driven by wireless data applications, as well as users' expectation of wire-equivalent quality wireless service.

Providing such high-rate high-quality wireless services is extremely challenging due to the inherent harsh wireless propagation environment. Compared to wired communication, wireless communication faces two fundamental problems that make fast and reliable wireless connection difficult to achieve, namely, interference and fading (variation of the channel strength over time and frequency due to the small-scale effect of multipath fading, as well as larger-scale fading effects such as path loss via distance attenuation and shadowing by obstacles such as tall buildings and mountains). In addition, wireless communication is required to carefully address the resource management problem, i.e. how to efficiently allocate and utilize power and spectrum (two principle resources in wireless communication).

Responding to these challenges, multiple-input multiple-output (MIMO) antenna systems were proposed independently by Telatar and Foschini and Gans. By introducing multiple antennas at both sides of the communication link, MIMO systems are able to substantially increase date rate and improve reliability without extra spectrum and power resources. The remarkable prospect of MIMO systems has not only sparked huge research interests in the research community, but also attracted enormous attentions from the industry and has led to practical implementation in real communication systems. For instance, MIMO technology has already been incorporated into various industry standards, i.e., wireless LAN IEEE 802.11n standard, wireless MAN IEEE 802.16e, Third Generation Partnership Project Long Term 4 Evolution (3GPP-LTE) Release 8. In general, MIMO technology is likely to become a prominent feature of future wireless communication systems.

The huge potential of MIMO technology has sparked a surge of research activities, which greatly strengthen our understanding of the fundamental limits and performance of MIMO channels. However, most of these research works are based on a relatively simple channel model, for instance, the channel is assumed to be a single random matrix and is subjected to Rayleigh fading or Rician fading.

On the other hand, the increasing popularity of MIMO technology calls for a better understanding of the performance of MIMO systems operating in more practical environments. Motivated by this, this thesis looks into several general and practical channel models, such as Nakagami-m MIMO fading channels, double-scattering MIMO channels, multi-keyhole MIMO channels, and AF dual-hop MIMO channels, and investigates the fundamental capacity limits of these channels, as well as the performance of certain popular signal processing schemes. The objective of the thesis is to enhance our understanding of MIMO systems operating in these general MIMO channels, and to derive a set of new analytical results for understanding the performance of these advanced MIMO systems.

1.3 OBJECTIVES & CONTRIBUTIONS

The objective is to propose a capacity difference metric that does not require the absolute values of the actual received SNRs in a RC, which cannot be estimated in common RC setups with vector network analyzers. However, capacity is a system-level metric that is less intuitive to antenna engineers who would prefer a power-related measure, such as the diversity gain. Moreover, since SM is the primary mechanism for increasing the spectral efficiency of MIMO systems, it is important to consider it explicitly in antenna design.

This thesis introduces multiplexing efficiency as a power-related metric for the SM mode of operation in MIMO systems and derives its approximate closed-form expression. The unique features of the expression are both its simplicity and the valuable insights it offers with respect to the performance impact of non ideal behaviors of multiple antennas. An example application of the metric is demonstrated for two realistic mobile terminal prototypes.

1.4 OUTLINE OF THESIS

Chapter 1 gives introduction of the dissertation containing background of MIMO, motivation behind the work and objectives to be achieved.

Chapter 2 is about MIMO system background and its basic concepts which includes MIMO system model, channel transmission schemes, capacity of MIMO channels, channel state information (CSI) and benefits of MIMO systems.

Chapter 3 describes regarding various modes of application of MIMO systems, their basis of classification and also describes trade-off between the two modes i.e. spatial diversity & spatial multiplexing.

Chapter 4 gives details of spatial diversity techniques exploited in MIMO systems and other concepts related with it e.g. multipath, fading; types of fading channel, rayleigh fading channel, system Capacity etc.

Chapter 5 describes another mode of MIMO application i.e. spatial multiplexing technique which consists of shannon's law, MIMO spatial multiplexing and multiplexing efficiency metric.

Chapter 6 illustrates simulation results and performance analysis using mathematical expressions and experimental data. Both were considered for a case study of a 2X2 MIMO system.

Chapter 7 explains the conclusions derived from this work and a view on current & future prospects for MIMO system and technology in various applications in wireless communication.

Chapter 2 MIMO SYSTEM

BACKGROUND

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2.1 BASIC MIMO CONCEPT

The concept of MIMO was first introduced by Jack Winters in 1987 for two basic communication systems. The first was for communication between multiple mobiles and a base station with multiple antennas and the second for communication between two mobiles each with multiple antennas. Hence, one can attribute the concept of MIMO and ad-hoc networking using multiple antennas at both ends, be it for the same unit or using multiple units to this paper. Subsequently, the papers of Foschini presented the analytical basis of MIMO systems and proposed two suitable architectures for its realization known as vertical BLAST, and diagonal BLAST. The basic motive was to increase the data rate in a constrained spectrum. The initial application of MIMO was envisaged for indoor WLAN, fixed wireless access networks, wireless local loop, and building-to-building wireless communications. Later other applications were proposed such as metropolitan voice/data wireless networks (UMTS, EDGE, and 4th generation networks), very high speed fixed and mobile wireless (point to multipoint), acoustic communications, and broadcast systems (HDTV). In principle, MIMO aims to separate data streams occupying the same bandwidth relying on the de- correlation of the multiple received signals in the presence of multipath. Therefore, the fundamental analysis of MIMO systems is based on the assumption of independent flat Rayleigh fading and constrained total power. In addition, the data are transmitted in bursts, such that the channel can be assumed quasistationary and that the channel is known at the receiver through the transmission of a training sequence but not necessarily at the transmitter. The training sequence enables the receiver to acquire adequate knowledge of the channel coefficients to extract the multiple data streams. The required training interval grows approximately linearly with the number of transmit antennas. To maximize the overall transmission rate, the number of transmit antennas is chosen such that half of the interval is used for training and half the interval for data transmission. Channel knowledge at the transmitter is generally considered to be beneficiary in the sense that the transmitter can optimize its transmission on the 'good channels', adaptively. In the case of time division duplex channels this requires the channel to be stationary. Hence, this approach is not necessarily practical since it requires the channel coefficients to be fed back to the transmitter at the rate at which the channel is changing.

In addition to the stationary requirement, in the case of frequency division duplex channels, the coefficients would have to be transmitted on a different frequency. Since FDD channels are not reciprocal, the feedback approach might not be optimum. While adaptive MIMO can give higher channel capacity, its practical application needs the proper estimation of the coherent time of the channel, particularly in outdoor environments where high Doppler shifts are expected and can be on the order of 35 Hz at 2 GHz even for a stationary user. To overcome the fast feedback constraint some researchers propose the feedback of the spatial mean of the channel capacity in the case of correlated channels. MIMO is a narrowband concept where the assumption of flat fading holds, and therefore, the majority of the channel capacity expressions are given for the narrowband case. Essential to this assumption is the measurement of the coherent bandwidth of the channel. The wideband case or the frequency selective channel is seen to provide diversity gain and hence higher capacity.

2.2 MIMO SYSTEM MODEL & CHANNEL

MIMO System model

The main characteristic of MIMO (Multiple Input Multiple Output) systems is the use of multiple antennas at both ends of the link to improve the communication performance. If we define N as the number of transmit antennas and M as the number of receive antennas the system can be described as in Figure 2.1.

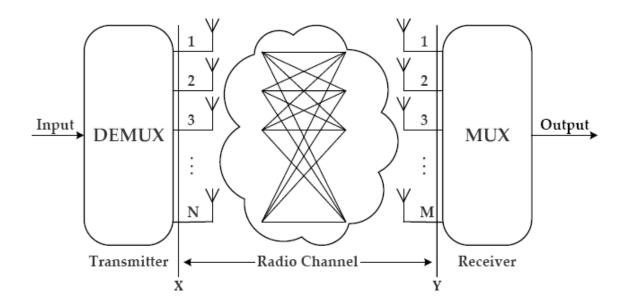


Figure 2.1: MIMO system with N transmit antennas and M receive antennas

When one frame X of L modulated and coded symbols is transmitted from the n=1...N transmitter antennas, the received signal at the m=1...M receiver antennas is expressed by

$$Y_s = H_s X_s + N_s \tag{2.1}$$

Where $s = 1, \dots L$

On their way from the transmitter to the receiver, the transmitted waves suffer changes due to the phenomena which characterize the wireless channel. The most important ones are listed below:

The effects of both transmit and receive antennas radiation patterns.

- Reflection occurs when the signal encounters obstacles similar to smooth surfaces of walls or hills.
- > Transmission occurs when the signal is absorbed by walls, doors and by the atmosphere.

- Scattering occurs when the signal hits small objects as leaves and branches of the trees or rough surfaces such as buildings.
- Diffraction happens when the signal is addressed at the edge of an impenetrable body, such as building rooftops and hilltops.

In order to facilitate the comprehension of these phenomena, Figure 2.2 is presented below:

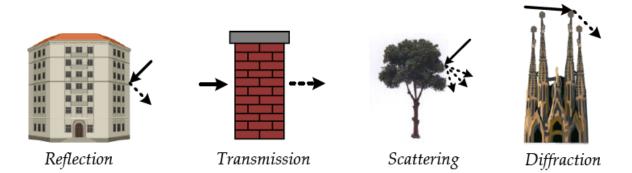


Figure 2.2 Wireless channel phenomena

MIMO channel

Since in MIMO system each of the receiver antennas detects all of the transmitted signals, there are $N \times N$ independent propagation paths, where there are N transmitter and N receiver antennas. This allows the channel to be represented as an N×N matrix. For 3 transmitter and 3 receiver antennas channel matrix

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$
(2.2)

Where h_{ij} represents the path from transmit antenna i to receiver antenna j.

Each of the elements in the channel matrix is an independent propagation path. The transmitted signal can be represented as a vector, as can the received signal. Hence, the system can be represented as the following equation

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \Leftrightarrow \underline{r} = \underline{\underline{H}} \cdot \underline{s} + \underline{n}$$
(2.3)

Where, r =received signal vector, H=Channel Matrix, s=Transmitted signal vector, n=noise.

The transmitted signals in the vector r are complex signals, as are the channel matrix values and the received signals in vector s. The complex form in each of the elements in the vectors represents the power of the signal and its phase delay. The complex form of the elements of the channel matrix H represents the attenuation and phase delay associated with that propagation path.

2.3 MIMO TRANSMISSION SCEMES

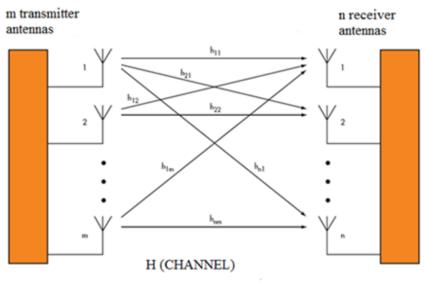


Figure 2.3 MIMO Transfer Matrix

The figure 2.3 above demonstrates how data is transmitted in a MIMO system. First, the data stream is broken down (de-multiplexed) into m equal rate data streams, where m is the number of transmitting antennas. Each of the lower bit rate sub data streams are transmitted from one of the antennas. All are transmitted at the same time and at the same frequency, therefore they mix together in the channel. Since all sub streams are being transmitted at the same frequency, it is very spectrally efficient.

Each of the receive antennas picks up all of the transmitted signals superimposed upon one another. If the channel H is a sufficiently rich scattering environment, each of the superimposed signals will have propagated over slightly different paths and hence will have differing spatial signatures. The spatial signatures exist due to the spatial diversity at both ends of the link, and therefore create independent propagation channels. Each transmit receive antenna pair can be treated as parallel sub channels (i.e. a single-input single-output (SISO) channel). Since the data is being transmitted over parallel channels, one channel for each antenna pair, the channel capacity increases in proportion to the number of transmitter-receiver pairs.

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2.4 CAPACITY OF MIMO SYSTEMS

It was shown by C. Shannon in 1948 that the throughput is limited when the reliable transmission in noisy channel is considered. The commonly used measure of the potential of the channel to transmit data is the capacity. It is the maximal transmission rate which is possible in the unit bandwidth with arbitrary low bit error rate. Hence, the capacity is the upper bound of the spectral efficiency achievable in the specific radio channel. For the definition of the capacity, neither the coding scheme nor the modulation is specified. It is the theoretic limit of the transmission rate with coding block assumed to be infinitely long. Shannon showed that the capacity C of the channel with additive white Gaussian noise is limited to:

$$C = \log_2\left(1 + SNR\right) \tag{2.4}$$

Where SNR is the signal-to-noise ratio at the receive antenna. Capacity unit is bit/s/Hz.

However, in the case of a system with multiple antennas, the Shannon's limit should be extended. It was proven that the capacity of the MIMO channel is equal to:

$$C = \log_2 det \left(I_m + \frac{\rho}{n} H H^* \right)$$
(2.5)

Where \mathbf{I}_{m} is m x m identity matrix, ρ is the ratio of the total transmit power to the noise power, n and m are the numbers of Transmitter and Receiver antennas and \mathbf{H} and $* \mathbf{H}$ are the channel transfer matrix and its transpose conjugate version, respectively. The capacity from equation (2.5) is sometimes calculated as:

$$C = \sum_{i=1}^{l} \log_2\left(1 + \frac{\rho}{n} \lambda_i^2\right)$$
(2.6)

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Where l is equal to the rank of the matrix **H** and λ_1 , λ_2 , λ_3 λ_1 are the singular values of **H** (nonnegative square roots of the eigen values of the matrix **HH***). In the case when the transfer functions of the MIMO sub-channels are not correlated, e.g. in a richly scattered environment, l is maximal and is equal to min [n, m]. This case will be assumed unless it is stated otherwise. It should be noted that average signal-to-noise ratio at the Receiver antennas can be calculated as:

$$SNR = \frac{\rho \sum_{i=1}^{m} \sum_{j=1}^{n} |h_{ij}|^2}{n.m}$$
(2.7)

The equations (2.5) and (2.6) present the capacity in most popular instance: when the channel transfer matrix is known only at the receiver side. In this case, the capacity of MIMO channel grows proportionally to 1. The capacity is the most important measure of the MIMO channel – it determines the possibility of the radio channel for the data transmission.

In the channels with fading, the notion of capacity is not convenient to describe the radio channel. When a deep fade occurs, no data can be transmitted.

According to the definition, capacity of such a channel is equal to zero. Thus, instead of the capacity, two other notions are usually used. They are e-outage capacity and ergodic capacity. The former is suitable when the changes of the channel characteristics are slow and a deep fade could be very long. In this case, the time can be divided into short periods and the capacity can be calculated for each of these periods. Then, the cumulative distributive function is calculated over these values of capacity. On this basis, the e-outage capacity is defined as a capacity that cannot be achieved by e % of time. So, the capacity of the radio channel is lower than a given value with the probability of e.

In the channels where the fading is rapid, the expectation value of the capacity is usually calculated. Again, the channel can be in a deep fade, but these periods are short and the loss of data can be compensated by the appropriate joint coding and interleaving. This expectation value is called ergodic capacity.

2.5 CHANNEL STATE INFORMATION (CSI)

Generally, the knowledge about the radio channel, also called channel state information (CSI), can be used for two purposes. On its basis, the transmitter can adapt the signal to the radio channel. On the other hand, the receiver uses the channel knowledge to decode the received signal.

The most frequently considered case of a MIMO system is when CSI is known only at the receiver. This knowledge is essential for proper detection of the data symbols. The simplest decoding algorithm can be thought of as a channel transfer matrix inversion and calculation of n variables (n transmitted data signals) – like solving the set of n equations. In practice, the decoding algorithm is more sophisticated and usually matches the coding scheme. Moreover, because of the noise and channel variations in time, the receiver does not know the radio channel perfectly. It additionally decreases the channel capacity.

The channel knowledge at the receiver is crucial for the whole transmission.

However, if the matrix \mathbf{H} is known only at the receiver, the transmitter will treat all the transmitted data signals in the same way and will allocate the equal power to all of them. In many cases, such a strategy is very ineffective, as some of these data signals are very strongly attenuated during the transmission. When the matrix \mathbf{H} is known also at the transmitter side, the signals can be adapted to the radio channel.

The greatest power is allocated to those data signals which are the least attenuated in the radio channel. This algorithm is called water-filling or water-pouring, because the power is "poured" into the radio sub-channels accordingly to their gains. The Water-filling is explained with details in the section 1.4.5 in the context of spatial multiplexing techniques.

The channel state information at the transmitter and the water-filling algorithm allow increasing the channel capacity in comparison with the case when the channel is known only at the receiver. However, this advantage converges to zero with the SNR increasing. As the water-

filling algorithm needs calculating the singular values of matrix \mathbf{H} , it is computationally complicated. So, it is rather not worthwhile in the high SNR region.

The MIMO systems with the channel knowledge only at the transmitter are rarely considered. Some information can be found in. In the last case, the channel is known neither at the transmitter not at the receiver. There exist some non-coherent and blind detection techniques. Such techniques can be useful especially for the fast varying radio channels where the training sequences should be transmitted very frequently to track the channel properly. Generally, blind detection is based on the exploiting the information about the statistics of the channel or received signals and the properties of the input signals, i.e. the finite number of symbols in the constellation.

2.6 BENEFITS OF MIMO SYSTEMS

The introduction of multiple antennas into communication systems has offered extra degree of freedom which can be exploited to provide various gains over conventional SISO systems, i.e., array gain (or power gain), diversity gain and multiplexing gain. In the following, we give a brief account of these gains.

Array Gain

Array gain is defined as the improvement of average SNR at the receiver by coherently combining the signals from multiple transmitters or receivers. For instance, in a single-input multiple-output (SIMO) system, the signal at the receiver is expressed as

$$y = hx + n$$

where h is the channel, and n is the noise $E\{nn^H\} = \sigma^2 I$. It is easy to show that the OC vector

is given by $\frac{h^{R}}{\|h\|}$, resulting the received SNR as $\frac{\|h\|^{2}}{\sigma^{2}}$, which clearly indicates the advantage when compared with the SISO SNR σ^{2} . It is important to note that realization of array gain requires channel state information (CSI) at the transmitter or receiver.

Diversity Gain

Fading is the most prominent feature of a wireless communication channel, and diversity is an efficient means of combating channel fading. The general principle behind diversity is that the overall link reliability can be improved by observing multiple independent copies of the transmitted signal at the receiver.

The diversity gain is usually measured in terms of how fast the bit error rate of a communication system decays with the increase of the SNR, i.e., the error exponent. Diversity gain in SISO systems can be obtained in time or frequency domain, i.e. by repeating the same message several times, which however incurs a penalty in terms of date rate. In multiple antenna systems, another form of diversity is available, namely, spatial diversity, which includes receive diversity and transmit diversity. In contrast to the temporal and frequency diversity, the realization of spatial diversity does not incur any penalty in data rate, instead, it provides an array gain introduced earlier. Receive diversity can be obtained in a system with multiple receive antennas by smartly combining the multiple independent copies observed at the individual antenna. However, transmit diversity is generally much difficult to exploit since it requires sophisticated coding schemes. The most popular approach to realize transmit diversity is the so called Space-Time Coding (STC), which performs coding across space (transmit antennas) and time to extract diversity.

Multiplexing Gain

Multiplexing gain is the most outstanding advantage of MIMO systems, and is defined as the linear increase in data rate without additional power or spectrum expenditure. Unlike array gain or diversity gain, which can be realized by either SIMO or multiple-input single-output (MISO) systems, multiplexing gain requires multiple antennas at both the transmitter and receiver ends.

The basic principle is to split a high-rate input data sequence into multiple lower-rate sequences, which are then modulated and independently sent in parallel via each of the transmit antennas, while the receiver employs appropriate signal processing technique to undo the mixing of the MIMO channel to detect the signals corresponding to each of the transmitted data streams. Multiplexing gain and diversity gain can be achieved simultaneously by appropriate coding, in fact, there exists a fundamental tradeoff between the multiplexing gain and diversity gain for a given system as first discovered already, since which, the design of efficient and practical coding schemes achieving the optimal diversity-multiplexing tradeoff curve has been an extremely active area of research.

Chapter 3 MODES OF APPLICATION

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3.1 BASIS OF CLASIFICATION

Depending on the signal-to-noise ratios (SNRs) of the received signals, different MIMO modes are required to optimize the system performance. For the low-SNR regime, diversity techniques are applied to mitigate fading, and the performance gain is typically expressed as diversity gain (in decibels, dB). Such a measure is convenient for antenna engineers since performance improvement is translated into a tangible power gain or, equivalently, an increase in coverage area. On the other hand, higher SNRs facilitate the use of spatial multiplexing (SM), i.e., the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice.

3.2 DIVERSITY V/S MULTIPLEXING GAIN

There are two main challenges for future wireless communication systems. First, there is a huge gap between the throughput in cable and wireless systems. Radio communication networks' users and clients expect high throughput, comparable with cable networks. However, in wireless systems, there is the problem of limited bandwidth. The wireless network cannot use the whole radio frequency bandwidth because of the interference with other radio systems. Therefore, limited bandwidth is assigned to the particular wireless network. In order to extend the throughput, the spectral efficiency should be increased. It could be done, e.g. using multilevel modulations, but it results in the higher requirements for SNR. Thus, the solution is a system with multiple antennas which allows enlarging the throughput and keeping the same bandwidth and SNR. In the MIMO (n, m) system, $l = \min [n, m]$ independent signals can be transmitted, so the spectral efficiency grows l times –there is spatial multiplexing gain equal to l.

The second challenge is the phenomenon of fading: the effect of variations of signal power at the receiver. Large-scale (slow) fading is caused by changes in signal attenuation when the terrain obstacles block some propagation paths between the transmitter and the receiver. On the other hand, small-scale (fast) fading is the effect of the constructive and destructive interference between the replicas of the transmitted signal which arrive to the receiver by different paths. Slow, as well as fast fading can be the result of the movement of the transmitter, receiver or objects in the surroundings of the wireless system. Also, the changes in the atmosphere can cause the large-scale fading effects. Because of fading, the transmission in radio channel cannot be reliable. In some time periods, the outage occurs: the signal attenuation in the radio channel is very strong and there is huge bit error rate (BER) during the data transmission.

To overcome the problem of fading, the diversity is applied to a radio system.

The data is transmitted by two or more different, independent ways. The same or correlated signals can be sent in different frequency bands or in different time periods. At the receiver, these signals can be combined or just the best signal is selected.

The multiple antennas can provide the additional kind of diversity to the system. The encoded signals are simultaneously transmitted from multiple Transmitter antennas or received by multiple Receiver antennas – it is called space diversity. In the MIMO (n, m) system, there are $n \times m$ different ways to transmit the data signal. If the characteristics of different sub-channels are uncorrelated, the effect of the fading can be overcome. When a sub-channel is faded, the others can provide good propagation conditions. The maximal diversity gain in MIMO (n, m) system is equal to $n \times m$, because it is the maximal number of independent sub-channels.

However, the MIMO system cannot provide the full diversity and multiplexing gain at the same time. It is the matter of coding which aspect of the MIMO system will be exploited. The MIMO system can maximize the transmission rate by sending many independent information streams simultaneously or protect the transmission from the errors caused by fading. Also, the compromise between these two strategies is possible. Switching between the coding schemes achieving the diversity or multiplexing gain can be realized during the transmission. Yet, it is always the tradeoff.

Chapter 4 SPATIAL DIVERSITY TECHNIQUE

4.1 INTRODUCTION

To overcome the problem of fading, the diversity is applied to a radio system. The data is transmitted by two or more different, independent ways. The same or correlated signals can be sent in different frequency bands or in different time periods. At the receiver, these signals can be combined or just the best signal is selected.

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4.2 MULTIPATH

In wireless telecommunications, multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. When a signal is sent between a transmitter and a receiver there are many possible paths for it to take. It almost always takes more than one of them; this is referred to as multipath. Multipath results in multiple copies of the same transmitted signal arriving at the receiver, at different times. Causes of multipath include atmospheric ducting, ionosphere reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings.

The effects of multipath include constructive and destructive interference, and phase shifting of the signal. This causes Rayleigh fading. The standard statistical model of this gives a distribution known as the Rayleigh distribution. To carry out any simulation, the multipath environment needs to be modeled.

4.3 FADING

The term fading is used to describe the rapid fluctuations of the amplitudes, phases, or multipath delays of a signal over a short period of time or distance. The fading may vary with time, geographical position and/or radio frequency, and is often modeled as a random process. It is caused by interference between multiple versions of the transmitted signal which arrive at the receiver at slightly different times. Hence, the resultant signal at the receiver has a wide-varying amplitude and phase. There are a few factors that contribute to this multipath effect, namely shadowing, diffraction, reflection and refraction. These effects also cause another phenomenon known as attenuation.

Attenuation is the decrease of power contained within a signal between the times it is transmitted to the time it is received. The magnitude of the attenuation depends on a number of different factors, such as the distance between the transmitter and the receiver and the density of the obstructions, like buildings and trees, medium through which the signal needs to travel. In short, the effects of multipath are rapid changes in signal strength over a small travel distance or time interval, random frequency modulation due to varying Doppler shifts on different multipath signals and time dispersion caused by multipath propagation delays. The multipath components combine at the receiver which causes the signal to distort, to fade or even to strengthen at times. A fading channel is a communication channel that experiences fading.

In wireless systems, fading may be either due to multipath propagation, referred to as multipath induced fading, or due to shadowing from obstacles affecting the wave propagation, sometimes referred to as shadow fading. Shadowing happens when an object blocks the path between the transmitter and receiver entirely. It is most severe in built-up areas where buildings are in the direct path of the signal. The receiver is still able to pick up the signal because it is reflected and refracted around the buildings. The power in the signal will be weaker; however, than if there was no scattering.

Refraction occurs when a signal passes through a medium other than air. It causes the signal to distort just as light is distorted when it travels through water. This effect is prevalent when signals have to travel to high altitudes, as the different layers in the atmosphere cause refraction. These effects can be seen in figure 4.1.

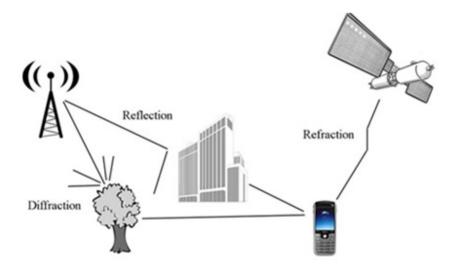


Figure 4.1: Signal reflection due to various obstacles

4.3.1 SLOW VERSUS FAST FADING

The terms slow and fast fading refer to the rate at which the magnitude and phase change imposed by the channel on the signal changes. The coherence time is a measure of the minimum time required for the magnitude change of the channel to become uncorrelated from its previous value.

Slow fading: It arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver. The amplitude change caused by shadowing is often modeled using a log-normal distribution with a standard deviation according to the log-distance path loss model.

Fast fading: It occurs when the coherence time of the channel is small relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel varies considerably over the period of use. In a fast-fading channel, the transmitter may take advantage of the variations in the channel conditions using time diversity to help increase robustness of the communication to a temporary deep fade. Although a deep fade may temporarily erase some of the information transmitted, use of an error-correcting code coupled with successfully transmitted bits during other time instances (interleaving) can allow for the erased bits to be recovered. In a slow-fading channel, it is not possible to use time diversity because the transmitter sees only a single realization of the channel within its delay constraint. A deep fade therefore lasts the entire duration of transmission and cannot be mitigated using coding.

Coherence Time and Doppler Spread: The coherence time of the channel is related to a quantity known as the Doppler spread of the channel. When a user (or reflectors in its environment) is moving, the user's velocity causes a shift in the frequency of the signal transmitted along each signal path. This phenomenon is known as the Doppler shift. Signals travelling along different paths can have different Doppler shifts, corresponding to different rates of change in phase. The difference in Doppler shifts between different signal components contributing to a single fading channel tap is known as the Doppler spread. Doppler spread, D_S , is a measure of the spectral broadening.

Doppler spectrum can be measured by sending a single sinusoidal tone of frequency fc and viewing the received signal spectrum, which have components from fc - fd to fc + fd, with fd being the Doppler shift. Doppler shift depends on the relative velocity and angle of movements. Coherence time Tc is the time domain dual of Doppler spread and is widely chosen as 0.423 / fm, with fm being the maximum Doppler shift given by (Velocity / λ). If the Doppler spread (DS) is far smaller than the baseband signal bandwidth or alternatively, if the coherence time of the channel is greater than the symbol transmission period, then, the channel is considered as a slow fading channel. In general, coherence time is inversely related to Doppler spread, typically expressed as $T_c\!=\!k\,/\,D_S$

Where, T_c is the coherence time, D_s is the Doppler spread, and k is a constant taking on values in the range of 0.25 to 0.5.

4.3.2 FLAT VERSUS FREQUENCY-SELECTIVE FADING

As the carrier frequency of a signal is varied, the magnitude of the change in amplitude will vary. The coherence bandwidth measures the separation in frequency after which two signals will experience uncorrelated fading.

Flat fading: In flat fading, the coherence bandwidth of the channel is larger than the bandwidth of the signal. Therefore, all frequency components of the signal will experience the same magnitude of fading.

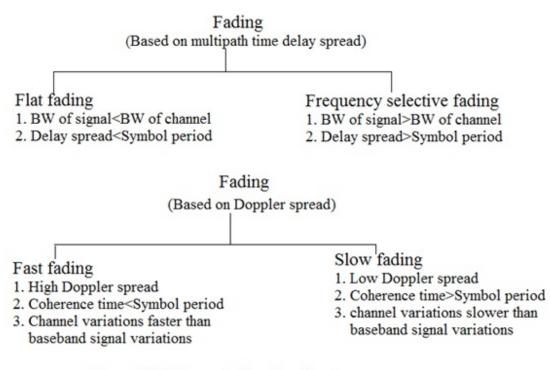
Frequency-selective fading: In frequency-selective fading, the coherence bandwidth of the channel is smaller than the bandwidth of the signal. Different frequency components of the signal therefore experience fading. Frequency-selective fading channels are also dispersive, in that the signal energy associated with each symbol is spread out in time. This causes transmitted symbols that are adjacent in time to interfere with each other. Equalizers are often deployed in such channels to compensate for the effects of the inter symbol interference.

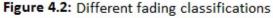
Coherence Bandwidth and Delay Spread: Time dispersive nature of the channel is described using the Coherence Bandwidth (*Bc*) and Delay Spread ($\sigma\tau$). The rms delay spread and coherence bandwidth are inversely proportional to one another, with their exact relationship depending on the exact multipath structure, i.e. on the power delay profile. The delay spread is a natural phenomenon caused by reflected and scattered propagation paths, while the coherence bandwidth is a defined relation derived from the rms delay spread. Coherence bandwidth indicates the range of frequencies over which the channel can be considered as flat, i.e., all the frequency components of the signal undergo equal gain and linear phase.

4.4 TYPES OF FADING CHANNELS

Fading channel models are often used to model the effects of electromagnetic transmission of information over the air in cellular networks and broadcast communication. Fading channel models are also used in underwater acoustic communications to model the distortion caused by the water. Mathematically, fading is usually modeled as a time-varying random change in the amplitude and phase of the transmitted signal.

Type of fading experienced by the signal going through a channel depends on the nature of the signal and the characteristics of the channel. The relation between bandwidth and symbol period of the signal on one hand and rms delay spread and Doppler spread of the channel on the other hand, determine what type of fading we are faced with. It is clear that we can have four distinct fading types which are summarized in Figure 4.2.





4.5 RAYLEIGH FADING CHANNEL

Rayleigh Fading Channel is a statistical model of the communication channel through which a radio signal propagates. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade, according to a Rayleigh distribution. Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component.

In probability theory and statistics, the Rayleigh distribution is a continuous probability distribution. A Rayleigh distribution is often observed when the overall magnitude of a vector is related to its directional components. One example where the Rayleigh distribution naturally arises is when wind speed is analyzed into its orthogonal 2-dimensional vector components. Assuming that the magnitude of each component is uncorrelated and normally distributed with equal variance, then the overall wind speed (vector magnitude) will be characterized by a Rayleigh distribution.

The Rayleigh probability density function is
$$f(x; \sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}$$
 (4.1)

and cumulative distribution function is $F(x) = 1 - e^{-x^2/2\sigma^2}$ (4.2)

For $x \in [0, \infty)$, σ - standard deviation

The nth path in a multipath Rayleigh fading channel is represented by

$$s(t) = \operatorname{Re}[s_{l}(t)e^{j2\pi f_{c}t}]$$
(4.3)

If the paths are summed to get the received signal for a multipath fading channel, it can be expressed as

$$x(t) = \sum_{n} \alpha_{n}(t) s[t - \tau_{n}(t)]$$
(4.4)

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where $\alpha_n(t)$ is the attenuation factor and $\alpha_n(t)$ the propagation delay each for the nth path of the received signal and s(t) is the transmitted signal.

4.6 SYSTEM CAPACITY

4.6.1 SISO, SIMO and MISO System Capacity

For a memory less SISO (single input single output) system the capacity is given by

$$C_{SISO} = \log_2(1 + \rho |h|^2)$$
(4.5)

Where ρ is the SNR at receiver antenna, and *h* is the normalized complex gain of the channel.

With N_R receiver antennas, the single-input-multiple-output (SIMO) system capacity is

$$C_{SIMO} = \log_2(1 + \rho \sum_{m=1}^{N_R} |h_m|^2)$$
(4.6)

Where h_m is the gain for $m^{th} R_X$ antenna.

However, if N_T transmitter antennas are used, a multiple input single input (MISO) is achieved. The capacity is given by

$$C_{MISO} = \log_2(1 + \frac{\rho}{N_T} \sum_{n=1}^{N_R} |h_n|^2)$$
(4.7)

Where h_n is the gain for nth T_X antenna. In order to ensure transmitter power restriction, SNR is normalized by N_T .

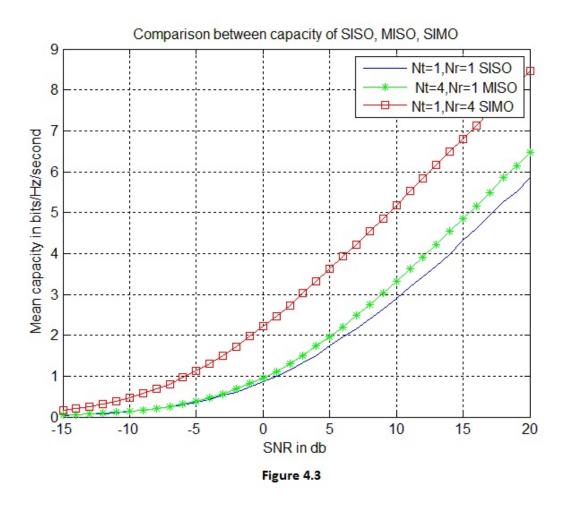


Figure 4.3 illustrates the capacity comparison of SISO, SIMO and MISO system versus SNR. From the figure, we can see that the SIMO and MISO channels improve the capacity compared with the SISO channel by exploiting more antennas. However, the SIMO and MISO channels can only offer a logarithmic increase in capacity with the number of antennas. It is clear that $C_{MISO} < C_{SIMO}$ when the channel information is not available at the transmitter.

4.6.2 MIMO System Capacity

In this section, I consider the case when the channel characteristics H is random MIMO channel according to a Rayleigh distribution in a quasi-static channel. The elements of H are taken to be identically distributed complex Gaussian variables with zero mean and unit variance.

We have taken a uniform correlation matrix model for transmit and receive branches Correlation between the sub-channels of the MIMO channel is the most important factor affecting the channel capacity. Each realization of the channel has a maximum channel capacity, depending on whether the channel state information is known or unknown to the transmitter.

a) Capacity of deterministic MIMO channel: Consider a MIMO channel with M_T transmitter antennas and M_R receiver antennas in wireless communication. The total transmitted power is denoted by P, and noise power of each received antennas is denoted by δ^2 , signal-to-noise ratio (SNR) is denoted by $\gamma = P / \delta^2$.

The capacity of the deterministic MIMO channel is given by

$$C = \log_2[\det(I_m + \frac{\gamma}{M_T}Q)]$$
(4.8)

Where $m = min (M_T, M_R)$, $I_m = m^*m$ identity matrix and matrix Q is given by

$$Q = \begin{cases} H^{H}H, M_{R} < M_{T} \\ HH^{H}, M_{R} \ge M_{T} \end{cases}$$
(4.9)

Using the eigen decomposition of matrix Q, channel capacity can be written as

$$C = \sum_{i=1}^{r} \log_2\left(1 + \frac{r}{M_T}\lambda_i\right)$$
(4.10)

Where r is the rank of the channel and λ_i (i = 1, 2, ..., r) are the positive eigen values of Q. Above equation expresses the capacity of the MIMO channel as a sum of the capacities of r SISO channels each having power gain of λ_i (i = 1, 2, ..., r).

b) Capacity of random channel: Random channel is a real-life situation encountered, for example, in wireless LANs with high data rates and low fades rates. Since the channel is random, the information rate associated with it is also random. The ergodic capacity of a MIMO channel is the ensemble average of the information rate over the distribution of the elements of the

channel matrix H. The significance of the ergodic capacity is that in an ergodic channel, we can signal at the rate defined by ergodic capacity with vanishing error assuming we use asymptotically optimal Codebooks. When the channel state information is unknown to the transmitter the ergodic capacity is given by

$$C = E\left\{\sum_{i=1}^{r} \log_2\left(1 + \frac{r}{M_T}\lambda_i\right)\right\}$$
(4.11)

It is possible by various means to learn the channel state information at the transmitter. In such an event the capacity can be increased by resorting to the so-called "waterfilling principle", by assigning various levels of transmitted power to various transmitting antennas. This power is assigned on the basis that the better the channel gets, the more power it gets and vice versa. The power allocated to the *i* th sub-channel is given by

$$p_i = \left(\mu - \frac{\delta^2}{\lambda_i}\right)^+ \qquad (i = 1, 2, \cdots, r)$$
(4.12)

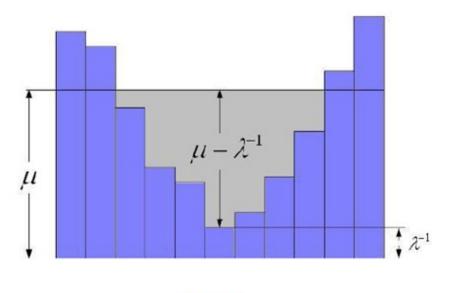
Where a^+ is the max (a, 0), μ satisfies $\sum_{i=1}^r p_i = P$, P is the total power.

So, when the channel state information (CSI) is known to the transmitter, the ergodic capacity is the ensemble average of the capacity achieved, and is given by

$$C = \sum_{i=1}^{r} \log_2(1 + \lambda_i \frac{p_i}{\delta^2})$$
(4.13)

4.6.3 MIMO System Capacity with Water Filling Algorithm

When the channel knowledge is absent at the transmitter, the individual sub-channels are not accessible. If the channel has no preferred direction and is completely unknown to the transmitter, equal power allocation across the transmit antenna array is logical. When the transmitter has perfect knowledge of the channel, the water filling method is used to optimize the transmitted signal power.





The principle of the Water filling theorem sees the division of total power in such a way that a greater portion goes to the channels with higher gain and less or evens none to the channels with small gains. The solution is given below

$$C_{WF} = \sum_{i=1}^{m} \log_2(\mu \lambda_i)^+$$
(4.14)

Where μ is chosen from the water filling algorithm, μ satisfies $\sum_{i=1}^{r} p_i = P$, P is the total power.

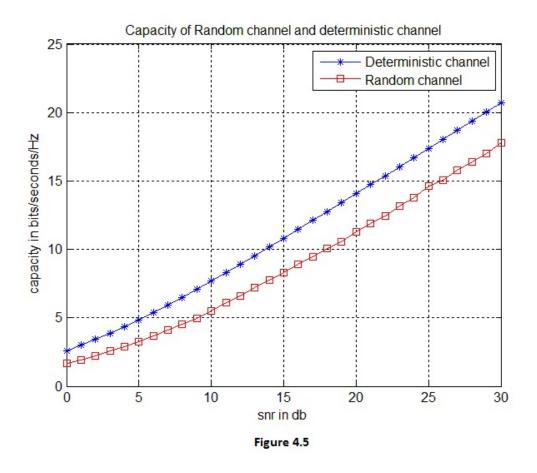
$$= \sum_{i=1}^{m} (\mu - \lambda_{i})^{+}$$
(4.15)

Where ()⁺ denotes taking only those terms which are positive and $\lambda 1, \ldots, \lambda m$ are the eigen values of W with m = min(N_T, N_R). Compared with the equal power scheme, water filling has a significant advantage especially at low SNR. However, this advantage decreases as SNR is increased.

4.6.4 Comparison Between Capacity Of Deterministic Channel And Random Channel

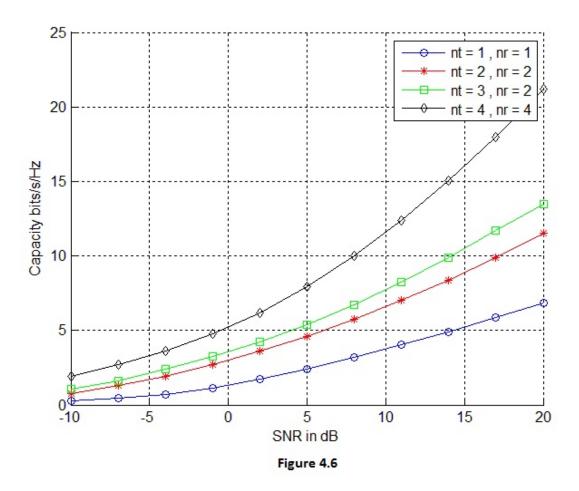
Capacity of a Rayleigh flat fading MIMO channel with $M_T = M_R = 2$ and the channel state information is unknown to the transmitter is calculated for random and deterministic channel.

Figure 4.5 shows the channel capacity in both cases (deterministic and random channel) for different values of signal to noise ratio. For deterministic channel, capacity of channel is the maximum mutual information between input and output of channel. It is clear from the figure-9 capacity of deterministic channel is higher than capacity of random channel, but sometime capacity for both channel may be equal also.



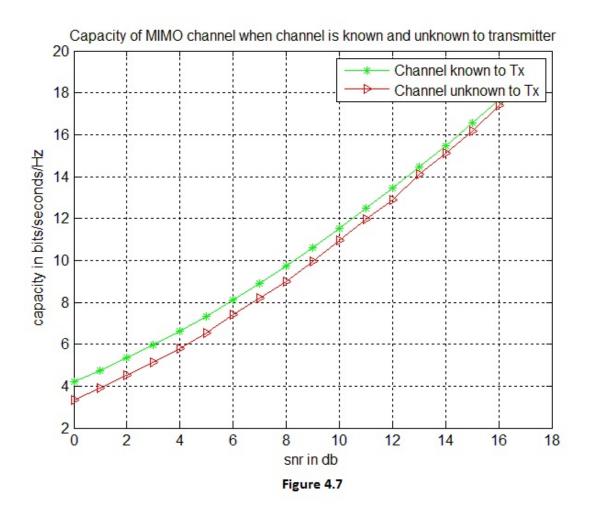
4.6.5 Capacity of MIMO Channel

Figure 4.6 shows the ergodic capacity over different system configurations as a function of signal to noise ratio. Note that, the ergodic capacity increases on increasing signal to noise ratio and with increasing number of transmitter and receiver antennas. It is clear from the figure-10 in case of multiple inputs multiple output capacity is higher than case where single input single output antenna is available.



4.6.6 Capacity of MIMO Channel When CSI is Known and Unknown to Transmitter

Ergodic capacity of an independent and identically distributed (IID) flat fading MIMO channel with $M_T = M_R = 4$ is shown in figure 4.7 when the channel state information is known and unknown to the transmitter. If the channel is known to the transmitter, the optimal Water-filling power allocation strategy is found. From figure 4.7, it is clear that for lower value of signal to noise ratio capacity is higher in case of MIMO channel when CSI is known to transmitter. But for higher values of SNR the difference between ergodic capacity for the case when CSI is known and unknown to transmitter is decreases.



Chapter 5

SPATIAL MULTIPLEXING TECHNIQUE

5.1 SHANNON'S LAW

As with many areas of science, there a theoretical boundary, beyond which it is not possible to proceed. This is true for the amount of data that can be passed along a specific channel in the presence of noise. The law that governs this is called Shannon's Law, named after the man who formulated it. This is particularly important because MIMO wireless technology provides a method not of breaking the law, but increasing data rates beyond those possible on a single channel without its use.

Shannon's law defines the maximum rate at which error free data can be transmitted over a given bandwidth in the presence of noise. It is usually expressed in the form:

$$C = W \log_2(1 + SNR) \tag{5.1}$$

Where C is the channel capacity in bits per second, W is the bandwidth in Hertz, and SNR is the Signal to Noise Ratio.

From this it can be seen that there is an ultimate limit on the capacity of a channel with a given bandwidth. However before this point is reached, the capacity is also limited by the signal to noise ratio of the received signal.

In view of these limits many decisions need to be made about the way in which a transmission is made. The modulation scheme can play a major part in this. The channel capacity can be increased by using higher order modulation schemes, but these require a better signal to noise ratio than the lower order modulation schemes. Thus a balance exists between the data rate and the allowable error rate, signal to noise ratio and power that can be transmitted.

While some improvements can be made in terms of optimizing the modulation scheme and improving the signal to noise ratio, these improvements are not always easy or cheap and they are invariably a compromise, balancing the various factors involved. It is therefore necessary to look at other ways of improving the data throughput for individual channels. MIMO is one way in which wireless communications can be improved and as a result it is receiving a considerable degree of interest.

5.2 MIMO SPATIAL MULTIPLEXING

To take advantage of the additional throughput capability, MIMO utilises several sets of antennas. In many MIMO systems, just two are used, but there is no reason why further antennas cannot be employed and this increases the throughput. In any case for MIMO spatial multiplexing the number of receive antennas must be equal to or greater than the number of transmit antennas.

To take advantage of the additional throughput offered, MIMO wireless systems utilize a matrix mathematical approach. Data streams $t_1, t_2,...,t_n$ can be transmitted from antennas 1, 2,...,n. Then there are a variety of paths that can be used with each path having different channel properties. To enable the receiver to be able to differentiate between the different data streams it is necessary to use. These can be represented by the properties h_{12} , travelling from transmit antenna one to receive antenna 2 and so forth. In this way for a three transmit, three receive antenna system a matrix can be set up:

$$r_{1} = h_{11}t_{1} + h_{21}t_{2} + h_{31}t_{3}$$

$$r_{2} = h_{12}t_{1} + h_{22}t_{2} + h_{32}t_{3}$$

$$r_{3} = h_{13}t_{1} + h_{23}t_{2} + h_{33}t_{3}$$
(5.2)

Where r_1 = signal received at antenna 1, r_2 is the signal received at antenna 2 and so forth.

In matrix format this can be represented as:

$$[R] = [H] \times [T]$$
 (5.3)

To recover the transmitted data-stream at the receiver it is necessary to perform a considerable amount of signal processing. First the MIMO system decoder must estimate the individual channel transfer characteristic hij to determine the channel transfer matrix. Once all of this has been estimated, then the matrix [H] has been produced and the transmitted data streams can be reconstructed by multiplying the received vector with the inverse of the transfer matrix.

$$[T] = [H]^{-1} x [R]$$
(5.4)

This process can be likened to the solving of a set of N linear simultaneous equations to reveal the values of N variables.

In reality the situation is a little more difficult than this as propagation is never quite this straightforward, and in addition to this each variable consists of an ongoing data stream; this nevertheless demonstrates the basic principle behind MIMO wireless systems.

5.3 MULTIPLEXING EFFICIENCY METRIC

Considering an $\mathbb{M} \times \mathbb{M}$ MIMO channel, the instantaneous channel capacity with no channel information at the transmitter (i.e., equal transmit power allocation) can be expressed as

$$C = \log_2 \left(I_M + \frac{\rho_T}{M} H H^H \right)$$
(5.5)

where the SNR is defined by $\rho_T = \frac{P_T}{\sigma_n^2}$. P_T denotes the transmit power and σ_n^2 is the noise power at the receiver. Since the interest here is in antenna design, the reference propagation environment of independent and identically distributed (i.i.d.) Rayleigh fading channel H_W is assumed, i.e., the entries of H_W are zero-mean circularly symmetric complex Gaussian random variables. Without loss of generality, the case of receive antennas is examined. Then, the MIMO channel is given by

$$H = R^{1/2} H_W$$
 (5.6)

Where R denotes the receive correlation matrix that fully describes the effects of the antennas on the channel, i.e., it characterizes the efficiency, efficiency imbalance, and correlation among the receive antennas. Specifically

$$R = \Lambda^{1/2} \overline{R} \Lambda^{1/2}$$
(5.7)

Where \overline{R} is a normalized correlation matrix is whose diagonal elements are 1, and the $(i,j)^{th}$ $(i \neq j)$ element $\overline{R}(i,j)$ denotes the complex correlation coefficient between the 3-D radiation patterns of the i^{th} and j^{th} antenna ports. Λ denotes a diagonal matrix given by

$$\Lambda = Diag[\eta_1, \eta_2, \eta_3, \dots, \eta_M]$$
(5.8)

where η_i is the total efficiency of the i^{th} antenna port.

In order to obtain a reliable estimate of the multiplexing capability of the antennas, it is noted that at high SNRs, the instantaneous capacity of (5.5) can be written as

$$C = C_0 + \log_2 \det(R) \tag{5.9}$$

where C_0 denotes the capacity of the ideal i.i.d. Rayleigh channel at high SNR

$$C_0 = \log_2 det\left(\frac{\rho_T}{M} H_W H_W^H\right)$$
(5.10)

which is achieved with ideal antennas in uniform 3-D angular power spectrum (APS), i.e., $\mathbf{R} = \mathbf{I}_{\mathbf{M}}$. Ideal antennas are 100% efficient and completely orthogonal to one another in radiation pattern (either in space and/or polarization). The high SNR approximation is also used in to obtain capacity difference for comparing different multiple-antenna terminals. By assuming the noise floor and propagation loss to be the same when measuring the channel in the RC for different terminals, it is shown that these parameters do not appear in the capacity difference metric at sufficiently high SNR.

Since $\log_2 det(\Lambda) \leq 0$ and $\log_2 det(\overline{R}) \leq 0$ in (5.9), non-ideal antenna effects will result in a constant degradation in the channel capacity over SNR, relative to that of the i.i.d. channel. In order to translate this capacity gap into a power related measure, we can apply the following equality:

$$det(R) = det(det(R)^{1/M} I_M)$$
(5.11)

to (5.9), which can then be rewritten as

$$C \approx \log_2 det\left(\frac{\rho_T}{M} det(R)^{1/M} H_W H_W^H\right)$$
(5.12)

Comparing (5.12) to (5.10), we conclude that at high SNRs, the capacity \boldsymbol{C} in (5.9) with non-ideal antennas is equivalent to that of ideal antennas in i.i.d. channel with the SNR

$$\rho_0 = \rho_T \, det(R)^{1/M} \tag{5.13}$$

In this context, the multiplexing efficiency is defined as

$$\eta_{mux} = \frac{\rho_0}{\rho_T} \le 1 \tag{5.14}$$

or equivalently

$$\eta_{mux}(dB) = \rho_0 - \rho_T \le 0$$
(5.15)

which measures the loss of efficiency in SNR (or power, assuming the noise power σ_n^2 is the same) when using a real multiple antenna prototype in an i.i.d. channel (with SNR ρ_T) to achieve the same capacity as that of an ideal array in the same i.i.d. channel (with SNR ρ_0). For high SNRs, η_{mux} is readily obtained from (5.13), i.e.,

$$\tilde{\eta}_{mux} = \lim_{\rho_{T \to \infty}} \eta_{mux} = det(\bar{R})^{\frac{1}{M}}$$
(5.16)

Substituting (5.7) into (5.16), we can rewrite

$$\tilde{\eta}_{max} = det(\Lambda \bar{R})^{1/M} = \left(\prod_{k=1}^{M} \eta_k\right)^{1/M} det(\bar{R})^{1/M}$$
(5.17)

which shows that the multiplexing efficiency is determined by the product of the geometric mean (or the arithmetic mean in Db scale) of the antenna efficiencies and a correlation induced term $det(\bar{R})^{\frac{1}{2n}}$. The geometric mean term is intuitive in that the overall efficiency should come in between the efficiencies of the constituent antennas. For the correlation induced term, its impact

can be understood in that, as the correlation among the ports increases, the condition number of \overline{R} increases. This in turn decreases both $det(\overline{R})$ and $det(\overline{R})^{\frac{1}{M}}$. In other words, a higher ρ_T is needed in order for its capacity to match that of the i.i.d. case with SNR ρ_0 .

In general, the definition (5.14) is still valid, even when the high SNR assumption is not satisfied. However, the resulting expression for η_{max} is more involved and is a function ρ_T of (or equivalently ρ_0). In other words, the constant capacity gap seen in (5.9) may not apply.

The procedure for deriving the exact η_{mux} for a given instantaneous realization of H_w is given as follows.

• Equate the i.i.d. capacity C_{iid} (of SNR ρ_0) with (1), and by the property of determinant

• Since the function $\log_2(.)$ is monotonic

$$det\left(I_{M} + \frac{\rho_{0}}{M} H_{W}H_{W}^{H}\right) = det\left(I_{M} + \frac{\rho_{T}}{M} RH_{W}H_{W}^{H}\right)$$

$$(5.19)$$

• Introduce (5.14) in (5.19) and solve for η_{mux} numerically.

Whereas η_{max} in (5.16) does not depend on either the channel realization or the exact SNR η_{max} , is influenced by both factors. In practice, MIMO performance is typically characterized by ergodic capacity, which is calculated from a large number of Monte Carlo realizations of the channel matrix H. Thus, it is more appropriate to derive η_{max} based on the ergodic capacity. This is achieved by first taking the expectation on both sides of (5.18)

$$E\{C_{iid}\} = E\{C\}$$

$$(5.20)$$

However, there is no known exact closed-form solution for (5.20). Instead, for a given SNR ρ_T , the ergodic capacity for the non-ideal antennas is calculated from Monte Carlo simulations, and the required SNR ρ_0 for the ideal antennas to offer the same ergodic capacity can then be obtained by a parametric search (e.g., by decreasing ρ_0 progressively from a starting guess of $\rho_0 = \rho_T$). Hence, η_{max} can be calculated numerically from the given ρ_T and the corresponding solution ρ_0 using (5.14).

One way to get around this cumbersome approach is to take the upper bound from Jensen's inequality on both sides of (5.20), which after some manipulations yields

$$(1 + \rho_0)^M = \det(I_M + \rho_T R)$$
(5.21)

Then, substituting into (5.21) and solving for, we obtain

$$\eta_{mux} = \frac{\left(det(I_M + \rho_T R)^{1/M} - 1\right)}{\rho_T}$$
(5.22)

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which can be shown to converge to (5.16) in the limit of high SNRs. However, it is noted that the closed-form solution (5.22) is obtained using the upper bounds for Jensen's inequality, which is in fact a loose bound. Moreover, the calculation involves taking the roots of a polynomial in and does not readily offer useful insights into the impact of non-ideal multiple antennas, as is possible with (5.17). In addition, since is deterministic, it can be shown that the high SNR solution (5.16) is equally valid for ergodic capacity, in spite of having been derived based on instantaneous capacity.

Chapter 6

SIMULATION RESULTS & PERFORMANCE ANALYSIS

6.1 CASE STUDY: 2X2 MIMO

For two receive antennas, the antenna efficiency and normalized correlation matrices of $R = \Lambda^{1/2} \overline{R} \Lambda^{1/2}$ are given by

$$\Lambda = \begin{bmatrix} \eta_1 & 0\\ 0 & \eta_2 \end{bmatrix} \tag{6.1}$$

$$\bar{R} = \begin{bmatrix} 1 & r \\ r^* & 1 \end{bmatrix}$$
(6.2)

where denotes the complex correlation coefficient between the two antennas. The approximate closed-form expression (5.16) at high SNR can then be written as

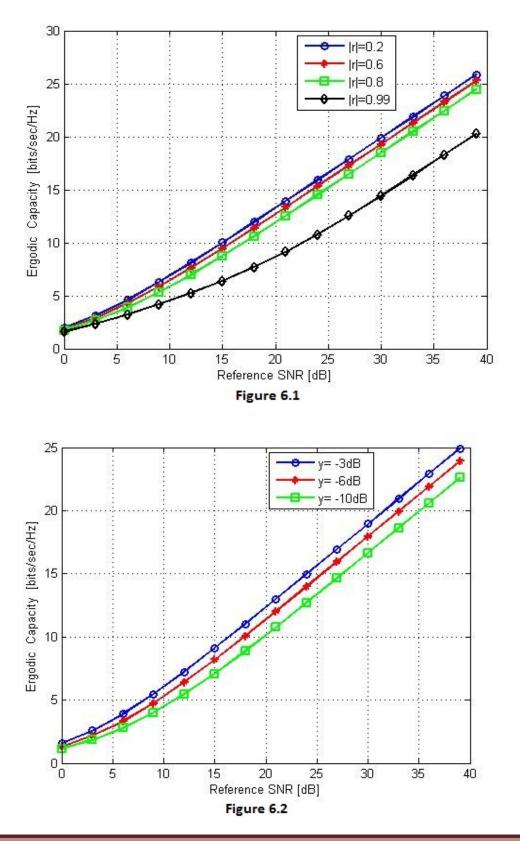
$$\hat{\eta}_{mux} = \sqrt{\eta_1 \eta_2 (1 - |r|^2)} \tag{6.3}$$

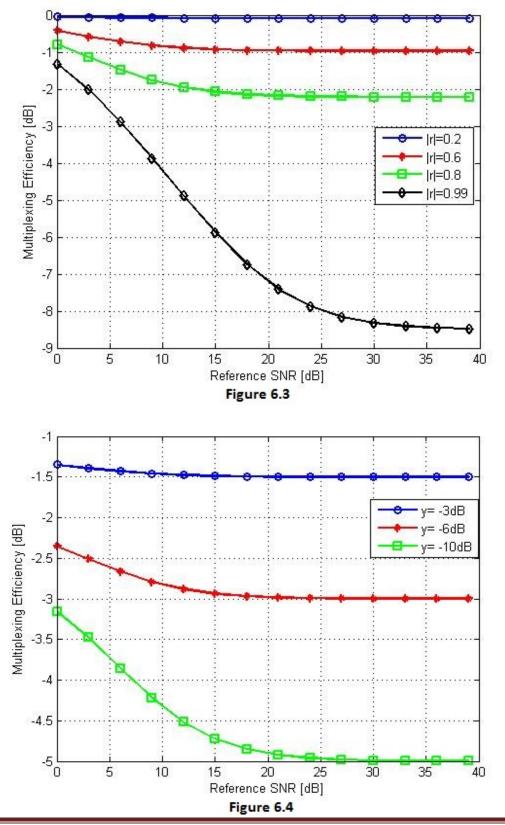
For comparison, the expression (5.22) for η_{max} that is derived based on the upper bound of ergodic capacity and without assuming high SNR can be simplified as

$$\eta_{mux} = \frac{(\sqrt{\eta_1 \eta_2 (1 - |r|^2) \rho_T^2 + (\eta_1 + \eta_2) \rho_T + 1} - 1)}{\rho_T}$$
(6.4)

It is easily verified that (6.4) converges to η_{max} high SNRs.

As mentioned earlier, the impact of correlation and efficiency imbalance η_{max} on can be studied separately using (5.17), which is illustrated as solid markers in Fig. 6.3 and 6.4. For comparison, the impact of correlation and efficiency imbalance on (6.4) are also given for different SNRs. From Fig. 6.3, it is observed that the multiplexing efficiency is relatively insensitive to low to moderate values of correlation, with the decrease in efficiency of lower than 1 dB for correlation of up to 0.6. However, as the correlation increases beyond 0.6, the multiplexing efficiency decreases more severely. This observation is consistent with the rule of thumb that the influence of correlation on diversity gain becomes significant for correlation of above 0.7. In addition, the rate of convergence of (6.4) to the limiting value η_{max} with SNR decreases significantly when the correlation is increased. Nevertheless, convergence is achieved at 30 dB SNR even for the highly unlikely extreme correlation of 0.99. This indicates that the approximate closed-form expression of η_{mux} is accurate for practical prototypes (as is confirmed by the later examples) at commonly used reference SNR values, e.g. $\rho_0 = 20$ dB. In any case, Fig. 6.3 also reveals that the approximate solution is a conservative estimate, which gives a lower bound to the exact η_{mux} . Fig. 6.4 confirms that at sufficiently high SNR and with |r| = 0, the multiplexing efficiency is the arithmetic average of the individual antenna efficiencies (in dB scale), as indicated by (5.17).





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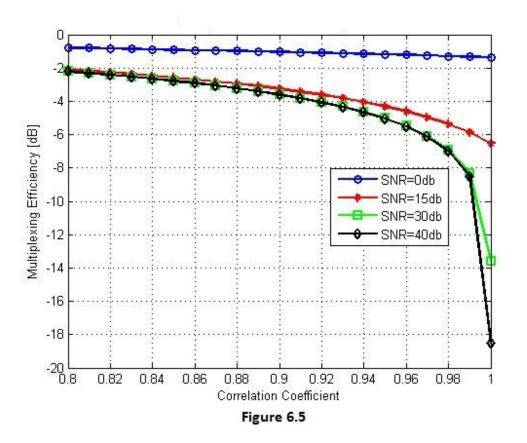


Figure 6.1 and figure 6.2 shows ergodic capacity versus reference SNR with respect to changes in

(a) Antenna correlation |r| ($\eta_1 = \eta_2 = 1$), in figure 6.1 and

(b) Efficiency imbalance (r = 0 and $\eta_1 = 1$, $\eta_2 = \gamma \text{ in } \Lambda$), in figure 6.2

While figure 6.3 and figure 6.4 shows multiplexing efficiency versus reference SNR with respect to changes in

(a) Antenna correlation |r| ($\eta_1 = \eta_2 = 1$), in figure 6.3 and

(b) Efficiency imbalance (r = 0 and $\eta_1 = 1$, $\eta_2 = \gamma$ in Λ), in figure 6.4.

Figure 6.5 shows multiplexing efficiency versus antenna correlation [r] with respect to changes in reference SNR keeping its levels at 0 dB, 15 dB, 30 dB and 40 dB. This curve is in a way reproduction of figure 6.3 by changing the horizontal scale to see the continuous variations in multiplexing efficiency with respect to antenna correlation while keeping SNR constant at various levels.

6.2 EXPERIMENTAL RESULTS

(Based On Data Provided By Other Experimenter)

To illustrate the effectiveness of the proposed metric for characterizing MIMO capability, two realistic mobile terminal prototypes are evaluated (see Fig. 6.6). Each of the test prototypes is fully equipped as a normal mobile terminal and has two well matched antennas operating in the 2.45-GHz frequency band. The antennas for prototype "P1" is intentionally equipped with a dual-feed planar inverted-F antenna (PIFA) to achieve high correlation (for the purpose of testing), whereas prototype "P2" is designed with spatially separated ceramic chip antennas for low correlation. The characteristics of the antenna prototypes, including measured efficiencies and magnitudes of the complex pattern correlation under uniform 3-D APS, are summarized in Table 6.1. As can be seen in Table 6.1, P1 suffers from much higher correlation, lower efficiency, and slightly higher efficiency imbalance as compared to P2.

Table 6.1

PERFORMANCE CHARACTERISTICS OF PROTOTYPES P1 AND P2

		P1	P2
Correlation r		0.80	0.19
Efficiency	η_1	$-4.7\mathrm{dB}$	$-3.9\mathrm{dB}$
	η_2	$-5.2\mathrm{dB}$	$-4.2\mathrm{dB}$
Multiplexing Efficiency η_{mux}		-7.2 dB	$-4.2\mathrm{dB}$



Photograph of the two terminal prototypes P1 and P2. Figure 6.6

As mentioned earlier, channel capacity is the conventional metric for evaluating and comparing the multiplexing capability of different antenna prototypes. Fig. 6.7 presents the ergodic capacity, which is calculated based on the antenna parameters given in Table 6.1. Although the figure clearly shows that P2 has a higher ergodic capacity than P1, the absolute difference expressed in bits/s/Hz does not lend itself to a convenient interpretation and offers no direct insight into the relative influence of antenna efficiency, efficiency imbalance, and correlation. In Fig. 6.8, the multiplexing efficiency η_{mux} of the two prototypes is illustrated. It is observed that the multiplexing efficiency for fully equipped terminal prototypes (as given in Table 6.1). On the other hand, P1 has a significantly lower multiplexing efficiency of -7 dB. According to Table 6.1, the lower average antenna efficiency of P1 (5 dB) contributes to a loss of 1 dB in multiplexing efficiency with respect to that of P2. The correlation coefficient of 0.8 is responsible for a further 2-dB loss, which can be seen from the results shown in Fig. 6.3.

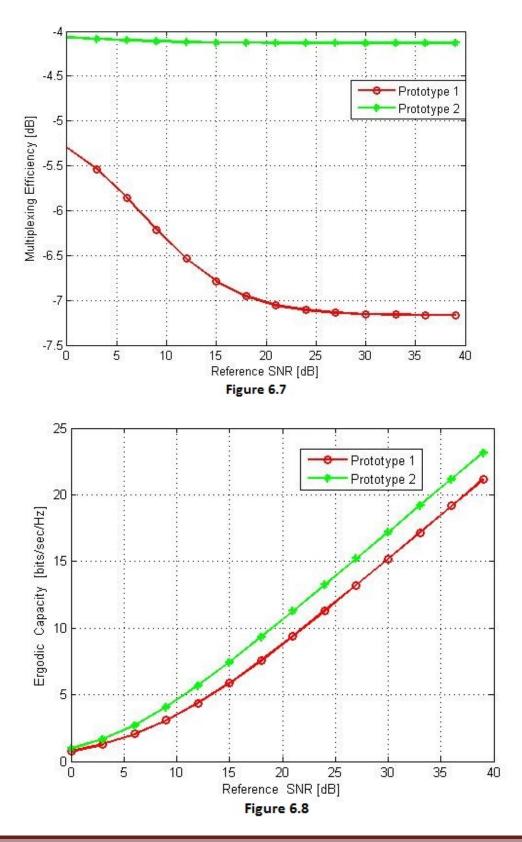


Figure 6.7 and figure 6.8 shows comparison of multiplexing efficiency of P1 and P2. The curves in figure 6.7 denote η_{max} derived from the capacity upper bounds (21) while curves in figure 6.8 denote ergodic capacity in bits/sec/Hz.

Conclusion & Future Work

7.1 CONCLUSION

In this thesis, multiplexing efficiency is proposed as a simple and intuitive metric for evaluating the effectiveness of MIMO antenna terminals operating in the SM mode. Instead of comparing the ergodic capacity, the metric quantifies the performance in terms of absolute efficiency. An example highlights its utility to antenna engineers in identifying and addressing critical design parameters, which will likewise be a useful metric for testing MIMO terminals with different antenna characteristics.

7.2 A VIEW ON CURRENT AND FUTURE PROSPECTS FOR MIMO

It is part of the "natural" evolution of systems that a first version is standardized and implemented, further versions following, improving specific aspects of the standard, including additional features, toward an increase of performance in various directions. These improvements can be dealing with very specific aspects of the system, hence, not being directly usable for other systems, or can be the result of integrating some general technology into the system, thus, taking advantage of a broader knowledge and of a larger number of suppliers. The use of MIMO in cellular networks and WLANs is clearly in the latter case.

Sometimes, the implementation of features in a system requires only the effort of infrastructure manufacturers, in the sense that these features do not affect mobile terminals; the use of diversity in a base station is a good example of this situation. However, many other times, a joint work and collaboration between infrastructure and mobile terminals manufacturers is required and essential, in order to achieve the desired target. Obviously, the latter situation increases the complexity of system improvement. In the case of MIMO, the latter rule also applies, since it requires the use of multiple antennas in both base station/access point and mobile terminal.

The increase of system performance can also be seen from two different perspectives. On one hand, some features are totally independent of users, being fully controlled by operators; the use of half-rate in GSM is a good example, among many others. On the other hand, many performance parameters do depend on the type of terminal users have, and on how they use it; of course, this creates problems in taking full advantage of the technology and in achieving the goals of optimum/maximum performance. Again, MIMO is the latter perspective.

It is clear that, although MIMO is a very powerful technology for the increase of performance (i.e., capacity, data rate, quality of service, etc.) of mobile and wireless systems, really profiting from it can be a complex and nonguaranteed process. Still, the potential of MIMO is so high that, regardless of these barriers, it is being implemented in current upgrades of these systems; for example, LTE, WiMAX, and Wi-Fi.

Nowadays, major manufacturers for cellular networks infrastructure already deliver MIMO products for base stations; four antennas are already made available, and trials with eight antennas have also been announced. However, the situation is quite different as far as mobile terminals are concerned: at this end of the link, terminals are severely size constrained and the space available to locate multiple antennas is often desperately limited. Fortunately, there is a lot of work being done on co-location of multiple (e.g., 2) antennas in mobile phones with a relatively low correlation, especially for orthogonal linear. On the other hand, one should not look into this problem by taking only (current) mobile phones as user's terminals, other devices being real alternatives as well; personal computers (to be used not only for Wi-Fi, but also for cellular networks) may include two or four antennas in a relatively decoupled way (there are a number of publications on this topic), and even other hypothesis that do not present space problems can be considered; for example, the use of cars as the "interface" between the mobile phone and wave propagation (this is already a reality today, for high-end cars).

So, although there may currently be some real problems in the implementation of MIMO in mobile and wireless systems, by taking advantage of multiple antennas at both ends of the link, the conditions for the efficient deployment of MIMO systems will definitely be favorable in the near future. The implementation of MIMO in IEEE 802.11 WLANs, i.e., Wi-Fi, has been introduced in the "802.11n version" of the standard [IEE09b], enabling (at the physical layer) data rates up to 600 Mbps with a 40MHz bandwidth, in a 4x4 MIMO configuration. Changes are described in the transmitter, at the Physical Layer Convergence Procedure and frame format, and at the signal processing architecture, in the receiver, where system performance depends on the implemented algorithms and the solution is not unique, looking into signal detection, synchronization, carrier frequency offset estimation, channel estimation, and decoder. A comparison of performance results is done, with and without space time block coding, using combinations of one and two antennas at both transmitter and receiver (i.e., from SISO to MIMO); MIMO does improve system performance, enabling gains in signal to noise ratio, frame error ratio, data rate, or other parameters, depending on the perspective used for the analysis. Gains can be as high as 10 dB in signal to noise ratio, or the increase in data rate of almost an order of magnitude, when comparing MIMO with SISO.

The implementation of MIMO in IEEE 802.16 cellular networks, that is, Wi-MAX also possible. In the recent standard [IEE09a], up to four antennas are foreseen, but most of the proposed schemes apply only to two, like the two schemes that are mandatory for use in downlink, but are optional for uplink, i.e., Space Time Block Coding and Spatial Multiplexing; still, a combination of these two schemes is being considered. Again, MIMO improves system performance, as expected. Additionally, a brief analysis of the transceiver architecture is performed for mobile terminals, showing that it is possible to incorporate a second antenna, together with its circuitry, given both size and cost issues.

LTE has been designed having MIMO in mind from the beginning [3gp09]. The effort in standardization was focused on specifying efficient schemes for downlink single user MIMO (that is, no indications are given for a transmission to multiple users such as broadcast), while for uplink the system was designed on the assumption that the mobile terminal front-end comprises a single signal chain; hence, for the latter, the support of single user MIMO is limited to adaptive transmit antenna switching; multiuser schemes are not excluded but are left to manufacturers implementation. The techniques and schemes considered in LTE for MIMO are not that much different from the ones taken in the previous systems, namely because they all share basic

aspects of the multiple access technique. Detailed information is given on several implementation aspects, and LTE-Advanced is considered as well. Additionally, the use of Space Division Multiplexing together with MIMO is addressed, namely the problems related to detection algorithms and their mapping in the context of the standard. It is concluded that the gap between theoretical algorithms and practical implementations is huge, but also that there are ways to bridge this gap. Also, the importance of scalable or flexible solutions is stressed, since worst-case design is not an acceptable solution for complex algorithms implementation in mobile terminals; a scalable architecture can support a high throughput by relying on simple algorithms when the channel is favorable, and may be able to be reconfigured in order to exploit much more complex detection strategies when the channel is worse.

However, MIMO development and usage is not going to be finished by the standardization of the previously mentioned systems, and one may see quite a number of developments in the future, some of them listed in the following text.

Due to the fact that the UHF band has excellent propagation characteristics for mobile and wireless communications, the race on higher data rates has not implied (yet) the move to higher frequency bands, and basically all mobile and wireless systems working today use this band. Moreover, with the digital dividend (the use of part of the spectrum released from broadcast systems to mobile and wireless ones), the pressure to go higher in frequency has decreased. The achievement of higher data rates are usually obtained by more efficient multiple access techniques and modulation schemes, better management of the radio resources, and so on, rather than by investigating in the area of propagation and channels. MIMO was the exception confirming the rule, i.e., by taking advantage of the randomness of the propagation channel, it was possible to profit from the "parallel channels," and therefore, to enable higher data rates.

Nevertheless, one cannot avoid going up in frequency, if higher data rates are to be obtained, especially because efficiency and performance boundaries are being attained today. The knowledge of propagation and channels up to the UHF band is almost complete, but beyond that (including mm waves and up to the THz band) there is still a lot of work to be done, and a thorough characterization of MIMO in these high frequency bands is still to be performed.

Mobile communications gave rise many years ago to the fears from (electromagnetic) radiation. Basically, the extremely fast introduction of a new technology in the mass market, without a proper explanation of the system behavior, generated a lot of health concerns, which are still present today. One can currently envision that environmental concerns, which are already of key importance nowadays ("green communications" is in the agenda of many initiatives, projects and fora), will become even more important, with a huge impact on systems and networks, concerning their development, deployment and operation. Therefore, energy efficiency has to be (is already being) taken into account in mobile and wireless communications. MIMO can play an important role in this matter, through a better exploitation of the spatial/polarization degrees of freedom toward more efficient communications, at a lower transmission power. Moreover, by combining MIMO with beam forming, one can further decrease the transmission power (given the increase in the corresponding antenna gain); hence, providing additional contribution to increase energy efficiency.

Another trend regards mobile terminals. The evolution has been such that one can foresee that users may carry just an RF SIM card for their identification by the network, using any terminal/device at hand for communication, ranging from a PC to a "TV screen," encompassing terminals embedded in cars and in the office, and reaching spectacles as the replacement of today's mobile phones. Therefore, short range communications, in the vicinity of a person's body, will play a major role. The exploration of MIMO in such distance ranges has not deserved much attention, most probably due to the low randomness of the propagation channel, as well as to the difficulties in having more than one antenna available at either end of the link. Still, given the development of technology (in antenna design, signal processing, nanodevices, etc.), together with an increase in frequency band, one can easily envision that the conditions for deploying MIMO in these kinds of applications may arise in the near future.

Propagation in not so usual environments needs to be studied as well, for example: several indoor scenarios, with a differentiation between business and residential environments; for personal systems, in-, on- and off-body propagation; for car communications, propagation in between cars, from a heavy traffic situation in a motorway (addressing high speed as well) to a

city street; public transport scenarios, including buses, trains, planes, and boats, among others. The characterization of all these scenarios will allow one to estimate how far they are appropriate for MIMO technologies, and how to optimize the location of antennas.

Location based services are already popular nowadays, where a user can know his/her location via the mobile phone, and take advantage of them to navigate or to find a nearest shop, among other applications. Basically, this means that the user is accessing the network for location purposes. However, one can invert this concept, that is, the network knowing where users are, and take advantage of it. For example, by exploiting users' terminals (and other devices) as channel sensors, the network can establish a geographical map of channel conditions, hence, of channel quality, and with that information forecast services availabilities in an efficient way (specifically for each user). The creation of such geographical channel mapping for better MIMO usage (i.e., enabling the system to know where and when to use MIMO, and then, increasing data rate accordingly), in a given scenario (from indoors to streets, including bodies and cars, and many others), would definitely increase the overall system performance.

Machine-to-machine communications (recently renamed as the Internet of Things) no longer requires a dedicated introduction. The increasing number of applications of this type of communications, where the user has no intervention, is obvious, and definitely will surpass human communications (voice or data) in terms of traffic volume, as data has already surpassed voice in terms of exchanged number of information bits. This also extends to car-to-car and carto-infrastructure communications, and other types of systems. This means that one will have devices all over the surroundings, at any location, many times with low power consumption requirements, communicating not only among each other but also to a control or information network. Again, MIMO can play an important role in this type of communication, and conditions for deployment should be explored.

Sensors can be considered as subset of the Internet of Things, not only for personal use (e.g., on the body), but also through myriad devices that start being implanted in our surroundings (e.g., in cars or houses). Being a subset of the Internet of Things, the same type of problems apply, but one can envision in this case that power consumption and efficiency of

communications will play enhanced roles. This opens a wide range of study cases, by extending the MIMO concept to the use of different multiple antennas, located at somehow random locations (e.g., buttons on clothes); hence, creating the need to analyze system behavior under those conditions.

In summary, the possibilities for deploying and amplifying MIMO technologies in the future are immense, bringing to the conclusion that the exploitation of these technologies is just in its infancy.

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