

A  
MAJOR PROJECT  
ON

# OPTIMUM POSITION OF HORIZONTAL STIFFENERS IN WEB PANELS

Submitted in partial fulfillment of the requirement  
for the award of the degree

**Master of Engineering**

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**CIVIL ENGINEERING  
(Structural Engineering)**

*Submitted By*

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**2010**

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**DECLARATION**

I hereby declare that the work which is embodied in this major project entitled “OPTIMUM POSITION OF HORIZONTAL STIFFENERS IN WEB PANELS” is authentic record of my own work carried out in partial fulfillment of the requirements for the award of Master of Civil Engineering (Structural Engineering) under the guidance and supervision of Dr. A. K. Gupta, Professor and Head of Department, Delhi College of Engineering, Delhi. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

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**NARINDER VERMA**

## **ABSTRACT**

An analysis based on energy method is carried out in this work to determine the optimum location of the longitudinal stiffener, which is applicable to all plated structures including the plate girders. This analysis gives an interactive equation between the plate buckling coefficient ( $k$ ), aspect ratio ( $\beta$ ), proportional stiffener-plate ratio ( $\delta$ ), stiffener placement ratio ( $c/b$ ) and relative flexural rigidity ( $\gamma$ ). This equation also includes a numerical factor ( $\alpha$ ) which decides cases into pure bending or pure compressive or intermediate case. In this study pure bending case is used. The above analysis is furthered to draw 3D plots using MATLAB to demonstrate the variation of the behavior between various parameters, using three at a time and keeping others constant. The plots are then inferred upon either to make some variables constant or to make conclusions. Comparisons of various interactive plots indicate the dependency of stiffener location on the plate-stiffener geometrical properties and modifying the codal provisions of stiffener location as one-fifth of the width of plate.

The analysis work is further extended to compare calculated values with experimental work of various analysts and codal provisions. The comparative study indicated similarity in the predicted and experimental results, thereby authenticating the usage of energy method modeling on experimental predictions.

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## LIST OF SYMBOLS

a	Length of Web Panel
A	$(1+\beta^2)+2\gamma\sin\pi c/b$
$a_{mn}$	Numerical Coefficient
b	Depth of Web Panel
B	$-2\beta^2(1-\alpha c/b) \sin\pi c/b$
$b_f$	Width of flange
c	Position of Longitudinal Stiffener
C	$2\gamma\sin\pi c/b$
D	Flexural Rigidity of Unit Width of the Web Plate ( $=Eh^3/12(1-\nu^2)$ )
D'	$-\beta^2 16\alpha/9\pi^2 - 2\beta^2(1-\alpha c/b) \sin 2\pi c/b$
E	Young's Modulus
$f_{yf}$	Yield stress of Web
$f_{yw}$	Yield Stress of Flange
G	$2\gamma\sin\pi c/b$
h	Thickness of Web Plate
H	$-\beta^2 16\alpha/9\pi^2 - 2\beta^2(1-\alpha c/b) \sin\pi c/b$
I	$(1+4\beta^2)+2\gamma\sin 2\pi c/b$
J	$-2\beta^2(1-\alpha c/b) \sin 2\pi c/b$
k	Plate Buckling Coefficient
m	Number of Half waves in x-direction
n	Number of Half waves in y-direction
$N_o$	Intensity of compressive force at $y=0$
$N_x$	Compressive Force per unit Length of Edge
P	Compressive Force acting on Stiffener
$P_{exp}$	Experimental Ultimate load
$P_{pr}$	Theoretical Patch Load as per Energy Method
$P_{pr BS5400}$	Ultimate Patch Load as per BS5400
$s_s$	Length of Patch Load

$t_f$	Thickness of Flange
$T_p$	Work Done by Compressive Force acting on Stiffener
$T_s$	Work Done by External Forces during Buckling
$t_{st}$	Stiffener Thickness
$U_p$	Strain energy of bending of plate
$U_s$	Strain energy of bending of Stiffener
$w$	Deflection of the plate in z-direction
$\alpha$	Numerical Factor
$\beta$	Plate aspect ratio
$\gamma$	Relative Flexural Rigidity
$\delta$	Proportional Plate-Stiffener Ratio
$\nu$	Poisson's Ratio
$\sigma_{cr}$	Critical Compressive Stress

## **INTRODUCTION**

### **1.1 GENERAL:**

Societies have used iron for about 5000 years. The Iron Age began somewhat before 1000B.C. in Western Asia and Egypt. The use of iron to advance the goals of the society had been almost a function of the ability of the society to transform the raw material into a serviceable product. The element iron occurs in nature in a combined state and it is the impurities with which it is combined that cause the product to be variable in its characteristics and usually quite brittle. The use of iron was immeasurably increased with the development of its most common alloy, steel.

The extensive use of iron or steel had to wait for an economical production process till the mid- nineteenth century when Sir Henry Bessemer developed the Bessemer convertor, which in the heating process removed undesirable elements including excessive amounts of carbon and subsequently introduced desirable quantities of principally carbon and magnesium. Hence Steel Age was born.

Today the iron and steel industry is the basic of key industry for any country. The unique position of iron among the metals may be attributed to its abundance and to wide range of properties that can be imparted to it by various treatments and alloying it with various amounts of other elements.

Steel has become the predominant material for the construction of bridges, buildings, towers and other structures. Steel exhibits desirable physical properties that make it one of the most versatile structural materials in use. Properties that make it material of choice for various structures such as steel bridges, high rise buildings, towers, etc are –strength, uniformity, light weight, ease of use, speed of erection, prefabrication, demountability, elasticity, toughness, addition to existing structures and last but not the least Ductility.

Structural steel is predominantly iron, which is combined with small portions of other minerals to create an alloy that serves the intended function. Over 200 types of steel are available for engineering design community. Steel an alloy of iron contains 0.1 to 1.1% of carbon which distinguishes it from its counterparts cast iron and wrought iron. Other elements such as manganese, silicon, chromium, iron, nickel and molybdenum may be added to it to form steels of various properties according to the requirement.

Various shapes of structural steel products are available of interest to designers which include beam sections, channel sections, angles, T-bars, bulb angles, plates, strips, flats, square bars and round bars.

Beams are the most common member elements used to transfer loads horizontally to supports. The easiest of the shapes of beams is the rectangular section. The rectangular shape is flexurally inefficient because much of the material is located near the middle of the beam is largely ineffective. So the most efficient section, that is, the I-section was developed which were narrow and stocky.

Various shapes of structural steels can be connected together to fulfill the demands of a designer. The various connection types available for steel sections are riveting, bolting and welding. Although recently adhesives have also come as an option to us but it is in stage of development and is undergoing testing procedures.

Welding involves connecting two pieces of metal by heating to a plastic or fluid stage (with or without pressure), so that fusion occurs. Today several welding processes are available to join various metals and alloys.

It is known that beams are structural members that support loads which are applied transverse to their longitudinal axis. When the loading is heavier and span is large designer has the following choices:

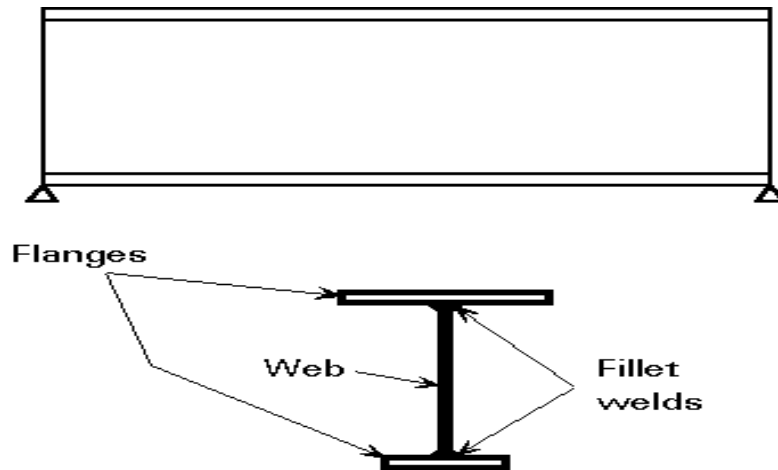
- a) Use two or more regularly available rolled beam sections, side by side (compound sections).
- b) Use of cover-plated beam i.e., weld a plate of adequate thickness to increase the bending resistance of flange.
- c) Use of fabricated plate girder, which provides freedom (within limits) to choose size of web and flanges.
- d) Use a steel truss or a steel concrete composite truss.

Compound sections are usually uneconomical and do not satisfy deflection limitation. Cover-plated rolled I-beam is advantageous where rolled section is marginally inadequate. A truss girder involves high cost of fabrication and erection, problems of vibration and impact, and requires high vertical clearance.

The plate girders are I-beam built up from plates using riveting or welding. They are deep flexural members used to carry loads that cannot be economically carried by rolled beams or trusses. They provide unique flexibility in fabrication. They are economical as they

provide exact amount of steel required at each section along the length of the plate girder, that is, it can be shaped to bending moment curve itself. They are used in gantry girders in industrial buildings to carry rails for large capacity overhead travelling crane, in power plant buildings to support bunkers, in bridges, etc.

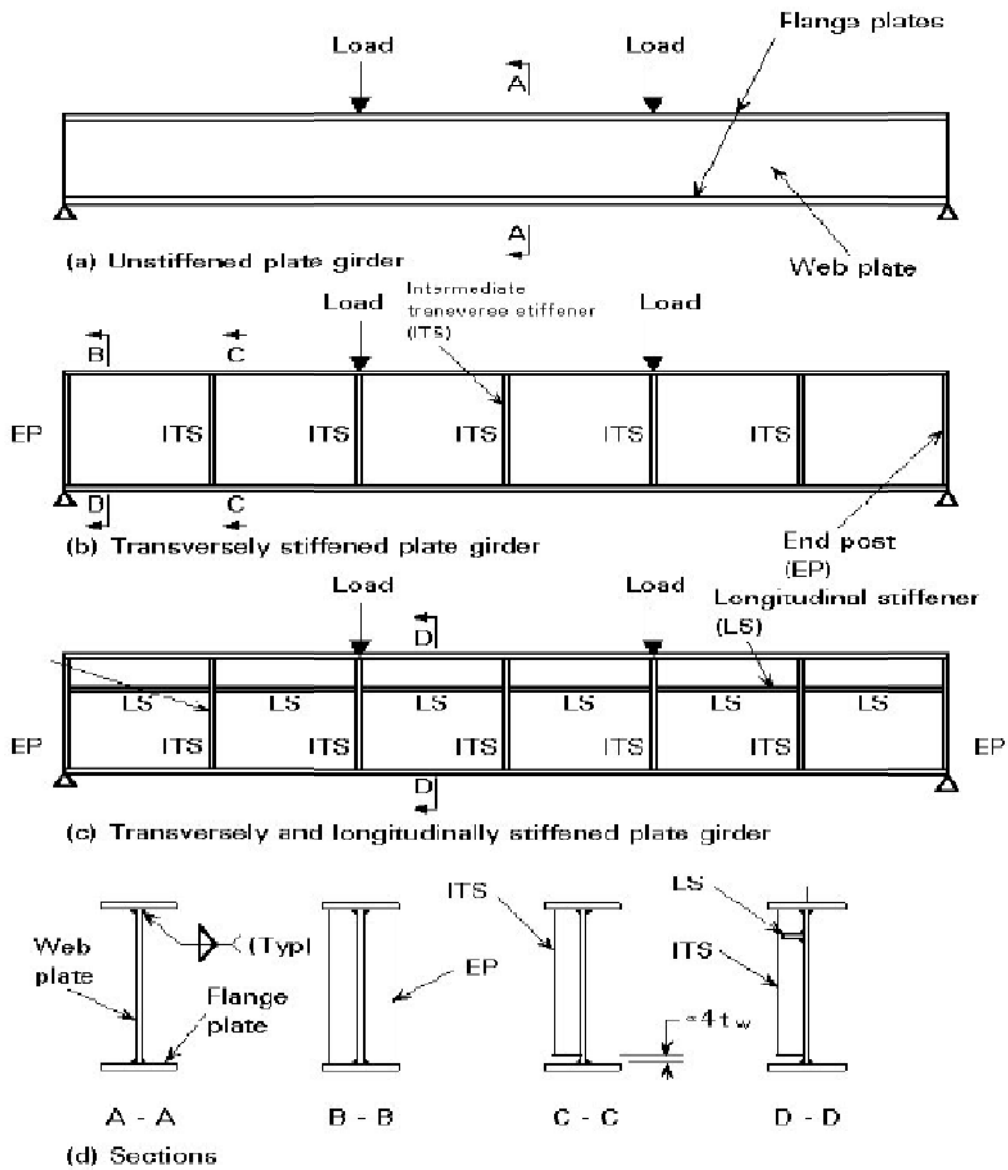
The elements of a plate girder in simplest form include Web plate, Flange plates and Weld or connection between web and flanges



**Fig. 1.1 Plate girder composed of three plates**

Other elements of plate girder which are required as per site considerations are –

- Cover plates on flanges
- Bearing stiffeners or end posts
- Intermediate transverse stiffeners
- Longitudinal stiffeners
- Web splices
- Flange splices
- End bearing or end connections.



**Fig. 1.2 Stiffened and Unstiffened plate Girders**

From the view point of plate stability, proportioning of various elements of plate girders is essential. Better understanding of plate girder behavior, high strength steels and improved welding techniques have combined to make plate girders economical. The deeper the analysis process of a plate girder is done the more is it comfortable for the designer to design an efficient and economical section.

The proportioning of a plate girder is facilitated by:

- i. Reduction of flange thickness (or width) in zone of applied moment.
- ii. Thicker web in zone of high shear.

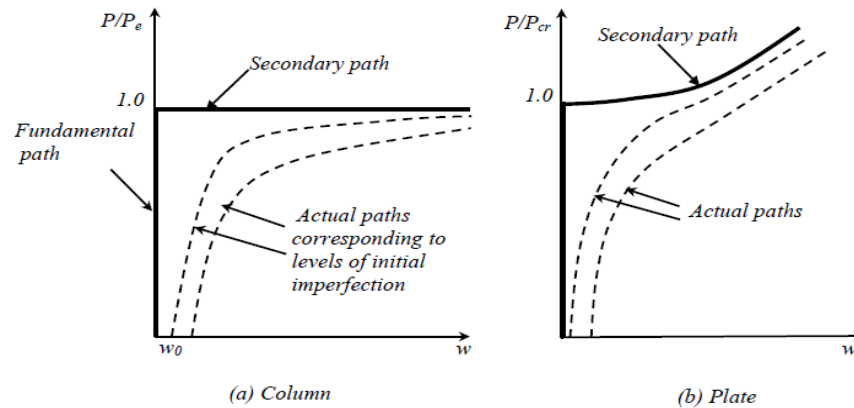
In plate girders the primary function of the flanges is to resist bending moment by developing axial compressive and tensile stresses. The web plates resist the shear. For making the cross section efficient in resisting the in plane bending, it is required that the maximum material is placed as far as possible from the neutral axis. The axial force in the flange decreases, as the depth of the girder increases. Thus a smaller cross-section would suffice than if smaller depth is chosen. However, this would mean that the web would be deep and thin, which in situations may lead to premature failure of thin web due to web buckling in shear. Hence there is a choice between the thin web provided with vertical and horizontal stiffeners and a thicker web requiring no stiffening, thus proportioning of various elements of plate girder become inevitable.

For better understanding of a plate girder, understanding the behaviour of web plates is necessary. It is interesting to compare the stability of a column and a plate. In the case of an ideal column, as the axial load is increased, the lateral displacement remains zero until the attainment of the critical buckling load (Euler load). The plot of the axial load versus lateral displacement, gives a line along the load axis up to  $P = P_{cr} = P_e$  (Fig. 1.3a). This is called the fundamental path. When the axial load becomes equal to the Euler's buckling load, the lateral displacement increases indefinitely, at constant load. This is the secondary path, which bifurcates from the fundamental path at the buckling load. The secondary path for column represents neutral equilibrium. For practical columns, which have initial imperfections, there is a smooth transition from the stable to neutral equilibrium paths as shown by the dashed line in Fig. 1.3(a).

The fundamental path for a perfectly flat plate is similar to that of an ideal column. At the critical buckling load, this path bifurcates into a secondary path as shown in Fig. 1.3(b). The secondary path reflects the ability of the plate to carry loads higher than the elastic critical load. Unlike columns, the secondary path for a plate is stable. Therefore, elastic buckling of a plate need not be considered as collapse. However, plates having one free edge and simply supported along the other edges (outstands), have very little post-buckling strength.

Actual failure load of the columns and plates are reached when the yielding spreads from the supported edges triggering collapse and thereafter the unloading occurs.

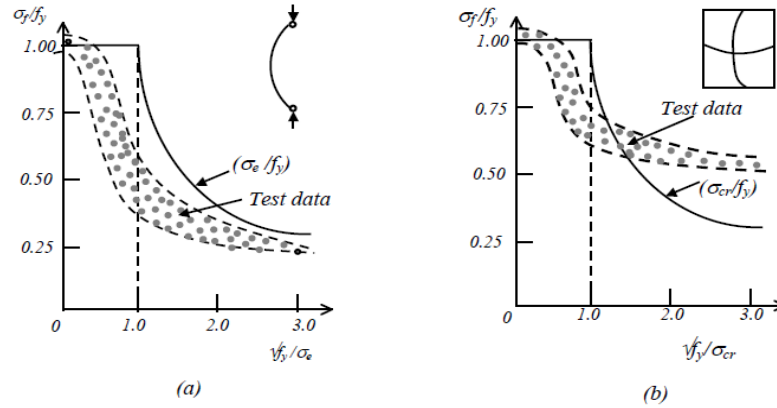




**Fig. 1.3 Load versus Out-of-plane Displacement Curves**

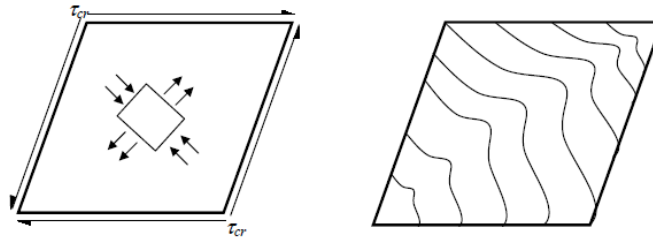
Plate strength curves can also be constructed similar to the column strength curves (Fig. 1.4). In the case of ideal columns with low slenderness (*i.e.* stocky columns), failure is expected by squashing at the yield stress. On the other hand, if the ideal column is slender, failure will be by buckling at or near the Euler load. Tests on practical columns indicate that failure always occurs below the failure load of an ideal column of the same slenderness. If the column is stocky, then the yield stress provides an upper bound and if the column is slender then the buckling stress provides an upper bound. Also the scatter in the test results is considerable particularly in the range of intermediate slenderness ratios ( $\sqrt{f_y/\sigma_e} = 1.0$ ).

In the case of a flat plate simply supported on all four sides, failure is expected by squashing if the  $b/t$  ratio is less than the limiting value. Similarly, for  $b/t$  ratios larger than the limiting value, failure after buckling at the critical buckling stress may be expected. However, tests on practical plates indicate that for large  $b/t$  ratios, the failure stress is substantially greater than the critical buckling stress. This is due to the post-buckling behaviour, which is unique to plates. The load from the middle strips gets transferred to the edges and the plate continues to carry higher load in stable post-buckling range, until the edges reach the yield stress. As with columns, the scatter in the test results is considerable in the range of intermediate  $b/t$  ratios (at  $f_y/\sigma_{cr} = 1.0$ ).



**Fig. 1.4 Column and Plate Strength Curves (INSDAG Vol.II:2001 )**

Rectangular plates loaded in shear such as web plates in a plate girder, also tend to buckle. Consider a plate loaded in shear in its own plane as shown in Fig.1.5. A square element in the plate (Fig.1.6), whose edges are oriented at  $45^\circ$  to the plate edges, experiences tensile stresses on two opposite edges and compressive stresses on the other two edges. The compressive stress can cause local buckling and as a result the plate develops waves perpendicular to them.

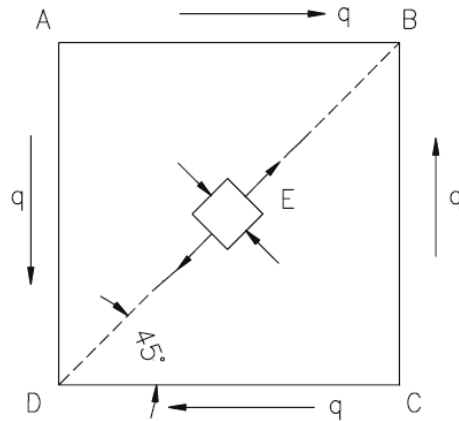


**Fig. 1.5 Shear buckling of a plate**

The critical shear stress at which this form of buckling occurs is given by the same formula as that for plate buckling under compression, except that the value for the buckling coefficient  $k$  is different.

Plates buckled in shear also can support additional loads. By drawing imaginary diagonals on the plate, the diagonal which gets loaded in compression, buckles and cannot support additional load. However, the diagonal in tension continues to take more loads and the plate becomes like a triangular truss with only tension diagonals. This is called **tension field action**.

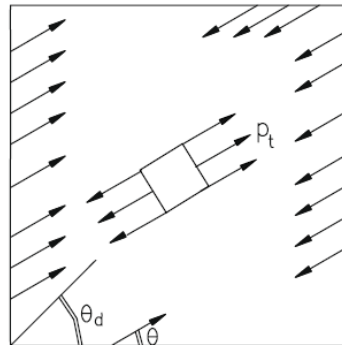
When a web plate is subjected to shear, the structural behaviour can be visualised by considering the effect of complementary shear stresses generating diagonal tension and diagonal compression.



**Fig.1.6 Unbuckled shear Panel**

Consider an element  $E$  in equilibrium inside a square web plate subject to a shear stress  $q$ . The requirements of equilibrium result in the generation of complementary shear stresses as shown in Fig. 1.6. This result in the element being subjected to principal compression along the direction  $AC$  and tension along the direction of  $BD$ . As the applied loading is incrementally enhanced, with corresponding increases in  $q$ , very soon, the plate will buckle along the direction of compressive diagonal  $AC$ . The plate will lose its capacity to any further increase in compressive stress; the corresponding shear stress in the plate is the “critical shear stress”  $q_{cr}$ .

The compression diagonal ( $AC$ ) is unable to resist any more loading beyond the one corresponding to the elastic critical stress. Once the web has lost its capacity to sustain increase in compressive stresses, a new load-carrying mechanism is developed. Applications of any further increases in the shear load are supported by a *tensile membrane field*, anchored to the boundaries, viz. the top and bottom flanges and the adjacent stiffener members on either side of the web.



**Fig 1.7 Post Buckled Behaviour**

Thus the buckling of plate is the major process that leads to tension field action and the consequent behaviour of plate girder.

Exact analysis of extremely complex behavior has been possible for vertical or transverse stiffeners but that has not been done taking into account the longitudinal stiffeners. Although methods have been developed for web panels having vertical stiffeners under bending or shear, the shear behaviour of panels having horizontal as well as vertical stiffeners taken together need greater attention particularly for long span bridges.

## **1.2 PROBLEM STATEMENT:**

To reduce the self weight (and the corresponding self weight bending moments), the web thickness should be limited to slender proportions (the web proportions are generally expressed as web slenderness ratio). Slender webs (having large slenderness ratio) would buckle at relatively low values of applied shear loading.

Efficient and economical design usually results in slender members. Hence advantage must be taken of the post buckling behaviour of the web i.e., ability of the girder to withstand transverse loads considerably in excess of the load at which the web buckles under shear. A girder of high strength to weight ratio can be analysed incorporating the post buckling strength of web. This would be particularly important where the reduction of self weight is of prime importance as in long bridges.

As there is negligible amount of work done for analysis of ultimate strength of plate girders having horizontal stiffeners along with vertical stiffeners. So the study here has been done with the ultimate strength of plate girders having both vertical and horizontal stiffeners and the optimization of position of the longitudinal stiffeners.

## **1.3 AIM AND OBJECTIVE:**

The aim and objective of this study are as follows:

1. To study the effect of providing horizontal stiffeners in web of plate girder in addition to transverse stiffeners, generally provided in the design.
2. To find the optimum position of horizontal stiffeners in web panels leading to economy of the structure.
3. To find the ultimate strength of the web panel in shear having vertical and transverse stiffeners.

## **LITERATURE REVIEW**

### **2.1 GENERAL :**

The primary functions of the web plate in a plate girder are to maintain the relative distance between the top and bottom flanges and to resist the introduced shearing force. In most practical ranges of span lengths for which a plate girder is designed, the induced shearing force is relatively low as compared with the axial forces in the flanges resulting from flexure. As a result, the thickness of the web plate is generally much smaller than that of the flanges. Consequently, the web panel buckles at a relatively low value of the applied shear loading. The webs are often reinforced with transverse stiffeners to increase their buckling strength, and web design involves finding a combination of an optimum plate thickness and stiffener spacing that renders economy in terms of the material and fabrication cost. The design methods of plate girder webs are divided into two categories:

- i. Allowable stress design based on elastic buckling as a limiting condition;
- ii. Strength design based on ultimate strength, including post-buckling as limit state.

Web buckling due to shear is essentially a local buckling phenomenon. Depending upon geometry, the web plate is capable of carrying additional loads considerably in excess of that at which the web starts to buckle, due to post buckling strength. Taking advantage of this reserved strength, a plate girder of high strength/weight ratio can be designed.

Wilson (1886) is credited with the first study of the postbuckling strength of the plate girder web panels. As Wagner (1931) first presented a uniform diagonal tension theory for aircraft structures with very thin web panels and rigid flanges, many researchers have studied the tension field action for plate girders, as summarized in Structural Stability Research Council-SSRC (1998).

**2.2 CLASSICAL THEORIES OF PLATE GIRDER WEB PANELS AS PER SSRC:**


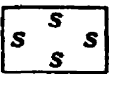
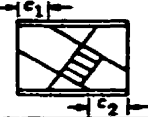
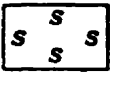
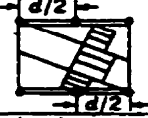
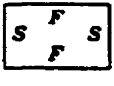

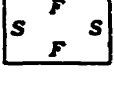
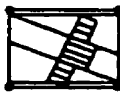
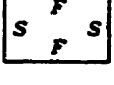
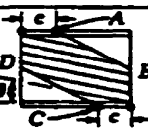
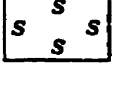

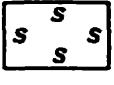


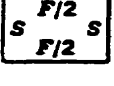

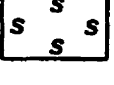
Investigator	Mechanism	Web Buckling Edge Support	Unequal Flanges	Longitudinal Stiffener	Shear and Moment
Basler (1963-a)			Immaterial	Yes, Cooper (1965)	Yes
Takeuchi (1964)			Yes	No	No
Fujii (1968, 1971)			Yes	Yes	Yes
Komatsu (1971)			No	Yes, at mid-depth	No
Chem and Ostapenko (1969)			Yes	Yes	Yes
Porter et al. (1975)			Yes	Yes	Yes
Hoglund (1971-a, b)			No	No	Yes
Herzog (1974-a, b)		Web buckling component neglected	Yes, in evaluating c	Yes	Yes
Sharp and Clark (1971)			No	No	No
Steinhardt and Schroter (1971)			Yes	Yes	Yes

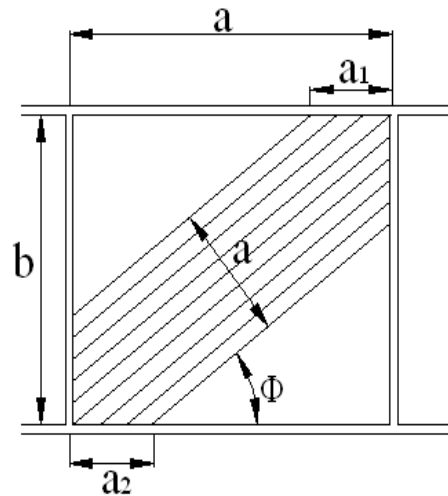
Fig. 2.1

**Basler and Thurlimann (1961) :**

They carried out extensive studies on the post buckling behaviour of plate girder web panels. They assumed that tension field develops only in parts of the web and that flanges are too flexible to support normal stresses induced by the inclined tension field. In other words, yield zones form away from flanges and merely transverse stiffeners act as anchors.

**Tekeuchi (1964):**

He suggested a modification of Basler's approach by proposing the tension field model assuming that tension field extends to distances  $a_1$  and  $a_2$  into the top and bottom flanges, respectively. Distances  $a_1$  and  $a_2$  were suggested to be proportional to the moment of inertia of the flanges about their own axes. This was an attempt to include the effect of the flanges on the ultimate shear strength. He was the first to make an allowance for the effect of flange stiffness on the yield zone of web plates.



**Fig. 2.2 Tekeuchi's Model**

**Fujii (1968):**

He presented a tension field model in which the tension field was assumed to be uniform in the panel and controlled by the vertical web stresses needed to develop three-hinge plastic beam mechanisms in the flanges. In other words he put forward that the criterion for ultimate strength is not only yielding of web plate but also plastic hinge mechanism of flanges, formed by plastic hinges at both ends supported by vertical stiffeners and at mid-span. He recommends calculating shear buckling coefficient considering the web fixed along flanges and simply supported along vertical stiffeners. This theory gave better agreement with the results of the tests conducted.

**Komatsu (1971):**

He gave formula for four mode failure. Failure in the first mode occurs in the manner as shown in the table above, where the inner band yields under the combined action of buckling stress and the post buckling tension field, where the smaller tension in the outer

bands is the value that can be supported by the girder flange as a beam mechanism with the interior hinge at the distance  $c$  determined by an empirical formula based on tests. The inclination of the yield band is determined so as to maximize the shear, as in Basler's solution, but the optimum inclination must be determined by trial. In the second mode, which is a limiting case of the first mode, the interior hinge develops at midpanel, and the web yields uniformly throughout the panel. In the third mode of failure the flanges are assumed to remain elastic while allowing complete yielding of the web. An optimum value of the tension-field inclination must also be found by trial for this case. The fourth case is a limiting case in which a Wagner field develops along with a panel mechanism of the flanges.

**Chern and Ostapenko (1971):**

They proposed the proposed tension field is as shown in the table above, where the principal band is determined by yielding, taking into account the stress that exists at buckling and a panel mechanism to develop in the flanges with both ends fixed.

**Porter et al. (1971):**

They assumed that inclined tension fields only develop in a limited portion, but that flanges do not contribute to the post-buckling strength by absorbing normal stresses from tension fields; and that as a result, girders collapse when plastic hinges form in their flanges.

**Hoglund (1971):**

He developed a theory for transversely stiffened and unstiffened plate girders. He used the system of diagonal tension and compression bars to model web plates. He gave most conservative results for slender webs because he based the axial force reduction in flange plastic moment on the largest moment in the panel.

**Herzog (1974):**

He assumed that the diagonal tension field develops in a limited portion of the web. He proposed limiting value for web width to thickness as a function of area ratio of web and flange there by incorporating the difference of flange restraint to web with change



of area ratio. Stress distribution at ultimate state given were an improvement in Basler's theory as no portion of the web have stress equal to zero.

**Sharp and Clark (1971):**

They assumed intuitively the boundary condition to be halfway between the simply supported and fixed condition.

**Steinhardt and Schroter (1971) :**

They suggested that tension field band is in the direction of the panel diagonal and its boundaries intersect the mid panel points of the flanges. The tension-field loading on the flange is assumed to vary sinusoidally with a maximum value at the stiffeners.

**Rockey and Skaloud (1971) :**

They presented the experimental results which indicated that the actual mode of failure of a girder panel under shear depends on the flange rigidity. They have suggested that plate girders fail in shear when a section of the web plate yields and plastic hinges form in the flanges, thereby permitting a shear mechanism to occur.

**2.3 RECENT STUDIES:**

**Marsh (1982):**

He suggested a different model in which, after initial buckling, the uniform shear distribution along the boundary changes, with increasing shear stress toward the tension corner, but with no stress normal to any boundary.

**Marsh (1988):**

He suggested that the shear capacity of a web with high slenderness ratio, having flanges with small bending rigidity, is reached when the web yields in shear in the tension corner, with no diagonal tension or stress normal to the boundary. With impractically heavy flanges, after first yielding of the web, the capacity increases as a result of the creation of force normal to the boundary by the bending strength of the flange, thereby developing

tension. In the extreme, the capacity of the frame itself in shear is added to the total capacity of the panel.

**Lee and Yoo (1998, 1999):**

They found that even a laterally supported panel (panel simply supported at all the edges) without any external anchors was able to develop post-buckling strength quantitatively close to those observed in tests of ordinary plate girders.

**Lee et al. (2002, 2003):**

They reported that the intermediate transverse stiffeners were not necessarily subjected to the direct compression.

**Lee and Yoo (2006):**

If out-of-plane deflections are restrained along the edges of a rectangular panel by means of simple supports, an external anchor system is not necessary for the development of practically meaningful post buckling strength. This is possible only because the diagonal compression increases near the edges after elastic shear buckling. Diagonal compression continuously increases near the edges of panels after buckling, which is contrary to the fundamental assumption adopted previously. Due to this increase in the diagonal compression, the normal stress perpendicular to the edge (fundamental assumption) is not necessary for equilibrium. Hence, a simply supported panel is able to develop post-buckling strength with no external anchor system. In the post-buckling stage, an increase in the diagonal compression is possible near the simply supported edges because the axial rigidity in the direction of the compression diagonal is not rapidly reduced, as in the center of the web. The axial rigidity in the direction of diagonal compression does not undergo a rapid reduction near a simply supported edge, as the out-of-plane deformations are restrained by the edge. As the intermediate transverse stiffeners are not subjected to the large axial compressive force predicted by the Basler model although they are subjected to some compression by virtue of their continuity with the web, the requirement for the area of the transverse stiffener developed by Basler (1961) is irrelevant. All forces developed during post-buckling are self-equilibrated within the web panel. This means that even end panels can develop.

**Lee and Yoo (2009):**

When two opposite edges of a simply supported panel are provided with rigid anchors that are capable of keeping the edges from moving the panel inwards, an anchoring mechanism can fully develop resulting in an enormous increment of the post buckling strength over the simply supported case. However, it is found that the rigid anchors cannot exert their potential capability unless the edges are laterally supported. This is because large deformations taking place across the laterally unsupported edges trigger a premature loss of out-of-plane stability prior to developing a complete anchoring mechanism. The lateral supports contribute to the post buckling strength in two ways. They not only have their own post buckling mechanism unveiled by Yoo and Lee (2006), but also help the anchoring mechanism develop to its potential capacity by preventing a premature failure caused by out-of-plane instability. Even with the flanges that are heavy enough to function as the rigid anchor, the anchoring mechanism cannot completely develop unless the flanges are supported by incompressible transverse stiffeners. When the transverse stiffeners are incompressible, even ordinary flanges are capable of substantially contributing to the post buckling strength through the anchoring mechanism. The primary reason why the anchoring mechanism by the flanges contributes little to the post buckling strength in ordinary plate girders is that the transverse stiffeners used are axially too flexible to be treated as incompressible. Axial forces developed in the transverse stiffeners attached to ordinary plate girders due to the anchoring mechanism are negligibly small. Utilization of the flange anchoring mechanism in practical designs is beyond the realm of possibility because it requires an unimaginably high axial stiffness of the transverse stiffeners.

**2.4 STIFFENERS:**

In large steel structures, concentrated forces introduced transversely to girders or beams are common. In general, such a concentrated force acting perpendicular to the girder web is denoted as patch load or point load. A simple solution to this problem is to use transverse web stiffeners, which are designed to transform the concentrated force to shear stresses in the web. This solution is reliable if the load is fixed in position as it usually is in buildings, but it is not necessarily the most economical design. As the labor cost keeps increasing more than material costs, there is a trend to avoid manually fitted details, such as

transverse stiffeners, at the expense of making the web thicker. The thickness of the webs depends on the accuracy with which the ultimate strength of the girder web can be estimated.

If the concentrated load is free to move along the girder, it is not possible to solve the problem with vertical stiffeners. This is the case, for instance, for crane and bridge girders during incremental launching. For such cases, it is necessary to design for the concentrated load to be introduced without bearing stiffeners, and good estimates of the ultimate strength are required for safety reasons. In addition, there is a trend in the construction of composite bridges to cast the bridge deck before launching (Dauner et al. 2000). This reduces the cost of the deck but at the same time it substantially increases the weight during launching, and the magnitude of the concentrated load to be resisted increases accordingly.

Specifically in large bridge girders, it is common to use longitudinal stiffeners. The purpose of longitudinal stiffening may be to increase the resistance to concentrated loading but usually they are introduced to increase the resistance to shear and/or bending. In the first case, it is of interest to know where to place the longitudinal stiffener for the best effect. In the second case, the location is governed by other considerations but it is still of interest to account for its effect. Some design procedures (Bergfelt 1979; Janus et al. 1988; Markovic and Hajdin 1992; Kutmanova and Skaloud 1992; BSI 2000) exist but they do not consider all relevant parameters, which unfortunately are many. The purpose of the present investigation is to fill this gap by studying the effects of longitudinal stiffening and to propose an improved design procedure for its consideration.

In general, two parameters are used to describe the capacity of a stiffener to resist buckling, namely the flexural rigidity and the torsional rigidity of the stiffener. These rigidities are expressed relative to the plate stiffness is the flexural rigidity of a unit width of the web plate. Herein, the second moment of area of the longitudinal stiffener is calculated for an effective cross section with respect to its centroidal axis parallel with the web plate.

It was believed (Bergfelt 1979, 1983; Galea et al. 1987; Janus et al. 1988; Markovic and Hajdin 1992) that the most important parameter for longitudinal stiffening for patch loading was the relative position of the stiffener. However, the most recent experimental results reported by Dubas and Tschamper (1990), Carretero and Lebet (1998), and Walbridge and Lebet (2001) have shown a larger influence of the slenderness ratio of the directly loaded panel; i.e., the panel formed between the loaded flange and the stiffener. The

research performed by Dubas and Tschamper (1990), and Walbridge and Lebet (2001) also demonstrated the influence of the type of longitudinal stiffener (open or closed), which can be expressed by the ratio torsional-to-flexural rigidity of the stiffener. A larger torsional rigidity may lead to a larger increase of the ultimate resistance to concentrated loading. But also, an increase only in the flexural rigidity changes the buckling pattern of the web increasing the strength of the girder accordingly.

Graciano (2002) presented a comprehensive review of experimental and numerical studies regarding the influence of longitudinal stiffening on the ultimate strength of plate girder webs subjected to concentrated loads. In addition to the relevance of the parameters mentioned above, the review recognized two principal buckling modes, a global and a local one, of the girder web. In the global mode, the web plate buckles together with the stiffener. The local mode is characterized by buckling of the directly loaded panel.

## **2.5 OBJECTIVE OF PRESENT STUDY:**

As defined by most codes the optimum location of the longitudinal stiffener is not always 0.2 of the width of the plate from the compression flange. So the present study aims at determining an efficient location of vertical stiffener lying in the compressive region during pure bending of the plate depending on the relative proportions of plate and stiffener.

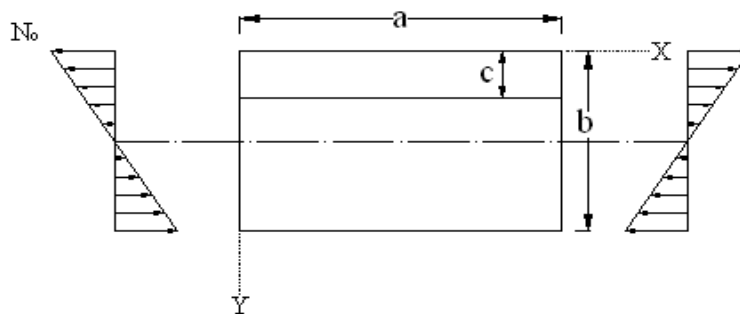
## ANALYSIS FOR OPTIMUM LOCATION OF LONGITUDINAL PLATE STIFFENER

### 3.1 GENERAL:

Stiffened plates are used in plate girders to counteract the effect of stresses occurring due to external loading. Interest in stiffened plate construction has been widespread in recent years due to economic and structural benefits. The advantage of stiffening a plate lies in achieving an economical, light weight design of the structure. While the stiffening elements add negligible weight to the overall structure, their influence on strength and stability is enormous.

Stability of stiffened plates has been a topic of interest for many years. Due to its complexity and the many parameters involved, a complete understanding of all aspects of behavior is not fully realized. Extensive literature has dealt with stability of stiffened plates under compression while limited literature has dealt with the behaviour of stiffened plates under non uniform compression. One of the major difficulties of stiffening plates under non uniform compression, e.g. deep plate girders under pure bending, is the placement of the stiffener, since the compressive forces vary across the plate width. If the plate, for example, is subjected to pure bending, half of the plate width will be subjected to linearly varying compression and the other half to linearly varying tension. If the stiffener is located at the centerline of the plate, the work done by the applied forces on the stiffener will be zero. However this is not the efficient location of the longitudinal stiffener. Another alternative location of the longitudinal stiffener is within the compression region.

### 3.2 OPTIMUM LOCATION OF LONGITUDINAL GIRDER USING ENERGY METHOD:



Consider a simply supported rectangular plate having length 'a' and width 'b' with longitudinal stiffener at a distance 'c' from y=0 and along whose sides x=0 and x=a distributed forces, acting in the middle plane of the plate, are applied, their intensity being given by the equation

$$N_x = N_o \left(1 - \alpha \frac{y}{b}\right)$$

where  $N_o$  is the intensity of compressive force acting at the edge  $y=0$  and  $\alpha$  is numerical factor. By changing  $\alpha$ , we can obtain various particular cases. If  $\alpha=0$ , we obtain the case of uniformly distributed compressive force. For  $\alpha < 2$ , there will be combination of bending and tension and for  $\alpha > 2$ , there will be a similar combination of bending and compression.

The deflection of buckled plate simply supported on all sides can be taken in the form of the double trigonometric series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad \dots(1)$$

The corresponding strain energy of bending of the plate is

$$\Delta U_p = \frac{\pi^4 D ab}{2 \cdot 4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 \quad \dots(2)$$

Assuming a general case of several longitudinal ribs and denoting by  $EI$  the flexural rigidity of a rib at the distance  $c$  from the edge  $y=0$ , we find that the strain energy of bending of the rib, when buckled together with the plate, is

$$\begin{aligned} \Delta U_s &= \frac{EI}{2} \int_0^a \left( \frac{\partial^2 w}{\partial x^2} \right)_{y=c}^2 dx \\ \Delta U_s &= \frac{\pi^4 EI}{4a^3} \sum_{m=1}^{\infty} m^4 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \quad \dots(3) \end{aligned}$$

The work done during buckling by the compressive forces  $N_x$  acting on the plate is

$$\Delta T_p = \frac{1}{2} \int_0^a \int_0^b N_o \left(1 - \alpha \frac{y}{b}\right) \left( \frac{\partial w}{\partial x} \right)^2 dx dy \quad \dots(4)$$

Substituting the expression for  $w$  from equation (1) and observing that

$$\begin{aligned} \int_0^b y \sin \frac{i\pi y}{b} \sin \frac{j\pi y}{b} dy &= \frac{b^2}{4} && \text{for } i=j \\ \int_0^b y \sin \frac{i\pi y}{b} \sin \frac{j\pi y}{b} dy &= 0 && \text{for } i \neq j \text{ and } i \pm j \text{ an even number} \end{aligned}$$

$$\int_0^b y \sin \frac{i\pi y}{b} \sin \frac{j\pi y}{b} dy = -\frac{4b^2}{\pi^2} \frac{ij}{(i^2-j^2)^2} \quad \text{for } i \neq j \text{ an odd number}$$

Thus the work done by external forces during buckling is given by

$$\Delta T_p = \frac{N_o ab}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \frac{m^2 \pi^2}{a^2} - \frac{N_o \alpha a}{2} \sum_{m=1}^{\infty} \frac{m^2 \pi^2}{a^2} \left[ \frac{b^2}{4} \sum_{n=1}^{\infty} a_{mn}^2 - \frac{8b^2}{\pi^2} \sum_{n=1}^{\infty} \sum_i \frac{a_{mn} a_{mi} n i}{(n^2 - i^2)^2} \right] \quad \dots\dots(5)$$

where for i only such numbers are taken that n±i is always odd.

The work done during buckling by compressive force P acting on a stiffener is

$$\Delta T_s = \frac{P}{2} \int_0^a \left( \frac{\partial w}{\partial x} \right)_{y=c}^2 dx = \frac{P \pi^2 a}{2 a^2} \sum_{m=1}^{\infty} m^2 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \quad \dots\dots(6)$$

The problem of calculating the optimum location can be classified into two cases:

1. When the stiffener has sufficient out of plane rigidity to force the plate to buckle locally and form the nodal line.
2. If the stiffener buckles with the plate in an overall mode. (special case of case-1)

**Case-1. When the stiffener has sufficient out of plane rigidity to force the plate to buckle locally and form the nodal line.**

For this case the general equation for calculating critical stress is

$$\begin{aligned} \Delta U_p + \Delta U_s &= \Delta T_p + \Delta T_s \\ \frac{\pi^4 D ab}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 + \frac{\pi^4 EI}{4a^3} \sum_{m=1}^{\infty} m^4 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \\ &= \frac{N_o ab}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \frac{m^2 \pi^2}{a^2} - \frac{N_o \alpha a}{2} \sum_{m=1}^{\infty} \frac{m^2 \pi^2}{a^2} \left[ \frac{b^2}{4} \sum_{n=1}^{\infty} a_{mn}^2 - \frac{8b^2}{\pi^2} \sum_{n=1}^{\infty} \sum_i \frac{a_{mn} a_{mi} n i}{(n^2 - i^2)^2} \right] \\ &+ \frac{P \pi^2 a}{2 a^2} \sum_{m=1}^{\infty} m^2 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \quad \dots\dots(7) \end{aligned}$$

Using the notations

$$\frac{a}{b} = \beta; \quad \frac{EI}{bD} = \gamma; \quad \frac{P}{bN_x} = \frac{A_s}{bh} = \delta; \quad \frac{(N_o)_{cr}}{h} = \sigma_{cr}$$

where bh is the cross sectional area and A<sub>s</sub> that of the stiffener.



$$\begin{aligned}
& \frac{\pi^2 D}{b^2 h \beta^2} \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} (m^2 + n^2 \beta^2)^2 + 2\gamma \sum_{m=1}^{\infty} m^4 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \right\} \\
& = \sigma_{cr} \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 m^2 - \frac{\alpha}{2} \sum_{m=1}^{\infty} m^2 \left[ \sum_{n=1}^{\infty} a_{mn}^2 - \frac{32}{\pi^2} \sum_{n=1}^{\infty} \sum_i \frac{a_{mn} a_{mi} n i}{(n^2 - i^2)^2} \right] \right. \\
& \quad \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \sum_{m=1}^{\infty} m^2 \left( a_{m1} \sin \frac{\pi c}{b} + a_{m2} \sin \frac{2\pi c}{b} + \dots \right)^2 \right\} \\
& \frac{\pi^2 D}{b^2 h \beta^2} \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} (m^2 + n^2 \beta^2)^2 + 2\gamma \sum_{m=1}^{\infty} m^4 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right)^2 \right\} \\
& = \sigma_{cr} \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 m^2 - \frac{\alpha}{2} \sum_{m=1}^{\infty} m^2 \left[ \sum_{n=1}^{\infty} a_{mn}^2 - \frac{32}{\pi^2} \sum_{n=1}^{\infty} \sum_i \frac{a_{mn} a_{mi} n i}{(n^2 - i^2)^2} \right] \right. \\
& \quad \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \sum_{m=1}^{\infty} m^2 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right)^2 \right\}
\end{aligned}$$

$\sigma_{cr} =$

$$\frac{\pi^2 D}{b^2 h \beta^2} \frac{\left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} (m^2 + n^2 \beta^2)^2 + 2\gamma \sum_{m=1}^{\infty} m^4 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right)^2 \right\}}{\left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 m^2 - \frac{\alpha}{2} \sum_{m=1}^{\infty} m^2 \left[ \sum_{n=1}^{\infty} a_{mn}^2 - \frac{32}{\pi^2} \sum_{n=1}^{\infty} \sum_i \frac{a_{mn} a_{mi} n i}{(n^2 - i^2)^2} \right] \right.} \dots \dots (8)$$

$$\left. \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \sum_{m=1}^{\infty} m^2 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right)^2 \right\} \right.$$

Equating to zero the derivatives of this expression with respect to  $a_{mn}$  we obtain a system of homogeneous linear equations as below:

$$\begin{aligned}
& \frac{\pi^2 D}{b^2 h} \left\{ a_{mn} (m^2 + n^2 \beta^2)^2 + 2\gamma m^4 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right) \right\} \\
& - \beta^2 \sigma_{cr} \left\{ a_{mn} m^2 - \frac{\alpha}{2} m^2 \left[ a_{mn} - \frac{16}{\pi^2} \sum_i \frac{a_{mi} n i}{(n^2 - i^2)^2} \right] + 2\delta \left( 1 - \alpha \frac{c}{b} \right) m^2 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right) \right\} \\
& = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\pi^2 D}{b^2 h} \left\{ a_{mn} (m^2 + n^2 \beta^2)^2 + 2\gamma m^4 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right) \right\} \\
& - \beta^2 \sigma_{cr} \left\{ a_{mn} m^2 - \frac{\alpha}{2} m^2 \left[ a_{mn} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{mi} n i}{(n^2 - i^2)^2} \right] \right. \\
& \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) m^2 \left( \sum_{p=1}^{\infty} a_{mp} \sin \frac{p\pi c}{b} \right) \right\} = 0 \quad \dots \dots (9)
\end{aligned}$$

These are the homogeneous linear equations in  $a_{11}, a_{12}, a_{13}, \dots$  which will be satisfied by putting  $a_{11}, a_{12}, a_{13}, \dots$  equal to zero, which corresponds to flat form of equilibrium of the plate. To get for the coefficients  $a_{11}, a_{12}, a_{13}, \dots$  solutions different from zero, which indicates the possibility of buckling of the plate, the determinant of the equations (9) must be zero. In this way an equation for calculating the critical values of compressive stresses is obtained. The calculation can be made by successive approximations. We begin by taking all the coefficients except  $a_{11}, a_{12}, a_{13}, a_{14}, a_{15}$  as zero.

Assuming that the reinforced plate buckles into one half-wave and we take  $m=1$ .

Putting  $\frac{\sigma_{cr} b^2 h}{\pi^2 D} = k$ ; equation (9) changes to

$$\begin{aligned}
& \left\{ a_{1n} (1 + n^2 \beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\
& - \beta^2 k \left\{ a_{1n} - \frac{\alpha}{2} \left[ a_{1n} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{1i} n i}{(n^2 - i^2)^2} \right] + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\
& = 0 \quad \dots \dots (10)
\end{aligned}$$

**Putting n=1**

$$\left\{ a_{11} (1 + \beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{11} - \frac{\alpha}{2} \left[ a_{11} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{1i} i}{(1-i^2)^2} \right] + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ = 0$$

$$\left\{ a_{11} (1 + \beta^2)^2 + 2\gamma \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} + a_{15} \sin \frac{5\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{11} - \frac{\alpha}{2} \left[ a_{11} - \frac{16}{\pi^2} \left( \frac{a_{12}^2}{(1-i^2)^2} + \frac{a_{14}^4}{(1-i^2)^2} \right) \right] \right\} \\ + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} \right. \\ \left. + a_{15} \sin \frac{5\pi c}{b} \right) \left. \right\} = 0$$

$$a_{11} \left[ (1 + \beta^2)^2 + 2\gamma \sin \frac{\pi c}{b} - \beta^2 k + \beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{\pi c}{b} \right] \\ + a_{12} \left[ 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{16\alpha}{9\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{2\pi c}{b} \right] \\ + a_{13} \left[ 2\gamma \sin \frac{3\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{3\pi c}{b} \right] \\ + a_{14} \left[ 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{32\alpha}{225\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{4\pi c}{b} \right] \\ + a_{15} \left[ 2\gamma \sin \frac{5\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{5\pi c}{b} \right] \\ = 0 \quad \dots\dots (11)$$

**Putting n=2**

$$\left\{ a_{12} (1 + 4\beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{12} - \frac{\alpha}{2} \left[ a_{12} - \frac{16}{\pi^2} \sum_i \frac{a_{1i} 2i}{(n^2 - i^2)^2} \right] + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ = 0$$

$$\left\{ a_{12} (1 + 4\beta^2)^2 + 2\gamma \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} + a_{15} \sin \frac{5\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{12} - \frac{\alpha}{2} \left[ a_{12} - \frac{16}{\pi^2} \left[ \frac{a_{11} 2}{9} + \frac{a_{13} 6}{25} + \frac{a_{15} 10}{441} \right] \right] \right. \\ \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} \right. \right. \\ \left. \left. + a_{15} \sin \frac{5\pi c}{b} \right) \right\} = 0$$

$$a_{11} \left[ 2\gamma \sin \frac{\pi c}{b} - \beta^2 k \frac{16\alpha}{9\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{\pi c}{b} \right] \\ + a_{12} \left[ (1 + 4\beta^2)^2 + 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k + \beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{2\pi c}{b} \right] \\ + a_{13} \left[ 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k \frac{48\alpha}{25\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{3\pi c}{b} \right] \\ + a_{14} \left[ 2\gamma \sin \frac{4\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{4\pi c}{b} \right] \\ + a_{15} \left[ 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k \frac{80\alpha}{441\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{5\pi c}{b} \right] \\ = 0 \quad \dots \dots (12)$$

**Putting n=3**

$$\left\{ a_{13} (1 + 9\beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{13} - \frac{\alpha}{2} \left[ a_{13} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{1i} 3i}{(9-i^2)^2} \right] + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ = 0$$

$$\left\{ a_{13} (1 + 9\beta^2)^2 + 2\gamma \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} + a_{15} \sin \frac{5\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{13} - \frac{\alpha}{2} \left[ a_{13} - \frac{16}{\pi^2} \left( \frac{a_{12} 6}{25} + \frac{a_{14} 12}{49} \right) \right] \right. \\ \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} \right. \right. \\ \left. \left. + a_{15} \sin \frac{5\pi c}{b} \right) \right\} = 0$$

$$a_{11} \left[ 2\gamma \sin \frac{\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{\pi c}{b} \right] \\ + a_{12} \left[ 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{48\alpha}{25\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{2\pi c}{b} \right] \\ + a_{13} \left[ (1 + 9\beta^2)^2 + 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k + \beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{3\pi c}{b} \right] \\ + a_{14} \left[ 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{96\alpha}{49\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{4\pi c}{b} \right] \\ + a_{15} \left[ 2\gamma \sin \frac{5\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{5\pi c}{b} \right] \\ = 0 \quad \dots \dots (13)$$

**Putting n=4**

$$\left\{ a_{14} (1 + 16\beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{14} - \frac{\alpha}{2} \left[ a_{14} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{1i} 4i}{(16 - i^2)^2} \right] \right. \\ \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} = 0$$

$$\left\{ a_{14} (1 + 16\beta^2)^2 \right. \\ \left. + 2\gamma \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} + a_{15} \sin \frac{5\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{14} - \frac{\alpha}{2} \left[ a_{14} - \frac{16}{\pi^2} \left( \frac{a_{11} 4}{225} + \frac{a_{13} 12}{49} + \frac{a_{15} 20}{81} \right) \right] \right. \\ \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} \right. \right. \\ \left. \left. + a_{15} \sin \frac{5\pi c}{b} \right) \right\} = 0$$

$$a_{11} \left[ 2\gamma \sin \frac{\pi c}{b} - \beta^2 k \frac{32\alpha}{225\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{\pi c}{b} \right] \\ + a_{12} \left[ 2\gamma \sin \frac{2\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{2\pi c}{b} \right] \\ + a_{13} \left[ 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k \frac{96\alpha}{49\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{3\pi c}{b} \right] \\ + a_{14} \left[ (1 + 16\beta^2)^2 + 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k + \beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{4\pi c}{b} \right] \\ + a_{15} \left[ 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k \frac{160\alpha}{81\pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{5\pi c}{b} \right] \\ = 0 \quad \dots \dots (14)$$

Putting  $n=5$

$$\left\{ a_{15} (1 + 25\beta^2)^2 + 2\gamma \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right. \\ \left. - \beta^2 k \left\{ a_{15} - \frac{\alpha}{2} \left[ a_{15} - \frac{16}{\pi^2} \sum_i^{\infty} \frac{a_{1i} 5i}{(25 - i^2)^2} \right] \right. \right. \\ \left. \left. + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( \sum_{p=1}^{\infty} a_{1p} \sin \frac{p\pi c}{b} \right) \right\} \right\} = 0$$

$$\left\{ a_{15} (1 + 25\beta^2)^2 \right. \\ \left. + 2\gamma \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} + a_{15} \sin \frac{5\pi c}{b} \right) \right\} \\ - \beta^2 k \left\{ a_{15} - \frac{\alpha}{2} \left[ a_{15} - \frac{16}{\pi^2} \left( \frac{a_{12} 10}{441} + \frac{a_{14} 20}{81} \right) \right] \right\} \\ + 2\delta \left( 1 - \alpha \frac{c}{b} \right) \left( a_{11} \sin \frac{\pi c}{b} + a_{12} \sin \frac{2\pi c}{b} + a_{13} \sin \frac{3\pi c}{b} + a_{14} \sin \frac{4\pi c}{b} \right. \\ \left. + a_{15} \sin \frac{5\pi c}{b} \right) \left. \right\} = 0$$

$$a_{11} \left[ 2\gamma \sin \frac{\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{\pi c}{b} \right] \\ + a_{12} \left[ 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{80 \alpha}{441 \pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{2\pi c}{b} \right] \\ + a_{13} \left[ 2\gamma \sin \frac{3\pi c}{b} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{3\pi c}{b} \right] \\ + a_{14} \left[ 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{160 \alpha}{81 \pi^2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{4\pi c}{b} \right] \\ + a_{15} \left[ (1 + 25\beta^2)^2 + 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k + \beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left( 1 - \alpha \frac{c}{b} \right) \sin \frac{5\pi c}{b} \right] \\ = 0 \quad \dots \dots (15)$$

Taking equations (11) to (15) and equating their determinant to zero, we get

$$\begin{vmatrix}
 \begin{bmatrix} (1 + \beta^2)^2 + 2\gamma \sin \frac{\pi c}{b} - \beta^2 k \\ +\beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{16\alpha}{9\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{2\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{3\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{3\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{32\alpha}{225\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{4\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{5\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{5\pi c}{b} \end{bmatrix} \\
 \begin{bmatrix} 2\gamma \sin \frac{\pi c}{b} - \beta^2 k \frac{16\alpha}{9\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{\pi c}{b} \end{bmatrix} & \begin{bmatrix} (1 + 4\beta^2)^2 + 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \\ +\beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{2\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k \frac{48\alpha}{25\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{3\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{4\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{4\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k \frac{80\alpha}{441\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{5\pi c}{b} \end{bmatrix} \\
 \begin{bmatrix} 2\gamma \sin \frac{\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{48\alpha}{25\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{2\pi c}{b} \end{bmatrix} & \begin{bmatrix} (1 + 9\beta^2)^2 + 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k \\ +\beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{3\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{96\alpha}{49\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{4\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{5\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{5\pi c}{b} \end{bmatrix} \\
 \begin{bmatrix} 2\gamma \sin \frac{\pi c}{b} - \beta^2 k \frac{32\alpha}{225\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{2\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{2\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{3\pi c}{b} - \beta^2 k \frac{96\alpha}{49\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{3\pi c}{b} \end{bmatrix} & \begin{bmatrix} (1 + 16\beta^2)^2 + 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \\ +\beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{4\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k \frac{160\alpha}{81\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{5\pi c}{b} \end{bmatrix} \\
 \begin{bmatrix} 2\gamma \sin \frac{\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{2\pi c}{b} - \beta^2 k \frac{80\alpha}{441\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{2\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{3\pi c}{b} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{3\pi c}{b} \end{bmatrix} & \begin{bmatrix} 2\gamma \sin \frac{4\pi c}{b} - \beta^2 k \frac{160\alpha}{81\pi^2} \\ -2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{4\pi c}{b} \end{bmatrix} & \begin{bmatrix} (1 + 25\beta^2)^2 + 2\gamma \sin \frac{5\pi c}{b} - \beta^2 k \\ +\beta^2 k \frac{\alpha}{2} - 2\beta^2 k \delta \left(1 - \alpha \frac{c}{b}\right) \sin \frac{5\pi c}{b} \end{bmatrix}
 \end{vmatrix} = 0$$

--- (16)



The equation obtained above has following variables- $\alpha$ ,  $\delta$ ,  $\beta$ ,  $\gamma$ ,  $c/b$  and  $k$ . To find the exact location of the stiffener from the above equation, maximum value of 'k' in reference to the ratio 'c/b' is arrived at. It is not possible to solve the above equation in one go, as it has mathematical complications. These complications result because of availability of only 2D and 3D plots at our disposal to check the variations.

The above equation also reflects a  $5 \times 5$  matrix which is arranged in a way that it can be converted into smaller square matrices and increasing the matrix size after  $2 \times 2$  leads to same result, that is, the optimum location remains the same.

To find the exact location of longitudinal stiffener variation of  $k$  and  $c/b$  are compared in view of a third variable and thereby inferences are drawn. This can be done either by plotting a 2D graph between  $k$  and  $c/b$  keeping values of other variables fixed in steps, making it cumbersome and lengthy process, or plot a 3D graph between three variables and keeping other variables fixed. The 3D graph can be more efficiently prepared using latest software like MATLAB which helps in drawing these graphs easily and further helps in the solution of cumbersome equations.

This study deals with only pure bending case i.e.,  $\alpha=2$ . The transitions from pure bending case to pure compression case need to be taken in future studies as and when required.

### **3.3 PLOTS BETWEEN 'k', 'c/b' AND 'β' :**

Taking  $\gamma=10$  and  $\delta=0.1$ , the implications of variation of 'k' and 'c/b' taking the plate aspect ratio 'β' into consideration is observed.

#### **A. Plot of 1x1 matrix:**

In the formulation of  $1 \times 1$  matrix it is assumed that the plate buckles in one half wave with nodal line parallel in the x-direction and one half wave in the y-direction. The 3D plot showing the variation between the three quantities 'k' 'c/b' and 'β' is shown in figures 3.1, 3.1a, 3.1b, 3.1c.

#### **Inference:**

The above variation plots show the 3D plot for 'k' 'c/b' and 'β' and the various planar views for clarity and easy understanding. From above figures it can be seen that maximum compression occurs in the zone of aspect ratio less than 1. It attains a minimum at  $a/b=1$  and thereafter increases with the aspect ratio. It is also clear that maximum compressive stresses occurs at  $c/b=0.5$ . This means that for plates in one half buckling mode and of small aspect ratio we can provide the longitudinal stiffener at the middle.

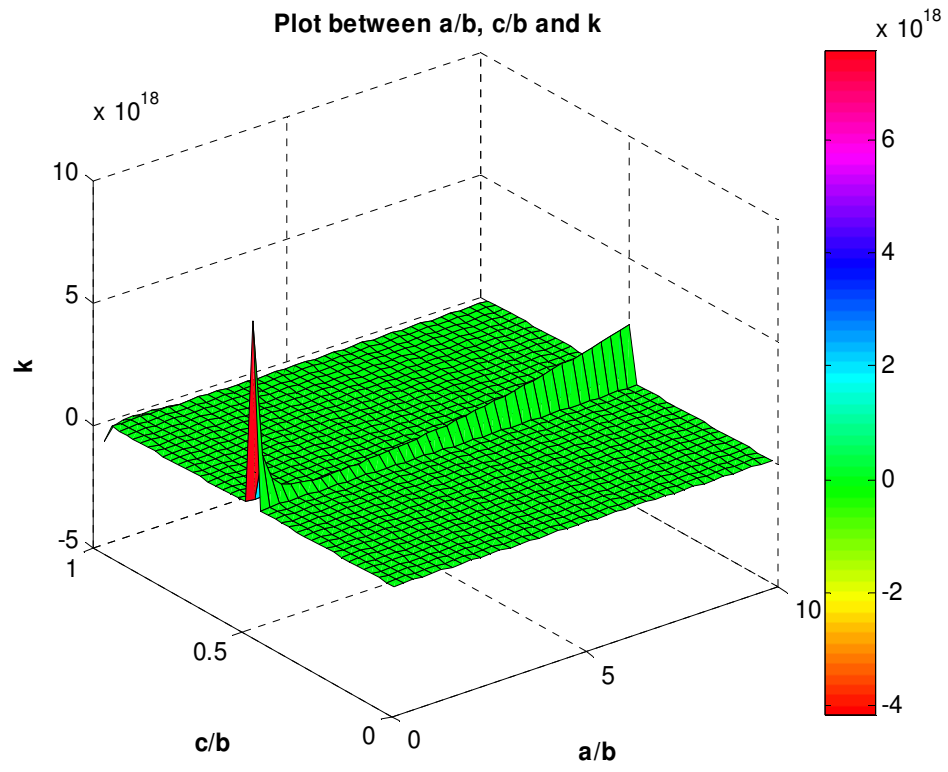


Fig. 3.1

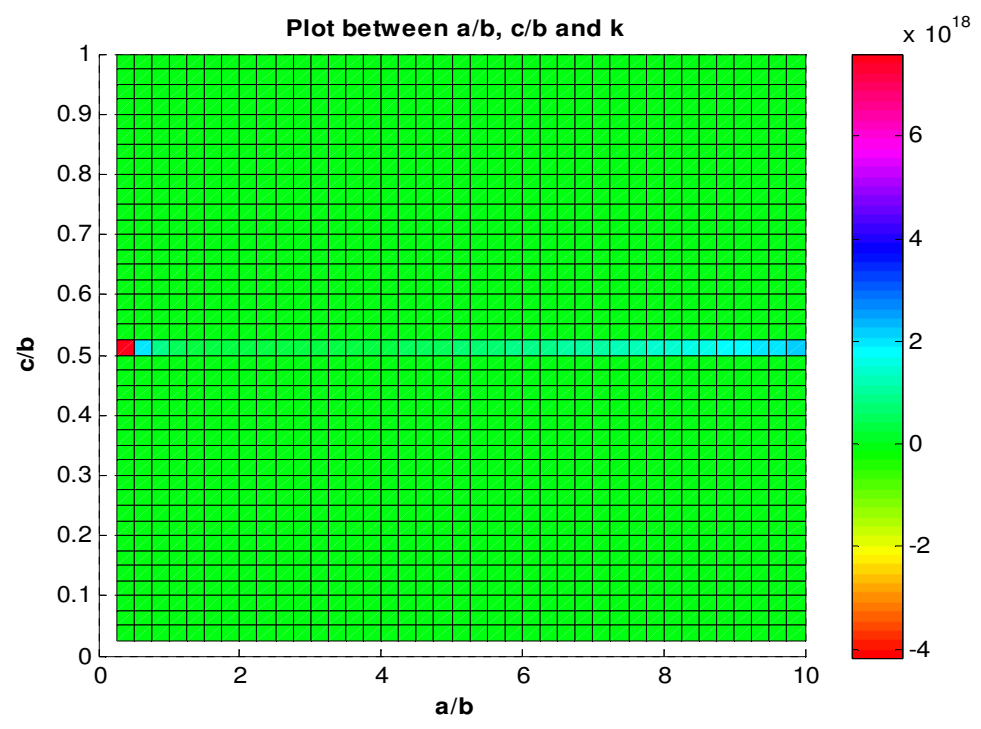


Fig.3.1a

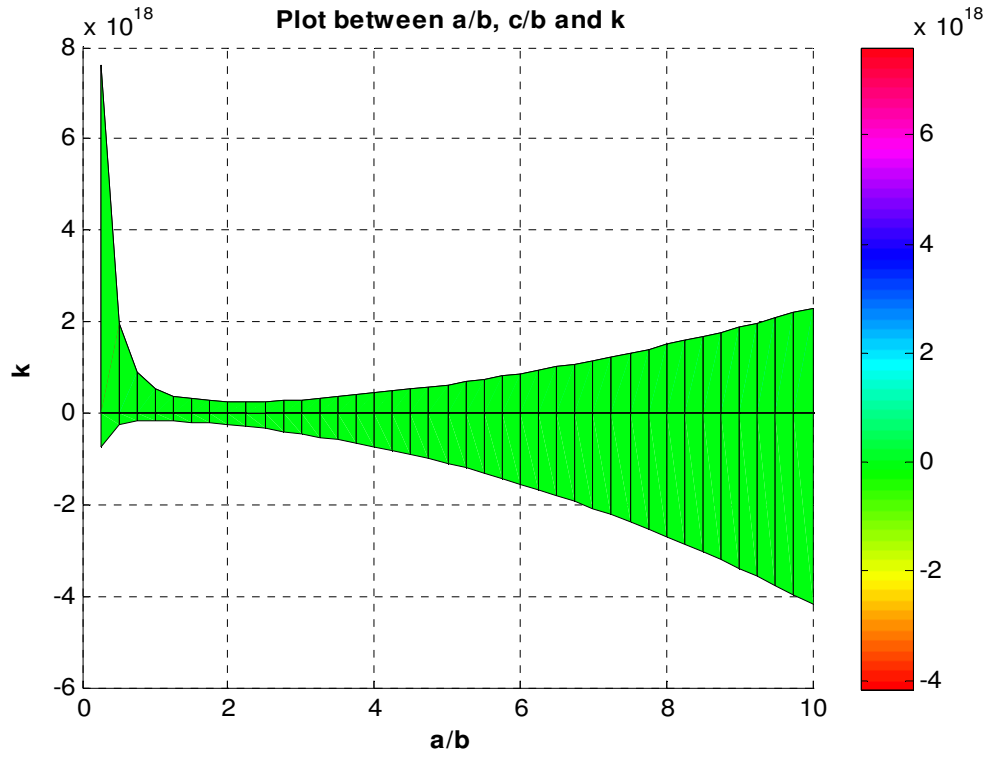


Fig.3.1b

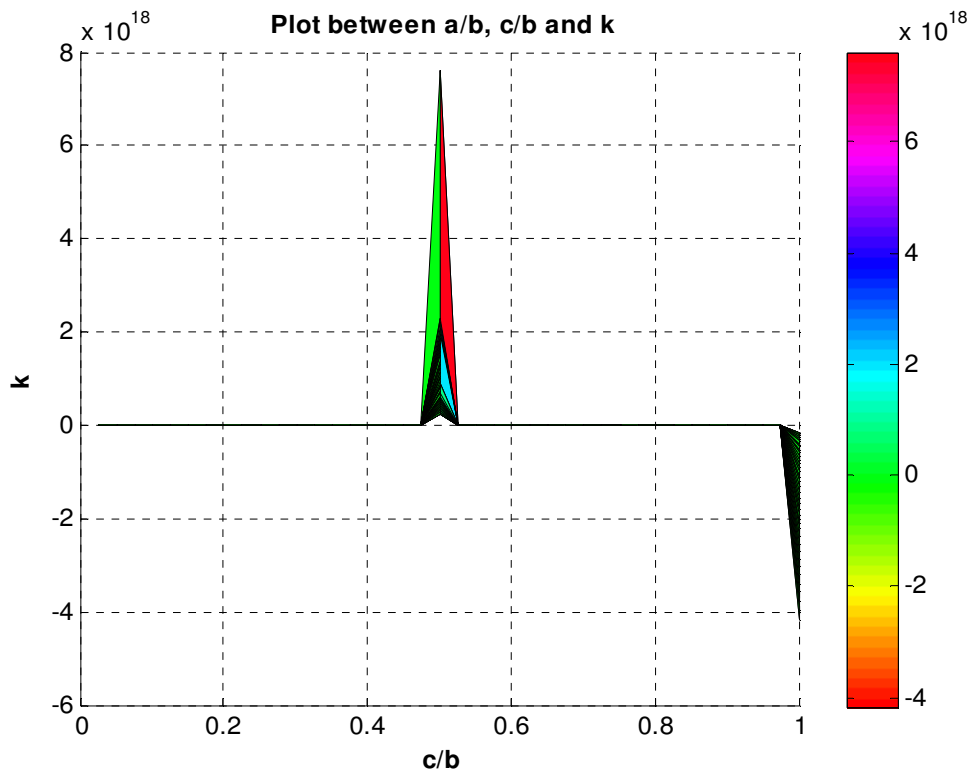


Fig. 3.1c

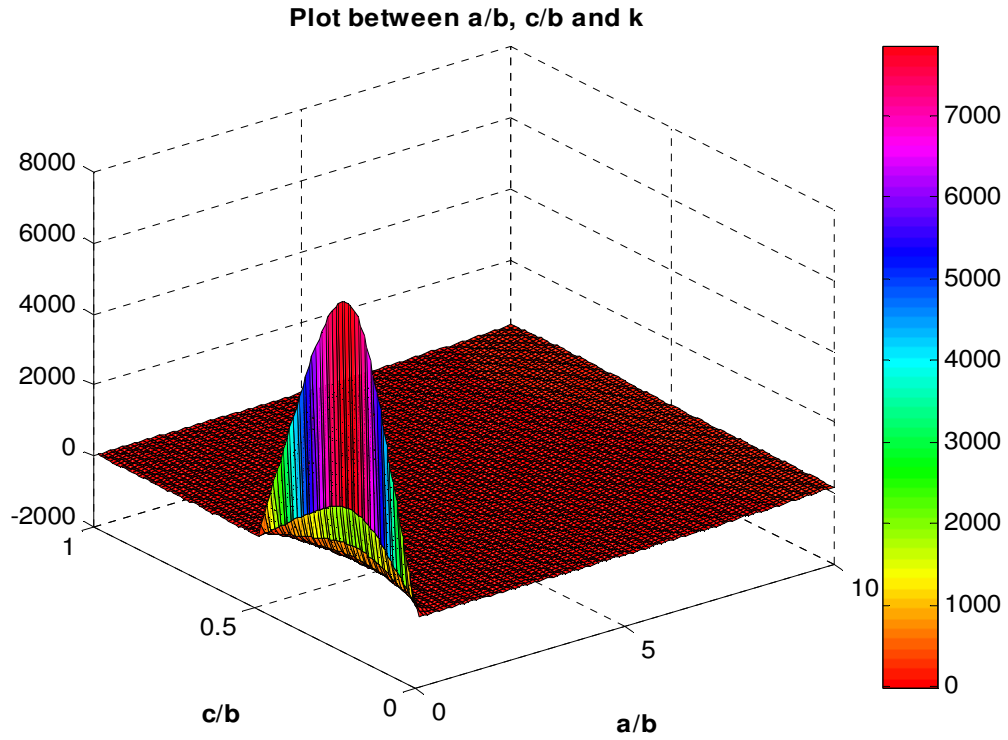
## **B. Plot of 2×2 matrix:**

The second case is of 2×2 matrix. In the formulation of this matrix it is assumed that the plate buckles in two half waves with nodal line parallel in the x-direction and two half waves in the y-direction. The 3D plot showing the variation between the three quantities 'k', 'c/b' and 'β' is shown in figures 3.2, 3.2a, 3.2b and 3.2c.

### **Inference:**

The above variation plots show the 3D plot for 'k', 'c/b' and 'β' and the various planar views for clarity and easy understanding. From above figures it is seen that maximum compression occurs in the zone of aspect ratio less than 1. It attains a minimum at  $a/b=1$  and thereafter increases with the aspect ratio. It is also clear that maximum compressive stresses occurs at  $c/b=0.21$ . This means that for plates in one half buckling mode and of small aspect ratio we can provide the longitudinal stiffener at 0.21b.

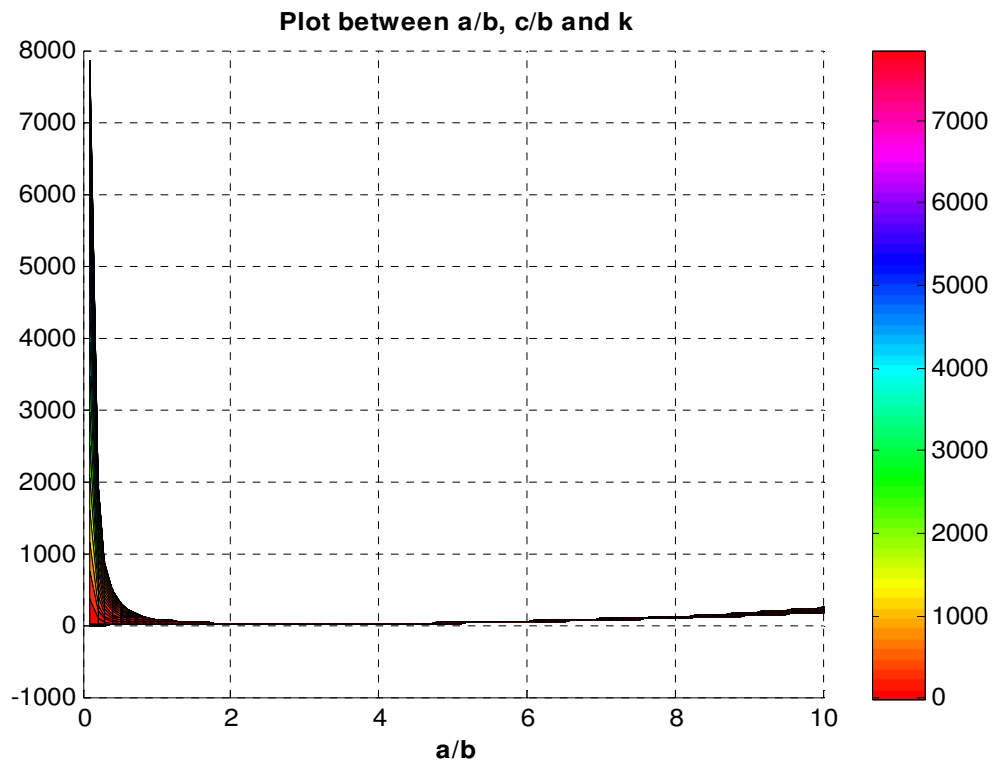
The above two inferences are the as a result of constant values of  $\gamma$  and  $\delta$ . Also it can be concluded that at aspect ratio of 1, the compressive stresses are minimum. For lower aspect ratios, the compressive stresses increase at a rapid rate, while as the aspect ratio increases from 1, the compressive stresses again starts increasing. Thus we fix one of the variables viz., aspect ratio,  $a/b$  as 1 i.e., assuming square web panel.



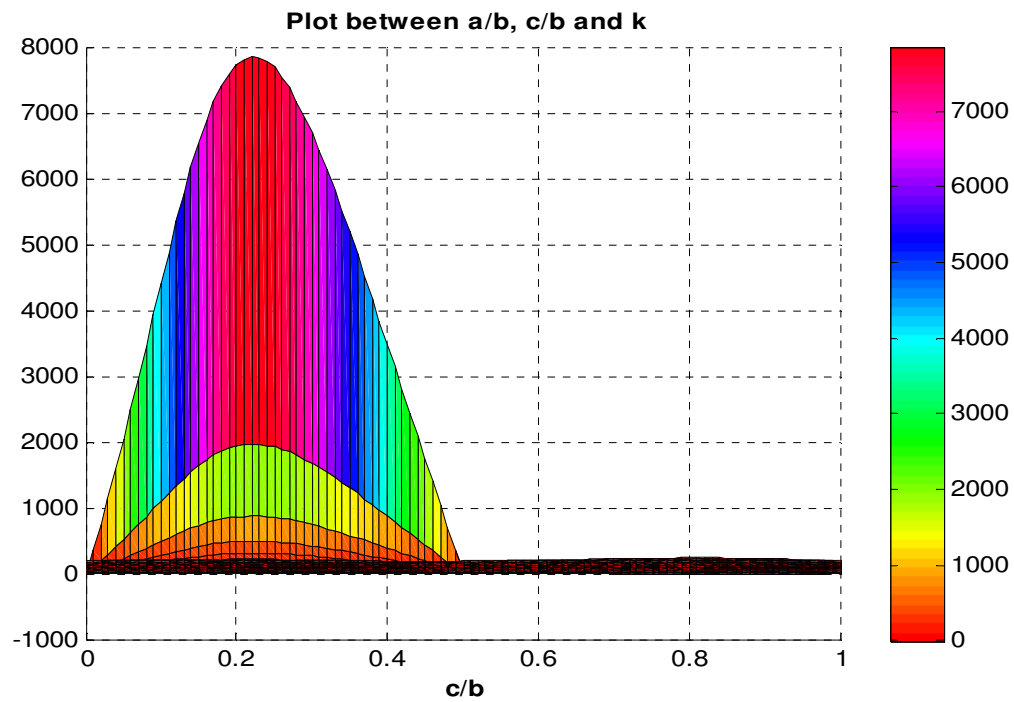
**Fig.3.2**



**Fig.3.2a**



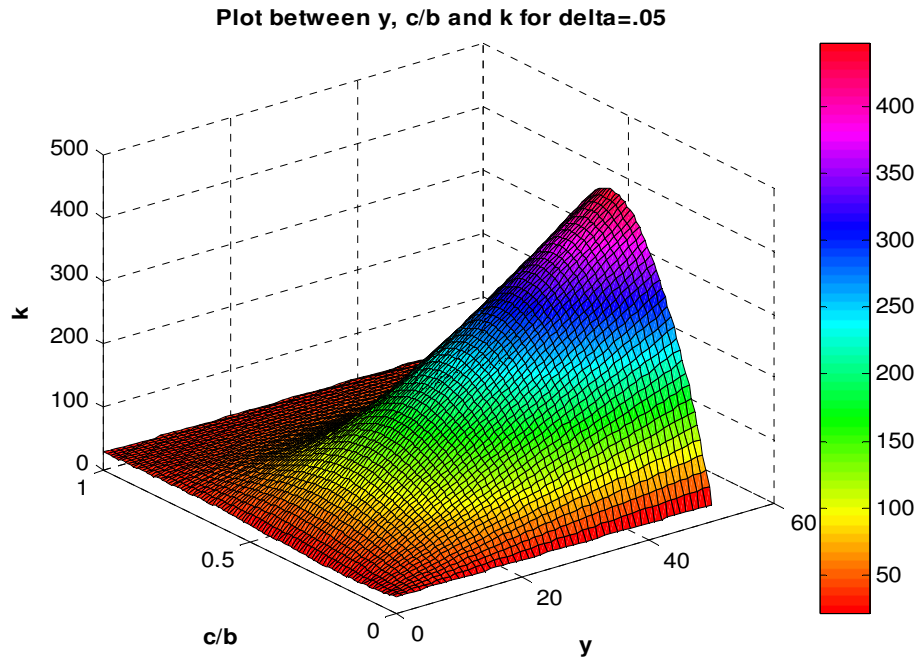
**Fig.3.2b**



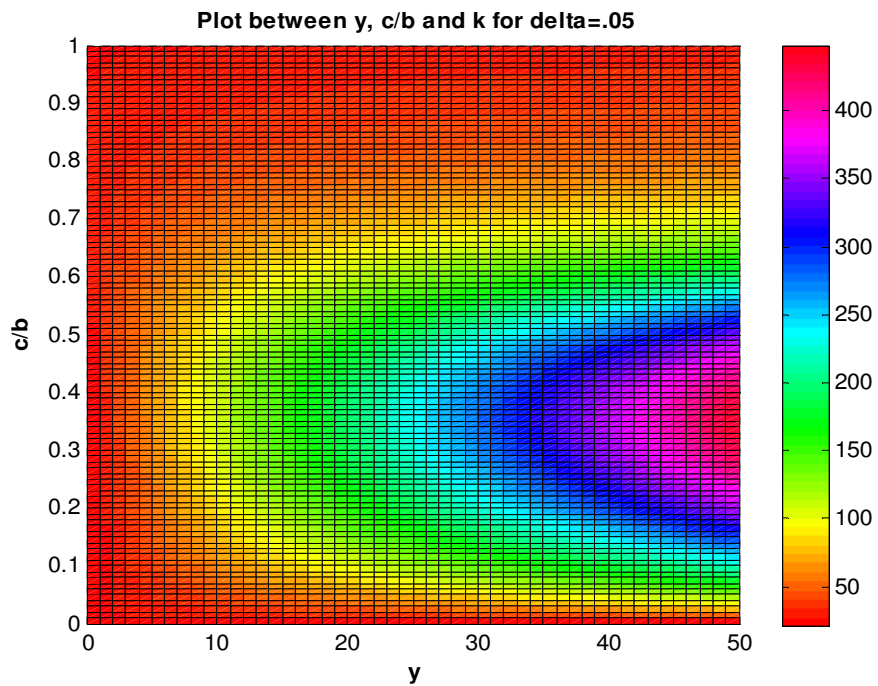
**Fig.3.2c**

### 3.4 PLOT BETWEEN 'k', 'c/b' AND 'γ' :

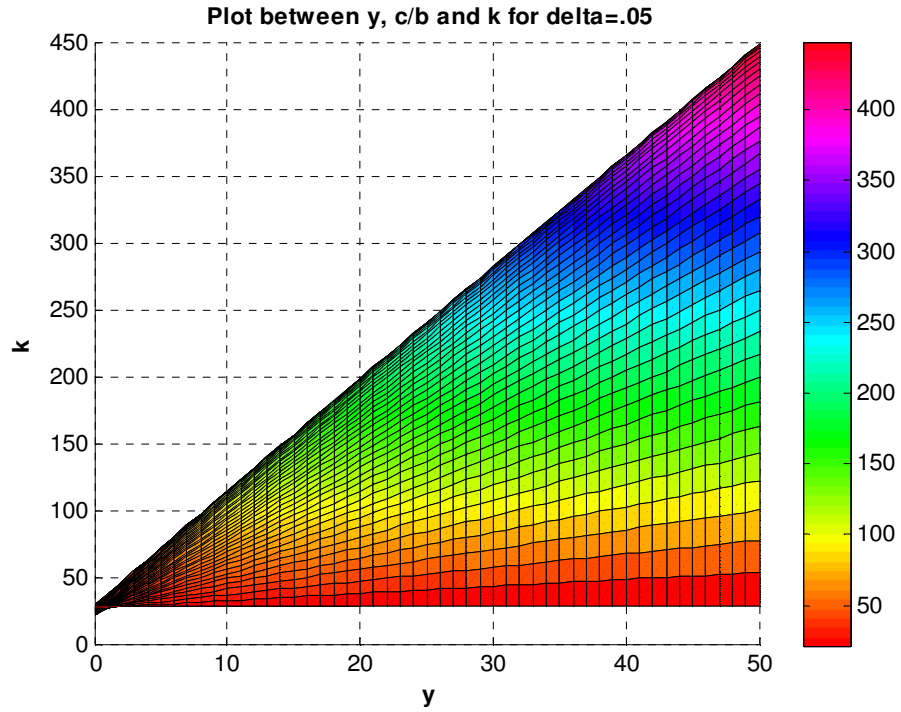
Since the second approximation gives a greater accuracy, we will carry further calculations with the  $2 \times 2$  matrix system. The plot showing the variation between 'k', c/b and 'γ' considering  $a/b=1$  and  $\delta=0.05$  is shown in figures 3.3, 3.3a, 3.3b and 3.3c.



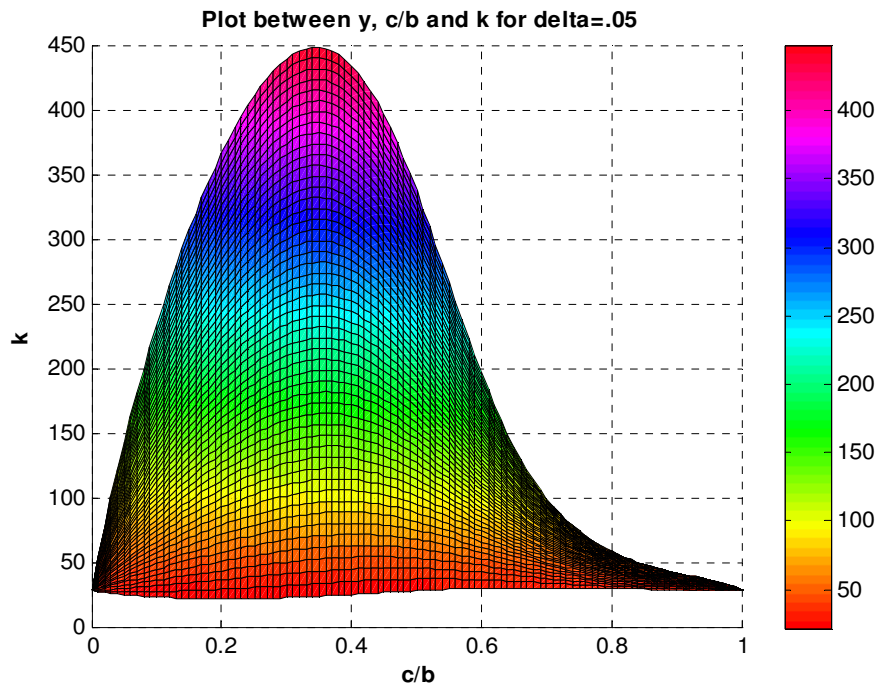
**Fig.3.3**



**Fig.3.3a**



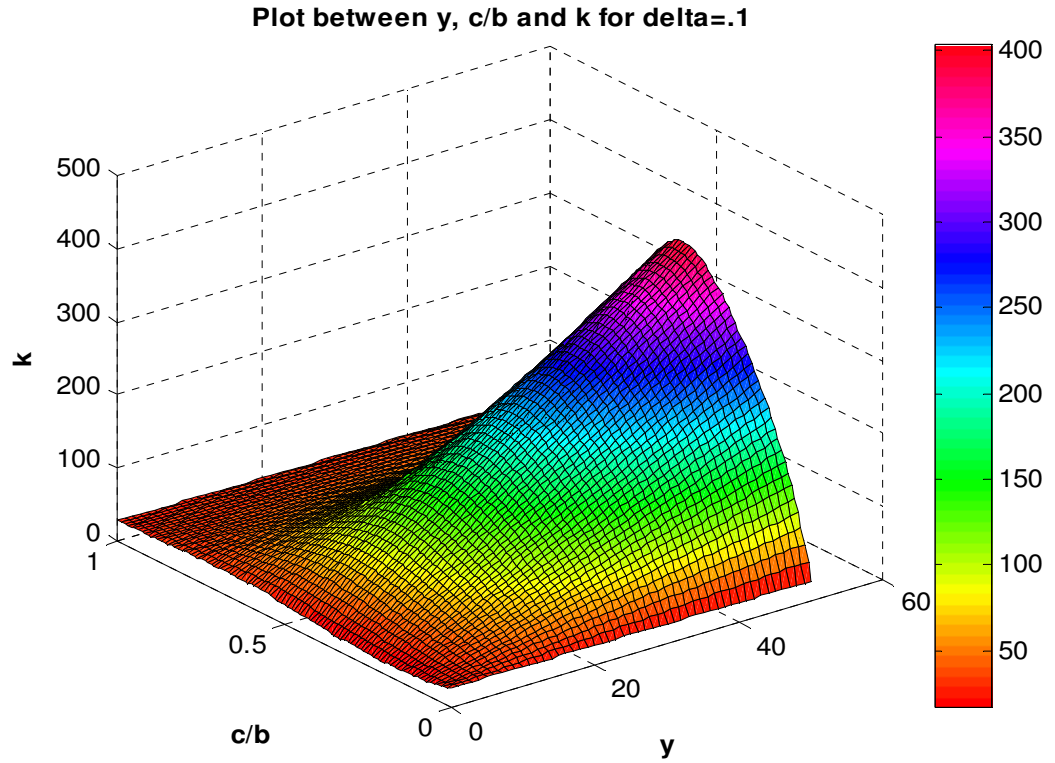
**Fig.3.3b**



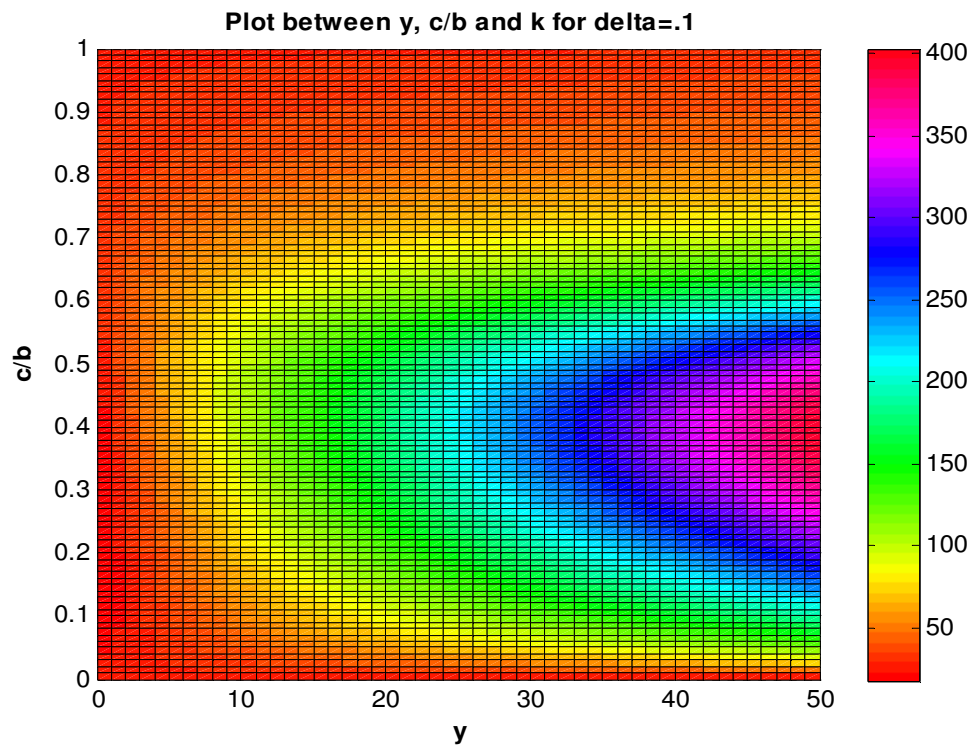
**Fig.3.3c**

The plot showing the variation between ' $k$ ',  $c/b$  and ' $\gamma$ ' considering  $a/b=1$  and  $\delta=0.10$  is shown in figures 3.4, 3.4a, 3.4b and 3.4c.

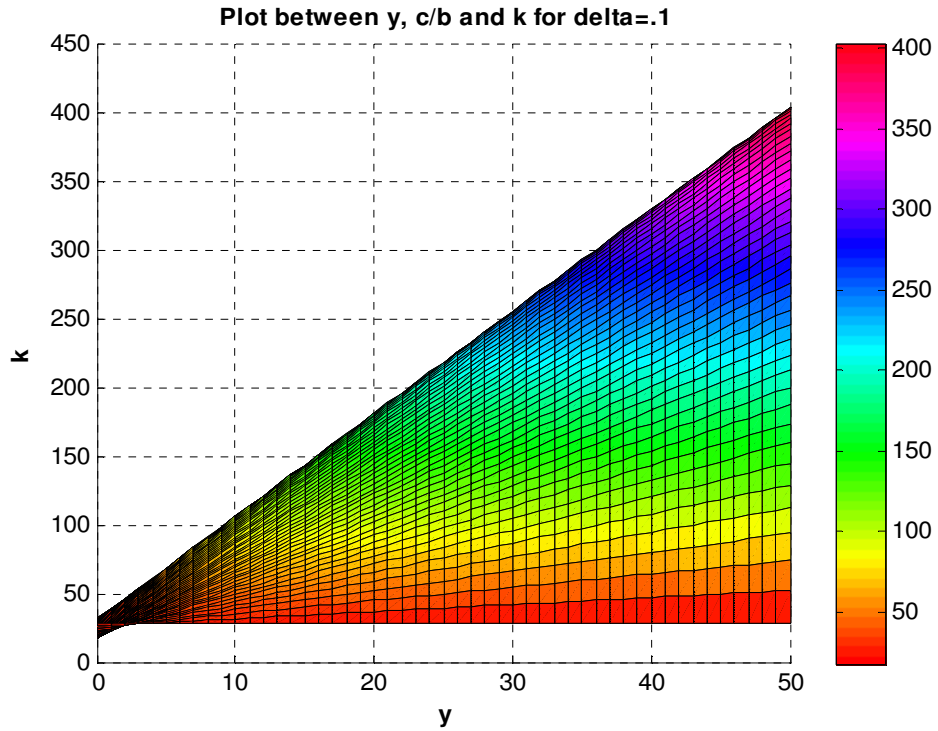




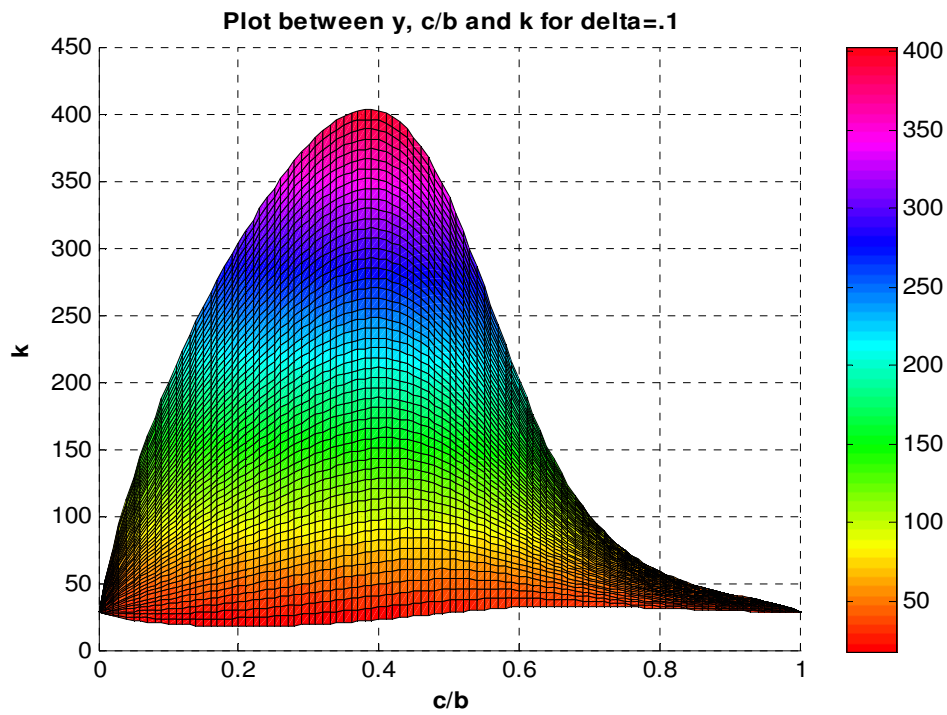
**Fig.3.4**



**Fig.3.4a**

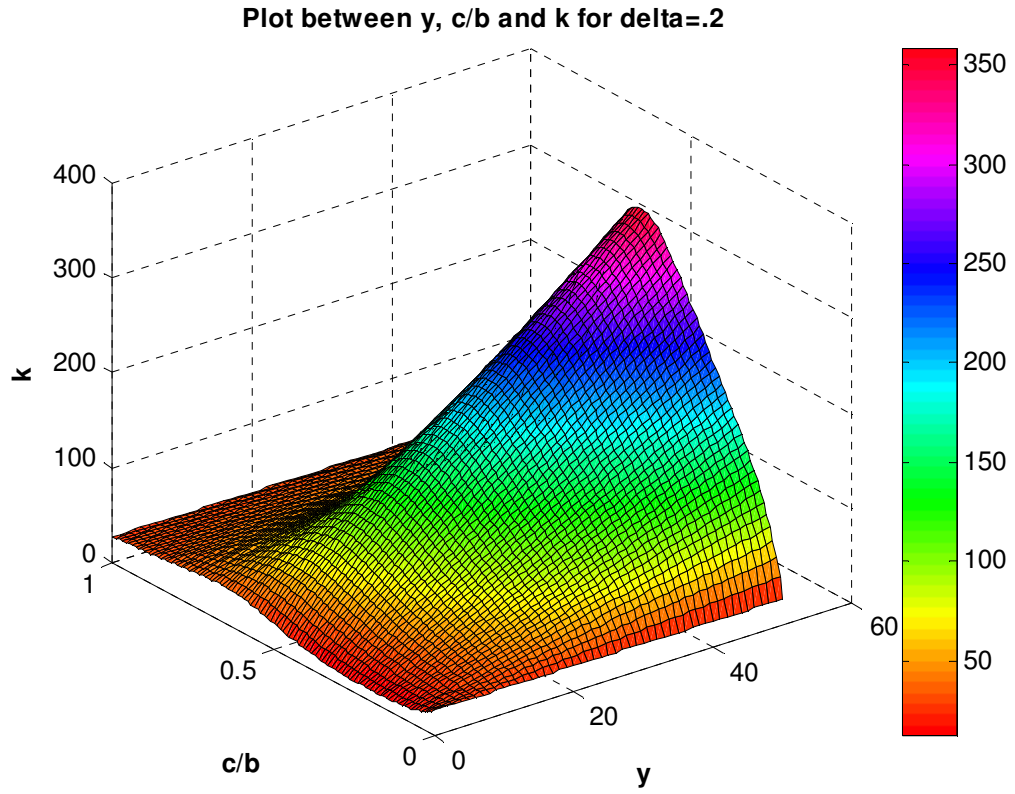


**Fig.3.4b**

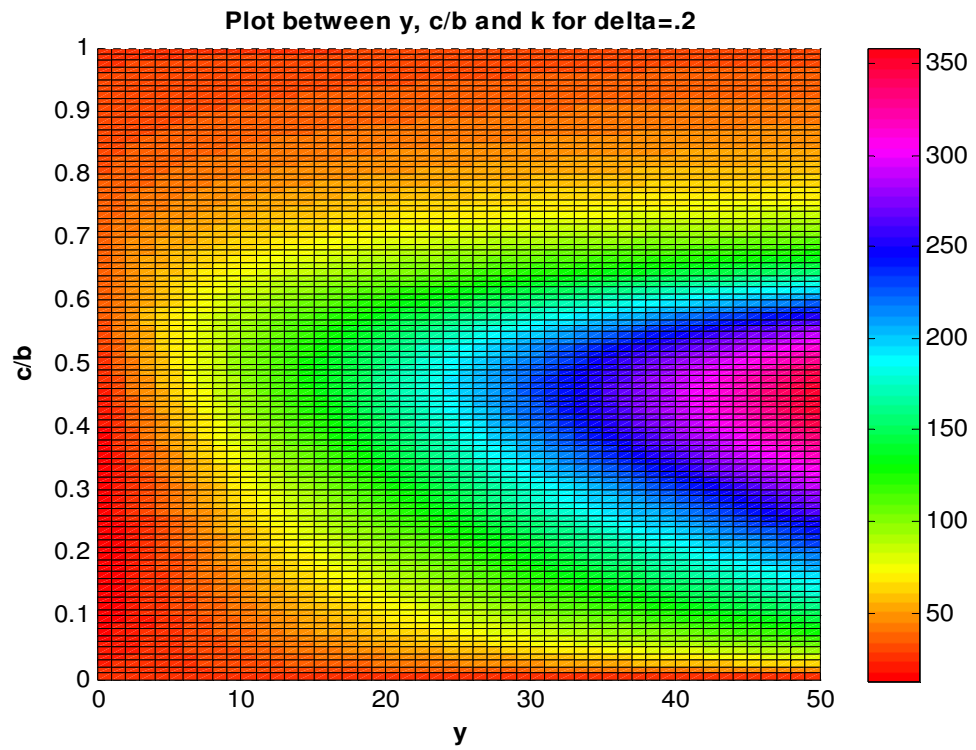


**Fig.3.4c**

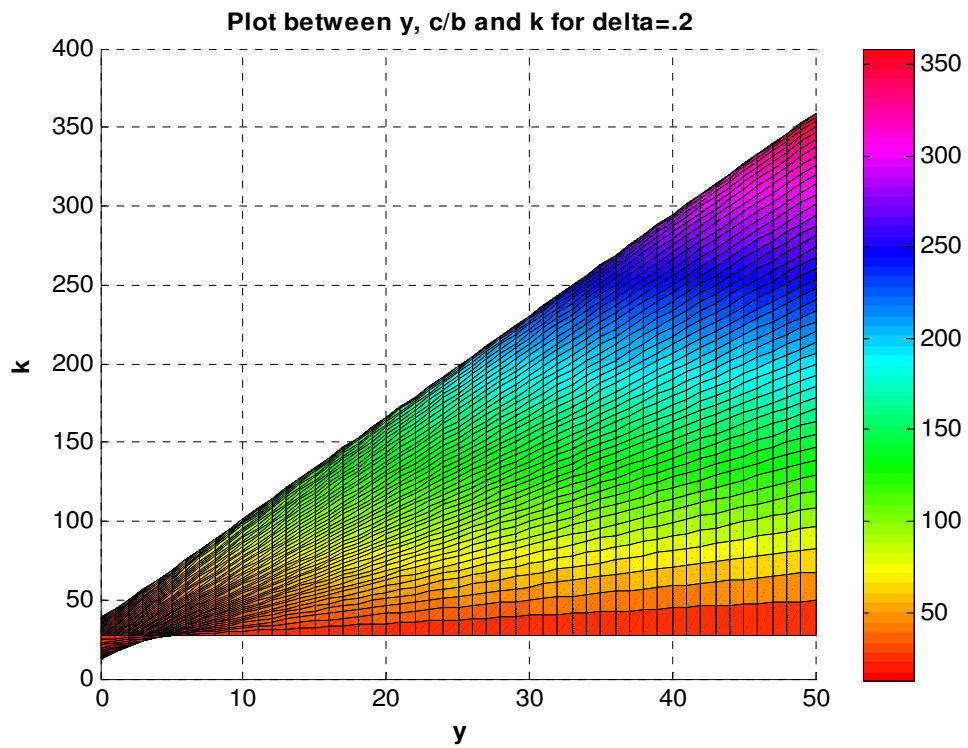
The plot showing the variation between 'k', c/b and 'γ' considering a/b=1 and δ=0.2 is shown in figures 3.5, 3.5a, 3.5b and 3.5c.



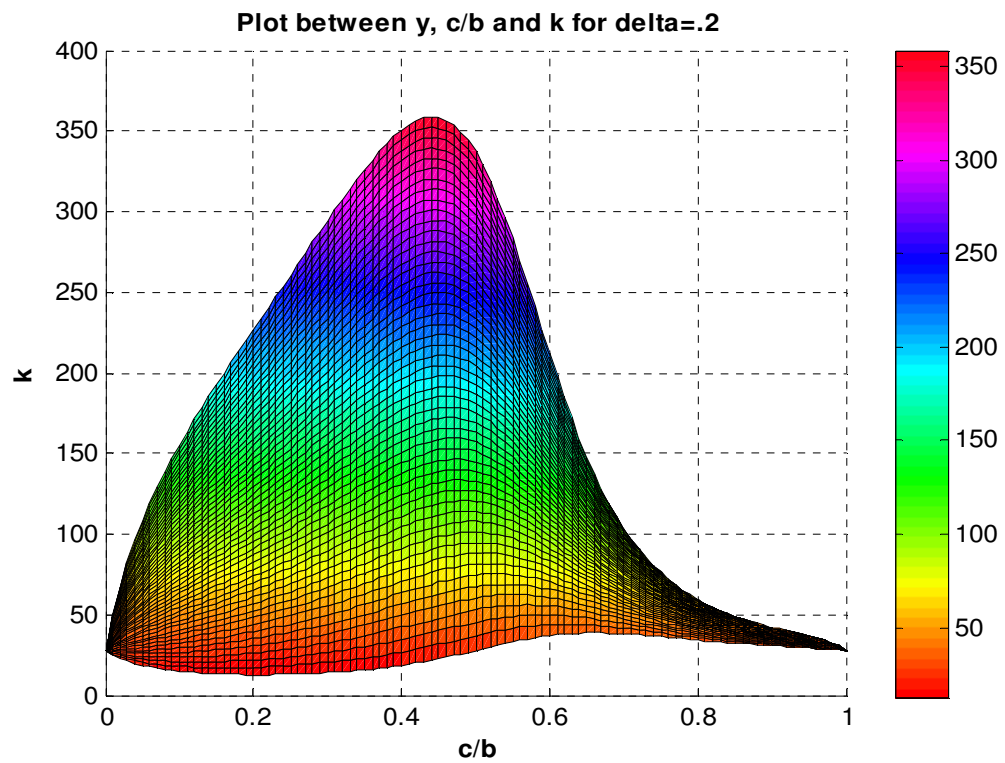
**Fig.3.5**



**Fig.3.5a**



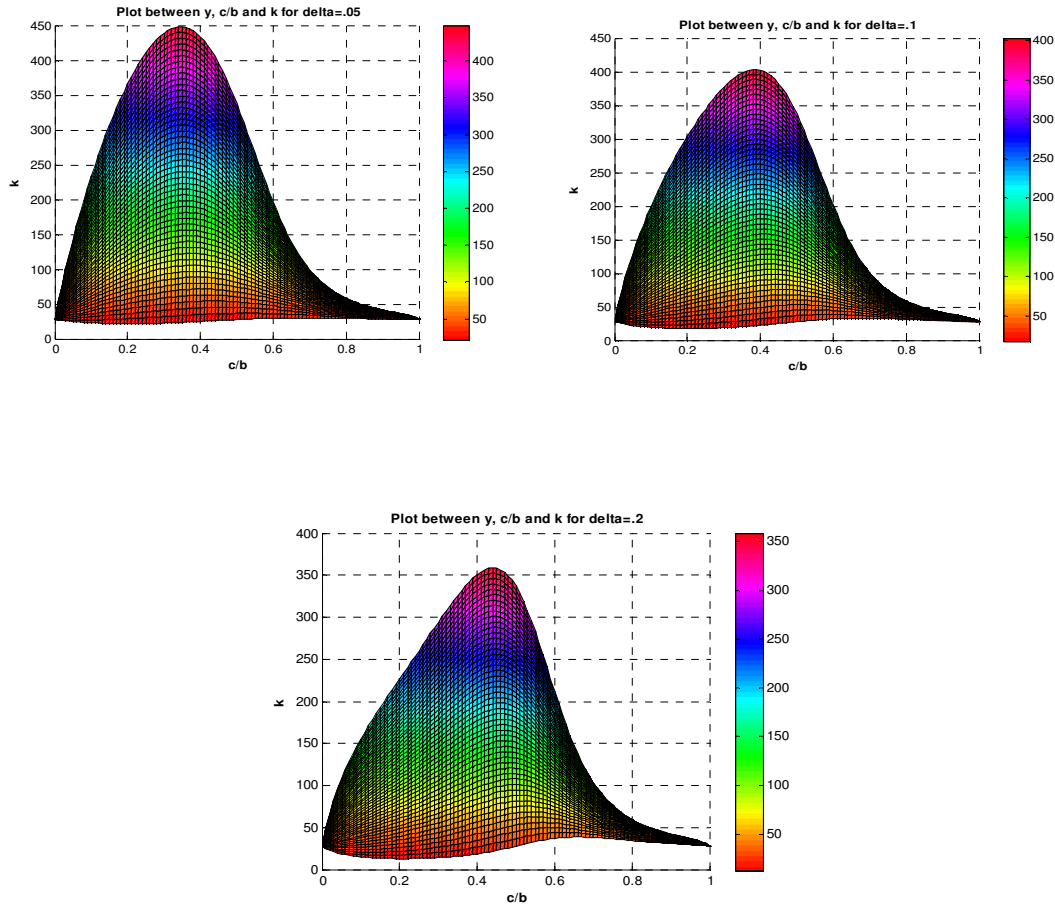
**Fig.3.5b**



**Fig.3.5c**

### 3.5 COMPARISON CHARTS AND THEIR INFERENCES

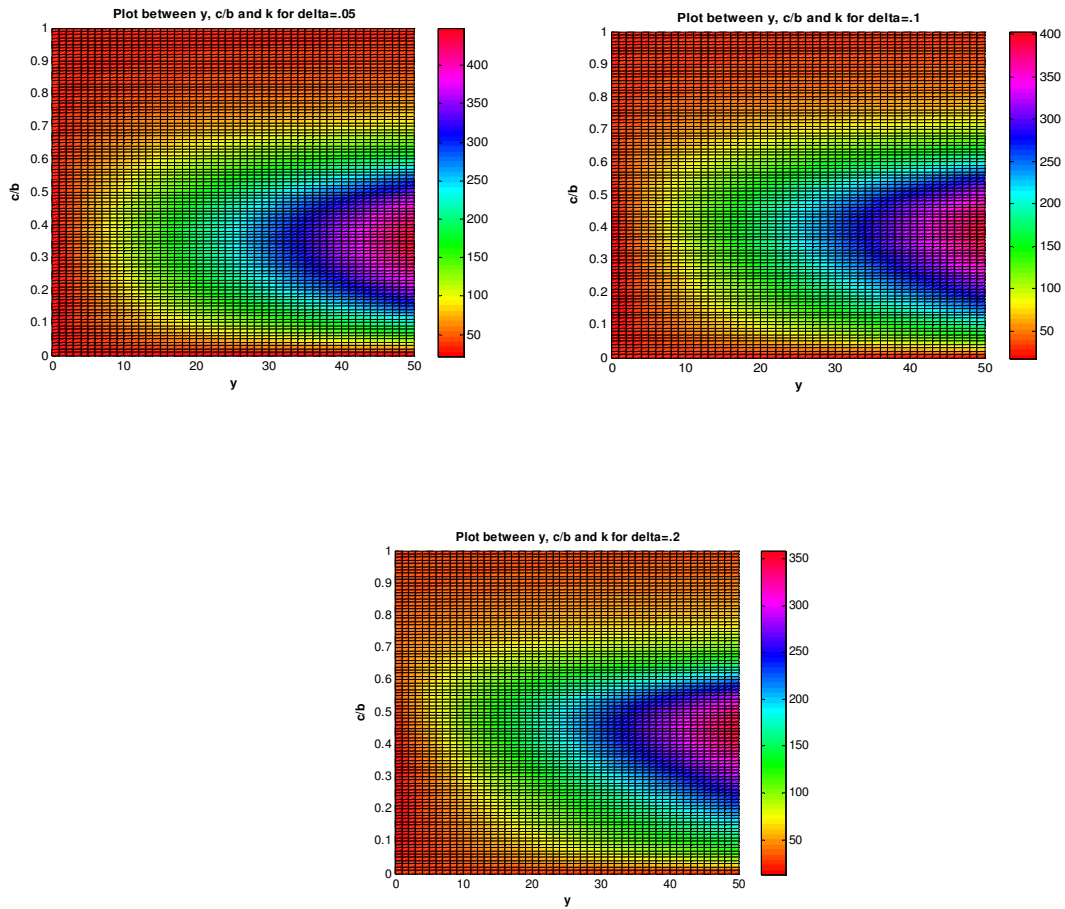
Fig. 3.3c, Fig. 3.4c and Fig. 3.5c are put together in fig 3.6 below for better observation and comparison.



**Fig.3.6**

It is observed from fig.3.6 that as  $\delta$  increases, peak for maximum compressive stress moves towards the centre of the plate, meaning thereby that as the proportionate ratio of longitudinal stiffener to the plate, increases the optimum position of the longitudinal stiffener moves towards the centre of the plate.

Similarly Fig. 3.3a, Fig. 3.4a and Fig. 3.5a are combined together in fig 3.7 below for better comparison.



**Fig. 3.7**

It is seen from Fig. 3.7, that as flexural rigidity parameter,  $\gamma$  increases, the maximum value of compressive stress moves away from the centre of the plate that is towards top of the plate.

## **COMPARATIVE STUDY OF TEST RESULTS** **WITH PREDICTED VALUES**

### **4.1 GENERAL:**

In this chapter a comparison of the predicted values with experimental values has been made. The various experimental values of ultimate load on plate girders with longitudinal stiffener as done by various authors to study the effect of plate-stiffener geometrical properties on location of longitudinal stiffener has been studied and then compared with the results of theoretical energy method modeling. The comparative study has demonstrated that the energy method modeling has not been entirely satisfactory for full range of experimental results involving longitudinal stiffening and patch loading. A very large influence of the stiffener and its properties is observed for some stiffener location.

### **4.2 EXPERIMENTAL INVESTIGATIONS ON LONGITUDINAL STIFFENING FOR PATCH LOADING**

#### **4.2.1 Experimental results by Walbridge and Lebet:**

Walbridge and Lebet (2001) conducted tests using steel webs . The experiments were conducted with five plate girders, two having trapezoidal stiffeners and three with flat plates. The aim was to investigate the behaviour a longitudinal stiffener on patch loading resistance of plate girder webs. Geometry and material data of the panels are shown in Table-A1, Appendix.

#### **4.2.1 Experimental results by Carretero and Lebet:**

Carretero and Lebet (1998) presented the results of a set of experiments, conducted using steel webs in composite plate girders (concrete-steel). The experiments were conducted with six plate girders, each similar to a continuous beam used in bridge construction, and composed of six panels. The aim was to investigate the behaviour of thin slender webs in girders subjected to concentrated loads, and especially the influence of a longitudinal stiffener on patch loading resistance of plate girder webs. All tests were done with trapezoidal stiffeners. Geometry and material data of the panels are shown in Table-A1, Appendix.

#### **4.2.2 Experimental results by Dubas and Tschamper:**

The results of this experimental investigation were presented in a series of articles and reports. Dubas and Tschamper (1990) studied the interaction between bending moments and patch loading. Additional experiments were conducted in order to investigate the influence of open section and closed section longitudinal stiffeners on the ultimate load of plate girders subjected to patch loading. Eight tests were done with flat plates and eight with V-shaped stiffeners. Data concerning the plate girders are shown in Table-A1, Appendix.

#### **4.2.3 Experimental results by Bergfelt:**

The tests performed by Bergfelt (1983) included ten welded plate girders with open section stiffeners (flat plates). The results and details for the experimental plate girders are shown in Table-A1, Appendix.

In this set of experiments the size of the stiffener was the same because its influence was not the aim of the study. Instead the influences of the stiffener position and of the panel aspect ratio were investigated.

#### **4.2.4 Experimental results by Janus, Karnikova and Skaloud:**

An extensive investigation of the influence of the longitudinal stiffeners with open cross section (flat plates) was performed at the Building research Institute of the Czech Technical University in Prague (1988, 1992). This research work comprised more than 100 experimental test girders and represents one of the largest databases regarding longitudinal stiffening for patch loading. In this group of experiments the influence of several parameters were investigated, among them the relative position of the longitudinal stiffener, its size (and relative flexural rigidity) and its detailing (one or double sided). Dimensions and material data of the plate girders are shown in Table-A1, Appendix.

#### **4.2.5 Additional Experimental studies:**

The results from experimental studies by various authors have been grouped here. Only stiffeners with open cross section (flat plates) were used in these investigations. Data concerning the plate girders are shown in Table-A1, Appendix.

The tests were conducted by Rockey et al. (1978), Bergfelt (1979), Markovic and Hajdin (1992), Dogaki et al. (1990), Galae et al. (1987) and Shimizu et al. (1987). The girders were subjected to patch loading and a moderate bending moment. Tests were made by Salkar (1992) in order to study the influence of eccentricities in the application of a patch load.



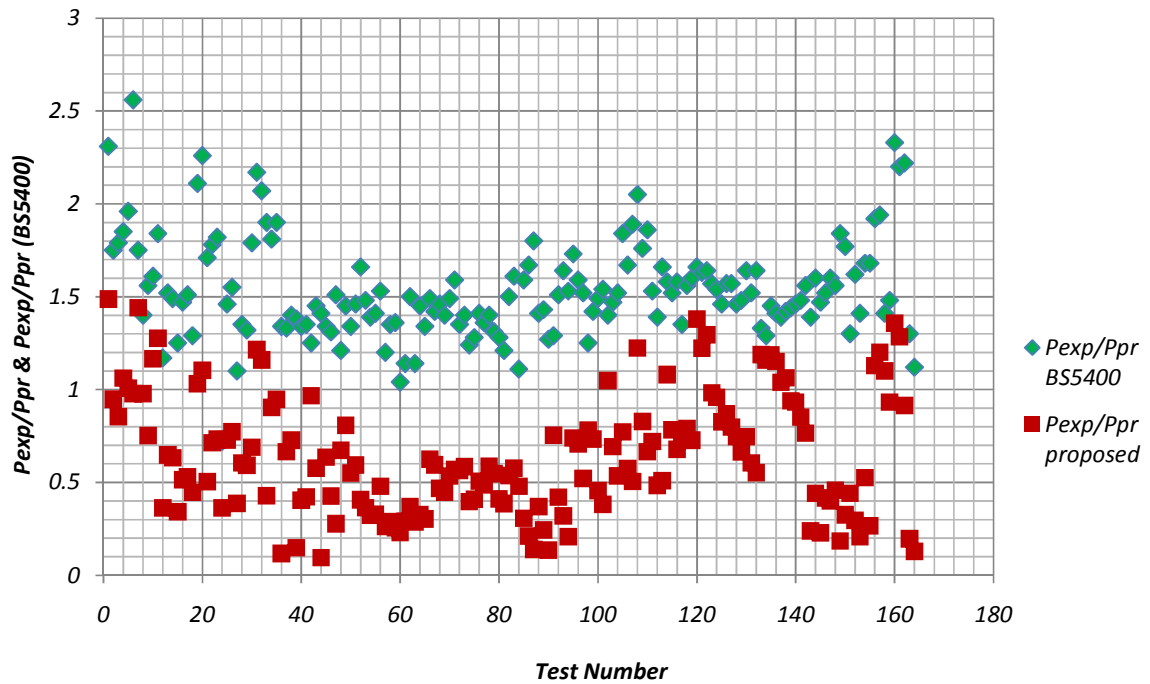
### **4.3 PREDICTED VALUES:**

The predicted values as obtained from the theoretical energy method modeling were evaluated by using various dimensional and material data of the various plate girders used by various authors for experimental investigations on various longitudinally stiffened plate girders for patch loading, in equation (16). The dimensional and material data were used to evaluate various constant and coefficients of k in equation (16). While calculating the ultimate load predictions the elasticity modulus E was taken as 210,000 MPa and Poisson's Ratio equal to 0.3. These values were then used to find value of k by transferring data in matrix form from Excel sheet to MATLAB. The determinant of equation (16) was then solved by MATLAB to give value of k. This value was transferred back to Excel and then put in equation  $\frac{\sigma_{cr} b^2 h}{\pi^2 D} = k$  to find the value of  $\sigma_{cr}$ . The predicted value  $P_{cr}$  obtained from energy method were then be obtained by multiplying  $\sigma_{cr}$  with thickness of the web. The calculations and results have been provided in Table-A1,A2 andA3, Appendix.

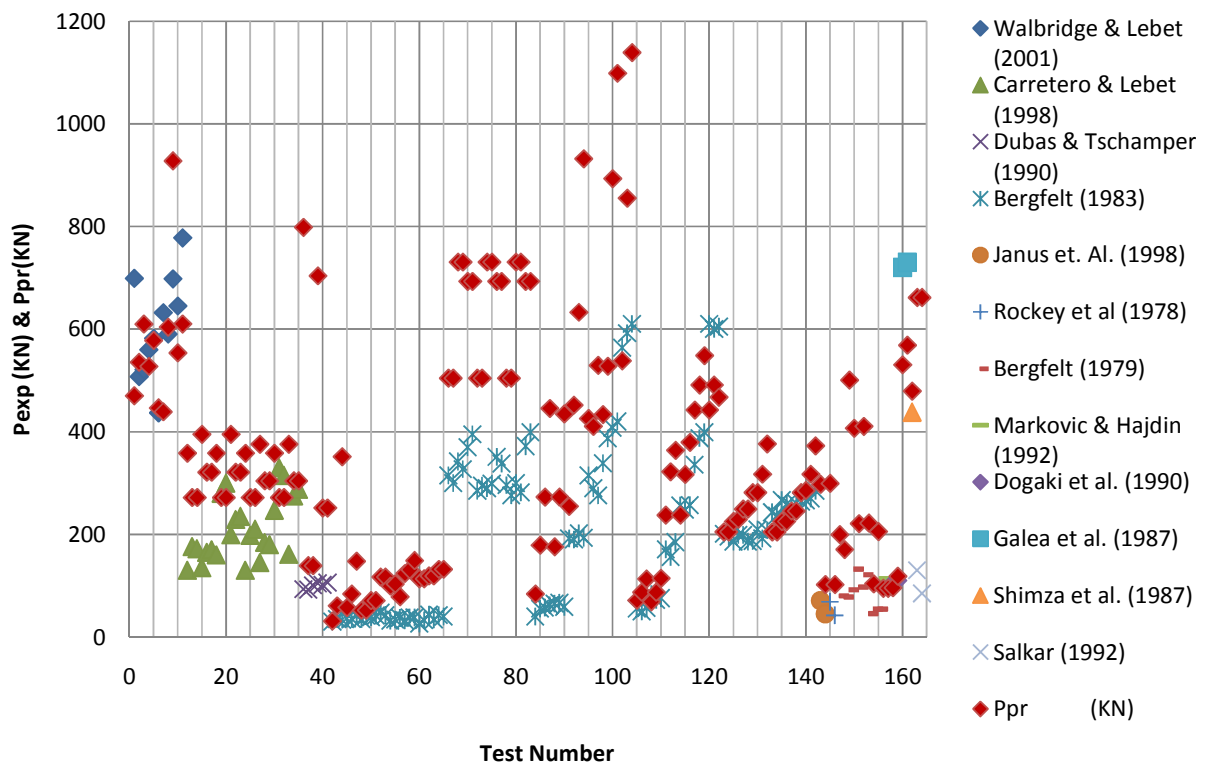
### **4.4 COMPARISON OF THEORETICAL PREDICTIONS AND TEST RESULTS:**

A summary of experimental investigations dealing with the ultimate resistance of longitudinally stiffened plate girders to concentrated loads is given in Table-A1, Appendix. Data from 12 groups of test results with a total of 164 girders were collected from a literature study, of which 145 girders have open section stiffeners and the remaining 19 have closed section stiffeners.

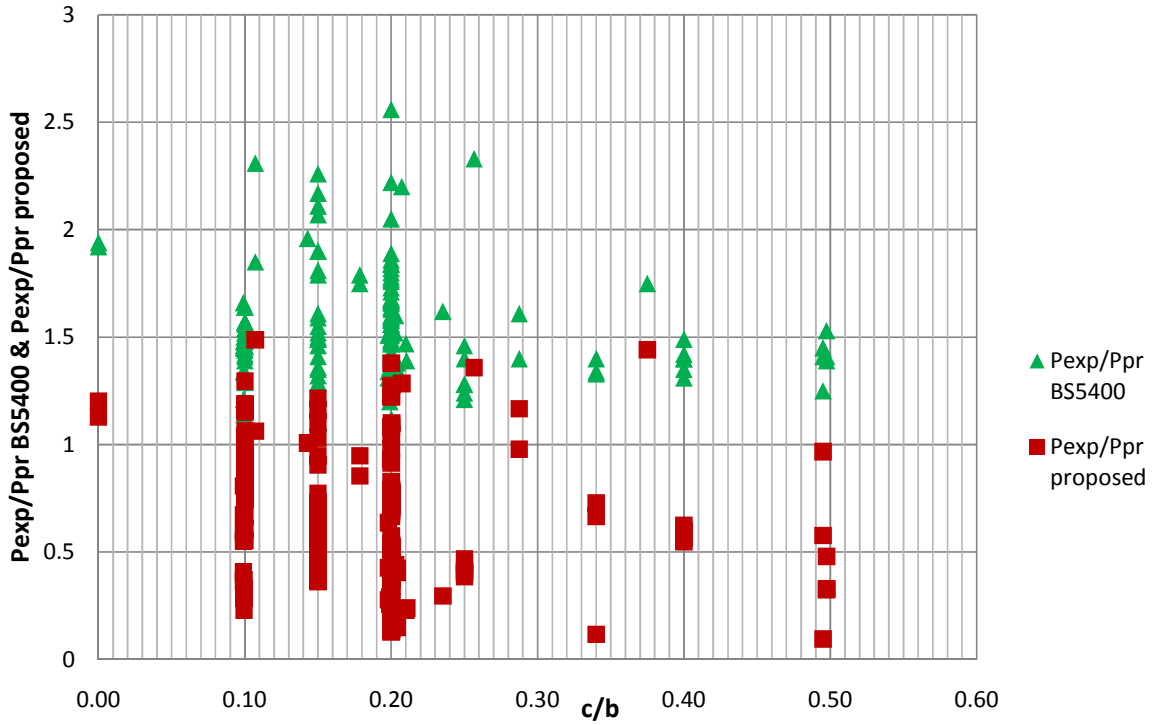
Plots comparing experimental values with theoretical ultimate loads based on BS5400 and energy method were made as a function of plate-stiffener ratio (c/b) and relative flexural rigidity ( $\gamma$ ).



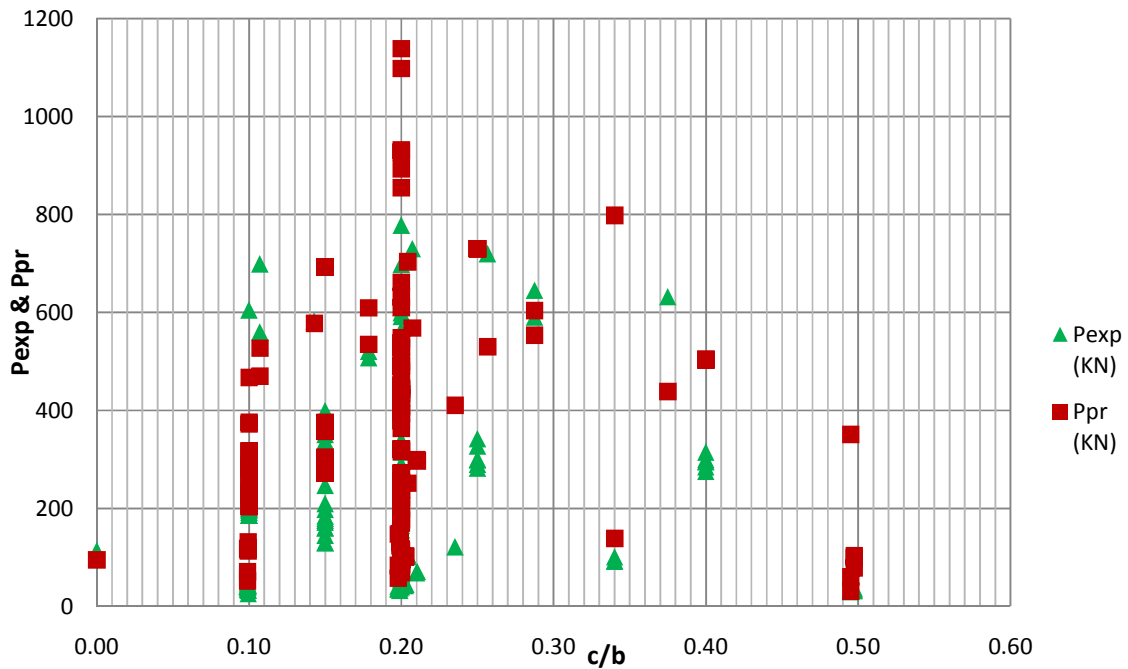
**Fig.4.1 Comparison of experimental with BS5400 and predicted values according to Energy Method Proposed**



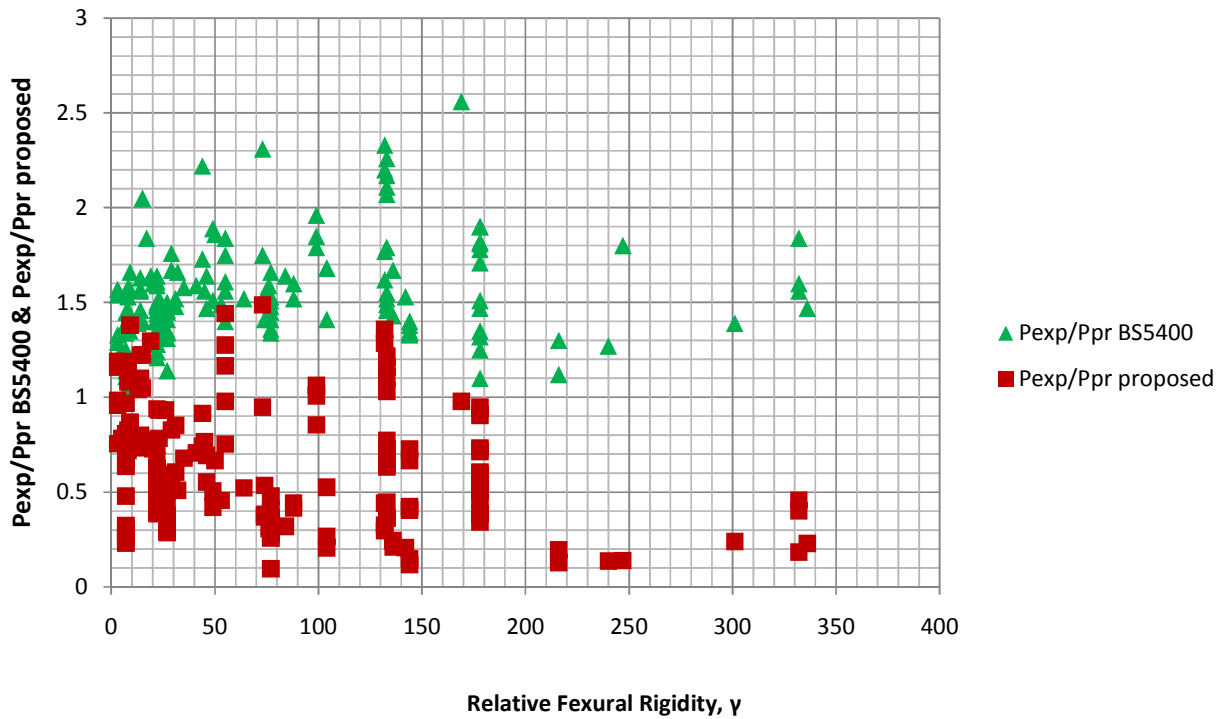
**Fig.4.2 Plot of experimental and predicted values according to Energy Method Proposed**



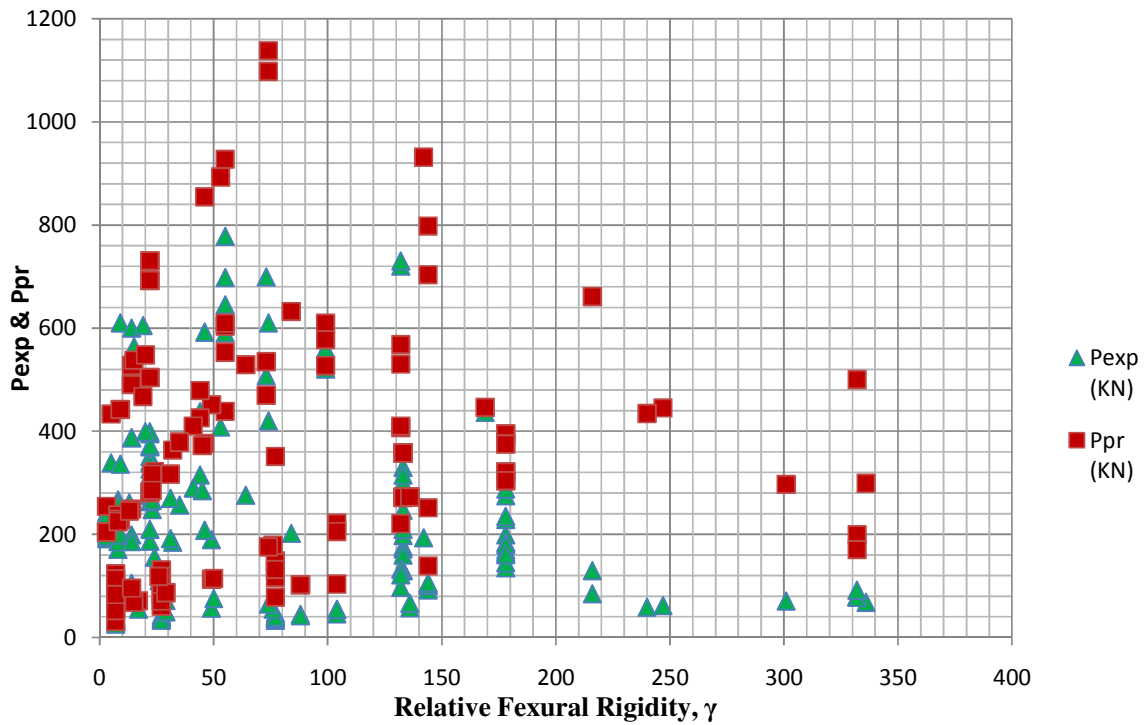
**Fig.4.3 Comparison of experimental with BS5400 and predicted values according to Energy Method Proposed as a function of plate stiffener aspect ratio ( $c/b$ )**



**Fig.4.4 Plot of experimental and predicted values according to Energy Method Proposed as a function of plate stiffener aspect ratio ( $c/b$ )**



**Fig.4.5 Comparison of experimental with BS5400 and predicted values according to Energy Method Proposed as a function of Relative Flexural Rigidity ( $\gamma$ )**



**Fig.4.6 Plot of experimental and predicted values according to Energy Method Proposed as a function of Relative Flexural Rigidity ( $\gamma$ )**

From the figures above and the statistical data obtained as mean value, standard deviation and coefficient of variation in Table-A1, Annexure, it is clear that the theoretical modeling by energy method gives quite high scatter (coefficient of variation), which can be interpreted as a sign that the models are incomplete in the sense that all relevant parameters are not included. However, there is marked similarity in the results obtained experimentally and theoretically.

Fig. 4.1 marks similar trends of comparative experimental data with BS5400 and the predicted values. The plot shows that theoretical predictions based on BS5400 are more conservative.

Fig. 4.2 leads to interpretation that excluding a few, the energy method predicts a visibly little higher ultimate load for almost all the experiments.

Fig. 4.3 again shows a similar trend during the comparative study of experimental data with BS5400 and predicted values.

Fig. 4.4 shows that the ultimate load for plate stiffener aspect ratio( $c/b$ ) less than 0.3 registers almost the same value for experimental as well as predicted data.

Fig. 4.5 too leads to same trend as other comparative plots showing the conservative behaviour of BS5400

Fig. 4.6 shows that the ultimate load for relative flexural rigidity ( $\gamma$ ) less than 200 predicts almost similar value for the experimental and theoretical energy method modeling.

#### **4.5 CRITICAL COMMENTS:**

In general it has been found that energy method gives a closer data to experimental values than BS5400, which runs on conservative side. The ultimate load values found experimentally agree with the theoretical energy method modeling for plate stiffener aspect ratio ( $c/b$ ) less than 0.3 and relative flexural rigidity ( $\gamma$ ) less than 200. The rest of the values may differ due to incomplete modeling because of the absence of some other relevant parameters.

## **CONCLUSIONS**

### **5.1 SUMMARY:**

This study included the energy method analysis of web plates and comparison of the predicted results with the experimental data obtained by various analysts as well as BS5400. The predictions of this method were found to be quite close to experimental values presented by various analysts and this can be used for experimental predictions. The analysis studied the behavior of web plates under pure bending, which can be further extended to include compressive stresses and other factors.

### **5.2 CONCLUSION BASED ON THEORETICAL ENERGY METHOD MODELING:**

This study investigated the behavior of web plates under pure bending. As a first stage an energy formulation for the analysis was presented. These formulations were then used to plot the variations between compressive stresses and plate stiffener geometrical properties to arrive at the maximum compressive stresses peaks, thus defining the optimum location. It was shown that the optimum location for the stiffener depends on the relative proportions of the plate and is not always at 0.2b location as recommended by various design specifications.

### **5.3 CONCLUSION BASED ON COMPARISON OF ENERGY METHOD ANALYSIS WITH EXPERIMENTAL DATA AND BS5400:**

The comparative study of the experimental data with codal analysis and theoretically predicted values of energy method lead to the following observations:

- i. Codal provision gives us conservative results.
- ii. Energy method is particularly useful in predicting for plate stiffener aspect ratio( $c/b$ ) less than 0.3 and relative flexural rigidity ( $\gamma$ ) less than 200.

In general, it has been found that energy method gives values closer to experimental values than BS5400, which runs on conservative side. The ultimate load values found

experimentally agree with the theoretical energy method modeling for plate stiffener aspect ratio( $c/b$ ) less than 0.3 and relative flexural rigidity ( $\gamma$ ) less than 200. The rest of the values may differ due to incomplete modeling because of the absence of some other relevant parameters.

#### **5.4 SCOPE FOR FURTHER RESEARCH**

This research work can be further carried to include various other parametrical aspects like

- i. The torsional buckling of plates.
- ii. The compressive stresses.

**APPENDIX**

**TABLE-A1 Summary of Test Results for girders with Longitudinal Stiffener Subjected to Patch Loading**

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	$f_{yw}$ (MPa)	$t_f$ (mm)	$b_f$ (mm)	$f_{yf}$ (MPa)	$s_s$ (mm)	c (mm)	$t_{st}$ (mm)	$b_{st}$ (mm)	$\gamma$	$P_{exp}$ (KN)	$\frac{P_{exp}}{P_{pr}}$ BS540 0	$\frac{P_{exp}}{P_{pr}}$ proposed	$P_{pr}$ (KN)
1	Panel 2-C1	5	1000	700	392	20	225	355	200	75	5	60	73	699.1	2.31	1.49	469.95
2	Panel 3-C2	5	1000	700	392	20	225	355	200	125	5	60	73	507.4	1.75	0.95	535.13
3	Panel 4-C2	5	1000	700	392	20	225	355	200	125	10	80	99	520.6	1.79	0.85	609.40
4	Panel 5-C3	5	1000	700	392	20	225	355	200	75	10	80	99	559.9	1.85	1.06	527.05
5	Panel 6-C3	5	1000	700	392	20	225	355	200	100	10	80	99	582.1	1.96	1.01	577.65
6	Panel 1-2	4	1200	800	405	10	160	371	300	160	5	60	169	436.5	2.56	0.98	446.17
7	Panel 2-2	6	1200	800	447	15	200	364	300	300	5	60	55	632.1	1.75	1.44	438.55
8	Panel 4-4	6	1200	800	483	20	300	399	300	230	5	60	55	590.3	1.4	0.98	603.63
9	Panel 4-6	6	1800	800	483	20	300	399	300	160	5	60	55	698	1.56	0.75	927.45
10	Panel 5-1	6	1050	800	483	20	300	399	200	230	5	60	55	645.1	1.61	1.17	553.29
11	Panel 6-2	6	1050	800	483	20	300	399	200	160	5	60	55	777.9	1.84	1.28	609.76
12	VT07-1	3.8	2480	1000	375	8.35	150	296	40	150	2	80	133	130	1.17	0.36	357.83
13	VT07-2	3.8	1760	1000	375	8.35	150	296	40	150	2	80	133	176	1.52	0.65	271.51
14	VT07-3	3.8	1760	1000	375	8.35	150	296	40	150	2	80	133	172	1.49	0.63	271.51
15	VT07-4	3.8	2480	1000	375	8.35	150	281	40	200	2	90	178	135	1.25	0.34	394.57
16	VT07-5	3.8	1760	1000	375	8.35	150	281	40	200	2	90	178	165	1.47	0.51	320.75
17	VT07-6	3.8	1760	1000	375	8.35	150	281	40	200	2	90	178	170	1.51	0.53	320.75
18	VT08-1	3.8	2480	1000	358	8.3	150	292	240	150	2	80	133	160	1.29	0.45	357.83
19	VT08-2	3.8	1760	1000	358	8.3	150	292	240	150	2	80	133	280	2.11	1.03	271.51
20	VT08-3	3.8	1760	1000	358	8.3	150	292	240	150	2	80	133	300	2.26	1.10	271.51
21	VT08-4	3.8	2480	1000	358	8.3	150	328	240	200	2	90	178	199	1.71	0.50	394.57
22	VT08-5	3.8	1760	1000	358	8.3	150	328	240	200	2	90	178	229	1.78	0.71	320.75
23	VT08-6	3.8	1760	1000	358	8.3	150	328	240	200	2	90	178	235	1.82	0.73	320.75



S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	f <sub>yw</sub> (MPa)	t <sub>f</sub> (mm)	b <sub>f</sub> (mm)	f <sub>yf</sub> (MPa)	s <sub>s</sub> (mm)	c (mm)	t <sub>st</sub> (mm)	b <sub>st</sub> (mm)	γ	P <sub>exp</sub> (KN)	P <sub>exp</sub> /P <sub>pr</sub> BS5400	P <sub>exp</sub> /P <sub>pr</sub> proposed	P <sub>pr</sub> (KN)
24	VT09-1	3.8	2480	1000	371	12	150	286	40	150	2	80	133	130	0.72	0.36	357.83
25	VT09-2	3.8	1760	1000	371	12	150	286	40	150	2	80	133	198	1.46	0.73	271.51
26	VT09-3	3.8	1760	1000	371	12	150	286	40	150	2	80	133	210	1.55	0.77	271.51
27	VT09-4	3.8	2480	1000	371	12	150	283	40	150	2	90	178	145	1.1	0.39	375.26
28	VT09-5	3.8	1760	1000	371	12	150	283	40	150	2	90	178	184	1.35	0.61	304.03
29	VT09-6	3.8	1760	1000	371	12	150	283	40	150	2	90	178	180	1.32	0.59	304.03
30	VT10-1	3.8	2480	1000	380	12	150	282	240	150	2	80	133	247	1.79	0.69	357.83
31	VT10-2	3.8	1760	1000	380	12	150	282	240	150	2	80	133	330	2.17	1.22	271.51
32	VT10-3	3.8	1760	1000	380	12	150	282	240	150	2	80	133	315	2.07	1.16	271.51
33	VT10-4	3.8	2480	1000	380	12	150	275	240	150	2	90	178	161	1.9	0.43	375.26
34	VT10-5	3.8	1760	1000	380	12	150	275	240	150	2	90	178	275	1.81	0.90	304.03
35	VT10-6	3.8	1760	1000	380	12	150	275	240	150	2	90	178	288	1.9	0.95	304.03
36	731	3	3000	735	252	12	250	277	40	250	6	60	144	93.3	1.34	0.12	798.02
37	732	3	1100	735	252	12	250	277	40	250	6	60	144	92.4	1.33	0.67	138.81
38	733	3	1100	735	252	12	250	277	120	250	6	60	144	101	1.4	0.73	138.81
39	734	3	3000	735	252	12	250	277	40	150	6	60	144	104.7	1.38	0.15	703.49
40	735	3	1100	735	252	12	250	277	40	150	6	60	144	101.8	1.34	0.41	251.30
41	736	3	1100	735	252	12	250	277	120	150	6	60	144	106.3	1.35	0.42	251.30
42	TG 1-1	2	505	505	236	5	50	439	50	250	5	12	7	30	1.25	0.97	31.03
43	TG 1-2	2	505	505	239	5	50	439	50	250	5	20	27	35	1.45	0.58	60.66
44	TG 1-3	2	505	505	231	5	50	453	50	250	5	30	77	33.5	1.41	0.10	351.10
45	TG 2-1	2	505	505	234	5	50	453	50	100	5	12	7	36.5	1.34	0.64	57.33
46	TG 2-2	2	505	505	232	5	50	446	50	100	5	20	27	35.6	1.31	0.43	83.46

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	$f_{yw}$ (MPa)	$t_f$ (mm)	$b_f$ (mm)	$f_{yf}$ (MPa)	$s_s$ (mm)	c (mm)	$t_{st}$ (mm)	$b_{st}$ (mm)	$\gamma$	$P_{exp}$ (KN)	$P_{exp}/P_{pr}$ BS5400	$P_{exp}/P_{pr}$ proposed	$P_{pr}$ (KN)
47	TG 2-3	2	505	505	233	5	50	458	50	100	5	30	77	41	1.51	0.28	147.62
48	TG 3-1	2	505	505	236	5	50	485	50	50	5	12	7	35	1.21	0.67	51.97
49	TG 3-1	2	505	505	234	5	50	466	50	50	5	12	7	42	1.45	0.81	51.97
50	TG 3-2	2	505	505	239	5	50	467	50	50	5	20	27	39	1.34	0.55	70.73
51	TG 3-2	2	505	505	232	5	50	471	50	50	5	20	27	42	1.46	0.59	70.73
52	TG 3-3	2	505	505	231	5	50	461	50	50	5	30	77	47.5	1.66	0.41	116.69
53	TG 3-3	2	505	505	233	5	50	481	50	50	5	30	77	42.5	1.48	0.36	116.69
54	TG 11-1	2	1005	502.5	199	5	50	293	100	250	5	12	7	32	1.39	0.32	98.96
55	TG 11-2	2	1005	502.5	210	5	50	472	100	250	5	20	27	34	1.41	0.33	103.31
56	TG 11-3	2	1005	502.5	215	5	50	476	100	250	5	30	77	37.5	1.53	0.48	78.29
57	TG 12-1	2	1005	502.5	204	5	50	295	100	100	5	12	7	32.5	1.2	0.26	123.64
58	TG 12-2	2	1005	502.5	218	5	50	461	100	100	5	20	27	38	1.35	0.29	131.04
59	TG 12-3	2	1005	502.5	218	5	50	470	100	100	5	30	77	38.2	1.36	0.26	148.80
60	TG 13-1	2	1005	502.5	191	5	50	303	100	50	5	12	7	26	1.04	0.23	113.36
61	TG 13-1	2	1005	502.5	204	5	50	293	100	50	5	12	7	33	1.14	0.29	113.36
62	TG 13-2	2	1005	502.5	210	5	50	475	100	50	5	20	27	44	1.5	0.37	118.75
63	TG 13-2	2	1005	502.5	218	5	50	469	100	50	5	20	27	34	1.14	0.29	118.75
64	TG 13-3	2	1005	502.5	215	5	50	478	100	50	5	30	77	43	1.45	0.33	131.75
65	TG 13-3	2	1005	502.5	218	5	50	473	100	50	5	30	77	40	1.34	0.30	131.75
66	TG 31-1	6	622.5	500	256	12	120	242	62	200	5	40	22	315	1.49	0.62	504.09
67	TG 31-1	6	622.5	500	256	12	120	242	62	200	5	40	22	300	1.42	0.60	504.09
68	TG 31-2	6	622.5	500	256	12	120	242	62	125	5	40	22	342	1.46	0.47	730.29
69	TG 31-2	6	622.5	500	256	12	120	242	62	125	5	40	22	327	1.4	0.45	730.29

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	$f_{yw}$ (MPa)	$t_r$ (mm)	$b_r$ (mm)	$f_{yf}$ (MPa)	$s_s$ (mm)	c (mm)	$t_{st}$ (mm)	$b_{st}$ (mm)	$\gamma$	$P_{exp}$ (KN)	$P_{exp}/P_{pr}$ BS5400	$P_{exp}/P_{pr}$ proposed	$P_{pr}$ (KN)
70	TG 31-3	6	622.5	500	256	12	120	242	62	75	5	40	22	370	1.49	0.53	692.64
71	TG 31-3	6	622.5	500	256	12	120	242	62	75	5	40	22	395	1.59	0.57	692.64
72	TG 32-1	6	622.5	500	256	12	120	242	62	200	5	40	22	285	1.35	0.57	504.09
73	TG 32-1	6	622.5	500	256	12	120	242	62	200	5	40	22	295	1.4	0.59	504.09
74	TG 32-2	6	622.5	500	256	12	120	242	62	125	5	40	22	290	1.24	0.40	730.29
75	TG 32-2	6	622.5	500	256	12	120	242	62	125	5	40	22	299	1.28	0.41	730.29
76	TG 32-3	6	622.5	500	256	12	120	242	62	75	5	40	22	351	1.41	0.51	692.64
77	TG 32-3	6	622.5	500	256	12	120	242	62	75	5	40	22	338	1.36	0.49	692.64
78	TG 33-1	6	622.5	500	256	12	120	242	62	200	5	40	22	296	1.4	0.59	504.09
79	TG 33-1	6	622.5	500	256	12	120	242	62	200	5	40	22	276	1.31	0.55	504.09
80	TG 33-2	6	622.5	500	256	12	120	242	62	125	5	40	22	300	1.28	0.41	730.29
81	TG 33-2	6	622.5	500	256	12	120	242	62	125	5	40	22	282	1.21	0.39	730.29
82	TG 33-3	6	622.5	500	256	12	120	242	62	75	5	40	22	372	1.5	0.54	692.64
83	TG 33-3	6	622.5	500	256	12	120	242	62	75	5	13.3	22	399	1.61	0.58	692.64
84	TG 021-0	2.4	500	500	224	5.1	100	292	50	100	5.2	31.5	7	40	1.11	0.48	83.48
85	TG 021-1	2.2	500	500	238	6.1	119.9	309	50	100	5.2	40.5	76	55	1.59	0.31	178.82
86	TG 021-2	2.2	500	500	238	6	119.8	309	50	100	5	50.2	136	57.5	1.67	0.21	272.39
87	TG 021-3	2.2	500	500	238	6	120.2	309	50	100	5.3	30.9	247	62	1.8	0.14	445.28
88	TG 022-1	2.2	500	500	238	11.7	119.9	239	50	100	5.7	40.5	74	65	1.41	0.37	175.69
89	TG 022-2	2.2	500	500	238	11.9	119.3	239	50	100	5	50.1	136	66.5	1.43	0.24	272.39
90	TG 022-3	2.2	500	500	238	11.9	119.7	239	50	100	5.1	17.2	240	59	1.27	0.14	434.38
91	TG 041-0	4.4	500	500	362	8.5	118.6	262	50	100	5.1	40.7	3	192	1.29	0.75	254.33

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	f <sub>yw</sub> (MPa)	t <sub>f</sub> (mm)	b <sub>f</sub> (mm)	f <sub>yf</sub> (MPa)	s <sub>s</sub> (mm)	c (mm)	t <sub>st</sub> (mm)	b <sub>st</sub> (mm)	γ	P <sub>exp</sub> (KN)	P <sub>exp</sub> /P <sub>pr</sub> BS5400	P <sub>exp</sub> /P <sub>pr</sub> proposed	P <sub>pr</sub> (KN)
92	TG 041-1	4	500	500	360	8.4	119.3	262	50	100	8.1	50	49	190	1.51	0.42	451.59
93	TG 041-2	4	500	500	360	7.8	119.1	262	50	100	8.1	60.5	84	202	1.64	0.32	632.41
94	TG 041-3	4	500	500	360	8.5	119.2	262	50	100	8.5	39.3	142	193.5	1.53	0.21	931.36
95	TG 042-1	4	500	500	360	20	120.9	285	50	100	7.8	50.7	44	315	1.73	0.74	425.70
96	TG 042-2	4	500	500	360	20	120.6	285	50	100	8.4	60.4	41	290	1.59	0.71	410.14
97	TG 042-3	4	500	500	360	20	120.4	285	50	100	8.4	22.8	64	276	1.52	0.52	529.16
98	TG 061-0	5.6	500	500	426	11.9	120.2	239	50	100	7.9	32.2	5	339	1.25	0.78	433.35
99	TG 061-1	5.6	500	500	426	12.3	89.7	277	50	100	10	51.1	14	387	1.42	0.73	527.59
100	TG 061-2	5.5	500	500	455	12.3	89.7	277	50	100	11	61.1	53	408	1.49	0.46	892.93
101	TG 061-3	5.5	500	500	426	12.1	89.4	277	50	100	9.9	32.4	74	420	1.54	0.38	1098.09
102	TG 062-1	5.6	500	500	426	30.4	99	254	50	100	10	50.7	15	564	1.4	1.05	537.95
103	TG 062-2	5.6	500	500	426	30.5	100	254	50	100	10	60.5	46	592	1.47	0.69	854.67
104	TG 062-3	5.6	500	500	244	30	100.1	254	50	100	10	16.6	74	610	1.52	0.54	1138.38
105	TG 121-1	2	500	500	244	6	119.6	274	50	100	5.2	20.2	17	55	1.84	0.77	71.25
106	TG 121-2	2	500	500	244	6	119.9	274	50	100	5.2	24.9	29	50	1.67	0.58	86.95
107	TG 121-3	2	500	500	244	6.1	119.9	274	50	100	5.2	15.8	49	57	1.89	0.50	112.90
108	TG 122-1	2	500	500	244	12.1	120.7	254	50	100	5.1	20.2	15	84	2.05	1.22	68.62
109	TG 122-2	2	500	500	244	12.1	120.7	254	50	100	5.2	25.2	29	72	1.76	0.83	86.95
110	TG 122-3	2	500	500	244	12.1	120.8	254	50	100	5.1	20.2	50	76	1.86	0.67	114.19
111	TG 141-1	4	500	500	283	8.4	120.1	294	50	100	8.1	30.6	8	171	1.53	0.72	237.27
112	TG 141-2	4	500	500	283	8.5	120.2	294	50	100	8.3	34.4	24	156	1.39	0.48	321.70
113	TG 141-3	4	500	500	283	8.3	120.4	294	50	100	8.1	19.8	32	185	1.66	0.51	363.40

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	$f_{yw}$ (MPa)	$t_f$ (mm)	$b_f$ (mm)	$f_{yf}$ (MPa)	$s_s$ (mm)	c (mm)	$t_{st}$ (mm)	$b_{st}$ (mm)	$\gamma$	$P_{exp}$ (KN)	$P_{exp}/P_{pr}$ BS5400	$P_{exp}/P_{pr}$ proposed	$P_{pr}$ (KN)
114	TG 142-1	4	500	500	283	20.3	120.9	270	50	100	8.2	30.2	8	256.5	1.58	1.08	237.27
115	TG 142-2	4	500	500	283	20.4	120.9	270	50	100	8.4	35.9	23	248	1.52	0.78	316.47
116	TG 142-3	4	500	500	396	20.2	120.7	270	50	100	8	25	35	257	1.58	0.68	379.00
117	TG 161-1	5.4	500	500	396	12.4	90.7	272	50	100	10	30.2	9	336	1.35	0.76	442.17
118	TG 161-2	5.4	500	500	396	12.3	90.8	272	50	100	11	34.7	14	387.5	1.56	0.79	490.58
119	TG 161-3	5.4	500	500	396	12.4	90.7	272	50	100	11	25.6	20	399	1.6	0.73	548.13
120	TG 162-1	5.4	500	500	396	30.4	99.6	269	50	100	10	30.3	9	610	1.66	1.38	442.17
121	TG 162-2	5.4	500	500	396	30.6	99.3	269	50	100	11	33.9	14	600	1.63	1.22	490.58
122	TG 162-3	5.4	500	500	396	30.6	100.2	269	50	50	11	16	19	605	1.64	1.30	467.11
123	TG 241-1	4.1	500	500	304	8.3	120.33	278	50	50	5.2	16	3	201	1.57	0.98	204.43
124	TG 241-1	4.1	500	500	304	8.3	120.4	278	50	50	5.1	20.8	3	196	1.54	0.96	204.43
125	TG 241-2	4.1	500	500	304	8.2	120.8	278	50	50	8.1	21.5	8	186	1.46	0.83	224.97
126	TG 241-2	4.1	500	500	304	8.1	120.6	278	50	50	8.1	25.5	9	199	1.57	0.87	229.04
127	TG 241-3	4.1	500	500	304	8.2	120.3	278	50	50	8.3	25.5	14	199	1.57	0.80	249.24
128	TG 241-3	4.1	500	500	304	8.2	120.8	278	50	50	8.2	30.3	14	186	1.46	0.75	249.24
129	TG 241-4	4.1	500	500	304	8.1	120.7	278	50	50	8.2	30.4	22	187	1.48	0.66	281.24
130	TG 241-4	4.1	500	500	304	8.4	120.6	278	50	50	8.2	35.1	22	210	1.64	0.75	281.24
131	TG 241-5	4.1	500	500	304	8.1	120.7	278	50	50	7.9	40.5	31	192	1.52	0.61	316.95
132	TG 241-6	4.1	500	500	304	8.2	120.7	278	50	50	8.1	15.4	46	208	1.64	0.55	376.09
133	TG 242-1	4.1	500	500	304	19.7	118.2	244	50	50	5.2	15.8	3	243	1.33	1.19	204.43
134	TG 242-1	4.1	500	500	304	19.7	118.6	244	50	50	5.2	20.6	3	237	1.29	1.16	204.43
135	TG 242-2	4.1	500	500	304	19.8	118.5	244	50	50	8.1	20.4	8	267	1.45	1.19	224.97

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	f <sub>yw</sub> (MPa)	t <sub>f</sub> (mm)	b <sub>f</sub> (mm)	f <sub>yf</sub> (MPa)	s <sub>s</sub> (mm)	c (mm)	t <sub>st</sub> (mm)	b <sub>st</sub> (mm)	γ	P <sub>exp</sub> (KN)	P <sub>exp</sub> /P <sub>pr</sub> BS5400	P <sub>exp</sub> /P <sub>pr</sub> proposed	P <sub>pr</sub> (KN)
136	TG 242-2	4.1	500	500	304	19.9	118.4	244	50	50	8.1	24.6	8	259	1.41	1.15	224.97
137	TG 242-3	4.1	500	500	304	19.8	118.6	244	50	50	8.3	24.5	13	255	1.39	1.04	245.22
138	TG 242-3	4.1	500	500	304	19.9	118.7	244	50	50	8.3	30.2	13	261	1.42	1.06	245.22
139	TG 242-4	4.1	500	500	304	19.7	118.6	244	50	50	8.3	30.9	22	264	1.44	0.94	281.24
140	TG 242-4	4.1	500	500	304	19.6	118.3	244	50	50	8.2	35.1	23	266	1.46	0.93	285.22
141	TG 242-5	4.1	500	500	304	19.6	118.3	244	50	50	8	40.4	31	270	1.48	0.85	316.95
142	TG 242-6	4.1	500	500	304	19.6	118.4	244	50	50	8.1	60	45	285	1.56	0.77	372.16
143	R2	2.1	802	798	266	15.55	300.5	286	40	168	6.1	40	301	71	1.39	0.24	297.06
144	R4	2	800	798	266	5.07	120.4	285	40	162	4	60	88	45	1.6	0.44	101.94
145	R22 SS	2	800	800	266	15	300	295	40	168	6	60	336	68.5	1.47	0.23	299.02
146	R42 SS	2	800	800	266	5	120	285	40	162	6	60	88	42.5	1.52	0.42	101.98
147	A12 S	2	2500	800	300	15	300	295	40	160	6	60	332	80	1.6	0.40	199.35
148	A14 S	2	1200	800	300	15	300	295	40	160	6	60	332	78	1.56	0.46	170.17
149	A16 S	2	600	800	300	15	300	295	40	160	6	60	332	92	1.84	0.18	500.36
150	A22 S	3	2500	800	245	12	250	265	40	160	6	60	132	132.6	1.77	0.33	406.45
151	A24 S	3	1200	800	245	12	250	265	40	160	6	60	132	97.5	1.3	0.44	220.83
152	A26 S	3	600	680	245	12	250	265	40	160	6	60	132	121.4	1.62	0.30	410.26
153	A32 S	2	2200	680	354	5	120	290	40	136	4	40	104	45.8	1.41	0.21	221.74
154	A34 S	2	1020	680	354	5	120	290	40	136	4	40	104	54.4	1.68	0.53	103.48
155	A36 S	2	510	680	354	5	120	290	40	136	4	40	104	54.7	1.68	0.27	205.22
156	F3-6-2/1	3	500	500	242	5.9	150	308	50	-	-	-	-	107	1.92	1.13	94.83
157	F3-6-3/1	3	500	500	242	5.9	150	308	50	-	-	-	-	114	1.94	1.20	94.83

S.No.	Number in Reference	h (mm)	a (mm)	b (mm)	$f_{yw}$ (MPa)	$t_f$ (mm)	$b_f$ (mm)	$f_{yf}$ (MPa)	$s_s$ (mm)	c (mm)	$t_{st}$ (mm)	$b_{st}$ (mm)	$\gamma$	$P_{exp}$ (KN)	$P_{exp}/P_{pr}$ BS5400	$P_{exp}/P_{pr}$ proposed	$P_{pr}$ (KN)
158	MODEL 4	3.2	897	899	270	8	181.2	266	90	180	4.7	29.7	14	105.42	1.41	1.10	95.80
159	MODEL 5	3.2	892	901	270	8	180.4	266	90	180	4.7	38.2	26	110.36	1.48	0.93	118.30
160	P2	6	1780	1274	279	40	230	244	690	327	12	110	132	720	2.33	1.36	530.13
161	P3	6	1780	1274	286	40	230	267	690	264	12	110	132	730	2.2	1.28	568.27
162	EL1	6	1000	1000	325	9	300	235.2	300	200	6	80	44	438.2	2.22	0.91	478.91
163	TEST4	3.2	635	635	303	12.7	152.4	303	127	127	4.8	74.4	216	130	1.3	0.20	661.38
164	TEST5	3.2	635	635	275	6.35	152.4	275	127	127	4.8	74.4	216	85	1.12	0.13	661.38
<b>mean</b>														226.03	1.54	0.65	356.26
<b>standard deviation</b>														179.64	0.27	0.32	228.00
<b>coefficient of variation</b>														0.79	0.17	0.50	0.64

**TABLE-A2 Summary of Constants and Coefficients of k for Equation (16)**

S.No.	Number in Reference	$\beta$	$\delta$	$\frac{D}{=Et_w^3/12(1-\nu^2)}$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$\frac{2\beta^2(1-\alpha c/b)}{\sin\pi c/b}$	$\frac{2\beta^2(1-\alpha c/b)}{\sin 2\pi c/b}$	$\beta^2 16\alpha/9\pi^2$
1	Panel 2-C1	1.43	0.09	2403846	9.68	0.11	9.25	83.97	48.22	91.03	1.06	2.00	0.74
2	Panel 3-C2	1.43	0.09	2403846	9.68	0.18	9.25	83.97	77.68	131.54	1.40	2.36	0.74
3	Panel 4-C2	1.43	0.23	2403846	9.68	0.18	9.25	83.97	105.34	178.39	1.40	2.36	0.74
4	Panel 5-C3	1.43	0.23	2403846	9.68	0.11	9.25	83.97	65.40	123.45	1.06	2.00	0.74
5	Panel 6-C3	1.43	0.23	2403846	9.68	0.14	9.25	83.97	85.91	154.80	1.26	2.28	0.74
6	Panel 1-2	1.50	0.09	1230769	4.75	0.20	10.56	100.00	198.67	321.46	1.59	2.57	0.81
7	Panel 2-2	1.50	0.06	4153846	10.68	0.38	10.56	100.00	101.63	77.78	1.04	0.80	0.81
8	Panel 4-4	1.50	0.06	4153846	10.68	0.29	10.56	100.00	86.38	106.96	1.50	1.86	0.81
9	Panel 4-6	2.25	0.06	4153846	10.68	0.20	36.75	451.56	64.66	104.62	3.57	5.78	1.82
10	Panel 5-1	1.31	0.06	4153846	10.68	0.29	7.41	62.26	86.38	106.96	1.15	1.42	0.62
11	Panel 6-2	1.31	0.06	4153846	10.68	0.20	7.41	62.26	64.66	104.62	1.22	1.97	0.62
12	VT07-1	2.48	0.04	1055231	2.74	0.15	51.13	655.44	120.76	215.20	3.91	6.97	2.22
13	VT07-2	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
14	VT07-3	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
15	VT07-4	2.48	0.05	1055231	2.74	0.20	51.13	655.44	209.25	338.58	4.34	7.02	2.22
16	VT07-5	1.76	0.05	1055231	2.74	0.20	16.79	179.30	209.25	338.58	2.18	3.54	1.12
17	VT07-6	1.76	0.05	1055231	2.74	0.20	16.79	179.30	209.25	338.58	2.18	3.54	1.12
18	VT08-1	2.48	0.04	1055231	2.74	0.15	51.13	655.44	120.76	215.20	3.91	6.97	2.22
19	VT08-2	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
20	VT08-3	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
21	VT08-4	2.48	0.05	1055231	2.74	0.20	51.13	655.44	209.25	338.58	4.34	7.02	2.22
22	VT08-5	1.76	0.05	1055231	2.74	0.20	16.79	179.30	209.25	338.58	2.18	3.54	1.12
23	VT08-6	1.76	0.05	1055231	2.74	0.20	16.79	179.30	209.25	338.58	2.18	3.54	1.12



S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$2\beta^2(1-\alpha c/b) \sin\pi c/b$	$2\beta^2(1-\alpha c/b) \sin 2\pi c/b$	$\beta^2 16\alpha/9\pi^2$
24	VT09-1	2.48	0.04	1055231	2.74	0.15	51.13	655.44	120.76	215.20	3.91	6.97	2.22
25	VT09-2	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
26	VT09-3	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
27	VT09-4	2.48	0.05	1055231	2.74	0.15	51.13	655.44	161.62	288.01	3.91	6.97	2.22
28	VT09-5	1.76	0.05	1055231	2.74	0.15	16.79	179.30	161.62	288.01	1.97	3.51	1.12
29	VT09-6	1.76	0.05	1055231	2.74	0.15	16.79	179.30	161.62	288.01	1.97	3.51	1.12
30	VT10-1	2.48	0.04	1055231	2.74	0.15	51.13	655.44	120.76	215.20	3.91	6.97	2.22
31	VT10-2	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
32	VT10-3	1.76	0.04	1055231	2.74	0.15	16.79	179.30	120.76	215.20	1.97	3.51	1.12
33	VT10-4	2.48	0.05	1055231	2.74	0.15	51.13	655.44	161.62	288.01	3.91	6.97	2.22
34	VT10-5	1.76	0.05	1055231	2.74	0.15	16.79	179.30	161.62	288.01	1.97	3.51	1.12
35	VT10-6	1.76	0.05	1055231	2.74	0.15	16.79	179.30	161.62	288.01	1.97	3.51	1.12
36	731	4.08	0.16	519231	3.16	0.34	311.87	4575.02	252.44	243.03	9.34	8.99	6.00
37	732	1.50	0.16	519231	3.16	0.34	10.50	99.19	252.44	243.03	1.26	1.21	0.81
38	733	1.50	0.16	519231	3.16	0.34	10.50	99.19	252.44	243.03	1.26	1.21	0.81
39	734	4.08	0.16	519231	3.16	0.20	311.87	4575.02	172.26	276.10	11.79	18.90	6.00
40	735	1.50	0.16	519231	3.16	0.20	10.50	99.19	172.26	276.10	1.59	2.54	0.81
41	736	1.50	0.16	519231	3.16	0.20	10.50	99.19	172.26	276.10	1.59	2.54	0.81
42	TG 1-1	1.00	0.06	153846	2.98	0.50	4.00	25.00	14.00	0.44	0.02	0.00	0.36
43	TG 1-2	1.00	0.10	153846	2.98	0.50	4.00	25.00	53.99	1.68	0.02	0.00	0.36
44	TG 1-3	1.00	0.15	153846	2.98	0.50	4.00	25.00	153.98	4.79	0.02	0.00	0.36
45	TG 2-1	1.00	0.06	153846	2.98	0.20	4.00	25.00	8.16	13.26	0.70	1.14	0.36
46	TG 2-2	1.00	0.10	153846	2.98	0.20	4.00	25.00	31.47	51.15	0.70	1.14	0.36

S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$\frac{2\beta^2(1-\alpha c/b)}{\sin\pi c/b}$	$\frac{2\beta^2(1-\alpha c/b)}{\sin 2\pi c/b}$	$\beta^2 16\alpha/9\pi^2$
47	TG 2-3	1.00	0.15	153846	2.98	0.20	4.00	25.00	89.74	145.86	0.70	1.14	0.36
48	TG 3-1	1.00	0.06	153846	2.98	0.10	4.00	25.00	4.28	8.16	0.49	0.93	0.36
49	TG 3-1	1.00	0.06	153846	2.98	0.10	4.00	25.00	4.28	8.16	0.49	0.93	0.36
50	TG 3-2	1.00	0.10	153846	2.98	0.10	4.00	25.00	16.53	31.47	0.49	0.93	0.36
51	TG 3-2	1.00	0.10	153846	2.98	0.10	4.00	25.00	16.53	31.47	0.49	0.93	0.36
52	TG 3-3	1.00	0.15	153846	2.98	0.10	4.00	25.00	47.13	89.74	0.49	0.93	0.36
53	TG 3-3	1.00	0.15	153846	2.98	0.10	4.00	25.00	47.13	89.74	0.49	0.93	0.36
54	TG 11-1	2.00	0.06	153846	3.01	0.50	25.00	289.00	14.00	0.22	0.04	0.00	1.44
55	TG 11-2	2.00	0.10	153846	3.01	0.50	25.00	289.00	54.00	0.84	0.04	0.00	1.44
56	TG 11-3	2.00	0.15	153846	3.01	0.50	25.00	289.00	154.00	2.41	0.04	0.00	1.44
57	TG 12-1	2.00	0.06	153846	3.01	0.20	25.00	289.00	8.19	13.29	2.82	4.57	1.44
58	TG 12-2	2.00	0.10	153846	3.01	0.20	25.00	289.00	31.60	51.25	2.82	4.57	1.44
59	TG 12-3	2.00	0.15	153846	3.01	0.20	25.00	289.00	90.13	146.16	2.82	4.57	1.44
60	TG 13-1	2.00	0.06	153846	3.01	0.10	25.00	289.00	4.31	8.19	1.97	3.75	1.44
61	TG 13-1	2.00	0.06	153846	3.01	0.10	25.00	289.00	4.31	8.19	1.97	3.75	1.44
62	TG 13-2	2.00	0.10	153846	3.01	0.10	25.00	289.00	16.61	31.60	1.97	3.75	1.44
63	TG 13-2	2.00	0.10	153846	3.01	0.10	25.00	289.00	16.61	31.60	1.97	3.75	1.44
64	TG 13-3	2.00	0.15	153846	3.01	0.10	25.00	289.00	47.36	90.13	1.97	3.75	1.44
65	TG 13-3	2.00	0.15	153846	3.01	0.10	25.00	289.00	47.36	90.13	1.97	3.75	1.44
66	TG 31-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
67	TG 31-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
68	TG 31-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56
69	TG 31-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56

S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$2\beta^2(1-\alpha c/b) \sin\pi c/b$	$2\beta^2(1-\alpha c/b) \sin 2\pi c/b$	$\beta^2 16\alpha/9\pi^2$
70	TG 31-3	1.25	0.07	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
71	TG 31-3	1.25	0.07	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
72	TG 32-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
73	TG 32-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
74	TG 32-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56
75	TG 32-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56
76	TG 32-3	1.25	0.07	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
77	TG 32-3	1.25	0.07	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
78	TG 33-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
79	TG 33-1	1.25	0.07	4153846	27.33	0.40	6.50	51.84	41.85	25.86	0.59	0.36	0.56
80	TG 33-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56
81	TG 33-2	1.25	0.07	4153846	27.33	0.25	6.50	51.84	31.11	44.00	1.10	1.55	0.56
82	TG 33-3	1.25	0.07	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
83	TG 33-3	1.25	0.02	4153846	27.33	0.15	6.50	51.84	19.98	35.60	0.99	1.76	0.56
84	TG 021-0	1.00	0.14	265846	4.37	0.20	4.00	25.00	8.23	13.31	0.71	1.14	0.36
85	TG 021-1	1.00	0.19	204769	3.67	0.20	4.00	25.00	89.34	144.56	0.71	1.14	0.36
86	TG 021-2	1.00	0.23	204769	3.67	0.20	4.00	25.00	159.88	258.69	0.71	1.14	0.36
87	TG 021-3	1.00	0.15	204769	3.67	0.20	4.00	25.00	290.37	469.82	0.71	1.14	0.36
88	TG 022-1	1.00	0.21	204769	3.67	0.20	4.00	25.00	86.99	140.76	0.71	1.14	0.36
89	TG 022-2	1.00	0.23	204769	3.67	0.20	4.00	25.00	159.88	258.69	0.71	1.14	0.36
90	TG 022-3	1.00	0.08	204769	3.67	0.20	4.00	25.00	282.14	456.51	0.71	1.14	0.36
91	TG 041-0	1.00	0.09	1638154	14.70	0.20	4.00	25.00	3.53	5.71	0.71	1.14	0.36
92	TG 041-1	1.00	0.20	1230769	12.15	0.20	4.00	25.00	57.60	93.20	0.71	1.14	0.36

S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$2\beta^2(1-\alpha c/b) / \sin\pi c/b$	$2\beta^2(1-\alpha c/b) / \sin 2\pi c/b$	$\beta^2 16\alpha/9\pi^2$
93	TG 041-2	1.00	0.25	1230769	12.15	0.20	4.00	25.00	98.75	159.78	0.71	1.14	0.36
94	TG 041-3	1.00	0.17	1230769	12.15	0.20	4.00	25.00	166.93	270.10	0.71	1.14	0.36
95	TG 042-1	1.00	0.20	1230769	12.15	0.20	4.00	25.00	51.73	83.69	0.71	1.14	0.36
96	TG 042-2	1.00	0.25	1230769	12.15	0.20	4.00	25.00	48.20	77.99	0.71	1.14	0.36
97	TG 042-3	1.00	0.10	1230769	12.15	0.20	4.00	25.00	75.24	121.74	0.71	1.14	0.36
98	TG 061-0	1.00	0.09	3377231	23.81	0.20	4.00	25.00	5.88	9.51	0.71	1.14	0.36
99	TG 061-1	1.00	0.18	3377231	23.81	0.20	4.00	25.00	16.46	26.63	0.71	1.14	0.36
100	TG 061-2	1.00	0.24	3199519	22.97	0.20	4.00	25.00	62.31	100.81	0.71	1.14	0.36
101	TG 061-3	1.00	0.12	3199519	22.97	0.20	4.00	25.00	86.99	140.76	0.71	1.14	0.36
102	TG 062-1	1.00	0.18	3377231	23.81	0.20	4.00	25.00	17.63	28.53	0.71	1.14	0.36
103	TG 062-2	1.00	0.22	3377231	23.81	0.20	4.00	25.00	54.08	87.50	0.71	1.14	0.36
104	TG 062-3	1.00	0.06	3377231	23.81	0.20	4.00	25.00	86.99	140.76	0.71	1.14	0.36
105	TG 121-1	1.00	0.11	153846	3.04	0.20	4.00	25.00	19.98	32.34	0.71	1.14	0.36
106	TG 121-2	1.00	0.13	153846	3.04	0.20	4.00	25.00	34.09	55.16	0.71	1.14	0.36
107	TG 121-3	1.00	0.08	153846	3.04	0.20	4.00	25.00	57.60	93.20	0.71	1.14	0.36
108	TG 122-1	1.00	0.10	153846	3.04	0.20	4.00	25.00	17.63	28.53	0.71	1.14	0.36
109	TG 122-2	1.00	0.13	153846	3.04	0.20	4.00	25.00	34.09	55.16	0.71	1.14	0.36
110	TG 122-3	1.00	0.10	153846	3.04	0.20	4.00	25.00	58.78	95.11	0.71	1.14	0.36
111	TG 141-1	1.00	0.12	1230769	12.15	0.20	4.00	25.00	9.40	15.22	0.71	1.14	0.36
112	TG 141-2	1.00	0.14	1230769	12.15	0.20	4.00	25.00	28.21	45.65	0.71	1.14	0.36
113	TG 141-3	1.00	0.08	1230769	12.15	0.20	4.00	25.00	37.62	60.87	0.71	1.14	0.36
114	TG 142-1	1.00	0.12	1230769	12.15	0.20	4.00	25.00	9.40	15.22	0.71	1.14	0.36
115	TG 142-2	1.00	0.15	1230769	12.15	0.20	4.00	25.00	27.04	43.75	0.71	1.14	0.36

S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$2\beta^2(1-\alpha c/b) \sin\pi c/b$	$2\beta^2(1-\alpha c/b) \sin 2\pi c/b$	$\beta^2 16\alpha/9\pi^2$
116	TG 142-3	1.00	0.10	1230769	12.15	0.20	4.00	25.00	41.14	66.57	0.71	1.14	0.36
117	TG 161-1	1.00	0.11	3028154	22.14	0.20	4.00	25.00	10.58	17.12	0.71	1.14	0.36
118	TG 161-2	1.00	0.14	3028154	22.14	0.20	4.00	25.00	16.46	26.63	0.71	1.14	0.36
119	TG 161-3	1.00	0.10	3028154	22.14	0.20	4.00	25.00	23.51	38.04	0.71	1.14	0.36
120	TG 162-1	1.00	0.11	3028154	22.14	0.20	4.00	25.00	10.58	17.12	0.71	1.14	0.36
121	TG 162-2	1.00	0.14	3028154	22.14	0.20	4.00	25.00	16.46	26.63	0.71	1.14	0.36
122	TG 162-3	1.00	0.07	3028154	22.14	0.10	4.00	25.00	11.74	22.34	0.49	0.94	0.36
123	TG 241-1	1.00	0.04	1325404	12.76	0.10	4.00	25.00	1.85	3.53	0.49	0.94	0.36
124	TG 241-1	1.00	0.05	1325404	12.76	0.10	4.00	25.00	1.85	3.53	0.49	0.94	0.36
125	TG 241-2	1.00	0.08	1325404	12.76	0.10	4.00	25.00	4.94	9.40	0.49	0.94	0.36
126	TG 241-2	1.00	0.10	1325404	12.76	0.10	4.00	25.00	5.56	10.58	0.49	0.94	0.36
127	TG 241-3	1.00	0.10	1325404	12.76	0.10	4.00	25.00	8.65	16.46	0.49	0.94	0.36
128	TG 241-3	1.00	0.12	1325404	12.76	0.10	4.00	25.00	8.65	16.46	0.49	0.94	0.36
129	TG 241-4	1.00	0.12	1325404	12.76	0.10	4.00	25.00	13.60	25.86	0.49	0.94	0.36
130	TG 241-4	1.00	0.14	1325404	12.76	0.10	4.00	25.00	13.60	25.86	0.49	0.94	0.36
131	TG 241-5	1.00	0.16	1325404	12.76	0.10	4.00	25.00	19.16	36.44	0.49	0.94	0.36
132	TG 241-6	1.00	0.06	1325404	12.76	0.10	4.00	25.00	28.43	54.08	0.49	0.94	0.36
133	TG 242-1	1.00	0.04	1325404	12.76	0.10	4.00	25.00	1.85	3.53	0.49	0.94	0.36
134	TG 242-1	1.00	0.05	1325404	12.76	0.10	4.00	25.00	1.85	3.53	0.49	0.94	0.36
135	TG 242-2	1.00	0.08	1325404	12.76	0.10	4.00	25.00	4.94	9.40	0.49	0.94	0.36
136	TG 242-2	1.00	0.10	1325404	12.76	0.10	4.00	25.00	4.94	9.40	0.49	0.94	0.36
137	TG 242-3	1.00	0.10	1325404	12.76	0.10	4.00	25.00	8.03	15.28	0.49	0.94	0.36
138	TG 242-3	1.00	0.12	1325404	12.76	0.10	4.00	25.00	8.03	15.28	0.49	0.94	0.36

S.No.	Number in Reference	$\beta$	$\delta$	$D = Et_w^3/12(1-\nu^2)$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$2\beta^2(1-\alpha c/b) / \sin\pi c/b$	$2\beta^2(1-\alpha c/b) / \sin 2\pi c/b$	$\beta^2 16\alpha/9\pi^2$
139	TG 242-4	1.00	0.13	1325404	12.76	0.10	4.00	25.00	13.60	25.86	0.49	0.94	0.36
140	TG 242-4	1.00	0.14	1325404	12.76	0.10	4.00	25.00	14.21	27.04	0.49	0.94	0.36
141	TG 242-5	1.00	0.16	1325404	12.76	0.10	4.00	25.00	19.16	36.44	0.49	0.94	0.36
142	TG 242-6	1.00	0.24	1325404	12.76	0.10	4.00	25.00	27.81	52.90	0.49	0.94	0.36
143	R2	1.01	0.15	178096	1.31	0.21	4.04	25.40	369.76	583.58	0.72	1.13	0.36
144	R4	1.00	0.15	153846	1.19	0.20	4.02	25.20	104.79	168.38	0.71	1.14	0.36
145	R22 SS	1.00	0.23	153846	1.19	0.21	4.00	25.00	411.87	650.89	0.71	1.12	0.36
146	R42 SS	1.00	0.23	153846	1.19	0.20	4.00	25.00	104.57	168.22	0.71	1.14	0.36
147	A12 S	3.13	0.23	153846	1.19	0.20	115.90	1605.00	390.29	631.50	6.89	11.15	3.52
148	A14 S	1.50	0.23	153846	1.19	0.20	10.56	100.00	390.29	631.50	1.59	2.57	0.81
149	A16 S	0.75	0.23	153846	1.19	0.20	2.44	10.56	390.29	631.50	0.40	0.64	0.20
150	A22 S	3.13	0.15	519231	2.67	0.20	115.90	1605.00	155.18	251.08	6.89	11.15	3.52
151	A24 S	1.50	0.15	519231	2.67	0.20	10.56	100.00	155.18	251.08	1.59	2.57	0.81
152	A26 S	0.88	0.18	519231	3.69	0.24	3.16	16.93	177.86	262.87	0.56	0.82	0.28
153	A32 S	3.24	0.12	153846	1.64	0.20	131.50	1837.71	122.26	197.82	7.38	11.95	3.77
154	A34 S	1.50	0.12	153846	1.64	0.20	10.56	100.00	122.26	197.82	1.59	2.57	0.81
155	A36 S	0.75	0.12	153846	1.64	0.20	2.44	10.56	122.26	197.82	0.40	0.64	0.20
156	F3-6-2/1	1.00	0.00	519231	6.83	0.00	4.00	25.00	0.00	0.00	0.00	0.00	0.36
157	F3-6-3/1	1.00	0.00	519231	6.83	0.00	4.00	25.00	0.00	0.00	0.00	0.00	0.36
158	MODEL 4	1.00	0.05	630154	2.40	0.20	3.98	24.82	16.47	26.64	0.70	1.14	0.36
159	MODEL 5	0.99	0.06	630154	2.39	0.20	3.92	24.21	30.54	49.43	0.69	1.12	0.35
160	P2	1.40	0.17	4153846	4.21	0.26	8.71	77.59	190.55	263.77	1.37	1.90	0.70
161	P3	1.40	0.17	4153846	4.21	0.21	8.71	77.59	159.98	254.52	1.39	2.20	0.70

S.No.	Number in Reference	$\beta$	$\delta$	$\frac{D}{=Et_w^3/12(1-\nu^2)}$	$P_{pr}/k$	$c/b$	$(1+\beta^2)^2$	$(1+4\beta^2)^2$	$2\gamma\sin\pi c/b$	$2\gamma\sin 2\pi c/b$	$\frac{2\beta^2(1-\alpha c/b)}{\sin\pi c/b}$	$\frac{2\beta^2(1-\alpha c/b)}{\sin 2\pi c/b}$	$\beta^2 16\alpha/9\pi^2$
162	EL1	1.00	0.08	4153846	6.83	0.20	4.00	25.00	51.73	83.69	0.71	1.14	0.36
163	TEST4	1.00	0.18	630154	4.82	0.20	4.00	25.00	253.92	410.86	0.71	1.14	0.36
164	TEST5	1.00	0.2	630154	4.82	0.20	4.00	25.00	253.92	410.86	0.71	1.14	0.36

**TABLE-A3 Data Exchange Sheet for Excel-MATLAB link**

S.No.	Number in Reference	A	B	C	D'	G	H	I	J	k
1	Panel 2-C1	57.47	-1.06	48.22	-2.73	48.22	-1.79	174.99	-2.00	-69.33
2	Panel 3-C2	86.92	-1.40	77.68	-3.10	77.68	-2.13	215.51	-2.36	-78.94
3	Panel 4-C2	114.59	-1.40	105.34	-3.10	105.34	-2.13	262.36	-2.36	-89.90
4	Panel 5-C3	74.64	-1.06	65.40	-2.73	65.40	-1.79	207.42	-2.00	-77.75
5	Panel 6-C3	95.16	-1.26	85.91	-3.01	85.91	-2.00	238.77	-2.28	-85.22
6	Panel 1-2	209.23	-1.59	198.67	-3.38	198.67	-2.40	421.46	-2.57	-117.54
7	Panel 2-2	112.19	-1.04	101.63	-1.61	101.63	-1.85	177.78	-0.80	-51.35
8	Panel 4-4	96.95	-1.50	86.38	-2.67	86.38	-2.31	206.96	-1.86	-70.67
9	Panel 4-6	101.41	-3.57	64.66	-7.60	64.66	-5.39	556.18	-5.78	-108.59
10	Panel 5-1	93.80	-1.15	86.38	-2.04	86.38	-1.77	169.22	-1.42	-64.78
11	Panel 6-2	72.07	-1.22	64.66	-2.59	64.66	-1.84	166.88	-1.97	-71.39
12	VT07-1	171.89	-3.91	120.76	-9.18	120.76	-6.12	870.64	-6.97	-130.56
13	VT07-2	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
14	VT07-3	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
15	VT07-4	260.38	-4.34	209.25	-9.23	209.25	-6.55	994.02	-7.02	-143.97
16	VT07-5	226.04	-2.18	209.25	-4.65	209.25	-3.30	517.88	-3.54	-117.03
17	VT07-6	226.04	-2.18	209.25	-4.65	209.25	-3.30	517.88	-3.54	-117.03
18	VT08-1	171.89	-3.91	120.76	-9.18	120.76	-6.12	870.64	-6.97	-130.56
19	VT08-2	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
20	VT08-3	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
21	VT08-4	260.38	-4.34	209.25	-9.23	209.25	-6.55	994.02	-7.02	-143.97
22	VT08-5	226.04	-2.18	209.25	-4.65	209.25	-3.30	517.88	-3.54	-117.03
23	VT08-6	226.04	-2.18	209.25	-4.65	209.25	-3.30	517.88	-3.54	-117.03
24	VT09-1	171.89	-3.91	120.76	-9.18	120.76	-6.12	870.64	-6.97	-130.56
25	VT09-2	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
26	VT09-3	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
27	VT09-4	212.75	-3.91	161.62	-9.18	161.62	-6.12	943.45	-6.97	-136.92
28	VT09-5	178.41	-1.97	161.62	-4.62	161.62	-3.08	467.31	-3.51	-110.93
29	VT09-6	178.41	-1.97	161.62	-4.62	161.62	-3.08	467.31	-3.51	-110.93
30	VT10-1	171.89	-3.91	120.76	-9.18	120.76	-6.12	870.64	-6.97	-130.56
31	VT10-2	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
32	VT10-3	137.55	-1.97	120.76	-4.62	120.76	-3.08	394.50	-3.51	-99.07
33	VT10-4	212.75	-3.91	161.62	-9.18	161.62	-6.12	943.45	-6.97	-136.92
34	VT10-5	178.41	-1.97	161.62	-4.62	161.62	-3.08	467.31	-3.51	-110.93
35	VT10-6	178.41	-1.97	161.62	-4.62	161.62	-3.08	467.31	-3.51	-110.93
36	731	564.30	-9.34	252.44	-14.99	252.44	-15.34	4818.06	-8.99	-343.37



S.No.	Number in Reference	A	B	C	D'	G	H	I	J	k
37	732	262.93	-1.26	252.44	-2.02	252.44	-2.06	342.22	-1.21	-59.73
38	733	262.93	-1.26	252.44	-2.02	252.44	-2.06	342.22	-1.21	-59.73
39	734	484.12	-11.79	172.26	-24.91	172.26	-17.80	4851.12	-18.90	-302.69
40	735	182.75	-1.59	172.26	-3.35	172.26	-2.39	375.28	-2.54	-108.13
41	736	182.75	-1.59	172.26	-3.35	172.26	-2.39	375.28	-2.54	-108.13
42	TG 1-1	18.00	-0.02	14.00	-0.36	14.00	-0.38	25.44	0.00	-20.64
43	TG 1-2	57.99	-0.02	53.99	-0.36	53.99	-0.38	26.68	0.00	40.35
44	TG 1-3	157.98	-0.02	153.98	-0.36	153.98	-0.38	29.79	0.00	233.54
45	TG 2-1	12.16	-0.70	8.16	-1.50	8.16	-1.06	38.26	-1.14	-38.14
46	TG 2-2	35.47	-0.70	31.47	-1.50	31.47	-1.06	76.15	-1.14	-55.52
47	TG 2-3	93.74	-0.70	89.74	-1.50	89.74	-1.06	170.86	-1.14	-98.19
48	TG 3-1	8.28	-0.49	4.28	-1.29	4.28	-0.85	33.16	-0.93	-34.57
49	TG 3-1	8.28	-0.49	4.28	-1.29	4.28	-0.85	33.16	-0.93	-34.57
50	TG 3-2	20.53	-0.49	16.53	-1.29	16.53	-0.85	56.47	-0.93	-47.05
51	TG 3-2	20.53	-0.49	16.53	-1.29	16.53	-0.85	56.47	-0.93	-47.05
52	TG 3-3	51.13	-0.49	47.13	-1.29	47.13	-0.85	114.74	-0.93	-77.62
53	TG 3-3	51.13	-0.49	47.13	-1.29	47.13	-0.85	114.74	-0.93	-77.62
54	TG 11-1	39.00	-0.04	14.00	-1.44	14.00	-1.48	289.22	0.00	-65.50
55	TG 11-2	79.00	-0.04	54.00	-1.44	54.00	-1.48	289.84	0.00	-68.38
56	TG 11-3	179.00	-0.04	154.00	-1.44	154.00	-1.48	291.41	0.00	-51.82
57	TG 12-1	33.19	-2.82	8.19	-6.01	8.19	-4.26	302.29	-4.57	-81.84
58	TG 12-2	56.60	-2.82	31.60	-6.01	31.60	-4.26	340.25	-4.57	-86.73
59	TG 12-3	115.13	-2.82	90.13	-6.01	90.13	-4.26	435.16	-4.57	-98.49
60	TG 13-1	29.31	-1.97	4.31	-5.19	4.31	-3.41	297.19	-3.75	-75.03
61	TG 13-1	29.31	-1.97	4.31	-5.19	4.31	-3.41	297.19	-3.75	-75.03
62	TG 13-2	41.61	-1.97	16.61	-5.19	16.61	-3.41	320.60	-3.75	-78.60
63	TG 13-2	41.61	-1.97	16.61	-5.19	16.61	-3.41	320.60	-3.75	-78.60
64	TG 13-3	72.36	-1.97	47.36	-5.19	47.36	-3.41	379.13	-3.75	-87.20
65	TG 13-3	72.36	-1.97	47.36	-5.19	47.36	-3.41	379.13	-3.75	-87.20
66	TG 31-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89
67	TG 31-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89
68	TG 31-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
69	TG 31-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
70	TG 31-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
71	TG 31-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
72	TG 32-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89

S.No.	Number in Reference	A	B	C	D'	G	H	I	J	k
73	TG 32-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89
74	TG 32-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
75	TG 32-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
76	TG 32-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
77	TG 32-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
78	TG 33-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89
79	TG 33-1	48.35	-0.59	41.85	-0.92	41.85	-1.15	77.70	-0.36	-36.89
80	TG 33-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
81	TG 33-2	37.62	-1.10	31.11	-2.11	31.11	-1.65	95.84	-1.55	-53.44
82	TG 33-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
83	TG 33-3	26.48	-0.99	19.98	-2.31	19.98	-1.54	87.44	-1.76	-50.69
84	TG 021-0	12.23	-0.71	8.23	-1.50	8.23	-1.07	38.31	-1.14	-38.18
85	TG 021-1	93.34	-0.71	89.34	-1.50	89.34	-1.07	169.56	-1.14	-97.33
86	TG 021-2	163.88	-0.71	159.88	-1.50	159.88	-1.07	283.69	-1.14	-148.26
87	TG 021-3	294.37	-0.71	290.37	-1.50	290.37	-1.07	494.82	-1.14	-242.36
88	TG 022-1	90.99	-0.71	86.99	-1.50	86.99	-1.07	165.76	-1.14	-95.63
89	TG 022-2	163.88	-0.71	159.88	-1.50	159.88	-1.07	283.69	-1.14	-148.26
90	TG 022-3	286.14	-0.71	282.14	-1.50	282.14	-1.07	481.51	-1.14	-236.43
91	TG 041-0	7.53	-0.71	3.53	-1.50	3.53	-1.07	30.71	-1.14	-34.61
92	TG 041-1	61.60	-0.71	57.60	-1.50	57.60	-1.07	118.20	-1.14	-74.35
93	TG 041-2	102.75	-0.71	98.75	-1.50	98.75	-1.07	184.78	-1.14	-104.12
94	TG 041-3	170.93	-0.71	166.93	-1.50	166.93	-1.07	295.10	-1.14	-153.35
95	TG 042-1	55.73	-0.71	51.73	-1.50	51.73	-1.07	108.69	-1.14	-70.09
96	TG 042-2	52.20	-0.71	48.20	-1.50	48.20	-1.07	102.99	-1.14	-67.53
97	TG 042-3	79.24	-0.71	75.24	-1.50	75.24	-1.07	146.74	-1.14	-87.13
98	TG 061-0	9.88	-0.71	5.88	-1.50	5.88	-1.07	34.51	-1.14	-36.40
99	TG 061-1	20.46	-0.71	16.46	-1.50	16.46	-1.07	51.63	-1.14	-44.32
100	TG 061-2	66.31	-0.71	62.31	-1.50	62.31	-1.07	125.81	-1.14	-77.76
101	TG 061-3	90.99	-0.71	86.99	-1.50	86.99	-1.07	165.76	-1.14	-95.63
102	TG 062-1	21.63	-0.71	17.63	-1.50	17.63	-1.07	53.53	-1.14	-45.19
103	TG 062-2	58.08	-0.71	54.08	-1.50	54.08	-1.07	112.50	-1.14	-71.80
104	TG 062-3	90.99	-0.71	86.99	-1.50	86.99	-1.07	165.76	-1.14	-95.63
105	TG 121-1	23.98	-0.71	19.98	-1.50	19.98	-1.07	57.34	-1.14	-46.92
106	TG 121-2	38.09	-0.71	34.09	-1.50	34.09	-1.07	80.16	-1.14	-57.26
107	TG 121-3	61.60	-0.71	57.60	-1.50	57.60	-1.07	118.20	-1.14	-74.35
108	TG 122-1	21.63	-0.71	17.63	-1.50	17.63	-1.07	53.53	-1.14	-45.19

S.No.	Number in Reference	A	B	C	D'	G	H	I	J	k
109	TG 122-2	38.09	-0.71	34.09	-1.50	34.09	-1.07	80.16	-1.14	-57.26
110	TG 122-3	62.78	-0.71	58.78	-1.50	58.78	-1.07	120.11	-1.14	-75.21
111	TG 141-1	13.40	-0.71	9.40	-1.50	9.40	-1.07	40.22	-1.14	-39.07
112	TG 141-2	32.21	-0.71	28.21	-1.50	28.21	-1.07	70.65	-1.14	-52.97
113	TG 141-3	41.62	-0.71	37.62	-1.50	37.62	-1.07	85.87	-1.14	-59.83
114	TG 142-1	13.40	-0.71	9.40	-1.50	9.40	-1.07	40.22	-1.14	-39.07
115	TG 142-2	31.04	-0.71	27.04	-1.50	27.04	-1.07	68.75	-1.14	-52.11
116	TG 142-3	45.14	-0.71	41.14	-1.50	41.14	-1.07	91.57	-1.14	-62.40
117	TG 161-1	14.58	-0.71	10.58	-1.50	10.58	-1.07	42.12	-1.14	-39.95
118	TG 161-2	20.46	-0.71	16.46	-1.50	16.46	-1.07	51.63	-1.14	-44.32
119	TG 161-3	27.51	-0.71	23.51	-1.50	23.51	-1.07	63.04	-1.14	-49.52
120	TG 162-1	14.58	-0.71	10.58	-1.50	10.58	-1.07	42.12	-1.14	-39.95
121	TG 162-2	20.46	-0.71	16.46	-1.50	16.46	-1.07	51.63	-1.14	-44.32
122	TG 162-3	15.74	-0.49	11.74	-1.30	11.74	-0.85	47.34	-0.94	-42.20
123	TG 241-1	5.85	-0.49	1.85	-1.30	1.85	-0.85	28.53	-0.94	-32.04
124	TG 241-1	5.85	-0.49	1.85	-1.30	1.85	-0.85	28.53	-0.94	-32.04
125	TG 241-2	8.94	-0.49	4.94	-1.30	4.94	-0.85	34.40	-0.94	-35.26
126	TG 241-2	9.56	-0.49	5.56	-1.30	5.56	-0.85	35.58	-0.94	-35.89
127	TG 241-3	12.65	-0.49	8.65	-1.30	8.65	-0.85	41.46	-0.94	-39.06
128	TG 241-3	12.65	-0.49	8.65	-1.30	8.65	-0.85	41.46	-0.94	-39.06
129	TG 241-4	17.60	-0.49	13.60	-1.30	13.60	-0.85	50.86	-0.94	-44.07
130	TG 241-4	17.60	-0.49	13.60	-1.30	13.60	-0.85	50.86	-0.94	-44.07
131	TG 241-5	23.16	-0.49	19.16	-1.30	19.16	-0.85	61.44	-0.94	-49.67
132	TG 241-6	32.43	-0.49	28.43	-1.30	28.43	-0.85	79.08	-0.94	-58.94
133	TG 242-1	5.85	-0.49	1.85	-1.30	1.85	-0.85	28.53	-0.94	-32.04
134	TG 242-1	5.85	-0.49	1.85	-1.30	1.85	-0.85	28.53	-0.94	-32.04
135	TG 242-2	8.94	-0.49	4.94	-1.30	4.94	-0.85	34.40	-0.94	-35.26
136	TG 242-2	8.94	-0.49	4.94	-1.30	4.94	-0.85	34.40	-0.94	-35.26
137	TG 242-3	12.03	-0.49	8.03	-1.30	8.03	-0.85	40.28	-0.94	-38.43
138	TG 242-3	12.03	-0.49	8.03	-1.30	8.03	-0.85	40.28	-0.94	-38.43
139	TG 242-4	17.60	-0.49	13.60	-1.30	13.60	-0.85	50.86	-0.94	-44.07
140	TG 242-4	18.21	-0.49	14.21	-1.30	14.21	-0.85	52.04	-0.94	-44.70
141	TG 242-5	23.16	-0.49	19.16	-1.30	19.16	-0.85	61.44	-0.94	-49.67
142	TG 242-6	31.81	-0.49	27.81	-1.30	27.81	-0.85	77.90	-0.94	-58.32
143	R2	373.80	-0.72	369.76	-1.50	369.76	-1.08	608.98	-1.13	-283.21
144	R4	108.81	-0.71	104.79	-1.50	104.79	-1.07	193.58	-1.14	-107.15

S.No.	Number in Reference	A	B	C	D'	G	H	I	J	k
145	R22 SS	415.87	-0.71	411.87	-1.48	411.87	-1.07	675.89	-1.12	-315.09
146	R42 SS	108.57	-0.71	104.57	-1.50	104.57	-1.07	193.22	-1.14	-107.46
147	A12 S	506.19	-6.89	390.29	-14.66	390.29	-10.41	2236.51	-11.15	-210.06
148	A14 S	400.85	-1.59	390.29	-3.38	390.29	-2.40	731.50	-2.57	-179.31
149	A16 S	392.73	-0.40	390.29	-0.84	390.29	-0.60	642.06	-0.64	-527.24
150	A22 S	271.07	-6.89	155.18	-14.66	155.18	-10.41	1856.08	-11.15	-190.35
151	A24 S	165.74	-1.59	155.18	-3.38	155.18	-2.40	351.08	-2.57	-103.42
152	A26 S	181.02	-0.56	177.86	-1.10	177.86	-0.84	279.80	-0.82	-163.32
153	A32 S	253.75	-7.38	122.26	-15.72	122.26	-11.15	2035.53	-11.95	-198.61
154	A34 S	132.82	-1.59	122.26	-3.38	122.26	-2.40	297.82	-2.57	-92.69
155	A36 S	124.70	-0.40	122.26	-0.84	122.26	-0.60	208.38	-0.64	-183.81
156	F3-6-2/1	4.00	0.00	0.00	-0.36	0.00	-0.36	25.00	0.00	-27.76
157	F3-6-3/1	4.00	0.00	0.00	-0.36	0.00	-0.36	25.00	0.00	-27.76
158	MODEL 4	20.46	-0.70	16.47	-1.49	16.47	-1.06	51.46	-1.14	-44.31
159	MODEL 5	34.46	-0.69	30.54	-1.47	30.54	-1.04	73.64	-1.12	-54.84
160	P2	199.26	-1.37	190.55	-2.60	190.55	-2.07	341.36	-1.90	-98.84
161	P3	168.70	-1.39	159.98	-2.91	159.98	-2.09	332.11	-2.20	-105.96
162	EL1	55.73	-0.71	51.73	-1.50	51.73	-1.07	108.69	-1.14	-70.09
163	TEST4	257.92	-0.71	253.92	-1.50	253.92	-1.07	435.86	-1.14	-216.09
164	TEST5	257.92	-0.71	253.92	-1.50	253.92	-1.07	435.86	-1.14	-216.09

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