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NUMERICAL ANALYSIS OF THE SINGLE STEP DISCONTINUITY IN THE TWO DIMENSIONAL DIELECTRIC WAVE GUIDE

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Abstract—The two dimensional step discontinuity of the junction of two different dielectric rectangular waveguides has been solved using integral equation arising from the field matching of the discrete modes and the continuous spectrum. Accurate numerical solution has been obtained using Ritz-Galerkin variational approach with appropriate sets of expanding functions. The results in the form of scattering parameters for varying step ratio's have been depicted graphically. Computed results from generalized integral expressions are found to be in excellent agreement with results obtained in one dimensional case.

1. INTRODUCTION

Step discontinuities in planar dielectric waveguides are commonly used in integrated circuits ranging from sub-millimeter to optical frequencies. The step discontinuity in dielectric slab guides is, in fact, a basic one for several components such as distributed feedback lasers, gratings, transformers, antenna feeds, and others. It is thus important to have accurate and reliable theoretical predictions of the behavior of this discontinuity. The need for theoretical models is made more acute by the difficulties of tuning integrated components once they are built. In addition, theoretical understanding of the **scattering properties** of a single step discontinuity provides considerable insight into the role played by the continuous spectrum in discontinuity problems. Because of the unboundedness of the structure and the presence of continuous spectrum, the analysis of the discontinuity is more difficult than that of closed waveguides. For problems with step discontinuity, some authors have replaced the unbounded configuration by bounded [1]. Rozzi [2] presented a rigorous analysis of arbitrarily large steps based on Ritz-Galerkin approach with appropriate sets of expanding functions. In an another approach a novel integration formulation [3] is derived for handling the discontinuity problem. In all these analysis they have used one dimensional structure with same refractive index distribution for waveguide I & II (fig. 1). In this paper we have extended the analysis for two dimensional case based on the rigorous analysis of Rozzi and also calculated the scattering parameters for different refractive index ratio.

2. ANALYSIS OF DISCONTINUITY

The diffraction problem at an abrupt discontinuity can be solved by using a field matching technique, which requires field description on either side of the discontinuity in terms of modes. The complete field propagating in an open slab waveguide can be resolved into a finite set of surface wave modes and a continuum of radiative modes. In the following we considered a two dimensional dielectric waveguide, excited by transverse electric (TE) waves with the transverse field components E_y & H_x. The two dimensional discontinuity structure considered is shown in Figure 1. Because of the symmetry about z-axis we have shown only half portion. The refractive index distribution of waveguide I ($z \leq 0$) is n_1 & waveguide II ($z \geq 0$) is n_3 and outside the guide is n_2. Here $n_1^2 = \varepsilon_1$, $n_2^2 = \varepsilon_2$, $n_3^2 = \varepsilon_3$ are dielectric constants of dielectric waveguide I, air, and waveguide II, respectively.

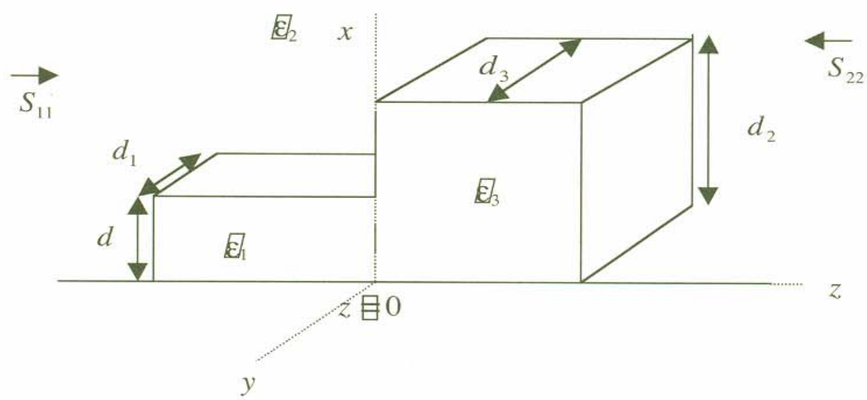

Figure 1. Two dimensional dielectric waveguide with single step discontinuity.

2.1. Scattering Matrix Formulation (TE Case)

For a TE mode excitation with variation along the x-y direction, all five components exist and they are related as shown in Figure 1.

$$E_x = 0 \qquad H_x = \frac{1}{j\omega\mu}\left(\frac{\partial}{\partial x^2} + \omega^2\mu\varepsilon\right)\varphi \tag{1a}$$

$$E_y = -\frac{\partial\varphi}{\partial z} \qquad H_y = \frac{1}{j\omega\mu}\frac{\partial^2\varphi}{\partial x\partial y} \tag{1b}$$

$$E_z = \frac{\partial\varphi}{\partial y} \qquad H_z = \frac{1}{j\omega\mu}\frac{\partial^2\varphi}{\partial x\partial y} \tag{1c}$$

These field components are related as

$$H_x(x, y, z) = \frac{1}{j\omega\mu}\frac{\partial}{\partial z}E_y(x, y, z) = \frac{\beta}{\omega\mu}E_y(x, y, z) \tag{1d}$$

Where ω, μ, β are angular frequency, permeability and propagation constant respectively. $E_y(x, y, z)$ may be expressed as a modal expression

$$E_y = \left\{\sum_k a_k(x, y)\varphi_k(x, y) + \int_0^\infty\int_0^\infty b(k_x, k_y)\phi(x, k_x : y, k_y)dk_x dk_y\right\} e^{j(\omega t - \beta z)} \tag{2}$$

$a_k(x, y)$ & $b(k_x, k_y)$ are unknown amplitudes of the surface and continuum modes respectively. In the following we consider the problem for the even TE case.

Surface wave modes

The surface mode function $\varphi_k(x,y)$ has been expressed for a symmetrical waveguide with dimensions d & d_1 as

$$\begin{aligned}\varphi(x,y) = A\cos\kappa x\cos\kappa_1 y \quad &: x \le d,\ y \le d_1\\ A\cos\kappa d e^{\gamma(x-d)}\cos\kappa_1 d_1 e^{\gamma_1(y-d_1)} \quad &: x \ge d,\ y \ge d_1\end{aligned} \tag{3}$$

Where A is the normalization constant such that

$$\int_0^\infty\int_0^\infty \varphi^2(x,y)dxdy = 1 \tag{4}$$

$$A = \sqrt{\frac{4}{dd_1 + \frac{1}{\gamma\gamma_1}}} \tag{5}$$

Furthermore, the transverse propagation constants and wavenumbers $\gamma, \gamma_1, \kappa, \kappa_1$ satisfy the eigen value equations

$$\begin{aligned}\kappa\tan\kappa d &= \gamma\\ \kappa_1\tan\kappa_1 d_1 &= \gamma_1\end{aligned} \tag{6}$$

as well as the conservation of wave numbers with k_0 free space wave number

$$\begin{aligned}-\gamma^2 - \gamma_1^2 + \beta^2 &= n_2^2 k_0^2 \quad \text{in air}\\ \kappa^2 + \kappa_1^2 + \beta^2 &= n_1^2 k_0^2 \quad \text{in the slab waveguide}\end{aligned} \tag{7}$$

Continuous modes

The normalized mode functions pertaining to the continuum are given as

$$\begin{aligned}\phi(x,y) = \sqrt{\frac{2}{\pi}}\frac{1}{c}\cos qx\sqrt{\frac{2}{\pi}}\frac{1}{c}\cos q_1 y \quad &\text{in the slab } (x \le d,\ y \le d_1)\\ \sqrt{\frac{2}{\pi}}\cos[k_x(x-d)+\alpha]\sqrt{\frac{2}{\pi}}\cos[k_y(y-d_1)+\alpha_1] \quad &\text{in the air}\end{aligned} \tag{8}$$

Where

$$c = \sqrt{1 + \frac{\nu^2}{k_x^2}\sin^2(qd)} \qquad c_1 = \sqrt{1 + \frac{\nu_1^2}{k_y^2}\sin^2(q_1 d_1)} \tag{9}$$

$$\tan\alpha = \frac{q}{k_x}\tan qd \qquad \tan\alpha_1 = \frac{q_1}{k_x}\tan q_1 d_1$$

normalized or effective frequencies are

$$\nu = \sqrt{(n_1^2 - n_2^2)}k_0 d \;\&\; \nu_1 = \sqrt{(n_1^2 - n_2^2)}k_0 d_1$$

The transverse wave numbers q, q_1, k_x and k_y satisfy the relations

$$\beta^2 = n_1^2 k_0^2 - q^2 - q_1^2 = n_2^2 k_0^2 - k_x^2 - k_y^2 \qquad (10)$$

Clearly, for $0 < k_x^2 + k_y^2 < n_2^2 k_0^2$, β is real and positive and the continuum modes in this range propagate to give the radiation modes, otherwise β is negative and imaginary, and modes are evanescent, thus representing the reactive part of the continuum.

Surface wave modes

The surface mode function $\varphi_k(x, y)$ has been expressed for a symmetrical waveguide with dimensions d_2 & d_3 as

$$\begin{aligned} \varphi(x,y) &= A_1 \cos\kappa_2 x \cos\kappa_3 y && : x \le d_2,\ y \le d_3 \\ A_1 \cos\kappa_2 d_2 e^{\gamma_2(x-d_2)} &\cos\kappa_3 d_3 e^{\gamma_3(y-d_3)} && : x \ge d_2,\ y \ge d_3 \end{aligned} \qquad (11)$$

Where A_1 is the normalization constant such that

$$\int_0^\infty \int_0^\infty \varphi^2(x,y)\,dx\,dy = 1 \qquad (12)$$

$$A_1 = \sqrt{\frac{4}{d_2 d_3 + \dfrac{1}{\gamma_2\gamma_3}}} \qquad (13)$$

Furthermore, the transverse propagation constants and wavenumbers $\gamma_2, \gamma_3, \kappa_2, \kappa_3$ satisfy the eigen value equations

$$\begin{aligned} \kappa_2 \tan\kappa_2 d_2 &= \gamma_2 \\ \kappa_3 \tan\kappa_3 d_3 &= \gamma_3 \end{aligned} \qquad (14)$$

as well as the conservation of wave numbers

$$\begin{aligned} -\gamma_2^2 - \gamma_3^2 + \beta_1^2 &= n_2^2 k_0^2 \quad \text{in air} \\ \kappa_2^2 + \kappa_3^2 + \beta_1^2 &= n_3^2 k_0^2 \quad \text{in the slab waveguide} \end{aligned} \qquad (15)$$

Continuous modes

The normalized mode functions pertaining to the continuum are given as

$$\phi(x,y) = \sqrt{\frac{2}{\pi}}\frac{1}{c_2}\cos q_2 x\sqrt{\frac{2}{\pi}}\frac{1}{c_3}\cos q_3 y \quad \text{in the slab } (x \le d_2,\ y \le d_3)$$
$$\sqrt{\frac{2}{\pi}}\cos[k_x(x-d_2)+\alpha_2]\sqrt{\frac{2}{\pi}}\cos[k_y(y-d_3)+\alpha_3] \text{ in the air} \tag{16}$$

Where

$$c_2 = \sqrt{1+\frac{\nu_2^2}{k_x^2}\sin^2(q_2 d_2)} \qquad c_3 = \sqrt{1+\frac{\nu_3^2}{k_y^2}\sin^2(q_3 d_3)} \tag{17}$$

$$\tan\alpha_2 = \frac{q_2}{k_x}\tan q_2 d_2 \qquad \tan\alpha_3 = \frac{q_3}{k_y}\tan q_3 d_3$$

normalized or effective frequencies are

$$\nu_2 = \sqrt{(n_3^2 - n_2^2)}k_0 d_2 \ \&\ \nu_3 = \sqrt{(n_3^2 - n_2^2)}k_0 d_3$$

The transverse wave numbers q_2, q_3, k_x and k_y satisfy the relations

$$\beta_1^2 = n_3^2 k_0^2 - q_2^2 - q_3^2 = n_2^2 k_0^2 - k_x^2 - k_y^2 \tag{18}$$

Clearly, for $0 < k_x^2 + k_y^2 < n_2^2 k_0^2$, β_1 is real and positive and the continuum modes in this range propagate to give the radiation modes, otherwise β_1 is negative and imaginary, and modes are evanescent, thus representing the reactive part of the continuum.

2.2. Scattering Formulation for the Single Step Discontinuity, Magnetic Field Formulation

Let us consider a steady-state and source free problem, with two different semi-infinite two dimensional waveguides forming a step discontinuity at $z = 0$ (Fig. 1). The incident field considered here will be composed of surface waves only. For now, let us assume that there are n_j surface modes which are capable of propagating in guide I (left), and n_r surface modes which can propagate in guide II (right), with the total number of propagating surface waves given by

$$n_i = n_j + n_r \tag{19}$$

Continuity of electric field $E_y(x, y, z)$ and the magnetic field $H_x(x, y, z)$ at $z = 0$ is expressed as

$$\begin{aligned} &E_y(x, y, 0) \\ &= \sum_{k=1}^{n_j}(V_k^i + V_k^r)\varphi_k(x, y) + \int_0^\infty\int_0^\infty V^I(k_x, k_y)\phi^I(x; k_x, y; k_y)dk_x dk_y \\ &= \sum_{k=n_j+1}^{n_i}(V_k^i + V_k^r)\varphi_k(x, y) + \int_0^\infty\int_0^\infty V^{II}(k_x, k_y)\phi^{II}(x; k_x, y; k_y)dk_x dk_y \end{aligned} \tag{20}$$

$$\begin{aligned} &H_x(x, y, 0) \\ &= \sum_{k=1}^{n_j} -\frac{1}{z_k}(V_k^i - V_k^r)\varphi_k(x, y) + \int_0^\infty\int_0^\infty \frac{V^I(k_x, k_y)\phi^I(x; k_x, y; k_y)}{Z^I(k_x, k_y)}dk_x dk_y \\ &= \sum_{k=n_j+1}^{n_i} -\frac{1}{z_k}(-V_k^i + V_k^r)\varphi_k(x, y) - \int_0^\infty\int_0^\infty \frac{V^{II}(k_x, k_y)\phi^{II}(x; k_x, y; k_y)}{Z^{II}(k_x, k_y)}dk_x dk_y \end{aligned} \tag{21}$$

Since a scattering formulation is sought, the incident V_k^i and reflected V_k^r surface wave amplitudes are made explicit where as $V^I(k_x, k_y)$ and $V^{II}(k_x, k_y)$ represent the amplitudes of the (scattered) continuum fields in guide I and II, respectively. It is interesting to note that matching of magnetic fields $H_x(x, y, z)$ at $z = 0$ is equivalent to matching the gradient of electric fields $E_y(x, y, z)$ at $z = 0$. By using orthogonality of the modal functions in (21), one may express the unknown modal amplitudes in terms of the magnetic field $H_x(x, y, 0)$ as

$$V_k^r = V_k^i + s_k z_k \int_0^\infty\int_0^\infty \varphi_k(x, y)H_x(x, y, 0)dxdy \tag{22}$$

$$V^{II}(k_x, k_y) = \int_0^\infty\int_0^\infty \phi^{II}(x; k_x, y; k_y)z(k_x, k_y)H_x(x, y, 0)dxdy \tag{23}$$

where

$$\begin{aligned} s_k &= 1 \quad k < n_j \\ &= -1 \quad k > n_j \end{aligned} \tag{24}$$

Upon substituting the above expressions into Eq. (20) and rearranging, one obtains

$$\sum_{k=1}^{n_j} s_k V_k^i \varphi_k(x,y) = \int_0^\infty \int_0^\infty Z(x,y,x',y') H_x(x',y') dx' dy' \qquad (25)$$

where

$$Z(x,y,x',y') = \frac{1}{2} \sum_{k=1}^{n_i} z_k \varphi_k(x,y) \varphi_k(x',y')$$
$$+\frac{1}{2} \int_0^\infty \int_0^\infty \left[\begin{array}{l} Z^I(k_x,k_y)\phi^I(x;k_x,y;k_y)\phi^I(x';k_x,y';k_y)+ \\ Z^{II}(k_x,k_y)\phi^{II}(x;k_x,y;k_y)\phi^{II}(x';k_x,y';k_y) \end{array} \right] dk_x dk_y \quad (26)$$

The summation on the left hand side of Eq. (25) represents the total incident electric field impinging on either side of the discontinuity. The term $Z(x,y,x',y')$ is a Green's function witch may be viewed as a "impedance" of the step discontinuity, $[G(x,x')$ represents the impedance at some point x caused by the unit impulse source at the point $x']$ then the scattered field due to magnetic field $H_x(x',y')$ over entire range 0 to infinity is given by above formula. The linear relationship between scattered (reflected and transmitted) and incident modal amplitudes in Eq. (22) suggests that we may obtain a scattering matrix formulation of the form

$$V^r = SV^i \qquad (27)$$

Let first consider the case of single surface wave incident on the discontinuity. Since the amplitude of the incident surface mode is arbitrary, it is possible to set

$$V_j^i = 1 \quad V_{k\neq j}^i = 0$$

and Eq. (27) reduces to

$$V_k^r = S_{kj} \qquad (28)$$

Let the corresponding scattered magnetic field be $h_j(x,y)$ then according to (25) we have

$$s_j V_k^i \varphi_j(x,y) = \int_0^\infty \int_0^\infty Z(x,y,x',y') h_j(x',y') dx' dy' \qquad (29)$$

In the above equation, $h_j(x,y)$ is the unknown function to be determined by discretization of Eq. (29) by means of a Ritz-Galerkin procedure. In this procedure an orthonormal basis functions set, $f(x,y)$, is introduced in the interval $0 < x,\ y <$ infinity and the magnetic field is represented as

$$h_j(x,y) = \sum_{n=0}^{\infty} \lambda_{nj} f_n(x,y) \tag{30}$$

By using the above Equation in (29) and testing the latter eq. With the weight functions $f_k(x,y)$, we obtain

$$s_j V_j^i \int_0^{\infty}\int_0^{\infty} f_k(x,y)\varphi_j(x,y)dxdy$$
$$= \sum_{n=0}^{\infty} \lambda_{nj} \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} f_k(x,y)Z(x,y,x',y')f_n(x,y)dxdydx'dy' \tag{31}$$

$$Q_{kj} = \int_0^{\infty}\int_0^{\infty} f_k(x,y)\varphi_j(x,y) \tag{32}$$

$$Z_{kn} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} f_k(x,y)Z(x,y,x',y')f_n(x,y)dxdydx'dy' \tag{33}$$

And of the vectors

$$Q_j = \begin{bmatrix} Q_{1j} \\ Q_{2j} \\ \cdot \\ \cdot \\ Q_{nj} \end{bmatrix} \qquad \lambda_j = \begin{bmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \cdot \\ \cdot \\ \lambda_{nj} \end{bmatrix} \tag{34}$$

can be written in matrix form as

$$\lambda_j = s_j Z^{-1} Q_j \tag{35}$$

The scattering matrix is obtained from (22), (28), (35) as

$$\begin{aligned} S_{kj} &= \delta_{kj} + s_k z_k \int_0^{\infty}\int_0^{\infty} \varphi_k(x,y) \sum_{n=0}^{\infty} \lambda_{nj} f_n(x,y) dxdy \\ &= \delta_{kj} + s_k s_j z_k Q_k^T Z^{-1} Q_j \end{aligned} \tag{36}$$

where δ_{kj} is the kronecker delta and Q_k^T denotes transposition of Q_k matrix.

Giving the transformation ratio

$$n = \frac{1}{\sqrt{z_k}} \tag{38}$$

where z_k is the characteristic impedance of the k-th surface mode. This means that the new incident $(\overline{V}^i)$ and reflected $(\overline{V}^r)$ wave amplitudes are related to the old values by the linear transformation.

$$\overline{V}^i = \frac{V^i}{n} = V^i Z^{-\frac{1}{2}} \tag{39}$$

$$\overline{V}^r = \frac{V^r}{n} = V^r Z^{-\frac{1}{2}} \tag{40}$$

Which by using $V^r = SV^i$ implies

$$\overline{V}^r Z^{\frac{1}{2}} = S Z^{\frac{1}{2}} \overline{V}^i \tag{41}$$

$$\overline{V}^r = Z^{-\frac{1}{2}} S Z^{\frac{1}{2}} \overline{V}^i \tag{42}$$

The normalized scattering matrix is obtained by inserting above equations into $V^r = SV^i$ Thus giving

$$\overline{S} = Z^{-\frac{1}{2}} S Z^{\frac{1}{2}} \tag{43}$$

$$\overline{V}^r = \overline{S}\,\overline{V}^i \tag{44}$$

or in the form of (36) as

$$\overline{S}_{kj} = \delta_{kj} + s_k s_j \sqrt{z_k z_j} Q_k^T Z^{-1} Q_j \tag{45}$$

which now displays the symmetry required by the reciprocity of the junction.

Owing to the orthogonality of modes, the scattering formulation between continuous modes and surface modes can be easily derived

$$S_{j,k'_x,k'_y} = s_k s'_k \sqrt{z_j z(k'_x, k'_y)} Q_j^T Z^{-1} Q(k'_x, k'_y) \tag{46}$$

2.3. Radiation Loss at the Step

The radiation loss at the step is a consequence of excitation of the radiative part of the continuum modes from the incident guided field, i.e., from the surface mode. However, unlike in closed wave guides, radiation modes are also propagating causing interaction with neighboring circuit elements. When this is the case, radiation modes

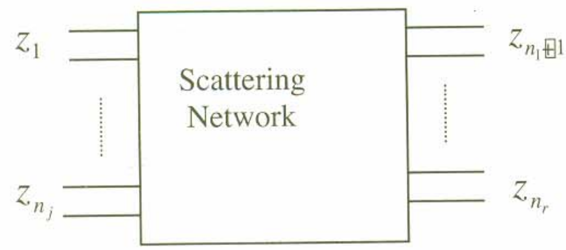

Figure 2. Generalized 2n-port scattering network.

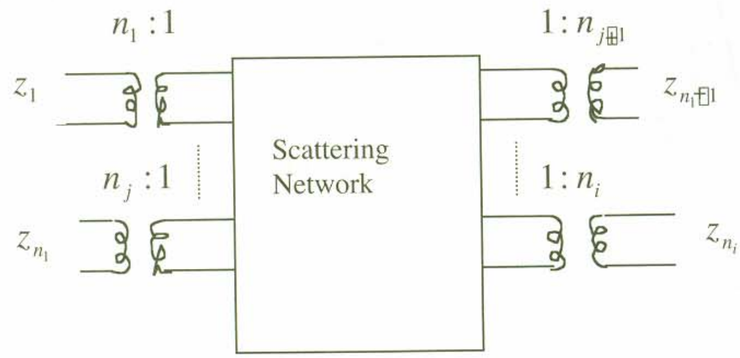

Figure 3. Normalized scattering network with embedded ideal transformers.

The above equation specifies the scattering coefficient of the incident j^{th} surface mode to the k^{th} surface mode. In the Ritz-Galarkin approach, the infinite column matrices Q_k and Q_j and square matrix Z are replaced by their finite truncations $(0 < n < N)$

By the careful choice of expanding functions, the oscillations in the solution will decrease rapidly with increasing order and convergence is quickly achieved. The step discontinuity is therefore represented by a generalized $(n_j + n_r)$ port scattering network as shown in the Fig. 2.

However, the step discontinuity is a reciprocal junction and its scattering matrix ought to be symmetrical this will only be so if all ports are terminated by the same matching impedance and these are all normalized to unity. Hence, the normalized scattering matrix is obtained by introducing ideal transformers connected at each ports as shown in Figure 3. Such that

$$\overline{z} = 1 = n^2 z_k \tag{37}$$

must also be considered accessible. The difference in power between the incident surface mode(s) and the scattered surface mode(s) provides the amount of radiation loss. Since the power loss to radiation is given by

$$P_{rad} = 1 - \frac{1}{n} \sum_{i} \sum_{w} S_{iw} s_{iw}^{*} \tag{47}$$

Where n is the total number of incident surface waves. For loss less junction or step the P_{rad} is zero. If the junction or step is lossy then $P_{rad} > 0$.

2.4. Choice of Basis Functions for TE Case

Most of the energy of the field in the waveguides is carried by surface waves. Modal functions as given by Eq. (3), shows that the surface waves have a cosine shape in the guide and an exponential decay outside. This suggests a possible appropriate choice of the expanding functions for symmetric fields as: **Cosine-Laguerre (7)** hybrid set **Cosine-Laguerre** hybrid set for two dimensional case

$$C_m(x, y) = \sqrt{\frac{e_m}{d}} \cos \frac{m\pi x}{d} \sqrt{\frac{e_m}{d_1}} \cos \frac{m\pi y}{d_1} \tag{48}$$

where $e_m = 1$ for $m = 0$ and $e_m = 2$ for $m \neq 0$, for the region $0 < x < d$, $0 < y < d_1$ (inside the slab)

$$L_n(x - d, y - d_1) = \frac{1}{\sqrt{xo}} L_n\left(\frac{x - d}{xo}\right) e^{-\frac{x-d}{2xo}} \frac{1}{\sqrt{yo}} L_n\left(\frac{y - d_1}{yo}\right) e^{-\frac{y-d_1}{2yo}} \tag{49}$$

for the region $x > d$, $y > d_1$ (outside the slab).

$L_n\left(\frac{x-d}{xo}\right) L_n\left(\frac{y-d_1}{yo}\right)$ are the general Laguerre polynomials with an arbitrary scale variable xo & yo. The above functions are orthogonal in their intervals of definition so that

$$\int_0^d \int_0^{d_1} C_m(x, y) C_n(x, y) dx dy = \delta_{mn} \tag{50}$$

$$\int_d^{\infty} \int_{d_1}^{\infty} L_m(x - d, y - d_1) L_n(x - d, y - d_1) = \delta_{mn}$$

However, in order to compute Z_{kn}, overlapping integrals between the basis functions and the modal fields must be evaluated. The overlapping integral with surface modes inside the slab is given by

$$Q_m = \int_0^d \int_0^{d_1} f_m(x,y)\varphi(x,y)dxdy = \int_0^d \int_0^{d_1} A\cos\kappa x\cos\kappa_1 ydxdy$$
$$= A\sqrt{e_m d}(-1)^m\left[\frac{\kappa d\sin\kappa d}{(\kappa d)^2-(m\pi)^2}\right]A\sqrt{e_m d_1}(-1)^m\left[\frac{\kappa_1 d_1\sin\kappa_1 d_1}{(\kappa_1 d_1)^2-(m\pi)^2}\right] \tag{51}$$

while the overlapping integral in air is given by

$$\begin{aligned}\overline{Q}_m &= \int_d^\infty \int_{d_1}^\infty \varphi(x,y)L_m(x-d,y-d_1)dxdy \\ &= A\sqrt{xo}\cos\kappa d(qxo-0.5)^m(qxo+0.5)^{-m-1} \\ &\quad A\sqrt{xo}\cos\kappa_1 d_1(q_1xo-0.5)^m(q_1xo+0.5)^{-m-1}\end{aligned} \tag{52}$$

The overlapping integrals with continuum modes inside the slab are given by

$$Q_m(k_x,k_y) = \int_0^d \int_0^{d_1} \phi(x;k_x,y;k_y)C_m(x,y)dxdy$$
$$= \sqrt{\frac{2e_m}{\pi d}}\frac{(-1)^m}{c}\left[\frac{qd\sin qd}{(qd)^2-(m\pi)^2}\right]\sqrt{\frac{2e_m}{\pi d_1}}\frac{(-1)^m}{c_1}\left[\frac{q_1 d_1\sin q_1 d_1}{(q_1 d_1)^2-(m\pi)^2}\right] \tag{53}$$

$$\begin{aligned}\overline{Q}_m(k_x,k_y) &= \int_d^\infty \int_{d_1}^\infty \phi(x;k_x,y;k_y)L_m(x-d,y-d_1)dxdy \\ &= \sqrt{\frac{1}{2\pi xo}}(-1)^m\left(\frac{1}{4}+k_x^2xo^2\right)^m\left[e^{ja}\left(\frac{1}{2}-jk_xxo\right)^{-2m-1}\right. \\ &\quad \left.+e^{-ja}\left(\frac{1}{2}+jk_xxo\right)^{-2m-1}\right]\sqrt{\frac{1}{2\pi yo}}(-1)^m\left(\frac{1}{4}+k_y^2yo^2\right)^m \\ &\quad \left[e^{ja_1}\left(\frac{1}{2}-jk_yyo\right)^{-2m-1}+e^{-ja_1}\left(\frac{1}{2}+jk_yyo\right)^{-2m-1}\right]\end{aligned} \tag{54}$$

In above equation (54) j represents the imaginary.

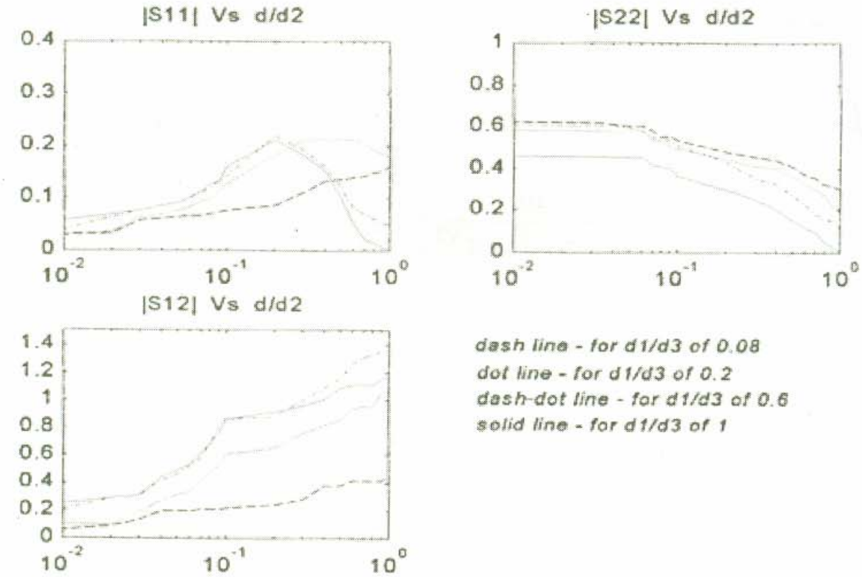

Figure 4. Scattering parameters $|S_{11}|$, $|S_{22}|$, $|S_{12}|$, for varying step ratio d/d_2 with different values of d_1/d_3 step ratio. The geometric dimensions are $k_0d_2 = 1$, $k_0d_3 = 1$, $\varepsilon_1 = \varepsilon_3 = 5$, $\varepsilon_2 = 1$.

3. NUMERICAL RESULTS

It is now interesting to observe the scattering behavior of the step discontinuity. All above formulas are valid for the waveguide II with dimensions d_2 & d_3 in x and y direction respectively. Figure 4. is illustrates the variation of scattering parameters for varying width d and d_1 of the smaller guide. Since the example refers to a slab with fairly high refractive index, radiation losses are low for small steps. The numerically calculated scattering parameters using math-CAD software are well matching with the results calculated by Rozzi (2) for $d_1 = d_3$ and $\varepsilon_1 = \varepsilon_3$ indicating the one dimensional case.

In case of two dimensional the variation in the scattering parameters as compared to the one dimension for different step ratio's is due to the variation in the impedance. The variation in the d as well as d_1 both in x and y direction makes the changes in the β value inturn the impedance. This is because β is related with Z as $Z = \frac{\omega\mu}{\beta}$. From the graph it is obvious the scattering parameters have more value as compared to one dimension. This is because of the discontinuity in both the direction increases the magnitude of the scattering parameters. The transmission coefficient also decreases for increasing step size, as energy leaks out in the form of radiation.

Table 1. Scattering parameters for fixed step ratio's $d/d_2 = d_1/d_3 = 0.6$ with $\varepsilon_1 = 5$, $\varepsilon_2 = 1$ and different values of ε_3 of waveguide II for the structure shown in Fig. 1.

ε_3	$\lvert S_{11}\rvert$	$\lvert S_{22}\rvert$	$\lvert S_{12}\rvert$
2	0.17	0.696	2.45
3	0.16	0.688	1.85
4	0.15	0.502	1.45
5	0.14	0.256	1.25

It is also apparent that more power is radiated (calculated from Equation. (47)) for waves incident from left i.e. from the thinner guide. The physical interpretation is that, the guided mode being closer to the dielectric cutoff, less reflected power is captured by the reflected guided wave and lost through backward radiation. In addition, it is interesting to note that S_{11} also remains fairly small since for small values of d & d_1 the surface waves is no longer guided.

Above table shows the variation of scattering parameters for different values of ε_3. It is observed that as the ratio of refractive index for waveguide I & II is small, there will be less scattering of wave at the step discontinuity.

4. CONCLUSION

In this paper a rigorous analysis of discontinuity in two dimensional dielectric waveguide has been presented. The validity of the generalized integral expressions has been tested for the one dimensional case. The variation in curves is due to the discontinuity in two dimension as compared to one dimension. Further the analysis is extended for different refractive indices n_1 & n_3 for the structure shown in Fig. 1 and we can conclude that there will be a reduction in the scattering parameters values for decrement in refractive index ratio at the step discontinuity. This fact is particularly useful in the optical frequency region where the refractive-index variation is usually small.

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