A NEW APPROCH TO CONTROL PEAK TO AVERAGE POWER FOR OFDM SIGNALS

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By

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CERTIFICATION

This is to certify that this thesis dissertation titled ***"A NEW APPROACH TO CONTROL PEAK TO AVERAGE POWER FOR OFDM SIGNALS"*** submitted by Priyanka Agarwal roll no. 01/E&C/04, towards the partial requirement for the award of Degree of Master Of Engineering, in Electronics & Communications Department, by Delhi University, New Delhi, is a record of bonafide work carried out and completed under my supervision and guidance during the academic session 2004-2006. The matter contained in this thesis has not been submitted elsewhere for award of any other degree.

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Abstract

The telecommunications' industry is in the midst of a veritable explosion in wireless technologies. Once exclusively military, satellite and cellular technologies are now commercially driven by ever more demanding consumers, who are ready for seamless communication from their home to their car, to their office, or even for outdoor activities. With this increased demand comes a growing need to transmit information wirelessly, quickly, and accurately. To address this need, communications engineer have combined technologies suitable for high rate transmission with forward error correction techniques. The latter are particularly important as wireless communications channels are far more hostile as opposed to wire alternatives, and the need for mobility proves especially challenging for reliable communications. For the most part, Orthogonal Frequency Division Multiplexing (OFDM) is the standard being used throughout the world to achieve the high data rates necessary for data intensive applications that must now become routine.

Orthogonal Frequency Division Multiplexing (OFDM) is a Multi-Carrier Modulation technique in which a single high rate data-stream is divided into multiple low rate data-streams and is modulated using sub-carriers which are orthogonal to each other. Some of the main advantages of OFDM are its multi-path delay spread tolerance and efficient spectral usage by allowing overlapping in the frequency domain. Also one other significant advantage is that the modulation and demodulation can be done using IFFT and FFT operations, which are computationally efficient.

The peak to average (amplitude) ratio (PAR) of the time domain envelope is an important parameter at the physical layer of the communication system using OFDM signaling. The signal must maintain a specified average energy level in the channel to obtain the desired system error rate. The peak signal level relative to that average defines the maximum dynamic range that must be accommodated by the components in the signal flow path to support the desired average. The primary components of concern are

the Digital to Analog Converter (DAC) and the power amplifier. A secondary concern with the peak signal level in the OFDM signal is the interference susceptibility of other shared channel signals to these peaks.

In this thesis, we will provide an overview of the OFDM technique, discuss its advantages, state some of the current applications, and briefly introduce a number of techniques for controlling the PAR of an OFDM signal set. The control mechanism should not alter the spectra of the signal, since the amplitude and phase of the numerous spectral components contains the modulation content of the signal. Control mechanisms that do modify the spectral components of the signal introduce additive interference that impacts error rate performance. We will illustrate a control mechanism named tone reservation, which will not modify the spectrum but will provide the better control on PAR. Although, this technique is not a new one but we are providing a new approach to implement it.

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1. ***Introduction***

Orthogonal frequency division multiplexing (OFDM) is nowadays widely used for achieving high data rates as well as combating multipath fading in wireless communications. In this multi-carrier modulation scheme dividing a single wideband stream into several smaller transmits data or narrowband parallel bit streams. Each narrowband stream is modulated onto an individual carrier. The narrowband channels are orthogonal to each other, and are transmitted simultaneously. In doing so, the symbol duration is increased proportionately, which reduces the effects of inter-symbol interference (ISI) induced by multipath Rayleigh-faded environments. The spectra of the sub-carriers overlap each other, making OFDM more spectral efficient as opposed to conventional multi-carrier communication schemes.

OFDM, first discussed in the mid-1960s and later patented in 1970, is a popular method of high-speed data transmission. Early on, OFDM's main appeal was that high-speed equalization was not necessary because data was sent in parallel on different sub-carriers. OFDM was also touted for its ability to fully use the available bandwidth, combat impulsive noise and mitigate the effects of multipath fading.

1.1 Motivation

Orthogonal Frequency Division Multiplexing (OFDM) is the standard being used throughout the world to achieve the high data rates necessary for data intensive applications that must now become routine. A particularly attractive feature of OFDM systems is that they are capable of operating without a classic equalizer, when communicating over depressive transmission media, such as wireless channels, while conveniently accommodation the time- and frequency-domain channel quality fluctuations of the wireless channel.

1.2 Thesis organization

After discussing importance of high rate data transmission in various field and representing OFDM is as solution in *Introductory* chapter, we tried our best to organize this thesis in very systematic way which includes these chapters-

Chapter 2: In this chapter of the thesis, we commence my detail discourse by demonstration that OFDM modems can be efficiently implemented by invoking the Fast Fourier transform (FFT). A number of basic OFDM design issues are discussed in an accessible style. OFDM Synchronization is not highly required for the thesis but we referred here to give the brief idea of complete OFDM field. This chapter also highlights the applications in which OFDM is already implemented.

Chapter 3: One of the main problems associated with the implementation of multi-carrier communication systems is the high Peak to Average power Ratio (PAPR or PAR), requiring highly linear power amplifiers. This chapter include definition of PAPR & near about the entire peak to average power reduction techniques.

Chapter 4: Starting from the basic OFDM mathematical formulation to the PAR problem, this chapter includes all the necessary equations, which must be considered for the better understanding of the this field. This part also introduces the technique which we will illustrate in next section with problem demonstration and simulation, named – Tone Reservation Technique with entirely new approach.

Chapter 5: The most important and all the essence of the project work lies in this section, where demonstration of the problem of PAR, and an entirely different approach to implement Tone Reservation Technique is illustrated. All the work illustrated here is implemented on MATLAB 7.

Chapter 6: Conclusion symbolizes the whole work & so this section is having some key words, which reflect my labour & dedication for this project work under the guidance of my guide.

Last but not the least, *References* give completeness to my thesis and my project work.

1.3 Research Survey

OFDM, First discussed in the mid-1960s and later patented in 1970, is a popular method of high-speed data transmission. In 1971 Weinstein and Ebert introduced the idea of using the discrete Fourier transform in the modulation/demodulation process [15]. Prior to this breakthrough, OFDM systems were prohibitively complex because arrays of sinusoidal generators and coherent demodulators were necessary in the implementation. With special-purpose fast Fourier transform (FFT) chips, the entire OFDM system could be implemented digitally.

More recently, OFDM has been implemented in mobile wideband data transmission (IEEE 802.11a, Hyper LAN II), high-bit-rate digital subscriber lines (HDSL), asymmetric digital subscriber lines (ADSL), very high-speed digital subscriber lines (VHDSL), digital audio broadcasting (DAB), digital television and high-definition television (HDTV). It is also implemented for the IEEE 802.16 Wi-MAX standard and its predecessor multicarrier multipoint distribution service (MMDS).

Despite the widespread acceptance of OFDM, it has its drawbacks. One drawback is that OFDM systems are not robust again carrier frequency estimation errors. Even small carrier offsets destroy the orthogonality between the subcarriers causing drastic error rate increases [16]. The second drawback is that OFDM signals suffer from large envelope variations. Such variations are problematic because practical communication systems are peak power limited. Thus, envelope peaks require a system to accommodate an instantaneous signal power that is larger than the signal average power, necessitating either low operating power efficiencies or power amplifier (PA) saturation. This problem is named as Peak to average Power Reduction.

PAR reduction was first of interest for radar and speech synthesis applications. In radar, PAR reduction was important because radar systems are peak-power limited just like communications systems. In speech synthesis applications, peaky signals lead to a “hard sounding computer" voice [17]. In simulating human speech this characteristic is not desirable1. However, in both radar and speech synthesis, maintaining a certain spectral shape is of interest. Therefore, PAR reductions in these fields can be done per spectral shape and are specified by frequency-domain phase sequences. Some examples of low-PAR sequences are the Newmann phase sequence, the Rudin-Shapiro phase sequences, and Galios phase sequences. These phase sequences all produce low PAR time-domain sequences for a wide variety of spectral masks. But this is not sufficient for OFDM where *random* data is modulated in the frequency domain. As a matter of fact, it is impossible to find a phase sequence that produces a globally low PAR for all data sequences. Another difference between communications PAR reduction and radar of speech PAR reduction is that communications systems, out of necessity, must occasionally distort or clip signals that exceed some peak threshold. Because some sort of distortion will occur, communications PAR reduction scheme are not limited to being distortion- less.

Clipping was the first idea but cause spectral distortion and SNR decrease. There have been several proposals that aim at mitigating the spectral regrowth and BER increases due to clipping. Notably, Kim and Stuber in [18] present an iterative receiver design that tries to undo the clipping effect to increase the BER. Companding is a composite word that combines compress and expand. It was first used as a technique to expand the dynamic range of DACs [19] and was later adopted as a perspective PAR reduction technique. Time to time many techniques came as Selective Coding, Partial Transmit Sequence, Tone Reservation, Tone Injection [7] [9] [11] [11].

Tone reservation was first introduced in [20]. In that paper, the authors implemented a projection onto convex sets (POCS) method for solving [21]. POCS is an active field with many applications like fillter design, array signal processing, electron microscopy, speckle interferometry, topography, spectral estimation, neural networks [21], and PAR reduction. Basically, the idea is to find an element that exists in two sets.

Later, Tellado and Cioffi discussed the idea of tone reservation [13] [22]. They provided a detailed analysis of how to solve [21] as a linear programming problem that has an exact solution (the POCS method is suboptimal). The linear programming solution can be reached with complexity $O[N \log N]$. They also detailed a (suboptimal) gradient search method in [22] with complexity $O[N]$.

2. ***Basics of OFDM***

OFDM is a Multi-Carrier Modulation technique in which a high rate bit-stream is split into (say) N parallel bit-streams of lower rate and each of these are modulated using one of N orthogonal sub-carriers.

2.1 Evolution of OFDM

OFDM owes its origin to Frequency Division Multiplexing (FDM). In FDM, each of the several low rate user signals is modulated with a separate carrier and transmitted in parallel. Thus the separation of the users is in the frequency domain. In-order to be able to easily demodulate each user signal, the carriers are spaced sufficiently apart from each other. Moreover, guard band has to be provided between 2 adjacent carriers so that realizable filters can be designed. Hence the spectral efficiency is very low.

2.1.1 OFDM verses single carrier transmission

The performance of OFDM system depends on several factors, such as the modulation scheme used, the amount of multipath, and the level of noise in the signal. However if we look at the performance of OFDM with just AWGN then the performance of OFDM is the same as that of a single carrier coherent transmission using the same modulation scheme. If we look at just a single OFDM subcarrier, then this is the same as a single carrier transmission. If we use QAM modulation, the transmitted amplitude and phase is held constant over the period of the symbol and is set based on the modulation scheme and the transmitted data. This transmitted vector is then updated at the start of each symbol. This results in a sinc frequency response, which is required for OFDM.

The best thing to demodulate a single carrier is to use a coherent matched receiver, which can be implemented by mixing the signal to DC using IQ mixer. Then we will get an IQ output that describes the amplitude and phase of the received modulated carrier. The amplitude and phase of the transmitted signal is constant during the symbol period, so we can demodulate the signal by IQ demodulation after taking the average of the received IQ vector over the entire symbol. The demodulation of an OFDM signal is performed in exactly the same manner. In the receiver a FFT is used to estimate the amplitude and phase of each subcarrier. The FFT perform the same operation as the matched receiver for the single carrier transmission, except now for a bank of subcarriers. From this we can say that in AWGN, OFDM will have the same performance as a single carrier transmission.

However, most propagation environments suffer from effects of multipath propagation. For a given bandwidth, the symbol rate for a single carrier transmission is very high, where as for OFDM signal it is N times lower, where N is the number of subcarriers used. The lower symbol rate leads to low ISI. Also using guard period at the start of each symbol removes any ISI shorter than its length. If the guard period is long enough, then all the ISI can be removed.

Multipath propagation results in frequency selective fading that leads to fading of individual subcarriers. Forward error correction is used in OFDM to compensate for the subcarriers suffer from severe fading. The performance of the OFDM system will be determined by the noise seen at the receiver. How ever, the performance of a single carrier transmission will degrade rapidly in the presence of multipath.

Let's compare this parallel transmission scheme with a single high rate data transmission. The results of the comparison are tabulated in Table 1.

Transmission method	**Parallel**	**Serial**
Symbol time	Ts	Ts/N
Rate	1/Ts	N/Ts
Total BW required	2*N/Ts + N*0.1/Ts (Assume Guard band = 0.1/Ts)	2*N/Ts
Susceptibility to ISI	Less	More

Table 1: Comparison of Parallel and Serial Transmission Schemes.

It is of interest to note that before equalizers were developed, the parallel transmission method was the means of achieving high data rates over a dispersive channel, in spite of its high cost and relative bandwidth inefficiency [1].

From Table 1, we see that the major disadvantages of the parallel transmission scheme are that its bandwidth inefficient and that several modulators and demodulator blocks are required.

In OFDM, these problems are overcome by

- Using orthogonal sub-carriers instead of widely spaced sub-carriers (i.e., carriers with guard band between them).
- Using IFFT and FFT algorithms for implementing the modulation and demodulation operations.

2.2 Orthogonality Principle

Orthogonal Frequency Division Multiplexing (OFDM) is simply defined as a form of multi-carrier modulation where the carrier spacing is carefully selected so that each sub carrier is orthogonal to the other sub carriers. Two signals are orthogonal if their dot product is zero. That is, if you take two signals multiply them together and if their integral over an interval is zero, then two signals are orthogonal in that interval.

Orthogonality can be achieved by carefully selecting carrier spacing, such as letting the carrier spacing be equal to the reciprocal of the useful symbol period. As the sub carriers are orthogonal, the spectrum of each carrier has a null at the center frequency of each of the other carriers in the system. This results in no interference between the carriers, allowing them to be spaced as close as theoretically possible. Mathematically, suppose we have a set of signals Ψ, where Ψ_p is the *p*-th element in the set.

$$\int \psi_p(t)\psi_q(t)^*\,dt = \begin{cases} k & for \quad p \neq q \\ 0 & for \quad p = q \end{cases}$$

Where * indicates the complex conjugate and interval [a, b], is a symbol period**.** Since the carriers are orthogonal to each other the nulls of one carrier coincides with the peak of another sub carrier. As a result it is possible to extract the sub carrier of interest.

2.3 OFDM Signal

OFDM is similar to FDM technique except that the 'N' sub-carriers are made orthogonal to each other over the OFDM symbol (frame) duration Ts. By orthogonality of the carriers, we mean that the carrier frequencies satisfy the following requirement –

$$f_k = f_0 + \frac{k}{T_s}, k = 1,2,\ldots, N-1$$ --------->(1)

The above requirement translated to the time domain, means that there must be integer number of cycles of each carrier over the duration Ts. Refer figure 2.

Figure 1: Sub-carriers in the OFDM Spectrum.

From equation (1) and figure 1, we see that the carriers satisfy in the frequency domain, the famous 'Nyquist criterion for zero Inter-symbol Interference in time domain' [1]. Hence we can conclude that there will zero Inter-Carrier Interference in the OFDM system.

Figure 2: Sub-carriers within an OFDM Symbol (Time Domain).

2.2.1 OFDM Modulation

The OFDM signal can, in general, be represented as the sum of 'N' separately modulated orthogonal sub-carriers [15] -

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} d_{n,k} g_k (t - nT_s) \text{ ------------->(2)}$$

Where $g_k(t), k = 0,1,..., N-1$ represent the 'N' carriers and are given by,

$$g_k(t) = e^{j2\pi f_k t}, \; t \in [0,T_s) \text{ ----------> (3)}$$

Figure 3: OFDM Modulation – Block Diagram [23].

In equation (2), $d_{n,k}$ stands for the symbol that modulates the kth carrier in the nth signaling interval and each signaling interval is of duration Ts. From equation (2), we see that 'N' symbols are transmitted in Ts time interval. The symbol sequence $d_{n,k}$ is obtained by converting a serial symbol sequence of rate N/Ts (symbol duration = Ts/N) into 'N' parallel symbol sequences of rate 1/Ts (each with symbol duration Ts).

As mentioned previously, the sub-carrier frequencies satisfy the following requirement -

$$f_k = f_0 + \frac{k}{T_s}, k = 1,2,..., N-1 \text{ -------------->(4)}$$

Figures [1],[2] show the time domain and frequency domain representation of the 'N' orthogonal sub-carriers in an OFDM symbol.

The signal transmitted in the 'n'th signaling interval (of duration Ts) is defined as the 'n'th OFDM frame, i.e.,

$$F_n(t) = \sum_{k=0}^{N-1} d_{n,k} g_k (t - nT_s) \text{ ------------>(5)}$$

Thus we see that the ‘n’th OFDM frame $F_n(t)$ consists of ‘N’ symbols, each modulating one of the ‘N’ orthogonal sub-carriers.

2.2.2 OFDM Demodulation

Since the carriers are orthogonal with each other, it follows that the scalar product

$$< g_k(t), g_i(t) >_{T_s} = \int_{T_s} g_k(t) g_i^*(t) dt = T_s \delta(k-i) \text{ ------->(6)}$$

Thus, the orthogonality of the carriers can be used to demodulate each of the sub-carriers (without Inter-Carrier Interference) as follows –

$$d'_{n,k} = \frac{1}{T_s} \int_{nT_s}^{(n+1)T_s} s(t) g_k(t) dt \text{ ----------->(7)}$$

If there is zero Inter-Frame Interference, then the above expression reduces to

$$d'_{n,k} = \frac{1}{T_s} \int_{nT_s}^{(n+1)T_s} F_n(t) g_k(t) dt = d_{n,k} \text{ ------------->(8)}$$

Thus we are able to perfectly demodulate each sub-carrier in the transmitted signal and get back the transmitted symbol sequence.

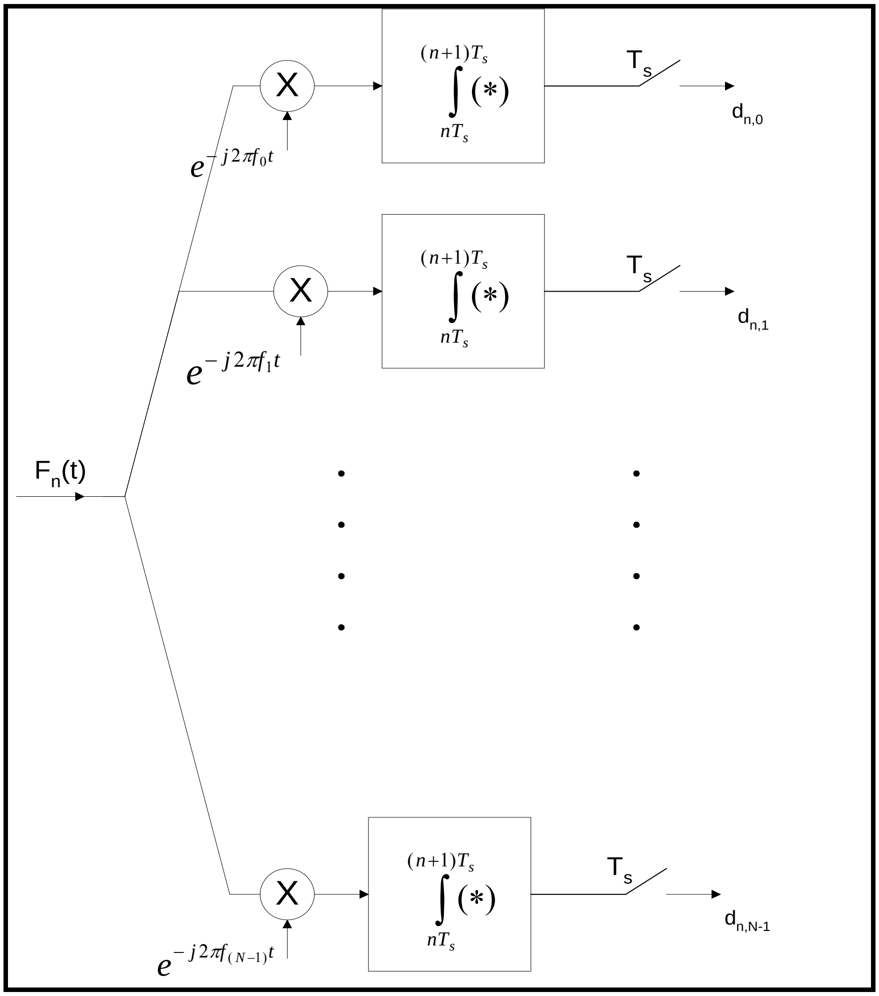

Figure 4: OFDM Demodulation – Block Diagram [23].

2.2.3 OFDM Modulation as IFFT

From equations (5), (7) we see that 'N' modulators and 'N' demodulators are required at the transmitter and the receiver respectively. The number of sub-carriers 'N' in OFDM systems is usually of the order of 100's implying that the transmitter and receiver blocks become bulky and expensive to build. Also the oscillators (for generating the carrier frequencies) have temperature instability and other problems.

In [5], the Discrete Fourier Transform is used to solve the modulation and demodulation complexities discussed above. In the following, we show that the modulation process can be achieved by the IFFT operation.

If we sample the OFDM frame represented by equation (5) at a rate 'N/Ts', the resulting discrete-time signal is

$$F_n^m = \sum_{k=0}^{N-1} d_{n,k} g_k (t - nT_s) @ t = (n + \frac{m}{N})T_s \,,\ m = 0,1,...,N-1 \text{---------->(9)}$$

Expanding the above equation, we see that

$$F_n^m = e^{j2\pi f_0 \frac{m}{N} T_s} \sum_{k=0}^{N-1} d_{n,k} e^{j2\pi \frac{mk}{N}} \,,\ m = 0,1,...,N-1 \text{----------->(10)}$$

If we assume $f_0 = 0$, then the above equation reduces to

$$F_n^m = \sum_{k=0}^{N-1} d_{n,k} e^{j2\pi \frac{mk}{N}}, m = 0,1,...,N-1 \text{------------->(11)}$$

The above equation can be expressed in terms of the IFFT as,

$$F_n^m = N \cdot IFFT(d_{n,k}) \text{------------->(12)}$$

Applying the FFT operation on both sides of the above equation, w e get

$$FFT(F_n^m) = N \cdot FFT(IFFT(d_{n,k})) = d_{n,k} \text{----------->(13)}$$

Thus the OFDM modulation and demodulation can be accomplished using the computationally efficient operations - IFFT and FFT respectively.

2.3 Guard Time and Cyclic Extension

One of the main advantages of OFDM is its immunity to multi-path delay spread that causes Inter-symbol Interference (ISI) in wireless channels. Since the symbol duration is made larger (by converting a high data rate signal into 'N' low rate signals), the effect of delay spread is reduced by the same factor. Guard Time is introduced in-order to eliminate the ISI almost completely. Making the guard time duration larger than that of the estimated delay spread in the channel does this. If the guard period is left empty, the orthogonality of the sub-carriers no longer holds, i.e., Inter-Carrier Interference (ICI) comes into picture. In-order to eliminate both the ISI as well as the ICI, the OFDM symbol is cyclically extended into the guard period. This preserves the orthogonality of the sub-carriers by ensuring that the delayed versions of the OFDM symbol always have an integer number of samples within the FFT interval [1].

Thus we can eliminate ISI and ICI by cyclically extending the OFDM symbol into the guard period and making sure that the guard time duration is larger than the delay spread.

In order to appreciate the theory introduced above, we generated OFDM symbols using MATLAB [16]. Here, we generated random binary bit sequence and applied the sequence to a serial to parallel converter. I took 64 bits at a time and applied a 128-point IFFT to it. The first 64 outputs of the IFFT is taken and plotted by taking absolute value. Cyclic extension is then done assuming a guard time of 10 samples.

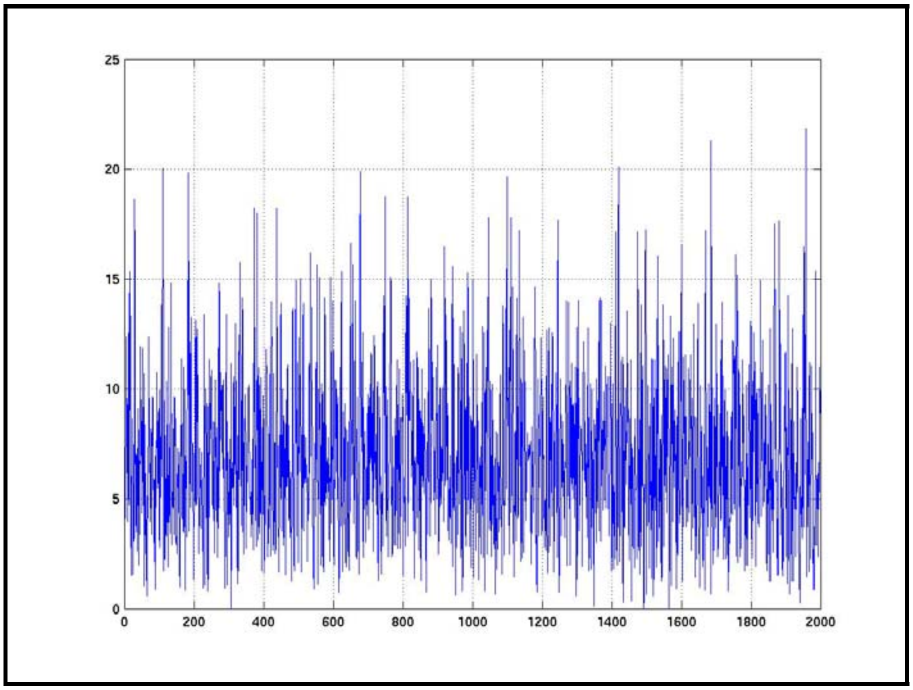

Figure 5: Generated OFDM Symbol Sequence (each symbol contains 74 samples).

2.4 Windowing

The power spectral density of the OFDM symbol (refer equation) falls off slowly, according to a sinc function. Even if the number of sub-carriers is increased, the fall off rate becomes rapid only in the beginning, but slows down after the 3dB bandwidth. In-order to make the power spectral density go down much more rapidly, time domain window is applied to the OFDM symbol. Windowing smoothens the amplitude variations due to phase changes at symbol boundaries. Usually raised cosine windowing is applied and is given by [1]

$$w(t) = \left\{ \begin{array}{l} 0.5 + 0.5\cos(\pi + t\pi/(\beta T_s), 0 \le t \le \beta T_s \\ 1.0, \beta T_s \le t \le T_s \\ 0.5 + 0.5\cos((t - T_s)\pi/(\beta T_s)), T_s \le t \le (1+\beta)T_s \end{array} \right\} \text{------>(14)}$$

In the above equation, β stands for the roll-off factor and T_s is the symbol interval, which is shorter than the total symbol duration since the adjacent symbols are allowed to overlap. As the roll-off factor is increased, the spectrum falls off rapidly, but the delay tolerance is decreased.

Instead of windowing, filtering can also be done to reduce the out-of-band spectrum. When filtering is applied, the rippling effects introduced on the envelope of the OFDM symbol should be kept minimum as it reduces the delay-spread tolerance of OFDM. It should also be noted that filtering is more computationally intensive compared to windowing and hence usually windowing is preferred.

2.5 Design of an OFDM System

The design of an OFDM system requires a trade-off between various parameters as like in all communication system design. Usually, the input parameters to the design are the bit rate, available bandwidth and the maximum delay spread introduced by the channel. The design involves calculation of symbol duration, guard time, number of sub-carriers and the modulation and coding schemes among others.

2.5.1 Guard Time and Symbol Duration

As said previously, the guard time is made longer than the maximum delay spread introduced by the channel. But the guard time cannot be made very large since no information bits are transmitted during the guard time. As a rule of thumb, the guard time should be at least 2-4 times the root-mean-squared delay spread [1].

The symbol duration must be fixed in such a way that the overhead associated with the guard time is minimal. This can be achieved by making the symbol duration much longer than the guard time. However large symbol duration means more number of sub-carriers and thus causes implementation complexities and increased peak-to-average power problems. Thus a practical design choice for the symbol duration is around 5-6 times the guard time [1].

2.5.2 Number of Sub-carriers

Once the symbol duration is fixed, the spacing between the sub-carriers can be obtained as the inverse of the symbol duration minus the guard time. The number of the sub-carriers can then be calculated as the ratio of the available bandwidth to the carrier spacing.

2.5.3 Modulation and Coding Schemes

The decision of which modulation and coding technique to use depends on various issues. The decision significantly overlaps with the design of the number of sub-carriers discussed above and usually a back-and-forth approach is followed. For example, if the number of bits that are to be assigned to each sub-carrier is known, then the modulation and coding for each sub-carrier can be designed based on this. On the other hand, if the modulation and coding are specified, then the number of sub-carriers can be determined.

2.5.4 Example Design of OFDM system

In this section, we consider a typical design problem with the following requirements [1]

- Bit Rate : 20Mbps
- Maximum Delay Spread : 200ns
- Available Bandwidth : 15MHz

Guard Time:

From the design considerations discussed previously, we see that a guard time of 4*200ns = 800ns is a reasonable choice.

Symbol Duration:

Let the OFDM symbol duration to be 6 times the guard time, i.e., OFDM symbol duration = 6*800ns = 4.8μs.

Sub-carrier Spacing:

Sub-carrier spacing = 1/(symbol duration – guard time) = 250KHz.

Modulation and Coding:

To transmit 20Mbps data, the number of bits to be transmitted in an OFDM symbol is = 20Mbps*4.8μs = 96 bits/OFDM symbol. Now, we can go for

- 16-QAM with rate (1/2) coding for each sub-carrier so that there are 2 bits per symbol per sub-carrier. Thus we see that 48 sub-carriers are required in this case.
- QPSK with rate (3/4) coding for each sub-carrier so that there are 1.5 bits per symbol per sub-carrier. Thus in this case, we need 64 sub-carriers.

But in the latter case, 64 sub-carriers require a bandwidth of 64*250KHz = 16MHz which is greater than the available bandwidth of 15MHz. Hence the first one is a good choice in the sense that it satisfies the constraints.

2.6 Block Diagram of an OFDM System

Having dealt with the important concepts of an OFDM system, I provide here the block diagram of the entire system. I briefly describe the system details.

At the transmitter, the user information bit sequence is first subjected to channel encoding to reduce the probability of error at the receiver due to the channel effects. Usually, convolution encoding is preferred. Then the bits are mapped to symbols. Usually, the bits are mapped into the symbols of either 16-QAM or QPSK. The symbol sequence is converted to parallel format and IFFT (OFDM modulation) is applied and the sequence is once again converted to the serial format. Guard time is provided between the

OFDM symbols and the guard time is filled with the cyclic extension of the OFDM symbol. Windowing is applied to the OFDM symbols to make the fall-off rate of the spectrum steeper. The resulting sequence is converted to an analog signal using a DAC and passed on to the RF modulation stage. The resulting RF modulated signal is, then, transmitted to the receiver using the transmit antennas. Here, directional beamforming can be achieved using antenna array, which allows for efficient spectrum reuse by providing spatial diversity.

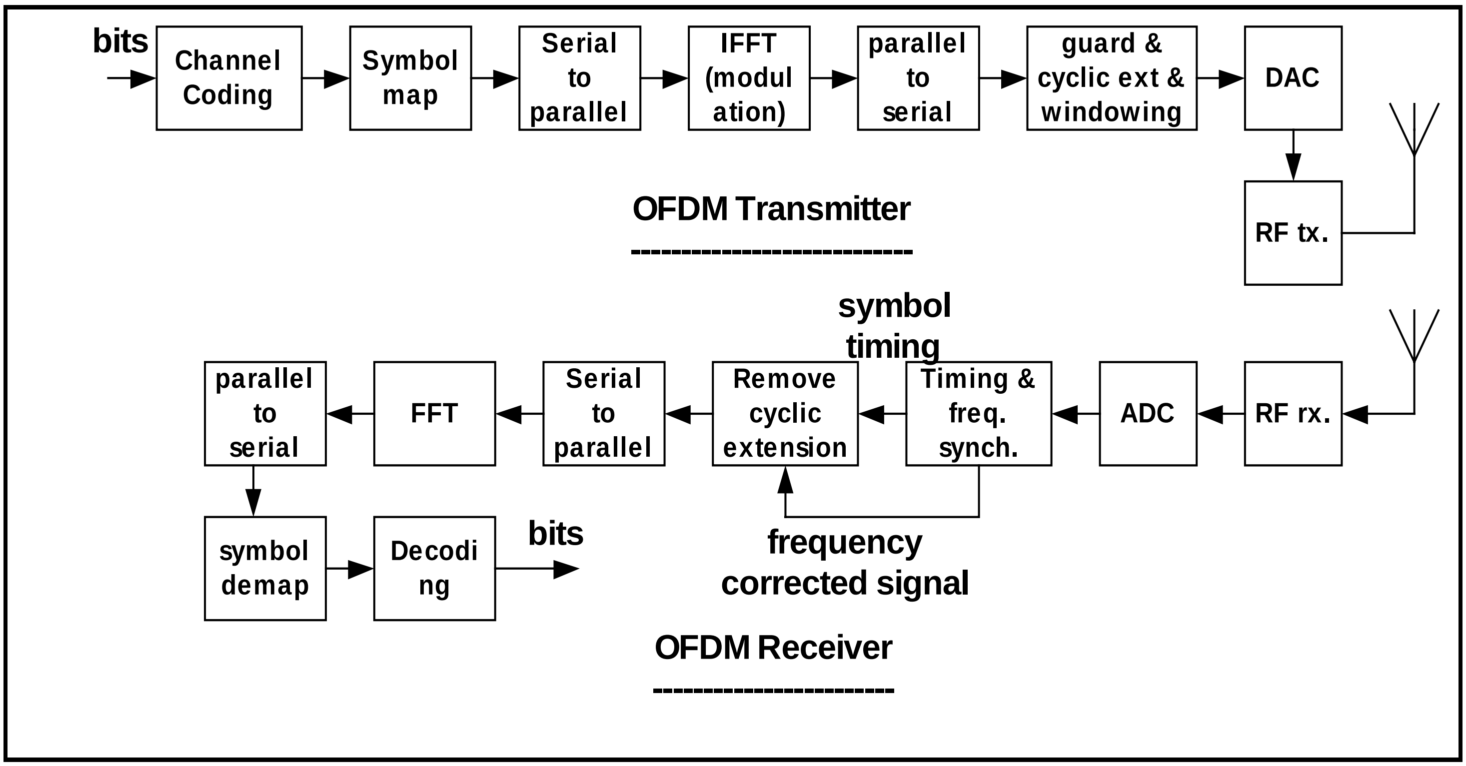

Figure 6: OFDM System Block Diagram [1].

At the receiver, first RF demodulation is performed. Then, the signal is digitized using an ADC and timing and frequency synchronization are performed. Synchronization will be dealt with in the later sections. The guard time is removed from each OFDM symbol and the sequence is converted to parallel format and FFT (OFDM demodulation) is applied. The output is then serialized and symbol de-mapping is done to get back the coded bit sequence. Channel decoding is, then, done to get the user bit sequence.

2.7 Advantages of OFDM

OFDM has several advantages over single carrier modulation systems and these make it a viable alternative for CDMA in future wireless networks. In this section, I will discuss some of these advantages.

2.7.1 Multipath Delay Spread Tolerance

OFDM is highly immune to multipath delay spread that causes inter-symbol interference in wireless channels. Since the symbol duration is made larger (by converting a high data rate signal into ‘N’ low rate signals), the effect of delay spread is reduced by the same factor. Also by introducing the concepts of guard time and cyclic extension, the effects of inter-symbol interference (ISI) and inter-carrier interference (ICI) is removed completely.

2.7.2 Immunity to Frequency selective fading Channels

If the channel undergoes frequency selective fading, then complex equalization techniques are required at the receiver for single carrier modulation techniques. But in the case of OFDM the available bandwidth is split among many orthogonal narrowly spaced sub-carriers. Thus the available channel bandwidth is converted into many narrow flat-fading sub-channels. Hence it can be assumed that the sub-carriers experience flat fading only, though the channel gain/phase associated with the sub-carriers may vary. In the receiver, each sub-carrier just needs to be weighted according to the channel gain/phase encountered by it. Even if some sub-carriers are completely lost due to fading, proper coding and interleaving at the transmitter can recover the user data.

2.7.3 High Spectral Efficiency

OFDM achieves high spectral efficiency by allowing the sub-carriers to overlap in the frequency domain. At the same time, to facilitate inter-carrier interference free demodulation of the sub-carriers, the sub-carriers are made orthogonal to each other. If the number of sub-carriers in 'N', the total bandwidth required is

$$BW_{total} = \frac{(N+1)}{T_s} \text{-------->(15)}$$

For large values of N, the total bandwidth required can be approximated as

$$BW_{total} \approx \frac{N}{T_s} \text{---------->(16)}$$

On the other hand, the bandwidth required for serial transmission of the same data is

$$BW_{total} = \frac{2N}{T_s} \text{--------->(17)}$$

Thus we achieve a spectral gain of nearly 100% in OFDM compared to the single carrier serial transmission case.

2.7.4 Efficient Modulation and Demodulation

Modulation and Demodulation of the sub-carriers is done using IFFT and FFT methods respectively, which are computationally efficient. By performing the modulation and demodulation in the digital domain, the need for highly frequency stable oscillators is avoided.

2.7.5 Frequency Diversity

A technique, MC-CDMA, which is based on a combination of DS-CDMA and OFDM has been introduced recently [6],[7].

2.8 Synchronizations of OFDM

Time and frequency synchronization are very important for the OFDM based communication system. Without correct frequency synchronization the orthogonality will not exist leading to and increase in BER. Without correct timing synchronization it is not possible to identify start of frames.

The condition for orthogonality is that the OFDM subcarriers have an integer number of cycles within the FFT interval. Once that is lost due to frequency offset, Inter Channel Interference is introduced into the system.

While the OFDM system can handle a timing error of a maximum of the guard interval, the system performance and robustness against delay spread decreases with timing error. So the system has to be synchronized in time also.

2.8.1 Synchronization using cyclic extension

The cyclic extension of the OFDM symbol can be used for synchronization purposes. When the symbol is taken and the guard time part of the symbol is correlated with the end of the symbol we get a correlation peak. The frequency offset is estimated by averaging the correlation over the guard time period and then estimating the phase.

The Correlator output is

$$y(n) = \sum_{0}^{N_G} x(n)x(n-N) \text{ ----------->(20)}$$

The correlation peaks corresponding to the guard time are as shown below.

Figure 10: Correlation peaks achieved using Cyclic Extension.

2.8.2 Synchronization using training sequences

The cyclic extension method is only used for blind synchronization where the data is not known. But when it is possible to send a training sequence as in the case of a packet transmission system an easier way of synchronization would be to implement a matched filter at the receiver.

The output of the matched filter will have correlation peaks, from which the timing and frequency synchronization can be achieved.

2.9 Applications of OFDM

Orthogonal Frequency Division Multiplexing is a new technology whose applications just being explored. The primary applications are in multimedia push technology and in wireless LAN.

5.1 Wireless LAN Applications

5.1.1 HIPERLAN/2

HIPERLAN/2 is a Wireless LAN application defined by the ETSI. HIPERLAN/2 handles data rates between 6 Mbit/s to 54 Mbit/s. HIPERLAN/2 provide a DLC layer on top of which an IP based broadband network can be implemented. The PHY layer of HIPERLAN/2 is based on the Orthogonal Frequency Division Multiplexing (OFDM) modulation scheme. The Numerical Values of OFDM parameters in HIPERLAN are given below:

Parameter	Value
Sampling rate f_s=1/T	20 MHz
Symbol part duration T_U	64*T 3,2 µs
Cyclic prefix duration T_{CP}	16*T, 0,8 µs (mandatory); 8*T, 0,4 µs (optional)
Symbol interval T_S	80*T, 4,0 µs (T_U+T_{CP}) ; 72*T, 3,6 µs (T_U+T_{CP})
Number of data sub-carriers N_{SD}	48
Number of pilot sub-carriers N_{SP}	4
Sub-carrier spacing .Δ_f	0,3125 MHz (1/TU)
Spacing between the two outmost sub-carriers	16,25 MHz (N_{ST}*Δf)

Table 2: HiperLAN/2 Parameters

5.1.2 IEEE802.11

The IEEE 802.11 committee has a standard similar to the HIPERLAN its OFDM parameters are as shown below:

Data Rate	6,9,12,18,24,36,48,54 Mbps
Modulation	BPSK, QPSK, 16-QAM, 64 QAM
Coding Rate	1/2, 2/3,3/4
No of Sub-Carriers	52
No of pilots	4
OFDM Symbol Duration	4 us
Guard Interval	800 ns
Sub-Carrier Spacing	312.5 kHz
3 dB bandwidth	16.56 MHz
Channel Spacing	20 MHz

Table 3: IEEE 802.11 Parameters

5.2 Digital Audio Broadcasting (DAB)

Digital Audio Broadcasting is a new multimedia push technology, with a good sound quality and better spectrum efficiency. This is achieved by the use of OFDM technology.

The DAB system samples audio at a sample rate of 48 kHz and a resolution of 22bits. Then the data is compressed to between 32 and 384 KBPS. A rate ¼ convolution code is used with constraint length 7. The total data rate is about 2.2Mbps. The frame time is 24ms. QPSK modulation is performed at the transmitter.

The advantage of using OFDM for DAB is that the OFDM suffers very little from delay spread and also that the OFDM system has high spectral efficiency.

5.3 Digital Video Broadcasting (DVB)

Digital Video Broadcasting (DVB) is an ETSI standard for broadcasting Digital Television over satellites, cables and thorough terrestrial (wireless) transmission.

Terrestrial DVB operates in either of 2 modes called 2k and 8k modes with 1705 carriers and 6817 carriers respectively. It uses QPSK,16 QAM or 64 QAM subcarrier modulation. It also uses pilot subcarriers for recovering amplitude and phase for coherent demodulation.

3. ***Peak to Average Power Ratio (PAPR)in OFDM systems***

Amongst the major downsides to designing an OFDM based Wireless System is the issue of Peak to Average Power Ratio.

The OFDM signal can be modeled as the sum of N independent complex variables with variable phases. When all the variables line up in phase a peak of the sum of amplitudes is formed. This leads to amplification difficulties at the transmitter. This leads to high out-of-band radiation in case of non-linear power amplifiers. The non-linearity arises mainly because of the large input power range.

I now discuss the Cause of the Peak to Average Power Ratio problem and will analyze a few of the Peak to Average Power Ratio reduction techniques.

3.1 Distribution of PAPR

When the number of sub-carriers in an OFDM system is high, conventional OFDM signals can be regarded as Gaussian noise like signals; their variable amplitude is approximately Rayleigh-distributed, and the power distribution has a cumulative distribution function given by

$F(z) = 1-e^{-z}$ ------->(18)

The probability that the PAPR is below some threshold is given by

$P(PAPR \le z) = F(z)^N = (1-e^{-z})^N$--------->(19)

The distribution is shown in figure (7).

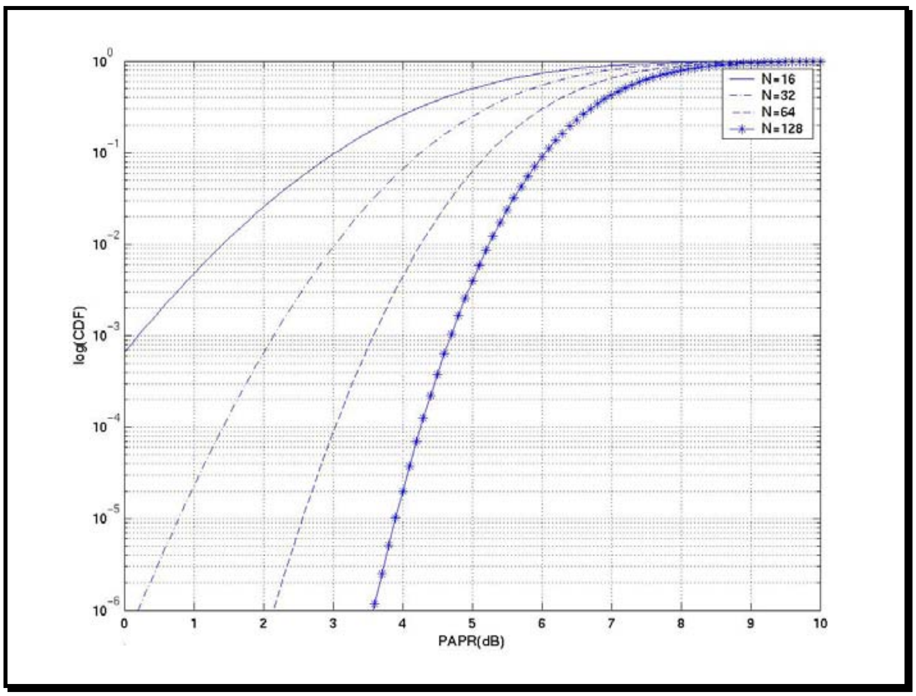

Figure 7: Cumulative Distribution Function of PAPR.

3.2 Effect of PAPR

The effects of the large Peak to Average Power ratio are:

1. The power amplifiers at the transmitter need to have a large linear range of operation. When considering a system with a transmitting power amplifier, the nonlinear distortions and peak amplitude limiting introduced by the High Power amplifier (HPA) will produce inter-modulation between the different carriers and introduce additional interference into the system. This additional interference leads to an increase in the Bit Error Rate (BER) of the system. One way to avoid such non-linear distortion and keep low BER is by forcing the amplifier to work in its linear region. Unfortunately such solution is not power efficient and thus not suitable for wireless communication.

2. The Analog to Digital converters and Digital to Analog converters need to have a wide dynamic range and this increases complexity.

3.3 PAPR Reduction Techniques

The PAPR reduction techniques broadly fall into one of the three techniques:

a) Signal Distortion
b) Coding
c) Scrambling and Carrier selection

3.3.1 Clipping

Clipping is a way of reducing PAPR by very simply limiting the maximum amplitude of the OFDM signal to a desirable maximum. This way of handling PAPR has a few disadvantages to it. Clipping alters the frequency spectrum and introduces out of band radiation.

This is clearly seen in the following graph(Figure 8), here an OFDM like symbol is simulated by a weighted sum of sines and cosines. Then the signal is peak limited and the spectrum of both plotted as well as the difference in their spectrum. Clipping can be viewed as windowing with a rectangular window that has a value 1 if the symbol value is less than the threshold and value less than 1 if greater than threshold. The out of band spectrum can be decreased by the use of non-rectangular windows.

Figure 8: Effects of Clipping on OFDM Spectrum.

But windowing also has some disadvantages in the sense that it acts on the whole of the signal instead of only on those points above threshold. This actually reduces the Bit Error Rate.

3.3.2 Peak Cancellation

This is method in which a time shifted and scaled version of a reference function (ex. Sinc function) is subtracted from the original signal to reduce the peak power of at least one signal sample. The advantage of this process is that, as the process is linear it does not introduce any out of band radiation.

Usually the Sinc function used is also windowed to have a finite length. The windowing function used is usually a Raised cosine type window. The process is clearly shown in the following graphs, where the peaks are identified and then they are eliminated by sinc functions at the location of the peak.

Figure 9: Peak Cancellation Technique.

3.3.3 Pulse shaping

The OFDM symbols are independent identically distributed Gaussian random variables. This correlation characteristic is responsible for the high variability of the OFDM signal, which is the reason for high PAPR.

Pulse shaping is the usage of time waveform of the different sub-carriers to create the appropriate correlation that reduces the PAPR of the multi-carrier signal. This method reduces PAPR without affecting bandwidth efficiency or needing side information.

In this technique each sub-carrier pulse of OFDM scheme has a different shape and all these pulse shapes are derived from the same pulse shape. The technique is referred to as pulse shaping. It has been shown that using this technique it is possible to design a set of time waveforms of OFDM systems that reduce the PAPR of the transmitted signal and improve its power spectrum simultaneously. The method avoids the use of an extra Inverse Fast Fourier Transformations (IFFTs). It works with arbitrary number of sub carriers and any type of base band modulation used. The implementation complexity of the proposed technique is by far much low compared to previously published methods. The technique is also very flexible and it can be used for OFDM and Discrete Multi-Tone (DMT) transmissions. This method has the potential of reducing the PAPR of the OFDM signal without affecting the bandwidth efficiency of the system and does not require any side information.

3.3.4 **Forward Error Correction Coding**

In order that the effect of greater distortion of symbols of high PAPR be reduced we apply forward error coding across several OFDM symbols. We therefore spread the probability and thereby reduce the probability of error. Here the errors caused by symbols of high PAPR value are corrected by the surrounding symbols.

3.3.5 PAPR Reduction codes

By using the property that only a small fraction of the OFDM symbols have a bad PAPR. This can be used to our advantage by coding to generate and OFDM signal that has the OFDM PAPR below some desirable level. The PAPR of a binary or poly-phase sequence of length can be as large as N, but if the sequence is constrained to be a member of a Golay complementary pair then its PAPR is atmost 2 [11].

Golay complementary sequences are sequence pairs for which the sum of auto-correlation functions is zero for all delay shifts unequal to zero.

3.3.6 Signal Scrambling

Here the input sequence is scrambled by a certain number of scrambling sequences and then the output sequence with the lowest PAPR is transmitted. Here we do not actually reduce the PAPR of the sequence instead we just transmit the sequence that has the lowest PAPR.

Usually constellation points that lead to high-magnitude time signals are generated by correlated bit patterns (for example, a long string of ones or zeros). Therefore, by scrambling the input bit streams, we may reduce the probability of large peaks generated by those bit patterns. The method is to form four code words in which the first two bits are 00,01,10 and 11 respectively. The message bits are first scrambled by four fixed cyclically in equivalent m-sequences. Then the one with the lowest PAPR is selected and one of the pair of bits defined earlier is appended at the beginning of the selected sequence. At the receiver, these first two bits are used to select the suitable descrambler. PAPR is typically reduced to 2% of the maximum possible value while incurring negligible redundancy in a practical system. However, an error in the bits that encode the choice of scrambling sequence may lead to long propagation of decoding errors.

3.3.7 Carrier Interferometry OFDM

Carrier Interferometry OFDM (CI/OFDM) is an OFDM technique that reduces PAPR while not increasing the system complexity. CI/OFDM exploits the frequency diversity of the fading channel by the transmission of each bit over each of the N carriers, and the seperability of each bit is maintained through carefully selected phase offsets [10].

The signal modulated using CI/OFDM has the same average value as seen in OFDM but has a significantly lower peak value. One more advantage of the CI/OFDM is that the reduction in PAPR is achieved without any redundancy or loss of throughput.

The reduction of PAPR is achieved by ensuring that the bits never add coherently as the CI/OFDM separates them is phase.

3.3.8 Adaptive Sub-carrier Selection

This scheme involves significant improvement of BER by neglecting weak sub-carriers and reduction of out of band radiation by using non-information carrying carriers [8].

Since all sub-carriers are orthogonal, we exclude those with the lowest SNR. The consequences of doing that are a lower data rate but an enormously reduced BER.

Then these carriers, which do not carry any information, are used to reduce the PAPR of the whole signal.

3.3.9 Selected Mapping

Here we transmit one of N statistically independent OFDM frames that represent the same information. Now if the selected frame has the minimum PAPR for transmission the probability of Peak amplitude greater than threshold decreases [9].

These independent frames are designed by multiplying each term of the OFDM frame by vectors P(n). Then all the N frames are converted to time domain and the one with the lowest PAR is selected for transmission.

In order to recover data the receiver needs highly reliable Information about which P (n) was used. This can be either transmitted as side information to the user or by coding [12].

3.3.10 Tone Reservation

Tone reservation is the scheme where the transmitter doesn't send data on a small set of carriers that are optimized for PAPR reduction. Tone injection is the scheme where the sub-symbol constellation is replicated by translating vectors. Now the choice is made for points with low PAPR.

In this method, the basic idea is to reserve a small set of tones for PAR reduction. Fortunately, the problem of computing the values for these reserved tones that minimize the PAR can be formulated as a convex problem and can be solved exactly. The amount of PAR reduction depends on the number of reserved tones, their locations within the frequency vector, and the amount of complexity. This method describes an additive method for reducing PAR in multi-carrier transmission, and shows that reserving a small fraction of tones leads to large reductions in PAR even with simple O(N) algorithms at the transmitter, and with no additional complexity at the receiver. When the number of tones N is small, the set of tones reserved for PAR reduction may represent a non-negligible fraction of the available bandwidth and can result in a reduction in data rate.
TR method has the following advantage:

1) No need for side information

2) Less complex- just one time IFFT operation is needed. But multiple iteration operations are needed after IFFT operation.

3) No special receiver operation is needed.

3.3.11 Tone injection

This is another additive method, which achieves PAR reduction of multi-carrier signals with no rate loss. The basic idea is to increase the constellation size so that each of the points in the original basic constellation can be mapped into several equivalent points in the expanded constellation. If these duplicate signal points are spaced by $D=\rho Md\sqrt{M}$, where M is the constellation size, and $\rho \geq 1$. The BER will not increase and the only addition to the standard receiver is a modulo-D addition after the FFT. Since each information unit can be mapped into several equivalent constellation points, these extra degrees of freedom can be exploited for PAR reduction. The method is called tone injection, as substituting the points in the basic constellations for the new points in the larger constellation is equivalent to injecting a tone of the appropriate frequency and phase in the multi-carrier symbol.

4. <u>PAPR Reduction Model</u>

I, examine a number of techniques for controlling the PAR of an OFDM signal set. The goal of PAR control techniques is to reduce the peak-to-average ratio of the time domain signal formed by the random QAM modulation of the separate spectral terms comprising the signal. The control mechanism should not alter the spectra of the signal, since the amplitude and phase of the numerous spectral components contains the modulation content of the signal. Control mechanisms that do modify the spectral components of the signal introduce additive interference that impacts error rate performance. At first glance, the requirement to modify the time envelope but not modify the spectrum that caused the time signal seems like an impossible task. The seminal work by José Tellado and John Cioffi at Stanford offer an enlightened solution to this problem[22].

To understand the technique briefly, we have to give a look to some mathematical formulation in multicarrier modulation.[13].

The continuous time representation of the multicarrier system is given by the equation

Where m is the symbol index, ω(t) is the rectangular window function(for simplicity) over the interval [0, *T*].

The cyclic prefix is the periodic extension of the system over the interval [-*Tcp,0*] resulting the interval [-*Tcp,T*].

ω_{cp}(t) is the rectangular window over the interval [-*Tcp, T*].

Moreover, with this representation, computing X(t) requires a continuous time Fourier Transform (CTFT), which is very hard to implement with analog components and can only be approximated by using an Inverse Discrete Fourier Transform (IDFT) as in figure12.

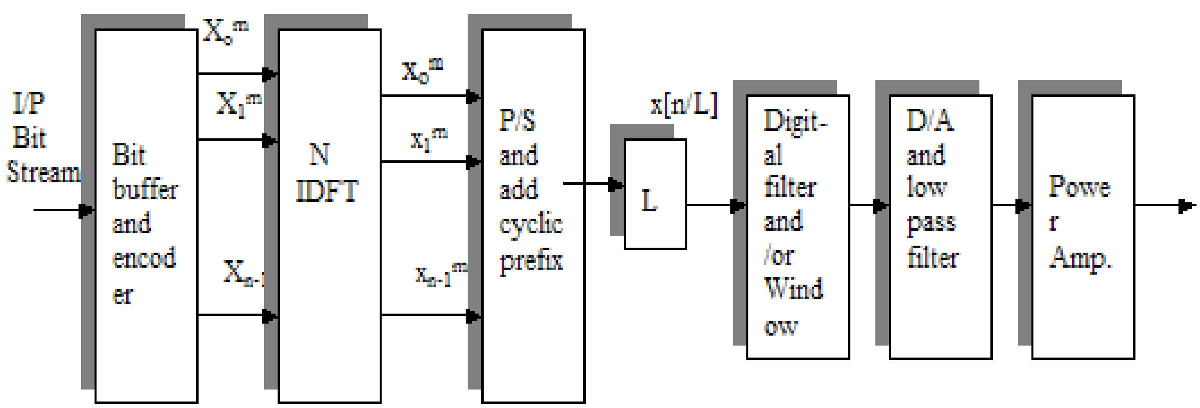

Figure 12: OFDM transmitter block diagram

The m-th block of encoded bits is mapped into the complex-valued OFDM of QAM constellation points

Which is transformed via an IDFT into T/N-aced discrete –time vector

& can be expressed as,

ω(n) is discrete time window of height 1 over the interval [0, *N*]

Since sampling is needed in practical designs, we will introduce the notation $x[N/L]$ to denote over sampling by a factor L. Several different over sampling strategies of $X^m(n)$ can also be defined. The oversampled function can be expressed as,
If the spectral confinement is performed on the discrete time sequence $x[n/L]$ with the digital filter $p[n/L]$, the filtered signal is expressed as

$$Z_F\,[n/L] = x[n/L] * p[n/L]$$

Where * is denotes convolution.

PAPR of OFDM system where tones share the same constellation can be expressed as,

Since the X^m are independent and N is typically large, from the Central Limit Theorem, the symbol samples $X^m\,[n]$ can be consider as Guassian.
Thus,

Moreover, it can be noted that CP does not affect the original symbol PAR, since it is merely a repetition of part of the signal. The PAR expression then becomes:

And signal is oversampled by factor L

But,

$$\max_n \left|x^m(n/L)\right|^2 = \max_n \left|\sum_{n=0}^{N-1} x_k^m e^{j2\pi kn/LN}\right|^2 \leq \sum_{k=0}^{N-1} \max_{x_k^m \in \{x\}} \left|x_k^m\right|^2 = N^2 \max_{x_k^m \in \{x\}} \left|x_k^m\right|$$

While for Parseval theorem,

$$E\{\left|x^m(n/L)\right|^2\} = \sum_{k=0}^{N-1} E\{\left|x_k^m\right|^2 \},$$

Thus if all subcarriers share the same constellation, *m*-th symbol PAR has a simple upper bound:

4.1 PAPR Reduction Model

Additive PAR reduction methods -The methods in this class reduce the data signal by adding a reduction signal to it, x[n] = x[n] + c[n], These two signals are then added to form a PAR-reduced signal x[n], which should have a low amplitude. The construction of the reduction signal c[n] can be done in different ways. What is important is that the receiver can decode the data signal as it was before the addition. In order for the resulting signal x[n] to be bounded in amplitude, the reduction signal c[n] must follow the shape of the data signal. If the data signal has a high peak at one location, the reduction signal must have a low value at this point. This signal has to approximate the main look of the peaks, not the whole data signal, and it also needs to avoid the creation of any unwanted peaks in new locations.[14]

X[n]

X[n]

C[n]

Figure 13: PAPR Reduction model

4.2 Definition of PAPR for reduced signals

The PAR of the reduced signal $x[n]$ = x[n]+$c[n]$ should include the reduction signal in the denominator. However, if this were the case, a decrease in PAR level could be caused by a increase in the total power, without any reduction of the peak power. So PAR can be defined as

$$PAR = \frac{\max\left|x[n] = c[n]\right|^2}{E\left|x[n]\right|^2}$$

This is *not* a true peak-to-average ratio of a signal, since it is a function of the signal both before and after reduction. This definition will, however, describe how much average signal power it is possible to use under a certain peak power constraint. If this PAR level can be lowered, we can use a higher average signal to a fixed peak level.

Denote the tone set with the PAR reduction reserved tones with $\mathcal{U}$={i1,i2,…..i_u}, where $\mathcal{U}$ is the number of reserved reduction tones. The reduction signal c[n] is built up by the sine and cosine components of these tones, c[n] = IDFT(C[k]), where C[k] $\neq$ 0 only if k $\in \mathcal{U}$ or N - k $\in \mathcal{U}$. Now c[n] can be written as,

$$c[n] = \frac{1}{\sqrt{N}} \sum_{k \in \mathcal{U}} C[k] e^{j\frac{2\pi nk}{N}} + C[N-k] e^{j\frac{2\pi n(N-k)}{N}}$$

Additive reduction techniques:

These techniques can be formulated as,

Where the frequency vector C^m =[C_o^m, C_1^m,……..C_{N-1}^m] or equaivalently, the time domain sequence $C^m = c^m[n/L]$ are the PAR reduction signals. Additive model for PAR reduction can be illustrated by the structure in Figure 14.

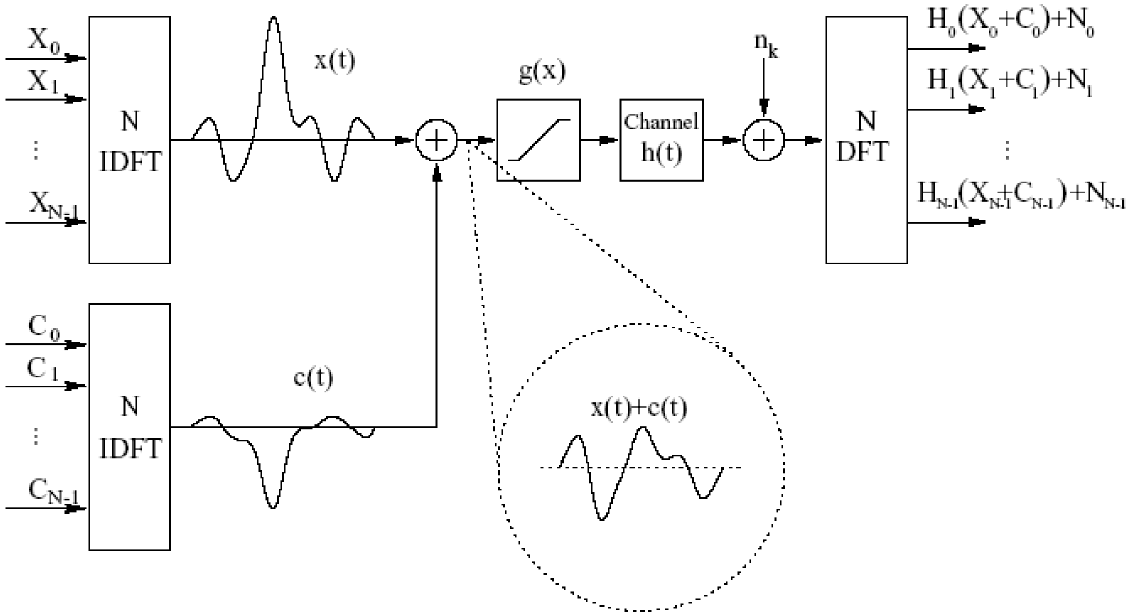

Figure 14: Additive Reduction technique structure

For these structures to be effective methods for PAR reduction, they must satify most of the following list of desirable properties:

1. The PAR reduction signal must achieve significant reductions of the PAR of the combined signal xm[n/L] + cm[n/L]. The PAR of this combined signal is defined as:

6. The receiver must decode X^m efficiently from the combined vector $X^m + C^m$ without degrading the performance of the modem. Preferably the transmitter should not need to communicate side information for canceling C^m for each symbol.

7. Since the transmitter must compute the PAR reduction signal for many transmit symbols, efficient algorithms must exist.

8. These additive signals should not reduce the data throughput significantly.

9. These PAR reduction structures should not prevent using coding techniques (block codes, convolutional codes, turbo codes, etc..)

Some additional desirable properties include the option for constraints on the combined transmit power, $E\{|X + C|^2\}$, or on each of the individual carriers, e.g. $|X_k^m + C_k^m|^2$.

4.3 Tone Reservation Technique

Recently, there has been a variety of creative methods on how to generate multi-carrier symbols with low PAR but none of these proposed methods is able to achieve simultaneously a large reduction in PAR with low complexity, with low coding overhead, without performance degradation and without transmitter receiver symbol handshake.

Josh Tallodo and John M. Ciffo first propose this technique [22]. Soon this technique is considered as the effective distortion less technique to reduce peak to average power ratio. This method is based on adding a symbol dependent, time domain signal to the original OFDM symbol to reduce its peaks. The transmitter does not send data on a small subset of carriers, which are optimized for PAR reduction. This technique can achieve 3db or 6db of PAR reduction for al loss in data rate of less than .2% and 5% respectively.

For the tone reservation technique, which is an additive reduction technique

$$x_k^m * c_k^m = 0 ------ (6.11)$$

where,

Data signal x_k^m & corelation signal c_k^m occupy the Disjoint Frequency Bins.

5. A New Approach for PAR Control in OFDM Signals

The peak to average (amplitude) ratio (PAR) of the time domain envelope is an important parameter at the physical layer of the communication system using OFDM signaling. The signal must maintain a specified average energy level in the channel to obtain the desired system error rate. The peak signal level relative to that average defines the maximum dynamic range that must be accommodated by the components in the signal flow path to support the desired average. The primary components of concern are the Digital to Analog Converter (DAC) and the power amplifier. A secondary concern with the peak signal level in the OFDM signal is the interference susceptibility of other shared channel signals to these peaks.

I, examine a number of techniques for controlling the PAR of an OFDM signal set. The goal of PAR control techniques is to reduce the peak-to-average ratio of the time domain signal formed by the random QAM modulation of the separate spectral terms comprising the signal. The control mechanism should not alter the spectra of the signal, since the amplitude and phase of the numerous spectral components contains the modulation content of the signal. Control mechanisms that do modify the spectral components of the signal introduce additive interference that impacts error rate performance. At first glance, the requirement to modify the time envelope but not modify the spectrum that caused the time signal seems like an impossible task. The seminal work by José Tellado and John Cioffi [22] at Stanford offer an enlightened solution to this problem. Here I am demonstrating the Problem of PAR and an effective solution to reduce it with all simulation on MATLAB 7 platform.

5.1 OFDM model without any PAR control*:*

Figure 5.1 shows a series of figures associated with an OFDM frame containing 401 frequency bins of 16 QAM modulated data. The time series of the OFDM frame is formed by a 1024-point transform so that the sample rate is more than twice the spectral support. We would obtain approximately the same figures if we had performed a 512-point transform on the 401 frequency bins and then performed a 1-to-2 interpolator to raise the output sample rate. The upper left corner shows the parametric complex envelope of the time series with two concentric circles. The inner circle represents the variance of the series, which has been normalized to a nominal value of 1.0. The outer circle is the value of the peak amplitude found in that particular realization. One or more samples are touching this outer circle. The figure in the upper right is the magnitude of the time series formed in this frame. It too contains a pair of indicator levels. The lower level is the sample variance of the time series while the upper level is the peak value found in this time series. The figure in the lower left is the entire set of 491 constellations of the 401 modulated frequencies overlaid on the same figure. Since there has not yet been a signal perturbation, the constellation points have not been disturbed and are mapped on top of each other. The figure in the lower right is the FFT power spectrum of the frame. There is no side-lobe structure present in the spectrum as there would be if the cyclic prefix were present or if the sample rate were raised. We do see the three different magnitudes distributed over the 401 occupied frequency bins.

Figure 5.1 Single OFDM Frame: Parameterized Complex Envelope, Magnitude of Complex Time Series, Constellation Set, and Spectrum of 401 Frequency Bins Embedded in a 1024-Point FFT

5.2 OFDM model for PAR control by clipping

Figure 5.2 presents a comparable set of figures for a similarly constructed OFDM frame. The slight difference, is that the outer ring in the upper left figure and the top level in the upper right level now represent a clipping level to the complex envelope signal. The level selected for this demonstration is 2.5. The probability of exceeding 2.5 for a unit variance Gaussian process is $1.7*10^{-3}$ or 1/575, and since the frame contains 401 independent samples, nearly every frame will have a sample value that crosses the clipping threshold. The time series in the upper right has five local peaks, tagged with markers that have crossed the clip threshold. The output of the clipper sets input levels exceeding the clip threshold to the clip level. The constellation diagram shows the scattering caused by the interference resulting from the minor amount of clipping. The spectrum shows the splattering of energy into the adjacent spectral region as well as the disturbance to the in-band spectrum.

Figure 5.2 Clipped OFDM Frame: Parameterized Complex Envelope, Magnitude of Complex Time Series, Constellation Set, and Spectrum of 401 Frequency Bins Embedded in a 1024-Point FFT

Figure 5.3 shows histograms formed for the input and output of the clip function for 5000 OFDM frames formed as 1024-point FFTs. Also shown in the complementary cumulative distribution presented with a dB (log scale). The clip level of 2.5 or 8.0 dB is clearly marked. We see that in these 5000 frames, the probability exceeding 11.0 dB (3.55 σ) or 3-dB above the clipping level is $3.0 \bullet 10^{-6}$.

Figure 5.3 Histogram of Input and Output of Clipping Operator and Probability of PAR Exceeding Indicated Level.

The process of clipping the peaks of the OFDM time series is defined below in eq (5.1).

$$\begin{aligned} s_2(n) &= s_1(n) && : if\ L_{CLIP} - |s_1(n)| > 0 \\ &= L_{CLIP} * sign(s_1(n)) && : if\ L_{CLIP} - |s_1(n)| < 0 \end{aligned} \tag{5.1}$$

The effect of this operator on an input signal is shown in figure 5.4.

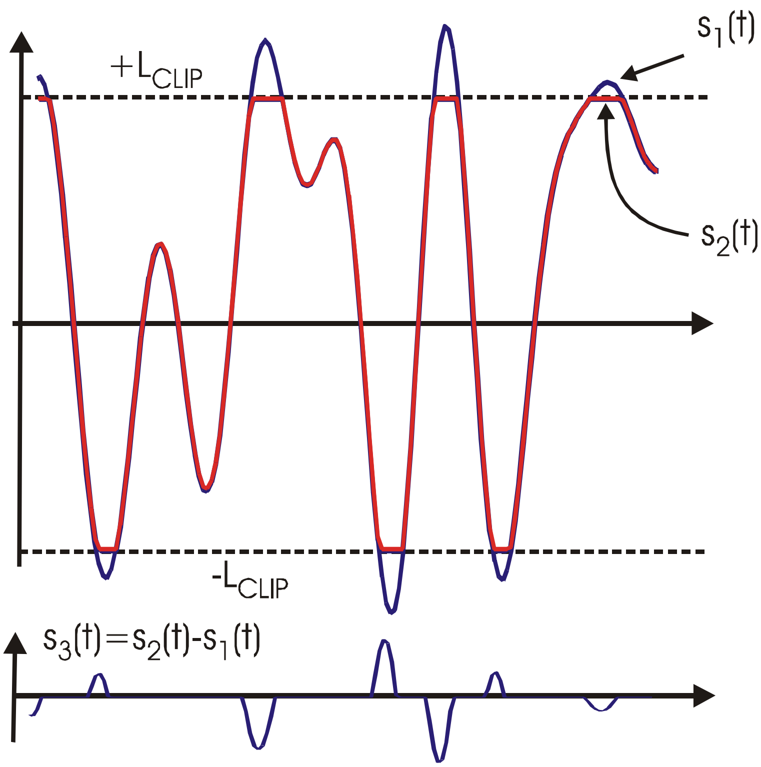

Figure 5.4 Input Signal $s_1(n)$, Clipped Output signal $s_2(n)$, and Difference Signal $s_3(n)$

The transfer function of the clipping operator is the memoryless nonlinearity shown in figure 5.5.

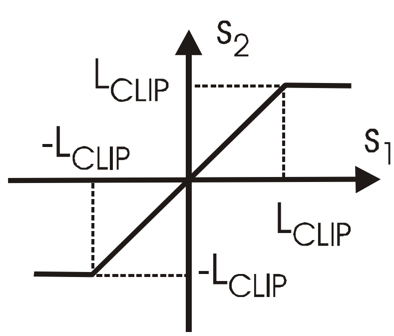

Figure 5.5 Transfer Function of Clipping Operator as a Memoryless Nonlinearity

The Clipping operator is identical to forming the difference between the input signal and a second signal equal to the part we want to eliminate. This second signal is defined as in eq 5.2.

$$\begin{aligned} s_3(n) &= s_1(n) - L_{CLIP} &&: if\ [s_1(n) - L_{CLIP}] > 0 \\ &= 0 &&: if\ [s_1(n) - L_{CLIP}] < 0 \end{aligned} \tag{5.2}$$

The transfer function of the alternate operator is a second memoryless nonlinearity shown in figure 7.6.

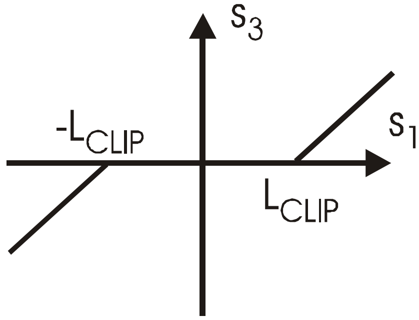

Figure 5.6 Transfer Function of Alternate Operator as a Memoryless Nonlinearity

How the two forms of the memoryless nonlinearity perform their function can be visualized with the aid of figure 7.7 as either a signal dependent multiplicative operator or as a signal dependent subtractive operator.

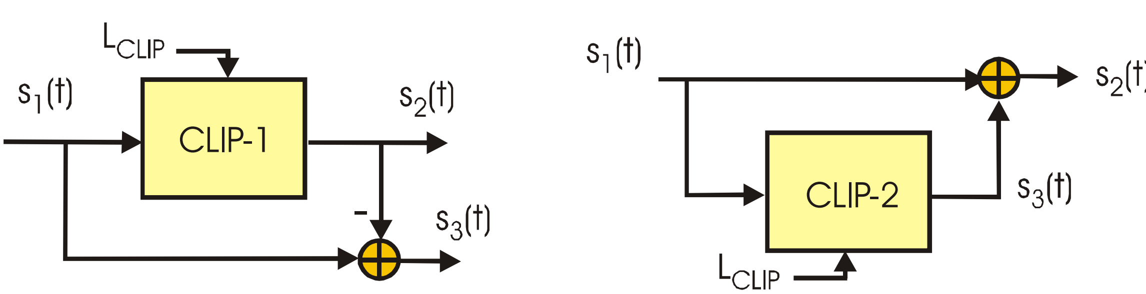

Figure 5.7 Clipping as an Multiplicative Operator and as a Subtractive Operator

5.3 OFDM model for PAR control by algorithm

The PAR control scheme can be modeled as a variant of the signal dependent subtractive operator. Looking at figure 4, we see that the clip-2 operator forms the signal s_3 from the signal s_1 that is to be added to the input signal s_1 to obtain the signal with controlled peaks s_2.

The task I, address is that of forming estimates of s_3 that we choose to call s_4 with desired spectral characteristics. Figure 8 shows the modified block diagram of the PAR control algorithm.

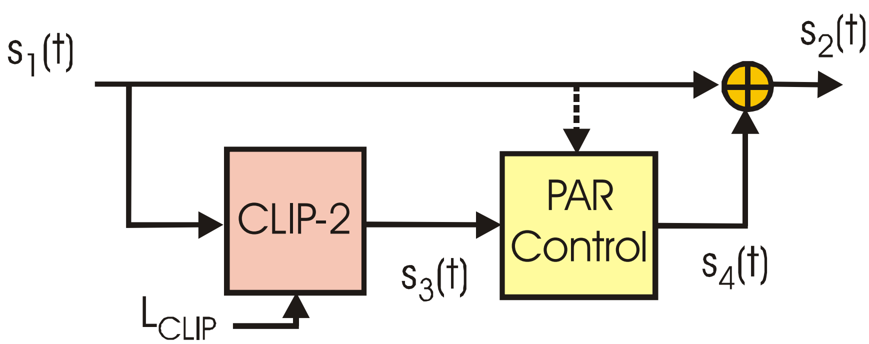

Figure 5.8 PAR Control Algorithm as a Subtractive Time Domain Operator

The primary characteristic we desire of signal s_4 is that it be orthogonal to s_1. There are a number of ways to manage this feat of apparent magic. The most obvious technique is to apply spectral components to s_4 that are orthogonal to the spectral components in s_1. The method proposed by Tellado and Cioffi is to reserve a pre assigned subset of the spectral bins at the input of the IFFT to build the PAR control signal with the remaining bins allocated to data modulation. The receiver has knowledge of the reserved frequency bins and knowing that they carry no modulation components they are simply discarded at the output of the FFT demodulator. This technique is called Tone Reservation. The structure of this process is suggested in figure 5.9. To first order, the PAR control algorithm projects the desired canceling signal $s_3(n)$ of the reserved basis sat of spectral lines to form the $s_4(n)$ the canceling signal component orthogonal to the modulation signal $s_1(n)$.

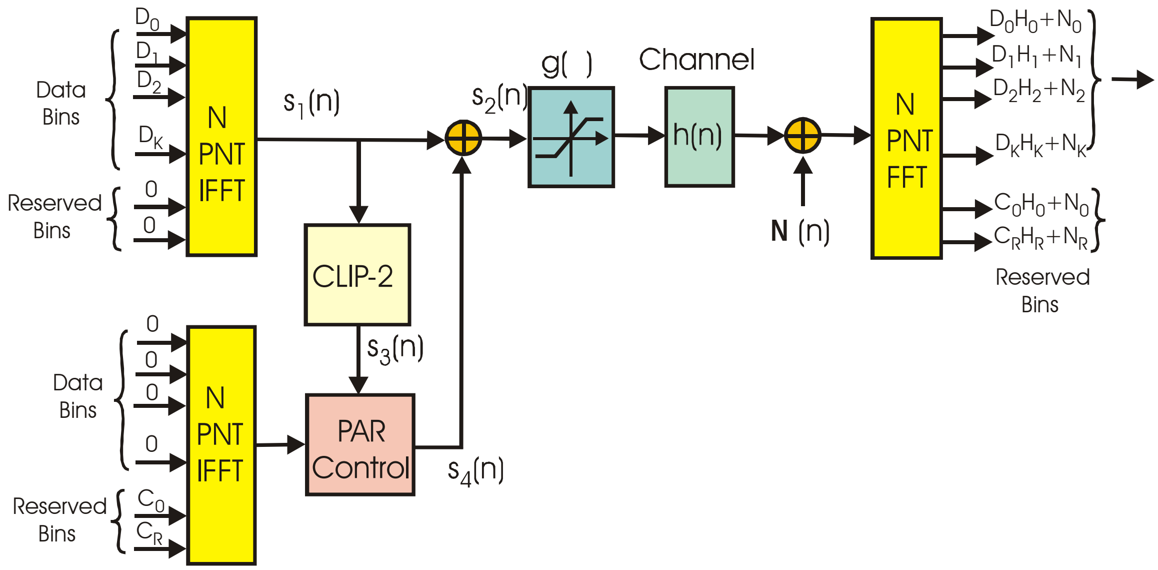

Figure 5.9 Tone Reservation Model of PAR control

To a first approximation, the process of projecting the canceling signal component on the reserved bin subspace is trivial, as I will show in a moment. It is instructive to consider the projection of a single data point on the subspace formed by the reserved tones. To cancel the single point the subspace projection must form an approximation to a time-domain unit impulse. If we successfully form a good approximation to the unit impulse then and set of canceling points can be formed by translated and scaled versions of the approximation. The problem now becomes one of approximating a unit impulse with a sparse subset, the reserved bins of the IFFT. We must resolve the question of how many bins, and how are they distributed? This is precisely the problem of forming beam patterns with a sparse array of antennae elements. To obtain a narrow main-lobe width in the approximation signal the signal must have large bandwidth. Thus we must avoid a cluster of closely spaced spectral terms and distribute the reserved spectral terms over the maximum bandwidth. A uniform distribution of spectral elements across the allowable spectral span will generate a comb function, a periodically spaced set of equal amplitude sin(x)/x functions.

Figure 5.10 Frequency and Time Function Formed by 11 Frequency Bins With Compact, Uniformly, and Randomly Distributed Spectral Positions

I conclude that the distribution should span maximum bandwidth to obtain a narrow main-lobe time function and be random to avoid periodicity in the time function. Figure 5.10 demonstrates the three frequency-time relationships just described for a distribution of 7.11 frequencies spanning 201 bins. As expected the random distribution offers the best approximation to a band limited impulse. The impulse is accompanied by a selection of side-lobes of various amplitudes and offset positions. We now seek a waveform formed with a specified number of spectral bins that achieves the minimum value of the maximum side-lobe level. We must also address the question of how does this minimum value vary with the number of frequencies we allocate to the spectral span. We conducted a Monte Carlos search consisting of 5000 realizations of time series to determine the minimum value of the maximum side-lobe for a range 5-to20 occupied bins in a span of 101 frequencies imbedded in a 256 point IFFT, Figure 7.11 presents the results of this search. We see that side-lobe amplitudes on the order of 0.5 are achievable with 10 bins out of 100. Examine figure 5.11, where the side-lobes for that random waveform have amplitude 0.6.

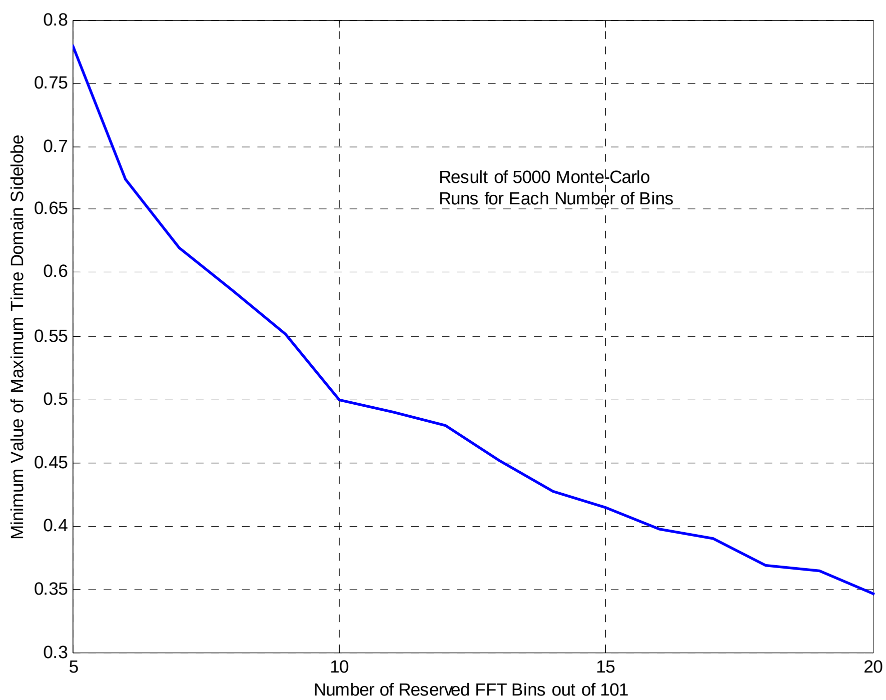

Figure 5.11 Minimum Value of Maximum Side-Lobe for Time Series Built with Random Distributions of Reserved FFT Bins

The minimum of the maximum side-lobe levels obtained for a fixed number of reserved frequency bins increases as the size of the IFFT increases. Thus as the size of the IFFT increases, we must increase the number of frequency bins required to obtain a specified minimum level of the maximum side-lobe. The number of required frequencies changes very slowly with increases in transform and spectral width. Table 1 below lists the number of frequency bins as a function of the transform size required to obtain side-lobes equal to or below amplitude 0.5.

Number of Bins	Minimum of Maximum Side-Lobe	Number of Frequency Bins	Size of IFFT
10	0.4865	101	256
13	0.4845	201	512
16	0.4990	401	1024

Table 4: **Number of Bins Required to Obtain Side-Lobe Level of 0.5 for Various Size Transforms**

Figures 5.13 through 5.15 show the result of Monte Carlos runs of 2000 trials of random bin placement in transforms of length 256, 512, and 1024 points with frequency spans of 101, 201, and 401 possible bins. The experiment was repeated a number of times till a realization with side-lobes below 0.5 were formed in the experiment. The figures show the sample distribution of the maximum side-lobe levels for the successful run, the locations of the spectral bins that achieved the minimum side-lobe for the run, and the time series formed by those spectral bins.

Doctor Heal Thyself:

Figures 5.16 and 5.17 show the result of an interesting experiment. The experiment applied the side-lobe reduction algorithm to the side-lobes of the time series used for side-lobe correction in the PAR reduction algorithm. This is an application of the advice, “Doctor heal thyself”. The initial series has the lowest valued maximum side-lobe found by random search and the final series has improved side-lobe levels. The spectral content of the corrections to the amplitude of the side-lobes is confined to the same reserved frequency bins used in the PAR reduction algorithm. Figure 5.16 is the initial spectral position and assignment with equal amplitude, real valued spectral values for the sequence that obtained the minimum valued maximum side-lobe. Also shown is the time series along with a threshold pulled from the next figure and finally the sample values and locations of the side-lobes in the time series that exceed the indicated threshold.

Figure 5.17 presents the same figures after the PAR algorithm has obtained the minimum side-lobes for the initial spectral assignment. The PAR algorithm forms a near-equal ripple time series from the initial time series by varying the amplitude and phase of the initial equal amplitude spectral values. In a number of experiments we found side-lobe improvements on the order of 10% to 20 % relative to initial time series identified by the random search.

10 Bins in 101: peak side-lobe = 0.486

[-4 12 13 14 18 23 29 33 36 43]

Figure 5.13 Histogram of Maximum Side-Lobe Levels in run of 2000 Random Placements of 10 Frequencies in Span of 101 Locations for 256 Point IFFT, Spectral Distribution for Minimum Realization, and Time Series with Minimum Maximum Side-Lobes

13 Bins in 201: peak side-lobe = 0.485

[-70 -56 -44 -29 -22 -21 -11 9 18 20 39 64]

Figure 5.14 Histogram of Maximum Side-Lobe Levels in run of 2000 Random Placements of 13 Frequencies in Span of 201 Locations for 512 Point IFFT, Spectral Distribution for Minimum Realization, and Time Series with Minimum Maximum Side-Lobes

16 Bins in 401: peak side-lobe = 0.4990

[-195 -178 -156 -138 -80 -70 -24 19 54 79 83 137 142 149 174 175]

Figure 5.15 Histogram of Maximum Side-Lobe Levels in run of 2000 Random Placements of 16 Frequencies in Span of 401 Locations for 1024 Point IFFT, Spectral Distribution for Minimum Realization, and Time Series with Minimum Maximum Side-Lobes

Figure 5.16 Initial Amplitude Assignment of 10 Spectral Lines with Positions Determined by 2000 Monte Carlos Random Frequency Assignmnents in Search for Minimum Value of Maximum Side-Lobe, Corresponding Time Series with Maximum Side-Lobe Amplitude of 0.481, and Peak Errors Relative to Threshold of Amplitude 0.43.

Figure 5.17 Final Amplitude Assignment of 10 Spectral Lines with Positions Determined by 2000 Monte Carlos Random Frequency Assignmnents in Search for Minimum Value of Maximum Side-Lobe, Corresponding Time Series with Maximum Side-Lobe Amplitude of 0.48 Dropped to Threshold, and Peak Errors Relative to Threshold of Amplitude 0.43.

Once the spectral distribution and spectral amplitude selection process is complete the time signal performing the PAR control is known and fixed. The PAR algorithm stores this waveform and performs cyclic shifts of the waveform to position the central peak at the various locations that require side-lobe control. The one or more cyclically shifted control signals are scaled by the measured excess at the peak positions and then subtracted from the OFDM signal to obtain the PAR controlled version of the signal.

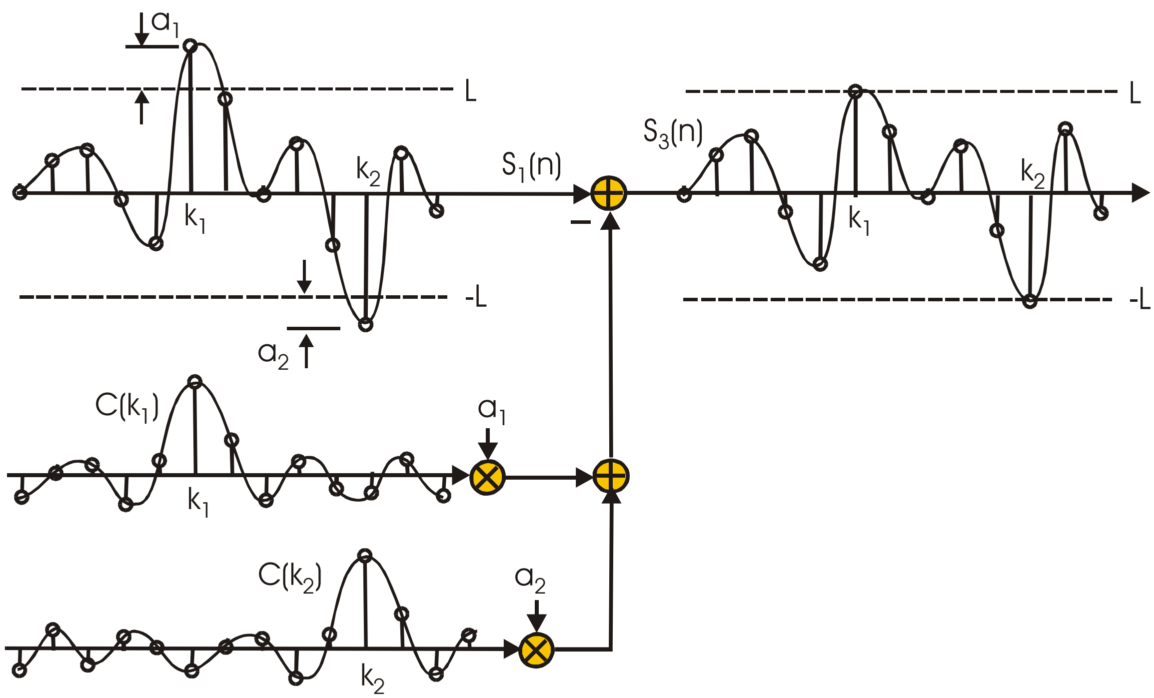

Figure 5.18 Visualization of PAR Control by Adding Orthogonal Canceling Series Formed from Reserved Frequency Bins to OFDM Side-Lobes that Exceed Threshold Test

Figure 5.18 illustrates the canceling process used in the PAR control algorithm. Figure 5.19 is a signal flow graph of the algorithm. The Single Pass is performed in the top segment with additional passes performed in the lower segment. The tasks are described as follows. The first task is to locate positions, if any, of OFDM samples that exceed the test threshold level L. The second task is to determine the amplitude to be removed from each offending sample. The third task is to form the correction series as a weighted sum of circularly shifted canceling signals. The fourth task is to subtract a portion of the composite canceling sequence; the size of the portion is related to the number of iterations in the algorithm. If there are additional iterations, the final task is to iterate the previous tasks, seeking new extrema that may now cross the threshold as a result of the previous correction while continuing to suppress previously identified extrema. Note that the prototype canceling series is computed once and saved as two successive cycles for ease of extracting cyclic shifted versions to form the composite canceling sequence.

Figure 5.19 Signal Flow Graph of PAR Control Algorithm

Figure 5.20 Input and Histogram, and Probability of Crossing PAR level for Single Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 16-Reserved Frequencies, Peak Side-Lobe = 0.46

Figures 5.20 show the histogram at the input and output of a single pass PAR reduction algorithm using the waveform built by the reserved frequencies. Also shown is the probability of crossing the indicated levels after the application of the PAR algorithm. Note that initially, the PAR algorithm reduces the peak amplitudes to the desired amplitude of 2.5 or 8-dB relative to normalized unit variance signal. As the amplitude of the required correction increase, the side-lobes of the correcting waveform starts to interact with side-lobes in the time signal to obtain summed secondary side-lobes that exceed the design threshold level. The single pass correction algorithm can maintain a 1.6 dB (20%) reduction in side-lobes when the side-lobes of the correcting term are on the order of 0.5.

Figure 5.21 Input and Histogram, and Probability of Crossing PAR level for Single Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 32-Reserved Frequencies, Peak Side-Lobe = 0.306

Figure 5.21 is similar to figure 5.20 except that the number of reserved frequencies in the OFDM signal set is 32 out of 401 (8%) rather than 16 out of 401 (4%). In exchange for the greater number of frequency terms in the canceling wave-shape, the peak side-lobe level has gone from 0.46 to 0.31 allowing the canceling series to have less interaction with the OFDM waveform during the additive canceling. Here too the side-lobes do eventually interact when the size of the terms being controlled are sufficiently large. For this canceling sequence the difference between the controlled and the raw peak amplitudes is on the order of 1.9 dB.

Figure 5.22 Histogram of Maximum Side-Lobe Levels in run of 2000 Random Placements of 64 Frequencies in Span of 401 Locations for 1024 Point IFFT, Spectral Distribution for Minimum Realization, and Time Series with Minimum Maximum Side-Lobes

64 Bins in 401: Pk side-lobe = 0.238

[-177 -171 -165 -160 -157 -146 -144 -131 -129 -126 -125 -114 -114 ....
-111 -108 -99 -98 -95 -87 -86 -76 -69 -59 -49 -38 -33 ....
-28 -26 -19 -18 -16 -15 -9 15 16 17 19 24 27 ....
39 48 49 64 68 78 88 103 108 110 115 123 131 ....
132 137 151 157 158 158 160 164 167 172 189 193]

Figure 5.23 Input and Histogram, and Probability of Crossing PAR level for Single Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 64-Reserved Frequencies, Peak Side-Lobe = 0.19

Figure 5.22 is similar to earlier figures, showing histogram and frequency allocation for random placement of 64 frequencies in a span of 401 frequencies. Also shown is the time series corresponding to the minimum maximum side-lobe of amplitude 0.238. This series was then subjected to the PAR reduction algorithm and the complex amplitudes adjusted to obtain a maximum side-lobe of 0.19.

Figure 5.23 shows the input and output histograms of the single pass PAR algorithm using the optimized canceling series. The controlled peak OFDM time series has peak values 2.8 dB below the raw OFDM time series.

Table 5 shows the amount of PAR improvement for the single pass PAR algorithm at signal levels corresponding to probability of error of 10^{-6} for different number of reserved frequency bins.

Number of Reserved Bins	16 out of 401	32 out of 401	64 out of 401
Percentage of Occupied Bins	4%	8%	15%
PAR Improvement	1.6 dB	1.9 dB	2.8 dB

Table 5: Comparison of PAR Reduction for Single Pass PAR Algorithm for Different Percent of Reserved Frequency Bins

Figure 5.24 shows the input and output histograms of the **two pass** PAR algorithm using the optimized canceling series formed with 64 reserved frequencies. The controlled peak OFDM time series has peak values 3.0 dB below the raw OFDM time series.

Figure 5.25 shows the input and output histograms of the **four pass** PAR algorithm using the optimized canceling series formed with 64 reserved frequencies. The controlled peak OFDM time series has peak values 3.4 dB below the raw OFDM time series.

Figure 5.26 shows the input and output histograms of the **two pass** PAR algorithm using the optimized canceling series formed with 16 reserved frequencies.. The controlled peak OFDM time series has peak values 2.7 dB below the raw OFDM time series.

Figure 5.27 shows the input and output histograms of the **four pass** PAR algorithm using the optimized canceling series formed with 16 reserved frequencies.. The controlled peak OFDM time series has peak values 3.3 dB below the raw OFDM time series.

Table 6 shows the amount of PAR improvement for the one, two, and three pass PAR algorithms at signal levels corresponding to probability of error of 10^{-6} for 16 reserved frequency bins.

Figure 5.24 Input and Histogram, and Probability of Crossing PAR level for Two Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 64-Reserved Frequencies, Peak Side-Lobe = 0.19

Figure 5.25 Input and Histogram, and Probability of Crossing PAR level for Four Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 64-Reserved Frequencies, Peak Side-Lobe = 0.19

Figure 5.26 Input and Histogram, and Probability of Crossing PAR level for Two-Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 16-Reserved Frequencies, Peak Side-Lobe = 0.46

Figure 5.27 Input and Histogram, and Probability of Crossing PAR level for Four-Pass Subtractive PAR Control of 401 Frequencies in 1024 Pont IFFT with 16- Reserved Frequencies, Peak Side-Lobe = 0.46

One pass		1.6 dB
Two pass		2.7 dB
Four pass		3.3 dB

Table 6: Comparison of PAR Reduction for PAR Algorithm with Different Number of Passes for 16-Reserved Frequencies.

6. ***Results & Conclusion***

To summarize, this major project gives the detail knowledge of a current key issue in the field of communications named Orthogonal Frequency Division Multiplexing (OFDM). This project highlights the main & burning problem in OFDM field i.e. Peak to Average Ratio Reduction (PAR). To fully support my thesis work, I simulated the entire work on MATLAB7. First I developed an OFDM system model then try to reduce PAR by implementing clipping. Then an entirely different approach, which is an additive reduction technique, is introduced. An algorithm is developed before simulation (Flow chart is given) and technique is successfully implemented.

The performance of this algorithm shows that it is a distortion-less technique and much better than clipping and any other techniques. In four passes we have reached to 3.3db reduction in PAR. Key conclusions are reached regarding the Probability of crossing the particular PAR level for fixed value of reserved Frequencies. We are now not forced to increase the number of reserved frequencies to decrease the PAR level to a particular value. This is definitely a key achievement.

The system model developed is quite flexible and can be easily modified and/or extended to study the performance of this scheme in conjunction with better equalization algorithms and error corresponding coding. From the study of the system, it can be concluded that we are able to reduce the PAR to some extent by using an easily implemented reduction technique. Although, it is totally theoretical approach but I hope that the day, when it is practically implemented will surely come in Future.

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ABBREVIATIONS

ADC:	Analog to digital converter
BER:	Bit error rate
BPSK:	Binary phase shift key
BW:	Band width
CNR:	Carrier to noise ratio
COFDM:	Coded orthogonal frequency division multiplexing
CDMA:	Code division multiplex access
CW:	Carrier wave
DAC:	Digital to analog converter
DVB:	Digital Audio Broadcasting
DVB-T:	DVB- Terrestrial

FFT: Fast Fourier Transform

FDM: Frequency division multiplexing

IFFT: Inverse Fast Fourier Transform

ICI: Inter carrier interference

ISI: Inter symbol interference

LAN: Local Area Network

OFDM: Orthogonal frequency division multiplexing

PAR: Peak to average reduction ratio

QAM: Quadrature Amplitude Multiplexing

QPSK: Quadrature Phase shift keying

16-QAM: 16 –state Quadrature Amplitude modulation

RF: Radio Frequency

SNR: Signal to noise ration

TR: Tone Reservation

1 ***MATLAB CODE***

Function Files:

```
% Defination of all constants used

b = 4;                               % Number of bits in message
N = 43;                              % Length of Data
half_N = floor(N/2);                 % Bin Center
fftLength = 128;                     % Spectrum Length
fftMidPoint = (fftLength/2) + 1;     % Spectrum Center
flowLength = ti_f1;                  % Number of frames
```

```
th = ti_f2;                    % Value of Threshold



function [sig] = generateSignal(indx, ti_f3, ti_f4);
vector = zeros(1, 128);
bins = (ones(1, 43) .* ti_f4);
bins(indx) = (ones(1,length(indx)) .* ti_f3);
vector(43 : 85) = bins;
vector = fftshift(vector);
signal = fft(vector, 128);
max_indx = find(abs(signal) == max(abs(signal)));
if length(max_indx) > 1
   max_indx = max_indx(2);
end

signal = signal ./ signal(max_indx);
signal = fliplr(signal);
sig = [signal signal];



function [bindata] = bindata(i, bits)
% Generate random binery data
% Useage <bindata(i, NumBits)>, where 'i' is the number of data and
% 'NumBits' is the number of bits in each data

bindata = round(rand(i, bits));



function [circle] = circle(r);
```

```
theta = 0:2*pi/2000:2*pi;
cir = r*exp(j*theta);
plot(cir, 'r-');
axis square;



function [data] = Data(i,type_flag)

%   This function returns digitally modulated data.
%   i.e. data = Data(Length_of_data, Type_of_modulation).
%   Length_of_data is the length of data.
%   Type_of_modulation takes values from 'BPSK','4QAM','16QAM' or '64QAM'.


if nargin < 2
   type_flag = 'BPSK';
end
type_flag = upper(type_flag);

if strcmp(type_flag, 'BPSK')
   data = 2*(floor(2*rand(1,i))- 0.5)+j*0.0000;
end

if strcmp(type_flag, '4QAM')
   I = 2*(floor(2*rand(1,i))- 0.5);
   Q = 2*(floor(2*rand(1,i))- 0.5);
   data = I + j*Q;
end

if strcmp(type_flag,'16QAM')
```

```
    I = 2*(floor(4*rand(1,i))- 1.5)/3;
    Q = 2*(floor(4*rand(1,i))- 1.5)/3;
    data = I + j*Q;
end

if strcmp(type_flag,'64QAM')
    I = 2*(floor(8*rand(1,i))- 2.5);
    Q = 2*(floor(8*rand(1,i))- 2.5);
    data = I + j*Q;
end



%This command generates 10 unique random numbers between 1 and 43
%The random numbers are stored in an array called "indx" file name index.m
while(1)
    indx(1) = round(43 * rand);
    if indx(1) == 0;
        continue;
    else
        break;
    end
end

while(1)
    indx(2) = round(43 * rand);
    if indx(2) == 0 | indx(2) == indx(1)
        continue;
    else
        break;
    end
```

```
end

while(1)
    indx(3) = round(43 * rand);
    if indx(3) == 0 | indx(3) == indx(1) | indx(3) == indx(2)
        continue;
    else
        break;
    end
end

while(1)
    indx(4) = round(43 * rand);
    if indx(4) == 0 | indx(4) == indx(1) | indx(4) == indx(2) | indx(4) == indx(3)
        continue;
    else
        break;
    end
end


while(1)
    indx(5) = round(43 * rand);
    if indx(5) == 0 | indx(5) == indx(1) | indx(5) == indx(2) | indx(5) == indx(3) | indx(5)
== indx(4)
        continue;
    else
        break;
    end
end
```

```
while(1)
    indx(6) = round(43 * rand);
    if indx(6) == 0 | indx(6) == indx(1) | indx(6) == indx(2) | indx(6) == indx(3) | indx(6)
== indx(4) | indx(6) == indx(5)
        continue;
    else
        break;
    end
end

while(1)
    indx(7) = round(43 * rand);
    if indx(7) == 0 | indx(7) == indx(1) | indx(7) == indx(2) | indx(7) == indx(3) | indx(7)
== indx(4) | indx(7) == indx(5) | indx(7) == indx(6)
        continue;
    else
        break;
    end
end

while(1)
    indx(8) = round(43 * rand);
    if indx(8) == 0 | indx(8) == indx(1) | indx(8) == indx(2) | indx(8) == indx(3) | indx(8)
== indx(4) | indx(8) == indx(5) | indx(8) == indx(6) | indx(8) == indx(7)
        continue;
    else
        break;
    end
end

while(1)
```

```
    indx(9) = round(43 * rand);
    if indx(9) == 0 | indx(9) == indx(1) | indx(9) == indx(2) | indx(9) == indx(3) | indx(9)
== indx(4) | indx(9) == indx(5) | indx(9) == indx(6) | indx(9) == indx(7) | indx(9) ==
indx(8)
        continue;
    else
        break;
    end
end

while(1)
    indx(10) = round(43 * rand);
    if indx(10) == 0 | indx(10) == indx(1) | indx(10) == indx(2) | indx(10) == indx(3) |
indx(10) == indx(4) | indx(10) == indx(5) | indx(10) == indx(6) | indx(10) == indx(7) |
indx(10) == indx(8) | indx(10) == indx(9)
        continue;
    else
        break;
    end
end


indx = unique(indx);




% Loads all files and variables file name loadFiles.m

load ti_f1;
load ti_f2;
load ti_f3;
```

```
load ti_f4;
load indx;



function[seq] = lookup(c_data);

mc_data = c_data;% * 3;

re = real(mc_data);
im = imag (mc_data);

if (abs(re) < 2 & abs(im) < 2)

   if (sign(re) == -1 & sign(im) == -1)

      seq = [0 0 0 0];
   else if (sign(re) == -1 & sign(im) == 1)

         seq = [0 1 0 0];
      else if (sign(re) == 1 & sign(im) == 1)
            seq = [1 1 0 0];
         else
            seq = [1 0 0 0];
         end
      end
   end
end


if (abs(re) > 2 & abs(im) < 2)
```

```
    if (sign(re) == -1 & sign(im) == -1)

        seq = [0 0 1 0];
    else if (sign(re) == -1 & sign(im) == 1)

            seq = [0 1 1 0];
        else if (sign(re) == 1 & sign(im) == 1)
                seq = [1 1 1 0];
            else
                seq = [1 0 1 0];
            end
        end
    end
end


if (abs(re) < 2 & abs(im) > 2)

    if (sign(re) == -1 & sign(im) == -1)

        seq = [0 0 0 1];
    else if (sign(re) == -1 & sign(im) == 1)

            seq = [0 1 0 1];
        else if (sign(re) == 1 & sign(im) == 1)
                seq = [1 1 0 1];
            else
                seq = [1 0 0 1];
            end
        end
```

```
    end
end

if (abs(re) > 2 & abs(im) > 2)

    if (sign(re) == -1 & sign(im) == -1)

        seq = [0 0 1 1];
    else if (sign(re) == -1 & sign(im) == 1)

            seq = [0 1 1 1];
        else if (sign(re) == 1 & sign(im) == 1)
                seq = [1 1 1 1];
            else
                seq = [1 0 1 1];
            end
        end
    end
end




function [tone] = make_tone(nn, fftLength, sig);

startAddress = (fftLength + 2) - nn;
stopAddress = startAddress + fftLength - 1;



tone = sig(startAddress : stopAddress);
```

```
function[m_data] = mapDataQPSK(seq);


p = [0 0
     0 1
     1 0
     1 1];

if seq == p(1,:)

    I = -3;
    Q = -3;
    m_data = (I + j * Q);% / 3;

else if seq == p(2,:)

        I = -3;
        Q = 3;
        m_data = (I + j * Q);% / 3;

    else if seq == p(3,:)

            I = 3;
            Q = -3;
            m_data = (I + j * Q);% / 3;

        else if seq == p(4,:)

                I = 3;
```

```
            Q = 3;
            m_data = (I + j * Q);% / 3;
        end
    end
  end
end



%file name test.m
counter = 0;
for i = 1:10

  for ii = 1:43
    p = [real(xx(i,ii)) imag(yy(i,ii)) real(xx(i,ii)) imag(yy(i,ii))];
    disp(real(p);

    pause;
  end
end



function [xline] = xline(x, y);
plot(x, y, 'r-');





function [yline] = yline(x, y);
plot(x, y, 'r');
```

```
% filename imp_bin.m
clear all;
close all;
clc;

%%%%%

N = 43;                         % Length of Data
half_N = floor(N/2);              % Bin Center
fftLength = 128;
fftMidPoint = (fftLength/2)+1;
flowLength = 500;                % Number of frames
trials = 5;

%%%%%

sl = 10;

for i = 1:10000

   vec = zeros(1, 128);

   fr_bins = zeros(1, 43);

   GenRandIndx;

   fr_bins(indx) = 1;

   vec((fftMidPoint - half_N) : (fftMidPoint + half_N)) = fr_bins;
```

```
subplot(121);
stem(vec, 'r'); grid on;

signal = fftshift((fft(vec, 128)));

max_sig = find(abs(signal) == max(abs(signal)));

if length(max_sig) > 1
   max_sig = max_sig(2);
end

signal = signal ./ signal(max_sig);

subplot(122);
plot(abs(signal)); grid on;
hold on;

temp = signal;

pk = find(abs(temp) == 1);

if length(pk) > 1
   if pk(1) > pk(2)
      pk = pk(1)
   else
      pk = pk(2);
   end
end

temp(pk -2 : pk + 2) = 0;
```

```
    if max(abs(temp)) < sl
        sel_vec = vec;
        sel_sig = signal;
        sel_indx = indx;
        sl = max(abs(temp))
    end

    plot(1:130, sl, 'r.');
    hold off;
    axis([0 140 0 1]);

    pause(0.0000001);
end

sigl = [sel_sig sel_sig];

% save sigl sigl;
% save sel_indx sel_indx;
% save sel_vec sel_vec;

indx = unique(sel_indx)

save indx;
```

1.1 Main Program File:

```
clear all;

close all;

clc;


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%
tiGUI;


waitfor(tiGUI);


loadFiles;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%
constants;
sig = generateSignal(indx, ti_f3, ti_f4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%


for i = 1 : flowLength

   msg = bindata(N, b);
   store_msg(i,:) = reshape(msg', 1, (4*N));

   for ii = 1 : N
      data(ii) = mapData16QAM(msg(ii,:));
   end

   len = length(indx);
   msg_qpsk = bindata(len, 2);
   store_msg_qpsk(i,:) = reshape(msg_qpsk', 1, (len*2));

   for ii = 1 : len
      temp(ii) = mapDataQPSK(msg_qpsk(ii,:));
   end

   data(indx) = temp;

   store_data(i,:) = data;
   store_data_rb(i,:) = data(indx);
```

```
    e_frame = zeros(1, fftLength);
    e_frame((fftMidPoint - half_N - 1):(fftMidPoint + half_N - 1)) = data;

    frame = fftshift(e_frame);

    store_frame(i,:) = frame;

    tx_data(i,:) = ifft(frame, fftLength);
    store_tx_data = tx_data;

end

save tx_complex_data.mat store_data;
save tx_complex_data_rb.mat store_data_rb;


f1 = figure(1);
Maximize(f1);

for i = 1 : flowLength

    xx = store_data(i,:);
    xx(indx) = 0 + j*0;
    yy = store_data_rb(i,:);

    subplot(2, 3, 1);
    plot(xx, 'bo'); grid on;
    title('Undistorted signal constellation (Unreserved bins)');
    hold on;
    axis([-4 4 -4 4]);
```

```
    subplot(2, 3, 4);
    plot(yy, 'bo'); grid on;
    hold on;
    title('Undistorted signal constellation (Reserved bins)');
    axis([-4 4 -4 4]);

    pause(0.0000001);

end



data = reshape(tx_data, 1, (flowLength*fftLength));

std_data = std(data);

data = data ./ std_data;

sigma = std(data);
threshold = th * sigma;


h_data = abs(data);

[nb, xx] = hist(h_data, 1000);


nb_temp = nb ./ sum(nb);
nb = filter([1 0], [1 -1], nb_temp);
nb = 1 - nb;
```

```
subplot(1, 3, 3);
semilogy(xx, abs(nb), 'r'); grid on;
hold on;
semilogy(xx(1:100:1000), abs(nb(1:100:1000)), 'ro'); grid on;

axis([0 4 (10^(-5)) 1]);
xlabel('Sigma (Standard Diviation)');
ylabel('Probability of occurance of peak');
title('Occurrence of peak Vs Sigma');
hold on;

% pause;

ustd_data = reshape(data, flowLength, fftLength);



f2 = figure(2);
Maximize(f2);

for i = 1 : flowLength

   data = ustd_data(i,:);
   clf;

   subplot(2, 3, 3);
   plot(-0.5:1/128:0.5-1/128, fftshift(20*log10(abs(fft(data,128))))); grid on;
   axis([-0.5 0.5 0 50]);
   title('Spectrum of the undistorted signal');

   subplot(2, 3, 1);
   title('Undistorted signal with the cancelling signal');
```

```
line([0 fftLength], [threshold threshold], 'color', 'r');
hold on;

plot(abs(data)); grid on;
axis([0 128 -2 4]);
hold off;


subplot(2, 3, 2);
circle(1);
hold on;

plot(data); grid on;
hold on;

circle(threshold);
axis([-3 3 -3 3]);
title('Undistorted complex data');
hold off;



for count = 1 : 3

   node = find(abs(data) > threshold);

   if length(node) == 0

      clp_data = data;

   else
```

```
mm = length(node);

for ii = 1 : mm
   nn = node(ii);

   delta = abs(data(nn)) - threshold;

   tone_t = make_tone(nn, fftLength, sig);

   tone = delta * tone_t;



   subplot(2, 3, 1);
   hold on;
   plot(((-1) * abs(tone)), 'r')

   ang_data = angle(data(nn));

   tone = tone * exp(j * ang_data);

   clp_data = data - tone;

   subplot(2, 3, 4);
   title('Distorted signal with cancelling signal');
   plot(-1 * abs(tone), 'r');
   hold on;

   plot(abs(clp_data), 'g'); grid on;
   hold on;

   line([0 128], [threshold threshold], 'color', 'r');
```

```
            hold on;
            axis([0 128 -2 4]);

            data = clp_data;

        end
    end
end

subplot(2, 3, 5);
circle(1);
hold on;

plot(clp_data, 'k'); grid on;
hold on;

circle(threshold);
axis([-3 3 -3 3]);
title('distorted complex data');
hold off;



subplot(2, 3, 1);
hold off;

subplot(2, 3, 4);
plot(abs(clp_data), 'k'); grid on;
hold on;

line([0 128], [threshold threshold], 'color', 'r');
```

```
    axis([0 128 -2 4]);
    hold on;

    line([0 128], [0 0], 'color', 'g');
    hold off;

    subplot(2, 3, 6);
    plot(-0.5:1/128:0.5-1/128, fftshift(20*log10(abs(fft(clp_data,128)))), 'k'); grid on;
    axis([-0.5 0.5 0 50]);
    title('Spectrum of the distorted signal');

    store_clp_data(i,:) = clp_data;

    pause;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
data = reshape(store_clp_data, 1, flowLength * fftLength);
no_ustd_data = data .* std_data;
rx_data = reshape(no_ustd_data, flowLength, fftLength);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure(1);
for i = 1 : flowLength

    data = fft(rx_data(i,:), 128);
    data = fftshift(data);

    rx = [data((fftMidPoint - half_N - 1):(fftMidPoint + half_N - 1))];
```

```
    store_rx_data(i,:) = rx;



    subplot(2, 3, 5);
    plot(rx(indx), 'ko'); grid on;
    title('Distorted signal constellation (Reserved Bins)');
    axis([-4 4 -4 4]);
    hold on;

    rx(indx) = 0;

    subplot(2, 3, 2);
    plot(rx, 'ko'); grid on;
    title('Distorted signal constellation (Unreserved bins)');
%     axis([-4 4 -4 4]);
    hold on;

    pause(0.00000001);
end

save rx_complex_data.mat store_rx_data;



dd = reshape(store_clp_data, 1, flowLength * fftLength);
h_clp_data = abs(dd);



[nb2, xx2] = hist(h_clp_data, 1000);



nb2_temp = nb2 ./ sum(nb2);
```

```
nb2 = filter([1 0], [1 -1], nb2_temp);
nb2 = 1 - nb2;

subplot(1, 3, 3);
semilogy(xx2, abs(nb2), 'k'); grid on;
hold on;
semilogy(xx2(1:100:1000), abs(nb2(1:100:1000)), 'k*');

axis([0 4 (10^(-5)) 1]);

legend('', 'Undistorted Signal', '', 'Distorted Signal');
hold off;


%---------------------------------------End of Program-----------------------------------
```