STUDY OF ENVELOPES FOR SEISMIC RESPONSE VECTORS

A Dissertation Submitted in Partial Fulfilment of the Requirements for Award of the Degree of

MASTER OF ENGINEERING IN STRUCTURAL ENGINEERING

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CERTIFICATE

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Abstract

In this project, the Analysis of Building frames by the Time-history and the Response Spectrum Method in SAP2000 v 11 (Advanced) has been carried out.

IS 1893 - 2002/ IS 456 - 2000 recommended to take peak values of responses such as Axial force P, Moments M_x and M_y to design RCC column sections, this leads to rectangular response envelop when earthquake are applied along the principal axes. In this approach upto 50 % extra reinforcement get provide.

In this project present study on approach developed by Charles Menun and and Armen Der Kiureghian which consider combination of responses P, Mx and My at one instant this leads to elliptical response envelop, when earthquake are applied along principal axes and Supreme envelop (combination of all the elliptical envelops for full range of angle between principal direction of earthquake and structure) when principal direction of earthquake is not known in advance.

This procedure is based upon the response spectrum method and the Penzien and Watabe (1975) characterization of multicomponent ground motions. It inherits a number of assumptions implicit in these methods. Specifically, the response spectrum method is based on stationary random vibrations theory and entails approximations involving peak factors (Der Kiureghian 1981), whereas the Penzien-Watabe idealization assumes the principal directions of the ground motion remain fixed for the duration of the strong motion. Naturally, these assumptions and approximations affect the accuracy of this procedure.

In First phase of research work, we generate all the data required to produce that Envelop and Supreme Envelop that bounds a vector of seismic responses when for case when the principal direction of earthquake w.r.t. to structure is known and when the direction is unknown, respectively. The comparison of elliptical envelop with Time-history results both before and after adding static contribution has been carried out. Then it is demonstrated significance of that envelop over traditionally used Rectangular envelop in design of columns.

It has been observed that when the structure is subjected to only one component of ground motion directed along the U1-axis (i.e., load case 1), only small differences exist between the reinforcement ratios predicted by the these approaches. This is because when the building is loaded in only one direction, the resulting responses are strongly correlated, and the critical combination of responses predicted by the elliptical envelope lies near one of the corners of the rectangular envelope. This is similar to 2D Frame analysis in which Axial force and Moments are almost proportionally correlated.

When the second component of ground motion is included in the analysis, significant differences appear in the computed reinforcement ratios by Rectangular and elliptical envelop developed. The introduction of the second component of ground motion causes the correlation between the responses to weaken. The results obtained upon introducing the third component of ground motion directed vertically along the $\it U3$ -axis differ only marginally from the results obtained for 2-component results. This is because the vertical component of ground motion does not contribute substantially to the responses of the example building (the symmetric envelop having no vertical irregularity).

In the second phase of analysis we analyzed building frames (with vertical irregularities) and try to understand behavior of building frames under vertical component of earthquake. For this analysis we took data of Northridge earthquake at Sylmar, CA - County Hospital, January 17, 1994, which is recognized as earthquake having magnitude of vertical component as large as that of the horizontal component. We did analysis with two different model first which is used in developing above mention approach and second model we divided all the beams in two parts (as per recommended by S. H. Ju, C. W. Liu, and K. Z. Wu in their paper published in ASCE, named 3D analysis of building under vertical component of earthquake).

We have also study an approach given by Martin R. Button, Edward L. Willons. They have shown that estimation of combined stress directly by mean maximum values of axial forces and moments is incorrect. A correct estimate of stress is obtained only if equation is evaluated for each mode shapes and maximum stress can then be found from combining these by the CQC method of modal combination (because both axial force and moment does not peaks simultaneously) and shown that how to evaluate critical values of different response of structure for earthquake loading for full range of angle between principal direction of structure axis and earthquake component in computer run. We find out modal values of responses like axial forces and moments for column section in SAP2000 v 11 (Advanced). We develop *Stress envelop* which enclose pure axial stress and stress due to moment at different points on column section. We have done comparison of results from both the approaches.

CONTENTS

Certificate Acknowledgement Abstract		i ii iii	
	omenclature	V	
	able of Contents	Vi	
16	able of Contents	V	
Ch	napters		
1.	Introduction	Page	
	1.1 General	1	
	1.2 Dynamic Analysis of Buildings	1	
	1.2.1 Modelling	1	
	1.2.1.1 Lumped Mass Modelling	2	
	1.2.1.2 Consistent Mass Modelling	2	
	1.2.2 Seismic analysis methods for Building	2	
	1.2.3.1 Equivalent Static method	2	
	1.2.3.2 Response spectrum method	3	
	1.2.3.3 Time – History Methods	3	
	1.2.3.4 Envelop for Seismic Response Vector	6	
	1.3 Dynamic analysis for Multi – Component Earthquake	7	
	1.4 Dynamic Analysis of structure for Vertical Component of Earthquake	8	
	1.5 Scope of work	11	
	1.6 Organization of Report	11	
2.	Literature Review	13	
	2.1 Response Spectrum Method	13	
	2.1.1 Modal Combination Rules	15	
	2.1.2 Direction Combinations Rules (orthogonal effect)	19	
	2.2 Previous Works	23	
	2.3 Recommendation for Dynamic Analysis for Multi-Component Earthquake Sp		
	2.4 3D Analysis of Building under Vertical Component of Earthquake	26	
	2.5 Characteristic of 3-Dimensional Earthquake Ground Motion	27	
	2.6 Software Review (Dynamic Analysis in SAP2000)	28	
	2.6.1 SAP2000 Analysis Features	28	
	2.6.2 Structural Analysis and Design	28	
	2.6.3 About Manuals	29	
	2.6.4 Dynamic Analysis in SAP	29	
	2.6.4.1 Degrees of Freedom	29	
	2.6.4.2 Degree of Freedom Output	32	
	2.6.4.3 Mass Source	33	
	2.6.4.4 Material Damping	35	
	2.6.4.5 Constraints	36	
	2.6.4.6 Analysis Cases	38	
	2.6.4.6.1 Case name	39	

_		
	2.6.4.6.2 Analysis type	39
	2.6.4.6.3Loads applied	39
	2.6.4.6.4 Running Analysis Cases	39
	2.6.4.6.5 Modal Analysis	39
	2.6.4.6.6 Modal Analysis Output	42
	2.6.4.6.7 Response-Spectrum Analysis	46
	2.6.4.6.8 Linear Time-History Analysis	48
	2.6.5 Loading on Elements	50
	2.6.5.1 Frame Distributed Loads	50
	2.6.5.2 Trapezoidal Loads	51
	2.6.5.3 Area Loads - Gravity (All)	51
	2.6.5.4 Area Loads - Surface Pressure (All)	52
	2.6.5.5 Area Uniform load	52
	2.6.6 Joint Element Output Conventions	53
	2.6.7 SAP Output	56
	2.6.7.1Assembled Joint Mass Output	53
	2.6.7.2 Displacement Output	53
	2.6.7.3 Force Output	53
	2.6.7.4Element Joint Force Output	54
	2.6.7.5 Constraint Output	54
	2.6.8 Accessing the Assembled Stiffness and Mass Matrices	54
3.	Methodology	59
	3.1 Charles Menun and Armen Der Kiureghian Approach	59
	3.2 Martin R. Button, Edward L. Willons Approach	63
4.	Numerical Problem	68
	4.1 Numerical problem 1	68
	4.2 Numerical problem 2	93
	4.3 Numerical problem 3	114
	4.4 Numerical problem 4	139
5.	Results & Discussion	150
6.	Conclusion & Future Scope	152
	6.1 Conclusion	152
	6.2 Future Scope	152
	Appendix 1	I
	References	IX

CHAPTER 1

Introduction

1.1 General

In earthquake engineering, the response spectrum method is conventionally used to estimate the maximum values of responses acting in linear structures subjected to seismic ground motions. Response Spectrum method gives individual peak responses are obtained using a prescribed set of response. These modal maxima are then combined in an appropriate manner to estimate the maximum values of the responses of interest by Combination rules that properly account for correlations between modal responses and between components of ground motion. The method bounds individual responses in an average sense. The 1996 edition of the "Blue Book" issued by the Structural Engineering Association of California observes that response spectrum analysis "is the preferred method of dynamic analysis for most buildings".

The Modal Time-History analysis uses the method of mode superposition to find responses of structure. In this method earthquake ground motion is directly applied at base of structure. The basic mode superposition approach is restricted to linearly elastic analysis; this produces the complete time history response of joint displacements and member forces. There have been major disadvantages in the use of this approach is that the method produces a large amount of output information that can require a significant amount of computational effort to conduct all possible design checks as a function of time.

1.2 Dynamic Analysis of Buildings

1.2.1 Modeling

How well the computed response agrees with the actual response of a structure during an earthquake depends primarily on the quality of the structural idealization. If the structure experiences no structural damage, good estimates of the response during the earthquake can usually be computed from an equivalent linear model with viscous damping. If the computed natural periods and modes and the estimated damping ratios represent the properties of the structure during the earthquake, the model analysis procedure will accurately predict linear response.

However, if the structure experiences no structural damage, good estimates of the response during the earthquake can usually be computed from an equivalent linear model with viscous damping. If the computed natural periods and modes and the estimated damping ratios represent the properties of the structure during the earthquake, the model analysis procedure will accurately predict linear response.

The usual situation, however, is different in the sense the natural periods and modes are computed from an idealization of the structure. It is the quality of this idealization that determines the accuracy of response. Therefore, only those structural and nonstructural elements that contribute to the mass and stiffness of the structure at the amplitude of motion expected during the earthquake should be included in the structural idealization and their stiffness properties should be determined using realistic assumptions.

The dynamic analysis is all about inertia forces and this is due to mass of structure hence modeling mass of actual structure is very important. The mass is distributed throughout an actual structure, but it can be idealized by the Lumped Mass Method in which mass is concentrated at the nodes of the discretized structure or by the Consistent mass method.

1.2.1.1 Lumped Mass Modelling

Such a lumped mass modeling is conventionally used and satisfactory. The mass representation can be simplified for multistory building because of the constraining effect of the floor slabs or floor diaphragms. Each floor diaphragms is usually assumed to be rigid in its own plane but is flexible in bending in the vertical direction, which is a reasonable representation of the true behavior of several types of floor systems.

1.2.1.2 Consistent Mass Modelling

In this type of modeling we evaluate mass coefficients corresponding to the nodal coordinates of structures by procedure similar to determination of the stiffness coefficients. The mass matrix coming out of this modeling is not diagonal matrix like one in the lumped mass modeling. In this modeling assignment to point mass includes rotational effect. This type of modeling is ideal representation of mass of structures.

1.2.2 Seismic analysis methods for Building

The Equivalent static method, Response spectrum method and Time – History method are conventional seismic analysis methods recommended for building in various codes.

1.2.2.1 Equivalent Static method

In Equivalent static method we determine equivalent static lateral forces, which can be directly apply on various floor of building to find internal forces due to lateral forces. First we determine lumped mass at each floor, mass of elements between two floors is divided equally to both floors. We determine fundamental natural periods of building by empirical expression given in code for moment resisting frames. For that natural period we determine design horizontal acceleration spectrum value. The base shear is distributed to various floors as per expression given in code. The vertical distribution of equivalent static lateral force in building

frames is parabolic. Various codes recommend this method of analysis for regular buildings, which do not have any kind of irregularities (Geometric, stiffness or mass irregularities).

1.2.2.2 Response spectrum method

Response spectrum method is consisting of Eigen value and Eigen vectors determination, Eigen values gives natural frequencies of various modes and Eigen vectors are mode shapes. Modal participation vectors are calculated which contribution of various modes to response of structure. In this method design lateral force are determined for each mode shapes then these modal maxima are then combined in an appropriate manner to estimate the maximum values of the responses of interest. Combination rules that properly account for correlations between modal responses and between components of ground motion are well developed [e.g., Singh and Chu (1976), Der Kiureghian (1981), Gupta (1990), Singh and Maldonado (1991), Der Kiureghian and Nakamura (1993), Menun and Der Kiureghian (1998)].

The closeness of the frequencies that is typical of most three dimensional building structures that are designed to resist earthquakes from both directions equally. Because of the small mass eccentricity, which is normal in real structures, the fundamental mode shape has x, y, as well as torsion components. Therefore, the model represents a very common three-dimensional building system. Also, note that there *is not a mode shape in a particular given direction*, as is implied in many building codes and some text books on elementary dynamics. For closely spaced modes it is recommended to use CQC method for combining modal responses.

1.2.2.3 Time – History method

Time- history analysis is used to determine the dynamic response of a structure to arbitrary loading. The dynamic equilibrium equations to be solved are given by:

$$Ku(t) + C\dot{u}(t) + M\ddot{u}(t) = r(t)$$
1.1

Where, K is the stiffness matrix; C is the damping matrix; M is the diagonal mass matrix; u(t), $\dot{u}(t)$, and $\ddot{u}(t)$ are the displacements, velocities, and accelerations with respect to the ground of the structure; and r is the applied load. If the load includes ground acceleration, the displacements, velocities, and accelerations are relative to this ground motion.

Modal Time History

Modal super position provides a highly efficient and accurate procedure for performing time-history analysis. Closed-form integration of the modal equations is used to compute the response, assuming linear variation of the time functions between the input data time points. Therefore, numerical instability problems are never encountered, and the time increment may be any sampling value that is deemed fine enough to capture the maximum response values.

One-tenth of the time period of the highest mode is usually recommended; however, a larger value may give an equally accurate sampling if the contribution of the higher modes is small.

The modes used are those computed by undamped free-vibration Modes (eigenvectors problem) or the load-dependent Ritz-vector. If all of the spatial load vectors, pi, are used as starting load vectors for Ritz-vector analysis, then the Ritz vectors will all ways pro duce more accurate results than if the same number of eigenvectors is used. Since the Ritz-vector algorithm is faster than the eigenvector algorithm, the former is recommended for time-history analyses.

It is up to us to determine if the Modes calculated are adequate to represent the timehistory response to the applied load. One should check:

- That enough Modes have been computed
- That the Modes cover an adequate frequency range
- That the dynamic load (mass) participation mass ratios are adequate for the load cases and/or Acceleration Loads being applied
- That the modes shapes adequately represent all desired deformations

In Linear modal time – history method principal coordinates are defined such that when equation of motion are expressed in those coordinate, the equation of motion get uncoupled.

Nonlinear Modal Time-History Analysis

The method of nonlinear time-history analysis method developed by Wilson (Ibrahimbegovic and Wilson, 1989; Wilson, 1993). The method is extremely efficient and is designed to be used for structural systems which are primarily linear elastic, but which have a limited number of predefined nonlinear elements. For the FNA method, all nonlinearity is restricted to the Link/Support elements. A short description of the method follows.

The dynamic equilibrium equations of a linear elastic structure with predefined nonlinear Link/Support elements subjected to an arbitrary load can be written as:

$$Ku(t) + C\dot{u}(t) + M\ddot{u}(t) + r_N(t) = r(t)$$
1.2

Where, K_L is the stiffness matrix for the linear elastic elements (all elements except the Links/Supports); C is the proportional damping matrix; M is the diagonal mass matrix; r_N is the vector of forces from the nonlinear degrees of freedom in the Link/Support elements; u(t), $\dot{\mathbf{u}}(t)$, and $\ddot{u}(t)$ are the relative displacements, velocities, and accelerations with respect to the ground; and r is the vector of applied loads.

Direct-Integration time history

Direct integration of the full equations of motion with out the use of modal super position is available in SAP2000. While modal superposition is usually more accurate and efficient, direct-integration does offer the following advantages for linear problems:

- Full damping that couples the modes can be considered
- Impact and wave propagation problems that might excite a large number of modes may be more efficiently solved by direct integration.
- For non linear problems, direct integration also allows consideration of more types of nonlinearity that does modal super position.

Direct integration results are extremely sensitive to time-step size in a way that is not true for modal superposition. You should always run your direct-integration analyses with decreasing time-step sizes until the step size is small enough that results are no longer affected by it. In particular, you should check stiff and localized response quantities. For example, a much smaller time step may be required to get accurate results for the axial force in a stiff member than for the lateral displacement at the top of a structure.

Time Integration Parameters

A variety of common methods are available for performing direct-integration time-history analysis. Since these are well documented in standard textbooks, we will not describe them further here, except to suggest that you use the default "Hilber-Hughes-Tayloralpha" (HHT) method, unless you have a specific preference for a different method.

The HHT method uses a single parameter called alpha. This parameter may take values between 0 and -1/3.

For alpha = 0, the method is equivalent to the Newmark method with gamma = 0.5 and beta = 0.25, which is the same as the average acceleration method (also called the trapezoidal rule.) Using alpha = 0 offers the highest accuracy of the available methods, but may permit excessive vibrations in the higher frequency modes, i.e., those modes with periods of the same order as or less than the time-step size. For more negative values of alpha, the higher frequency modes are more severely damped. This is not physical damping, since it decreases as smaller time-steps are used. However, it is often necessary to use a negative value of alpha to encourage a nonlinear solution to con verge. For best results, use the smallest time step practical, and select alpha as close to zero as possible. Try different values of alpha and time-step size to be sure that the solution is not too dependent upon these parameters.

Nonlinear Direct-Integration Time-History Analysis

In this method we do direct integration of the full equations of motion without the use of modal super position. While modal super position is usually more accurate and efficient, direct-integration does offer the following advantages:

- Full damping that couples the modes can be considered
- Impact and wave propagation problems that might excite a large number of modes may be more efficiently solved by direct integration
- All types of nonlinearity avail able in SAP2000 may be included in a nonlinear direct integration analysis.

Direct integration results are extremely sensitive to time-step size in a way that is not true for modal superposition. We should always run our direct-integration analyses with decreasing time-step sizes until the step size is small enough that results are no longer affected by it.

Transient Analysis vs. Periodic Analysis

Transient analysis considers the applied load as a one-time event, with a beginning and end. Periodic analysis considers the load to repeat in definitely, with all transient response damped out. Periodic analysis is only available for linear modal time-history analysis.

1.2.2.4 Envelop for Seismic Response Vector

The conventional response spectrum method is ideally suited to the design or analysis of structural elements that are controlled by the maximum value of a single response quantity (e.g., a beam governed by the maximum bending moment) because it gives maxima of individual responses. For members in which the simultaneous action of multiple seismic responses must be considered (e.g., a column subjected to axial load and bending moments), the critical combination of responses may not coincide with the maximum value of any of the responses.

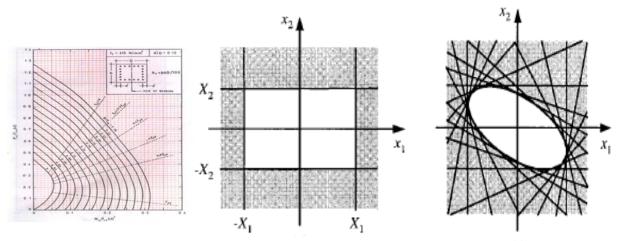


Fig. 1.1(a) Cross Sections of Moment
-Axial Capacity Surfaces for
Columns SP-16

Fig. 1.1(b) Rectangular Envelop

Fig.1.1(c) Elliptical Envelopes

In the current practice, it is common to use the response-spectrum-based estimates of the individual response maxima to construct a rectangular envelope Fig. 1.1(b) for this purpose. A response-spectrum-based procedure for predicting the envelope that bounds a vector of seismic responses is developed by **Charles Menun** and **Armen Der Kiuregian**. When the principal directions along which the ground motion components are uncorrelated are known, response spectrum method is used to estimate the peak value any response $x_{o1}(t)$ of the projection of the response vector along the direction of x_1 -axis and in direction other than x_1 -axis hence an envelop will develop which limits responses. The envelope is shown to be elliptical in shape. For the case when the orientation of the principal axes is unknown, a "supreme" envelope is derived, which corresponds to the most critical orientation of the axes. This envelope can then be superimposed over the capacity surface Fig. 1.1(a) of the member (commonly known as the interaction diagram) to determine the critical combination of the responses.

This procedure is based upon the response spectrum method and the Penzien and Watabe (1975) characterization of multicomponent ground motions; consequently, it inherits a number of assumptions and approximations implicit in these methods. Specifically, the response spectrum method is based on stationary random vibrations theory and entails approximations involving peak factors (Der Kiureghian 1981), whereas the Penzien-Watabe idealization assumes the principal directions of the ground motion remain fixed for the duration of the strong motion. Naturally, these assumptions and approximations affect the accuracy of the proposed procedure.

1.3 Dynamic analysis for Multi – Component Earthquake

A well-designed structure should be capable of equally resisting earthquake motions from all possible directions. One option in existing design codes for buildings and bridges requires that members be designed for "100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction." Other codes and organizations require the use of 40 percent rather than 30 percent. However, they give no indication on how the directions are to be determined for complex structures. For structures that are rectangular and have clearly defined principal directions, these "percentage" rules yield approximately the same results as the SRSS method.

For complex three-dimensional structures, such as non-rectangular buildings, curved bridges, arch dams or piping systems, the direction of the earthquake that produces the maximum stresses in a particular member or at a specified point is not apparent. For time history input, it is possible to perform a large number of dynamic analyses at various angles of input to check all points for the critical earthquake directions. Such an elaborate study could conceivably produce a different critical input direction for each stress evaluated. However, the cost of such a study would be prohibitive.

It is reasonable to assume that motions that take place during an earthquake have one principal direction. Or, during a finite period of time when maximum ground acceleration occurs, a principal direction exists. For most structures, this direction is not known and for most geographical locations cannot be estimated. Therefore, the only rational earthquake design criterion is that the structure must resist an earthquake of a given magnitude from any possible direction. In addition to the motion in the principal direction, a probability exists that motion normal to that direction will occur simultaneously. In addition, because of the complex nature of three-dimensional wave propagation, it is valid to assume that these normal motions are statistically independent.

Based on those assumptions, a statement of the design criterion is "a structure must resist a major earthquake motion of magnitude S1 for all possible angles θ and at the same point in time resist earthquake motions of magnitude S2 at 900 to the angle θ ."

A Design criterion implies that a large number of different analyses must be conducted in order to determine the maximum design forces. In approach given by **Edward Wilson** and **Martin R. Button**, it is shown that maximum values for all members can be exactly evaluated from one computer run for given seismic loading. Furthermore, these evaluated values are independent of selection of reference axis.

1.4 Dynamic Analysis of structure for Vertical Component of Earthquake

The effect of the vertical component of earthquakes has been investigated only recently. Ariaratnam and Leung (1990) analyzed a 2D six-story frame building and concluded that the vertical acceleration causes the reduction of the column stiffness and increases the lateral displacement. Sadeghvaziri and Foutch (1991) pointed out that vertical vibration leads to the instability of columns.

Extensive damage, which appeared to have been caused by vertical ground motion, was observed in numerous structures after the January 17, 1994, Northridge earthquake in the United States and the January 17, 1995, Hyogoken earthquake in Japan. The feature of these two earthquakes is that the magnitudes of the vertical component are as large as that of the horizontal component 10–50 km from the earthquake center. The effect of the vertical component of earthquakes has been investigated only recently; hence, research on this topic is limited. Iyengar and Shinozuka (1972) used a cantilever beam to investigate the effect of the vertical component of earthquakes.

Ariaratnam and Leung (1990) analyzed a 2D six-story frame building and concluded that the vertical acceleration causes the reduction of the column stiffness and increases the lateral displacement. Sadeghvaziri and Foutch (1991) pointed out that vertical vibration leads to the instability of columns. The effects of the vertical component of earthquake motion were studied by Gupta and Hutchinson (1994) using a simple lumped-mass model of a single story

building resting on a rigid foundation. The aforementioned investigations involve structural analyses of 1D or 2D cases with limited degrees of freedom.

A systematic study of 3D analyses of buildings under the vertical component of earthquake appears to be absent in the literature. **S. H. Ju, C. W. Liu,** and **K. Z. Wu** Studied number of structures and gave recommendation for correct analysis of 3D Analysis of earthquake under vertical earthquake.

Based on current research, it would seem that there is some disagreement as to whether vertical accelerations have a significant impact on the lateral response of a structure in a seismic event. Some research indicates that vertical accelerations are significant and other research indicates the opposite. Furthermore, most research on vertical accelerations has focused on specific buildings and so it is also not clear which types of structures would be influenced by vertical accelerations.

The existing literature on vertical accelerations, many studies have shown that vertical acceleration magnitudes can be as great or greater than the horizontal accelerations. Abrahamson and Litchiser (1989) showed that the ratio of vertical acceleration to horizontal acceleration (V/H) was dependent on the magnitude of the event and the distance from the site to the source. Bozorgnia et al. (1995) also noted that the V/H ratio was dependent on the site distance and earthquake magnitude. They also pointed out that the V/H acceleration response spectra ratio is dependent on the period of the structure in question. For short period structures, the ratio can be much greater than one, but for long period structures, the ratio is typically much less than one.

Not only does site distance and period affect vertical accelerations, but soil conditions also affect them. Amirbekian and Bolt (1998) showed that the vertical acceleration component tends to be higher than the horizontal in alluvial basins. There have also been many studies investigating the effects of vertical accelerations on structures. Some have concluded that vertical accelerations do not significantly affect structural response. Maison and Kasai (1997), for example, analytically tested a thirteen story steel moment frame which was damaged in the Northridge earthquake and showed that the lateral displacement of the simulated structure was very similar with and without vertical accelerations included. Furthermore, the major emphasis of their research was to examine connection failures that occurred in the building during the earthquake. They noted that increased gravity loads actually reduced damage to the connections. They reasoned that this was because the gravity loads caused a compressive preloading of the bottom flanges of the beams. They further reasoned that perhaps the lack of gravity loads in buildings under construction during the Northridge earthquake explained why more of those buildings experienced connection damage. So, it would seem that real structures with gravity loads are less likely to be adversely affected by vertical accelerations.

Jennings and Husid (1968) and Takizawa and Jennings (1980), using a small and focused range of models and parameters, argued that vertical accelerations have a negligible effect on the lateral response of structures.

While there are many studies indicating that vertical accelerations do not significantly affect structural response, there are also many which show that vertical accelerations do affect structural response. First, there are many studies that indicate that vertical accelerations increase the damage to various structural components in a building. Anderson and Bertero (1973), for example, analyzed a ten story steel moment frame subjected to horizontal and vertical ground motions from the San Fernando earthquake of 1971. From the study, they concluded that the ductility requirements at critical regions would be inaccurate if vertical accelerations were not included. In the upper stories, for example, the ductility requirements were increased by fifty per cent by including vertical accelerations. Saadeghvaziri (1988) and Saadeghvaziri and Foutch (1991) examined the effects of vertical and horizontal accelerations on bridges. They noted that vertical accelerations cause severe fluctuations in the axial loads in bridge piers and that these fluctuations result in highly erratic hysteresis loops. Moreover, the vertical accelerations tend to increase the shear demand and reduce the shear capacity in bridge piers. The increased axial loads stiffened the columns, which in turn caused the columns to attract more force. At the same time, the increased axial load reduced the shear capacity of the piers. Also, the increased axial loads reduced the ductility of the columns. Saadeghvaziri and Foutch speculated that the severe damage to bridges in some earthquakes may be primarily caused by vertical accelerations.

Hartetal (1995) examined a six story special steel moment resisting frame subjected to both vertical and horizontal accelerations as part of the SAC Steel Project. Hart's research focused on the effects of tributary mass, both in the vertical and horizontal directions. The mass distribution caused a great deal of scatter in the axial forces in the columns. Hart noted that the effect of vertical accelerations was most pronounced in the interior columns, which are primarily designed to withstand axial loads. In some situations, Hart found that seismic axial forces could be twice the dead load. Higazy et al. (1996) examined reinforced concrete (RC) beam-column connections under vertical accelerations. They found that should vertical accelerations cause a RC connection to go into tension, the shear capacity of the joint was reduced by eighteen to fifty percent. Also, the confinement in the joint core was rendered ineffective in the event of significant tension. Como et al. (2003) examined the likelihood that vertical accelerations would cause axial shortening in some columns and therefore increased rotations at beam ends. A second category of vertical acceleration research has focused on how vertical accelerations contribute to lateral displacements.

lyengar and Shinozuka (1972) examined the effects of vertical accelerations on tall buildings. They modeled tall buildings as distributed mass cantilevers. They concluded that vertical accelerations considerably increased the tip deflection, base shear, and base moment for the cantilever models studied.

Lin and Shin (1980) examined the effects of vertical accelerations on single degree of freedom inverted pendulum elastic structures. They noted that the vertical accelerations had the potential to amplify lateral response. Then, Shin and Lin (1982) performed a similar study involving single-degree-of-freedom-inverted pendulums except the systems were hysteretic rather than elastic. They examined systems with both 0.1 and 0.5 post-yield stiffness ratios, and found that for hysteretic models, the vertical accelerations had more of an effect on the lateral displacement than in the elastic models. Moreover, the lower post-yield stiffness ratio model was more influenced by vertical accelerations than the higher. Shin and Lin also noted that the vertical accelerations had a significant impact on the amount of residual deformations in their models.

Ariaratnam and Leung (1990) analyzed a six story multiple degree of freedom building using random vibrations in both the vertical and horizontal directions. They concluded that both gravity loads and vertical inertial loads can increase the lateral displacement when structures are subjected to earthquakes.

1.5 Scope of work

In present work, we studied the new approach developed by Charles Menun and Armen Der Kiuregian to bound responses which occurs simultaneously for which the member has to be design. We performed dynamic analysis of building frames in SAP2000 and derived elliptical envelop.

In different numerical problem we compare elliptical envelop with time history analysis results and we demonstrated significance of this approach for three dimensional earthquake input. Since it is well stabilized now that there are certain types of building which are significantly influenced by vertical component of earthquake hence we can not neglect it. Here we study significance of adding vertical component of earthquake with more correct way of modeling. Finally we compare approach given by Charles Menun and Armen Der Kiuregian (April,2000) with another approach given by Edward L.Willson and Martin R. Button (July 1981).

1.6 Organization of Report

This report has been divided into five chapters:

- Chapter 1 gives a general introduction to conventional dynamic analysis methods, dynamic analysis of buildings.
- Chapter 2 elaborates the literature review carried out to study the various aspects of response spectrum method, modal combination rules and characteristic of three dimensional earthquakes.
- Chapter 3 describes the methodology for deriving elliptical envelop and supreme envelop when principal direction of earthquake is known or unknown, respectively (Approach

Developed by Charles Menun and Armen Der Kiuregian). Procedure for evaluating critical responses of structure by approach given by Edward L.Willson and Martin R. Button (July 1981).

- Chapter 4 defines all the four problems we used to study Envelop for Seismic Response Vectors.
- Chapter 5 shows conclusion, result discussion and future scope of this work.
- Appendix 1 gives verification of input data we used in analysis.

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CHAPTER 2

Literature Review

2.1 Response Spectrum Method

Response spectrum method gives peak value of response of any structure. Peak value of response determined directly from the response spectrum for ground motion without carrying out time history analysis using smooth design spectra that are average of several earthquakes. For SDF systems it gives accurate results but for MDF results are not exact i.e. not identical to time history analysis.

The peak value of the nth-mode contribution $r_n(t)$ to response r(t) can be obtained from the earthquake response spectrum or design spectrum.

$$r_{no} = r_n^{st} A_n 2.1$$

The algebraic sign of r_{no} is the same as that of r_n^{st} because A_n is positive by definition. Where r_n^{st} = modal static response and A_n = pseudo-acceleration.

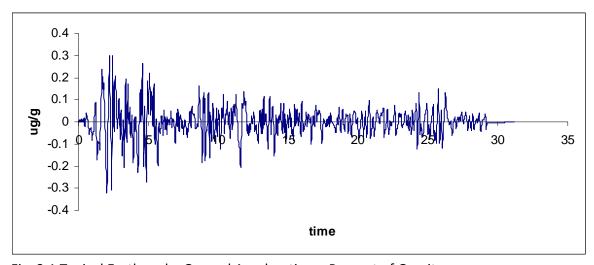


Fig. 2.1 Typical Earthquake Ground Acceleration – Percent of Gravity

North-south Component of Horizontal ground acceleration, El Centro, May 18, 1940

Provided on site, in digital form by University of California, Berkeley

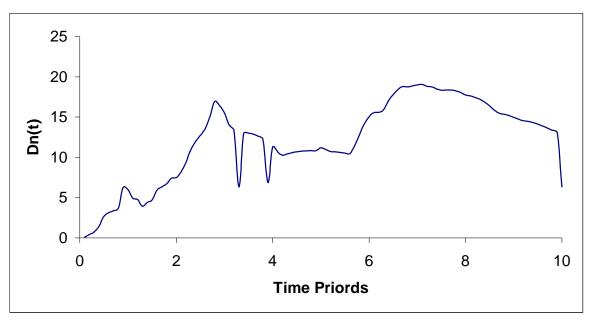


Fig. 2.2 Relative Displacement Spectrum $\,D_{n\,{
m max}}\left(t
ight)\,$ Inches As calculated in Excel for $\,\zeta=.02\,$ by Central interpolation method

It is apparent that all response spectrum curves represent the properties of the earthquake at a specific site and are not a function of the properties of the structural system. After estimation is made of the linear viscous damping properties of the structure, a specific response spectrum curve is selected.

Design spectra are not uneven curves as shown in above because they are intended to be the average of many earthquakes. At the present time, many building codes specify design spectra in the form shown in below,

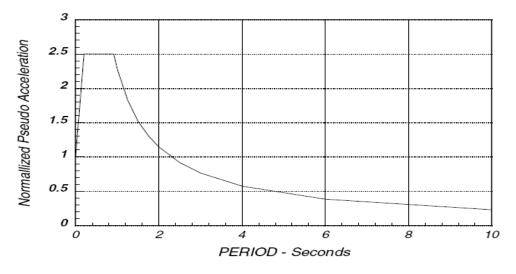


Fig. 2.3 Typical Design Spectrum

Two major problems must be solved to obtain an approximate response spectrum solution to three component earthquake. First, for each direction of ground motion, maximum peak forces and displacements must be estimated. Second, after the response for the three orthogonal directions has been solved, it is necessary to estimate the maximum response from the three components of earthquake motion acting at the same time.

Response spectrum method is restricted to linear elastic analysis in which the damping properties can only be estimated with a low degree of confidence. The use of nonlinear spectra, which is common, has very little theoretical background, and this approach should not be applied in the analysis of complex three-dimensional structures. For such structures, true nonlinear time-history response should be used.

2.1.1 Modal Combination Rules

It is not possible to determine exact value of the peak value of the total response, because the modal response attain their peaks at different time instant and the combine peak at yet different time instant.

Approximation must be introduce in combining the peak modal response r_{no} determined from the earthquake response spectrum because no information is available when these peak modal value occurs.

Absolute Sum Method

The most conservative method that is used to estimate a peak value r_o of displacement or force within a structure is to use the sum of the absolute of the modal response values. This approach assumes that the maximum modal values for all modes occur at the same point in time. For N no. of DOFs,

$$r_o \le \sum_{n=1}^{N} \left| r_{no} \right| \tag{2.2}$$

Square Root of sum of Square Method (SRSS)

Another very common approach is to use the Square Root of the Sum of the Squares, SRSS, on the maximum modal values to estimate the values of displacement or forces. The SRSS method assumes that all of the maximum modal values are statistically independent. This modal combination rule provides excellent response estimate for structure with well-separated natural frequencies. For three-dimensional structures in which a large number of frequencies are almost identical, this assumption is not justified.

$$r_o \square \left(\sum_{n=1}^N r_{no}^2\right)^{1/2}$$
 2.3

Complete Quadratic Combination Method

The Complete Quadratic Combination technique is described by Wilson, Der Kiureghian, and Bayo (1981). This is the default method of modal combination. The CQC method takes into account the statistical coupling between closely spaced Modes caused by modal damping. Increasing the modal damping increases the coupling between closely-spaced modes. If the damping is zero for all Modes, this method degenerates to the SRSS method.

The relatively new method of modal combination is the Complete Quadratic Combination, CQC, method (J. Pezein and M. Watabe ,1975) that was first published in 1981. It is based on random vibration theories and has found wide acceptance by most engineers and has been incorporated as an option in most modern computer programs for seismic analysis.

$$r_o \square \left(\sum_{i=1}^N \sum_{n=1}^N \rho_{no} r_{io} r_{no} \right)$$
 2.4

Where, ρ_{no} = correlation coefficient. ρ_{no} varies between 0 to 1. the above equation can be written as

$$r_{o} \leq \left(\sum_{n=1}^{N} r_{no}^{2} + \sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in} r_{io} r_{no}\right)^{1/2}$$
 2.5

The first summation on right side is identical to SRSS combination rule. the double summation include all the cross terms may be +ve or –ve depending on sign of static response r_i^{st} and r_n^{st} .

Several formulation for the peak response to earthquake excitation were published , some of these are identical but differ in the mathematical expressions given for the correlation coefficient.

The earliest was of E. Rosenblueth and J. Elorduy(1969).

$$\rho_{in} = \frac{1}{1 + \left(\left(\omega_{i}\sqrt{1 - \zeta_{i}^{2}}\right) - \omega_{n}\sqrt{1 - \zeta_{n}^{2}}\right)}$$

$$(\zeta'_{i}\omega_{i} + \zeta'_{n}\omega_{n})$$
2.6

Where,
$$\zeta'_n = \zeta_n + \frac{2}{\omega_n s}$$

S = duration of the strong phase of the earthquake

Equation shows that $\rho_{in}=\rho_{ni}$, $0\leq\rho_{in\leq}\leq1$, and $\rho_{in}=1$ for i=n. For system with the same damping ratio in all modes subjected to earthquake excitation with duration s long enough.

$$\rho_{in} = \frac{\zeta^2 (1 + \beta_{in})^2}{(1 - \beta_{in})^2 + 4\zeta^2 \beta_{in}}$$
2.7

The equation for the correlation coefficient due to Der Kiureghian (1981) is

$$\rho_{in} = \frac{8\sqrt{\zeta_{n}\zeta_{i}} \left(\beta_{in}\zeta_{i} + \zeta_{n}\right) \beta^{1.5}_{in}}{\left(1 - \beta_{in}^{2}\right)^{2} + 4\zeta_{n}\zeta_{i}\beta_{in} \left(1 + \beta_{in}^{2}\right) + 4\left(\zeta_{i}^{2} + \zeta_{n}^{2}\right)\beta_{in}^{2}}$$
2.8

This equation also implies that $\rho_{in}=\rho_{ni}$, $0\leq\rho_{in\leq}\leq1$ and $\rho_{in}=1$ for i=n. For equal frequencies and equal damping ratio.

$$\rho = \frac{8\zeta^{2} (1 + \beta_{in}) \beta_{in}^{3/2}}{(1 - \beta_{in}^{2})^{2} + 4\zeta^{2} \beta_{in} (1 + \beta_{in})^{2}}$$
2.9

It is now clear that the SRSS rule applies to structures with well-separated natural frequencies of those modes that contribution significantly to the response. The SRSS and CQC rule are based on random vibration theory; the assumptions used indicate that the modal combination rules would be most accurate for earthquake excitation that contains a wide band of frequencies with long periods of the structures, which are not too lightly damped. In particular, these modal combination rules will become less accurate for short-duration impulsive grounds motions and are nor recommended for ground motions that contain many cycles of essentially harmonic excitation.

It should be pointed out that a method similar to the CQC was first proposed by Rosenblueth and Elorduly (1969). Their method, which has a somewhat heuristic basis, has a more complicated cross-modal terms involving the duration of earthquake as will as the modal frequencies and damping values. This method has unfortunately been neglected or misrepresented over the past several years. For example, the NRC Regulatory Guide (Revision 1,1976) recommends it for structures with closely spaced modes, however, it wrongly specifies the cross-modal as being always positive. This will result in overly conservative response estimates in some applications.

It should also be pointed out that the SRSS method gives good results for some structures subjected to two-directional seismic input, even when the modal frequencies are closely spaced. It can be shown that this is due to canceling of the cross-modal terms corresponding to the two directions of input. This phenomenon, however, is not generally true. For example,

when the two components of input are of different intensities, or when the three-dimensional structure is highly asymmetric, the cross-modal terms would still be significant and, therefore, the SRSS method will lead to erroneous results.

It is strongly recommended that 'the use of the SRSS method for seismic response analysis of structure be immediately discontinued. SRSS method may dramatically overestimate the required design forces in some structural elements or it may significantly underestimate the forces in other elements. The CQC method is based on fundamental theories of random vibration and consistently yields accurate results when compared to time-history analyses' by E. L. Wilson, A. Der Kiuregian and E. P. Bayo (1981).

The SRSS and CQC modal combination rule are based on random vibration theory. Thus the modal combination rules are intended for use when the excitation is characterized by a design response spectrum, based on the response spectra for many earthquake excitations.

The SRSS and CQC modal combination rule used in conjunction with mean of peak values of response to individual excitation. The error in estimation of peak value may be on either side, conservative or unconsevative.

Most three dimensional building structures that are designed to resist earthquakes from both directions equally have the closeness of the frequencies. The time history results are exact. Shows that, the SRSS method under-estimate the exact values in the direction of the loads and overestimate responses normal to the loads. The sum of the absolute values, grossly over-estimates all results. The CQC method produces very realistic values that are close to the exact time history solution.

General Modal Combination Method

The General Modal Combination technique is the complete modal combination procedure described by Equation 3.31 in Gupta (1990). The GMC method takes into account the statistical coupling between closely- spaced Modes similarly to the CQC method, but also includes the correlation between modes with rigid- response con tent.

Increasing the modal damping in creases the coupling between closely- spaced modes. In addition, the GMC method re quires you to specify two frequencies, f1 and f2, which define the rigid- response content of the ground motion. These must satisfy 0< f1< f2. The rigid-response parts of all modes are assumed to be perfectly correlated.

The GMC method assumes no rigid response be low frequency f1, full rigid response above frequency f2, and an interpolated amount of rigid response for frequencies between f1 and f2.

Frequencies f1 and f2 are properties of the seismic in put, not of the structure. Gupta de fines f1 as:

$$f1 = \frac{S_{A\text{max}}}{2\Pi S_{V\text{max}}}$$
 2.10

Where, *S* Amax is the maxi mum spectral acceleration and *SV*max is the maxi mum spectral velocity for the ground motion considered. The de fault value for f1 is unity.

Gupta defines f2 as:

$$f2 = \frac{1}{3}f1 + \frac{2}{3}fr$$
 2.11

Where, fr is the rigid frequency of the seismic in put, i.e., that frequency above which the spectral acceleration is essentially constant and equal to the value at zero period (in finite frequency). Others have de fined f2 as:

$$f 2 = fr 2.12$$

The default value for f2 is zero, indicating in finite frequency. For the de fault value of f2, the GMC method gives results similar to the CQC method.

NRC Ten-Percent Method

This is the Ten-Per cent method of the U.S. Nu clear Regulatory Commission *Regulatory Guide 1.92*.

The Ten-Per cent method as sums full, positive coupling between all modes whose frequencies differ from each other by 10% or less of the smaller of the two frequencies. Modal damping does not affect the coupling.

NRC Double-Sum Method

This is the Double-Sum method of the U.S. Nuclear Regulatory Commission *Regulatory Guide 1.92*.

The Double-Sum method as sums a positive coupling between all modes, with correlation coefficients that depend upon damping in a fashion similar to the CQC and GMC methods, and that also depend upon the duration of the earth quake. You specify this duration as parameter td as part of the Analysis Cases definition.

2.1.2 Direction Combinations Rules (orthogonal effect)

For each displacement, force, or stress quantity in the structure, modal combination produces a single, positive result for each direction of acceleration. These directional values for a given response quantity are combined to pro duce a single, positive result.

SRSS Method

This method is invariant with respect to coordinate sys tem, i.e., the results do not depend upon your choice of coordinate sys tem when the given response - spectrum curves are the same. This is the recommended method for directional combination, and is the default.

Absolute Sum Method

This method is usually over- conservative. This approach assumes that the maximum modal values for all modes occur at the same point in time.

Scaled Absolute Sum Method

This Method combines the directional results by the scaled absolute sum method. Here, the directional results are combined by taking the maximum, over all directions, of the sum of the absolute values of the response in one direction plus dirf times the response in the other directions.

For example, if factor 0.3 is used to scale, the spectral response, *R*, for a given displacement, force, or stress would be:

$$R = \max(\overline{R1}, \overline{R2}, \overline{R3})$$
 2.12

where:

$$\overline{R1} = R1 + 0.3(R2 + R3)$$
 $\overline{R2} = R2 + 0.3(R1 + R3)$
 $\overline{R3} = R3 + 0.3(R1 + R2)$
2.13

Where R1, R2, and R3 are the modal- combination values for each direction. The results obtained by this method will vary depending upon the coordinate system you choose. Results obtained using scaled factor = 0.3 are comparable to the SRSS method (for equal in put spectra in each direction), but may be as much as 8% unconservative or 4% over- conservative, depending upon the coordinate system. Larger values of scale factor tend to produce more conservative results.

CQC3 Method

Menun and Der Kiureghian presented the CQC3 method for the combination of the effects of orthogonal spectrum. It is reasonable to assume that motions that take place during an earthquake have one principal direction. Or, during a finite period of time when maximum ground acceleration occurs, a principal direction exists. For most structures, this direction is not known and for most geographical locations cannot be estimated. Therefore, the only rational

earthquake design criterion is that the structure must resist an earthquake of a given magnitude from any possible direction. In addition to the motion in the principal direction, a probability exists that motion normal to that direction will occur simultaneously. In addition, because of the complex nature of three-dimensional wave propagation, it is valid to assume that these normal motions are statistically independent.

Based on those assumptions, a statement of the design criterion is "a structure must resist a major earthquake motion of magnitude S1 for all possible angles θ and at the same point in time resist earthquake motions of magnitude S2 at 90 to the angle θ as shown in fig.2.4."

The stated design criterion implies that a large number of different analyses must be conducted to determine the maximum design forces and stresses. It is shown that maximum values for all members can be exactly evaluated from one computer run in which two global dynamic motions are applied. Furthermore, the maximum member forces calculated are invariant with respect to the selection system.

Recently, Menun and Der Kiureghian(1998) presented the CQC3 method for the combination of the effects of orthogonal spectrum.

$$S_2 = aS_1$$
 2.14

Where, a is a number between 0 and 1.0.

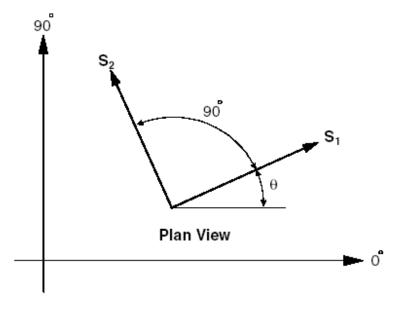


Fig. 2.4 Definition of Earthquake Spectra Input

The fundamental CQC3 equation for the estimation of a peak value is:

$$F = \left[F_0^{2+} a^2 F_{90}^2 - \left(F_0^2 - F_{90}^2 \right) \sin^2 \theta + 2 \left(1 - a^2 \right) F_{0-90} \sin \theta \cos \theta + F_z^2 \right]^{\frac{1}{2}}$$
 2.15

Where,

$$F_0^2 = \sum_{n} \sum_{m} f_{on} \rho_{nm} f_{om}$$

$$F_{90}^2 = \sum_{n} \sum_{m} f_{90n} \rho_{nm} f_{90m}$$

$$F_{0-90} = \sum_{n} \sum_{m} f_{0n} \rho_{nm} f_{90m}$$

$$F_Z^2 = \sum_{n} \sum_{m} f_{zn} \rho_{nm} f_{zm}$$

in which F_0 and F_{90} are the modal values produced by 100 percent of the lateral spectrum applied at 0 and 90 degrees respectively and f z n is the modal response from the vertical spectrum which can be different from the lateral spectrum.

It is important to note that for equal spectra a = 1, the value F is not a function of θ and the selection of the analysis reference system is arbitrary. Or,

$$F_{MAX} = \sqrt{\left(F_0^2 + F_{90}^2 + F_z^2\right)}$$
 2.16

This indicates that it is possible to conduct only one analysis with any reference system, and the resulting structure will have all members that are designed to equally resist earthquake motions from all possible directions. This method is acceptable by most building codes.

For a =1, the CQC3 method reduces to the SRSS method. However, this can be over conservative because real ground motions of equal value in all directions have not been recorded. Therefore, it is necessary to calculate the critical angle that produces the maximum response. Differentiation of Equation

$$\theta_{cr} = \frac{1}{2} \tan^{-1/2} \left[\frac{2F_{0-90}}{F_0^2 - F_{90}^2} \right]$$
 2.17

Two roots exist for Equation that must be checked in order that the following equation is maximum:

$$F = \left[F_0^{2+} a^2 F_{90}^2 - \left(F_0^2 - F_{90}^2 \right) \sin^2 \theta_{cr} + 2 \left(1 - a^2 \right) F_{0-90} \sin \theta_{cr} \cos \theta cr + F_z^2 \right]^{\frac{1}{2}}$$
 2.18

At the present time, no specific guidelines have been suggested for the value of a. Reference [Menun, C., and A. Der Kiureghian. 1998. "A Replacement for the 30 % Rule for

Multicomponent Excitation," Earthquake Spectra. Vol. 13, Number 1. February.] presented an example with values *a* between 0.50 and 0.85.

The previously presented theory clearly indicates that the CQC3 combination rule, with a equal to 1.0, is identical to the SRSS method and produces results for all structural systems that are not a function of the reference system used by the engineer.

It can be shown that the 100/30 combination method produces moments that are not symmetric, whereas the SRSS combination method produces logical and symmetric moments. If a structural engineer wants to be conservative, the results of the SRSS directional combination rule or the input spectra can be multiplied by an additional factor greater than one. One should not try to justify the use of the 100/40 percentage rule because it is conservative in "most cases." For complex Three-dimensional structures, the use of the 100/40 or 100/30 percentage rule will produce member designs that are not equally resistant to earthquake motions from all possible directions.

Recommendations on Orthogonal Effects (Edward L. Willson, Three Dimensional Static & Dynamic Analysis of Stucture)

'For three-dimensional response spectra analyses, it has been shown that the "design of elements for 100 percent of the prescribed seismic forces in one direction plus 30 or 40 percent of the prescribed forces applied in the perpendicular direction" is dependent on the user's selection of the reference system. These commonly used "percentage combination rules" are empirical and can underestimate the design forces in certain members and produce a member design that is relatively weak in one direction. It has been shown that the alternate building code approved method, in which an SRSS combination of two 100 percent spectra analyses with respect to any user-defined orthogonal axes, will produce design forces that are not a function of the reference system. Therefore, the resulting structural design has equal resistance to seismic motions from all directions. The CQC3 method should be used if a value of a less than 1.0 can be justified. It will produce realistic results that are not a function of the user-selected reference system'

2.2 PREVIOUS WORK

A.K. Gupta and M.P. Singh (1977) and Anastassiadis (1993) (in French) has develop similar kind of elliptical envelop when principal direction of earthquake ground motion is knowm in advanced.

2.2.1 A.K. Gupta and M.P. Singh Approach (Design of Column Section Subjected to three component of earthquake)

In this approach A.K. Gupta and M.P. Singh gave procedure for the seismic design of column section subjected to combination of axial force and moments which probalistically can occur simultaneously.

They have shown that combination of values comes form Response Spectrum Method $\pm N$, $\pm M_x$ and $\pm M_y$ are probable maximum values of axial force and moment about x and y, this procedure may be overly conservative. They suggested procedure which provides combination of N, M_x and M_y which probalitacally occurs simultaneously.

They have given equation of envelop

$$H^{rs} F^{r} F^{s} = 1$$
 2.19

Above equation represents an ellipsoid in three-dimensional (N $_x$, $_y$) space which is stated as an 'interaction ellipsoid' in their research paper. It is bounded by the planes N = $_x$ + m, M, and M, are stated as SRSS values of N, M, and M, respectively.

Where, $H^{rs}G^{st} = \delta^{rt}$

Equation is equivalent to stating that the 3 x 3 matrix [H] is the inverse of matrix [G]. Matrix [G] is as follows,

$$[G] = \begin{bmatrix} g11 & g12 & g13 \\ & g22 & g23 \\ Sym & & g33 \end{bmatrix},$$

$$\begin{split} &g11 = \varepsilon_{mn} \; N_{im} \; N_{in} \; , & g12 = \varepsilon_{mn} \; N_{im} \; N_{xin} \; , \\ &g13 = \varepsilon_{mn} \; N_{im} \; N_{yin} \; , & g22 = \varepsilon_{mn} \; N_{xim} \; N_{xin} \; , \\ &g23 = \varepsilon_{mn} \; N_{xim} \; N_{yin} \; , & g33 = \varepsilon_{mn} \; N_{yim} \; N_{yin} \; , & 2.20 \end{split}$$

The matrix [H] is as follows,

$$[H] = \begin{bmatrix} h11 & h12 & h13 \\ & h22 & h23 \\ Sym & h33 \end{bmatrix}$$

$$\begin{array}{l} h11 = (g_{22} \ g_{33}^- \ g_{23}^2)/D \ , \\ h12 = (g_{13} \ g_{23}^- \ g_{12} \ g_{22})/D, \\ h13 = (g_{12} \ g_{23}^- \ g_{13} \ g_{22})/D, \\ h22 = (g_{11} \ g_{33}^- \ g_{13}^2)/D, \\ h23 = (g_{12} \ g_{13}^- \ g_{11} \ g_{23})/D, \\ h33 = (g_{11} \ g_{22}^- \ g_{12}^2)/D \\ and \ D = g_{11} (g_{22} \ g_{33}^- \ g_{23}^2) + g_{12} (g_{13} \ g_{23}^- \ g_{12} \ g_{33}) + g_{13} (g_{12} \ g_{23}^- \ g_{13} \ g_{22}) \end{array} \ 2.21$$

They have described combined stress in column section σ as follows,

$$\sigma = N/A \pm Mx/Zx \pm My/Zy$$
, or

$$\sigma = C^r F^r$$
 2.22

Where, $C^1 = 1/A$, $C^2 = \pm 1/Zx$, $C^3 = \pm 1/Zy$ and $F^1 = N$, $F^2 = Mx$ and $F^3 = My$

For given mode and component of earthquake they described,

$$\sigma_{in} = C_r F_{in}^r \qquad 2.23$$

The maximum response is taken to be the square root of the sum of square of the response obtained in each direction due to contribution of each modes of vibration for three component of earthquake.

$$\sigma_e = C^r C^s G^{rs}$$
 2.24

where, $G^{rs} = E_{mn} F^{r}_{im} F^{s}_{in}$, F^{r}_{im} and F^{s}_{in} are contribution due to i^{th} mode shape and n^{th} component of earthquake.

They have shown that it is sufficient to design the section such that the interaction surface of the section completely includes the interaction ellipsoid. If design is carried out for the SRSS forces and moments, the design may be unnecessarily conservative, because the SRSS axial force and moments do not occurs simultaneously. If the section is subjected to axial force and biaxial moments, the interaction ellipsoid represents the seismic loading.

The design process simply shown consist of providing a section with an integration diagram or surface which would completely include the appropriate interaction ellipsoid. It is stated about this approach that it will results in the most economical design of the section. However, here economics is not a major consideration, the usual method of design that is considering rectangular envelop results from SRSS values will still gives safe section.

2.3 Recommendation for 3D Dynamic Analysis for Multi-Component Earthquake Spectra (Martin R. Button, Edward L. Willons)

A well-designed structure should be capable of equally resisting earthquake motions from all possible directions. One option in existing design codes for buildings and bridges requires that members be designed for "100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction." Other codes and organizations require the use of 40 percent rather than 30 percent. However, they give no indication on how the directions are to be determined for complex structures. For structures that are rectangular and have clearly defined principal directions, these "percentage" rules yield approximately the same results as the SRSS method.

For complex three-dimensional structures, such as non-rectangular buildings, curved bridges, arch dams or piping systems, the direction of the earthquake that produces the maximum stresses in a particular member or at a specified point is not apparent. For time history input, it is possible to perform a large number of dynamic analyses at various angles of input to check all points for the critical earthquake directions. Such an elaborate study could conceivably produce a different critical input direction for each stress evaluated. However, the cost of such a study would be prohibitive.

It is reasonable to assume that motions that take place during an earthquake have one principal direction. Or, during a finite period of time when maximum ground acceleration occurs, a principal direction exists. For most structures, this direction is not known and for most geographical locations cannot be estimated. Therefore, the only rational earthquake design criterion is that the structure must resist an earthquake of a given magnitude from any possible direction. In addition to the motion in the principal direction, a probability exists that motion normal to that direction will occur simultaneously. In addition, because of the complex nature of three-dimensional wave propagation, it is valid to assume that these normal motions are statistically independent.

Based on those assumptions, a statement of the design criterion is "a structure must resist a major earthquake motion of magnitude S1 for all possible angles θ and at the same point in time resist earthquake motions of magnitude S2 at 900 to the angle θ ."

A Design criterion implies that a large number of different analyses must be conducted in order to determine the maximum design forces. In that approach it is shown that maximum values for all members can be exactly evaluated from one computer run for given seismic loading. Furthermore, these evaluated values are independent of selection of reference axis.

2.4 3D Analysis of Building under Vertical Component of Earthquake (S. H. Ju, C. W. Liu, & K. Z. Wu)

The effect of the vertical component of earthquakes has been investigated only recently. Ariaratnam and Leung (1990) analyzed a 2D six-story frame building and concluded that the vertical acceleration causes the reduction of the column stiffness and increases the lateral displacement. Sadeghvaziri and Foutch (1991) pointed out that vertical vibration leads to the instability of columns.

Lumped mass schemes were investigated in this study. Moreover, dividing the main girder connected between two columns into one, two, and three two-node beam elements was also investigated.

• The errors from the lumped mass scheme are usually larger than those from the consistent mass scheme. Moreover, finer meshes always produce smaller errors.

- Taller buildings often produce smaller errors. To demonstrate this result, a uniform 1D bar with mass density γ , elastic modulus E, and length L is used to model the building column. One end of this bar is fixed and another is free. The minimum frequency of this bar is. This frequency is independent of the cross section, but dependent on the bar length. When the column line is longer, the minimum frequency of this column line in the axial direction is lower. Generally, lower frequencies are more important to calculate the response of a structure. When the building height increases, the response will gradually be produced from columns but not from girders; thus the mesh type of girders is less and less important while the building is taller and taller.
- Smaller errors are caused by mesh-2 with the lumped mass scheme and mesh-1 with the consistent mass scheme and the maximum error is only 2.0%. Therefore, these two types of analyses are accurate enough for building analyses under the vertical component of earthquakes. The accuracy of mesh-1 with the lumped mass scheme is also acceptable, because only the error of five-story buildings is a little large. For mesh-0 with both mass schemes, the errors are too large, and so they should not be used.
- In the structural analysis of buildings, floor slabs are usually assumed to be rigid in their planes, and the floor stiffness is neglected in the normal direction of a floor plane. This assumption was demonstrated to be adequate for buildings without shear walls under horizontal earthquakes.

However, for buildings under the vertical earthquake loading, the influence of the floor stiffness with varying thickness & with 5, 10, 15, and 20 stories were analyzed under the vertical component of The results indicate that the differences between the two methods are almost proportional to the floor thickness, when the floor thickness is smaller than or equal to 12 cm, the differences between the two methods (ie., rigid-floor and flexible-floor analyses method) are not large. Because neglecting the vertical stiffness of the floor slab causes building girders to carry more vertical loading, the traditional building analysis

Method (i.e., rigid in the floor plane and neglecting floor stiffness in the vertical direction) should be conservative. Thus, this traditional method is still suitable for the building analysis under the vertical component of earthquakes, if the floor thickness is not too thick, such as thinner than 15 cm.

The vertical effective mass ratio is suggested to be 80% for the building analysis under the vertical component of earthquakes. Using a mass ratio over 80% will include significantly higher mode shapes but does not lead to greater accuracy.

2.5 Characteristic of 3-Dimensional Earthquake Ground Motion

Joseph Penzien and Mokoto Watable (August 1975) introduces an orthogonal set of earthquake ground motion along which the component of variances have maximum, minimum And intermediate values and the covariance equals to zero. Corresponding axes are defined which yields maximum, minimum values for covariance. The orthogonal transformation evolved are identical in form to those used in transformation of stresses. They also shown that major principal axis points in the general direction of the epicenter and the minor principal axis is

nearly vertical. They concluded in their work that artificially generated components of ground motion are need not to be correlated statistically provided they are directed along principal axes.

2.6 SOFTWARE REVIEW (Dynamic Analysis in SAP2000 v 11 (Advanced))

Introduction to SAP 2000

SAP2000 is the latest and most powerful version of the well-known SAP series of structural analysis programs. SAP2000 is a full-featured program, which can be used for the simplest problems or the most complex projects.

2.6.1 SAP2000 Analysis Features

The SAP2000 structural analysis program offers the following features:

- Static and dynamic analysis
- Linear and nonlinear analysis
- Dynamic seismic analysis and static pushover analysis Methods of Dynamic Analysis of Concrete Gravity Dam & Introduction to SAP 2000 Major Project Report 58
- Vehicle live-load analysis for bridges
- Geometric nonlinearity, including P-delta and large-displacement effects
- Staged (incremental) construction
- Buckling analysis
- Frame and shell structural elements, including beam-column, truss,
- Membrane and plate behavior
- Two-dimensional plane and axisymmetric solid elements
- Three-dimensional solid elements
- Nonlinear link and spring elements
- Multiple coordinate systems
- Many types of constraints
- A wide variety of loading options
- Alpha-numeric labels
- Large capacity
- Highly efficient and stable solution algorithms

These features and many more, make SAP2000 the state-of-the-art in structural analysis programs.

2.6.2 Structural Analysis and Design

The following general steps are required to analysis and design a structure using SAP2000:

• Create or modify a model that numerically defines the geometry, properties,

- Loading, and analysis parameters for the structure
- Perform an analysis of the model
- Review the results of the analysis
- Check and optimize the design of the structure

This is usually an iterative process that may involve several cycles of the above sequence of steps. All of these steps can be performed seamlessly using the SAP2000 graphical user interface. Methods of Dynamic Analysis of Concrete Gravity Dam & Introduction to SAP 2000

2.6.3 About Manuals

Sap 2000 provides following manuals, which are design to help users quickly become productive with sap 2000.

- Sap 2000 getting started, which introduce the concepts of the structural model, the graphical user interface, and working with data tables.
- SAP2000 Basic Analysis Reference, which gives an introduction to the fundamental concepts underlying the structural model and the analysis techniques used by SAP2000
- Sap 2000 Analysis Reference, containing information about the advanced modeling and analysis features of the program.
- Sap 2000 Introductory Tutorial, which is intended to provide for users with hands-on experience using the modeling, analysis and design features of SAP2000.
- Sap 2000 design manuals, containing detailed design features specific to supported design codes
- Sap 2000 Verification Manual, containing examples showing the capabilities and verifying the accuracy of the analytical features of the program.

2.6.4 Dynamic Analysis in SAP

2.6.4.1 Degrees of Freedom

The Plane element activates the three translational degrees of freedom at each of its connected joints. Rotational degrees of freedom are not activated.

The deflection of the structural model is governed by the displacements of the joints. Every joint of the structural model may have up to six displacement components:

- The joint may translate along its three local axes. These Translation are denoted U1, U2 & U3.
- The joint may rotate about its three local axes. These rotations are denoted R1, R2, and R3.

These six displacement components are known as the degrees of freedom of the joint. In the usual case where the joint local coordinate system is parallel to the global system, the degrees of freedom may also be identified as UX, UY, UZ, RX, RY and RZ, according to which global axes are parallel to which local axes. The joint local degrees of freedom are illustrated in Figure 2.1.

In addition to the regular joints that you explicitly define as part of your structural model, the program automatically creates master joints that govern the behavior of any Constraints that you may have defined. Each master joint has the same six degrees of freedom as do the regular joints. Each degree of freedom in the structural model must be one of the following types:

- Active the displacement is computed during the analysis
- Restrained the displacement is specified, and the corresponding reaction is computed during the analysis
- Constrained the displacement is determined from the displacements at other degrees of freedom
- Null the displacement does not affect the structure and is ignored by the analysis
- Unavailable the displacement has been explicitly excluded from the analysis These different types of degrees of freedom are described in the following paragraph.

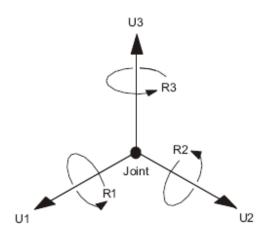


Fig. 2.5 The Six Displacement Degrees of Freedom in the Joint Local Coordinate System

Available and Unavailable Degrees of Freedom

You may explicitly specify the global degrees of freedom that are available to every joint in the structural model. By default, all six degrees of freedom are available to every joint. This de fault should generally be used for all three-dimensional structures. For certain planar structure, how ever, you may wish to re strict the available degrees of freedom. For example, in the X-Y plane: a planar truss needs only UX and UY; a planar frame needs only UX, UY, and RZ; and a planar grid or flat plate needs only UZ, RX, and RY.

The degrees of freedom that are not specified as being available are called unavailable degrees of freedom. Any stiffness, loads, mass, Restraints, or Constraints that are applied to the unavailable degrees of freedom are ignored by the analysis. The available degrees of freedom are always referred to the global coordinate system, and they are the same for every joint in the model. If any joint local coordinate systems are used, they must not couple available degrees of freedom with the unavailable degrees of freedom at any joint. For example, if the available degrees of freedom are UX, UY, and RZ, then all joint local coordinate systems must have one local axis parallel to the global Z axis.

Restrained Degrees of Freedom

If the displacement of a joint along any one of its available degrees of freedom is known, such as at a Support point, that degree of freedom is restrained. The known value of the displacement may be zero or non-zero, and may be different in different Load Cases. The force along the re strained degree of freedom that is required to impose the specified restraint displacement is called the reaction, and is determined by the analysis. Unavailable degrees of freedom are essentially restrained. However, they are excluded from the analysis and no reactions are computed, even if they are non-zero.

Constrained Degrees of Freedom

Any joint that is part of a Constraint or Weld may have one or more of its available degrees of freedom constrained. The program automatically creates a master joint to govern the behavior of each Constraint, and a master joint to govern the behavior of each set of joints that are connected together by a Weld. The displacement of a constrained degree of freedom is then computed as a linear combination of the displacements along the degrees of freedom at the corresponding master joint.

If a constrained degree of freedom is also restrained, the restraint will be applied to the constraint as a whole.

Active Degrees of Freedom

All available degrees of freedom that are neither constrained nor restrained must be either active or null. The program will automatically determine the active degrees of freedom as follows:

- If any load or stiffness is applied along *any* translational degree of freedom at a joint, then all available translational degrees of freedom at that joint are made active unless they are constrained or re strained.
- If any load or stiffness is applied along *any* rotational degree of freedom at a joint, then all available rotational degrees of freedom at that joint are made active unless they are constrained or re strained.

• All degrees of freedom at a master joint that govern constrained degrees of freedom are made active.

Its translational degrees of freedom activated. A joint that is connected to a Frame, Shell, or Link/Support element or to any rotational spring will have all of its rotational degrees of freedom activated. An exception is a Frame element with only truss- type stiffness, which will not activate rotational degrees of freedom.

Every active degree of freedom has an associated equation to be solved. If there are *N* active degrees of freedom in the structure, there are *N* equations in the system, and the structural stiffness matrix is said to be of order *N*. The amount of computational effort required to perform the analysis increases with *N*.

The load acting along each active degree of freedom is known (it may be zero). The corresponding displacement will be determined by the analysis.

If there are active degrees of freedom in the system at which the stiffness is known to be zero, such as the out- of- plane translation in a planar- frame, these must either be restrained or made unavailable. Otherwise, the structure is unstable and the solution of the static equations will fail.

Null Degrees of Freedom

The available degrees of freedom that are not restrained, constrained, or active, are called the null degrees of freedom. Because they have no load or stiffness, their displacements and reactions are zero, and they have no effect on the rest of the structure. The pro gram automatically excludes them from the analysis.

Joints that have no elements connected to them typically have all six degrees of freedom null. Joints that have only solid- type elements (Plane, Asolid, and Solid) connected to them typically have the three rotational degrees of freedom null.

2.6.4.2 Degree of Freedom Output

A table of the types of degrees of freedom present at every joint in the model is printed in the Analysis output (.OUT) file. The degrees of freedom are listed for all of the regular joints, as well as for the master joints created automatically by the program. For Constraints, the master joints are identified by the labels of their corresponding Constraints. For Welds, the master joint for each set of joints that are welded together is identified by the label of one of the welded joints. Joints are printed in alpha-numeric order of the labels.

The type of each of the six degrees of freedom at a joint is identified by the following symbols:

- (A) Active degree of freedom
- (-) Re strained degree of freedom

- (+) Constrained degree of freedom
- () Null or unavailable degree of freedom

The degrees of freedom are always referred to the local axes of the joint. They are identified in the output as U1, U2, U3, R1, R2, and R3 for all joints. However, if all regular joints use the global coordinate system as the local system (the usual situation), then the degrees of freedom for the regular joints are identified as UX, UY, UZ, RX, RY, and RZ.

2.6.4.3 Mass Source

The total mass of the element is equal to the integral over the plane of the element of the mass density multiplied by the thickness. The total mass is apportioned to the joints in a manner that is proportional to the diagonal terms of the consistent mass matrix. The total mass is applied to each of the three translational degrees of freedom (UX, UY, and UZ) even when the element contributes stiffness to only two of these degrees of freedom.

In a dynamic analysis, the mass of the structure is used to compute inertial forces. Normally, the mass is obtained from the elements using the mass density of the material and the volume of the element. This automatically produces lumped (uncoupled) masses at the joints. The element mass values are equal for each of the three translational degrees of freedom. No mass moments of inertia are produced for the rotational degrees of freedom. This approach is adequate for most analyses.

It is often necessary to place additional concentrated masses and/or mass moments of inertia at the joints. These can be applied to any of the six degrees of freedom at any of the joints in the structure.

For computational efficiency and solution accuracy, SAP2000 always uses lumped masses. This means that there is no mass coupling between degrees of freedom at a joint or between different joints. These uncoupled masses are always referred to the local coordinate system of each joint. Mass values along restrained degrees of freedom are ignored.

Inertial forces acting on the joints are related to the accelerations at the joints by a 6x6 matrix of mass values. These forces tend to oppose the accelerations. In a joint local coordinate system, the inertia forces and moments *F1*, *F2*, *F3*, *M1*, *M2* and *M3* at a joint are given by:

$$\begin{cases}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
F_5 \\
F_6
\end{cases} = -
\begin{bmatrix}
u_1 & 0 & 0 & 0 & 0 & 0 \\
u_2 & 0 & 0 & 0 & 0 \\
u_3 & 0 & 0 & 0 & 0 \\
u_1 & 0 & 0 & 0 & 0 \\
v_1 & 0 & 0 & 0 & 0 \\
v_1 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 0 & 0 & 0 \\
v_3 & 0 & 0 & 0 & 0 \\
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v_7 & 0 & 0 & 0$$

where $\ddot{u}1$, $\ddot{u}2$, $\ddot{u}3$, $\ddot{r}1$, $\ddot{r}2$ and $\ddot{r}3$ are the translational and rotational accelerations at the joint, and the terms **u1**, **u2**, **u3**, **r1**, **r2**, and **r3** are the specified mass values. $\ddot{u}1$ Uncoupled joint masses may instead be specified in the global coordinate system, in which case they are transformed to the joint local coordinate system. Coupling terms will be generated during this transformation in the following situation:

- The joint local coordinate system directions are not parallel to global coordinate directions, and
- The three translational masses or the three rotational mass moments of inertia are not equal at a joint.

These coupling terms will be discarded by the program, resulting in some loss of accuracy. For this reason, it is recommended that you choose joint local coordinate systems that are aligned with the principal directions of translational or rotational mass at a joint, and then specify mass values in these joint local coordinates.

Mass values must be given in consistent mass units (W/g) and mass moments of inertia must be in WL2/g units. Here W is weight, L is length, and g is the acceleration due to gravity. The net mass values at each joint in the structure should be zero or positive. See Figure 7 for mass moment of inertia formulations for various planar configurations.

In SAP Mass definition can be defined as following options:

From Element and Additional Masses option. This is the default. With this option, the program uses the following mass specifications:

- Mass density specified for materials
- Additional line mass assigned to frame or cable objects
- Additional area mass assigned to <u>area objects</u>
- Mass specified for link properties
- Mass assigned directly to the joints

From Loads option: With this option, mass may be calculated from a scaled combination of load cases (see *Define Mass Multiplier for Loads*, below). The absolute value of the net load acting in the global Z direction is divided by the acceleration due to gravity, in the current units, and is used for the mass in the three translational directions.

From Element and Additional Masses and Loads option: This option is a combination of the other two options wherein mass is defined from mass specifications and from scaled combinations of load cases.

Define Mass Multiplier for Loads: If the From Loads or From Element and Additional Masses and Loads option was selected for *Mass Definition*, enter a combination of one or more load

cases in the table using the Add, Modify, and Delete buttons. Those load cases will be scaled by the specified factors and then are added together to determine the mass. Do not include the acceleration as a result of gravity in the scale factors; the program will do that automatically.

2.6.4.4 Material Damping

You may specify material damping to be used in dynamic analyses. Different types of damping are available for different types of analysis cases. Material damping is a property of the material and affects all Analysis cases of a given type in the same way. You may specify additional damping in each analysis case. Because damping has such a significant affect upon dynamic response, you should use care in defining your damping parameters.

Modal Damping

The material modal damping available in SAP2000 is stiffness weighted, and is also known as composite modal damping. It is used for all response-spectrum and modal time-history analyses. For each material you may specify a material modal damping ratio, r, The damping ratio, r, contributed to mode l by element j of this material is given by

$$r_{ij} = \frac{r\phi_i^{\mathrm{T}} \mathbf{K}_j \phi_i}{K_i}$$
 2.26

where i is mode shape for mode I, $\mathbf{K}j$ is the stiffness matrix for element j, and Ki is the modal stiffness for mode I given by

$$K_{i} = \sum_{j} \phi_{i}^{\mathrm{T}} \mathbf{K}_{j} \phi_{i}$$

$$2.27$$

Summed over all elements, *j*, in the model.

Viscous Proportional Damping

Viscous proportional damping is used for direct-integration time-history analyses. For each material, you may specify a mass coefficient, cM, and a stiffness coefficient, cK. The damping matrix for element j of the material is computed as:

$$C_j = c_M M_j + c_K K_j$$
 2.28

Hysteretic Proportional Damping

Hysteretic proportional damping is used for steady-state and power-spectral-density analyses. For each material, you may specify a mass coefficient, dM, and a stiffness coefficient, dM. The hysteretic damping matrix for element j of the material is computed as:

$$\mathbf{D}_{j} = d_{M}\mathbf{M}_{j} + d_{K}\mathbf{K}_{j}$$
 2.29

2.6.4.5 Constraints

Overview

A constraint consists of a set of two or more constrained joints. The displacements of each pair of joints in the constraint are related by constraint equations. The types of behavior that can been forced by constraints are:

• Rigid-body behavior, in which the constrained joints translate and rotate together as if connected by rigid links.

The types of rigid behavior that can be modeled are:

- Rigid Body: fully rigid for all displacements
- Rigid Diaphragm: rigid for membrane behavior in a plane
- Rigid Plate: rigid for plate bending in a plane
- Rigid Rod: rigid for ex tension along an axis
- Rigid Beam: rigid for beam bending on an axis
- Equal-displacement behavior, in which the translations and rotations are equal at the constrained joints
- Symmetry and antisymmetry conditions The use of constraints reduces the number of equations in the system to be solved and will usually result in increased computational efficiency. Most constraint types must be defined with respect to some fixed coordinate system. The coordinate system may be the global coordinate system or an alternate coordinate system, or it may be automatically determined from the locations of the constrained joints. The Local Constraint does not use a fixed coordinate system, but references each joint using its own joint local coordinate system. Welds are used to connect together different parts of the model that were defined separately. Each Weld consists of a set of joints that *may* be joined. The program searches for joints in each Weld that share the same location in space and constrains them to act as a single joint.

Diaphragm Constraint

A Diaphragm Constraint causes all of its constrained joints to move together as a planar Diaphragm that is rigid against membrane deformation. Effectively, all constrained joints are connected to each other by links that are rigid in the plane, but do not affect out-of-plane (plate) deformation.

This Constraint can be used to:

- Model concrete floors (or concrete-filled decks) in building structures, which typically have very high in-plane stiffness
- Model Diaphragms in bridge superstructures

The use of the Diaphragm Constraint for building structures eliminates the numerical-accuracy problems created when the large in-plane stiffness of a floor diaphragm is modeled with membrane elements. It is also very useful in the lateral (horizontal) dynamic analysis of buildings, as it results in a significant reduction in the size of the eigenvalue problem to be solved. See Figure 10 (page 54) for an illustration of a floor Diaphragm.

Joint Connectivity

Each Diaphragm Constraint connects a set of two or more joints together. The joints may have any arbitrary location in space, but for best results all joints should lie in the plane of the constraint. Otherwise, bending moments may be generated that are re strained by the Constraint, which unrealistically stiffens the structure. If this happens, the constraint forces reported in the Analysis results may not be in equilibrium.

Local Coordinate System

Each Diaphragm Constraint has its own local coordinate system, the axes of which are denoted 1, 2, and 3. Local axis 3 is always nor mal to the plane of the constraint. The program arbitrarily chooses the orientation of axes 1 and 2 in the plane. The actual orientation of the planar axes is not important since only the normal direction affects the constraint equations.

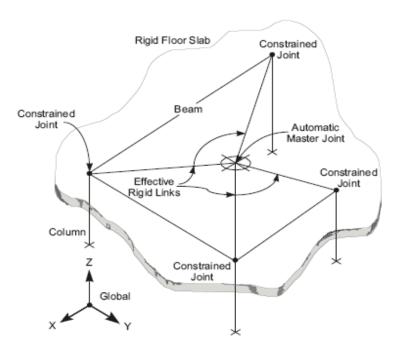


Fig. 2.6 Diaphragm Constraint

Constraint Equations

The constraint equations re late the displacements at any two constrained joints (sub scripts I and j) in a Diaphragm Constraint. These equations are expressed in terms of in-plane translations (u1 and u2), the rotation (r3) about the normal, and the in-plane coordinates (x1 and x2), all taken in the Constraint local coordinate system:

$$u_{1j} = u_{1i} - r_{3i} \Delta x_2$$

$$u_{2j} = u_{2i} + r_{3i} \Delta x_1$$

$$r_{3i} = r_{3j}$$
2.30

Automatic Master Joints

The program automatically creates an internal master joint for each explicit Constraint, and a master joint for each internal Body Constraint that is generated by a Weld. Each master joint governs the behavior of the corresponding constrained joints. The displacement at a constrained degree of freedom is computed as a linear combination of the displacements of the master joint.

2.6.4.6 Analysis Cases

An Analysis Case defines how the loads are to be applied to the structure (e.g., statically or dynamically), how the structure responds (e.g., linearly or nonlinearly), and how the analysis is to be performed (e.g., modal or by direct-integration.). User may as many named analysis cases of any type that he wishes. For analysis the model, user may select which cases are to be run and may also selectively delete results for any analysis case. The results of linear analyses may be superposed, i.e., added together after analysis. By default SAP perform linear static analysis and Modal analysis assuming default loadings.

The available types of linear analysis are:

- Static analysis
- Modal analysis for vibration modes, using eigenvectors or Ritz vectors
- Response-spectrum analysis for seismic response
- Time-history dynamic response analysis
- Buckling-mode analysis
- Moving-load analysis for bridge vehicle live loads
- Harmonic steady-state analysis

Each different analysis performed is called an Analysis Case. For each Analysis Case user has to define following type of information:

2.6.4.6.1 Case name

This name must be unique across all Analysis Cases of all types. The case name is used to call analysis results displacements, stresses, etc.), for creating Combinations, and sometimes for use by other dependent Analysis Cases.

2.6.4.6.2 Analysis type

This indicate the type of analysis (static, response-spectrum, time history, etc.), as well as available options for that type (linear, nonlinear, etc.).

2.6.4.6.3Loads applied

For most types of analysis, specify the Load Cases that are to be applied to the structure. Additional data may be required, depending upon the type of analysis being defined.

2.6.4.6.4 Running Analysis Cases

After defining a structural model and one or more Analysis Cases, run the Analysis Cases to get results for display, output, and design purposes. When an analysis is run, the program converts the object-based model to finite elements, and performs all calculations necessary to determine the response of the Methods of Dynamic Analysis of Concrete Gravity Dam & introduction to SAP 2000 structure to the loads applied in the Analysis Cases. The analysis results are saved for each case for subsequent use.

2.6.4.6.5 Modal Analysis

Modal Analysis is used to determine the vibration modes of a structure. These modes are useful to under stand the behavior of the structure. They can also be used as the basis for modal super position in response-spectrum and modal time-history Analysis Cases.

Overview

A modal Analysis is defined by creating an Analysis Case and setting its type to "Modal". You can de fine multiple modal Analysis Cases, resulting in multiple sets of modes. There are two types of modal Analysis to choose from when defining a modal Analysis Case:

- Eigenvector Analysis deter mines the undamped free-vibration mode shapes and frequencies of the system. These natural modes provide an excellent insight into the behavior of the structure.
- Ritz-vector Analysis seeks to find modes that are excited by a particular loading. Ritz vectors can pro vide a better basis than do eigenvectors when used for response-spectrum or time-history analyses that are based on modal super position.

Modal Analysis is always linear. A modal Analysis Case may be based on the stiffness of the full unstressed structure, or upon the stiffness at the end of a nonlinear Analysis Case (non linear static or nonlinear direct-integration time-history). By using the stiffness at the end of a nonlinear case, you can evaluate the modes under P-delta or geometric stiffening conditions, at different stages of construction, or following a significant non linear excursion in a large earthquake.

Eigenvector Analysis

Eigenvector analysis determines the undamped free- vibration mode shapes and frequencies of the system. These natural Modes pro vide an excellent in sight into the behavior of the structure. They can also be used as the basis for response- spectrum or time- history analyses, al though Ritz vectors are recommended for this purpose. Eigen vector analysis involves the solution of the generalized eigen value problem:

$$[K - \Omega^2 M] \Phi = 0$$
 2.31

where K is the stiffness matrix, M is the diagonal mass matrix, 2 is the diagonal matrix of eigen values, and 2 is the matrix of corresponding eigen vectors (mode shapes). Each eigen value- eigenvector pair is called a natural Vibration Mode of the structure. The Modes are identified by numbers from 1 to n in the order in which the modes are found by the pro gram. The eigen value is the square of the circular frequency, 2, for that Mode (unless a frequency shift is used, see below). The cyclic frequency, f, and period, f, of the Mode are related to f

$$T = \frac{1}{f}$$
 and $f = \frac{\omega}{2\pi}$ 2.32

You may specify the number of modes to be found, a convergence tolerance, and the frequency range of interest. These parameters are de scribed in the following sub topics.

Number of Modes

You may specify the maximum and minimum number of modes to be found. The program will not calculate more than the specified maximum number of modes. This number includes any static correction modes requested. The program may compute fewer modes if there are fewer mass degrees of freedom, all dynamic participation targets have been met, or all modes within the cut off frequency range have been found.

The program will not calculate fewer than the specified minimum number of modes, unless there are fewer mass degrees of freedom in the model. A mass degree of freedom is any *active* degree of freedom that possesses translational mass or rotational mass moment of inertia. The mass may have been as signed directly to the joint or may come from connected elements.

Only the modes that are actually found will be available for use by any subsequent responsespectrum or modal time-history analysis cases.

Ritz-Vector Analysis

Re search has indicated that the natural free- vibration mode shapes are not the best basis for a mode- superposition analysis of structures subjected to dynamic loads. It has been demonstrated (Wilson, Yuan, and Dickens, 1982) that dynamic analyses based on a special set of load- dependent Ritz vectors yield more ac cu rate results than the use of the same number of natural mode shapes. The algorithm is detailed in Wilson (1985).

The reason the Ritz vectors yield excellent results is that they are generated by taking into account the spatial distribution of the dynamic loading, whereas the direct use of the natural mode shapes neglects this very important information.

In addition, the Ritz-vector algorithm automatically includes the advantages of the proven numerical techniques of static condensation, Guyan reduction, and static correction due to higher-mode truncation.

The spatial distribution of the dynamic load vector serves as a starting load vector to initiate the procedure. The first Ritz vector is the static displacement vector corresponding to the starting load vector. The remaining vectors are generated from a recurrence relationship in which the mass matrix is multiplied by the previously obtained Ritz vector and used as the load vector for the next static solution. Each static solution is called a generation cycle.

When the dynamic load is made up of several in dependent spatial distributions, each of these may serve as a starting load vector to generate a set of Ritz vectors. Each generation cycle creates as many Ritz vectors as there are starting load vectors. If a generated Ritz vector is redundant or does not excite any mass degrees of freedom, it is discarded and the corresponding starting load vector is removed from all subsequent generation cycles.

Standard eigen solution techniques are used to orthogonalize the set of generated Ritz vectors, resulting in a final set of Ritz-vector Modes. Each Ritz-vector Mode consists of a mode shape and frequency. The full set of Ritz-vector Modes can be used as a basis to rep re sent the dynamic displacement of the structure.

When a sufficient number of Ritz- vector Modes have been found, some of them may closely approximate natural mode shapes and frequencies. In general, however, Ritz- vector Modes do not represent the intrinsic character is tics of the structure in the same way the natural Modes do. The Ritz- vector modes are biased by the starting load vectors.

You may specify the number of Modes to be found, the starting load vectors to be used, and the number of generation cycles to be performed for each starting load vector.

Number of Modes

You may specify the max i mum and minimum number of modes to be found. The pro gram will not calculate more than the specified maxi mum number of modes. The pro gram may compute fewer modes if there are fewer mass degrees of freedom, all dynamic participation targets have been met, or the maximum number of cycles has been reached for all loads. The program will not calculate fewer than the specified minimum number of modes, unless there are fewer mass degrees of freedom in the model.

A mass degree of freedom is any *active* degree of freedom that possesses translational mass or rotational mass moment of inertia. The mass may have been assigned directly to the joint or may come from connected elements. Only the modes that are actually found will be available for use by any subsequent response-spectrum or modal time-history Analysis cases.

2.6.4.6.6 Modal Analysis Output

Various properties of the Vibration Modes are available as Analysis results. This information is the same regardless of whether you use eigen vector or Ritz- vector analysis, and is described as follows,

Periods and Frequencies

The following time- properties are printed for each Mode:

- Period, T, in units of time
- Cyclic frequency, f, in units of cycles per time; this is the inverse of T
- Circular frequency, in units of radians per time;
- Eigen value, in units of radians- per- time squared

Participation Factors

The modal participation factors are the dot products of the three Acceleration Loads with the modes shapes. The participation factors for Mode n corresponding to Acceleration Loads in the global X, Y, and Z directions are given by:

$$f_{xn} = \varphi_n^T m_x$$

$$f_{yn} = \varphi_n^T m_y$$

$$f_{zn} = \varphi_n^T m_z$$
2.33

where *n* is the mode shape and *mx*, *my*, and, *mz* are the unit Acceleration Loads. These factors are the generalized loads acting on the Mode due to each of the Acceleration Loads. These values are called "factors" because they are related to the mode shape and to a unit acceleration. The modes shapes are each normalized, or scaled, with respect to the mass matrix such that:

$$\varphi_n^T M \varphi_n = 1$$
 2.34

The actual magnitudes and signs of the participation factors are not important. What is important is the relative values of the three factors for a given Mode. Participating Mass Ratios The participating mass ratio for a Mode provides a measure of how important the Mode is for computing the response to the Acceleration Loads in each of the three global directions. Thus it is useful for determining the accuracy of response spectrum analyses and seismic time-history analyses. The participating mass ratio provides *no information* about the accuracy of time-history analyses subjected to other loads.

The participating mass ratios for Mode *n* corresponding to Acceleration Loads in the global X, Y, and Z directions are given by:

$$r_{xn} = \frac{(f_{xn})^2}{M_x}$$

$$r_{yn} = \frac{(f_{yn})^2}{M_y}$$

$$r_{zn} = \frac{(f_{zn})^2}{M_z}$$

2.35

where fxn, fyn, and fzn are the participation factors de fined in the previous sub topic; and Mx, My, and Mz are the total un re strained masses acting in the X, Y, and Z directions. The participating mass ratios are expressed as percent ages.

The cumulative sums of the participating mass ratios for all Modes up to Mode n are printed with the individual values for Mode n. This provides a simple measure of how many Modes are required to achieve a given level of ac cu racy for ground acceleration loading.

If all eigen Modes of the structure are present, the participating mass ratio for each of the three Acceleration Loads should generally be 100%. However, this may not be the case in the presence of Asolid elements or certain types of Constraints where symmetry conditions prevent some of the mass from responding to translational accelerations.

Static and Dynamic Load Participation Ratios

The static and dynamic load participation ratios provide a measure of how adequate the calculated modes are for representing the response to time-history analyses. These two measures are printed in the output file for each of the following spatial load vectors:

- The three unit Acceleration Loads
- Three rotational acceleration loads
- All Load Cases specified in the definition of the modal Analysis Case
- All non linear deformation loads, if they are specified in the definition of the modal Analysis Case

The Load Cases and Acceleration Loads represent spatial loads that you can explicitly specify in a modal time-history analysis, whereas the last rep resents loads that can act implicitly in a *non linear* modal time-history Analysis. The load participation ratios are expressed as percent ages.

Static Load Participation Ratio

The static load participation ratio measures how well the calculated modes can represent the response to a given static load. This measure was first presented by Wilson (1997). For a given spa tial load vector p, the participation factor for Mode n is given by

$$f_n = \varphi_n^T p \tag{2.36}$$

where n is the mode shape (vector) of Mode n. This factor is the generalized load acting on the Mode due to load p. Note that f n is just the usual participation factor when p is one of the unit Acceleration loads. The static participation ratio for this mode is given by

$$r_n^S = \frac{\left(\frac{f_n}{\omega_n}\right)^2}{u^T p}$$
2.37

where u is the static solution given by Ku. This ratio gives the fraction of the total strain energy in the ex act static solution that is contained in Mode n.

Finally, the cumulative sum of the static participation ratios for all the calculated modes is printed in the output file:

$$R^{S} = \sum_{n=1}^{N} r_{n}^{S} = \frac{\sum_{n=1}^{N} \left(\frac{\varphi_{n}^{T} p}{\omega_{n}}\right)^{2}}{u^{T} p}$$
2.38

strain energy in the exact static solution that is captured by the *N* modes. When solving for static solutions using quasi- static time- history analysis, the value of *RS* should be close to 100% for any ap plied static Loads, and also for all non linear de formation loads if the analysis is non linear.

Note that when Ritz- vectors are used, the value of RS will always be 100% for all starting load vectors. This may not be true when eigen vectors are used. In fact, even using all possible eigen vectors will not give 100% static participation if load p acts on any mass less degrees of freedom.

Dynamic Load Participation Ratio

The dynamic load participation ratio measures how well the calculated modes can represent the response to a given dynamic load. This measure was developed for SAP2000, and it is an extension of the concept of participating mass ratios. It is assumed that the load acts only on degrees of freedom with mass. Any portion of load vector p that acts on mass less degrees of freedom cannot be represented by this measure and is ignored in the following discussion. For a given spatial load vector p, the participation factor for Mode n is given by

$$f_n = \varphi_n^T p \tag{2.39}$$

where is the mode shape for Mode n. Note that f n is just the usual participation factor when p is one of the unit Acceleration loads.

The dynamic participation ratio for this mode is given by

$$r_n^D = \frac{\left(f_n\right)^2}{a^T p}$$

2 40

where a is the Acceleration given by Ma. The Acceleration a is easy to calculate since M is diagonal. The values of a and p are taken to be zero at all mass less degrees of freedom. Note that the denominate or can also be represented as a T Ma. Finally, the cumulative sum of the dynamic participation ratios for all the calculated modes is printed in the output file:

where, N is the number of modes found. When p is one of the unit Acceleration loads, r D is the usual mass participation ratio, and RD is the usual cumulative mass participation ratio. When RD is 100%, the calculated modes should be capable of exactly representing the solution to any time-varying application of spatial load p. If RD is less than 100%, the ac cu racy of the solution will depend upon the frequency content of the time-function multiplying load p. Normally it is the high frequency response that is not captured when RD is less than 100%. The dynamic load participation ratio only measures how the modes capture the spatial characteristics of p, not its temporal characteristics. For this reason, RD serves only as a qualitative guide as to whether enough modes have been computed. You must still examine the response to each different dynamic loading with varying number of modes to see if enough modes have been used.

$$R^{D} = \sum_{n=1}^{N} r_{n}^{D} = \frac{\sum_{n=1}^{N} \left(\varphi_{n}^{T} p \right)^{2}}{a^{T} p}$$
2.41

Modal Definition and Analysis Results Tabular Data

Model definition data include all input components of the structural model (properties, objects, assignments, loads, analysis cases, design settings, etc.), as well as any options you have selected, and named result definitions you have created. Model definition data are always available, whether or not analyses have been run or design has been performed. These tables

can be edited, displayed, exported, imported, and printed by using definition data tables onscreen in the graphical user interface, or export and import in one of the following formats:

- Microsoft Access database
- Microsoft Excel spreadsheet
- Plain (ASCII) text

Analysis results data include the deflections, forces, stresses, energies, and other response quantities that can be produced in the graphical user interface. These data are only available for analysis cases that have actually been run. Analysis results tables can not be edited or imported, but displayed, exported, and printed on-screen in the graphical user interface, or export and import in one of the following formats:

- Microsoft Access database
- Microsoft Excel spreadsheet
- Plain (ASCII) text

2.6.4.6.7 Response-Spectrum Analysis

Response-spectrum analysis is a statistical type of analysis for the determination of the likely response of a structure to seismic loading.

Local Coordinate System

Each Spec has its own response- spectrum local coordinate system used to define the directions of ground acceleration loading. The axes of this local system are denoted 1, 2, and 3. By default these correspond to the global X, Y, and Z directions, respectively.

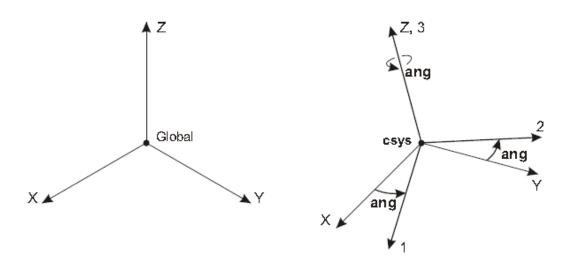


Fig. 2.8 Definition of Response Spectrum Local coordinate system

You may change the orientation of the local coordinate system by specifying:

- A fixed coordinate system csys (the default is zero, indicating the global co ordinate system)
- A coordinate angle, ang (the default is zero)

The local 3 axis is always the same as the Z axis of co ordinate system csys. The local 1 and 2 axes coincide with the X and Y axes of csys if angle ang is zero. Otherwise, ang is the angle from the X axis to the local 1 axis, measured counter clockwise when the +Z axis is pointing toward you.

Response-Spectrum Curve

The response- spectrum curve for a given direction is defined by digitized points of pseudo-spectral acceleration response versus period of the structure. The shape of the curve is given by specifying the name of a Function. All values for the abscissa and ordinate of this Function must be zero or positive. If no Function is specified, a constant function of unit acceleration value for all periods is assumed. You may specify a scale factor of the multiply the ordinate (pseudo spectral acceleration response) of the function. This is often needed to convert values given in terms of the acceleration due to gravity to units con sis tent with the rest of the model.

If the response- spectrum curve is not defined over a period range large enough to cover the Vibration Modes of the structure, the curve is extended to larger and smaller periods using a constant acceleration equal to the value at the nearest defined period.

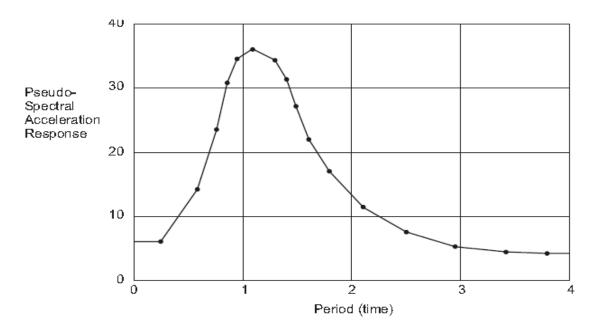


Fig.2.9 Pseudo Response Spectra Define Response Spectrum Functions

Response Spectrum Functions form and add, modify, or delete a response-spectrum function. A response-spectrum function is simply a list of period versus spectral-acceleration values. In SAP2000, the acceleration values in the function are assumed to be normalized; that is, the functions themselves are not assumed to have units. Instead, the units are associated with a scale factor that multiplies the function and that is specified when the response-spectrum analysis case is defined.

Note: By default, the program pre-defines a unit constant response-spectrum function. This function.

2.6.4.6.8 Linear Time-History Analysis

Time-history analysis is a step-by-step analysis of the dynamical response of a structure to a specified loading that may vary with time. The analysis may be linear or nonlinear. This Chapter de scribes time-history analysis in general and linear time-history analysis in particular.

Ground Displacement Load

The Ground Displacement Load is used to apply specified displacements (translations and rotations) at the *grounded end* of joint restraints and spring supports. Displacements may be specified in a fixed coordinate system (global or alternate coordinates) or the joint local coordinate system. The specified values are shown in Figure 8 (page 43). All displacements at a joint are transformed to the joint local coordinate system and added together. Restraints may be considered as *rigid* connections between the joint degrees of freedom and the ground. Springs may be considered as *flexible* connections between the joint degrees of freedom and the ground.

Ground displacements do not act on one-joint Link/Support Elements. To apply ground displacements through a nonlinear support, use a two-joint Link/Support element, restrain one end, and apply ground displacement to the restrained end. It is very important to understand that ground displacement load applies to the ground, and does not affect the structure unless the structure is supported by restraints or springs in the direction of loading

The load, r(t), applied in a given time-history case may be an arbitrary function of space and time. It can be writ ten as a finite sum of spatial load vectors, pi, multiplied by time functions, fi, as:

$$r(t) = \sum_{i} f_i(t) p_i$$
 2.42

The program uses Load Cases and/or Acceleration Loads to represent the spatial load vectors. The time functions can be arbitrary functions of time or periodic functions such as those produced by wind or sea wave loading.

If Acceleration Loads are used, the displacements, velocities, and accelerations are all measured relative to the ground. The time functions associated with the Acceleration Loads mx, my, and mz are the corresponding components of uniform ground acceleration, \ddot{u}_{gx} , \ddot{u}_{gy} and \ddot{u}_{gz} .

Defining the Spatial Load Vectors

To de fine the spatial load vector, *pi*, for a single term of the loading sum of Equation 1, you may specify either:

- The label of a Load Case using the parameter load, or
- An Acceleration Load using the parameters csys, ang, and acc, where:
- csys is a fixed co ordinate system (the de fault is zero, indicating the global co ordinate system)
- ang is a co ordinate angle (the de fault is zero)
- acc is the Acceleration Load (U1, U2, or U3) in the acceleration local co ordinate system as de fined be low

Each Acceleration Load in the loading sum may have its own acceleration local coordinate system with local axes de noted 1, 2, and 3. The local 3 axis is always the same as the Z axis of co ordinate system csys. The local 1 and 2 axes coincide with the X and Y axes of csys if angle ang is zero. Otherwise, ang is the angle from the X axis to the local 1 axis, measured counter clock wise when the +Z axis is pointing toward you. The response- spectrum local axes are always referred to as 1, 2, and 3. The global Acceleration Loads mx, my, and mz are transformed to the local co ordinate system for loading.

It is generally recommended, but not required, that the same co ordinate system be used for all Acceleration Loads applied in a given time-history case. Load Cases and Acceleration Loads may be mixed in the loading sum.

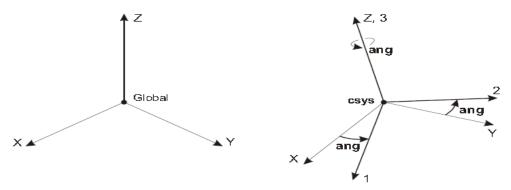


Fig. 2.9 Definition of History Acceleration Local Co-ordinate system

The label of a Function, using the parameter function, that defines the shape of the time variation (the de fault is zero, indicating the built- in ramp function defined above). A scale factor, sf, that multiplies the ordinate values of the Function (the default is unity)

- A time- scale factor, tf, that multiplies the time (abscissa) values of the Function (the default is unity)
- An arrival time, at, when the Function be gins to act on the structure (the default is zero)

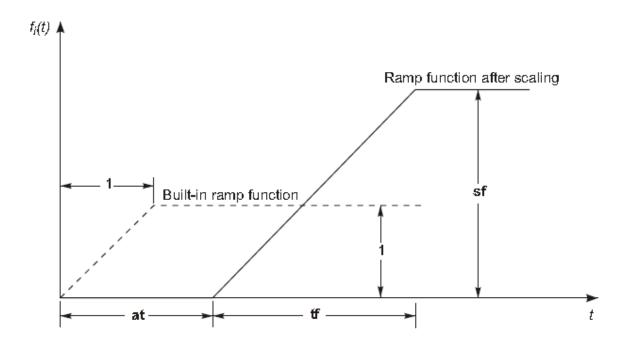


Fig. 2.10 Definition of History Acceleration Local Co-ordinate system

2.6.5 Loading on Elements

2.6.5.1 Frame Distributed Loads

Distributed (Uniform and Trapezoidal) forces and moments can be assigned along the length of frame/cable objects. The loads may be as simple or as complicated as required. Loads are specified in force-per-length or moment-per-length units.

Frame Distributed Loads forms:-

Load Case Name :- Choose the load case to which the load(s) is being assigned Units. Choose the units to be used.

Load Type and Direction :- Specify the load type (Forces or Moments) and coordinate system and the load direction.

Note: The Gravity direction is taken to be in the negative Global Z direction. To enter a load in the gravity direction, enter a positive number if the direction is set to Gravity and a negative number if the direction is set to Global Z.

Options Specify how the load will be applied :-

Add to Existing Loads: Adds the specified distributed load to the frame object. If one or more distributed load assignments have already been made, this option increases the existing load, assuming a positive load is being specified.

Replace Existing Loads: Replaces the currently specified distributed load, if any, with the new load. If no assignment exists, the new assignment is still made. This is the default option.

Delete Existing Loads: Deletes any and all distributed load assignments made to the selected object(s). When this option is selected, the items in the *Load Type and Direction, Trapezoidal Loads*, and *Uniform Load* areas of the form are ignored when the OK button is clicked. Note that the default option is *Replace* and that the program defaults to this every time the form is opened.

2.6.5.2 Trapezoidal Loads:-

For trapezoidal loads, enter up to four locations and the corresponding load values in the appropriate units. The load will change abruptly at the first and fourth locations, and vary smoothly at the second and third locations. If more than four locations are needed, perform more assignment operations, using the *Add to Existing Loads* option.

Relative Distance from End-I or Absolute Distance from End-I. Specify if the distance from End I is relative or absolute for the location(s) specified in the *Trapezoidal Loads* area of the form.

Uniform Loads. For uniform loads (i.e., constant over the full length of the selected object), enter the load value in the Uniform Load field. If uniform and trapezoidal loads are specified at the same time, they are additive.

2.6.5.3 Area Loads - Gravity (All)

Area Loads command is a way to add a factored self weight of the members as a force in any of the global directions, whereas the self-weight load itself acts equally on all elements of the structure and always in the global -Z direction. It is recommended that the actual self weight of the structure be included in the definition of the static load cases.

Area GravityLoads form :-

Load Case Name. Select the load case to which the gravity load is being added.

Gravity Multipliers:-Provide gravity multipliers in the global X, Y or Z directions. Coordinate System. Select the coordinate system for the load from the drop-down list.

Specify how the load is to be applied:-

Add to Existing Loads: Adds the specified gravity load to the area object. If one or more load assignments have already been made, this option increases the load, assuming that a positive load is being applied.

Replace Existing Loads: Replaces the currently specified gravity load, if any, with the new load. If no assignment exists, the new assignment is still made. This is the default option.

Delete Existing Loads: Deletes any and all gravity load assignments made to the selected area object(s). When this option is selected, the items in the *Gravity Multiplier* area of the form are ignored when the OK button is clicked.

2.6.5.4 Area Loads - Surface Pressure (All)

Surface pressure always acts normal to the area element face. Positive pressures are directed towards the interior of the element. The pressure may be constant over the face or interpolated from values given at the joints. The values at the joints are obtained from joint patterns, and need not be the same for different faces. Joint patterns can be used to easily apply hydrostatic pressure. Assign surface Pressure Loads form to areas as follows:

Load Case Name:- Select the load case to which this load is being added.

Pressure area:- Choose if the pressure will be applied by element or by joint pattern.

By Element:- When this option is selected, specify a pressure value in the *Pressure* edit box. By Joint Pattern:- When this option is selected, choose a joint pattern from the *Pattern* dropdown list and then specify a multiplier in the *Multiplier* edit box.

By Face:- Select the element Face from the drop-down list.

Options. Specify a load application option.

Add to Existing Loads:- Adds the specified pressure load to the area object. If one or more pressure load assignments have already been made, this option increases the existing load, assuming a positive load is being specified.

Replace Existing Loads:- Replaces the currently specified pressure load, if any, with the new load. If no assignment exists, the new assignment is still made. This is the default option.

Delete Existing Loads:- Deletes any and all pressure load assignments made to the selected area object(s). When this option is selected, the items in the *Pressure* area of the form are ignored when the OK button is clicked.

2.6.5.5 Area Uniform load

Uniform load is used to apply uniformly distributed forces to the mid surfaces of area elements. The direction of the loading may be specified in a fixed coordinate system (global or user-defined system) or in the element local coordinate system.

Uniform Loads form:-

Load Case Name. Select the load case to which the uniform load is being added.

Units. Verify that the units are appropriate for specifying the load. Use the drop-down list to change them, if necessary.

Load:- Specify the load value as forces per unit area in the *Load* edit box.

Coord System:- Select the coordinate system from the *Coord System* drop-down list.

Direction:- Select the load direction from the *Direction* drop-down list.

Note: The Gravity direction is taken to be in the negative Global Z direction. To enter a load in the gravity direction, enter a positive number if the direction is set to Gravity and a negative number if the direction is set to Global Z.

Options :- Specify how the load is to be applied.

Add to Existing Loads:- Adds the specified uniform load to the area object. If one or more uniform load assignments have already been made, this option increases the uniform load, assuming that a positive load is being specified.

Replace Existing Loads:- Replaces the currently specified uniform load if any, with the new load. If no assignment has been made, the new assignment is still made. This is the default option.

Delete Existing Loads:- Deletes any and all uniform load assignments made to the selected area object(s). When this option is selected, the items in the *Uniform Load* area of the form are ignored when the OK button is clicked.

2.6.6 Joint Element Output Conventions

Frame Element Internal Forces Output Conventions in SAP. The frame element internal forces are:

- P, the axial force
- V2, the shear force in the 1-2 plane
- V3, the shear force in the 1-3 plane
- T, the axial torque (about the 1-axis)
- M2, the bending moment in the 1-3 plane (about the 2-axis)
- M3, the bending moment in the 1-2 plane (about the 3-axis)

These internal forces and moments are present at every cross section along the length of the frame element may be requested as part of analysis output.

The sign convention is illustrated in Figure . Positive internal forces and axial torque acting on a positive 1 face are oriented in the positive direction of the element local co ordinate axes. Positive internal forces and axial torque acting on a negative face are oriented in the negative

direction of the element local co ordinate axes. A positive 1 face is one whose out ward nor mal (pointing away from element) is in the positive local 1 direction.

Positive bending moments cause compression at the positive 2 and 3 faces and tension at the negative 2 and 3 faces. The positive 2 and 3 faces are those faces in the positive local 2 and 3 directions, respectively, from the neutral axis.

For each load case and load combination the frame element internal forces and moments are computed and reported at each frame element output station. For frame element output displayed in a tabular form on the computer screen, printed to a printer or printed to a file, the locations of the output stations are identified by the absolute distance to the station measured from the i-end of the element.

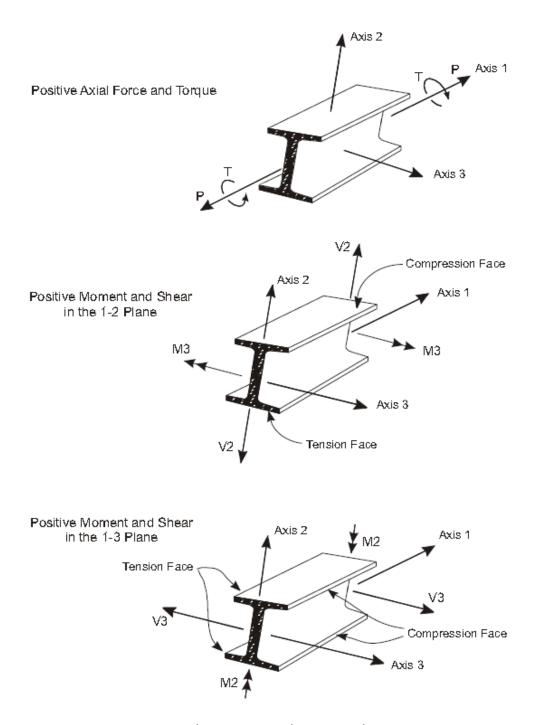


Fig.2.11 Frame Element Internal Forces and Moments

This sign convention can be described by defining the concept of positive and negative faces of an element. Consider a section cut through the element in the 2-3 plane. At this section the positive 1 face is the face whose outward normal (arrow that is perpendicular to the section and pointing away from the section) is in the positive local 1 direction. At this same section the

negative one face is one whose outward normal is in the negative local 1 direction. The positive 2 and 3 faces are those faces with outward normal in the positive local 2 and 3 directions, respectively, from the neutral axis.

Positive internal forces (P, V2 and V3) and positive axial torque (T) acting on a positive 1 face are oriented in the positive direction of the corresponding element local coordinate axis. For example, when V2 acting on a positive 1 face is positive, it is oriented in the direction of the positive local 2-axis. Positive M2 bending moments cause compression on the positive 3 face and tension on the negative 3 face. Positive M3 bending moments cause compression on the positive 2 face and tension on the negative 2 face.

When end offsets along the length of the frame element are present, the internal forces and moments are output at the faces of the supports rather than the ends of the element. No output is produced within the end offset length. The right-hand rule applies in the figure for determining the sense of the moments shown by the double arrows.

2.6.7 SAP Output

2.6.7.1Assembled Joint Mass Output

You can re quest assembled joint masses as part of the analysis results. The mass at a given joint includes the mass assigned directly to that joint as well as a portion of the mass from each element connected to that joint. Mass at re strained degrees of freedom is set to zero. All mass assigned to the elements is apportioned to the connected joints, so that this table represents the total *unrestrained* mass of the structure. The masses are always referred to the local axes of the joint.

2.6.7.2 Displacement Output

You can request joint displacements as part of the analysis results on a case by case basis. For dynamic analysis cases, you can also request velocities and accelerations. The output is always referred to the local axes of the joint.

2.6.7.3 Force Output

You can request joint support forces as part of the analysis results on a case by case basis. These Support forces are called reactions, and are the sum of all forces from restraints, springs, or one-joint Link/Support elements at that joint. The reactions at joints not sup ported will be zero. The forces and moments are always referred to the local axes of the joint. The values reported are always the forces and moments that *act on* the joints. Thus a positive value of joint force or moment tends to cause a positive value of joint translation or rotation along the corresponding degree freedom.

2.6.7.4Element Joint Force Output

The element joint forces are concentrated forces and moments acting at the joints of the element that represent the effect of the rest of the structure upon the element and that cause the deformation of the element. The moments will always be zero for the solid-type elements: Plane, Asolid, and Solid. A positive value of force or moment tends to cause a positive value of translation or rotation of the element along the corresponding joint degree of freedom. Element joint forces must not be confused with internal forces and moments which, like stresses, act within the volume of the element. For a given element, the vector of element joint forces, f, is computed as:

$$f = K u - r$$

where K is the element stiffness matrix, u is the vector of element joint displacements, and r is the vector of element applied loads as apportioned to the joints. The element joint forces are always referred to the local axes of the individual joints. They are identified in the out put as F1, F2, F3, M1, M2, and M3.

2.6.7.5 Constraint Output

For each Body, Diaphragm, Plate, Rod, and Beam Constraint having more than two constrained joints, the following information about the Constraint and its master joint is printed in the output file:

- The translational and rotational local coordinate systems for the master joint
- The total mass and mass moments of inertia for the Constraint that has been applied to the master joint
- The center of mass for each of the three translational masses The degrees of freedom are indicated as U1, U2, U3, R1, R2, and R3. These are referred to the two local coordinate systems of the master joint.

2.6.8 Accessing the Assembled Stiffness and Mass Matrices

When using the advanced equation solver, you may request that the program produce the assembled stiffness and mass matrices in the form of text files. This can be done for a single linear static, modal, or buck ling analysis case. To get the stiffness and mass matrices for a nonlinear case, define a linear case that uses the stiffness from the final state of the desired nonlinear case. The assembled matrices are provided in five text files that have the same name as the model file, but with the following extensions and contents:

- Extension .TXA: This file includes the counts of the number of joints and equations in the model, and also describes the format and contents of the other four files.
- Extension .TXE: This file gives the equation numbers for each degree of freedom (DOF) at each joint. Equation numbers are positive for active DOF that are present in the stiffness and mass matrices, negative for constrained DOF that are computed as linear combinations of active DOF, and zero for re strained or null DOF.
- Extension .TXC: This file de fines the constraint equations, and is only present if there are constraints in the model.
- Extension .TXK: This file gives the lower half of the symmetric stiffness matrix.
- Extension .TXM: This file gives the lower half of the symmetric mass matrix.

Each of the latter four files contains a single header line that begins with "Note:" and defines the data columns. All subsequent lines provide Tab-delimited data for easy import into texted or spread sheet programs.

Chapter 3

Methodology

We followed the methodology given by *Charles Menun and Armen Der Kiureghian* in deriving Elliptical envelop, when orientation of Principle direction of earthquake is known. Also we develop Supreme envelop when orientation of principle direction of earthquake is unknown, it is more general case because for most structures, this direction is not known and for most geographical locations cannot be estimated. They have analyzed the building frames in DRAIN 3DX and developed envelop but we have done it with SAP 2000 v 11 (Advanced). In the another approach *Martin R. Button, Edward L. Willons* have shown that estimation of combined stress directly by mean maximum values of axial forces and moments is incorrect. A correct estimate of stress is obtained only of equation is evaluated for each mode shapes and maximum stress can then be found from combining these by the CQC method of modal combination and shown that how to evaluate critical values of different response of structure for earthquake loading for full range of angle between principal direction of structure axis and earthquake component in computer run. We find out modal values of responses like axial forces and moments for column section in SAP2000 v 11 (Advanced).

3.1 Charles Menun and Armen Der Kiureghian Approach

3.1.1 Peak Value of an Individual Response Component due to a 3D Ground Motion

For N-degree-of-freedom linear and classically damped structure with $n \le N$ significant modes. When two horizontal components and one vertical component of ground motion are acting simultaneously, the response-spectrum-based estimate of the peak response X_r is

$$X_{r}^{2} = \sum_{k=1}^{3} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{r}^{T} \phi_{i} \gamma_{ki}) (q_{r}^{T} \phi_{j} \gamma_{kj}) \rho_{ij} S_{ki} S_{kj}$$
3.1

where ϕ_i and $\gamma_{ki} = \varphi_i^T M I_k / \varphi_i^T M \varphi_i = i^{th}$ mode shape and modal participation factor associated with the input in direction k, respectively; i.e., 'Displacement of mass resulting from static application of a unit ground displacement', $\mathbf{q}_r = \mathbf{f}$ function of the stiffness and unreformed geometry of the structure and superscript T denotes the vector transpose. $S_{ki} = \max(s_{ki})$ is oscillator response having corresponding to i th mode and those associate with k principal direction of ground motion, It should be noted that the conventional response spectrum is actually defined as $S_t = \max |s_t(t)|$. However, in the present paper, the definition is $S_t = \max[s_t(t)]$. The difference between the two definitions is normally negligible (i.e., <5%). Use of the conventional response spectrum in the present analysis will produce results that are on the conservative side by no more than 5%. ρ_{ij} is correlation factor depends on the modal combination rule used for the analysis. In this paper, the complete quadratic combination rule

is recommended, for which ρ_{ij} is a function of the frequencies and damping ratios of modes i and j. This rule applies when the principal directions, along which the ground motion components are uncorrelated, coincide with the structure axes.

 z_1, z_2 = structure axes $z_1', z_2' = \text{major and intermediate principal directions of the ground motion}$ z_2' Footprint of the structure

Fig. 3.1 Structure axis and Principal direction of the ground motion

More generally, when the principal directions of ground motion make a counterclockwise angle θ with the structure axes in the horizontal plane, as in Fig. 3.1, the rule takes on the form (Smeby and Der Kiureghian 1985; Menun and Der Kiureghian 1998).

$$X_{r}^{2}(\theta) = \sum_{k=1}^{3} \sum_{i=1}^{n} \sum_{j=1}^{n} (q_{r}^{T} \phi_{i} \gamma_{ki}) (q_{r}^{T} \phi_{j} \gamma_{kj}) \rho_{ij} S_{ki} S_{kj} - \sum_{k=1}^{2} \sum_{l=1}^{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (-1)^{k+1} (q_{r}^{T} \phi_{i} \gamma_{ki}) (q_{r}^{T} \phi_{j} \gamma_{kj}) \rho_{ij} S_{ki} S_{kj} \sin^{2} \theta$$

$$- \sum_{k=1}^{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (-1)^{k} (q_{r}^{T} \phi_{i}) (q_{r}^{T} \phi_{j}) (\gamma_{1i} \gamma_{2j} + \gamma_{2i} \gamma_{1j}) \rho_{ij} S_{ki} S_{kj} \sin \theta \cos \theta$$

$$3.2$$

In which the participation factors are defined with respect to the structure axes, and the response spectrum ordinates are those associated with the principal directions of the ground motion. The rule for a single component of ground motion can be obtained as a special case of (9) by setting $\theta=0$ and $S_{2i}=S_{3i}=0$ for all modes. Hence, for the remainder of this paper, only the general case of multicomponent excitation is considered.

when θ is specified, the envelope derived in this study is an ellipsoid in the m-dimensional response space that is inscribed within this conservative rectangular envelope. For the subsequent development of the elliptical envelope, convenient matrix form of above equation is defined.

$$X_{r}^{2} = q_{r}^{T} \Phi \left[\sum_{k=1}^{3} (\Gamma_{k} S_{k} R S_{k}^{T} \Gamma_{k}^{T}) - \sum_{k=1}^{2} \sum_{l=1}^{2} (-1)^{k+1} (\Gamma_{l} S_{k} R S_{k}^{T} \Gamma_{l}^{T}) \sin^{2}\theta - \sum_{k=1}^{2} (-1)^{k} (\Gamma_{1} S_{k} R S_{k}^{T} \Gamma_{2}^{T} + (\Gamma_{2} S_{k} R S_{k}^{T} \Gamma_{1}^{T}) \sin\theta \cos\theta \right] \Phi^{T} q_{r}$$
and
$$X_{r}^{2} = q_{r}^{T} \left[Z_{1} \sin^{2}\theta - Z_{2} \sin^{2}\theta + Z_{3} \sin\theta \cos\theta \right] q_{r}$$
3.3

Where

$$Z_{1} = \Phi \left[\sum_{k=1}^{3} (\Gamma_{k} S_{k} R S_{k}^{T} \Gamma_{k}^{T}) \right] \Phi^{T}$$

$$Z_{2} = \Phi \left[\sum_{k=1}^{2} \sum_{l=1}^{2} (-1)^{k+1} (\Gamma_{l} S_{k} R S_{k}^{T} \Gamma_{l}^{T}) \right] \Phi^{T}$$

$$Z_{3} = \Phi \left[\sum_{k=1}^{2} (-1)^{k} (\Gamma_{1} S_{k} R S_{k}^{T} \Gamma_{2}^{T} + (\Gamma_{2} S_{k} R S_{k}^{T} \Gamma_{1}^{T}) \sin \theta \cos \theta \right] \Phi^{T}$$

$$Z = Z_{1} + Z_{2} \sin^{2} \theta + Z_{3} \sin \theta \cos \theta$$
3.4

In above form of equation, $S_k = diag[S_{ki}]$ and $\Gamma_k = diag[\gamma_{ki}]$ are $n \times n$ diagonal matrices for k = 1, 2, 3, the $n \times n$ correlation matrix $\mathbf{R} = [\rho_{ki}]$, and the $N \times n$ modal matrix $\phi = [\phi_1, \phi_2, \phi_3,\phi_n]$, θ is as described previously with Fig. 3.1.

Response Matrix (Novel to this Approach)

One can easily verify that **Z** is an $N \times N$ symmetric matrix. For a given structure and set of response spectra, and for known principal directions of the ground motion, **Z** is known and is identical for all response quantities. In the following analysis, we also make use of the cross term for two response quantities $x_r(t)$ and $x_r(t)$, which we define as

$$X_{rs} = q_r^T Z q_s 3.5$$

Collecting 3.5 for all response pairs, we define the m x m "response matrix"

$$X = Q^T Z Q 3.6$$

Individual element of X are the squared peak value of the individual response quantities (i.e., $X_{rr} = r = 1, 2, \ldots, m$). We note that these quantities are exactly those that are computed in the conventional response spectrum method. The off diagonal elements $X_{rs} = q_r^T Z q_s$, $r \neq s$, which are novel to this, approach, are related to the covariance between the responses $x_r(t)$ and $x_s(t)$.

The algebraic sign of X_{rs} indicates whether the responses $x_r(t)$ and $x_s(t)$ are positively or negatively correlated. If $x_r(t)$ and $x_s(t)$ are uncorrelated, $X_{rs}=0$; whereas if $x_r(t)$ is proportional to $x_s(t)$, the two responses are perfectly correlated and $|X_{rs}|=X_rX_s$.

For the bounding hyperplane defined by the unit normal vector α and distance X_{α} from the origin of the response space, and a unit vector β having an acute angle γ with α .

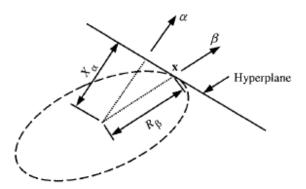


Fig. 3.2 Hyperplane defined by unit normal vector α and x in dir. of unit vector β

A convenient expression for developing the elliptical envelope derived is as follows

$$x = X\alpha/(\alpha^T X\alpha)^{1/2}$$
 3.7

This expression provides the Cartesian coordinates of a point on the envelope corresponding to a selected α . Note that the expression does not require inverting \mathbf{X} . Furthermore, due to the scaling involved, the vector a need not have a unit length.

3.1.2 Two-Component Vector Process

When m=2, a convenient closed-form solution for the eigenvectors and eigenvalues of **X** can be derived and used to define the envelope. Let Ψ denote the counterclockwise angle between the coordinate axes (x_1, x_2) and the principal axes (x'_1, x'_2) of the ellipse, as shown in Fig. 3.3. It follows that

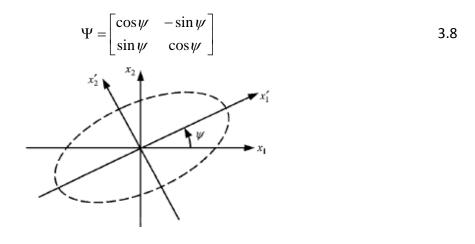


Fig. 3.3 Principal axis of response spectrum based envelop

The principal axes of this ellipsoid coincide with the coordinate system defined by the columns of Ψ , which were identified as the eigenvectors of \mathbf{X} ; that is, the eigenvectors of \mathbf{X} are the principal axes of the envelope. In addition, the length of the r^{th} semiaxis is λ_r , which is the square root of the r^{th} eigenvalue of \mathbf{X} . λ_1 and λ_2 , the lengths of the semiaxes of the ellipse are

$$\lambda_1^2 = X_1^2 \cos^2 \psi + X_2^2 \sin^2 \psi + 2X_{12} \sin \psi \cos \psi$$
 3.9

$$\lambda_2^2 = X_1^2 \sin^2 \psi + X_2^2 \cos^2 \psi - 2X_{12} \sin \psi \cos \psi$$
 3.10

To obtain the angle Ψ , we have to solve the eigenvalue equation

$$\psi = \frac{1}{2} \tan^{-1} \left(\frac{2X_{12}}{X_1^2 - X_2^2} \right)$$
 3.11

Hence, the size and orientation of the elliptical envelope in a 2D response space are completely defined by 3.9, 3.10 and 3.11 in terms of the elements of the response matrix **X**.

$$X = \begin{bmatrix} X_1^2 & X_{12} \\ X_{21} & X_2^2 \end{bmatrix}$$
 3.12

3.1.3 Three-Component Vector Process

When m=3 the solution for the eigenvectors and eigenvalues of X cannot be derived in similar way (with help of Eulerian angles) because we can not associate a vector with a finite rotation represented by an orthogonal (radius preserving) transformation. The direction would be the axis of rotation, and the magnitude would be the angle of rotation, or something related to it. But, since the rotations, in three dimensions, were not commutative, i.e., $Rot_1 \times Rot_2 \times Rot_1$, neither Rot_1 nor Rot_2 can be "true" vectors. On the other hand, for infinitesimal rotations, this association of a rotation with a vector is perfectly plausible. We have to use slightly difficult process of finding elliptical envelop coordinates by considering unit vector α for whole range 0 to 360 (Eq. 3.7).

3.1.4 Envelop for Earthquake Response Vector for any Direction of Earthquake (Supreme Envelop)

The size and orientation of the elliptical envelope are shown functions of the orientation of the principal axes of ground motion θ . In the preceding paragraph, θ was assumed to be known. Normally, however, this information is not available during the design phase of a structure. In such cases, it is prudent to consider all possible values that θ can assume.

That is, the envelope used for design should bound all the elliptical envelopes obtained for $0 \le \theta < 2\pi$.

For a specified direction α in the response space and a given orientation θ of the principal axes of ground motion, the distance X_{α} to the hyperplane defined by

$$X_{\alpha}^{2} = \alpha^{t} X_{S} \alpha$$
 3.13

Where,

$$X_{s} = Q^{t} \left[\left(Z_{1} + \frac{1}{2} Z_{2} \right) - \frac{1}{2} Z_{2} \left(-\frac{\alpha^{T} Q^{T} Z_{2} Q \alpha}{H} \right) + \frac{1}{2} Z_{3} \left(\frac{\alpha^{T} Q^{T} Z_{3} Q \alpha}{H} \right) \right] Q$$

For specified α , the value of θ that maximize X_{α} is found by differentiating the above equation w.r.t. θ . The result is

Where,

$$\cos 2\theta = \left(-\frac{\alpha^T Q^T Z_2 Q \alpha}{H}\right)$$

$$\sin 2\theta = \left(-\frac{\alpha^T Q^T Z_3 Q \alpha}{H}\right)$$

$$H = \left[(\alpha^T Q^T Z_2 Q \alpha)^2 + \alpha^T Q^T Z_3 Q \alpha)^2\right]$$
3.14

Substituting 3.14 into 3.13 yields

$$X_{\alpha} = \alpha^{T} Q^{t} \left[\left(Z_{1} + \frac{1}{2} Z_{2} \right) - \frac{1}{2} Z_{2} \left(-\frac{\alpha^{T} Q^{T} Z_{2} Q \alpha}{H} \right) + \frac{1}{2} Z_{3} \left(\frac{\alpha^{T} Q^{T} Z_{3} Q \alpha}{H} \right) \right] Q \alpha = \alpha^{T} X_{s} \alpha \qquad 3.15$$

The Eq. 3.15 defines the distance to the hyperplane tangent to the supreme envelope with α as its unit normal vector. This expression is analogous to 3.7 shown earlier for the elliptical envelope. However, the matrix **Xs** here is dependent upon α , and, as a result, the supreme envelope is not an ellipsoid. In a manner similar to the derivation that led to Eq. 3.7, one can easily understand that points **x** on the supreme envelope satisfy the relation.

$$x = \frac{X_s \alpha}{(\alpha^T X_s \alpha)^{1/2}}$$
 3.16

One can use this relation to compute points on the supreme envelope by varying α . Each of these points corresponds to a value of θ that can be computed in terms of α Eq. 3.14.

3.2 Martin R. Button, Edward L. Willons Approach

The design criterion is "a structure must resist a major earthquake motion of magnitude S1 for all possible angles θ and at the same point in time resist earthquake motions of magnitude S2 at 90o to the angle θ ."

A Design criterion implies that a large number of different analyses must be conducted in order to determine the maximum design forces. In that approach it is shown that maximum values for all members can be exactly evaluated from one computer run for given seismic loading. Furthermore, these evaluated values are independent of selection of reference axis.

In that approach we have to apply input spectra S1 and S2 in an arbitrary angle θ . In order to simplify the analysis we can assume the minor input spectrum is some fraction of major input spectrum. i.e.

$$S_2 = a S_1$$
 3.17

At the present time, no specific guidelines have been suggested for the value of a. An example with values a=0.85 is presented (Based on the observations reported by Penzien and Watabe (1975), it is assumed that $S_2=0.85S_1$).

The first step in analysis is to evaluate the force f_0 due to S1 applied at an angle 0 and forces f_{90} due to S1 applied at angle 90. This involved standard response spectrum analysis in which the contribution of each mode of vibration is combined by CQC method.

The resulting internal force f_1 due to S_1 applied at an angle θ is as follows,

$$f_1 = \pm f_0 \cos \theta \pm f_{90} \sin \theta \qquad 3.18$$

An additional contribution to the internal forces the internal force due to S_2 = a S_1 at angle (0 +90) is

$$f_2 = \pm a f_0 \cos \theta \pm a f_{90} \sin \theta \qquad 3.19$$

An estimate of total forces is given by

$$F = \sqrt{(f_1^2 + f_2^2)}$$
 3.20

And the angle at which this force will be maximum is given by

$$\tan(2\theta_{cr}) = \frac{2f_0 f_{90} (1 + a^2)}{(f_1^2 - f_2^2)(1 - a^2)}$$
3.21

In this equation sign of both numerator and denominator should be considered. Accordingly all internal forces have different critical angle. Each internal force quantity can have different critical =earthquake input angle. For three dimensional earthquake input where the vertical component of earthquake S3 is considered in addition to S2 and S1, the equation for critical angle is not altered. Since it has been shown that one principal axis of recorded three – dimensional earthquake motions is very close to vertical, only equation for resultant internal force need to be modified by the addition of the term due to S3.

Martin R. Button, Edward L. Willons have shown that estimation of combined stress directly by mean maximum values of axial forces and moments is incorrect. Combined stress for section of column ,

$$f = P/A + M_y z/I_y + M_z y/I_z$$
 3.22

Where, P, M_y and M_z are axial force and moment about x and y axis A, I_y and I_z are cross section and moment of inertias ,respectively.

A correct estimate of stress is obtained only if equation is evaluated for each mode shapes and maximum stress can then be found from combining these by the CQC method of modal combination.

A well-designed structure should be capable of equally resisting earthquake motions from all possible directions. One option in existing design codes for buildings and bridges requires that members be designed for "100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction." Other codes and organizations require the use of 40 percent rather than 30 percent. However, they give no indication on how the directions are to be determined for complex structures. For structures that are rectangular and have clearly defined principal directions, these "percentage" rules yield approximately the same results as the SRSS method.

For complex three-dimensional structures, such as non-rectangular buildings, curved bridges, arch dams or piping systems, the direction of the earthquake that produces the maximum stresses in a particular member or at a specified point is not apparent. For time history input, it is possible to perform a large number of dynamic analyses at various angles of input to check all points for the critical earthquake directions. Such an elaborate study could conceivably produce a different critical input direction for each stress evaluated. However, the cost of such a study would be prohibitive.

It is reasonable to assume that motions that take place during an earthquake have one principal direction. Or, during a finite period of time when maximum ground acceleration occurs, a principal direction exists. For most structures, this direction is not known and for most geographical locations cannot be estimated. Therefore, the only rational earthquake design criterion is that the structure must resist an earthquake of a given magnitude from any possible direction. In addition to the motion in the principal direction, a probability exists that motion

normal to that direction will occur simultaneously. In addition, because of the complex nature of three-dimensional wave propagation, it is valid to assume that these normal motions are statistically independent.

Chapter 4

NUMERICAL STUDY 1

This Numerical Study is aim to Compare Elliptical Envelop with Time – History Analysis results. In this exercise 4 - storey building frame has been solved in SAP2000 v 11(Advanaced), the modeling is similar to the model used by C. Menun and A. Der Kiureghian in their demonstration of this approach. The model and static loading is shown in Fig. 4.1 and 4.2.

The lateral force resisting system of this structure is consisting of three moment frames. To approximate stiffness of floor diaphragm each panel of floor is modeled by two diagonal beam elements.

In this problem Modal analysis has been performed in SAP to have Natural frequency, mode shapes and modal participation factors. SAP2000 v 11 (Advanced) also give the no. of equation it is using for calculation in .TXE file. These data are arranged in compatible form to derive elliptical envelop in Excel.

In tables 4.1 to 4.5 data which are exported and calculated from SAP to derive envelop are shown. Table 4.1 showing dynamic properties of structure which are basic characteristics of any structure. One notes the closeness of the frequencies that is typical of most three dimensional building structures that are designed to resist earthquakes from both directions. Displacement response spectra ordinate and Acceleration response spectra ordinate corresponding to time periods of different modes shown in table 4.2. Displacement response spectra ordinate are require to derive envelop. However, we can input only acceleration response spectra in SAP. In Table 4.3 modal participation factor in X and Y direction are shown these are defined w.r.t. structure axis. In table 4.4 mode shapes are shown, these are results of Modal Analysis in SAP, it normalizes modes so that modal mass have unit value. We have arranged mode shape in order of increasing D.O.F. no. In Table 4.5 Correlation Coefficient matrix for Complete Quadratic Combination rule are shown which is used in deriving elliptical envelop. We calculated it from frequency ratios available to us, we can get it verified it by .OUT SAP file of model but it will give us only first 10 correlation coeifficient.

The Response Spectrum Analysis is performed to verify the diagonal terms of Response Matrix (Novel to this approach) which are nothing but peak values of responses given by Response Spectrum Method. The values are shown in table 4.6. For this analysis the Actual Response Spectra of Northridge Earthquake (Jan. 17, 1994) (Major Principal Component and Intermediate Principal Component) are applied in U1 and U2 direction with θ = 0. By default these correspond to X and Y direction respectively. We have included 12 mode shapes so that for both the horizontal component of ground motion 99% participating mass was included.

Simultaneously, Linear Modal Time – History Analysis of this model has been performed, for two horizontal principle ground motion components of Northridge earthquake (Jan. 17, 1994). A plot between P-M (2-2) and P-M (3-3) exported in Excel format. The values of Axial force and moments which occurs simultaneously have been plotted to compare with Elliptical envelop. An elliptical envelop derived through this new approach, which are enclosing values of axial force and moment occurring simultaneously are shown in figure 4.3 and Fig. 4.4.

Up to above derivations of the elliptical envelopes, only time-varying responses arising from seismic excitation of the structure were considered. In general, however, static loads are present that cause time-invariant components x_o to act concurrently with dynamic responses $\mathbf{x}(t)$. Because the structure is linear, $\mathbf{x}(t)$ and x_o can be added to yield a total response vector that varies in time about point x_o in the response space. The size and orientation of the two envelopes are unaffected by the presence of these static responses. The resulting elliptical envelop is shown in Fig. 4.5 and Fig. 4.6.

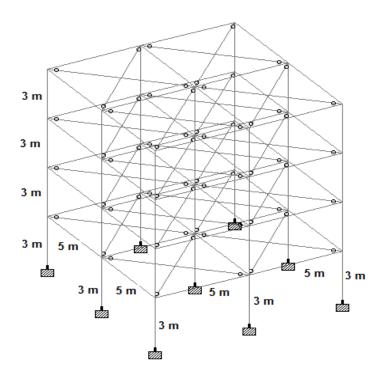


Fig 4.1 Geometry & Model of Structure used in analysis

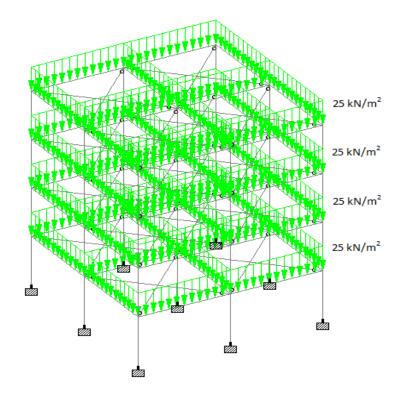


Fig 4.2 Loading details on Structure Model

Modal Analysis Results

Table 4.1 Modal Periods, Frequency, Participating Mass Ratio and Modal mass and stiffness

Mode	Period	Frequency	Damping	Sum of Modal Mass Participation		ModalMass	ModalStiff
Shape	Sec	Cyc/sec	Ratio	in U1 dir.%	in U2 dir.%	KN-m-s2	KN-m
1	0.864628	1.15657	0.05	0.84809	0.00021	1	52.80823307
2	0.864628	1.15657	0.05	0.84830	0.84830	1	52.80823307
3	0.759648	1.31640	0.05	0.84830	0.84830	1	68.41242987
4	0.27698	3.61036	0.05	0.94931	0.85267	1	514.5900729
5	0.27698	3.61036	0.05	0.95369	0.95369	1	514.5900729
6	0.246032	4.06452	0.05	0.95369	0.95369	1	652.1957037
7	1.59E-01	6.29296	0.05	0.97580	0.96772	1	1563.400425
8	0.158908	6.29296	0.05	0.98984	0.98984	1	1563.400425
9	1.44E-01	6.96056	0.05	0.98984	0.98984	1	1912.703891
10	1.16E-01	8.63096	0.05	0.99998	0.98985	1	2940.8854
11	1.16E-01	8.63096	0.05	0.99999	0.99999	1	2940.8854
12	1.06E-01	9.44379	0.05	0.99999	0.99999	1	3520.88947

Table 4.2 Response Spectra Ordinates

Modes	Acceleration	n Response	Displaceme	ent Response		
	(SAP	Uses)	(To Derive Envelop)			
	m/sec2	m/sec2	m	m		
1	6.39079547	6.937911266	0.12101892	0.131379349		
2	6.39079547	6.937911266	0.12101892	0.131379349		
3	7.17881974	8.086061909	0.10493444	0.1181958		
4	13.9173705	10.24650723	0.02704555	0.01991198		
5	13.9173705	10.24650723	0.02704555	0.01991198		
6	12.7442713	10.47682395	0.01954056	0.016063927		
7	9.77578201	8.544269976	0.0062529	0.005465183		
8	9.77578201	8.544269976	0.0062529	0.005465183		
9	9.32192174	7.700440611	0.00487369	0.004025945		
10	8.49392625	6.161008841	0.00288822	0.00209495		
11	8.49392625	6.161008841	0.00288822	0.00209495		
12	8.19696281	5.608886333	0.00232809	0.001593031		

Table 4.3 Modal Participation Factors

Modes	Period	UX	UY
	Sec	KN-s2	KN-s2
1	0.864628	-32.7045	-0.5179
2	0.864628	-0.5179	32.7045
3	0.759648	0.0000	0.0000
4	0.27698	-11.2870	-2.3488
5	0.27698	-2.3488	11.2870
6	0.246032	0.0000	0.0000
7	1.59E-01	-5.2812	4.2073
8	0.158908	4.2073	5.2812
9	1.44E-01	0.0000	0.0000
10	1.16E-01	-3.5761	0.1141
11	1.16E-01	-0.1141	-3.5761
12	1.06E-01	0.0000	0.0000

Table 4.4 Mode shape Output from SAP (Mass orthogonal sets

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
1	MODAL	LinModal	1	0	0	0	0	0	0
1	MODAL	LinModal	2	0	0	0	0	0	0
1	MODAL	LinModal	3	0	0	0	0	0	0
1	MODAL	LinModal	4	0	0	0	0	0	0
1	MODAL	LinModal	5	0	0	0	0	0	0
1	MODAL	LinModal	6	0	0	0	0	0	0
1	MODAL	LinModal	7	0	0	0	0	0	0
1	MODAL	LinModal	8	0	0	0	0	0	0
1	MODAL	LinModal	9	0	0	0	0	0	0
1	MODAL	LinModal	10	0	0	0	0	0	0
1	MODAL	LinModal	11	0	0	0	0	0	0
1	MODAL	LinModal	12	0	0	0	0	0	0
2	MODAL	LinModal	1	9.83E-03	1.60E-04	2.52E-04	-4.76E-05	2.98E-03	-1.50E-06
2	MODAL	LinModal	2	1.52E-04	-9.83E-03	-2.44E-04	2.98E-03	4.68E-05	-1.55E-06
2	MODAL	LinModal	3	9.52E-03	-9.52E-03	5.67E-15	2.68E-03	2.68E-03	1.94E-03
2	MODAL	LinModal	4	2.70E-02	5.67E-03	-4.55E-04	-1.18E-03	5.61E-03	-1.92E-05
2	MODAL	LinModal	5	0.00558063	-2.70E-02	2.98E-04	5.60E-03	1.16E-03	-2.93E-05
2	MODAL	LinModal	6	-2.63E-02	2.63E-02	-4.74E-12	-5.04E-03	0.00504366	-4.93E-03
2	MODAL	LinModal	7	2.90E-02	-2.31E-02	2.77E-05	7.44E-04	9.34E-04	-1.28E-04
2	MODAL	LinModal	8	-2.34E-02	-2.92E-02	-2.44E-04	9.37E-04	-7.48E-04	-1.45E-05
2	MODAL	LinModal	9	-0.0349813	3.50E-02	-2.64E-10	-8.13E-04	-8.13E-04	-5.94E-03
2	MODAL	LinModal	10	2.88E-02	-6.27E-04	-3.83E-04	-1.26E-04	-5.31E-03	-1.31E-04
2	MODAL	LinModal	11	1.21E-03	2.88E-02	-4.08E-04	5.32E-03	-2.13E-04	1.22E-04
2	MODAL	LinModal	12	-2.69E-02	2.69E-02	-7.93E-09	4.74E-03	4.74E-03	-4.24E-03
3	MODAL	LinModal	1	2.27E-02	3.63E-04	4.14E-04	-4.30E-05	2.72E-03	-1.99E-06
3	MODAL	LinModal	2	0.00035716	-2.27E-02	-4.01E-04	2.72E-03	4.32E-05	-2.06E-06
3	MODAL	LinModal	3	2.16E-02	-2.16E-02	-1.42E-15	2.38E-03	2.38E-03	4.27E-03
3	MODAL	LinModal	4	3.50E-02	0.0073251	-1.10E-03	6.16E-04	-2.95E-03	-2.23E-05
3	MODAL	LinModal	5	7.23E-03	-3.50E-02	7.23E-04	-2.95E-03	-6.10E-04	-3.41E-05
3	MODAL	LinModal	6	-3.32E-02	3.32E-02	-4.05E-12	2.79E-03	2.79E-03	-6.08E-03
3	MODAL	LinModal	7	-2.99E-03	2.38E-03	5.13E-05	-7.05E-03	-8.87E-03	1.16E-05
3	MODAL	LinModal	8	2.41E-03	3.02E-03	-4.53E-04	-8.95E-03	7.15E-03	1.31E-06
3	MODAL	LinModal	9	4.52E-03	-4.52E-03	6.08E-10	1.00E-02	1.00E-02	7.27E-04
3	MODAL	LinModal	10	-3.57E-02	7.92E-04	-5.97E-04	-1.82E-05	-1.29E-04	1.53E-04
3	MODAL	LinModal	11	-1.49E-03	-3.58E-02	-6.37E-04	1.28E-04	9.98E-06	-1.44E-04
3	MODAL	LinModal	12	3.38E-02	-3.38E-02	-3.20E-09	-6.63E-05	-6.63E-05	5.21E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
4	MODAL	LinModal	1	3.29E-02	5.26E-04	4.94E-04	-2.97E-05	1.86E-03	-2.90E-06
4	MODAL	LinModal	2	5.17E-04	-3.29E-02	-4.79E-04	1.86E-03	2.93E-05	-3.00E-06
4	MODAL	LinModal	3	3.09E-02	-3.09E-02	-2.36E-14	1.58E-03	1.58E-03	6.10E-03
4	MODAL	LinModal	4	4.58E-03	9.56E-04	-1.64E-03	1.82E-03	-8.70E-03	-2.32E-06
4	MODAL	LinModal	5	9.51E-04	-4.58E-03	1.08E-03	-8.69E-03	-1.80E-03	-3.53E-06
4	MODAL	LinModal	6	-3.55E-03	3.55E-03	-3.08E-12	7.85E-03	7.85E-03	-6.33E-04
4	MODAL	LinModal	7	-2.68E-02	2.13E-02	1.26E-04	2.44E-03	3.07E-03	1.08E-04
4	MODAL	LinModal	8	2.15E-02	2.70E-02	-1.11E-03	3.08E-03	-2.46E-03	1.22E-05
4	MODAL	LinModal	9	3.21E-02	-3.21E-02	-8.69E-10	-3.68E-03	-3.68E-03	5.29E-03
4	MODAL	LinModal	10	2.83E-02	-6.35E-04	-7.85E-04	9.41E-05	5.23E-03	-1.20E-04
4	MODAL	LinModal	11	1.17E-03	2.84E-02	-8.37E-04	-5.23E-03	2.40E-04	1.13E-04
4	MODAL	LinModal	12	-2.72E-02	2.72E-02	7.03E-10	-4.49E-03	-4.49E-03	-4.18E-03
5	MODAL	LinModal	1	3.87E-02	6.18E-04	5.18E-04	-1.42E-05	9.18E-04	-3.23E-06
5	MODAL	LinModal	2	6.09E-04	-3.87E-02	-5.02E-04	9.18E-04	1.49E-05	-3.33E-06
5	MODAL	LinModal	3	3.60E-02	-3.60E-02	-4.78E-14	7.05E-04	7.05E-04	7.11E-03
5	MODAL	LinModal	4	-3.26E-02	-6.81E-03	-1.86E-03	1.32E-03	-6.36E-03	2.02E-05
5	MODAL	LinModal	5	-6.73E-03	3.25E-02	1.22E-03	-6.35E-03	-1.32E-03	3.08E-05
5	MODAL	LinModal	6	3.13E-02	-3.13E-02	7.07E-12	5.39E-03	5.39E-03	5.73E-03
5	MODAL	LinModal	7	1.78E-02	0.01417827	1.79E-04	7.34E-03	9.23E-03	-7.20E-05
5	MODAL	LinModal	8	-1.43E-02	-1.80E-02	-1.58E-03	9.28E-03	-7.40E-03	-8.15E-06
5	MODAL	LinModal	9	-2.20E-02	0.02198138	-3.19E-10	-1.01E-02	-1.01E-02	-3.65E-03
5	MODAL	LinModal	10	-1.08E-02	2.47E-04	-1.07E-03	-2.71E-04	0.01000877	4.54E-05
5	MODAL	LinModal	11	-4.45E-04	-1.08E-02	-1.14E-03	1.00E-02	-3.68E-04	-4.26E-05
5	MODAL	LinModal	12	1.07E-02	-1.07E-02	3.94E-09	8.58E-03	8.58E-03	1.65E-03
6	MODAL	LinModal	1	0	0	0	0	0	0
6	MODAL	LinModal	2	0	0	0	0	0	0
6	MODAL	LinModal	3	0	0	0	0	0	0
6	MODAL	LinModal	4	0	0	0	0	0	0
6	MODAL	LinModal	5	0	0	0	0	0	0
6	MODAL	LinModal	6	0	0	0	0	0	0
6	MODAL	LinModal	7	0	0	0	0	0	0
6	MODAL	LinModal	8	0	0	0	0	0	0
6	MODAL	LinModal	9	0	0	0	0	0	0
6	MODAL	LinModal	10	0	0	0	0	0	0
6	MODAL	LinModal	11	0	0	0	0	0	0
6	MODAL	LinModal	12	0	0	0	0	0	0

7 MODAL LinModal 1 9.84E-03 1.56E-04 2.48E-04 -2.90E-05 2.98E-03 1.15E-08 7 MODAL LinModal 2 1.56E-04 -9.84E-03 3.93E-06 1.83E-03 4.72E-05 7.29E-07 7 MODAL LinModal 4 2.72E-02 5.63E-03 1.09E-14 1.68E-03 -1.26E-05 7 MODAL LinModal 5 5.66E-03 -2.71E-02 8.20E-05 2.99E-03 1.17E-03 -1.26E-05 7 MODAL LinModal 6 -3.96E-11 2.63E-02 2.34E-01 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 6 -3.96E-11 2.63E-02 2.34E-04 -1.29E-05 -9.48E-04 -3.25E-05 7 MODAL LinModal 9 -4.90E-09 3.52E-02 1.36E-09 8.44E-05 2.53E-09 -6.47E-03 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -1.65E-05 3.11E-04 -7.27E-										
7 MODAL LinModal 3 4.78E-13 -9.53E-03 1.09E-14 1.68E-03 -1.82E-14 1.92E-03 7 MODAL LinModal 4 2.72E-02 5.63E-03 3-94E-04 6.23E-04 5.63E-03 2.63E-06 7 MODAL LinModal 5 5.66E-03 -2.71E-02 -8.20E-05 2.99E-03 1.17E-03 -1.26E-05 7 MODAL LinModal 6 -3.96E-11 2.63E-02 2.34E-11 -2.83E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 8 -2.36E-02 -2.93E-02 -1.31E-04 -1.62E-05 -7.55E-04 -4.08E-05 7 MODAL LinModal 10 2.98E-02 -3.31E-04 -1.62E-05 -7.55E-04 -4.08E-05 7 MODAL LinModal 10 2.98E-02 -3.1E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 12 2.51E-08 0.02724617 -2.49E-08 2.84E-03 -7.92E-09	7	MODAL	LinModal	1	9.84E-03	1.56E-04	2.48E-04	-2.90E-05	2.98E-03	1.15E-08
7 MODAL LinModal 4 2.72E-02 5.63E-03 -3.94E-04 -6.23E-04 5.63E-03 2.63E-06 7 MODAL LinModal 5 5.66E-03 -2.71E-02 -8.20E-05 2.99E-03 1.17E-03 -1.26E-05 7 MODAL LinModal 6 -3.96E-02 2.23E-02 2.38E-04 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 8 -2.36E-02 -2.93E-02 1.31E-04 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -1.92E-05 9.5E-04 -4.08E-05 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 -7.0E-05 8 MODAL LinModal 1 2.27E-03 3.60E-04 4.09E-08 <th< td=""><td>7</td><td>MODAL</td><td>LinModal</td><td>2</td><td>1.56E-04</td><td>-9.84E-03</td><td>3.93E-06</td><td>1.83E-03</td><td>4.72E-05</td><td>-7.29E-07</td></th<>	7	MODAL	LinModal	2	1.56E-04	-9.84E-03	3.93E-06	1.83E-03	4.72E-05	-7.29E-07
7 MODAL LinModal 5 5.66E-03 -2.71E-02 -8.20E-05 2.99E-03 1.17E-03 -1.26E-05 7 MODAL LinModal 6 -3.96E-01 2.63E-02 2.34E-11 -2.83E-03 -1.72E-13 -5.12E-03 7 MODAL LinModal 7 2.96E-02 -2.33E-02 1.65E-04 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 8 -2.36E-02 -2.93E-02 -1.31E-04 -1.62E-05 -7.55E-04 -4.08E-05 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 7.70E-05 8 MODAL LinModal 1 2.27E-02 -1.65E-05 3.11E-03 -1.75E-04 7.70E-05 8 MODAL LinModal 1 2.27E-02 6.48E-06 1.81E-03 4.31E-05 <	7	MODAL	LinModal	3	4.78E-13	-9.53E-03	1.09E-14	1.68E-03	-1.82E-14	1.92E-03
7 MODAL LinModal 6 -3.96E-11 2.63E-02 2.34E-11 -2.83E-03 -1.72E-13 -5.12E-03 7 MODAL LinModal 7 2.96E-02 -2.33E-02 1.65E-04 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 8 -2.36E-02 -2.93E-02 1.36E-09 4.49E-05 -7.55E-04 -4.08E-05 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 -7.70E-05 7 MODAL LinModal 11 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 5.46E-15 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 3.60E-04 -2.27E-02 5.46E-15 1.59E-03 <td< td=""><td>7</td><td>MODAL</td><td>LinModal</td><td>4</td><td>2.72E-02</td><td>5.63E-03</td><td>-3.94E-04</td><td>-6.23E-04</td><td>5.63E-03</td><td>2.63E-06</td></td<>	7	MODAL	LinModal	4	2.72E-02	5.63E-03	-3.94E-04	-6.23E-04	5.63E-03	2.63E-06
7 MODAL LinModal 7 2.96E-02 -2.38E-02 1.65E-04 -1.29E-05 9.48E-04 -3.25E-05 7 MODAL LinModal 8 -2.36E-02 -2.93E-02 -1.31E-04 -1.62E-05 -7.55E-04 -4.08E-05 7 MODAL LinModal 10 -2.98E-02 -9.31E-04 -5.18E-09 9.44E-05 5.547E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 -7.02E-05 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 -7.92E-09 -4.78E-03 8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 9.81E-07 8 MODAL LinModal 3 -2.50E-02 2.78E-03 <	7	MODAL	LinModal	5	5.66E-03	-2.71E-02	-8.20E-05	2.99E-03	1.17E-03	-1.26E-05
7 MODAL LinModal 8 -2.36E-02 -2.93E-02 -1.31E-04 -1.62E-05 -7.55E-04 -4.08E-03 7 MODAL LinModal 9 -4.90E-09 3.52E-02 1.36E-09 8.44E-05 2.53E-09 -6.47E-03 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 -7.0E-05 7 MODAL LinModal 12 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 4.22Fe-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 <td< td=""><td>7</td><td>MODAL</td><td>LinModal</td><td>6</td><td>-3.96E-11</td><td>2.63E-02</td><td>2.34E-11</td><td>-2.83E-03</td><td>-1.72E-13</td><td>-5.12E-03</td></td<>	7	MODAL	LinModal	6	-3.96E-11	2.63E-02	2.34E-11	-2.83E-03	-1.72E-13	-5.12E-03
7 MODAL LinModal 9 -4,90E-09 3.52E-02 1.36E-09 8.44E-05 2.53E-09 -6.47E-03 7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 7.70E-05 7 MODAL LinModal 12 2.51E-08 0.02724617 -2.49E-08 2.84E-03 -7.92E-09 -4.78E-03 8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-03 2.72E-03 1.55E-08 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1	7	MODAL	LinModal	7	2.96E-02	-2.33E-02	1.65E-04	-1.29E-05	9.48E-04	-3.25E-05
7 MODAL LinModal 10 2.98E-02 -9.31E-04 -5.18E-04 -9.93E-05 -5.47E-03 -2.46E-06 7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 7.70E-05 7 MODAL LinModal 12 2.51E-08 0.02724617 -2.49E-08 2.84E-03 -7.92E-09 -4.78E-03 8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.4E-04 3.79E-04 -1.82E-03 3.14E-06 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.	7	MODAL	LinModal	8	-2.36E-02	-2.93E-02	-1.31E-04	-1.62E-05	-7.55E-04	-4.08E-05
7 MODAL LinModal 11 9.51E-04 2.92E-02 -1.65E-05 3.11E-03 -1.75E-04 7.70E-05 7 MODAL LinModal 12 2.51E-08 0.02724617 -2.49E-08 2.84E-03 -7.92E-09 -4.78E-03 8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal	7	MODAL	LinModal	9	-4.90E-09	3.52E-02	1.36E-09	8.44E-05	2.53E-09	-6.47E-03
7 MODAL LinModal 12 2.51E-08 0.02724617 -2.49E-08 2.84E-03 -7.92E-09 -4.78E-03 8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 -6.37E-04 -1.51E-05 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.7	7	MODAL	LinModal	10	2.98E-02	-9.31E-04	-5.18E-04	-9.93E-05	-5.47E-03	-2.46E-06
8 MODAL LinModal 1 2.27E-02 3.60E-04 4.09E-04 -2.87E-05 2.72E-03 1.55E-08 8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-0	7	MODAL	LinModal	11	9.51E-04	2.92E-02	-1.65E-05	3.11E-03	-1.75E-04	7.70E-05
8 MODAL LinModal 2 3.60E-04 -2.27E-02 6.48E-06 1.81E-03 4.31E-05 -9.81E-07 8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 7.20E-03 4.14E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.4	7	MODAL	LinModal	12	2.51E-08	0.02724617	-2.49E-08	2.84E-03	-7.92E-09	-4.78E-03
8 MODAL LinModal 3 -8.01E-14 -2.16E-02 5.46E-15 1.59E-03 -1.69E-13 4.29E-03 8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 <t< td=""><td>8</td><td>MODAL</td><td>LinModal</td><td>1</td><td>2.27E-02</td><td>3.60E-04</td><td>4.09E-04</td><td>-2.87E-05</td><td>2.72E-03</td><td>1.55E-08</td></t<>	8	MODAL	LinModal	1	2.27E-02	3.60E-04	4.09E-04	-2.87E-05	2.72E-03	1.55E-08
8 MODAL LinModal 4 3.52E-02 7.28E-03 -9.46E-04 3.79E-04 -2.96E-03 3.14E-06 8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal <td>8</td> <td>MODAL</td> <td>LinModal</td> <td>2</td> <td>3.60E-04</td> <td>-2.27E-02</td> <td>6.48E-06</td> <td>1.81E-03</td> <td>4.31E-05</td> <td>-9.81E-07</td>	8	MODAL	LinModal	2	3.60E-04	-2.27E-02	6.48E-06	1.81E-03	4.31E-05	-9.81E-07
8 MODAL LinModal 5 7.32E-03 -3.50E-02 -1.97E-04 -1.82E-03 -6.17E-04 -1.51E-05 8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08	8	MODAL	LinModal	3	-8.01E-14	-2.16E-02	5.46E-15	1.59E-03	-1.69E-13	4.29E-03
8 MODAL LinModal 6 5.24E-12 3.32E-02 2.07E-11 1.72E-03 2.03E-11 -6.38E-03 8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 <td< td=""><td>8</td><td>MODAL</td><td>LinModal</td><td>4</td><td>3.52E-02</td><td>7.28E-03</td><td>-9.46E-04</td><td>3.79E-04</td><td>-2.96E-03</td><td>3.14E-06</td></td<>	8	MODAL	LinModal	4	3.52E-02	7.28E-03	-9.46E-04	3.79E-04	-2.96E-03	3.14E-06
8 MODAL LinModal 7 -3.06E-03 2.41E-03 3.08E-04 -3.80E-03 -9.04E-03 3.30E-06 8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 <t< td=""><td>8</td><td>MODAL</td><td>LinModal</td><td>5</td><td>7.32E-03</td><td>-3.50E-02</td><td>-1.97E-04</td><td>-1.82E-03</td><td>-6.17E-04</td><td>-1.51E-05</td></t<>	8	MODAL	LinModal	5	7.32E-03	-3.50E-02	-1.97E-04	-1.82E-03	-6.17E-04	-1.51E-05
8 MODAL LinModal 8 2.43E-03 3.02E-03 -2.45E-04 -4.78E-03 7.20E-03 4.14E-06 8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15	8	MODAL	LinModal	6	5.24E-12	3.32E-02	2.07E-11	1.72E-03	2.03E-11	-6.38E-03
8 MODAL LinModal 9 1.88E-08 -4.55E-03 9.23E-10 5.65E-03 1.03E-09 8.18E-04 8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03	8	MODAL	LinModal	7	-3.06E-03	2.41E-03	3.08E-04	-3.80E-03	-9.04E-03	3.30E-06
8 MODAL LinModal 10 -3.69E-02 1.15E-03 -8.45E-04 -1.46E-07 -1.45E-04 2.97E-06 8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04	8	MODAL	LinModal	8	2.43E-03	3.02E-03	-2.45E-04	-4.78E-03	7.20E-03	4.14E-06
8 MODAL LinModal 11 -1.18E-03 0.03614074 -2.70E-05 4.59E-06 -4.63E-06 -9.31E-05 8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11	8	MODAL	LinModal	9	1.88E-08	-4.55E-03	9.23E-10	5.65E-03	1.03E-09	8.18E-04
8 MODAL LinModal 12 -3.54E-08 -3.42E-02 -1.68E-08 -1.05E-04 -1.10E-08 5.93E-03 9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal	8	MODAL	LinModal	10	-3.69E-02	1.15E-03	-8.45E-04	-1.46E-07	-1.45E-04	2.97E-06
9 MODAL LinModal 1 3.29E-02 5.21E-04 4.88E-04 -2.01E-05 1.86E-03 2.31E-08 9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal	8	MODAL	LinModal	11	-1.18E-03	0.03614074	-2.70E-05	4.59E-06	-4.63E-06	-9.31E-05
9 MODAL LinModal 2 5.21E-04 -3.29E-02 7.73E-06 1.27E-03 2.95E-05 -1.46E-06 9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2	8	MODAL	LinModal	12	-3.54E-08	-3.42E-02	-1.68E-08	-1.05E-04	-1.10E-08	5.93E-03
9 MODAL LinModal 3 -4.24E-13 0.03086878 -7.58E-15 1.07E-03 -1.48E-14 6.14E-03 9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 <td< td=""><td>9</td><td>MODAL</td><td>LinModal</td><td>1</td><td>3.29E-02</td><td>5.21E-04</td><td>4.88E-04</td><td>-2.01E-05</td><td>1.86E-03</td><td>2.31E-08</td></td<>	9	MODAL	LinModal	1	3.29E-02	5.21E-04	4.88E-04	-2.01E-05	1.86E-03	2.31E-08
9 MODAL LinModal 4 4.60E-03 9.51E-04 -1.41E-03 1.17E-03 -8.74E-03 3.33E-07 9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	2	5.21E-04	-3.29E-02	7.73E-06	1.27E-03	2.95E-05	-1.46E-06
9 MODAL LinModal 5 9.58E-04 -4.57E-03 -2.93E-04 -5.61E-03 -1.82E-03 -1.60E-06 9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	3	-4.24E-13	0.03086878	-7.58E-15	1.07E-03	-1.48E-14	6.14E-03
9 MODAL LinModal 6 1.19E-10 3.54E-03 -1.01E-11 5.08E-03 1.23E-11 -6.71E-04 9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	4	4.60E-03	9.51E-04	-1.41E-03	1.17E-03	-8.74E-03	3.33E-07
9 MODAL LinModal 7 -2.73E-02 2.15E-02 6.98E-04 1.56E-03 3.13E-03 2.83E-05 9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	5	9.58E-04	-4.57E-03	-2.93E-04	-5.61E-03	-1.82E-03	-1.60E-06
9 MODAL LinModal 8 2.18E-02 2.70E-02 -5.56E-04 1.95E-03 0.00249124 3.56E-05 9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	6	1.19E-10	3.54E-03	-1.01E-11	5.08E-03	1.23E-11	-6.71E-04
9 MODAL LinModal 9 2.46E-09 -3.22E-02 -1.97E-09 -2.31E-03 -3.62E-09 5.84E-03 9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	7	-2.73E-02	2.15E-02	6.98E-04	1.56E-03	3.13E-03	2.83E-05
9 MODAL LinModal 10 2.93E-02 -9.13E-04 -1.12E-03 6.53E-05 5.35E-03 -2.32E-06 9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	8	2.18E-02	2.70E-02	-5.56E-04	1.95E-03	0.00249124	3.56E-05
9 MODAL LinModal 11 9.34E-04 2.86E-02 -3.57E-05 -2.05E-03 1.71E-04 7.28E-05	9	MODAL	LinModal	9	2.46E-09	-3.22E-02	-1.97E-09	-2.31E-03	-3.62E-09	5.84E-03
	9	MODAL	LinModal	10	2.93E-02	-9.13E-04	-1.12E-03	6.53E-05	5.35E-03	-2.32E-06
9 MODAL LinModal 12 -2.08E-08 2.75E-02 7.56E-11 -2.01E-03 7.61E-09 0.0047567	9	MODAL	LinModal	11	9.34E-04	2.86E-02	-3.57E-05	-2.05E-03	1.71E-04	7.28E-05
	9	MODAL	LinModal	12	-2.08E-08	2.75E-02	7.56E-11	-2.01E-03	7.61E-09	0.0047567

10 MODAL LinModal 1 3.88E-02 6.14E-04 5.12E-04 8.50E-06 9.19E-04 2.58E-08 10 MODAL LinModal 3 2.56E-13 3.36E-02 1.80E-14 3.98E-04 1.45E-05 1.63E-06 10 MODAL LinModal 4 -3.27E-02 -6.77E-03 -1.59E-03 6.84E-04 -6.39E-03 -2.86E-06 10 MODAL LinModal 5 -6.81E-03 3.26E-02 -3.31E-04 3.29E-03 -1.33E-03 1.37E-05 10 MODAL LinModal 6 1.23E-10 -3.14E-02 -1.36E-11 2.85E-03 -2.45E-12 6.02E-03 10 MODAL LinModal 7 1.82E-02 -1.43E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 4.71E-03 9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-05 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 -1.16E-08 8.78E-03 11 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 -1.16E-08 8.78E-03 11 MODAL LinModal 12 0.01E-03 0.00 0.00 0.00 0.00 11 MODAL LinModal 2 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 3 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 4 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 5 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 5 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 7 0.00 0.00 0.00 0.00 0.00 11 MODAL LinModal 12 0.00 0.00 0.00 0.00 0.00 12 MODAL LinModal 13 0.95E-03 1.52E-04 2.48E-03 2.98E-03 1.55E-05 1.50E-06 12 MODAL LinModal 14 0.00 0.00 0.00 0.00 0.00 13 MODAL LinModal 14 0.00 0.00 0.00 0.00 0.00 0.00 14 MODAL LinModal 15 0.50E-03 0.52E-03 0.52E-03 0.52E-03 0.52E-03 0.52E-0		1			1					
10 MODAL LinModal 3 -2.56E-13 -3.60E-02 -1.80E-14 3.98E-04 1.06E-13 7.16E-03 10 MODAL LinModal 4 -3.27E-02 -6.77E-03 -1.59E-03 6.84E-04 -6.39E-03 -2.86E-06 10 MODAL LinModal 5 -6.81E-03 3.26E-02 -3.31E-04 -3.29E-03 -1.33E-03 1.37E-05 10 MODAL LinModal 6 1.23E-10 -3.14E-02 -1.36E-11 2.85E-03 -2.45E-12 6.02E-03 10 MODAL LinModal 7 1.82E-02 -1.43E-02 9.74E-04 3.20E-03 9.39E-03 -1.88E-05 10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.66E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 -1.14E-04 -1.03E-02 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 -1.16E-08 1.87E-03 11 MODAL LinModal 2 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	1	3.88E-02	6.14E-04	5.12E-04	-8.50E-06	9.19E-04	2.58E-08
10 MODAL LinModal 4 -3.27E-02 -6.77E-03 -1.59E-03 6.84E-04 -6.39E-03 -2.86E-06 10 MODAL LinModal 5 -6.81E-03 3.26E-02 -3.31E-04 -3.29E-03 -1.33E-03 1.37E-05 10 MODAL LinModal 6 1.23E-10 -3.14E-02 -1.36E-11 2.85E-03 -2.45E-12 6.0ZE-03 10 MODAL LinModal 7 1.82E-02 -1.43E-02 9.74E-04 3.20E-03 9.39E-03 -1.88E-05 10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.0ZE-03 9.39E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 -9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.09E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 2 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	2	6.14E-04	-3.87E-02	8.11E-06	5.37E-04	1.45E-05	-1.63E-06
10 MODAL LinModal 5 -6.81E-03 3.26E-02 -3.31E-04 -3.29E-03 -1.33E-03 1.37E-05 10 MODAL LinModal 6 1.23E-10 -3.14E-02 -1.36E-11 2.85E-03 -2.45E-12 6.02E-03 10 MODAL LinModal 7 1.82E-02 -1.43E-02 9.74E-04 3.20E-03 9.39E-03 1.88E-05 10 MODAL LinModal 8 -1.45E-02 -1.43E-02 9.74E-04 4.02E-03 9.39E-03 1.88E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 4.71E-03 9.79E-10 4.01E-03 1.00E-05 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 4.66E-05 3.59E-03 -3.28E-04 2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 12 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	3	-2.56E-13	-3.60E-02	-1.80E-14	3.98E-04	1.06E-13	7.16E-03
10 MODAL LinModal 6 1.23E-10 -3.14E-02 -1.36E-11 2.85E-03 -2.45E-12 6.02E-03 10 MODAL LinModal 7 1.82E-02 -1.43E-02 9.74E-04 3.20E-03 9.39E-03 -1.88E-05 10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 -9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 2.34E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 2 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	4	-3.27E-02	-6.77E-03	-1.59E-03	6.84E-04	-6.39E-03	-2.86E-06
10 MODAL LinModal 7 1.82E-02 -1.43E-02 9.74E-04 3.20E-03 9.39E-03 -1.88E-05 10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 -9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 3.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 12 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	5	-6.81E-03	3.26E-02	-3.31E-04	-3.29E-03	-1.33E-03	1.37E-05
10 MODAL LinModal 8 -1.45E-02 -1.79E-02 -7.76E-04 4.02E-03 -7.48E-03 -2.36E-05 10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 -9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 12 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	6	1.23E-10	-3.14E-02	-1.36E-11	2.85E-03	-2.45E-12	6.02E-03
10 MODAL LinModal 9 -6.32E-09 2.20E-02 7.47E-10 -4.71E-03 -9.79E-10 -4.01E-03 10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 1 0	10	MODAL	LinModal	7	1.82E-02	-1.43E-02	9.74E-04	3.20E-03	9.39E-03	-1.88E-05
10 MODAL LinModal 10 -1.12E-02 3.49E-04 -1.46E-03 -1.14E-04 -1.03E-02 8.71E-07 10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 1 0 0 0 0 0 0 0 0 0	10	MODAL	LinModal	8	-1.45E-02	-1.79E-02	-7.76E-04	4.02E-03	-7.48E-03	-2.36E-05
10 MODAL LinModal 11 -3.57E-04 -1.09E-02 -4.66E-05 3.59E-03 -3.28E-04 -2.73E-05 10 MODAL LinModal 12 1.01E-08 -1.08E-02 1.96E-08 3.41E-03 1.16E-08 1.87E-03 11 MODAL LinModal 1 0 0 0 0 0 0 11 MODAL LinModal 2 0 0 0 0 0 0 11 MODAL LinModal 3 0 0 0 0 0 0 11 MODAL LinModal 5 0 0 0 0 0 0 0 11 MODAL LinModal 7 0 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 0 11 MODAL LinModal 11 0 <t< td=""><td>10</td><td>MODAL</td><td>LinModal</td><td>9</td><td>-6.32E-09</td><td>2.20E-02</td><td>7.47E-10</td><td>-4.71E-03</td><td>-9.79E-10</td><td>-4.01E-03</td></t<>	10	MODAL	LinModal	9	-6.32E-09	2.20E-02	7.47E-10	-4.71E-03	-9.79E-10	-4.01E-03
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11 MODAL LinModal 1 0 0 0 0 0 0 11 MODAL LinModal 2 0 0 0 0 0 0 11 MODAL LinModal 3 0 0 0 0 0 0 11 MODAL LinModal 4 0 0 0 0 0 0 11 MODAL LinModal 5 0 0 0 0 0 0 11 MODAL LinModal 6 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 11 MODAL LinModal 10 0 0 0 0 0 0 11 MODAL LinModal 11 0 0 0 0 0 0 0 12	10	MODAL	LinModal	11	-3.57E-04	-1.09E-02	-4.66E-05	3.59E-03	-3.28E-04	-2.73E-05
11 MODAL LinModal 2 0 0 0 0 0 0 11 MODAL LinModal 4 0 0 0 0 0 0 11 MODAL LinModal 5 0 0 0 0 0 0 11 MODAL LinModal 6 0 0 0 0 0 0 11 MODAL LinModal 7 0 0 0 0 0 0 0 11 MODAL LinModal 8 0	10	MODAL	LinModal	12	1.01E-08	-1.08E-02	1.96E-08	3.41E-03	1.16E-08	1.87E-03
11 MODAL LinModal 3 0 0 0 0 0 0 11 MODAL LinModal 4 0 0 0 0 0 0 11 MODAL LinModal 5 0 0 0 0 0 0 11 MODAL LinModal 6 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 11 MODAL LinModal 9 0 0 0 0 0 0 11 MODAL LinModal 10 0 0 0 0 0 0 11 MODAL LinModal 11 0	11	MODAL	LinModal	1	0	0	0	0	0	0
11 MODAL LinModal 4 0 0 0 0 0 0 11 MODAL LinModal 5 0 0 0 0 0 0 0 11 MODAL LinModal 6 0 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 0 11 MODAL LinModal 9 0	11	MODAL	LinModal	2	0	0	0	0	0	0
11 MODAL LinModal 5 0 0 0 0 0 0 11 MODAL LinModal 6 0 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 0 11 MODAL LinModal 9 0 <td< td=""><td>11</td><td>MODAL</td><td>LinModal</td><td>3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></td<>	11	MODAL	LinModal	3	0	0	0	0	0	0
11 MODAL LinModal 6 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 11 MODAL LinModal 9 0 0 0 0 0 0 11 MODAL LinModal 10 0 0 0 0 0 0 11 MODAL LinModal 11 0 0 0 0 0 0 0 11 MODAL LinModal 12 0	11	MODAL	LinModal	4	0	0	0	0	0	0
11 MODAL LinModal 7 0 0 0 0 0 0 11 MODAL LinModal 8 0 0 0 0 0 0 11 MODAL LinModal 10 0 0 0 0 0 0 11 MODAL LinModal 11 0 0 0 0 0 0 11 MODAL LinModal 11 0 0 0 0 0 0 0 11 MODAL LinModal 12 0	11	MODAL	LinModal	5	0	0	0	0	0	0
11 MODAL LinModal 8 0 0 0 0 0 0 11 MODAL LinModal 9 0 0 0 0 0 0 0 11 MODAL LinModal 10 0 0 0 0 0 0 0 11 MODAL LinModal 11 0 <	11	MODAL	LinModal	6	0	0	0	0	0	0
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11 MODAL LinModal 10 0 0 0 0 0 0 11 MODAL LinModal 11 0 0 0 0 0 0 0 11 MODAL LinModal 12 0 0 0 0 0 0 0 12 MODAL LinModal 1 9.83E-03 1.52E-04 2.44E-04 -4.68E-05 2.98E-03 1.55E-06 12 MODAL LinModal 2 1.60E-04 -9.83E-03 2.52E-04 2.98E-03 4.76E-05 -1.50E-06 12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05	11	MODAL	LinModal	8	0	0	0	0	0	0
11 MODAL LinModal 11 0 0 0 0 0 0 0 11 MODAL LinModal 12 0 0 0 0 0 0 0 12 MODAL LinModal 1 9.83E-03 1.55E-04 2.44E-04 -4.68E-05 2.98E-03 1.55E-06 12 MODAL LinModal 2 1.60E-04 -9.83E-03 2.52E-04 2.98E-03 4.76E-05 -1.50E-06 12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03	11	MODAL	LinModal	9	0	0	0	0	0	0
11 MODAL LinModal 12 0 0 0 0 0 0 12 MODAL LinModal 1 9.83E-03 1.52E-04 2.44E-04 -4.68E-05 2.98E-03 1.55E-06 12 MODAL LinModal 2 1.60E-04 -9.83E-03 2.52E-04 2.98E-03 4.76E-05 -1.50E-06 12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04	11	MODAL	LinModal	10	0	0	0	0	0	0
12 MODAL LinModal 1 9.83E-03 1.52E-04 2.44E-04 -4.68E-05 2.98E-03 1.55E-06 12 MODAL LinModal 2 1.60E-04 -9.83E-03 2.52E-04 2.98E-03 4.76E-05 -1.50E-06 12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 <	11	MODAL	LinModal	11	0	0	0	0	0	0
12 MODAL LinModal 2 1.60E-04 -9.83E-03 2.52E-04 2.98E-03 4.76E-05 -1.50E-06 12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09	11	MODAL	LinModal	12	0	0	0	0	0	0
12 MODAL LinModal 3 -9.52E-03 -9.52E-03 -7.60E-14 2.68E-03 -2.68E-03 1.94E-03 12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04	12	MODAL	LinModal	1	9.83E-03	1.52E-04	2.44E-04	-4.68E-05	2.98E-03	1.55E-06
12 MODAL LinModal 4 2.70E-02 5.58E-03 -2.98E-04 -1.16E-03 5.60E-03 2.93E-05 12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04	12	MODAL	LinModal	2	1.60E-04	-9.83E-03	2.52E-04	2.98E-03	4.76E-05	-1.50E-06
12 MODAL LinModal 5 5.67E-03 -2.70E-02 -4.55E-04 5.61E-03 1.18E-03 -1.92E-05 12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	3	-9.52E-03	-9.52E-03	-7.60E-14	2.68E-03	-2.68E-03	1.94E-03
12 MODAL LinModal 6 0.02628285 2.63E-02 2.70E-12 -5.04E-03 5.04E-03 -4.93E-03 12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	4	2.70E-02	5.58E-03	-2.98E-04	-1.16E-03	5.60E-03	2.93E-05
12 MODAL LinModal 7 2.92E-02 -2.34E-02 2.44E-04 7.48E-04 9.37E-04 1.45E-05 12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	5	5.67E-03	-2.70E-02	-4.55E-04	5.61E-03	1.18E-03	-1.92E-05
12 MODAL LinModal 8 -2.31E-02 -2.90E-02 2.77E-05 9.34E-04 -7.44E-04 -1.28E-04 12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	6	0.02628285	2.63E-02	2.70E-12	-5.04E-03	5.04E-03	-4.93E-03
12 MODAL LinModal 9 3.50E-02 3.50E-02 -1.16E-09 -8.13E-04 8.13E-04 -5.94E-03 12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	7	2.92E-02	-2.34E-02	2.44E-04	7.48E-04	9.37E-04	1.45E-05
12 MODAL LinModal 10 0.02883927 -1.21E-03 -4.08E-04 -2.13E-04 -5.32E-03 1.22E-04 12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	8	-2.31E-02	-2.90E-02	2.77E-05	9.34E-04	-7.44E-04	-1.28E-04
12 MODAL LinModal 11 6.27E-04 0.02882064 3.83E-04 5.31E-03 -1.26E-04 1.31E-04	12	MODAL	LinModal	9	3.50E-02	3.50E-02	-1.16E-09	-8.13E-04	8.13E-04	-5.94E-03
	12	MODAL	LinModal	10	0.02883927	-1.21E-03	-4.08E-04	-2.13E-04	-5.32E-03	1.22E-04
12 MODAL LinModal 12 2.69E-02 2.69E-02 -9.94E-09 4.74E-03 -4.74E-03 -4.24E-03	12	MODAL	LinModal	11	6.27E-04	0.02882064	3.83E-04	5.31E-03	-1.26E-04	1.31E-04
	12	MODAL	LinModal	12	2.69E-02	2.69E-02	-9.94E-09	4.74E-03	-4.74E-03	-4.24E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
13	MODAL	LinModal	1	2.27E-02	3.57E-04	4.01E-04	-4.32E-05	2.72E-03	2.06E-06
13	MODAL	LinModal	2	3.63E-04	-2.27E-02	4.14E-04	2.72E-03	4.30E-05	-1.99E-06
13	MODAL	LinModal	3	-2.16E-02	-2.16E-02	-8.28E-14	2.38E-03	-2.38E-03	4.27E-03
13	MODAL	LinModal	4	0.03496447	7.23E-03	-7.23E-04	6.10E-04	-2.95E-03	3.41E-05
13	MODAL	LinModal	5	7.33E-03	-3.50E-02	-1.10E-03	-2.95E-03	-6.16E-04	-2.23E-05
13	MODAL	LinModal	6	3.32E-02	3.32E-02	-2.66E-12	2.79E-03	-2.79E-03	-6.08E-03
13	MODAL	LinModal	7	-3.02E-03	2.41E-03	4.53E-04	-7.15E-03	-8.95E-03	-1.32E-06
13	MODAL	LinModal	8	2.38E-03	2.99E-03	5.13E-05	-8.87E-03	7.05E-03	1.16E-05
13	MODAL	LinModal	9	-4.52E-03	-4.52E-03	-1.67E-09	1.00E-02	-1.00E-02	7.27E-04
13	MODAL	LinModal	10	-3.58E-02	1.49E-03	-6.37E-04	9.98E-06	-1.28E-04	-1.44E-04
13	MODAL	LinModal	11	-7.92E-04	-3.57E-02	5.97E-04	1.29E-04	-1.82E-05	-1.53E-04
13	MODAL	LinModal	12	-3.38E-02	-3.38E-02	-7.40E-09	-6.63E-05	6.63E-05	5.21E-03
14	MODAL	LinModal	1	0.03290705	5.17E-04	4.79E-04	-2.93E-05	1.86E-03	3.00E-06
14	MODAL	LinModal	2	5.26E-04	-3.29E-02	4.94E-04	1.86E-03	2.97E-05	-2.90E-06
14	MODAL	LinModal	3	-3.09E-02	-3.09E-02	-2.20E-14	1.58E-03	-1.58E-03	6.10E-03
14	MODAL	LinModal	4	4.58E-03	9.51E-04	-1.08E-03	1.80E-03	-8.69E-03	3.53E-06
14	MODAL	LinModal	5	9.56E-04	-4.58E-03	-0.001643	-8.70E-03	-1.82E-03	-2.31E-06
14	MODAL	LinModal	6	3.55E-03	3.55E-03	-6.55E-12	7.85E-03	-7.85E-03	-6.33E-04
14	MODAL	LinModal	7	-2.70E-02	2.15E-02	1.11E-03	2.46E-03	3.08E-03	-1.22E-05
14	MODAL	LinModal	8	2.13E-02	2.68E-02	1.26E-04	3.07E-03	-2.44E-03	1.08E-04
14	MODAL	LinModal	9	-0.032076	-3.21E-02	-6.63E-10	-3.68E-03	3.68E-03	5.29E-03
14	MODAL	LinModal	10	2.84E-02	-1.17E-03	-8.37E-04	2.40E-04	5.23E-03	1.13E-04
14	MODAL	LinModal	11	6.35E-04	2.83E-02	7.85E-04	-5.23E-03	9.41E-05	1.20E-04
14	MODAL	LinModal	12	2.72E-02	2.72E-02	1.20E-09	-4.49E-03	4.49E-03	-4.18E-03
15	MODAL	LinModal	1	3.87E-02	6.09E-04	5.02E-04	-1.49E-05	9.18E-04	3.33E-06
15	MODAL	LinModal	2	6.18E-04	0.03874577	5.18E-04	9.18E-04	1.42E-05	-3.23E-06
15	MODAL	LinModal	3	-3.60E-02	-3.60E-02	4.58E-14	7.05E-04	-7.05E-04	7.11E-03
15	MODAL	LinModal	4	-3.25E-02	-6.73E-03	-1.22E-03	1.32E-03	-6.35E-03	-3.08E-05
15	MODAL	LinModal	5	-6.81E-03	3.26E-02	-1.86E-03	-6.36E-03	-1.32E-03	2.02E-05
15	MODAL	LinModal	6	-3.13E-02	-3.13E-02	4.76E-12	5.39E-03	-5.39E-03	5.73E-03
15	MODAL	LinModal	7	1.80E-02	-1.43E-02	1.58E-03	7.40E-03	9.28E-03	8.15E-06
15	MODAL	LinModal	8	-1.42E-02	0.01783347	1.79E-04	9.23E-03	-7.34E-03	-7.20E-05
15	MODAL	LinModal	9	0.02198138	2.20E-02	2.13E-09	-1.01E-02	1.01E-02	-3.65E-03
15	MODAL	LinModal	10	-1.08E-02	4.45E-04	-1.14E-03	-3.68E-04	-1.00E-02	-4.26E-05
15	MODAL	LinModal	11	-2.47E-04	-1.08E-02	1.07E-03	0.0100088	-2.71E-04	-4.54E-05
15	MODAL	LinModal	12	-1.07E-02	-1.07E-02	7.50E-09	8.58E-03	-8.58E-03	1.65E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
16	MODAL	LinModal	1	0	0	0	0	0	0
16	MODAL	LinModal	2	0	0	0	0	0	0
16	MODAL	LinModal	3	0	0	0	0	0	0
16	MODAL	LinModal	4	0	0	0	0	0	0
16	MODAL	LinModal	5	0	0	0	0	0	0
16	MODAL	LinModal	6	0	0	0	0	0	0
16	MODAL	LinModal	7	0	0	0	0	0	0
16	MODAL	LinModal	8	0	0	0	0	0	0
16	MODAL	LinModal	9	0	0	0	0	0	0
16	MODAL	LinModal	10	0	0	0	0	0	0
16	MODAL	LinModal	11	0	0	0	0	0	0
16	MODAL	LinModal	12	0	0	0	0	0	0
17	MODAL	LinModal	1	9.84E-03	1.56E-04	3.93E-06	-4.72E-05	1.83E-03	-7.29E-07
17	MODAL	LinModal	2	1.56E-04	-9.84E-03	-2.48E-04	2.98E-03	2.90E-05	-1.15E-08
17	MODAL	LinModal	3	9.53E-03	-9.19E-13	-5.59E-14	6.10E-14	1.68E-03	1.92E-03
17	MODAL	LinModal	4	2.71E-02	5.66E-03	-8.20E-05	-1.17E-03	2.99E-03	-1.26E-05
17	MODAL	LinModal	5	5.63E-03	-2.72E-02	3.94E-04	5.63E-03	6.23E-04	-2.63E-06
17	MODAL	LinModal	6	-2.63E-02	-6.46E-11	2.54E-11	1.54E-11	-2.83E-03	-5.12E-03
17	MODAL	LinModal	7	2.93E-02	-2.36E-02	-1.31E-04	7.55E-04	-1.62E-05	-4.08E-05
17	MODAL	LinModal	8	-2.33E-02	0.02963616	-1.65E-04	9.48E-04	1.29E-05	3.25E-05
17	MODAL	LinModal	9	-3.52E-02	-1.14E-08	4.69E-09	3.60E-09	8.44E-05	-6.47E-03
17	MODAL	LinModal	10	2.92E-02	-9.51E-04	1.65E-05	-1.75E-04	-3.11E-03	-7.70E-05
17	MODAL	LinModal	11	9.31E-04	2.98E-02	-5.18E-04	5.47E-03	-9.93E-05	-2.46E-06
17	MODAL	LinModal	12	-2.72E-02	-5.90E-08	-9.28E-09	-1.15E-09	2.84E-03	-4.78E-03
18	MODAL	LinModal	1	2.27E-02	3.60E-04	6.48E-06	-4.31E-05	1.81E-03	-9.81E-07
18	MODAL	LinModal	2	3.60E-04	-2.27E-02	-4.09E-04	2.72E-03	2.87E-05	-1.55E-08
18	MODAL	LinModal	3	2.16E-02	-2.28E-14	-4.76E-14	-1.66E-13	1.59E-03	4.29E-03
18	MODAL	LinModal	4	3.50E-02	7.32E-03	-1.97E-04	6.17E-04	-1.82E-03	-1.51E-05
18	MODAL	LinModal	5	7.28E-03	-3.52E-02	9.46E-04	-2.96E-03	-3.79E-04	-3.14E-06
18	MODAL	LinModal	6	0.0332065	-1.02E-10	2.00E-11	-1.11E-11	1.72E-03	-6.38E-03
18	MODAL	LinModal	7	-3.02E-03	2.43E-03	-2.45E-04	-7.20E-03	-4.78E-03	4.14E-06
18	MODAL	LinModal	8	2.41E-03	3.06E-03	-3.08E-04	-9.04E-03	3.80E-03	-3.30E-06
18	MODAL	LinModal	9	4.55E-03	-2.54E-08	3.41E-09	-2.41E-09	5.65E-03	8.18E-04
18	MODAL	LinModal	10	-3.61E-02	1.18E-03	2.70E-05	-4.62E-06	-4.59E-06	9.30E-05
18	MODAL	LinModal	11	-1.15E-03	-3.69E-02	-8.45E-04	1.45E-04	-1.46E-07	2.97E-06
18	MODAL	LinModal	12	3.42E-02	3.91E-08	-4.94E-09	-1.18E-08	-1.05E-04	5.93E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
19	MODAL	LinModal	1	3.29E-02	5.21E-04	7.73E-06	-2.95E-05	1.27E-03	-1.46E-06
19	MODAL	LinModal	2	5.21E-04	-3.29E-02	-4.88E-04	1.86E-03	2.01E-05	-2.31E-08
19	MODAL	LinModal	3	3.09E-02	4.61E-13	6.60E-15	1.89E-14	1.07E-03	6.14E-03
19	MODAL	LinModal	4	4.57E-03	9.58E-04	-2.93E-04	1.82E-03	-5.61E-03	-1.60E-06
19	MODAL	LinModal	5	9.51E-04	-4.60E-03	1.41E-03	-8.74E-03	-1.17E-03	-3.33E-07
19	MODAL	LinModal	6	-3.54E-03	3.56E-11	-1.10E-11	-3.58E-12	5.08E-03	-6.71E-04
19	MODAL	LinModal	7	-2.70E-02	2.18E-02	-5.56E-04	2.49E-03	1.95E-03	3.56E-05
19	MODAL	LinModal	8	2.15E-02	2.73E-02	-6.98E-04	3.13E-03	-1.56E-03	-2.83E-05
19	MODAL	LinModal	9	3.22E-02	8.08E-09	-1.71E-09	-3.34E-09	-2.31E-03	5.84E-03
19	MODAL	LinModal	10	2.86E-02	-9.34E-04	3.57E-05	1.71E-04	2.05E-03	-7.28E-05
19	MODAL	LinModal	11	9.13E-04	2.93E-02	-1.12E-03	-5.35E-03	6.53E-05	-2.32E-06
19	MODAL	LinModal	12	-2.75E-02	3.83E-08	4.38E-10	1.21E-08	-2.01E-03	-4.76E-03
20	MODAL	LinModal	1	3.87E-02	6.14E-04	8.11E-06	-1.45E-05	5.37E-04	-1.63E-06
20	MODAL	LinModal	2	6.14E-04	-3.88E-02	-5.12E-04	9.19E-04	8.50E-06	-2.58E-08
20	MODAL	LinModal	3	3.60E-02	-7.71E-14	6.44E-14	5.12E-14	3.98E-04	7.16E-03
20	MODAL	LinModal	4	-3.26E-02	-6.81E-03	-3.31E-04	1.33E-03	-3.29E-03	1.37E-05
20	MODAL	LinModal	5	-6.77E-03	3.27E-02	1.59E-03	-6.39E-03	-6.84E-04	2.86E-06
20	MODAL	LinModal	6	3.14E-02	-8.69E-11	-1.20E-11	2.16E-11	2.85E-03	6.02E-03
20	MODAL	LinModal	7	1.79E-02	-1.45E-02	-7.76E-04	7.48E-03	4.02E-03	-2.36E-05
20	MODAL	LinModal	8	-1.43E-02	-1.82E-02	-9.74E-04	9.39E-03	-3.20E-03	1.88E-05
20	MODAL	LinModal	9	-2.20E-02	-4.08E-09	-2.54E-09	2.67E-09	-4.71E-03	-4.01E-03
20	MODAL	LinModal	10	-1.09E-02	3.57E-04	4.66E-05	-3.28E-04	0.00358531	2.73E-05
20	MODAL	LinModal	11	-3.49E-04	-1.12E-02	-1.46E-03	1.03E-02	-1.14E-04	8.71E-07
20	MODAL	LinModal	12	1.08E-02	-5.47E-08	6.64E-09	1.20E-08	3.41E-03	1.87E-03
21	MODAL	LinModal	1	0	0	0	0	0	0
21	MODAL	LinModal	2	0	0	0	0	0	0
21	MODAL	LinModal	3	0	0	0	0	0	0
21	MODAL	LinModal	4	0	0	0	0	0	0
21	MODAL	LinModal	5	0	0	0	0	0	0
21	MODAL	LinModal	6	0	0	0	0	0	0
21	MODAL	LinModal	7	0	0	0	0	0	0
21	MODAL	LinModal	8	0	0	0	0	0	0
21	MODAL	LinModal	9	0	0	0	0	0	0
21	MODAL	LinModal	10	0	0	0	0	0	0
21	MODAL	LinModal	11	0	0	0	0	0	0
21	MODAL	LinModal	12	0	0	0	0	0	0

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
22	MODAL	LinModal	1	1.29E-03	1.89E-02	-1.26E-12	-3.62E-03	2.47E-04	1.04E-15
22	MODAL	LinModal	2	1.89E-02	-1.29E-03	1.86E-13	2.47E-04	3.62E-03	-2.89E-15
22	MODAL	LinModal	3	1.57E-13	3.20E-12	-2.51E-12	-1.44E-13	8.73E-14	-3.60E-03
22	MODAL	LinModal	4	-5.28E-03	-5.30E-02	7.19E-10	6.04E-03	-6.01E-04	1.59E-15
22	MODAL	LinModal	5	-5.30E-02	5.28E-03	-4.13E-10	-6.01E-04	-6.04E-03	4.92E-13
22	MODAL	LinModal	6	-5.87E-10	-1.09E-09	3.59E-10	3.04E-11	-7.83E-11	-9.84E-03
22	MODAL	LinModal	7	-5.89E-02	4.38E-02	1.61E-09	-1.23E-04	-1.66E-04	1.12E-11
22	MODAL	LinModal	8	4.38E-02	5.89E-02	7.02E-09	-1.66E-04	1.23E-04	-5.81E-11
22	MODAL	LinModal	9	2.73E-09	1.90E-08	-4.91E-09	-1.31E-09	1.86E-10	1.31E-02
22	MODAL	LinModal	10	-5.86E-02	4.84E-03	2.33E-08	5.28E-04	6.40E-03	-9.10E-11
22	MODAL	LinModal	11	4.84E-03	5.86E-02	1.97E-08	6.40E-03	-5.28E-04	-2.43E-10
22	MODAL	LinModal	12	-5.97E-09	-5.89E-08	0.05603657	2.43E-09	-1.18E-09	-1.05E-10
23	MODAL	LinModal	1	3.01E-03	4.40E-02	-1.31E-12	-3.60E-03	2.46E-04	1.18E-15
23	MODAL	LinModal	2	4.40E-02	-3.01E-03	2.49E-13	2.46E-04	3.60E-03	8.26E-16
23	MODAL	LinModal	3	4.86E-13	1.47E-12	-2.61E-12	1.05E-13	-1.31E-13	-8.37E-03
23	MODAL	LinModal	4	-6.83E-03	0.06865499	7.74E-10	-3.79E-03	3.78E-04	2.17E-12
23	MODAL	LinModal	5	-6.87E-02	6.83E-03	-4.88E-10	3.78E-04	3.79E-03	-2.26E-12
23	MODAL	LinModal	6	-1.04E-09	-1.40E-10	2.77E-10	-2.28E-10	6.51E-11	-1.27E-02
23	MODAL	LinModal	7	5.87E-03	-4.36E-03	4.85E-10	7.31E-03	9.84E-03	-9.16E-13
23	MODAL	LinModal	8	-4.36E-03	-5.87E-03	5.96E-09	9.84E-03	-7.31E-03	-8.84E-11
23	MODAL	LinModal	9	-4.82E-10	1.61E-08	-2.30E-09	2.15E-09	-9.03E-10	-1.25E-03
23	MODAL	LinModal	10	7.19E-02	-5.94E-03	4.45E-08	2.35E-05	2.85E-04	2.95E-10
23	MODAL	LinModal	11	-5.94E-03	-7.19E-02	1.43E-08	2.85E-04	-2.35E-05	-1.97E-10
23	MODAL	LinModal	12	-7.48E-09	-2.41E-08	0.10517682	-9.30E-09	3.17E-09	-6.85E-11
24	MODAL	LinModal	1	4.37E-03	6.40E-02	5.74E-14	-2.54E-03	1.73E-04	1.37E-15
24	MODAL	LinModal	2	6.40E-02	-4.37E-03	-5.84E-15	1.73E-04	2.54E-03	-2.39E-15
24	MODAL	LinModal	3	-2.02E-12	1.67E-12	1.93E-14	-2.56E-14	7.80E-14	-1.21E-02
24	MODAL	LinModal	4	-7.84E-04	-7.88E-03	-1.39E-10	-1.14E-02	1.13E-03	-2.45E-13
24	MODAL	LinModal	5	-7.88E-03	7.84E-04	4.68E-11	1.13E-03	0.011385864	-4.61E-14
24	MODAL	LinModal	6	3.52E-10	1.86E-09	-1.40E-10	-1.43E-11	1.15E-10	-1.30E-03
24	MODAL	LinModal	7	5.37E-02	-3.99E-02	-4.69E-10	-3.08E-03	-4.15E-03	1.53E-11
24	MODAL	LinModal	8	-3.99E-02	-5.37E-02	2.70E-11	-4.15E-03	3.08E-03	-4.09E-11
24	MODAL	LinModal	9	-1.06E-08	-6.77E-09	2.07E-09	1.49E-10	-7.61E-11	-1.19E-02
24	MODAL	LinModal	10	-5.68E-02	4.69E-03	1.89E-09	-3.46E-04	-4.19E-03	-2.62E-12
24	MODAL	LinModal	11	4.69E-03	5.68E-02	1.44E-09	-4.19E-03	3.46E-04	2.44E-10
24	MODAL	LinModal	12	3.13E-08	6.14E-08	0.14135915	2.58E-09	4.20E-10	-2.32E-11

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
25	MODAL	LinModal	1	3.88E-02	6.14E-04	-1.96E-14	-8.51E-06	5.37E-04	-6.88E-16
25	MODAL	LinModal	2	6.14E-04	-3.88E-02	-1.38E-13	5.37E-04	8.51E-06	5.08E-16
25	MODAL	LinModal	3	-1.17E-13	-2.42E-13	-3.96E-13	-2.01E-14	-2.51E-14	7.16E-03
25	MODAL	LinModal	4	-3.27E-02	-6.81E-03	-1.05E-09	6.89E-04	-3.31E-03	1.17E-12
25	MODAL	LinModal	5	-6.81E-03	3.27E-02	-5.42E-11	-3.31E-03	-6.89E-04	1.58E-12
25	MODAL	LinModal	6	4.68E-11	9.37E-12	-4.33E-12	9.19E-12	5.72E-12	6.04E-03
25	MODAL	LinModal	7	1.82E-02	-1.45E-02	2.59E-09	3.25E-03	4.08E-03	-3.68E-11
25	MODAL	LinModal	8	-1.45E-02	-1.82E-02	8.35E-09	4.08E-03	-3.25E-03	7.50E-11
25	MODAL	LinModal	9	3.11E-09	-1.28E-08	4.01E-10	1.71E-09	3.07E-10	-4.08E-03
25	MODAL	LinModal	10	-1.12E-02	3.58E-04	1.19E-08	-1.18E-04	-3.71E-03	-2.19E-10
25	MODAL	LinModal	11	-3.58E-04	-1.12E-02	-2.26E-09	3.71E-03	-1.18E-04	-8.41E-11
25	MODAL	LinModal	12	-1.76E-08	-2.58E-08	-2.19E-08	2.24E-09	-2.02E-09	1.94E-03
26	MODAL	LinModal	1	0	0	0	0	0	0
26	MODAL	LinModal	2	0	0	0	0	0	0
26	MODAL	LinModal	3	0	0	0	0	0	0
26	MODAL	LinModal	4	0	0	0	0	0	0
26	MODAL	LinModal	5	0	0	0	0	0	0
26	MODAL	LinModal	6	0	0	0	0	0	0
26	MODAL	LinModal	7	0	0	0	0	0	0
26	MODAL	LinModal	8	0	0	0	0	0	0
26	MODAL	LinModal	9	0	0	0	0	0	0
26	MODAL	LinModal	10	0	0	0	0	0	0
26	MODAL	LinModal	11	0	0	0	0	0	0
26	MODAL	LinModal	12	0	0	0	0	0	0
27	MODAL	LinModal	1	9.84E-03	1.56E-04	-3.93E-06	-4.72E-05	1.83E-03	7.29E-07
27	MODAL	LinModal	2	1.56E-04	-9.84E-03	2.48E-04	0.0029803	2.90E-05	1.15E-08
27	MODAL	LinModal	3	-9.53E-03	7.76E-14	1.87E-13	-2.52E-14	-1.68E-03	1.92E-03
27	MODAL	LinModal	4	0.0270777	5.66E-03	8.20E-05	-1.17E-03	2.99E-03	1.26E-05
27	MODAL	LinModal	5	5.63E-03	-2.72E-02	-3.94E-04	5.63E-03	6.23E-04	2.63E-06
27	MODAL	LinModal	6	2.63E-02	-6.20E-11	4.97E-11	-3.39E-12	2.83E-03	-5.12E-03
27	MODAL	LinModal	7	2.93E-02	-2.36E-02	1.31E-04	7.55E-04	-1.62E-05	4.08E-05
27	MODAL	LinModal	8	-2.33E-02	-2.96E-02	1.65E-04	9.48E-04	1.29E-05	-3.25E-05
27	MODAL	LinModal	9	3.52E-02	-1.29E-08	-2.44E-09	-1.10E-09	-8.44E-05	-6.47E-03
27	MODAL	LinModal	10	2.92E-02	-9.51E-04	-1.65E-05	-1.75E-04	-3.11E-03	7.70E-05
27	MODAL	LinModal	11	9.31E-04	0.02979488	5.18E-04	5.47E-03	-9.93E-05	2.46E-06
27	MODAL	LinModal	12	2.72E-02	-8.71E-10	-7.77E-09	4.63E-10	-2.84E-03	-4.78E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
28	MODAL	LinModal	1	0.02272711	3.60E-04	-6.48E-06	-4.31E-05	1.81E-03	9.81E-07
28	MODAL	LinModal	2	3.60E-04	-2.27E-02	4.09E-04	2.72E-03	2.87E-05	1.55E-08
28	MODAL	LinModal	3	-2.16E-02	-9.95E-14	2.04E-13	1.28E-13	-1.59E-03	4.29E-03
28	MODAL	LinModal	4	3.50E-02	7.32E-03	1.97E-04	6.17E-04	-1.82E-03	1.51E-05
28	MODAL	LinModal	5	7.28E-03	-3.52E-02	-9.46E-04	-2.96E-03	-3.79E-04	3.14E-06
28	MODAL	LinModal	6	3.32E-02	5.61E-11	4.85E-11	-3.02E-11	-1.72E-03	-6.38E-03
28	MODAL	LinModal	7	-3.02E-03	2.43E-03	2.45E-04	-7.20E-03	-4.78E-03	-4.14E-06
28	MODAL	LinModal	8	2.41E-03	3.06E-03	3.08E-04	-9.04E-03	3.80E-03	3.30E-06
28	MODAL	LinModal	9	-4.55E-03	4.79E-09	-2.09E-09	-1.99E-09	-5.65E-03	8.18E-04
28	MODAL	LinModal	10	-3.61E-02	1.18E-03	-2.70E-05	-4.62E-06	-4.59E-06	-9.30E-05
28	MODAL	LinModal	11	-1.15E-03	-3.69E-02	8.45E-04	1.45E-04	-1.46E-07	-2.97E-06
28	MODAL	LinModal	12	-3.42E-02	-2.10E-08	2.25E-09	2.30E-09	1.05E-04	5.93E-03
29	MODAL	LinModal	1	3.29E-02	5.21E-04	-7.73E-06	-2.95E-05	1.27E-03	1.46E-06
29	MODAL	LinModal	2	5.21E-04	-3.29E-02	4.88E-04	1.86E-03	2.01E-05	2.31E-08
29	MODAL	LinModal	3	-3.09E-02	-8.92E-13	3.24E-15	1.88E-14	0.00106878	6.14E-03
29	MODAL	LinModal	4	4.57E-03	9.58E-04	2.93E-04	1.82E-03	-5.61E-03	1.60E-06
29	MODAL	LinModal	5	9.51E-04	-4.60E-03	-1.41E-03	-8.74E-03	-1.17E-03	3.33E-07
29	MODAL	LinModal	6	3.54E-03	2.05E-10	5.62E-12	-1.91E-12	-5.08E-03	-6.71E-04
29	MODAL	LinModal	7	-2.70E-02	2.18E-02	5.56E-04	2.49E-03	1.95E-03	-3.56E-05
29	MODAL	LinModal	8	2.15E-02	2.73E-02	6.98E-04	3.13E-03	-1.56E-03	2.83E-05
29	MODAL	LinModal	9	-3.22E-02	-6.88E-10	1.85E-09	3.60E-09	2.31E-03	5.84E-03
29	MODAL	LinModal	10	0.02862809	-9.34E-04	-3.57E-05	1.71E-04	2.05E-03	7.28E-05
29	MODAL	LinModal	11	9.13E-04	2.93E-02	1.12E-03	-5.35E-03	6.53E-05	2.32E-06
29	MODAL	LinModal	12	2.75E-02	-1.97E-08	4.77E-09	-4.55E-09	2.01E-03	-4.76E-03
30	MODAL	LinModal	1	3.87E-02	6.14E-04	-8.11E-06	-1.45E-05	5.37E-04	1.63E-06
30	MODAL	LinModal	2	6.14E-04	-3.88E-02	5.12E-04	9.19E-04	8.50E-06	2.58E-08
30	MODAL	LinModal	3	-3.60E-02	-2.98E-13	-2.17E-13	-9.56E-14	-3.98E-04	7.16E-03
30	MODAL	LinModal	4	-0.0325535	-6.81E-03	3.31E-04	1.33E-03	-3.29E-03	-1.37E-05
30	MODAL	LinModal	5	-6.77E-03	3.27E-02	-1.59E-03	-6.39E-03	-6.84E-04	-2.86E-06
30	MODAL	LinModal	6	-3.14E-02	1.07E-10	-5.48E-11	9.15E-12	-2.85E-03	6.02E-03
30	MODAL	LinModal	7	1.79E-02	-1.45E-02	7.76E-04	7.48E-03	4.02E-03	2.36E-05
30	MODAL	LinModal	8	-1.43E-02	-1.82E-02	9.74E-04	9.39E-03	-3.20E-03	-1.88E-05
30	MODAL	LinModal	9	2.20E-02	-1.83E-08	5.71E-10	2.27E-09	4.71E-03	-4.01E-03
30	MODAL	LinModal	10	-1.09E-02	3.57E-04	-4.66E-05	-3.28E-04	-3.59E-03	-2.73E-05
30	MODAL	LinModal	11	-3.49E-04	-1.12E-02	1.46E-03	1.03E-02	-1.14E-04	-8.71E-07
30	MODAL	LinModal	12	-1.08E-02	9.55E-09	-2.50E-09	-4.23E-09	-3.41E-03	1.87E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
31	MODAL	LinModal	1	0	0	0	0	0	0
31	MODAL	LinModal	2	0	0	0	0	0	0
31	MODAL	LinModal	3	0	0	0	0	0	0
31	MODAL	LinModal	4	0	0	0	0	0	0
31	MODAL	LinModal	5	0	0	0	0	0	0
31	MODAL	LinModal	6	0	0	0	0	0	0
31	MODAL	LinModal	7	0	0	0	0	0	0
31	MODAL	LinModal	8	0	0	0	0	0	0
31	MODAL	LinModal	9	0	0	0	0	0	0
31	MODAL	LinModal	10	0	0	0	0	0	0
31	MODAL	LinModal	11	0	0	0	0	0	0
31	MODAL	LinModal	12	0	0	0	0	0	0
32	MODAL	LinModal	1	9.83E-03	1.52E-04	-2.44E-04	-4.68E-05	2.98E-03	-1.55E-06
32	MODAL	LinModal	2	1.60E-04	-9.83E-03	-2.52E-04	2.98E-03	4.76E-05	1.50E-06
32	MODAL	LinModal	3	9.52E-03	9.52E-03	3.31E-14	-2.68E-03	2.68E-03	1.94E-03
32	MODAL	LinModal	4	0.0270224	5.58E-03	2.98E-04	-1.16E-03	5.60E-03	-2.93E-05
32	MODAL	LinModal	5	5.67E-03	0.02704097	4.55E-04	5.61E-03	1.18E-03	1.92E-05
32	MODAL	LinModal	6	-2.63E-02	-2.63E-02	-5.54E-12	5.04E-03	-5.04E-03	-4.93E-03
32	MODAL	LinModal	7	2.92E-02	-2.34E-02	-2.44E-04	7.48E-04	9.37E-04	-1.45E-05
32	MODAL	LinModal	8	-2.31E-02	-2.90E-02	-2.77E-05	9.34E-04	-7.44E-04	1.28E-04
32	MODAL	LinModal	9	-3.50E-02	0.03498124	1.48E-09	8.13E-04	-8.13E-04	-5.94E-03
32	MODAL	LinModal	10	2.88E-02	-1.21E-03	4.08E-04	-2.13E-04	-5.32E-03	-1.22E-04
32	MODAL	LinModal	11	6.27E-04	2.88E-02	-3.83E-04	5.31E-03	-1.26E-04	-1.30E-04
32	MODAL	LinModal	12	-2.69E-02	-2.69E-02	-7.55E-09	-4.74E-03	4.74E-03	-4.24E-03
33	MODAL	LinModal	1	2.27E-02	3.57E-04	-4.01E-04	-4.32E-05	2.72E-03	-2.06E-06
33	MODAL	LinModal	2	3.63E-04	0.02272773	-4.14E-04	2.72E-03	4.30E-05	1.99E-06
33	MODAL	LinModal	3	2.16E-02	2.16E-02	2.56E-14	-2.38E-03	2.38E-03	4.27E-03
33	MODAL	LinModal	4	0.03496447	7.23E-03	7.23E-04	6.10E-04	-2.95E-03	-3.41E-05
33	MODAL	LinModal	5	7.33E-03	-3.50E-02	1.10E-03	-2.95E-03	-6.16E-04	2.23E-05
33	MODAL	LinModal	6	-3.32E-02	-3.32E-02	-5.55E-12	-2.79E-03	2.79E-03	-6.08E-03
33	MODAL	LinModal	7	-3.02E-03	2.41E-03	-4.53E-04	-7.15E-03	-8.95E-03	1.31E-06
33	MODAL	LinModal	8	2.38E-03	2.99E-03	-5.13E-05	-8.87E-03	7.05E-03	-1.16E-05
33	MODAL	LinModal	9	4.52E-03	4.52E-03	1.21E-09	-1.00E-02	1.00E-02	7.27E-04
33	MODAL	LinModal	10	-3.58E-02	1.49E-03	6.37E-04	9.98E-06	-1.28E-04	1.44E-04
33	MODAL	LinModal	11	-7.92E-04	0.03574715	-5.97E-04	1.29E-04	-1.82E-05	1.53E-04
33	MODAL	LinModal	12	3.38E-02	3.38E-02	-8.54E-09	6.63E-05	-6.63E-05	5.21E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
34	MODAL	LinModal	1	3.29E-02	5.17E-04	-4.79E-04	-2.93E-05	1.86E-03	-3.00E-06
34	MODAL	LinModal	2	5.26E-04	-3.29E-02	-4.94E-04	1.86E-03	2.97E-05	2.90E-06
34	MODAL	LinModal	3	3.09E-02	3.09E-02	-2.05E-14	-1.58E-03	1.58E-03	6.10E-03
34	MODAL	LinModal	4	4.58E-03	9.51E-04	1.08E-03	1.80E-03	-8.69E-03	-3.53E-06
34	MODAL	LinModal	5	9.56E-04	-4.58E-03	1.64E-03	-8.70E-03	-1.82E-03	2.31E-06
34	MODAL	LinModal	6	0.0035505	-3.55E-03	3.00E-14	-7.85E-03	7.85E-03	-6.33E-04
34	MODAL	LinModal	7	-2.70E-02	0.02153805	-1.11E-03	2.46E-03	3.08E-03	1.22E-05
34	MODAL	LinModal	8	2.13E-02	2.68E-02	-1.26E-04	3.07E-03	-2.44E-03	-1.08E-04
34	MODAL	LinModal	9	3.21E-02	3.21E-02	-1.04E-10	3.68E-03	-3.68E-03	5.29E-03
34	MODAL	LinModal	10	2.84E-02	-1.17E-03	8.37E-04	2.40E-04	5.23E-03	-1.13E-04
34	MODAL	LinModal	11	6.35E-04	2.83E-02	-7.85E-04	-5.23E-03	9.41E-05	-1.20E-04
34	MODAL	LinModal	12	0.0271986	-2.72E-02	8.01E-10	4.49E-03	-4.49E-03	-4.18E-03
35	MODAL	LinModal	1	3.87E-02	6.09E-04	-5.02E-04	-1.49E-05	9.18E-04	-3.33E-06
35	MODAL	LinModal	2	6.18E-04	-3.87E-02	-5.18E-04	9.18E-04	1.42E-05	3.23E-06
35	MODAL	LinModal	3	3.60E-02	0.03599302	-5.62E-14	-7.05E-04	7.05E-04	7.11E-03
35	MODAL	LinModal	4	-3.25E-02	-6.73E-03	1.22E-03	1.32E-03	-6.35E-03	3.08E-05
35	MODAL	LinModal	5	-6.81E-03	3.26E-02	1.86E-03	-6.36E-03	-1.32E-03	-2.02E-05
35	MODAL	LinModal	6	3.13E-02	3.13E-02	3.12E-12	-5.39E-03	5.39E-03	5.73E-03
35	MODAL	LinModal	7	1.80E-02	-1.43E-02	-1.58E-03	7.40E-03	9.28E-03	-8.15E-06
35	MODAL	LinModal	8	-1.42E-02	-1.78E-02	-1.79E-04	9.23E-03	-7.34E-03	7.20E-05
35	MODAL	LinModal	9	-2.20E-02	-2.20E-02	-1.04E-09	1.01E-02	-1.01E-02	-3.65E-03
35	MODAL	LinModal	10	-1.08E-02	4.45E-04	1.14E-03	-3.68E-04	-1.00E-02	4.26E-05
35	MODAL	LinModal	11	-2.47E-04	-1.08E-02	-1.07E-03	1.00E-02	-2.71E-04	4.54E-05
35	MODAL	LinModal	12	1.07E-02	1.07E-02	7.86E-09	-8.58E-03	8.58E-03	1.65E-03
36	MODAL	LinModal	1	0	0	0	0	0	0
36	MODAL	LinModal	2	0	0	0	0	0	0
36	MODAL	LinModal	3	0	0	0	0	0	0
36	MODAL	LinModal	4	0	0	0	0	0	0
36	MODAL	LinModal	5	0	0	0	0	0	0
36	MODAL	LinModal	6	0	0	0	0	0	0
36	MODAL	LinModal	7	0	0	0	0	0	0
36	MODAL	LinModal	8	0	0	0	0	0	0
36	MODAL	LinModal	9	0	0	0	0	0	0
36	MODAL	LinModal	10	0	0	0	0	0	0
36	MODAL	LinModal	11	0	0	0	0	0	0
36	MODAL	LinModal	12	0	0	0	0	0	0

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
37	MODAL	LinModal	1	9.84E-03	1.56E-04	-2.48E-04	-2.90E-05	2.98E-03	-1.15E-08
37	MODAL	LinModal	2	1.56E-04	-9.84E-03	-3.93E-06	1.83E-03	4.72E-05	7.29E-07
37	MODAL	LinModal	3	-4.68E-13	9.53E-03	2.37E-13	-1.68E-03	-2.56E-15	1.92E-03
37	MODAL	LinModal	4	2.72E-02	5.63E-03	3.94E-04	-6.23E-04	5.63E-03	-2.63E-06
37	MODAL	LinModal	5	5.66E-03	-0.0270777	8.20E-05	2.99E-03	1.17E-03	1.26E-05
37	MODAL	LinModal	6	-2.40E-11	-2.63E-02	3.21E-11	2.83E-03	-1.95E-11	-5.12E-03
37	MODAL	LinModal	7	2.96E-02	-2.33E-02	-1.65E-04	-1.29E-05	9.48E-04	3.25E-05
37	MODAL	LinModal	8	0.0236098	-2.93E-02	1.31E-04	-1.62E-05	-7.55E-04	4.08E-05
37	MODAL	LinModal	9	9.64E-10	0.03517265	5.45E-09	-8.44E-05	-2.40E-09	-6.47E-03
37	MODAL	LinModal	10	2.98E-02	-9.31E-04	5.18E-04	-9.93E-05	-5.47E-03	2.46E-06
37	MODAL	LinModal	11	9.51E-04	2.92E-02	1.65E-05	0.0031111	-1.75E-04	-7.70E-05
37	MODAL	LinModal	12	-1.59E-08	-2.72E-02	-1.12E-08	-2.84E-03	6.87E-09	-4.78E-03
38	MODAL	LinModal	1	2.27E-02	3.60E-04	-4.09E-04	-2.87E-05	2.72E-03	-1.55E-08
38	MODAL	LinModal	2	3.60E-04	-2.27E-02	-6.48E-06	1.81E-03	4.31E-05	9.81E-07
38	MODAL	LinModal	3	9.32E-14	2.16E-02	2.64E-13	-1.59E-03	2.14E-13	4.29E-03
38	MODAL	LinModal	4	3.52E-02	7.28E-03	9.46E-04	3.79E-04	-2.96E-03	-3.14E-06
38	MODAL	LinModal	5	7.32E-03	-3.50E-02	1.97E-04	-1.82E-03	-6.17E-04	1.51E-05
38	MODAL	LinModal	6	-1.52E-10	-3.32E-02	4.45E-11	-1.72E-03	-6.13E-12	-6.38E-03
38	MODAL	LinModal	7	-3.06E-03	2.41E-03	-3.08E-04	-3.80E-03	-9.04E-03	-3.30E-06
38	MODAL	LinModal	8	2.43E-03	3.02E-03	2.45E-04	-4.78E-03	7.20E-03	-4.14E-06
38	MODAL	LinModal	9	-1.27E-08	4.55E-03	6.85E-09	0.0056502	1.22E-09	8.18E-04
38	MODAL	LinModal	10	-3.69E-02	1.15E-03	8.45E-04	-1.47E-07	-1.45E-04	-2.97E-06
38	MODAL	LinModal	11	-1.18E-03	-3.61E-02	2.70E-05	4.59E-06	-4.62E-06	9.31E-05
38	MODAL	LinModal	12	3.92E-08	3.42E-02	-7.20E-09	1.05E-04	1.35E-08	5.93E-03
39	MODAL	LinModal	1	3.29E-02	5.21E-04	-4.88E-04	-2.01E-05	1.86E-03	-2.31E-08
39	MODAL	LinModal	2	5.21E-04	-3.29E-02	-7.73E-06	1.27E-03	2.95E-05	1.46E-06
39	MODAL	LinModal	3	9.52E-13	3.09E-02	1.73E-14	-1.07E-03	-1.12E-14	6.14E-03
39	MODAL	LinModal	4	4.60E-03	9.51E-04	1.41E-03	1.17E-03	-8.74E-03	-3.33E-07
39	MODAL	LinModal	5	9.58E-04	-4.57E-03	2.93E-04	-5.61E-03	-1.82E-03	1.60E-06
39	MODAL	LinModal	6	-5.37E-11	-3.54E-03	6.35E-12	-5.08E-03	7.99E-12	-6.71E-04
39	MODAL	LinModal	7	-2.73E-02	2.15E-02	-6.98E-04	1.56E-03	3.13E-03	-2.83E-05
39	MODAL	LinModal	8	2.18E-02	2.70E-02	5.56E-04	1.95E-03	-2.49E-03	-3.56E-05
39	MODAL	LinModal	9	1.50E-08	3.22E-02	6.96E-10	2.31E-03	2.87E-09	5.84E-03
39	MODAL	LinModal	10	2.93E-02	-9.13E-04	1.12E-03	6.53E-05	5.35E-03	2.32E-06
39	MODAL	LinModal	11	9.34E-04	0.02862809	3.57E-05	-2.05E-03	1.71E-04	-7.28E-05
39	MODAL	LinModal	12	5.15E-08	-2.75E-02	3.08E-09	2.01E-03	-1.18E-08	-4.76E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
40	MODAL	LinModal	1	3.88E-02	6.14E-04	-5.12E-04	-8.50E-06	9.19E-04	-2.58E-08
40	MODAL	LinModal	2	6.14E-04	-3.87E-02	-8.11E-06	5.37E-04	1.45E-05	1.63E-06
40	MODAL	LinModal	3	2.25E-14	0.03599185	-2.61E-13	-3.98E-04	-1.93E-13	7.16E-03
40	MODAL	LinModal	4	-3.27E-02	-6.77E-03	1.59E-03	6.84E-04	-6.39E-03	2.86E-06
40	MODAL	LinModal	5	-6.81E-03	0.0325535	3.31E-04	-3.29E-03	-1.33E-03	-1.37E-05
40	MODAL	LinModal	6	-7.07E-11	0.03135665	-4.74E-11	-2.85E-03	-1.54E-12	6.02E-03
40	MODAL	LinModal	7	1.82E-02	-1.43E-02	-9.74E-04	3.20E-03	9.39E-03	1.88E-05
40	MODAL	LinModal	8	-1.45E-02	-1.79E-02	7.76E-04	4.02E-03	-7.48E-03	2.36E-05
40	MODAL	LinModal	9	8.97E-09	-2.20E-02	-7.06E-09	4.71E-03	-6.64E-10	-4.01E-03
40	MODAL	LinModal	10	0.0111883	3.49E-04	1.46E-03	-1.14E-04	-1.03E-02	-8.72E-07
40	MODAL	LinModal	11	-3.57E-04	-1.09E-02	4.66E-05	3.59E-03	-3.28E-04	2.73E-05
40	MODAL	LinModal	12	-4.66E-08	1.08E-02	6.04E-09	-3.41E-03	-2.03E-08	1.87E-03
41	MODAL	LinModal	1	0	0	0	0	0	0
41	MODAL	LinModal	2	0	0	0	0	0	0
41	MODAL	LinModal	3	0	0	0	0	0	0
41	MODAL	LinModal	4	0	0	0	0	0	0
41	MODAL	LinModal	5	0	0	0	0	0	0
41	MODAL	LinModal	6	0	0	0	0	0	0
41	MODAL	LinModal	7	0	0	0	0	0	0
41	MODAL	LinModal	8	0	0	0	0	0	0
41	MODAL	LinModal	9	0	0	0	0	0	0
41	MODAL	LinModal	10	0	0	0	0	0	0
41	MODAL	LinModal	11	0	0	0	0	0	0
41	MODAL	LinModal	12	0	0	0	0	0	0
42	MODAL	LinModal	1	9.83E-03	1.60E-04	-2.52E-04	-4.76E-05	2.98E-03	1.50E-06
42	MODAL	LinModal	2	1.52E-04	-9.83E-03	2.44E-04	2.98E-03	4.68E-05	1.55E-06
42	MODAL	LinModal	3	-9.52E-03	9.52E-03	3.96E-14	-2.68E-03	-2.68E-03	1.94E-03
42	MODAL	LinModal	4	2.70E-02	5.67E-03	4.55E-04	-1.18E-03	5.61E-03	1.92E-05
42	MODAL	LinModal	5	5.58E-03	-2.70E-02	-2.98E-04	5.60E-03	1.16E-03	2.93E-05
42	MODAL	LinModal	6	2.63E-02	-2.63E-02	7.28E-12	5.04E-03	5.04E-03	-4.93E-03
42	MODAL	LinModal	7	2.90E-02	-2.31E-02	-2.77E-05	0.0007435	9.34E-04	1.28E-04
42	MODAL	LinModal	8	-2.34E-02	-2.92E-02	2.44E-04	9.37E-04	-7.48E-04	1.45E-05
42	MODAL	LinModal	9	3.50E-02	0.03498124	-1.67E-10	8.13E-04	8.13E-04	-5.94E-03
42	MODAL	LinModal	10	2.88E-02	-6.27E-04	3.83E-04	-1.26E-04	-5.31E-03	1.31E-04
42	MODAL	LinModal	11	1.21E-03	2.88E-02	4.08E-04	5.32E-03	-2.13E-04	-1.22E-04
42	MODAL	LinModal	12	0.0268944	-2.69E-02	-9.56E-09	-4.74E-03	-4.74E-03	-4.24E-03

Joint	OutputCase	CaseType	Modes	U1	U2	U3	R1	R2	R3
Text	Text	Text		m	m	m	Radians	Radians	Radians
43	MODAL	LinModal	1	0.02272773	3.63E-04	-4.14E-04	-4.30E-05	2.72E-03	1.99E-06
43	MODAL	LinModal	2	3.57E-04	-2.27E-02	4.01E-04	2.72E-03	4.32E-05	2.06E-06
43	MODAL	LinModal	3	-2.16E-02	0.02156562	4.28E-14	-2.38E-03	-2.38E-03	4.27E-03
43	MODAL	LinModal	4	3.50E-02	7.33E-03	1.10E-03	6.16E-04	-2.95E-03	2.23E-05
43	MODAL	LinModal	5	7.23E-03	-3.50E-02	-7.23E-04	-2.95E-03	-6.10E-04	3.41E-05
43	MODAL	LinModal	6	3.32E-02	-3.32E-02	2.59E-12	-2.79E-03	-2.79E-03	-6.08E-03
43	MODAL	LinModal	7	-2.99E-03	2.38E-03	-5.13E-05	-7.05E-03	-8.87E-03	-1.16E-05
43	MODAL	LinModal	8	2.41E-03	3.02E-03	4.53E-04	-8.95E-03	7.15E-03	-1.31E-06
43	MODAL	LinModal	9	-4.52E-03	4.52E-03	4.18E-10	-1.00E-02	-1.00E-02	7.27E-04
43	MODAL	LinModal	10	-3.57E-02	7.92E-04	5.97E-04	-1.82E-05	-1.29E-04	-1.53E-04
43	MODAL	LinModal	11	-1.49E-03	-3.58E-02	6.37E-04	1.28E-04	9.98E-06	1.44E-04
43	MODAL	LinModal	12	-3.38E-02	3.38E-02	-5.89E-09	6.63E-05	6.63E-05	5.21E-03
44	MODAL	LinModal	1	3.29E-02	5.26E-04	-4.94E-04	-2.97E-05	1.86E-03	2.90E-06
44	MODAL	LinModal	2	5.17E-04	0.03290705	4.79E-04	1.86E-03	2.93E-05	3.00E-06
44	MODAL	LinModal	3	-3.09E-02	3.09E-02	6.52E-15	-1.58E-03	-1.58E-03	6.10E-03
44	MODAL	LinModal	4	4.58E-03	9.56E-04	1.64E-03	1.82E-03	-8.70E-03	2.32E-06
44	MODAL	LinModal	5	9.51E-04	-4.58E-03	-1.08E-03	-8.69E-03	-1.80E-03	3.53E-06
44	MODAL	LinModal	6	3.55E-03	-3.55E-03	1.74E-12	-7.85E-03	-7.85E-03	-6.33E-04
44	MODAL	LinModal	7	-2.68E-02	2.13E-02	-1.26E-04	2.44E-03	3.07E-03	-1.08E-04
44	MODAL	LinModal	8	2.15E-02	2.70E-02	1.11E-03	3.08E-03	-2.46E-03	-1.22E-05
44	MODAL	LinModal	9	-0.032076	3.21E-02	9.18E-10	3.68E-03	3.68E-03	5.29E-03
44	MODAL	LinModal	10	2.83E-02	-6.35E-04	7.85E-04	9.41E-05	5.23E-03	1.20E-04
44	MODAL	LinModal	11	1.17E-03	2.84E-02	8.37E-04	-5.23E-03	2.40E-04	-1.13E-04
44	MODAL	LinModal	12	2.72E-02	-2.72E-02	1.88E-09	4.49E-03	4.49E-03	-4.18E-03
45	MODAL	LinModal	1	3.87E-02	6.18E-04	-5.18E-04	-1.42E-05	9.18E-04	3.23E-06
45	MODAL	LinModal	2	6.09E-04	-3.87E-02	5.02E-04	9.18E-04	1.49E-05	3.33E-06
45	MODAL	LinModal	3	-3.60E-02	3.60E-02	-3.45E-14	-7.05E-04	-7.05E-04	7.11E-03
45	MODAL	LinModal	4	-3.26E-02	-6.81E-03	1.86E-03	1.32E-03	-6.36E-03	-2.02E-05
45	MODAL	LinModal	5	-6.73E-03	3.25E-02	-1.22E-03	-6.35E-03	-1.32E-03	-3.08E-05
45	MODAL	LinModal	6	-3.13E-02	3.13E-02	-7.88E-12	-5.39E-03	-5.39E-03	5.73E-03
45	MODAL	LinModal	7	1.78E-02	-1.42E-02	-1.79E-04	7.34E-03	9.23E-03	7.20E-05
45	MODAL	LinModal	8	-0.0143363	-1.80E-02	1.58E-03	9.28E-03	-7.40E-03	8.15E-06
45	MODAL	LinModal	9	2.20E-02	0.02198135	-6.73E-10	0.0101148	1.01E-02	-3.65E-03
45	MODAL	LinModal	10	-1.08E-02	2.47E-04	1.07E-03	-2.71E-04	-1.00E-02	-4.54E-05
45	MODAL	LinModal	11	-4.45E-04	-1.08E-02	1.14E-03	0.0100119	-3.68E-04	4.26E-05
45	MODAL	LinModal	12	-1.07E-02	1.07E-02	5.38E-09	-8.58E-03	-8.58E-03	1.65E-03

Table 4.5 Correlation Coefficients (CQC Rule)

Correla	Correlaton Coefficient matrix												
Mode	1	2	3	4	5	6	7	8	9	10	11	12	
1	1.0000	1.0000	0.3726	0.0059	0.0059	0.0046	0.0020	0.0020	0.0017	0.0012	0.0012	0.0010	
2	1.0000	1.0000	0.3726	0.0059	0.0059	0.0046	0.0020	0.0020	0.0017	0.0012	0.0012	0.0010	
3	0.3726	0.3726	1.0000	0.0079	0.0079	0.0060	0.0025	0.0025	0.0021	0.0014	0.0014	0.0012	
4	0.0059	0.0059	0.0079	1.0000	1.0000	0.4150	0.0295	0.0295	0.0208	0.0111	0.0111	0.0089	
5	0.0059	0.0059	0.0079	1.0000	1.0000	0.4150	0.0295	0.0295	0.0208	0.0111	0.0111	0.0089	
6	0.0046	0.0046	0.0060	0.4150	0.4150	1.0000	0.0478	0.0478	0.0315	0.0154	0.0154	0.0120	
7	0.0020	0.0020	0.0025	0.0295	0.0295	0.0478	1.0000	1.0000	0.4950	0.0893	0.0893	0.0553	
8	0.0020	0.0020	0.0025	0.0295	0.0295	0.0478	1.0000	1.0000	0.4950	0.0893	0.0893	0.0553	
9	0.0017	0.0017	0.0021	0.0208	0.0208	0.0315	0.4950	0.4950	1.0000	0.1761	0.1761	0.0952	
10	0.0012	0.0012	0.0014	0.0111	0.0111	0.0154	0.0893	0.0893	0.1761	1.0000	1.0000	0.5518	
11	0.0012	0.0012	0.0014	0.0111	0.0111	0.0154	0.0893	0.0893	0.1761	1.0000	1.0000	0.5518	
12	0.0010	0.0010	0.0012	0.0089	0.0089	0.0120	0.0553	0.0553	0.0952	0.5518	0.5518	1.0000	

Table 4.6 Response Spectrum Method Results & Derived Envelop

TABLE: Elemen	t Forces	s - Frames				
Frame	Joint	CaseType	StepType	Р	M2	M3
Element No.	No	No Text		KN	KN-m	KN-m
1	1	LinRespSpec	Max	1409.407766	824.5920751	1025.785
1	2 LinRespSpec		Max	1409.407766	412.5759822	507.7506

Response Matrix for P-M (2-2)

1409.40775	662.626192
662.6261918	411.428175

Response Matrix for P-M (3-3)

1409.408	777.0537664
777.0538	506.9453777

Deriving Elliptical Envelop From Response Matrix

When m=2, a convenient closed-form solution for the eigenvectors and eigenvalues of **X** can be derived and used to define the envelope. Let Ψ denote the counterclockwise angle between the coordinate axes (x_1, x_2) and the principal axes (x'_1, x'_2) of the ellipse, as shown in Fig. 3.3. It follows that

$$\Psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$
 4.1

The principal axes of this ellipsoid coincide with the coordinate system defined by the columns of Ψ , which were identified as the eigenvectors of \mathbf{X} ; that is, the eigenvectors of \mathbf{X} are the principal axes of the envelope. In addition, the length of the r^{th} semiaxis is λ_r , which is the square root of the r^{th} eigenvalue of \mathbf{X} . λ_1 and λ_2 , the lengths of the semiaxes of the ellipse are

$$\lambda_1^2 = X_1^2 \cos^2 \psi + X_2^2 \sin^2 \psi + 2X_{12} \sin \psi \cos \psi$$
 4.2

$$\lambda_2^2 = X_1^2 \sin^2 \psi + X_2^2 \cos^2 \psi - 2X_{12} \sin \psi \cos \psi$$
 4.3

To obtain the angle Ψ , we have to solve the eigenvalue equation

$$\psi = \frac{1}{2} \tan^{-1} \left(\frac{2X_{12}}{X_1^2 - X_2^2} \right)$$
 4.4

Hence, the size and orientation of the elliptical envelope in a 2D response space are completely defined by 3.9, 3.10 and 3.11 in terms of the elements of the response matrix \mathbf{X} .

$$X = \begin{bmatrix} X_1^2 & X_{12} \\ X_{12} & X_2^2 \end{bmatrix}$$
 4.5

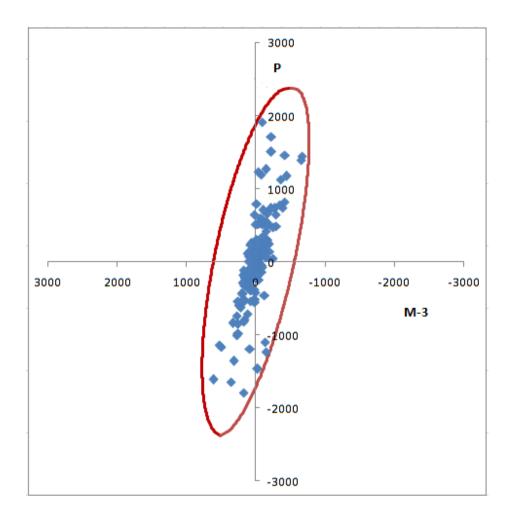


Fig 4.3 Elliptical Envelop Enclosing P – M3 occurring simultaneously

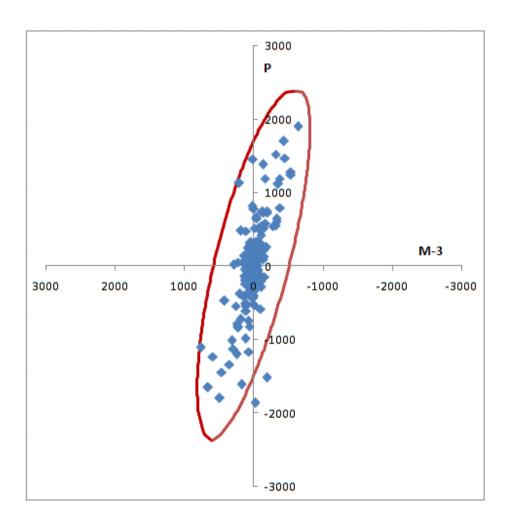


Fig 4.4 Elliptical Envelop Enclosing P – M2 occurring simultaneously

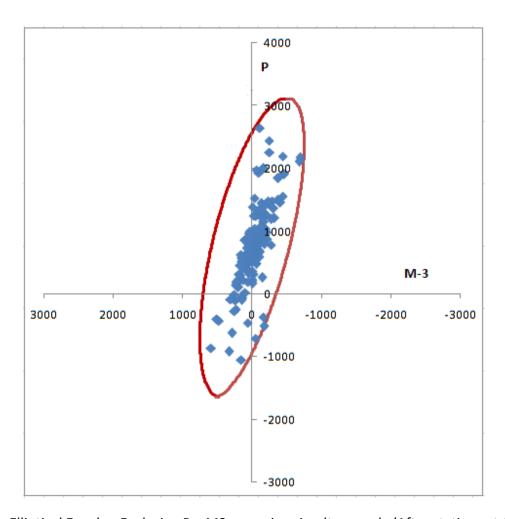


Fig 4.5 Elliptical Envelop Enclosing P – M3 occurring simultaneously (After static contribution)

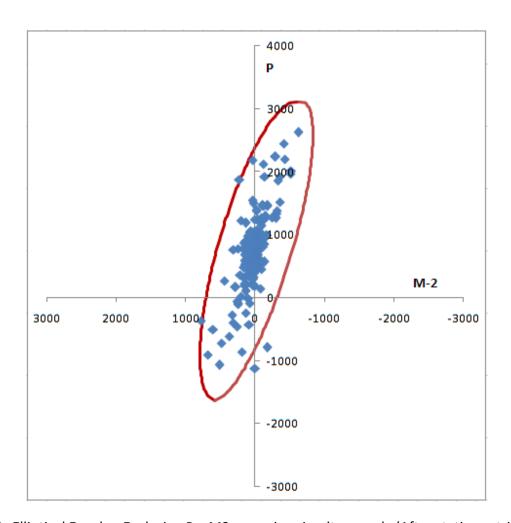


Fig 4.6 Elliptical Envelop Enclosing P – M2 occurring simultaneously (After static contribution)

NUMERICAL STUDY 2

This Numerical Study is aim to show that whether or not the use of the elliptical envelope, rather than the rectangular envelope commonly used in the current practice, has any significant effect on the design of structural elements subjected to interacting forces. In this exercise 4 - storey building frame model has been solved with Response spectrum method to derive rectangular envelop and with new approach to derive elliptical envelop. Supreme envelop that bounds responses of structure when principle direction of earthquake is not known in advance has been derive. The model and loading is same as previous problem.

In this problem Modal analysis has been performed in SAP to have Natural frequency, mode shapes and modal participation factor. . SAP2000 v 11 (Advanced) also give the no. of equation it is using for calculation in .TXE file. These data are arranged in compatible form to derive elliptical envelop in Excel.

We have performed Response Spectrum Analysis to verify the diagonal terms of Response Matrix, which are nothing but peak values of responses given by Response Spectrum Method. The values are shown in table 4.6. The Design Response Spectra as given in IS 1893 – 2002 (part 1)(Criteria for Earthquake Resistance Design of Structures) are applied in U1, U2 and U3 direction, by default in SAP these direction are in global X, Y and Z direction of model, these are also principal axes of our model (Angle between principal axes of structure and earthquake component, $\theta = 0$). We have to factored forces due to second and third component added subsequently by 0.3 as recommended in code, we also need to apply scale factor 2/3 to design response spectra when it is applied in U3 i.e. vertically downwards direction (Load cases are given in table 4.7). We compared required reinforcement ratio values comes from rectangle and elliptical envelop on subsequently adding earthquake component in all the three directions.

We have considered 75 mode shapes both in Response spectrum analysis and in deriving elliptical envelop so that for both the horizontal component and vertical component, 99% participating mass can be included.

We have performed above analysis for full range of θ , so that we can get supreme envelop. This envelop averagely encloses simultaneous occurrence of Axial forces and Moments in any column, when principal direction of earthquake is not known in advance. We have compared required reinforcement ratio values comes from rectangle and supreme envelop.

The contribution of static forces arising from dead loads is included in this analysis. Static loads are cause time-invariant components to act concurrently. Because the structure is linear,

these can be added to yield a total response vector that varies in time about point in the response space. The size and orientation of the two envelopes are unaffected by the presence of these static responses.

Using moment-axial capacity surfaces computed in SP – 16, Design Aids for Reinforced Concrete to IS 456. and the elliptical envelope, the required reinforcement are computed. We have taken circular column section so that problem reduces to uniaxial bending, otherwise for non-axsymmetric reinforced column section the neutral axis is generally not parallel to resultant axis of bending. In fact determination of N.A. is laborious process of trial and error. The reinforcement ratio is compared to the maximum reinforcement ratio required when the rectangular envelope is used to design the column.

The required reinforcement ratios for column A and B for different load case when the elliptical envelopes and when rectangular envelopes are used are summarized in Table 4.2. The amount of additional reinforcement required if the rectangular envelope is used to compute reinforcement ratio instead of the elliptical envelope is expressed as the percentage. It is showing the significance of using this new approach over conventional approach.

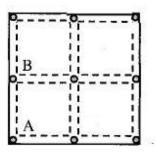


Fig 4.7 Example 4-Storey RCC Building (Typical Floor plan showing Column A & B)

Table 4.7 Dynamic Load Cases considered

Lood	Response Spectra in Principal Direction								
Load Cases	Major	Intermediate	Minor						
1	S_1	0	0						
2	S_1	S_1							
3	S ₁	S ₁	0.67\$1						

Modal Analysis Results

Table 4.8 Modal Periods, Frequency, Participating Mass Ratio and Modal mass and Modal stiffness

Modo	Period	Frequency	Damping	Sum of Mo	odal Mass Pa	rticipation	ModalMass	ModalStiff
Mode Shape				in U1	in U2	in U3	•	
Shape	Sec	Cyc/sec	Ratio	dir.%	dir.%	dir.%	KN-m-s2	KN-m
1	0.86462751	7.26693	0.05	0.848087	0.000213	0.000000	1	310.3160561
2	0.86462751	7.26693	0.05	0.848300	0.848300	0.000000	1	310.3160561
3	0.75964793	8.27118	0.05	0.848300	0.848300	0.000000	1	370.3385458
4	0.27698048	22.68458	0.05	0.949315	0.852674	0.000000	1	3048.571074
5	0.27698048	22.68458	0.05	0.953689	0.953689	0.000000	1	3048.571074
6	0.24603161	25.53812	0.05	0.953689	0.953689	0.000000	1	3622.845038
7	1.59E-01	39.53986	0.05	0.975804	0.967725	0.000000	1	9380.300587
8	0.15890763	39.53986	0.05	0.989839	0.989839	0.000000	1	9380.300587
9	1.44E-01	43.73447	0.05	0.989839	0.989839	0.000000	1	11087.88671
10	1.16E-01	54.22993	0.05	0.999980	0.989850	0.000000	1	17791.62233
11	1.16E-01	54.22993	0.05	0.999990	0.999990	0.000000	1	17791.62233
12	1.06E-01	59.33708	0.05	0.999990	0.999990	0.000000	1	18800.97155
13	9.88E-02	63.58040	0.05	0.999990	0.999990	0.316836	1	18800.97155
14	8.35E-02	75.23340	0.05	0.999990	0.999991	0.316836	1	18800.97155
15	8.35E-02	75.23340	0.05	0.999991	0.999991	0.316836	1	18800.97155
16	8.27E-02	76.01866	0.05	0.999991	0.999991	0.316836	1	18800.97155
17	8.26E-02	76.08052	0.05	0.999991	0.999991	0.736858	1	18800.97155
18	6.89E-02	91.14421	0.05	0.999991	0.999991	0.736858	1	18800.97155
19	6.81E-02	92.22482	0.05	0.999991	0.999992	0.736858	1	18800.97155
20	6.81E-02	92.22482	0.05	0.999992	0.999992	0.736858	1	18800.97155
21	6.74E-02	93.19361	0.05	0.999992	0.999992	0.893982	1	18800.97155
22	3.66E-02	171.50515	0.05	0.999992	0.999992	0.910530	1	18800.97155
23	3.51E-02	179.07037	0.05	0.999992	0.999992	0.910530	1	18800.97155
24	3.47E-02	181.06436	0.05	0.999992	0.999992	0.910530	1	18800.97155
25	3.46E-02	181.35370	0.05	0.999993	0.999992	0.910530	1	18800.97155
26	3.46E-02	181.35370	0.05	0.999993	0.999993	0.910530	1	18800.97155
27	3.42E-02	183.48813	0.05	0.999993	0.999993	0.910530	1	18800.97155
28	3.42E-02	183.80884	0.05	0.999993	0.999994	0.910530	1	18800.97155
29	3.42E-02	183.80884	0.05	0.999994	0.999994	0.910530	1	18800.97155
30	3.38E-02	186.14773	0.05	0.999994	0.999994	0.910530	1	18800.97155
31	3.36E-02	186.82388	0.05	0.999994	0.999996	0.910530	1	18800.97155
32	3.36E-02	186.82388	0.05	0.999996	0.999996	0.910530	1	18800.97155
33	3.31E-02	189.81778	0.05	0.999998	0.999996	0.910530	1	18800.97155
34	3.31E-02	189.81778	0.05	0.999998	0.999998	0.910530	1	18800.97155
35	3.17E-02	198.11233	0.05	0.999998	0.999998	0.910530	1	18800.97155
36	3.13E-02	200.66284	0.05	0.999998	0.999998	0.910530	1	18800.97155

N/Landa	Period	Frequency	Damping	Sum of Mo	odal Mass Pa	rticipation	ModalMass	ModalStiff
Mode Shape				in U1	in U2	in U3		
Shape	Sec	Cyc/sec	Ratio	dir.%	dir.%	dir.%	KN-m-s2	KN-m
37	3.08E-02	203.91320	0.05	0.999998	0.999998	0.910530	1	18800.97155
38	3.04E-02	206.36361	0.05	0.999999	0.999999	0.910530	1	18800.97155
39	3.04E-02	206.36361	0.05	0.999999	0.999999	0.910530	1	18800.97155
40	3.04E-02	206.50489	0.05	0.999999	0.999999	0.910530	1	18800.97155
41	3.04E-02	206.52494	0.05	0.999999	0.999999	0.910530	1	18800.97155
42	3.04E-02	206.61732	0.05	0.999999	0.999999	0.951858	1	18800.97155
43	2.84E-02	221.04188	0.05	0.999999	0.999999	0.951858	1	18800.97155
44	2.81E-02	223.73069	0.05	0.999999	0.999999	0.951858	1	18800.97155
45	2.76E-02	227.59635	0.05	0.999999	0.999999	0.951858	1	18800.97155
46	2.73E-02	230.09805	0.05	0.999999	0.999999	0.951858	1	18800.97155
47	2.46E-02	254.96571	0.05	0.999999	0.999999	0.951858	1	18800.97155
48	2.46E-02	255.17456	0.05	0.999999	0.999999	0.951858	1	18800.97155
49	2.46E-02	255.17456	0.05	0.999999	0.999999	0.951858	1	18800.97155
50	2.46E-02	255.45719	0.05	0.999999	0.999999	0.977078	1	18800.97155
51	2.41E-02	260.60040	0.05	0.999999	0.999999	0.980672	1	18800.97155
52	2.39E-02	262.59440	0.05	0.999999	0.999999	0.980672	1	18800.97155
53	2.39E-02	262.59440	0.05	0.999999	0.999999	0.980672	1	18800.97155
54	2.37E-02	264.63934	0.05	0.999999	0.999999	0.980672	1	18800.97155
55	2.37E-02	264.63934	0.05	0.999999	0.999999	0.980672	1	18800.97155
56	2.35E-02	267.86448	0.05	0.999999	0.999999	0.980672	1	18800.97155
57	2.35E-02	267.86448	0.05	0.999999	0.999999	0.980672	1	18800.97155
58	2.33E-02	270.20238	0.05	0.999999	0.999999	0.980672	1	18800.97155
59	2.33E-02	270.20238	0.05	0.999999	0.999999	0.980672	1	18800.97155
60	2.26E-02	278.00113	0.05	0.999999	0.999999	0.980672	1	18800.97155
61	2.25E-02	279.48944	0.05	0.999999	0.999999	0.980676	1	18800.97155
62	2.23E-02	281.88150	0.05	0.999999	0.999999	0.980678	1	18800.97155
63	2.21E-02	283.92898	0.05	0.999999	0.999999	0.980680	1	18800.97155
64	2.17E-02	288.91374	0.05	0.999999	0.999999	0.980682	1	18800.97155
65	2.16E-02	291.06979	0.05	0.999999	0.999999	0.980682	1	18800.97155
66	2.13E-02	294.60884	0.05	0.999999	0.999999	0.980685	1	18800.97155
67	2.11E-02	298.18028	0.05	0.999999	0.999999	0.980685	1	18800.97155
68	2.01E-02	311.99034	0.05	0.999999	0.999999	0.980685	1	18800.97155
69	2.01E-02	311.99034	0.05	0.999999	0.999999	0.980685	1	18800.97155
70	2.01E-02	313.30975	0.05	0.999999	0.999999	0.980685	1	18800.97155
71	2.01E-02	313.30975	0.05	0.999999	0.999999	0.980685	1	18800.97155
72	2.00E-02	314.01443	0.05	0.999999	0.999999	0.980685	1	18800.97155
73	2.00E-02	314.01443	0.05	0.999999	0.999999	0.980685	1	18800.97155
74	2.00E-02	314.08767	0.05	0.999999	0.999999	0.980685	1	18800.97155
75	2.00E-02	314.17667	0.05	0.999999	0.999999	0.990315	1	18800.97155

Table 4.9 Modal Participation Factors

Modes	Period	UX	UY	UZ
Shapes	Sec	KN-sec ²	KN-sec ²	KN-sec ²
1	0.864628	-3.27E+01	-5.18E-01	-1.21E-12
2	0.864628	-5.18E-01	3.27E+01	-2.14E-13
3	0.759648	-3.22E-12	-2.36E-12	-3.30E-11
4	0.27698	-1.13E+01	-2.35E+00	-1.15E-07
5	0.27698	-2.35E+00	1.13E+01	-3.32E-09
6	0.246032	-1.62E-11	-1.03E-10	-6.06E-09
7	1.59E-01	-5.28E+00	4.21E+00	1.04E-07
8	0.158908	4.21E+00	5.28E+00	4.43E-07
9	1.44E-01	-4.42E-09	-1.63E-08	-4.36E-07
10	1.16E-01	-3.58E+00	1.14E-01	2.02E-07
11	1.16E-01	-1.14E-01	-3.58E+00	-3.47E-07
12	1.06E-01	8.32E-09	-5.29E-08	1.20E-06
13	9.88E-02	7.66E-10	-1.52E-08	-2.00E+01
14	8.35E-02	1.51E-02	3.09E-02	-2.06E-06
15	8.35E-02	-3.09E-02	1.51E-02	-1.78E-06
16	8.27E-02	-3.51E-09	-2.16E-09	-4.96E-07
17	8.26E-02	-3.43E-09	-1.30E-09	2.30E+01
18	6.89E-02	-1.43E-09	2.28E-08	-3.66E-06
19	6.81E-02	-2.56E-03	4.25E-02	-8.08E-07
20	6.81E-02	-4.25E-02	-2.56E-03	-2.76E-06
21	6.74E-02	-2.28E-08	1.10E-07	1.41E+01
22	3.66E-02	1.17E-07	-9.56E-07	4.57E+00
23	3.51E-02	6.42E-08	2.81E-08	1.02E-04
24	3.47E-02	-4.34E-07	-3.10E-06	6.62E-05
25	3.46E-02	2.09E-02	3.10E-03	7.29E-08
26	3.46E-02	-3.10E-03	2.09E-02	2.63E-05
27	3.42E-02	-7.23E-08	-3.10E-07	4.78E-05
28	3.42E-02	-7.89E-03	-4.31E-02	2.88E-05
29	3.42E-02	-4.31E-02	7.89E-03	-1.68E-05
30	3.38E-02	1.78E-07	1.92E-06	-4.37E-05
31	3.36E-02	3.04E-03	5.13E-02	-2.96E-05
32	3.36E-02	-5.13E-02	3.04E-03	2.45E-05
33	3.31E-02	5.06E-02	-4.85E-03	1.19E-05
34	3.31E-02	4.85E-03	5.06E-02	5.02E-05
35	3.17E-02	2.19E-07	2.36E-07	-3.02E-05
36	3.13E-02	-4.77E-07	4.53E-07	-4.47E-05
37	3.08E-02	-2.13E-07	-5.50E-07	4.48E-05
38	3.04E-02	1.51E-02	-1.21E-02	-1.73E-05

Modes	Period	UX	UY	UZ
Shapes	Sec	KN-sec ²	KN-sec ²	KN-sec ²
39	3.04E-02	-1.21E-02	-1.51E-02	-3.28E-05
40	3.04E-02	-4.71E-08	5.17E-08	3.40E-07
41	3.04E-02	1.05E-07	-3.31E-07	-1.40E-05
42	3.04E-02	-9.55E-08	6.88E-08	7.22E+00
43	2.84E-02	2.44E-07	-2.94E-07	-1.33E-04
44	2.81E-02	-4.73E-08	1.05E-08	3.03E-05
45	2.76E-02	3.31E-07	3.32E-08	-3.46E-05
46	2.73E-02	2.75E-07	-5.73E-07	-5.06E-05
47	2.46E-02	5.23E-08	-2.22E-07	-4.35E-05
48	2.46E-02	-8.57E-03	-2.03E-02	1.30E-05
49	2.46E-02	2.03E-02	-8.57E-03	-9.86E-05
50	2.46E-02	-1.45E-08	4.16E-07	-5.64E+00
51	2.41E-02	9.83E-08	2.67E-07	-2.13E+00
52	2.39E-02	-6.14E-03	-7.21E-04	3.61E-05
53	2.39E-02	-7.21E-04	6.14E-03	1.80E-05
54	2.37E-02	2.07E-03	-9.06E-03	1.91E-05
55	2.37E-02	-9.06E-03	-2.07E-03	2.58E-05
56	2.35E-02	-1.69E-03	8.91E-03	3.75E-05
57	2.35E-02	8.91E-03	1.69E-03	-4.14E-05
58	2.33E-02	1.83E-03	2.07E-03	-1.83E-06
59	2.33E-02	-2.07E-03	1.83E-03	-6.16E-05
60	2.26E-02	-1.78E-07	2.32E-07	1.46E-02
61	2.25E-02	1.50E-08	6.60E-08	-7.23E-02
62	2.23E-02	-3.20E-08	2.27E-07	-4.99E-02
63	2.21E-02	-3.06E-08	-1.56E-07	-4.47E-02
64	2.17E-02	1.30E-07	-1.91E-07	5.52E-02
65	2.16E-02	-6.65E-08	-2.69E-08	-4.90E-04
66	2.13E-02	-2.62E-07	4.65E-07	5.79E-02
67	2.11E-02	-2.62E-07	6.15E-07	-3.49E-03
68	2.01E-02	-5.77E-04	1.86E-03	5.45E-05
69	2.01E-02	1.86E-03	5.77E-04	-3.24E-05
70	2.01E-02	-4.53E-03	-2.13E-03	9.02E-05
71	2.01E-02	-2.13E-03	4.53E-03	7.29E-05
72	2.00E-02	-8.88E-03	-4.10E-03	-2.97E-06
73	2.00E-02	4.10E-03	-8.88E-03	3.47E-05
74	2.00E-02	-3.14E-07	2.62E-07	6.34E-05
75	2.00E-02	4.41E-07	-1.17E-07	3.48E+00

Table 4.10 Response Spectra Ordinates for Load Case 1

	Acce	leration Resp	onse	Displacement Response		
Modes		(SAP Uses)		(To Derive Envelop)		
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp
	m/sec ²	m/sec ²	m/sec ²	m	m	m
1	1.76400	0.00000	0.00000	0.03340	0.00000	0.00000
2	1.76400	0.00000	0.00000	0.03340	0.00000	0.00000
3	1.76400	0.00000	0.00000	0.02578	0.00000	0.00000
4	2.20500	0.00000	0.00000	0.00428	0.00000	0.00000
5	2.20500	0.00000	0.00000	0.00428	0.00000	0.00000
6	2.20500	0.00000	0.00000	0.00338	0.00000	0.00000
7	2.20500	0.00000	0.00000	0.00141	0.00000	0.00000
8	2.20500	0.00000	0.00000	0.00141	0.00000	0.00000
9	2.20500	0.00000	0.00000	0.00115	0.00000	0.00000
10	2.20500	0.00000	0.00000	0.00075	0.00000	0.00000
11	2.20500	0.00000	0.00000	0.00075	0.00000	0.00000
12	2.20500	0.00000	0.00000	0.00063	0.00000	0.00000
13	2.18942	0.00000	0.00000	0.00054	0.00000	0.00000
14	1.98692	0.00000	0.00000	0.00035	0.00000	0.00000
15	1.98692	0.00000	0.00000	0.00035	0.00000	0.00000
16	1.97550	0.00000	0.00000	0.00034	0.00000	0.00000
17	1.97461	0.00000	0.00000	0.00034	0.00000	0.00000
18	1.79403	0.00000	0.00000	0.00022	0.00000	0.00000
19	1.78335	0.00000	0.00000	0.00021	0.00000	0.00000
20	1.78335	0.00000	0.00000	0.00021	0.00000	0.00000
21	1.77398	0.00000	0.00000	0.00020	0.00000	0.00000
22	1.36669	0.00000	0.00000	0.00005	0.00000	0.00000
23	1.34621	0.00000	0.00000	0.00004	0.00000	0.00000
24	1.34110	0.00000	0.00000	0.00004	0.00000	0.00000
25	1.34037	0.00000	0.00000	0.00004	0.00000	0.00000
26	1.34037	0.00000	0.00000	0.00004	0.00000	0.00000
27	1.33503	0.00000	0.00000	0.00004	0.00000	0.00000
28	1.33424	0.00000	0.00000	0.00004	0.00000	0.00000
29	1.33424	0.00000	0.00000	0.00004	0.00000	0.00000
30	1.32856	0.00000	0.00000	0.00004	0.00000	0.00000
31	1.32695	0.00000	0.00000	0.00004	0.00000	0.00000
32	1.32695	0.00000	0.00000	0.00004	0.00000	0.00000
33	1.31993	0.00000	0.00000	0.00004	0.00000	0.00000
34	1.31993	0.00000	0.00000	0.00004	0.00000	0.00000
35	1.30159	0.00000	0.00000	0.00003	0.00000	0.00000
36	1.29626	0.00000	0.00000	0.00003	0.00000	0.00000
37	1.28966	0.00000	0.00000	0.00003	0.00000	0.00000

	Acce	leration Resp	onse	Displacement Response		
Modes		(SAP Uses)		(To Derive Envelop)		
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp
	m/sec ²	m/sec ²	m/sec ²	m	m	m
38	1.28482	0.00000	0.00000	0.00003	0.00000	0.00000
39	1.28482	0.00000	0.00000	0.00003	0.00000	0.00000
40	1.28454	0.00000	0.00000	0.00003	0.00000	0.00000
41	1.28450	0.00000	0.00000	0.00003	0.00000	0.00000
42	1.28432	0.00000	0.00000	0.00003	0.00000	0.00000
43	1.25807	0.00000	0.00000	0.00003	0.00000	0.00000
44	1.25355	0.00000	0.00000	0.00003	0.00000	0.00000
45	1.24724	0.00000	0.00000	0.00002	0.00000	0.00000
46	1.24327	0.00000	0.00000	0.00002	0.00000	0.00000
47	1.20803	0.00000	0.00000	0.00002	0.00000	0.00000
48	1.20776	0.00000	0.00000	0.00002	0.00000	0.00000
49	1.20776	0.00000	0.00000	0.00002	0.00000	0.00000
50	1.20740	0.00000	0.00000	0.00002	0.00000	0.00000
51	1.20098	0.00000	0.00000	0.00002	0.00000	0.00000
52	1.19856	0.00000	0.00000	0.00002	0.00000	0.00000
53	1.19856	0.00000	0.00000	0.00002	0.00000	0.00000
54	1.19611	0.00000	0.00000	0.00002	0.00000	0.00000
55	1.19611	0.00000	0.00000	0.00002	0.00000	0.00000
56	1.19233	0.00000	0.00000	0.00002	0.00000	0.00000
57	1.19233	0.00000	0.00000	0.00002	0.00000	0.00000
58	1.18965	0.00000	0.00000	0.00002	0.00000	0.00000
59	1.18965	0.00000	0.00000	0.00002	0.00000	0.00000
60	1.18102	0.00000	0.00000	0.00002	0.00000	0.00000
61	1.17942	0.00000	0.00000	0.00002	0.00000	0.00000
62	1.17690	0.00000	0.00000	0.00001	0.00000	0.00000
63	1.17477	0.00000	0.00000	0.00001	0.00000	0.00000
64	1.16972	0.00000	0.00000	0.00001	0.00000	0.00000
65	1.16759	0.00000	0.00000	0.00001	0.00000	0.00000
66	1.16416	0.00000	0.00000	0.00001	0.00000	0.00000
67	1.16078	0.00000	0.00000	0.00001	0.00000	0.00000
68	1.14844	0.00000	0.00000	0.00001	0.00000	0.00000
69	1.14844	0.00000	0.00000	0.00001	0.00000	0.00000
70	1.14732	0.00000	0.00000	0.00001	0.00000	0.00000
71	1.14732	0.00000	0.00000	0.00001	0.00000	0.00000
72	1.14672	0.00000	0.00000	0.00001	0.00000	0.00000
73	1.14672	0.00000	0.00000	0.00001	0.00000	0.00000
74	1.14666	0.00000	0.00000	0.00001	0.00000	0.00000
75	1.14659	0.00000	0.00000	0.00001	0.00000	0.00000

Table 4.11 Response Spectra Ordinates for Load Case 2

	Acce	leration Resp	onse	Displacement Response		
Modes		(SAP Uses)		(To Derive Envelop)		
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp
	m/sec ²	m/sec ²	m/sec ²	m	m	m
1	1.76400	1.76400	0.00000	0.03340	0.03340	0.00000
2	1.76400	1.76400	0.00000	0.03340	0.03340	0.00000
3	1.76400	1.76400	0.00000	0.02578	0.02578	0.00000
4	2.20500	2.20500	0.00000	0.00428	0.00428	0.00000
5	2.20500	2.20500	0.00000	0.00428	0.00428	0.00000
6	2.20500	2.20500	0.00000	0.00338	0.00338	0.00000
7	2.20500	2.20500	0.00000	0.00141	0.00141	0.00000
8	2.20500	2.20500	0.00000	0.00141	0.00141	0.00000
9	2.20500	2.20500	0.00000	0.00115	0.00115	0.00000
10	2.20500	2.20500	0.00000	0.00075	0.00075	0.00000
11	2.20500	2.20500	0.00000	0.00075	0.00075	0.00000
12	2.20500	2.20500	0.00000	0.00063	0.00063	0.00000
13	2.18942	2.18942	0.00000	0.00054	0.00054	0.00000
14	1.98692	1.98692	0.00000	0.00035	0.00035	0.00000
15	1.98692	1.98692	0.00000	0.00035	0.00035	0.00000
16	1.97550	1.97550	0.00000	0.00034	0.00034	0.00000
17	1.97461	1.97461	0.00000	0.00034	0.00034	0.00000
18	1.79403	1.79403	0.00000	0.00022	0.00022	0.00000
19	1.78335	1.78335	0.00000	0.00021	0.00021	0.00000
20	1.78335	1.78335	0.00000	0.00021	0.00021	0.00000
21	1.77398	1.77398	0.00000	0.00020	0.00020	0.00000
22	1.36669	1.36669	0.00000	0.00005	0.00005	0.00000
23	1.34621	1.34621	0.00000	0.00004	0.00004	0.00000
24	1.34110	1.34110	0.00000	0.00004	0.00004	0.00000
25	1.34037	1.34037	0.00000	0.00004	0.00004	0.00000
26	1.34037	1.34037	0.00000	0.00004	0.00004	0.00000
27	1.33503	1.33503	0.00000	0.00004	0.00004	0.00000
28	1.33424	1.33424	0.00000	0.00004	0.00004	0.00000
29	1.33424	1.33424	0.00000	0.00004	0.00004	0.00000
30	1.32856	1.32856	0.00000	0.00004	0.00004	0.00000
31	1.32695	1.32695	0.00000	0.00004	0.00004	0.00000
32	1.32695	1.32695	0.00000	0.00004	0.00004	0.00000
33	1.31993	1.31993	0.00000	0.00004	0.00004	0.00000
34	1.31993	1.31993	0.00000	0.00004	0.00004	0.00000
35	1.30159	1.30159	0.00000	0.00003	0.00003	0.00000
36	1.29626	1.29626	0.00000	0.00003	0.00003	0.00000
37	1.28966	1.28966	0.00000	0.00003	0.00003	0.00000

	Acce	Acceleration Response			Displacement Response			
Modes		(SAP Uses)		(To Derive Envelop)				
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp		
	m/sec ²	m/sec ²	m/sec ²	m	m	m		
38	1.28482	1.28482	0.00000	0.00003	0.00003	0.00000		
39	1.28482	1.28482	0.00000	0.00003	0.00003	0.00000		
40	1.28454	1.28454	0.00000	0.00003	0.00003	0.00000		
41	1.28450	1.28450	0.00000	0.00003	0.00003	0.00000		
42	1.28432	1.28432	0.00000	0.00003	0.00003	0.00000		
43	1.25807	1.25807	0.00000	0.00003	0.00003	0.00000		
44	1.25355	1.25355	0.00000	0.00003	0.00003	0.00000		
45	1.24724	1.24724	0.00000	0.00002	0.00002	0.00000		
46	1.24327	1.24327	0.00000	0.00002	0.00002	0.00000		
47	1.20803	1.20803	0.00000	0.00002	0.00002	0.00000		
48	1.20776	1.20776	0.00000	0.00002	0.00002	0.00000		
49	1.20776	1.20776	0.00000	0.00002	0.00002	0.00000		
50	1.20740	1.20740	0.00000	0.00002	0.00002	0.00000		
51	1.20098	1.20098	0.00000	0.00002	0.00002	0.00000		
52	1.19856	1.19856	0.00000	0.00002	0.00002	0.00000		
53	1.19856	1.19856	0.00000	0.00002	0.00002	0.00000		
54	1.19611	1.19611	0.00000	0.00002	0.00002	0.00000		
55	1.19611	1.19611	0.00000	0.00002	0.00002	0.00000		
56	1.19233	1.19233	0.00000	0.00002	0.00002	0.00000		
57	1.19233	1.19233	0.00000	0.00002	0.00002	0.00000		
58	1.18965	1.18965	0.00000	0.00002	0.00002	0.00000		
59	1.18965	1.18965	0.00000	0.00002	0.00002	0.00000		
60	1.18102	1.18102	0.00000	0.00002	0.00002	0.00000		
61	1.17942	1.17942	0.00000	0.00002	0.00002	0.00000		
62	1.17690	1.17690	0.00000	0.00001	0.00001	0.00000		
63	1.17477	1.17477	0.00000	0.00001	0.00001	0.00000		
64	1.16972	1.16972	0.00000	0.00001	0.00001	0.00000		
65	1.16759	1.16759	0.00000	0.00001	0.00001	0.00000		
66	1.16416	1.16416	0.00000	0.00001	0.00001	0.00000		
67	1.16078	1.16078	0.00000	0.00001	0.00001	0.00000		
68	1.14844	1.14844	0.00000	0.00001	0.00001	0.00000		
69	1.14844	1.14844	0.00000	0.00001	0.00001	0.00000		
70	1.14732	1.14732	0.00000	0.00001	0.00001	0.00000		
71	1.14732	1.14732	0.00000	0.00001	0.00001	0.00000		
72	1.14672	1.14672	0.00000	0.00001	0.00001	0.00000		
73	1.14672	1.14672	0.00000	0.00001	0.00001	0.00000		
74	1.14666	1.14666	0.00000	0.00001	0.00001	0.00000		
75	1.14659	1.14659	0.00000	0.00001	0.00001	0.00000		

Table 4.12 Response Spectra Ordinates for Load Case 3

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
1	1.76400	1.76400	1.17589	0.03340	0.03340	0.02227	
2	1.76400	1.76400	1.17589	0.03340	0.03340	0.02227	
3	1.76400	1.76400	1.17589	0.02578	0.02578	0.01719	
4	2.20500	2.20500	1.46986	0.00428	0.00428	0.00286	
5	2.20500	2.20500	1.46986	0.00428	0.00428	0.00286	
6	2.20500	2.20500	1.46986	0.00338	0.00338	0.00225	
7	2.20500	2.20500	1.46986	0.00141	0.00141	0.00094	
8	2.20500	2.20500	1.46986	0.00141	0.00141	0.00094	
9	2.20500	2.20500	1.46986	0.00115	0.00115	0.00077	
10	2.20500	2.20500	1.46986	0.00075	0.00075	0.00050	
11	2.20500	2.20500	1.46986	0.00075	0.00075	0.00050	
12	2.20500	2.20500	1.46986	0.00063	0.00063	0.00042	
13	2.18942	2.18942	1.45947	0.00054	0.00054	0.00036	
14	1.98692	1.98692	1.32448	0.00035	0.00035	0.00023	
15	1.98692	1.98692	1.32448	0.00035	0.00035	0.00023	
16	1.97550	1.97550	1.31687	0.00034	0.00034	0.00023	
17	1.97461	1.97461	1.31628	0.00034	0.00034	0.00023	
18	1.79403	1.79403	1.19591	0.00022	0.00022	0.00014	
19	1.78335	1.78335	1.18878	0.00021	0.00021	0.00014	
20	1.78335	1.78335	1.18878	0.00021	0.00021	0.00014	
21	1.77398	1.77398	1.18254	0.00020	0.00020	0.00014	
22	1.36669	1.36669	0.91104	0.00005	0.00005	0.00003	
23	1.34621	1.34621	0.89739	0.00004	0.00004	0.00003	
24	1.34110	1.34110	0.89398	0.00004	0.00004	0.00003	
25	1.34037	1.34037	0.89349	0.00004	0.00004	0.00003	
26	1.34037	1.34037	0.89349	0.00004	0.00004	0.00003	
27	1.33503	1.33503	0.88994	0.00004	0.00004	0.00003	
28	1.33424	1.33424	0.88941	0.00004	0.00004	0.00003	
29	1.33424	1.33424	0.88941	0.00004	0.00004	0.00003	
30	1.32856	1.32856	0.88562	0.00004	0.00004	0.00003	
31	1.32695	1.32695	0.88454	0.00004	0.00004	0.00003	
32	1.32695	1.32695	0.88454	0.00004	0.00004	0.00003	
33	1.31993	1.31993	0.87987	0.00004	0.00004	0.00002	
34	1.31993	1.31993	0.87987	0.00004	0.00004	0.00002	
35	1.28450	0.00000	0.00000	0.00003	0.00003	0.00002	
36	1.28432	0.00000	0.00000	0.00003	0.00003	0.00002	
37	1.25807	0.00000	0.00000	0.00003	0.00003	0.00002	

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To Derive Envelop)			
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
38	1.30159	1.30159	0.86764	0.00003	0.00003	0.00002	
39	1.29626	1.29626	0.86409	0.00003	0.00003	0.00002	
40	1.28966	1.28966	0.85969	0.00003	0.00003	0.00002	
41	1.28482	1.28482	0.85646	0.00003	0.00003	0.00002	
42	1.28482	1.28482	0.85646	0.00003	0.00003	0.00002	
43	1.28454	1.28454	0.85628	0.00003	0.00003	0.00002	
44	1.28450	1.28450	0.85625	0.00003	0.00003	0.00002	
45	1.28432	1.28432	0.85613	0.00002	0.00002	0.00002	
46	1.25807	1.25807	0.83863	0.00002	0.00002	0.00002	
47	1.25355	1.25355	0.83562	0.00002	0.00002	0.00001	
48	1.24724	1.24724	0.83141	0.00002	0.00002	0.00001	
49	1.24327	1.24327	0.82876	0.00002	0.00002	0.00001	
50	1.20803	1.20803	0.80528	0.00002	0.00002	0.00001	
51	1.20776	1.20776	0.80510	0.00002	0.00002	0.00001	
52	1.20776	1.20776	0.80510	0.00002	0.00002	0.00001	
53	1.20740	1.20740	0.80486	0.00002	0.00002	0.00001	
54	1.20098	1.20098	0.80058	0.00002	0.00002	0.00001	
55	1.19856	1.19856	0.79896	0.00002	0.00002	0.00001	
56	1.19856	1.19856	0.79896	0.00002	0.00002	0.00001	
57	1.19611	1.19611	0.79733	0.00002	0.00002	0.00001	
58	1.19611	1.19611	0.79733	0.00002	0.00002	0.00001	
59	1.19233	1.19233	0.79481	0.00002	0.00002	0.00001	
60	1.19233	1.19233	0.79481	0.00002	0.00002	0.00001	
61	1.18965	1.18965	0.79302	0.00002	0.00002	0.00001	
62	1.18965	1.18965	0.79302	0.00001	0.00001	0.00001	
63	1.18102	1.18102	0.78727	0.00001	0.00001	0.00001	
64	1.17942	1.17942	0.78621	0.00001	0.00001	0.00001	
65	1.17690	1.17690	0.78452	0.00001	0.00001	0.00001	
66	1.17477	1.17477	0.78311	0.00001	0.00001	0.00001	
67	1.16972	1.16972	0.77974	0.00001	0.00001	0.00001	
68	1.16759	1.16759	0.77832	0.00001	0.00001	0.00001	
69	1.16416	1.16416	0.77603	0.00001	0.00001	0.00001	
70	1.16078	1.16078	0.77378	0.00001	0.00001	0.00001	
71	1.14844	1.14844	0.76555	0.00001	0.00001	0.00001	
72	1.14844	1.14844	0.76555	0.00001	0.00001	0.00001	
73	1.14732	1.14732	0.76480	0.00001	0.00001	0.00001	
74	1.14732	1.14732	0.76480	0.00001	0.00001	0.00001	
75	1.14672	1.14672	0.76441	0.00001	0.00001	0.00001	

Table 4.13 Response Spectrum Method Results & Derived Envelop (Load Case 1)

TABLE: Eleme	nt Forces - Fram						
Columns	Element	Joint	CaseType	StepType	Р	M2	M3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	443.8342	0.237466872	371.7235389
Α	1	2	LinRespSpec	Max	443.8342	0.212569975	205.3849347
В	5	6	LinRespSpec	Max	445.1653	2.20E-06	372.1361326
В	5	7	LinRespSpec	Max	445.1653	2.21E-06	205.7282773

Column A's Response Matrix for P-M3-M2

443.722208	296.3607233	8.612589815
296.360723	205.3812491	6.219430569
8.61258982	6.219430569	0.208762007

Column B's Response Matrix for P-M3-M2

445.052945	296.8817152	0.007287545
296.881715	205.7188758	0.007490786
0.00728754	0.007490786	2.1383E-06

Table 4.14 Response Spectrum Method Results & Derived Envelop (Load Case 2)

TABLE: Element Forces - Frames							
Columns		Joint	CaseType	StepType	Р	M2	М3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	505.9438	125.6994027	371.722634
Α	1	2	LinRespSpec	Max	505.9438	112.523762	205.3812347
В	5	6	LinRespSpec	Max	445.1529	178.0020995	372.1061326
В	5	7	LinRespSpec	Max	445.1529	178.8112	205.7172773

Column A's Response Matrix for P-M3-M2

505.921157	296.3982646	162.5517774
296.398265	205.3812809	7.091242257
162.551777	7.091242257	112.4921368

Column B's Response Matrix for P-M3-M2

445.052945	296.8817152	0.008080728
296.881715	205.7188758	0.007237009
0.00808073	0.007237009	178.0811112

Table 4.15 Response Spectrum Method Results & Derived Envelop (Load Case 3)

TABLE: Elemen							
Columns	Element	Joint	CaseType	StepType	Р	M2	М3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	513.737	125.6994027	371.7235389
Α	1	2	LinRespSpec	Max	513.737	112.523762	205.3823411
В	5	6	LinRespSpec	Max	475.4486	177.3250881	372.1361326
В	5	7	LinRespSpec	Max	475.4486	178.1311112	205.7282773

Column A's Response Matrix for P-M3-M2

513.725979	296.4012781	162.5572774
296.401278	205.3821711	7.116977009
162.557277	7.116977009	112.493762

Column B's Response Matrix for P-M3-M2

475.448583	296.838503	0.00575119
296.838503	205.7232608	0.007015151
0.00575119	0.007015151	178.0811112

Table 4.16 Element Forces for DEAD Load

Columns							
Columns	Element	Joint	CaseType	StepType	P	M2	M3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	DEAD		-743.5949	-10.1177	-10.1177
Α	1	2	DEAD		-729.7151	20.8731	20.8731
В	5	6	DEAD		-1158.2833	0.0000	-10.1931
В	5	7	DEAD		-1144.4035	0.0000	21.0913

Deriving Elliptical Envelop From Response Matrix

A convenient expression for developing the elliptical envelope derived is as follows

$$x = \frac{X\alpha}{\left(\alpha^T X\alpha\right)^{\frac{1}{2}}}$$
 4.6

This expression provides the Cartesian coordinates of a point on the envelope corresponding to a selected α . Note that the expression does not require inverting \mathbf{X} . Furthermore, due to the scaling involved, the vector α need not have a unit length. Since we have to find 3D Elliptical Envelop we determine α as follows

$$\alpha = \begin{bmatrix} r\cos\phi\cos\theta \\ r\cos\phi\sin\theta \\ r\sin\phi \end{bmatrix}$$
 4.7

When m=3 (Response Values) the solution for the eigenvectors and eigenvalues of **X** (Response Matrix) cannot be derived in similar way (with help of Eulerian angles) as we done for m=2 in the previous problem because we cannot associate a vector with a finite rotation represented by an orthogonal (radius preserving) transformation. The direction would be the axis of rotation, and the magnitude would be the angle of rotation, or something related to it. But, since the rotations, in three dimension, were not commutative, i.e.,Rot₁ x Rot₂ ≠ Rot₂ x Rot₁, neither Rot₁ nor Rot₂ can be "true" vectors. On the other hand, for infinitesimal rotations, this association of a rotation with a vector is perfectly plausible. We have to use slightly difficult process of finding elliptical envelop coordinates by considering unit vector α for whole range 0 to 360 as described above.

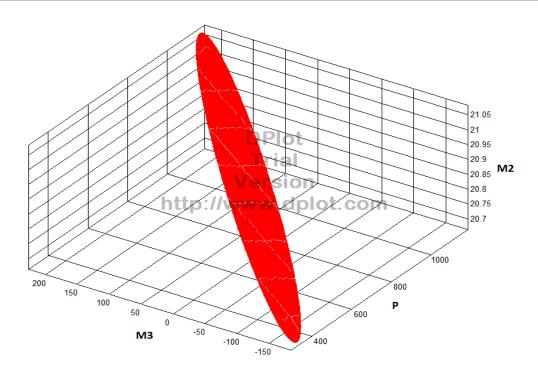


Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 1) (Column A)

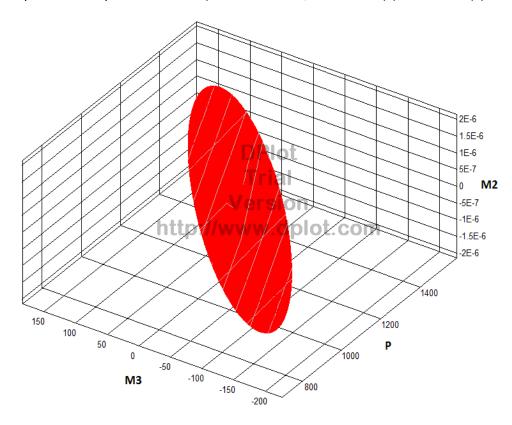


Fig 4.9 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 1) (Column B)

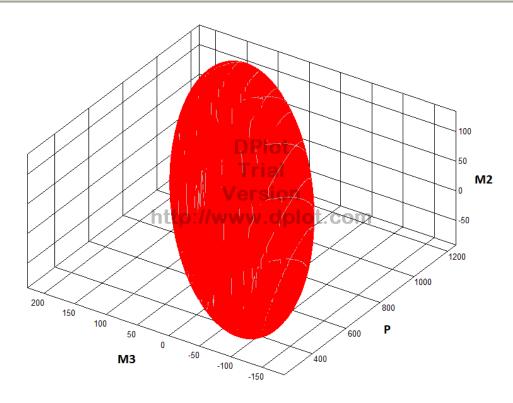


Fig 4.10 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 2)(Column A)

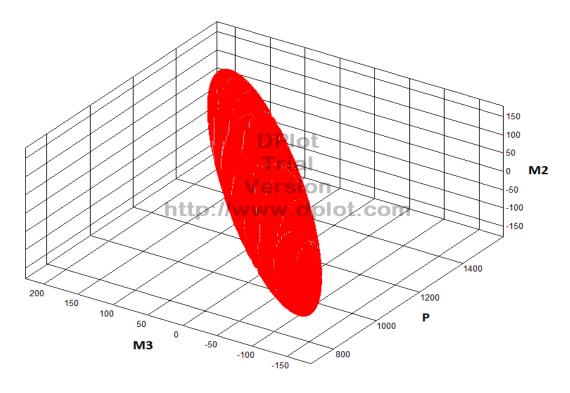


Fig 4.11 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 2)(Column B)

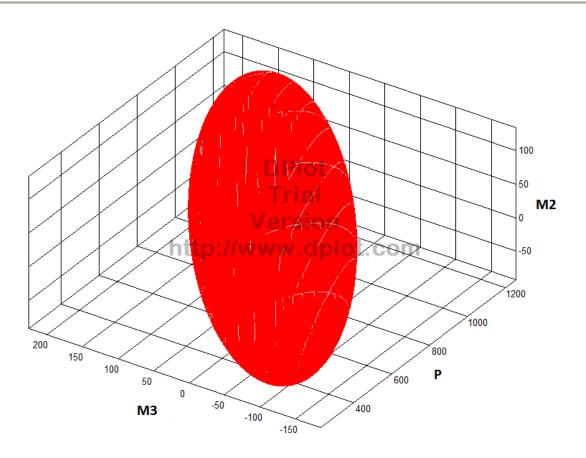


Fig 4.12 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 3)(Column A)

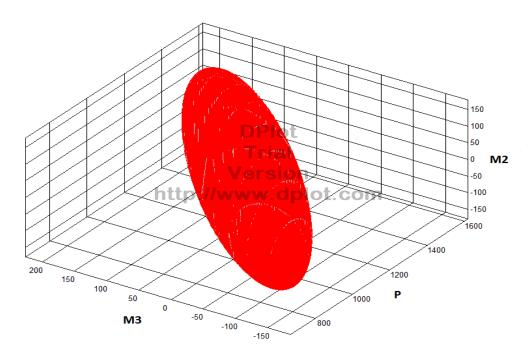


Fig 4.13 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 3)(Column B)

We have performed 24 Modal Analysis in SAP2000 to cover full range of θ from 0^0 to 360^0 with interval 15^0 . It is important to know that SAP2000 v 11 (Advanced) does not change Modal participation factor values w.r.t. structure axes as we change θ at MPF's usual output place but our influence matrix is changing. The following supreme envelops are after adding static contribution, hence can be used for calculating required reinforcement ratio for column A and B as marked in Fig. 4.7.

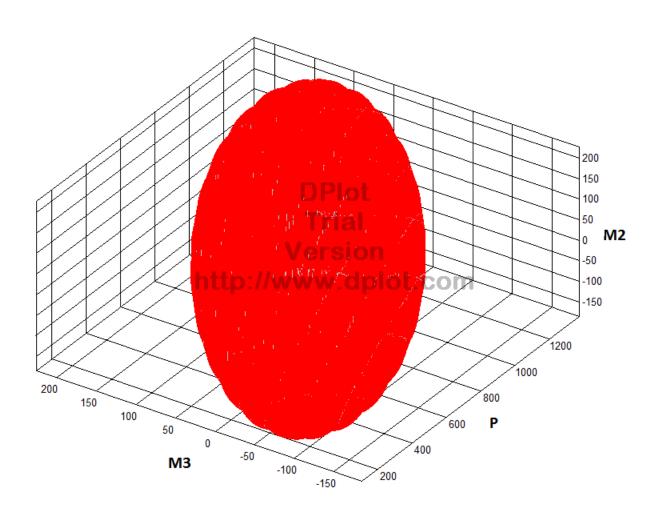


Fig 4.14 Supreme Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Column A)

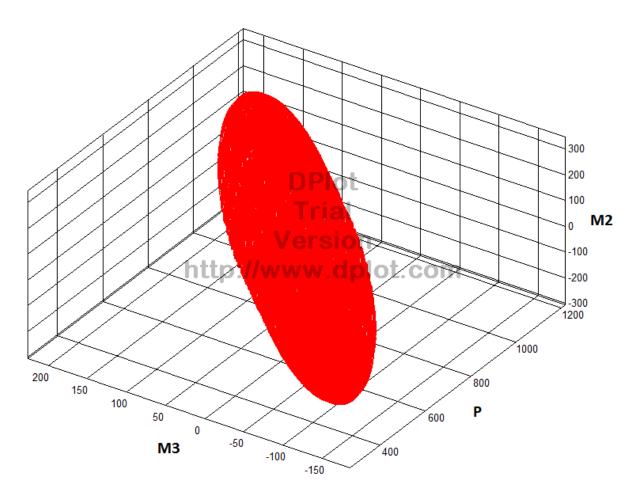


Fig 4.15 Supreme Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) Column B)

Table 4.17 Comparison of % Reinforcement from Conventional Rectangular Envelop and Elliptical Envelop

TABLE: Reinforcement Percent								
Columns	Load Case	Rectangular Envelop		Elliptica	l Envelop	% Difference		
	No	p/fck	р	p/fck	р	р		
Α	1	0.06	1.5	0.059	1.475	1.69		
	2	0.06	1.5	0.05	1.25	20.00		
	3	0.059	1.475	0.049	1.225	20.41		
В	1	0.0465	1.1625	0.044	1.1	5.68		
	2	0.063	1.575	0.058	1.45	8.62		
	3	0.071	1.775	0.051	1.275	39.22		

Table 4.18 Comparison of % Reinforcement from Conventional Rectangular Envelop and Supreme Envelop

TABLE: Reinforcement Precent								
Rectangular								
Columns	Load Case	Envelo	р	Supreme Envelop		% Difference		
	No	p/fck	р	p/fck	р	р		
Α	3	0.064	1.6	0.055	1.375	16.36		
В	3	0.06	1.5	0.056	1.4	7.14		

Numerical Study 3

In this exercise building frames (with vertical irregularities) has been analyzed and try to understand correlation between P, Mx and My of building frames under vertical component of earthquake and significance of using elliptical envelop instead of conventional rectangular envelop on adding third component in analysis. For this analysis data of Northridge earthquake at Sylmar, CA - County Hospital, January 17, 1994, which is recognized as earthquake having magnitude of vertical component as large as that of the horizontal component has been taken. Two different model first which is used in developing above mention approach and second model in which main girder into two parts (Mesh-1) and includes secondary beams in analysis (as per recommended by S. H. Ju, C. W. Liu, and K. Z.) has been analyzed.

The horizontal dimension of lateral force resisting system in first storey is half of that of ground storey (Vertical geometric irregularity). The lateral stiffness of ground storey is less than 70% of that of first storey. The loading and model of building frame has been taken for this object is shown in Fig. .

To approximate stiffness of floor diaphragm each panel of floor is modeled by two diagonal beam elements, but these diagonal beams should not affect modeling aspect of building frame for vertical component of earthquake. So that these beams mass factor modified from 1 to 0. SAP2000 allows us to change mass, weight, area, shear area and moment of inertias properties for frame objects. Hence diagonal element will not going to contribute in mass matrix of structure model.

In this problem Modal analysis has been performed in SAP to have Natural frequency, mode shapes and modal participation factor. SAP2000 v 11 (Advanced) also give the no. of equation it is using for calculation in .TXE file. These data are arranged in compatible form to derive elliptical envelop in Excel.

Response Spectrum Analysis has performed to verify the diagonal terms of Response Matrix, which are nothing but peak values of responses given by Response Spectrum Method. The values are shown in table 4.6. For this analysis the Actual Response Spectra of Northridge earthquake at Sylmar, CA - County Hospital, January 17, 1994 are applied in U1, U2 and U3 directions, by default in SAP these direction are in global X and Y direction of model, these are also principal axes of model (Angle between principal axes of structure and earthquake component, θ = 0). We have analyzed these models for all the three component of earthquake

adding subsequently, and compares reinforcement from rectangular and elliptical envelops for column A (As shown in Fig. 4.).

We have considered 35 mode shapes for Model-1 are considered both in Response spectrum analysis and in deriving elliptical envelop so that for both the horizontal component, 99% participating mass can be included and for vertical component of ground motion 80% participating mass can be included. We have considered 70 mode shapes for Model-2 are considered both in Response spectrum analysis and in deriving elliptical envelop so that for both the horizontal component, 99% participating mass can be included and for vertical component of ground motion 80% participating mass can be included. Hence we can keep the results comparable not effected by % if participating mass.

In tables 4.1 to 4.5 we show data which are collected and calculated from SAP to derive envelop. Table 4.1 showing dynamic properties of structure which are characteristic of any structure. One notes the closeness of the frequencies that is typical of most three dimensional building structures that are designed to resist earthquakes from both directions. Displacement response spectra ordinate and Acceleration response spectra ordinate corresponding to time periods of different modes shown in table 4.2. Displacement response spectra ordinate are require to derive envelop. However, we can input response spectra only in acceleration spectra form in SAP. In Table 4.3 modal participation factor in X and Y direction are shown. In table 4.4 mode shapes are shown, these are results of Modal Analysis in SAP, it normalizes modes so that modal mass have unit value. In Table 4.5 Correlation Coefficient for Complete Quadratic Combination rule are shown.

The contribution of static forces arising from dead loads is included in this analysis. Static loads are cause time-invariant components to act concurrently. Because the structure is linear, these can be added to yield a total response vector that varies in time about point in the response space. The size and orientation of the two envelopes are unaffected by the presence of these static responses.

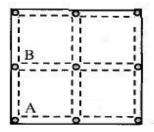


Fig 4.18 Example 4-Storey RCC Building (Typical Floor plan showing Column A)

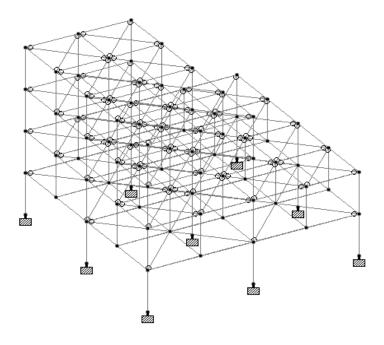


Fig 4.16 Model-1 for 4-Storey RCC Building

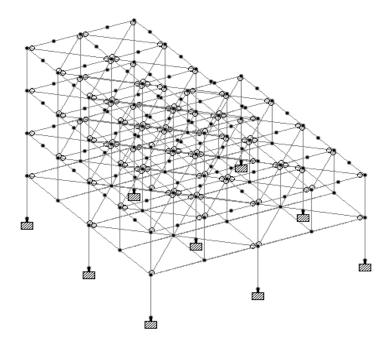


Fig 4.17 Model-1 for 4-Storey RCC Building

Modal Analysis Results

Table 4.19 Modal Periods, Frequency, % Participating Mass and Modal mass and stiffness (Model-1)

N 4l -	Period	Frequency	Damping	Sum of Mo	odal Mass Pa	rticipation	ModalMass	ModalStiff
Mode				in U1	in U2	in U3		
Shape	Sec	Cyc/sec	Ratio	dir.%	dir.%	dir.%	KN-m-s2	KN-m
1	0.15034451	30.21541	0.05	0.000000	0.576913	0.000000	1	1746.564325
2	0.14499296	30.73274	0.05	0.674746	0.576913	0.000002	1	1877.871654
3	0.11343555	31.41302	0.05	0.674746	0.685727	0.000002	1	3068.042534
4	0.06458073	34.34083	0.05	0.674746	0.946241	0.000002	1	9465.724522
5	0.06414651	34.58159	0.05	0.970458	0.946241	0.000012	1	9594.309657
6	0.05454447	34.68555	0.05	0.970458	0.972357	0.000012	1	13269.6157
7	4.35E-02	36.87453	0.05	0.970475	0.972357	0.520535	1	20863.93136
8	0.04057651	38.37914	0.05	0.970475	0.973095	0.520535	1	23977.86139
9	3.45E-02	42.23338	0.05	0.971485	0.973095	0.664837	1	33155.88946
10	3.31E-02	42.56319	0.05	0.971485	0.975123	0.664837	1	36042.65764
11	3.25E-02	44.33323	0.05	0.997195	0.975123	0.668766	1	37287.40503
12	3.18E-02	44.52796	0.05	0.997195	0.996706	0.668766	1	38956.42007
13	2.91E-02	45.51413	0.05	0.997195	0.997306	0.668766	1	46556.6158
14	2.89E-02	46.91959	0.05	0.997288	0.997306	0.684065	1	47211.69877
15	2.88E-02	48.61216	0.05	0.997288	0.997672	0.684065	1	47495.98709
16	2.71E-02	50.73715	0.05	0.997479	0.997672	0.727487	1	53680.03083
17	2.61E-02	51.14098	0.05	0.997479	0.997676	0.727487	1	58150.05737
18	2.37E-02	53.72896	0.05	0.997479	0.998724	0.727487	1	70415.99836
19	2.35E-02	55.83882	0.05	0.997510	0.998724	0.738182	1	71520.07827
20	2.26E-02	60.54063	0.05	0.997510	0.998881	0.738182	1	77592.28424
21	2.25E-02	62.39208	0.05	0.997908	0.998881	0.756320	1	78275.40549
22	2.20E-02	63.94837	0.05	0.999241	0.998881	0.759761	1	81780.95434
23	2.13E-02	67.22062	0.05	0.999241	0.998986	0.759761	1	86909.69024
24	2.06E-02	70.00739	0.05	0.999467	0.998986	0.768778	1	93293.11637
25	1.97E-02	70.12146	0.05	0.999502	0.998986	0.784100	1	101627.6296
26	1.96E-02	71.11567	0.05	0.999502	0.999482	0.784100	1	103251.8615
27	1.86E-02	183.48813	0.05	0.999503	0.999482	0.784107	1	113966.3306
28	1.79E-02	183.80884	0.05	0.999503	0.999518	0.784107	1	123092.66
29	1.65E-02	183.80884	0.05	0.999503	0.999518	0.784107	1	144695.0428
30	1.60E-02	186.14773	0.05	0.999508	0.999518	0.784250	1	153680.4712
31	1.56E-02	186.82388	0.05	0.999514	0.999518	0.809795	1	161442.8243
32	1.49E-02	186.82388	0.05	0.999515	0.999518	0.809836	1	178387.6388
33	1.43E-02	189.81778	0.05	0.999516	0.999518	0.811743	1	193485.1014
34	1.43E-02	189.81778	0.05	0.999516	0.999556	0.811743	1	194116.1163
35	1.41E-02	198.11233	0.05	0.999531	0.999556	0.832112	1	199659.6917

Table 4.20 Modal Periods, Frequency, % Participating Mass and Modal mass and stiffness (Model-2)

N 4l -	Period	Frequency	Damping	Sum of Mo	odal Mass Pa	rticipation	ModalMass	ModalStiff
Mode				in U1	in U2	in U3		
Shape	Sec	Cyc/sec	Ratio	dir.%	dir.%	dir.%	KN-m-s2	KN-m
1	0.15027692	6.65438	0.05	0.000000	0.585224	0.000000	1	1748.135734
2	0.14523626	6.88533	0.05	0.674051	0.585224	0.000003	1	1871.58519
3	0.11184434	8.94100	0.05	0.674051	0.686074	0.000003	1	3155.961819
4	0.06473668	15.44719	0.05	0.674051	0.947756	0.000003	1	9420.174283
5	0.06447966	15.50877	0.05	0.968263	0.947756	0.000025	1	9495.421052
6	0.05448752	18.35283	0.05	0.968263	0.970350	0.000025	1	13297.36677
7	4.31E-02	23.22508	0.05	0.968266	0.970350	0.555120	1	21294.83736
8	0.03988008	25.07517	0.05	0.968266	0.970870	0.555120	1	24822.62352
9	3.40E-02	29.37735	0.05	0.969446	0.970870	0.697107	1	34070.99922
10	3.33E-02	30.03589	0.05	0.996137	0.970870	0.702296	1	35615.63191
11	3.29E-02	30.42734	0.05	0.996137	0.989810	0.702296	1	36550.02797
12	3.20E-02	31.21644	0.05	0.996137	0.995739	0.702296	1	38470.38955
13	3.08E-02	32.49007	0.05	0.996137	0.995739	0.702296	1	41673.6012
14	2.96E-02	33.83500	0.05	0.996137	0.996692	0.702296	1	45195.16638
15	2.90E-02	34.46730	0.05	0.996138	0.996692	0.702296	1	46900.1627
16	2.77E-02	36.16542	0.05	0.996224	0.996692	0.717761	1	51635.30147
17	2.74E-02	36.43723	0.05	0.996224	0.996822	0.717761	1	52414.38384
18	2.68E-02	37.26676	0.05	0.996224	0.996822	0.717761	1	54828.08916
19	2.61E-02	38.35875	0.05	0.996700	0.996822	0.759655	1	58088.30275
20	2.59E-02	38.62537	0.05	0.996700	0.996859	0.759655	1	58898.62247
21	2.53E-02	39.51643	0.05	0.996700	0.998152	0.759655	1	61647.44849
22	2.52E-02	39.71959	0.05	0.996701	0.998152	0.759655	1	62282.94917
23	2.48E-02	40.30284	0.05	0.996701	0.998709	0.759655	1	64125.53419
24	2.46E-02	40.72348	0.05	0.996701	0.998732	0.759655	1	65471.06756
25	2.42E-02	41.32331	0.05	0.999259	0.998732	0.763369	1	67413.98144
26	2.39E-02	41.92798	0.05	0.999265	0.998732	0.763369	1	69401.31156
27	2.36E-02	42.41970	0.05	0.999270	0.998732	0.763398	1	71038.69995
28	2.31E-02	43.27210	0.05	0.999270	0.998740	0.763398	1	73922.35
29	2.31E-02	43.33230	0.05	0.999270	0.998747	0.763398	1	74128.15866
30	2.29E-02	43.71959	0.05	0.999270	0.998753	0.763398	1	75459.1645
31	2.25E-02	44.52453	0.05	0.999286	0.998753	0.764579	1	78263.3493
32	2.24E-02	44.57470	0.05	0.999346	0.998753	0.767825	1	78439.8376
33	2.24E-02	44.72766	0.05	0.999347	0.998753	0.767932	1	78979.08792
34	2.23E-02	44.94381	0.05	0.999347	0.999184	0.767932	1	79744.27807
35	2.22E-02	45.14502	0.05	0.999347	0.999198	0.767932	1	80459.90307
36	2.21E-02	45.20262	0.05	0.999347	0.999199	0.767932	1	80665.32997

5

Mode	Period	Frequency	Damping	Sum of Mo	odal Mass Pa	rticipation	ModalMass	ModalStiff
Shape				in U1	in U2	in U3		
Shape	Sec	Cyc/sec	Ratio	dir.%	dir.%	dir.%	KN-m-s2	KN-m
37	2.19E-02	45.56065	0.05	0.999347	0.999199	0.788618	1	81948.22862
38	2.19E-02	45.57962	0.05	0.999347	0.999349	0.788618	1	82016.48131
39	2.16E-02	46.27131	0.05	0.999350	0.999349	0.789007	1	84524.64733
40	2.16E-02	46.39726	0.05	0.999350	0.999361	0.789007	1	84985.40913
41	2.14E-02	46.75172	0.05	0.999356	0.999361	0.789763	1	86288.89737
42	2.13E-02	46.85573	0.05	0.999356	0.999364	0.789763	1	86673.2804
43	2.13E-02	46.89160	0.05	0.999356	0.999379	0.789763	1	86806.02402
44	2.12E-02	47.08859	0.05	0.999358	0.999379	0.789769	1	87536.90405
45	2.09E-02	47.79768	0.05	0.999358	0.999379	0.789814	1	90193.12068
46	2.09E-02	47.93038	0.05	0.999358	0.999380	0.789814	1	90694.59864
47	2.07E-02	48.29953	0.05	0.999365	0.999380	0.789825	1	92097.00655
48	2.06E-02	48.50362	0.05	0.999365	0.999387	0.789825	1	92876.95311
49	2.05E-02	48.82849	0.05	0.999392	0.999387	0.790058	1	94125.27671
50	2.05E-02	48.88342	0.05	0.999392	0.999387	0.790058	1	94337.1814
51	2.04E-02	48.94675	0.05	0.999405	0.999387	0.790175	1	94581.75587
52	2.04E-02	48.96300	0.05	0.999405	0.999387	0.790175	1	94644.57842
53	2.04E-02	49.10297	0.05	0.999405	0.999394	0.790175	1	95186.48132
54	2.03E-02	49.28699	0.05	0.999407	0.999394	0.790184	1	95901.27511
55	2.03E-02	49.38095	0.05	0.999407	0.999396	0.790184	1	96267.25472
56	2.01E-02	49.80936	0.05	0.999407	0.999396	0.790293	1	97944.87199
57	2.01E-02	49.85888	0.05	0.999407	0.999398	0.790293	1	98139.71818
58	2.00E-02	49.89428	0.05	0.999407	0.999413	0.790293	1	98279.13094
59	2.00E-02	49.92482	0.05	0.999407	0.999415	0.790293	1	98399.47051
60	1.99E-02	50.25523	0.05	0.999407	0.999415	0.790293	1	99706.21642
61	1.97E-02	50.71331	0.05	0.999407	0.999415	0.790298	1	101532.1495
62	1.97E-02	50.76418	0.05	0.999409	0.999415	0.790316	1	101735.9574
63	1.96E-02	50.96737	0.05	0.999411	0.999415	0.790323	1	102552.0312
64	1.94E-02	51.49388	0.05	0.999411	0.999415	0.790323	1	104681.7595
65	1.94E-02	51.50883	0.05	0.999411	0.999415	0.790331	1	104742.5439
66	1.94E-02	51.67836	0.05	0.999411	0.999415	0.790331	1	105433.1454
67	1.93E-02	51.81432	0.05	0.999412	0.999415	0.790331	1	105988.6612
68	1.93E-02	51.86079	0.05	0.999412	0.999416	0.790331	1	106178.828
69	1.88E-02	53.30280	0.05	0.999412	0.999416	0.790331	1	112165.641
70	1.86E-02	53.75963	0.05	0.999416	0.999416	0.822569	1	114096.5022

Table 4.21 Modal Participation Factors (Model-1)

Modes	Period	UX	UY	UZ
Shapes	Sec	KN-sec ²	KN-sec ²	KN-sec ²
1	0.150	4.79E-19	5.77E-01	7.81E-17
2	0.145	6.75E-01	2.13E-18	2.15E-06
3	0.113	2.53E-21	1.09E-01	3.02E-18
4	0.065	3.64E-17	2.61E-01	9.43E-16
5	0.064	2.96E-01	3.52E-17	1.01E-05
6	0.055	2.50E-17	2.61E-02	4.92E-16
7	0.043	1.76E-05	6.98E-17	5.21E-01
8	0.041	9.84E-18	7.38E-04	1.52E-16
9	0.035	1.01E-03	4.29E-16	1.44E-01
10	0.033	2.77E-18	2.03E-03	2.07E-15
11	0.033	2.57E-02	1.98E-16	3.93E-03
12	0.032	1.13E-17	2.16E-02	6.27E-15
13	0.029	7.07E-17	6.00E-04	5.57E-15
14	0.029	9.35E-05	7.05E-18	1.53E-02
15	0.029	3.51E-17	3.66E-04	1.63E-15
16	0.027	1.91E-04	1.69E-17	4.34E-02
17	0.026	2.91E-16	4.70E-06	4.33E-16
18	0.024	3.58E-18	1.05E-03	2.26E-15
19	0.023	3.09E-05	7.07E-17	1.07E-02
20	0.023	3.62E-15	1.57E-04	2.82E-14
21	0.022	3.98E-04	3.34E-19	1.81E-02
22	0.022	1.33E-03	2.49E-15	3.44E-03
23	0.021	2.19E-15	1.05E-04	1.36E-14
24	0.021	2.26E-04	7.31E-14	9.02E-03
25	0.020	3.44E-05	1.45E-13	1.53E-02
26	0.020	5.74E-15	4.96E-04	5.41E-15
27	0.019	1.26E-06	1.87E-14	6.61E-06
28	0.018	2.86E-15	3.53E-05	4.66E-15
29	0.017	3.27E-14	5.42E-07	1.86E-14
30	0.016	5.10E-06	8.79E-15	1.43E-04
31	0.016	6.23E-06	1.69E-14	2.55E-02
32	0.015	3.84E-07	1.00E-14	4.05E-05
33	0.014	1.68E-06	1.91E-15	1.91E-03
34	0.014	5.72E-15	3.81E-05	5.82E-13
35	0.014	1.48E-05	2.15E-14	2.04E-02

Table 4.22 Modal Participation Factors (Model-2)

Modes	Period	UX	UY	UZ
Shapes	Sec	KN-sec ²	KN-sec ²	KN-sec ²
1	0.150	5.07E-09	3.47E+01	-2.43E-08
2	0.145	-3.72E+01	6.93E-09	8.17E-02
3	0.112	-3.43E-07	1.44E+01	8.04E-07
4	0.065	-7.13E-08	2.32E+01	7.08E-08
5	0.064	2.46E+01	1.17E-07	2.11E-01
6	0.054	3.19E-07	-6.81E+00	9.59E-07
7	0.043	6.83E-02	-1.05E-06	3.38E+01
8	0.040	3.45E-06	-1.03E+00	-9.94E-06
9	0.034	-1.56E+00	-2.80E-06	1.71E+01
10	0.033	-7.40E+00	-6.46E-07	-3.26E+00
11	0.033	-2.70E-06	6.24E+00	3.65E-06
12	0.032	3.10E-06	-3.49E+00	-8.52E-06
13	0.031	1.85E-06	-2.90E-02	1.11E-05
14	0.030	-2.01E-06	1.40E+00	-9.86E-06
15	0.029	3.84E-02	9.12E-06	-7.53E-03
16	0.028	-4.22E-01	7.96E-06	-5.64E+00
17	0.027	7.55E-07	5.16E-01	-6.31E-05
18	0.027	1.67E-06	2.37E-03	-1.29E-05
19	0.026	9.88E-01	2.17E-06	-9.28E+00
20	0.026	3.45E-07	2.75E-01	2.26E-05
21	0.025	3.14E-06	-1.63E+00	-1.50E-05
22	0.025	4.71E-02	1.46E-05	-1.72E-02
23	0.025	9.07E-07	-1.07E+00	9.05E-06
24	0.025	7.30E-06	-2.16E-01	-1.35E-04
25	0.024	-2.29E+00	-2.25E-05	-2.76E+00
26	0.024	1.18E-01	1.01E-05	2.54E-02
27	0.024	1.01E-01	-1.60E-05	2.46E-01
28	0.023	-4.36E-05	1.31E-01	2.05E-04
29	0.023	-7.66E-06	1.18E-01	3.88E-05
30	0.023	3.98E-05	1.08E-01	-2.39E-04
31	0.022	1.82E-01	-1.20E-05	-1.56E+00
32	0.022	3.50E-01	-1.76E-05	-2.58E+00
33	0.022	-4.38E-02	3.52E-05	-4.70E-01
34	0.022	-4.01E-06	-9.42E-01	-9.62E-05
35	0.022	1.14E-05	-1.65E-01	-2.37E-04
36	0.022	-1.26E-06	-5.08E-02	2.67E-04

Modes	Period	UX	UY	UZ
Shapes	Sec	KN-sec ²	KN-sec ²	KN-sec ²
37	0.022	-2.48E-02	-1.06E-05	6.52E+00
38	0.022	-1.99E-05	5.56E-01	-2.55E-05
39	0.022	-7.77E-02	1.53E-05	8.94E-01
40	0.022	2.04E-05	1.54E-01	-3.65E-04
41	0.021	-1.07E-01	-6.96E-05	1.25E+00
42	0.021	6.38E-05	-7.42E-02	2.23E-04
43	0.021	2.52E-05	-1.78E-01	-1.25E-04
44	0.021	7.23E-02	1.80E-05	1.07E-01
45	0.021	-6.78E-03	4.73E-05	3.07E-01
46	0.021	2.74E-05	-4.08E-02	-8.81E-05
47	0.021	-1.21E-01	-1.28E-05	1.51E-01
48	0.021	3.43E-05	1.19E-01	-1.16E-04
49	0.020	2.35E-01	-1.30E-06	-6.92E-01
50	0.020	-9.07E-05	3.78E-03	9.57E-05
51	0.020	-1.65E-01	3.23E-06	4.90E-01
52	0.020	-1.66E-05	-2.73E-03	-1.62E-04
53	0.020	1.38E-05	-1.21E-01	2.24E-04
54	0.020	4.86E-02	-2.01E-05	1.39E-01
55	0.020	4.32E-05	6.52E-02	-1.82E-04
56	0.020	-1.44E-02	-2.87E-05	4.73E-01
57	0.020	-1.88E-05	-6.46E-02	5.72E-05
58	0.020	-2.86E-06	-1.72E-01	4.30E-05
59	0.020	-7.57E-06	7.60E-02	-1.39E-04
60	0.020	1.99E-05	5.07E-03	-2.14E-04
61	0.020	-3.01E-02	-1.18E-05	9.93E-02
62	0.020	-5.62E-02	1.97E-05	1.92E-01
63	0.020	6.44E-02	1.48E-05	1.23E-01
64	0.019	6.25E-06	-1.12E-02	4.31E-06
65	0.019	-2.30E-02	-9.72E-07	1.25E-01
66	0.019	-9.37E-06	1.15E-02	6.26E-05
67	0.019	4.36E-02	2.57E-06	1.45E-02
68	0.019	-7.79E-06	3.72E-02	1.25E-04
69	0.019	-8.71E-07	-1.96E-02	9.00E-06
70	0.019	-8.95E-02	-7.69E-06	-8.14E+00

Table 4.23 Response Spectra Ordinates for Load Case 1 (Model-1)

	Acce	leration Resp	onse	Displ	acement Resp	onse
Modes		(SAP Uses)		(To	Derive Envel	op)
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp
	m/sec ²	m/sec ²	m/sec ²	m	m	m
1	9.52078	0.00000	0.00000	0.00545	0.00000	0.00000
2	9.36142	0.00000	0.00000	0.00499	0.00000	0.00000
3	8.42167	0.00000	0.00000	0.00274	0.00000	0.00000
4	8.02157	0.00000	0.00000	0.00085	0.00000	0.00000
5	8.02157	0.00000	0.00000	0.00084	0.00000	0.00000
6	8.02157	0.00000	0.00000	0.00060	0.00000	0.00000
7	8.02157	0.00000	0.00000	0.00038	0.00000	0.00000
8	8.02157	0.00000	0.00000	0.00033	0.00000	0.00000
9	8.02157	0.00000	0.00000	0.00024	0.00000	0.00000
10	8.02157	0.00000	0.00000	0.00022	0.00000	0.00000
11	8.02157	0.00000	0.00000	0.00022	0.00000	0.00000
12	8.02157	0.00000	0.00000	0.00021	0.00000	0.00000
13	8.02157	0.00000	0.00000	0.00017	0.00000	0.00000
14	8.02157	0.00000	0.00000	0.00017	0.00000	0.00000
15	8.02157	0.00000	0.00000	0.00017	0.00000	0.00000
16	8.02157	0.00000	0.00000	0.00015	0.00000	0.00000
17	8.02157	0.00000	0.00000	0.00014	0.00000	0.00000
18	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000
19	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000
20	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000
21	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000
22	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000
23	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000
24	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000
25	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000
26	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000
27	8.02157	0.00000	0.00000	0.00007	0.00000	0.00000
28	8.02157	0.00000	0.00000	0.00007	0.00000	0.00000
29	8.02157	0.00000	0.00000	0.00006	0.00000	0.00000
30	8.02157	0.00000	0.00000	0.00005	0.00000	0.00000
31	8.02157	0.00000	0.00000	0.00005	0.00000	0.00000
32	8.02157	0.00000	0.00000	0.00004	0.00000	0.00000
33	8.02157	0.00000	0.00000	0.00004	0.00000	0.00000
34	8.02157	0.00000	0.00000	0.00004	0.00000	0.00000
35	8.02157	0.00000	0.00000	0.00004	0.00000	0.00000

Table 4.24 Response Spectra Ordinates for Load Case 1 (Model-2)

	Acce	leration Resp	onse	Displ	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)		
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp		
	m/sec ²	m/sec ²	m/sec ²	m	m	m		
1	9.51877	0.00000	0.00000	0.00545	0.00000	0.00000		
2	9.36866	0.00000	0.00000	0.00501	0.00000	0.00000		
3	8.37429	0.00000	0.00000	0.00265	0.00000	0.00000		
4	8.02157	0.00000	0.00000	0.00085	0.00000	0.00000		
5	8.02157	0.00000	0.00000	0.00084	0.00000	0.00000		
6	8.02157	0.00000	0.00000	0.00060	0.00000	0.00000		
7	8.02157	0.00000	0.00000	0.00038	0.00000	0.00000		
8	8.02157	0.00000	0.00000	0.00032	0.00000	0.00000		
9	8.02157	0.00000	0.00000	0.00024	0.00000	0.00000		
10	8.02157	0.00000	0.00000	0.00023	0.00000	0.00000		
11	8.02157	0.00000	0.00000	0.00022	0.00000	0.00000		
12	8.02157	0.00000	0.00000	0.00021	0.00000	0.00000		
13	8.02157	0.00000	0.00000	0.00019	0.00000	0.00000		
14	8.02157	0.00000	0.00000	0.00018	0.00000	0.00000		
15	8.02157	0.00000	0.00000	0.00017	0.00000	0.00000		
16	8.02157	0.00000	0.00000	0.00016	0.00000	0.00000		
17	8.02157	0.00000	0.00000	0.00015	0.00000	0.00000		
18	8.02157	0.00000	0.00000	0.00015	0.00000	0.00000		
19	8.02157	0.00000	0.00000	0.00014	0.00000	0.00000		
20	8.02157	0.00000	0.00000	0.00014	0.00000	0.00000		
21	8.02157	0.00000	0.00000	0.00013	0.00000	0.00000		
22	8.02157	0.00000	0.00000	0.00013	0.00000	0.00000		
23	8.02157	0.00000	0.00000	0.00013	0.00000	0.00000		
24	8.02157	0.00000	0.00000	0.00012	0.00000	0.00000		
25	8.02157	0.00000	0.00000	0.00012	0.00000	0.00000		
26	8.02157	0.00000	0.00000	0.00012	0.00000	0.00000		
27	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000		
28	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000		
29	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000		
30	8.02157	0.00000	0.00000	0.00011	0.00000	0.00000		
31	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000		
32	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000		
33	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000		
34	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000		
35	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000		

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
36	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000	
37	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000	
38	8.02157	0.00000	0.00000	0.00010	0.00000	0.00000	
39	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
40	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
41	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
42	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
43	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
44	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
45	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
46	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
47	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
48	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
49	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
50	8.02157	0.00000	0.00000	0.00009	0.00000	0.00000	
51	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
52	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
53	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
54	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
55	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
56	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
57	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
58	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
59	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
60	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
61	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
62	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
63	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
64	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
65	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
66	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
67	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
68	8.02157	0.00000	0.00000	0.00008	0.00000	0.00000	
69	8.02157	0.00000	0.00000	0.00007	0.00000	0.00000	
70	8.02157	0.00000	0.00000	0.00007	0.00000	0.00000	

Table 4.25 Response Spectra Ordinates for Load Case 2 (Model-1)

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
1	9.52078	8.07017	0.00000	0.00545	0.00462	0.00000	
2	9.36142	7.77387	0.00000	0.00499	0.00414	0.00000	
3	8.42167	6.02667	0.00000	0.00274	0.00196	0.00000	
4	8.02157	5.28280	0.00000	0.00085	0.00056	0.00000	
5	8.02157	5.28280	0.00000	0.00084	0.00055	0.00000	
6	8.02157	5.28280	0.00000	0.00060	0.00040	0.00000	
7	8.02157	5.28280	0.00000	0.00038	0.00025	0.00000	
8	8.02157	5.28280	0.00000	0.00033	0.00022	0.00000	
9	8.02157	5.28280	0.00000	0.00024	0.00016	0.00000	
10	8.02157	5.28280	0.00000	0.00022	0.00015	0.00000	
11	8.02157	5.28280	0.00000	0.00022	0.00014	0.00000	
12	8.02157	5.28280	0.00000	0.00021	0.00014	0.00000	
13	8.02157	5.28280	0.00000	0.00017	0.00011	0.00000	
14	8.02157	5.28280	0.00000	0.00017	0.00011	0.00000	
15	8.02157	5.28280	0.00000	0.00017	0.00011	0.00000	
16	8.02157	5.28280	0.00000	0.00015	0.00010	0.00000	
17	8.02157	5.28280	0.00000	0.00014	0.00009	0.00000	
18	8.02157	5.28280	0.00000	0.00011	0.00008	0.00000	
19	8.02157	5.28280	0.00000	0.00011	0.00007	0.00000	
20	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
21	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
22	8.02157	5.28280	0.00000	0.00010	0.00006	0.00000	
23	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
24	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
25	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
26	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
27	8.02157	5.28280	0.00000	0.00007	0.00005	0.00000	
28	8.02157	5.28280	0.00000	0.00007	0.00004	0.00000	
29	8.02157	5.28280	0.00000	0.00006	0.00004	0.00000	
30	8.02157	5.28280	0.00000	0.00005	0.00003	0.00000	
31	8.02157	5.28280	0.00000	0.00005	0.00003	0.00000	
32	8.02157	5.28280	0.00000	0.00004	0.00003	0.00000	
33	8.02157	5.28280	0.00000	0.00004	0.00003	0.00000	
34	8.02157	5.28280	0.00000	0.00004	0.00003	0.00000	
35	8.02157	5.28280	0.00000	0.00004	0.00003	0.00000	

Table 4.26 Response Spectra Ordinates for Load Case 2 (Model-2)

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
1	9.51877	8.06642	0.00000	0.00545	0.00461	0.00000	
2	9.36866	7.78734	0.00000	0.00501	0.00416	0.00000	
3	8.37429	5.93857	0.00000	0.00265	0.00188	0.00000	
4	8.02157	5.28280	0.00000	0.00085	0.00056	0.00000	
5	8.02157	5.28280	0.00000	0.00084	0.00056	0.00000	
6	8.02157	5.28280	0.00000	0.00060	0.00040	0.00000	
7	8.02157	5.28280	0.00000	0.00038	0.00025	0.00000	
8	8.02157	5.28280	0.00000	0.00032	0.00021	0.00000	
9	8.02157	5.28280	0.00000	0.00024	0.00016	0.00000	
10	8.02157	5.28280	0.00000	0.00023	0.00015	0.00000	
11	8.02157	5.28280	0.00000	0.00022	0.00014	0.00000	
12	8.02157	5.28280	0.00000	0.00021	0.00014	0.00000	
13	8.02157	5.28280	0.00000	0.00019	0.00013	0.00000	
14	8.02157	5.28280	0.00000	0.00018	0.00012	0.00000	
15	8.02157	5.28280	0.00000	0.00017	0.00011	0.00000	
16	8.02157	5.28280	0.00000	0.00016	0.00010	0.00000	
17	8.02157	5.28280	0.00000	0.00015	0.00010	0.00000	
18	8.02157	5.28280	0.00000	0.00015	0.00010	0.00000	
19	8.02157	5.28280	0.00000	0.00014	0.00009	0.00000	
20	8.02157	5.28280	0.00000	0.00014	0.00009	0.00000	
21	8.02157	5.28280	0.00000	0.00013	0.00009	0.00000	
22	8.02157	5.28280	0.00000	0.00013	0.00008	0.00000	
23	8.02157	5.28280	0.00000	0.00013	0.00008	0.00000	
24	8.02157	5.28280	0.00000	0.00012	0.00008	0.00000	
25	8.02157	5.28280	0.00000	0.00012	0.00008	0.00000	
26	8.02157	5.28280	0.00000	0.00012	0.00008	0.00000	
27	8.02157	5.28280	0.00000	0.00011	0.00007	0.00000	
28	8.02157	5.28280	0.00000	0.00011	0.00007	0.00000	
29	8.02157	5.28280	0.00000	0.00011	0.00007	0.00000	
30	8.02157	5.28280	0.00000	0.00011	0.00007	0.00000	
31	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
32	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
33	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
34	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
35	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)			Derive Envel		
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
·	m/sec ²	m/sec ²	m/sec ²	m	m	m	
36	8.02157	5.28280	0.00000	0.00010	0.00007	0.00000	
37	8.02157	5.28280	0.00000	0.00010	0.00006	0.00000	
38	8.02157	5.28280	0.00000	0.00010	0.00006	0.00000	
39	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
40	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
41	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
42	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
43	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
44	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
45	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
46	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
47	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
48	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
49	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
50	8.02157	5.28280	0.00000	0.00009	0.00006	0.00000	
51	8.02157	5.28280	0.00000	0.00008	0.00006	0.00000	
52	8.02157	5.28280	0.00000	0.00008	0.00006	0.00000	
53	8.02157	5.28280	0.00000	0.00008	0.00006	0.00000	
54	8.02157	5.28280	0.00000	0.00008	0.00006	0.00000	
55	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
56	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
57	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
58	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
59	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
60	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
61	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
62	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
63	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
64	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
65	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
66	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
67	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
68	8.02157	5.28280	0.00000	0.00008	0.00005	0.00000	
69	8.02157	5.28280	0.00000	0.00007	0.00005	0.00000	
70	8.02157	5.28280	0.00000	0.00007	0.00005	0.00000	

Table 4.27 Response Spectra Ordinates for Load Case 3 (Model-1)

	Acce	leration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	(qc	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
1	9.52078	8.07017	6.34005	0.00545	0.00462	0.00363	
2	9.36142	7.77387	6.24818	0.00499	0.00414	0.00333	
3	8.42167	6.02667	5.70646	0.00274	0.00196	0.00186	
4	8.02157	5.28280	5.47583	0.00085	0.00056	0.00058	
5	8.02157	5.28280	5.47583	0.00084	0.00055	0.00057	
6	8.02157	5.28280	5.47583	0.00060	0.00040	0.00041	
7	8.02157	5.28280	5.47583	0.00038	0.00025	0.00026	
8	8.02157	5.28280	5.47583	0.00033	0.00022	0.00023	
9	8.02157	5.28280	5.47583	0.00024	0.00016	0.00017	
10	8.02157	5.28280	5.47583	0.00022	0.00015	0.00015	
11	8.02157	5.28280	5.47583	0.00022	0.00014	0.00015	
12	8.02157	5.28280	5.47583	0.00021	0.00014	0.00014	
13	8.02157	5.28280	5.47583	0.00017	0.00011	0.00012	
14	8.02157	5.28280	5.47583	0.00017	0.00011	0.00012	
15	8.02157	5.28280	5.47583	0.00017	0.00011	0.00012	
16	8.02157	5.28280	5.47583	0.00015	0.00010	0.00010	
17	8.02157	5.28280	5.47583	0.00014	0.00009	0.00009	
18	8.02157	5.28280	5.47583	0.00011	0.00008	0.00008	
19	8.02157	5.28280	5.47583	0.00011	0.00007	0.00008	
20	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
21	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
22	8.02157	5.28280	5.47583	0.00010	0.00006	0.00007	
23	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006	
24	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006	
25	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005	
26	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005	
27	8.02157	5.28280	5.47583	0.00007	0.00005	0.00005	
28	8.02157	5.28280	5.47583	0.00007	0.00004	0.00004	
29	8.02157	5.28280	5.47583	0.00006	0.00004	0.00004	
30	8.02157	5.28280	5.47583	0.00005	0.00003	0.00004	
31	8.02157	5.28280	5.47583	0.00005	0.00003	0.00003	
32	8.02157	5.28280	5.47583	0.00004	0.00003	0.00003	
33	8.02157	5.28280	5.47583	0.00004	0.00003	0.00003	
34	8.02157	5.28280	5.47583	0.00004	0.00003	0.00003	
35	8.02157	5.28280	5.47583	0.00004	0.00003	0.00003	

Table 4.28 Response Spectra Ordinates for Load Case 3 (Model-2)

	Acce	eleration Resp	onse	Displacement Response			
Modes		(SAP Uses)		(To	Derive Envel	op)	
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp	
	m/sec ²	m/sec ²	m/sec ²	m	m	m	
1	9.51877	8.06642	6.33889	0.00545	0.00461	0.00363	
2	9.36866	7.78734	6.25236	0.00501	0.00416	0.00334	
3	8.37429	5.93857	5.67915	0.00265	0.00188	0.00180	
4	8.02157	5.28280	5.47583	0.00085	0.00056	0.00058	
5	8.02157	5.28280	5.47583	0.00084	0.00056	0.00058	
6	8.02157	5.28280	5.47583	0.00060	0.00040	0.00041	
7	8.02157	5.28280	5.47583	0.00038	0.00025	0.00026	
8	8.02157	5.28280	5.47583	0.00032	0.00021	0.00022	
9	8.02157	5.28280	5.47583	0.00024	0.00016	0.00016	
10	8.02157	5.28280	5.47583	0.00023	0.00015	0.00015	
11	8.02157	5.28280	5.47583	0.00022	0.00014	0.00015	
12	8.02157	5.28280	5.47583	0.00021	0.00014	0.00014	
13	8.02157	5.28280	5.47583	0.00019	0.00013	0.00013	
14	8.02157	5.28280	5.47583	0.00018	0.00012	0.00012	
15	8.02157	5.28280	5.47583	0.00017	0.00011	0.00012	
16	8.02157	5.28280	5.47583	0.00016	0.00010	0.00011	
17	8.02157	5.28280	5.47583	0.00015	0.00010	0.00010	
18	8.02157	5.28280	5.47583	0.00015	0.00010	0.00010	
19	8.02157	5.28280	5.47583	0.00014	0.00009	0.00009	
20	8.02157	5.28280	5.47583	0.00014	0.00009	0.00009	
21	8.02157	5.28280	5.47583	0.00013	0.00009	0.00009	
22	8.02157	5.28280	5.47583	0.00013	0.00008	0.00009	
23	8.02157	5.28280	5.47583	0.00013	0.00008	0.00009	
24	8.02157	5.28280	5.47583	0.00012	0.00008	0.00008	
25	8.02157	5.28280	5.47583	0.00012	0.00008	0.00008	
26	8.02157	5.28280	5.47583	0.00012	0.00008	0.00008	
27	8.02157	5.28280	5.47583	0.00011	0.00007	0.00008	
28	8.02157	5.28280	5.47583	0.00011	0.00007	0.00007	
29	8.02157	5.28280	5.47583	0.00011	0.00007	0.00007	
30	8.02157	5.28280	5.47583	0.00011	0.00007	0.00007	
31	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
32	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
33	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
34	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	
35	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007	

	Acce	leration Resp	onse	Displ	acement Resp	onse
Modes		(SAP Uses)		(To	Derive Envel	op)
Shapes	U1Acc	U2Acc	U3Acc	U1Disp	U2Disp	U3Disp
	m/sec ²	m/sec ²	m/sec ²	m	m	m
36	8.02157	5.28280	5.47583	0.00010	0.00007	0.00007
37	8.02157	5.28280	5.47583	0.00010	0.00006	0.00007
38	8.02157	5.28280	5.47583	0.00010	0.00006	0.00007
39	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
40	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
41	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
42	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
43	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
44	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
45	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
46	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
47	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
48	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
49	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
50	8.02157	5.28280	5.47583	0.00009	0.00006	0.00006
51	8.02157	5.28280	5.47583	0.00008	0.00006	0.00006
52	8.02157	5.28280	5.47583	0.00008	0.00006	0.00006
53	8.02157	5.28280	5.47583	0.00008	0.00006	0.00006
54	8.02157	5.28280	5.47583	0.00008	0.00006	0.00006
55	8.02157	5.28280	5.47583	0.00008	0.00005	0.00006
56	8.02157	5.28280	5.47583	0.00008	0.00005	0.00006
57	8.02157	5.28280	5.47583	0.00008	0.00005	0.00006
58	8.02157	5.28280	5.47583	0.00008	0.00005	0.00006
59	8.02157	5.28280	5.47583	0.00008	0.00005	0.00006
60	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
61	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
62	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
63	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
64	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
65	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
66	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
67	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
68	8.02157	5.28280	5.47583	0.00008	0.00005	0.00005
69	8.02157	5.28280	5.47583	0.00007	0.00005	0.00005
70	8.02157	5.28280	5.47583	0.00007	0.00005	0.00005

Table 4.29 Response Spectrum Method Results & Derived Envelop (Load Case 1) (Model-1)

TABLE: Elemei	nt Forces - Fram	es					
Columns	Element	Joint	CaseType	StepType	Р	M2	M3
Columns No	No	Text	Text	KN	KN-m	KN-m	
Α	1	1	LinRespSpec	Max	2380.979	166.424	2451.750
Α	1	2	LinRespSpec	Max	2380.979	341.678	1749.990

Table 4.30 Response Spectrum Method Results & Derived Envelop (Load Case 2) (Model-1)

TABLE: Element Forces - Frames							
Columns		Joint	CaseType	StepType	Р	M2	М3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	2911.93	2560.593425	2568.412659
Α	1	2	LinRespSpec	Max	2911.93	1797.59654	1788.569365

Table 4.31 Response Spectrum Method Results & Derived Envelop (Load Case 3) (Model-1)

TABLE: Element Forces - Frames							
Columns	Element	Joint	CaseType	StepType	Р	M2	М3
Columns No	No	Text	Text	KN	KN-m	KN-m	
Α	1	1	LinRespSpec	Max	2980.868	2561.837199	2569.480219
Α	1	2	LinRespSpec	Max	2980.868	1806.587278	1796.808178

Table 4.32 Response Spectrum Method Results & Derived Envelop (Load Case 1) (Model-2)

TABLE: Element Forces - Frames							
Columns		Joint	CaseType	StepType	Р	M2	М3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	2379.22	166.1429732	2450.056237
Α	1	2	LinRespSpec	Max	2379.22	340.3650485	1748.907327

Table 4.33 Response Spectrum Method Results & Derived Envelop (Load Case 2) (Model-2)

TABLE: Element Forces - Frames							
Columns	Element	Joint	CaseType	StepType	Р	M2	М3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	2921.953	2568.138308	2561.119007
Α	1	2	LinRespSpec	Max	2921.953	1804.649678	1784.367785

Table 4.34 Response Spectrum Method Results & Derived Envelop (Load Case 3) (Model-2)

TABLE: Element Forces - Frames							
Calumana	Element	Joint	CaseType	StepType	Р	M2	M3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	LinRespSpec	Max	3000.99	2569.408692	2562.098458
Α	1	2	LinRespSpec	Max	3000.99	1813.808215	1792.48906

Table 4.34 Element Forces for DEAD Load

Columns							
Columns	Element	Joint	CaseType	StepType	P	M2	M3
Columns	No	No	Text	Text	KN	KN-m	KN-m
Α	1	1	DEAD		-1464.034	-126.592	-122.825
Α	1	2	DEAD		-1432.804	284.190	283.147

Deriving Elliptical Envelop From Response Matrix

We have derived elliptical envelop in this exercise in similarly as last exercise.

- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 1) (Model-1)
- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 1) (Model-2)
- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 2) (Model-1)
- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 2) (Model-2)
- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 3) (Model-1)
- Fig 4.8 Elliptical Envelop For P-M2-M3 (P in kN and M2,M3 in kN-m) (Load Case 3) (Model-2)

NUMERICAL STUDY 4

This Exercise is aim to Compare Elliptical Envelop (approach-1) for seismic response vectors with an approach given by Martin R. Button, Edward L. Willons for evaluating critical values of each response for complete range of angle between principal direction of building structure axis and earthquake component (approach-2). They have also shown that estimation of combined stress directly by mean maximum values of axial forces and moments is incorrect. A correct estimate of combine stress is obtained only if equation is evaluated for each mode shapes and maximum stress can then be found from combining these by the CQC method of modal combination because both axial force and moment does not peaks simultaneously. We develop *Supreme Stress Envelop* by envelop approach-1, which encloses pure axial stress and stress due to moment at different points on column section. We have done comparison of results from both the approaches. The model and static loading is similar to the model we used in problem 1.

We have performed Modal analysis in SAP to have Natural frequency, mode shapes, modal participation factor and axial forces and moments at model level. We have derived *supreme stress envelop* for axial stress and stress due to moments in column A by approach-1. It will bounds the values of these stress occurring simultaneously.

We have performed Response Spectrum Analysis to verify the diagonal terms of Response Matrix, which are nothing but peak values of responses given by Response Spectrum Method. (In stress envelop case the diagonal terms are verified by peak values of axial stress and stress due to moment calculated from response spectrum analysis results of axial forces and moments of column). The values are shown in table 4.6. The Design Response Spectra as given in IS 1893 – 2002 (part 1)(Criteria for Earthquake Resistance Design of Structures) are applied in U1 and U2 direction, by default in SAP these direction are in global X and Y direction of model, these are also principal axes of our model (Angle between principal axes of structure and earthquake component, $\theta = 0$). We have to factored forces due to of second component by 0.085 as recommended in both approches.

We evaluated combine stresses on column section at modal level by results of Modal analysis done in SAP2000 and then combined these by CQC rule to have peak values (which is suggested correct approach of evaluating response spectrum values of combined stress). *These are considered as response spectrum values for further analysis*. We computed critical angle for these stress by an approach-2 and find out critical values of stress.

We have also evaluated combined stresses directly by response spectrum values of axial forces and moments on column A. We computed critical angle for these stress by an approach-2 and find out critical values of stress. We compared combined stress values by above two estimate in table 4. .

We have calculated maximum of maximum and minimum of minimum combined stresses by coordinates of supreme stress envelop. This gave values of maximum and minimum combined stresses for complete range of θ for selected column section (approach-1). We compared these value with critical values of maximum and minimum combines stress for that column estimated by approach-2 in table-4.

We have not considered vertical component of earthquake in analysis because in approach-2 for three dimensional earthquake input the equation of evaluating critical angle is not altered only estimate of total forces will have to change to add term due to vertical component.

We have included 12 mode shapes so that for both the horizontal component of ground motion 99% participating mass was included.

The contribution of static forces arising from dead loads is not included in this analysis. Static loads are cause time-invariant components to act concurrently with dynamic responses. Because the structure is linear, these can be added to yield a total response vector that varies in time about point in the response space. Neither the critical values comes from approach-2 will be affected by this fact nor size and orientation of the envelopes are unaffected by the presence or absent of these static responses.

In this exercise we took circular column section so that our distance of maximum stressed fiber (which we uses in calculating $Z_{max} = I/Y_{max}$ i.e. maximum section modulus for any section) remains constant.

Modal Analysis Results

Table 4.3777 Modal Periods, Frequency, Participating Mass Ratio and Modal mass and stiffness

Mode	Period	Frequency	Damping	Sum of Modal M	ass Participation	ModalMass	ModalStiff	
Shape	Sec	Cyc/sec	Ratio	in U1 dir.%	in U2 dir.%	KN-m-s2	KN-m	
1	0.864628	1.15657	0.05	0.84809	0.00021	1	52.80823307	
2	0.864628	1.15657	0.05	0.84830	0.84830	1	52.80823307	
3	0.759648	1.31640	0.05	0.84830	0.84830	1	68.41242987	
4	0.27698	3.61036	0.05	0.94931	0.85267	1	514.5900729	
5	0.27698	3.61036	0.05	0.95369	0.95369	1	514.5900729	
6	0.246032	4.06452	0.05	0.95369	0.95369	1	652.1957037	
7	1.59E-01	6.29296	0.05	0.97580	0.96772	1	1563.400425	
8	0.158908	6.29296	0.05	0.98984	0.98984	1	1563.400425	
9	1.44E-01	6.96056	0.05	0.98984	0.98984	1	1912.703891	
10	1.16E-01	8.63096	0.05	0.99998	0.98985	1	2940.8854	
11	1.16E-01	8.63096	0.05	0.99999	0.99999	1	2940.8854	
12	1.06E-01	9.44379	0.05	0.99999	0.99999	1	3520.88947	

Table 4.38 Response Spectra Ordinates

Modes	Acceleration	n Response	Displacement Response			
	(SAP	Uses)	(To Derive Envelop)			
	m/sec2	m/sec2	m	m		
1	6.39079547	6.937911266	0.12101892	0.131379349		
2	6.39079547	6.937911266	0.12101892	0.131379349		
3	7.17881974	8.086061909	0.10493444	0.1181958		
4	13.9173705	10.24650723	0.02704555	0.01991198		
5	13.9173705	10.24650723	0.02704555	0.01991198		
6	12.7442713	10.47682395	0.01954056	0.016063927		
7	9.77578201	8.544269976	0.0062529	0.005465183		
8	9.77578201	8.544269976	0.0062529	0.005465183		
9	9.32192174	7.700440611	0.00487369	0.004025945		
10	8.49392625	6.161008841	0.00288822	0.00209495		
11	8.49392625	6.161008841	0.00288822	0.00209495		
12	8.19696281	5.608886333	0.00232809	0.001593031		

Table 4.39 Modal Participation Factors

Modes	Period	UX	UY	
	Sec	KN-s2	KN-s2	
1	0.864628	-32.7045	-0.5179	
2	0.864628	-0.5179	32.7045	
3	0.759648	0.0000	0.0000	
4	0.27698	-11.2870	-2.3488	
5	0.27698	-2.3488	11.2870	
6	0.246032	0.0000	0.0000	
7	1.59E-01	-5.2812	4.2073	
8	0.158908	4.2073	5.2812	
9	1.44E-01	0.0000	0.0000	
10	1.16E-01	-3.5761	0.1141	
11	1.16E-01	-0.1141	-3.5761	
12	1.06E-01	0.0000	0.0000	

Table 4.40 Correlation Coefficients (CQC Rule)

Correlaton Coefficient matrix												
Mode	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	1.0000	0.3726	0.0059	0.0059	0.0046	0.0020	0.0020	0.0017	0.0012	0.0012	0.0010
2	1.0000	1.0000	0.3726	0.0059	0.0059	0.0046	0.0020	0.0020	0.0017	0.0012	0.0012	0.0010
3	0.3726	0.3726	1.0000	0.0079	0.0079	0.0060	0.0025	0.0025	0.0021	0.0014	0.0014	0.0012
4	0.0059	0.0059	0.0079	1.0000	1.0000	0.4150	0.0295	0.0295	0.0208	0.0111	0.0111	0.0089
5	0.0059	0.0059	0.0079	1.0000	1.0000	0.4150	0.0295	0.0295	0.0208	0.0111	0.0111	0.0089
6	0.0046	0.0046	0.0060	0.4150	0.4150	1.0000	0.0478	0.0478	0.0315	0.0154	0.0154	0.0120
7	0.0020	0.0020	0.0025	0.0295	0.0295	0.0478	1.0000	1.0000	0.4950	0.0893	0.0893	0.0553
8	0.0020	0.0020	0.0025	0.0295	0.0295	0.0478	1.0000	1.0000	0.4950	0.0893	0.0893	0.0553
9	0.0017	0.0017	0.0021	0.0208	0.0208	0.0315	0.4950	0.4950	1.0000	0.1761	0.1761	0.0952
10	0.0012	0.0012	0.0014	0.0111	0.0111	0.0154	0.0893	0.0893	0.1761	1.0000	1.0000	0.5518
11	0.0012	0.0012	0.0014	0.0111	0.0111	0.0154	0.0893	0.0893	0.1761	1.0000	1.0000	0.5518
12	0.0010	0.0010	0.0012	0.0089	0.0089	0.0120	0.0553	0.0553	0.0952	0.5518	0.5518	1.0000

Table 4.41 Response Spectrum Method Results (θ = 0)

TABLE: Eleme	nt Force	es - Frames				
Frame	Joint	CaseType	StepType	P	M2	M3
Element No.	No	Text	Text	KN	KN-m	KN-m
1	1	LinRespSpec	Max	610.7763	372.3028	339.3274
1	2	LinRespSpec	Max	610.7763	205.705	187.4854

Table 4.42 Response Spectrum Values for stresses (By Response Spectrum Results) (θ = 0)

TABLE: Elemen	nt Force	es - Frames				
Frame	Joint	CaseType	StepType	Р	M2	М3
Element No.	No	Text	Text	KN	KN-m	KN-m
1	1	LinRespSpec	Max	3.080062	33.12862	30.19437
1	2	LinRespSpec	Max	3.080062	18.30425	16.68302

Table 4.43 Derived Response Matrix for $f_{a^-}f_{m1}^-f_{m2}$ (θ = 0)

3.074515	6.271461	5.782797
6.271461	18.04116	0.743109
5.782797	0.743109	16.63313

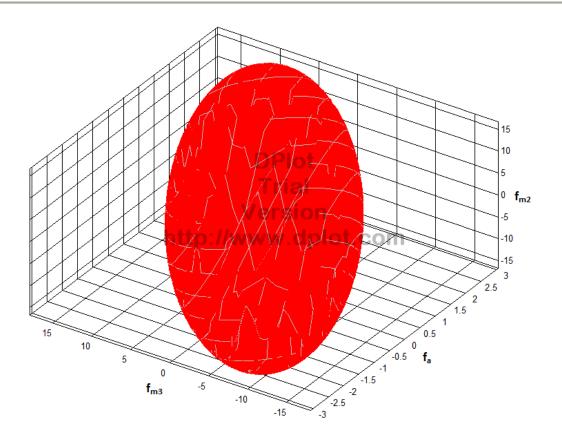


Fig 4.25 Stress Envelop For f_a - f_{m1} - f_{m2} (f_a , f_{m1} and f_{m2} in N/mm²) (θ = 0)

Table 4.44 Combined stresses (By Modal Analysis Results)

mer	Element Forces - Frames	ames			Joint Sperated	erated			Stresses		Combin	Combine Stress
int	Joint OutputCase CaseType StepType StepNum	CaseType	StepType	StepNum	Ь	M2	M3	Axial	Flexural2	Flexural2 Flexural3	Max	Min
Text	Text	Text	Text	Unitless	KN	KN-m	KN-m	N/mm2	N/mm2	N/mm2	N/mm2	N/mm2
1	1 MODAL	LinModal Mode	Mode	1	-411.75	5.50	-332,96	-2.10	0.49	-29.89	-31.50	27.31
1	1 MODAL	LinModal Mode	Mode	2	398.91	-332.96	-5.14	2.03	-29.89	-0.46	-28.32	32.38
1	1 MODAL	LinModal Mode	Mode	3	0.00	-334.84	-334.84	0.00	-29.80	-29.80	-59.59	59.59
1	1 MODAL	LinModal Mode	Mode	4	744.96	219.29	-1045.87	3.80	19.51	-93.06	-69.76	77.35
1	1 MODAL	LinModal Mode	Mode	5	-488.32	-1045.15	-215.85	-2.49	-93.00	-19.21	-114.69	109.72
1	1 MODAL	LinModal Mode	Mode	9	0.00	1035.65	1035.65	0.00	92.15	92.15	184.31	-184.31
1	1 MODAL	LinModal Mode	Mode	7	-45.25	-1082.17	-1361.59	-0.23	-96.29	-121.16	-217.68	217.22
1	1 MODAL	LinModal Mode	Mode	8	399.82	-1372.72	1096.13	2.04	-122.15	97.54	-22.57	26.65
1	1 MODAL	LinModal Mode	Mode	6	0.00	1656.25	1656.25	0.00	147.38	147.38	294.76	-294.76
1	1 MODAL	LinModal Mode	Mode	10	626.12	-36.32	-1646.31	3.19	-3.23	-146.49	-146.54	152.92
1	1 MODAL	LinModal Mode	Mode	11	667.39	1647.34	92'89-	3.40	146.59	-6.12	143.87	-137.07
1	1 MODAL	LinModal Mode	Mode	12	-0.01	1768.03	1768.03	0.00	157.32	157.32	314.65	-314.65
2	2 MODAL	LinModal Mode	Mode	1	411.75	3.07	-183.65	2.10	0.27	-16.34	-13.97	18.17
2	2 MODAL	LinModal Mode	Mode	2	-398.91	-183.64	-2.75	-2.03	-16.34	-0.24	-18.62	14.55
2	2 MODAL	LinModal Mode	Mode	3	0.00	-197.82	-197.82	0.00	-17.60	-17.60	-35.21	35.21
2	2 MODAL	LinModal Mode	Mode	4	-744.96	159.18	-759.19	-3.80	14.16	-67.56	-57.19	49.60
2	2 MODAL	LinModal Mode	Mode	5	488.32	-758.67	-156.69	2.49	-67.51	-13.94	-78.96	83.94
2	2 MODAL	LinModal Mode	Mode	9	0.00	777.75	27.777	0.00	69.21	69.21	138.41	-138.41
2	2 MODAL	LinModal Mode	Mode	7	45.25	-1044.15	-1313.82	0.23	-92.91	-116.91	-209.59	210.05
2	2 MODAL	LinModal Mode	Mode	8	-399.82	-1324.78	1057.91	-2.04	-117.88	94.14	-25.78	21.71
2	2 MODAL	LinModal Mode	Mode	6	0.00	1614.66	1614.66	0.00	143.68	143.68	287.36	-287.36
2	2 MODAL	LinModal Mode	Mode	10	-626.12	-42.76	-1917,94	-3.19	-3.81	-170.66	-177.66	171.28
2	2 MODAL	LinModal Mode	Mode	11	-667.39	1919.12	99'6/-	-3.40	170.77	-7.09	160.28	-167.08
2	2 MODAL	LinModal Mode	Mode	12	-0.01	1768.03	1768.03	0.00	157.32	157.32	314.65	-314.65

Table 4.45 Critical Combined stresses (By approach-2) (At Modal Level)

TABLE: f _{max} =	P/A + N	$M_y z/I_y + M_z y/$	lz				
Frame	Joint	Critical	f _o	f ₉₀	f_1	f_2	f _{Total}
		Angle					
Element		0					
No.	No.		N/mm ²	N/mm ²	N/mm ²	N/mm²	N/mm ²
1	2	-37.88637	16.09253	20.68192	0	5.47434866	5.474349
1	2				-25.4014	22.274402	33.78434
1	2				25.4014	-22.274402	33.78434
1	2				0	-5.4743487	5.474349

TABLE: f _{min} =	P/A - N	ly/ly - Mz / lz					
Frame	Joint	Critical	f _o	f ₉₀	f ₁	f ₂	f _{Total}
		Angle					
Element		0					
No.	No.		N/mm ²	N/mm ²	N/mm ²	N/mm³	N/mm ⁴
1	2	38.72506	20.44542	16.39456	26.2068	21.74355	34.0525872
1	2				-5.69446	0	5.69445803
1	2				5.694458	0	5.69445803
1	2				-26.2068	-21.7436	34.0525872

Table 4.46 Critical Combined stresses (By approach-2) (By Response Spectrum Analysis Results)

TABLE: Comb	ined Str	esses								
Frame	Joint	Critical	Critical	Response	f _o	f ₉₀	f ₁	f ₂	f _{Total}	Max
		Response	Angle	Values						Stresses
Element No.	No.	Value	0	At CA						N/mm ²
1	2	Р	45	P (kN)	443.83412	443.83412	627.67624	533.5248	823.7877	
1	2						-6.619E-07	0	6.619E-07	
1	2						6.619E-07	0	6.619E-07	
1	2						-627.67624	-533.5248	823.7877	
1	2			M-2 (kN-m)	16.092534	20.681918	145.37929	123.5724	190.80167	
1	2						145.07886	123.31703	190.40736	
1	2						-145.07886	-123.31703	190.40736	
1	2						-145.37929	-123.5724	190.80167	
1	2			M-3 (kN-m)	16.092534	20.681918	145.37929	123.5724	190.80167	
1	2						-145.07886	123.31703	190.40736	
1	2						145.07886	-123.31703	190.40736	
1	2						-145.37929	-123.5724	190.80167	38.152804
1	2	M-2	-0.059264	P (kN)	16.092534	20.681918	443.3748	376.86858	581.90304	
1	2						-444.29297	377.64902	583.10808	
1	2						444.29297	-377.64902	583.10808	
1	2						-443.3748	-376.86858	581.90304	
1	2			M-2 (kN-m)	16.092534	20.681918	0	174.57691	174.57691	
1	2						-0.4248824	174.57728	174.5778	
1	2						0.4248824	-174.57728	174.5778	
1	2						0	-174.57691	174.57691	
1	2			M-3 (kN-m)	16.092534	20.681918	205.3846	-1.86E-06	205.3846	
1	2						-205.38504	0.3611482	205.38535	
1	2						205.38504	-0.3611482	205.38535	
1	2						-205.3846	1.86E-06	205.3846	36.780775
1	2	M-3	0.0592637	P (kN)	16.092534	20.681918	444.29296	377.64902	583.10807	
1	2						-443.37481	376.86859	581.90304	
1	2						443.37481	-376.86859	581.90304	
1	2						-444.29296	-377.64902	583.10807	
1	2			M-2 (kN-m)	16.092534	20.681918	0.4248802	174.57728	174.5778	
1	2						-2.188E-06	174.57691	174.57691	
1	2						2.188E-06	-174.57691	174.57691	
1	2						-0.4248802	-174.57728	174.5778	
1	2			M-3 (kN-m)	16.092534	20.681918	205.38504	0.3611463	205.38535	
1	2						-205.3846	0	205.3846	
1	2						205.3846	0	205.3846	
1	2						-205.38504	-0.3611463	205.38535	36.780775

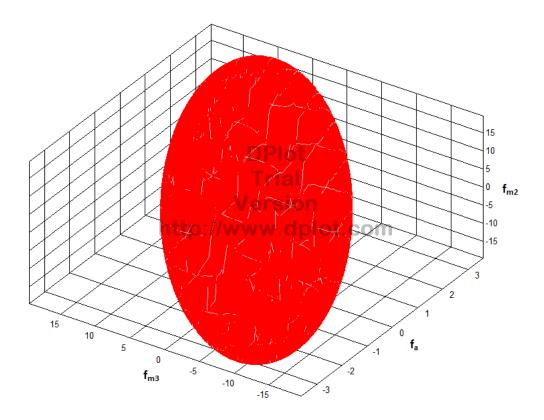


Fig 4.26 Supreme Stress Envelop For f_a - f_{m1} - f_{m2} (f_a , f_{m1} and f_{m2} in N/mm²)

Table 4.47 Combined stresses by Supreme stress envelop coordinates (approach-1)

TABLE: f _{combined}	= f _a +	$f_{m1} + f_{m2}$	
Frame	Joint		f _{combined}
Element No.	No.		N/mm²
1	2	Maximum of Max.	32.3475
1	2	Minimum of Min.	-32.3475

CHAPTER 6

Conclusions & Future Scope

6.1 Conclusion

From the results mentioned in previous chapter we can conclude following points:

- The envelop is effectively enclosing Time-History Analysis results of all the values of P and M i.e. axial forces and moments in column occurring simultaneously at any time instant. Hence it can be used for designing structures.
- The % reinforcement get provided by using conventional Rectangular envelop is 30-40% higher in comparison to Elliptical envelop in case of column like members which are design for both axial force and moments. Hence we should use this envelop to get economy in design.
- The modelling for vertical component of earthquake as followed in problem 3 is more correct and this is increasing significance of elliptical envelop we are using because it weakening correlation between axial force and moments in column.

6.2 Future Scope

- In present derivation of envelop we did matrix operation in Microsoft Excel, one can program this methelogy.
- We can repeat this analysis for different vertically irregular structure. So that we can do
 better comments on significance of elliptical envelop over conventional rectangular
 envelop.
- Here we have not taken care of size of beam section used for approximating floor diaphragm stiffness. One can use better recommendation and repeat whole analysis to see significance of elliptical envelop.
- It is now clear that this approach is resulting into significance saving in material for design of member like column which are simultaneously critical in axial force and moments coming on it. So it should be accommodated in codes and different analysis software.
- One can study other member which are critical in designing in two or more response of structure like bridge girder.

CHAPTER 6

Conclusions & Future Scope

6.1 Conclusion

From the results mentioned in previous chapter we can conclude following points:

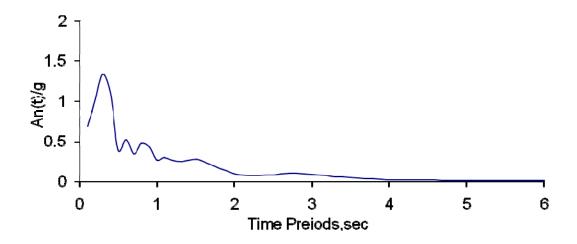
- The envelop is effectively enclosing Time-History Analysis results of all the values of P
 and M i.e. axial forces and moments in column occurring simultaneously at any time
 instant. Hence it can be used for designing structures.
- The % reinforcement get provided by using conventional Rectangular envelop is 30-40% higher in comparison to Elliptical envelop in case of column like members which are design for both axial force and moments. Hence we should use this envelop to get economy in design.
- The modelling for vertical component of earthquake as followed in problem 3 is more correct and this is increasing significance of elliptical envelop we are using because it weakening correlation between axial force and moments in column.

6.2 Future Scope

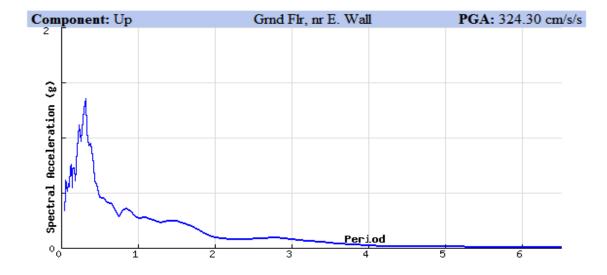
- In present derivation of envelop we did matrix operation in Microsoft Excel, one can program this methelogy.
- We can repeat this analysis for different vertically irregular structure. So that we can do
 better comments on significance of elliptical envelop over conventional rectangular
 envelop.
- Here we have not taken care of size of beam section used for approximating floor diaphragm stiffness. One can use better recommendation and repeat whole analysis to see significance of elliptical envelop.
- It is now clear that this approach is resulting into significance saving in material for design of member like column which are simultaneously critical in axial force and moments coming on it. So it should be accommodated in codes and different analysis software.
- One can study other member which are critical in designing in two or more response of structure like bridge girder.

Input Data Verification

We took Corrected Accelerogram data and corresponding response spectra available on site of www.consrv.ca.gov i.e. CSMIP (California Strong Motion Instrument Program) and derive Response spectra for all the three component of earthquake by Central Interpolation Method. Response Spectra Derived and Collected from site are compared below. For Northridge earthquake at Sylmar, CA - County Hospital, January 17, 1994, Spectra for UP Component.(All the spectra shown below are for 2% damping).



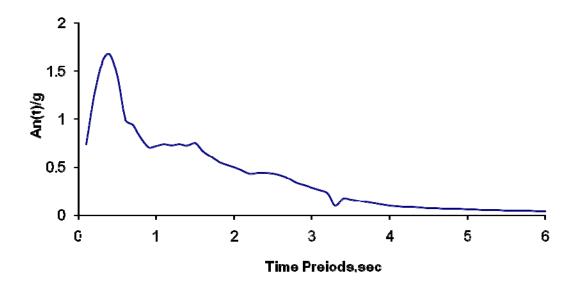
Pseudo – Acceleration Spectra (Calculated From Corrected Accelerogram, UP)



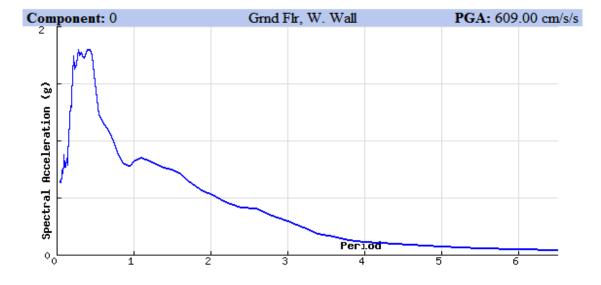
Response Spectra Available On site of California Strong Motion Instrumentation Program

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Spectra for 0 degree Component

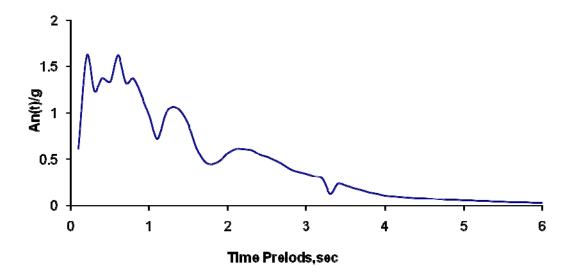


Pseudo – Spectra Acceleration (Calculated From Corrected Accelerogram, 0 Degree)

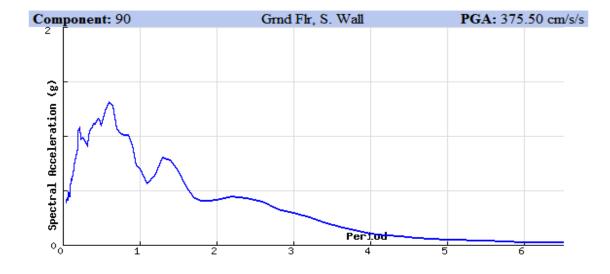


Response Spectra Available On site of California Strong Motion Instrumentation Program

Spectra for 90 Degree Component

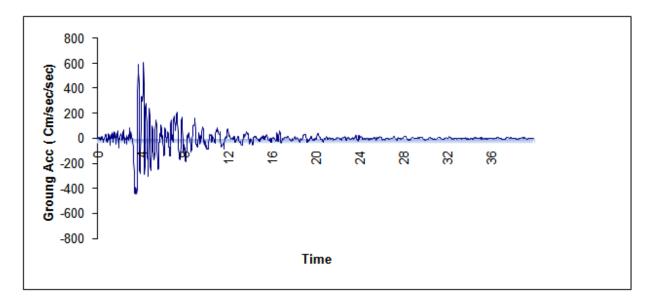


Pseudo – Spectra Acceleration (Calculated From Corrected Accelerogram, 90 Degree)

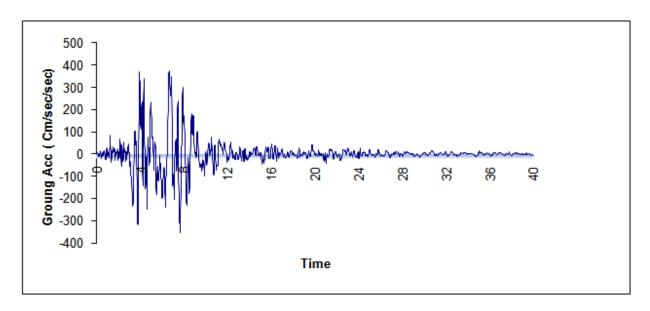


Response Spectra Available On site of California Strong Motion Instrumentation Program

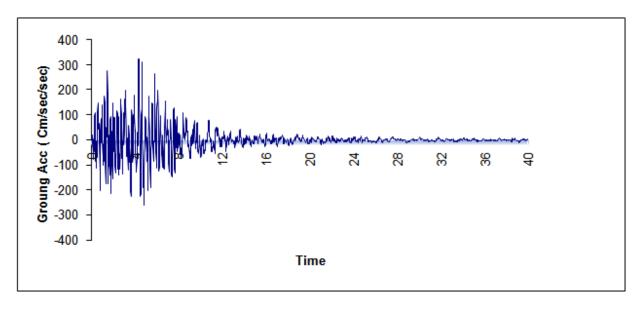
The Corrected Accelerogram for Northridge Earthquake (Jan. 17, 1994) at Sylmar, CA - County Hospital station form CSMIP site is available for 40 sec. For the 0 Degree component the ground acceleration record is as below,



Northridge Earthquake (Jan. 17, 1994) at Sylmar, CA - County Hospital (O Degree Component)



Northridge Earthquake (Jan. 17, 1994) at Sylmar, CA - County Hospital (90 Degree Component)



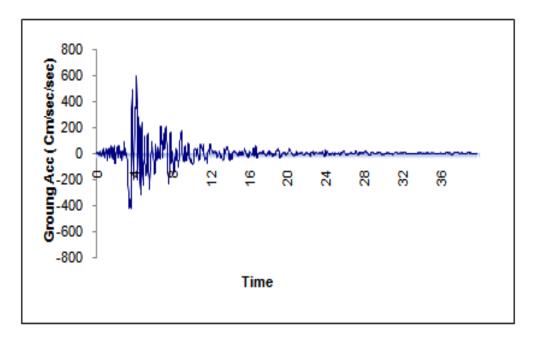
Northridge Earthquake (Jan. 17, 1994) at Sylmar, CA - County Hospital (UP Component)

These ground motion components recorded for each of these events are not necessarily uncorrelated, these records was first decomposed into uncorrelated components through the rotational transformation.

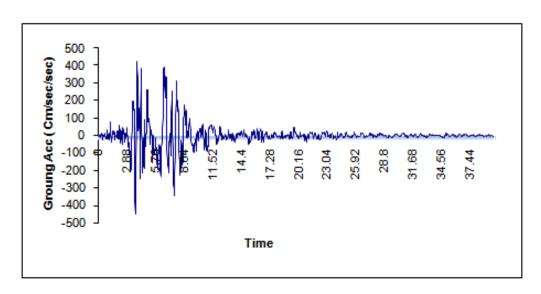
Where $\ddot{u}_{g1}(t)$ and $\ddot{u}_{g2}(t)$ = original recorded horizontal ground motions $\ddot{u}_{g1}(t)$ and $\ddot{u}_{g2}(t)$ = uncorrelated components; and ϕ required rotation, which is computed by Charles Menun and Armen Der Kiureghian as described by Penzien and Watabe (1975). For Northridge records ϕ = 0.243 rad were obtained. The transformed ground motion components $\ddot{u}_{g1}(t)$ and $\ddot{u}_{g2}(t)$ were then used as input for the Time - History analyses for comparing elliptical envelop with time history results. The response spectra associated with $\ddot{u}_{g1}(t)$ and $\ddot{u}_{g2}(t)$ were also computed and used to estimate the elliptical and rectangular envelopes. The vertical component of earthquake need not to modify because in paper "Characteristic if three dimensional earthquake spectra "by Penzien and Watabe (1975) it is shown that minor principal directions of earthquake remains nearly vertical.

As per above procedure, we calculated uncorrelated component of earthquake. The same is used for deriving Acceleration Response Spectra. These data are used for Response Spectrum

Analysis and Time – History Analysis in SAP. Uncorrelated ground acceleration principle component of Northridge earthquake in major principle direction is as below,

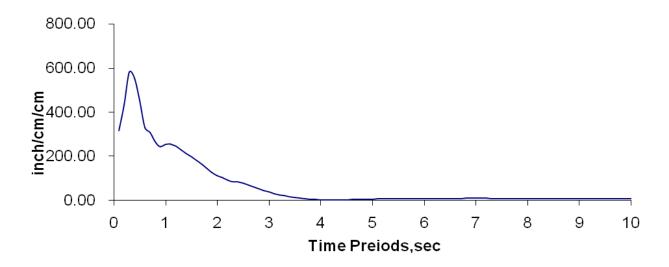


Northridge Earthquake (Jan. 17, 1994) (Major Principle Component)

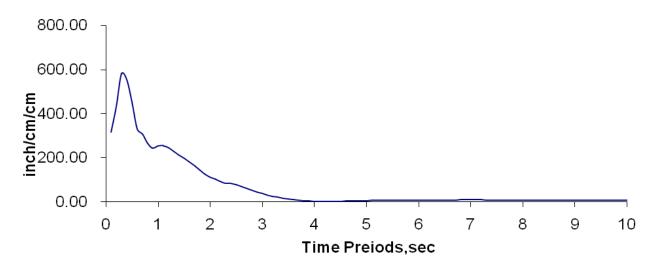


Northridge Earthquake (Jan. 17, 1994) (Intermediate Principle Component)

Acceleration Response Spectra derived from above time history is used for Response Spectrum in SAP and for developing Elliptical and Supreme Envelop (For 5% Damping).



Pseudo – Spectra Acceleration (Calculated From above transform Accelerogram, Major Principle Direction)



Pseudo – Spectra Acceleration (Calculated From above transform Accelerogram, Intermediate Principle Direction)

These Spectra are derived in $inch/Sec^2$. For Analysis in SAP we use 0.0254 Scale Factor to convert in m/Sec^2 . We input time histories in cm/Sec^2 and Scale Factor of 0.01 is used for SAP Time – History analysis. Conventionally for response spectrum displacement response spectrum ordinate is defined as $S_i = \max |s(t)|$, in present analysis we defined it as $S_i = \max [s_i(t)]$. Use of conventional response spectrum in the present analysis will produce results that are on conservative side by more than 5%. We have use conventional definition for response spectrum for data verification but used recommended definition for developing elliptical envelop and supreme envelop.

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