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RESEARCH PAPER

Strength and dilatancy of jointed rocks with granular fill

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Abstract It is well recognised that the strength of rock masses depends upon the strain history, extent of discontinuities, orientation of plane of weakness, condition of joints, fill material in closely packed joints and extent of confinement. Several solutions are available for strength of jointed rock mass with a set of discontinuities. There is a great multiplicity in the proposed relationships for the strength of jointed rocks. In the present study, the author conceives the effect of increasing stresses to induce permanent strains. This permanent strain appears as micro crack, macro crack and fracture. A fully developed network of permanent deformations forms joint. The joint may contain deposits of hydraulic and hydrothermal origin commonly known as gouge. The joint factor numerically captures varied engineering possibilities of joints in a rock mass. The joints grow as an effect of loading. The growth of the joints is progressive in nature. It increases the joint factor, which modifies the failure stresses. The dilatancy explains the progressive failure of granular media. Hence, a mutual relationship conjoins effectively the strength of jointed rock and a dilatancy-dependent parameter known as relative dilatancy. This study provides a simple and integral solution for strength of jointed rocks, interpreted in relation to the commonly used soil, and rock parameters, used for a realistic design of structure on rock masses. It has scope for prediction of an equivalent strength for triaxial and plane strain conditions for unconfined and confined rock masses using a simple technique.

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List of symbols

ϕ_{cn}, ϕ_{peak}	Angle of critical friction and peak internal friction, respectively (°)
ϕ_j	Equivalent friction angle for the jointed rocks (°)
ψ	Angle of dilatancy (°)
γ^p, ε^p	Plastic shear and plastic volumetric strain
A	Empirical constant and has a value of 3 for axe-symmetrical and 5 for plane strain case
C' and C	Initial confining pressure-dependant empirical fitting parameters for jointed rocks
c_g	A modification factor for gouge
d_a	Reference depth of joint (=sample diameter in mm)
d_j	Depth of joint in mm
D^p	Dilatancy as a function of plastic shear and volumetric strain
$d\varepsilon_v/d\varepsilon_1$	Ratio of changes in volumetric and axial strain
g_d	Correction factor depending upon the density of gouge in joint
I_r	relative dilatancy index
J_{d_j}	Joint depth parameter
J_f	Joint factor
J_{fg}	Joint factor corrected for gouge
J_n	Number of joints in the direction of loading (joints per metre length of the sample)

J_t	Gouge thickness parameter
L_{na}	Reference length (=1 m)
M and B	Empirical rock constants
n	Joint orientation parameter depending upon inclination of the joint plane [β (°)] with respect to the direction of loading
p	Mean confining pressure (kPa)
p_a and σ_a	Reference pressure (=1 kPa)
p_i	Initial mean confining pressure (kPa)
q	Shear stress (kPa)
Q_j and r_j	Empirical material fitting constants for gouge
r	Joint strength parameter
RAC	Ramamurthy–Arora criterion
R_D	Relative density of gouge
t	Thickness of gouge in the joint (mm)
t_a	Reference thickness of gouge in the joint (=1 mm)
λ	Empirical coefficient for joint factor
ξ	Empirical coefficient for dilatancy
α	Fitting constant
σ_1, σ_3	Major and minor principal stresses, respectively (kPa)
σ_{ci}, σ_{cj}, σ_{cjg}	Uniaxial compressive strength of intact, jointed and jointed rock with gouge respectively (kPa)
σ_{cr}	Strength ratio
S_{cr}	Strength reduction factor during shear along the gouge

1 Introduction

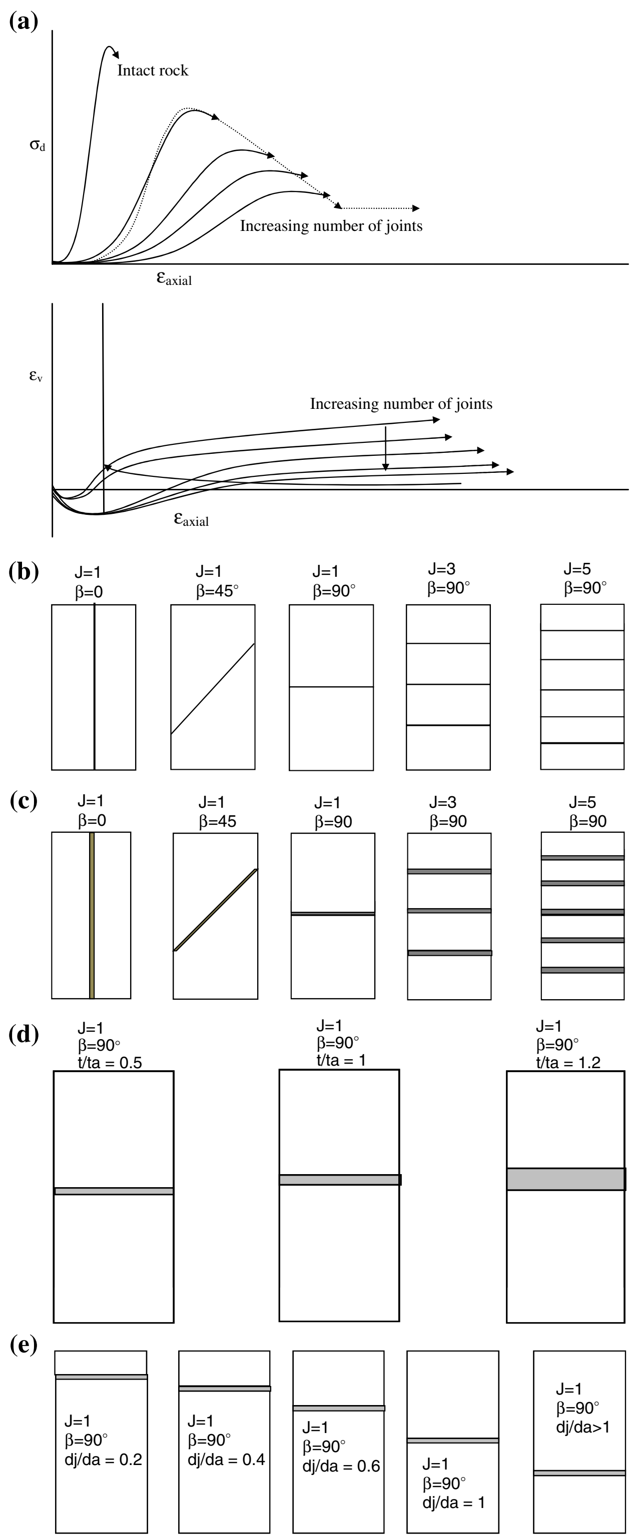
The strength of jointed rock mass is important for the design of structures built on rocks such as towers, bridge piers, tunnels, deep-seated nuclear and hazardous waste confinements and dams. The rock masses occur in nature with joints and varying amount of infill material commonly known as gouge. The in situ tests show indirect effect of an expanding network of micro cracks in rock mass indirectly incorporated through the effect of size [6, 7, 28] on rock mass compressive strength. Interestingly, a large network of micro fractures may have a similar effect of strength reduction as observed in the case of jointed rocks with a number of joints numerically simulated to similar strain history. At a stress around 10% of the failure stress, its redistribution begins. The deformation starts to be progressively non-linear. It is often classified as non-associated plastic flow.

Upon loading, rock masses experience early plasticity as accommodated in crack closure for intact rocks or joint closure for jointed rocks. Further deformations are elasto-plastic until the brittle failure takes place in the intact rocks. The rock masses with multiple joints conceal brittle failure largely as the joints tolerate large plastic deformations. Figure 1a shows a conceptual model of the stress–strain and volume–change plots for jointed rocks with increasing joints. As a result, the peak strength goes down and failure occurs at a higher strain. If the peaks of all the stress–strain diagrams are joined together by a smooth curve, the resulting plot incorporates the dilation of jointed rocks with increasing joint network. Thus, plastic flow becomes more prominent with number of joints. Figure 1a shows a sequel of failure points conjoined to illustrate a dilatancy pattern for jointed rocks. Dilatancy is a characteristic of material, which is associated with volume change during the process of deformation. The intact rocks experience fracturing whilst being subjected to shear at low to intermediate confining pressure, and pre-existing fractures that undergo shear displacement whilst being subjected to a relatively low normal stresses [10–12, 20, 21, 34]. There had been a little effort to map growth of joint network and dilatancy parameters as an effect of mechanical loadings. However, the difficulties associated with defining a model that adequately reflects dilatancy of rock masses has consistently attracted attention of early researchers in rock mechanics [10, 12, 20].

2 Review of literature and scope of study

According to Terzaghi [34], an intact rock has no joints or hair cracks. Normally joints are recognised as discontinuities at the boundary of the intact rock [2, 5, 14, 17, 18, 22, 23, 26, 29]. The discontinuities may exist with or without fragments of parent rock material deposited in the joints [3, 4, 19, 27, 32, 35, 36].

Hoek and Brown [17] and Barton [5] measured scale effects in uniaxial compressive strength of intact rock. Their criterion covers the sizes in the range of laboratory scale (50 mm) to field sample of certain size at which intact strength offsets the effect of network of micro cracks. The strength ratio drops to nearly half as the sample size increases to a certain size [36]. Hoek and Brown criterion [17, 18] considered uniaxial compressive strength of intact rock, and proposed a relation for rock mass rating (RMR) and geological strength index (GSI). This system does not directly consider the joint orientation. Further, the joint size is not included directly as a parameter in estimating either the RMR or GSI. However, the effect of joint size is indirectly considered in rock mass strength in terms of scale effects. The RMR includes the joint spacing and rock quality designation (RQD). Furthermore, RMR and GSI provide measure of qualitative assessment of rock



◀ **Fig. 1** **a** Stress–strain behaviour of rock mass with increasing joints. **b** Typical examples of discontinuity (joint) at varying joint inclination ($\beta°$) and joint number ($@1J{:}J_n = 26$). **c** Typical examples of discontinuity (gouge thickness) at varying joint inclination ($\beta°$) and joint number. **d** Typical variation gouge thickness factor (t/t_a) at a particular joint inclination ($\beta = 90°$) and joint number ($J = 1$). **e** Typical variations of joint depth factor (d_j/d_a) with gouge at a particular joint inclination ($\beta = 90°$)

mass strength. It shows that the rock mass strength criteria should be improved to have a capability of predicting rock mass strength under different extents of joint spread and unifying the scale dependence, anisotropy, and the effect of discontinuities. The past studies have left out scope for estimation of dilatancy of rock masses as one of the unresolved issues. Second, the strength of jointed rock was yet to relate a parameter estimated by direct measurements, which could connect it with dilatancy.

2.1 Definition of the problem and applications

In routine engineering applications, the relation of rock mass strength with dilatancy receives far a less attention, which is essentially because of the fact that many problems of rock mechanics are solved avoiding failure and secondly due to inherent difficulties in estimating dilatancy. The aim of present study is to advance a consistent model for jointed rocks, which connects the strength with dilatancy index frequently used for granular mass. It has potential application in rock excavations, tunnelling and foundations in rock masses. In view of the present trends in modelling, the purpose of this study is not to obtain highly accurate values of strength, but rather to focus on the issue that considers the affects of dilatancy in theoretical and practical engineering problems. A large ensemble of studies on this topic reveal that the dilatancy angle (ψ) and relative dilatancy index (I_r) are rarely taken in to consideration. Often, when it is considered, the approach is poorly drawn consisting of an associated flow rule ($\psi = \phi$) or non-associated flow rule ($\psi = 0$). Nevertheless, associated flow rule does not necessarily represent failure behaviour of rock masses.

The results of model studies on rocks and modified masses [11, 14, 21, 26] illustrated the possibility of varied failure modes owing to the highly intricate internal stress distribution within a jointed rock mass. With the progress of failure, there is a mutual adjustment of the micromechanical strength parameters with the mean effective confining pressure. So far, the strength behaviour of jointed rock mass has been quantified as a function of joint orientation, joint size, frequency, roughness and waviness of the joints. There are some difficulties in mapping these parameters for rock masses. Therefore, combined effect in terms of equivalent values adopted for joint factor or relative dilatancy may capture reasonably well the strength of jointed rock mass right form a state of low-confinement to heavily confined state using iterative inputs of the resultant strength.

2.2 Preliminary definitions

In this study, the experimental results considered uniaxial and triaxial testing of model material blocks having fully persistent joints as an effect of plastic strain history. The proposed technique is largely useful for jointed anisotropic mass, where failure would be progressive over and above experienced plasticity. The conventional strength evaluation methods applied for jointed rocks do not consider progressive failure. The author explains these effects based on the non-linear strength behaviour of the granular fill material and occurrence of progressive failure through the joints. Few novel observations presented here show the strength of jointed rocks as a compressed function of material characteristics of the rocks and joint factor as a function of dilatancy. The joint factor (J_f) is defined as a ratio of joint frequency (J_n), to the product of joint orientation parameter (n) and joint strength parameter (r) [2, 29].

$$J_f = J_n/nr \tag{1}$$

J_n Number of joints in the direction of loading (equal to number of joints per metre length of the sample)

N Orientation parameter related to inclination of joints (β) with the direction of major principal stress and

r Joint strength parameter depending on the joint condition (ϕ_j), which is equivalent friction angle along the joint plane so that the roughness of the surface is represented through this value (it is obtained by a shear test on the rock joint)

Theoretically, joint factor has the effects of already experienced dilatancy by the rock mass. Therefore, a relation between joint factor (J_f) and dilatancy (D^P) represents this effect as

$$J_f = f(\gamma^p, \varepsilon^p) \tag{2a}$$

$$J_f = f'(D^p) \tag{2b}$$

γ^p, ε^p Plastic shear and volumetric strain

D^p Dilatancy which is defined as a change in volume resulting from the shear distortion of an element in a material. It is a function of plastic shear strain and plastic volumetric strain. It may include the effects of damage in the rock material, if any. In order to consider the effect of damage in theoretical framework of plasticity, there is a need of components of damage (γ^{pd}, ε^{pd}) in Eq. 2a. Since the joint factor is, an empirical formulation for the effect of stress, it should experimentally capture all its consequences

Therefore, the experimentally obtained strength has been evaluated corresponding to a joint factor, and applied stress coupled with joint strength parameter. The constitutive relationships for peak angle of internal friction based on knowledge of compactness or relative density (R_D) of in-fill material, mean effective confining pressure (p'), and angle of critical friction (ϕ_{cn}), for different rock materials. The value of angle of critical friction (ϕ_{cn}) is the angle of shearing observed in a simple shear test on a joint loose enough to be in critical state with zero dilation. The granular material in the joint's rupture zone dilates fully to achieve the critical state at which shear deformation can continue without any volume change. The relative density (R_D) is considered conventionally as a ratio of difference of maximum void ratio (e_{max}) and natural state void ratio (e_n) to maximum void ratio (e_{max}) and minimum void ratio (e_{min}) of the gouge material [$R_D = (e_{max} - e_n)/(e_{max} - e_{min})$]. The effect of pore pressure is discounted having not considered in this analysis; therefore, mean effective confining pressure (p') is equal to mean confining pressure (p) [$p = (\sigma_1 + \sigma_2 + \sigma_3)/3$, where σ_1, σ_2, σ_3, are principal stresses].

The strength of jointed rock is often evaluated in terms of strength ratio. The primary aim of finding strength ratio relationship with joint factor is to get readily the strength of jointed rock by conducting single uniaxial compression test on the parent rock mass. The strength ratio (σ_{cr}) is defined as a ratio of strength of a jointed rock (σ_{cj}) with respect to uniaxial compressive strength of same sized intact rock (σ_{ci}) sample of the same parent material. If σ_{1cj}, σ_{2cj}, and σ_{3cj} are triaxial principal stresses at failure in the jointed rock and σ_{ci} is uniaxial compressive strength of the intact rock sample, then in the triaxial state, the strength ratio,

$$\sigma_{cr} = \left[\left(\sigma_{1cj} + \sigma_{2cj} + \sigma_{3cj}\right)/3\right]\left[(\sigma_{ci})/3\right] \tag{3}$$

$$= p/\left[(\sigma_{ci})/3\right] = 3p/(\sigma_{ci}) \tag{4}$$

In unconfined state, the strength ratio,

$$\sigma_{cr} = \left[(\sigma_{cj})/3\right]\left[(\sigma_{ci})/3\right] = \left[\sigma_{cj}\right]/\left[\sigma_{ci}\right] \tag{5}$$

The author analysed the test results with a focus on ways to determine the dilatancy jointed rocks with and without gouge in relation to the joint factor. Further, the author also examined the result in light of the Johnston's generalisation evolving from triaxial testing [22, 23]. The author conjoined these findings to propose a relationship for strength ratio incorporating dilatancy dependant joint factor.

3 Materials and methods

The author prepared the cylindrical cores of Kota sand stone, 38 mm diameter obtained from the block of rock

Table 1 Summary of experimental programme

Sample material	Rock mass; testing condition	Joint inclination (β) (in degrees)	Joint depth d_j/d_a (at mid-point of the vertical axis)	Joint thickness factor (t/t_a)	No. of tests
Kota sand stone	Intact; uniaxial	–	–	–	3
	Jointed; uniaxial	55, 60, 65, 70, 75, 80, 85, 90	1	0	27
	Jointed; uniaxial	55, 60, 65, 70, 75, 80, 85, 90	1	1	27
	Jointed; uniaxial	90	0.2, 0.4, 0.5, 0.6, 0.8, 1	0	6
	Jointed; uniaxial	90	0.2, 0.4, 0.5, 0.6, 0.8, 1	1	18
	Jointed; uniaxial	90	1	1, 1.4, 2, 3, 4, 5, 6	21
Plaster of Paris Arora [2]	Intact; uniaxial	–	–	–	3
	Jointed; triaxial	25, 30. 40, 50, 60, 70, 80, 90	1	0	27

using diamond impregnator core drills. The author considered the cylindrical cores of plaster of Paris in the mould of 38 mm internal diameter and 76 mm height. A disc cutting power saw finished the desired length of the core. The ends of the core specimen thus obtained were smoothened to meet the tolerance limit as specified by ISRM (1982). The height to the diameter ratio of the intact specimen was kept as two. The anisotropy was introduced in the rough broken joints at various inclinations by adopting a special technique of notching and breaking in the direction of desired orientation. It involved giving direction to breaking by means of creating grooves in the desired direction on the cylindrical sample using chisel and scale. Having given a direction to the groove, the sample was placed on a notch. With strike of the chisel along the groove on the sample, two pieces of a jointed specimen ware obtained. A thick paste of kaolin (specific gravity = 2.6, liquid limit = 48%, plasticity index = 24%) was prepared at water content of 27%. It was compressed inside the joints thus developed as a gouge in the specimen. The specimen was dried first in airtight desiccators then at 105°C in an electric oven. The uniaxial compression testing was carried out as per ISRM (1979) testing procedure. The Kota sand stone and plaster of Paris used in the present study had uniaxial compressive strength of 80,400 and 11,000 kPa. Table 1 shows summary of experimental programme used in this study. The typical examples of the jointed rock samples tested are shown in Fig. 1b–e.

3.1 Preliminary observations

The author considered an experimental programme for evaluation of strength behaviour of jointed rocks with and without gouge. The author conducted and considered a set of laboratory-controlled experiments. The objective was namely, to study the effect of thickness of gouge filled in the horizontal joints, the effect of orientation in the presence of gauge, the effect of frequency of horizontal joints with gouge material in it, the effect of location of joint with respect to the loading plane and the effect of confining pressure. The author also considered the results of experimental evaluation of jointed rock by various investigators [2, 22, 23, 29, 32, 35].

4 Interpretation of strength of jointed rocks and test results

During the past three decades, there has been extensive research on the strength of jointed rocks. The research on uniaxial and triaxial tests on intact and jointed specimens of plaster of Paris, Jamarani sandstone, and Agra sandstone [2, 29] indicated a relationship of strength ratio with intrinsic strength parameters of rock joints namely joint factor. Based on the results of uniaxial and triaxial tests of intact and jointed specimens, Ramamurthy and Arora [2, 29], proposed an empirical relation as shown in Eq. 6 for strength ratio (σ_{cr}) which later became popular as Ramamurthy–Arora criterion (RAC). As per RAC strength ratio is expressed as

$$\sigma_{cr} = \exp(\alpha J_f) \tag{6}$$

where J_f is joint factor as per Eq. 1 and σ_{cr} is strength ratio for jointed rocks. Factor 'α' is a fitting constant ($\alpha = -0.008$ [2, 29]). Various investigators interpreted [2, 19, 29, 32, 35] the values of the factor 'α' differently. The statistical variances [35] of RAC [2, 29], splitting sliding and rotational failures [32] for blank [29] and gouged joints having influence of joint inclination (β = 90–75° and 75–60°) [35] are shown in Table 2.

Barton [4] provided a basis for the joint friction angle considered as a function of the joint roughness coefficient (JRC), the joint wall compressive strength (JCS) and the basic friction angle of the joint surface and normal stress acting on the joint. The value of joint orientation parameter (n) is obtained numerically by taking the ratio of

Table 2 Empirical relationships for strength ratio

Model expressions	Coefficients	Data source	Reference	Remarks
$\sigma_{cr} = \exp(\alpha J_f)$	$\alpha = -0.008$	1. Yaji 2. Arora 3. Singh and Dev 4. Sharma	Arora [2] Ramamurthi and Arora [29]	1. Data mostly on PoP and sand stone 2. No data available for $J_f = 1$–13, $J_f > 800$ 3. No adjustment of r for mean effective stresses, density of gouge material and depth of joints with fill material 4. Bias of soft rock test results
$\sigma_{cr} = a + b \exp(-J_f/c)$ $\sigma_{cr} = b \exp(-J_f/c)$ $\sigma_{cr} = \exp(\alpha J_f)$	$a = 0.039$ $b = 0.893$ $c = 160.99$ @ $R^2 = 0.9961$ $b = 0.917$ $c = 179.26$ @ $R^2 = 0.9890$ $\alpha = -0.0065$ @ $R^2 = 0.9632$	1. Arora 2. Yaji 3. Einstien and Hirschfeld 4. Brown and Trollope 5. Brown 6. Roy	Jade and Sitharam [19]	1. Statistical re look on data 2. No data available for $J_f = 1$–13 3 Provides higher strength at all the joint factors compared to the preceding models 4. Bias of hard rock test results
$\sigma_{cr} = \exp(\alpha J_f)$	$\alpha = -0.0123$ @ Splitting/shearing $\alpha = -0.0180$ @ Sliding $\alpha = -0.0250$ @ Rotation	1. Yaji 2. Arora 3. Singh	Singh and Rao [32]	1. Experimental and statistical re look on data 2. No data available for $J_f = 1$–13 3. No adjustment of r for mean effective stresses, density of gouge material and depth of joints with fill material 4. Bias of orientation of joints
$\sigma_{cr} = \exp(\alpha J_f + \delta)$ (with granular fills)	$\alpha = -0.0250$, $\delta = 0.1$ @ $t/t_a < 5$ and $\beta = 90°$ $\alpha = -0.0140$, $\delta = -0.34$ @ $t/t_a < 5$ and $\beta = 90$–$75°$ $\alpha = -0.047$, $\delta = -2.2$ @ $t/t_a < 5$ and $\beta < 75°$	1. Arora 2. Trivedi	Trivedi [35] Arora and Trivedi [3]	1. Experimental re look on data 2. No data available for $J_f = 1$–13 3. No adjustment of r for mean effective stresses, density of gouge material and depth of joints with fill material 4. Bias of orientation of joints and clayey gauge
$\sigma_{cr} = \exp(\alpha J_f)$ (c_g applied for thickness depth and density adjustments of granular fills, $c_g = 1$ blank compact joints)	$\alpha = -0.008$ @ $\beta = 90$–$75°$ $R^2 = 0.9463$ $\alpha = -0.009$ @ $\beta = 90$–$75°$ $R^2 = 0.9934$		Trivedi and Arora [36]	1. Statistical relook on data 2. No adjustment of r for mean effective stresses, density of gouge material 3. Bias of gauge thickness and joint number

Table 2 continued

Model expressions	Coefficients	Data source	Reference	Remarks
$\sigma_{cr} = \exp(\lambda J_f/C)$	$\lambda = 0.025$–0.0415 $C = -1$ to -5	1. Johnston and Chew 2. Bolton	Present work	1. Parameter '*C*' interpreted from Johnston's data, Bolton's I_r, and *A* and modified joint factor, J_{fg}
$S_{cr} = \exp(AI_r/C)$	$A = 3$–5 $C = -1$ to -5 $I_r = \mathrm{RD}\,[Q_j - \ln(p'/p_a)] - r_j$	3. Arora 4. Trivedi		2. Relative dilatancy of soils integrated with jointed rocks with and without gouge for uniaxial and triaxial strength 3. Extended for plane strain cases

logarithmic of strength reduction $[\sigma_{cj@\beta=90}]$ at $\beta = 90°$ to logarithmic strength reduction $[\sigma_{cj@\beta=\beta n}]$ at the desired value of inclination angle of the joint ($\beta = \beta n$). These values of '*n*' were almost same irrespective of number of joints per unit length.

$$n = \log\left[\sigma_{cj@\beta=90}\right] / \log\left[\sigma_{cj@\beta=\beta n}\right] \quad (7)$$

The results of triaxial shear testing with inclination of plane of weakness for More-town phyllite, slate and Green river shale reported by Maclamore and Gray [27] indicated significant variation of strength ratio at low confining pressure. As a result, the orientation parameter appears as a composite parameter having combined effect of joint friction and inclination of joints as emerged from the studies of Maclamore and Gray [27] and Arora [2] and that of authors [3, 36]. The joint orientation parameter varies independent of joint frequency but not of the joint strength parameter. Figure 2a shows the results for plaster of Paris samples tested in triaxial test [2]. It shows that the effect of joint inclination upon strength ratio becomes insignificant with increasing initial confining pressure ratio $[p_i/\sigma_{ci}]$. It is defined as a ratio of initial mean effective confining pressure $[p_i = (\sigma_{1j} + \sigma_{2j} + \sigma_{3j})_i/3$, where $(\sigma_{1j}, \sigma_{2j}, \sigma_{3j})_i$ are initial principal stresses in the jointed rock] applied on the jointed rock in triaxial test to the uniaxial compressive strength of intact rock (σ_{ci}). An equivalence amongst joint orientation parameter (*n*) often referred as a number and joint inclination angle (β) is drawn for the equal strength reduction for jointed rocks with and without gouge [35] (Fig. 2b). In uniaxial state, there is a significant strength reduction due to the presence of gouge at the same joint inclination angle compared to the jointed rocks without gouge [35]. Further, if the confinement is increased significantly the strength reduction shall be indifferent to the presence of gouge of parent material. The presence of a gouge alters the joint strength according to its material properties, placement and thickness. In past, a simplistic view on adoption of joint strength parameter (*r*) has evolved. The values of '*r*' adopted for both blank joints and joints filled with gouge material, considered a constant value for joint strength parameter [2]. A constant value of joint strength parameter if used in Eq. 2a, it fails to capture the confining pressure and dilatancy effects of the joint material upon the friction amid the joints. Figure 2c shows variation of strength ratio with variation of joint strength parameter. Selecting a series of value for joint strength parameter (*r*), the strength ratio and joint factor fitting provides a few correlations for joint strength parameter (*r*). The experimental results of uniaxial compressive strength of rock masses from varied geological origins [2] are analysed by the author to examine a relationship for joint strength parameter in non-dimensional terms (Table 3). The compressive strength has an influence on values of joint strength parameter as per Eq. 8a, 8b.

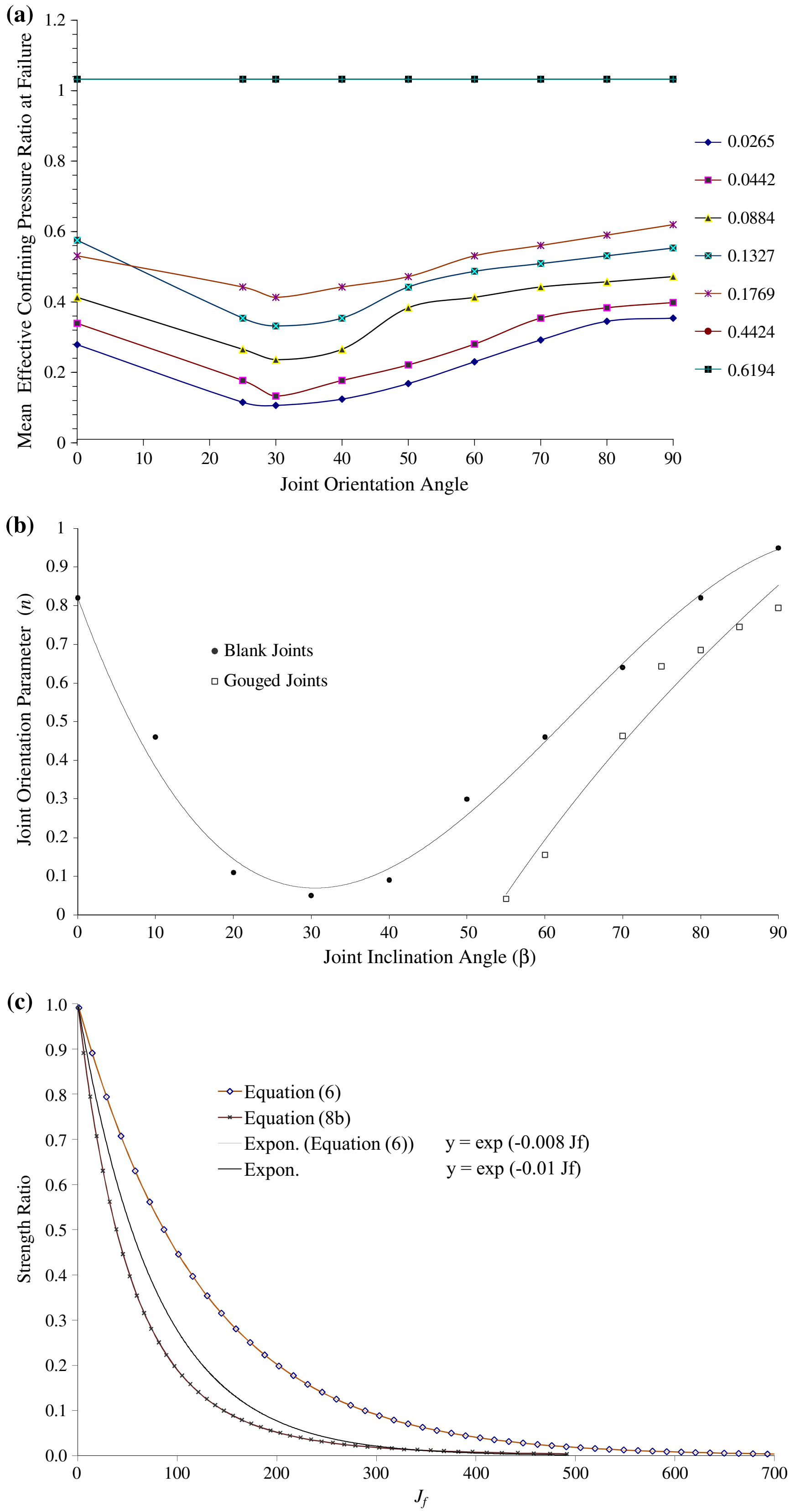


Fig. 2 **a** Effect of joint orientation on mean effective confining pressure ratio at failure. **b** Relationship of joint orientation parameter with joint inclination angle. **c** Strength ratio versus J_f at varying joint roughness factor

Table 3 Suggested values for r based on uniaxial compressive strength

Uniaxial compressive strength of intact rock[a] (σ_{ci}) (MPa)	Joint strength parameter[a] (r)	Uniaxial compressive strength of jointed rock[b] (σ_{cj}) (MPa)
2.5	0.30	1.77
5	0.45	3.97
15	0.60	12.61
25	0.70	21.55
45	0.80	39.51
65	0.90	57.91
100	1.00	90.12

[a] Arora [2], Ramamurthi and Arora [29]

[b] Interpreted by author

Table 4 Suggested values for fitting parameter for r based on uniaxial compressive strength

Fitting parameter	Intact rock[a]		Jointed rock[b]	
	a_{ci}	b_{ci}	a_{cj}	b_{cj}
Empirical values	0.182	0.130	0.171	0.192
R^2	0.990		0.991	

[a] Interpreted from data of Arora [2], Ramamurthi and Arora [29]

[b] Author

$$r_{ci} = a_{ci} \ln(\sigma_{ci}/\sigma_a) + b_{ci} \quad (8a)$$

$$r_{cj} = a_{cj} \ln(\sigma_{cj}/\sigma_a) + b_{cj} \quad (8b)$$

where σ_{cj} = Uniaxial compressive strength of jointed rock in kPa and $\sigma_a = 1$ kPa. a and b are fitting constants given in Table 4.

The Eq. 8b calls for a necessity of adjustments for joint strength parameter with a consideration of logarithmic of pressure on the joint material. Keeping the same values of joint number and orientation, if the joint strength parameter is adopted as per Eq. 8b, RAC [29] and a fixed r ($\phi_j = \pi/4$; $\tan \phi_j = 1$), respectively; a gradual drop in the strength ratio is observed (Fig. 2c).

This observation supports the effect of reduction of joint strength parameter leading to increasing joint factor and consequent strength reduction. In Hoek and Brown criterion [17, 18], the increasing confinements have similar effect of a non-linear strength reduction. There had been subjectivity in the interpretation of joint factor (Table 2) particularly in relation to joint strength parameter. Various investigators [2, 19, 29, 32, 35] considered it arbitrarily as a constant friction factor independent of dilatancy. In the present framework, the author correlated the resultant friction due to the joints with the dilatancy of the joint material.

4.1 Effect of gouge on joint factor

In the presence of gouge, the strength ratio followed a relationship with joint factor, which needed modification for depth of joints from loading plane and thickness of the gouge material. The author analysed the results of the uniaxial compression tests conducted on jointed Kota sand stone with and without gouge as shown in Fig. 3a. It shows that the strength ratio varies according to the joint depth factor (d_j/d_a) with respect to the loading plane, where d_j is depth of joint (mm) from the loading plane and d_a is the reference depth = diameter of the specimen (mm). There is a linear reduction in the strength of jointed rocks with the proximity of joints to the loading surface. The presence of clayey gouge tends to produce further reduction in the strength. However, if the distance of joints is at a depth more than a value of a non-dimensional joint depth parameter (J_{d_j}), it does not affect the strength ratio anymore. The author introduced a non-dimensional joint depth parameter (J_{d_j}) as a multiplication factor for the joint factor (J_f). Figure 3b shows that the values of joint depth parameter (J_{d_j}) may always be more than one. For joints located at a same depth relative to the mid height of the

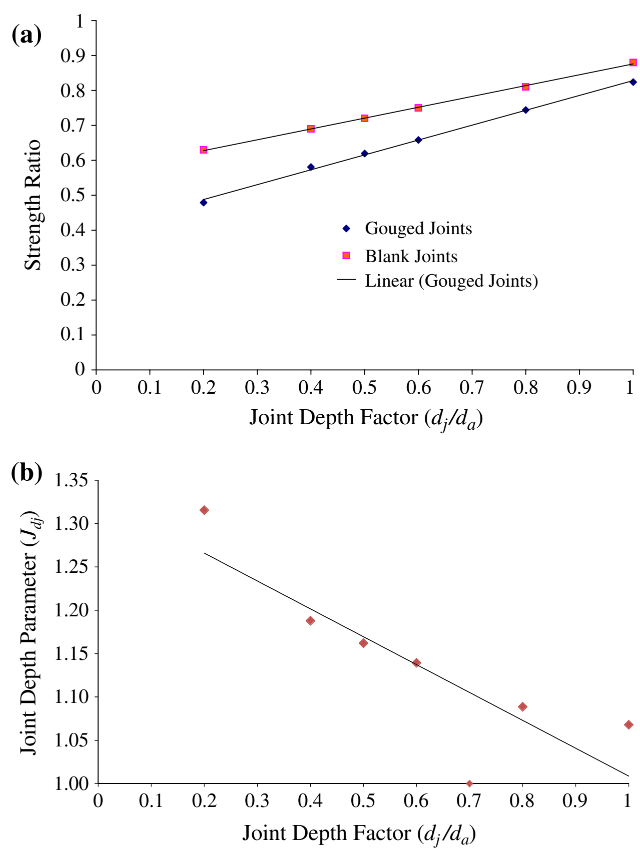


Fig. 3 **a** Strength ratio vs joint depth factor. **b** Joint depth parameter versus joint depth factor

sample, its value is taken as unity. Further, it does not remain a relevant factor for joints located at significant depths as the case may arise frequently in the field.

The author conducted analysis of the tests results on jointed Kota sand stone with varying gouge thickness (t), which indicated that increasing thickness (up to 3 mm) reduces strength of the jointed rocks. The increase in the thickness of gouge (beyond 3 mm) decreases the strength ratio to an extent when strength of jointed rock reaches the residual strength of multi-fractured rock mass ($\sigma_{cr} < 35\%$). Figure 4a shows the variation of strength ratio with gouge thickness factor (t/t_a). It was observed that initial increase in the gouge thickness factor (<2) the strength reduction in horizontally jointed samples is insignificant. This thickness uses packing of gaps in asperities on compression accompanied by initial plastic deformations. However, further increase in thickness of gouge material ($t/t_a = 2$–5), the strength drop is exponential as long as any further increase in thickness ($t/t_a > 5$), the strength of jointed rock reaches residual value. The author introduces a gouge thickness parameter (J_t) to incorporate the effect of thickness in non-dimensional form as shown in Fig. 4b. The gouge thickness parameter (J_t) varies as a function of gouge thickness factor (t/t_a). Beyond a value of gouge thickness parameter (J_t), the strength drop is observed at a lower rate compared to an initial drop.

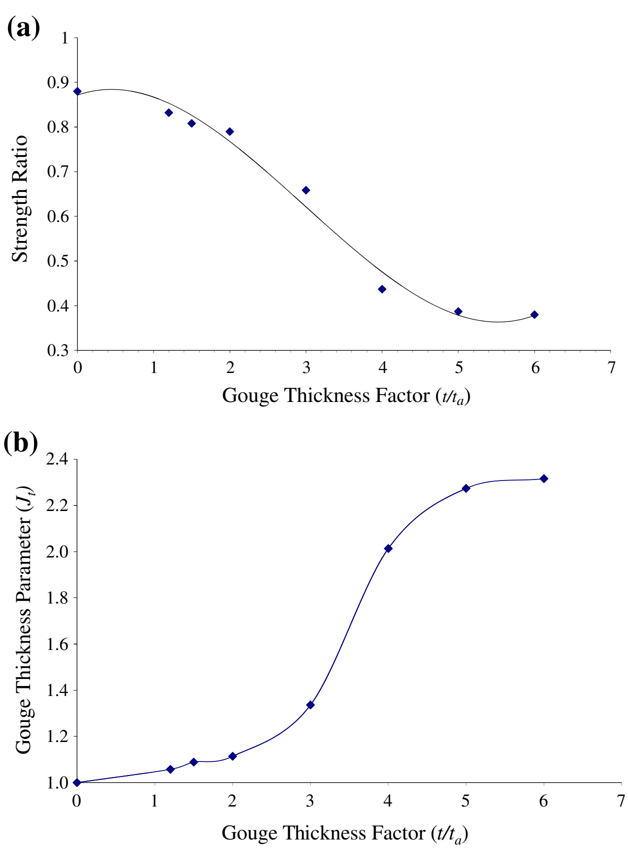


Fig. 4 **a** Strength ratio vs gouge thickness factor. **b** Gouge thickness parameter versus gouge thickness factor

Based on the results [3, 35, 36], the author modified the joint factor (J_f) as per Eq. 1 in terms of non-dimensional quantities as (J_{fg}) given in Eq. 9

$$J_{fg} = c_g(J_n L_{na}/nr) \tag{9}$$

where c_g is a modification factor for gouge,

$$c_g = J_{d_j} J_t / g_d \tag{10}$$

- J_{d_j} Correction for the depth of joint (joint depth parameter)
- J_t Correction for the thickness of gouge in joint (gouge thickness parameter)
- g_d Correction factor depending upon the compactness or relative density of gouge in joint, equal to unity for fully compacted joint fill. For clean compact joints, when no gouge is present c_g is equal to unity,
- J_n Number of joints per unit length in the direction of loading (joints per metre length of the sample),
- L_{na} reference length = 1 m.

The analysis of the experimental observation of strength ratio of jointed rocks with and without gouge with a large number of horizontal joints indicate that a scheme of corrections may converge the effect of gouge of parent rock material with blank joints upon mutual closure of the joints on compression. Figure 5 shows variation of strength ratio in uniaxial state using joint factor with and without the correction for presence of gouge. Figure 5 considers joint strength parameter as per a constant value of friction along the joints ($r = \tan\phi_j$). In the presence of filled up discontinuities of clayey gouge there appears a composite strength reduction compared to the blank joints. The fitting parameter observed ($\alpha = -0.009$) for clayey gouge in parallel joints ($\beta = 90°$) has a higher coefficient of regression ($R^2 = 0.993$) compared to the same data fitting for $\alpha = -0.008$ ($R^2 = 0.946$). Further, a high coefficient of regression ($R^2 = 0.99$) is anticipated for α ($\alpha = -0.008$) for the joints having gouge material of parent rock [2] in parallel joints ($\beta = 90°$). The strength ratio of jointed rocks with or without gouge of parent rock material indicates convergence of fitting parameter at high confinement pressure (Fig. 2a).

4.2 Model behaviour of intact rocks extended to jointed rocks

Johnston and Chiu [23] and Johnston [22] proposed a relationship in normalised form for intact rocks as

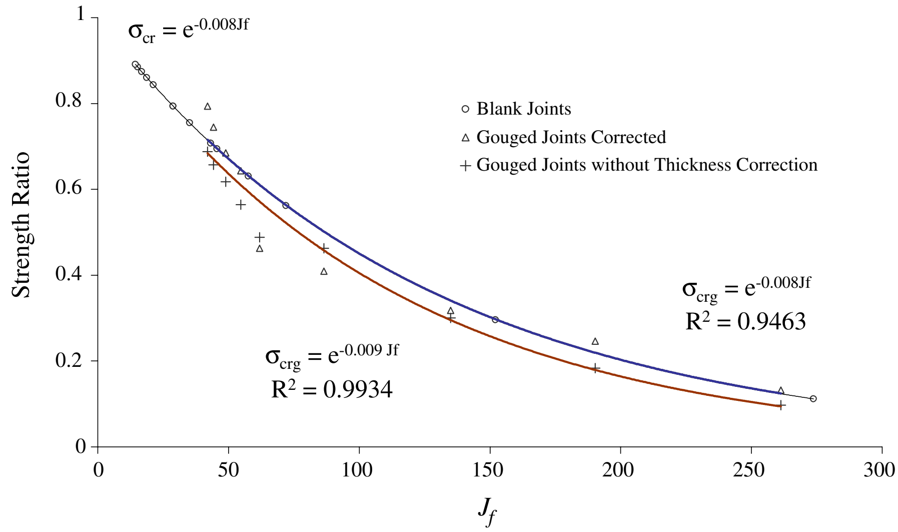


Fig. 5 Variation of strength ratio with corrected joint factor

$$\sigma_{1N} = [(M/B)\sigma_{3N} + 1]^B \tag{11}$$

where $\sigma_{1N} = \sigma_1/\sigma_{ci}$ and $\sigma_{3N} = \sigma_3/\sigma_{ci}$; σ_1,σ_3 are principal stresses and σ_{ci} is uniaxial compressive strength of rock sample. M and B are empirical rock constants. Upon simplification we get,

$$\sigma_{ci}/\sigma_{ti} = -M/B \tag{12}$$

where σ_{ti} = tensile strength of rock sample.

$$\text{If } B = 1; M = \tan^2(45 + \varphi/2) \tag{13}$$

This corresponds to linear Mohr-Coulomb failure criterion.

Griffith theory [16] predicts that uniaxial compressive strength at crack extension is eight times the uniaxial tensile strength. The frictional forces in the micro crack network tend to modify the ratio as per Eq. 12 beyond the limits of prediction of Griffith-theory.

With further investigations Johnston and Chew [23] and Johnston [22] proposed following relationships

$$B = 1 - 0.0172(\log \sigma_{ci})^2 \tag{14}$$

$$M = 2.065 + 0.276(\log \sigma_{ci})^2 \tag{15}$$

For a variety of rocks with micro fracture network and equivalent jointed rocks in confined state, the author numerically obtained the ratio as per Eq. 12. The ratios of compressive and tensile stresses indicate convergence of response in terms of strength ratios upon modification of joint factor as per Eqs. 6 and 9. It is an effect of the plastic deformations to produce an equivalent reduction in uniaxial compressive strength of jointed rocks. Accordingly, the Eqs. 14 and 15 are used for jointed rocks substituting σ_{cj} for σ_{ci}. Using Eqs. 11 and 12 in conjunction with the strength ratio shown in Fig. 5, the author integrated, the strength ratio of compressive and tensile strength, with the variation of joint factor.

The uniaxial compressive strength of jointed rock varies significantly upon the joint factor values. The concentrations of stresses tend to extend the failure surface, which in turn modifies the stress pattern. With the progress of failure, the joint factor increases. The progress of failure modifies the mean effective confining pressure. A comprehensive review of the literature and observations in regard to published test results indicate that dilatancy is highly dependent both on the plasticity already experienced by the material and confining stress in rocks [1, 41]. Since the logarithmic of stress has shown to effect linear reduction in peak angle of friction, Bolton [9] proposed (Billam [8], Vesic and Clough [40]) a relationship for maximum angle of dilatancy (ψ_{max}) as per Eq. 16a.

$$\psi_{max} = (-AR_D)\ln(p'/p_{cr}) \tag{16a}$$

It is implied that p' is mean effective confining pressure at failure, $-AR_D$ is a constant containing a factor of density and strain restrictions, and p_{cr} is a state of stress to eliminate dilation. Based on the generic observation of Eq. 16a, the behaviour of granular materials is broadly represented in Eq. 16b. Based on the experimental observations [22, 23] the author interpreted, following relationship (Eq. 16b) for angle of dilatancy of jointed rocks (ψ_j) and mean effective confining pressure ratio (p'/σ_{ci}) for jointed rocks.

$$\psi_j = C'\ln(p'/\sigma_{ci}) \tag{16b}$$

Equation 16b is simplified as

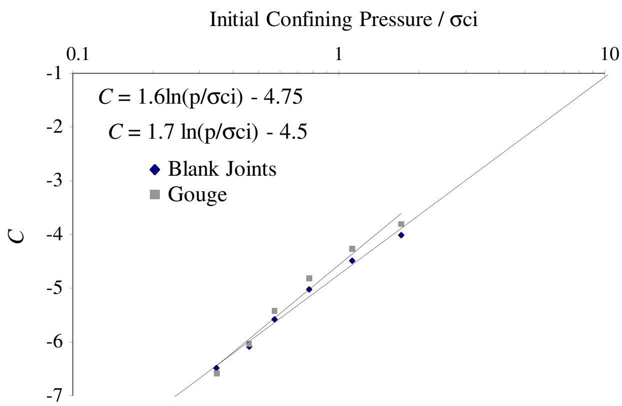


Fig. 6 Results of trials for evaluation of empirical factor C

$$\psi_j = C\ln(3p'/\sigma_{ci}) \tag{16c}$$

$$\psi_j = C\ln(\sigma_{cr}) \tag{16d}$$

Therefore, in triaxial condition at failure

$$\sigma_{cr} = \exp\left[\psi_j/C\right] \tag{17}$$

ψ_j = angle of dilation for jointed rock corresponding to a mean effective confining pressure at failure (p') relative to intact rock failure (σ_{ci}).

The author correlated, mean effective confining pressure through a simple numerical code to predict the relative dilatancy of jointed rocks with progression of failure and to provide values of empirical fitting parameter for jointed rocks 'C'. The empirical fitting parameter for jointed rocks 'C' is obtained by estimation of excess effective friction angle over and above angle of critical friction of the joint material with varying values of effective confining pressure for different initial confining pressure ratio (p_i/σ_{ci}). Figure 6 shows variation of initial confining pressure ratio (p_i/σ_{ci}) with the empirical fitting parameter 'C' for jointed rock.

The parameter 'C' further varies according to initial mean effective confining pressure (p_i') and uniaxial compressive strength of intact rocks (σ_{ci}). Since there is no effect of pore pressure considered further in this analysis, $p_i' = p_i$ and $p' = p$, respectively. The parameter 'C' contains very sensitive effects of plastic yielding, hardening and crushing characteristics of the joint material. The initial confining pressure ratio (p_i/σ_{ci}) and presence of a gouge material tends to transform empirical parameter 'C' due to yielding, hardening and crushing characteristics of the joint as shown in Fig. 6. The author numerically evaluated, through a simple analysis, to propose a relationship for 'C' dependent on initial confining pressure ratio (Table 5). Based on this analysis, it is found that,
For blank joints

$$C = 1.6\ln(p_i/\sigma_{ci}) - 4.75 \tag{18a}$$

For joints filled with clayey gouge

$$C = 1.77\ln(p_i/\sigma_{ci}) - 4.56 \tag{18b}$$

For $p_i/\sigma_{ci} \ll 1$; the limiting value of

$$C = -5 \tag{19a}$$

For $p_i/\sigma_{ci} \gg 1$; the limiting value of

$$C = -1 \tag{19b}$$

However, accounting for a decrease in strength ratio (σ_{cr}) due to an increase in the joint factor (Fig. 5) for the presence of gouge (J_{fg}), an equivalent strength reduction [exp ($-0.001J_f$)] is observed (with coefficient of regression R^2 amongst 0.99–0.94). It implies that 'C' has probable values between Eq. 18a and Eq. 18b. A value of α is proposed to be adopted for fully compacted joints (where the joint contacts are as close as $c_g = 1$). Henceforth, the following discussions refer to Eq. 18a as a guide for evaluation of empirical parameter 'C'.

4.3 Dilatancy of jointed rocks

Following the early work of D.W.Taylor [33] on dilatancy of soils, there were varied attempts [1, 15, 24, 25, 41] to model the strength and deformation behaviour of jointed rocks on similar grounds. In an effort to improve existing technique for obtaining significant rock mass property, Hoek and Brown [18] used his vast experience in numerical analysis of a variety of practical problems. Scrutinising the observed data of Hoek and Brown [18], a link between dilatancy angle

Table 5 Summary of numerical trials

Data/Model	Rock type	Test condition	Trial parameters	Trial function	Reference equations	Reference figures
Author	Jointed	Triaxial	J_f	σ_3/σ_{ci}, β, p'/σ_{ci}	–	Fig. 2a
RAC, author	Jointed	Uniaxial, triaxial	r	σ_{cj}/σ_a	Eq. 8b	Fig. 2c
RAC	Jointed	Uniaxial	J_f	σ_{cj}/σ_a	Eq. 6	Fig. 5
Johnston	Jointed	Triaxial	p_i/σ_{ci}	C	Eqs. 11–19a	Fig. 6
Author	Jointed	Triaxial	J_f	Strength ratio (plain strain)	Eq. 23	Fig. 7a
RAC/author	Jointed	Triaxial	J_f	Strength ratio (triaxial)	Eq. 23	Fig. 7b
Author	Jointed	Triaxial	I_r	Strength reduction factor	Eq. 24a	Fig. 7c

and friction angle can be found. Interestingly, designated rock mass quality is a good quality rock for $\psi = \phi/4$ and a poor quality rock for $\psi = 0$. Alejano and Alonso [1] reports the interesting fact about this approach that reflects the significant error induced in design calculation when a simple associated flow rule is considered. Significantly, it happens because of dilatancy consideration independent of confining pressure. Hoek and Brown [18] also suggested a transition from brittle to perfectly plastic rock masses for decreasing rock mass quality. The volume change behaviour of rock joint material is localised in the zone of shear for lower value of joint number, whilst it shall be distributed well throughout the rock mass for higher value of joint number. Therefore, an average value of volume change is inherently interpretive in nature. As an effect of volume change, during the shear, the value of joint strength parameter does not remain constant. During shear, the joints dilate to increase the joint factor as a compressed function of the state of stress, compactness of joints and material characteristics of the gouge, which in actual practise are difficult to measure but easier to predict if the concept of stress dilatancy [8, 9, 13, 30, 40] is used for the rock joints.

Bolton [9] proposed an empirical equation for dilatancy of soils

$$\psi_{\max} = AI_{\mathrm{r}} \tag{20}$$

where A is an empirical constant and has the value of three for axe-symmetrical and five for plane strain conditions; and I_{r} is a relative dilatancy index which is a function of dilatancy angle (ψ). The dilatancy for soils is a value of ψ considered over and above the angle of critical friction (ϕ_{cn}). ϕ_{cn} is constant volume friction angle also referred as angle of critical friction of joint material, which is a material characteristic. It is often approximated equal to residual state friction angle of the joint material. In case of rock masses, upon shearing it dilates and causes reduction in its strength below the intact rocks. Henceforth, the increasing values of Ir and ψ are essentially associated with a strength reduction.

According to Bolton [9], relative dilatancy for granular material is

$$I_{\mathrm{r}} = 10/3[-d\varepsilon_{\mathrm{v}}/d\varepsilon_1] \tag{21}$$

where ε_{v} is the volumetric strain in the zone of shear and ε_1 the axial strain.

Since the joint factor J_{f} is related to the joint strength parameter 'r' which is a derivative of joint friction, it is advanced for dilatancy of jointed rocks at very large value of joint factor that

$$\phi_{\mathrm{peak}} - \phi_{\mathrm{j}} = \lambda J_{\mathrm{f}} \tag{22}$$

where λ is an empirical fitting constant, ϕ_{peak} is peak angle of internal friction; ϕ_{j} is angle of internal friction for jointed rock. When J_{f} is very large (say $J_{\mathrm{f}} = 1{,}000$), the limiting value of $\phi_{\mathrm{j}} = \phi_{\mathrm{cn}}$ and then λ takes a typical value [=0.025] for axe-symmetrical case and a typical value [=0.0415] for plane strain. It indicates a reduction for peak friction angle (25° for axe-symmetrical and 41.1° for plane strain case). The values of λ are obtained by data substitution in the numerical trials for typical sand stone joints.

Equation 6 may be rewritten as

$$\sigma_{\mathrm{cr}} = \exp[(\lambda/C)J_{\mathrm{f}}] \tag{23}$$

For the boundary conditions of varying confining pressures, the strength ratio is plotted for plane strain and triaxial conditions in Fig. 7a, b, respectively. The coefficient of joint factor 'α' is amongst −0.005 to −0.025 for varied conditions of initial stresses in the rock masses in axe-symmetrical conditions. Figure 7a, b shows that confinement influences the joint factor first largely by means of orientation and then friction. This affect is more acute in directional strain regimes namely plane strain condition than in triaxial conditions (Fig. 7c).

The author defines in presence of gouge a strength reduction factor [S_{cr}] as a ratio of dilated to undilated rock mass strength. In fact, the strength is entirely controlled by the shear along the gouge. By using Eqs. 17, 20, 22 and 23, an expression for strength reduction [S_{cr}] due to dilatancy of gouge is obtained as,

$$S_{\mathrm{cr}} = \exp(AIr/C) \tag{24a}$$

$$S_{\mathrm{cr}} = \exp(\xi Ir) \tag{24b}$$

where

$$\xi = A/C \tag{24c}$$

Numerically, the strength reduction factor equals strength ratio if no gouge is present ($c_{\mathrm{g}} = 1$). Table 6 shows the limiting values of coefficients of joint factor and relative dilatancy index (namely, $\alpha = \lambda/C$ and $\xi = A/C$) with variation of initial confining pressure ratio. The value of coefficients of joint factor and relative dilatancy index, when initial confining pressure is 10% of uniaxial compressive strength ($p_{\mathrm{i}}/\sigma_{\mathrm{ci}} \ll 1$) of intact rock of parent material loaded through a circular footing system, is −0.005 and −0.6, respectively. However, the value of coefficients of joint factor and relative dilatancy index, when initial confining pressure is 2–3 times that of uniaxial compressive strength ($p_{\mathrm{i}}/\sigma_{\mathrm{ci}} \gg 1$) loaded through a circular footing system, is −0.025 and −3, respectively. Similarly under plane strain conditions, the value of coefficients of joint factor and relative dilatancy index, for low confinement ($p_{\mathrm{i}}/\sigma_{\mathrm{ci}} \ll 1$), is −0.0083 and −1, respectively. The value of coefficients of joint factor and relative dilatancy index, for high confining pressure ($p_{\mathrm{i}}/\sigma_{\mathrm{ci}} \gg 1$) in plane strain conditions is −0.0415 and −3, respectively.

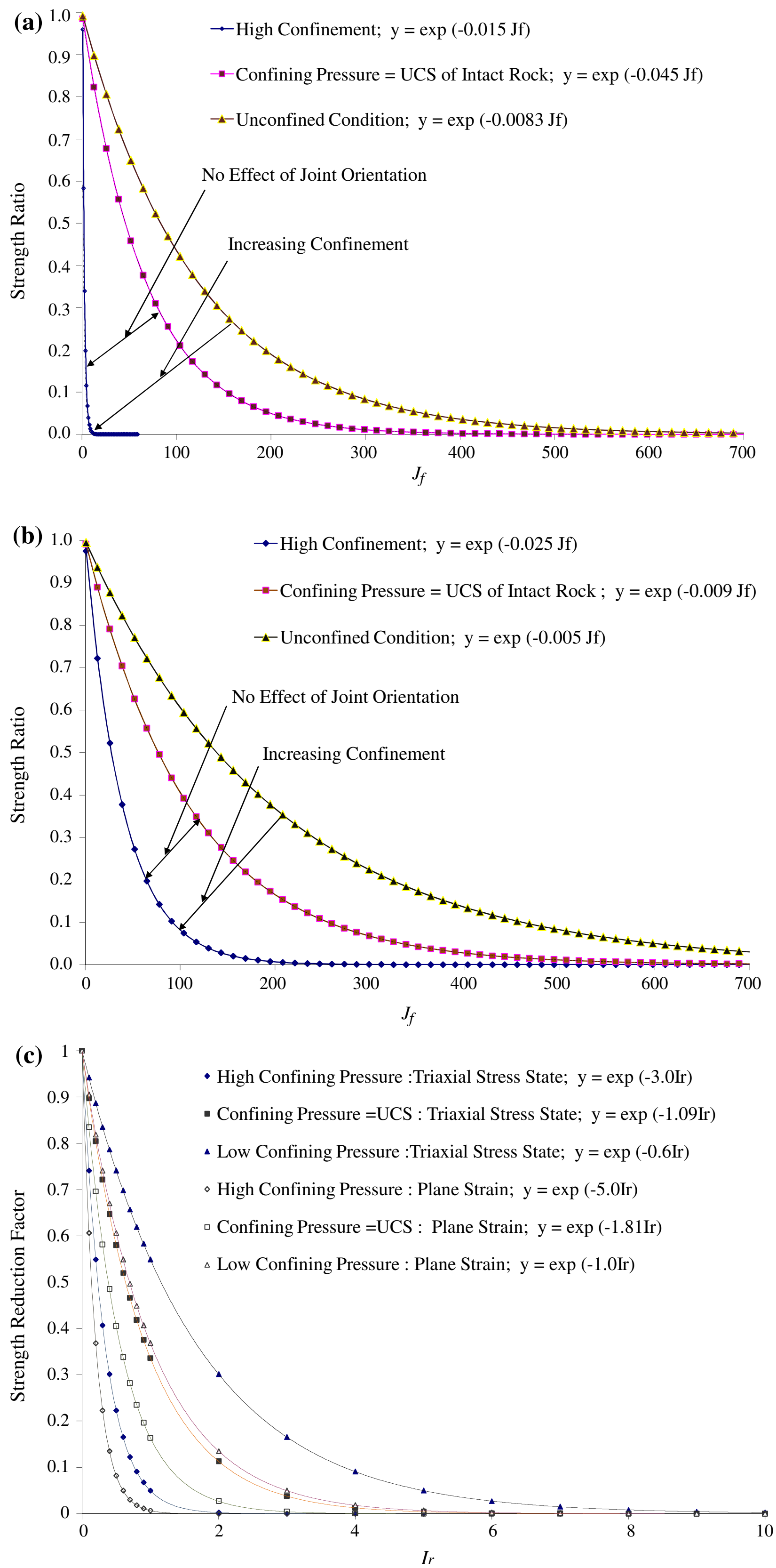


Fig. 7 **a** Variation of strength ratio with joint factor in plane strain case. **b** Variation of strength ratio with joint factor in triaxial state. **c** Variation of strength ratio with relative dilatancy index

Table 6 Empirical coefficients for jointed rocks

Initial confining pressure ratio	Recommended area of application	Empirical coefficients			
		λ	C	α	ξ
Triaxial case					
$p_i/\sigma_{ci} \gg 1$	Circular piers in deep rock deposits	0.0250	−1	−0.025	−3
$p_i/\sigma_{ci} = 1$	Circular rock sockets at moderate depth in jointed rocks		−2.75	−0.009	−1.09
$p_i/\sigma_{ci} \ll 1$	Circular rock sockets at shallow depth in jointed rocks, rock pillars in underground mine support		−5	−0.005	−0.6
Plane strain case					
$p_i/\sigma_{ci} \gg 1$	Deep seated slip through rock faults	0.0415	−1	−0.0415	−5
$p_i/\sigma_{ci} = 1$	Rectilinear sealing of nuclear and hazardous waste at moderate depth in rock deposits		−2.75	−0.0150	−1.81
$p_i/\sigma_{ci} \ll 1$	Rectangular (length/breadth ≥5) rock sockets at shallow depth in jointed rocks		−5	−0.0083	−1

Table 7 Values of material fitting parameter for gouge material

Joint material	Q_j	r_j
Quartz and feldspar[a]	10	1
Silty sand[b]	>8	<1
Limestone[a]	8	–
Ash[c]	7.7	–
Anthracite[a]	7	–
Chalk[a]	5.5	–

[a] Bolton [9], [b]Salgado et al. [31], [c]Trivedi and Sud [37, 39]

The useful estimates of I_r (for Eq. 24a) is made from the following relationships

$$I_r = \left[R_D\left\{Q_j - \ln(p'/p_a)\right\} - r_j\right] \quad (25)$$

where R_D is dimensionless relative density or index of compactness of joint gouge material, which is unity for compact joints and zero for open joints. p' is the mean effective confining pressure in kPa at failure; p_a is reference pressure, which equals to one kPa. Q_j and r_j are empirical material fitting constants with values of 10 and 1, respectively, for clean silica contacts [9] for sand stone joints. Incorporating Billam's [8] triaxial test data, Bolton [9] suggested that progressive crushing suppresses dilatancy in the soils with weaker grains, i.e. limestone, anthracite, and chalk, where Q_j values of 8, 7, and 5.5, respectively, may be adopted (Table 7). This may occur because of reduction of the critical mean confining pressure beyond which increase in mean confining pressure for a joint does not modify peak angle above the critical angle. The angle of critical friction (ϕ_{cn}) is morphological and mineralogical parameter of joint material. Therefore, from the knowledge of ϕ_{cn}, R_D, and p' the author interpreted the peak angle of friction of rock joints. The knowledge of material characteristics, and mean confining pressure correlated relative dilatancy of the joints with residual strength. Figure 7a–c may be used as design chart for numerical analysis of foundations in rock mass.

The author uses relative dilatancy as representative parameter for the progressive failure prediction of residual strength. The empirical relation based on the load tests on the granular media [39] show that the concept of progressive failure extended to rock masses is affected by the strain restriction, boundary conditions, the material characteristics of the granular mass and the ratio of settlement to least dimension of loading face [38, 39].

5 Final comments

The compiled data of Arora [2], Ramamurthy and Arora [29], Johnston and Chew [23], Johnston [22], Singh and Rao [32], Jade and Sitharam [35] and Trivedi and Arora [36] fall within the reasonable zone of prediction using the present relations. However, the relations described above should be applied with care owing to following reasons

1. Arora [2], Ramamurthy and Arora [29], Johnston and Chew [23], Johnston [22], Singh and Rao [32] and Jade and Sitharam [35] and Bolton [9] are essentially empirical in nature and the constants (Table 2) associated with them do not have any physical meaning.
2. Hoek and Brown [17, 18] developed an empirical strength criterion for intact rocks and then extended it to the rock masses. Johnston and Chew [23] developed strength criterion for intact rocks. In the present study, the author extended it to the rock masses. Ramamurthy and Arora [29] developed a strength criterion for a set of jointed rocks. In the present study, the author extended it

to the various rock masses. The process used for the development of strength criteria [17, 23] was one of pure trial and error. Apart from conceptual starting point provided by Griffith theory [16], there is no fundamental relationship between the empirical constants included in any of strength criterion and any physical characteristics of the rock [42]. Because of the empirical nature of strength criterion of rock masses, it is uncertain if it will adequately predict the behaviour of all the rock masses.

3. The strength ratio considers the strength of jointed and intact rock for the same size of sample therefore size effects are assumed discounted. However, in practise, the size effects may not be linearly discounted.
4. The failure as assumed for axe-symmetrical and plane strain case is similar to that in soil mechanics, which implicitly assumes that rock mass is isotropic and that continuum behaviour prevails. In practise, contrary to the assumption, the rock mass may be an-isotropic and the rock failure may be discontinuous.
5. The values of peak friction of the rock masses are estimated with greater certainty than the angle of critical friction contrary to the case of soils. Hence, dilatancy for soils is recognised as value of ψ considered over and above the angle of critical friction ϕ_{cn}, whilst in case of rock masses it is assumed to dilate and cause reduction in strength below the peak strength. Therefore, the increasing values of I_r and ψ for rock masses are essentially associated with a strength reduction.
6. Whilst gouge within the joints undergoes volume change, the strength reduction may accompany first, due to strain softening of gouge and second, the reduction in peak strength due to damage in the jointed rock mass. The strength reduction factor considers both the parts but largely the first. In order to isolate the effects of damage accompanying dilation of gouge, there is a need to fine-tune the results.
7. The purpose of the present relations is to provide a framework to handle dilatancy problem with confining pressure than to precisely predict strength of jointed rocks.

The limitations described above account for the difference between the proposed relationship for the strength ratio and the test data.

6 Conclusions

This paper describes an approach to find strength of jointed rocks with and without gouge in terms of empirically established joint factor (J_f). Historically, the joint factor is adopted in relation to a constant joint strength parameter (r), constant joint orientation parameter (n), constant number of joints (J_n), and modification factor for gouge (c_g) in terms of gouge thickness (t), compactness of fill material (g_d) and distance of joints from loading plane (d_j). These consideration bring forth multiplicity in interpretation of empirical joint factor and hence strength ratio (σ_{cr}). The joint strength parameter, joint orientation parameter, number of joints and modification factor for gouge (c_g) gets altered according to initial confining pressure (p_i) and stress conditions of plain strain and triaxial states.

The dependencies of joint factor on initial confining pressure provide input to a numerical technique to incorporate these effects in the strength ratio. Comparing the results of the proposed model with the test data indicated that multiplicity appearing in the interpretation of strength ratio is essentially due to dilatancy. This paper considers the effect of dilatancy in varied confinement conditions in plane strain and axe-symmetrical case on strength by coefficients (λ, C and ξ) of readily estimated joint factor, and clearly recognised relative dilatancy index (I_r) for granular soils. These coefficients are presented along with its potential area of application in rock mechanic designs. The main advantage of this model is to provide estimate of the strength of jointed rocks in terms of the already established parameters (J_f and I_r). The relationship for strength and relative dilatancy index tends to resolve the diversity in interpretation of the behaviour of jointed rocks and granular soils.

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