Bearing Capacity and Settlement of Footing Resting on Confined Loose Silty Sands

Rajeev Gupta
Lecturer, Department of Civil Engineerin
Thapar University, Patiala-147001, India
E-mail: rajeev.gupta@thapar.edu

Ashutosh Trivedi
Professor, Department of Civil Engineering, Delhi College of Engineering
(Faculty of Technology, University of Delhi), Bawana Road, Delhi-110042, India.
Email: prof.trivedi@yahoo.com

ABSTRACT

An experimental study on the influence of cell confinement on the bearing capacity and settlement of circular footing on silty sand was carried out. Laboratory experiments on clean sand and sand containing silt up to 25% were performed. Model cells with different diameters and heights have been used to confine the silty sand. The effect of the cell diameter, cell height and fines on bearing capacity and settlement were investigated with the help of an experimental programme using circular footing having a diameter of 0.15m. Initially, the response of a footing without confinement was determined and then compared with that of footing with confinement. The results indicate that the bearing capacity of circular footing decreases on the addition of fines and can be appreciably increased by soil confinement. The results also indicate that the settlement of circular footing can be appreciably decreased by soil confinement. It was interpreted that such confinement provides lateral displacement of soil underneath the footing. It leads to a significant improvement in the response of the footing. The cell–soil footing behaves as one unit for small cell diameters, while this pattern was no longer observed with large diameter cells. The recommended cell diameters that give the maximum ultimate bearing capacity improvement and less settlement are presented and discussed.

KEYWORDS: Silty soil; Model tests; Footings; Shallow foundations; Bearing capacity; Confinement; Settlement.

INTRODUCTION

The Indian subcontinent has vast deposits of silty sands along the bank of perennial rivers namely Indus, Ghaggar, Barinadi, Yammuna, and Ganga, where the river sands as are obtained with varied proportions of non plastic silts. In modern times many granular industrial byproducts with common range of specific gravity, unit weight and grain characteristics are often classified for sizes namely sand and silts (Trivedi and Sud, 2002). Over past fifty years there were intensive attempts to characterize sandy soil without fines (Feda, 1961; deBeer, 1965; Meyerhof, 1965; Brinch Hasen, 1970; Vesic 1973). However there were scant efforts to map the engineering behavior of silty sands. The authors observed that silty sands are largely found in earth crest in a low to medium density states with varied moisture. This material supports structural rafts and deep foundations for multistoried buildings, underground excavations, tunnels and pipeline. There is a need to characterize this granular media as an engineering material.

In recent years civil engineering professionals have adopted the practice of soil improvement by reinforcement, compaction, grouting etc. The decreasing availability of good construction sites has led to increased use of sites with marginal soil properties. In view of this, the requirement for in situ treatment of foundation soil to improve its bearing capacity and reducing the settlement has raised markedly. The soil confinement is one such method of improving bearing capacity and reducing the settlement of the footings resting on silty sands.

The more recent advancement in this field is to provide confinement to the soil by using metal cells. This novel technique of soil confinement, though successfully applied in some areas of geotechnical engineering, has not received much attention in foundation applications. Shallow foundations are generally designed to satisfy bearing capacity and settlement criteria. The bearing capacity criteria stipulate that there is adequate safety against bearing capacity failure beneath the foundation, and a factor of safety of three is generally used on the computed ultimate bearing capacity. Settlement criterion is to ensure that the settlement is within tolerable limit. It is commonly believe that the settlement criterion is more critical than the bearing capacity one in the design of shallow foundations. By limiting the total settlement, differential settlements and any subsequent distress to the structure are limited.

The structural measures for foundations are widely used in weak soil conditions to support column loads. Sometimes the excavation needs to be braced during foundation construction. One of the available solutions is to use side support to the excavation during construction. Due to the problems associated with the removal these supports, they are provided as part of the permanent structure. Accordingly it consists of two parts; it is to deals with the structural analysis of the footing if the side supports are used as end supports for the foundation (Sawwaf and Nazer, 2005). Secondly, the effect of these supports on the lateral movement of the soil underneath the foundation is to be investigated as the effect of the lateral confinement on the bearing capacity of the silty sands. While there are several solutions for the first problem, such as isolating the foundation from the side supports. But the effect of lateral confinement by these side supports on the foundation behavior is not well understood.

Swwaf and Nazer (2005) studied the effect of confinement on the bearing capacity of sand and have found an improvement in bearing capacity as high as 17 times as that without confinement.

Rajagopal et al. (1999) studied the strength of confined sand by carrying out a large number of triaxial compression test to study the influence of geocell confinement on the strength and stiffness behavior of granular soils.

Rea and Mitchell (1978) conducted a series of model plate load test on circular footings supported over sand filled square shaped paper grid cells was carried to identify different modes of failure and arrive at optimum dimensions of the cell.

Dash et al.(2001a) conducted a load test for a strip footing on homogeneous dense sand (relative density of 70%) beds, however, indicate that an 8 fold increase in bearing capacity could be achieved with the provision of geocell in the foundation sand. Dash et al.(2001b) conducted the model test results on a circular footing supported on a dense sand layer (relative density of 70%) overlying a soft clay bed show about a six-fold increase in bearing capacity with the provision of geocell in the overlying sand layer. The higher performance improvement due to geocell in the sand bed compared with that in the soft clay bed is attributed to the mobilization of higher passive force at the geocell walls and frictional resistance at the geogrid-soil surface.

In order to investigate the effect of confinement on bearing capacity and settlement characteristics of circular footing, the cells were instrumented in the laboratory. It was made of mild steel plate having a thickness of 0.94 mm and having different diameters. The cells were open at both the ends. It was modeled as a circular footing supported on a silty soil, which is surrounded by a mild steel cell having same soil outside. The tests were performed first without cells (un–confined case) below the footing and then with cells (confined case) and the results for bearing capacity and settlement corresponding to a constant pressure intensity of 100 kPa are compared. In cases where structures are very sensitive to settlement, soil confinement can be used to obtain the same allowable bearing capacity at a much lower settlement.

LABORATORY MODEL TESTS

Footing

A circular model footing of size 150 mm diameter and 10mm thick made of mild steel was used. The base was made rough by fixing a thin layer of sand onto the base of the model footing. The footing was placed on the surface of the sand bed and load was applied on it by a hand-operated hydraulic jack. The load transferred to the footing was measured by a pre-calibrated proving ring. The load was applied in increments. Each load increment was maintained constant till the footing settlement had stabilized. The settlement of the footing was measured by dial gauges placed on the footing.

Model test tank

Model tests were conducted in a test tank, having inside dimensions of 600 x 600 mm in plan and 600 mm deep. The size of the tank was decided by the size of the footing and the zone of influence. The test tank is made of steel and has arrangement to fix the proving ring with specially fabricated load device for applying the axial load to the footing as shown in Fig.1

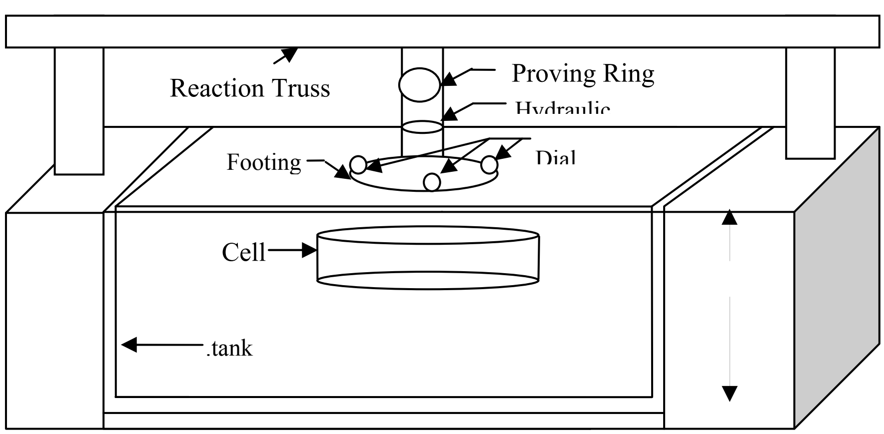

Figure 1: Line sketch of plate load test (free scale)

TEST MATERIAL

Ghaggar sand and Fine characteristics

Going through the preliminary survey reports of the geology of the area it was decided to obtain sand was from the bank of Ghaggar river which was processed later for specific uses like specific gravity, grain size distribution and load bearing and settlement characteristics.

Locally available Ghaggar sand after washing is used as the foundation bed and it is designated as clean sand. The physical properties of Ghaggar sand are presented in Table 1. The grain size distribution curve of Ghaggar sand is shown in Fig. 2 and is classified as SP according unified classification system. The non plastic fines which passes through IS 75μ sieve were used. The fines were prepared in the laboratory. Numbers of soil sample were taken from near by area and then wet analysis was carried to know the percentage of particles passing 75 μ sieves. After processing, silt was finalized for the preparation of fines which have a maximum amount particles passing 75 μ. A wet analysis was carried on the selected soil samples. The material which passes through 75 μ was collected in a container and allowed to settle. Then the passing material is dried in the oven and pulverized. The pulverized material was again sieved though 75 μ sieve. Then a hydrometer analysis was carried out as per IS: 2720(Part 4)-1985 to know the amount of clay particles. The amount of clay particles was found insignificant. The specific gravity of fines was 2.63. Fig. 2 shows the grain size analysis of clean sand and that of fines.

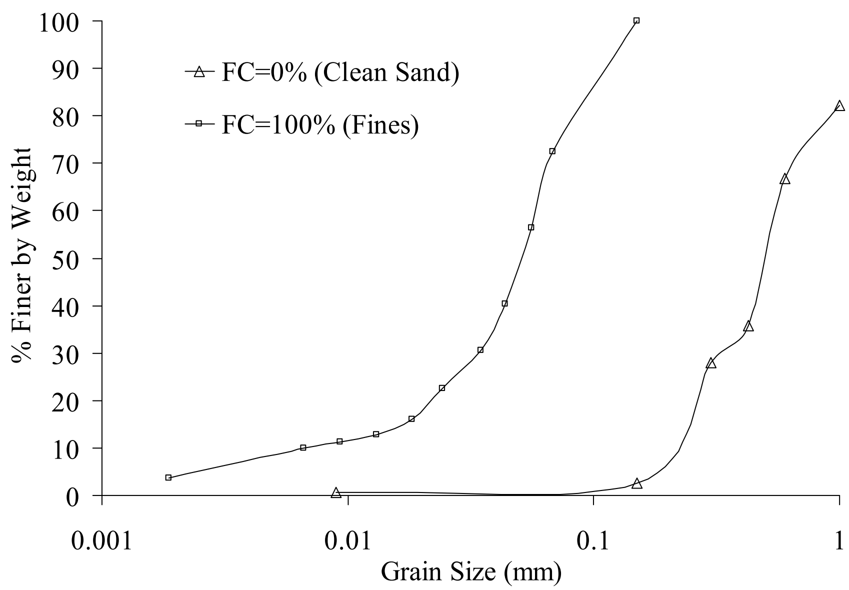

Figure 2: Grain size distribution of the sand and fines

Confining cells

The confining cells were made of mild steel plate with different diameters and heights. The internal diameter of cells used is 150, 225 and 300 mm. The heights of cell used are 75, 112.5 and 150 mm. The thickness of the all the confining cells is 0.94 mm. The tests were carried out by placing the confining box initially in position and then the sand bed was prepared by rainfall technique to get the desired relative density.

EXPERIMENTAL SETUP AND TEST PROGRAM

The footing was placed in position and the load was applied to it by the jack through the proving ring. The load was applied in small increments until failure occurred. Each load increment was maintained constant until the footing settlement had stabilized. The settlements of the footing were measured by dial gauges. The geometry of the soil, model footing and confining cell is shown in Fig.3. The test program consisted of carrying out of five series of tests (1-5) on circular footing to study the effect of soil confinement on the soil-foundation response as shown in Table 2. Initially, the test has been carried out under axial load on the footing resting on the unconfined bed. Then, each series of the tests were carried out under axial load to study the effect of one parameter while the other variables were kept constant. The studied variables are the cell height (h), cell diameter (d) and the effect of fines keeping footing diameter (D) constant for all the cases.

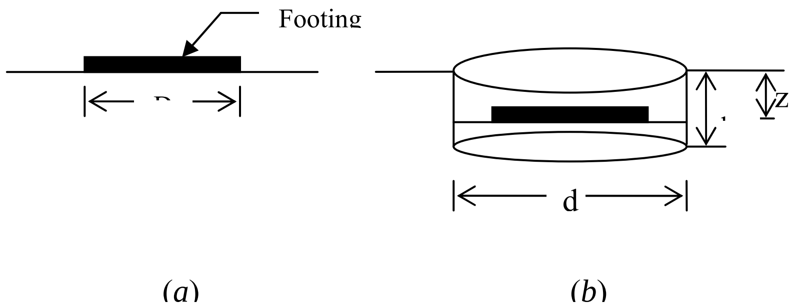

Figure 3: Geometric parameters of confined soil-foundation model; (a) Footing on silty sand ;(b) Footing on silty sand without confinement.

Table 1: Properties of Ghaggar sand

S. No.	Characteristics	Value
1	I. S. Classification	SP
2	D10	0.19 mm
3	D50	0.50mm
4	Uniformity coefficient (Cu)	2.9
5	Coefficient of curvature (Cc)	1.007
6	Minimum void ratio (e_{min})	0.51
7	Maximum void ratio (e_{max})	0.71
8	Specific gravity of Ghaggar sand (G)	2.67
9	Specific gravity of fines	2.63
10	Minimum dry density (γ_{min})	15.7 kN/m^3
11	Maximum dry density (γ_{max})	17.7 kN/m^3
12	Test density (dry state)	16.6 kN/m3
13	Relative density	51 % (App.)
14	Angle of internal friction (ϕ)	35.39^o

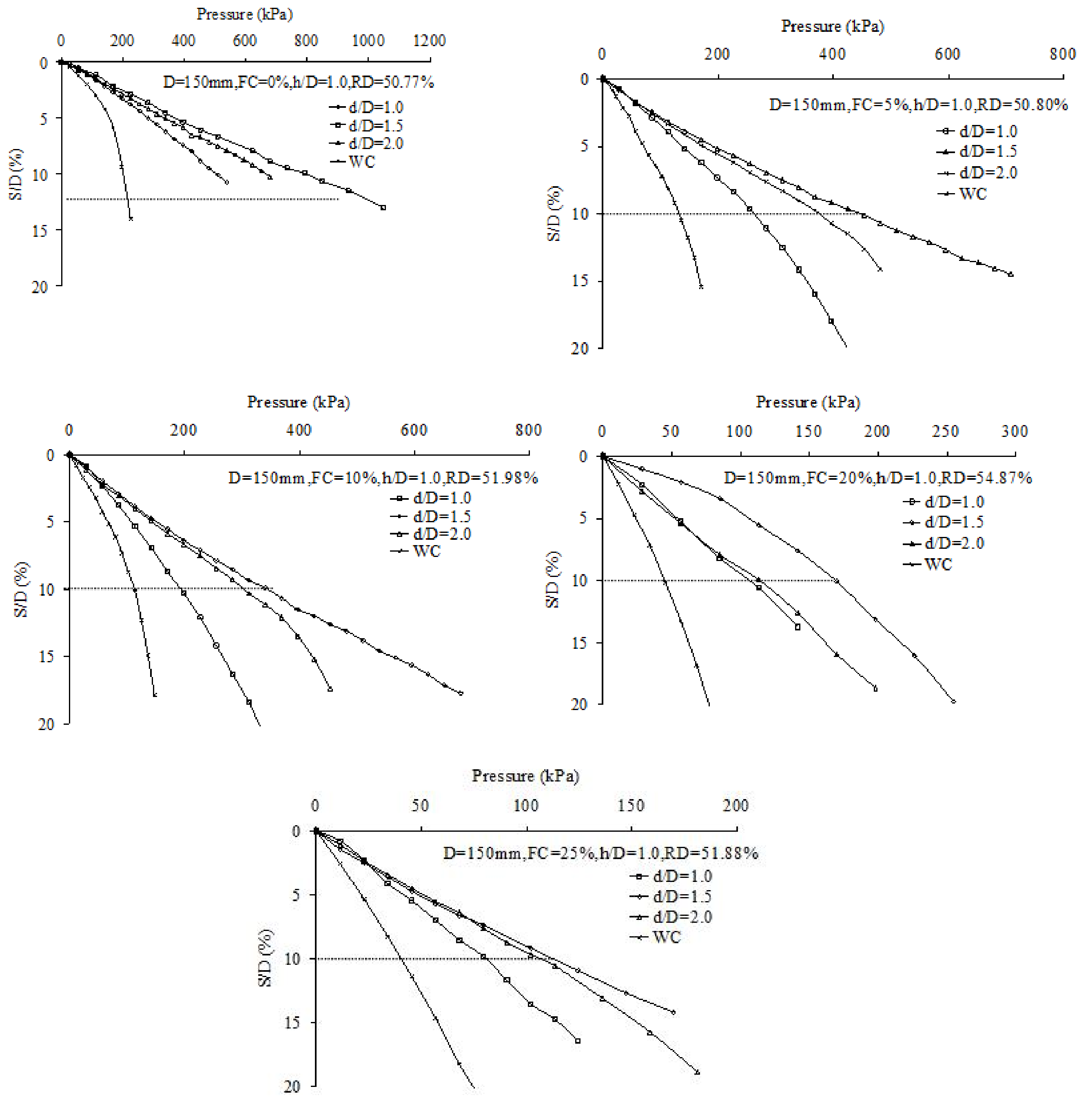

Figure 4: Variation of bearing pressure with (S/D) ratio for different (d/D) ratios for (a) FC=0%;(b) FC=5%; (c) FC=10%; (d) FC=20% and (e) FC=25%

Table 2: A Summary of Experimental Programme for Model plate load test

Test series	Constant parameters	Variable parameters	No of tests
1	Load tests on sand without confinement	FC=0%, 5%, 10%, 15%, 20%, 25%	06
2	*d*/*D* = 1.0	*h*/*D* =0.5, 0.75,1.0 FC=0%, 5%, 10%, 20%, 25%	15
3	*d*/*D* =1.5	*h*/*D* =0.5, 0.75,1.0 FC=0%, 5%, 10%, 20%, 25%	15
4	*d*/*D* = 2.0	*h*/*D* =0.5, 0.75,1.0 FC=0%, 5%, 10%, 20%, 25%	15
5	*d*/*D* = 1.5, *h*/*D* = 0.5, FC =10%	*z*/*h* =0.0, 0.2, 0.4, 0.6, 0.8	05

Table 3: Results of model scale footing resting on silty sands.

FC (%)	RD (%)	D (m)	Ultimate Bearing Pressure (kPa)
0	50.77	0.15	202.11
5	50.80	0.15	131.7
10	51.98	0.15	112.84
15	51.80	0.15	81.61
20	54.87	0.15	67.43
25	51.88	0.15	40.29

RESULTS AND DISCUSSION

The load–settlement relationship and the ultimate bearing capacity of the footing with and without confinement have been obtained. The bearing capacity improvement due to the soil confinement is represented using a non-dimensional factor, called the Improvement Factor **(**I_f). This factor is defined as the ratio of the ultimate bearing capacity with confinement to the ultimate bearing capacity in tests without confinement. In order to analyze settlement characteristics, settlement with and without confinement corresponding to a constant stress level of 100 kPa have been obtained for 0.15m footing diameter. The reduction in settlement due to the soil confinement is represented using a non-dimensional factor, called the settlement reduction factor (S_f). This factor is defined as the ratio of the settlement with confinement to the settlement in tests without confinement corresponding to a constant stress intensity of 100 kPa. The study was carried out for clean sand and sand with varying proportions of fines.

The footing settlement (*S*) is also expressed in non-dimensional form in terms of the footing diameter (*D*) as the settlement ratio, S_D (*S*/*D*, %). In the present study the ultimate capacity is interpreted as the bearing pressure, which produced a relative settlement of 10% of diameter of footing (the values across the dotted line, *S*/*D*=0.1). Although selecting to define q_{ult} at a relative settlement of *S*/*D* is due to (i) is convenience and ease, (ii) may actually be close to the average soil strain at failure, (iii) forces a fixed value at q_{ult} for comparison (iv) treats the displacement of all footing sizes at the same strain level (Trivedi and Sud 2005; Lutenegger and Adams 2007).

Effects of cell Diameter

In order to investigate the effect of cell diameter on the footing behavior, three cells with diameters of 150, 225 and 300 mm were used. Circular footing on clean sand and sand with increasing proportions of fines were tested to investigate the effect of confinement on bearing capacity and settlement. Typically, pressure-settlement responses observed from different series of tests are presented in Fig. 4-5. The pressure settlement responses show that there is no pronounced peak in the case of an unconfined soil bed, but slope of the pressure settlement curve tends to become steeper beyond a level of settlement of the footing diameter. This indicated soil failure. With the provision of cell, clear failure is not noticed even at larger percentage of settlement. Fig.6 shows the variation of Improvement factor with normalized cell diameter for different cell heights with a constant footing diameter of 150mm. A significant increase in the bearing capacity of the model footing supported on confined sand with the increase of normalized cell diameter d/D is observed up to about d/D ratio of 1.5; after which the improvement factor decreases with an increase in the d/D ratio. It can be seen that installations of confining cell appreciably reduce the settlement of the footing corresponding to constant pressure intensity. Fig.7 shows the variation of settlement reduction factor (S_f) with normalized cell diameter for a h/D=0.5 to 1.0 ratio. A significant decrease in the settlement of the model footing supported on confined silty sand with the increase of normalized cell diameter d/D is observed up to d/D ratio of 1.5; after which the settlement reduction factor (S_f) increases with an increase in the d/D ratio. While conducting the model tests, it was observed that as failure approached in tests carried out with small cell diameters, the cell and the soil within the cell behaved as one unit. In tests carried out with large cell diameters, this behavior was noticed initially, but as the load was increased it was no longer observed. Fig.6 also shows that using soil confinement could result in an improvement in bearing capacity as high as 3.94 times (FC=0%) more than that without soil confinement. It is clear that the best benefit of soil confinement could be obtained with a (d/D) ratio between1.0 to 2.0 with the maximum improvement in the bearing capacity at a (d/D) ratio of about 1.5 for different heights of confining cells. When the footing is loaded, such confinement resists the lateral displacements of soil particles underneath the footing and confines the soil leading to a significant decrease in the vertical settlement and hence improving the bearing capacity. For small cell diameters, as the pressure is increased, the plastic state is developed initially around the edges of the footing and then spreads downward and outward. The mobilized vertical friction between sand and the inside wall of the cell increases with the increase of the acting active earth pressure (Singh et al. 2007). This behavior is observed until the point when the system (the cell, sand and footing) starts to behave as one unit. The behavior is similar to that observed in deep foundations (piles and caissons) in which the bearing load increases due to the shear resistance of cell surface. This illustrates the increase of the bearing load with the increase of the cell diameter and cell height.

Effect of cell height

In order to investigate the effect of cell height on the footing response, tests were carried out using three different heights for each cell diameters. The variation of bearing pressure with settlement ratio is shown in Fig.5.From the figure it is clear that as the height of cell increases the bearing capacity increases and settlement decreases. The variation of improvement factor with normalized cell height (h/D) is shown in Fig.8 for different normalized cell diameters (d/D). The figure shows the same pattern of behavior for the different cell diameters. Increasing cell heights results in a greater improvement in the improvement factor. The increase in cell height results in the enlargement in the surface area of the cell–model footing leading to a higher bearing capacity. The slope of the improvement factor versus h/D curves for d/D ratios of 1.0 and 2.0 are less than the comparable slopes for d/D ratios of 1.5. This trend confirms the previous conclusion that the greatest benefit of cell confinement can be obtained at a d/D ratio of about 1.5.

Effect of fines

In order to analyze the effect of fines , a series of tests were carried out with all parameters held constant namely diameter of the cylinder, height of the cylinder except the percentage of fines content (Fig.9).Tests were conducted for different normalized diameter of the cell to the diameter of footing (d/D) values varied among 1 and 2 as shown in Table 2.Fig.9 shows the variation of bearing capacity with different percentages of fines content for varying cells with d/D of 1,1.5 and 2 for different h/D ratios. It is clear that increasing the percentage of fines for different values of d/D and h/D ratios, the bearing capacity decreases. It is due to the fact that as we increase the proportions of fine content the density increases along with the compressibility. The effect of compressibility offsets the improvement due increase in density. In other words, with the addition of fines settlement increases and the ultimate bearing carrying capacity decrease. Hence, in the presence of fines the failure criterion is governed by allowable settlement and the bearing capacity of the footing decreases.

Effect of the depth of embedment

In order to investigate the effect of side supports constructed to support soil cuts on the behavior when the foundation level is low, a series of tests were carried out (Series 5) (i.e. some times the footing is placed at low depth relative to top of the side support). All parameters namely diameters of the cell, height of the cell and proportion of fines held constant except the depth of the footing relative to the top of the cell (z). The cell diameters with d/D ratio equal to 1.5 was used. The normalized depths of the footing to the cell height (z/h) values varied among 0 to 0.8 for d/D equal to 1.5 were used. Fig.10 shows the variation of improvement factor with normalized embedded depth (z/h) for cells with d/D equal to 1.5 and h/D of 0.5. It is clear that varying the footing depth relative to the cell top has no effect on the behavior of cell–model footing. The difference between the maximum improvement factor (1.05) and the minimum value (0.90) is 0.15, which is caused by the slight disturbance that occurred in the sand beds while placing the footing within the cell. This can be explained as follows. For ordinary footings (without cellular support), increasing the foundation depth results in increasing the overburden pressure and hence increasing the bearing capacity. However, footing with cellular support the effect of overburden pressure is not significant. When the footing is loaded, it settles and the plastic state is developed until the point at which the soil – cell system behave as one unit. Therefore, increasing the embedment affects only the initial part of the behavior until that point after which the ultimate load depends on the surface area of the cell, which is constant. Hence, it can be concluded that the embedment of a footing in confined granular soil has no effect on the response of the footing–cell system.

CONCLUSIONS

The objective of this paper is to study the effect of lateral confinement on the behavior of footing on silty sand. Based on this experimental study, the following conclusions are drawn.

- Soil confinement has a significant effect on improving the behavior of circular footing supported on silty sands. The bearing capacity was found to increase by a factor of 3.94 as compared (FC=0%) to the unconfined case**.** This type of cells with different diameter could be easily manufactured and placed around the individual footings leading to a significant improvement in their response.

- In cases where structures are very sensitive to settlement, soil confinement can be used to obtain the same allowable bearing capacity at a much lower settlement.
- The improvement factor (I_f) and settlement reduction factor (S_f) is highly dependent on the d/D ratio (cell diameter/footing diameter). The optimum ratio is about 1.5 beyond which the improvement in bearing capacity decreases and settlement reduction factor increases as the ratio increases.
- Increasing the height of the confining cells, results in increasing the surface area of the cell-model footing, this transfers the footing load to deeper depth and leads to increase in improvement factor and decrease in settlement reduction factor.
- Bearing capacity of circular footing decreases and settlement increases on increase in proportions of fines.
- The embedded depth of the footing relative to the top of confining cell has insignificant effect on the response of footing–cell systems in the range of footing sizes and depths investigated in this study.

ACKNOWLEDGMENTS

The tests were performed in the Geotechnical Engineering Laboratory of Civil Engineering department, Thapar University, Patiala, India. The authors are thankful; to Prof. V. K. Sud for his valuable suggestions on conceptualization of this problem. The first author appreciates the support provided by Prof. M. Kumar HOD & the staff of the Department of Civil Engineering, Thapar University, Patiala.

LIST OF SYMBOLS

The following symbols are used in this paper:

ϕ	angle of internal friction;
C_c	coefficient of curvature for the sand;
C_u	uniformity coefficient;
d	cell diameter;
D	footing diameter;
D_{10}	effective size;
D_{50}	mean size(mm);
e_{max}	maximum void ratio;
e_{min}	minimum void Ratio;
FC	fines content;

G	specific gravity;
h	cell height;
qult	ultimate bearing capacity;
S	footing settlement;
S/D	settlement ratio;
Sf	settlement reduction factor;
WC	without cellular support;
γ_{max}	natural unit weight;
γ_{min}	natural unit weight;
γ_{nat}	natural unit weight;

REFERENCES

1. Brinch Hasen, J. **(1970)** "Revised and extended formula for bearing capacity." Bulletin No. 28, *Danish Tech. Inst.,* Copenhagen, 5-11.
2. deBeer, E. E. (1965) "Bearing capacity and settlement of shallow foundations on sand." Symposium on Bearing Capacity and Settlement of Foundations, Duke Univ. 15-33.
3. Feda, J. (1961) "Research on bearing capacity of loose soil." *Procs. 5th Int. Conf. Soil Mech & Found. Eng.,* Paris, 1, 635-642
4. Dash, S., N. Krishnaswamy and K. Rajagopal (2001 a) "Bearing capacity of strip footing supported on geocell-reinforced sand." Geotextile and Geomembrane, 19, 535-256.
5. Dash, S., K. Rajagopal and N. Krishnaswamy (2001b) "Strip footing on geocell reinforced sand beds with additional planar reinforcement." Geotextile and Geomembrane, 19, 529-538.
6. IS: 2720 **(**1985**)** "Methods of test for soils". Grain size analysis,Part 4
7. Lutenegger, A. J., and Adams, M. (2007) "Scale effects of shallow foundation bearing capacity on granular material." ASCE, 133 (10), 1192-1202.
8. Meyerhof, G.G. (1965) "Shallow foundations", J. of SMFD, ASCE, Vol. 91, SM2, 21-31.
9. Rajagopal, K., N. Krishnaswamy and G. Latha (1999) "Behavior of sand confined with single and multiple geocells", Geotextile and Geomembrane, 17, 171-184.
10. Rea, C. and J.K. Mitchell (1978) "Sand reinforcement using paper grid cells", Proc. Symposium on Earth Reinforcement, Pittsburg, ASCE, 644-663.
11. Sawwaf, M.E., and A. Nazer (2005) "Behavior of circular footing resting on confined granular soil." Journal of Geotechnical and Geoenvironmental Engineering, 131(3), 359-366.
12. Singh, V. K., A. Prasad and R.K. Aggarwal (2007) "Effect of Soil Confinement on Ultimate Bearing Capacity of Square Footing under Eccentric–Inclined Load." Electronic Journal of geotechnical Engineering, Vol.12,
13. Trivedi A., V. K. Sud (2002) "Grain characteristics and engineering properties of coal ash". Granular Matter, 4:3, 93-101.

14. Trivedi A., V. K. Sud (2005) “Ultimate bearing capacity of footings on coal ash”. Granular Matter, 7:4, 203-212

15. Vesic, A.S. (1973) “Analysis of ultimate loads of Shallow foundations.” Journal of Soil Mechanics and Foundation Division, Vol. 99(1), 45-73.

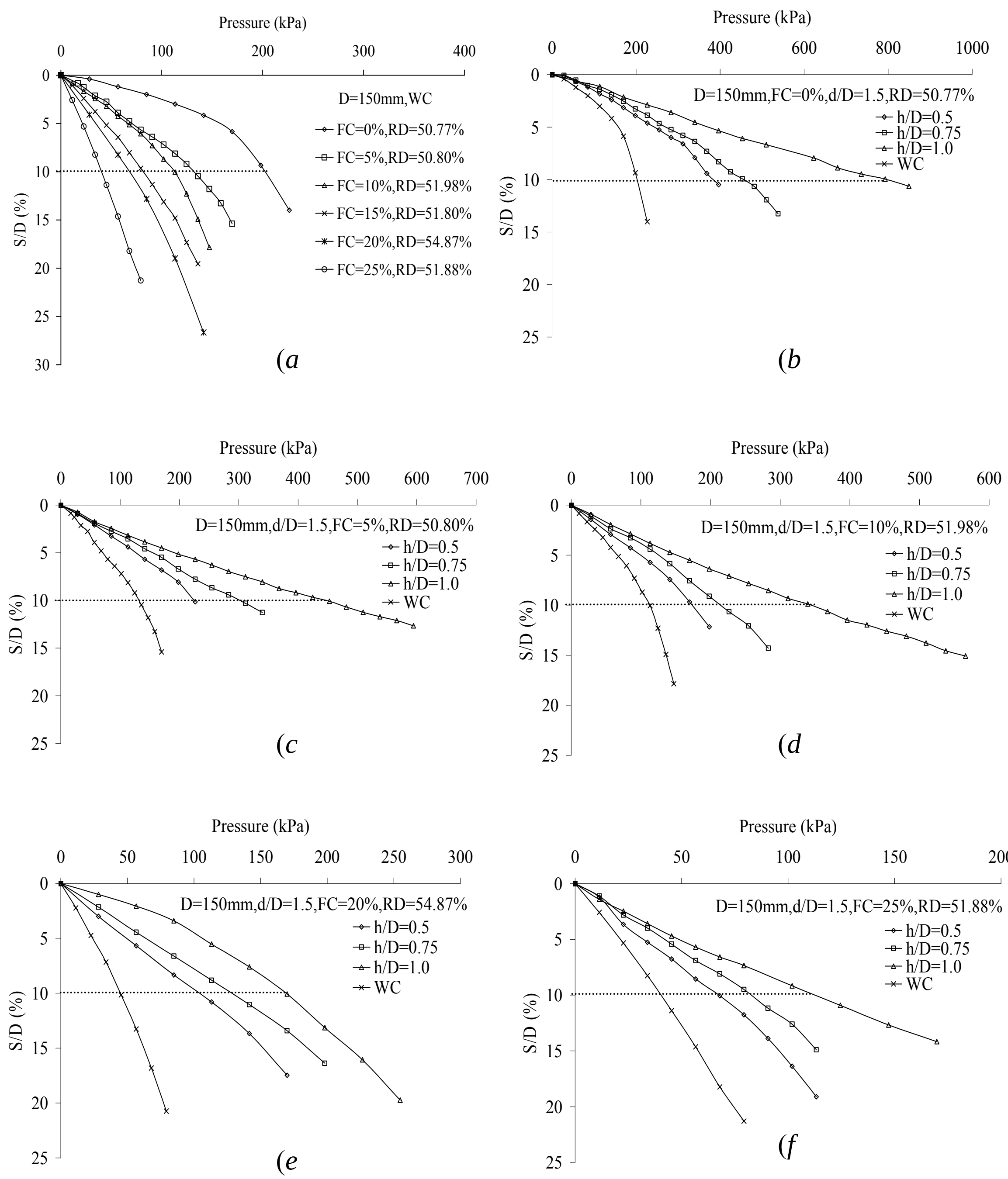

Figure 5: Variation of bearing pressure with (S/D) ratio for different ratios of (h/D) for proportions of fines: (a) WC ;(b) 0%**;** (c) 5% ;(d) 10%; (e) 20%; (f) 25%

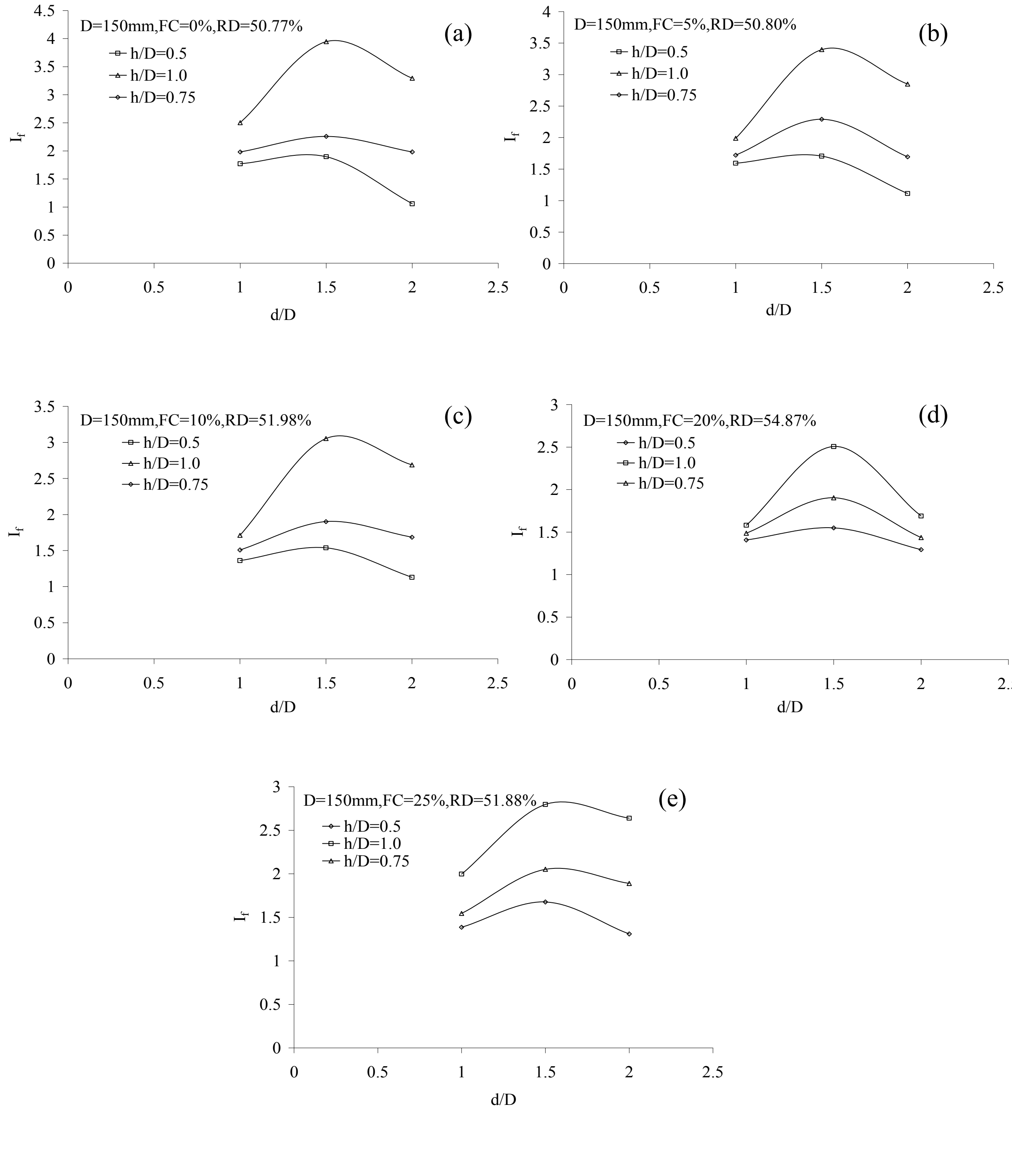

Figure 6: Variation of improvement factor with normalized cell diameters (d/D) for different cell heights for (a) FC=0 %;(b) FC=5%; (c) FC=10%; (d) FC=20% and (e) FC=25%

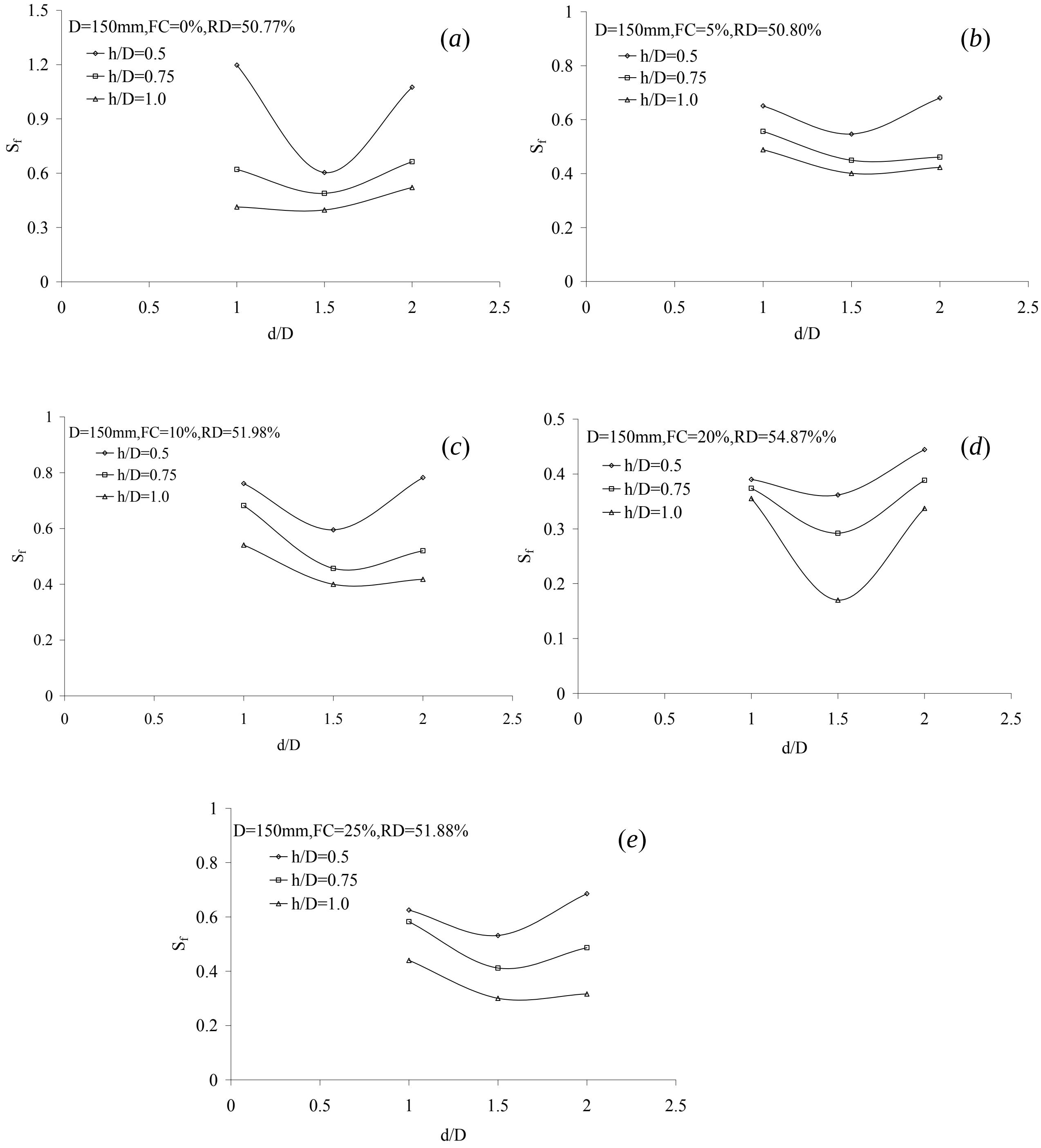

Figure 7: Variation of settlement reduction factor with normalized d/D ratio for different (h/D) ratios for (*a*) FC=0%; (*b*) FC=5%; (*c*) FC=10%; (*d*) FC=20% and (*e*) FC=25%

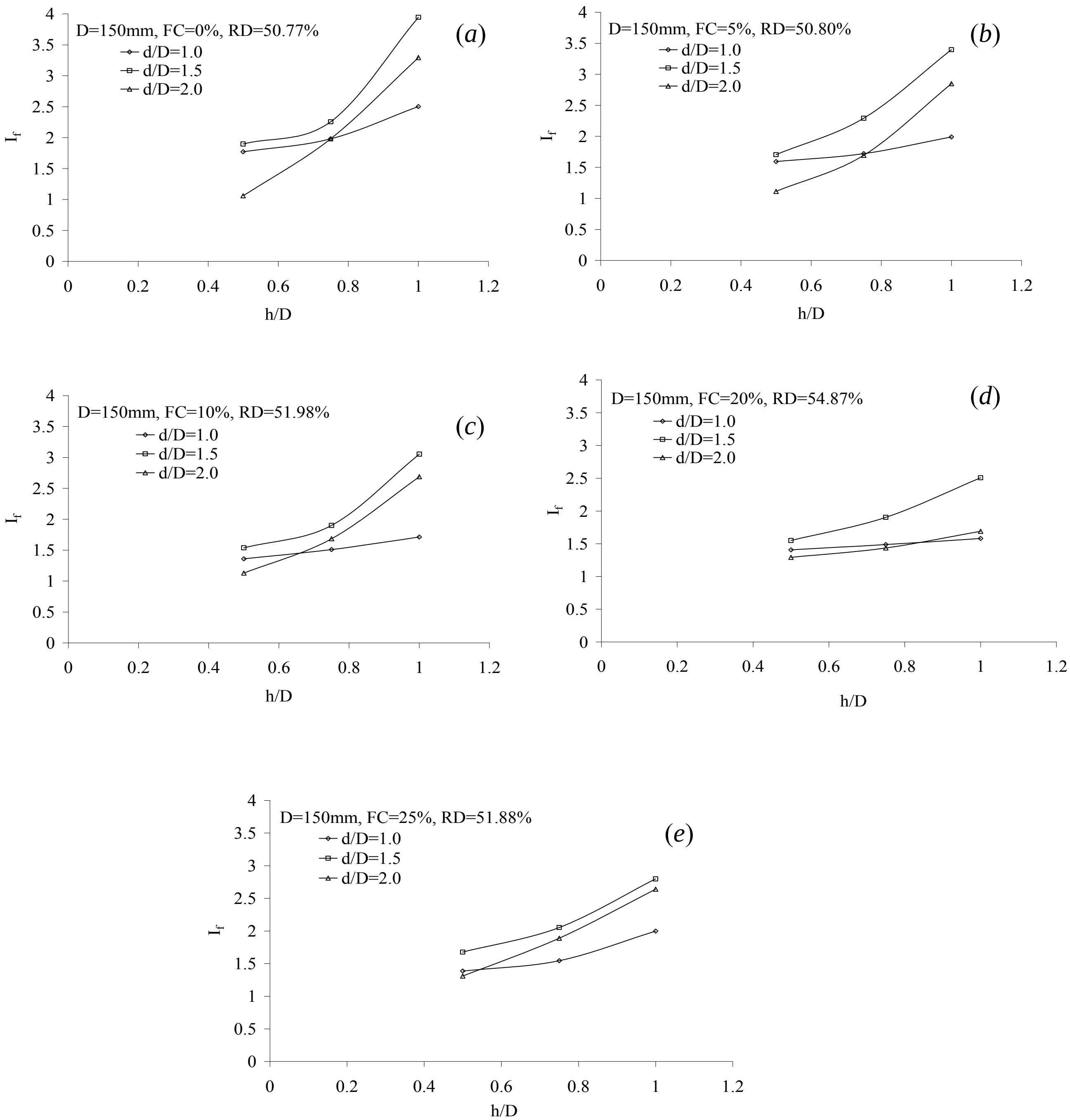

Figure 8: Variation of improvement factor with normalized h/D ratio for different (d/D) ratios: (*a*) FC=0 %; (*b*) FC=5%; (*c*) FC=10%; (*d*) FC=20% and (*e*) FC=25%

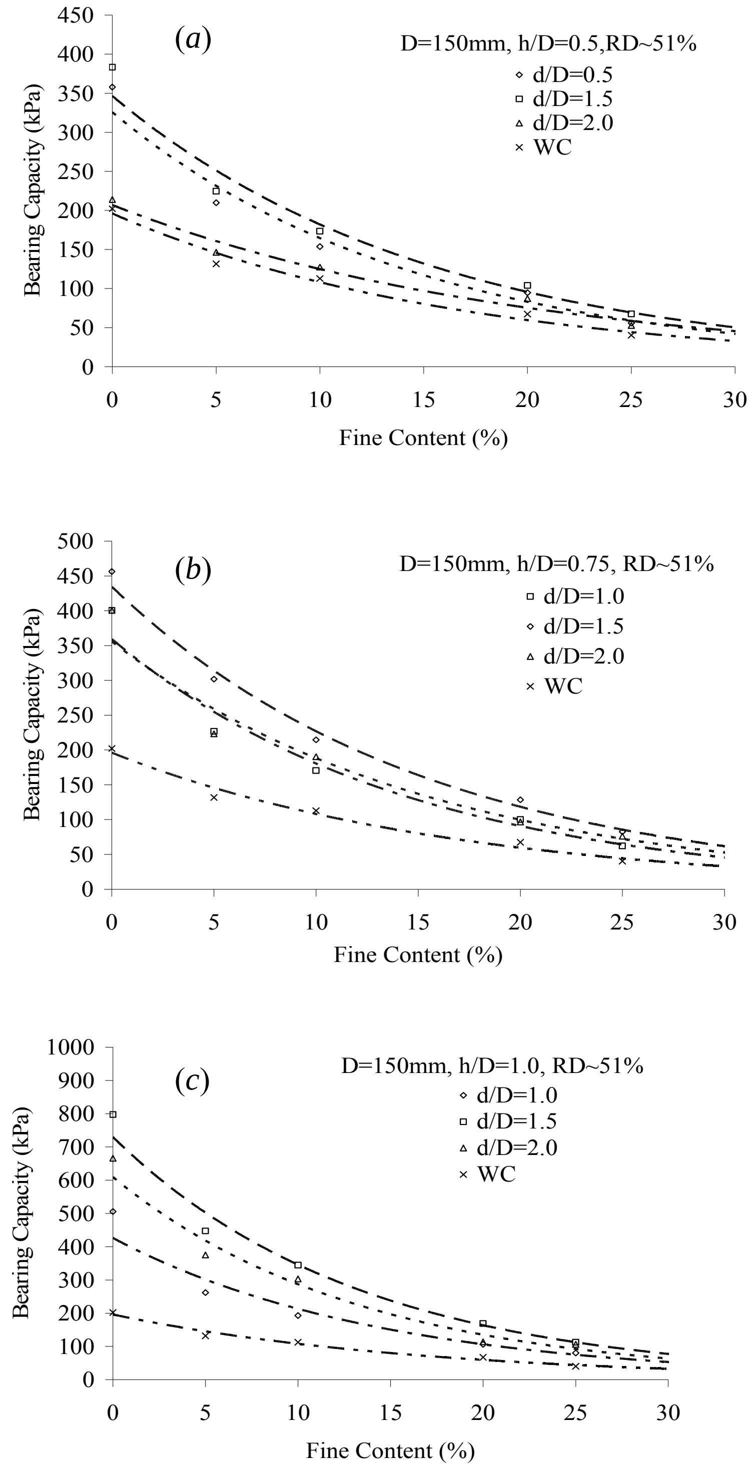

Figure 9: Variation of bearing capacity with different proportions of fines for different values of (*h*/*D*) ratio: (*a*) 0.5, (*b*) 0.75, (*c*) 1.0

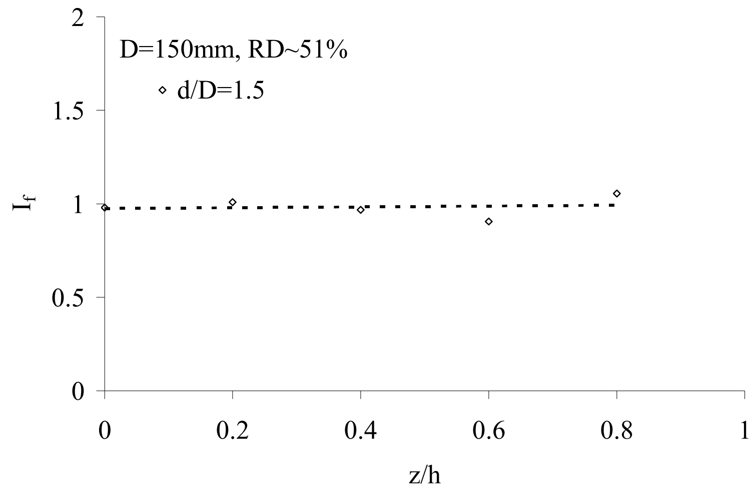

Figure 10: Variation of improvement factor with normalized embedded depth (*z*/*h*) for h/D = 0.5