

A  
*Dissertation*  
On  
**DISPLACEMENT AND STRAIN MEASUREMENT BY  
DIGITAL IMAGE CORRELATION**

*Submitted in partial fulfillment of the requirement  
for the award of Degree*

*Of*  
**Master of Engineering**

In  
**Structural Engineering**

By  
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## **CERTIFICATE**

This is to certify that this thesis titled “*Displacement And Strain Measurement By Digital Image Correlation*” submitted by **Dinesh Kumar Prasad**, University Roll No. **3210**, towards the fulfillment of the partial requirement for the award of Degree of Master Of Engineering, in Civil Engineering Department, Delhi College of Engineering, Delhi University, Delhi, is a record of bonafide work carried out and completed under my supervision and guidance during the academic session 2005-2007.

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## **Abstract**

The aim of this thesis is the measurement of a test specimen by image correlation methods for the purpose of extracting displacement measurements and subsequently strain measurements when the loads will be apply.

The measurement of displacements and displacement gradients (strains) has always been an important topic in the evaluation of material properties, such as material strengths or fracture parameters and in experimental stress analysis. Optical techniques such as moiré interferometry, holography and speckle inteferometry have been proven to be matured techniques to analyze macroscopic parameters and are being applied successfully in many different applications. However, all the interferometric techniques have stringent requirements for system's stability. Moreover, the processing of fringe patterns is laborious and time-consuming. This technical difficulty has raised many researchers' attention and computerized procedures have been developed to automate the processing of the data from the fringe patterns. In the last decade, a non-contacting optical technique, digital image correlation, has been developed by Sutton et al. (1983, 1986, 1988, 1991) and Bruck, et al. (1989). It was applied to measurement of displacements and strains.

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## Nomenclature

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1.	CCD	<i>Charged Coupled Device</i>
2.	u,v	<i>Displacements</i>
3.	$\theta$	<i>Rotation</i>
4.	$\epsilon_x$	<i>Strain along x</i>
5.	$\epsilon_y$	<i>Strain along y</i>
6.	FEM	<i>Finite Element Method</i>
7.	CFRP	<i>Carbon Fiber Reinforce Plastic</i>
8.	Z	<i>Level of Gray Scale Image</i>
9.	$\tilde{x}, \tilde{y}$	<i>Subset in the Deformed Image</i>
10.	$\forall$	<i>For all</i>
11.	E(f)	<i>mean</i>
12.	H(f)	<i>entropy</i>
13.	$P_r$	<i>probability distribution</i>
14.	$\otimes$	<i>binary operator for +, -, <math>\times</math> &amp; <math>\div</math></i>
15.	Var(f)	<i>Image variance</i>
16.	C	<i>Correlation coefficient</i>

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# CHAPTER 1

## INTRODUCTION

### 1. Introduction

Digital image correlation is an appealing optical method to measure deformation on an object surface based on the modern digital image processing technology and optical measurement. However, the traditional optical measurements, such as photo elasticity, moiré, holography, speckle interferometry, etc., which need to pre-treat the surface of object with spraying paint, coating and reproducing grid, and even more to make model, may give full-field information of object deformation from the interferometric fringe. Such techniques suffer from characteristic limitations and are commonly performed in the laboratory. In fact, the natural texture or the random artificial speckle on the object surface is the carrier of deformed information, and they keep one-to-one in accordance with the object's transformation.

Digital image correlation (DIC) is a relatively new technique for stress analysis that is gaining popularity and it doesn't have the limitations posed by the traditional methods. In this method, images of raw object(object in its original form) from before and during a test are used to determine the displacement field in the specimen. The strain field can be calculated from these displacements, or the displacements can be directly used depending on the situation (e.g. displacements can be used to determine the stress intensity factor in a fracture specimen).

A good overview and explanation of DIC can be found in Dally and Riley (2005) chapter 19. One of the first papers on this method is the one by Peters and Ranson (1982). The algorithm most commonly used in this method can be found in Sutton et al. (1986).

### 1.1. Background

In the early eighties of the 20th century, digital image correlation was first used by Yamaguchi, Peter and Ranson to focus on the determination of surface displacements. It was later formulated to improve the correlation algorithms with simple search method, coarse-fine grid method, optimum-seeking method, and Newton-Raphson iterative method, and enable displacement measurement to subpixel accuracy. One common drawback among these techniques is that they, in general measure the displacement field, rather than the often more interesting strain field. The general procedure has then been to differentiate the displacement

field to obtain the strain field. The approach is not as straightforward as it first appears to be because random errors in the displacement field are magnified, they will disturb the shape of the underlying strain field and lower the precision of strain. Though some researchers attempted to reduce the error of strain by eliminating the noise of displacement field firstly and then calculating the gradient of displacement, adopting the natural sample smooth method, or using the multi-displacement fields average method as well as exploiting the self-compensation technology, other researchers tried to extend the method to measure the specimen's mechanical strain and thermal strain in high-temperature situation, and to determinate micro-parameters of rock; only a little progress has been made to advance the strains precision at the micro-region. At the same time, low strain precision hinders the method application in the micro-region measurement.

## **1.2. Approach**

In the last decade, a non-contacting optical technique, digital image correlation, has been developed by M. A. Sutton et al. (1983, 1986, 1988, 1991) and Bruck, et al. (1989). It was applied to measurement of displacements and strains. The applications include microscopic strain measurements in electronic packaging, strain fields in polyurethane foam plastic materials and evaluation of their mechanical properties, and evaluation of thermal strain in the solder joints. This computer vision technique has the advantages of a simple system and direct sensing and thus avoids the laborious interpretation of interferometric fringes. The technique utilizes two similarly speckled images, which were captured by a solid state video camera, to represent the states of the object before and after deformation. By utilizing the concept of digitalization, one can characterize the image by the patterns of different levels of light intensity. Both of the digitized images are then correlated by an algorithm, based on their mutual correlation coefficient or other statistical functions, to find out the subtle differences between them.

## **1.3. Organization of the thesis**

For better understanding we have given various chapters on the proposed approach, organized in a systematic way as explained below:

### **Chapter 1**

Introduction presents a brief overview of Digital image Correlation, Background and the proposed approach,

## **Chapter 2.**

This chapter explains the research survey i.e, the present state of art for the research in digital image correlation.

## **Chapter 3**

Definitions, formation and presentation of digital image. Digital image formation, gives the knowledge of image formation, grey scale image. How digital image form from the real world image. And also gives idea about the image enhancement and its formulation

## **Chapter 4**

In this chapter we will discuss about the digital image correlation and its procedures

## **Chapter 5**

In this chapter, we are implementing the digital image correlation technique to find out the displacement and strain. For this technique we are using the Matlab 7.0 version. We take the image for the analysis by 2-mega pixel camera and second one through online.

## **Chapter 6**

We are discussing here about result, conclusion and future work

### 2.1. General

Digital Image Correlation technique is generally going to use for various purpose in different field. We can use this method in detection of metallic property of metal, in bio-mechanics .in civil engineering field etc. This is used by different organization for their own experiment and it is proved by them. In comparison to other method this is too accurate to measure displacement and strain.

### 2.2. Existing studies on digital image correlation technique:

**Jean-Noel PERIE, Sylvain CALLOCH, Christophe CLUZEL and Francois HILD**  
(September 2001)

The 'planar' digital image correlation technique needs a single CCD camera to acquire the surface patterns of a zone of a specimen in the un-deformed and deformed states. With these two images, one can determine in-plane displacement and strain fields. The digital image correlation technique used herein is based on Fast Fourier Transforms, which are very effective in reducing the computation cost. Its performance is assessed and discussed on artificial signals and in a real experimental situation. The technique is utilized to analyze experimental results of a plane shear experiment and validate a damage meso-model describing different degradations in a composite material.

**Mark C. Pitter, Chung W. See, Jason Y. L. Goh and Michael G. Somekh** UK (2002)

Subpixel digital image correlation has been applied to microscope images to analyze surface deformation. Non-integral pixel shifting and successive approximation are used to calculate the subpixel element of the sample displacement without introducing systematic interpolation errors. Although in-plane displacement precision of better than 2% of a pixel, or < 15 nm at x10 magnification, is shown to be achievable, the use of even moderate numerical aperture microscope objectives render the technique sensitive to errors or variations in sample focusing. The magnitude of this effect is determined experimentally and a focus

compensation method is described and demonstrated.

**Po-Chih Hung and A. S. Voloshin** (July-September 2003)

The proposed algorithm uses only fine search at the pixel level resolution and surface fitting for sub-pixel level. Two different specimens are investigated to explore the feasibility of this proposed algorithm. The results show that the experimental data are in good agreement with the theoretical solution. The proposed algorithm is found to be much faster than Newton-Raphson method with inferior, yet reasonable, accuracy for displacement and strain evaluation in the cases of uniaxial tension and disk under diametrical compression tests.

This research presents an analysis of an area-based image matching technique under geometric distortion. The translation and rotation baseline tests show the precision and limitations of this algorithm. However, it should be noted that with the existence of large distortions, the specified area is liable to lose its perceptiveness in recognizing the target area. In such case, the detection of displacements may wander off and hence the precision of this method would be reduced significantly. The inherent errors of using the proposed algorithm to correlate two speckled patterns were also analyzed. The experimental data from the tensile test show good agreement with the readings of strain gages in the range from 0.0029 to 0.046. This result indicates that for the uniform tension test, this method is a very convenient, fast, and efficient tool. Less accuracy was achieved in a disk under diametrical compression test. Therefore the proposed here approach may not be applicable when accurate data is required and the processing time is not a problem. However, the facts that the errors are not sensitive to the averaging technique and the relatively small consumption of the computational time in detecting the displacements and strains suggest that this method has a potential for online quality analysis where time is of essence.

**Jinlong Chen, Ganmin Xia, Kebin Zhou, Guisuo Xia, Yuwen Qin** (December 2003)

A method of two-step digital image correlation is well developed with more stable and reliable calculating technology, which consists of a simple searching method and an



iterative correlation method. This Digital Image Correlation method can not only improve the calculating speed and the measuring accuracy, but also simplify the process of the experiment. In order to further increase the sensitivity of the technique, the sub-pixel reconstruction is performed in subimage by utilizing the higher precision calculation of bicubic spline interpolation value method, and the accuracy of displacement is extended to better than 0.01 pixel; the strain resolution is limited to less than 0.0002 in micro-region. The above method is applied to quantify the micro-deformation of bimaterial sample and coating sample. The experimental results show that the method of two-step digital image correlation is a potential boon to investigations at extremely small-size scales

**Sandra Bergonnier, Francois Hild and St. Ephane Roux** (September, 2004 )

Mechanical compression and tearing tests are carried out on crimped glass wool samples. The displacement field is determined by using digital image correlation based on images taken at different stages of the mechanical tests. A multiscale algorithm is used to resolve accurately fine details of the displacement field. This technique reveals strain heterogeneities and further localisation in compression tests well below the peak stress. Crack formations are identified in tearing tests. Reliability and resolution of the displacement and strain fields are validated by using different window sizes in the correlation analysis

**Matteo Accardi , Lori Graham-Brady , Surendra Shah** (April 2006)

Digital image correlation (DIC) is a computer vision technique that is used to track the surface displacements of deforming materials. DIC is a fully non-destructive and non-contact measurement tool; all that is required to perform the technique are digital images of the specimen surface before deformation (reference image) and after deformation (deformed image). From these two images, DIC computes the displacement of a regular grid of points on the specimen surface. DIC has been widely used to examine the deformations of engineering materials, including concrete and related cement-based materials.

Digital image correlation (DIC) is an effective tool for the determination of fracture behavior of materials through the examination of surface displacements. Specifically, projects are currently underway to study the CFRP-concrete bond properties, and to study the

interfacial transition zone at the aggregate–cement paste bond in concrete. The CFRP project has yielded information on bond stress, local slip, fracture propagation, and an experimental constitutive law for the CFRP-substrate interface. The interface fracture project has shown the distributed softening at the interface that occurs ahead of macrocrack formation. Further research towards a bond constitutive law for the interface region is currently underway.

### **S. Mguil-Touchal, F. Morestin, M. Brunet**

Despite the powerful of current calculus codes by finite element or analytical, their reliability and validity must be justified by experimental tests. An original software for the displacement field measurement is presented in this paper. This software use the digital image correlation principle. Thanks to this technique, the precision of the measure could reach 1/60 th of a pixel and, it is possible to measure strains between  $5.10^{-5}$  and 0.8. Initially, this software was developed for measuring strains on a sheet testing in metal forming. But, it is revealed that this application field is more important : biomechanical field, geotechnic field, metal characterization, control tests.

This software could be used, nowadays, for all application which need the knowledge of displacement and strain field for a plane surface. Of course, the experimental conditions must be in such a way that any deterioration of the random aspect, deposed on the piece, will occurs. Lost, alteration or little modification of the speckle aspect will not guarantee the success of the measure. Finally, the purpose of this paper will be, first, to give clearly the mathematical principle of the displacement field determination by a correlation method. Then, various experimental tests using the correlation method, are presented. The powerful of this technique is show by their variety. This method of measure without contact can be an another tool for validating, in a short time, a numerical or analytical result

The various tests described in this paper underlines all the interest that one has to carry on this correlation technique for the displacement field measurement. This is very powerful technique : strains until 80% were appreciated between two images. Nevertheless, some researches till to be done. For example, the computation time must be reduced. More, it would be necessary to modify the correlation program in order to allow the choice of any study zone : inclined rectangles, form curves. An extension of this correlation technique for the 3 D strain measurement is now in labour. Finally, this correlation technique has to

become an another tool for measuring the displacement and/or strain field in certain hard cases. So it is a tool which it is necessary to know the capacities, the power, the rapidity, the limits... to give a result in order to use if it is necessary and for validating, in short time, a numerical or analytical result.

**Kenji Machida, Tokyo University of Science, Japan**

Fiber reinforced composites are heterogeneous and anisotropic. The applicability of the stress analysis methods on such heterogeneous and anisotropic materials is not well known. In the present study, an attempt is made to apply the digital image correlation method and the intelligent hybrid method to a carbon fiber reinforced plastic (CFRP) laminate. A material used is carbon/epoxy system. Laminate configuration is unidirectional. Tensile load is applied in off-axis (45 degrees) direction as well as longitudinal (0 degree) and transverse (90 degrees) directions on a CFRP laminate. Displacement, strain and stress fields due to the tensile loading in the CFRP unidirectional laminate are analyzed. The constitutive equation considering material anisotropy is built into the intelligent hybrid method used at the time of analyses. The validity of the algorithm is checked through comparison between results of the present method, experimental results from strain gauge method, and the analytical results from finite element method (FEM).

**J.M. Considine, C.T. Scott, R. Gleisner and J.Y. Zhu USDA Forest Service, Forest Products Laboratory, Madison, Wisconsin**

Digital image correlation was used to measure the full-field deformation of paperboard and handsheet tensile specimens. The correlation technique was able to accurately measure strain in regions 0.6 by 0.6 mm. Results showed the variation of strain to be much larger than has been previously reported. For machinemade paperboard tested in the cross-direction, the variation of strain increased throughout the tensile test and became erratic near failure, indicating many local failures. The measured strain distribution can be characterized by a Weibull function in agreement with weak-link failure theories. The analysis of a handsheet tensile specimen with a low-grammage region, approximately 4 mm wide, showed large negative strains near the region's edge.

- Digital image correlation can be an effective technique to examine local strain behavior of fibrous networks under stress.
- The variation of strain increases during straining as regions undergo plastic deformation and failure. In most cases, the variation of strain is a precursor to failure.
- The variation of strain measured by digital image correlation is much higher than previously reported. One reason for this larger variation is that digital image correlation uses smaller regions than those used in previous studies. Also, this technique determined strains from an entire region within the specimen, and not just selected regions.
- The distribution of strains changes as the global stress level increases. The shape can be characterized by a Weibull distribution in agreement with weak link failure theories.
- Regions of low grammage were shown to have the characteristic shape and size required to produce compressive stresses during tensile testing in agreement with classical mechanics.

## DEFINITION, FORMATION & REPRESENTATION OF DIGITAL IMAGE

### 3.1. Definition

#### Image

In common usage, an image (from Latin imago) or picture is an artifact that reproduces the likeness of some subject usually a physical object or a person.

A graphic representation or description of a scene, typically produced by an optical or electronic device. Common examples include remotely sensed data (e.g., satellite data), scanned data, and photographs. An image is stored as a raster data set of binary or integer values that represent the intensity of reflected light, heat or other range of values on the spectrum.

#### Digital image

An electronic photograph scanned from an original document, made up of a set of picture elements ("pixels"). Each pixel is assigned a tonal value (black, white, a shade of gray, or color) and is represented digitally in binary code (zeros and ones).

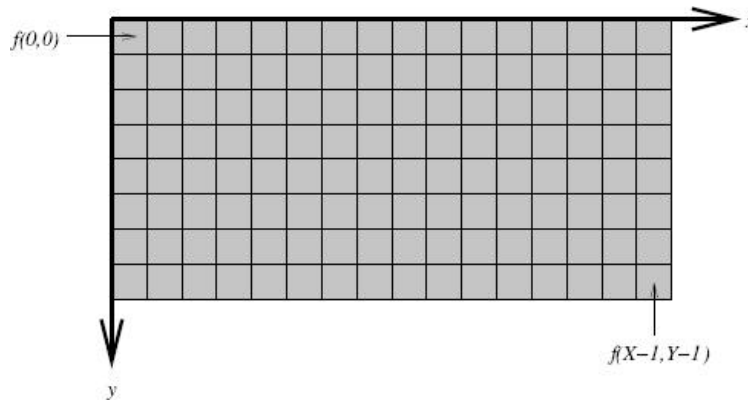
#### Correlation

Used to describe the observed relationship between instances of two events. A systematic pattern can be seen in the occurrences of events that are correlated. When the events involve numbers, a positive correlation means that as one increase, the other increases as well. A negative correlation means that as one increase, the other decreases. Correlation does NOT imply causation in any way. In other words, just because two events are correlated does not mean that one causes another, or has anything to do with the other - correlations deal only with observed instances of events, and any further conclusions cannot be inferred from correlation alone. Strong correlation, however, does often warrant further investigation to determine causation.

### 3.2. Digital Image Formation

A **digital image** is a discrete two-dimensional function,  $f(\mathbf{x}, \mathbf{y})$ , which has been quantized over its domain and range. Without loss of generality, it will be assumed that the

image is rectangular, consisting of  $Y$  rows and  $X$  columns. The **resolution** of such an image is written as  $X \times Y$ . By convention,  $f(0, 0)$  is taken to be the top left corner of the image, and  $f(X-1, Y-1)$  the bottom right corner. This is summarized in Figure 1.



**Fig 1: A rectangular digital image of resolution 16 x 8.**

Each distinct coordinate in an image is called a **pixel**, which is short for picture element. The nature of the output of  $f(x, y)$  for each pixel is dependent on the type of image. Most images are the result of measuring a specific physical phenomenon, such as light, heat, distance, or energy. The measurement could take any numerical form.

### 3.2.1. Grey Scale Image

A **grey scale** image measures light intensity only. Each pixel is a scalar proportional to the brightness. The minimum brightness is called black, and the maximum brightness is called white. A typical example is given in Figure 6. A colour image measures the intensity and chrominance of light. Each colour pixel is a vector of colour components. Common colour spaces are RGB (red, green and blue), HSV (hue, saturation, value), and CMYK (cyan, magenta, yellow, black), which is used in the printing industry. Pixels in a range image measure the depth of distance to an object in the scene. Range data is commonly used in machine vision applications.



**Figure 2: A typical grey scale image of resolution 512 x 512**

For storage purposes, pixel values need to be quantized. The brightness in grey scale images is usually quantized to  $Z$  levels, so  $f(x, y) \in \{0, 1, \dots, z-1\}$ . If  $Z$  has the form  $2^L$ , the image is referred to as having **L bits per pixel**. Many common grey scale images use 8 bits per pixel, giving **256** distinct grey levels. This is a rough bound on the number of different intensities the human visual system is able to discern. For the same reasons, each component in a color pixel is usually stored using 8 bits.

Medical scans often use 12-16 bits per pixel, because their accuracy could be critically important. Those images to be processed predominantly by machine may often use higher values of  $Z$  to avoid loss of accuracy throughout processing. Images not encoding visible light intensity, such as range data, may also require a larger value of  $Z$  to store sufficient distance information.

There are many other types of pixels. Some measure bands of the electromagnetic spectrum such as infra-red or radio, or heat, in the case of thermal images. Volume images are actually three-dimensional images, with each pixel being called a voxel. In some cases, volume images may be treated as adjacent two-dimensional image slices. Although this thesis deals with grey scale images, it is often straightforward to extend the methods to function with different types of images.

### **3.2.2. Image acquisition**

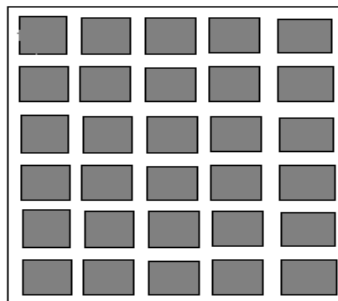
Image acquisition is the process of obtaining a digitized image from a real world source. Each step in the acquisition process may introduce random changes into the values of pixels in the image. These changes are called noise. Assume you want to send a photo of your new house to a friend over the Internet. This may be achieved by taking a photograph with a conventional camera, having the film made into a print, scanning the print into a computer, and finally emailing it to your friend. Figure 3. shows the many potential sources of noise.



**Figure 3: Noise may be introduced at each step in the acquisition process**

The air between the photographer and the house may contain dust particles which interfere with the light reaching the camera lens. The silver-halide crystals on the film vary in size and are discontinuous, resulting in film grain noise in the printing process [MJ66]. Most scanners use a CCD array to scan a row of the print, which may introduce photo-electronic noise. The scanner's CCD array is controlled by a fine stepper motor. This motor has some degree of vibration and error in its movement, which may cause pixels to be mis-aligned. The scanner also quantizes the CCD signal, introducing quantization noise [GN98]. Transmitting the image over the Internet is nearly always a bit preserving operation thanks to error checking in network protocols. However, an image transmitted to Earth from a remote space probe launched in the 1970's is almost guaranteed to contain errors.

### 3.2.3. Image Acquisition Using Sensor Array

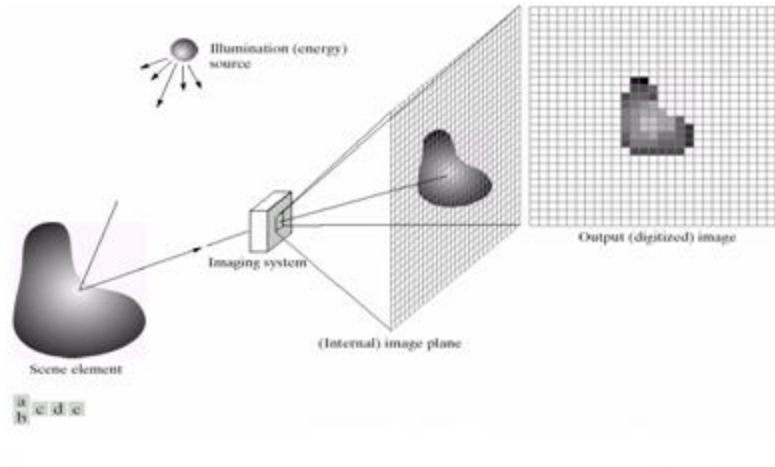


**Figure 4: Array sensor**

This figure shows individual sensors arranged in the form of a 2-D array. This type of arrangement found in digital cameras.



A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packed in a rugged array of 4000x4000 elements or more. CCD sensors are used widely in digital cameras and other light sensing instruments. The response of each sensor is energy projected on to the surface of the sensors.



**Figure 5: An example of the digital image acquisition process. (a) Energy (illumination) source. (b) an element of a scene. (c) Imaging system. (d) Projection of the scene on to the plane. (e) Digitized image.**

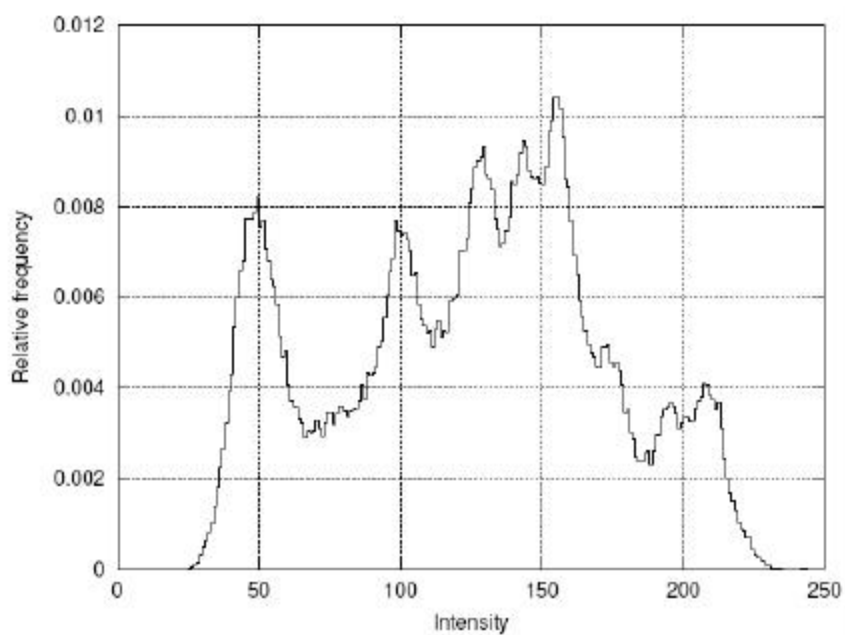
The principal manner in which array sensors are used in above figure. This figure shows the energy from an illumination source being reflected from a scene element. The first function performed by the imaging system shown in figure (c) is to collect the incoming energy and focus it on to an image plane. If the illumination is light; the front end of the imaging system is a lens, which projects the viewed scene on to the lens focal plane in the figure (d). The sensor array. This is coincident with the focal plane, produces output proportional to the integral of the light received at each sensor. Digital and analog circuitry sweeps these outputs and converts them to a video signal, which is then digitized by another section of the imaging system. The output is a digital image. as shown in figure (e).

### 3.2.4. Image statistics

#### 3.2.4.1. The histogram

A histogram plots the relative frequency of each pixel value that occurs in a grey scale image. Figure 6 shows the intensity histogram for the image from Figure 2. The histogram

provides a convenient summary of the intensities in an image, but is unable to convey any information regarding spatial relationships between pixels. In this example, the image does not contain many very low or very high intensity pixels. It is possible that peaks in the histogram correspond to objects in the image, but it is difficult to be certain without visually examining the image.



**Figure 6: The histogram for the greyscale image in Figure 2.**

### 3.2.4.2. The mean

The **image mean** is the average pixel value of an image. For a grey scale image this is equal to the average brightness or intensity. Let the image  $\mathbf{f}(\mathbf{x}, \mathbf{y})$  be referred to using the shorthand  $f$ . The mean of this image,  $\mathbf{E}[f]$ , may be calculated using Equation 1

$$\mathbf{E}[f] = \frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} f(x, y) \quad (1)$$

### 3.2.4.3. The variance

The image variance,  $\mathbf{Var} [f]$ , gives an estimate of the spread of pixel values around the image mean. It can be calculated using either Equation 2 or Equation 5. The latter has the

advantage of requiring only one pass through the image. The standard deviation is simply  $\sqrt{\text{Var}[f]}$ .

$$\text{Var}[f] = E[F - E[F]]^2 \quad (2)$$

$$= \frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} (f(x,y) - E[F])^2 \quad (3)$$

$$= \frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} (f(x,y) - \frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} (f(x,y)))^2 \quad (4)$$

$$\text{Var}[f] = E[f^2] - E[f]^2 \quad (5)$$

$$= (\frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} (f(x,y))^2) - (\frac{1}{YX} \sum_{y=0}^{Y-1} \sum_{x=0}^{X-1} (f(x,y)))^2 \quad (6)$$

### 3.2.4.4. The entropy

The image histogram may be considered a probability distribution over pixel values. For the case of a  $Z$  level grey scale image, the histogram entry for intensity  $z$  may be written as  $\text{Pr}(z)$ . The entropy of an image,  $f$ , is given by Equation 7. The units of entropy are bits when using logarithms to base 2

$$H(f) = - \sum_{z=0}^{Z-1} \text{Pr}(z) \log_2 \text{Pr}(z) \text{ bits} \quad (7)$$

The entropy has a maximum value of  $\log Z$  when all intensities occur with equal frequency, corresponding to a uniform histogram. It has a minimum value of 0 when all pixels have the same intensity. The entropy is one measure of the information content of an image. Because it is calculated from the histogram, it is unable to take spatial factors into consideration.

## 3.2.5. Image algebra

### 3.2.5.1. Image-scalar operations

Various useful arithmetical operations may be defined for images. Let  $\otimes$  represent the binary operator for addition, subtraction, multiplication, or division. Equation 8 shows

how to combine a scalar,  $c$ , and an image,  $g$ , to produce a new image,  $f$ . This is a pixel-wise operation -each pixel in  $f$  is operated on using  $\otimes$  with  $c$ , and the result put in  $f$ .

$$f = g \otimes c = \forall(x, y) f(x, y) = g(x, y) \otimes c \quad (8)$$

This idea could be used to enhance an image which is too dark. Consider the image in Figure 7(a) which uses 8 bits per pixel (256 levels), but only contains pixels with intensities from 64 to 191. One may consider enhancing it to use the full intensity range. This can be achieved using Equation 9, where  $\lfloor \cdot \rfloor$  Denotes integer truncation, and floating point precision is used for all pixels during the calculation. The result is given in Figure 7(b)

$$f = \lfloor \frac{g-64}{128} \times 255 \rfloor \quad (9)$$



**Figure 7: (a) low contrast image; (b) after enhancement.**

### 3.3. Digital image representation

*“Virtual image, a point or system of points, on one side of a mirror or lens, which, if it existed, would emit the system of rays which actually exists on the other side of the mirror or lens.”*

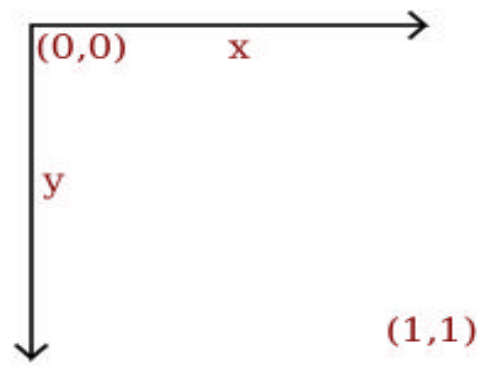
–Clerk Maxwell

### 3.3.1. Vector images

One way to describe an image using numbers is to declare its contents using position and size of geometric forms and shapes like lines, curves, rectangles and circles; such images are called vector images.

### 3.3.2. Coordinate system

We need a coordinate system to describe an image, the coordinate system used to place elements in relation to each other is called *user space*, since this is the coordinates the user uses to define elements and position them in relation to each other.



**Figure 8: Coordinate system**

The coordinate system used for all examples in this document has the origin in the upper left, with the x axis extending to the right and y axis extending downwards.

### 3.3.3. Sampling

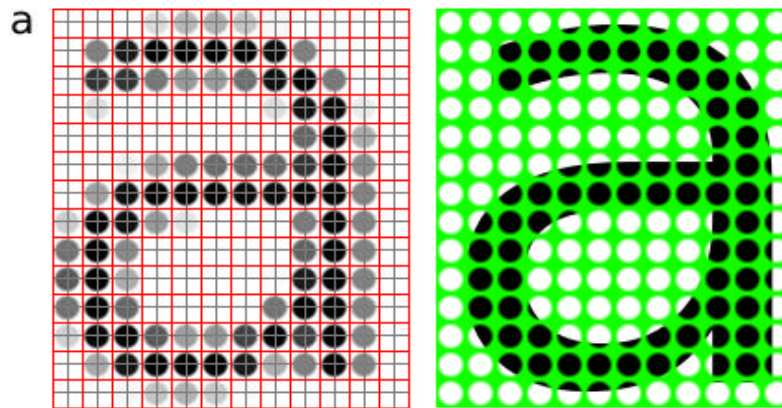
When measuring the value for a pixel, one takes the average color of an area around the location of the pixel. A simplistic model is sampling a square, this is called a box filter, a more physically accurate measurement is to calculate a weighted Gaussian average (giving the value exactly at the pixel coordinates a high weight, and lower weight to the area around it). When perceiving a bitmap image the human eye should blend the pixel values together, recreating an illusion of the continuous image it represents.

### 3.3.4. Image dimensions

The number of horizontal and vertical samples in the pixel grid is called *Image dimensions*, it is specified as width x height.

#### 3.3.4.1. Resolution

Resolution is a measurement of sampling density, resolution of bitmap images gives a relationship between pixel dimensions and physical dimensions. The most often used measurement is ppi, pixels per inch <sup>[2]</sup>.



**Figure9: Sampling grid**

A rasterized form of the letter 'a' magnified 16 times, where each pixel is represented as a circle instead of a square.

#### 3.3.4.2. Mega pixels

Mega pixels refer to the total number of pixels in the captured image, an easier metric is image dimensions which represent the number of horizontal and vertical samples in the sampling grid. An image with a 4:3 aspect ratio with dimension 2048x1536 pixels, contain a total of  $2048 \times 1536 = 3,145,728$  pixels; approximately 3 million, thus it is a 3 mega pixel image.

Dimensions	Mega pixels	Name	Comment
640x480	0.3	VGA	VGA
720x576	0.4	CCIR 601 DV PAL	Dimensions used for PAL DV, and PAL DVDs
768x576	0.4	CCIR 601 PAL full	PAL with square sampling grid ratio
800x600	0.4	SVGA	
1024x768	0.8	XGA	The currently (2004) most common computer screen dimensions.
1280x960	1.2		
1600x1200	2.1	UXGA	
1920x1080	2.1	1080i HDTV	interlaced, high resolution digital TV format.
2048x1536	3.1	2K	Typically used for digital effects in feature films.
3008x1960	5.3		
3088x2056	6.3		
4064x2704	11.1		

**Table 1. Common image dimensions**

#### **3.3.4.3. Scaling / Re-sampling**

When we need to create an image with different dimensions from what we have we scale the image. A different name for scaling is re-sampling, when re-sampling algorithms try to reconstruct the original continuous image and create a new sample grid.

#### **3.3.4.4. Reducing image dimensions**

The process of reducing the image dimensions is called *decimation*; this can be done by averaging the values of source pixels contributing to each output pixel.

#### **3.3.4.5 Increasing image dimensions**

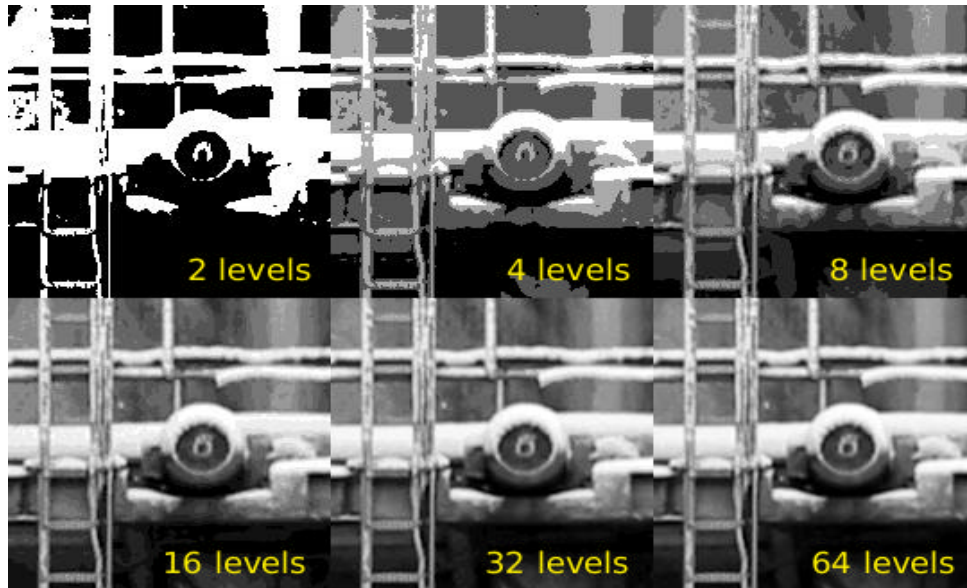
When we increase the image size we actually want to create sample points between the original sample points in the original raster, this is done by *interpolation* the values in the sample grid, effectively guessing the values of the unknown pixels <sup>[3]</sup>.

#### **3.3.4.6. Sample depth**

The values of the pixels need to be stored in the computers memory; this means that in the end the data ultimately need to end up in a binary representation, the spatial continuity of the image is approximated by the spacing of the samples in the sample grid. The values we can represent for each pixel is determined by the sample format chosen.

The same image width varying sample depths, note that high frequency areas (detailed areas) have an OK look earlier than low frequency areas.





**Figure 10: Sample depth**

**3.3.4.7. 8bit**

A common sample format is 8bit integers, 8bit integers can only represent 256 discrete values ( $2^8 = 256$ ), thus brightness levels are quantized into these levels.

**3.3.4.8. 12bit**

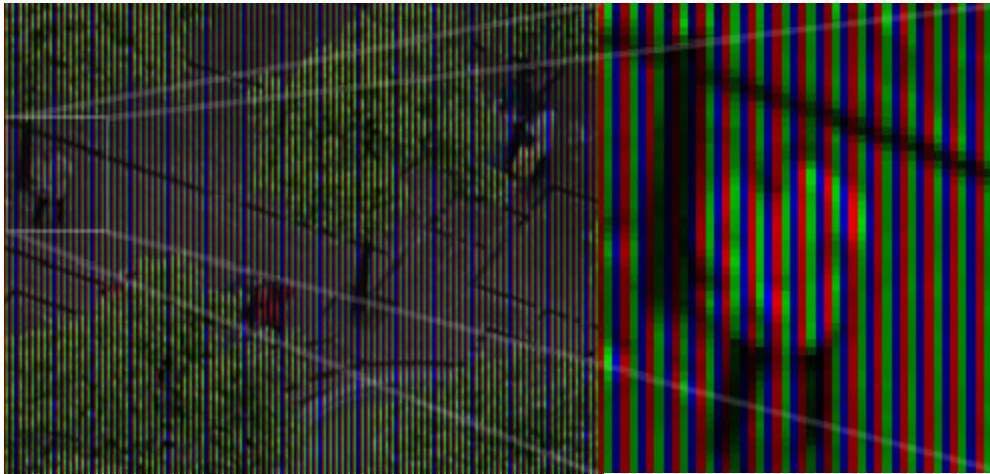
For high dynamic range images (images with detail both in shadows and highlights) 8bits 256 discrete values does not provide enough precision to store an accurate image. Some digital cameras operate with more than 8bit samples internally, higher end cameras (mostly SLRs) also provide RAW images that often are 12bit ( $2^{12} = 4096$ ).

**3.3.4.9. 16bit**

The PNG and TIF image formats supports 16bit samples, many image processing and manipulation programs perform their operations in 16bit when working on 8bit images to avoid quality loss in processing, the film industry in Hollywood often uses floating point values to represent images to preserve both contrast, and information in shadows and highlights.

### 3.3.4.10. Colors

The most common way to model color in Computer Graphics is the RGB color model, this corresponds to the way both CRT monitors and LCD screens/projectors reproduce color. Each pixel is represented by three values, the amount of red, green and blue. Thus an RGB color image will use three times as much memory as a gray-scale image of the same pixel dimensions.



**Figure 11: RGB bands**

Color image built up of bands of red, green and blue color (This is image illustrates how a laptop display is constructed, note that this image is preferred to be viewed on a computer screen, and not in print.)

One of the most common pixel formats used is **8bit rgb** where the red, green and blue values are stored interleaved in memory.

**DIGITAL IMAGE CORRELATION****4.1. Advantages and Disadvantages****4.1.1. Advantages—**

The digital image correlation technique is

- Non-contacting
- Insensitive to temperature changes (when specimen does not emit significant amounts of visible light)
- Full-field

**4.1.2. Disadvantage.—**

Some drawbacks of DIC is that it

- requires a specimen with a speckle pattern
- can only be used on flat surfaces
- has trouble near edges of the specimen
- requires optical access to the specimen
- is sensitive to light fluctuations
- requires a moderately large amount of computation time
- is sensitive to rigid body motion (this can be an advantages in some circumstances)
- does not provide full-field strain resolution better than ~0.1 %
- can require expensive equipment
- Typically only captures two-dimensional displacements on the surface (two camera DIC is in limited used for detecting out of plane displacements)

**4.2. Procedure**

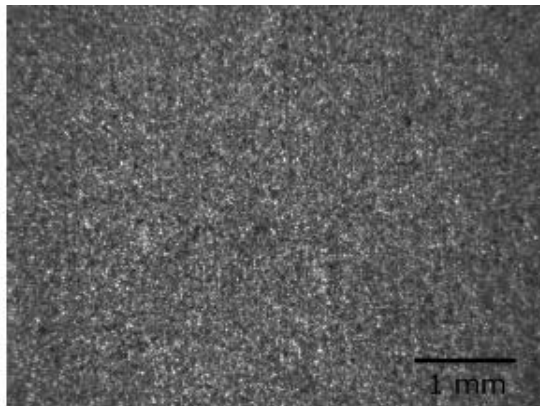
A random speckle pattern is placed on the specimen, and the scale is determined for the optical setup to be used. Throughout the test, images are taken at each desired load level. By comparing these images to a reference image with a DIC program, the displacement field

can be determined at each load level. The desired quantities (displacement or strain) are then used to determine what occurred during the test. In summary, the steps are as follows:

1. Generating a speckle pattern
2. Setup Equipment
3. Determine Scale and Take Reference Image
4. Capturing images throughout the test
5. Performing the correlation
6. Interpreting Results.

#### **4.2.1. Generating a speckle pattern.**

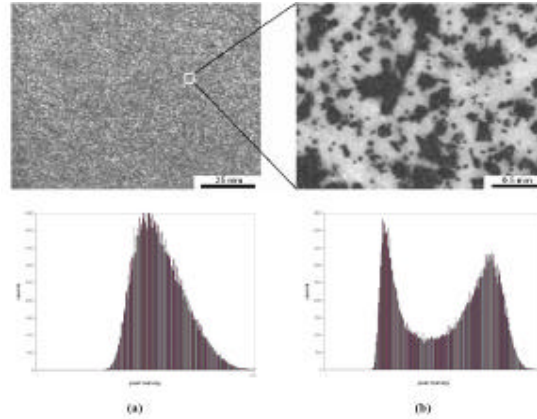
In the DIC technique, a random speckle pattern is placed on a specimen. The speckles will be tracked to determine displacements. For best results, a speckle pattern that looks like “white noise” on a television screen is desired such as that in Figure 12.



**Figure 12: Typical speckle pattern**

Since a speckle pattern that works well at one length scale is poor at other length scales, there are many different methods for creating a speckle pattern on a specimen. Depending on the material and the roughness of the specimen, the specimen surface itself may provide an adequate speckle pattern, but this is not typically the case. Some researchers have deposited powder onto specimens, but most commonly, an airbrush is used to deposit very small speckles of paint onto the specimen. At larger scales (image sizes on the order of 2 inches by 2 inches), it is possible to use spray paint. At very small scales, nanoparticles are sometimes used. For good correlation, a speckle pattern with a unimodal pixel intensity

distribution is desired, as opposed to a bimodal distribution (See Figure 13). A unimodal distribution means there are fewer sharp lines and more niqueness in the pattern. Since measuring sub-pixel displacements is important for obtaining useful data, gradual changes in pixel intensity allow for more accurate interpolation between pixel locations.



**Figure 13: (a) Good speckle pattern with unimodal pixel intensity distribution. (b) Bimodal distribution. Taken from Patel (2005)**

The pixel intensity distribution obviously depends on the speckle pattern, but lighting effects also play an important role in the final appearance of the pattern. Typically, two light sources are used in order to reduce shadows. An example of this is a Foster fiber optic light source with double gooseneck tubing (See Figure 14).



**Figure 14: Double gooseneck light source.**

A related technique called electronic speckle pattern interferometry (ESPI) actually uses the shadows of laser light as a speckle pattern for digital image correlation.

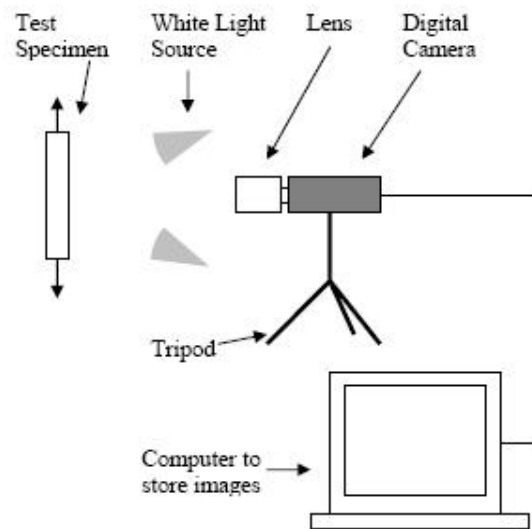
#### **4.2.2. Setup Equipment.**

The basic apparatus for digital image correlation investigations consists of:

- Digital CCD camera

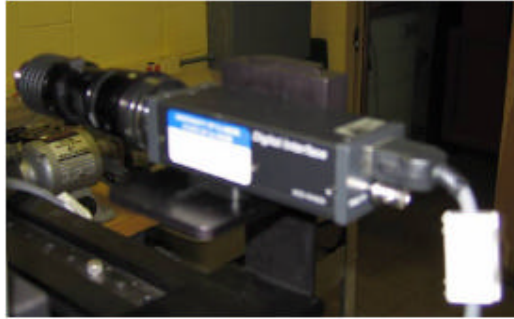
- Viewing/imaging optics
- Lighting source
- Stressing rig
- PC
- One camera for 2D measurements
- Two cameras for 3D measurements

After a good speckle pattern has been created, the specimen is placed in the load frame. Equipment is then moved into place so that the specimen's speckle pattern appears unimodal and in focus. The experimental setup is shown in Figure 15.



**Figure 15: Experimental Setup (Taken after Abanto-Bueno (2004)).**

It is necessary to have the proper lens for the magnification of interest. One lens does not typically work for all cases. A typical lens and camera are shown in Figure 16. Any digital camera can be used, but some cameras are better for certain applications than others. Monochrome cameras are typically better for DIC because most codes only use black and white data for correlation. The camera shown here is a monochrome xcd-900 ccd camera from Sony. It is approximately a 1 megapixel camera, it can be controlled by the computer, and it has a maximum frame rate of 7.5 frames/second.



**Figure 16: Long-distance microscope lens and charge coupled device (ccd) camera.**

The tripod is a very important part of the system. It is necessary to have a very stable, high quality tripod. It is beneficial to have a tripod that can move the camera by small amounts since this helps in focusing. A high quality tripod is shown in Figure 17. Typically, tripods do not need to be as robust as this, but if you have a \$30,000 tripod lying around, then use it.

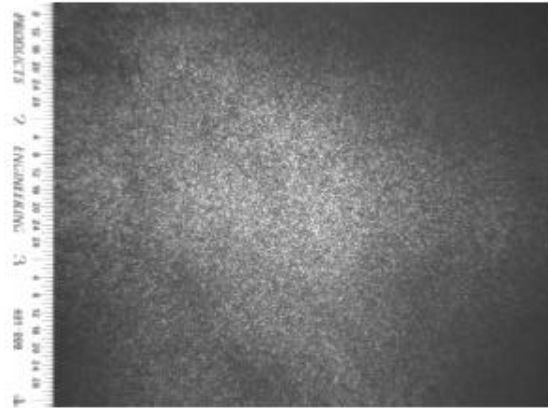


**Figure 17: High quality tripod.**

#### **4.2.3. Determine Scale and Take Reference Image.**

Once an acceptable speckle pattern has been created, the specimen is placed in the loading fixture. With the optics in the place they will be throughout the test, an image is captured to measure the scale of the image (pixels per distance); displacements cannot be determined without a scale. The scale is typically determined by placing some known dimension in the image such as a ruler or grid paper (see Figure 18). If the camera is at an angle compared to the specimen, the vertical and horizontal scales can be different. However, the camera and specimen are often aligned and only one length scale as the specimen width is needed. The scale is determined by capturing an image and measuring a known length in

number of pixels. This can be done with widely available imaging software such as ImageJ and Gimp (both are shareware).



**Figure 18: Take an image to determine scale.**

With the load at zero, a reference image is taken. Images taken throughout the test will be compared to the reference image. Comparing images to previous images taken under load introduces error and is to be avoided if possible.

#### **4.2.4. Capturing images throughout the test.**

The specimen is loaded and images are captured at desired times. For a slow moving test involving only a few images, the images can be captured manually. When timing is important or when many images will be captured, the camera is triggered with a computer program. When capturing images, motion in the room should be limited since ground vibrations can have an effect on displacements. Motion caused by people walking in the room can cause significant displacements. Fortunately, most of the undesirable motion is rigid body motion so it can be subtracted later.

##### **4.2.4.1. Performing the correlation.**

Writing a program to perform digital image correlation is complicated and tedious; however, if a program can be obtained, performing the correlation is relatively easy. Most often, an in-house code is used such as the programs used by the Lambros and Chasiotis research groups at the University of Illinois. Several digital image correlation programs are available free on the web, but these codes are often inaccurate. One commercial digital imaging program, called Vic2d, was



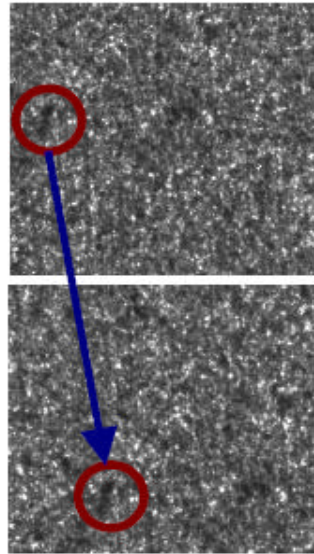
created by the Sutton research group at the University of South Carolina. Mike Sutton is renowned for his work on digital image correlation. Vic2d works very fast compared to most codes with good accuracy. The biggest drawback of Vic2d is that it is very expensive- around \$15,000, but this is a fair price compared to the cost of having a graduate student spend several years writing an in-house code. Also, since it is only an executable program, it lacks some flexibility. An area of the image is chosen for correlation; it is important to make sure that the area chosen from the reference image is also completely within the deformed image. Square regions in the reference image (called subsets) are compared to subsets in the deformed image. The subsets in the deformed image may be rotated and may have shape changes compared to the reference subsets. Several parameters are chosen that affect the accuracy and speed of the correlation. Typical correlation time for a pair of images is 10 minutes, but it can vary significantly; Correlation of one pair of images can take as little as one minute or as much as one day.

#### **4.2.4.2. Interpreting Results.**

The correlation program outputs a file containing displacements in the x and y direction, u and v, respectively for each point correlated. In some cases, the displacement fields are sufficient. In cases where strains are needed, they can be calculated from the displacement field. In most cases, strains are only accurate within about 0.1%; for this reason, digital image correlation works well for polymers. It is important to keep in mind that if the strains become very large (above 10% or so) then a code that uses large strain theory should be used. With displacement and/or strain fields calculated, an analysis of the test can begin.

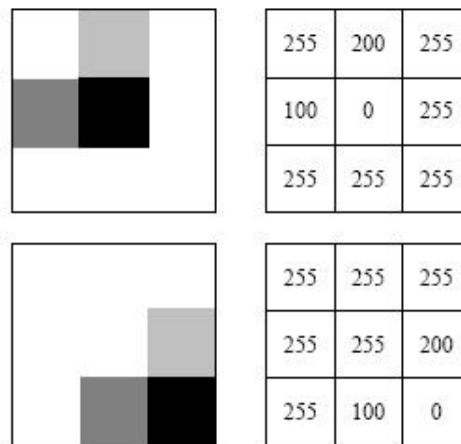
### **4.3 How DIC Works**

As described in the Procedure section, a reference image and a “deformed” image are captured. Digital image correlation can be performed visually as demonstrated by the following images:



**Figure 19: Visual demonstration of DIC.**

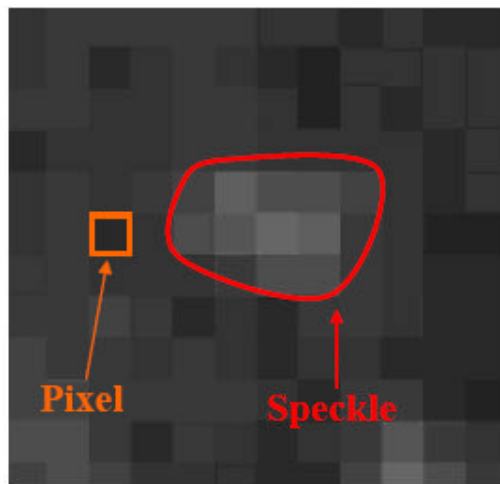
It is easy for your eye to do digital image correlation, but it is not as easy to make a computer do DIC. Each image is made up of tiny dots called pixels, and each pixel has a value corresponding to its brightness. Typically, 8-bit grayscale images are used, so each pixel has a value from 0 to 255 with 0 corresponding to pure black and 255 corresponding to pure white). Each image can be represented by a matrix of numbers. Figure 20 shows a simple 9-pixel image with its digital representation.



**Figure 20: Image in matrix format before and after motion (taken after Dally and Riley (2005)).**

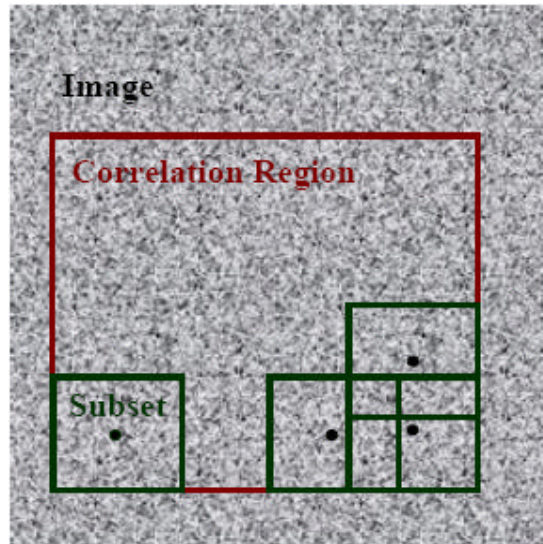
Say there is a pixel with intensity of 100 in the reference image. You can imagine looking for a pixel with a value of 100 in the deformed image. Since a typical image has over

one million pixels and since there are only 256 possible values, it is obvious that some values are going to be repeated several times; there might be hundreds of pixels that have intensity values of 100. To get something useful, we need a unique way to identify each point; to do this, we introduce regions of points called subsets. A subset is a square group of pixels in the reference image that we call one point. So each point represents a subset, not a pixel. To assign an intensity value to a subset, we use the average intensity of its pixels. A typical subset size is around 41 x 41 pixels, so you can see that the chance of two subsets having the same average intensity is very slim. A hypothetical 12 x 12 subset is shown in Figure 21. It is a smaller subset than would typically be used, but it is better to illustrate the concept.



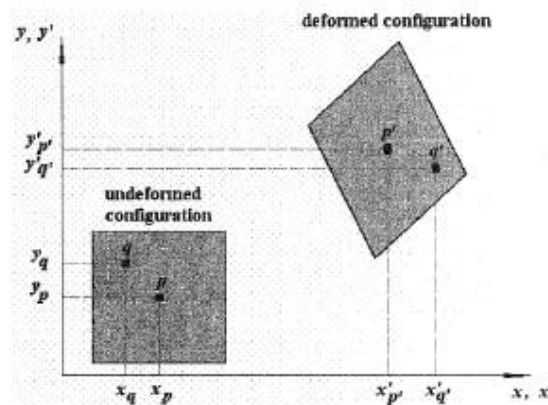
**Figure 21: A 12 x 12 subset. The relationship between a subset, pixel, and speckle.**

To correlate a pair of images, the correlation region in the reference image is divided into subsets (See Figure 22).



**Figure 22: Relationship between regions in DIC terminology. One point represents a subset. Subsets usually overlap to produce a fine mesh.**

With the subsets defined, regions are then sought in the deformed image that agree best with the appearance of the undeformed subset. The deformed subsets do not have to be squares; they can be rotated and stretched (See Figure 23).



**Figure 23: Subsets in the reference and deformed image (Taken from Abanto-Bueno (2004)).**

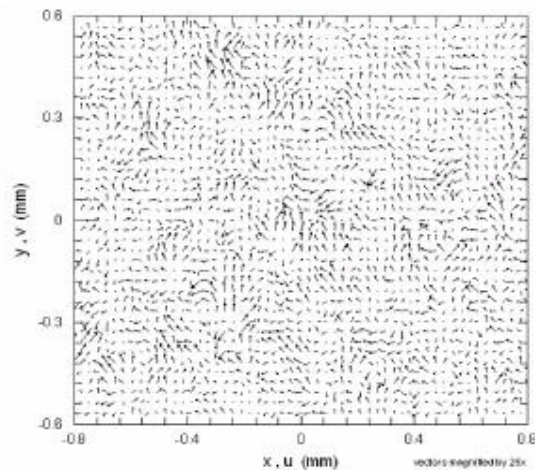
To complicate the issue, the pixel intensities are never constant; the values fluctuate even when nothing is moved. This is mainly due to the fact that, a light bulb does not emit light at a constant intensity; it flickers like a candle (but obviously not to such a large extent). Even though the individual pixel values fluctuate, the subset value does not change significantly.

## 4.4. Baseline Tests

Before using any digital image correlation code, it is necessary to verify that it works properly. This is done with a series of baseline tests. In baseline tests, a known deformation is created between images; then, the DIC code is run with these two images to determine whether the code works properly. Either theoretical deformations created by an image processing program or actual deformations imposed on the specimen can be used for baseline tests. Each method of applying deformations has advantages and disadvantages. Image processing programs can create deformations easily, can create very complex deformations, and can create known deformations accurately. Therefore, theoretical deformations are good for comparing DIC programs and for examining whether a code is working properly. Experimental deformations capture any errors or variation in the entire system; therefore, they are good for finding the capabilities of the system and for determining sources of experimental error like light variation and vibration.

### 4.4.1. Self-Correlation Test.

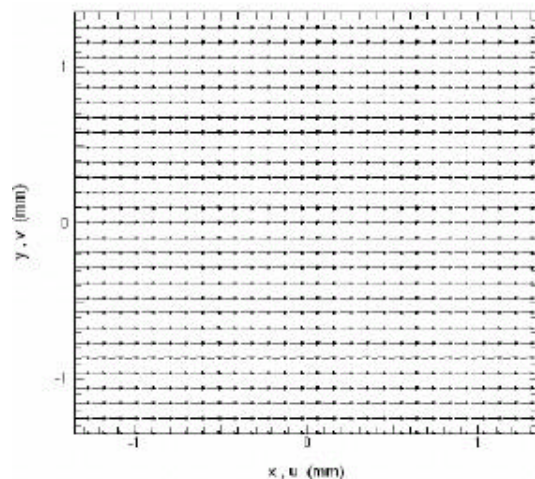
To verify that a code works, it should first be checked that the code accurately predicts no motion when the reference image is correlated with itself. A successful self-correlation test yields results such as those shown below.



**Figure 24: Vector plot of a self-correlation test (vectors magnified by 25x). Taken from Patel, 2003.**

### 4.4.2. Rigid Body Translation

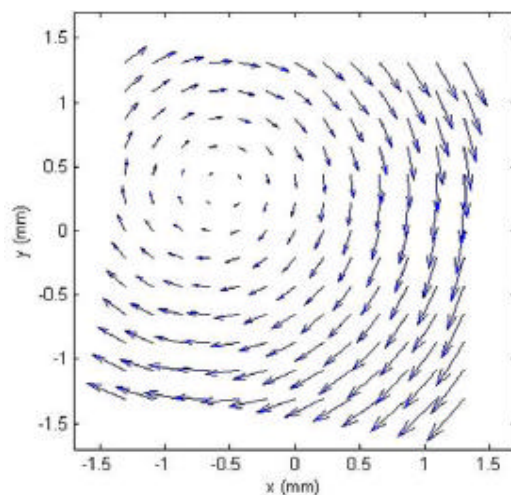
In this test, the second image is simply moved (without rotation) compared to the reference image. An acceptable code can predict translation within 1/20th of a pixel (even better if theoretical displacements are used). You can think of a pixel as a measurement like meters. Regardless of how large the scale in the image is (meters or nanometers), the accuracy of the code will be the same in units of pixels. The results of such a test are shown below.



**Figure 25: Vector plot of a rigid translation test of  $100\mu\text{m}$  to the right. Taken from Patel, 2003.**

#### 4.4.3. Rigid Body Rotation

In this test, the second image is rotated (without translation) compared to the reference image. The rotation does not have to be about the center of the correlation region, and if the DIC results are plotted, it is easy to find the center of rotation (See Figure 25).



**Figure 26: Vector plot of a rigid body rotation test (Rotation of 1 degree at a scale of 10).**

#### **4.4.4. More Complex Verification Tests**

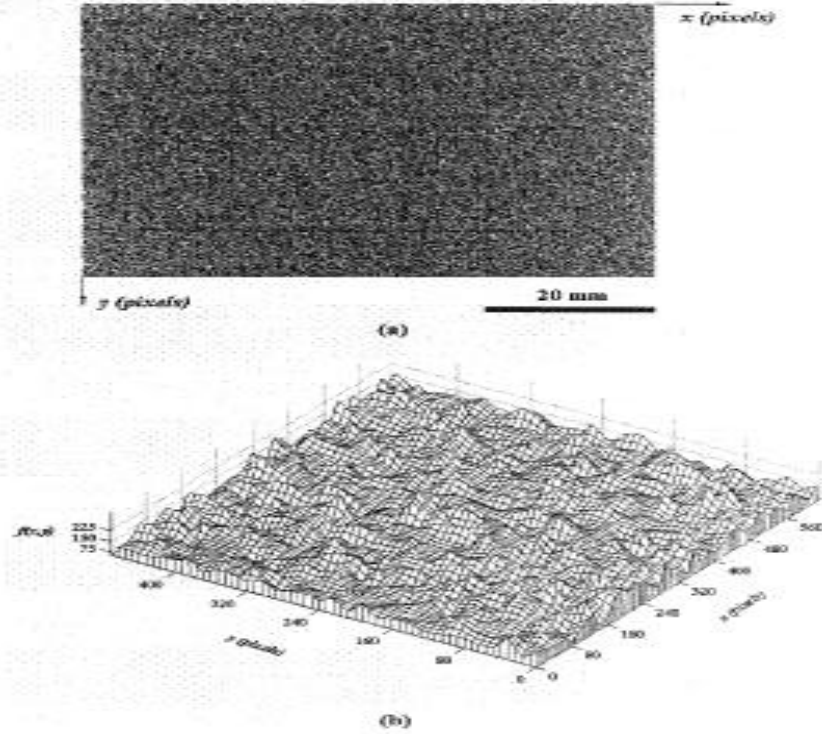
To examine the DIC code's behavior under different circumstances, more complex tests can be run such as combined tension and rotation, tests with strains, and tests with strain gradients. For example, to check a code with second order terms incorporated, tests with large strains must be used.

### **4.5. Correlation Algorithms**

#### **4.5.1. Interpolation**

Since pixel intensity values in each image are discrete points (recall earlier that images are represented as a matrix of numbers), interpolation is used to obtain values between pixel locations. Two-dimensional interpolation schemes that are typically used are bicubic interpolation, bicubic spline interpolation (See Dally and Riley (2005)), and quintic interpolation. If pixel intensity is plotted as height in a three-dimensional plot, interpolation turns the discrete heights into a smooth surface such as that shown in Figure 16b. These intermediate values allow displacement values to be measured with sub-pixel accuracy. For reasonable measurements, sub-pixel accuracy is a necessity.





**Figure 27: (a) A typical DIC image and (b) the representation of the image in a three dimensional plot of Pixel intensities (Taken from Abanto-Bueno (2004)).**

#### 4.5.2. Mapping Displacements

The location of a subset in the deformed image  $(\tilde{x}, \tilde{y})$  can be expressed in terms of its initial location by,

$$\begin{aligned}\tilde{x} &= x + u(x, y) \\ \tilde{y} &= y + v(x, y)\end{aligned}\tag{10}$$

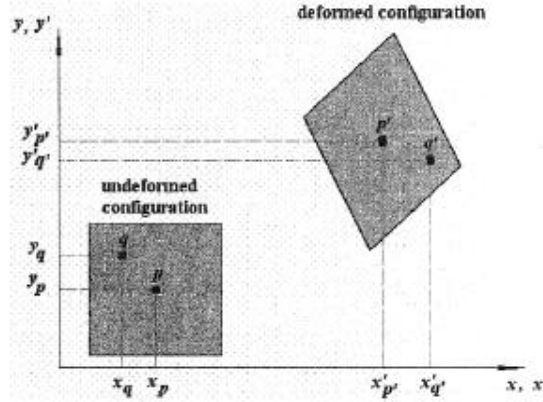
where  $(x, y)$  is the subset's original position and  $u$  and  $v$  are displacement fields in the  $x$  and  $y$  directions, respectively. Using a second order Taylor series expansion for  $u$  and  $v$  about the point  $(x_0, y_0)$ , (10) can be expressed as,



$$\tilde{x} = x_0 + u_0 + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \Delta y^2 + \frac{\partial^2 u}{\partial x \partial y} \Delta x \Delta y$$

$$\tilde{y} = y_0 + v_0 + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 v}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 v}{\partial x^2} \Delta y^2 + \frac{\partial^2 v}{\partial x \partial y} \Delta x \Delta y$$

where  $\Delta x = x - x_0$  and  $\Delta y = y - y_0$  (See Dally and Riley (1995) pg. 587).



**Figure 28: Subsets in the reference and deformed image (Taken from Abanto-Bueno (2004)).**

In situations with large strains and/or large strain gradients, the contributions from second order terms can be significant, but the first order expansion is adequate for most cases:

$$\begin{aligned} \tilde{x} &= x_0 + u_0 + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y \\ \tilde{y} &= y_0 + v_0 + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y \end{aligned} \quad (11)$$

The last two terms in each equation in (11) are called the displacement gradients; they are related to strains  $\epsilon_x$ ,  $\epsilon_y$ ,  $\gamma_{xy}$  and rotation,  $\bar{\omega}_{xy}$ , through the following relations:

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \bar{\omega}_{xy} &= \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \end{aligned}$$

Although strains and rotations can be calculated from displacement gradients, the main purpose of displacement gradients is to assist in giving more accurate displacement measurements by improving correlation. Usually, strain fields are calculated by differentiating the displacement field obtained through DIC. In (11), there are six parameters

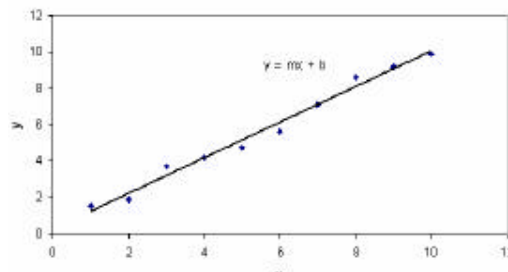
that can be adjusted for each subset to find the deformed subset that best matches the reference subset. The deformed subset location and shape will be adjusted until the best fit is found.

### 4.5.3. Correlation Coefficient

To determine how good of a match the deformed subset is to a reference subset, a value called the correlation coefficient is used. Different correlation coefficients are used by different programs. The most common is the least squares correlation coefficient,  $C$ , which is

$$C = \frac{\sum_{s_p=s} [g(s_p) - h(s_p, P)]^2}{\sum_{s_p=s} g^2(s_p)} \quad (12)$$

Where  $g$  represents the intensities in the reference subset and  $h$  represents the intensities in the deformed subset (both of these being after interpolation). A more detailed explanation of the correlation coefficient can be found in Dally and Riley (2005) on page 588. To summarize the meaning of (12): At each point in the subset, the reference and deformed intensities are subtracted and divided by the reference intensity. This gives a measurement of error at that point. To determine the total amount of error with the subset, the error at each point is summed. A correlation coefficient of zero means the reference and deformed subset are perfectly correlated. Any non-perfect correlation will have a correlation coefficient greater than zero. To find the deformed subset that corresponds to a reference subset, guesses are made for the six parameters in (11). Using these guesses, the correlation coefficient is calculated by comparing intensities in the reference subset to intensities in this moved/deformed/rotated subset. Using minimization techniques such as the Newton-Raphson method, the parameters are adjusted until the minimum correlation coefficient is found. This procedure is repeated for each subset being correlated (usually thousands of subsets), so obviously, correlation of a pair of images can take a significant amount of time. This minimization is similar to fitting a line to points (see Figure 29)



**Figure 29: Example of regression with 2 parameters.**

The Newton-Raphson method is very fast, but it requires accurate initial guesses for all six parameters. Since displacements,  $u$  and  $v$ , have a much larger effect on the correlation than the displacement gradients, the algorithm developed by Sutton et al. (1986) is often used. In this algorithm, a search is performed with displacement gradients set to zero (square regions are sought in the deformed image). In this part of the search, the search region is divided into a number of intervals, and the best-correlating interval is chosen. The chosen interval is then again divided into intervals and the procedure is repeated until sufficient accuracy is obtained. This part of the correlation program is called the coarsefine algorithm. The displacements found from the coarsefine part are then differentiated to approximate displacement gradients. These displacements and displacement gradients are then used as initial guesses in the Newton-Raphson part of the program. All six parameters are then adjusted to obtain optimal correlation.

#### **4.6. CCD camera and its working**

A CCD camera uses a small, rectangular piece of silicon rather than a piece of film to receive incoming light. This is a special piece of silicon called a charge-coupled device (CCD). This silicon wafer is a solid-state electronic component which has been micro-manufactured and segmented into an array of individual light-sensitive cells called "photosites." Each photosite is one element of the whole picture that is formed, thus it is called a picture element, or "pixel." The more common CCDs found in camcorders and other retail devices have a pixel array that is a few hundred photosites high by a few hundred photosites wide (e.g., 500x300, or 320x200), yielding tens of thousands of pixels. Since most CCDs are only about 1/4" or 1/3" square, each of the many thousands of pixels are only about 10 millionths of a meter (about 4 ten-thousandths of an inch) wide!

The CCD photosites accomplish their task of sensing incoming light through the photoelectric effect, which is a characterization of the action of certain materials to release an electron when hit with a photon of light. The electrons emitted within the CCD are fenced within nonconductive boundaries, so that they remain within the area of the photon strike. As long as light is allowed to impinge on a photosite, electrons will accumulate in that pixel. When the source of light is extinguished (e.g., the shutter is closed), simple electronic circuitry and a microprocessor or computer are used to unload the CCD array, count the electrons in each pixel, and process the resulting data into an image on a video monitor or other output media.

The difference between a CCD camcorder and an astronomical CCD camera is that a camcorder must take and display 60 sequential images per second to replicate motion and color from daylight scenes, while an astronomical camera is used to take long-duration exposures (from many seconds up to a few hours long) of very dim starlight to replicate an apparently motionless object. Camcorders make color images by merging the data taken simultaneously by groups of adjacent pixels covered by red, green, and blue filters.

Astronomical CCD cameras also can make color images, but these are made by post-exposure processing and merging of three separate exposures of an object made through red, green, and blue filters.

Finally, there are two characteristics of CCDs which are factors that must be considered in making a final astronomical image: 1) since they are electronic components CCDs are sensitive to heat within the camera as well as light from the object of interest and 2) the individual photosites in the CCD array may vary significantly in their sensitivity to both heat and light. First, this means that the electrons generated by heat rather than by light need to be subtracted from the final tally of electrons in each pixel so that a truer image can be rendered. This is called "dark subtraction." Second, the variance in electron depth across the CCD array due to inherent differences among the pixels needs to be leveled by dividing each pixel value by the array's average pixel value. This is called "flat fielding."

Dark subtraction is accomplished by subtracting a "dark frame" from the object image (called a "light frame"). The dark frame is created by taking an exposure while the CCD is maintained in complete darkness. This exposure must be the same duration as the light frame and be made with the CCD at the same temperature as during the light frame so that electrons generated during the dark frame replicate the heat-generated electrons present in the light frame.

Flat field images are made by taking a picture of an evenly illuminated scene, such as the sky at dusk or the flat gray interior of an observatory dome. The resultant image shows the inherent variances in pixel value across the CCD array due to differences in photosite sensitivity or to dust specks or vignetting in the optical system. Image processing softwares use mathematical algorithms to divide all pixel values in the flat field image by the array's average pixel value. The results are then correlated, pixel by pixel, against the array values in the light image to produce a better representation of the object of interest.

In the final stages of image production, the light frame (object image) is adjusted by first having an appropriate dark frame subtracted and then having an appropriate flat field

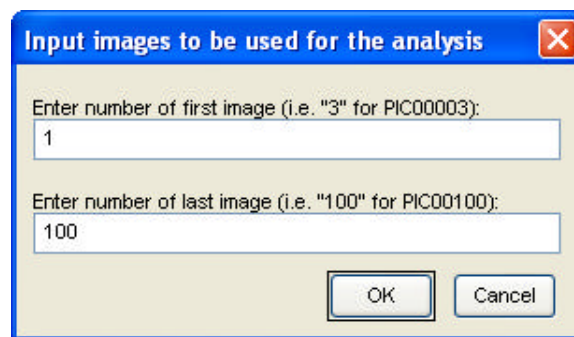
divided into the image. This process is called image calibration and results in a truer, less noisy image.

**IMPLEMENTATION OF DIGITAL IMAGE CORRELATION****5.1. Digital Image Tracking Through Matlab v7.0**

This implementation helps to perform a simple and fast analysis of images. First of all image format and the naming of files should be considered. The preferred image format is \*.tif and can be compressed with the packbits compression. JPEG or other image formats as well as MPEG video compression formats will not provide with sub pixel resolution since the images are processed to save as much space as possible. The script we use to create a list of images to process (filelist\_generator.m) is kind of limited to a certain format. Other softwares like irfanview ([www.irfanview.com](http://www.irfanview.com)) can be used to batch process a huge number of files for image format change and naming of files.

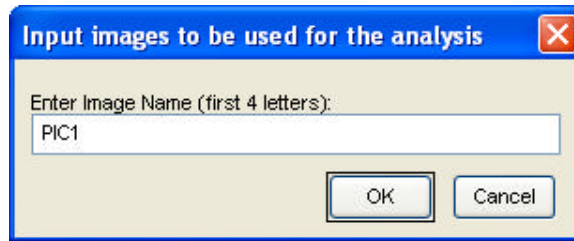
**5.2. Steps:****5.2.1. Filename list generation with filelist\_generator.m**

On running 'filelist\_generator' ; at the command line of Matlab. The following window should appear:



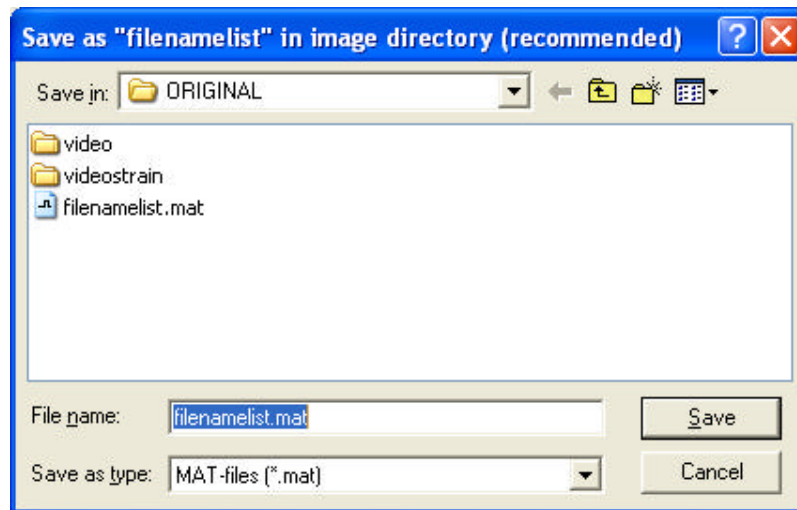
**Figure 30: Input of first and last image to create an image list with filelist\_generator.m**

The numbers will be the number at the end of each filename. After depositing these numbers in the dialog the next window will ask for the first 4 letters of the filenames.



**Figure 31: Input for the first 4 letters in `filelist_generator.m`**

The next step is to save the file name list into the folder with the images to process.

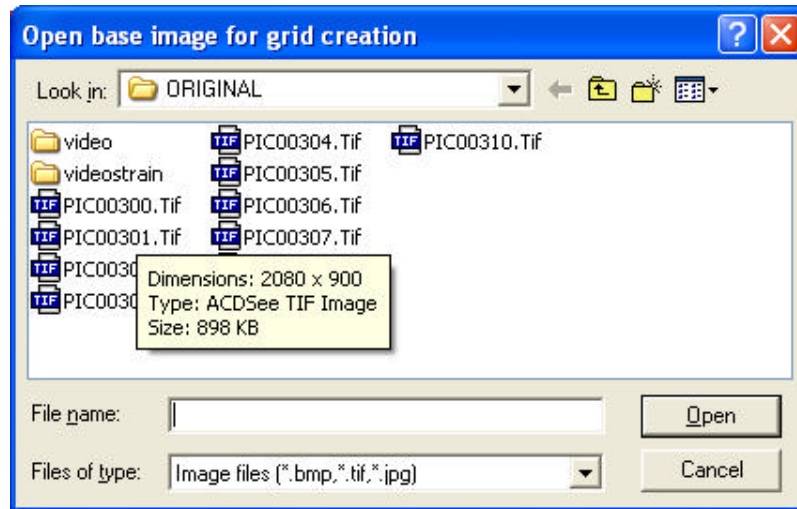


**Figure 32: Dialog to save the file name list into the folder with the images to analyze.**

### 5.2.2. Grid generation with `grid_generator.m` for correlation:

The user can always generate his own marker positions. Therefore the marker position in pixel has to be saved as a text based format where the  $x$ -position is saved as 'grid\_x.dat' and the  $y$ -position saved as 'grid\_y.dat'.

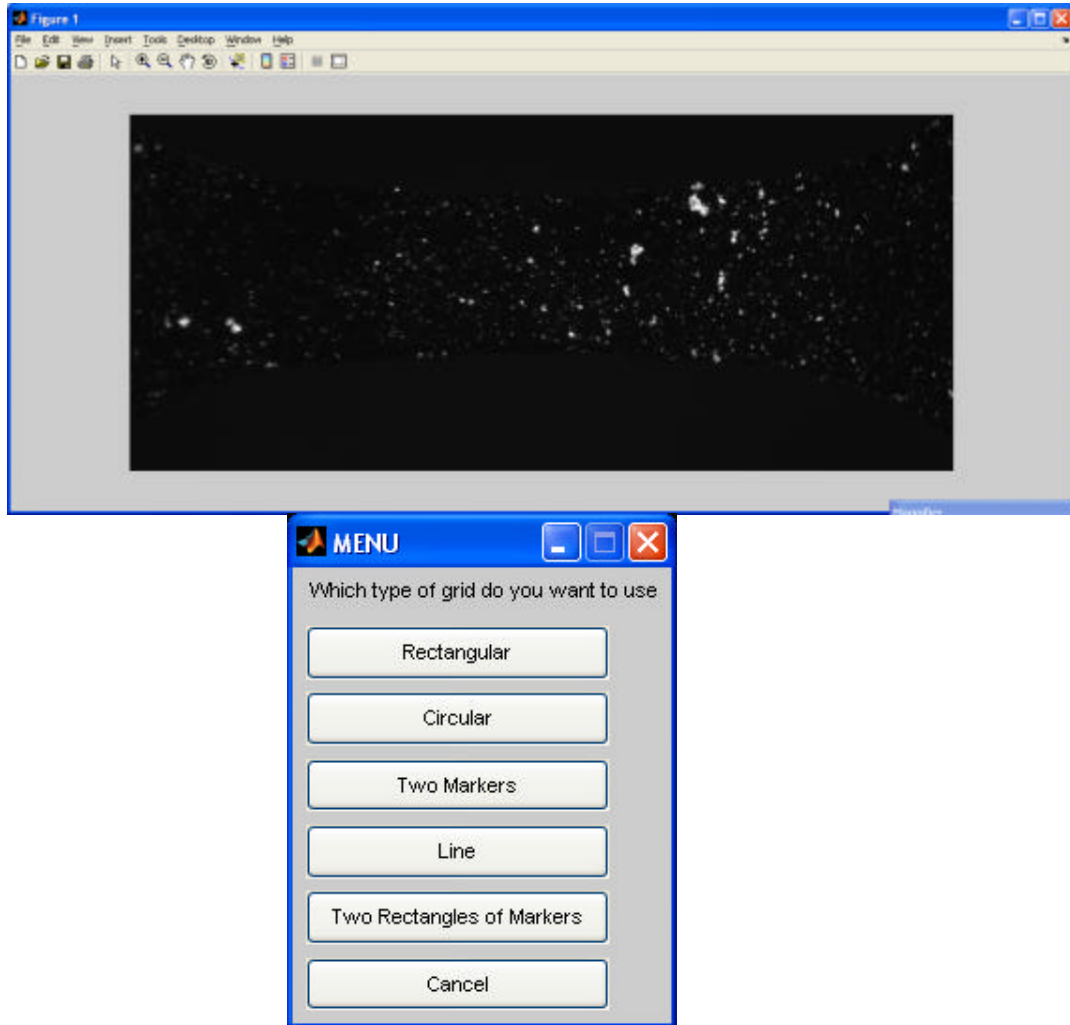
To start 'grid\_generator;' has to be typed and 'ENTER' is pressed at the command line of Matlab. The following window should appear:



**Figure 33: Dialog to open the first (base) image to generate a grid**

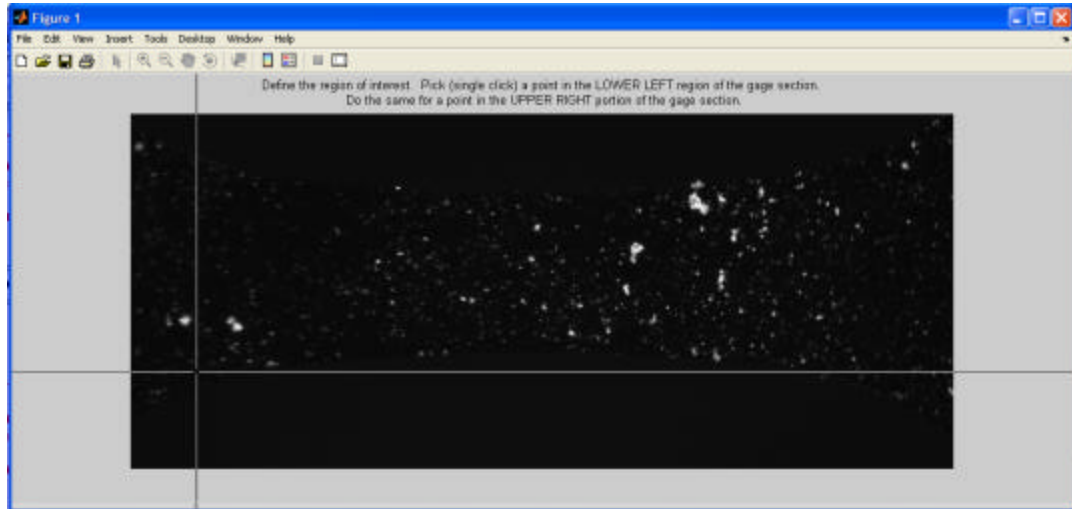
In this dialog the first (base) image can be selected in which the grid can be created. After selecting this base image, the image will be opened and a new dialog pops up to ask you what kind of grid should be used.





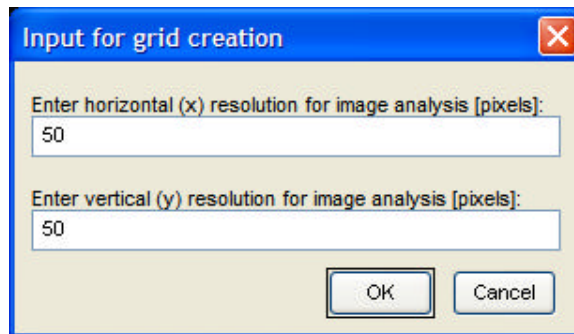
**Figure 34: The opened base image and the menu to select the grid type.**

The different types are a rectangular or circular grid, two markers, a line or two rectangles of markers. If you choose a rectangular grid type, the pointer will change from an arrow to a horizontal and a vertical line which will help you finding the right position. The idea is to click on the two diagonal positions which will define the outer dimensions of a box containing the grid.



**Figure 35: The horizontal and vertical lines allow an accurate positioning of the grid.**

The selected box will be shown in the image and a dialog will pop up to ask for the input of a raster point distance in x and y direction.



**Figure 36: Horizontal (x-direction) and vertical (y-direction) grid resolution with a default resolution of 50 pixel distance between raster points.**

The code will now generate the chosen grid and will plot it on top of the sample image. The last dialogue will ask if user wants to use the generated grid and save the grid\_x.dat and grid\_y.dat to be processed, if user wants to try again or if user wants to choose another grid type.



**Figure 37: The last menu will allow to accept the grid (which will be saved), try again or choose another grid type.**

### 5.2.3. Run correlation with `automate_image.m`:

The automation function is the central function and processes all markers and images. Therefore the 'Current directory' in Matlab has to be the folder where `automate_image.m` finds the `filenamelist.mat`, `grid_x.dat` and `grid_y.dat` as well as the images specified in 'filenamelist.mat'. '`automate_image;`' just has to be typed and 'ENTER' is pressed at the command line of Matlab.

At first, `automate_image.m` will open the first image in the `filenamelist.mat` and plot the grid as green crosses on top. The next step will need some time since all markers in that image have to be processed for the first image. After correlating image one and two the new raster positions will be plotted as red crosses. On top of the image and the green crosses. The next dialog will ask if user wants to continue with this correlation or cancel. If continue is pressed, '`automate_image.m`' will process all images in the '`filenamelist.mat`'. The time it will take to process all images will be plotted on the figure but can easily be estimated by knowing the raster point processing speed.

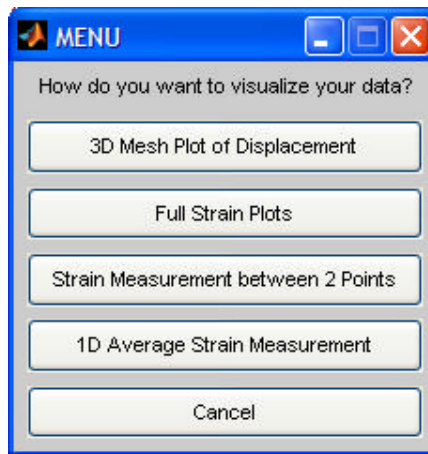
Depending on the number of images and markers being tracked, this process can take between seconds and days. For 100 images and 200 markers a decent computer should need 200 seconds. To get a better resolution the jobs can be run overnight (e.g. 6000 markers in 1000 images) with higher resolutions.

When all images are processed, `automate_image` will write the files `validx.mat`, `validy.mat`, `validx.txt` and `validy.txt`. The text files are meant to store the result in a format which can be accessed by other programs also in the future.

#### 5.2.4. Analyze the displacement with displacement.m:

As last part, the post processing is the most interesting and awarding step since here user actually can analyze the collected displacement data. The displacement.m function is a small collection of functions which allows to review the displacement field, calculate the strain or delete markers which were not correlated or tracked very well.

To start, 'displacement' is to be typed in the command window. A window will pop up asking for the validx.dat file which contains the x-displacement of all markers in all images, followed by a dialog for the validy.dat containing the y data. After 'displacement.m' has loaded both files, a new window pops up which allows to choose between different options.

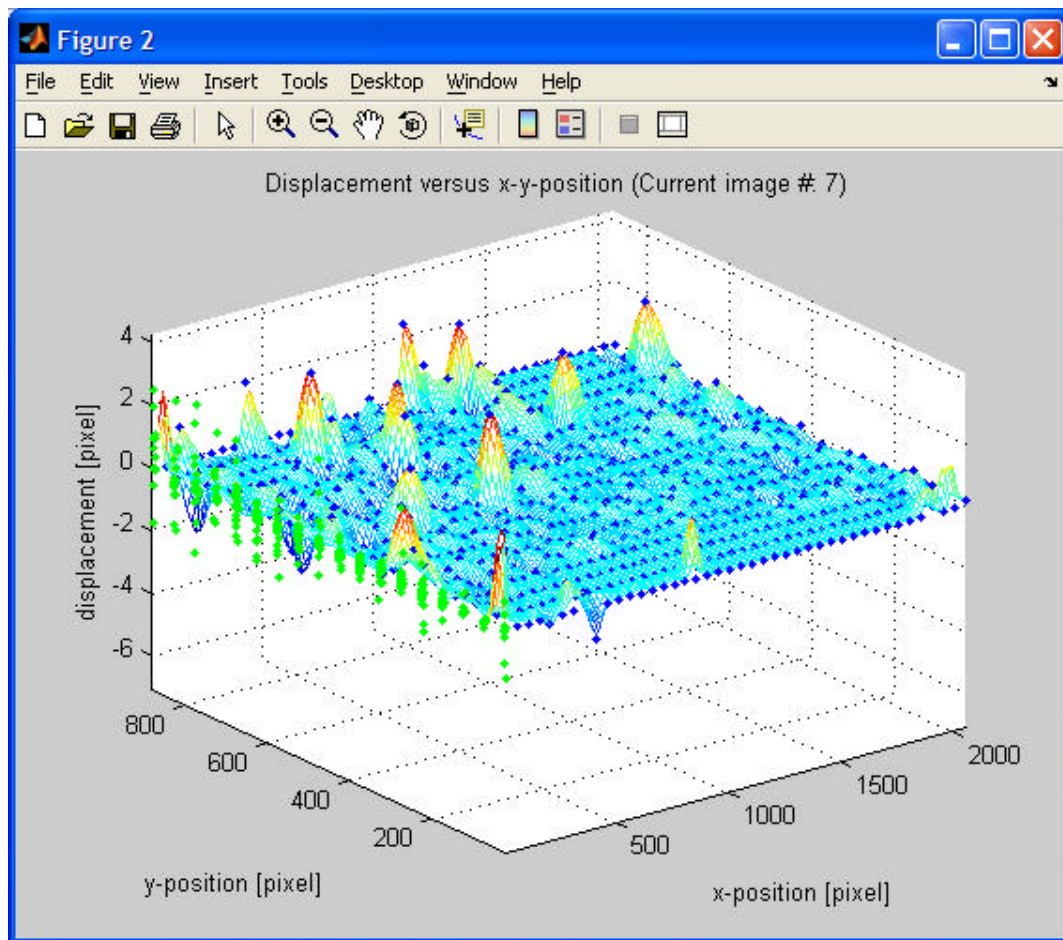


**Figure 38: The displacement.m function allows cleaning up the data set selecting parts of it plot the displacement or measure the strain in x- and y-direction.**

For a typical analysis you always have to delete some of the markers which did not all too well during the correlation or peak fitting step. This can happen e.g. due to marker movement during the test, changing light conditions or in case of the correlation technique due to the fact that the sample surface did not provide enough characteristics.

We start with clicking on the '3D Mesh Plot of Displacement' button which will bring up a new window and a dialog asking if you want to create a video. Clicking on 'yes' will create a new folder called video and the 3D displacement plot of each image will be saved as \*.jpg. The click on the no button will just start the 3D displacement plot. This part of

the 'displacement.m' allows you to watch displacement (z-axis) versus location (x- and y axis) for all images. To get a better 3-dimensional understanding all markers are projected as green dots on the plane normal to the y axis. The Image number will be shown in the plot. It has to be noted that the orientation of the strain depends on the orientation of the image during the correlation process. The xaxis in the plot is the horizontal direction in the image and the y-direction the perpendicular direction. The plotted displacement on the z-axis is always the x-displacement of the data contained in validx.mat and validy.mat.

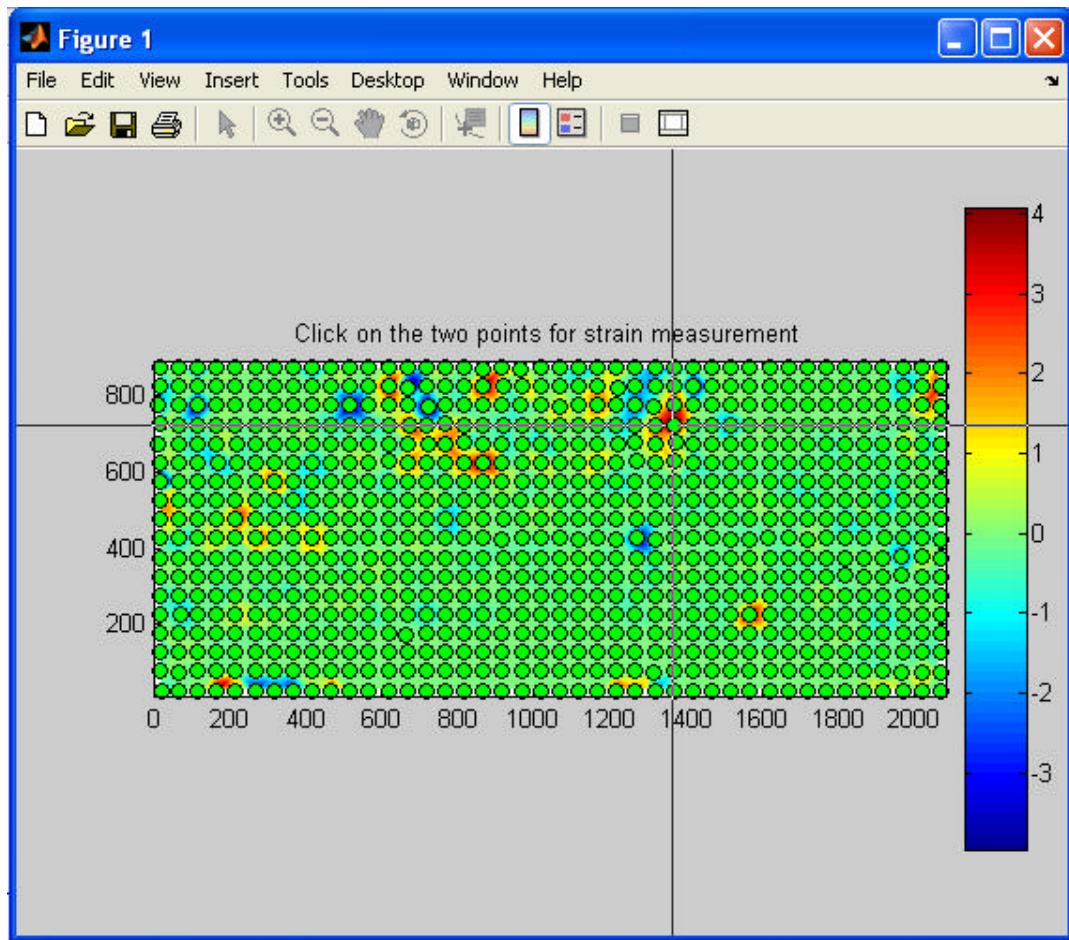


**Figure 39: 3D Displacement versus x- and y-position. The orientation of the x - axis is the horizontal in the analyzed image and the y-axis is the vertical.**

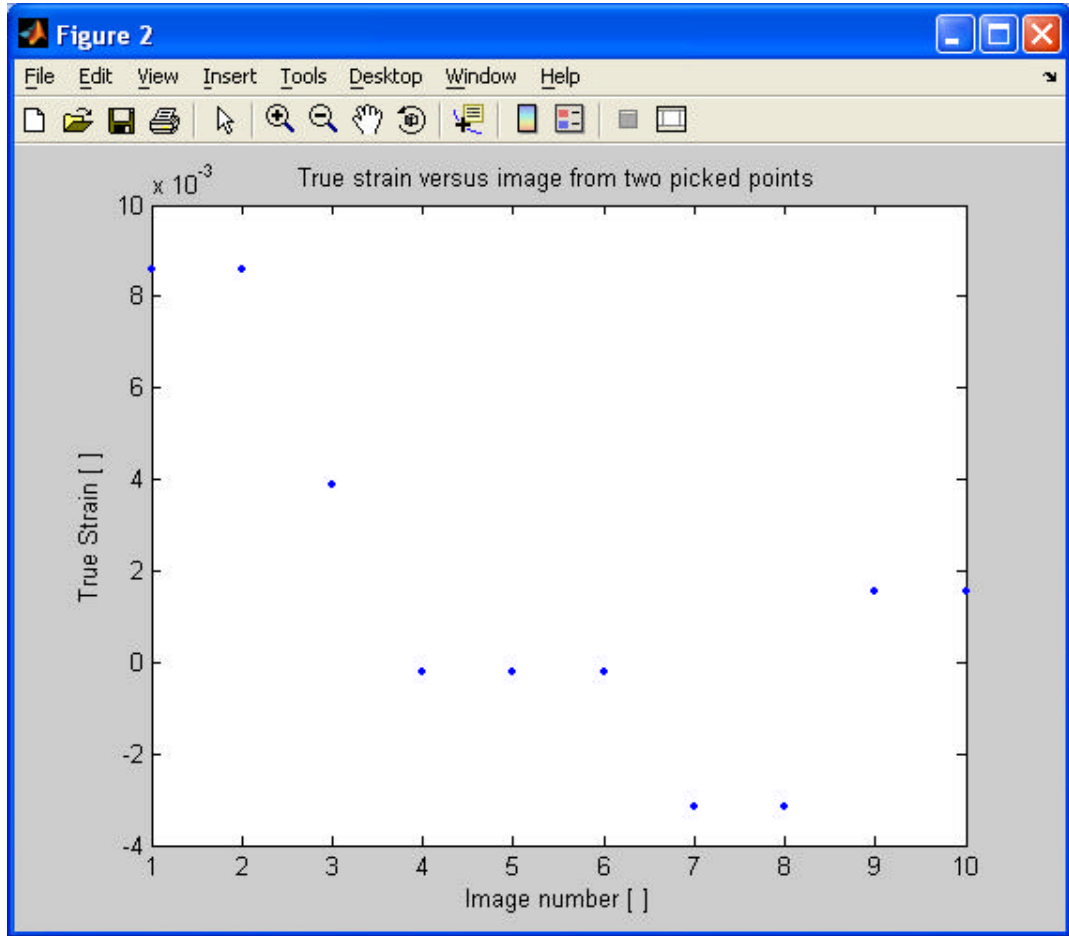
The way for Strain measurement is straight forward by just clicking on 'Strain Measurement between 2 Points' which will let user choose two points

After clicking on ‘Strain Measurement between 2 Points’ user will have to choose two points. The function will find the closest two points to where you clicked and plot the strain versus image number.

If user is not happy with the result user can still choose two other markers by clicking ‘Yes’ in the next dialog.

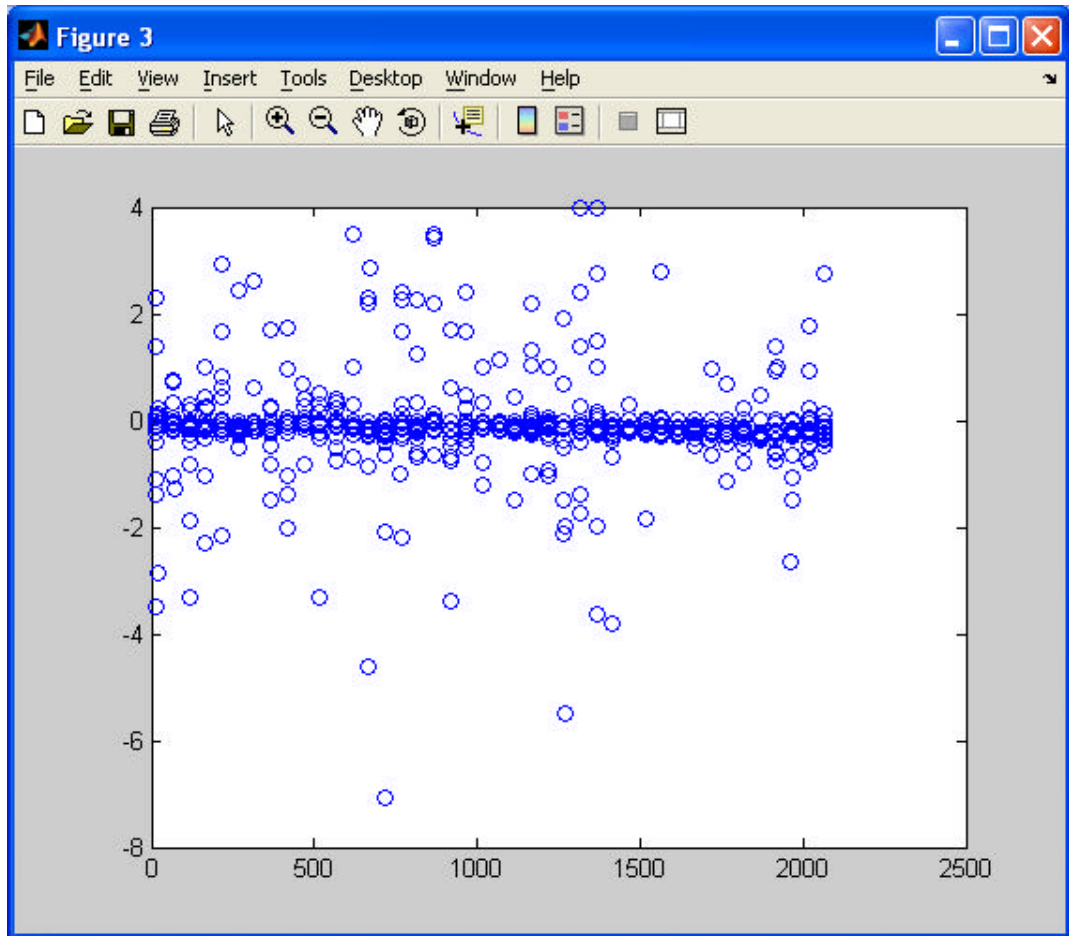


**Figure 40: From this window two points can be picked which will be used to measure the strain**



**Figure 41: Strain versus image number**

After clicking on the **'1D Average Strain Measurement'** button, the x-displacement versus x-direction will be plotted for each image and then fitted by a linear function. The slope is the true strain which will be plotted versus the image number after all images are processed.

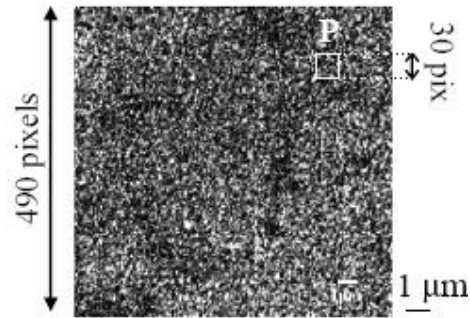


**Figure 42: The slope of the linear fit of the displacement versus position allows to plot the true strain versus image number.**



Lets see some more results of the above stated steps for Digital Image Correlation on another set of images. The images are of a sample of Ti-6Al-4V Titanium alloy.

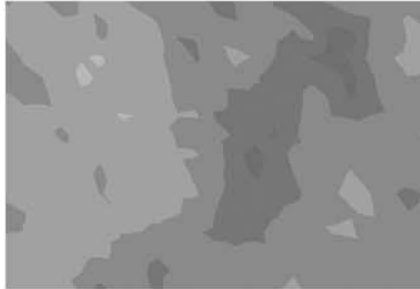
Below is the listing of images taken for analysis:



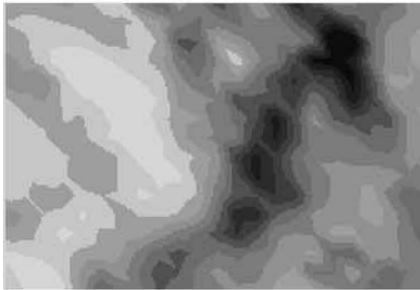
**Figure 43.A 30x30 pixel dimension of the above image has been taken for further magnification for DIC process.**



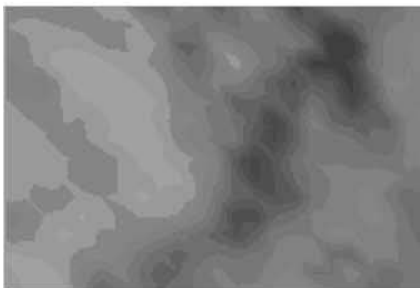
stress applied=500MPa



stress applied=940MPa



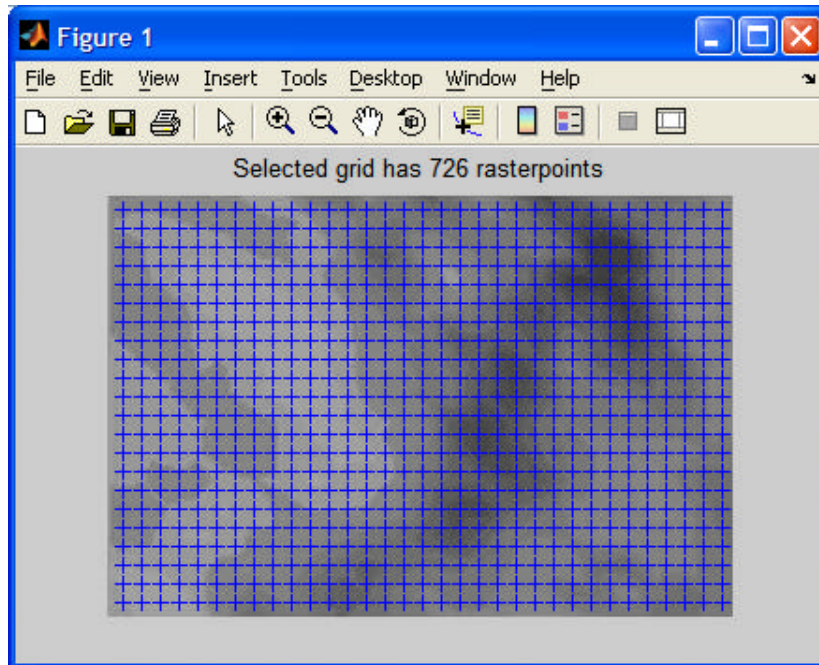
stress applied=945MPa



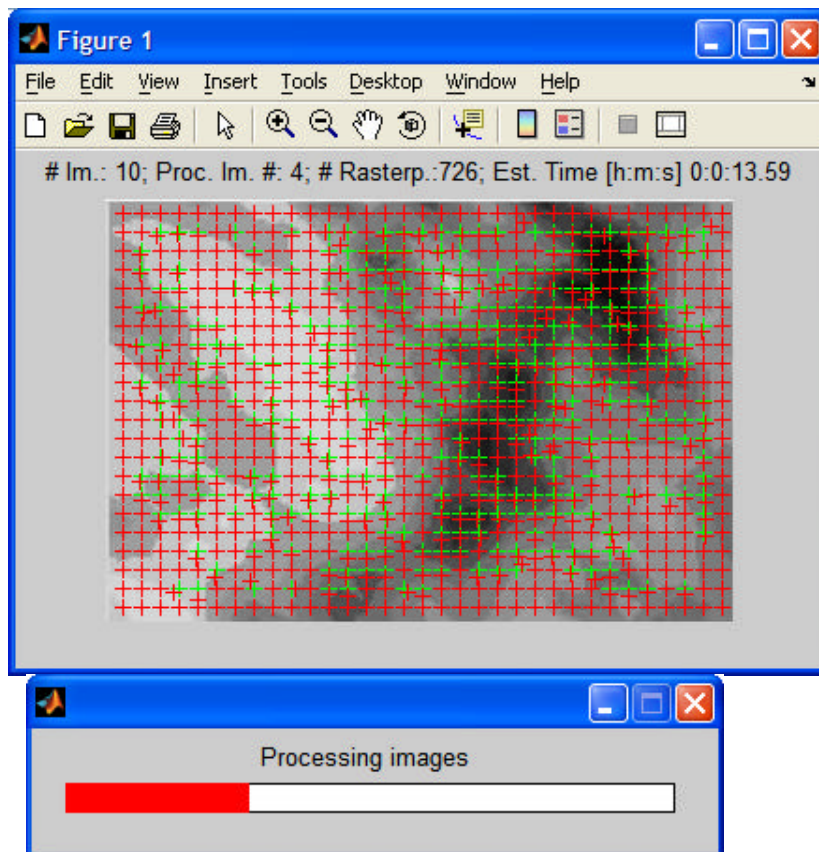
after unloading:residual strain

**Figure 44. In situ SEM in-plane strain measurements on Ti-6Al-4V material for different applied stress values.**

Lets see some more results of the above stated steps for Digital Image Correlation on another set of images of Ti-6Al-4V Titanium alloy taken.

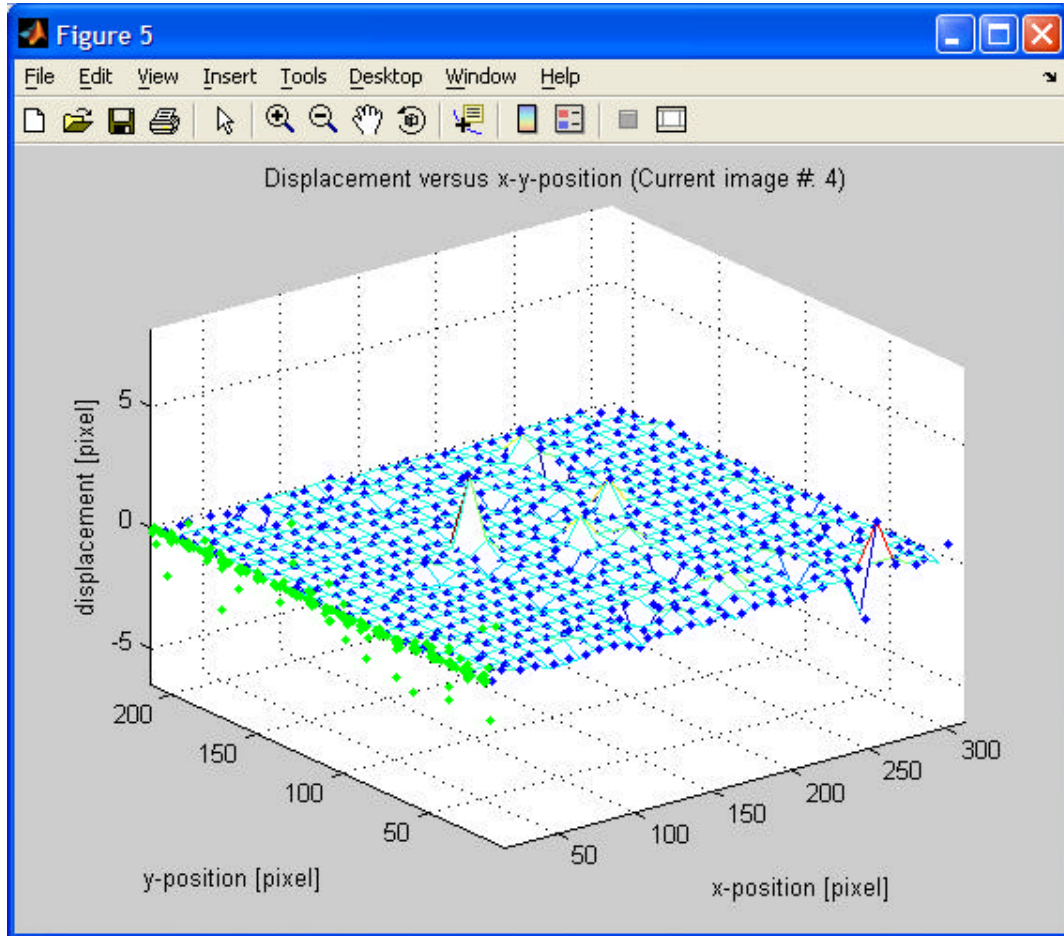


**Figure 45: Horizontal (x-direction) and vertical (y-direction) grid resolution with a default resolution of 10 pixel distance between raster points.**

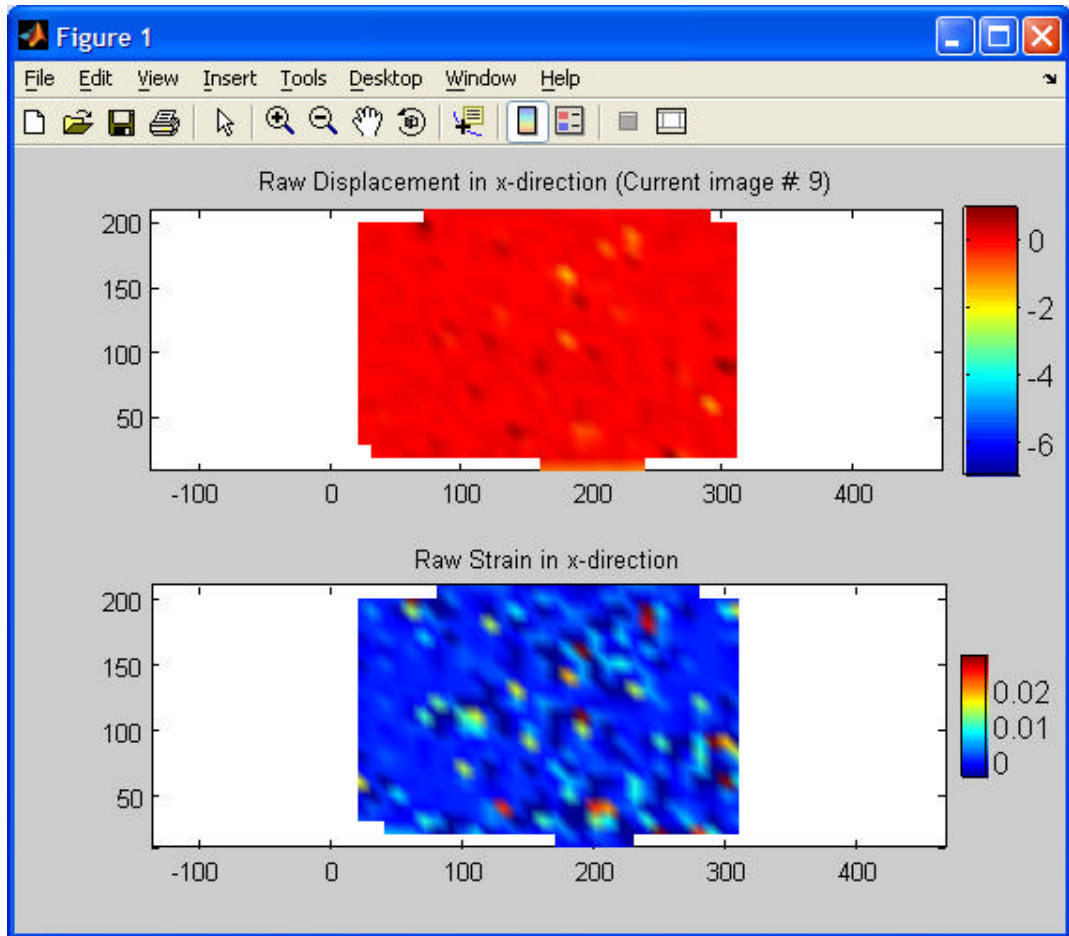


**Fig 46: Images being processed**

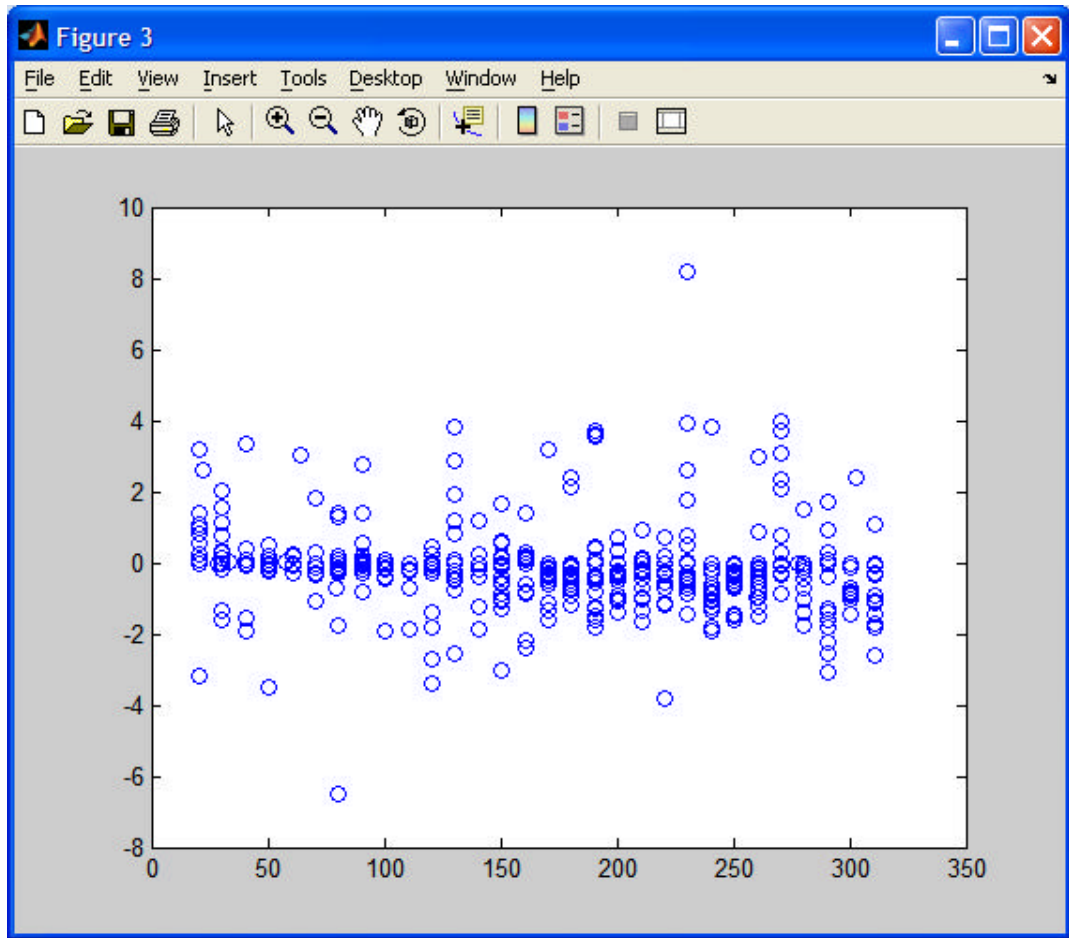
Automate\_image.m function is processing the list of files specified earlier in filenamelist.mat. the function opens the first image and plot the grid as green crosses on top. The next step will need some time since all markers in that image have to be processed for the first image. After correlating image one and two the new raster positions will be plotted as red crosses on top of the image and the green crosses.



**Figure 47: 3D Mesh Plot of Displacement**



**Figure 48: Full Strain Plots**



**Figure 49: 1D Average Strain Measurement**

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## Strain graph on Images

### Image# 1 After Removal of Load

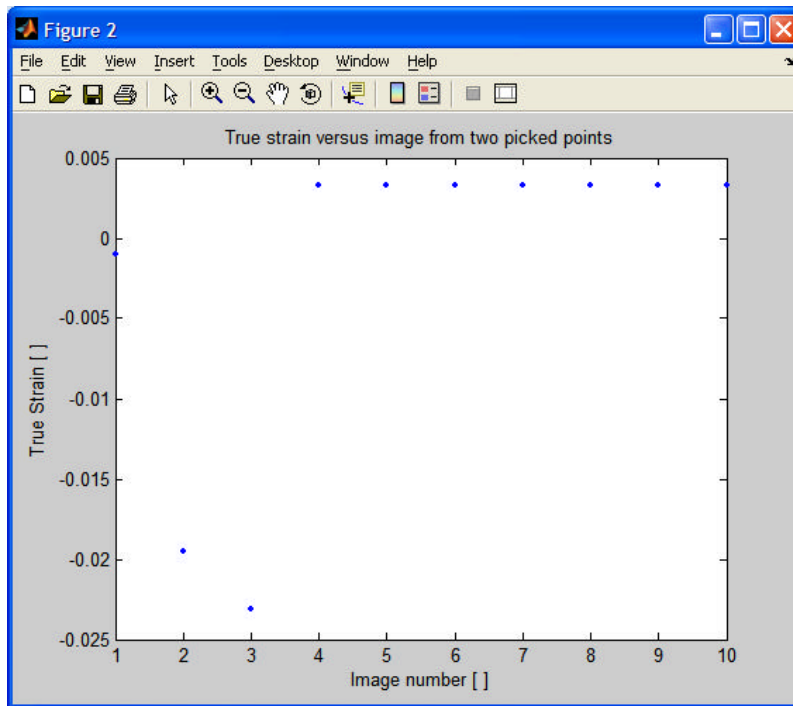
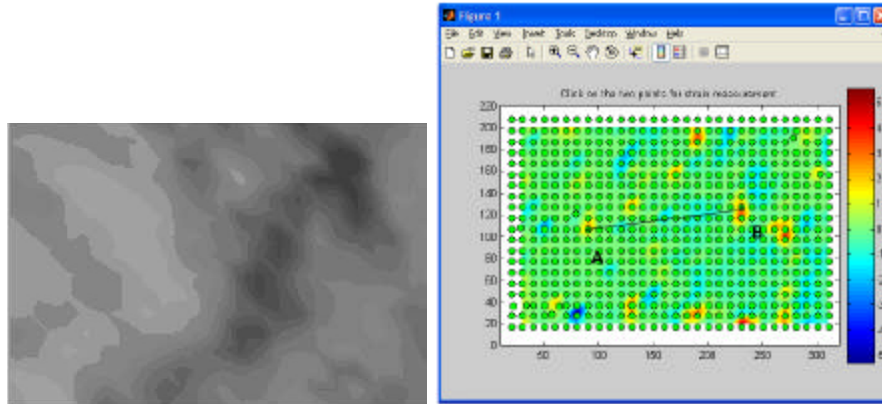
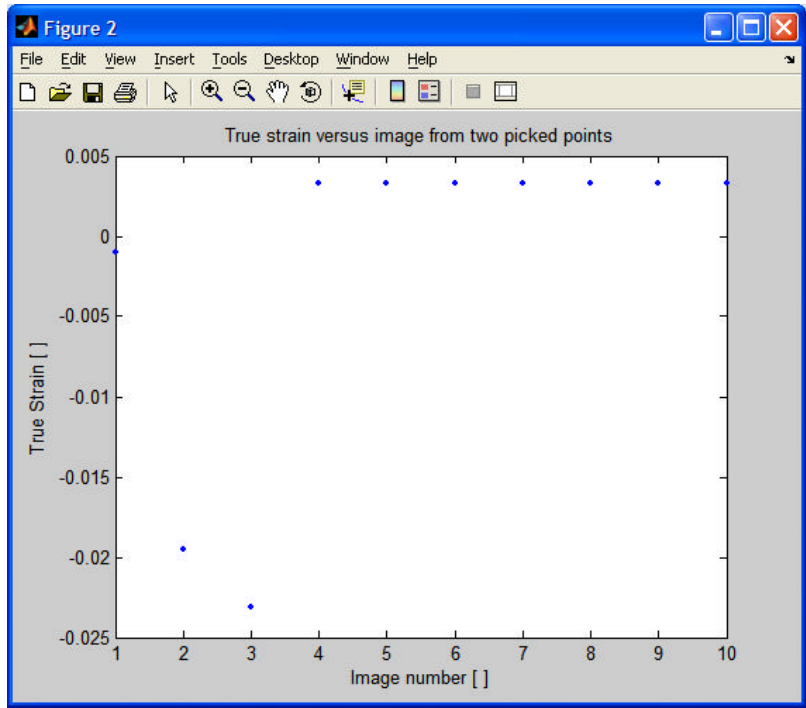
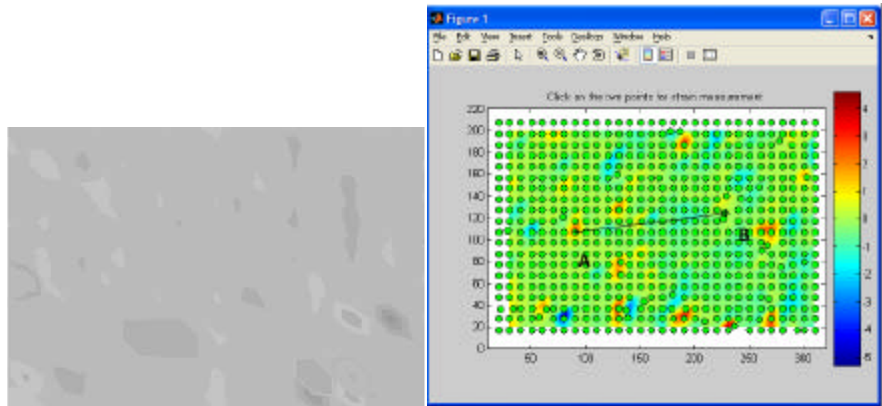


Figure 50. Strain calculated between the shown points = 0.00123

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**Image# 2: On Load 500MPa**

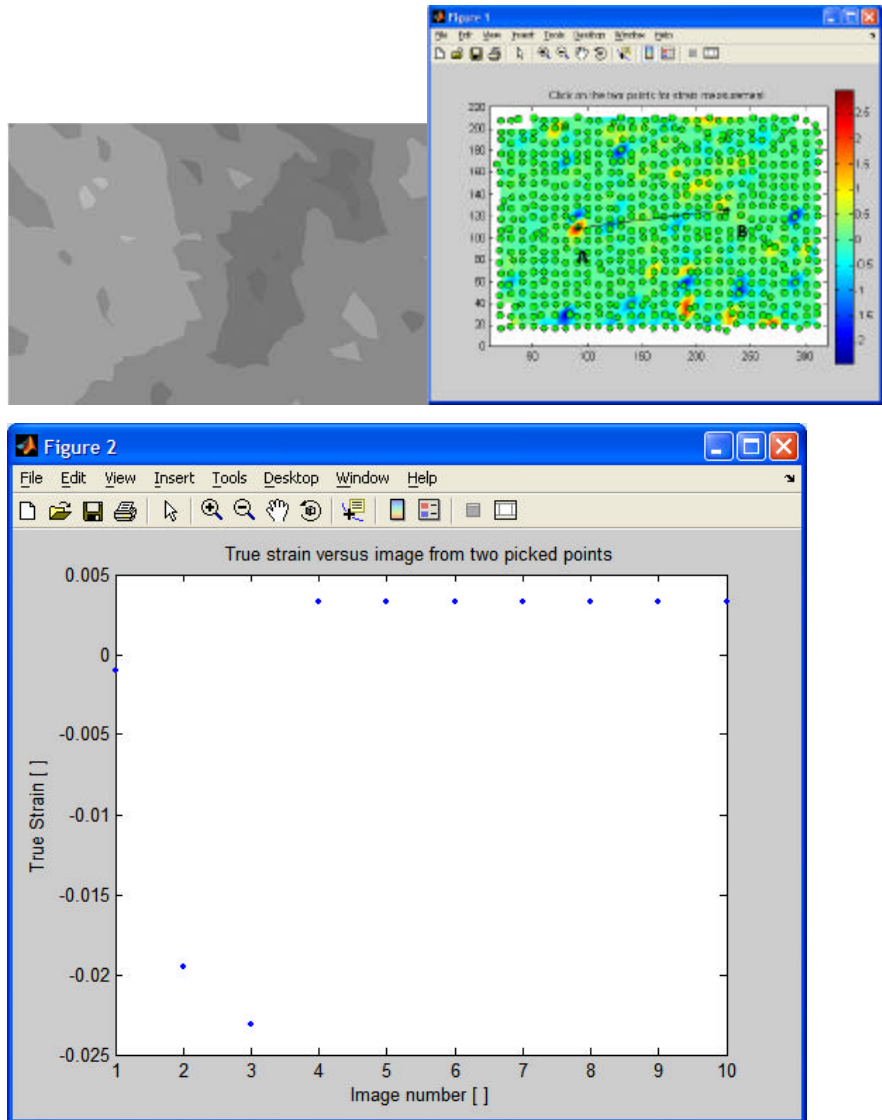


**Figure 51. Strain calculated between the shown points = 0.019**

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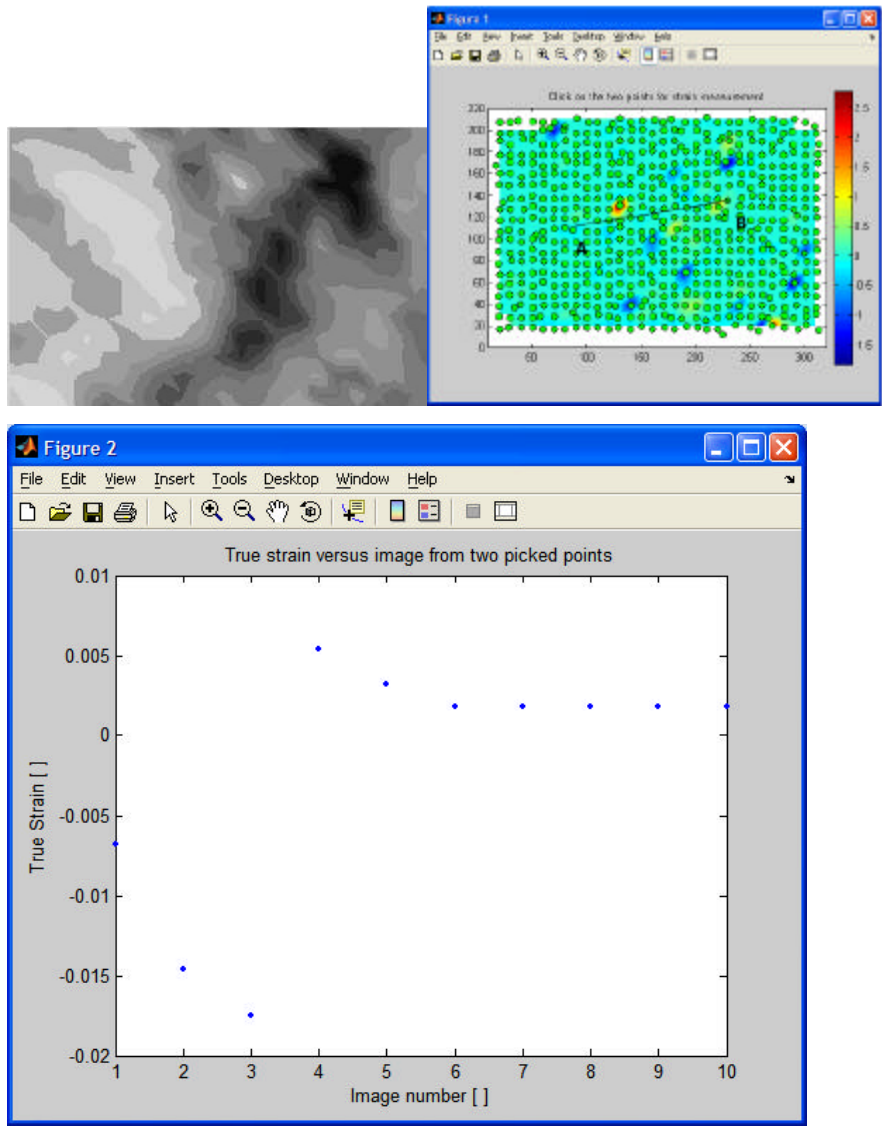
**Image# 3: On Load 940MPa**





**Figure 52. Strain calculated between the shown points = 0.0235**

**Image# 4: On Load 945MPa**



**Figure 53. Strain calculated between the shown points = 0.0058**

**Comparison of results:**

<b>Image Number</b>	<b>Load on Specimen</b>	<b>Strain calculated from the given data of specimen (Modulus of Elasticity=121 GPa)</b>	<b>True Strain calculated through Digital Image Correlation</b>
1.	500 MPa	0.00413	0.019
2.	940 MPa	0.00776	0.0235
3.	945 MPa	0.0078	0.0058

**6. Discussion and Conclusion****6.1. Discussion**

First, we must ensure that the random aspect is not subjected to any modifications during the tests. A partial loss or a change of this aspect will carry a wrong correlation. Afterwards, there is the lighting problem during the tests. A high contrast for the image is required. So, a suitable lighting of the study zone has to be set up. It has to assure, principally, an uniform lighting of the study zone.

Proper care has to be taken during the process. The preciseness of the result depends upon the ideal condition of equipments and methodology being employed. More is the care taken in things like illumination of object, camera mexapixel value, subsequent images being taken while object is getting distorted gradually under the force etc. yields more likely an accurate result.

A sample of 5000-10,000 images is a good count for strain measurement, higher number of images ensures more accuracy of result. Ideally the Matlab code we used for strain measurement requires to be left to run overnight processing images.

I haven't however taken a lot many images ideally otherwise suited for the illustration of the process due to lack of proper equipments. However the experiment wasn't done in ideal laboratory condition but the methodology and the Matlab Code I have illustrated in this thesis is up to standards and can be employed for Strain Measurement for practical purposes.

**6.2. Conclusion**

The various tests described in this paper underline all the interest that one has to carry on this correlation technique for the displacement field measurement. This is very powerful technique: strains until 80% were appreciated between two images. Nevertheless, some researches still to be done. For example, the computation time must be reduced. More, it would be necessary to modify the correlation program in order to allow the choice of any study zone: inclined rectangles, form curves. An extension of this correlation technique for the 3 D strain measurement is now in labour. Finally, this correlation technique has to become another tool for measuring the displacement and/or strain field in certain hard cases. So it is a tool which it is necessary to know the capacities, the power, the rapidity, the limits.

to give a result in order to use if it is necessary and for validating, in short time, a numerical or analytical result.

### **6.3. Future scope**

The methodology described here could have a great future if its done under ideal laboratory conditions and with proper equipments.

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