CHAPTER - 1

INTRODUCTION

In light of energy cost, environmental concerns, right-of-way restrictions and other legislative and cost problems, the construction of both generation facilities and in particular, new transmission lines have been delayed. In this connection, power electronics based FACTS controllers are increasingly being used to address the issue of better utilization of existing transmission corridors [1].

FACTS controllers enhance controllability and increase power transfer capability while maintaining sufficient steady state and transient margins. The FACTS Controllers achieve these objectives by controlling in a fast and effective way, the interrelated parameters that govern the operation of transmission systems including series impedance, shunt impedance, current, voltage, phase angle and the damping of power system oscillations [1-3].

Apart from the thyristor based controllers such as the Static VAR Compensator (SVC), the thyristor controlled series capacitor (TCSC), etc., the family of such equipments and include the voltage sourced converter (VSC) based devices such as the static compensator (STATCOM), the static synchronous series compensator (SSSC) and the unified power flow controller(UPFC). However, generally the performance of the VSC based controllers is superior to those of the thyristor based ones [1-4].

Amongst the VSC based Controllers, the STATCOMs are the earliest to be conceived and are currently installed in maximum numbers by the utilities [5]. The first STATCOM, with a rating of ± 100 MVAR, was commissioned in late 1995 at the Sullivan substation of the Tennessee Valley Authority (TVA) in U.S.A., jointly sponsored by the EPRI and the TVA, and manufactured by the Westinghouse Electric Corporation [1-5].

For proper utilization of STATCOMs in power system planning, operation and control, power flow solution of the network incorporating them is a necessity. As a consequence, the development of a suitable power flow model of a STATCOM has been challenge to power engineers worldwide [3].

The earliest algorithms for power flow solution of networks were based on the Gauss-Seidel method. They suffered, however, from relatively poor convergence characteristics [6-8]. Subsequently the Newton-Raphson (NR) algorithm was developed [9-10]. Subsequently, the development of the fast-decoupled power flow ushered in a new revolution in the field of power flow-flow [11]. In this case, the updating of matrices is no longer required and the computational burden is greatly reduced.

The first comprehensive, multi-control functional model of a STATCOM for power flow studies was reported in [12].a novel power injection model of a STATCOM for power flow and voltage stability studies, along with practical device limit constraints of the STATCOM, is addressed in [13].

In this thesis, an attempt has been made to develop a Newton power flow model of a STATCOM. This model can account for the coupling transformer resistance. Validation of the model is demonstrated on a simple four bus system followed by the IEEE 30-bus test system. It is observed that the model devoleped possesses excellent convergence characteristics.

CHAPTER – 2

FACTS CONTROLLERS

2.1Introduction

Traditionally, reactive compensation has been applied by fixed or mechanically switched circuit elements to improve steady state power transmission. The recovery from dynamic disturbances was accomplished by generous stability margins at the price of relatively poor system utilization. Moreover all plant components used in high voltage transmission to provide voltage and power flow were using electro –mechanical technology, which severely impaired the effectiveness of the intended control actions, particularly during fast changing operating conditions. Since the 1970s, energy cost, environmental restrictions, right of way difficulties, together with other legislative, social and cost problems, has delayed the construction of both generation facilities and new transmission lines. In this time period, there have also been profound changes in the industrial structure and significant geographic shifts of highly populated areas. The economic, social and legislative developments have demanded the review of traditional power transmission theory and practice, and the creation of new concepts that allow full utilization of existing power generation and transmission facilities without compromising system stability and security.

In the late 1980s, the Electric Power Research Institute (EPRI), the utility arm of North American utilities formulated the vision of the Flexible AC Transmission System (FACTS) in which various power electronic based controllers regulate power flow and transmission voltage and, through rapid control action, mitigate disturbances. Controllers used in high voltage transmission are grouped under the heading of FACTS and those used in low voltage distribution under the heading of Custom Power. The main objective of the FACTS is to increase the useable transmission capacity lines and control power flow over designated transmission routes.

In its most general expression, the FACTS concept is based on the substantial incorporation of power electronic devices and methods into the high-voltage side of the network, to make it electronically controllable. Many of the ideas upon which the foundation of FACTS rests evolved over a period of many decades. Nevertheless, FACTS, an integrated philosophy, is a novel concept that was brought to fruition during the 1980s.FACTS looks at ways of capitalizing on the many breakthroughs taking place in the area of high-voltage and high current power electronics, aiming at increasing the control of power flows in the high voltage side of the network during both steady-state and transient conditions.

The new Reality of making the power network electronically controllable has started to alter the way power plant equipment is designed and built as well as the thinking and procedures that go into the planning and operation of transmission and distribution networks. These developments may also affect the way energy transactions are conducted, as high-speed control of the path of the energy flow is now feasible. Owing to the many economical and technical benefits it promised, FACTS received the uninstinctive support of electrical equipment manufacturers, utilities, and research organizations around the world .

Several kinds of FACTS controllers have been commissioned in various parts of the world. The most popular are: load tap changers, phase-angle regulators, static VAR compensators, thyristor-controlled series compensators, inter phase power controllers, static compensators, and unified power flow controllers.

Early developments of the FACTS technology were in power electronic versions of the phase-shifting and tap-changing transformers. These controllers together with the electronic series compensator can be considered to belong to the first generation of FACTS equipment. The unified power flow controller, the static compensator, and the inter phase power controller are more recent developments. Their control capabilities and intended function are more sophisticated than those of the first wave of FACTS controllers. They may be considered to belong to a second generation of FACTS equipment. Shunt-connected thyristor-switched capacitors and thyristor-controlled reactors, as well as high-voltage direct-current (DC) power converters, have been in existence for many years, although their operational characteristics resemble those of FACTS controllers.

For most practical purposes the thyristor-based static VAR compensator (SVC) has made the rotating synchronous compensator redundant, except where an increase in the short circuit level is required along with fast-acting reactive power support. However, as power electronic technology continues to develop further, the replacement of the SVC by a new breed of static compensators based on the use of voltage source converters (VSCs) is looming. They are known as STATCOMs (static compensators) and provide all the functions that the SVC can provide but at a higher speed; it is more compact and requires only a fraction of the land required by an SVC installation. The STATCOM is essentially a VSC interfaced to the AC system through a shunt-connected transformer. The VSC is the basic building block of the new generation of power electronic controllers that have emerged from the FACTS and custom power initiatives.

2.2 Types of Facts Controllers

In high-voltage transmission, the most popular FACTS equipment are: the STATCOM, the unified power flow controller (UPFC) and the HVDC-VSC. At the low-voltage distribution level, the SVC provides the core of the following custom power equipment: the distribution STATCOM, the dynamic voltage restorer, and active filters.

There are two approaches to the realization of power electronics based FACTS controllers: one employs conventional thyristor switched capacitors and reactors, and quadrature tap changing transformers, and the other employs self commutated switching converters as synchronous voltage sources. The first approach has resulted in Static Var Compensators (SVC), the Thyristor Controlled Series Capacitor (TCSC), and the Thyristor Controlled Phase Shifter (TCPS). The second approach has produced the Static Synchronous Compensator (STATCOM), the Unified Power Flow Controller (UPFC), and the Interline Power Flow Controller. These two groups of FACTS devices have distinctly different operating and performance characteristics.

The thyristor controlled group employs capacitor and reactor banks with fast solid state switches in traditional shunt or series circuit arrangements. The thyristor switches controls the ON and OFF periods of the capacitor and reactor banks, thereby, in fact, realize variable active impedance. Except for losses, they cannot exchange real power with the AC system.

The Synchronous Voltage type FACTS controller group employs self commutated dc to ac converters, using GTO thyristor, which can internally generate capacitive and reactive power for transmission line compensation, without the use of AC capacitor or reactor banks. The converter, supported by a dc power supply, can also exchange real power with the ac system, in addition to the independently controllable reactive power. The converter based SVS can be used uniformly to control transmission line voltage, impedance, and angle by providing reactive shunt compensation, series compensation and phase shifting, or to control directly the real and reactive power flow in the line by forcing the necessary voltage across the series line impedance. When used for reactive shunt compensation, the SVS acts like an ideal Synchronous compensator being able to maintain the maximum capacitive output current at any system voltage down to zero. This V-I characteristics is superior to that obtainable with the conventional thyristor controlled SVC whose maximum current decrease linearly with the system voltage.

As a reactive series compensator, the SVS can provide controllable series capacitive compensation without the danger of sub synchronous resonance. Its capability to maintain the maximum compensating voltage independent of the line current, and to provide capacitive as well as inductive compensation, results in a much wider control range than possible with controlled series capacitor compensation. This makes it highly effective in power flow control, as well as in power oscillation damping.

The deployment of increasing number of FACTS controllers will make it necessary to reconceptualize the control of the transmission system as a dynamic entity in order to prevent undesirable interactions and obtain attainable maximum economic and operating benefits.

A number of FACTS controllers have been commissioned. Most of them perform a useful role during both steady-state and transient operation, but some are specifically designed to operate only under transient conditions, for instance, Hingorani’s sub synchronous resonance (SSR) damper.

The application of FACTS controllers to the solution of steady-state operating problems is outlined in Table 3.1.

Table 2.1 The role of FACTS (flexible alternating current transmission systems) controllers in power system operation

Operating problem	Corrective action	FACTS controller
Voltage limits:		
Low voltage at heavy load	Supply reactive power	STATCOM, SVC,
High voltage at low load	Absorb reactive power	STATCOM, SVC, TCR
High voltage following an outage	Absorb reactive power; prevent overload	STATCOM, SVC, TCR
Low voltage following an outage	Supply reactive power; prevent overload	STATCOM, SVC
Thermal limits:		
Transmission circuit overload	Reduce overload	TCSC, SSSC, UPFC, IPC, PS
Tripping of parallel circuits	Limit circuit loading	TCSC, SSSC, UPFC, IPC, PS
Loop flows:		
Parallel line load sharing	Adjust series reactance	IPC, SSSC, UPFC, TCSC, PS
Postfault power flow sharing	Rearrange network or use thermal limit actions	IPC, TCSC, SSSC, UPFC, PS
Power flow direction reversal	Adjust phase angle	IPC, SSSC, UPFC, PS

2.3 Controllers Based on Conventional Thyristors

Power electronic circuits using conventional thyristors have been widely used in power transmission applications since the early 1970s. The first applications took place in the area of HVDC transmission, but shunt reactive power compensation using fast controllable inductors and capacitors soon gained general acceptance. More recently, fast-acting series compensators using thyristors have been used to vary the electrical length of key transmission lines, with almost no delay, instead of the classical series capacitor, which is mechanically controlled. In distribution system applications, solid state transfer switches using thyristors are being used to enhance the reliability of supply to critical customer loads.

2.3.1 The Thyristor-controlled Reactor:

The main components of the basic TCR are shown in Figure 2.2(a). The controllable element is the antiparallel thyristor pair, Th1 and Th2, which conducts on alternate halfcycles of the supply frequency. The other key component is the linear (air-core) reactor of inductance L.

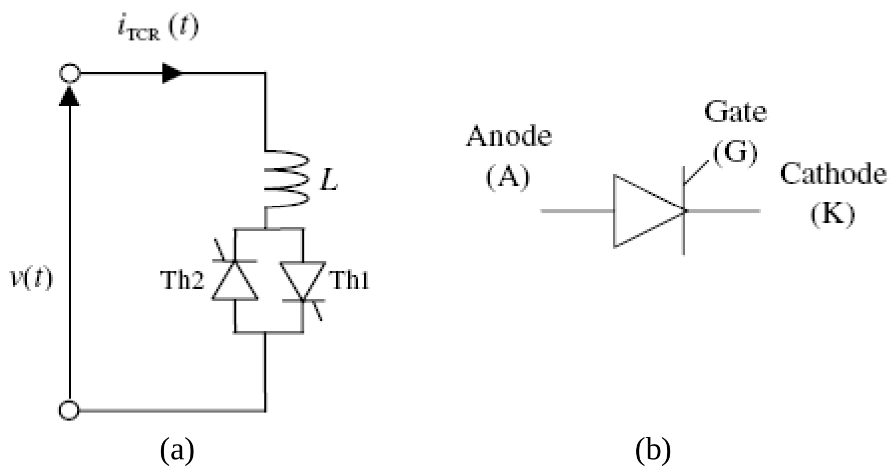


Figure 2.1 Thyristor-based circuit: (a) Basic thyristor-controlled reactor (TCR); (b) thyristor circuit symbol.

The thyristor circuit symbol is shown in Figure 2.1(b).The overall action of the thyristor controller on the linear reactor is to enable the reactor to act as a controllable susceptance, in the inductive sense, which is a function of the firing angle α However, this action is not trouble free, since the TCR achieves its fundamental frequency steady-state operating point at the expense of generating harmonic distortion, except for the condition of full conduction.

First, consider the condition when no harmonic distortion is generated by the TCR, which takes place when the thyristors are gated into conduction, precisely at the peaks of the supply voltage. The reactor conducts fully, and one could think of the thyristor controller as being short-circuited. The reactor contains little resistance and the current is essentially sinusoidal and inductive, lagging the voltage by almost 90°($\pi/2$). This is illustrated in Figure 2.2(a), where a fundamental frequency period of the voltage and current are shown.

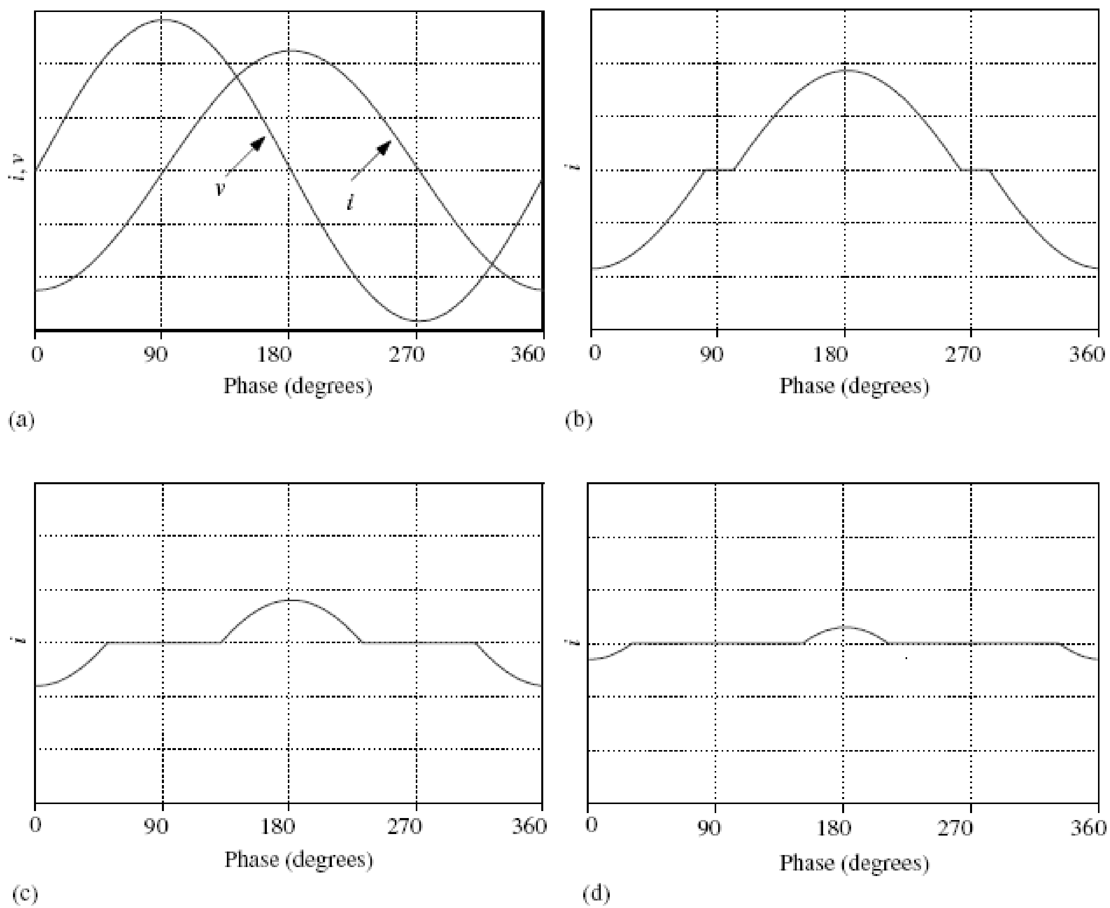


Figure 2.2 Current waveforms in the basic thyristor-controlled reactor: (a) $\alpha = 90^\circ$, $\sigma = 180^\circ$ (b) $\alpha = 100^\circ$, $\sigma = 160^\circ$ (c) $\alpha = 130^\circ$, $\sigma = 100^\circ$ (d) $\alpha = 150^\circ$, $\sigma = 60^\circ$

It should be mentioned that this condition corresponds to a firing angle α of $\pi/2$, which is the current zero-crossing measured with reference to the voltage zero-crossing. The relationship between the firing angle α and the conduction angle σ is given by

$$\sigma = 2(\pi - \alpha).$$

Partial conduction is achieved with firing angles in the range: $\pi/2 < \alpha < \pi$, in radians. This is illustrated in Figures 2.2(b)–2.2(d), where TCR currents, as a function of the firing angle are shown. Increasing the value of firing angle above $\pi/2$ causes the TCR current waveform to become nonsinusoidal, with its fundamental frequency component reducing in magnitude. This, in turn, is equivalent to an increase in the inductance of the reactor, reducing its ability to draw reactive power from the network at the point of connection.

Figure 2.3 shows a three-phase, delta-connected TCR. This topology uses six groups of thyristor and is commonly known as a six-pulse TCR.

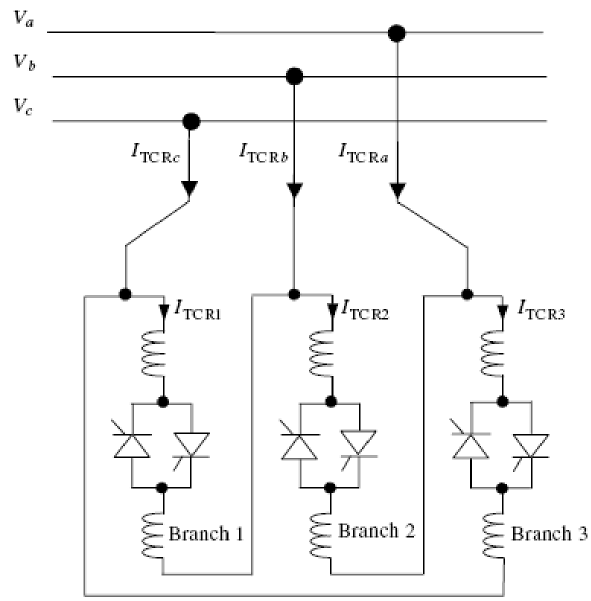


Figure 2.3 Three-phase thyristor-controlled reactor

2.3.2 The Static VAR Compensator:

In its simplest form, the SVC consists of a TCR in parallel with a bank of capacitors. From an operational point of view, the SVC behaves like a shunt-connected variable reactance, which either generates or absorbs reactive power in order to regulate the voltage magnitude at the point of connection to the AC network. It is used extensively to provide fast reactive power and voltage regulation support. The firing angle control of the thyristor enables the SVC to have almost instantaneous speed of response. A schematic representation of the SVC is shown in Figure 2.4, where a three-phase, three winding transformer is used to interface the SVC to a high-voltage bus. The transformer has two identical secondary windings: one is used for the delta-connected, six-pulse TCR and the other for the star-connected, three-phase bank of capacitors, with its star point floating. The three transformer windings are also taken to be star-connected, with their star points floating.

Similar to the TCR, no zero sequence current can flow in the SVC circuit as the star point of the bank of capacitors is not grounded.

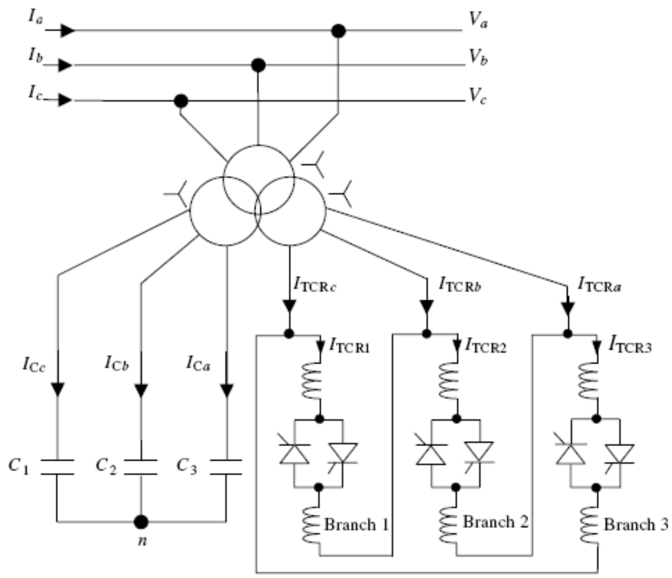


Figure 2.4 Representation of a three-phase static VAR compensator (SVC) comprising fixed capacitors and thyristor-controlled reactors (TCRs)

The positive sequence and negative sequence circuits contain equal impedances. It should be remarked that the positive sequence model of the SVC should also serve the purpose of representing a single-phase SVC.

2.3.3 The Thyristor Controlled Series Capacitor (TCSC) :

The TCSC varies the electrical length of the compensated transmission line with little delay. In other words TCSC consists of a series capacitor bank shunted by a thyristor controlled reactor in order to provide a smoothly variable series capacitive reactance. Owing to these characteristics, it may be used to provide fast active power flow regulation. It also increases the stability margin of the system and has proved very effective in damping SSR and power oscillations.

The working of TCSC is based on the concept of a non-linear series reactance, which is adjusted using Newton's algorithm to satisfy a specified active power flow across the variable reactance representing the TCSC. It is having a thyristor without the gate turn off capability. It is an alternative to SSSC above and like an SSSC, it is very important FASTS controller. A variable reactor like a thyristor controlled reactor is connected across a series capacitor. As the firing angle is advanced form 180 degrees to lesser values, the capacitive impedance increase. When the TCR firing angle is 90 degree, the reactor becomes fully conducting and the total impedance becomes inductive, because the reactor impedance is designed to be much lower than the series capacitor impedance. With 90 degree firing angle, TCSC helps in limiting the fault current.

The active power transfer P_{lm} across an impedance connected between nodes l and m is determined by the voltage magnitudes $|v_l|$ and $|v_m|$ the difference in voltage phase angles θ_l and θ_m and the transmission line resistance R_{lm} and reactance X_{lm}. In high voltage transmission lines, the reactance is much larger than the resistance and the following approximate equation may be used to calculate the active power transfer P_{lm}

$$P_{lm} = \frac{|V_l||V_m|}{X_{lm}} \sin(\theta_l - \theta_m)$$

If the electrical branch is a TCSC controller as opposed to a transmission line then P_{lm} is calculated using the following expression

$$P_{lm}^{reg} = \frac{|V_l||V_m|}{X_{TCSC}} \sin(\theta_l - \theta_m)$$

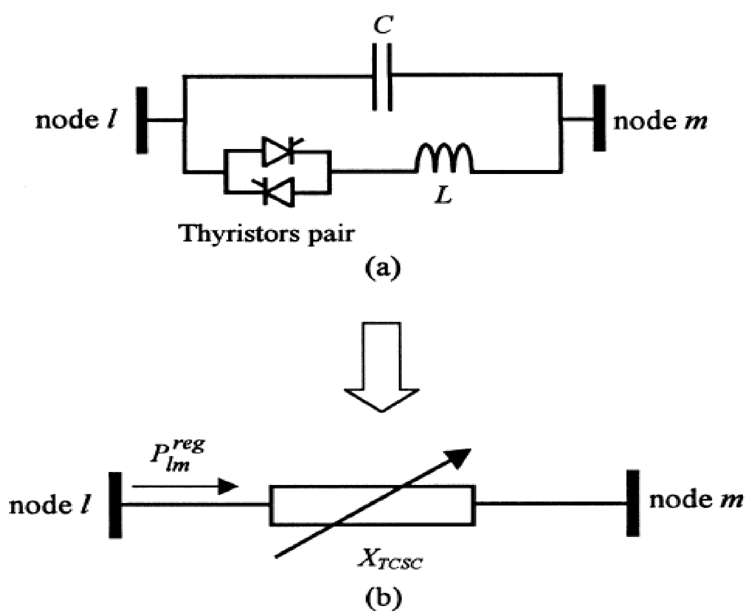


Fig 2.5 TCSC (a) structure formed by fixed capacitor and TCR ;
(b) variable reactance presentation

Where X_{TCSC} is the equivalent reactance of the TCSC controller which may be adjusted to regulate the transfer of active power across the TCSC, hence, P_{lm} becomes P_{lm}^{reg}

2.4 Power Electronic Controllers Based on Fully Controlled Semiconductor Devices:

Modern power system controllers based on power electronic converters are capable of generating reactive power with no need for large reactive energy storage elements, such as in SVC systems. This is achieved by making the currents circulate through the phases of the AC system with the assistance of fast switching devices.

The semiconductor devices employed in the new generation of power electronic converters are of the fully controlled type, such as the insulated gate bipolar transistor (IGBT) and the gate turn-off thyristor (GTO). Their respective circuit symbols are shown in Figure 2.6.

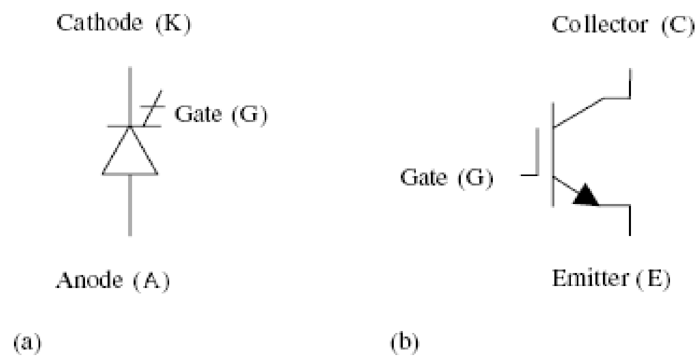


Figure 2.6 Circuit symbols for: (a) gate turn-off thyristor and (b) insulated gate bipolar transistor.

The GTO is a more advanced version of the conventional thyristor, with a similar Switched-on characteristic but with the ability to switch off at a time different from when the forward current falls naturally below the holding current level. Such added functionality has enabled new application areas in industry to be developed, even at bulk power transmission where nowadays it is possible to redirect active power at the megawatt level. However, there is room for improvement in GTO construction and design, where still large negative pulses are required to turn them off. At present, the maximum switching frequency attainable is in the order of 1 kHz.

The IGBT is one of the most well-developed members of the family of power transistors. It is the most popular device used in the area of AC and DC motor drives, reaching power levels of a few hundred kilowatts. Power converters aimed at power systems applications are beginning to make use of IGBTs owing to their increasing power-handling capability and relatively low conduction losses. Further progress is expected in IGBT and GTO technology and applications.

In DC–AC converters that use fully controlled semiconductors rather than conventional thyristors, the DC input can be either a voltage source (typically a capacitor) or current source (typically a voltage source in series with an inductor). With reference to this basic operational principle, converters can be classified as either voltage source converters (VSCs) or current source converters. For economic and performance reasons, most reactive power controllers are based on the VSC topology. The availability of modern semiconductors with relatively high voltage and current ratings, such as GTOs or IGBTs, has made the concepts of reactive compensation based on switching converters a certainty, even for substantial high-power applications.

A number of power system controllers that use VSCs as their basic building block are in operation in various parts of the world. The most popular are: STATCOMs, solid-state series controllers (SSSCs), the UPFC, and the HVDC-VSC.

2.4.1 The Voltage Source Converter (VSC):

There are several VSC topologies currently in use in actual power system operation and some others that hold great potential, including: the single-phase full bridge (H-bridge); the conventional three-phase, two-level converter; and the three-phase, three-level converter based on the neutral-point-clamped converter. Other VSC topologies are based on combinations of the neutral-point-clamped topology and multilevel-based systems.
Common aims of these topologies are: to minimize the operating frequency of the semiconductors inside the VSC and to produce a high-quality sinusoidal voltage waveform with minimum or no filtering requirements. By way of example, the topology of a conventional two-level VSC using IGBT switches is illustrated in Figure 2.7

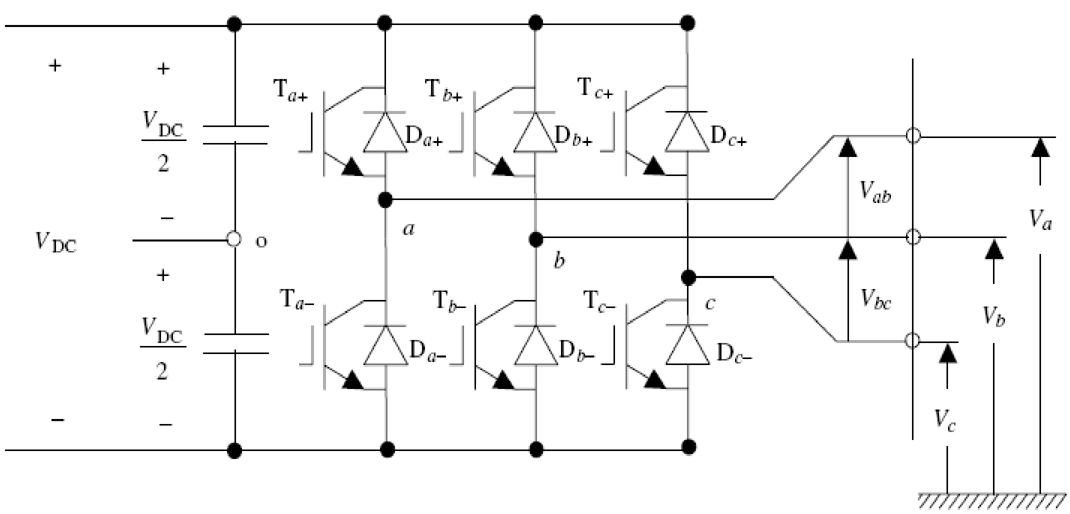


Figure 2.7 Topology of a three-phase, two-level voltage source converter (VSC) using insulated gate bipolar transistors

The VSC shown in Figure 2.7 comprises six IGBTs, with two IGBTs placed on each leg. Moreover, each IGBT is provided with a diode connected antiparallel to make provisions for possible voltage reversals due to external circuit conditions. Two equally sized capacitors are placed on the DC side to provide a source of reactive power.
Although not shown in the circuit of Figure 2.7 the switching control module is an integral component of the VSC . Its task is to control the switching sequence of the various semiconductor devices in the VSC, aiming at producing an output voltage waveform that is as

near to a sinusoidal waveform as possible, with high power controllability and minimum switching loss.

Current VSC switching strategies aimed at utility applications may be classified into two main categories:

- Fundamental frequency switching: the switching of each semiconductor device is limited to one turn-on and one turn-off per power cycle. The basic VSC topology shown in Figure 2.7 with fundamental frequency switching, yields a quasi-square-wave output, which has an unacceptable high harmonic content. It is current practice to use several six-pulse VSCs, arranged to form a multipulse structure, to achieve better waveform quality and higher power ratings.
- Pulse-width modulation (PWM): This control technique enables the switches to be turned on and off at a rate considerably higher than the fundamental frequency. The output waveform is chopped and the width of the resulting pulses is modulated. Undesirable harmonics in the output waveform are shifted to the higher frequencies, and filtering requirements are much reduced. Over the years, various PWM control techniques have been published, but the sinusoidal PWM scheme remains one of the most popular owing to its simplicity and effectiveness.

From the viewpoint of utility applications, both switching techniques are far from perfect. The fundamental frequency switching technique requires complex transformer arrangements to achieve an acceptable level of waveform distortion. Such a drawback is offset by its high semiconductor switch utilization and low switching losses; and it is, at present, the switching technique used in high-voltage, high-power applications. The PWM technique incurs high switching loss, but it is envisaged that future semiconductor devices will reduce this by a significant margin, making PWM the universally preferred switching technique, even for high-voltage and extra-high-voltage transmission applications.

Pulse-width modulation control :

The basic PWM control method can be explained with reference to Figure 2.8, in which a sinusoidal fundamental frequency signal is compared with a high-frequency triangular signal, producing a square-wave signal, which serves the purpose of controlling the firing of the individual valves of a given converter topology, such as the one shown in Figure 2.10. The sinusoidal and triangular signals, and their associated frequencies, are termed reference and carrier signals and frequencies, respectively. By varying the amplitude of the sinusoidal signal against the fixed amplitude of the carrier signal, which is normally kept at 1 p.u., the amplitude of the fundamental component of the resulting control signal varies linearly.

In Figures 2.8(a)–2.8(c), the carrier frequency f_s is taken to be 9 times the desired frequency f1.

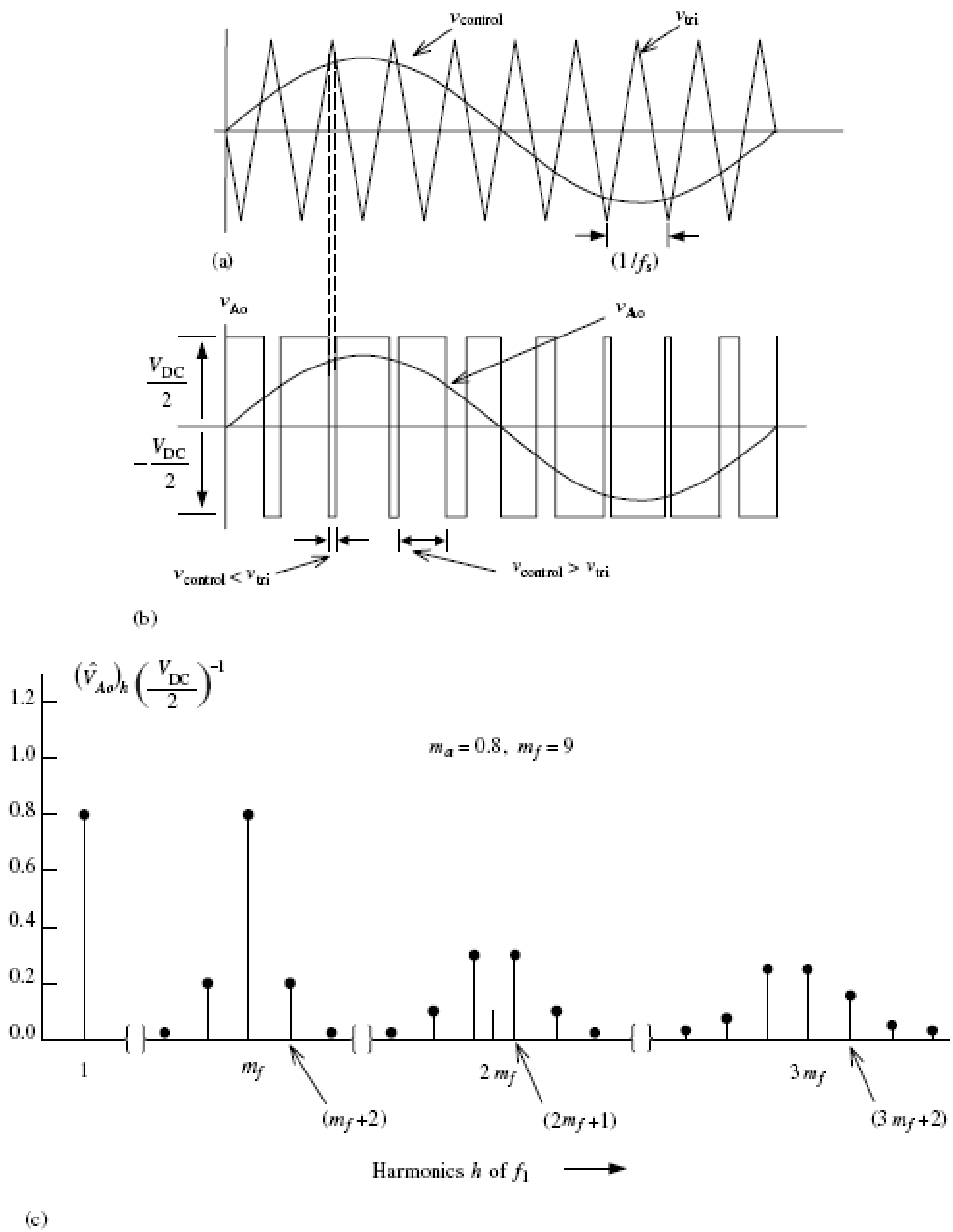


Figure 2.8 Operation of a pulse-width modulator: (a) comparison of a sinusoidal fundamental frequency with a high-frequency triangular signal; (b) resulting train of square-wave signals; (c) harmonic voltage spectrum

The width of the square wave is modulated in a sinusoidal manner, and the fundamental and harmonic components can be determined by means of Fourier analysis. To determine the magnitude and frequency of the resulting fundamental and harmonic terms, it is useful to use the concept of amplitude modulation ratio, ma, and frequency modulation ratio, mf:

$$m_a = \frac{V_{control}}{V_{tri}}$$

$$m_f = \frac{f_s}{f_1}$$

where $V_{control}$ is the peak amplitude of the sinusoidal (control) signal and V_{tri} is the peak Amplitude of the triangular (carrier) signal, which, for most practical purposes, is kept constant. With reference to the 'one-leg' converter shown in Figure 2.9, corresponding to one leg of the three-phase converter of Figure 2.8, the switches T_a+ and T_a- are controlled by straightforward comparison of $V_{control}$ and V_{tri}, resulting in the following output voltages:

$$V_{Ao} = \begin{cases} \frac{V_{DC}}{2} \text{ when } T_{a+} \text{ is on in response to } V_{control} > V_{tri}; \\ -\frac{V_{DC}}{2} \text{ when } T_{a-} \text{ is on in response to } V_{control} < V_{tri}; \end{cases}$$

The output voltage V_{Ao} fluctuates between $\frac{V_{DC}}{2}$ and $-\frac{V_{DC}}{2}$, as T_{a+} and T_{a-} are never off simultaneously, and is independent of the direction of i_o.

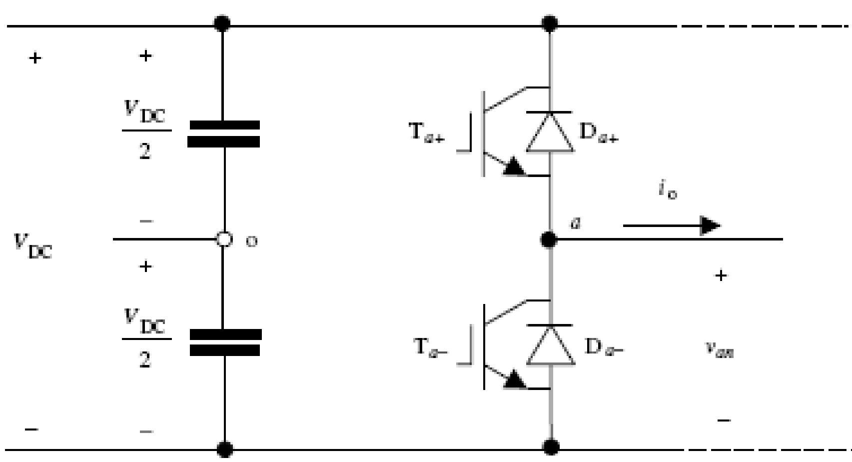


Figure 2.9.'One leg' voltage source converter (VSC).

The voltage V_{Ao} and its fundamental frequency component are shown in Figure 2.8(b), for the case of m_f = 9 and m_a = 0.8. The corresponding harmonic voltage spectrum, in normalized form, is shown in Figure 2.8(c). This is a case of linear voltage control, where m_a < 1, but this is not the only possibility. Two other forms of voltage control exist, namely, over modulation and square-wave modulation. The former takes place in the region 1 < m_a < 3:24 and the latter applies when m_a > 3:24.

Only the case of linear voltage control ($m_a < 1$) is of interest in this section. The peak amplitude of the fundamental frequency component is ma multiplied by $V_{DC}/2$, and the harmonics appear as sidebands, centered around the switching frequency and its multiples, following a well-defined pattern given by:

$$f_h = (\beta_{mf} \pm k)f_1$$

Harmonic terms exist only for odd values of β with even values of k. Conversely, even values of β combine with odd values of k. Moreover, the harmonic m_f should be an odd integer in order to prevent the appearance of even harmonic terms in V_{Ao}.

b). Principles of Voltage Source Converter Operation :

The interaction between the VSC and the power system may be explained in simple terms, by considering a VSC connected to the AC mains through a loss-less reactor, as illustrated in the single-line diagram shown in Figure 2.10(a). The premise is that the amplitude and the

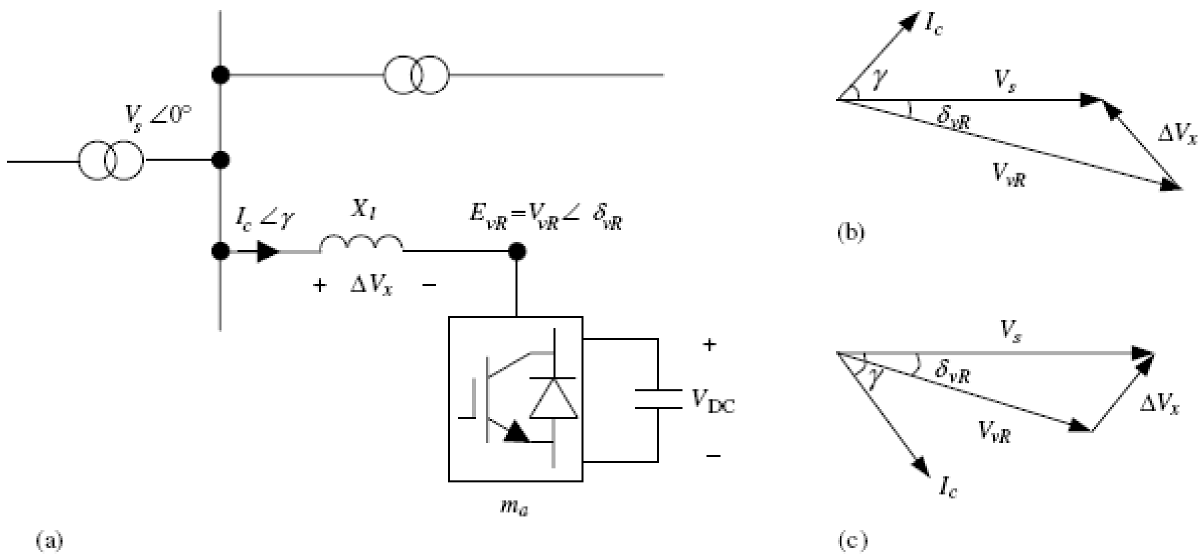


Figure 2.10 Basic operation of a voltage source converter (VSC): (a) VSC connected to a system bus. Space vector representation for (b) lagging operation and (c) leading operation

Phase angle of the voltage drop, ΔV_x, across the reactor, X_l, can be controlled, defining the amount and direction of active and reactive power flows through X_l. The voltage at the supply bus is taken to be sinusoidal, of value $V_s \angle 0^{0(1)}$, and the fundamental frequency component of the SVC AC voltage is taken to be $V_{vr} \angle_{vR}$. The positive sequence fundamental frequency vector representation is shown in Figures 2.10(b) and 2.10(c) for leading and lagging VAR compensation, respectively.

According to Figure 2.10, for both leading and lagging VAR, the active and the reactive powers can be expressed as

$$P = \frac{V_s V_{vr}}{X_1}\sin\delta_{vr}$$

$$Q = \frac{V_s^2}{X_1} - \frac{V_s V_{vr}}{X_1}\cos\delta_{vr} \qquad \text{..................(1)}$$

With reference to Figure 2.10 and Equation (1), the following observations are derived:

- The VSC output voltage V_{vR} lags the AC voltage source Vs by an angle δ_{vR}, and the input current lags the voltage drop across the reactor ΔVx by $\frac{\pi}{2}$. The active power flow between the AC source and the VSC is controlled by the phase angle δ_{vR}.
- Active power flows into the VSC from the AC source for lagging δ_{vR} ($\delta_{vR} > 0$) and flows out of the VSC from the AC source for leading δ_{vR} ($\delta_{vR} < 0$).
- The reactive power flow is determined mainly by the magnitude of the voltage source, Vs, and the VSC output fundamental voltage, V_{vR}. For V_{vR} > Vs, the VSC generates reactive power and consumes reactive power when V_{vR} < Vs.
- The DC capacitor voltage VDC is controlled by adjusting the active power flow that goes into the VSC. During normal operation, a small amount of active power must flow into the VSC to compensate for the power losses inside the VSC, and δ_{vR} is kept slightly larger than 0° (lagging).

2.4.2 The Static Synchronous Compensator(STATCOM) :

The STATCOM consists of one Voltage Source Converter (VSC) and its associated shunt-connected transformer. It is the static counterpart of the rotating synchronous condenser but it generates or absorbs reactive power at a faster rate because no moving parts are involved.

It is in general a solid-state switching converter capable of generating or absorbing independently controllable real and reactive power at its output terminals when it is fed from an energy source or energy-storage device at its input terminals. Specifically, the STATCOM considered in this chapter is a voltage-source converter that, from a given input of dc voltage, produces a set of 3-phase ac-output voltages, each in phase with and coupled to the corresponding ac system voltage through a relatively small reactance (which is provided by either an interface reactor or the leakage inductance of a coupling Transformer). The dc voltage is provided by an energy-storage capacitor.

A STATCOM can improve power-system performance in such areas as the following:

1. The dynamic voltage control in transmission and distribution systems;
2. The power-oscillation damping in power-transmission systems;
3. The transient stability;
4. The voltage flicker control; and
5. The control of not only reactive power but also (if needed) active power in the connected line, requiring a dc energy source.

Furthermore, a STATCOM does the following:

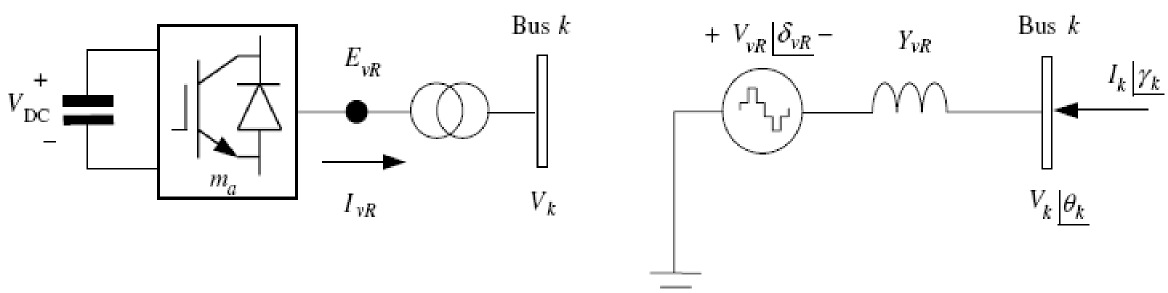
1. it occupies a small footprint, for it replaces passive banks of circuit elements by compact electronic converters;
2. it offers modular, factory-built equipment, thereby reducing site work and commissioning time; and
3. it uses encapsulated electronic converters, thereby minimizing its environmental impact.

A STATCOM is analogous to an ideal synchronous machine, which generates a balanced set of three sinusoidal voltages—at the fundamental frequency—with controllable amplitude and phase angle. This ideal machine has no inertia, is practically instantaneous, does not significantly alter the existing system impedance, and can internally generate reactive (both capacitive and inductive) power.

The Tennessee Valley Authority (TVA) installed the first ∓ 100-MVA STATCOM in 1995 at its Sullivan substation. The application of this STATCOM is expected to reduce the TVA's need for load tap changers, thereby achieving savings by minimizing the potential for transformer failure. This STATCOM aids in resolving the off-peak dilemma of overvoltages in the Sullivan substation area while avoiding the more labor- and space-intensive installation of an additional transformer bank. Also, this STATCOM provides instantaneous control—and therefore increased capacity—of transmission voltage, providing the TVA with greater flexibility in bulk-power transactions, and it also increases the system reliability by damping grids of major oscillations in this grid.

2.4.3 Principle of Operation

A STATCOM is a controlled reactive-power source. It provides the desired reactive- power generation and absorption entirely by means of electronic processing of the voltage and current waveforms in a voltage-source converter (VSC).



(a) (b)

Figure 2.11 Static compensator (STATCOM) system: (a) voltage source converter (VSC) connected to the AC network via a shunt-connected transformer; (b) shunt solid-state voltage source

A single-line STATCOM power circuit is shown in Fig. 2.11(a), where a VSC is connected to a utility bus through magnetic coupling. In Fig. 2.11(b), a STATCOM is seen as an adjustable voltage source behind a reactance—meaning that capacitor banks and shunt reactors are not needed for reactive-power generation and absorption, thereby giving a STATCOM a compact design, or small footprint, as well as low noise and low magnetic impact.

The exchange of reactive power between the converter and the ac system can be controlled by varying the amplitude of the 3-phase output voltage, *Es*, of the converter. That is, if the amplitude of the output voltage is increased above that of the utility bus voltage, E_t, then a current flows through the reactance from the converter to the ac system and the converter generates capacitive-reactive power for the ac system. If the amplitude of the output voltage is decreased below the utility bus voltage, then the current flows from the ac system to the converter and the converter absorbs inductive-reactive power from the ac system. If the output voltage equals the ac system voltage, the reactive-power exchange becomes zero, in which case the STATCOM is said to be in a floating state.

Adjusting the phase shift between the converter-output voltage and the ac system voltage can similarly control real-power exchange between the converter and the ac system. In other words, the converter can supply real power to the ac system from its dc energy storage if the converter-output voltage is made to lead the ac-system voltage. On the other hand, it can absorb real power from the ac system for the dc system if its voltage lags behind the ac-system voltage.

A STATCOM provides the desired reactive power by exchanging the instantaneous reactive power among the phases of the ac system. The mechanism by which the converter internally generates and/ or absorbs the reactive power can be understood by considering the relationship between the output and input powers of the converter. The converter switches connect the dc-input circuit directly to the ac-output circuit. Thus the net instantaneous power at the ac output terminals must always be equal to the net instantaneous power at the dc-input terminals (neglecting losses).

Assume that the converter is operated to supply reactive-output power. In this case, the real power provided by the dc source as input to the converter must be zero. Furthermore, because the reactive power at zero frequency (dc) is by definition zero, the dc source supplies no reactive power as input to the converter and thus clearly plays no part in the generation of reactive-output power by the converter. In other words, the converter simply interconnects the three output terminals so that the reactive-output currents can flow freely among them. If the terminals of the ac system are regarded in this context, the converter establishes a circulating reactive-power exchange among the phases. However, the real power that the converter exchanges at its ac terminals with the ac system must, of course, be supplied to or absorbed from its dc terminals by the dc capacitor.

Although reactive power is generated internally by the action of converter switches, a dc capacitor must still be connected across the input terminals of the converter. The primary need for the capacitor is to provide a circulating-current path as well as a voltage source. The magnitude of the capacitor is chosen so that the dc voltage across its terminals remains fairly constant to prevent it from contributing to the ripples in the dc current. The VSC-output voltage is in the form of a staircase wave into which smooth sinusoidal current from the ac system is drawn, resulting in slight fluctuations in the output power of the converter. However, to not violate the instantaneous power-equality constraint at its input and output terminals, the converter must draw a fluctuating current from its dc source. Depending on the converter configuration employed, it is possible to calculate the minimum capacitance required to meet the system requirements, such as ripple limits on the dc voltage and the rated-reactive power support needed by the ac system.

The VSC has the same rated-current capability when it operates with the capacitive- or inductive-reactive current. Therefore, a VSC having a certain MVA rating gives the STATCOM twice the dynamic range in MVAR (this also contributes to a compact design). A dc capacitor bank is used to support (stabilize) the controlled dc voltage needed for the operation of the VSC.

The reactive power of a STATCOM is produced by means of power-electronic equipment of the voltage-source-converter type. The VSC may be a 2- level or 3-level type, depending on the required output power and voltate. A number of VSCs are combined in a multi-pulse connection to form the STATCOM. In the steady state, the VSCs operate with fundamental-frequency switching to minimize converter losses. However, during transient conditions caused by line faults, a pulse width–modulated (PWM) mode is used to prevent the fault current from entering the VSCs. In this way, the STATCOM is able to withstand transients on the ac side without blocking.

2.4.4 The V-I Characteristic

A typical *V-I* characteristic of a STATCOM is depicted in Fig. 2.12. As can be seen, the STATCOM can supply both the capacitive and the inductive compensation and is able to independently control its output current over the rated maximum capacitive or inductive range irrespective of the amount of ac-system voltage. That is, the STATCOM can provide full capacitive-reactive power at any system voltage—even as low as 0.15 pu.

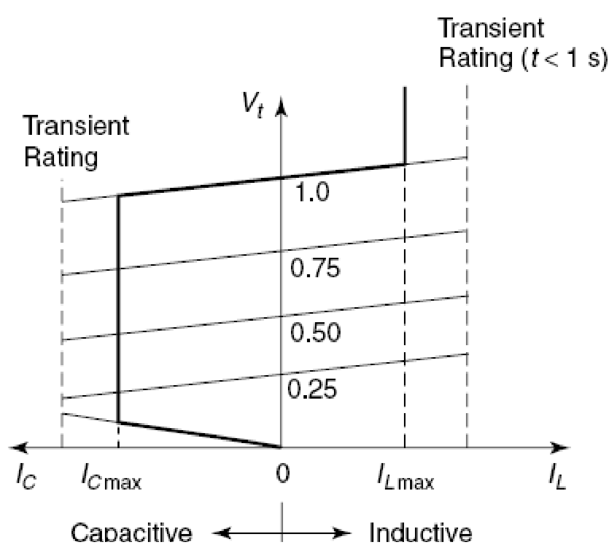


Figure 2.12 The *V-I* characteristic of the STATCOM

The characteristic of a STATCOM reveals strength of this technology: that it is capable of yielding the full output of capacitive generation almost independently of the system voltage (constant-current output at lower voltages). This capability is particularly useful for situations in which the STATCOM is needed to support the system voltage during and after faults where voltage collapse would otherwise be a limiting factor.

Figure 2.12 also illustrates that the STATCOM has an increased transient rating in both the capacitive- and the inductive-operating regions. The maximum attainable transient over current in the capacitive region is determined by the maximum current turn-off capability of the converter switches. In the inductive region, the converter switches are naturally commutated; therefore, the transient-current rating of the STATCOM is limited by the maximum allowable junction temperature of the converter switches.

In practice, the semiconductor switches of the converter are not lossless, so the energy stored in the dc capacitor is eventually used to meet the internal losses of the converter, and the dc capacitor voltage diminishes. However, when the STATCOM is used for reactive-power generation, the converter itself can keep the capacitor charged to the required voltage level. This task is accomplished by making the output voltages of the converter lag behind the ac-system voltages by a small angle (usually in the 0.18–0.28 range). In this way, the converter absorbs a small amount of real power from the ac system to meet its internal losses and keep the capacitor voltage at the desired level. The same mechanism can be used to increase or decrease the capacitor voltage and thus, the amplitude of the converter-output voltage to control the var generation or absorption.

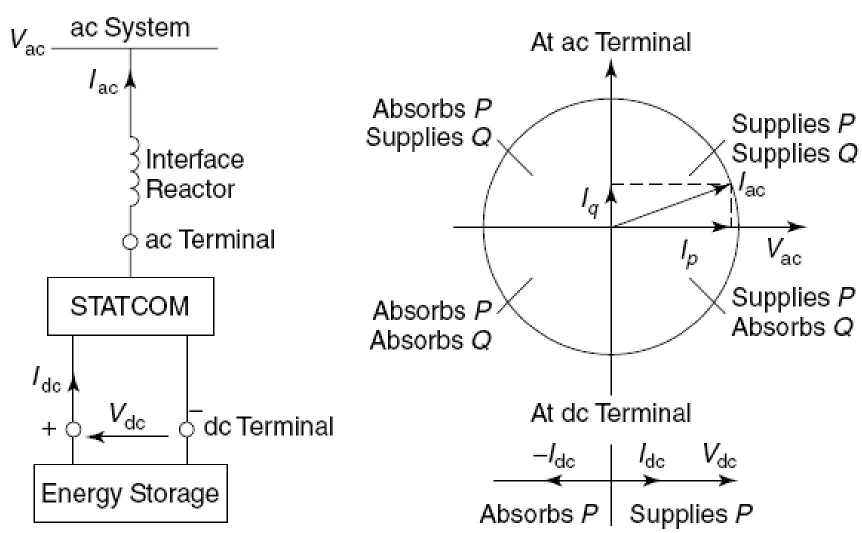


Figure 2.13 The power exchange between the STATCOM and the ac system

The reactive- and real-power exchange between the STATCOM and the ac system can be controlled independently of each other. Any combination of realpower generation or absorption with var generation or absorption is achievable if the STATCOM is equipped with an energy-storage device of suitable capacity, as depicted in Fig. 2.13. With this capability, extremely effective control strategies for the modulation of reactive- and real-output power can be devised to improve the transient- and dynamic-system-stability limits.

2.4.5 Static Synchronous Series Compensator(SSSC) :

A static synchronous generator operated without an external electric energy source as a series compensator whose output voltage is in quadrature with, and controllable independently of, the line current for the purpose of increasing or decreasing the overall reactive voltage drop across the line and there by controlling the transmitted electric power. For the purpose of steady-state operation, the SSSC performs a similar function to the static phase shifter. However, the SSSC is a far more versatile controller than the phase shifter because it does not draw reactive power from the AC system; it has its own reactive power provisions in the form of a DC capacitor. This characteristic makes the SSSC capable of regulating not only active but also reactive power flow or nodal voltage magnitude. The SSSC may include transiently rated energy storage or energy absorbing devices to enhance the dynamic behavior of the power system by additional temporary real power compensation, to increase or decrease momentarily, the overall voltage across the line.

A schematic representation of the SSSC and its equivalent circuit are shown in Figures 2.15(a) and 2.15(b), respectively.

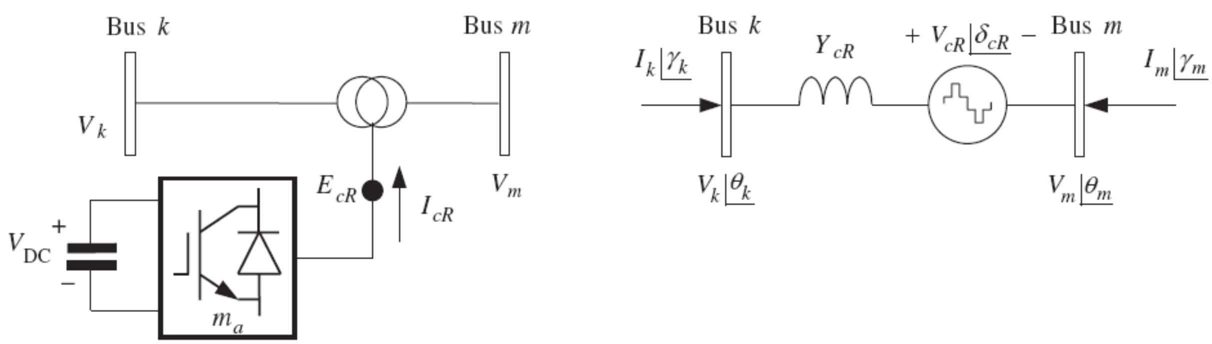


Figure 2.15 Solid state series compensator (SSSC) system: (a) voltage source converter (VSC) connected to the AC network using a series transformer and (b) series solid state voltage source

The series voltage source of the three-phase SSSC may be represented by

$$E_{cR} = V_{cR}(\cos\delta_{cR} + j\sin\delta_{cR})$$

The magnitude and phase angle of the SSSC model are adjusted by using any suitable iterative algorithm to satisfy a specified active and reactive power flow across the SSSC. Similar to the STATCOM, maximum and minimum limits will exist for the voltage magnitude V_{cR}, which are a function of the SSSC capacitor rating; the voltage phase angle δ_{cR} can take any value between 0 and 2π radians.

SSSC is one of the most important FACTS controllers. It is like a STATCOM, except that the output AC voltage is in series with the line. It can be based on voltage source converter or current source converter. Usually the injected voltage in series would be quite small compared to the line voltage, and the insulation to ground would be quite high. With an appropriate insulation between the primary and secondary of the transformer, the converter equipment is located at the ground potential until the entire converter equipment is located on a platform duly insulated from ground. The transformer ratio is tailored to the most economical converter design. Without an extra energy source, SSSC can only inject a variable voltage which, which is 90 degree leading or lagging the current. The primary of the transformer and hence the secondary as well as the converter has to carry the full line current including the fault current unless the converter is temporarily bypassed during severe line faults.

2.4.6 The Unified Power Flow Controller

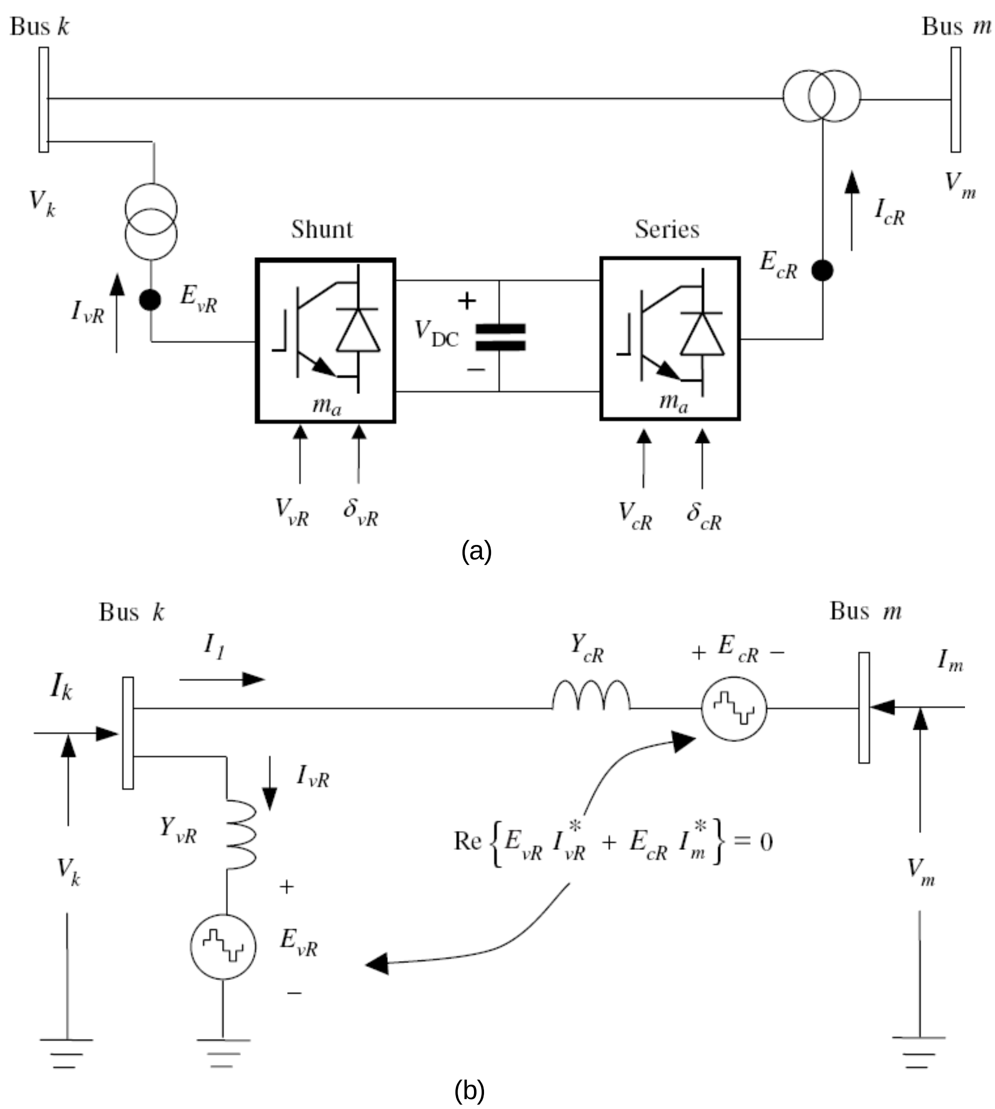


Figure 2.14 Unified power flow controller (UPFC) system: (a) two back-to-back voltage source converters (VSCs), with one VSC connected to the AC network using a shunt transformer and the second VSC connected to the AC network using a series transformer; (b) equivalent circuit based on solid-state voltage sources.

The UPFC may be seen to consist of two VSCs sharing a common capacitor on their DC side and a unified control system. A simplified schematic representation of the UPFC is given in Figure 2.14(a), together with its equivalent circuit, in Figure 2.14(b). The UPFC allows simultaneous control of active power flow, reactive power flow, and voltage magnitude at the UPFC terminals. Alternatively, the controller may be set to control one or more of these parameters in any combination or to control none of them.

The UPFC equivalent circuit shown in Figure 2.14(b) consists of a shunt-connected voltage source, a series-connected voltage source, and an active power constraint equation, which links the two voltage sources. The two voltage sources are connected to the AC system through inductive reactances representing the VSC transformers. In a three-phase UPFC, suitable expressions for the two voltage sources and constraint equation would be:

$$E_{vR} = V_{vR}(\cos\delta_{vR} + j\sin\delta_{vR})$$

$$E_{cR} = V_{cR}(\cos\delta_{cR} + j\sin\delta_{cR})$$

$$\mathrm{Re}\{-E_{vR}I_{vR}^{*} + E_{vR}I_{m}^{*}\} = 0$$

Similar to the shunt and series voltage sources used to represent the STATCOM the voltage sources used in the UPFC application would also have limits.

CHAPTER – 3

LOAD FLOW STUDIES

3.1 Introduction:

Power flow analysis is concerned with describing the operating state of an entire power system, by which we mean a network of generators, transmission lines, and loads that could represent an area as small as a municipality or as large as several states. Given certain known quantities—typically, the amount of power generated and consumed at different locations—power flow analysis allows one to determine other quantities. The most important of these quantities are the voltages at locations throughout the transmission system, which, for alternating current (a.c.), consist of both a magnitude and a time element or phase angle. Once the voltages are known, the currents flowing through every transmission link can be easily calculated. Thus the name power flow or load flow, as it is often called in the industry: given the amount of power delivered and where it comes from, power flow analysis tells us how it flows to its destination.

Owing mainly to the peculiarities of a.c., but also to the sheer size and complexity of a real power system—its elaborate topology with many nodes and links, and the large number of generators and loads—it turns out to be no mean feat to deduce what is happening in one part of the system from what is happening elsewhere, despite the fact that these happenings are intimately related through well-understood, deterministic laws of physics. Although we can readily calculate voltages and currents through the branches of small direct current (d.c.) circuits in terms of each other, even a small network of a handful of a.c. power sources and loads defies our ability to write down formulas for the relationships among all the variables: as a mathematician would say, the system cannot be solved analytically; there is no closed-form solution. We can only get at a numerical answer through a process of successive approximation or iteration. In order to find out what the voltage or current at any given point will be, we must in effect simulate the entire system.

Historically, such simulations were accomplished through an actual miniature d.c. model of the power system in use. Generators were represented by small d.c. power supplies, loads by resistors, and transmission lines by appropriately sized wires. The voltages and currents could be found empirically by direct measurement. To find out how much the current on line A would increase, for example, due to Generator X taking over power production from Generator Y, one would simply adjust the values on X and Y and go read the ammeter on line A. The d.c. model does not exactly match the behavior of the a.c. system, but it gives an approximation that is close enough for most practical purposes. In the age of computers, we no longer need to physically build such models, but can create them mathematically. With plenty of computational power, we can not only represent a d.c. system, but the a.c. system itself in a way that accounts for the subtleties of a.c. Such a simulation constitutes power flow analysis. Power flow answers the question, What is the present operating state of the system, given certain known quantities? To do this, it uses a mathematical algorithm of successive approximation by iteration, or the repeated application of calculation steps. These steps represent a process of trial and error that starts with assuming one array of numbers for the entire system, comparing the relationships among the numbers to the laws of physics, and then repeatedly adjusting the numbers until the entire array is consistent with both physical law and the conditions stipulated by the user. In practice, this looks like a computer program to which the operator gives certain input information about the power system, and which then provides output that completes the picture of what is happening in the system, that is, how the power is flowing. There are variations on what types of information are chosen as input and output, and there are also different computational techniques used by different programs to produce the output. Beyond the straightforward power flow program that simply calculates the variables pertaining to a single, existing system condition, there are more involved programs that analyze a multitude of hypothetical situations or system conditions and rank them according to some desired criteria; such programs are known as optimal power flow (OPF). Section 3.2 introduces the problem of power flow, showing how the power system is abstracted for the purpose of this analysis and how the known and unknown variables are defined. Section 3.3 explicitly states the equations used in power flow analysis and outlines a basic mathematical algorithm used to solve the problem, showing also what is meant by decoupled power flow.

3.2 The Power flow Problem:

3.2.1 Network Representation

In order to analyze any circuit, we use as a reference those points that are electrically distinct, that is, there is some impedance between them, which can sustain a potential difference. These reference points are called nodes. When representing a power system on a large scale, the nodes are called buses, since they represent an actual physical busbar where different components of the system meet. A bus is electrically equivalent to a single point on a circuit, and it marks the location of one of two things: a generator that injects power, or a load that consumes power. At the degree of resolution generally desired on the larger scale of analysis, the load buses represent aggregations of loads (or very large individual industrial loads) at the location where they connect to the high-voltage transmission system. Such an aggregation may in reality be a transformer connection to a subtransmission system, which in turn branches out to a number of distribution substations; or it may be a single distribution substation from which originate a set of distribution feeders . In any case, whatever lies behind the bus is taken as a single load for purposes of the power flow analysis.

The buses in the system are connected by transmission lines. At this scale, one does not generally distinguish among the three phases of an a.c. transmission line. Based on the assumption that, to a good approximation, the same thing is happening on each phase, the three are condensed by the model into a single line, making a so-called one-line diagram. Most power engineering textbooks provide a detailed justification of this important simplification, demonstrating that what we learn about the single “line” from our analysis can legitimately be extrapolated to all three phases. Indeed, a single line between two buses in the model may represent more than one three-phase circuit. Still, for this analysis, all the important characteristics of these conductors can be condensed into a single quantity, the impedance of the one line. Since the impedance is essentially determined by the physical characteristics of the conductors (such as their material composition, diameter, and length), it is taken to be constant. Note that this obviates the need for geographical accuracy, since the distance between buses is already accounted for within the line impedance, and the lines are drawn in whatever way they best fit on the page.

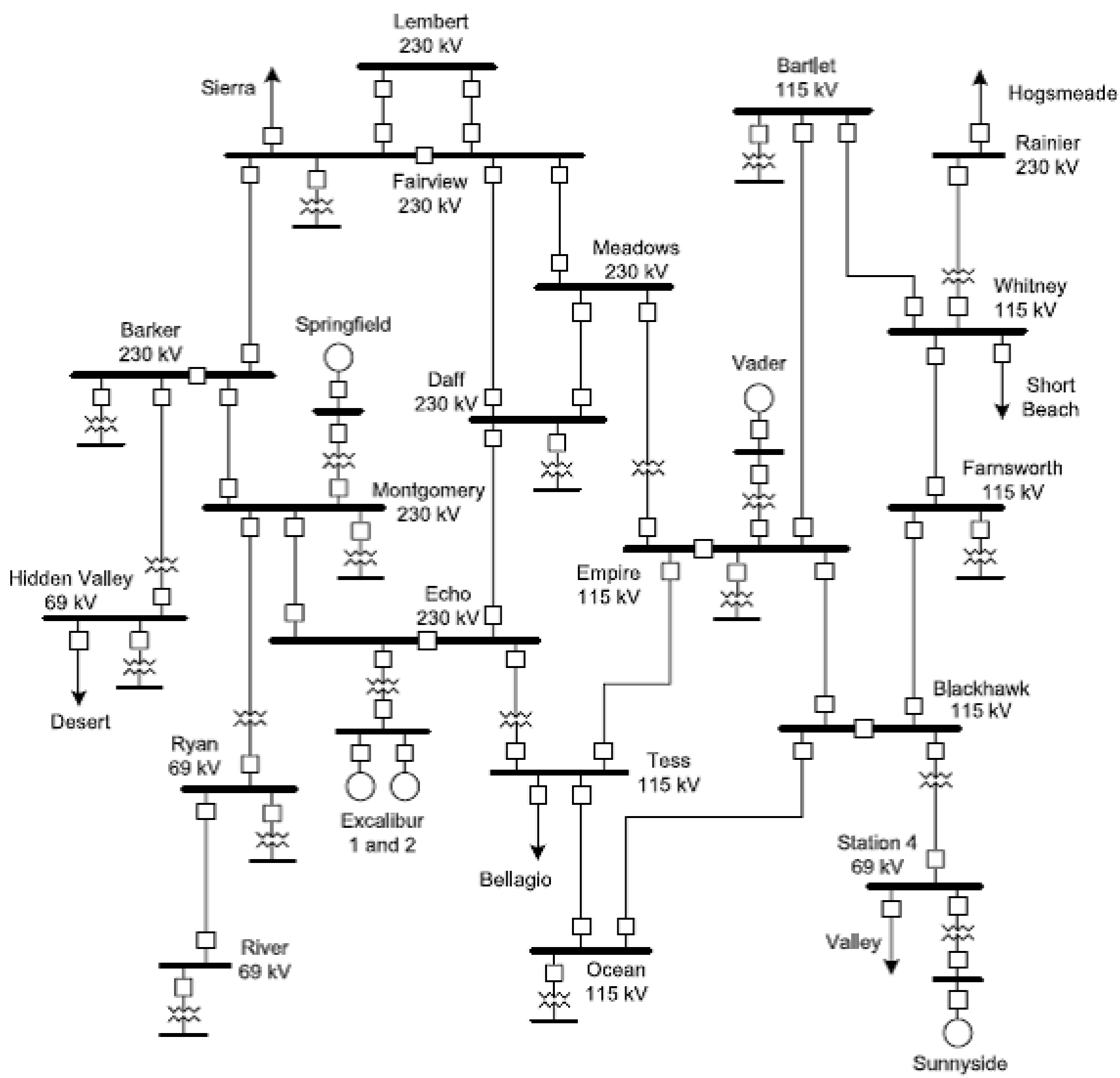


Figure 3.1 One-line diagram for a power system

Thus, the model so far represents the existing hardware of the power system, drawn as a network of buses connected by lines. An example of such a one-line diagram is shown in Figure 3.1. This topology or characteristic connection of the network may in reality be changed by switching operations, whereby, for example, an individual transmission line can be taken out of service. Such changes, of course, must be reflected by redrawing the one-line diagram, where now some lines may be omitted or assigned a new impedance value. For a given analysis run, though, the network topology is taken to be fixed

3.2.2 Choice of Variables

From the analysis of simple d.c. circuits, we are familiar with the notion of organizing the descriptive variables of the circuit into categories of "knowns" and "unknowns," whose relationships can subsequently be expressed in terms of multiple equations. Given sufficient information, these equations can then be manipulated with various techniques so as to yield numerical results for the hitherto unknowns. The conditions under which such a system of equations is solvable (meaning that it can yield unambiguous numerical answers) are straightforward: for each unknown quantity, there must be exactly one equation for each unknown. Each equation represents one statement relating one unknown variable to some set of known quantities. This set of equations must not be redundant: if any one equation duplicates information implied by the others, it does not tell us anything new and therefore does not count toward making the whole system solvable. If there are fewer equations than unknowns, we do not have enough information to decide which values the unknowns must take (in other words, the information given does not rule out a multiplicity of possibilities); if there are more equations than unknowns, the system is overspecified, meaning that some equations are either redundant or mutually contradictory. In order to determine whether an unambiguous, unique solution to a system of equations such as those describing an electric power system can be found, one must begin by taking an inventory of variables and information that translates into equations for those variables.

There are two basic quantities that describe the flow of electricity: voltage and current. Both voltage and current will vary from one location to another in a circuit, but they are everywhere related: the current through each circuit branch corresponds to the voltage or potential difference between the two nodes at either end, divided by the impedance of this branch. It is generally assumed that the impedances throughout the circuit are known, since these are more or less permanent properties of the hardware. Thus, if we are told the voltages at every node in the circuit, we can deduce from them the currents flowing through all the branches, and everything that is happening in the circuit is completely described. If one or more pieces of voltage information were missing, but we were given appropriate information about the current instead, we could still work backwards and solve the problem. In this sense, the number of variables in a circuit corresponds to the number of electrically distinct points in it: assuming we already know all the properties of the hardware, we need to be told one piece of information per node in order to figure out everything that's going on in a d.c. circuit.

For a.c. circuits, the situation is a bit more complicated, because we have introduced the dimension of time: unlike in d.c., where everything is essentially static (except for the instant at which a switch is thrown), with a.c. we are describing an ongoing oscillation or movement. Thus each of the two main variables, voltage and current, in an a.c. circuit really has two numerical components: a magnitude component and a time component. By convention, a.c. voltage and current magnitude are describes in terms of root-mean-squared (rms) values and their timing in terms of a phase angle, which represents the shift of the wave with respect to a reference point in time . To fully describe the voltage at any given node in an a.c. circuit, we must therefore specify two numbers: a voltage magnitude and a voltage angle. Accordingly, when we solve for the currents in each branch, we will again obtain two numbers: a current magnitude and a current angle. And when we consider the amount of power transferred at any point of an a.c. circuit, we again have two numbers: a real and a reactive component .

An a.c. circuit thus requires exactly two pieces of information per node in order to be completely determined. More than two, and they are either redundant or contradictory; fewer than two, and possibilities are left open so that the system cannot be solved.

A word of caution is necessary here: Owing to the nonlinear nature of the power flow problem, it may be impossible to find one unique solution because more than one answer is mathematically consistent with the given configuration. However, it is usually straightforward in such cases to identify the "true" solution among the mathematical possibilities based on physical plausibility and common sense. Conversely, there may be no solution at all because the given information was hypothetical and does not correspond to any situation that is physically possible. Still, it is true in principle—and most important for a general conceptual understanding— that two variables per node are needed to determine everything that is happening in the system.

Having discussed voltage and current, each with magnitude and angle, as the basic electrical quantities, which are known and which are unknown? In practice, current is not known at all; the currents through the various circuit branches turn out to be the last thing that we calculate once we have completed the power flow analysis. Voltage, as we will see, is known explicitly for some buses but not for others. More typically, what is known is the amount of power going into or out of a bus. Power flow analysis consists of taking all the known real and reactive power flows at each bus, and those voltage magnitudes that are explicitly known, and from this information calculating the remaining voltage magnitudes and all the voltage angles. This is the hard part. The easy part, finally, is to calculate the current magnitudes and angles from the voltages.

We know how to calculate real and reactive power from voltage and current: power is basically the product of voltage and current, and the relative phase angle between voltage and current determines the respective contributions of real and reactive power. Conversely, one can deduce voltage or current magnitude and angle if real and reactive power are given, but it is far more difficult to work out mathematically in this direction. This is because each value of real and reactive power would be consistent with many different possible combinations of voltages and currents. In order to choose the correct ones, we have to check each node in relation to its neighboring nodes in the circuit and find a set of voltages and currents that are consistent all the way around the system. This is what power flow analysis does.

3.2.3 Types of Buses:

Let us now articulate which variables will actually be given for each bus as inputs to the analysis. Here we must distinguish between different types of buses based on their actual, practical operating constraints. The two main types are generator buses and load buses, for each of which it is appropriate to specify different information. At the load bus, we assume that the power consumption is given— determined by the consumer—and we specify two numbers, real and reactive power, for each load bus. Referring to the symbols P and Q for real and reactive power, load buses are referred to as P,Q buses in power flow analysis.

At the generator buses we could in principle also specify P and Q. Here we run into two problems, however: the first has to do with balancing the power needs of the system, and the second with the actual operational control of generators. As a result, it turns out to be convenient to specify P for all but one generator, the slack bus, and to use the generator bus voltage, V, instead of the reactive power, Q, as the second variable. Generator buses are therefore called P,V buses.

3.2.4 Variables for Balancing Real Power:

Balancing the system means that all the generators in the system collectively must supply power in exactly the amount demanded by the load, plus the amount lost on transmission lines. This applies to both real and reactive power, but let us consider only real power first. If we tried to specify a system in which the sum of P generated did not match the P consumed, our analysis would yield no solution, reflecting the fact that in real life the system would lose synchronicity and crash. Therefore, for all situations corresponding to a stable operation of the system, and thus a viable solution of the power flow problem, we must require that real power generated and consumed matches up. Of course, we can vary the contributions from individual generators—that is, we can choose a different dispatch—so long as the sum of their P's matches the amount demanded by the system. As mentioned earlier, this total P must not only match the load demand, it must actually exceed that amount in order to make up for the transmission losses, which are the resistive I^2R energy losses .

Now we have a problem: How are we supposed to know ahead of time what the transmission losses are going to be? Once we have completed the power flow analysis, we will know what the current flows through all the transmission lines are going to be, and combining this information with the known line impedances will give us the losses. But we cannot tell a priori the amount of losses. The exact amount will vary depending on the dispatch, or amount of power coming from each generator, because a different dispatch will result in a different distribution of current over the various transmission paths, and not all transmission lines are the same. Therefore, if we were given a total P demanded at the load buses and attempted now to set the correct sum of P for all the generators, we could not do it.

The way to deal with this situation mathematically reflects the way it would be handled in actual operation. Knowing the total P demanded by the load, we begin by assuming a typical percentage of losses, say, 5%. We now dispatch all the generators in the system in some way so that the sum of their output approximately matches what we expect the total real power demand (load plus losses) to be: in this case, 105% of load demand. But since we do not yet know the exact value of the line losses for this particular dispatch (seeing that we have barely begun our power flow calculation), we will probably be off by a small amount. A different dispatch might, for example, result in 4.7% or 5.3% instead of 5% losses overall. We now make the plausible assumption that this uncertainty in the losses constitutes a sufficiently small amount of power that a single generator could readily provide it. So we choose one generator whose output we allow to adjust, depending on the system's needs: we allow it to "take up the slack" and generate more power if system losses are greater than expected, or less if they are smaller. In power flow analysis, this generator bus is appropriately labeled the slack bus, or sometimes swing bus.

Thus, as the input information to our power flow analysis, we specify P for one less than the total number of buses. What takes the place of this piece of information for the last bus is the requirement that the system remain balanced. This requirement will be built into the equations used to solve the power flow and will ultimately determine what the as yet unknown P of the slack bus has got to be. The blank space among the initial specifications for the slack bus, where P is not given, will be filled by another quantity, the voltage angle, which will be discussed later in this chapter, following the discussion of reactive power.

3.2.5 Variables for Balancing Reactive Power:

Analogous to real power, the total amount of reactive power generated throughout the system must match the amount of reactive power consumed by the loads.5 Whereas in the case of a mismatch of real power, the system loses synchronicity, a mismatch of reactive power leads to voltage collapse. Also analogous to real power transmission losses, there are reactive power losses. Reactive losses are defined simply as the difference between reactive power generated and reactive power consumed by the metered load.

Physically, these losses in Q reflect the fact that transmission lines have some reactance and thus tend to “consume” reactive power; in analogy to I^2R, we could call them I^2X losses. The term “consumption,” however, like the reactive power “consumption” by a load, does not directly imply an energy consumption in the sense of energy being withdrawn from the system. To be precise, the presence of reactive power does necessitate the shuttling around of additional current, which in turn is associated with some real I^2R losses “in transit” of a much smaller magnitude. The term “reactive losses” thus does not refer to any physical measure of something lost, but rather should be thought of as an accounting device. While real power losses represent physical heat lost to the environment and therefore always have to be positive, reactive losses on a given transmission link can be positive or negative, depending on whether inductive or capacitive reactance plays a dominant role.

In any case, what matters for both operation and power flow analysis is that Q, just like P, needs to be balanced at all times. Thus, just as for real power, all the generators in the system must generate enough reactive power to satisfy the load demand plus the amount that vanishes into the transmission lines.

This leaves us with the analogous problem of figuring out how much total Q our generators should produce, not knowing ahead of time what the total reactive losses for the system will turn out to be: as with real losses, the exact amount of reactive losses will depend on the dispatch. Operationally, though, the problem of balancing reactive power is considered in very different terms. When an individual generator is instructed to provide its share of reactive power, in practice this is not usually done by telling it to generate a certain number of MVAR . Instead, the generator is instructed to maintain a certain voltage magnitude at its bus. The voltage is continually and automatically adjusted through the generator’s field current , and is therefore a straightforward variable to control.

Their own bus voltage is in fact the one immediate measure available to the generators for determining whether the correct amount of reactive power is being generated: when the combined generation of reactive power by all the generators in the system matches the amount consumed, their bus voltage holds steady. Conversely, if there is a need to increase or decrease reactive power generation, adjusting the field current at one or more generators so as to return to the voltage set point will automatically accomplish this objective. The new value of MVAR produced by each generator can then be read off the dial for accounting purposes. Conveniently for power flow analysis, then, there is no need to know explicitly the total amount of Q required for the system. Specifying the voltage magnitude is essentially equivalent to requiring a balanced Q. In principle, we could specify P and Q for each generator bus, except for one slack bus assigned the voltage regulation (and thus the onus of taking up the slack of reactive power). For this “reactive slack” bus we would need to specify voltage magnitude V instead of Q, with the understanding that this generator would adjust its Q output as necessary to accommodate variations in reactive line losses. In practice, however, since voltage is already the explicit operational control variable, it is customary to specify V instead of Q for all generator buses, which are therefore called P,V buses. In a sense, this assignment implies that all generators share the "reactive slack," in contrast to the real slack that is taken up by only a single generator.

3.2.6 The Slack Bus

We have now, for our power flow analysis, three categories of buses: P,Q buses, which are generally load buses, but could in principle also be generator buses; P,V buses, which are necessarily generator buses (since loads have no means of voltage control); and then there is the slack bus, for which we cannot specify P, only V. What takes the place of P for the slack bus?

Real power balance manifests operationally as a steady frequency such as 60 Hz. A constant frequency is indicated by an unchanging voltage angle, which for this reason is also known as the power angle, at each generator. When more power is consumed than generated, the generators' rotation slows down: their electrical frequency drops, and their voltage angles fall farther and farther behind. Conversely, if excess power is generated, frequency increases and the voltage angles move forward. While generators are explicitly dispatched to produce a certain number of megawatts, the necessary small adjustments to balance real power in real-time are made (by at least one or more load-following generator) through holding the generator frequency steady at a specified value. Not allowing the frequency to depart from this reference value is equivalent to not letting the voltage angle increase or decrease over time.

In power flow analysis, the slack bus is the one mathematically assigned to do the load following. Its instructions, as it were, are to do whatever is necessary to maintain real power balance in the system. Physically, this would mean holding the voltage angle constant. The place of P will therefore be taken by the voltage angle, which is the variable that in effect represents real power balance. We can think of the voltage angle here as analogous to the voltage magnitude in the context of reactive power, where balance is achieved operationally by maintaining a certain voltage (magnitude) set point at the generator bus. Specifying that the bus voltage magnitude should be kept constant effectively amounts to saying that whatever is necessary should be done to keep the system reactive power balanced. Similarly, specifying a constant voltage angle at the generator bus amounts to saying that this generator should do whatever it takes to keep real power balanced.

We thus assign to the slack bus a voltage angle, which, in keeping with the conventional notation for the context of power flow analysis, we will call u (lowercase Greek theta). This u can be interpreted as the relative position of the slack bus voltage at time zero. Note that this u is exactly the same thing that is elsewhere called the power angle and labeled as d (delta).

What is important to understand here is that the actual numerical value of this angle has no physical meaning; what has physical meaning is the implication that this angle will not change as the system operates. The choice of a numerical value for u is a matter of convenience. When we come to the output of the power flow analysis, we will discover a voltage angle u for each of the other buses throughout the system, which is going to take on a different (constant) value for each bus depending on its relative contribution to real power. These numerical values only have meaning in relation to a reference: what matters is the difference between the voltage angle at one bus and another, which physically corresponds to the phase difference between the voltage curves, or the difference in the precise timing of the voltage maximum.9

We now conveniently take advantage of the slack bus to establish a system wide reference for timing, and we might as well make things simple and call the reference point “zero.” This could be interpreted to mean that the alternating voltage at the slack bus has its maximum at the precise instant that we depress the “start” button of an imaginary stopwatch, which starts counting the milliseconds (in units of degrees within a complete cycle of 1/60th second) from time zero. In principle, we could pick any number between 0 and 360 degrees as the voltage angle for the slack bus, but 0° is the simple and conventional choice.

3.2.7 Summary of Variables

To summarize, our three types of buses in power flow analysis are P,Q (load bus), P,V (generator bus), and θ,V (slack bus). Given these two input variables per bus, and knowing all the fixed properties of the system (i.e., the impedances of all the transmission links, as well as the a.c. frequency), we now have all the information required to completely and unambiguously determine the operating state of the system. This means that we can find values for all the variables that were not originally specified for each bus: θ and V for all the P,Q buses; θ and Q for the P,V buses; and P and Q for the slack bus. The known and unknown variables for each type of bus are tabulated later in the following paragraph for easy reference.

Table3.1 Varaibles in Power flow Analysis

Type of Bus	Variables Given (Knowns)	Variables Found (Unknowns)
Generator	Real power (P)	Voltage angle (θ)
	Voltage magnitude (V)	Reactive power (Q)
Load or generator	Real power (P)	Voltage angle (θ)
	Reactive power (Q)	Voltage magnitude (V)
Slack	Voltage angle (θ)	Real power (P)
	Voltage magnitude (V)	Reactive power (Q)

Once we know θ and V, the voltage angle and magnitude, at every bus, we can very easily find the current through every transmission link; it becomes a simple matter of applying Ohm’s law to each individual link. (In fact, these currents have to be found simultaneously in order compute the line losses, so that by the time the program announces θ’s and V’s, all the hard work is done.) Depending on how the output of a power flow program is formatted, it may state only the basic output variables, as in Table 3.1, it may explicitly state the currents for all transmission links in amperes; or it may express the flow on each transmission link in terms of an amount of real and reactive power flowing, in megawatts (MW) and MVAR.

3.3 Power flow Equations and Solution Methods:

3.3.1 Derivation of Power flow Equations

In Section 3.2, we stated the known and unknown variables for each of the different types of buses in power flow analysis. The power flow equations show explicitly how these variables are related to each other.

The complete set of power flow equations for a network consists of one equation for each node or branch point in this network, referred to as a bus, stating that the complex power injected or consumed at this bus is the product of the voltage at this bus and the current flowing into or out of the bus. Because each bus can have several transmission links connecting it to other buses, we must consider the sum of power entering or leaving by all possible routes. To help with the accounting, we will use a summation index i to keep track of the bus for which we are writing down the power equation, and a second index k to keep track of all the buses connected to i.

We express power in complex notation, which takes into account the two dimensionality— magnitude and time—of current and voltage in an a.c. system. The complex power S can be written in shorthand notation as

$$S = VI^*$$

where all variables are complex quantities and the asterisk denotes the complex conjugate of the current.

Recall that S represents the complex sum of real power P and reactive power Q, where P is the real and Q the imaginary component. While the real part represents a tangible physical measurement (the rate at which energy is transferred), the imaginary part can be thought of as the flippety component that oscillates. At different times it may be convenient to either refer to P and Q separately or simply to S as the combination.

In the most concise notation, the power flow equations can be stated as

$$S_i = V_i I_i$$

where the index i indicates the node of the network for which we are writing the equation.

Thus, the full set of equations for a network with n nodes would look like

$$\begin{gathered} S_1 = V_1 I_1 \\ S_2 = V_2 I_2 \\ \vdots \\ S_n = V_n I_n \end{gathered}$$

We can choose to define power as positive either going into or coming out of that node, as long as we are consistent. Thus, if the power at load buses is positive, that at generator buses is negative.

So far, these equations are not very helpful, since we have no idea what the I_i are. In order to mold the power flow equations into something we can actually work with, we must make use of the information we presumably have about the network itself. Specifically, we want to write down the impedances of all the transmission links between nodes. Then we can use Ohm's law to substitute known variables (voltages and impedances) for the unknowns (currents). Written in the conventional form, Ohm's law is $V = IZ$ (where Z is the complex impedance). However, when solving for the current I, this would require putting Z in the denominator:
$I = V/Z$. It is therefore tidier to use the inverse of the impedance, known as the admittance Y (where $Y = 1/Z$), so that Ohm's law becomes $I = VY$. Not only does this avoid the formatting issues of division, it also allows us to indicate the absence of a transmission link with a zero (for zero admittance) instead of infinity (for infinite impedance), which vastly improves the appearance of the imminent matrix algebra.

The complex admittance $Y = G + jB$, where G is the conductance and B the susceptance. The admittances of all the links in the network are summarized by way of an admittance matrix Y, where the lowercase $y_{ik} = g_{ik} + b_{ik}$ indicates that matrix element that connects nodes i and k. (Essentially, the matrix notation serves the purpose of tidy bookkeeping.)

The relationship $I = VY$ is what we wish to write down and substitute for every I_i that appears in the power flow equations. But now we face the next complication: the total current I_i coming out of any given node is in fact the sum of many different currents going between node i and all other nodes that physically connect to i. We will indicate these connected nodes with the index k, where k could theoretically include all nodes other than i in the network from 1 to n. In practice, fortunately, only a few of these will actually have links connecting to node i. For any node k that is not connected to i, $y_{ik} = 0$.

For the current between nodes i and k, we can now write

$$I_{ik} = V_k y_{ik}$$

or, expanding the admittance into its conductance and susceptance components,

$$I_{ik} = V_k(g_{ik} + b_{ik})$$

Note that if this equation stood by itself, we would want V_k to mean "the voltage difference between nodes i and k" rather than simply "the voltage at node k." However, we will see the voltage differences between nodes naturally arise out of the complex conjugate once we expand the terms; specifically, the voltage differences will appear in terms of differences between phase angles at each node.

To complete the statement $S_i = V_i \underline{I_i}$ for node i, we must now sum over all nodes k that could possibly be connected to i. This summation over the index k means accounting for all the current that is entering or leaving this one particular node, i, by way of the various links it has to other nodes k. (For the complete system of power flow equations, we also have to consider every value of the index i so as to consider power flow for every node, but we do not need to write this out explicitly.)

Thus,

$$S_i = V_i I_i^* = V_i \{\sum_{k=1}^{n} y_{ik} V_k\}^* \qquad \ldots\ldots..(3.1)$$

But

$$S_i = P_i + jQ_i$$

So,

$$P_i = \text{Real}\,[V_i \{\sum_{k=1}^{n} y_{ik} V_k\}^*] \qquad \ldots\ldots(3.2)$$

$$Q_i = \text{Imag}[V_i \{\sum_{k=1}^{n} y_{ik} V_k\}^*] \qquad \ldots\ldots.(3.3)$$

Now we expand the y's into g's and b's, noting that the complex conjugate gives us a minus sign in front of the jb:

$$S_i = V_i \sum_{k=1}^{n} (g_{ik} - jb_{ik}) V^*_k$$

After changing the order of terms to look more organized, we write the voltage phasors out in longhand, first as exponentials and then broken up into sines and cosines:

$$S_i = \sum_{k=1}^{n} |V_i||V_k| e^{j(\theta i-\theta k)}(g_{ik} - jb_{ik}) \qquad \ldots\ldots.(3.4)$$

$$Si = \sum_{k=1}^{n} |V_i||V_k| [\cos(\theta_i - \theta_k) + j\sin(\theta_i - \theta_k)](g_{ik} - jb_{ik}) \qquad \ldots\ldots.(3.5)$$

The term $(\theta_i - \theta_k)$ in this equation denote the difference in voltage angle between nodes i and k, where the minus sign came from having used the complex conjugate of I initially, which gave us the complex conjugate of V_k. This angle difference is not the same as the instantaneous difference in magnitude between the two voltages, but it is the correct measure of separation between the nodes for the purpose of calculating the power transferred.

The equation we now have for S_i entails the product of two complex quantities, written out in terms of their real and imaginary components. By cross-multiplying all the real and imaginary terms, we can separate the real and imaginary parts of the result S, which will be the familiar P and Q. Remembering that j times j gives -1, we obtain:

$$P_i = \sum_{k=1}^{n} |V_i||V_k| [g_{ik} \cos(\theta_i - \theta_k) + b_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| [g_{ik} \sin(\theta_i - \theta_k) - b_{ik} \cos(\theta_i - \theta_k)]$$

The complete set of power flow equations for a network of n nodes contains n such equations for S_i, or pairs of equations for P_i and Q_i. This complete set will account for every node and its interaction with every other node in the network.

3.3.2 Solution Methods

There is no analytical, closed-form solution for the set of power flow equations given in the preceding section. In order to solve the system of equations, we must proceed by a numerical approximation that is essentially a sophisticated form of trial and error.

To begin with, we assume certain values for the unknown variables. For clarity, let us suppose that these unknowns are the voltage angles and magnitudes at every bus except the slack, making them all P,Q buses (it turns out that having some P,V buses eases the computational volume in practice, but it does not help the theoretical presentation). In the absence of any better information, we would probably choose a flat start, where we assume the initial values of all voltage angles to be zero (the same as the slack bus) and the voltage magnitudes to be 100% of the nominal value, or 1 p.u. In other words, for lack of a better guess, we suppose that the voltage magnitude and angle profile across the system is completely flat.

We then plug these values into the power flow equations. Of course, we know they do not describe the actual state of the system, which was supposed to be consistent with the known variables (P's and Q's). Essentially, this will produce a contradiction: Based on the starting values, the power flow equations will predict a different set of P's and Q's than we stipulated at the beginning. Our objective is to make this contradiction go away by repeatedly inserting a better set of voltage magnitudes and angles: As our voltage profile matches reality more and more closely, the discrepancy between the P's and Q's, known as the mismatch, will shrink. Depending on our patience and the degree of precision we require, we can continue this process to reach some arbitrarily close approximation. This type of process is known as an iterative solution method, where "to iterate" means to repeatedly perform the same manipulation.

The heart of the iterative method is to know how to modify each guess in the right direction and by the right amount with each round of computation (iteration), so as to arrive at the correct solution as quickly as possible. Specifically, we wish to glean information from our equations that tells us which value was too high, which was too low, and approximately by how much, so that we can prepare a well-informed next guess, rather than blindly groping around in the dark for a better set of numbers. There are several standard techniques for doing this; the ones most commonly used in power flow analysis are the Newton–Raphson, the Gauss, and the Gauss–Seidel iterations.

3.3.3 Steps of G-S Algorithm :

Step 0. Formulate and Assemble Y_{bus} in Per Unit

Step 1. Assign Initial Guesses to Unknown Voltage Magnitudes and Angles

$$|V| = 1.0, \theta = 0$$

Step 2a. For Load Buses, Find form Eq. (1)

$$S_i = V_i I_i^* = V_i \{\sum_{k=1}^{n} y_{ik} V_k\}^*$$

$$S_i^* = V_i^* I_i = V_i^* \{\sum_{k=1}^{n} y_{ik} V_k\} \qquad \text{where } Si^* = P_i - jQ_i$$

$$= V_i^* Y_{ii} V_i + \left\{\sum_{k=1,k\neq i}^{n} V_i^* y_{ik} V_k\right\}$$

Finally we get,

$$V_i^{(t+1)} = \frac{\frac{Pi - jQi}{Vi^{*t}} - \left\{\sum_{k=1,k\neq i}^{n} y_{ik} V_k^t\right\}}{Y_{ii}}$$ where t = iteration no.

For voltage-controlled busses, Q_i and V_i can be calculated as

$$Q_i^{(t+1)} = -\,\mathrm{Imag}\left[V_i^{*t}\left\{V_i^{(t)}Yii + \left\{\sum_{k=1,k\neq i}^{n} Y_{ik} V(t)\right\}\right\}\right] \qquad \ldots\ldots(3.6)$$

Then,

$$V_i^{(t+1)} = \frac{\frac{Pi - jQi}{Vi^{*t}} - \left\{\sum_{k=1,k\neq i}^{n} y_{ik} V_k^t\right\}}{Y_{ii}} \qquad \ldots\ldots(3.7)$$

However, $|V_i|$ is specified for voltage-controlled busses. So $V_i^{(t+1)} = |V_{i,spec}| \angle \theta_i{}^{(t+1)}{}_{cal}$. In using Eqs. (3.4) and (3.5), one must remember to use the most recently calculated values of bus voltages in each iteration. So, for example, if there are five busses in the system being studied, and one has determined new values of bus voltages at busses 1–3, then during the determination of bus voltage at bus 4, one should use these newly calculated values of bus voltages at 1, 2, and 3; busses 4 and 5 will have the values from the previous iteration.

Step 2b. For Faster Convergence, Apply Acceleration Factor to Load Buses

$$V_{i,acc}^{(t+1)} = V_{i,acc}^{(t)} + \alpha(V_i^{(t)} - V_{i,acc}^{(t)})$$

where α is acceleration factor.

Step 3. Check Convergence

$$|Re|[V_i^{(t+1)}] - Re[V_i^{(t)}] \leq \varepsilon$$

That is, the absolute value of the difference of the real part of the voltage between successive iterations should be less than a tolerance value ε. Typically, $\varepsilon \leq 10^{-4}$, and also,

$$|Imag|[V_i^{(t+1)}] - Imag[V_i^{(t)}] \leq \varepsilon$$

That is, the absolute value of the difference of the imaginary value of the voltage should be less than a tolerance value ε. If the difference is greater than tolerance, return to Step 3. If the difference is less than tolerance, the solution has converged; go to Step 4.

Step 4. Find Slack Bus Power P_G and Q_G from Eqs. (3.2) and (3.3)

Step 5. Find All Line Flows

As the last step in any power-flow solution, one has to find the line flows. Line current I_{jj}, at bus i is defined positive in the direction i → j.

$$\mathbf{I_{ij} = I_s + I_{pi}(V_i^* - V_j^*)_{ys} + V_i Y_{pi}} \quad \text{........(3.8)}$$

Let S_{ij}, S_{ji} be line powers defined positive into the line at bus *i* and *j*, respectively

$$\mathbf{S_{ij}} = P_{ij} + jQ_{ij} = \mathbf{V_i I_{ij}^*} = \mathbf{V_i(V_i^* - V_j^*)Y_s^*} + |V_i|^2\mathbf{Y_{pi}}^* \quad \text{........(3.9)}$$

$$\mathbf{S_{ji}} = P_{ji} + jQ_{ji} = \mathbf{V_j I_{ji}^*} = \mathbf{V_j(Vj^* - V_i^*)Y_s^*} + |V_j|^2\mathbf{Y_{pi}}^* \quad \text{........(3.10)}$$

The power loss in line (*i - j*) is the algebraic sum of the power flows determined from (9) and (10).

$$\therefore \mathbf{S_{Lij} = S_{ij} + S_{ji}} \quad \text{........(3.11)}$$

3.3.4 Newton-Raphson (N-R) Method for Power-Flow Solution

The Newton-Raphson method enables us to replace the nonlinear set of power-flow equations of (1) with a linear set. We will show this after the basis for the method is explained. The Taylor series expansion of a function f(x) of a single variable, x, around the point (x- a) is given by

$$f(x) = f(a) + (x - a)\left(\frac{\partial f}{\partial X}\right)_{X=a} + \left(\frac{(X-a)^n}{2!}\frac{\partial^2 f}{\partial X^2}\right)_{X=a} + \ldots\ldots + \frac{(X-a)^n}{!n}\frac{\partial^n f}{\partial X^n} + Ћ_n \quad \text{.......(3.12)}$$

where $\left(\frac{\partial f}{\partial X}\right)_{X=a}$ value of the derivative evaluated at x = *a*.

The series converges if $\lim_{n\to\infty} Ћ_n = 0$.

If (x - a) ≪ 1 then we can neglect the higher-order terms and write (11) as

$$f(x) \approx f(a) + (x - a)\left(\frac{\partial f}{\partial x}\right)_{x=a} \quad \text{............(3.13)}$$

For a function of n variables, one can expand around the point: $(x_1 - a_1)$, $(x_2 \_ a_2)$, …$(x_n - a_n)$ with $(x_k - a_k) \ll 1$ and k = 1, 2, . . . , n. Then, Eq. (11) becomes

$$f(x_1, x_2, ..., x_n) \approx f(a_1, a_2, \ldots, a_n) + (x_1 - a_1)\frac{\partial f}{\partial x_1}\Big|_{a1} + (x_2 - a_2)\frac{\partial f}{\partial x_2}\Big|_{a2} + \ldots (x_n - a_n)\frac{\partial f}{\partial x_n}\Big|_{an}$$

…………(3.14)

Let us consider a set of nonlinear equations, each a function of n variables:

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= y_1 \\ f_2(x_1, x_2, \ldots, x_n) &= y_2 \\ f_n(x_1, x_2, \ldots, x_n) &= y_n \end{aligned}$$ …………(3.15)

or

$$f_k(x_1, x_2, \ldots, x_n) = y_k \qquad k = 1, 2, \ldots, n$$

Assume initial values $x_k^{(0)}$ and some correction, _Δx_k, which when added to .$x_k^{(0)}$ yield $x_k^{(1)}$. When $x_k^{(0)}$ are close to the solution, the Δx_k^s, are small.

Using the approximate Taylor's series, we have

$$f_k(x_1, x_2, \ldots, x_n) = f_k(x_1^{(0)}, x_2^{(0)}, \quad \ldots \quad, x_n^{(0)}) + \Delta x_1\frac{\partial f_k}{\partial x_1}\Big|_{x1^{(0)}} + \Delta x_2\frac{\partial f_k}{\partial x_2}\Big|_{x2^{(0)}} +$$

$$+ \quad \ldots \quad + \quad \Delta x_n\frac{\partial f_k}{\partial x_n}\Big|_{xn^{(0)}} = y_k \quad k = 1, 2, \ldots, n$$ ……….(3.16)

or, in matrix form,

$$\begin{bmatrix} y_1 - f_1(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\ y_2 - f_2(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \\ \vdots \\ y_n - f_n(x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}\Big|_{x_1^{(0)}} & \frac{\partial f_1}{\partial x_2}\Big|_{x_2^{(0)}} & \cdots & \frac{\partial f_1}{\partial x_n}\Big|_{x_n^{(0)}} \\ \frac{\partial f_2}{\partial x_1}\Big|_{x_1^{(0)}} & \frac{\partial f_2}{\partial x_1}\Big|_{x_2^{(0)}} & \cdots & \frac{\partial f_2}{\partial x_n}\Big|_{x_n^{(0)}} \\ \vdots & \vdots & \vdots & \\ \frac{\partial f_n}{\partial x_1}\Big|_{x_1^{(0)}} & \frac{\partial f_n}{\partial x_2}\Big|_{x_2^{(0)}} & \cdots & \frac{\partial f_n}{\partial x_n}\Big|_{x_n^{(0)}} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

……….(3.17)

or

$$[\Delta U]^{(0)} = [J]^{(0)}[\Delta X]^{(0)}$$ ……….(3.18)

Where [J] is the jacobian matrix

$$\therefore [\Delta X]^{(0)} = ([J]^{(0)})^{-1}\Delta U]^{(0)}$$ ……….(3.19)

To continue iteration, find $[X]^{(1)}$ from

$$[X]^{(1)} = [X]^{(0)} + [\Delta X]^{(0} \quad \text{……….(3.20)}$$

Generally,

$$[X]^{(k+1)} = [X]^{(k)} + [\Delta X]^{(k)} \quad \text{……….(3.21)}$$

where k = iteration number.

3.3.5 The Newton-Raphson Method Applied to Power-Flow Equations

The N-R method is typically applied on the real form of the power-flow equations:

$$P_i = \sum_{k=1}^{n} |V_i||V_k||y_{ik}| \cos(\theta_k - \theta_i + y_{ik}) = f_{ip} \quad \text{……….(3.22)}$$

$$Q_i = -\sum_{k=1}^{n} |V_i||V_k||y_{ik}| \sin(\theta_k - \theta\delta_i + y_{ik}) = f_{iq} \quad i = 1, \ldots, n \quad \text{……….(3.23)}$$

Assume, temporarily, that all busses, except bus 1, are of the "load" type. Thus, the unknown parameters consist of the (n - 1) voltage phasors, $V_2, \ldots, V_n$ In terms of real variables, these are:

Angles	$\theta_2, \theta_3, \ldots, \theta_n$	(n _ 1) variables
Magnitudes	$\|V_2\|, \|V_3\|, \ldots, \|V_n\|$	(n _ 1) variables

Rewriting Eq (3.16) for the power-flow equations,

$$\begin{pmatrix} \Delta P_2 \\ \vdots \\ \Delta Pn \\ \Delta Q_2 \\ \vdots \\ \Delta Q_n \end{pmatrix} = \begin{pmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_n} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_n} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial Q_n}{\partial \delta_2} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_2} & \cdots & \frac{\partial Q_n}{\partial V_n} \end{pmatrix} \begin{pmatrix} \Delta\theta_2 \\ \vdots \\ \Delta\theta_n \\ \Delta V_2 \\ \vdots \\ \Delta V_n \end{pmatrix}$$

……….(3.24)

Before proceeding any further, we need to account for voltage-controlled busses. For every voltage-controlled bus in the system, delete the corresponding row and column from the Jacobian matrix. This is done because the mismatch element for a voltage controlled bus is unknown.

Writing Eq. (24) in matrix form,

$$\Delta \mathbf{U}^{(0)} = \mathbf{J}^{(0)}\Delta \mathbf{X}^{(0)} \qquad \text{……….(3.25)}$$

Where

$\Delta \boldsymbol{U}^{(0)}$ = vector of power mismatches at initial guesses
$\boldsymbol{J}^{(0)}$ = the Jacobian matrix evalutated at the initial guesses
$\Delta \boldsymbol{X}^{(0)}$ = the error vector at the zeroth iteration

3.3.6. Elements of Jacobian Marrix for N.R. method

From Eq (3.1) the injected complex power at any bus in any metwork is,

$$S_i = V_i I_i^* = V_i \{\sum_{k=1}^{n} Y_{ik}V_k\}^*$$

$$S_i^* = V_i^* I_i = V_i^* \{\sum_{k=1}^{n} Y_{ik}V_k\} \qquad \text{……(3.25)}$$

Where $Si^* = P_i - j$

Taking

$$V_i = V_i(\cos\theta_i + j\sin\theta_i) \qquad \text{……(3.26)}$$

$$Y_{ik} = g_{ik} + jb_{ik} \qquad \text{……(3.27)}$$

$$V_k = V_k(\cos\theta_k + j\sin\theta_k) \qquad \text{……(3.28)}$$

From Eq. 3.26 – 3.28, Eq 3.25 can be rewritten as

$$Si^* = P_i - jQ_i = V_i^*(Y_{i1}V_1 + Y{=}V_2 + \ldots Y_{in}V_n)$$

$$= V_i^*Y_{i1}V_1 + V_i^*Y_{i2}V_2 + \ldots + V_i^*Y_{in}V_n$$

$$= Y_{i1} V_1V_i\{\cos(\theta_1 - \theta_i) + j\sin(\theta_1 - \theta_i\} + Y_{i2}V_2V_i\{\cos(\theta_2 - \theta_i) + j\sin(\theta_2 - \theta_i\} + \ldots + Y_{in}V_nV_i\{\cos(\theta_n - \theta_i) + j\sin(\theta_n - \theta_i\}$$

$$= V_1V_i[\{g_{i1}\cos((\theta_1 - \theta_i) - b_{i1}\sin(\theta_1 - \theta_i)\} + j\{-g_{i1}\sin(\theta_1 - \theta_i) + b_{i1}\cos(\theta_1 - \theta_i)\}] + \ldots + V_nV_i[\{g_{in}\cos(\theta_n - \theta_i) + b_{in}\sin(\theta_n - \theta_i)\} + j\{-g_{in}\sin(\theta_n - \theta_i) + b_{in}\cos(\theta_n - \theta_i)\}].$$

Separating in to real and imaginary gives P_i and Q_i as,

$$P_I = V_1V_i\{g_{i1}\cos((\theta_1 - \theta_i) - b_{i1}\sin(\theta_1 - \theta_i)\} + \ldots + V_nV_i\{g_{in}\cos(\theta_n - \theta_i) + b_{in}\sin(\theta_n - \theta_i)\} \quad \ldots\ldots(3.29)$$

$$Q_i = V_1V_i\{g_{i1}\sin(\theta_1 - \theta_i) + b_{i1}\cos(\theta_1 - \theta_i) + \ldots + V_nV_i\{g_{in}\sin(\theta_n - \theta_i) + b_{in}\cos(\theta_n - \theta_i)\} \quad \ldots(3.30)$$

Differentiation of Eq (5) and (6) gives Elements of jacobian matrix as,

Case (1) when $i \neq k$

$$\frac{\partial P_i}{\partial \theta_k} = V_iV_k(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k) \quad \ldots\ldots(3.31)$$

$$\frac{\partial Q_i}{\partial \theta_k} = -V_iV_k(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k) \quad \ldots\ldots(3.32)$$

$$\frac{\partial P_i}{\partial V_k} = V_i(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k) \quad \ldots\ldots(3.33)$$

$$\frac{\partial Q_i}{\partial V_k} = V_i(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k) \quad \ldots\ldots(3.34)$$

Case (2) when I =k

$$\frac{\partial P_i}{\partial \theta_i} = V_i \sum_{k=1}^{n}[V_k\{(-g_{ik}\sin(\theta_i - \theta_k) + b_{ik}\cos(\theta_i - \theta_k)\}] - V_i^2 b_{ii} \quad \ldots\ldots(3.35)$$

$$\frac{\partial P_i}{\partial V_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)\}] + V_i g_{ii} \quad \ldots\ldots(3.36)$$

$$\frac{\partial Q_i}{\partial \theta_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)\}] - V_i^2 g_{ii} \quad \ldots\ldots(3.37)$$

$$\frac{\partial Q_i}{\partial V_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)\}] - V_i b_{ii} \quad \ldots\ldots(3.38)$$

Hence, using Eq 3.30 to 3.38 we can determine elements of the jacobian matrix.

3.3.7 Algorithm of Newton-Raphson Method

Step 0. Formulate and Assemble Y_{bus} in Per Unit

Step 1. Assign Initial Guesses to Unknown Voltage Magnitudes and Angles for a Flat Start

$$|V| = 1.0, \theta = 0$$

Step 2. Determine the Mismatch Vector ΔU for Iteration k

Step 3. Determine the Jacobian Matrix J for Iteration k

Step 4. Determine Error Vector ΔX from Eq. (24)

Set X at iteration (k + 1): $X^{(k+1)} = X^{(k)} + \Delta X^{(k)}$. Check if the power mismatches are within tolerance. If so, go to Step 5. Otherwise, go back to Step 2.

Step 5. Find Slack Bus Power P_G and Q_G from Eqs. (3.2) and (3.3)

Step 6. Compute Line Flows Using Eqs. (3.9) and (3.10) and the Total Line Losses from Eq. (3.11)

3.3.8 Characteristics of the Newton-Raphson Load Flow

With sparse programming techniques and optimally ordered triangular factorization, the Newton method for solving load flow has become faster than other methods for large systems. The number of iterations is virtually independent of system size (from a flat voltage start and with no automatic adjustments) due to the quadratic characteristic of convergence. Most systems are solved in 2 + 5 iterations with no acceleration factors being necessary.

With good programming, the time per iteration rises nearly linearly with the number of system buses N , so that the overall solution time varies as N . One Newton iteration is equivalent to about seven Gauss-Seidel iterations. For a 500-bus system, the conventional Gauss-Seidel method takes about 500 iterations and the speed advantage of the Newton method is then 15:l. Storage requirements of the Newton method are greater, however, but increase linearly with system size. It is, therefore, attractive for large systems.

The Newton method is very reliable in system solving, given good starting approximations. Heavily loaded systems with phase shifts up to 90" can be solved. The methods not troubled by ill-conditioned systems and the location of slack bus is not critical. Due to the quadratic convergence of bus voltages, high accuracy (near exact solution) is obtained in only a few iterations. This is important for the use of load flow in short circuit and stability studies. The method is readily extended to include tap-changing transformers, variable constraints on bus voltages, and reactive and optimal power scheduling. Network modifications are easily made.

CHAPTER – 4

POWER FLOW ANALYSIS WITH STATCOM

4.1 Operation Principles of the STATCOM

As we have discussed in chapter 2 A STATCOM is usually used to control transmission voltage by reactive power shunt compensation. Typically, a STATCOM consists of a coupling transformer, an inverter and a DC capacitor, which is shown in Fig. 3.1. For such an arrangement, in ideal steady-state analysis, it can be assumed that the active power exchange between the AC system and the STATCOM can be neglected, and only the reactive power can be exchanged between them.

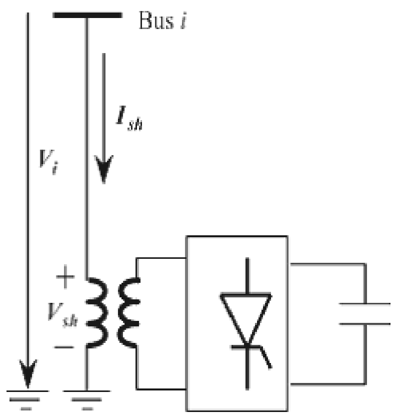


Fig. 3.1. STATCOM

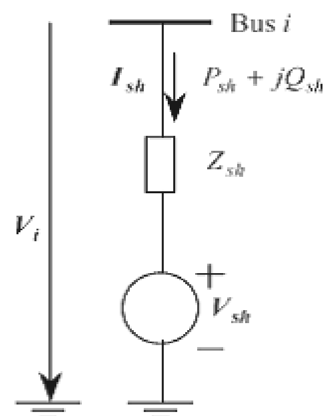


Fig.3.2.STATCOM equivalent circuit

4.2 Power flow Constraints of the STATCOM

Based on the operating principle shown in Fig. 3.1, the equivalent circuit of the STATCOM can be derived, which is given in Fig. 3.2. In the derivation, it is assumed that (a) harmonics generated by the STATCOM are neglected; (b) the system as well as the STATCOM are three phase balanced. Then the STATCOM can be equivalently represented by a controllable fundamental frequency positive sequence voltage source V_{sh}. In principle, the STATCOM output voltage can be regulated such that the reactive power of the STATCOM can be changed.

According to the equivalent circuit of the STATCOM shown in Fig. 3.2, suppose

$$\mathbf{V_{sh}} = V_{sh} \angle\theta_{sh}, \quad \mathbf{V_i} = Vi\angle\theta_i,$$

then the power flow constraints of the STATCOM are:

$$P_{sh} = V_i^2 g_{sh} - V_iV_{sh}(g_{sh}\cos(\theta_i - \theta_{sh}) + b_{sh}\sin(\theta_i - \theta_{sh})) \qquad (4.1)$$
$$Q_{sh} = -V_i^2 b_{sh} - V_iV_{sh}(g_{sh}\sin(\theta_i - \theta_{sh}) - b_{sh}\cos(\theta_i - \theta_{sh}))$$

Where $g_{sh} + jb_{sh} = 1/Z_{sh}$.

Operating constraint of the STATCOM – the active power exchange via the DC link is zero, which is described by

$$PE = Re(V_{sh}I_{sh}^{*}) = 0$$

Where

$$Re(V_{sh}I_{sh}^{*}) = V_{sh}^{2} g_{sh} - V_i V_{sh}(g_{sh} \cos(\theta_i - \theta_{sh}) - b_{sh} \sin(\theta_i - \theta_{sh}))$$

4.3 Multi-control functions of the STATCOM

In the practical applications of a STATCOM, it may be used for controlling one of the following parameters: (1) the voltage magnitude of the local bus, to which the STATCOM is connected, (2) the reactive power injection to the local bus, to which the STATCOM is connected, (3) the impedance of the STATCOM, (4) the current magnitude of the STATCOM while the current Ish leads the voltage injection V_{sh} by 90◦, (5) the current magnitude of the STATCOM, while the current I_{sh} lags the voltage injection V_{sh} by 90◦, (6) the voltage injection control, (7) the voltage magnitude at a remote bus, (8) the reactive power flow control, (9) the apparent power or current control of a local or remote transmission line. Among these control options, (1) control of the voltage of the local bus, which the STATCOM is connected to, is the well-recognized control function. The other control possibilities have not fully been investigated in power flow analysis. So, the mathematical descriptions of the only control functions, voltage control is presented.

4.4. Control function : bus voltage control

The bus control constraint is as follows:

$$V_i - V_{Speci} = 0$$

Or

$$F = V_i - V_{Speci} = 0$$

Where V_{Speci} is the bus voltage control reference.

4.5 Implementation of Voltage-Control Functional Model of STATCOM in Newton power flow

For a STATCOM, it has only one degree of freedom for control since the active power exchange with the DC link should be zero at any time. The Newton power flow equation including power mismatch constraints of buses i, j, k, and the STATCOM control constraints may be represented by

$$
\begin{pmatrix} \Delta P_2 \\ \vdots \\ \Delta P_3 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_n \\ \Delta PE \\ \Delta F_2 \end{pmatrix}
=
\begin{pmatrix}
\frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_n} & \frac{\partial P_2}{\partial V_{sh}} & \frac{\partial P_2}{\partial \theta_{sh}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\frac{\partial P_n}{\partial \delta_2} & \cdots & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_n} & \frac{\partial P_n}{\partial V_{sh}} & \frac{\partial P_n}{\partial \theta_{sh}} \\
\frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_n} & \frac{\partial Q_2}{\partial V_{sh}} & \frac{\partial Q_2}{\partial \theta_{sh}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\
\frac{\partial Q_n}{\partial \delta_2} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_2} & \cdots & \frac{\partial Q_n}{\partial V_n} & \frac{\partial Q_n}{\partial V_{sh}} & \frac{\partial Q_n}{\partial \theta_{sh}} \\
\frac{\partial PE}{\partial \delta_2} & \cdots & \frac{\partial PE}{\partial \delta_n} & \frac{\partial PE}{\partial V_2} & \cdots & \frac{\partial PE}{\partial V_n} & \frac{\partial PE}{\partial V_{sh}} & \frac{\partial PE_2}{\partial \theta_{sh}} \\
\frac{\partial F_2}{\partial \delta_2} & \cdots & \frac{\partial F_2}{\partial \delta_n} & \frac{\partial F_2}{\partial V_2} & \cdots & \frac{\partial F_2}{\partial V_n} & \frac{\partial F_2}{\partial V_{sh}} & \frac{\partial F_2}{\partial \theta_{sh}}
\end{pmatrix}
\begin{pmatrix} \Delta \theta_2 \\ \vdots \\ \Delta \theta_n \\ \Delta V_2 \\ \vdots \\ \Delta V_n \\ \Delta V_{sh} \\ \Delta \theta_{sh} \end{pmatrix}
$$

or

$$\Delta \mathbf{U} = \mathbf{J} . \Delta \mathbf{X}$$

where ΔP and ΔQ are the real and reactive power mismatches.

The STATCOM has two state variables θ_{sh} and V_{sh} and two equalities. The two equalities formulate the first two rows of the above Newton equation. The first equality is the active power balance equation described by

$$PE = Re(V_{sh} I_{sh}^*) = 0$$

while the second equality is the control constraint of the STATCOM, which is generally described by

$$F = V_i - V_{Speci} = 0$$

Elements of jacobian matrix can be calculated by taking partial derivatives of the corresponding equation.

3.3.6. Elements of Jacobian Marrix for N.R. method

From Eq (3.1) the injected complex power at any bus in any metwork is,

$$S_i = V_i I_i^* = V_i \{\textstyle\sum_{k=1}^{n} Y_{ik} V_k\}^*$$

$$S_i^* = V_i^* I_i = V_i^* \{\textstyle\sum_{k=1}^{n} Y_{ik} V_k\} \qquad \ldots\ldots(3.25)$$

Where $Si^* = P_i - j$

Taking

$$V_i = V_i(\cos\theta_i + j\sin\theta_i) \qquad \ldots\ldots(3.26)$$

$$Y_{ik} = g_{ik} + jb_{ik} \qquad \ldots\ldots(3.27)$$

$$V_k = V_k(\cos\theta_k + j\sin\theta_k) \qquad \ldots\ldots(3.28)$$

From Eq. 3.26 – 3.28, Eq 3.25 can be rewritten as

$$Si^* = P_i - jQ_i = V_i^*(Y_{i1}V_1 + Y{=}V_2 + \ldots Y_{in}V_n)$$

$$= V_i^*Y_{i1}V_1 + V_i^*Y_{i2}V_2 + \ldots + V_i^*Y_{in}V_n$$

$$= Y_{i1}V_1V_i\{\cos(\theta_1 - \theta_i) + j\sin(\theta_1 - \theta_i\} + Y_{i2}V_2V_i\{\cos(\theta_2 - \theta_i) + j\sin(\theta_2 - \theta_i\} + \ldots + Y_{in}V_nV_i\{\cos(\theta_n - \theta_i) + j\sin(\theta_n - \theta_i\}$$

$$= V_1V_i[\{g_{i1}\cos((\theta_1 - \theta_i) - b_{i1}\sin(\theta_1 - \theta_i)\} + j\{-g_{i1}\sin(\theta_1 - \theta_i) + b_{i1}\cos(\theta_1 - \theta_i)\}] + ..+V_nV_i[\{g_{in}\cos(\theta_n - \theta_i) + b_{in}\sin(\theta_n - \theta_i)\} + j\{-g_{in}\sin(\theta_n - \theta_i) + b_{in}\cos(\theta_n - \theta_i)\}].$$

Separating in to real and imaginary gives P_i and Q_i as,

$$P_I = V_1V_i\{g_{i1}\cos((\theta_1 - \theta_i) - b_{i1}\sin(\theta_1 - \theta_i)\} + \ldots + V_nV_i\{g_{in}\cos(\theta_n - \theta_i) + b_{in}\sin(\theta_n - \theta_i)\} \qquad \ldots\ldots(3.29)$$

$$Q_i = V_1V_i\{g_{i1}\sin(\theta_1 - \theta_i) + b_{i1}\cos(\theta_1 - \theta_i) + \ldots + V_nV_i\{g_{in}\sin(\theta_n - \theta_i) + b_{in}\cos(\theta_n - \theta_i)\} \qquad \ldots\ldots(3.30)$$

Differentiation of Eq (5) and (6) gives Elements of jacobian matrix as,

Case (1) when $i \neq k$

$$\frac{\partial P_i}{\partial \theta_k} = V_iV_k(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k) \qquad \ldots\ldots(3.31)$$

$$\frac{\partial Q_i}{\partial \theta_k} = -V_iV_k(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k) \qquad \ldots\ldots(3.32)$$

$$\frac{\partial P_i}{\partial V_k} = V_i(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k) \qquad \ldots\ldots(3.33)$$

$$\frac{\partial Q_i}{\partial V_k} = V_i(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k) \qquad \ldots\ldots(3.34)$$

Case (2) when I =k

$$\frac{\partial P_i}{\partial \theta_i} = V_i \sum_{k=1}^{n}[V_k\{(-g_{ik}\sin(\theta_i - \theta_k) + b_{ik}\cos(\theta_i - \theta_k)\}] - V_i^2 b_{ii} \quad ......(3.35)$$

$$\frac{\partial P_i}{\partial V_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)\}] + V_i g_{ii} \quad ......(3.36)$$

$$\frac{\partial Q_i}{\partial \theta_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)\}] - V_i^2 g_{ii} \quad ......(3.37)$$

$$\frac{\partial Q_i}{\partial V_i} = V_i \sum_{k=1}^{n}[V_k\{(g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)\}] - V_i b_{ii} \quad ......(3.38)$$

Hence, using Eq 3.30 to 3.38 we can determine elements of the jacobian matrix.

3.3.7 Algorithm of Newton-Raphson Method

Step 0. Formulate and Assemble Y_{bus} in Per Unit

Step 1. Assign Initial Guesses to Unknown Voltage Magnitudes and Angles for a Flat Start

$$|V| = 1.0, \theta = 0$$

Step 2. Determine the Mismatch Vector ΔU for Iteration k

Step 3. Determine the Jacobian Matrix J for Iteration k

Step 4. Determine Error Vector ΔX from Eq. (24)

Set X at iteration (k + 1): $X^{(k+1)} = X^{(k)} + \Delta X^{(k)}$. Check if the power mismatches are within tolerance. If so, go to Step 5. Otherwise, go back to Step 2.

Step 5. Find Slack Bus Power P_G and Q_G from Eqs. (3.2) and (3.3)

Step 6. Compute Line Flows Using Eqs. (3.9) and (3.10) and the Total Line Losses from Eq. (3.11)

3.3.8 Characteristics of the Newton-Raphson Load Flow

With sparse programming techniques and optimally ordered triangular factorization, the Newton method for solving load flow has become faster than other methods for large systems. The number of iterations is virtually independent of system size (from a flat voltage start and with no automatic adjustments) due to the quadratic characteristic of convergence. Most systems are solved in 2 + 5 iterations with no acceleration factors being necessary.

With good programming, the time per iteration rises nearly linearly with the number of system buses N , so that the overall solution time varies as N . One Newton iteration is equivalent to about seven Gauss-Seidel iterations. For a 500-bus system, the conventional Gauss-

Seidel method takes about 500 iterations and the speed advantage of the Newton method is then 15:l. Storage requirements of the Newton method are greater, however, but increase linearly with system size. It is, therefore, attractive for large systems.

The Newton method is very reliable in system solving, given good starting approximations. Heavily loaded systems with phase shifts up to 90" can be solved. The methods not troubled by ill-conditioned systems and the location of slack bus is not critical. Due to the quadratic convergence of bus voltages, high accuracy (near exact solution) is obtained in only a few iterations. This is important for the use of load flow in short circuit and stability studies. The method is readily extended to include tap-changing transformers, variable constraints on bus voltages, and reactive and optimal power scheduling. Network modifications are easily made.

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CHAPTER - 5

CASE STUDIES AND RESULTS

To test the convergence of the developed model, a number of case studies were carried out on a simple four bus system and the IEEE 30-bus test system. In each of these test systems, multiple STATCOMs were incorporated. In all the case studies, a convergence tolerance of 10^{-10} p.u has been chosen. The initial conditions for the STATCOM voltage source were chosen as $1.0\angle 0^0$ p.u. The coupling transformer impedance was taken as (0.01+j 0.1) p.u.. The case studies are elaborated below:

Case 1. This is a base case without STATCOM for a simple four bus system. A graph bas been plotted between mismatch vector and the no. of iterations as shown in fig. 5.1.

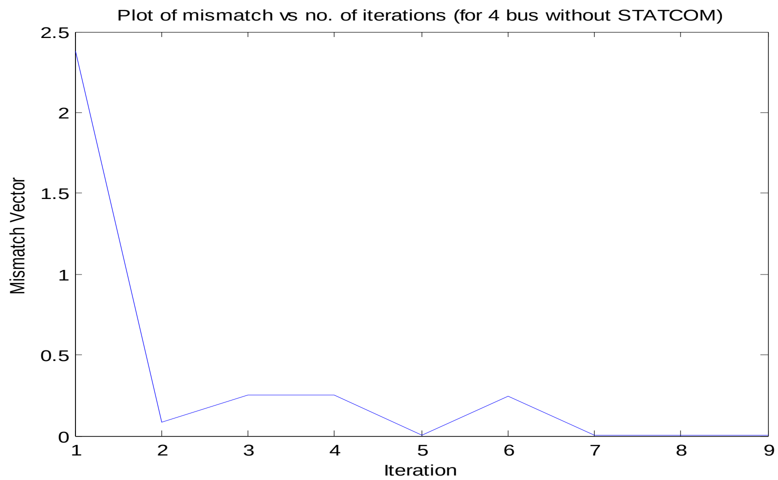

Fig. 5.1

It is clear from the graph that 9 iterations are required to satisfy our convergence criteria. Also voltages at different buses are shown in the table below.

Table 4.1

Bus No.	Voltage in p.u.
1	1.0000
2	0.9526
3	09498
4	.9700

From table 4.1, it is observed that among all the buses, voltage at bus no. 3 requires more compensation then other.

Case 2. It is similar to Case 1 except that there is a STATCOM installed in the bus no.3 at which voltage is lowest. Results are shown below.

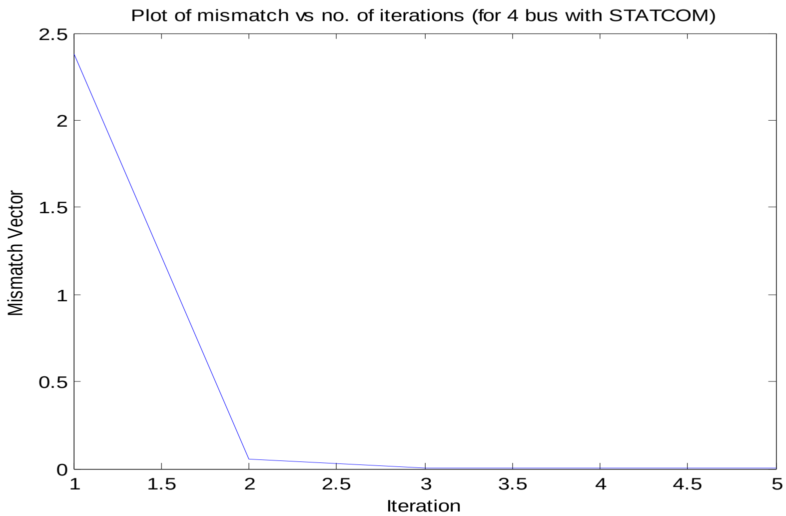

Fig. 5.2

The solution is obtained in five iterations This can be easily observed here that incorporation of the STATCOM improves the voltage profile at bus 3. In addition the voltages at bus numbers 2 and 4 also show an improvement due to the STATCOM at bus 3. It is also observed that the number of iterations is also reduced when STATCOM is incorporated.

Table 4.2

Bus No.	V(p.u.) without STATCOM	V(p.u.) with STATCOM
1	1.0000	1.0000
2	0.9526	0.9706
3	0.9648	1.0000
4	0.9700	1.0000

From table 4.2 we can observe that voltage profile of the system has been improved. Although, No. of iterations with installation of STATCOM is increased.

Table 4.3 shows that in order to improve voltage profile at bus no. 3, STATCOM supplies reactive power to the bus

Table 4.3

Statcom Bus	V_{sh}	θ_{sh}	Q_{sh}
2	1.1588	-3.0922	1.5896

Case 3. It is similar to Case 1 except that simple four bus system is replaced by IEEE 30 bus system. A plot between no. of iterations and mismatch vector has been shown.

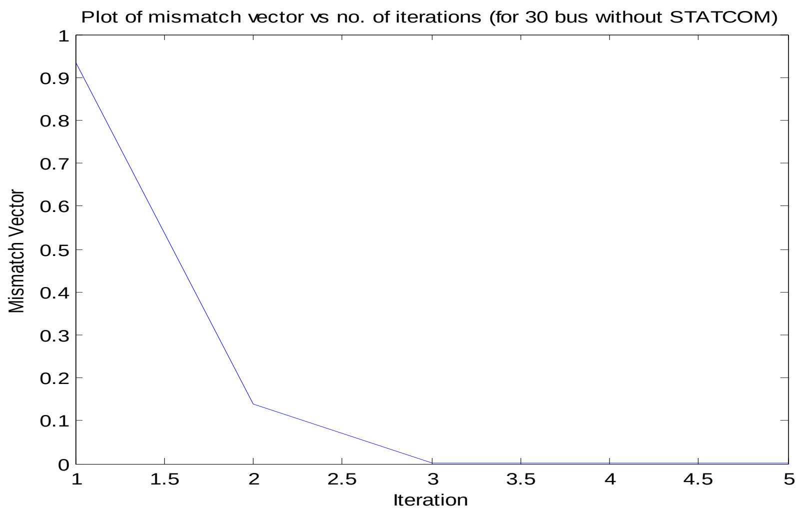

Fig 4.3

In this case No. of iterations are 5 only.

Voltage profile of the system is shown below.

Table 4.3

Bus no.	Voltage(pu)	Bus no.	Voltage(pu)
1	1.0600	16	1.0245
2	1.0430	17	1.0133
3	1.0186	18	1.0057
4	1.0091	19	1.0009
5	1.0100	20	1.0040
6	1.0085	21	1.0029
7	1.0014	22	1.0070
8	1.0100	23	1.0032
9	1.0339	24	0.9944
10	1.0163	25	0.9998
11	1.0720	26	0.9818
12	1.0432	27	1.0120
13	1.0610	28	1.0068
14	1.0258	29	0.9919
15	1.0191	30	0.9803

From table 4.3 it is observed that the bus voltage magnitudes at bus nos. 24,25,26,29 and 30 is low. Hence they require compensation.

Case 4. In this case we installed three STATCOMs at bus no. 26,29 and 30. Results are shown below.

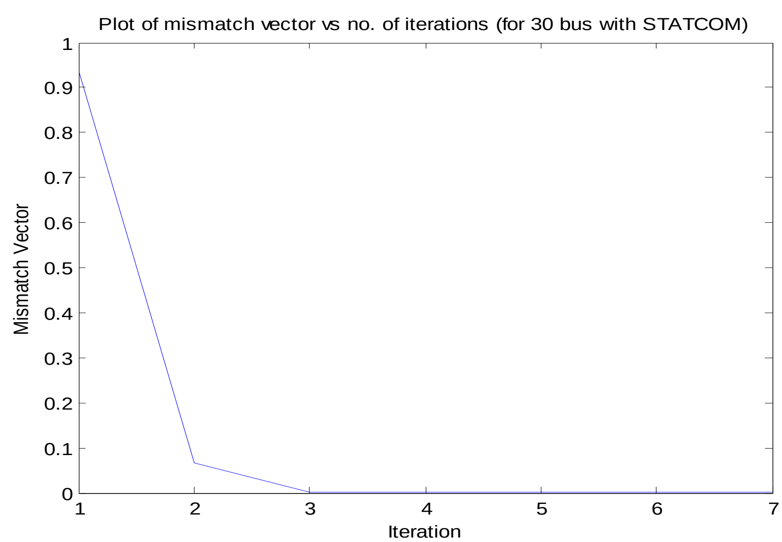

Fig 4.4

Here No. of iterations with STATCOM are slightly increased.

Table 4.4

Bus no.	V(with statcom)	Bus no.	V(with statcom)
1	1.0600	16	1.0314
2	1.0430	17	1.0201
3	1.0199	18	1.0127
4	1.0170	19	1.0079
5	1.0100	20	1.0108
6	1.0100	21	1.0099
7	1.0022	22	1.0144
8	1.0401	23	1.0103
9	1.0100	24	1.0028
10	1.0229	25	1.0113
11	1.0820	26	1.0
12	1.0502	27	1.0218
13	1.0710	28	1.0088
14	1.0329	29	1.0
15	1.0262	30	1.0

It is seen here that installation of STATCOM improves voltage profile of the system.

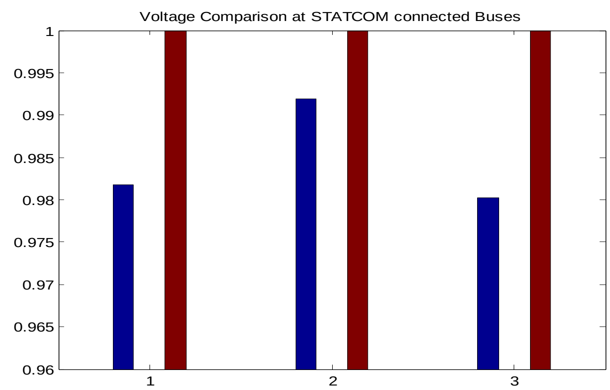

Fig 4.5

In Fig 4.5 the red and blue bars are used to denote bus voltages with and without STATCOMs respectively. Above bar graph shows improvement in voltages at 26,29 and 30 corrosponding to points 1,2,3 in the above figure.

In table 4.5 negative sign in Q_{sh} shows that STATCOM is absorbing reactive power from the bus at which it is connected (Bus no.26 and 30 in this case) because of higher voltage then their corrosponding bus . But at bus no. 29 V_{sh} is less then the bus voltage V_{29} hence it is supplying VARs to the bus.

Table 4.5

Statcom Bus	V_{sh}	θ_{sh}	Q_{sh}
26	1.0017	-16.8419	0.0168
29	0.9970	-16.8516	-0.0298
30	1.0041	-18.1154	0.0410

Case 5. In this case we installed five STATCOMs at bus no. 24,25,26,29 and 30. Results are shown below.

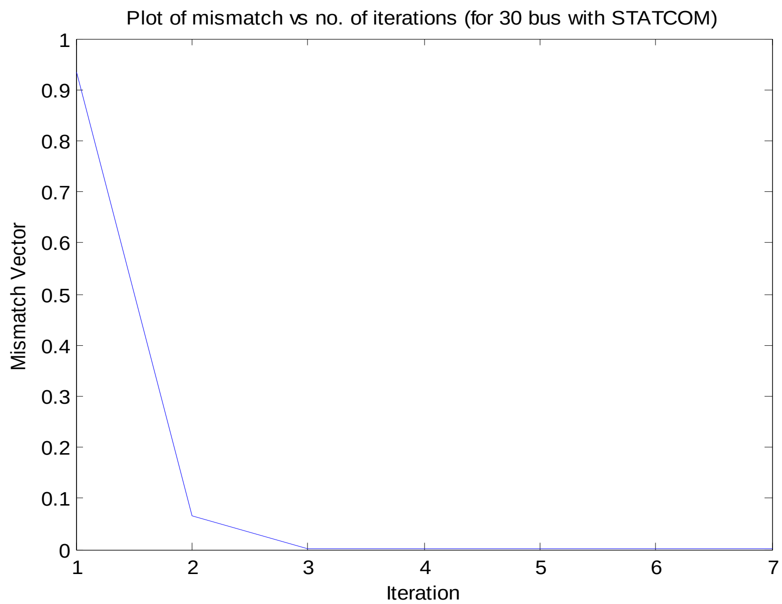

Fig 4.6

It is observed here that in both cases, 5 and 6 No. of iterations are same inrespective of no. of STATCOMs.

Table 4.6

Statcom Bus	V_{sh}	θ_{sh}	Q_{sh}
24	1.0016	-16.3187	0.0158
25	0.9915	-15.8861	-0.0847
36	1.0044	-17.0653	0.0464
29	0.9985	-16.9974	-0.0151
30	1.0036	-18.2252	0.0458

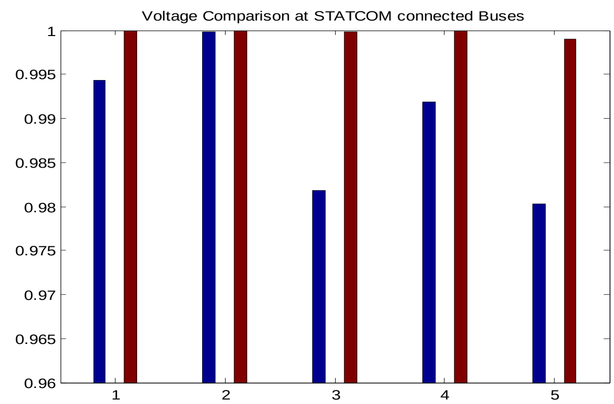

Fig 4.7

Above bar graph shows that there is a further improvement in voltage profile of the system with increased no. of STATCOMs. (5 STATCOMs in this case).

CHAPTER – 6

CONCLUSIONS AND SCOPE OF FUTURE WORK

In this thesis, the Newton power flow model of a STATCOM is developed. This model can account for the losses in the coupling transformer. Validity of the proposed model is demonstrated on a simple five bus system and the IEEE 30 bus test system with excellent convergence characteristics. It is observed that installation of the STATCOM(s) substantially improves the steady-state voltage profile of the buses to which they are installed.

Further improvement in the power flow model of the STATCOM can be achieved if the model is suitable for decoupling. A decoupled power flow model avoids updation of the Jacobian matrix at each iteration step. The Jacobian matrix would be rendered constant with all elements known beforehand.

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APPENDEX

IEEE 3O BUS DATA

BRANCH DATA FOLLOWS

FROM BUS	TO BUS	R	X	$B_{SH}/2$	TAP SETTING
1	2	0.0192	0.0575	0.0264	1
1	3	0.0452	0.1652	0.0204	1
2	4	0.0570	0.1737	0.0184	1
3	4	0.0132	0.0379	0.0042	1
2	5	0.0472	0.1983	0.0209	1
2	6	0.0581	0.1763	0.0187	1
4	6	0.0119	0.0414	0.0045	1
5	7	0.0460	0.1160	0.0102	1
6	7	0.0267	0.0820	0.0085	1
6	8	0.0120	0.0420	0.0045	1
6	9	0.0	0.2080	0.0	0.978
6	10	0.0	0.5560	0.0	0.969
9	11	0.0	0.2080	0.0	1
9	10	0.0	0.1100	0.0	1
4	12	0.0	0.2560	0.0	0.932
12	13	0.0	0.1400	0.0	1
12	14	0.1231	0.2559	0.0	1
12	15	0.0662	0.1304	0.0	1
12	16	0.0945	0.1987	0.0	1
14	15	0.2210	0.1997	0.0	1
16	17	0.0824	0.1923	0.0	1
15	18	0.1073	0.2185	0.0	1
18	19	0.0639	0.1292	0.0	1
19	20	0.0340	0.0680	0.0	1
10	20	0.0936	0.2090	0.0	1
10	17	0.0324	0.0845	0.0	1
10	21	0.0348	0.0749	0.0	1
10	22	0.0727	0.1499	0.0	1
21	23	0.0116	0.0236	0.0	1
15	23	0.1000	0.2020	0.0	1
22	24	0.1150	0.1790	0.0	1
23	24	0.1320	0.2700	0.0	1
24	25	0.1885	0.3292	0.0	1
25	26	0.2544	0.3800	0.0	1
25	27	0.1093	0.2087	0.0	1
28	27	0.0000	0.3960	0.0	0.968
27	29	0.2198	0.4153	0.0	1
27	30	0.3202	0.6027	0.0	1
29	30	0.2399	0.4533	0.0	1
8	28	0.0636	0.2000	0.0214	1
6	28	0.0169	0.0599	0.065	1

BUS DATA FOLLOWS

Bus	Type	Vsp	theta	PGi	QGi	PLi	QLi	Qmin	Qmax
1	1	1.06	0	0	0	0	0	0	0;
2	2	1.043	0	40	50.0	21.7	12.7	-40	50;
3	3	1.0	0	0	0	2.4	1.2	0	0;
4	3	1.0	0	0	0	7.6	1.6	0	0;
5	2	1.01	0	0	37.0	94.2	19.0	-40	40;
6	3	1.0	0	0	0	0.0	0.0	0	0;
7	3	1.0	0	0	0	22.8	10.9	0	0;
8	2	1.01	0	0	37.3	30.0	30.0	-10	40;
9	3	1.0	0	0	0	0.0	0.0	0	0;
10	3	1.0	0	0	0	5.8	2.0	0	0;
11	2	1.082	0	0	16.2	0.0	0.0	-6	24;
12	3	1.0	0	0	0	11.2	7.5	0	0;
13	2	1.071	0	0	10.6	0.0	0.0	-6	24;
14	3	1.0	0	0	0	6.2	1.6	0	0;
15	3	1.0	0	0	0	8.2	2.5	0	0;
16	3	1.0	0	0	0	3.5	1.8	0	0;
17	3	1.0	0	0	0	9.0	5.8	0	0;
18	3	1.0	0	0	0	3.2	0.9	0	0;
19	3	1.0	0	0	0	9.5	3.4	0	0;
20	3	1.0	0	0	0	2.2	0.7	0	0;
21	3	1.0	0	0	0	17.5	11.2	0	0;
22	3	1.0	0	0	0	0.0	0.0	0	0;
23	3	1.0	0	0	0	3.2	1.6	0	0;
24	3	1.0	0	0	0	8.7	6.7	0	0;
25	3	1.0	0	0	0	0.0	0.0	0	0;
26	3	1.0	0	0	0	3.5	2.3	0	0;
27	3	1.0	0	0	0	0.0	0.0	0	0;
28	3	1.0	0	0	0	0.0	0.0	0	0;
29	3	1.0	0	0	0	2.4	0.9	0	0;
30	3	1.0	0	0	0	10.6	1.9	0	0

LINE AND BUS DATA FOR SIMPLE 4 BUS SYSTEM

BRANCH DATA FOLLOWS

FROM	TO	R	X	$B_{SH}/2$
1	2	0.01008	0.0504	0.0512
1	3	0.0074	0.0372	0.0387
2	4	0.0074	0.0372	0.0387
3	4	0.0127	0.0636	0.0638

BUS DATA FOLLOWS

Bus	Type	Vsp	theta	PGi	QGi	PLi	QLi	Qmin	Qmax
1	1	1.0	0	0	0	50	0	0	0;
2	3	1.0	0	0	0	170	105	0	0;
3	3	1.0	0	0	0	200	123	0	0;
4	2	1.0	0	318	80	80	0	-40	40;