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Dispersion properties of photonic crystal fiber: comparison by scalar and fully vectorial effective index methods

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Abstract. The development of analysis and simulation of propagation characteristics of photonic crystal fiber (PCF) using scalar and fully vectorial effective index methods are described. As a result, we report how the fundamental space filling mode, guided mode and dispersion of the PCF depends on its structural parameters like its normalized air hole spacing, center-to-center spacing of the air holes in the photonic crystal or pitch and radius of the unit cell. Normalized frequency parameter V_{eff} as a function of normalized wavelength for various relative air hole sizes is obtained to estimate the dispersion characteristics of PCF. It is observed that wavelength of zero dispersion, ultraflattened dispersion response and high negative dispersion remarkably differ from two different effective index methods.

Key words: dispersion, fully vectorial effective index method (FVEIM), photonic crystal Fibers (PCF), scalar effective index method (SEIM), space filling modes (SFM), zero dispersion wavelength (ZDW)

1. Introduction

Photonic crystal materials attracted much attention in the past few years with the number of publications and patents increasing exponentially. Two-dimensional (2D) photonic crystal structures, like holey fibers and 2D slab-type photonic crystals, are probably the most advanced and fast developing areas owing to mature fabrication methods and envisioned broad applications. One of the most important applications of photonic crystals is the design of novel waveguides known as photonic crystal fibers (PCFs) or holey fiber (HF) or micro structured fibers. PCFs are single material optical fibers with an array of periodic air holes across the cross-section running down its entire length. By leaving a single lattice site without an air hole, a localized region of higher refractive index is formed. This localized region acts as a waveguide core in which light can be trapped along the axis of the fiber. Photonic crystal with periodicity in transverse direction supports mode propagating along the longitudinal direction. These modes are referred as space filling modes (SFM) because they are infinitely extended in transverse direction. In the effective index method, a single material having refractive index equal to the modal index of fundamental space filling mode replaces the photonic crystal cladding.

A considerable amount of interest has been generated in PCFs during the last few years, due to its single-mode operation over extended range of operating wavelengths, large mode area, soliton propagation and continuum generation and overall controllable dispersion. In optical communication, dispersion plays an important role as it determines the information carrying capacity of the fiber. Therefore, it becomes important to study the dispersion properties of PCF. Many modeling techniques have been applied to study its propagation characteristics, which include the effective index method (Birks *et al.* 1997; Knight *et al.* 1998; Varshney *et al.* 2003), plane wave expansion method (Ferrando *et al.* 1999, 2000; Johnson and Joannopoulos, 2001), localized function method (Mogilevstev *et al.* 1999; Monro *et al.* 1999), finite element method (Brechet *et al.* 2000), finite difference time domain method (Qiu 2001), multiple method (White *et al.* 2001) and finite difference frequency domain method (Zhu and Brown 2002) etc.

In this paper, the dispersion properties of PCFs have been analyzed and compared using the scalar effective index method and fully vectorial effective index method. Previously, dispersion properties such as wavelength of zero dispersion nearly zero ultra flattened dispersion, and anomalous dispersion was obtained by the finite element method, plane wave expansion method, and localized function method, respectively. The effective index method has also been used to study the propagation characteristics of PCFs. This is because the effective index method is a simple numerical technique that qualitatively provides the same modal properties of PCFs as obtained by other numerical techniques. Further, it is shown that the fundamental properties of PCFs such as effective index and dispersion parameter can be understood from classical optical fiber theories (Koshiba and Saitoh 2004), where scalar effective index method is used. However, this method is applicable for weekly guiding regime and in case of PCFs, where large index contrast exists between air and silica reduces the applicability of SEIM. Therefore, very recently, fully vectorial effective index method for the calculation of effective index and dispersion parameters has been employed (Li *et al.* 2004). However, this paper lacks the detailed studies of dispersion in terms of zero dispersion wavelengths, nearly zero ultraflattened dispersion and the high negative dispersion provided by PCFs. Therefore, in this paper we report how the fully vectorial effective index method is applied to compare the dispersion properties, such as high negative chromatic dispersion and nearly zero ultra flattened dispersion of PCF.

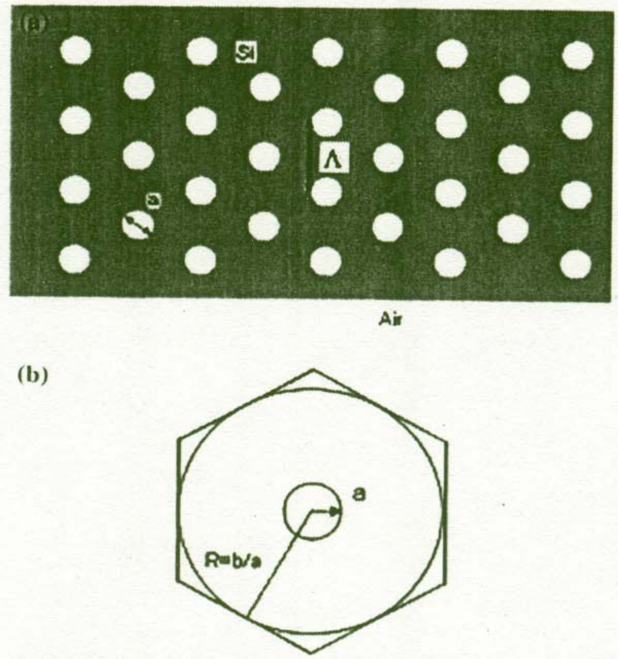

Fig. 1. (a) Schematic diagram of PCF and its parameters. (b) The equivalent circular unit cell of a hexagonal.

The dispersion properties of silica-based PCFs are analyzed to compare: (i) zero dispersion at any wavelength, (ii) nearly zero ultra flattened dispersion and (iii) a very high negative chromatic dispersion for various designs of PCFs using both SEIM and FVEIM. It is shown that a wavelength of zero dispersion and high negative dispersion is remarkably differing from these two effective index methods.

1.1. SCALAR EFFECTIVE INDEX METHOD (SEIM)

In scalar effective index method, one starts form the Scalar wave equation

$$\lfloor \nabla_t + (k^2 n^2 - \beta^2) \rfloor \psi = 0 \tag{1}$$

where ∇_t is the transverse Laplacian operator in cylindrical coordinates, $k = 2\pi/\lambda, \lambda$ is free space wavelength, n is material index and β is the propagation constant. Using boundary condition that field must be continuous at boundary of air-hole and silica and it must vanish at the outer boundary of circular unit cell i.e. ψ and $\frac{\partial\psi}{\partial r}\Big|_{R=1}$ must be continuous and $\frac{\partial\psi}{\partial r}\Big|_{R=\frac{b}{a}} = 0$, where $R = r/a$, a being the radius of the air hole and r is the normal co-ordinate to the boundary 'S' and $b = \Lambda\sqrt{\frac{\sqrt{3}}{2\pi}}$ is defined as a radius of the outer circle, which is obtained by equating the filling fraction of hexagonal unit cell and its circular approximation as shown in Figure 1(b). Therefore equation for inner and outer areas of the air hole is

$$\psi_1 = AI_0(WR) \qquad \text{Air Hole} \tag{2a}$$

$$\psi_2 = BJ_0(UR) + CY_0(UR) \qquad \text{Silica region} \tag{2b}$$

Applying boundary conditions and making use of Bessel functions an eigen value equation for evaluating the effective index n_{FSM} is obtained,

$$BJ_1(u) + CY_1(u) = 0, \tag{3}$$

where B and C are the constants given by

$$B = \frac{A}{J_0(U)}\left[I_0(W) - \frac{WI_1(W)J_0(U) - UJ_1(U)I_0(W)}{U(J_1(U)Y_0(U) - J_0(U)Y_1(U))}\right] \tag{4}$$

$$C = \frac{A[WI_1(W)J_0(U) + UJ_1(U)I_0(W)]}{U[J_1(U)Y_0(U) - J_0(U)Y_1(U)]} \tag{5}$$

with parameters U, W and u as follows

$$U = k_0 a\sqrt{n_s^2 - n_{cl}^2}$$

$$W = k_0 a\sqrt{n_{cl}^2 - n_a^2} \tag{6}$$

$$u = k_0 b\sqrt{n_s^2 - n_{cl}^2}$$

n_s and n_a are the refractive indices of pure silica and air, respectively. The modal indices of fundamental space filling mode is obtained and hence n_{cl} is determined. Thereafter, the PCF is assumed to be a step index fiber as shown in Figure 1(b).

1.2. FULLY VECTORIAL EFFECTIVE INDEX METHOD

In Full Vectorial Effective index method (FVEIM), both the effective cladding index method and the effective index of the guided mode of the PCF are calculated using fully vectorial equations. The electromagnetic fields in the optical fibers are expressed in cylindrical coordinates as

$$\vec{E} = E(r,\theta)e^{j(\omega t-\beta z)}, \vec{H} = H(r,\theta)e^{j(\omega t-\beta z)} \tag{7}$$

Substituting into Maxwell's equation we will get the two sets of wave equations.

$$[\nabla_t + (k^2n^2 - \beta^2)]\begin{pmatrix} E_z \\ H_z \end{pmatrix} = 0 \tag{8}$$

Where, symbols carry their usual meaning. Solving above Maxwell equation we will get the modal indices of fundamental space filling mode.

$$\left(\frac{P_1'(U)}{UP_1(U)} + \frac{I_1'(W)}{WI_1(W)}\right)\left(n_s^2\frac{P_1'(U)}{UP_1(U)} + n_a^2\frac{I_1'(W)}{WI_1(W)}\right) = \left(\frac{1}{U^2} + \frac{1}{W^2}\right)\left(\frac{\beta}{k}\right)^2, \tag{9}$$

where, $P_1(U)$ is defined as

$$P_1(U) = J_1(U)Y_1(u) - Y_1(U)J(u) \tag{10}$$

U, W, u are defined as in Equation (6) and the primes denote differentiation with respect to the argument. In order to calculate β (propagation constant) for the PCF, the hexagonal unit cell is approximated by a circular one of radius b and hence the propagation constant of the guided mode will be calculated.

2.1. INDEX GUIDED MODE

The equivalent step index fiber consists of core and cladding regions having refractive indices $n_{co}(=n_s)$ and n_{cl}, respectively. From the obtained values of cladding index using both the stated effective index method methods, we will get the cladding index and hence, calculate the index-guiding mode for different fiber parameters, which are shown in Figure 2. The scalar solution obtained for the fundamental mode is given by

$$\begin{aligned} \psi &= AJ_1(U_{eff}R) \quad R<1 \\ &= BK_1(W_{eff}R) \quad R<1 \end{aligned} \tag{11}$$

where A and B are constants and the eigenvalue equation obtained is the similar to the eigenvalue equation of step index except the waveguide parameters which are

$$U_{eff} = k_0 r_c\sqrt{n_s^2 - n_{eff}^2} \tag{12}$$

$$W_{eff} = k_0 r_c\sqrt{n_{eff}^2 - n_{cl}^2} \tag{13}$$

$$V_{eff} = k_0 r_c\sqrt{n_s^2 - n_{cl}^2} \tag{14}$$

n_{eff} is the effective index of the guided mode and r_c is the radius of the core.
The characteristic equation obtained from the FVEIM is written as

$$\left(\frac{J_1'(U_{\rm eff})}{U_{\rm eff}J_1(U_{\rm eff})}+\frac{K_1'(W_{\rm eff})}{W_{\rm eff}K_1(W_{\rm eff})}\right)\left(n_{\rm cl}^2\frac{J_1'(U_{\rm eff})}{U_{\rm eff}J_1(U_{\rm eff})}+n_{\rm eff}^2\frac{K_1'(W_{\rm eff})}{W_{\rm eff}K_1(W_{\rm eff})}\right)$$
$$=\left(\frac{1}{U_{\rm eff}^2}+\frac{1}{W_{\rm eff}^2}\right)^2\left(\frac{\beta}{k}\right)^2 \tag{15}$$

where, $n_{\rm eff}$ is the effective index of the fundamental mode and $n_{\rm cl}$ is the effective cladding index obtained from Equation (9).

In Figure 2(a), calculations have been done by using PCF parameters, center-to-center spacing of the air holes in the photonic crystal or pitch $\Lambda = 2.3$ μm, core radius $r_c = 0.64\Lambda$, air hole diameter $d = 0.69$ μm. In later figure i.e. Figure 2(b) we change the air hole diameter $d = 1.035$ μm. It is evident from the figure that the difference in the values of fundamental space filling mode and the guided mode index calculated by these methods are

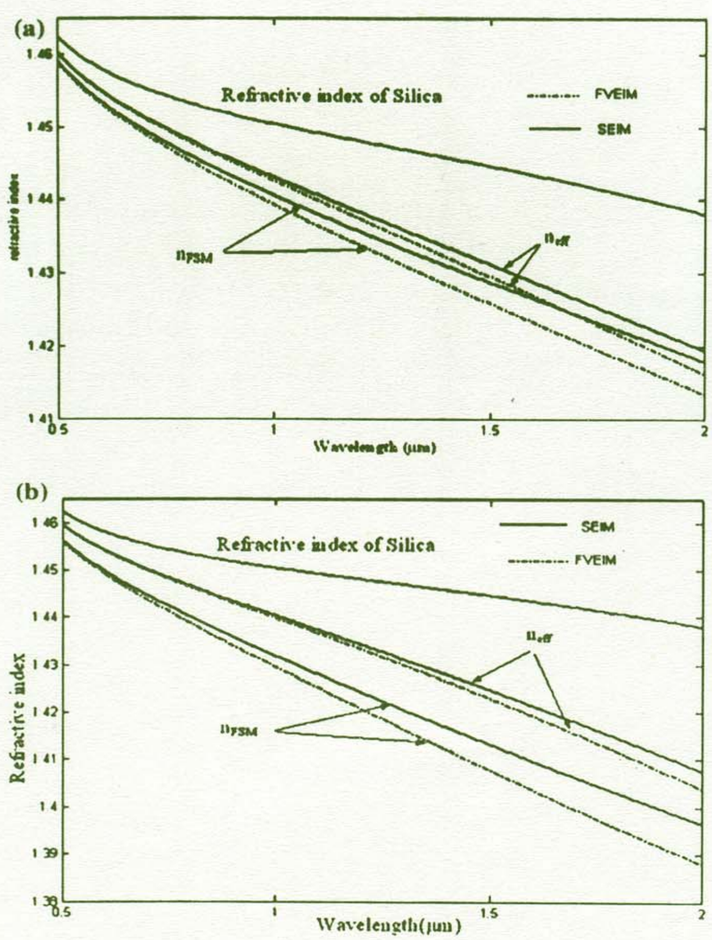

Fig. 2. (a) Effective index of the guided mode as a function of wavelength for normalized air-hole sizes of the PCF with parameters $\Lambda = 2.3$μm and $r_c = 0.64\Lambda$, $d/\Lambda = 0.30$. (b) Effective index of the guided mode as a function of wavelength for normalized air-hole size of the PCF with parameters $\Lambda = 2.3$ μm and $r_c = 0.64\Lambda$, $d/\Lambda = 0.45$.

significantly different. We concluded from the figures that, as the air filling fraction increases the credibility of the SEIM reduces since the difference between the refractive index of the silica and the refractive index of the fundamental space filling mode increases, which violets the foremost requirement of the weakly guided approximation.

Figure 3 shows the variation of effective normalized frequency parameter for a PCF using FVEIM and SEIM Figure 3(b) shows that just by changing any one parameter of PCF (e.g., Λ d or r_c we can design an endlessly single mode fiber. Viewing the Figure 3 it is being observed that the normalized cut off frequency calculated by both the stated method is quantitatively different. Using FVEIM we get smaller cut off normalized wavelength than by SEIM.

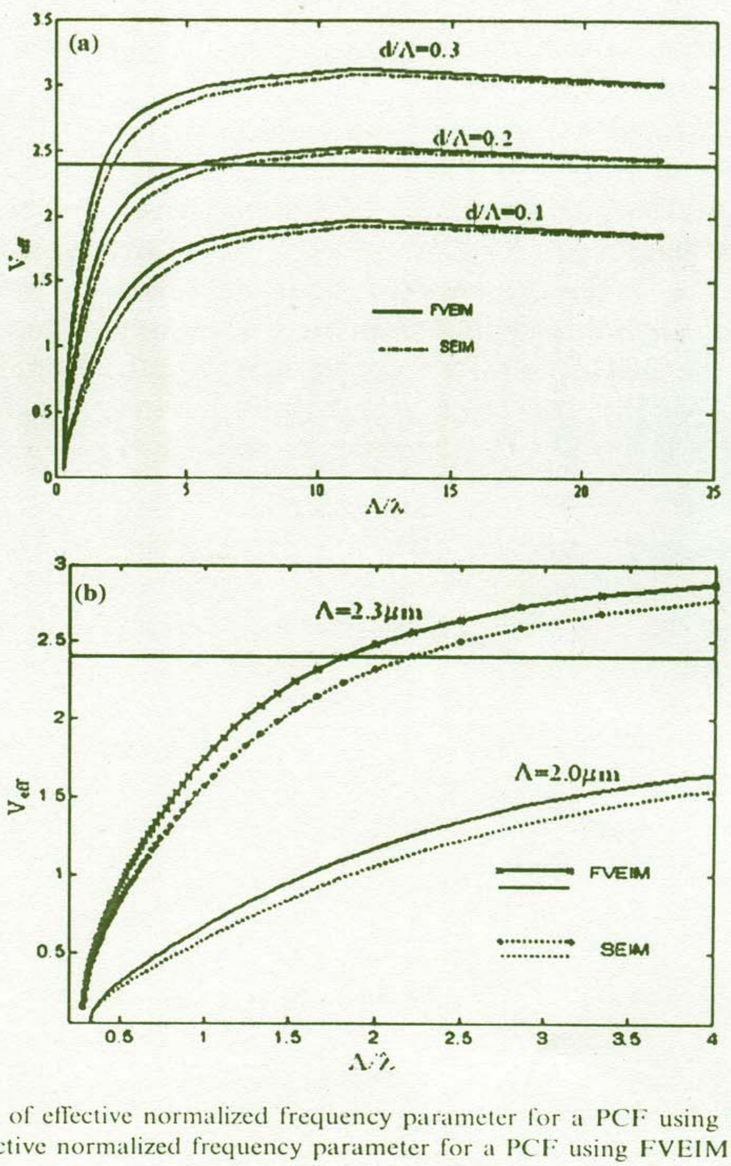

Fig. 3. (a) Variation of effective normalized frequency parameter for a PCF using FVEIM and SEIM. (b) Variation of effective normalized frequency parameter for a PCF using FVEIM and SEIM keeping $d/\Lambda = 0.3$.

2.2. DISPERSION CALCULATION

The dispersion characteristics of PCF are investigated by taking into account the refractive index of pure silica by means of the Sellemier formula, while the index of air is assumed constant. The dispersion D is given as $D = -\frac{\lambda}{c}\frac{d^2 n_e}{d\lambda^2}$, where n_e is the effective index of the guided mode, which has been calculated (Okamoto 2000) from scalar effective index method and fully vectorial effective index method and λ (in μm) is the free space wavelength. Figure 4 shows the dispersion behavior, calculated from both the above stated methods, as a function of wavelength for different normalized air holes. From the figure we can state that dispersion curve obtained from SEIM and FVEIM are not same. For the relative air hole size $d/\Lambda = 0.3$ the curve is flattened near zero unlike as obtained from FVEIM for the same PCF parameters.

2.2.1. *Zero dispersion wavelengths*

The dispersion (ps/nm-km) for different structures of PCF is obtained and shown in Figure 4. Figure depicts that one can design PCFs by choosing appropriate cladding parameters to obtain a desired zero dispersion wavelength (ZDW). Variation of ZDW of PCF with normalized air hole size is represented in Fig. 5 for the PCF of same parameters. It is clear from the figure that as the air hole size increases the wavelength of zero dispersion also increases. For the SEIM, zero-dispersion wavelength is smaller than that of obtained from FVEIM. It is clear from Fig. 5 that the results are significantly different for the large air fill fraction as the contrast of refractive index

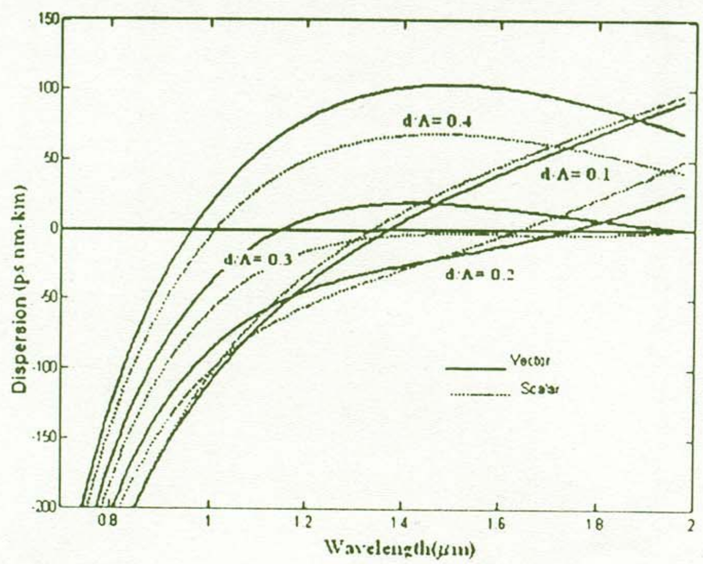

Fig. 4. Comparison of group velocity dispersion as a function of wavelength for different relative air-hole sizes.

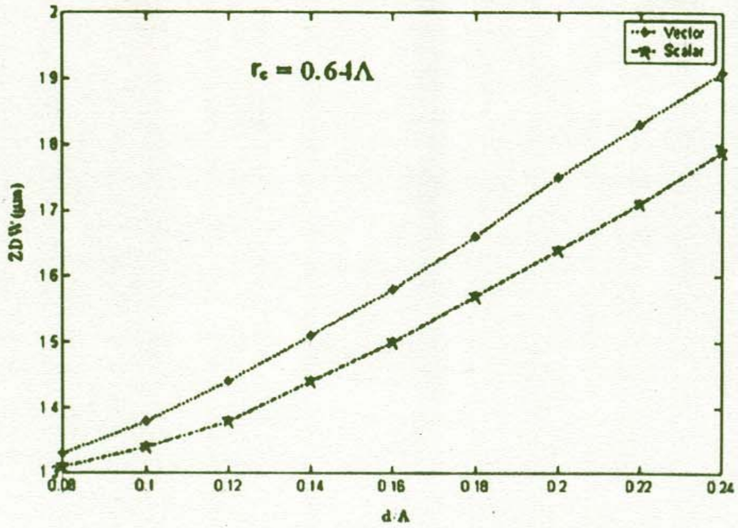

Fig. 5. Comparison of zero-dispersion wavelength as a function of relative air hole size.

increases; this is because the fact that the effective index of cladding is wavelength dependent. At short wavelength, the modal field remains confined to the silica region, but at longer wavelengths the effective cladding index decreases.

2.2.2. *Nearly zero ultra flattened dispersion*

From Figure 4 it is clear that dispersion properties obtained by using two methods (SEIM and FVEIM) are not the same. Dispersion curve for the PCF of parameters, hole-to-hole distance i.e. $\Lambda = 2.3$ μm, air hole diameter $d = 0.69$ μm and core diameter $2r_c = 3$ μm is flattened near zero using SEIM. Earlier, nearly zero ultra flattened dispersion is obtained for silica based PCF with these design parameters and it is reported that the ultraflattened response is in the order of $(+0.05, -0.25)$ ps/nm-km (Sinha and Varshney 2003) using scalar effective index method. However, with these design parameters, nearly zero ultra flattened dispersion response is not achievable using FVEIM. We can obtaine even more flattened dispersion behavior of PCF using FVEIM with design parameters, $\Lambda = 2.3$ μm, $d = 0.59$ μm, $2r_c = 3$ μm and is in the order of $(+0.038, -0.13)$ ps/nm-km, as shown in Figure 6. More flattened response means more applicability in DWDM based optical communications systems.

2.2.3 *High negative dispersion*

Chromatic dispersion in single-mode fibers causes light pulses to spread, limiting the data transmission rate and length of optical-fiber links. To overcome these limits, various techniques are used to suppress the dispersion

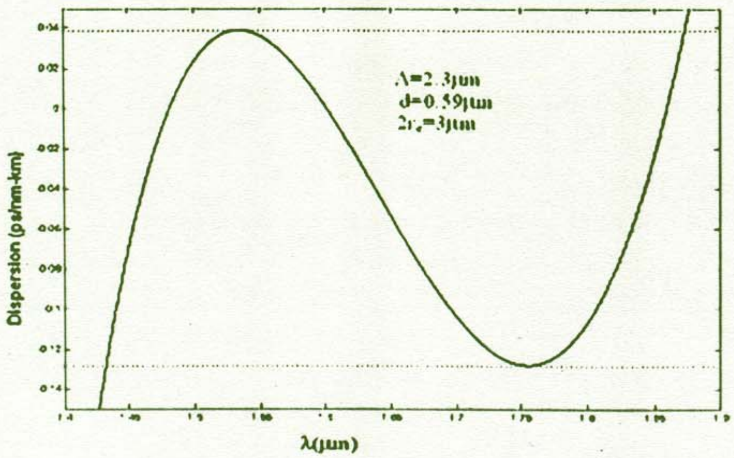

Fig. 6. Nearly zero ultraflattened dispersion response in wavelength range of 1.3–1.7 μm.

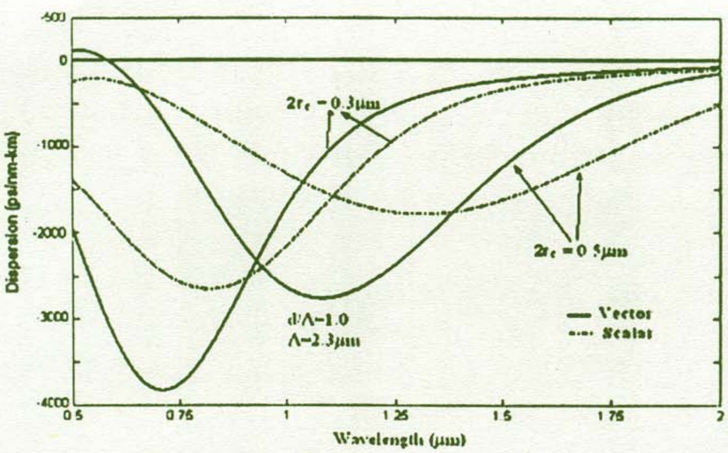

Fig. 7. High negative chromatic dispersion in photonic crystal fibers for different core diameters using two different numerical techniques.

effects. Dispersion can be compensated by a short length of a special type of optical fibers, such as dispersion compensating fibers (DCFs), with a dispersion of opposite sign so that the net dispersion of two fibers in series becomes zero. The higher the dispersion coefficient of the compensating fiber, the smaller will be the required length of the compensating fiber. A large index contrast between core and cladding permits a high dispersion over a broad wavelength range. Since the negative dispersion is obtained because of the large index contrast due to which the foremost requirement for the scalar approximation will be no more in picture, which leads us to a non-reliable values of the negative dispersion offered by PCF. It is evident from the Fig. 7 that the dispersion obtained from FVEIM is more negative than that of obtained from SEIM at lower wavelength.

3. Conclusion

The results on dispersion characteristics of PCFs, so obtained, have been compared with the two standard methods namely SEIM and FVEIM for the same geometrical and wave guiding parameters. For lower wavelengths, there is negligible difference in the results obtained by using above stated methods, however, as wavelength increases difference becomes apparent. The discrepancy observed at higher wavelength values is due to the fact that the refractive index of the silica and the effective cladding index are wavelength dependent. The above results show that the scalar effective index method is applicable only for the PCFs whose effective refractive index difference between core and cladding is less. It has been found that wavelength of zero dispersion and high negative dispersion characteristics of PCFs obtained from SEIM and FVEIM differ significantly.

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