Institute of Physics Publishing
Semiconductor Science and Technology
Semicond. Sci. Technol. 18 (2003) 292–296
PII: S0268-1242(03)52479-8

Design parameters of a tunable semiconductor multiple quantum well electron wave filter

Shalini Garg, R K Sinha and K L Deori

Department of Applied Physics, Delhi College of Engineering, Faculty of Technology, University of Delhi, Bawana Road, Delhi-110042, India

E-mail: dr_rk_sinha@yahoo.com

Received 14 August 2002, in final form 9 January 2003
Published 26 February 2003
Online at stacks.iop.org/SST/18/292

Abstract

In this paper, we discuss the use of a semiconductor multiple quantum well (MQW) electron wave filter for multi-channel communication applications. This bandpass-type electron wave energy filter, made of GaAs and $Ga_{0.55}Al_{0.45}As$, is designed for different numbers of layers and cavities for a pass wavelength of 10 nm. We report on the calculated values of the design parameters, such as the passband wavelength, 3-dB bandwidth, quality factor and passband loss for the electron wave filter. The design parameters are also evaluated for a tunable MQW electron wave filter when varying the angle at which the electron wave is incident on the input layer of the filter.

1. Introduction

Low-dimensional systems have revolutionized semiconductor physics. They rely on the technology of heterostructures, where the composition of a semiconductor can be changed on a nanometre scale. For example, a sandwich of GaAs between two layers of $Al_xGa_{1-x}As$ acts like an elementary quantum well. Semiconductor quantum wells are ultrafine-layered media whose layer thicknesses are in the range of one or a few atomic layers. Corresponding to the wave propagation in a waveguide, the electrons propagate through a narrow channel with a width of the order of the de Broglie wavelength and with a length less than the phase coherence length of electrons with sufficiently smooth boundaries. Such channels that guide coherent electron waves from one end to the other are known as electron waveguides. Due to their ballistic nature, the electrons act as waves. Electron wave devices are a new class of semiconductor devices based on the use of electron waves. Both faster and smaller than existing semiconductor devices [1–3], these structures find applications in electronic and optical devices and can be combined into guided electron wave integrated circuits. Such circuits could become the 'next generation of semiconductor technology', i.e. nanotechnology. As a result, there is a growing interest in the design and development of devices based on electron wave propagation in semiconductor quantum wells, quantum wires or quantum dot structures utilizing interference and diffraction phenomena of electron waves [4].

The analysis of the quantum interference devices such as electron waveguide couplers [5], switches, filters and diffraction gratings [6] is carried out with the help of the coupled mode theory [7] for electron waves. Analogous to the optical thin-film filter [8], an electron wave energy filter [9] consists of multiple thin layers of semiconductor material having different potential energies. It reflects one or more spectral bands or lines and transmits others, while maintaining a nearly zero coefficient of absorption for all wavelengths of interest. An interference filter [10] may be a high-pass, low-pass, bandpass, or band rejection filter. In this paper, we describe the theory of reflection and refraction of electron waves from an interface separating two media and from a stack of layers. This study is then applied to finding the design parameters such as passband wavelength, 3-dB bandwidth, quality factor and passband loss for a bandpass $GaAs/Ga_{0.55}Al_{0.45}As$ multiple quantum well (MQW) electron wave filter for the filter configurations having a variable number of layers and cavities for the pass wavelength of 10 nm. Also, these design parameters are evaluated for the different thickness values of the film layer. It is observed that the filter configuration can be made tunable by varying the angle of incidence. As this angle of incidence is increased, the wavelength of peak transmission decreases. Thus, depending

0268-1242/03/040292+05$30.00

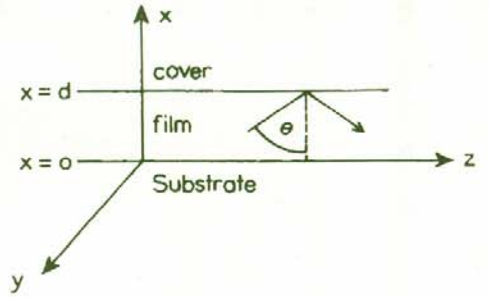

Figure 1. An electron wave slab waveguide.

upon the requirement, the pass wavelength and the angle of incidence is selected. These MQW energy filters can in turn be utilized to design other electron wave devices such as transmission filters, reflectors, beamsplitters, multiplexers, demultiplexers, ballistic transistors, etc., for multi-channel communication systems.

2. Reflectivity and transmissivity of the semiconductor MQW structure

A semiconductor quantum well can be designed to confine electron waves and act as an electron waveguide. The electron waveguide serves as a basic building block for the development of high-speed electronic devices such as electron waveguide couplers, electron switches [11, 12], electron waveguide filters, etc. The steady-state motion of the electron wave is governed by the well-known Schrödinger equation

$$\left(-\frac{\hbar^2}{2m^*}\nabla^2 + V(x)\right)\Psi(x, y, z) = E\Psi(x, y, z) \qquad (1)$$

where $\Psi(x, y, z)$ is the electron wavefunction corresponding to the electron energy E. m^* is the effective mass of the electron. The solutions are then

$$\Psi(x, y, z) = \Psi(x, y)\,\mathrm{e}^{ikz} \qquad (2)$$

where k denotes the wave vector.

A slab waveguide [13] is shown in figure 1. Using standard waveguide terminology, the three regions are denoted substrate (s), film (f) and cover (c), and the direction perpendicular to the waveguide surfaces is x. The thickness of the waveguide is d. The direction of guided mode propagation is z. The angle of incidence of the two plane wave components that constitute the guided wave is the zig-zag angle θ.

The magnitude of the electron wave vector based on equation (2) in any of the three regions may be expressed as

$$k = [2m^*(E - V)]^{1/2}/\hbar \qquad (3)$$

where m^* is the electron effective mass, E is the total energy and V is the electron potential energy. An electron of total energy E is incident upon the film–cover interface. The kinetic energy of the incident electron is

$$E - V_f = \frac{\hbar^2}{2m_f^*}\left(k_{fx}^2 + k_{fy}^2\right) = \frac{\hbar^2}{2m_f^*}k_f^2 \qquad (4)$$

where $\hbar$ is the Planck constant divided by 2π, m_f^* is the electron effective mass in the film region. k_{fx} and k_{fy} are the components of the incident electron wave vector normal and parallel to the boundary, respectively, and k_f is the magnitude of the incident electron wave vector, which is given by

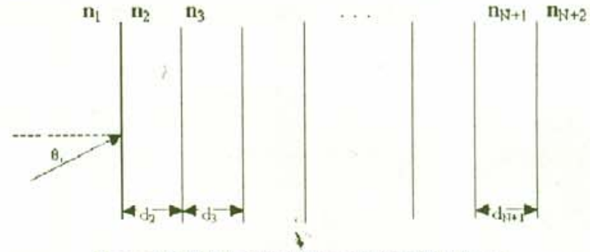

Figure 2. MQW structure consisting of N layers.

$$k_f = [2m_f^*(E - V_f)]^{1/2}/\hbar. \qquad (5)$$

The kinetic energy of the transmitted electron is

$$E - V_c = \frac{\hbar^2}{2m_c^*}\left(k_{cx}^2 + k_{cy}^2\right) = \frac{\hbar^2}{2m_c^*}k_c^2 \qquad (6)$$

where k_{cx} and k_{cy} are the components of the transmitted electron wave vector normal and parallel to the boundary, respectively, and k_c is the magnitude of the transmitted wave vector in the cover region which is given by

$$k_c = [2m_c^*(E - V_c)]^{1/2}/\hbar. \qquad (7)$$

The kinetic energy of the reflected electron is

$$E - V_f = \frac{\hbar^2}{2m_f^*}\left(k_{fx}^2 + k_{fy}^2\right) = \frac{\hbar^2}{2m_f^*}k_f^2 \qquad (8)$$

where k_f is the magnitude of the reflected wave vector.

The electron amplitude transmittivity and the corresponding fraction of the electron current transmitted are respectively given by

$$t_e = \frac{2(E/m_i^*)^{1/2}\cos\theta_i}{(E/m_i^*)^{1/2}\cos\theta_i + [(E - V)/m_i^*]^{1/2}\cos\theta_t} \qquad (9)$$

and

$$T_e = \left[\frac{(E - V)_t}{(E - V)_i}\right]^{1/2}|t_e|^2. \qquad (10)$$

The electron amplitude reflectivity and the corresponding fraction of the electron current reflected are respectively given by

$$r_e = \frac{(E/m_i^*)^{1/2}\cos\theta_i - [(E - V)/m_i^*]^{1/2}\cos\theta_t}{(E/m_i^*)^{1/2}\cos\theta_i + [(E - V)/m_i^*]^{1/2}\cos\theta_t} \qquad (11)$$

and

$$R_e = |r_e|^2. \qquad (12)$$

We next consider an arrangement as shown in figure 2. It shows an electron wave in the region having the refractive index, n_1, incident at an angle θ upon a stack of N layers of differing electron potential energy. The amplitudes incident upon ($\psi_{i,n}$) and reflected from ($\psi_{r,n}$) the nth layer may be expressed in terms of the amplitudes incident upon ($\psi_{i,n+1}$) and reflected from ($\psi_{r,n+1}$) the $(n+1)$th layer [14], as

$$\begin{bmatrix}\psi_{i,n}\\ \psi_{r,n}\end{bmatrix} = \frac{1}{t_n}\begin{bmatrix}1 & r_n\\ r_n & 1\end{bmatrix} \times \begin{bmatrix}\exp[\,jk_n d_n(\cos\theta_n)] & 0\\ 0 & \exp[-jk_n d_n(\cos\theta_n)]\end{bmatrix}\begin{bmatrix}\psi_{i,n+1}\\ \psi_{r,n+1}\end{bmatrix} \qquad (13)$$

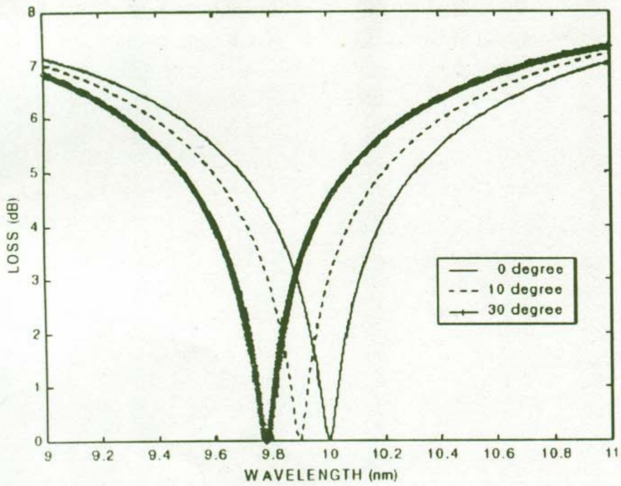

Figure 7. Spectral response of the BPF at different angles of incidence.

Table 5. Observations of the spectral response of BPF at different angles of incidence.

S No	Angle of tilt (°)	Wavelength of peak transmission
1	0	10.000 nm
2	10	9.894 nm
3	30	9.775 nm

configuration, substrate|/LHLHL(2H)LHLHL/|cover, for the pass wavelength of 10 nm. Table 5 gives the corresponding values of the wavelength of minimum loss due to reflection.

It is observed from figure 7 that, at normal incidence, the minimum loss due to reflection occurs at the pass wavelength for which the filter is designed, i.e. 10 nm. As the angle of incidence increases, the passbands shift to shorter wavelengths. At a tilt of 20°, the wavelength of minimum loss is observed at 9.894 nm, and at a tilt of 30° the wavelength of peak transmission is at 9.775 nm. Thus, the filter can be made tunable.

4. Conclusion

Mathematical expressions have been given for the design of an electron wave interference filter using the $GaAs/Ga_{0.55}Al_{0.45}As$ semiconductor heterostructure system. The electron wave interference effect on the reflection characteristics of the filter is demonstrated in terms of parameters such as pass wavelength, 3-dB bandwidth, quality factor and passband loss. We have carried out a comparative study of the spectral response of the electron wave filter for different cavity lengths, different numbers of cavities, different numbers of layers and the different thicknesses of the cavity layer and the results are shown in tables 1–4. Also, a variation in the wavelength of minimum loss due to reflection is observed for different angles of incidence. As the angle of incidence increases, the passband shifts to the shorter wavelength sides. Thus, by varying the angle of incidence, the filter can be made tunable. This makes the filter useful for multi-channel communication applications. Extensive research and development efforts are under way to realize the tunable electron wave interference filter experimentally, since such a device may serve as one of the basic building blocks of future guided wave integrated circuits. Furthermore, the influence of the graded index nature of the electron waveguides on the design parameters may be investigated. The bandwidth of the proposed device may be controlled by introducing a phase shift in the reflector layers. Electron waveguide based devices having a graded index nature with symmetric and asymmetric structures can be analysed to obtain enhanced filter characteristics.

References

[1] Weisbuch C and Vinter B 1991 *Quantum Semiconductor Structures* (Boston, MA: Academic)
[2] Ferry D K and Goodnick S M 2000 *Transport in Nanostructures* (Cambridge: Cambridge University Press)
[3] Garg S, Sinha R K and Deori K L 2002 Nanostructure devices based on electron waveguides *IETE J. Tech. Rev.* **19** 269
[4] Tsukada N, Wieck A D and Ploog K 1990 Proposal of novel electron wave coupled devices *Appl. Phys. Lett.* **56** 2527
[5] Alamo A Del and Eugster C C 1995 Dual electron waveguide devices: the quest for electron directional coupling *Japan. J. Appl. Phy.* **34** 4439
[6] Henderson G N, Glytsis E N and Gaylord T K 1991 Electron wave diffraction by semiconductor gratings: rigorous analysis and design parameters *Appl. Phys. Lett.* **59** 440
[7] Sarangan A M and Huang W P 1994 A coupled mode theory for electron wave directional couplers *IEEE J. Quantum Electron.* **30** 2803
[8] Macleod H A 1986 *Thin Film Optical Filters* (New York: Macmillan)
[9] Gaylord T K and Brennan K F 1989 Electron wave optics in semiconductors *J. Appl. Phys.* **65** 814
[10] Guthrie D K *et al* 1998 Measurement of the zero-bias electron transmittance as a function of energy for half- and quarter-electron-wavelength semiconductor quantum interference filters *Appl. Phys. Lett.* **72** 374
[11] Garg S, Sinha R K and Deori K L 2002 Switching characteristics of voltage controlled electron waveguides *Sci. Lett. National Academy of Sciences, India* **25** 36
[12] Lewen R *et al* 2002 High frequency characterization of a GaInAs/InP electronic waveguide T-branch switch *J. Appl. Phys.* **91** 2398
[13] Gaylord T K, Glytsis E N and Brennan K F 1989 Semiconductor quantum wells as electron wave slab waveguides *J. Appl. Phys.* **66** 15
[14] Gaylord T K, Henderson G N and Glytsis E N 1993 Application of electromagnetic formalism to quantum-mechanical electron-wave propagation in semiconductors *J. Opt. Soc. Am.* B **10** 333
[15] Ma C 1994 Coupling properties in periodic waveguides and in multiple quantum well waveguides *IEEE J. Quantum Electron.* **30** 2811
[16] Suzuki T *et al* 2002 Fabrication and optical characterization of five-layer asymmetric coupled quantum well *Japan. J. Appl. Phys.* **41** 2701
[17] Adachi S 1985 GaAs, AlAs and $Al_xGa_{1-x}As$: material parameters for use in research and device applications *J. Appl. Phys.* **58** R1
[18] Born M and Wolf E 1980 *Principles of Optics* 6th edn (Oxford: Pergamon)

where t_n is the transmittivity, r_n is the reflectivity, k_n is the wave-vector magnitude, d_n is the thickness, and θ_n is the propagation angle at the nth layer. Generalizing the above analysis for N films and for oblique incidence permits the analysis of propagation of electron waves in multi-layer structures [15, 16] and will be useful to design electron wave energy filters analogous to optical dielectric thin-film filters.

3. Design parameters of a semiconductor MQW electron wave filter

Generally, an interference bandpass filter (BPF) consists of alternating layers of H and L, film, alternating layers of H and L, ..., film, alternating layers of H and L. Here, H and L indicate high and low index layers, respectively, with a common thickness of quarter wavelength. Also, the film consists of $2n$ 'H' or $2n$ 'L' where n is an integer and determines the length of the cavity. In order to maximize the reflectivity at each boundary, the values of the mole fraction, x, of the two alternating layers in the $Ga_{1-x}Al_xAs$ MQW system are taken to be far apart. Corresponding to an optical thin-film interference filter, an electron wave interference filter is designed by following the analogies of electron waves in quantum mechanics [9] to optical waves in electromagnetics. First, the electron pass kinetic energy for the electron pass wavelength, $\lambda_{e,0} = 10$ nm in the input region, is calculated by

$$(E - V)_0 = h^2/2m_0^*\lambda_{e,0}^2 \qquad (14)$$

where m_0^* is the electron effective mass for the $j = 0$ region. Secondly, kinetic energy for each jth layer is calculated by

$$(E - V)_j = (n_j/S)^2 m_j^* \qquad (15)$$

where $(E - V)_j$, n_j and m_j^* are the kinetic energy, refractive index and effective mass, respectively, of the jth layer. S is the scaling parameter and is given by

$$S = n_0/[(E - V)_0/m_0^*]^{1/2} \qquad (16)$$

where n_0 is refractive index in the input region in the optical design. Thirdly, the quarter wavelength thickness, d_j of the jth layer is given by

$$d_j = h/2^{5/2}\{[m_j^*(E - V)_j]^{1/2}\}. \qquad (17)$$

The kinetic energies and the thickness values of the different layers of the MQW stack are calculated for a pass wavelength of 10 nm. A fraction of the electron energy is reflected at each of the barrier interfaces and represents the loss. If the thickness of the film is of half-wavelength for the desired wavelength to be filtered, then other wavelengths will be attenuated by destructive interference. An interference filter requires a very high value of transmittivity over a small region of wavelength so that at the output of the filter there is a very small spread in wavelength.

In this paper, we simulate a MQW electron wave filter consisting of alternating layers of GaAs ($n = 3.6260$) and $Ga_{0.55}Al_{0.45}As$ ($n = 3.3137$) [17] surrounded by the substrate and cover regions made of $Ga_{0.55}Al_{0.45}As$. The effective masses used are $m^*(GaAs) = 0.067m_0$ and $m^*(Ga_{0.55}Al_{0.45}As) = 0.10435m_0$ where m_0 is the free electron mass. Here H is the region of high refractive index, i.e. GaAs, and L is the region of low refractive index, i.e. $Ga_{0.55}Al_{0.45}As$.

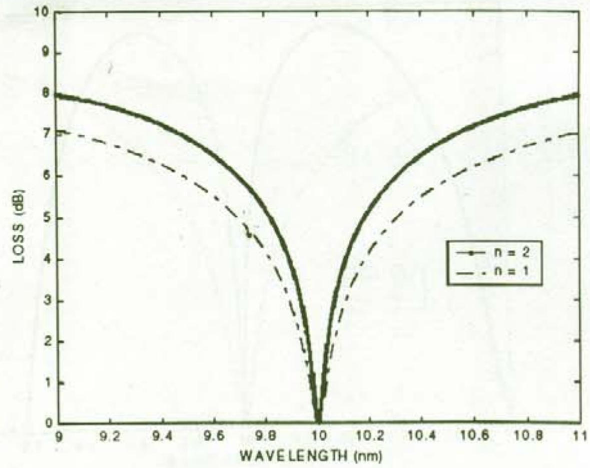

Figure 3. Spectral response of the BPF according to the cavity length.

Table 1. Observations of the spectral response of BPFs according to cavity length.

S No	Cavity length	3-dB bandwidth	Quality factor
1	2H ($n = 1$)	0.0502 nm	1.99×10^2
2	4H ($n = 2$)	0.0300 nm	3.33×10^2

The reflection characteristics for the different filter configurations are plotted using equation (12), i.e. the fraction of electron current reflected. We have used MATLAB-5.3 software on a Pentium III equipped personal computer. We concentrated mainly on obtaining the precise design parameters rather than the computation time required. The design parameters which control the reflection characteristics are the value of pass wavelength, 3-dB bandwidth or full width at half maximum (FWHM), quality factor and passband loss. The 3-dB bandwidth or FWHM [18], in the case of the pattern in transmitted light, is the width between the points on either side of a maximum where the intensity has fallen to half its maximum value. It is a measure of the sharpness of the fringes. Another parameter that determines the performance of the device is the quality factor of the filter, which is given by the ratio of the passband wavelength to the 3-dB bandwidth. The quality factor is a measure of selectivity.

Figure 3 shows the spectral response of an 11-layer, 1-cavity BPF according to cavity length (n) for a filter configuration 'substrate|/LHLHL(2nH)LHLHL/|cover' and a pass wavelength of 10 nm. The numerical values of the design parameters, i.e. 3-dB bandwidth and quality factor, are obtained from figure 3 and are given in table 1. It is observed that, the longer the cavity length, the sharper the cut-off characteristics and the narrower the 3-dB bandwidth.

Figure 4 shows the spectral response of 2H-cavity BPFs with one and two cavities for a filter configuration, substrate|/AHA/|cover, where the symbol A represents the layers LHLHL(2H)LHLHL for the pass wavelength of 10 nm. The numbers of layers are 11 and 23, respectively. As is evident from table 2, the increase in the number of cavities leads to a corresponding decrease in the 3-dB bandwidth.

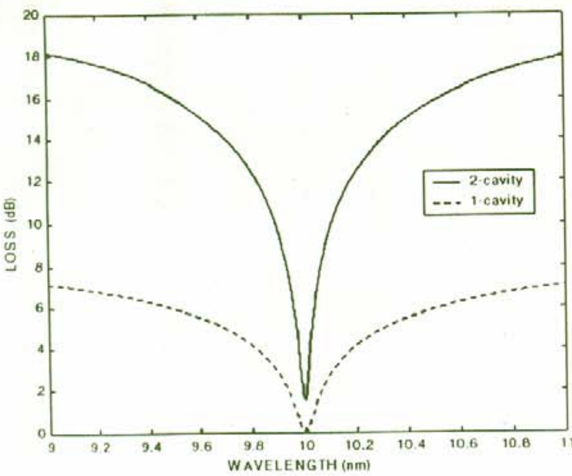

Figure 4. Spectral response of 1-cavity and 2-cavity BPFs.

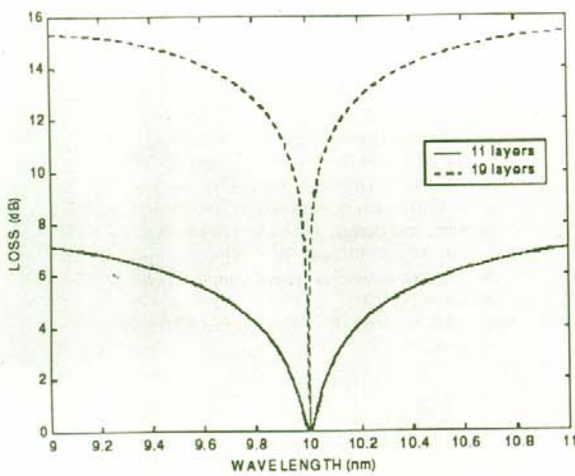

Figure 5. Spectral response of 11-layer and 19-layer BPFs.

Table 2. Observations of the spectral response of 1-cavity and 2-cavity BPFs.

S No	Filter configurations	3-dB bandwidth	Quality factor
1	Substrate|/A/|cover	0.0500 nm	2.00×10^2
2	Substrate|/AHA/|cover	0.0160 nm	6.25×10^2

Figure 5 shows the spectral response for a 1-cavity ($n = 1$) BPF filter configuration substrate|/A/|cover, where the symbol A represents the layers LHLHL(2H)LHLHL in one case and the layers LHLHLHLHL(2H)LHLHLHLHL in the other case for the pass wavelength of 10 nm. The corresponding design parameters are shown in table 3. Figure 5 shows that, as the number of layers increase, the cut-off characteristic sharpness increases and there is a sharp decrease in the value of 3-dB bandwidth. This makes the filter efficient with a better quality factor.

Figure 6 shows the spectral response for a 1-cavity ($n = 1$), 7-layer BPF configuration having different thicknesses of the cavity layer for the pass wavelength of 10 nm. The corresponding values of the design parameters are given in table 4, where d is the quarter-wavelength thickness of the primary cavity layer.

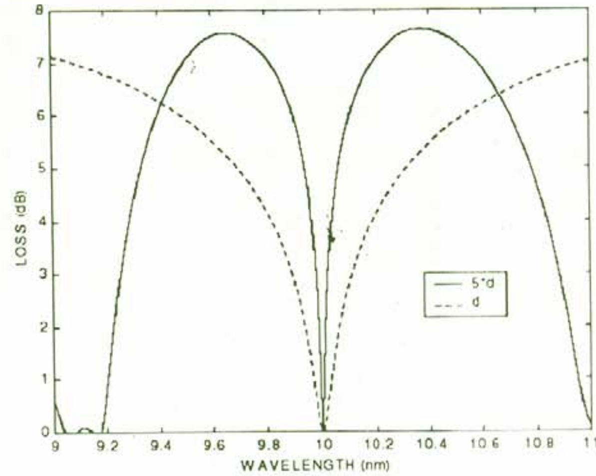

Figure 6. Spectral response of the BPF according to the thickness of the cavity layer.

Table 3. Observations of the spectral response of 11-layer and 19-layer BPFs.

S No	No of layers	3-dB bandwidth	Quality factor
1	11	0.0500 nm	2.00×10^2
2	19	0.0006 nm	1.67×10^4

Table 4. Observations of the spectral response of the BPF according to the thickness of the cavity layer.

S No	Thickness	3-dB bandwidth	Quality factor
1	d	0.0500 nm	2.00×10^2
2	5^*d	0.0100 nm	10.0×10^2

As can be seen from figure 6, although the loss due to reflection is zero at 10 nm for both the cavity layer thicknesses, the variation of loss due to reflection of the required range of the spectrum is much smaller when the thickness has a multiple factor of one. It is observed that, at 9.1 nm, the loss again decreases to a very small value for 5^*d, thus opening another window for the wavelength to transmit. This is not the requirement of the filter and so for this reason a film of thickness d should be used rather than a film of thickness 5^*d. In all the above cases, a passband loss of less than 0.1 dB is obtained. We also note that the FWHM obtained for an electron wave interference filter in [9] was 2.2 Å or 0.22 nm. Compared with this result, we have obtained FWHM values ranging from 0.05 nm to as low as 0.0006 nm for the different filter configurations.

Until now, the design parameters have been calculated for normal incidence. The filter can be made tunable by varying the angle of incidence. As the angle of incidence increases, a shift in pass wavelength towards smaller wavelength values is observed. Now, we change the angle at which the electron wave is incident on the input layer of the filter. Figure 7 shows the spectral response of the 1-cavity ($n = 1$), 11-layer BPF according to angle of incidence for a filter