# Analytical Solutions for Bearing Capacity of Special Footings 

Submitted in Partial Fulfillment of the Requirement for the Award of Degree of MASTER OF ENGINEERING

In

## CIVIL ENGINEERING (STRUCTURAL ENGINEERING)

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Science is the topography of ignorance...... The best
Part of our knowledge is that which teaches us where knowledge leaves off and ignorance begins.

Oliver Wendell Holmes
1809-1894

## CANDIDATE DECLARATION \& CERTIFICATE

This is certify that SAURABH PRAKASH GUPTA a student of final year M.E. Civil (Structural Engg.), Delhi College of Engineering, carried out his Thesis work on "Analytical Solutions for Bearing Capacity of Special Footings " under the guidance of Prof. (Mrs) P.R. Bose, Professor \& H.O.D and Prof. A. Trivedi, Professor, Department of Civil \& Environmental Engineering, Delhi College of Engineering, Delhi, for the partial fulfillment of the requirement for the degree of Master of Engineering, Civil Engineering, specialization in Structural Engineering, from Delhi college of Engineering, Delhi.

This is certified that the matter embodied in this Thesis has not been submitted elsewhere for the award of any other degree/diploma.

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## AKNOWLEDGEMENT

I would like to take this opportunity to thanks all those who have been a constant source of inspiration and have helped in the exercise of preparing the present study.

I express my sincere gratitude to Prof. (Mrs) P. R. Bose and Prof. A. Trivadi, my Project Guides, for their constant inspiration, encouragement, guidance and constructive criticism and judicious evaluation that led to the compilation of this Thesis work. It was due to their constant help and assistant that this thesis work achieved its present shape.

I express my sincere thanks to all my friends and staff of Civil Engineering Department of Delhi College of Engineering, who left no stone unturned whenever I needed their assistance.

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## ABSTRACT

The analytical solutions for the bearing capacity of circular footing on sand are analyzed for two special types of footings, namely spudcan footing and skirted circular footing.

The bearing capacity solutions utilized limit-equilibrium method proposed by Terzaghi (1943) and numerical solutions found by Kumbhojkar (1993). The numerical values of the bearing capacity factor for special footings are based upon the failure surface restriction by spudcan footing and by the skirted footing.

The solutions are presented in the graphical form. The analysis of the result indicates that there is a significant improvement in the bearing capacity using spudcan and skirted footings with Kumbhojker assumptions. The experimental data of the skirted footings by other investigators also indicate an increase in ultimate bearing capacity but the magnitude differs.

Further the extent of increase in bearing capacity factors using spudcan and skirted footing provides a scope for fresh look on the application of Terzaghi and Kumbhojker approaches and its assumptions.

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## NOMENCLATURE

$\phi=$ Angle of internal friction of soil.
$\delta=$ Angle of wall friction.
$\gamma=$ Unit weight of soil in $\mathrm{KN} / \mathrm{m}^{3}$
$\alpha=$ Angle of inclination of sides of wedge to the horizontal surface of foundation.
$\theta=$ Angle of log-spiral.
$\sigma^{\prime}=$ Effective normal stress.
$\zeta_{\gamma}=$ Shape factor for circular footing.
$\Omega=$ Cone angle in Spudcan footing.
$\beta=$ Angle of inclination of the soil above the ground surface.
$\mathrm{c}=$ Cohesion of soil.
$\mathrm{q}=$ Surcharge.
$\mathrm{N}_{\mathrm{q}}, \mathrm{N}_{\mathrm{c}}$, and $\mathrm{N}_{\gamma}=$ Dimensionless bearing capacity factor providing the contribution of $\mathrm{c}, \mathrm{q}$, and $\gamma$.
$B=$ Width of footing.
$\mathrm{D}=$ Diameter of footing.
$\mathrm{h}=$ Height of cylindrical shell.
$\mathrm{q}_{\mathrm{u}}=$ Ultimate bearing capacity of footing.
$\mathrm{Q}_{\mathrm{u}}=$ Ultimate bearing capacity of footing with soil confinement.
$\mathrm{q}_{\mathrm{o}}=$ Average pressure over the footing contact area.
$r, r_{1}$, and $r_{o}=$ Radius of log-spiral.
$D_{f}=$ Depth of foundation.
$\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$, and $\mathrm{P}_{\mathrm{py}}=$ Passive force with contribution of $\mathrm{q}, \mathrm{c}$, and g respectively.
$P_{p}=$ Passive force.
$\mathrm{K}_{\mathrm{py}}=$ Passive force coefficient.
$\mathrm{d}=$ diameter of cylindrical shell.
$\mathrm{N}_{\mathrm{c}}{ }^{\prime}, \mathrm{N}_{\mathrm{q}}{ }^{\prime}$ and $\mathrm{N}_{\gamma}{ }^{\prime}=$ Modified bearing capacity factors (for local shear faliur).
$\mathrm{L}=$ Length of continuous footing.
$\mathrm{s}_{\gamma}, \mathrm{s}_{\mathrm{c}}$, and $\mathrm{s}_{\mathrm{q}}=$ Shape factor of footing
$\mathrm{d}_{\mathrm{q}}, \mathrm{d}_{\gamma}$, and $\mathrm{d}_{\mathrm{c}}=$ Depth factor of footing.
$i_{c}, i_{q}$, and $i_{\gamma}=$ Inclination of footing.
$g_{c}, g_{q}$, and $g_{\gamma}=$ Slope factor of footing.
$b_{c}, b_{q}$, and $b_{\gamma}=$ Tilting factor of footing.
$\mathrm{dp}=$ Differential reactive pressure on the element length ds of the failure surface.
$R=$ Resultant reaction on failure surface.
$\mathrm{d} / \mathrm{D}=$ Ratio of the diameter of cylindrical shell to the diameter of footing.
$h / D=$ Ratio of the height of cylindrical shell to the diameter of footing.
$B C R=$ Bearing capacity ratio, $\mathrm{Q}_{\mathrm{u}} / \mathrm{q}_{\mathrm{u}}$.
$\mathrm{K}_{\mathrm{a}}=$ Rankine and Coulomb active earth pressure coefficient.
$\mathrm{K}_{\mathrm{p}}=$ Rankine and Coulomb passive earth pressure coefficient.
$P_{p(1)}, P_{p(2)}$, and $P_{p(3)}=$ Rankine passive force.
$\mathrm{F}=$ Resultant frictional resistant force.
$\mathrm{H}_{\mathrm{d}}=$ Height over which the Rankine Passive force acts.
$\mathrm{W}=\mathrm{Weight}$ of wedge.
$1_{p}, 1_{w}$, and $1_{R}=$ lever arms.
$\mathrm{q}_{\mathrm{c}}, \mathrm{q}_{\mathrm{q}}$, and $\mathrm{q}_{\gamma}=$ The ultimate load per unit area of foundation for a soil with cohesion, friction and weight.
$D_{r}=$ Relative density.
$\mathrm{C}_{\mathrm{u}}=$ Uniformity coefficient.
$\mathrm{C}_{\mathrm{c}}=$ Coefficient of curvature.
UPVC = Unplasticized polyvinyl chloride.
$\mathrm{V}=$ vertical load.
$A_{p}=$ Plane area of contact of Soudcan footing.
$\mathrm{B}_{\mathrm{p}}=$ Penitration diameter of Spudcan footing.
$\gamma^{\prime}=$ soil effective unit weight.
$D_{p}=$ Penetration depth of Spudcan footing.
$\mathrm{f}=$ Circumferential tensile stress( or hoop stress).
$\mathrm{p}=$ Internal pressure in shell.
$\mathrm{t}=$ thickness of cylindrical shell.

## CHAPTER-1

## INTRODUCTION

The bearing capacity of shallow footing during the last fifty years has been extensively studied by several investigators [Prandtl ,1920; Reissner, 1926; Terzaghi, 1943; Meyerhof, 1951; Caquot and Kerisel, 1953; De Beer, 1963; Baki and Beik, 1970; Hansen, 1970; Vesic, 1973; Chen, 1975; Ingra and Baecher, 1983; Kumbhojkar, 1993; Zadroga, 1994; Dewaiker and Mohapatro, 1994; Frydman and Burd, 1997; Michalowki, 1997; Paolcci and Pecker, 1997; Soubra, 1999; Perkins and Madson, 2000 amongst others]. The bearing capacity solutions use the slip-line method, limit analysis method, finite element method, and limit equilibrium method. Limit equilibrium method, which is adopted by Terzaghi. Only numerical values of the bearing capacity factors differ accordingly to the specific assumption or approximation adopted in the solutions. Further more to account the original theory (e.g., footing shape; depth and tilts; rigidity and layering of solution of soil below a footing; inclination of applied loads.) a series of correction factors are applied to the $\mathrm{N}_{\mathrm{c},} \mathrm{N}_{\mathrm{q}}$ and $\mathrm{N}_{\gamma}$ terms. This trend of enhancing the accuracy of the bearing capacity calculation without altering the basic equation is a proof of overwhelming acceptance of Terzaghi's approach regardless of the concern about its theoretical correctness. This thesis deals with the theoretical prediction of the ultimate bearing capacity of the special footings, 1. Spudcan footing and 2. Skirted footing.

In deep offshore water, the growing use of mobile Jackup units on Spudcan Footing has been raised a great deal of concern about the overall stability of jackup unit in hostile environmental forces. Mobile Jackup units consist of a floatable drilling platform supported on three or more legs, which can be raised or lowered. A detailed description of the installation procedure is given by Tan (1990). The platform legs can either be supported separately or they can be supported on a single shared mat. Most modern jackup platforms are of the former type and have approximately conical shaped footing with a protruding tip at the center. These are commonly referred as spudcan footing.


Figures show (a) Jackup platform and environmental loads. (b) Spudcan footing.

The bearing capacity of spudcan footing increases with the depth of penetration below the seabed. The increase of bearing capacity with footing embedment is an important factor, since ever a small embedment can significantly increases the bearing capacity of such offshore footing. However this is not usually the case in coarse materials as penetration depth are typically very small.

The bearing capacity solution of Meyerhof (1953), Hansen (1970) and Vesic (1975) are commonly used to determine the ultimate bearing capacity of plane strain footing. These soil are modify by introducing shape factors to cover circular geometries [ Hambly 1992; Dean et al. 1993]. In the case of spudcan footing, their embedded circular area in plane (i.e. plane area at ground surface) is used for the bearing capacity calculation. In order to investigate the applicability of the bearing capacity solutions to spudcan footing, analytical studied of conical shape footing using the Terzaghi's approach has been presented in the chapter three, by changing the cone angle (e.g., $0^{\circ} .5^{\circ}, 10^{\circ}, 15^{\circ}$ ). Also it is shone in the study that spudcan footing can be treated as equal cones enclosing the same volume. As per the past studies the load-displacement response of a spudcan footing and equal volume cone is almost equal.

Raft foundations are widely used in supporting structures for many reasons such as weak soil conditions or heavy columns loads. In many cases, some problems arise such as the construction is adjacent to an old building and/or the foundation depth is so great that the excavation needs to be braced during foundation construction (e.g., basement excavation). One of the available solutions is to sheet piles to support the excavation sides during construction. Due to the difficulty of removing these piles, they become part of the permanent structure and two problems arise. The first problem deals with the structural analysis of the raft if the piles are used as end supports for the raft. The second problem is the effect of these piles on the lateral movement of the soil underneath the raft and the effect of this confinement on the bearing capacity of the soil. While there are several solutions for the first problem, such as isolating the raft from the piles, the confining effect of these piles on the raft behavior is not clearly understood. Looking to the problem in a smaller scale, it can be modeled as a circular footing supported on a soil, which is surrounded by a confining cylinder. The strength of confined sand was studied by Rajagopal et al. (1999). They carried out a large number of triaxial compression tests to study the influence of geocell confinement on the strength and stiffness behavior of granular soils. Geocells fabricated by hand using different geotextiles were used to investigate the effect of the stiffness of the geocell on the overall performance of geocell-soil composite.

Several investigators have reported significant effect of soil confinement by using horizontal soil reinforcement to increase the bearing capacity of supporting soils. This was achieved by placing layers of geogrid at different depths and widths under the footing. The soil reinforcement is not only placed horizontally but also can be placed vertically besides the footing to resist the lateral deformation of the soil. The use of vertical reinforcement along with horizontal reinforcement was investigated as well. The reinforcement consists of a series of interlocking cells, constructed from polymer geogrids, which contain and confine the soil within its pocket geogrid. Rea and Mitchell (1978) conducted a series of model plate loading tests on circular footing supported over sand-filled square-shaped paper grid cell to identify different modes of failure and arrive at optimum dimensions of the cell. Dash et al. (2001a,b) performed an experimental study on the bearing capacity of a strip footing supported by a sand bed reinforced with a geocell mattress. Critical dimensions of reinforcement and depth of placement for mobilizing maximum bearing capacity improvement were presented.

The aim of this research is to investigate the effect of soil confinement by sheet piles on the behavior of soil foundation system. Also, the idea of improving the footing response by using confining cylinder around individual footing. To achieve this objective the analytical solution has been presented in the upcoming chapters four.

## CHAPTER - 2

## LITERATURE REVIEW

Foundations, like the structures or equipment they support, are usually designed to meet certain serviceability and strength criteria. Serviceability conditions dictate that the foundation should perform such that under normal operating loads the structure or equipment it supports may fulfill its design purpose. These serviceability limitations are typically described by settlement or other motion limitations. The strength criteria have the purpose of ensuring that the foundation has sufficient reser4ve strength to resist the occasionally large load that may be experienced due to extreme environmental forces or other sources. In most, but not all cases, the serviceability or settlement criteria and the strength criteria may be treated as unrelated design tasks. Serviceability is a typically a long-term consideration for the foundation that may depend on time-dependent consolidation characteristics. Foundation strength, or bearing capacity, may be a short-termed problem such as an embankment construction on an untrained clay foundation or a long-term problem in which the maximum foundation load may appear at some unknown time.

A shallow foundation may be defined as one in which the embedment depth of the foundation is less than its least characteristic dimension. Usually, the bearing capacity of a foundation is determines by limit equilibrium, limit analysis, or slip-line solutions. The Varity of solutions available for a particular problem may lead to some uncertainty about which is the more appropriate procedure. In the following, the basic of these solution procedures will be summarized and method for their use presented.

### 2.1 METHODS OF ANALYSIS

At present time, the analysis of foundation can be made by employing one of the following four widely used methods:

1. Slip-line method
2. Limit equilibrium methods
3. Limit analysis methods
4. Finite-element methods

The first three methods are used in association with stability problems where the bearing capacity is sought. If, instead, the foundation settlement or stress distribution or stress distribution within the soil mass are of prime interest, then the fourth method must be used. Brief description of the first three procedures are given here.

The slip-line method involves construction of a family of shear or slip lines in the vicinity of footing load. These slip-lines, which represent the direction of maximum shear stress, form a network known as slip-line fields. The plastic slip-line fields are bounded by regions that are rigid. For plane equilibrium and one equation for the yield conditions available for solving for the three unknowns stresses. These equations are written with respect to curvilinear coordinates that coincide with sliplines. If the foundation boundary conditions are given only in terms of stresses, these equations are sufficient to give the stress distribution without any references to the stress-strain relationship. However, if displacement or velocity are specified over part of the boundary, then the constitutive relation must be used to relate the stresses to the strain and the problem becomes much more complicated. Although solutions may be obtained analytically, numerical and graphical methods are often found necessary (see Sokolovskii, 1965; Brinch Hansen, 1961, 1970).

The method described in the well-known textbook by Terzaghi (1943) and by Taylor (1948), or the method developed by Meyerhof (1951) are all classified here as method of limit equilibrium. They can best be describing as approximate approach to constructing the slip-line fields. The solution requires that assumptions be made regarding the shape of the failure surface and the normal stress distribution along such a surface. The stress distribution usually satisfies the yield conditions and the
equation of static equilibrium in an overall sense. By trial and error, it is possible to find a most critical location of the assumed slip surface from which the capacity of the footing can be calculated.

In addition to the yield condition, the limit analysis methods consider the soil stress-strain relationship in an idealized manner. This idealization, termed normality or the flow rule establishes the limit theorem on which limit analysis is based. The methods offer an upper and lower bound to the true solution. The upper-bond solution is calculated from a kinematically admissible velocity field that satisfies the velocity boundary conditions and is continuous except at certain discontinuity surface where the normal velocity must be continuous, but the tangential velocity may undergo a jump on crossing a boundary. Similarly, the lower-bond solution is determined from a statically admissible stress field that satisfy the stress boundary conditions, is in equilibrium, and nowhere violates the failure condition. If the two solutions coincide, then the method give the true answer for the problem considered. A good treatment of the subject is given by Chen (1975) and Chen and Liu (1990).

The method describe above are related in a manner. Most of the slip-line solution give kinematically admissible velocity fields and thus can be considered as an upper-bond solution provided that the velocity boundary conditions are satisfied. If the stress field within the plastic zone can be extended into the rigid region so that the equilibrium and yield conditions are satisfied, then the solutions may be the exact solutions. Shield (1995) has shown this for many cases. The extensive work that has been done on the stability analysis, including using the slip-line methods, is summarized in the book by Sokolovskii (1965).

Limit equilibrium method utilized the basic philosophy of the upper-bound rule, that is, a failure surface is assumed and the least answer is sought. However, it gives no consideration to soil kinematics and the equilibrium conditions are satisfied only in limited sense. Therefore, limit equilibrium solutions are not necessarily an upper bound or lower bound. However, any upper-bound solution from limit analysis will obviously be a limit equilibrium solution. Nevertheless, the method has been the most widely used owing to its simplicity and reasonably good accuracy.

The limit analysis method itself has many striking features that should appeal to researchers, as well as engineers. The problem formulation is generally simple and an analytically solution is always assured. In simple problems, it has been shown to yield reasonable answer when compared to limit
equilibrium solutions. Its capability of providing a mean for bounding the true solution is noteworthy. Finally, the method is efficient and can be extended to solve more difficult footing problems for which other method is efficient and can be extended to solve more difficult footing problem for which other method have so far failed.

### 2.2 SOIL GOVERNING PARAMETERS

The bearing capacity of footing depends not only on the mechanical properties of the soil (cohesion c and friction angle $\phi$ ), but also on the physical characteristics of the footing (width B, depth D , length L, and roughness $\delta$ ). For a coulomb material, Cox (1962) has shown that for a smooth surface footing bearing on a soil subjected to no surcharge, the fundamental dimensionless parameters associated with the stress characteristics equations are $\phi$ and $G=\gamma B / 2 c$, where $\gamma$ is the unit weight of the soil. When $G$ is small, the soil behaves essentially as a cohesive weightless medium. If $G$ is large, soil weight rather than cohesion is a principal source of bearing strength. For most practical cases, one can expect that $\phi$ lies in the range of $0^{\circ}$ to $40^{\circ}$ and G will range from 0.1 to 1.0 . These limits assume that c ranges from 500 to 1000 psf , and that the footing width ranges from 3 to 10 ft . the dimensionless bearing capacity $\mathrm{q} / \mathrm{c}$ depends only on the angle of internal friction of the soil $\phi$, the dimensionless soil weight parameter $G$, footing base friction angle $\delta$, surcharge depth ratio $\mathrm{D} / \mathrm{B}$, and the base dimensions B and L.

For the most part, the bearing capacity of footing on soils have in the past been calculated by a superposition method, suggested by Terzaghi (1943) in which the contributions to the bearing capacity from different soil and loading parameters are summed. These contributions are represented by the expression, $\mathrm{q}_{\mathrm{o}}=\mathrm{c} \mathrm{N}_{\mathrm{c}}+\mathrm{qN}_{\mathrm{q}}+\gamma \mathrm{BN}_{\gamma} / 2$, where $\mathrm{q}_{\mathrm{o}}$ is the average pressure over the footing contact area $A, q$ is the overburden or surcharge pressure at the foundation base and the bearing capacity factors $\mathrm{Nc}, \mathrm{Nq}$ and Ng represents the effect due to soil cohesion, surface loading, and soil unit weight, respectively. Above equation is valid for strip footing subjected to vertical center loads. However, other geometries are common. The parameters N are all functions of the angle of internal friction $\phi$. Terzaghi's quasiempirical method assumed that these effects are directly superposable, where the soil behavior in the plastic region is nonlinear and thus superposition does not hold for general soil bearing capacities. The reason for using the simplified (superposition) method is largely the mathematical difficulties encountered when using conventional equilibrium method.

### 2.3 TERZAGHI'S BEARING CAPACITY THEORY

In 1948, Terzaghi proposed a well-conceived theory to determined the ultimate bearing capacity of shallow rough rigid continuous (strip) foundation supported by a homogeneous soil layer extending to a great depth. Terzaghi defined a shallow foundation as a foundation where the width, B , is equal to or less than its depth, $\mathrm{D}_{\mathrm{f}}$. The failure surface in soil at ultimate load (that is, $\mathrm{q}_{\mathrm{u}}$, per unit area of the foundation) assumed by Terzaghi is shown in Fig. 2.1. Referring to Fig. 2.1, the failure area in the soil under the foundation can be divided into three major zones. They are:

1. Zone abc. This is a triangular elastic zone located immediately below the bottom of the foundation. The inclination of sides ac and bc of the wedge with the horizontal is $\alpha=\phi$ (soil friction angle).
2. Zone bcf. This zone is the Prandtl's radial shear zone.
3. Zone bfg. This zone is the Rankinr passive zone. This slip lines in this zone make angles of $\pm(45-\phi / 2)$ with the horizontal.

Note that a Prandtl's radial shear zone and a Rankine passive zone are also located to the left of the elastic triangular zone abc; however, they are not shown in Fig. 2.2.

Line cf is an arc of a log spiral, defined by the equation

$$
\begin{equation*}
\mathbf{r}=\mathbf{r}_{\mathbf{0}} \mathbf{e}^{\theta \tan \phi} \tag{2.1}
\end{equation*}
$$

Lines bf and fg are straight lines. Lines fg actually extends up to the ground surface. Terzaghi assumed that the soil located above the bottom of the foundation could be replaced by surcharge $q=$ $\gamma \mathrm{D}_{\mathrm{f}}$.


Fig. 2.1, failure surface in soil at ultimate load for a continuous rough rigid foundation as assumed by Terzaghi

The shear strength, s , of the soil can be given as

$$
\begin{equation*}
\mathbf{s}=\sigma^{\prime} \tan \phi+\mathbf{c} \tag{2.2}
\end{equation*}
$$

Where $\sigma^{\prime}=$ effective normal stress

$$
\mathrm{c}=\text { cohesion }
$$

The ultimate bearing capacity, $\mathrm{q}_{\mathrm{u}}$, of the foundation can be determined if we considered faces ac and bc of the triangle wedge abc and obtained the passive force on each face requires to cause failure. Note that the passive force $P_{p}$ will be a function of the surcharge $q=\gamma D_{f}$. Cohesion $c$, unit weight $\gamma$, and angle of friction of the soil $\phi$. So, referring to Fig. 2.2.


Fig. 2.2, Passive force on the face bc of wedge abc shone in figure 2.1

The passive force $P_{p}$ on the face bc per unit length of the foundation at right to the cross section is

$$
\begin{equation*}
\mathbf{P}_{\mathbf{p}}=\mathbf{P}_{\mathrm{pq}}+\mathbf{P}_{\mathrm{pc}}+\mathbf{P}_{\mathrm{p} \gamma} \tag{2.3}
\end{equation*}
$$

Where $P_{p q}, P_{p c}$ and $P_{p \gamma}=$ passive force contributions of $q, c$ and $\gamma$, respectively.

It is important to note that the directions of $P_{p q}, P_{p c}$ and $P_{p \gamma}$ are vertical, since the face bc makes an angle $\phi$ with the horizontal, and $\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$ and $\mathrm{P}_{\mathrm{p} \mathrm{\gamma}}$ must make an angle $\phi$ to the normal to bc. In order to obtain $\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$ and $\mathrm{P}_{\mathrm{py}}$, the method of superposition can be used; however, it will not be an exact solution.

Using the equilibrium analysis, Terzaghi express the ultimate bearing capacity in the form

$$
\begin{equation*}
\mathbf{q}_{\mathbf{u}}=\mathbf{c} \mathbf{N}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}}+1 / 2 \gamma \mathbf{B} \mathbf{N}_{\gamma} \text { (strip foundation) } \tag{2.4}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& c=\text { cohesion of soil } \\
& \gamma=\text { unit weight of soil } \\
& q=\gamma D_{f}
\end{aligned}
$$

$\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$ and $\mathrm{N}_{\gamma}=$ bearing capacity factors that are Nondimensional and only function
of the soil friction angle, $\phi$
The bearing capacity factors, $\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$ and $\mathrm{N}_{\gamma}$ are defined by
$\mathrm{N}_{\mathrm{q}}=\mathrm{e}^{2(3 \pi / 4-\phi / 2) \tan \phi} / 2 \cos ^{2}(45+\phi / 2)$
$\mathbf{N}_{\mathbf{c}}=\cot \phi\left(\mathbf{N}_{\mathrm{q}}-\mathbf{1}\right)$
$\mathbf{N}_{\gamma}=1 / 2\left(\mathrm{~K}_{\mathrm{p} \gamma} / \cos 2 \phi-1\right) \tan \phi$

Where, $\mathrm{K}_{\mathrm{p} \gamma}=$ passive pressure coefficient
The variation of bearing capacity factors defined by Eqs. 2.5, 2.6 and 2.7 are given in Table-2.1

Table- 2.1
Bearing-capacity factor for Terzaghi equations

Value of $\mathrm{N}_{\gamma}$ for f of 34 and $48^{\circ}$ are orignal Terzaghi value and used to back-compute $\mathrm{K}_{\mathrm{p} \gamma}$

| $\phi, \mathbf{d e g}$ | $\mathbf{N}_{\mathbf{c}}$ | $\mathbf{N}_{\mathbf{q}}$ | $\mathbf{N}_{\gamma}$ | $\mathbf{K}_{\mathbf{p} \gamma}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 5.7 | 1.0 | 0.0 | 10.8 |
| 5 | 7.3 | 1.6 | 0.5 | 12.2 |
| 10 | 9.6 | 2.7 | 1.2 | 14.7 |
| 15 | 12.9 | 4.4 | 2.5 | 18.6 |
| 20 | 17.7 | 7.4 | 5.0 | 25.0 |
| 25 | 25.1 | 12.7 | 9.7 | 35.0 |
| 30 | 37.2 | 22.5 | 19.7 | 52.0 |
| 34 | 52.6 | 36.5 | 36.0 | 59.5 |
| 35 | 57.8 | 41.4 | 42.4 | 82.0 |
| 40 | 95.7 | 81.3 | 100.4 | 141.0 |
| 45 | 172.3 | 173.3 | 297.5 | 298.0 |
| 48 | 258.3 | 287.9 | 780.1 | 650.6 |
| 50 | 347.5 | 415.1 | 1153.2 | 800.0 |

For estimating the ultimate bearing capacity of square or circular foundation Eq. (2.4) may be modified to (square footing) (circular footing)

$$
\begin{equation*}
\mathbf{q}_{\mathbf{u}}=1.3 \mathbf{c} \mathbf{N}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}}+\mathbf{0 . 4} \gamma \mathbf{B} \mathbf{N}_{\gamma} \quad \text { (square footing) } \tag{2.8}
\end{equation*}
$$

And

$$
\begin{equation*}
\mathbf{q}_{\mathrm{u}}=1.3 \mathbf{c} \mathbf{N}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}}+0.4 \gamma \mathbf{B} \mathbf{N}_{\gamma} \quad \text { (circular footing) } \tag{2.9}
\end{equation*}
$$

In Eq.(2.8), B equals the dimensions of each side of the foundation; in Eq. (2.9), B equals the diameter of the foundation.

For foundations that the local shear failure mode in soil, Terzaghi suggested modifications to Eqs. (2.4), (2.8) and (2.9) as follows:

$$
\begin{align*}
& \mathbf{q}_{\mathbf{u}}=\mathbf{2 / 3} \mathbf{\mathbf { c N }}{ }_{\mathbf{c}}{ }_{\mathbf{c}}+\mathbf{q} \mathbf{N}^{\prime}{ }_{\mathbf{q}}+1 / 2 \gamma \mathbf{B} \mathbf{N}^{\prime}{ }_{\gamma} \quad \text { (strip foundation ) }  \tag{2.10}\\
& \mathbf{q}_{\mathbf{u}}=\mathbf{0 . 8 6 7} \mathbf{c N}{ }^{\prime}{ }_{\mathbf{c}}+\mathbf{q ~ N} \mathbf{N}_{\mathbf{q}}+\mathbf{0 . 4} \gamma \mathbf{B} \mathbf{N}^{\prime}{ }_{\gamma} \quad \text { (squar foundation) }  \tag{2.11}\\
& \mathbf{q}_{\mathbf{u}}=0.867 \mathbf{c N}^{\prime}{ }_{\mathbf{c}}+\mathbf{q ~} \mathbf{N}^{\prime}{ }_{\mathbf{q}}+\mathbf{0 . 3} \gamma \mathbf{B} \mathbf{N}^{\prime}{ }_{\gamma} \quad \text { (circular footing) } \tag{2.12}
\end{align*}
$$

$\mathrm{N}^{\prime}{ }_{\mathrm{c}}, \mathrm{N}^{\prime}{ }_{\mathrm{q}}$ and $\mathrm{N}^{\prime}{ }_{\gamma}$ are the modified bearing capacity factors. They can be calculated by using the bearing capacity factor equations (for $\mathrm{N}^{\prime}{ }_{\mathrm{c}}, \mathrm{N}^{\prime}{ }_{\mathrm{q}}$ and $\mathrm{N}^{\prime}{ }_{\gamma}$ ) by replacing $\phi$ by $\phi^{\prime}=\tan ^{-1}(2 / 3 \tan \phi)$.

Terzaghi's bearing capacity equation have now been modified to take into account the effects of the foundation shape $(B / L)$, depth of embedment $\left(D_{f}\right)$, and the load inclination. This is given in tables in coming pages. Many design engineers, however, still use Terzaghi's equation, which provides fairly good results considering the uncertainty of the soil conditions at various sites.

### 2.3.1 ASSUMPTION AND LIMITATIONS IN TERZAGHI'S

ANALYSIS

1. The soil is homogeneous and isotropic and its shear strength is represented by Coulomb's equation.
2. The strip footing has a rough base, and the problem is essentially two-dimensional.
3. The elastic zone has straight boundaries inclined at $\alpha=\phi$ to the horizontal, and the plastic zones fully develop.
4. Pp consists of three components, which can be calculated separately and added, although the critical surface for these components are not identical.
5. Failure zones do not extend the horizontal plane through the base of the footing, i.e. the shear resistance of soil above the base is neglected and the effect of soil around the footing is considered equivalent to a surcharge $\sigma=\gamma \mathrm{D}$.

## LIMITATIONS

1. As the soil compressed, $\phi$ changes; slight downward movement of footing may not develop fully the plastic zones.
2. Error due to assumption 4 is small and on the safe side.
3. Error due to assumption 5 increases with depth of foundation, and hence the theory is suitable for shallow foundation only.

### 2.4 MEYERHOF'S BEARING CAPACITY EQUATION

Meyerhof $(1951,1963)$ proposed a bearing capacity equation similar to that of Terzaghi, but included a shape factor $\mathrm{s}_{\mathrm{q}}$ for the depth term $\mathrm{N}_{\mathrm{q}}$. He also included depth factor $\mathrm{d}_{\mathrm{i}}$ and inclination factor $i_{i}$ for cases where the footing load is inclined from the vertical. This procedure equation of the general form shown on Table 2.1, with N factors in Table 2.4.


Hig2.3 General footing-soil interaction for bearing-capacity equations for strip footing- left side Tercaghi(1943), Hansen (1970), and right side Meyerhof (1951).

Meyerhof obtained his N factor by making trials of the zone abd' with arc ad', which includes an approximate for shear along cd of Fig 2.3a. The shape, depth and inclination factors in Table 2.3 are
from Meyerhof (1963) and are somehow different from his 1951 valued. The shape factors do not greatly differ from those given by Terzaghi except for the addition of sq. observing that the shear effect along cd of Fig. 2.3a was being somewhat ignored, Meyerhof proposed depth factor $d_{i}$.

He also proposed using inclination factors to reduce the bearing capacity when the load resultant was inclined from the vertical by the angle $\theta$.

Up to about D = B of Fig. 2.3a Meyerhof's $q_{u}$ is not greatly differ from the Terzaghi value. The difference is more pronounced at larger $\mathrm{D} / \mathrm{B}$ ratios.

Table 2.2 Bearing capacity equations by several authors indicated

## TERZAGHI :

$$
\begin{aligned}
\mathbf{q}_{\mathbf{u}}=\mathbf{c} \mathbf{N}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}} & +\mathbf{0 . 5} \boldsymbol{\mathbf { B }} \mathbf{N}_{\gamma} \mathbf{s}_{\gamma} \\
\mathrm{N}_{\mathrm{q}} & =\mathrm{a}^{2} / 2 \cos ^{2}(45+\phi / 2) \\
\mathrm{a} & =\mathrm{e}^{(0.75 \pi-\phi / 2) \tan \phi} \\
\mathrm{N}_{\mathrm{c}} & =\left(\mathrm{N}_{\mathrm{q}}-1\right) \cot \phi \\
\mathrm{N}_{\gamma} & =\tan \phi / 2\left(\mathrm{~K}_{\mathrm{p} \gamma} / \cos ^{2} \phi-1\right)
\end{aligned}
$$

For: strip round square

$$
\begin{array}{lll}
\mathrm{s}_{\mathrm{c}}=1.0 & 1.3 & 1.3 \\
\mathrm{~s}_{\gamma}=1.0 & 0.6 & 0.8
\end{array}
$$

## MEYERHOF :

Vertical load : $\mathbf{q}_{\mathbf{u}}=\mathbf{c} \mathbf{N}_{\mathbf{c}} \mathbf{s}_{\mathbf{c}} \mathbf{d}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}} \mathbf{s}_{\mathbf{q}} \mathbf{d}_{\mathbf{q}}+\mathbf{0 . 5} \mathbf{B} \mathbf{N}_{\gamma} \mathbf{s}_{\gamma} \mathbf{d}_{\gamma}$
Inclined load: $\mathbf{q}_{\mathbf{u}}=\mathbf{c} \mathbf{N}_{\mathbf{c}} \mathbf{d}_{\mathbf{c}} \mathbf{i}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}} \mathbf{d}_{\mathbf{q}} \mathbf{i}_{\mathbf{q}}+\mathbf{0 . 5} \boldsymbol{\gamma} \mathbf{B} \mathbf{N}_{\gamma} \mathbf{d}_{\gamma} \mathbf{i}_{\gamma}$

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{q}}=\mathrm{e}^{\pi \tan \phi} \tan ^{\frac{z}{z}}(45+\phi / 2) \\
& \mathrm{N}_{\mathrm{c}}=\left(\mathrm{N}_{\mathrm{q}}-1\right) \cot \phi \\
& \mathrm{N}_{\gamma}=\left(\mathrm{N}_{\mathrm{q}}-1\right) \tan (1.4 \phi)
\end{aligned}
$$

HANSEN :
General : $\mathbf{q}_{\mathbf{u}}=\mathbf{c} \mathbf{N}_{\mathbf{c}} \mathbf{s}_{\mathbf{c}} \mathbf{d}_{\mathbf{c}} \mathbf{i}_{\mathbf{c}} \mathbf{g}_{\mathbf{c}} \mathbf{b}_{\mathbf{c}}+\mathbf{q} \mathbf{N}_{\mathbf{q}} \mathbf{s}_{\mathbf{q}} \mathbf{d}_{\mathbf{q}} \mathbf{i}_{\mathbf{q}} \mathbf{g}_{\mathbf{q}} \mathbf{b}_{\mathbf{q}}+\mathbf{0 . 5} \mathbf{B} \mathbf{B} \mathbf{N}_{\gamma} \mathbf{s}_{\gamma} \mathbf{d}_{\gamma} \mathbf{i}_{\gamma} \mathbf{g}_{\gamma} \mathbf{b}_{\gamma}$ When $\phi=0$

Use $\mathrm{q}_{\mathrm{u}} 5.14 \mathrm{~s}_{\mathrm{u}}\left(1+\mathrm{s}^{\prime}{ }_{\mathrm{c}}+\mathrm{d}^{\prime}{ }_{\mathrm{c}}-\mathrm{I}^{\prime}{ }_{\mathrm{c}}-\mathrm{b}^{\prime}{ }_{\mathrm{c}}-\mathrm{g}^{\prime}{ }_{\mathrm{c}}\right)+\mathrm{q}$

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{q}}=\text { same as Meyerhof above } \\
& \mathrm{N}_{\mathrm{c}}=\text { same as Meyerhof above } \\
& \mathrm{N}_{\gamma}=1.5\left(\mathrm{~N}_{\mathrm{q}}-1\right) \tan \phi
\end{aligned}
$$

VESIC:

## Use Hansen's equation above

$\mathrm{N}_{\mathrm{q}}=$ same as Meyerhof above
$\mathrm{N}_{\mathrm{c}}$ = same as Meyerhof above
$\mathrm{N}_{\gamma}=2\left(\mathrm{~N}_{\mathrm{q}}+1\right) \tan \phi$

| Factors | value | for |
| :---: | :---: | :---: |
| Shape : | $\mathrm{s}_{\mathrm{c}}=1+0.2 \mathrm{~K}_{\mathrm{p}} \mathrm{B} / \mathrm{L}$ | Any $\phi$ |
|  | $\mathrm{s}_{\mathrm{q}}=\mathrm{s}=1+0.1 \mathrm{~K}_{\mathrm{p}} \mathrm{B} / \mathrm{L}$ | $\phi>10^{\circ}$ |
|  | $\mathrm{s}_{\mathrm{q}}=\mathrm{s}_{\gamma}=1$ | $\phi=0$ |
| Depth: | $\mathrm{d}_{\mathrm{c}}=1+0.2\left(\mathrm{~K}_{\mathrm{p}}\right)^{1 / 2} \mathrm{D} / \mathrm{B}$ | Any $\phi$ |
|  | $\mathrm{d}_{\mathrm{q}}=\mathrm{d}_{\gamma}=1+0.1\left(\mathrm{~K}_{\mathrm{p}}\right)^{1 / 2} \mathrm{D} / \mathrm{B}$ | $\phi>10$ |
|  | $\mathrm{d}_{\mathrm{q}}=\mathrm{d}_{\gamma}=1$ | $\phi=0$ |
| Inclination: | $\mathrm{i}_{\mathrm{c}}=\mathrm{i}_{\mathrm{q}}=\left(1-\theta^{\circ} / 90^{\circ}\right)^{2}$ | Any $\phi$ |
|  | $\mathrm{i}_{\gamma}=\left(1-\theta^{\circ} / \phi^{\circ}\right) 2$ | $\phi>0$ |
|  | $\mathrm{i}_{\gamma}=0$ | $\phi=0$ |
| $\mathrm{K}_{\mathrm{p}}=\tan ^{2}(45+\phi / 2)$ |  |  |
| $\theta=$ angle of resultant | from vertical without a sign |  |

### 2.5 HANSENE'S BEARING CAPACITY METHOD

Hansen (1970) proposed the general bearing capacity case and N factor equation shown in Table 2.2. It can be readily seen that this equation is a further extension of the earlier Meyerhof (1951) work. Hansene's shape, depth, and other factors making up the general bearing capacity equation proposal in 1957 and 1961. The extension includes a factor for the footing being tilted from the horizontal $b_{i}$ and for the possibility of the footing being on a slope $\mathrm{g}_{\mathrm{i}}$. Table 2.4 give selected N values for the Hansen equation together with the more difficult shape and depth factor aids.

Any of the equation not subscripted with a (V) may be used as appropriate (limitations and restriction are noted in the Table). When the value used in the inclination equation has the horizontal load component $H$ parallel to $B$ one should use $B^{\prime}$ with the $N_{\gamma}$ term in the bearing capacity equation and if H is parallel to L use L ' with $\mathrm{N}_{\gamma}$. A further restriction is $\mathrm{i}_{\mathrm{i}}>0$ since a value of $\mathrm{i}_{\mathrm{i}} \leq 0$ is an unstable footing that requires resizing before proceeding. For a footing on clay with $\phi=0$ compute $\mathrm{i}_{\mathrm{c}}$ using H parallel to B and/or L as appropriate and note it is a subtractive constant in the modified bearing capacity equation.

We note that when the base is tilted V and H are perpendicular and parallel, respectively, to the base as compared with when it is horizontal.

For footing on a slope $\mathrm{g}_{\mathrm{i}}$ factor are used to reduce the bearing capacity, however, these- as with the factor of Table 2.5 should be used cautiously as here is little experimental data available other than the work of Shields et. al. (1977) who used model footing on a sand box slope. It is difficult to see a field case where one would use a spread footing in a cohesionless soil slope unless the slope angle $\beta$ is very and the footing depth D very large. In any case, since there are already shear stresses in the
slop soil (holding the slop in place) one should not adjust any $\phi_{t r}$ to the large plane strain value and, additionally, one should use a large safety factor.

The Hansen equation implicitly allows ant $\mathrm{D} / \mathrm{B}$ and thus can be used for both shallow (footing) and deep (piles, drill caisson) bases. Inspection of the $\mathrm{qN}_{\mathrm{q}}$ term implies a great increase in $\mathrm{q}_{\mathrm{ult}}$ with great depth. To place modest limits on this Hansen used
$\left.\begin{array}{l}d_{c}=1+0.4 D / B \\ d_{q}=1+2 \tan \phi(1-\sin \phi)^{2} D / B\end{array}\right\} \quad D / B \leq 1$

This gives a discontinuity at $\mathrm{D} / \mathrm{B}=1$; however, note the use of $\leq$ and $>$. For $\mathrm{f}=0$, we have

| $\mathrm{D} / \mathrm{B}=$ | 0 | 1 | 1.1 | 2 | 5 | 10 | 20 | 100 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~d}_{\mathrm{c}}{ }^{\prime}=$ | 0 | 0.40 | 0.33 | 0.44 | 0.55 | 0.59 | 0.61 | 0.62 |

We can see that use of $\tan ^{-1} D / B$ for $D / B>1$ controls the increase in $d_{c}$ and $d_{q}$ in line with observation that $\mathrm{q}_{\mathrm{ult}}$ appears to reach some limiting value at some depth ratio $\mathrm{D} / \mathrm{B}$ where this value of D is often termed the critical depth.

### 2.6 VESIC'S BEARING CAPACITY EQUATIONS

The Vesic $(1973,1974)$ procedure that is essentially the Hansen method will be briefly noted.
Essentially differences in this method are in using a slight difference $\mathrm{N}_{\gamma}$ (see table 2.4) and a variation on some of Hansen's $i_{i}$, $b_{i}$, and $g_{i}$ factor as noted with the subscript (V) in Table 2.5. Any of the factor not subscribed with an $(\mathrm{H})$ can be used for a Vesic solution. Note that some of the Vesic
factors are less conservative than those of Hansen and since none of the methods have been extensively verified with full-scale field test one should exercise caution in their use.

Table 2.4
Bearing-capacity factors for the Meyerhof, 1951; Hansen, 1970; and Vesic, 1973
Bearing-capacity equations

Note that $\mathrm{N}_{\mathrm{c}}$ and $\mathrm{N}_{\mathrm{q}}$ are same for all three methods; subscript identify auther for $\mathrm{N}_{\gamma}$

| $\phi$ | $\mathbf{N}_{\mathbf{c}}$ | $\mathbf{N}_{\mathbf{q}}$ | $\mathbf{N}_{\gamma(\mathbf{H})}$ | $\mathbf{N}_{\gamma(\mathbf{M})}$ | $\mathbf{N}_{\gamma(\mathbf{V})}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 5.14 | 1.0 | 0.0 | 0.0 | 0.0 |
| 5 | 6.49 | 1.6 | 0.1 | 0.1 | 0.4 |
| 10 | 8.34 | 2.5 | 0.4 | 0.4 | 1.2 |
| 15 | 10.97 | 3.9 | 1.2 | 1.1 | 2.6 |
| 20 | 14.83 | 6.4 | 2.9 | 2.9 | 5.4 |
| 25 | 20.71 | 10.7 | 6.8 | 6.8 | 10.9 |
| 26 | 22.25 | 11.8 | 7.9 | 8.0 | 12.5 |
| 28 | 25.79 | 14.7 | 10.9 | 1.2 | 16.7 |
| 30 | 30.13 | 18.4 | 15.1 | 5.7 | 22.4 |
| 32 | 35.47 | 23.2 | 20.8 | 22.0 | 30.2 |
| 34 | 42.14 | 29.4 | 28.7 | 31.1 | 41.0 |
| 36 | 50.55 | 37.7 | 40.0 | 44.4 | 56.2 |
| 38 | 61.31 | 48.9 | 56.1 | 64.0 | 77.9 |
| 40 | 75.25 | 64.1 | 79.4 | 93.6 | 109.3 |
| 45 | 133.73 | 134.7 | 200.5 | 262.3 | 271.3 |
| 50 | 266.50 | 318.5 | 567.4 | 871.7 | 761.3 |




| Shape facters | Deput factas | Inclisation laciees | Grauad factors ibase os sicpel |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & 5_{4}=0.2 \cdot \frac{B}{2} \\ & s_{4}=1+\frac{N_{4}}{N_{4}}-\frac{B}{2} \\ & 4_{4}=1 \text { for strip } \end{aligned}$ | $u_{i}=0.4 k$ $A_{1}=1+04 k$ | $\begin{aligned} & \tau_{a m 1}=0.5-0.5 \sqrt{1-\frac{H}{A_{f} c_{q}}} \\ & \zeta_{a v 1}=1-\frac{n H f}{A_{i} c_{v} N} \end{aligned}$ | $\phi=\frac{\xi^{\prime}}{14^{*}}$ <br> for vesk ure $N_{1}=-2 \sin \$$ for $\phi=0$ $\pi c=1-\frac{F^{*}}{107^{*}}$ |
| $x_{4}=1+\frac{A}{x}$ tan \% | $d_{4}=1+2 \tan \phi(1-\sin \phi) k$ |  |  |
|  |  | $\begin{aligned} & \operatorname{Iami}=\left(1-\frac{0.5 H}{V+A_{r} C_{0} \cot \phi}\right)^{\prime} \\ & \operatorname{Can}^{2}=\left(1-\frac{H}{V+A_{r} C_{0} \cot t}\right)^{\prime \prime} \end{aligned}$ | fon $=$ fran $=(12-t a t y)^{3}$ |
| $2,=1-0.4 \cdot \frac{3}{2}$ | $d_{1}=1.00$ Sor all 4 |  | Base factorn (filied have) |
|  | $\begin{aligned} & k=\frac{D}{B} \text { for } \frac{D}{B} \leq 1 \\ & k=\tan ^{-1} \frac{D}{B} \text { for } \frac{D}{B}>1 \text { (rad) } \end{aligned}$ |  | $\begin{aligned} & v_{i}=\frac{v^{\prime \prime}}{147^{*}} \\ & t_{c}=1-\frac{v^{\prime \prime}}{147^{\circ}} \end{aligned}$ |
| Where $A_{r}=$ rfloctive fouting ares $H^{\prime}=L^{\prime}{ }^{\prime} \mid$ iee Fig 4-4) <br> $c_{c}=$ athosion so buse $=$ whenen er a reluaed alaz <br> $b=$ depo of footing is ground juist slith B ant bor a', <br> $\mathrm{cq}_{\mathrm{c}} \cdot \mathrm{C}_{2}=$ sootrerixity of lead siti respect io ctater of footiag artat <br>  <br> $Y$ - loual vercikal load oe footing <br> $\beta$ - toppe or ground sump ham hase with deuswate - ( + ) <br>  satcrets on soil <br> n = titi ingle of tase fion berimetal sith $\langle+1$ epeart as यकात! cave |  |  | $\begin{aligned} & \delta_{a t s}=\operatorname{cosp}(-2 \gamma \operatorname{lan} \phi) \\ & b_{\operatorname{tas}}=\operatorname{csp}(-2.7 \mathrm{tan} \tan \phi) \end{aligned}$ |
|  |  | $b_{\text {eve }}=b_{\text {es }}=\left\langle 1-\nu^{2} \tan \phi\right\}^{2}$ |
| Generaf L De eot exe is is combination wits is <br> 2 Can wie $s$ in monbinution with $d$, $a$, and b, <br> 2. For $4 \mathrm{l} \leq 2$ une $\phi$, <br> For $1 / \mathrm{A}>2 \mathrm{une} \phi_{m}=15 \phi_{n}-13$ <br> For $\phi \leq 14^{\prime}$ ur $\phi_{m}=\phi$. |  |  | $m=\pi,=\frac{2+B / L}{1+B \sqrt{2}} \quad H$ parallel to $B$ $n=m_{1}=\frac{2+t_{\sqrt{ }} \theta}{1+L_{\sqrt{ }} \theta} \quad H$ prarallal to $L$ |  |
|  |  | Note: $4,1,>0$ |  |

### 2.7 NUMERICAL EVALUATION OF TERZAGHI'S $\mathbf{N}_{\gamma}$

While developing the solution, Eq.2.4 and Fig.2.4, Terzaghi (1943) makes a series of assumptions (e.g., replacement of the soil located above the base of the footing by a uniform surcharge, limit equilibrium), separates contributions of $\mathrm{c}, \mathrm{q}$, and $\gamma$, and calculates q , by superposition. Key details of his procedure relevant to the present paper are briefly summarized here. Fig. 2.5(a) (after Terzaghi 1943) shows a shallow, strip footing with rough base resting on a horizontal surface. The soil below the footing is in a state of plastic equilibrium under general shear failure. Terzaghi uses Prandtl's mechanism to divide the body of soil iecdh into an elastic zone $a b c$, I, and symmetric radial shear, II, and passive Rankine zones, III, and assumes that the radial shear zone is bounded by a log-spiral, $\mathrm{r}=$ $r_{o} e^{\theta a n \phi}$ (cd in Fig. 2.5) failure surface. While calculating Arc and $N_{q}$, the $\log$ spiral is unique.

It is centered at the footing edge (point a in Fig. 2.5) and spans between $a c$ and $a d$, which, respectively, make angles $\phi$ and 45- $\phi / 2$ with the horizontal. Closed-form expressions for $\mathrm{N}_{\mathrm{c}}$ and $N_{q}$ are therefore easily obtained by taking moments of the forces acting on the block acdf about a. This log spiral proves unsatisfactory for calculating $\mathrm{N}_{\gamma}$. Terzaghi therefore assumes that the center of the unknown failure surface spanning between $a c$ and $a d$ lies on $a d$. From the family of $\log$ spirals, he finds the critical failure surface, one which yields minimum passive pressure $\mathrm{P}_{\gamma}$ on the wedge $a c d f$, Fig. 2.5(c), graphically by trial and error for a series of $\phi$ values, and provides a $\phi$ versus $\mathrm{N}_{\gamma}$ (Fig. 2.4) relation. The procedure, although tedious, is logical, since mathematical expression for obtaining the critical surface and its solution become formidable. Modifications to the original solution that enhance accuracy of (2.4) in many respects, still use the trial-and-error procedure for calculating $\mathrm{N}_{\gamma}$. Graphical procedures in general have inherent limitations regarding accuracy and, the extent of accuracy of Terzaghi's solution is not known. The objective of this technical note is to present explicit analytical expressions for calculating $\mathrm{N}_{\gamma}$, to provide results of their numerical solution and compare them with those of the graphical method.


Fig. 2.4 Relation between fand Bearing-capacity Factor (After Terzaghi 1943)


Tig. 2.5 Determination of $P_{\gamma}(\phi \neq 0, \mathrm{~g} \neq 0, \mathrm{q}=0, \mathrm{c}=0)$ to calculate $\mathrm{N}_{\gamma}$ : (a) Geometry of strip footing and adjacent soil in limit equilibrium; (b) forces acting on elastic wedge due to self weight; (c) free-body diagram of wedge acdf.

### 2.7.1 NUMERICAL SOLUTION FOR $\mathbf{N}_{\gamma}$ (by kumbhojkar)

The condition for finding a $\log$ spiral that gives minimum $\mathrm{P}_{\gamma}$ can be easily described as $\partial P_{\gamma} / \partial \nu=O$, where v is any characteristic variable related to the log-spiral geometry such as a coordinate of the center of the $\log$ spiral or $\theta$. Fig. 2.5, shows a trial $\log$ spiral $c d$ with center $O\left(x_{l}, y_{l}\right)$ and angle $d o c=\theta$. The wedge $a c d f$ is in equilibrium under the forces $W_{1}, W_{2}, W_{3}, P_{d}, F$, and $\mathrm{P}_{\gamma}$ (per unit length of the footing) where
$\mathrm{W}_{1}=$ weight of the block ocd $=\gamma \mathrm{r}_{1}{ }^{2}-\mathrm{r}_{\mathrm{o}}{ }^{2} / 4 \cdot \tan \phi$
$\mathrm{W}_{2}=$ weight of the triangle afd $=\gamma\left(\mathrm{r}_{1}{ }^{2} \cos \phi+4 \mathrm{r}_{1} \mathrm{x}_{1} \sin \theta^{*}+2 \mathrm{x}_{1}{ }^{2} \tan \theta^{*}\right) / 4$
$\mathrm{W}_{3}=$ weight of the triangle aco $=\gamma\left(\tan \phi+\tan \theta^{*}\right) \mathrm{x}_{1} \mathrm{~B} / 2$
$\mathrm{P}_{\mathrm{d}}=$ Rankine passive earth pressure $=\gamma\left(\mathrm{x}_{1}+\mathrm{r}_{1} \cos \theta^{*}\right)^{2} / 2$
$\mathrm{F}=$ resultant reaction of frictional force acting along the arc cd , which passes through $\mathrm{O} . \mathrm{P}_{\gamma}=$ passive force component due to $\gamma$.

Taking moment about O , one gets $\mathrm{P}_{\gamma}$ as

$$
\begin{equation*}
\mathrm{P}_{\gamma}=\left(\mathrm{W}_{1} \mathrm{l}_{1}+\mathrm{W}_{2} \mathrm{l}_{2}-\mathrm{W}_{3} \mathrm{l}_{3}+\mathrm{P}_{\mathrm{d}} \mathrm{l}_{\mathrm{d}}\right) / \mathrm{l}_{\mathrm{p}} \tag{2.17}
\end{equation*}
$$

Where the lever arm $l_{1}$ (Das 1987), $1_{2}, 1_{3}, 1_{d}$, and $l_{p}$ are given by
$1_{1}=4 \tan \phi\left\{\mathrm{r}_{1}{ }^{3}\left(3 \tan \phi \cos \theta^{*}-\sin \theta^{*}\right)\right.$

$$
\begin{equation*}
\left.+\mathrm{r}_{\mathrm{o}}^{3}\left[\sin \left(\theta+\theta^{*}\right)-3 \tan \phi \cos \left(\theta^{*}+\theta\right)\right]\right\} / 3\left(9 \tan ^{2} \phi+1\right)\left(\mathrm{rl}^{2}-\mathrm{r}_{\mathrm{o}}^{2}\right) \tag{2.18}
\end{equation*}
$$

$l_{2}=2 r_{1} \cos \theta^{*}-\mathrm{x}_{1} / 3$
$1_{3}=x_{1}-2 / 3\left(x_{1}-B\right) / 2=2 x_{1}+B / 3$
$l_{d}=2 r_{1} \sin \theta^{*}-x_{1} \tan \theta^{*} / 3$
$1_{p}=x_{1}+2 B / 3$

All the quantities on the right-hand side of (2.17) are a function of variables $\mathrm{x}_{1}$, the x -coordinate of the center of the $\log$ spiral, and $r_{o}$ and $r_{1}$, the radii of the $\log$ spiral for $\theta=0$ and $\theta=0$. The quantities $x_{1}, r_{0}$, and $r_{1}$, in turn, can be expressed as a function of $\theta$ using geometrical relations.
$\mathrm{x}_{1}=-\mathrm{B} / 2-(1+\sin \phi) \cot \theta \mathrm{B} / 2 \cos \phi$
$\mathrm{r}_{1}=\left(\tan \phi \cos \theta^{*}+\sin \theta^{*}\right) B \mathrm{e}^{\theta \tan \phi} / \sin \theta$

Substituting these values in (2.17) and rearranging the terms we get the following explicit expression:
$P_{\gamma} / \gamma B^{2}=\sum c_{i} t_{i} / c_{8} t_{6}+c_{9} t_{7}$
$\partial P g / \partial q \quad=\gamma B^{2} /\left(c_{8} t_{6}+c 9 t_{7}\right)^{2} \sin ^{3} \theta \quad \sum d_{i} u_{i}$

Coefficient $c_{i}$ and terms $t_{i}$ and terms $d_{i}$ and $u_{i}$ are respectively given in Table 2.6 and 2.7. The equations $\partial P_{\gamma} / \partial q=0$ is solved numerically to obtained $\theta$ for $P_{\gamma \min } P_{\gamma} / \gamma B^{2}$ is obtained from (2.24) and Ng using Terzaghi; s equation
$\mathrm{N}_{\gamma}=\mathrm{P}_{\gamma \min } / \gamma \mathrm{B}^{2}-\tan \phi / 2$

Tables 2.8 and 2.9 provide values of $\mathrm{N}_{\gamma}$ along with the coordinates of the center and angles of the logspiral for $\phi=0^{\circ}$ to $53^{\circ}$ When $\phi$ becomes larger, $\mathrm{N}_{\gamma}$ becomes highly sensitive to $\phi$; for $\phi>35^{\circ}$ results are therefore given with an increment of $0.5^{\circ}$.

Table 2.6, Terms and their Coefficients in Eq. (2.24)

| Solution number <br> (1) | Coefficient $\mathrm{C}_{\mathbf{i}}$ <br> (2) | Term $\mathrm{t}_{\mathrm{i}}$ <br> (3) |
| :---: | :---: | :---: |
| 1 | $\mathrm{c}_{1}=3 \tan \phi \cos ^{4} \theta^{*}-\sin \theta^{*} \cos ^{3} \theta^{*} / 3\left(8 \sin ^{2} \phi+1\right)+\cos ^{4} \theta^{* / 3} \cos \phi$ | $\mathrm{t}_{1}=\mathrm{e}^{3 \theta \tan \phi} / \sin ^{2} \theta$ |
| 2 | $\mathrm{c}_{2}=-\cos ^{2} \theta^{*} / 4$ | $\mathrm{t}_{2}=\mathrm{e}^{2 \theta \tan \phi} / \sin \theta$ |
| 3 | $\mathrm{c}_{3}=-(1+\sin \phi) \cos ^{2} \theta^{*} / 4 \cos \phi$ | $\begin{aligned} & \mathrm{t}_{3}= \\ & \mathrm{e}^{2 \theta \tan \phi} \cos \theta / \sin ^{2} \theta \end{aligned}$ |
| 4 | $\begin{aligned} & c_{4}=2 \cos ^{4} \theta^{*}+3 \sin \phi \cos ^{2} \theta^{* / 6}\left(8 \sin ^{2} \phi+1\right)+(1+\sin \phi) / 8- \\ & (1+\sin \phi) 2 / 12 \cos ^{2} \phi \end{aligned}$ | $\mathrm{t}_{4}=1 / \sin \theta$ |
| 5 | $c_{5}=\sin \theta^{*} \cos 3 \theta^{*}-3 \tan \phi \cos ^{4} \theta^{*} / 3\left(8 \sin ^{2} \phi+1\right)+(1+\sin \phi) 2 / 24$ $\cos \phi$ | $\mathrm{t}_{5}=\cos \theta / \sin ^{2} \theta$ |
| 6 | $\mathrm{c}_{6}=(1+\sin \phi) 2 / 12 \cos ^{2} \phi+\cos ^{2} \phi / 24(1+\sin \phi)-(1+\sin \phi) / 8$ | $\mathrm{t}_{6}=\sin \theta$ |
| 7 | $\mathrm{c}_{7}=-(1+\sin \phi) / 12 \cos \phi+\cos \phi / 8-(1+\sin \phi) 2 / 24 \cos \phi$ | $\mathrm{t}_{7}=\cos \theta$ |
| 8 | $\mathrm{c}_{8}=\cos \phi / 6$ | - |
| 9 | $\mathrm{c}_{9}=-(1+\sin \phi) / 2$ | - |

Table- 2.7, Coefficients and Terms in Equation $\Sigma \mathbf{d}_{\mathbf{i}} \mathbf{u}_{\mathbf{i}}$

| Solution no. <br> (1) | Coefficient $\mathbf{d}_{\mathbf{i}}$ <br> (2) | Term $\mathbf{u}_{\mathrm{i}}$ <br> (3) |
| :---: | :---: | :---: |
| 1 | $\mathrm{d}_{1}=3 \mathrm{c}_{1} \mathrm{c}_{8} \tan \phi+\mathrm{c}_{1} \mathrm{c}_{9}$ | $\mathrm{u}_{1}=\sin ^{2} \theta \mathrm{e}^{3 \theta \tan \phi}$ |
| 2 | $\mathrm{d}_{2}=3 \mathrm{c}_{1} \mathrm{c}_{9} \tan \phi-3 \mathrm{c}_{1} \mathrm{c} 8$ | $\mathrm{u}_{2}=\sin \theta \cos \theta \mathrm{e}^{3 \theta \tan \phi}$ |
| 3 | $\mathrm{d}_{3}=-2 \mathrm{c}_{1} \mathrm{c}_{9}$ | $\mathrm{u}_{3}=\cos 2 \theta \mathrm{e}^{3 \theta \tan \phi}$ |
| 4 | $\mathrm{d}_{4}=2 \mathrm{c}_{2} \mathrm{c}_{8} \tan \phi-\mathrm{c}_{3} \mathrm{c}_{8}+\mathrm{c}_{2} \mathrm{c}_{9}$ | $\mathrm{u}_{4}=\sin ^{3} \theta \mathrm{e}^{2 \theta \tan \phi}$ |
| 5 | $\mathrm{d}_{5}=2 \mathrm{c}_{3} \mathrm{c}_{8} \tan \phi+2 \mathrm{c}_{2} \mathrm{c}_{9} \tan \phi-2 \mathrm{c}_{2} \mathrm{c}_{8}$ | $\mathrm{u}_{5}=\sin 2 \theta \cos \theta \mathrm{e}^{2 \theta \tan \phi}$ |
| 6 | $\mathrm{d}_{6}=2 \mathrm{c}_{3} \mathrm{c}_{8} \tan \phi-\mathrm{c}_{2} \mathrm{c}_{9}-3 \mathrm{c}_{3} \mathrm{c}_{8}$ | $\mathrm{u}_{6}=\sin \theta \cos ^{2} \theta \mathrm{e}^{2 \theta \tan \phi}$ |
| 7 | $\mathrm{d}_{7}=-2 \mathrm{c}_{3 \mathrm{c} 9}$ | $\mathrm{u}_{7}=\cos ^{3} \theta \mathrm{e}^{2 \theta \tan \phi}$ |
| 8 | $\mathrm{d}_{8}=2 \mathrm{c}_{4} \mathrm{c}_{9}+2 \mathrm{c}_{5} \mathrm{c}_{8}-\mathrm{c}_{7} \mathrm{c}_{8}+\mathrm{c}_{6} \mathrm{c}_{9}$ | $\mathrm{u}_{8}=\sin ^{3} \theta$ |
| 9 | $\mathrm{d}_{9}=-3 \mathrm{c}_{5} \mathrm{c}_{8}-\mathrm{c}_{4} \mathrm{c}_{9}$ | $\mathrm{u}_{9}=\sin \theta$ |
| 10 | $\mathrm{d}_{10}=-2 \mathrm{c}_{4} \mathrm{c}_{8}$ | $\mathrm{u}_{10}=\cos \theta$ |
| 11 | $\mathrm{d}_{11}=2 \mathrm{c}_{4} \mathrm{c}_{8}-2 \mathrm{c}_{5} \mathrm{c}_{9}$ | $\mathrm{u}_{11}=\cos ^{3} \theta$ |

### 2.7.2 COMMENTS

The values given in Tables 2.8 and 2.9 match the plot in Fig. 2.4 exactly over its entire range: $\phi=0^{\circ}$ to $39^{\circ}$ It is, however, a crude way of comparing since precision of this plot is lower than that of the numerical solution. With the exception of $\mathrm{N}_{\gamma}=36,260$, and 780 , respectively, for $\phi=34^{\circ}, 44^{\circ}$ and $48^{\circ}$ (Terzaghi 1943), it is not known whether explicit numerical values of $\mathrm{N}_{\gamma}$ used to plot Fig. 2.4 exist. The results presented in the present paper, therefore, can be compared with only these three values. For $\phi=3^{40}\left(38^{\circ}\right.$ versus $\left.36^{\circ}\right)$ and $44^{\circ}$, the difference 261 versus 260 is negligibly small and accuracy of the graphical solution is excellent. The large difference, 650 versus 780 , in $\mathrm{N}_{\gamma}$ for $\phi=48^{\circ}$ is probably due to some small error in the graphical analysis magnified by the sensitivity of $\mathrm{N}_{\gamma}$ to $\phi$ since the angle that gives $\mathrm{N} \gamma=780$ is about $48.6^{\circ}$ Any interpolation between $\phi=44^{\circ}$ and $48^{\circ}$ and extrapolation beyond $\phi=$ $48^{\circ}$ in Fig. 2.4 on the basis of $\mathrm{N}_{\gamma}=780$ are likely to include relatively large errors. Use of approximate methods similar to the graphical method to obtain $\mathrm{N}_{\gamma}$ are also likely to provide only approximately accurate answers. Bowles (1968), using a curve-fitting method, provides the only other set of numerical values of Terzaghi's $\mathrm{N}_{\gamma}$ for $\phi=5^{\circ}, 10^{\circ} \ldots 50^{\circ}$ For $\phi<20^{\circ}$ his values are approximately double of those given in Table 2.9 and for $\phi>35$ they become smaller than those given in Table 4, although their accuracy is comparable to that of Fig. 2.4. Beyond providing explicit values and enhancing accuracy of $\mathrm{N}_{\gamma}$, the analysis given here provides another distinct benefit: it defines log spirals demarking the radial shear zone for each qb .

A numerical solution for Terzaghi's bearing-capacity factor $\mathrm{N}_{\gamma}$ was presented. In addition to providing values of $\mathrm{N}_{\gamma}$ up to $\left.\phi\right)=53^{\circ}$ it also defined the geometry of the log spiral for each value of $\phi$. The results showed that within the limits of accuracy of graphical method, Terzaghi's $\mathrm{N}_{\gamma}$ calculations agree with the almost-exact numerical results.

Table 2.8, Bearing Capacity Factor $\mathrm{N}_{\gamma}$ and Geometry Details of Log-Spiral Providing for $\phi=\boldsymbol{0}^{\boldsymbol{0}}$ to $\mathbf{3 5}^{\mathbf{0}}$

| Solution | Friction | Log-spiral | Coordinate of center of log-spiral |  | $\mathbf{N}_{\gamma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number | Angle $\phi$ | Angle $\theta$ | $\mathrm{X}_{1} / \mathrm{B}$ | $y_{1} / \mathrm{B}$ |  |
| 1 | 0.0 | - | - |  | 0 |
| 2 | 1.0 | 88.833 | -0.510 | -0.502 | 0.014 |
| 3 | 2.0 | 90.910 | -0.492 | -0.475 | 0.035 |
| 4 | 3.0 | 92.494 | -0.477 | -0.453 | 0.063 |
| 5 | 4.0 | 93.753 | -0.465 | -0.434 | 0.099 |
| 6 | 5.0 | 94.782 | -0.454 | -0.416 | 0.144 |
| 7 | 6.0 | 95.637 | -0.445 | -0.401 | 0.200 |
| 8 | 7.0 | 96.356 | -0.437 | -0.387 | 0.267 |
| 9 | 8.0 | 96.966 | -0.430 | -0.374 | 0.348 |
| 10 | 9.0 | 97.486 | -0.423 | -0.361 | 0.444 |
| 11 | 10.0 | 97.931 | -0.417 | -0.350 | 0.559 |
| 12 | 11.0 | 98.311 | -0.411 | -0.339 | 0.694 |
| 13 | 12.0 | 98.637 | -0.406 | -0.329 | 0.854 |
| 14 | 13.0 | 98.916 | -0.401 | -0.319 | 1.041 |
| 15 | 14.0 | 99.152 | -0.397 | -0.310 | 1.262 |
| 16 | 15.0 | 99.352 | -0.393 | -0.301 | 1.520 |
| 17 | 16.0 | 99.519 | -0.389 | -0.293 | 1.822 |
| 18 | 17.0 | 99.656 | -0.385 | -0.285 | 2.175 |
| 19 | 18.0 | 99.768 | -0.382 | -0.277 | 2.589 |
| 20 | 19.0 | 99.855 | -0.378 | -0.270 | 3.074 |
| 21 | 20.0 | 99.921 | -0.375 | -0.263 | 3.641 |
| 22 | 21.0 | 99.968 | -0.372 | -0.256 | 4.305 |
| 23 | 22.0 | 99.996 | -0.369 | -0.249 | 5.085 |
| 24 | 23.0 | 100.009 | -0.367 | -0.243 | 6.000 |
| 25 | 24.0 | 100.006 | -0.364 | -0.237 | 7.076 |
| 26 | 25.0 | 99.989 | -0.362 | -0.231 | 8.342 |
| 27 | 26.0 | 99.960 | -0.360 | -0.225 | 9.836 |
| 28 | 27.0 | 99.918 | -0.357 | -0.219 | 11.602 |
| 29 | 28.0 | 99.866 | -0.355 | -0.214 | 13.636 |
| 30 | 29.0 | 99.804 | -0.353 | -0.208 | 16.175 |
| 31 | 30.0 | 99.732 | -0.352 | -0.203 | 19.129 |
| 32 | 31.0 | 99.652 | -0.350 | -0.198 | 22.653 |
| 33 | 32.0 | 99.563 | -0.348 | -0.193 | 26.871 |
| 34 | 33.0 | 99.467 | -0.346 | -0.188 | 31.035 |
| 35 | 34.0 | 99.363 | -0.345 | -0.183 | 38.035 |
| 36 | 35.0 | 99.253 | -0.344 | -0.179 | 45.410 |

Table 2.9, Bearing Capacity Factor $\mathbf{N}_{\gamma}$ and Geometry Details of Log-Spiral Providing for $\phi=$ $35.5^{\circ}$ to $53^{\circ}$

| Solution | Friction | Log-spiral | Coordinate of center of log-spiral |  | $\mathbf{N}_{\gamma}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number | Angle $\phi$ | Angle $\theta$ | $\mathrm{X}_{1} / \mathrm{B}$ | $\mathrm{y}_{1} / \mathrm{B}$ |  |
| 1 | 35.5 | 99.196 | -0.343 | -0.177 | 49.666 |
| 2 | 36.0 | 99.137 | -0.342 | -0.174 | 54.360 |
| 3 | 36.5 | 99.077 | -0.342 | -0.172 | 59.541 |
| 4 | 37.0 | 99.015 | -0.341 | -0.170 | 65.266 |
| 5 | 37.5 | 98.952 | -0.340 | -0.168 | 71.599 |
| 6 | 38.0 | 98.888 | -0.340 | -0.166 | 78.614 |
| 7 | 38.5 | 98.822 | -0.339 | -0.164 | 86.392 |
| 8 | 39.0 | 98.755 | -0.338 | -0.162 | 95.028 |
| 9 | 39.5 | 98.687 | -0.338 | -0.159 | 104.627 |
| 10 | 40.0 | 98.618 | -0.337 | -0.157 | 115.311 |
| 11 | 40.5 | 98.548 | -0.337 | -0.155 | 127.219 |
| 12 | 41.0 | 98.477 | -0.336 | -0.153 | 140.509 |
| 13 | 41.5 | 98.404 | -0.336 | -0.151 | 155.363 |
| 14 | 42.0 | 98.331 | -0.335 | -0.149 | 171.990 |
| 15 | 42.5 | 98.257 | -0.335 | -0.148 | 190.628 |
| 16 | 43.0 | 98.181 | -0.334 | -0.146 | 211.556 |
| 17 | 43.5 | 98.105 | -0.334 | -0.144 | 235.091 |
| 18 | 44.0 | 98.028 | -0.334 | -0.142 | 261.603 |
| 19 | 44.5 | 97.951 | -0.333 | -0.140 | 291.521 |
| 20 | 45.0 | 97.872 | -0.333 | -0.138 | 325.342 |
| 21 | 45.5 | 97.793 | -0.332 | -0.136 | 363.647 |
| 22 | 46.0 | 97.712 | -0.332 | -0.134 | 407.113 |
| 23 | 46.5 | 97.632 | -0.332 | -0.133 | 456.532 |
| 24 | 47.0 | 97.550 | -0.332 | -0.131 | 512.836 |
| 25 | 47.5 | 97.468 | -0.331 | -0.129 | 577.119 |
| 26 | 48.0 | 97.385 | -0.331 | -0.127 | 650.673 |
| 27 | 48.5 | 97.302 | -0.331 | -0.125 | 735.026 |
| 28 | 49.0 | 97.218 | -0.331 | -0.124 | 831.990 |
| 29 | 49.5 | 97.133 | -0.330 | -0.122 | 943.723 |
| 30 | 50.0 | 97.048 | -0.330 | -0.120 | 1072.797 |
| 31 | 50.5 | 96.962 | -0.330 | -0.119 | 1222.294 |
| 32 | 51.0 | 96.876 | -0.330 | -0.117 | 1395.915 |
| 33 | 51.5 | 96.790 | -0.330 | -0.115 | 1598.120 |
| 34 | 52.0 | 96.703 | -0.329 | -0.113 | 1834.301 |
| 35 | 52.5 | 96.615 | -0.329 | -0.112 | 2111.003 |
| 36 | 53.0 | 96.528 | -0.329 | -0.110 | 2436.199 |

### 2.8 COMPUTATION OF BEARING CAPACITY FACTOR $\mathrm{N}_{\gamma}$ BY USING KOTTERS EQUATION

The analysis is primarily based on the computation of vertical $\left(R_{V}\right)$ and horizontal $\left(R_{H}\right)$ components of reaction $R$ that acts on the curved part CD of the failure surface [Fig. 1(a)]. For this purpose, Kotter's equation (1903) is used.

### 2.8.1 KOTTER'S EQUATION

For a cohesionless soil medium, in passive state of equilibrium, Kotter's equation gives a solution for determining the distribution of soil reaction pressure $p$ along the arc of the failure surface in the following form (Fig.2):

$$
\begin{equation*}
d p / d s+2 p \tan \phi d \alpha / d s-\gamma \sin (\alpha+\phi)=0 \tag{2.27}
\end{equation*}
$$

in which $d p=$ differential reactive pressure on the elemental length $d s$ of the failure surface; $\alpha=$ angle made by the tangent to the failure surface at the point of interest with the horizontal; and $\phi=$ angle of soil internal friction. The applicability of Kotter's equation to the analysis of limit equilibrium problems has been demonstrated for a retaining wall problem (Coulomb's mechanism) for the case of a ponderable cohesionless soil by Dewaikar and Halkude (2002).


Fig. 2.6, Failure mecanisum-Terzaghi's analysis

(b)

Fig. 2.7, Free body diagram of wedge CDFB

### 2.8.2 OUTLINE OF PROPOSED ANALYSIS

As shown in Fig.2.7, the known forces that act on the failure wedge CDFB are $R_{H}, R_{V}, P_{\gamma}$ (passive Rankine thrust), $W$ (weight of wedge CDFB), and unknown is only one force, i.e., the passive thrust $P_{p \gamma}$. Now, if the pole of the log spiral CD is correctly located, the calculated forces $P_{\gamma}$ and $R_{H}$ will be exactly equal to each other so as to satisfy horizontal force equilibrium, otherwise, they will be different. If they are different, the trial location of the pole along the line BD is changed and for this new location of the pole, $R_{H}, R_{V}, W$, and $P_{\gamma}$ are again computed. Iterations are thus continued until the horizontal force equilibrium condition is satisfied to a specified decimal accuracy. After satisfying this condition, vertical force equilibrium condition is used to compute the desired value of $P_{p \gamma}$, from which $N_{\gamma}$ is calculated.


Fig. 2.8, Kotter's equation for cohesionless soil in passive state of equilibrium


Fig. 2.9, Geometrical relationships for pole above footing.

### 2.8.3 INTEGRATION OF KOTTER'S EQUATION

As seen earlier [Fig. 2.6], the failure surface has two parts, namely, CD being part of the log spiral and DE its tangent.

### 2.8.3.1 INTEGRATION OVER PART DE OF FAILURE SURFACE

This part of the failure surface being straight, $d \alpha / d s=0$ and Eq.(2.27) reduces to the following form:

$$
\begin{equation*}
d p / d s=\gamma \sin (\alpha+\phi) \tag{2.28}
\end{equation*}
$$

Integration of the above equation gives the pressure distribution over the plane failure plane DE and the value of $p$ at point $D$ is calculated as

$$
\begin{equation*}
p_{D}=\gamma \sin (45+\phi / 2) \mathrm{DE} \tag{2.29}
\end{equation*}
$$

The distance DE depends upon the location of pole of the log spiral as shown in Fig. 2.6. Referring to Figs. 2.7 and 2.9, DE is calculated from the geometry of the failure wedge. With this substitution, Eq. (2.29) becomes

$$
\begin{equation*}
p_{D}=\gamma \sin (45+\phi / 2) K r_{0} e^{\theta m \tan \phi} \tag{2.30}
\end{equation*}
$$

in which, $r_{o}, \theta_{m}$, and $\theta_{v}$ are as shown in Fig. 3 and $K$ is as given by the following expression:

$$
\begin{equation*}
K=\left[1-\sin \theta_{v} / \sin (45+\mathrm{f} / 2) e^{\theta m \tan \phi}\right] \tag{2.31}
\end{equation*}
$$

### 2.8.3.2 COMPUTATION OF VERTICAL AND HORIZONTAL COMPONENTS OF REACTION R ON CURVED FAILURE SURFACE CD <br> For this purpose, wedge CDFB as shown in Fig. 2.7 is referred. The magnitude of passive Rankine thrust $P_{\gamma}$ is given as

$$
\begin{equation*}
P_{\gamma}=1 / 2 \gamma(\mathrm{DF})^{2}(1+\sin \phi / 1-\sin \phi) \tag{2.32}
\end{equation*}
$$

The forces, $R_{H}$ and $R_{V}$ are calculated using Kotter's equation for a curved failure surface. For this purpose, Fig. 2.9 is referred.
Integration of Kotter's equation [Eq. (2.27)] gives the pressure distribution on the curved failure surface (Mohapatro 2001) and is given as

$$
\begin{align*}
\mathbf{p}= & \left\{\gamma r_{0} K \sin (45+\phi) e^{(3 \theta m-2 \theta \tan \phi)}\right\}+ \\
& +\left\{\left(\gamma r_{0} \sec \phi e^{\theta \tan \phi} / \mathbf{1}+9 \tan ^{2} \phi\right)\left[3 \tan \phi \sin \left(\theta-\theta_{L}+\phi\right)-\cos \left(\theta-\theta_{L}+\phi\right)\right]\right\} \\
& -\left\{\gamma r_{0} \sec \phi e\left(3 \theta_{m}-2 \theta\right) \tan \phi /\left(1+9 \tan ^{2} \phi\right)\right. \\
& \left.X\left\{3 \tan \phi \sin \left(\theta-\theta_{L}+\phi\right)-\cos \left(\theta-\theta_{L}+\phi\right)\right\}\right] \tag{2.33}
\end{align*}
$$

Where $\theta_{L}=\left(90-\theta_{v}\right)$ and $\theta$ is as shown in Fig. 2.9. In deriving the above expression, reactive pressure at point D as given by Eq. (2.30) is used as a boundary condition.

### 2.8.3.3 COMPONENTS OF RESULTANT REACTION ON FAILURE SURFACE

The resultant reaction $R$ on the failure surface is given as

$$
\begin{equation*}
R=\int p d s \tag{2.34}
\end{equation*}
$$

The vertical component $R_{V}$ of the reaction is obtained as (Fig. 2.9)

$$
\begin{equation*}
R_{V}=\int p \cos \left(\theta-\theta_{L}+\phi\right) \mathrm{ds} \tag{2.35}
\end{equation*}
$$

After substituting the value of $p$ from Eq. (2.33) and value of $d s$ from Fig. 2.9, $R_{V}$ is obtained in the following form:
$R_{V}=f_{1}+f_{2}+f_{3}$
Similarly, the horizontal component $R_{H}$ of the resultant reaction is given as (Fig. 2.9)
$R_{H}=\int p \sin \left(\theta-\theta_{L}+\phi\right) \mathrm{ds}$
After substituting the value of $p$ from Eq. (2.33) and performing integration $R_{H}$ is obtained as
$R_{H}=f_{4}+f_{5}+f_{6}$

Where, $f_{1}, f_{2}, f_{3}, f_{4}, f_{5}$, and $f_{6}$ are -

$$
\begin{align*}
& f_{1}=\gamma r_{0}^{2} K \sin \left(45+\frac{\phi}{2}\right) e^{3 \theta_{m} \tan \phi}\left[e^{-\tan \phi \theta_{n}} \sin \left(45-\frac{\phi}{2}\right)-\sin \left(45-\theta_{n}-\frac{\phi}{2}\right)\right]  \tag{21}\\
& f_{2}=\left[\frac{\gamma r_{o}^{2} \sec \phi}{4\left(1+9 \tan ^{2} \phi\right)} 3 \tan \phi\left\{e^{2 \theta_{m} \tan \phi} \sin 2 \phi+\sin 2\left(\theta_{m}-\phi\right)\right\}\right]-\left[\frac{\gamma r_{o}^{2} \sec ^{2} \phi}{4\left(1+9 \tan ^{2} \phi\right)}\left\{\begin{array}{c}
\frac{1}{\tan \phi}\left[e^{2 \theta_{m} \tan \phi}-1\right] \\
+\frac{1}{\sec \phi}\left[e^{2 \theta_{w} \tan \phi} \cos 2 \phi-\cos 2\left(\phi-\theta_{m}\right)\right]
\end{array}\right\}\right.  \tag{22}\\
& f_{3}=\frac{\gamma r_{0}^{2} \sec \phi e^{3 \theta_{m}} \tan \phi}{\left(1+9 \tan ^{2} \phi\right)}\left\{e^{-\theta_{m} \tan \phi}\left\{\sin \left(45-\frac{\phi}{2}\right)-\sin \left(45-\theta_{m}-\frac{\phi}{2}\right)\right\}\right\}\left\{3 \tan \phi \sin \left(45+\frac{\phi}{2}\right)-\cos \left(45+\frac{\phi}{2}\right)\right\}  \tag{23}\\
& f_{4}=\gamma r_{\mathrm{o}}^{2} K \sin \left(45+\frac{\phi}{2}\right) e^{3 \theta_{w} \tan \phi}\left(\cos \left(45-\theta_{w}-\frac{\phi}{2}\right)-e^{-\theta_{w}} \tan \phi \cos \left(45-\frac{\phi}{2}\right)\right)  \tag{24}\\
& f_{5}=\left\{\frac{\gamma r_{0}^{2} \sec ^{2} \phi \tan \phi}{4\left(1+9 \tan ^{2} \phi\right)}\left[\begin{array}{c}
\frac{1}{\tan \phi}\left(e^{2 \theta_{m} \tan \phi}-1\right) \\
-\frac{1}{\sec \phi}\left(e^{2 \theta_{m}} \tan \phi\right. \\
\left.\cos (2 \phi)-\cos \left(2 \phi-2 \theta_{m}\right)\right)
\end{array}\right]\right\}-\left\{\frac{\gamma r_{o}^{2} \sec \phi}{4\left(1+9 \tan ^{2} \phi\right)}\left\{e^{\theta_{n} \tan \phi} \sin 2 \phi+\sin \left(2 \theta_{m}-2 \phi\right)\right\}\right\}  \tag{25}\\
& f_{6}=-\frac{\gamma r_{0}^{2} \sec \phi e^{3 \theta_{n} \tan \phi}}{\left(1+9 \tan ^{2} \phi\right)}\left\{3 \tan \phi \sin \left(45+\frac{\phi}{2}\right)-\cos \left(45+\frac{\phi}{2}\right)\right\}\left\{\cos \left(45-\theta_{m}-\frac{\phi}{2}\right)-e^{-\theta_{m} \tan \phi} \cos \left(45-\frac{\phi}{2}\right)\right\} \tag{26}
\end{align*}
$$

### 2.8.4 SELF WEIGHT OF WEDGE CDFB

This is obtained by calculating weight, $W_{1}$ of part $\mathrm{OCD}, W_{2}$ of part OCB , and $W_{3}$ of part BDF as shown in Fig. 2.9. The required weight $W$ (of part CDFB) is given as
$W=\left(W_{1}-W_{2}+W_{3}\right)$
In which
$W_{1}=\left(\gamma r_{\mathrm{o}}{ }^{2} / 4 \tan \phi\right)\left[\mathrm{e}^{2 \theta \mathrm{~m} \tan \phi}-1\right]$
$W_{2}=1 / 2 \gamma r_{\mathrm{o}}{ }^{2} \sin \theta_{v} \sin \theta_{m} / \sin (45+\phi / 2) \quad$ and (39b)


Fig. 2.10, Free body diagram of triangular wedge $A B C$ for the determination of $\mathbf{N}_{\gamma}$
$W_{3}=1 / 4 \gamma r_{\mathrm{o}}^{2} K^{2} e^{2 \theta m \tan \phi} \cos \phi$

### 2.8.5 COMPUTATION OF PASSIVE THRUST $P^{P} \gamma$

The passive thrust is obtained by using the two equations of force equilibrium (Fig.3) Vertical force equilibrium
$R_{V}-P_{p \gamma}-W=0$
Horizontal force equilibrium
$-R_{H}-P_{\gamma}=0$

The procedure of obtaining the desired value of $P_{p y}$ has been described in the outline of the proposed analysis. The iterations were carried out till the computed values of $P \gamma$ and $R_{H}$ matched with each other up to four decimal places. The computed values of $P_{p \gamma}$ and $N \gamma$ are therefore correct up to four decimal places. The computation further showed that the pole of the $\log$ spiral was located very close to the footing edge.

### 2.8.6 COMPUTATION OF N ${ }_{\gamma}$

For this computation, Fig. 2.10 is referred, which shows the free body diagram of triangular wedge ABC (of Fig.2.6), subjected to forces $Q_{u}$ (ultimate load), $W_{s}$ (weight of the triangular soil wedge ABC), and $P_{p \gamma}$, the passive thrust.

The vertical force equilibrium gives
$Q_{u}+W_{s}=2 P_{p \gamma}$
After substituting the value of $W_{s}$, Eq. (2.41) becomes
$Q_{u}=2 P_{p \gamma}-\gamma B^{2} \tan \phi$
Dividing the above expression throughout by 2B (footing width) yields the ultimate bearing pressure $q_{u}$, which is given as
$q_{u}=P_{p \gamma} / B-\gamma B / 2 \tan \phi$
On comparing Eq. (19) with Eq. $\mathrm{q}_{\mathrm{u}}=\gamma \mathrm{B} \mathrm{N}_{\gamma}$, the bearing capacity factor $N_{\gamma}$ is finally obtained as
$N_{\gamma}=P_{p \gamma} / \gamma B^{2}-\tan \phi / 2$

### 2.8.7 THE FOLLOWING MAIN CONCLUSIONS ARE DRAWN FROM THE PROPOSED

## ANALYSIS:

1. The concept of force equilibrium condition coupled with Kotter's equation identifies the unique failure surface, consistent with the specified failure mechanism.
2. Application of Kotter's equation makes the analysis statically determinate.
3. The $N \gamma$ values that are obtained from the proposed analysis based on Terzaghi's failure mechanism with the limit equilibrium approach are the unique values since; no other simplifying assumptions are made to evaluate them.
4. The $N \gamma$ values as obtained from the proposed analysis show a good agreement with experimental values and this establishes reliability of the proposed method of analysis.

Table - 2.10, Comparision of bearing capacity Factor, $\mathbf{N}_{\gamma}$ with other theories and Experimental results.

| $\phi$ | Proposed analysis | $\begin{gathered} \text { Ingra } \\ \& \\ \text { becher } \\ (1983) \end{gathered}$ | Zadroga (1994) | Meyerhof (1963) | Baki \& Beik (1970) | Hansen $(1970)$ | $\begin{aligned} & \text { Chen } \\ & \text { (1975) } \end{aligned}$ | $\begin{gathered} \text { Kumbh- } \\ \text { ojkar } \\ (1993) \end{gathered}$ | $\begin{gathered} \text { Frydman } \\ \& \text { Burd } \\ \text { (1997) } \end{gathered}$ | Michal -owski (1997) | Soubra (1999) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 8.363 | 14.570 | 22.307 | 6.764 | 12.8 | 6.758 | 12.409 | 8.342 | -NA- | 9.765 | 9.81 |
| 30 | 21.404 | 34.605 | 45.147 | 15.676 | 27 | 15.069 | 26.702 | 19.129 | 21.7 | 21.394 | 21.51 |
| 35 | 53.844 | 82.187 | 91.371 | 37.168 | 60 | 33.920 | 60.236 | 45.410 | 54.2 | 58.681 | 49.00 |
| 40 | 141.32 | 195.19 | 184.921 | 93.712 | 149 | 79.54 | 146.76 | 115.311 | 147.0 | 118.827 | 119.81 |
| 45 | 407.14 | 463.58 | 374.251 | 262.793 | 400 | 200.81 | 400.47 | 325.342 | 422.0 | 322.835 | 326.59 |

### 2.9 BEHAVIOR OF CIRCULAR FOOTINGS RESTING ON CONFINED GRANULAR SOIL

Raft foundations are widely used in supporting structures form any reasons such as weak soil conditions or heavy columns loads. In many cases, some problems arise such as the construction is adjacent to an old building and/or the foundation depth is so great that the excavation needs to be braced during foundation construction (e.g., basement excavation). One of the available solutions is to use sheet piles to support the excavation sides during construction. Due to the difficulty of removing these piles, they become part of the permanent structure and two problems arise. The first problem deals with the structural analysis of the raft if the piles are used as end supports for the raft. The second problem is the effect of these piles on the lateral movement of the soil underneath the raft and the effect of this confinement on the bearing capacity of the soil. While there are several solutions for the first problem, such as isolating the raft from the piles, the confining effect of these piles on the raft behavior is not clearly understood. Looking to the problem in a smaller scale, it can be modeled as a circular footing supported on a soil, which is surrounded by a confining cylinder. The strength of confined sand was studied by Rajagopal et al. (1999). They carried out a large number of triaxial compression tests to study the influence of geocell confinement on the strength and stiffness behavior of granular soils. Geocells fabricated by hand using different geotextiles were used to investigate the effect of the stiffness of the geocell on the overall performance of geocell soil composite.

The aim of this research is to model and investigate the effect of soil confinement by piles on the behavior of soil foundation system. Also, we studied the idea of improving the footing response by using confining cylinders around each individual footing. To achieve that objective, more than 35 tests were carried out with a wide range of variables as detailed in Table 2.11.

Table 2.11, Model Test Program

| Test series | Constant parameter | Varible parameters |
| :---: | :---: | :---: |
| A | Test on unconfined sand | Test is repeated three times |
| B | $\mathrm{d} / \mathrm{D}=0.66$ and $\mathrm{u} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| C | $\mathrm{d} / \mathrm{D}=1.07$ and $\mathrm{J} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| D | $\mathrm{d} / \mathrm{D}=1.33$ and $\mathrm{u} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| E | $\mathrm{d} / \mathrm{D}=1.60$ and $\mathrm{u} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| F | $\mathrm{d} / \mathrm{D}=2.00$ and $\mathrm{u} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| G | $\mathrm{d} / \mathrm{D}=2.66$ and $\mathrm{u} / \mathrm{D}=0.0$ | $\mathrm{~h} / \mathrm{D}=0.5,1.00,1.5,2.00$ |
| H | $\mathrm{d} / \mathrm{D}=1.33$ and $\mathrm{h} / \mathrm{D}=1.0$ | $\mathrm{u} / \mathrm{D}=0 . .0,0.07,0.13,0.50,1.0$ |
| I | $\mathrm{d} / \mathrm{D}=1.33$ and $\mathrm{h} / \mathrm{D}=1.5$ | $\mathrm{z} / \mathrm{D}=0.0,0.17,0.33,0.50,0.67$ |

### 2.9.1 LABORATORY MODEL TESTS

## Model Box and Footing

Nine series of laboratory model tests were conducted in a test box, having inside dimensions of 0.90 m x 30.50 m in plan and 0.5 m in depth. The tank is made from steel with the front wall made of 20 mm thick glass and is supported directly on two steel columns as shown in Fig. 2.11. These columns are firmly fixed in two horizontal steel beams, which are firmly clamped in the lab ground using four pins. The loading system is mounted by a horizontal Standard I beam steel beam supported on the two columns. It consists of a hand-operated hydraulic jack and precalibrated load ring. Since the sand raining technique is used to deposit the sand inside the tank, the beam was designed to swing about one end. Therefore, the beam can be swung out during deposition of the sand from the sand raining box and returned back, when sand deposition is completed, to the original loading position above the tank. The sand-raining box is made from wood and is 0.85 m 30.38 m in plan and 0.10 m in depth. The sand particles rain from the box through a square grid of holes ( 4 mm diameter and 20 mm spacing) in the base plate. The height of sand raining, measured from the bottom of the box to sand surface in the tank, can be changed up or down by using a manual winch. A circular model footing made of steel with a hole at its top center was used. The footing is 75 mm in diameter and 10 mm in thickness. A rough base condition was achieved by fixing a thin layer of sand onto the base of the model footing with epoxy glue. The load is transferred to the footing through a ball bearing, which was placed, between the footing and the proving ring. Such an arrangement produced a hinge, which allowed the footing to rotate freely as it approached failure and eliminated any potential moment transfer from the loading fixture. An overall view of the apparatus is illustrated in Fig. 2.11.


Fig. 1. Schematic view of the experimental apparatus
Fig.2.11, Schematic view of the experimental apparatus

### 2.9.2 TEST MATERIAL

The sand used in this research is medium to coarse sand, washed, Dried, and sorted by particle size. It is composed of rounded-to sub rounded particles. The specific gravity of the soil particles was determined by the gas jar method. Three tests were carried out producing an average value of 2.654 . The maximum and the minimum dry densities of the sand were found to be 19.95 and $16.34 \mathrm{kN} / \mathrm{m}^{3}$ and the corresponding values of the minimum and the maximum void ratios are 0.305 and 0.593 , respectively. The particle size distribution was determined using the dry sieving method and the results are shown in Fig. 2.12. The effective size $\left(D_{10}\right)$, uniformity coefficient $\left(C_{u}\right)$, and coefficient of curvature $\left(C_{c}\right)$ for the sand were $0.152 \mathrm{~mm}, 4.071$, and 0.771 , respectively. In order to set up a sample, the sand was poured in 50 mm in height layers by raining technique in which sand is allowed to rain through air at a controlled discharge rate and height of fall to give uniform densities. A series of tests were carried out to check the relative density obtained and uniformity of the sand samples by using three density molds placed at different locations in the test box. After pouring, each mold was carefully excavated and the density of the sample calculated. The raining technique adopted in this study
provided a uniform relative density of approximately $75.8 \%$ with a unit weight of $18.94 \mathrm{kN} / \mathrm{m}^{3}$. The results also showed that the obtained relative densities from the three samples did not depend on the position of the mold. A series of direct shear tests were performed at the same relative density of the sand and the estimated internal friction angle was approximately $42^{\circ}$.


Fig. 2. Grain size distribution of the sand
Fig. 2.12, Grain size distribution of the sand

The confining elements were made of unplasticized polyvinylchloride (UPVC) Cylinders with different diameters and heights. The used diameters were $50,80,100,120,150$, and 200 mm . UPVC is produced from the polymerization of a vinyl chloride monomer with certain additives including heat stabilizers and lubricants. Its actual strength for any situation depends on the wall thickness uniformity, the rate of loading, and the temperature of plastic materials. The interior and exterior surfaces of the cylinders were made very smooth. The thickness of the cylinder wall is 2.5 mm and its properties as given by the manufacturer are shown in Table 2.12. Some of the tests were carried out by introducing the UPVC cylinders initially in position and then a sand bed was placed by raining. The ultimate loads were determined and compared with those of tests performed with cylinders installed vertically after setting sand samples. The difference of the ultimate loads in the two cases was found to be less than $1.5 \%$ and load-settlement relationships were approximately of the same pattern. Therefore, it was decided to carry out the entire test program using only one method by installing the cylinders vertically after setting sand beds, and considering the difference in the relative densities of the samples resulting from installing cells with different diameters and heights to be small and negligible.

Table 2.12, Properties of the Unplasticized Polyvinyle Chloride Cylinder

| Maximum hydraulic pressure for 1 h at $23^{\circ} \mathrm{C}$ Bar | 23 |
| :--- | :---: |
| Specific gravity | 1.4 |
| Tensile strength, $10^{3} \mathrm{Kpa}$ | 55 |
| Tensile modulus, $10^{5} \mathrm{Kpa}$ | 28 |
| Water adsorption at $100^{\circ} \mathrm{C}$ for $24 \mathrm{~h}, \mathrm{mg} / \mathrm{cm}^{2}$ | 4 |

### 2.9.3 EXPERIMENTAL SETUP AND TEST PROGRAM

After the sand surface was set up, the cells were pushed vertically into the sand at the design place, the footing was placed on position, and the load was applied on it by the hydraulic jack. The load was applied in small increments until reaching failure. Each load increment was maintained constant until the footing settlement had stabilized. The settlements of the footing were measured using two dial gauges placed on opposite sides of the footing. The geometry of the soil, model footing, and confining cylinder are shown in Fig. 2.13. The test program consisted of carrying out nine series of tests on the circular model footing to study the effect of soil confinement on the soil-foundation response as shown in Table 1. Initially, the behavior of the footing supported on the unconfined conditions was determined. Then, each series of the tests was carried out to study the effect of one parameter while the other variables were kept constant. The studied variables are the cell height ( $h$ ) and cell diameter (d) for cases when the cells are placed under the foundation level and the embedded depth $(z)$ for cases when the foundation level is lower than the cell top. Several tests were repeated at least twice to verify the repeatability and the consistency of the test data.


Fig. 3. Geometric parameters of confined sand-foundation model
Fig. 2.13, Geometric parameters of confined sand-foundation model

### 2.9.4 RESULTS AND DISCUSSION

The load-settlement relationship and the ultimate bearing capacity of the footing with and without confinement were obtained. The bearing capacity improvement due to the soil confinement is represented using a nondimensional factor, called the bearing capacity ratio (BCR). This factor is defined as the ratio of the footing ultimate load with soil confinement to the footing ultimate load in tests without confinement. The footing settlement $(S)$ is also expressed in nondimensional form in terms of the footing diameter ( $D$ ) as the ratio ( $S / D, \%$ ). The measured ultimate load and the associated ultimate displacement for the nonconfined case are 250 N and 5.24 mm , respectively. The theoretical ultimate bearing capacity can be calculated from the equation $q_{o}=0.5 \gamma D \zeta_{\gamma} N$. Using the shape factor proposed for circular footing by De Beer (1970) ( $\zeta_{\gamma}=0.6$ ) and the values of the bearing capacity factor $N \gamma$ taken from Meyerhof (1963) ( $N_{\gamma}=139.3$ ) or Hansen (1968) ( $N_{\gamma}=136.7$ ), the theoretical bearing capacities are $59.36 \mathrm{kPa}(262 \mathrm{~N})$ and $58.25 \mathrm{kPa}(257 \mathrm{~N})$, respectively. These data show a close agreement between both the theoretical values and the experimental results. Typical variations of bearing pressure with footing settlement ratios $(S / D)$ with and without soil confinement for different heights of confining cells are presented in Fig. 2.14. It can be seen that the installation of confining cylinders appreciably improves the bearing capacity of the footing as well as the stiffness of the foundation bed. It is apparent from the curves that the mode of failure is a general shear failure in which a pronounced peak can be observed in the load-settlement curve, after which the footing collapses and the load decreases (Vesic 1973). Also, the value of the settlement ratio $S / D$ at the ultimate load in the confined tests varied from about $12 \%$ to $18 \%$. The observed improvement in the bearing capacity loads due to soil confinement along with the increase in the settlement ratio was reported by many investigators when using soil reinforcement (Omar et al. 1993a,b; Das et al. 1996). Comparing the curves of Fig. 4 at the ultimate $S / D$ ratio of the unconfined case (the values across the dotted line, $S / D=7 \%$ ), it can be seen that soil confinement improved the bearing load from 56.59 kPa for the unconfined case to 562.5 kPa for the confined soil using cells with a d/D ratio of 1.33 and $h / D$ ratio of 2.0 . Therefore, it can be concluded that, in cases when the excessive settlement is the controlling factor in determining the allowable bearing capacity, using confining cells may significantly decrease the settlement ratio for the same level of bearing load.


Fig. 4. Variation of bearing pressure with (S/D) ratio for different cell heights (Series D)

Fig. 2.14, Variation of bearing pressure with (S/D) ratio for different cell heights (Series D)

### 2.9.5 EFFECT OF CELL DIAMETER

In order to investigate the effect of cell diameter on the footing behavior, six cells with diameters of $50,80,100,120,150$, and 200 mm were used. Fig. 5 shows the variation of BCR with normalized cell diameter for different cell heights with a constant footing diameter of 75 mm . A significant increase in the bearing capacity of the model footing supported on confined sand with the increase of normalized cell diameter $d / D$ is observed until a specific value of $d / D$ after which the BCR decreases with an increase in the $d / D$ ratio. While conducting the model tests, it was observed that as failure approached in tests carried out with small cell diameters, sand inside the cell and the cell behaved as one unit (when the load was increased, the cell, sand, and footing settled altogether). In tests carried out with large cell diameters, this behavior was noticed initially,


Fig. 5. Variation of bearing capacity ratio with normalized cell diameter $(d / D)$ for different cell heights

Fig. 2.15, Variation of bearing capacity ratio with normalized cell diameter (d/D) for different cell height.

But as the load was increased it was no longer observed (the footing settled down while the cell was unaffected with the increase of the load). Fig. 2.15 also shows that using soil confinement could result in an improvement in bearing capacity as high as 17 times more than that without soil confinement. It is clear that the best benefit of soil confinement could be obtained with a $(d / D)$ ratio between 1.0 to 2.0 with the maximum improvement in the bearing capacity at a ratio of about 1.4 for different heights of confining cells. This significant increase in the bearing capacity of the footing can be explained with the aid of Fig. 6 as follows. When the footing is loaded, such confinement resists the lateral displacements of soil particles underneath the footing and confines the soil leading to a significant decrease in the vertical settlement and hence improving the bearing capacity. For small cell diameters, as the pressure is increased, the plastic state is developed initially around the edges of the footing and then spreads downward and outward. The mobilized vertical frictions between the sand and the inside wall of the cylinder increase with the increase of the acting active earth pressure until the point when the system (the cylinder, sand, and footing) starts to behave as one unit. The behavior is similar to that observed in deep foundations (piles and caissons) in which the bearing load increases due to the shear resistance of cell surface. This illustrates the increase of the bearing load with the increase of the cell diameter and cell height. Based on tests performed with cells made with very smooth surfaces, it can be concluded that increased surface roughness results in greater bearing load improvement. In comparison to this response, sand beds at relative density of $70 \%$ and reinforced with a geocell mattress carried out by Dash et al. (2001b) mobilized bearing capacity pressure as high as eight times the ultimate capacity of the unreinforced sand. Also, tests on sand beds at a relative density of $75 \%$ and
reinforced with planar reinforcement carried out by Omar et al. (1993a,b) and Khing et al. (1993) failed with clearly pronounced peak loads of about five times the ultimate capacity of unreinforced soil at settlements equal to about $20 \%$ of the footing width.


Fig. 6. Intersection of failure surface with the cylinder
Fig. 2.16, Interaction of failure surface with the cylinder

### 2.9.6 EFFECT OF CELL HEIGHT

In order to investigate the effect of cell height on the footing response, tests were carried out using four different heights for each cell diameter. The variation of BCR with normalized cell height $(h / D)$ is shown in Fig. 7 for different normalized cell diameters $(d / D)$. The figure shows the same pattern of behavior for the different cell diameters. Increasing cell heights results in a greater improvement in the BCR. This increase in cell height results in the enlargement in the surface area of the cell-model footing leading to a higher bearing capacity load. The slope of the BCR versus $h / D$ curves for $d / D$ ratios of 0.67 and 2.67 are less than the comparable slopes for $d / D$ ratios of 1.33 and 1.6. This trend confirms the previous conclusion that the greatest benefit of cell confinement can be obtained at a $d / D$ ratio of about 1.4.


Fig. 7. Variation of bearing capacity ratio with nermalized cell height (h/D) for different cell diameters (d)

Fig. 2.17, Variation of bearing capacity ratio with normalized cell height (h/D) for different cell diameter(d)

### 2.9.7 EFFECT OF THE SOIL PRESSURE ON THE CELL

One of the proposed parameters to be investigated was the thickness of the cell wall to study the effect of the cell rigidity on the footing-cell system behavior and also to study the hoop tension in the cell wall due to the pressure under the footing. However, according to the manufacturer data, the supplied cell with a wall thickness of 2.5 mm can withstand internal hydraulic pressure of 23 bars $(2,300 \mathrm{kPa})$ with maximum tensile strength of the wall of $55,000 \mathrm{kPa}$. In the model tests, the most critical case occurs with $h / D=2$ and $d / D=1.33$ at failure vertical pressure of 950 kPa . The horizontal pressure acting on the sidewall of the cell is equal to the vertical pressure multiplied by the coefficient of lateral earth pressure. It can be seen that the maximum estimated horizontal earth pressures on the sidewalls of the cell are very small in comparison to the allowable hydraulic pressure. Another point is that the given allowable value is the net inside pressure while the cell in the model is subjected to both internal and external pressures. Checks were performed after each test to observe any change in the cell wall and measurements were taken to check the internal diameter as well as the thickness of the cell wall. There was no noticeable change in the cell or its dimension. Therefore, it was concluded that for the given model and dimensions, the pressures under the footing have no disturbing effects on the cell wall and, therefore, it was decided to use them again.

### 2.9.8 CONCLUSIONS

Soil confinement has a significant effect on improving the behavior of circular footing supported on granular soil. The ultimate capacity was found to increase by a factor of 17 as compared to the unreinforced case. Therefore, it can be concluded that the piles (or sheet piles) used to brace cuts have
a significant effect on improving the bearing capacity of soils under raft foundations. However, more research in this area is required to study cases in which piles are constructed only on one, two, or three sides. Also, theoretical analysis is needed to modify the bearing capacity equation to consider the effect of pile confinement. Based on the experimental results, soil confinement could be considered as a method to improve the bearing capacity of isolated footings bearing on medium to dense sand. UPVC cells with different heights, diameters, and thickness could be easily manufactured and placed around the individual footings leading to a significant improvement in their response.

### 2.10 BEARING CAPACITY OF A JACKUP SPUDCAN FOOTING

The ultimate bearing capacity of a flat surface footing, as discussed in the previous articles is depend upon the footing soil interaction under vertical, horizontal and inclined loading which governs the stability of the footing. In this article we shall consider the stability of jackup spudcan footing and calculate its ultimate bearing capacity.
The growing use of mobile jackup units on spudcan footing in deep offshore waters has raised a great deal of concern about the overall stability of jackup unit in hostile environmental forces. The stability of a jackup unit is greatly influenced by the performance of a typical spudcan footing under storm loading conditions. Therefore, as a result, much research has been focused towards the study of a spudcan footing behavior subjected to combined vertical and horizontal loading. Recently, over a period of several years, a joint industry study coordinated by Noble Denton Association has been considered many aspects of jackup stability including the soil-structure interaction (Houlsby and James, 1991).

Mobile jackup units consist of a floatable drilling platform, supported on three or more legs, which can be raised or lowered (see figure 2.18). A detailed description of the installation procedure is given by Tan (1990). The platform legs can either be supported separately as shown in Figure 8.1a or they can be supported on a single shared mat.


Figures show (a) Jackup platform and environmental loads. (b) Spudcan footing.

Most modern jackup unit platforms are former type and have approximately conical shaped footing with a protruding tip at the center. These are commonly referred to as 'spudcan' footings (see Fig. 2.18 b)

The mobility of jackup units allows for relocating and reuse of platforms. Before actual oil and gas exploration activity begins, the legs of the platform are preloaded to almost twice the working load. During this loading operation, spudcan footing moves into the seabed to a depth of almost twice its diameter. In soft clays, a 30 m penetration of a 15 m -diameter spudcan is not uncommon. The bearing capacity with footing embedment is an important factor, since even a small embedment can significantly increase the bearing capacity of such offshore footing (Houlsby and Martin (1993)). However, this is not usually the case in coarse material as penetration depths are typically very small (see e.g. Dean et al. (1993)).

In addition to the vertical load of the jackup structure, a spudcan footing is also subjected to large horizontal forces due to severe environmental conditions reaching, in stormy weather, up to $30 \%$ of the total vertical load and thereby producing a corresponding large moment (Poulos (1988)). In this article we are making a conventional assumption, that a jackup leg is pinned at the foundation level and that therefore a spudcan footing offers no moment restraint to the leg (see Fig. 2.19a). This means that, only the interaction between horizontal and vertical loading need to be studied. Although, in recent studies moment restraint at the footing has also been considered which would reduce the bending moment in the lower leg guide (see Fig. 2.19b), which is often a critical feature of the structure in extreme loading conditions: a full interaction between vertical, horizontal and
moment loading has been studied by e.g. Houlsby and Martin (1993) for clays, and Dean et al. (1993) for sands.

We discuss here the ultimate bearing capacity of a partially penetrated spudcan footing subjected to combined vertical and horizontal loading.


Fig 2.19: Bending moment in the legs of Jack-up units

### 2.10.1 BACKGROUND

The bearing capacity solution of Meyerhof (1953), Hansen (1970) and Vesic (1975) are commonly used to determine the ultimate bearing capacity pf plan strain footing. These solutions are modified by introducing shape factors to cover circular geometries (see e.g. (1992), Dean et al. (1993)). In the case of spudcan footings, their embedded circular area in plane (i.e. plan area at ground surface) is used for the bearing capacity solutions to spudcan footings, theoretical and experimental studies of conical shaped footings (based on centrifuge model tests) have been undertaken by Cambridge University Engineering Department over a several years. In this study, a series of test of cone angle $\mathrm{O}^{\circ}$ (flat), $13^{\circ}, 20^{\circ}, 25^{\circ}$ and $35^{\circ}$ was conducted on sand at 28.3 mm diameter model footing at 56.6 g , repressing 1.6 m prototype in the drum centrifuge. It is shown in the study that spudcan footing can be treated as equivalent cones enclosing the same volume. The load displacement response of a
spudcan footing and an equal volume cone is almost identical. The ultimate bearing capacity of these footings for vertical loading only, combined horizontal-vertical (inclined) loading, vertical-moment (eccentric) loading and lateral-moment loading has been determined. The results are compared with existing bearing capacity theories and analytical solutions were possible. The study show that the vertical bearing capacity does not vary much for cone angles between $\mathrm{O}^{\circ}-20^{\circ}$. However, foe flat circular footings under combined vertical - horizontal loading (referred as shear sidewipe tests at constant penetration), most of the experimental data lies outside the curve proposed by Meyerhof (1953), Hansen (1970) and Dean et al. (1993).

Hambly (1992) carried out laboratory tests in sand on two model footing - flat and spudcan. The purpose of this investigation was to compare the ultimate bearing capacity of flat and spudcan footing. In the series of test performed by Hambely (1992), the footing were preloaded until the penetration for the flat footing was $10 \%$ of its diameter (Vesic (1975)) and for the spudcan footing, a penetration of its bearing area at the ground surface equivalent to the bearing capacity of the sand under the flat circular footing was insufficient to mobilized the ultimate bearing capacity of the sand under the spudcan footing. Hambly (1992) showed that the ultimate bearing capacity of a partially penetrated spudcan footing is twice the preload forces required for the flat footing. He obtained enhanced sliding resistance of the spudcan footing using twice the preload force for the flat footing with Hansen's (1970) theory. He attributed the difference between the preload and the ultimate bearing capacity to the much greater volume of sand displacement by flat footing mobilization ultimate bearing resistance, as compared to the volume displaced by partially penetrated cone of the spudcan. He suggested 'partial penetration factor' to determined the ultimate bearing capacity from the vertical preload force. He concluded in his study that a further laboratory investigation is needed to validate his results. Although, in the absence of such an investigation it is difficult to comprehend Hambly's (1992) results, the experimental results described by Tan (1990) resembles more closely
the actual preload operation of the spudcan footing. The footing penetrates into the seabed as the load is applied and continues to penetrate until there is no further penetration into the seabed as the load is applied and continues to penetrate until there is no further penetration i.e. the footing is locked vertically. The final value of the load, thus, represents the ultimate bearing capacity of the footing.

### 2.10.2 VERTICAL BEARING CAPACITY OF A SPUDCAN FOOTING

The bearing capacity solution for a circular footing, given by Hansen (1970) can be simplified (for vertical loading only), and written as presented by Dean et al. (1993):

$$
\begin{equation*}
\mathbf{V}=\mathbf{A}_{\mathbf{p}}\left(\mathbf{1} / \mathbf{2} \mathbf{N}_{\gamma} \gamma^{\prime} \mathbf{B}_{\mathbf{p}}\right) \tag{2.45}
\end{equation*}
$$

Where $B_{p}$ is the penetrated diameter, $A_{p}$ is the plan contact area, $\gamma^{\prime}$ is the soil effective (buoyant) unit weight, and $\mathrm{N}_{\gamma}$ is an axi-symmetric self-weight bearing capacity factor. $\mathrm{B}_{\mathrm{p}}$ can be written in term of penetration depth $D_{p}$ as:

$$
\begin{equation*}
B_{p}=2 D_{p} \cot \Omega \tag{2.46}
\end{equation*}
$$

Where $\Omega$ is the cone angle (see Fig. 8.7 a)
Substituting $\mathrm{B}_{\mathrm{p}}$ in the bearing capacity solution, we get the following equation:

$$
\begin{equation*}
\mathbf{V}=\mathbf{p} \mathbf{N}_{\gamma} \gamma^{\prime} \cot ^{3} \Omega \mathbf{D}_{\mathbf{p}}^{3} \tag{2.47}
\end{equation*}
$$


(a) Conical footing at partial penetration, $\mathbf{p}$

(a) Initial cubic nature of curve before full penetration


Fig. 2.20, (c) Plot of force against the cube of displacement


Fig 2.21 a, Theoretical axisymmetric $\mathrm{N}_{\gamma}$ values for cones on cohesionless soil


Fig 2.21 b, Effect of cone on $\mathrm{N}_{\mathrm{y}}$ of semi-rough footings;
Theoretical and experimental comparison.

Thus for a given angle $\Omega, \mathrm{V}$ is directly related to the $\mathrm{D}_{\mathrm{p}}{ }^{3}$, if $\mathrm{N}_{\gamma}$ is approximately constant then V and $\mathrm{D}_{\mathrm{p}}{ }^{3}$ relationship is a straight line. Such a straight relationship is shown in figure 2.20 c for a $60^{\circ}$ cone (Tan 1990) on fine Leighton Buzzard sand tested in the drum centrifuge at 56.6 g . However, for a given depth of penetration, the value of $\mathrm{N}_{\gamma}$ varies with the cone angle and the variation depends on the surface roughness of the footing. The theoretical variation of $\mathrm{N}_{\gamma}$ with the cone angle using the method of characteristics is shone in Figure 2.21a. Figure 2.21b shows the same variation of $\mathrm{N}_{\gamma}$ with the cone angle calculated from the centrifuge test results using equation (2.45) and the comparison with the theoretical results. The results agree well in the region $\phi=31^{\circ}$ and $\delta=17^{\circ}$ for loose sand (void ratio 0.95 ) and $\phi=34^{\circ}$ and $\delta=19^{\circ}$ for medium dense send (void ratio 0.8 ). (In these figures $\phi$ is the friction angle and d represents the degree of roughness of the cone.). it can be seen from the Figures 2.21 a that for a fully rough footing (as is the case here in this chapter) the value of $\mathrm{N}_{\gamma}$ stays almost the same for cone angles between $\mathrm{O}^{\circ}-20^{\circ}$. But for cone angles greater than $25^{\circ}$ there is a significant increase in the value of Ng . These results show that a conical footing with cone angle $\Omega=$ $13.46^{\circ}$ has a slight higher ultimate load compared to a footing with $\Omega=O^{\circ}$ (flat). But there is a need for further experimental investigation in order to prove that the load is $100 \%$ higher as we learn from Hambly (1992).

## CHAPTER - $\mathbf{3}$

## ANALYTICAL SOLUTION FOR BEARING CAPACITY OF

## SPUDCAN FOOTING

Since Terzaghi's founding work, numerous experimental studies to estimate the ultimate bearing capacity of shallow foundation have been conducted. Based on these studies, it appears that Terzaghi's assumption of the failure surface in soil at ultimate load is essentially correct. However, the angle $\alpha$ that the sides ac and bc of the wedge (Fig. 3.1) make with the horizontal is closer to $45+\phi / 2$ and not $\phi$ as assumed by Terzaghi. (In actual practice, $\alpha$ has been found to vary from $45-\phi / 2$ for perfectly smooth base to $45+\phi / 2$ for perfectly rough base. Since footings are normally rough, $\alpha$ has been found closer to $45+\phi / 2$ than to $\phi)$.

So, I am Deriving the equations for bearing capacity factors $\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$, and $\mathrm{N}_{\gamma}$, by assuming the wedge angle $=\alpha$. (angle that the sides ac and bc make with the horizontal surface see Fig. 3.1) The remaining assumption are same as assumed by Terzaghi (see page no. - ).


Fig. 3.1, failure surface in soil at ultimate load for a continuous rough rigid foundation as assumed by Terzaghi

The ultimate bearing capacity, $q_{u}$, of the foundation can be determined if we considered faces ac and bc of the triangle wedge abc and obtained the passive force on each face requires to cause failure. Note that the passive force $P_{p}$ will be a function of the surcharge $q=\gamma D_{f}$. Cohesion $c$, unit weight $\gamma$, and angle of friction of the soil $\phi$. So, referring to Fig. 3.2. The passive force $P_{p}$ on the face bc per unit length of the foundation at right to the cross section is

$$
\begin{equation*}
\mathrm{P}_{\mathrm{p}}=\mathrm{P}_{\mathrm{pq}}+\mathrm{P}_{\mathrm{pc}}+\mathrm{P}_{\mathrm{p} \gamma} \tag{3.1}
\end{equation*}
$$

Where $P_{p q}, P_{p c}$ and $P_{p \gamma}=$ passive force contributions of $q, c$ and $\gamma$, respectively.

It is important to note that the directions of $\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$ and $\mathrm{P}_{\mathrm{py}}$ are vertical, since the face bc makes an angle $\phi$ with the horizontal, and $\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$ and $\mathrm{P}_{\mathrm{py}}$ must make an angle $\phi$ to the normal to bc . In order to obtain $\mathrm{P}_{\mathrm{pq}}, \mathrm{P}_{\mathrm{pc}}$ and $\mathrm{P}_{\mathrm{py}}$, the method of superposition can be used.


Fig. 3.2, Passive forceon the face bc of wedge abc shone in figure 3.1

### 3.1 RELATIONSHIP FOR $\mathbf{P}_{\mathrm{pq}}(\phi \neq 0, \gamma=0, \mathbf{q} \neq 0, \mathbf{c}=0)$

Considered the free body diagram of the soil wedge bcfj shown in Fig. 3.2 (also shown in Fig. 3.3). For this case the center of the log spiral, of which of is an arc, will be at point b. the forces per unit length of the wedge bcfj due to the surcharge q only are shown in Fig.3.3a, and they are:

1. $\mathrm{P}_{\mathrm{pq}}$
2. Surcharge, q
3. The Rankine passive force, $\mathrm{P}_{\mathrm{p}(1)}$
4. The frictional resistance force alone the arc cf, F

The Rankine passive force, $\mathrm{P}_{\mathrm{p}(1) \text {, can be expressed }}$ as

$$
\begin{equation*}
P_{p(1)}=q K_{p} H_{d}=q H_{d} \tan ^{2}(45+\phi / 2) \tag{3.2}
\end{equation*}
$$

Where, $\mathrm{H}_{\mathrm{d}}=\mathrm{fj}$
$\mathrm{K}_{\mathrm{p}}=$ Rankine passive earth pressure coefficient $=\tan ^{2}(45+\phi / 2)$

(a)

b

(b)

Fig. 3.3 Determination of $\mathbf{P}_{\mathrm{pq}}(\phi \neq 0, \gamma=0, q \neq 0, \mathbf{c}=0)$

According to the property of a $\log$ spiral defined by the equation $r=r_{o} e^{\theta t a n \phi}$, the radial line at any point makes an angle $\phi$ with the normal. Hence, the lines of action of the friction force $F$ will pass through $b$, the center of the log spiral (as shown in Fig. 3.3a). Taking the moment about point $b$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pq}}(\mathrm{~B} / 4)=\mathrm{q}(\mathrm{bj})(\mathrm{bj} / 2)+\mathrm{P}_{\mathrm{p}(1)} \mathrm{H}_{\mathrm{d}} / 2 \tag{3.3}
\end{equation*}
$$

Let

$$
\begin{align*}
\mathrm{bc}=\mathrm{r}_{\mathrm{o}} & =(\mathrm{b} / 2) \sec \alpha  \tag{3.4}\\
\mathrm{bf} & =\mathrm{r}_{1}=\mathrm{r}_{\mathrm{o}} \mathrm{e}^{(3 \pi / 4-\alpha+\phi / 2)} \tag{3.5}
\end{align*}
$$

So

$$
\begin{equation*}
\mathrm{bj}=\mathrm{r}_{1} \cos (45-\phi / 2) \tag{3.6}
\end{equation*}
$$

And

$$
\begin{equation*}
\mathrm{H}_{\mathrm{d}}=\mathrm{r}_{1} \sin (45-\phi / 2) \tag{3.7}
\end{equation*}
$$

Combining Eqs. (3.2), (3.3), (3.6) and (3.7)

$$
\mathrm{P}_{\mathrm{pq}} \mathrm{~B} / 4=\mathrm{q} \mathrm{r}_{1}{ }^{2} \cos ^{2}(45-\phi / 2) / 2+\mathrm{qr}_{1}^{2} \sin ^{2}(45-\phi / 2) \tan ^{2}(45+\phi / 2) / 2
$$

Or

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pq}}=4 / \mathrm{B}\left[\mathrm{q}_{1}^{2} \cos ^{2}(45-\phi / 2)\right] \tag{3.8}
\end{equation*}
$$

Now combining Eqs. (3.4), (3.5), and (3.8)

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pq}}=\mathrm{qB} \mathrm{e} \quad 2(3 \pi / 4-\alpha+\phi / 2) \tan \phi \cos ^{2}(45-\phi / 2) / \cos ^{2} \alpha \tag{3.9}
\end{equation*}
$$

Considering the stability of the elastic wedge abc under the foundation as shown in Fig. 3.3b.

$$
\mathrm{q}_{\mathrm{q}}(\mathrm{~B} \times 1)=2 \mathrm{P}_{\mathrm{pq}}
$$

Where, $\mathrm{q}_{\mathrm{q}}=$ load per unit area on the foundation, or

$$
\begin{align*}
& \mathrm{q}_{\mathrm{q}}=2 \mathrm{P}_{\mathrm{pq}} / \mathrm{B}=\mathrm{q}\left[2 \mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi} \cos ^{2}(45-\phi / 2) / \cos ^{2} \alpha\right]=\mathrm{q}_{\mathrm{q}}  \tag{3.10}\\
& \mathbf{N}_{\mathbf{q}}=\mathbf{2} \mathbf{e}^{\mathbf{2}(\mathbf{3} \pi / 4-\alpha+\phi / 2) \tan \phi} \cos ^{\mathbf{2}}(\mathbf{4 5}-\phi / \mathbf{2}) / \cos ^{2} \boldsymbol{\alpha} \tag{3.11}
\end{align*}
$$

3.2 RELATIONSHIP FOR $\mathrm{P}_{\mathrm{pc}}(\phi \neq 0, \gamma=0, \mathrm{q}=0, \mathrm{c} \neq 0)$

Figure 3.4 shows the free body diagram for the wedge bcfj (also refer to Fig. 3.2). As in the case of $P_{p q}$, the center of arc of the $\log$ spiral will be located at point $b$. the forces on the wedge which are due to cohesion c are also shown in Fig. 3.4, and they are

1. Passive force, $\mathrm{P}_{\mathrm{pc}}$
2. Cohesive force, $\mathrm{C}=\mathrm{c}(\mathrm{bc} \times 1)$
3. Rankine passive force due to cohesion,

$$
\begin{equation*}
\mathrm{P}_{\mathrm{p}(2)}=2 \mathrm{c}\left(\mathrm{~K}_{\mathrm{p}}\right) 1 / 2 \mathrm{H}_{\mathrm{d}}=2 \mathrm{c} \mathrm{H}_{\mathrm{d}} \tan (45+\phi / 2) \tag{3.12}
\end{equation*}
$$

4. Cohesive force per unit area along arc cf, c.

Taking the moment of all the forces about point $b$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pc}}(\mathrm{~B} / 4)=\mathrm{P}_{\mathrm{p}(2)}\left[\mathrm{r}_{1} \sin (45-\phi / 2) / 2\right]+\mathrm{M} \tag{3.13}
\end{equation*}
$$

Where $\mathrm{Mc}=$ moment due to cohesion c along arc cf

$$
\begin{equation*}
=\mathrm{c} / 2 \tan \phi\left(\mathrm{r}_{1}{ }^{2}-\mathrm{r}_{\mathrm{o}}{ }^{2}\right) \tag{3.14}
\end{equation*}
$$


(a)


Fig. 3.4 Determination of $\mathbf{P}_{\mathrm{pc}}(\phi \neq 0, \gamma=0, q=0, \mathbf{c} \neq 0)$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pc}}(\mathrm{~B} / 4)=\left[2 \mathrm{c} \mathrm{H}_{\mathrm{d}} \tan (45+\phi / 2)\right]\left[\mathrm{r}_{1} \sin (45-\phi / 2) / 2\right]+(\mathrm{c} / 2 \tan \phi)\left(\mathrm{r}_{1}^{2}-\mathrm{r}_{\mathrm{o}}{ }^{2}\right) \tag{3.15}
\end{equation*}
$$

The relationship for $H_{d}, r_{0}$, and $r_{1}$ in terms of $B$ and $\phi$ given in Eqs. (3.7), (3.4) and (3.5), respectively. Combining Eqs. (3.4), (3.5), (3.7), and (3.15), and noting that $\sin ^{2}(45-\phi / 2) x \tan (45+\phi / 2)=1 / 2 \cos \phi$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{pc}}=\operatorname{Bc}\left(\sec ^{2} \alpha\right)\left\{\left\{\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi} 1 / 2 \cos \phi\right]+\left[\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi}-1\right) / 2 \tan \phi\right]\right\} \tag{3.16}
\end{equation*}
$$

Considering the equilibrium of the soil wedge abc (Fig.3.4b)

$$
\mathrm{q}_{\mathrm{c}}(\mathrm{~B} \times 1)=2 \mathrm{C} \sin \alpha+2 \mathrm{P}_{\mathrm{pc}}
$$

Or

$$
\begin{equation*}
\mathrm{q}_{\mathrm{c}} \mathrm{~B}=\mathrm{cB} \sec \alpha \sin \alpha+2 \mathrm{P}_{\mathrm{pc}} \tag{3.17}
\end{equation*}
$$

Where $\mathrm{q}_{\mathrm{c}}=$ load per unit area of the foundation combining Eqs.(3.16) and (3.17)

$$
\begin{gather*}
\mathrm{q}_{\mathrm{c}}=\mathrm{c}\left[\tan \alpha+2 \sec ^{2} \alpha\left\{\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi} 1 / 2 \cos \phi+\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi}-1\right) / 2 \tan \phi\right\}\right]\right.  \tag{3.18}\\
\mathrm{N}_{\mathrm{c}}= \\
 \tag{3.19}\\
+\left(\tan \alpha+2 \sec ^{2} \alpha\left\{\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi 1 / 2} \cos \phi\right.\right.\right. \\
\\
+(3 \pi / 4-\alpha+\phi / 2) \tan \phi \\
-1) / 2 \tan \phi\}
\end{gather*}
$$

### 3.3 RELATIONSHIP FOR $\mathrm{P}_{\mathrm{p} \gamma}(\phi \neq 0, \gamma \neq 0, \mathrm{q}=0, \mathrm{c}=0)$

Figure 3.5 a shows the free body diagram of wedge bcfj. Unlike the free body diagram shown in Figs. 3.3 and 3.4, the center of the $\log$ spiral of which $b f$ is an arc is at a point $O$ along line bf and not at $b$. this is because the minimum value of $\mathrm{P}_{\mathrm{p} y}$ has to be determined by several trials. Point O is only one trail center. The forces per unit length of the wedge that need to be considered are:

1. Passive force, $\mathrm{P}_{\mathrm{p} \gamma}$
2. The weight of wedge bcfj, W
3. The resultant of the frictional resistance force acting along arc cf, F
4. The Rankine passive force, $\mathrm{P}_{\mathrm{p}(3)}$

The Rankine passive force $\mathrm{P}_{\mathrm{p}(3)}$ can be given the relation

$$
\begin{equation*}
P_{p(3)}=1 / 2 \gamma H_{d}{ }^{2} \tan ^{2}(45+\phi / 2) \tag{3.20}
\end{equation*}
$$

Also note the line of action of force F will pass through O . taking the moment about O

$$
\begin{equation*}
\mathrm{P}_{\mathrm{p} \gamma} \mathrm{l}_{\mathrm{p}}=\mathrm{W} 1_{\mathrm{w}}+{ }_{\mathrm{Pp}(3)} \mathrm{l}_{\mathrm{R}} \tag{3.21}
\end{equation*}
$$

Also note that the line of action of force F will pass through O .
Taking the moment about O .

(a)

(b)

Fig. 3.5 Determination of $\mathbf{P}_{\mathbf{p} \gamma}(\phi \neq \mathbf{0}, \gamma \neq \mathbf{0}, \mathbf{q}=\mathbf{0}, \mathbf{c}=\mathbf{0})$

$$
\mathrm{P}_{\mathrm{py}} \mathrm{l}_{\mathrm{p}}=\mathrm{W} 1_{\mathrm{p}}+\mathrm{P}_{\mathrm{p}(3)} \mathrm{l}_{\mathrm{R}}
$$

Or

$$
\begin{equation*}
\mathrm{P}_{\mathrm{p} \mathrm{\gamma}}=1 / \mathrm{l}_{\mathrm{p}}\left[\mathrm{~W} 1_{\mathrm{w}}+\mathrm{P}_{\mathrm{p}(3)} \mathrm{l}_{\mathrm{R}}\right] \tag{3.22}
\end{equation*}
$$

If numbers of trail of this type are made by changing the location at the center of $\log$ spiral O along line bf, then the minimum value of $\mathrm{P}_{\mathrm{py}}$ can be determined
Considered the stability of wedge abc as shown in Fig. 3.5, we can write that

$$
\begin{equation*}
\mathrm{q}_{\gamma} \mathrm{B}=2 \mathrm{P}_{\mathrm{p} \gamma}-\mathrm{W}_{\mathrm{w}} \tag{3.23}
\end{equation*}
$$

Where, $\quad q_{\gamma}=$ force per unit area of the foundation $\mathrm{W}_{\mathrm{w}}=$ weight of wedge abc

However,

$$
\begin{equation*}
\mathrm{W}_{\mathrm{w}}=\mathrm{B}^{2} / 4 \gamma \tan \alpha \tag{3.24}
\end{equation*}
$$

So

$$
\begin{equation*}
\mathrm{q}_{\gamma}=1 / \mathrm{B}\left(2 \mathrm{P}_{\mathrm{p} \gamma}-\mathrm{B}^{2} / 4 \gamma \tan \alpha\right) \tag{3.25}
\end{equation*}
$$

The passive force $\mathrm{P}_{\mathrm{p} \gamma}$ can be expressed in the form

$$
\begin{equation*}
\mathrm{P}_{\mathrm{p} \mathrm{\gamma}}=1 / 2 \gamma \mathrm{~h}^{2} \mathrm{~K}_{\mathrm{p} \mathrm{\gamma}}=1 / 2 \gamma(\mathrm{~B} \tan \alpha / 2)^{2} \mathrm{~K}_{\mathrm{p} \gamma}=1 / 8 \gamma \mathrm{~B}^{2} \mathrm{~K}_{\mathrm{p} \mathrm{\gamma}} \tan ^{2} \alpha \tag{3.26}
\end{equation*}
$$

Where $\mathrm{K}_{\mathrm{py}}=$ passive earth pressure coefficient

Substituting Eq. (2.28) into Eq. (2.27)

$$
\begin{align*}
& \mathrm{q}_{\gamma}=1 / \mathrm{B}\left(1 / 4 \gamma \mathrm{~B}^{2} \mathrm{~K}_{\mathrm{p} \gamma} \tan ^{2} \alpha-\mathrm{B}^{2} / 4 \mathrm{~g} \tan \alpha\right) \\
&=1 / 2 \gamma \mathrm{~B}\left(1 / 2 \mathrm{~K}_{\mathrm{p} \gamma} \tan ^{2} \alpha-\tan \alpha / 2\right)=1 / 2 \gamma \mathrm{~B} \mathrm{~N}  \tag{3.27}\\
& \gamma \tag{3.28}
\end{align*}
$$

So, the new ultimate bearing capacity factors for angle of wedge $=\alpha$ are

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{q}}=2 \mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi} \cos ^{2}(45-\phi / 2) / \cos ^{2} \alpha \\
& \begin{aligned}
\mathrm{N}_{\mathrm{c}}=\left[\tan \alpha+2 \sec ^{2} \alpha\left\{\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi} 1 / 2 \cos \phi\right.\right.\right.
\end{aligned} \\
& \left.\left.+\left(\mathrm{e}^{2(3 \pi / 4-\alpha+\phi / 2) \tan \phi}-1\right) / 2 \tan \phi\right\}\right]
\end{aligned}
$$

- Now the bearing capacity factors for $\alpha=\phi$, are same as given by Trezaghi (see page no ).
- If we take $\alpha=45+\phi / 2$ (as assumed by Meyerhof bearing capacity theory) see the difference in values of bearing capacity factors $\mathrm{N}_{\mathrm{q}}, \mathrm{N}_{\mathrm{c}}$ and $\mathrm{N}_{\gamma}$ in comparison to Terzaghi's values, shown in TABLES and in FIGURES.

TABLE-1, COMPARISION OF BEARING CAPACITY FACTOR N ${ }_{q}$

| ANGLE OF INTERNAL FRICTION |  | TERZAGHI$\mathbf{N}_{\mathbf{q}}$ | MEYERHOF |
| :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ |  | $\mathrm{N}_{\mathrm{q}}$ |
| DEGREE | RADIANS | FOR $\alpha=\phi$ | FOR $\alpha=45+\phi / 2$ |
| 1 | 0.0175 | 1.10 | 2.19 |
| 2 | 0.0349 | 1.22 | 2.39 |
| 3 | 0.0524 | 1.35 | 2.62 |
| 4 | 0.0698 | 1.49 | 2.87 |
| 5 | 0.0873 | 1.64 | 3.14 |
| 6 | 0.1047 | 1.81 | 3.43 |
| 7 | 0.1222 | 2.00 | 3.76 |
| 8 | 0.1396 | 2.21 | 4.12 |
| 9 | 0.1571 | 2.44 | 4.51 |
| 10 | 0.1745 | 2.69 | 4.94 |
| 11 | 0.1920 | 2.98 | 5.42 |
| 12 | 0.2094 | 3.29 | 5.95 |
| 13 | 0.2269 | 3.63 | 6.53 |
| 14 | 0.2443 | 4.02 | 7.17 |
| 15 | 0.2618 | 4.45 | 7.88 |
| 16 | 0.2793 | 4.92 | 8.67 |
| 17 | 0.2967 | 5.45 | 9.54 |
| 18 | 0.3142 | 6.04 | 10.52 |
| 19 | 0.3316 | 6.70 | 11.60 |
| 20 | 0.3491 | 7.44 | 12.80 |
| 21 | 0.3665 | 8.26 | 14.14 |
| 22 | 0.3840 | 9.19 | 15.64 |
| 23 | 0.4014 | 10.23 | 17.32 |
| 24 | 0.4189 | 11.40 | 19.21 |
| 25 | 0.4363 | 12.72 | 21.32 |
| 26 | 0.4538 | 14.21 | 23.71 |
| 27 | 0.4712 | 15.90 | 26.40 |
| 28 | 0.4887 | 17.81 | 29.44 |
| 29 | 0.5061 | 19.98 | 32.89 |
| 30 | 0.5236 | 22.46 | 36.80 |
| 31 | 0.5411 | 25.28 | 41.26 |
| 32 | 0.5585 | 28.52 | 46.35 |
| 33 | 0.5760 | 32.23 | 52.18 |
| 34 | 0.5934 | 36.50 | 58.88 |
| 35 | 0.6109 | 41.44 | 66.59 |
| 36 | 0.6283 | 47.16 | 75.50 |
| 37 | 0.6458 | 53.80 | 85.84 |
| 38 | 0.6632 | 61.55 | 97.87 |
| 39 | 0.6807 | 70.61 | 111.91 |
| 40 | 0.6981 | 81.27 | 128.39 |
| 41 | 0.7156 | 93.85 | 147.79 |
| 42 | 0.7330 | 108.75 | 170.75 |
| 43 | 0.7505 | 126.50 | 198.03 |
| 44 | 0.7679 | 147.74 | 230.62 |
| 45 | 0.7854 | 173.29 | 269.75 |
| 46 | 0.8029 | 204.19 | 317.00 |
| 47 | 0.8203 | 241.80 | 374.41 |
| 48 | 0.8378 | 287.85 | 444.60 |
| 49 | 0.8552 | 344.64 | 530.99 |
| 50 | 0.8727 | 415.15 | 638.11 |

TABLE-2, COMPARISION OF BEARING CAPACITY FACTOR $\mathbf{N}_{\mathbf{c}}$

| ANGLE OF INTERNAL FRICTION |  | TERZAGHI | MEYERHOF |
| :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | $\mathrm{N}_{\mathrm{c}}$ | $\mathbf{N}_{\mathbf{c}}$ |
| DEGREE | RADIANS | FOR $\alpha=\phi$ | FOR $\alpha=45+\phi / 2$ |
| 1 | 0.0175 | 5.997 | 9.741 |
| 2 | 0.0349 | 6.300 | 10.228 |
| 3 | 0.0524 | 6.624 | 10.746 |
| 4 | 0.0698 | 6.968 | 11.298 |
| 5 | 0.0873 | 7.337 | 11.886 |
| 6 | 0.1047 | 7.730 | 12.515 |
| 7 | 0.1222 | 8.151 | 13.186 |
| 8 | 0.1396 | 8.602 | 13.904 |
| 9 | 0.1571 | 9.086 | 14.673 |
| 10 | 0.1745 | 9.605 | 15.498 |
| 11 | 0.1920 | 10.163 | 16.383 |
| 12 | 0.2094 | 10.763 | 17.334 |
| 13 | 0.2269 | 11.410 | 18.358 |
| 14 | 0.2443 | 12.108 | 19.460 |
| 15 | 0.2618 | 12.861 | 20.650 |
| 16 | 0.2793 | 13.676 | 21.935 |
| 17 | 0.2967 | 14.559 | 23.325 |
| 18 | 0.3142 | 15.517 | 24.831 |
| 19 | 0.3316 | 16.558 | 26.465 |
| 20 | 0.3491 | 17.690 | 28.241 |
| 21 | 0.3665 | 18.925 | 30.175 |
| 22 | 0.3840 | 20.272 | 32.283 |
| 23 | 0.4014 | 21.746 | 34.586 |
| 24 | 0.4189 | 23.361 | 37.107 |
| 25 | 0.4363 | 25.135 | 39.871 |
| 26 | 0.4538 | 27.085 | 42.908 |
| 27 | 0.4712 | 29.236 | 46.252 |
| 28 | 0.4887 | 31.612 | 49.942 |
| 29 | 0.5061 | 34.242 | 54.023 |
| 30 | 0.5236 | 37.162 | 58.547 |
| 31 | 0.5411 | 40.411 | 63.575 |
| 32 | 0.5585 | 44.036 | 69.176 |
| 33 | 0.5760 | 48.090 | 75.435 |
| 34 | 0.5934 | 52.637 | 82.447 |
| 35 | 0.6109 | 57.754 | 90.326 |
| 36 | 0.6283 | 63.528 | 99.208 |
| 37 | 0.6458 | 70.067 | 109.254 |
| 38 | 0.6632 | 77.495 | 120.653 |
| 39 | 0.6807 | 85.966 | 133.637 |
| 40 | 0.6981 | 95.663 | 148.482 |
| 41 | 0.7156 | 106.807 | 165.522 |
| 42 | 0.7330 | 119.669 | 185.167 |
| 43 | 0.7505 | 134.580 | 207.915 |
| 44 | 0.7679 | 151.950 | 234.383 |
| 45 | 0.7854 | 172.285 | 265.333 |
| 46 | 0.8029 | 196.219 | 301.720 |
| 47 | 0.8203 | 224.549 | 344.741 |
| 48 | 0.8378 | 258.285 | 395.913 |
| 49 | 0.8552 | 298.718 | 457.173 |
| 50 | 0.8727 | 347.509 | 531.016 |

FIG. 3.6, COMPARISION OF BEARING CAPACITY FACTOR $\mathbf{N}_{\mathrm{q}}$ FOR $\alpha=\phi$ AND $\alpha=45+\phi / 2$


FIG. 3.7, COMPARISON OF BEARING CAPACITY FACTOR $\mathrm{N}_{\mathrm{c}}$


TABLE-3, COMPARISION OF BEARING CAPACITY FACTOR $\mathbf{N}_{\gamma}$

| ANGLE OF <br> FRICTION | INTERNAL | $\theta$ MINIMUM VALUE | TERZAGHI | $\theta$ MINIMUM VALUE | MEYERHOF |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | FOR $\alpha=\Phi$ | $\mathrm{N}_{\gamma}$ | FOR $\alpha=45+\Phi / 2$ | $\mathbf{N}_{\gamma}$ |
| DEGREE | RADIANS | DEGREE | FOR $\alpha=\varnothing$ | DEGREE | FOR $\alpha=45+\Phi / 2$ |
| 1 | 0.0175 | 88.833 | 0.014 | 96.950 | 115.162 |
| 2 | 0.0349 | 90.910 | 0.035 | 96.944 | 133.327 |
| 3 | 0.0524 | 92.494 | 0.063 | 96.932 | 154.466 |
| 4 | 0.0698 | 93.753 | 0.099 | 96.912 | 179.101 |
| 5 | 0.0873 | 94.782 | 0.144 | 96.882 | 207.857 |
| 6 | 0.1047 | 95.637 | 0.200 | 96.856 | 241.479 |
| 7 | 0.1222 | 96.356 | 0.267 | 96.821 | 280.861 |
| 8 | 0.1396 | 96.966 | 0.348 | 96.781 | 327.076 |
| 9 | 0.1571 | 97.486 | 0.444 | 96.736 | 381.415 |
| 10 | 0.1745 | 97.931 | 0.559 | 96.688 | 445.440 |
| 11 | 0.1920 | 98.311 | 0.694 | 96.636 | 521.042 |
| 12 | 0.2094 | 98.637 | 0.854 | 96.580 | 610.518 |
| 13 | 0.2269 | 98.916 | 1.041 | 96.522 | 716.668 |
| 14 | 0.2443 | 99.152 | 1.262 | 96.461 | 842.918 |
| 15 | 0.2618 | 99.352 | 1.520 | 96.397 | 993.471 |
| 16 | 0.2793 | 99.519 | 1.822 | 96.330 | 1173.503 |
| 17 | 0.2967 | 99.656 | 2.175 | 96.262 | 1389.411 |
| 18 | 0.3142 | 99.768 | 2.589 | 96.191 | 1649.132 |
| 19 | 0.3316 | 99.855 | 3.074 | 96.119 | 1962.557 |
| 20 | 0.3491 | 99.921 | 3.641 | 96.044 | 2342.052 |
| 21 | 0.3665 | 99.968 | 4.305 | 95.968 | 2803.157 |
| 22 | 0.3840 | 99.996 | 5.085 | 95.890 | 3365.481 |
| 23 | 0.4014 | 100.009 | 6.000 | 95.812 | 4053.881 |
| 24 | 0.4189 | 100.006 | 7.076 | 95.731 | 4900.029 |
| 25 | 0.4363 | 99.989 | 8.342 | 95.650 | 5944.474 |
| 26 | 0.4538 | 99.960 | 9.836 | 95.567 | 7239.218 |
| 27 | 0.4712 | 99.918 | 11.602 | 95.484 | 8852.421 |
| 28 | 0.4887 | 99.866 | 13.693 | 95.400 | 10871.438 |
| 29 | 0.5061 | 99.804 | 16.175 | 95.314 | 13411.662 |
| 30 | 0.5236 | 99.732 | 19.129 | 95.229 | 16624.905 |
| 31 | 0.5411 | 99.652 | 22.653 | 95.142 | 20712.570 |
| 32 | 0.5585 | 99.563 | 26.871 | 95.055 | 25943.690 |
| 33 | 0.5760 | 99.467 | 31.935 | 94.967 | 32680.244 |
| 34 | 0.5934 | 99.363 | 38.035 | 94.879 | 41412.971 |
| 35 | 0.6109 | 99.253 | 45.410 | 94.790 | 52812.471 |
| 36 | 0.6283 | 99.137 | 54.360 | 94.701 | 67802.810 |
| 37 | 0.6458 | 99.015 | 65.266 | 94.612 | 87668.530 |
| 38 | 0.6632 | 98.888 | 78.614 | 94.523 | 114211.831 |
| 39 | 0.6807 | 98.755 | 95.028 | 94.433 | 149985.865 |
| 40 | 0.6981 | 98.618 | 115.311 | 94.343 | 198644.894 |
| 41 | 0.7156 | 98.477 | 140.509 | 94.253 | 265476.022 |
| 42 | 0.7330 | 98.331 | 171.990 | 94.162 | 358216.664 |
| 43 | 0.7505 | 98.181 | 211.556 | 94.072 | 488327.751 |
| 44 | 0.7679 | 98.028 | 261.603 | 93.982 | 673004.157 |
| 45 | 0.7854 | 97.872 | 325.342 | 93.891 | 938395.634 |
| 46 | 0.8029 | 97.712 | 407.113 | 93.801 | 1324847.070 |
| 47 | 0.8203 | 97.550 | 512.836 | 93.711 | 1895564.156 |
| 48 | 0.8378 | 97.385 | 650.673 | 93.620 | 2751193.652 |
| 49 | 0.8552 | 97.218 | 831.990 | 93.530 | 4054810.739 |
| 50 | 0.8727 | 97.048 | 1072.797 | 93.440 | 6075589.081 |

FIG.3.8, VARIATION OF TERZAGHI N ${ }_{\gamma}$ WITH $\phi$, FOR $\alpha=\phi$


FIG. 3.9, VARIATION OF N WITH $\phi$, FOR $\alpha=45+\phi / 2$


FIG. 3.10, VARIATION OF TERZAGHI'S NWITH MINIMUM ANGLE OF LOG SPIRAL $\theta$


FIG. 3.11, VARIATION OF $N_{\gamma}$ FOR MINMUM ANGLE OF LOG SPIRAL $\theta$,

$$
\text { IF } \alpha=45+\phi / 2
$$



- Now applying the Terzaghi's modified bearing capacity factors formulas (Eq. 3.11, 3.19, and 3.28) for calculating the value of $\mathrm{N}_{\mathrm{q}}, \mathrm{N}_{\mathrm{C}}$, and $\mathrm{N}_{\gamma}$ for "SPUDCAN footing".
- SPUDCAN footing is type of conical shaped footing with protruding tip at the center (see Fig.3.12). It is use in the Mobile jackup units of a floatable platform (use in offshore structures). For details see Literature review page no.-


Fig. 3.12

- In the SPUDCAN footings, their embedded circular area in plane is used for the bearing capacity calculations. The SPUDCAN footing is treated as equivalent cone enclosing the same volume.

In my study the series of cone angles $\Omega=0^{\circ}$ (Flat), $5^{\circ}, 10^{\circ}$, and $15^{\circ}$ is use for calculating the bearing capacity factors $\mathrm{N}_{\mathrm{q}}, \mathrm{N}_{\mathrm{c}}$, and $\mathrm{N}_{\gamma}$.

Also two approaches is used

1. The wedge angle, $\alpha=\Omega+\phi$.
2. And, $\alpha=\Omega+[45+\phi / 2]$. See Fig. 3.13


Fig. 3.13 Failure surface in soil at ultimate load for spudcan footing as assumed in the proposed analysis

All the values of bearing capacity factors $\mathrm{N}_{\mathrm{q}}, \mathrm{N}_{\mathrm{c}}$, and $\mathrm{N}_{\gamma}$ are calculated in the Tables for different cone angles, $\Omega=5^{\circ}, 10^{\circ}$, and $15^{\circ}$ and there variation are shown in the Figures.

TABLE-3.4, COMPARISION OF BEARING CAPACITY FACTOR $\mathrm{N}_{\mathrm{q}}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+\phi$ AND $\Omega=5^{\circ}, 10^{\circ}$, AND $15^{\circ}$.

| ANGLE OF INTERNAL FRICTION |  | TERZAGHI | $\Omega=5^{\circ}$ | $\Omega=10^{\circ}$ | $\Omega=15^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | $\mathrm{N}_{\mathrm{q}}$ | $\mathbf{N}_{\mathrm{q}}$ | $\mathrm{N}_{\mathrm{q}}$ | $\mathrm{N}_{\mathrm{q}}$ |
| DEGREE | RADIANS | IF $\alpha=\phi$ | IF $\alpha=\Omega+$ | $\alpha=\Omega+$ | $\alpha=\Omega+\phi$ |
| 1 | 0.0175 | 1.10 | 1.11 | 1.14 | 1.18 |
| 2 | 0.0349 | 1.22 | 1.23 | 1.26 | 1.31 |
| 3 | 0.0524 | 1.35 | 1.36 | 1.39 | 1.45 |
| 4 | 0.0698 | 1.49 | 1.50 | 1.53 | 1.60 |
| 5 | 0.0873 | 1.64 | 1.65 | 1.69 | 1.76 |
| 6 | 0.1047 | 1.81 | 1.83 | 1.87 | 1.95 |
| 7 | 0.1222 | 2.00 | 2.02 | 2.06 | 2.15 |
| 8 | 0.1396 | 2.21 | 2.23 | 2.28 | 2.38 |
| 9 | 0.1571 | 2.44 | 2.46 | 2.52 | 2.62 |
| 10 | 0.1745 | 2.69 | 2.72 | 2.78 | 2.90 |
| 11 | 0.1920 | 2.98 | 3.00 | 3.07 | 3.21 |
| 12 | 0.2094 | 3.29 | 3.31 | 3.40 | 3.54 |
| 13 | 0.2269 | 3.63 | 3.66 | 3.76 | 3.92 |
| 14 | 0.2443 | 4.02 | 4.05 | 4.16 | 4.34 |
| 15 | 0.2618 | 4.45 | 4.48 | 4.60 | 4.81 |
| 16 | 0.2793 | 4.92 | 4.96 | 5.09 | 5.33 |
| 17 | 0.2967 | 5.45 | 5.50 | 5.64 | 5.91 |
| 18 | 0.3142 | 6.04 | 6.09 | 6.26 | 6.55 |
| 19 | 0.3316 | 6.70 | 6.76 | 6.94 | 7.28 |
| 20 | 0.3491 | 7.44 | 7.50 | 7.71 | 8.09 |
| 21 | 0.3665 | 8.26 | 8.34 | 8.57 | 9.00 |
| 22 | 0.3840 | 9.19 | 9.27 | 9.54 | 10.03 |
| 23 | 0.4014 | 10.23 | 10.33 | 10.63 | 11.18 |
| 24 | 0.4189 | 11.40 | 11.51 | 11.85 | 12.48 |
| 25 | 0.4363 | 12.72 | 12.84 | 13.23 | 13.95 |
| 26 | 0.4538 | 14.21 | 14.35 | 14.79 | 15.61 |
| 27 | 0.4712 | 15.90 | 16.05 | 16.56 | 17.50 |
| 28 | 0.4887 | 17.81 | 17.99 | 18.57 | 19.65 |
| 29 | 0.5061 | 19.98 | 20.19 | 20.86 | 22.10 |
| 30 | 0.5236 | 22.46 | 22.69 | 23.46 | 24.90 |
| 31 | 0.5411 | 25.28 | 25.56 | 26.44 | 28.10 |
| 32 | 0.5585 | 28.52 | 28.83 | 29.86 | 31.79 |
| 33 | 0.5760 | 32.23 | 32.60 | 33.79 | 36.04 |
| 34 | 0.5934 | 36.50 | 36.93 | 38.32 | 40.95 |
| 35 | 0.6109 | 41.44 | 41.93 | 43.55 | 46.64 |
| 36 | 0.6283 | 47.16 | 47.73 | 49.63 | 53.27 |
| 37 | 0.6458 | 53.80 | 54.48 | 56.71 | 61.01 |
| 38 | 0.6632 | 61.55 | 62.34 | 64.98 | 70.09 |
| 39 | 0.6807 | 70.61 | 71.56 | 74.69 | 80.78 |
| 40 | 0.6981 | 81.27 | 82.39 | 86.12 | 93.42 |
| 41 | 0.7156 | 93.85 | 95.18 | 99.64 | 108.44 |
| 42 | 0.7330 | 108.75 | 110.35 | 115.72 | 126.36 |
| 43 | 0.7505 | 126.50 | 128.42 | 134.91 | 147.87 |
| 44 | 0.7679 | 147.74 | 150.06 | 157.95 | 173.81 |
| 45 | 0.7854 | 173.29 | 176.12 | 185.76 | 205.30 |
| 46 | 0.8029 | 204.19 | 207.66 | 219.52 | 243.76 |
| 47 | 0.8203 | 241.80 | 246.07 | 260.76 | 291.04 |
| 48 | 0.8378 | 287.85 | 293.15 | 311.47 | 349.59 |
| 49 | 0.8552 | 344.64 | 351.25 | 374.26 | 422.64 |
| 50 | 0.8727 | 415.15 | 423.47 | 452.61 | 514.56 |

TABLE-3.5, COMPARISION OF BEARING CAPACITY FACTOR N ${ }_{C}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+\phi$ AND $\Omega=5^{\circ}, \mathbf{1 0}^{\circ}$, AND $15^{\circ}$.

| ANGLE OF INTERNAL FRICTION |  | TERZAGHI | $\Omega=5^{\circ}$ | $\Omega=10^{\circ}$ | $\Omega=15^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | Nc | Nc | Nc | Nc |
| DEGREE | RADIANS | IF $\alpha=\phi$ | IF $\alpha=\Omega+$ | $\alpha=\Omega+$ | $\alpha=\Omega+\phi$ |
| 1 | 0.0175 | 5.997 | 5.954 | 5.999 | 6.133 |
| 2 | 0.0349 | 6.300 | 6.260 | 6.311 | 6.457 |
| 3 | 0.0524 | 6.624 | 6.585 | 6.644 | 6.803 |
| 4 | 0.0698 | 6.968 | 6.932 | 6.999 | 7.172 |
| 5 | 0.0873 | 7.337 | 7.303 | 7.378 | 7.567 |
| 6 | 0.1047 | 7.730 | 7.699 | 7.784 | 7.989 |
| 7 | 0.1222 | 8.151 | 8.123 | 8.218 | 8.441 |
| 8 | 0.1396 | 8.602 | 8.578 | 8.683 | 8.927 |
| 9 | 0.1571 | 9.086 | 9.065 | 9.182 | 9.448 |
| 10 | 0.1745 | 9.605 | 9.588 | 9.718 | 10.008 |
| 11 | 0.1920 | 10.163 | 10.150 | 10.295 | 10.611 |
| 12 | 0.2094 | 10.763 | 10.754 | 10.915 | 11.261 |
| 13 | 0.2269 | 11.410 | 11.406 | 11.585 | 11.963 |
| 14 | 0.2443 | 12.108 | 12.109 | 12.307 | 12.721 |
| 15 | 0.2618 | 12.861 | 12.869 | 13.088 | 13.542 |
| 16 | 0.2793 | 13.676 | 13.691 | 13.933 | 14.431 |
| 17 | 0.2967 | 14.559 | 14.582 | 14.850 | 15.396 |
| 18 | 0.3142 | 15.517 | 15.548 | 15.845 | 16.445 |
| 19 | 0.3316 | 16.558 | 16.598 | 16.927 | 17.588 |
| 20 | 0.3491 | 17.690 | 17.740 | 18.106 | 18.834 |
| 21 | 0.3665 | 18.925 | 18.986 | 19.392 | 20.196 |
| 22 | 0.3840 | 20.272 | 20.347 | 20.797 | 21.687 |
| 23 | 0.4014 | 21.746 | 21.835 | 22.336 | 23.321 |
| 24 | 0.4189 | 23.361 | 23.467 | 24.025 | 25.118 |
| 25 | 0.4363 | 25.135 | 25.259 | 25.881 | 27.096 |
| 26 | 0.4538 | 27.085 | 27.230 | 27.925 | 29.279 |
| 27 | 0.4712 | 29.236 | 29.405 | 30.182 | 31.693 |
| 28 | 0.4887 | 31.612 | 31.808 | 32.678 | 34.370 |
| 29 | 0.5061 | 34.242 | 34.469 | 35.447 | 37.344 |
| 30 | 0.5236 | 37.162 | 37.425 | 38.524 | 40.657 |
| 31 | 0.5411 | 40.411 | 40.714 | 41.954 | 44.359 |
| 32 | 0.5585 | 44.036 | 44.386 | 45.786 | 48.505 |
| 33 | 0.5760 | 48.090 | 48.494 | 50.080 | 53.163 |
| 34 | 0.5934 | 52.637 | 53.104 | 54.905 | 58.413 |
| 35 | 0.6109 | 57.754 | 58.293 | 60.345 | 64.348 |
| 36 | 0.6283 | 63.528 | 64.152 | 66.497 | 71.082 |
| 37 | 0.6458 | 70.067 | 70.790 | 73.477 | 78.748 |
| 38 | 0.6632 | 77.495 | 78.335 | 81.427 | 87.508 |
| 39 | 0.6807 | 85.966 | 86.944 | 90.513 | 97.560 |
| 40 | 0.6981 | 95.663 | 96.804 | 100.941 | 109.142 |
| 41 | 0.7156 | 106.807 | 108.143 | 112.957 | 122.547 |
| 42 | 0.7330 | 119.669 | 121.238 | 126.866 | 138.133 |
| 43 | 0.7505 | 134.580 | 136.430 | 143.040 | 156.349 |
| 44 | 0.7679 | 151.950 | 154.139 | 161.942 | 177.750 |
| 45 | 0.7854 | 172.285 | 174.887 | 184.148 | 203.035 |
| 46 | 0.8029 | 196.219 | 199.327 | 210.381 | 233.091 |
| 47 | 0.8203 | 224.549 | 228.281 | 241.556 | 269.048 |
| 48 | 0.8378 | 258.285 | 262.792 | 278.839 | 312.362 |
| 49 | 0.8552 | 298.718 | 304.194 | 323.727 | 364.924 |
| 50 | 0.8727 | 347.509 | 354.209 | 378.165 | 429.216 |

FIG. 3.14, Comparision of $\mathrm{N}_{\mathrm{q}}$ for diffrent Spudcan footing angles $\left(5^{\circ}\right.$, $10^{\circ}, \& 15^{\circ}$ ) FOR $\alpha=\Omega+\phi$


FIG. 3.15, COMPARISION OF Nc FOR DIFFERENT SPUDCAN FOOTING ANGLES (5º, $\left.10^{\circ}, \& 15^{\circ}\right)$ FOR $\alpha=\Omega+\phi$


TABLE- 3.6, COMPARISION OF BEARING CAPACITY FACTOR $\mathrm{N}_{\gamma}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+\phi$ AND $\Omega=5^{\circ}, \mathbf{1 0}^{\circ}$, AND $15^{\circ}$.

| $\phi$ | $\Omega$ | $\alpha=\Omega+\phi$ | $\theta$ | $\mathrm{N}_{\gamma}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 6.000 | 93.486 | 0.076 |
| 5 | 5 | 10.000 | 97.572 | 0.358 |
| 10 | 5 | 15.000 | 99.015 | 1.161 |
| 15 | 5 | 20.000 | 99.487 | 2.985 |
| 20 | 5 | 25.000 | 99.455 | 7.115 |
| 25 | 5 | 30.000 | 99.089 | 16.750 |
| 30 | 5 | 35.000 | 98.526 | 40.543 |
| 35 | 5 | 40.000 | 97.823 | 104.559 |
| 40 | 5 | 45.000 | 96.162 | 298.868 |
| 50 | 5 | 55.000 | 95.856 | 2548.325 |


| 1 | 10 | 11.000 | 85.919 | 0.244 |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 10 | 15.000 | 94.849 | 0.831 |
| 10 | 10 | 20.000 | 97.822 | 2.375 |
| 15 | 10 | 25.000 | 98.864 | 5.953 |
| 20 | 10 | 30.000 | 99.100 | 14.453 |
| 25 | 10 | 35.000 | 98.895 | 35.754 |
| 30 | 10 | 40.000 | 98.419 | 93.779 |
| 35 | 10 | 45.000 | 97.765 | 271.817 |
| 40 | 10 | 50.000 | 96.995 | 917.546 |
| 50 | 10 | 60.000 | 95.259 | 9856.548 |


| 1 | 15 | 16.000 | 91.903 | 0.699 |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 15 | 20.000 | 96.168 | 1.815 |
| 10 | 15 | 25.000 | 98.076 | 4.868 |
| 15 | 15 | 30.000 | 98.680 | 12.267 |
| 20 | 15 | 35.000 | 98.664 | 31.111 |
| 25 | 15 | 40.000 | 98.292 | 83.136 |
| 30 | 15 | 45.000 | 97.699 | 244.436 |
| 35 | 15 | 50.000 | 96.962 | 836.397 |
| 40 | 15 | 55.000 | 96.134 | 3586.521 |
| 50 | 15 | 65.000 | 94.347 | 12456.363 |

FIG. 3.16, COMPARISION OF Ng FOR DIFFERENT SPUDCAN FOOTING ANGLES $\left(5^{\circ}, 10^{\circ}, 15^{\circ}\right)$ FOR $\alpha=\Omega+\phi$


## TABLE- 3.7, COMPARISION OF BEARING CAPACITY FACTOR $\mathrm{N}_{\mathrm{q}}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+45+\phi / 2$ AND $\Omega=5^{0}, 10^{\circ}, 15^{\circ}$

| ANGLE OF INTERNAL FRICTION |  | TERZAGHI | $\Omega=5^{\circ}$ | $\Omega=10^{\circ}$ | $\Omega=15^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | Nq | $\mathrm{N}_{\mathrm{q}}$ | $\mathrm{N}_{\mathrm{q}}$ | $\mathrm{N}_{\mathrm{q}}$ |
| DEGREE | RADIANS | IF $\alpha=\phi$ | IF $\alpha=\Omega+45+\phi / 2$ | IF $\alpha=\Omega+45+\phi / 2$ | IF $\alpha=\Omega+45+\phi / 2$ |
| 1 | 0.0175 | 1.10 | 2.65 | 3.33 | 4.39 |
| 2 | 0.0349 | 1.22 | 2.90 | 3.65 | 4.82 |
| 3 | 0.0524 | 1.35 | 3.17 | 4.00 | 5.30 |
| 4 | 0.0698 | 1.49 | 3.47 | 4.38 | 5.83 |
| 5 | 0.0873 | 1.64 | 3.80 | 4.81 | 6.41 |
| 6 | 0.1047 | 1.81 | 4.17 | 5.28 | 7.06 |
| 7 | 0.1222 | 2.00 | 4.56 | 5.79 | 7.77 |
| 8 | 0.1396 | 2.21 | 5.00 | 6.36 | 8.56 |
| 9 | 0.1571 | 2.44 | 5.49 | 6.99 | 9.45 |
| 10 | 0.1745 | 2.69 | 6.02 | 7.68 | 10.43 |
| 11 | 0.1920 | 2.98 | 6.61 | 8.45 | 11.52 |
| 12 | 0.2094 | 3.29 | 7.26 | 9.30 | 12.74 |
| 13 | 0.2269 | 3.63 | 7.98 | 10.25 | 14.10 |
| 14 | 0.2443 | 4.02 | 8.77 | 11.30 | 15.62 |
| 15 | 0.2618 | 4.45 | 9.66 | 12.48 | 17.34 |
| 16 | 0.2793 | 4.92 | 10.64 | 13.78 | 19.26 |
| 17 | 0.2967 | 5.45 | 11.73 | 15.24 | 21.42 |
| 18 | 0.3142 | 6.04 | 12.94 | 16.88 | 23.86 |
| 19 | 0.3316 | 6.70 | 14.29 | 18.71 | 26.62 |
| 20 | 0.3491 | 7.44 | 15.81 | 20.76 | 29.75 |
| 21 | 0.3665 | 8.26 | 17.50 | 23.07 | 33.30 |
| 22 | 0.3840 | 9.19 | 19.39 | 25.68 | 37.35 |
| 23 | 0.4014 | 10.23 | 21.52 | 28.62 | 41.97 |
| 24 | 0.4189 | 11.40 | 23.92 | 31.95 | 47.26 |
| 25 | 0.4363 | 12.72 | 26.62 | 35.72 | 53.33 |
| 26 | 0.4538 | 14.21 | 29.67 | 40.02 | 60.33 |
| 27 | 0.4712 | 15.90 | 33.12 | 44.91 | 68.42 |
| 28 | 0.4887 | 17.81 | 37.04 | 50.51 | 77.81 |
| 29 | 0.5061 | 19.98 | 41.49 | 56.92 | 88.74 |
| 30 | 0.5236 | 22.46 | 46.58 | 64.30 | 101.52 |
| 31 | 0.5411 | 25.28 | 52.39 | 72.80 | 116.52 |
| 32 | 0.5585 | 28.52 | 59.05 | 82.64 | 134.21 |
| 33 | 0.5760 | 32.23 | 66.72 | 94.07 | 155.17 |
| 34 | 0.5934 | 36.50 | 75.56 | 107.39 | 180.15 |
| 35 | 0.6109 | 41.44 | 85.80 | 122.97 | 210.06 |
| 36 | 0.6283 | 47.16 | 97.69 | 141.27 | 246.09 |
| 37 | 0.6458 | 53.80 | 111.55 | 162.86 | 289.79 |
| 38 | 0.6632 | 61.55 | 127.77 | 188.45 | 343.13 |
| 39 | 0.6807 | 70.61 | 146.83 | 218.93 | 408.74 |
| 40 | 0.6981 | 81.27 | 169.33 | 255.41 | 490.10 |
| 41 | 0.7156 | 93.85 | 195.99 | 299.32 | 591.86 |
| 42 | 0.7330 | 108.75 | 227.74 | 352.48 | 720.39 |
| 43 | 0.7505 | 126.50 | 265.76 | 417.24 | 884.45 |
| 44 | 0.7679 | 147.74 | 311.52 | 496.68 | 1096.33 |
| 45 | 0.7854 | 173.29 | 366.91 | 594.79 | 1373.55 |
| 46 | 0.8029 | 204.19 | 434.36 | 716.92 | 1741.61 |
| 47 | 0.8203 | 241.80 | 517.05 | 870.20 | 2238.36 |
| 48 | 0.8378 | 287.85 | 619.11 | 1064.30 | 2921.52 |
| 49 | 0.8552 | 344.64 | 746.01 | 1312.45 | 3881.55 |
| 50 | 0.8727 | 415.15 | 905.06 | 1633.03 | 5265.14 |

## TABLE- 3.8, COMPARISION OF BEARING CAPACITY FACTOR $N_{C}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+45+\phi / 2$ AND $\Omega=5^{0}, 10^{0}, 15^{\circ}$

| ANGLE OF INTERNAL FRICTION |  | TERZAGHIS | $\Omega=5^{\circ}$ | $\Omega=10^{\circ}$ | $\Omega=15^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | $\phi$ | $\mathrm{N}_{\mathrm{c}}$ | $\mathrm{N}_{\mathrm{c}}$ | $\mathbf{N}_{\mathrm{c}}$ | $\mathbf{N}_{\mathrm{c}}$ |
| DEGREE | RADIANS | IF $\alpha=\phi$ | IF $\alpha=\Omega+45+\phi / 2$ | IF $\alpha=\Omega+45+\phi / 2$ | IF $\alpha=\Omega+45+\phi / 2$ |
| 1 | 0.0175 | 5.997 | 11.342 | 13.647 | 17.131 |
| 2 | 0.0349 | 6.300 | 11.927 | 14.386 | 18.127 |
| 3 | 0.0524 | 6.624 | 12.551 | 15.177 | 19.198 |
| 4 | 0.0698 | 6.968 | 13.218 | 16.024 | 20.351 |
| 5 | 0.0873 | 7.337 | 13.930 | 16.933 | 21.595 |
| 6 | 0.1047 | 7.730 | 14.691 | 17.908 | 22.938 |
| 7 | 0.1222 | 8.151 | 15.507 | 18.956 | 24.390 |
| 8 | 0.1396 | 8.602 | 16.381 | 20.084 | 25.963 |
| 9 | 0.1571 | 9.086 | 17.320 | 21.299 | 27.668 |
| 10 | 0.1745 | 9.605 | 18.328 | 22.611 | 29.521 |
| 11 | 0.1920 | 10.163 | 19.414 | 24.027 | 31.536 |
| 12 | 0.2094 | 10.763 | 20.583 | 25.560 | 33.733 |
| 13 | 0.2269 | 11.410 | 21.844 | 27.221 | 36.131 |
| 14 | 0.2443 | 12.108 | 23.206 | 29.024 | 38.753 |
| 15 | 0.2618 | 12.861 | 24.679 | 30.982 | 41.627 |
| 16 | 0.2793 | 13.676 | 26.276 | 33.115 | 44.783 |
| 17 | 0.2967 | 14.559 | 28.007 | 35.440 | 48.254 |
| 18 | 0.3142 | 15.517 | 29.889 | 37.980 | 52.082 |
| 19 | 0.3316 | 16.558 | 31.936 | 40.759 | 56.312 |
| 20 | 0.3491 | 17.690 | 34.169 | 43.806 | 60.997 |
| 21 | 0.3665 | 18.925 | 36.607 | 47.153 | 66.200 |
| 22 | 0.3840 | 20.272 | 39.274 | 50.838 | 71.992 |
| 23 | 0.4014 | 21.746 | 42.198 | 54.903 | 78.458 |
| 24 | 0.4189 | 23.361 | 45.409 | 59.397 | 85.697 |
| 25 | 0.4363 | 25.135 | 48.942 | 64.377 | 93.826 |
| 26 | 0.4538 | 27.085 | 52.839 | 69.909 | 102.983 |
| 27 | 0.4712 | 29.236 | 57.146 | 76.070 | 113.332 |
| 28 | 0.4887 | 31.612 | 61.918 | 82.950 | 125.070 |
| 29 | 0.5061 | 34.242 | 67.218 | 90.654 | 138.433 |
| 30 | 0.5236 | 37.162 | 73.118 | 99.305 | 153.707 |
| 31 | 0.5411 | 40.411 | 79.704 | 109.051 | 171.237 |
| 32 | 0.5585 | 44.036 | 87.075 | 120.065 | 191.447 |
| 33 | 0.5760 | 48.090 | 95.351 | 132.553 | 214.857 |
| 34 | 0.5934 | 52.637 | 104.670 | 146.765 | 242.112 |
| 35 | 0.6109 | 57.754 | 115.195 | 162.998 | 274.017 |
| 36 | 0.6283 | 63.528 | 127.125 | 181.614 | 311.581 |
| 37 | 0.6458 | 70.067 | 140.693 | 203.051 | 356.088 |
| 38 | 0.6632 | 77.495 | 156.181 | 227.848 | 409.176 |
| 39 | 0.6807 | 85.966 | 173.930 | 256.666 | 472.965 |
| 40 | 0.6981 | 95.663 | 194.354 | 290.323 | 550.222 |
| 41 | 0.7156 | 106.807 | 217.958 | 329.840 | 644.603 |
| 42 | 0.7330 | 119.669 | 245.362 | 376.500 | 761.003 |
| 43 | 0.7505 | 134.580 | 277.331 | 431.927 | 906.062 |
| 44 | 0.7679 | 151.950 | 314.819 | 498.191 | 1088.934 |
| 45 | 0.7854 | 172.285 | 359.019 | 577.958 | 1322.454 |
| 46 | 0.8029 | 196.219 | 411.433 | 674.691 | 1624.975 |
| 47 | 0.8203 | 224.549 | 473.973 | 792.933 | 2023.316 |
| 48 | 0.8378 | 258.285 | 549.085 | 938.712 | 2557.655 |
| 49 | 0.8552 | 298.718 | 639.933 | 1120.113 | 3289.942 |
| 50 | 0.8727 | 347.509 | 750.639 | 1348.121 | 4318.943 |

FIG. 3.17, COMPARISION OF Nq FOR DIFFERENT SPUDCAN FOOTING ANGLE $\left(5^{\circ}, 10^{\circ}\right.$, AND $\left.15^{\circ}\right)$ FOR $\alpha=\Omega$ $+45+\phi / 2$


FIG. 3.18, COMPARISION OF Nc FOR DIFFERENT SPUDCAN FOOTING
ANGLES $\left(5^{\circ}, 10^{\circ}\right.$, AND $\left.15^{\circ}\right)$ FOR $\alpha=\Omega+45+\phi / 2$


TABLE- 3.9, COMPARISION OF BEARING CAPACITY FACTOR $\mathrm{N}_{\gamma}$ FOR SPUDCAN FOOTING FOR $\alpha=\Omega+45+\phi / 2$ AND $\Omega=5^{\circ}, \mathbf{1 0}^{\circ}$, AND $15^{\circ}$.

| $\phi$ | $\Omega$ | $\alpha=\Omega+45+\phi / 2$ | $\theta$ | $\mathrm{~N}_{\gamma}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 50.500 | 96.460 | 371.240 |
| 5 | 5 | 52.500 | 96.279 | 727.028 |
| 10 | 5 | 55.000 | 95.973 | 1765.905 |
| 12 | 5 | 56.000 | 95.833 | 2565.796 |
| 15 | 5 | 57.500 | 95.608 | 4604.157 |
| 20 | 5 | 60.000 | 95.205 | 13196.229 |
| 25 | 5 | 62.500 | 94.778 | 42882.508 |
| 30 | 5 | 65.000 | 94.337 | 50845.244 |
| 40 | 5 | 70.000 | 93.439 | 63258.334 |
| 45 | 5 | 72.500 | 92.992 | 72458.107 |
| 50 | 5 | 75.000 | 92.549 | 84582.025 |


| 1 | 10 | 55.500 | 95.814 | 1525.784 |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 10 | 57.500 | 95.551 | 3388.915 |
| 10 | 10 | 60.000 | 95.173 | 9592.506 |
| 15 | 10 | 62.500 | 94.761 | 24580.265 |
| 20 | 10 | 65.000 | 94.328 | 48584.356 |
| 25 | 10 | 67.500 | 93.885 | 62466.128 |
| 30 | 10 | 70.000 | 93.438 | 78480.652 |
| 40 | 10 | 75.000 | 92.549 | 88155.699 |
| 50 | 10 | 80.000 | 91.992 | 93254.111 |


| 1 | 15 | 60.500 | 95.053 | 9074.841 |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 15 | 62.500 | 94.737 | 19256.365 |
| 10 | 15 | 65.000 | 94.316 | 29248.316 |
| 15 | 15 | 67.500 | 93.879 | 46586.215 |
| 20 | 15 | 70.000 | 93.435 | 55879.100 |
| 25 | 15 | 72.500 | 92.991 | 65482.322 |
| 30 | 15 | 75.000 | 92.549 | 86265.215 |
| 40 | 15 | 80.000 | 92.115 | 98125.125 |
| 50 | 15 | 85.000 | 91.987 | 112485.321 |

FIG. 3.19, COMPARISION OF N ${ }_{\gamma}$ FOR DIFFERENT SPUDCAN FOOTING ANGLES $\left(5^{\circ}, 10^{\circ}\right.$, AND $\left.15^{\circ}\right)$ FOR $\alpha=45+\phi / 2$


## CHAPTER - 4

## ANALYTICAL SOLUTION FOR BEARING CAPACITY OF

## SKIRTED FOOTING

### 4.1 INTRODUCTION

This chapter presents the results of Analytical solutions for the influence of soil confinement on the behavior of circular footing resting on granular soil. Skirts with different heights and diameters were used to confine the sand. The ultimate bearing capacity of circular footing supported on confining sand bed was studied. The studied parameter includes the skirt height and skirt diameter. Initially, the response of a nonconfined case (bearing capacity by using Terzaghi's formulation) was determined and then compared with that of confined soil. The results were than analyzed to study the effect of each parameter. The results indicate that the bearing capacity of circular footing can be appreciably increased by soil confinement. It was concluded that such reinforcement (skirts) resist lateral displacement of soil underneath the footing leading to a significant improvement in the response of the footing. For small skirt diameters, the skirt-soil footing behaves as one unit (deep foundation), while this pattern of behavior was no longer observed with large skirt diameters. The recommended skirt heights, depth and diameter that give the maximum bearing capacity improvement are presented and discussed.

### 4.2 MODELING DETAILS

The geometry of the soil, model footing, and confining cylindrical skirt are shown in the Fig. (4.1). The confining cylindrical skirt is assumed to be made of the steel shell with different diameters and heights. The used diameters are 100, 107, 133, 160 and 200 centimeter and heights of $50,100,150$, 250 and 300 centimeters. The interior and exterior surfaces of the cylindrical skirt are assumed to be very smooth and rough (by considering the angle of wall friction) for studying both the cases for
calculating bearing capacity. The thickness of the cylindrical skirt wall is assumed to be 1 centimeter. The model footing is circular with diameter of 100 centimeter.

The analytical solution is carrying out to study the effect of soil confinement on bearing capacity of foundation by varying height and diameters of cylindrical skirts as shown in Table 4.1.

fig. 4.1

## TABLE 4.1

| Constant parameters |  |  |
| :--- | :---: | :---: |
| 1. | variables parameter |  |
| 2. | d/D $=1.00$ | h/D $=0.5,1.00,1.50,2.00,2.50,3.00$ |
| 3. | d/D $=1.07$ | h/D $=0.5,1.00,1.50,2.00,2.50,3.00$ |
| 4. | d/D $=1.33$ | h/D $=0.5,1.00,1.50,2.00,2.50,3.00$ |
| 5. | d/D $=2.00$ | h/D $=0.5,1.00,1.50,2.00,2.50,3.00$ |
|  |  | h/D $=0.5,1.00,1.50,2.00,2.50,3.00$ |

### 4.3 PROPERTIES OF THE MATERIAL

## COHESIONLESS SOIL (SAND)

- Medium to coarse having
- Unit weight $(\gamma)$

$$
=18 \mathrm{~N} / \mathrm{m}^{3}
$$

- Void ratio $=0.35$ to 0.45
- Angle of internal friction $(\phi)=42,38$, and 34 degree
- Angle of wall friction $(\delta) \quad=0,22$, and 25 degree


## THIN CYLINDRICAL STEEL SHELL

- Thickness, t
$=1 \mathrm{~cm}$
- Height, h
- Diameter, d $=1.00,1.07,1.33,1.60$, and 2.00 meter
- Permissible tensile stress for the steel shell $=100 \mathrm{~N} / \mathrm{mm}^{2}=100000 \mathrm{Kpa}$


### 4.4 PROPOSED ANALYSIS \& FORMULATION

In calculation of bearing capacity of skirted foundation two approaches are PROPOSED
First, the cylindrical skirts of different diameter and heights as shown in Table-3.1 above is used for calculating the bearing capacity (see Fig. 4.2). It is called NORMAL APPROACH.

Second, as per Terzaghi's theory of bearing capacity (as explained in literature review) the path of the log-spiral has taken as the boundary for the height of the cylindrical shell. Means at every change of diameter the height is also varies with path of log-spiral. If we provide the height beyond the line of log-spiral this will acts as factor of safety for calculating bearing capacity (see Fig.4.3). It is called TERZAGH'S APPROACH.


Fig. 4.2, This figure shows Normal approach. The h \& d varies as shown in Table-4.1, used for calculating bearing capacity of footing


Fig. 4.3 This figure shows Terzaghi's approach, the height of the shell varies from $\mathrm{bO}_{1}$ to $\mathrm{a}_{5} f$ as per the path of the Log-spiral. If the height is provide beyond the Path of log-spiral it will acts as the factor of safety.

The analysis here is done for the cylindrical shell of very smooth surface ( $\delta=0$ ) (exterior and interior both) and also for rough surface ( $\delta \neq 0$ ).

- For very smooth surface of cylindrical shell the angle of wall friction $(\delta)=0$, as assumed by Rankine in his theory of earth pressure for calculating the Active and Passive ( $K_{a} \& K_{p}$ ) earth pressure. So, I will call the smooth surface cylindrical shell as RANKINE WALL.
- And for the rough surface of cylindrical shell the angle of wall friction $(\delta) \neq 0$, for that the Coulomb has given the formulas for calculating the active and passive earth pressure for cohesionless soil ( $\mathrm{K}_{\mathrm{a}} \& \mathrm{~K}_{\mathrm{p}}$ ) in his wedge theory. so, I will call the rough surface cylindrical shell as COULOMB WALL.


### 4.4.1 CACULATION OF $K_{a}$ AND $K_{p}$

- $\underline{K}_{\mathrm{a}} \& \mathrm{~K}_{\mathrm{p}}$ FOR RANKINE WALL $(\delta=0)$

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{a}}=\tan ^{2}(45-\phi / 2) \\
& \mathrm{K}_{\mathrm{p}}=\tan ^{2}(45+\phi / 2)
\end{aligned}
$$

- $\underline{K}_{\underline{a}} \& K_{p}$ FOR COULOMB WALL $(\delta \neq 0)$

$$
\left.\mathrm{K}_{\mathrm{a}}=\frac{\sin ^{2}(\alpha+\phi)}{\sin ^{2} \alpha \sin (\alpha-\delta)\left\{\sqrt{1+} \frac{\sin (\phi+\delta) \sin (\phi-\beta)}{\sin (\alpha-\delta) \sin (\alpha+\beta)}\right.}\right\}^{2}
$$

$$
\mathrm{K}_{\mathrm{p}}=\frac{\sin ^{2}(\alpha-\phi)}{\sin ^{2} \alpha \sin \left(\alpha+\delta\left\{\sqrt{\left.1-\sqrt{\sin (\phi+\delta) \sin (\phi+\beta)} \frac{\sin (\alpha+\delta) \sin (\alpha+\beta)}{}\right\}^{2}}\right.\right.}
$$

Where,
$\phi=$ angle of internal friction of wall.
$\delta=$ angle of wall friction.
$\beta=$ angle of inclination of the soil above the ground surface, in our case it is zero degree.
$\alpha=$ angle of inclination of wall, in our case it is 90 degree.
All the values of $\mathrm{K}_{\mathrm{a}}$ and $\mathrm{K}_{\mathrm{p}}$ are given in Table-4.2, for different values of $\phi$ and $\delta$.

TABLE - 4.2

| $\phi$ | $\mathbf{K}_{\mathbf{a}}$ <br> For <br> $\delta=0^{\circ}$ | $\mathbf{K}_{\mathbf{a}}$ <br> For <br> $\delta=22^{\circ}$ | $\mathbf{K}_{\mathbf{a}}$ <br> For <br> $\delta=25^{\circ}$ | $\mathbf{K}_{\mathbf{p}}$ <br> For <br> $\delta=0^{\circ}$ | $\mathbf{K}_{\mathbf{p}}$ <br> For <br> $\delta=22^{\circ}$ | $\mathbf{K}_{\mathbf{p}}$ <br> For <br> $\delta=25^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 42 | 0.198 | 0.183 | 0.183 | 5.044 | 15.776 | 19.758 |
| 38 | 0.238 | 0.217 | 0.217 | 4.204 | 11.466 | 13.901 |
| 34 | 0.238 | 0.254 | 0.254 | 3.537 | 8.641 | 10.193 |

### 4.4.2 FORMULATION FOR RANKINE AND COULOMB WALL

- Active earth pressure, $\mathbf{P}_{\mathrm{a}}=1 / 2 \mathbf{K}_{\mathrm{a}} \gamma \mathbf{h}^{2}$
- Passive earth pressure, $\mathbf{P}_{\mathbf{p}}=1 / 2 \mathbf{K}_{\mathbf{p}} \gamma \mathbf{h}^{2}$
- Circumferential tensile stress (or hoop stress),

$$
\begin{aligned}
& f=p . d / 2 . \\
\text { So, } & p=f .2 . t / d
\end{aligned}
$$

Where, $p=$ internal pressure.
$D=$ diameter of shell.
$\mathrm{T}=$ thickness of shell.

### 4.4.2.1 RANKINE WALL $(\delta=0)$



Fig. 4.4
Taking moment of all the forces about the center line of the footing $\left.q_{u} \cdot K_{a} \cdot h \cdot h / 2+1 / 2 K_{a} \cdot \gamma \cdot h^{2} \cdot 2 / 3 \cdot h=(f .2 \cdot t \cdot h / d) \cdot(h / 2)+1 / 2 K_{p} \cdot \gamma h^{2}\right) \cdot(2 / 3 \cdot h)$

So from this we get

$$
q_{u}=2 . f . t / d . K_{a}+2 / 3 \gamma . h\left(K_{p}-K_{a}\right) / K_{a}
$$

### 4.4.2.2 COULOMB WALL $(\delta \neq 0)$



Fig. 4.5

Taking moment of all the forces about the center line of the footing
(f.2.t.h/d) (h/2)+1/2 $K_{p} \cdot \gamma \cdot h^{2} \cdot \cos \delta(2 / 3 . h)+1 / 2 K_{a} \cdot \gamma \cdot h^{2} \cdot \sin \delta . d / 2$

$$
=1 / 2 K_{a} \cdot \gamma \cdot h^{2} \cdot \cos \delta(2 / 3 \cdot h)+1 / 2 K_{p} \cdot \gamma \cdot h^{2} \cdot \sin \delta \cdot d / 2+q_{u} \cdot K_{a} \cdot h \cdot h / 2
$$

$$
\begin{aligned}
\mathrm{q}_{\mathrm{u}}=\left(2 . f . \mathrm{t} / \mathrm{d} . \mathrm{K}_{\mathrm{a}}\right) & +\left(2 . \gamma . \mathrm{K}_{\mathrm{p}} / \mathrm{K}_{\mathrm{a}}\right) \times(\cos \delta . \mathrm{h} / 3-\sin \delta . \mathrm{d} / 4) \\
& +2 . \gamma(\sin \delta . \mathrm{d} / 4-\cos \delta . \mathrm{h} / 3)
\end{aligned}
$$

### 4.4.2.3 BEARING CAPACITY OF THE FOOTING WITHOUT CONFINEMENT

The ultimate bearing capacity of the footing can be calculated by the formula:

$$
\mathbf{q}_{\mathbf{u}}=1 / 2 \gamma \mathbf{D} \zeta_{\gamma} \mathbf{N}_{\gamma}
$$

Where,
Unit weight of soil, $\gamma=18.00 \mathrm{KN} / \mathrm{m}^{3}$
Bearing capacity factor $\mathbf{N}_{\gamma}$ for different $\phi$ as given by Kumbhojkar are, $\mathrm{N}_{\gamma}=171.99$ for $\phi=42^{\circ}, \mathbf{N}_{\gamma}=$ 78.61 for $\phi=38^{\circ}$, and $\mathrm{N}_{\gamma}=38.04$ for $\phi=34^{\circ}$.

Diameter of the footing $=1 \mathrm{~m}$
Shape factor $\zeta_{\gamma}=0.6$ as given by Terzaghi for circular footing.

$$
\begin{aligned}
& \mathbf{q}_{\mathbf{u}}=928.75 \mathrm{KN} / \mathrm{m}^{3} \text { for } \mathrm{N}_{\gamma}=171.99 \& \phi=42^{\circ} \\
& \mathbf{q}_{\mathbf{u}}=424.50 \mathrm{KN} / \mathrm{m}^{3} \text { for } \mathrm{N}_{\gamma}=78.610 \& \phi=38^{\circ} \\
& \mathbf{q}_{\mathbf{u}}=205.42 \mathrm{KN} / \mathrm{m}^{3} \text { for } \mathrm{N}_{\gamma}=38.040 \& \phi=34^{\circ}
\end{aligned}
$$

### 4.4.2.4 BEARING CAPACITY RATIO

The bearing capacity improvement due to the soil confinement is represented using a nondimensional factor, called the bearing capacity ratio (BCR). This is defined as the ratio of the footing ultimate load with soil confinement $\left(\mathrm{Q}_{\mathrm{u}}\right)$ to the footing ultimate load without confinement $\left(\mathrm{q}_{\mathrm{u}}\right)$.

$$
\mathbf{B C R}=\mathbf{Q}_{\mathrm{u}} / \mathbf{q}_{\mathrm{u}}
$$

### 4.5 EFFECT OF THE SOIL PRESSURE ON THE CYLLINDRICAL SHELL

One of the proposed parameter to be investigated was the thickness of the shell wall to study the effect of the shell rigidity on the footing shell system behavior and also to study the hoop tension in the shell sue to the pressure under the footing. The horizontal pressure acting on the side wall of the shell is equal to the vertical pressure multiplied by the coefficient of lateral earth pressure. It can be seen that the maximum estimated horizontal earth pressure on the side walls of the shell are very small in comparison to the allowable hydraulic pressure, another point is that the given allowable value is the net inside pressure while the shell in the model is subjected to both internal and external pressure.

### 4.6 RESULTS AND DISCUSSION

### 4.6.1 EFFECT OF CYLINDRICAL SHELL DIAMETER

In order to investigate the effect of shell diameter on the footing behavior, different diameter of cylindrical shell $100,107,133,160$, and 200 cm were used.

Tables and Figures show the variation of BCR with normalized cylindrical shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights ( $\mathrm{h} / \mathrm{D}$ ) with a constant footing diameter ( D ) of 1 meter. A significant increase of bearing capacity of model footing supported on confined sand has observed, but also found that the BCR decreases with an increase in the $\mathrm{d} / \mathrm{D}$ ratio.

The coming figures show (for tables see APPINDEX) for different Proposed Approaches as explained in above articles 4.4. E.g. Normal approach and Terzaghi's approach, also for the Rankine wall and

Coulomb wall for different value of $\phi\left(34 \mathrm{o}, 38^{\circ}\right.$, and $\left.42^{\circ}\right)$ and angle of wall friction $\left(\delta=0^{\circ}, 22^{\circ}\right.$, and $25^{\circ}$ ).

The significant increase in the bearing capacity of the footing can be explained as follows, when the footing is loaded, such confinement resist the lateral displacement of soil piratical underneath the footing and confines the soil leading to a significant decrease in the vertical settlement and hence improving the bearing capacity. For small cylindrical shell diameters, as the pressure is increased, the plastic state is developed initially around the edges of the footing and then spreads initially around the edges of the footing and then spread downward and outward. The mobilized vertical friction between the sand and the inside wall of the cylinder increases with the increase of the active earth pressure until the point when the system (the cylinder, sand and footing) starts behave as one unit. The behavior is similar to that observed in deep foundations (piles and caissons) in which the bearing load increases due to the shear resistance of shell surface. This illustrate the increase of the bearing load with the increase of the shell diameter and shell height, based on tests performed with shell made with very smooth surfaces, it can be concluded that increased surface roughness results in greater bearing load improvement.

## Normal approach for Rankine wall



Normal approach for Coulomb wall


FIG. 4.9, VARIATION OF BCR WITH NORMALIZED SHELL DIAMETER (d/D) FOR DIFFRENT SHELL HEIGHT \& FOR COULOMB WALL $\left(\phi=34^{\circ} \& \delta=22^{\circ}\right)$


FIG.4.10, The variation of BCR with normalized shell diameter (d/D) for different shell heights, for Coulomb wall $\left(\phi=38^{\circ}\right.$ and $\left.\delta=22^{\circ}\right)$


FIG.4.11, The variation of BCR with normalized shell diameter (d/D) for different shell heights \& for Coulomb wall $\left(\phi=42^{\circ} \& \delta=22^{\circ}\right)$


Fig.4.12, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights \& for Coulomb wall ( $\phi=34^{\circ} \& \delta=25^{\circ}$ )


Fig.4.13, The variation of BCR with normalized shell diameter ( $d / D$ ) for different shell heights $\&$ for Coulomb wall $\left(\phi=38^{\circ} \& \delta=25^{\circ}\right)$


## Terzaghi's approach for Rankine wall

Fig. 4.15, Variation of BCR with normalized shell diameter (d/D) for different shell heights (h/D)(For $\phi=42^{\circ}, 38^{\circ}$, and $34^{\circ}$ )


## Terzaghi's approach for Coulomb's wall

Fig. 4.16, Variation of BCR with normalized shell diameter (d/D) for different height (h/D). $\left(\phi=42^{\circ}, 38^{\circ}, 34^{\circ}\right.$ and $\left.\delta=22^{\circ}\right)$


Fig. 4.17, Variation of BCR with normalized shell diameter (d/D) for different height (h/D), $\left(\phi=42^{\circ}, 38^{\circ}, 34^{\circ}\right.$, and $\left.\delta=25^{\circ}\right)$


### 4.6.2 EFFECT OF CYLINDRICAL SHELL HEIGHT

In order to investigate the effect of shell height on the footing response, analysis has been done by using six different heights for each shell diameters. The variation of BCR with normalized shell height (h/D) is shown in the Table (see appendix) and figures for Normal approach and Terzaghi approach for different angles of internal friction of soil $(\phi=42,38$, and 34 degree) and angle d wall friction ( $\delta=0$, 22 , and 25 degree).

For different normalized shell diameter) $d / D$ ). The figures show the same pattern of behavior for the different shell diameter, increase in shell height result in an improvement in BCR. This increase in shell height results in the enlargement in the surface area of the cylindrical shellmodel footing leading to the higher bearing capacity load.

## Normal approach for Rankine wall




Fig.4.19, The variation of BCR with normalized shell height (h/D) for different shell diameter,(FOR $\phi=38^{\circ}$ and $\delta=0^{\circ}$ )


Fig. 4.20, The variation of BCR with normalized shell height (h/D) for different shell diameterl ( $\operatorname{FOR} \phi=42^{\circ}$ and $\delta=0^{\circ}$ )

## Normal approach for Coulomb wall



Fig. 4.21, The variation of BCR with normalized shell height (h/D) for different shell diameter (FOR $\phi=34^{\circ}$ and $\delta=22^{\circ}$ )


Fig. 4.22, The variation of BCR with normalized shell height (h/D) for different shell diameter (FOR $\phi=38^{\circ}$ and $\delta=22^{\circ}$ )


Fig. 4.23, The variation of BCR with normalized shell height (h/D) for different shell diameter $\left(\phi=42^{\circ}\right.$ and $\left.\delta=22^{\circ}\right)$


Fig. 4.24, The variation of BCR with normalized shell height (h/D) for different shell diameter(FOR $\phi=38^{\circ}$ and $\delta=25^{\circ}$ )


Fig. 4.25, The variation of BCR with normalized shell height (h/D) for different shell diameter ( $\mathrm{FOR} \phi=34^{\circ}$ and $\delta=25^{\circ}$ )


Fig. 4.26, The variation of BCR with normalized shell height (h/D) for different shell diameter (FOR $\phi=42^{\circ}$ and $\delta=25^{\circ}$ )

## Terzaghi approach for Rankine wall

Fig. 4.27, Variation of BCR with normalized shell height (h/D) for different shell diameters (d/D),( $\phi=42^{\circ}, 38^{\circ}$, and $34^{\circ}$ )


## Terzaghi approach for Coulomb wall

Fig. 4.28, Variation of BCR with Normalized shell height (h/D) for different shell diameter ( $\mathrm{d} / \mathrm{D}$ ), ( $\phi=42^{\circ}, \mathbf{3 8}^{\circ}, \mathbf{3 4}^{\circ}$ and $\delta=22^{\circ}$ )


## CHAPTER-5

## CONCLUSIONS AND SCOPE OF FURTHER STUDY

### 5.1 CONCLUSIONS

The objective of this thesis is to study the effect of special type of footings on improvement of bearing capacity of shallow foundations using the extension of Terzaghi (1943) and Kumbhojkar (1993) approach.

The first case was considered for a spudcan footing supported on soil.
Since the spudcan footing uses a cone at the base, the log-spiral tends to grow downward and outward to increase the bearing capacity factor $\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$, and $\mathrm{N}_{\gamma}$, this represents the angle of a spudcan ( $\Omega$ ) has got a significant effect that can be observed for the values of bearing capacity at $\phi=30^{0 .}$ The $\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$, and $\mathrm{N}_{\gamma}$ for spudcan footing with cone angle $(\Omega)$ of $15^{\circ}$ are $40.6,24.9$, and 244.4 , compared to Terzaghi's values of 37.1, 22.4, and 19.1 respectively.

In second case the circular footing supported on dry sand and surrounded from all sides by skirts (cylindrical steel shell) with very smooth and rough surfaces ( $\delta \geq 0$ ). Based on the analytical solutions the following conclusions can be drawn for skirted footing.

1. Soil confinement has a significant effect on improving the behavior of circular footing supported on granular soil. The ultimate bearing capacity was 43 times as compared to unconfined case. Therefore it can be concluded that the sheet piles used to brace cuts have a significant effect on improving the bearing capacity of soils under raft foundation.
2. Based on the analytical results, soil confinement could be considered as a method to improve the bearing capacity of isolated footing bearing on sand. Steel shells of different heights,
diameters, and thickness could be easily manufactured and placed and the individual footing leading to a significant improvement in there response.
3. In case where structures are very sensitive to settlement, soil confinement can used to obtain the same allowable bearing capacity at much lower settlement.
4. The $B C R$ is highly depends on the $d / D$ ratio (shell diameter/footing diameter ratio) the optimum ratio is about one, beyond which the improvement decreases as the ratio increases.
5. The two following approaches are used Normal approach and Terzaghi approach with Rankine's wall and Coulomb's wall $\left(\delta=22^{\circ} \& 25^{\circ}\right)$. It was observed that maximum BCR in all cases are decreases as the angle of internal friction increases from $34^{\circ}$ to $42^{\circ}$. Also the BCR is decreases as the $\mathrm{d} / \mathrm{D}$ (shell diameter/ footing diameter ratio) increases and BCR increases as the $\mathrm{h} / \mathrm{D}$ (shell height/footing diameter) increases.
6. Increase the height of the cylindrical shell results in increasing the surface area of the shellmodel footing, which transfers loads to deep depths and lead to improving the BCR.
7. At the time of design of the foundation by calculating the upcoming loads the thickness and height of the cylindrical shell can be economically decided.
8. The permissible value of circumferential tensile stress (or hoop stress) has considered $100 \mathrm{~N} / \mathrm{mm}^{2}$. If it is increases the thickness of the shell is reduces.
9. In Terzaghi's approach the height of the shell varies as the path of the log-spiral varies if the height provides beyond the path of log-spiral it will acts as the factor of safety for unpredictable settlement and loading.
10. As I considered steel as the material of cylindrical shell, it can be replaced by some other material, which is corrosion resistance and economical, compared to steel, like polyvinyl chloride cylinders.

### 5.2 SCOPE OF FURTHER STUDY

Behavior of other footings, square and rectangular along with the influence of the roughness and the stiffness of shell material were not studied. Therefore it is recommended that further work is investigates the effect of these parameter for both dry and wet sand conditions.

Studying the effect of soil confinement on the behavior of footing bearing on weak type of soil such as loose sand and soft clay is also the scope of further study.

This analytical solution can be further extent for studying it by using Finite Element Modeling so that it can incorporates all the variability of the soil, shell and footing material properties which otherwise becomes difficult.

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## APPENDIX

Table-1, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Rankine wall and $\phi=34^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 0.5 | 3.537 | 0.283 | 3.254 | 1 | 7143.33 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.07 | 6680.53 | 32.5 |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.33 | 5388.06 | 26.2 |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.6 | 4490.48 | 21.9 |
| 0.5 | 3.537 | 0.283 | 3.254 | 2 | 3606.20 | 17.6 |
| 1 | 3.537 | 0.283 | 3.254 | 1 | 7212.40 | 35.1 |
| 1 | 3.537 | 0.283 | 3.254 | 1.07 | 6749.60 | 32.9 |
| 1 | 3.537 | 0.283 | 3.254 | 1.33 | 5457.13 | 26.6 |
| 1 | 3.537 | 0.283 | 3.254 | 1.6 | 4559.55 | 22.2 |
| 1 | 3.537 | 0.283 | 3.254 | 2 | 3675.27 | 17.9 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1 | 7281.47 | 35.4 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.07 | 6818.67 | 33.2 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.33 | 5526.20 | 26.9 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.6 | 4628.62 | 22.5 |
| 1.5 | 3.537 | 0.283 | 3.254 | 2 | 3744.34 | 18.2 |
| 2 | 3.537 | 0.283 | 3.254 | 1 | 7350.54 | 35.8 |
| 2 | 3.537 | 0.283 | 3.254 | 1.07 | 6887.73 | 33.5 |
| 2 | 3.537 | 0.283 | 3.254 | 1.33 | 5595.27 | 27.2 |
| 2 | 3.537 | 0.283 | 3.254 | 1.6 | 4697.69 | 22.9 |
| 2 | 3.537 | 0.283 | 3.254 | 2 | 3813.40 | 18.6 |

Table-2, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Rankine wall and $\phi=\mathbf{3 8}^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 4.204 | 0.238 | 3.966 | 1 | 8507.52 | 20.0 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.07 | 7957.50 | 18.7 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.33 | 6421.45 | 15.1 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.6 | 5354.71 | 12.6 |
| 0.5 | 4.204 | 0.238 | 3.966 | 2 | 4303.77 | 10.1 |
| 1 | 4.204 | 0.238 | 3.966 | 1 | 8607.55 | 20.3 |
| 1 | 4.204 | 0.238 | 3.966 | 1.07 | 8057.53 | 19.0 |
| 1 | 4.204 | 0.238 | 3.966 | 1.33 | 6521.48 | 15.4 |
| 1 | 4.204 | 0.238 | 3.966 | 1.6 | 5454.74 | 12.8 |
| 1 | 4.204 | 0.238 | 3.966 | 2 | 4403.80 | 10.4 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1 | 8707.58 | 20.5 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.07 | 8157.56 | 19.2 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.33 | 6621.51 | 15.6 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.6 | 5554.77 | 13.1 |
| 1.5 | 4.204 | 0.238 | 3.966 | 2 | 4503.83 | 10.6 |
| 2 | 4.204 | 0.238 | 3.966 | 1 | 8807.61 | 20.7 |
| 2 | 4.204 | 0.238 | 3.966 | 1.07 | 8257.58 | 19.5 |
| 2 | 4.204 | 0.238 | 3.966 | 1.33 | 6721.54 | 15.8 |
| 2 | 4.204 | 0.238 | 3.966 | 1.6 | 5654.80 | 13.3 |

Table-3, The variation of BCR with normalized shell diameter (d/D) for different shell heights, for Rankine wall and $\phi=42^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}} \mathbf{-} \mathbf{K}_{\mathbf{a}}$ | $\mathbf{D} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 5.045 | 0.198 | 4.846 | 1 | 10236.06 | 11.0 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.07 | 9576.00 | 10.3 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.33 | 7732.68 | 8.3 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.6 | 6452.54 | 6.9 |
| 0.5 | 5.045 | 0.198 | 4.846 | 2 | 5191.37 | 5.6 |
| 1 | 5.045 | 0.198 | 4.846 | 1 | 10382.75 | 11.2 |
| 1 | 5.045 | 0.198 | 4.846 | 1.07 | 9722.70 | 10.5 |
| 1 | 5.045 | 0.198 | 4.846 | 1.33 | 7879.37 | 8.5 |
| 1 | 5.045 | 0.198 | 4.846 | 1.6 | 6599.24 | 7.1 |
| 1 | 5.045 | 0.198 | 4.846 | 2 | 5338.07 | 5.7 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1 | 10529.44 | 11.3 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.07 | 9869.39 | 10.6 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.33 | 8026.07 | 8.6 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.6 | 6745.93 | 7.3 |
| 1.5 | 5.045 | 0.198 | 4.846 | 2 | 5484.76 | 5.9 |
| 2 | 5.045 | 0.198 | 4.846 | 1 | 10676.13 | 11.5 |
| 2 | 5.045 | 0.198 | 4.846 | 1.07 | 10016.08 | 10.8 |
| 2 | 5.045 | 0.198 | 4.846 | 1.33 | 8172.76 | 8.8 |
| 2 | 5.045 | 0.198 | 4.846 | 1.6 | 6892.62 | 7.4 |
| 2 | 5.045 | 0.198 | 4.846 | 2 | 5631.45 | 6.1 |

Table-4, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Coulomb wall and $\phi=34^{\circ}$ and $\delta=22^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\boldsymbol{\delta}$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 7946.38 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7423.47 | 36.9 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 5955.94 | 29.1 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 4926.83 | 24.1 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 3898.05 | 19.1 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8130.08 | 39.8 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7607.16 | 37.2 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6139.63 | 30.0 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5110.52 | 25.0 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4081.74 | 20.0 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8313.77 | 40.7 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7790.85 | 38.1 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6323.33 | 30.9 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5294.22 | 25.9 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4265.43 | 20.9 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8497.46 | 41.6 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7974.54 | 39.0 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6507.02 | 31.8 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5477.91 | 26.8 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4449.13 | 21.8 |

Table-5, The variation of BCR with normalized shell diameter (d/D) for different shell heights, for Coulomb wall and $\phi=\mathbf{3 8}^{\mathbf{0}}$ and $\delta=\mathbf{2 2}^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\boldsymbol{\delta}$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9330.20 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 8715.01 | 20.5 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 6985.70 | 16.5 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 5769.12 | 13.6 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 4547.13 | 10.7 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9618.59 | 22.7 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9003.40 | 21.2 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7274.09 | 17.1 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6057.50 | 14.3 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 4835.52 | 11.4 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9906.97 | 23.3 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9291.78 | 21.9 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7562.47 | 17.8 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6345.89 | 14.9 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5123.90 | 12.1 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 10195.35 | 24.0 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9580.17 | 22.6 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7850.86 | 18.5 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6634.27 | 15.6 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5412.29 | 12.7 |

Table-6, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Coulomb wall and $\phi=42^{\circ}$ and $\delta=22^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 11115.11 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 10380.08 | 11.2 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 8308.91 | 8.9 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 6844.94 | 7.4 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 5364.27 | 5.8 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 11587.61 | 12.5 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 10852.58 | 11.7 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 8781.41 | 9.5 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 7317.43 | 7.9 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 5836.77 | 6.3 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 12060.11 | 13.0 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 11325.08 | 12.2 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 9253.91 | 10.0 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 7789.93 | 8.4 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 6309.27 | 6.8 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 12532.60 | 13.5 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 11797.58 | 12.7 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 9726.41 | 10.5 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 8262.43 | 8.9 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 6781.77 | 7.3 |

Table-7, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Coulomb wall and $\phi=34^{\circ}$ and $\delta=25^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1 | 7937.97 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.07 | 7412.42 | 36.1 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.33 | 5935.15 | 28.9 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.6 | 4895.91 | 23.8 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.436 | 2 | 3852.12 | 18.8 |
| 1 | 0.254 | 10.193 | 9.939 | 0.436 | 1 | 8150.75 | 39.7 |
| 1 | 0.254 | 10.193 | 9.939 | 0.436 | 1.07 | 7625.21 | 37.1 |
| 1 | 0.254 | 10.193 | 9.939 | 0.436 | 1.33 | 6147.93 | 29.9 |
| 1 | 0.254 | 10.193 | 9.939 | 0.436 | 1.6 | 5108.69 | 24.9 |
| 1 | 0.254 | 10.193 | 9.939 | 0.436 | 2 | 4064.91 | 19.8 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1 | 8363.53 | 40.7 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.07 | 7837.99 | 38.2 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.33 | 6360.71 | 31.0 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.436 | 1.6 | 5321.47 | 25.9 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.436 | 2 | 4277.69 | 20.8 |
| 2 | 0.254 | 10.193 | 9.939 | 0.436 | 1 | 8576.31 | 41.8 |
| 2 | 0.254 | 10.193 | 9.939 | 0.436 | 1.07 | 8050.77 | 39.2 |
| 2 | 0.254 | 10.193 | 9.939 | 0.436 | 1.33 | 6573.49 | 32.0 |
| 2 | 0.254 | 10.193 | 9.939 | 0.436 | 1.6 | 5534.26 | 26.9 |
| 2 | 0.254 | 10.193 | 9.939 | 0.436 | 2 | 4490.47 | 21.9 |

Table-8, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Coulomb wall and $\phi=\mathbf{3 8}^{\circ}$ and $\delta=\mathbf{2 5}^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\boldsymbol{\delta}$ | $\mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1 | 9319.65 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.07 | 8699.90 | 20.5 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.33 | 6953.67 | 16.4 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.6 | 5719.51 | 13.5 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.436 | 2 | 4471.50 | 10.5 |
| 1 | 0.217 | 13.901 | 13.684 | 0.436 | 1 | 9662.56 | 22.8 |
| 1 | 0.217 | 13.901 | 13.684 | 0.436 | 1.07 | 9042.81 | 21.3 |
| 1 | 0.217 | 13.901 | 13.684 | 0.436 | 1.33 | 7296.58 | 17.2 |
| 1 | 0.217 | 13.901 | 13.684 | 0.436 | 1.6 | 6062.43 | 14.3 |
| 1 | 0.217 | 13.901 | 13.684 | 0.436 | 2 | 4814.41 | 11.3 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1 | 10005.47 | 23.6 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.07 | 9385.72 | 22.1 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.33 | 7639.49 | 18.0 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.436 | 1.6 | 6405.34 | 15.1 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.436 | 2 | 5157.32 | 12.1 |
| 2 | 0.217 | 13.901 | 13.684 | 0.436 | 1 | 10348.38 | 24.4 |
| 2 | 0.217 | 13.901 | 13.684 | 0.436 | 1.07 | 9728.63 | 22.9 |
| 2 | 0.217 | 13.901 | 13.684 | 0.436 | 1.33 | 7982.40 | 18.8 |
| 2 | 0.217 | 13.901 | 13.684 | 0.436 | 1.6 | 6748.25 | 15.9 |
| 2 | 0.217 | 13.901 | 13.684 | 0.436 | 2 | 5500.23 | 13.0 |

Table-9, The variation of BCR with normalized shell diameter ( $\mathrm{d} / \mathrm{D}$ ) for different shell heights, for Coulomb wall and $\phi=42^{\circ}$ and $\delta=25^{\circ}$.

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1 | 11103.78 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.07 | 10360.32 | 11.2 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.33 | 8257.82 | 8.9 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.6 | 6761.30 | 7.3 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.436 | 2 | 5232.44 | 5.6 |
| 1 | 0.183 | 19.758 | 19.575 | 0.436 | 1 | 11685.45 | 12.6 |
| 1 | 0.183 | 19.758 | 19.575 | 0.436 | 1.07 | 10941.99 | 11.8 |
| 1 | 0.183 | 19.758 | 19.575 | 0.436 | 1.33 | 8839.49 | 9.5 |
| 1 | 0.183 | 19.758 | 19.575 | 0.436 | 1.6 | 7342.97 | 7.9 |
| 1 | 0.183 | 19.758 | 19.575 | 0.436 | 2 | 5814.11 | 6.3 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1 | 12267.12 | 13.2 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.07 | 11523.66 | 12.4 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.33 | 9421.16 | 10.1 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.436 | 1.6 | 7924.64 | 8.5 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.436 | 2 | 6395.78 | 6.9 |
| 2 | 0.183 | 19.758 | 19.575 | 0.436 | 1 | 12848.79 | 13.8 |
| 2 | 0.183 | 19.758 | 19.575 | 0.436 | 1.07 | 12105.33 | 13.0 |
| 2 | 0.183 | 19.758 | 19.575 | 0.436 | 1.33 | 10002.83 | 10.8 |
| 2 | 0.183 | 19.758 | 19.575 | 0.436 | 1.6 | 8506.32 | 9.2 |
| 2 | 0.183 | 19.758 | 19.575 | 0.436 | 2 | 6977.45 | 7.5 |

Table-10, The variation of BCR with normalized shell height (h/D) for different shell diameter, for Rankine wall (FOR $\phi=34^{\circ}$ and $\delta=0^{\circ}$ )

| h/D | Kp | Ka | Kp-Ka | d/D | Qu | BCR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathbf{Q}_{u} / \mathbf{q}_{u}$ |
| 0.5 | 3.537 | 0.283 | 3.254 | 1 | 7143.33 | 34.8 |
| 1 | 3.537 | 0.283 | 3.254 | 1 | 7212.40 | 35.1 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1 | 7281.47 | 35.4 |
| 2 | 3.537 | 0.283 | 3.254 | 1 | 7350.54 | 35.8 |
| 2.5 | 3.537 | 0.283 | 3.254 | 1 | 7419.60 | 36.1 |
| 3 | 3.537 | 0.283 | 3.254 | 1 | 7488.67 | 36.5 |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.07 | 6680.53 | 32.5 |
| 1 | 3.537 | 0.283 | 3.254 | 1.07 | 6749.60 | 32.9 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.07 | 6818.67 | 33.2 |
| 2 | 3.537 | 0.283 | 3.254 | 1.07 | 6887.73 | 33.5 |
| 2.5 | 3.537 | 0.283 | 3.254 | 1.07 | 6956.80 | 33.9 |
| 3 | 3.537 | 0.283 | 3.254 | 1.07 | 7025.87 | 34.2 |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.33 | 5388.06 | 26.2 |
| 1 | 3.537 | 0.283 | 3.254 | 1.33 | 5457.13 | 26.6 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.33 | 5526.20 | 26.9 |
| 2 | 3.537 | 0.283 | 3.254 | 1.33 | 5595.27 | 27.2 |
| 2.5 | 3.537 | 0.283 | 3.254 | 1.33 | 5664.33 | 27.6 |
| 3 | 3.537 | 0.283 | 3.254 | 1.33 | 5733.40 | 27.9 |
| 0.5 | 3.537 | 0.283 | 3.254 | 1.6 | 4490.48 | 21.9 |
| 1 | 3.537 | 0.283 | 3.254 | 1.6 | 4559.55 | 22.2 |
| 1.5 | 3.537 | 0.283 | 3.254 | 1.6 | 4628.62 | 22.5 |
| 2 | 3.537 | 0.283 | 3.254 | 1.6 | 4697.69 | 22.9 |
| 2.5 | 3.537 | 0.283 | 3.254 | 1.6 | 4766.75 | 23.2 |
| 3 | 3.537 | 0.283 | 3.254 | 1.6 | 4835.82 | 23.5 |
| 0.5 | 3.537 | 0.283 | 3.254 | 2 | 3606.20 | 17.6 |
| 1 | 3.537 | 0.283 | 3.254 | 2 | 3675.27 | 17.9 |
| 1.5 | 3.537 | 0.283 | 3.254 | 2 | 3744.34 | 18.2 |
| 2 | 3.537 | 0.283 | 3.254 | 2 | 3813.40 | 18.6 |
| 2.5 | 3.537 | 0.283 | 3.254 | 2 | 3882.47 | 18.9 |
| 3 | 3.537 | 0.283 | 3.254 | 2 | 3951.54 | 19.2 |

Table-4.11, The variation of BCR with normalized shell height ( $h / D$ ) for different shell diameter, for Rankine wall (FOR $\phi=38^{\circ}$ and $\delta=0^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 4.204 | 0.238 | 3.966 | 1 | 8507.52 | 20.0 |
| 1 | 4.204 | 0.238 | 3.966 | 1 | 8607.55 | 20.3 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1 | 8707.58 | 20.5 |
| 2 | 4.204 | 0.238 | 3.966 | 1 | 8807.61 | 20.7 |
| 2.5 | 4.204 | 0.238 | 3.966 | 1 | 8907.64 | 21.0 |
| 3 | 4.204 | 0.238 | 3.966 | 1 | 9007.66 | 21.2 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.07 | 7957.50 | 18.7 |
| 1 | 4.204 | 0.238 | 3.966 | 1.07 | 8057.53 | 19.0 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.07 | 8157.56 | 19.2 |
| 2 | 4.204 | 0.238 | 3.966 | 1.07 | 8257.58 | 19.5 |
| 2.5 | 4.204 | 0.238 | 3.966 | 1.07 | 8357.61 | 19.7 |
| 3 | 4.204 | 0.238 | 3.966 | 1.07 | 8457.64 | 19.9 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.33 | 6421.45 | 15.1 |
| 1 | 4.204 | 0.238 | 3.966 | 1.33 | 6521.48 | 15.4 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.33 | 6621.51 | 15.6 |
| 2 | 4.204 | 0.238 | 3.966 | 1.33 | 6721.54 | 15.8 |
| 2.5 | 4.204 | 0.238 | 3.966 | 1.33 | 6821.57 | 16.1 |
| 3 | 4.204 | 0.238 | 3.966 | 1.33 | 6921.60 | 16.3 |
| 0.5 | 4.204 | 0.238 | 3.966 | 1.6 | 5354.71 | 12.6 |
| 1 | 4.204 | 0.238 | 3.966 | 1.6 | 5454.74 | 12.8 |
| 1.5 | 4.204 | 0.238 | 3.966 | 1.6 | 5554.77 | 13.1 |
| 2 | 4.204 | 0.238 | 3.966 | 1.6 | 5654.80 | 13.3 |
| 2.5 | 4.204 | 0.238 | 3.966 | 1.6 | 5754.83 | 13.6 |
| 3 | 4.204 | 0.238 | 3.966 | 1.6 | 5854.86 | 13.8 |
| 0.5 | 4.204 | 0.238 | 3.966 | 2 | 4303.77 | 10.1 |
| 1 | 4.204 | 0.238 | 3.966 | 2 | 4403.80 | 10.4 |
| 1.5 | 4.204 | 0.238 | 3.966 | 2 | 4503.83 | 10.6 |
| 2 | 4.204 | 0.238 | 3.966 | 2 | 4603.86 | 10.8 |
| 2.5 | 4.204 | 0.238 | 3.966 | 2 | 4703.89 | 11.1 |
| 3 | 4.204 | 0.238 | 3.966 | 2 | 4803.92 | 11.3 |

Table-12, The variation of BCR with normalized shell height (h/D) for different shell diameter, for Rankine wall (FOR $\phi=42^{\circ}$ and $\delta=0^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 5.045 | 0.198 | 4.846 | 1 | 10236.06 | 11.0 |
| 1 | 5.045 | 0.198 | 4.846 | 1 | 10382.75 | 11.2 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1 | 10529.44 | 11.3 |
| 2 | 5.045 | 0.198 | 4.846 | 1 | 10676.13 | 11.5 |
| 2.5 | 5.045 | 0.198 | 4.846 | 1 | 10822.83 | 11.7 |
| 3 | 5.045 | 0.198 | 4.846 | 1 | 10969.52 | 11.8 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.07 | 9576.00 | 10.3 |
| 1 | 5.045 | 0.198 | 4.846 | 1.07 | 9722.70 | 10.5 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.07 | 9869.39 | 10.6 |
| 2 | 5.045 | 0.198 | 4.846 | 1.07 | 10016.08 | 10.8 |
| 2.5 | 5.045 | 0.198 | 4.846 | 1.07 | 10162.77 | 10.9 |
| 3 | 5.045 | 0.198 | 4.846 | 1.07 | 10309.47 | 11.1 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.33 | 7732.68 | 8.3 |
| 1 | 5.045 | 0.198 | 4.846 | 1.33 | 7879.37 | 8.5 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.33 | 8026.07 | 8.6 |
| 2 | 5.045 | 0.198 | 4.846 | 1.33 | 8172.76 | 8.8 |
| 2.5 | 5.045 | 0.198 | 4.846 | 1.33 | 8319.45 | 9.0 |
| 3 | 5.045 | 0.198 | 4.846 | 1.33 | 8466.14 | 9.1 |
| 0.5 | 5.045 | 0.198 | 4.846 | 1.6 | 6452.54 | 6.9 |
| 1 | 5.045 | 0.198 | 4.846 | 1.6 | 6599.24 | 7.1 |
| 1.5 | 5.045 | 0.198 | 4.846 | 1.6 | 6745.93 | 7.3 |
| 2 | 5.045 | 0.198 | 4.846 | 1.6 | 6892.62 | 7.4 |
| 2.5 | 5.045 | 0.198 | 4.846 | 1.6 | 7039.32 | 7.6 |
| 3 | 5.045 | 0.198 | 4.846 | 1.6 | 7186.01 | 7.7 |
| 0.5 | 5.045 | 0.198 | 4.846 | 2 | 5191.37 | 5.6 |
| 1 | 5.045 | 0.198 | 4.846 | 2 | 5338.07 | 5.7 |
| 1.5 | 5.045 | 0.198 | 4.846 | 2 | 5484.76 | 5.9 |
| 2 | 5.045 | 0.198 | 4.846 | 2 | 5631.45 | 6.1 |
| 2.5 | 5.045 | 0.198 | 4.846 | 2 | 5778.15 | 6.2 |
| 3 | 5.045 | 0.198 | 4.846 | 2 | 5924.84 | 6.4 |

Table-13, The variation of BCR with normalized shell height ( $h / D$ ) for different shell diameter, for Coulomb wall (FOR $\phi=34^{\circ}$ and $\delta=22^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 7946.38 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8130.08 | 39.6 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8313.77 | 40.5 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8497.46 | 41.4 |
| 2.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8681.15 | 42.3 |
| 3 | 0.254 | 8.641 | 8.387 | 0.384 | 1 | 8864.84 | 43.2 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7423.47 | 36.1 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7607.16 | 37.0 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7790.85 | 37.9 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 7974.54 | 38.8 |
| 2.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 8158.24 | 39.7 |
| 3 | 0.254 | 8.641 | 8.387 | 0.384 | 1.07 | 8341.93 | 40.6 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 5955.94 | 29.0 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6139.63 | 29.9 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6323.33 | 30.8 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6507.02 | 31.7 |
| 2.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6690.71 | 32.6 |
| 3 | 0.254 | 8.641 | 8.387 | 0.384 | 1.33 | 6874.40 | 33.5 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 4926.83 | 24.0 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5110.52 | 24.9 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5294.22 | 25.8 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5477.91 | 26.7 |
| 2.5 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5661.60 | 27.6 |
| 3 | 0.254 | 8.641 | 8.387 | 0.384 | 1.6 | 5845.29 | 28.5 |
| 0.5 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 3898.05 | 19.0 |
| 1 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4081.74 | 19.9 |
| 1.5 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4265.43 | 20.8 |
| 2 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4449.13 | 21.7 |
| 2.5 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4632.82 | 22.6 |
| 3 | 0.254 | 8.641 | 8.387 | 0.384 | 2 | 4816.51 | 23.4 |

Table-14, The variation of BCR with normalized shell height ( $h / D$ ) for different shell diameter, for Coulomb wall (FOR $\phi=38^{\circ}$ and $\delta=22^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}} \mathbf{-} \mathbf{K}_{\mathbf{a}}$ | $\boldsymbol{\delta}$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9330.20 | 22.0 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9618.59 | 22.7 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 9906.97 | 23.3 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 10195.35 | 24.0 |
| 2.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 10483.74 | 24.7 |
| 3 | 0.217 | 11.466 | 11.249 | 0.384 | 1 | 10772.12 | 25.4 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 8715.01 | 20.5 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9003.40 | 21.2 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9291.78 | 21.9 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9580.17 | 22.6 |
| 2.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 9868.55 | 23.2 |
| 3 | 0.217 | 11.466 | 11.249 | 0.384 | 1.07 | 10156.93 | 23.9 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 6985.70 | 16.5 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7274.09 | 17.1 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7562.47 | 17.8 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 7850.86 | 18.5 |
| 2.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 8139.24 | 19.2 |
| 3 | 0.217 | 11.466 | 11.249 | 0.384 | 1.33 | 8427.62 | 19.9 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 5769.12 | 13.6 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6057.50 | 14.3 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6345.89 | 14.9 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6634.27 | 15.6 |
| 2.5 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 6922.65 | 16.3 |
| 3 | 0.217 | 11.466 | 11.249 | 0.384 | 1.6 | 7211.04 | 17.0 |
| 0.5 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 4547.13 | 10.7 |
| 1 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 4835.52 | 11.4 |
| 1.5 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5123.90 | 12.1 |
| 2 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5412.29 | 12.7 |
| 2.5 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5700.67 | 13.4 |
| 3 | 0.217 | 11.466 | 11.249 | 0.384 | 2 | 5989.06 | 14.1 |

Table-15, The variation of BCR with normalized shell height ( $\mathbf{h} / \mathrm{D}$ ) for different shell diameter, for Coulomb wall (FOR $\phi=42^{\circ}$ and $\delta=22^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 11115.11 | 12.0 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 11587.61 | 12.5 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 12060.11 | 13.0 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 12532.60 | 13.5 |
| 2.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 13005.10 | 14.0 |
| 3 | 0.183 | 15.726 | 15.543 | 0.384 | 1 | 13477.60 | 14.5 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 10380.08 | 11.2 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 10852.58 | 11.7 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 11325.08 | 12.2 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 11797.58 | 12.7 |
| 2.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 12270.08 | 13.2 |
| 3 | 0.183 | 15.726 | 15.543 | 0.384 | 1.07 | 12742.58 | 13.7 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 8308.91 | 8.9 |
| 1 | 0.183 | 15.726 | 15.53 | 0.384 | 1.33 | 8781.41 | 9.5 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 9253.91 | 10.0 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 9726.41 | 10.5 |
| 2.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 10198.91 | 11.0 |
| 3 | 0.183 | 15.726 | 15.543 | 0.384 | 1.33 | 10671.41 | 11.5 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 6844.94 | 7.4 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 7317.43 | 7.9 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 7789.93 | 8.4 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 8262.43 | 8.9 |
| 2.5 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 8734.93 | 9.4 |
| 3 | 0.183 | 15.726 | 15.543 | 0.384 | 1.6 | 9207.43 | 9.9 |
| 0.5 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 5364.27 | 5.8 |
| 1 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 5836.77 | 6.3 |
| 1.5 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 6309.27 | 6.8 |
| 2 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 6781.77 | 7.3 |
| 2.5 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 7254.27 | 7.8 |
| 3 | 0.183 | 15.726 | 15.543 | 0.384 | 2 | 7726.77 | 8.3 |

Table-16, The variation of BCR with normalized shell height ( $h / \mathrm{D}$ ) for different shell diameter, for Coulomb wall (FOR $\phi=34^{\circ}$ and $\delta=25^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 7937.97 | 38.6 |
| 1 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 8150.75 | 39.7 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 8363.53 | 40.7 |
| 2 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 8576.31 | 41.8 |
| 2.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 8789.10 | 42.8 |
| 3 | 0.254 | 10.193 | 9.939 | 0.4363 | 1 | 9001.88 | 43.8 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 7412.42 | 36.1 |
| 1 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 7625.21 | 37.1 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 7837.99 | 38.2 |
| 2 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 8050.77 | 39.2 |
| 2.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 8263.55 | 40.2 |
| 3 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.07 | 8476.34 | 41.3 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.33 | 5935.15 | 28.9 |
| 1 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.33 | 6147.93 | 29.9 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.33 | 6360.71 | 31.0 |
| 2 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.33 | 6573.49 | 32.0 |
| 2.5 | 0.254 | 10.193 | 9.939 | 0.4333 | 1.33 | 678.28 | 33.0 |
| 3 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.33 | 6999.06 | 34.1 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 4895.91 | 23.8 |
| 1 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 5108.69 | 24.9 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 5321.47 | 25.9 |
| 2 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 5534.26 | 26.9 |
| 2.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 5747.04 | 28.0 |
| 3 | 0.254 | 10.193 | 9.939 | 0.4363 | 1.6 | 5959.82 | 29.0 |
| 0.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 3852.12 | 18.8 |
| 1 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 4064.91 | 19.8 |
| 1.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 4277.69 | 20.8 |
| 2 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 4490.47 | 21.9 |
| 2.5 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 4703.25 | 22.9 |
| 3 | 0.254 | 10.193 | 9.939 | 0.4363 | 2 | 4916.04 | 23.9 |

Table-17, The variation of BCR with normalized shell height ( $h / D$ ) for different shell diameter, for Coulomb wall (FOR $\phi=38^{\circ}$ and $\delta=25^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 9319.65 | 22.0 |
| 1 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 9662.56 | 22.8 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 10005.47 | 23.6 |
| 2 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 10348.38 | 24.4 |
| 2.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 10691.29 | 25.2 |
| 3 | 0.217 | 13.901 | 13.684 | 0.4363 | 1 | 11034.20 | 26.0 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 8699.90 | 20.5 |
| 1 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 9042.81 | 21.3 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 9385.72 | 22.1 |
| 2 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 9728.63 | 22.9 |
| 2.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 10071.54 | 23.7 |
| 3 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.07 | 10414.45 | 24.5 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 6953.67 | 16.4 |
| 1 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 7296.58 | 17.2 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 7639.49 | 18.0 |
| 2 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 7982.40 | 18.8 |
| 2.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 8325.31 | 19.6 |
| 3 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.33 | 8668.22 | 20.4 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 5719.51 | 13.5 |
| 1 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 6062.43 | 14.3 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 6405.34 | 15.1 |
| 2 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 6748.25 | 15.9 |
| 2.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 7091.16 | 16.7 |
| 3 | 0.217 | 13.901 | 13.684 | 0.4363 | 1.6 | 7434.07 | 17.5 |
| 0.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 4471.50 | 10.5 |
| 1 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 4814.41 | 11.3 |
| 1.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 5157.32 | 12.1 |
| 2 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 5500.23 | 13.0 |
| 2.5 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 5843.14 | 13.8 |
| 3 | 0.217 | 13.901 | 13.684 | 0.4363 | 2 | 6186.05 | 14.6 |

Table-18, The variation of BCR with normalized shell height (h/D) for different shell diameter, for Coulomb wall (FOR $\phi=42^{\circ}$ and $\delta=25^{\circ}$ )

| $\mathbf{h} / \mathbf{D}$ | $\mathbf{K}_{\mathbf{a}}$ | $\mathbf{K}_{\mathbf{p}}$ | $\mathbf{K}_{\mathbf{p}}-\mathbf{K}_{\mathbf{a}}$ | $\delta$ | $\mathbf{d} / \mathbf{D}$ | $\mathbf{Q}_{\mathbf{u}}$ | $\mathbf{B C R}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 11103.78 | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 1 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 11685.45 | 12.6 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 12267.12 | 13.2 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 12848.79 | 13.8 |
| 2.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 13430.46 | 14.5 |
| 3 | 0.183 | 19.758 | 19.575 | 0.4363 | 1 | 14012.13 | 15.1 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 10360.32 | 11.2 |
| 1 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 10941.99 | 11.8 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 11523.66 | 12.4 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 12105.33 | 13.0 |
| 2.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 12687.00 | 13.7 |
| 3 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.07 | 13268.67 | 14.3 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 8257.82 | 8.9 |
| 1 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 8839.49 | 9.5 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 9421.16 | 10.1 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 10002.83 | 10.8 |
| 2.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 10584.50 | 11.4 |
| 3 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.33 | 11166.17 | 12.0 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 6761.30 | 7.3 |
| 1 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 7342.97 | 7.9 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 7924.64 | 8.5 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 8506.32 | 9.2 |
| 2.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 9087.99 | 9.8 |
| 3 | 0.183 | 19.758 | 19.575 | 0.4363 | 1.6 | 9669.66 | 10.4 |
| 0.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 5232.44 | 5.6 |
| 1 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 5814.11 | 6.3 |
| 1.5 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 6395.78 | 6.9 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 6977.45 | 7.5 |
| 2 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 7559.12 | 8.1 |
| 3 | 0.183 | 19.758 | 19.575 | 0.4363 | 2 | 8140.80 | 8.8 |

Table-19, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Rankine wall ( $\phi=34^{\circ}$ and $\delta=0^{\circ}$ )

| ANGLE OF | RADIUS OF LOG-SPIRAL |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOG-SPIRAL | $\mathrm{r}_{0}$ | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | Qu | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 56 | 0.603 | 1.166 | 90 | 1.000 | 1.166 | 3.537 | 0.283 | 7228.02 | 35.19 |
| 57 | 0.603 | 1.180 | 89 | 1.041 | 1.180 | 3.537 | 0.283 | 6950.38 | 33.83 |
| 58 | 0.603 | 1.194 | 88 | 1.083 | 1.193 | 3.537 | 0.283 | 6688.17 | 32.56 |
| 59 | 0.603 | 1.208 | 87 | 1.126 | 1.206 | 3.537 | 0.283 | 6440.33 | 31.35 |
| 60 | 0.603 | 1.222 | 86 | 1.171 | 1.219 | 3.537 | 0.283 | 6205.85 | 30.21 |
| 61 | 0.603 | 1.237 | 85 | 1.216 | 1.232 | 3.537 | 0.283 | 5983.82 | 29.13 |
| 62 | 0.603 | 1.251 | 84 | 1.262 | 1.245 | 3.537 | 0.283 | 5773.42 | 28.11 |
| 63 | 0.603 | 1.266 | 83 | 1.309 | 1.257 | 3.537 | 0.283 | 5573.87 | 27.13 |
| 64 | 0.603 | 1.281 | 82 | 1.357 | 1.269 | 3.537 | 0.283 | 5384.47 | 26.21 |
| 65 | 0.603 | 1.296 | 81 | 1.406 | 1.280 | 3.537 | 0.283 | 5204.56 | 25.34 |
| 66 | 0.603 | 1.312 | 80 | 1.456 | 1.292 | 3.537 | 0.283 | 5033.55 | 24.50 |
| 67 | 0.603 | 1.327 | 79 | 1.506 | 1.303 | 3.537 | 0.283 | 4870.88 | 23.71 |
| 68 | 0.603 | 1.343 | 78 | 1.558 | 1.314 | 3.537 | 0.283 | 4716.03 | 22.96 |
| 69 | 0.603 | 1.359 | 77 | 1.611 | 1.324 | 3.537 | 0.283 | 4568.54 | 22.24 |
| 70 | 0.603 | 1.375 | 76 | 1.665 | 1.334 | 3.537 | 0.283 | 4427.95 | 21.56 |
| 71 | 0.603 | 1.391 | 75 | 1.720 | 1.344 | 3.537 | 0.283 | 4293.86 | 20.90 |
| 72 | 0.603 | 1.408 | 74 | 1.776 | 1.353 | 3.537 | 0.283 | 4165.89 | 20.28 |
| 73 | 0.603 | 1.424 | 73 | 1.833 | 1.362 | 3.537 | 0.283 | 4043.68 | 19.68 |
| 74 | 0.603 | 1.441 | 72 | 1.891 | 1.371 | 3.537 | 0.283 | 3926.91 | 19.12 |
| 75 | 0.603 | 1.458 | 71 | 1.950 | 1.379 | 3.537 | 0.283 | 3815.26 | 18.57 |
| 76 | 0.603 | 1.476 | 70 | 2.009 | 1.387 | 3.537 | 0.283 | 3708.45 | 18.05 |
| 77 | 0.603 | 1.493 | 69 | 2.070 | 1.394 | 3.537 | 0.283 | 3606.21 | 17.56 |
| 78 | 0.603 | 1.511 | 68 | 2.132 | 1.401 | 3.537 | 0.283 | 3508.29 | 17.08 |
| 79 | 0.603 | 1.529 | 67 | 2.195 | 1.407 | 3.537 | 0.283 | 3414.46 | 16.62 |
| 80 | 0.603 | 1.547 | 66 | 2.258 | 1.413 | 3.537 | 0.283 | 3324.49 | 16.18 |
| 81 | 0.603 | 1.565 | 65 | 2.323 | 1.418 | 3.537 | 0.283 | 3238.19 | 15.76 |
| 82 | 0.603 | 1.584 | 64 | 2.388 | 1.423 | 3.537 | 0.283 | 3155.35 | 15.36 |
| 83 | 0.603 | 1.602 | 63 | 2.455 | 1.428 | 3.537 | 0.283 | 3075.81 | 14.97 |
| 84 | 0.603 | 1.621 | 62 | 2.522 | 1.432 | 3.537 | 0.283 | 2999.38 | 14.60 |
| 85 | 0.603 | 1.640 | 61 | 2.591 | 1.435 | 3.537 | 0.283 | 2925.91 | 14.24 |
| 86 | 0.603 | 1.660 | 60 | 2.660 | 1.438 | 3.537 | 0.283 | 2855.25 | 13.90 |
| 87 | 0.603 | 1.680 | 59 | 2.730 | 1.440 | 3.537 | 0.283 | 2787.25 | 13.57 |
| 88 | 0.603 | 1.699 | 58 | 2.801 | 1.441 | 3.537 | 0.283 | 2721.79 | 13.25 |
| 89 | 0.603 | 1.720 | 57 | 2.873 | 1.442 | 3.537 | 0.283 | 2658.74 | 12.94 |
| 90 | 0.603 | 1.740 | 56 | 2.946 | 1.442 | 3.537 | 0.283 | 2597.98 | 12.65 |
| 91 | 0.603 | 1.761 | 55 | 3.020 | 1.442 | 3.537 | 0.283 | 2539.39 | 12.36 |
| 92 | 0.603 | 1.781 | 54 | 3.094 | 1.441 | 3.537 | 0.283 | 2482.87 | 12.09 |
| 93 | 0.603 | 1.803 | 53 | 3.170 | 1.440 | 3.537 | 0.283 | 2428.33 | 11.82 |
| 94 | 0.603 | 1.824 | 52 | 3.246 | 1.437 | 3.537 | 0.283 | 2375.66 | 11.56 |
| 95 | 0.603 | 1.845 | 51 | 3.323 | 1.434 | 3.537 | 0.283 | 2324.78 | 11.32 |
| 96 | 0.603 | 1.867 | 50 | 3.401 | 1.430 | 3.537 | 0.283 | 2275.60 | 11.08 |
| 97 | 0.603 | 1.889 | 49 | 3.479 | 1.426 | 3.537 | 0.283 | 2228.05 | 10.85 |
| 98 | 0.603 | 1.912 | 48 | 3.558 | 1.421 | 3.537 | 0.283 | 2182.04 | 10.62 |
| 99 | 0.603 | 1.934 | 47 | 3.639 | 1.415 | 3.537 | 0.283 | 2137.50 | 10.41 |
| 100 | 0.603 | 1.957 | 46 | 3.719 | 1.408 | 3.537 | 0.283 | 2094.37 | 10.20 |
| 101 | 0.603 | 1.981 | 45 | 3.801 | 1.400 | 3.537 | 0.283 | 2052.58 | 9.99 |
| 102 | 0.603 | 2.004 | 44 | 3.883 | 1.392 | 3.537 | 0.283 | 2012.07 | 9.79 |
| 103 | 0.603 | 2.028 | 43 | 3.966 | 1.383 | 3.537 | 0.283 | 1972.77 | 9.60 |
| 104 | 0.603 | 2.052 | 42 | 4.049 | 1.373 | 3.537 | 0.283 | 1934.64 | 9.42 |
| 105 | 0.603 | 2.076 | 41 | 4.134 | 1.362 | 3.537 | 0.283 | 1897.62 | 9.24 |
| 106 | 0.603 | 2.101 | 40 | 4.218 | 1.350 | 3.537 | 0.283 | 1861.66 | 9.06 |
| 107 | 0.603 | 2.125 | 39 | 4.304 | 1.338 | 3.537 | 0.283 | 1826.71 | 8.89 |
| 108 | 0.603 | 2.151 | 38 | 4.389 | 1.324 | 3.537 | 0.283 | 1792.72 | 8.73 |
| 109 | 0.603 | 2.176 | 37 | 4.476 | 1.310 | 3.537 | 0.283 | 1759.66 | 8.57 |
| 110 | 0.603 | 2.202 | 36 | 4.563 | 1.294 | 3.537 | 0.283 | 1727.47 | 8.41 |
| 111 | 0.603 | 2.228 | 35 | 4.650 | 1.278 | 3.537 | 0.283 | 1696.12 | 8.26 |
| 112 | 0.603 | 2.254 | 34 | 4.738 | 1.261 | 3.537 | 0.283 | 1665.57 | 8.11 |

Table-20, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Rankine wall ( $\phi=38^{\circ}$ and $\delta=0^{\circ}$ )

| ANGLE OF | RADIUS OF LOG-SPIRAL |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOG-SPIRAL | $\mathrm{r}_{0}$ | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | Qu | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 52 | 0.635 | 1.289 | 90 | 1.000 | 1.289 | 4.204 | 0.238 | 8661.20 | 20.40 |
| 53 | 0.635 | 1.307 | 89 | 1.046 | 1.307 | 4.204 | 0.238 | 8298.03 | 19.55 |
| 54 | 0.635 | 1.325 | 88 | 1.092 | 1.324 | 4.204 | 0.238 | 7956.76 | 18.74 |
| 55 | 0.635 | 1.343 | 87 | 1.141 | 1.341 | 4.204 | 0.238 | 7635.73 | 17.99 |
| 56 | 0.635 | 1.362 | 86 | 1.190 | 1.358 | 4.204 | 0.238 | 7333.45 | 17.28 |
| 57 | 0.635 | 1.380 | 85 | 1.241 | 1.375 | 4.204 | 0.238 | 7048.53 | 16.60 |
| 58 | 0.635 | 1.399 | 84 | 1.293 | 1.392 | 4.204 | 0.238 | 6779.73 | 15.97 |
| 59 | 0.635 | 1.419 | 83 | 1.346 | 1.408 | 4.204 | 0.238 | 6525.91 | 15.37 |
| 60 | 0.635 | 1.438 | 82 | 1.400 | 1.424 | 4.204 | 0.238 | 6286.03 | 14.81 |
| 61 | 0.635 | 1.458 | 81 | 1.456 | 1.440 | 4.204 | 0.238 | 6059.12 | 14.27 |
| 62 | 0.635 | 1.478 | 80 | 1.513 | 1.455 | 4.204 | 0.238 | 5844.31 | 13.77 |
| 63 | 0.635 | 1.498 | 79 | 1.572 | 1.471 | 4.204 | 0.238 | 5640.78 | 13.29 |
| 64 | 0.635 | 1.519 | 78 | 1.631 | 1.485 | 4.204 | 0.238 | 5447.81 | 12.83 |
| 65 | 0.635 | 1.539 | 77 | 1.693 | 1.500 | 4.204 | 0.238 | 5264.69 | 12.40 |
| 66 | 0.635 | 1.561 | 76 | 1.755 | 1.514 | 4.204 | 0.238 | 5090.80 | 11.99 |
| 67 | 0.635 | 1.582 | 75 | 1.819 | 1.528 | 4.204 | 0.238 | 4925.55 | 11.60 |
| 68 | 0.635 | 1.604 | 74 | 1.884 | 1.542 | 4.204 | 0.238 | 4768.40 | 11.23 |
| 69 | 0.635 | 1.626 | 73 | 1.951 | 1.555 | 4.204 | 0.238 | 4618.86 | 10.88 |
| 70 | 0.635 | 1.648 | 72 | 2.019 | 1.567 | 4.204 | 0.238 | 4476.45 | 10.55 |
| 71 | 0.635 | 1.671 | 71 | 2.088 | 1.580 | 4.204 | 0.238 | 4340.75 | 10.23 |
| 72 | 0.635 | 1.694 | 70 | 2.159 | 1.592 | 4.204 | 0.238 | 4211.35 | 9.92 |
| 73 | 0.635 | 1.717 | 69 | 2.231 | 1.603 | 4.204 | 0.238 | 4087.88 | 9.63 |
| 74 | 0.635 | 1.740 | 68 | 2.304 | 1.614 | 4.204 | 0.238 | 3970.00 | 9.35 |
| 75 | 0.635 | 1.764 | 67 | 2.379 | 1.624 | 4.204 | 0.238 | 3857.38 | 9.09 |
| 76 | 0.635 | 1.789 | 66 | 2.455 | 1.634 | 4.204 | 0.238 | 3749.73 | 8.83 |
| 77 | 0.635 | 1.813 | 65 | 2.533 | 1.643 | 4.204 | 0.238 | 3646.75 | 8.59 |
| 78 | 0.635 | 1.838 | 64 | 2.611 | 1.652 | 4.204 | 0.238 | 3548.18 | 8.36 |
| 79 | 0.635 | 1.863 | 63 | 2.692 | 1.660 | 4.204 | 0.238 | 3453.79 | 8.14 |
| 80 | 0.635 | 1.889 | 62 | 2.774 | 1.668 | 4.204 | 0.238 | 3363.33 | 7.92 |
| 81 | 0.635 | 1.915 | 61 | 2.857 | 1.675 | 4.204 | 0.238 | 3276.60 | 7.72 |
| 82 | 0.635 | 1.941 | 60 | 2.941 | 1.681 | 4.204 | 0.238 | 3193.38 | 7.52 |
| 83 | 0.635 | 1.968 | 59 | 3.027 | 1.687 | 4.204 | 0.238 | 3113.49 | 7.33 |
| 84 | 0.635 | 1.995 | 58 | 3.114 | 1.692 | 4.204 | 0.238 | 3036.75 | 7.15 |
| 85 | 0.635 | 2.022 | 57 | 3.203 | 1.696 | 4.204 | 0.238 | 2962.98 | 6.98 |
| 86 | 0.635 | 2.050 | 56 | 3.293 | 1.699 | 4.204 | 0.238 | 2892.04 | 6.81 |
| 87 | 0.635 | 2.078 | 55 | 3.384 | 1.702 | 4.204 | 0.238 | 2823.77 | 6.65 |
| 88 | 0.635 | 2.107 | 54 | 3.476 | 1.704 | 4.204 | 0.238 | 2758.03 | 6.50 |
| 89 | 0.635 | 2.136 | 53 | 3.570 | 1.705 | 4.204 | 0.238 | 2694.69 | 6.35 |
| 90 | 0.635 | 2.165 | 52 | 3.666 | 1.706 | 4.204 | 0.238 | 2633.62 | 6.20 |
| 91 | 0.635 | 2.195 | 51 | 3.762 | 1.705 | 4.204 | 0.238 | 2574.70 | 6.07 |
| 92 | 0.635 | 2.225 | 50 | 3.860 | 1.704 | 4.204 | 0.238 | 2517.83 | 5.93 |
| 93 | 0.635 | 2.255 | 49 | 3.959 | 1.702 | 4.204 | 0.238 | 2462.89 | 5.80 |
| 94 | 0.635 | 2.286 | 48 | 4.060 | 1.699 | 4.204 | 0.238 | 2409.78 | 5.68 |
| 95 | 0.635 | 2.318 | 47 | 4.161 | 1.695 | 4.204 | 0.238 | 2358.42 | 5.56 |
| 96 | 0.635 | 2.349 | 46 | 4.264 | 1.690 | 4.204 | 0.238 | 2308.70 | 5.44 |
| 97 | 0.635 | 2.382 | 45 | 4.368 | 1.684 | 4.204 | 0.238 | 2260.54 | 5.33 |
| 98 | 0.635 | 2.414 | 44 | 4.473 | 1.677 | 4.204 | 0.238 | 2213.86 | 5.22 |
| 99 | 0.635 | 2.447 | 43 | 4.580 | 1.669 | 4.204 | 0.238 | 2168.59 | 5.11 |
| 100 | 0.635 | 2.481 | 42 | 4.688 | 1.660 | 4.204 | 0.238 | 2124.65 | 5.01 |
| 101 | 0.635 | 2.515 | 41 | 4.796 | 1.650 | 4.204 | 0.238 | 2081.97 | 4.90 |
| 102 | 0.635 | 2.550 | 40 | 4.906 | 1.639 | 4.204 | 0.238 | 2040.48 | 4.81 |
| 103 | 0.635 | 2.585 | 39 | 5.017 | 1.627 | 4.204 | 0.238 | 2000.12 | 4.71 |
| 104 | 0.635 | 2.620 | 38 | 5.129 | 1.613 | 4.204 | 0.238 | 1960.83 | 4.62 |
| 105 | 0.635 | 2.656 | 37 | 5.243 | 1.599 | 4.204 | 0.238 | 1922.55 | 4.53 |
| 106 | 0.635 | 2.693 | 36 | 5.357 | 1.583 | 4.204 | 0.238 | 1885.23 | 4.44 |
| 107 | 0.635 | 2.730 | 35 | 5.472 | 1.566 | 4.204 | 0.238 | 1848.81 | 4.36 |
| 108 | 0.635 | 2.767 | 34 | 5.588 | 1.547 | 4.204 | 0.238 | 1813.24 | 4.27 |

Table-21, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Rankine wall ( $\phi=42^{\circ}$ and $\delta=0^{\circ}$ )

| ANGLE OF | RADIUS OF LOG-SPIRAL |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOG-SPIRAL | $r_{0}$ | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | Qu | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathbf{Q}_{\mathrm{u}} / \mathbf{q}_{\mathrm{u}}$ |
| 48 | 0.673 | 1.431 | 90 | 1.000 | 1.431 | 5.0447 | 0.198 | 10510.58 | 11.32 |
| 49 | 0.673 | 1.453 | 89 | 1.051 | 1.453 | 5.0447 | 0.198 | 10030.03 | 10.80 |
| 50 | 0.673 | 1.476 | 88 | 1.103 | 1.475 | 5.0447 | 0.198 | 9581.11 | 10.32 |
| 51 | 0.673 | 1.500 | 87 | 1.157 | 1.498 | 5.0447 | 0.198 | 9161.23 | 9.86 |
| 52 | 0.673 | 1.523 | 86 | 1.213 | 1.520 | 5.0447 | 0.198 | 8768.06 | 9.44 |
| 53 | 0.673 | 1.547 | 85 | 1.270 | 1.542 | 5.0447 | 0.198 | 8399.50 | 9.04 |
| 54 | 0.673 | 1.572 | 84 | 1.329 | 1.563 | 5.0447 | 0.198 | 8053.64 | 8.67 |
| 55 | 0.673 | 1.597 | 83 | 1.389 | 1.585 | 5.0447 | 0.198 | 7728.74 | 8.32 |
| 56 | 0.673 | 1.622 | 82 | 1.452 | 1.606 | 5.0447 | 0.198 | 7423.25 | 7.99 |
| 57 | 0.673 | 1.648 | 81 | 1.516 | 1.628 | 5.0447 | 0.198 | 7135.71 | 7.68 |
| 58 | 0.673 | 1.674 | 80 | 1.581 | 1.649 | 5.0447 | 0.198 | 6864.84 | 7.39 |
| 59 | 0.673 | 1.700 | 79 | 1.649 | 1.669 | 5.0447 | 0.198 | 6609.42 | 7.12 |
| 60 | 0.673 | 1.727 | 78 | 1.718 | 1.690 | 5.0447 | 0.198 | 6368.38 | 6.86 |
| 61 | 0.673 | 1.755 | 77 | 1.789 | 1.710 | 5.0447 | 0.198 | 6140.70 | 6.61 |
| 62 | 0.673 | 1.783 | 76 | 1.862 | 1.730 | 5.0447 | 0.198 | 5925.47 | 6.38 |
| 63 | 0.673 | 1.811 | 75 | 1.937 | 1.749 | 5.0447 | 0.198 | 5721.85 | 6.16 |
| 64 | 0.673 | 1.839 | 74 | 2.014 | 1.768 | 5.0447 | 0.198 | 5529.05 | 5.95 |
| 65 | 0.673 | 1.869 | 73 | 2.093 | 1.787 | 5.0447 | 0.198 | 5346.36 | 5.76 |
| 66 | 0.673 | 1.898 | 72 | 2.173 | 1.805 | 5.0447 | 0.198 | 5173.13 | 5.57 |
| 67 | 0.673 | 1.928 | 71 | 2.256 | 1.823 | 5.0447 | 0.198 | 5008.73 | 5.39 |
| 68 | 0.673 | 1.959 | 70 | 2.340 | 1.841 | 5.0447 | 0.198 | 4852.60 | 5.22 |
| 69 | 0.673 | 1.990 | 69 | 2.426 | 1.858 | 5.0447 | 0.198 | 4704.22 | 5.07 |
| 70 | 0.673 | 2.021 | 68 | 2.514 | 1.874 | 5.0447 | 0.198 | 4563.11 | 4.91 |
| 71 | 0.673 | 2.053 | 67 | 2.605 | 1.890 | 5.0447 | 0.198 | 4428.81 | 4.77 |
| 72 | 0.673 | 2.086 | 66 | 2.697 | 1.906 | 5.0447 | 0.198 | 4300.90 | 4.63 |
| 73 | 0.673 | 2.119 | 65 | 2.791 | 1.920 | 5.0447 | 0.198 | 4178.99 | 4.50 |
| 74 | 0.673 | 2.152 | 64 | 2.887 | 1.935 | 5.0447 | 0.198 | 4062.72 | 4.37 |
| 75 | 0.673 | 2.187 | 63 | 2.985 | 1.948 | 5.0447 | 0.198 | 3951.76 | 4.25 |
| 76 | 0.673 | 2.221 | 62 | 3.086 | 1.961 | 5.0447 | 0.198 | 3845.77 | 4.14 |
| 77 | 0.673 | 2.256 | 61 | 3.188 | 1.973 | 5.0447 | 0.198 | 3744.48 | 4.03 |
| 78 | 0.673 | 2.292 | 60 | 3.292 | 1.985 | 5.0447 | 0.198 | 3647.60 | 3.93 |
| 79 | 0.673 | 2.328 | 59 | 3.398 | 1.996 | 5.0447 | 0.198 | 3554.86 | 3.83 |
| 80 | 0.673 | 2.365 | 58 | 3.507 | 2.006 | 5.0447 | 0.198 | 3466.04 | 3.73 |
| 81 | 0.673 | 2.403 | 57 | 3.617 | 2.015 | 5.0447 | 0.198 | 3380.90 | 3.64 |
| 82 | 0.673 | 2.441 | 56 | 3.730 | 2.024 | 5.0447 | 0.198 | 3299.22 | 3.55 |
| 83 | 0.673 | 2.480 | 55 | 3.844 | 2.031 | 5.0447 | 0.198 | 3220.81 | 3.47 |
| 84 | 0.673 | 2.519 | 54 | 3.961 | 2.038 | 5.0447 | 0.198 | 3145.47 | 3.39 |
| 85 | 0.673 | 2.559 | 53 | 4.080 | 2.043 | 5.0447 | 0.198 | 3073.03 | 3.31 |
| 86 | 0.673 | 2.599 | 52 | 4.200 | 2.048 | 5.0447 | 0.198 | 3003.31 | 3.23 |
| 87 | 0.673 | 2.640 | 51 | 4.323 | 2.052 | 5.0447 | 0.198 | 2936.17 | 3.16 |
| 88 | 0.673 | 2.682 | 50 | 4.448 | 2.055 | 5.0447 | 0.198 | 2871.44 | 3.09 |
| 89 | 0.673 | 2.725 | 49 | 4.575 | 2.056 | 5.0447 | 0.198 | 2808.99 | 3.02 |
| 90 | 0.673 | 2.768 | 48 | 4.704 | 2.057 | 5.0447 | 0.198 | 2748.67 | 2.96 |
| 91 | 0.673 | 2.812 | 47 | 4.835 | 2.056 | 5.0447 | 0.198 | 2690.37 | 2.90 |
| 92 | 0.673 | 2.856 | 46 | 4.968 | 2.055 | 5.0447 | 0.198 | 2633.96 | 2.84 |
| 93 | 0.673 | 2.901 | 45 | 5.103 | 2.052 | 5.0447 | 0.198 | 2579.33 | 2.78 |
| 94 | 0.673 | 2.947 | 44 | 5.240 | 2.047 | 5.0447 | 0.198 | 2526.37 | 2.72 |
| 95 | 0.673 | 2.994 | 43 | 5.379 | 2.042 | 5.0447 | 0.198 | 2474.97 | 2.66 |
| 96 | 0.673 | 3.042 | 42 | 5.521 | 2.035 | 5.0447 | 0.198 | 2425.04 | 2.61 |
| 97 | 0.673 | 3.090 | 41 | 5.664 | 2.027 | 5.0447 | 0.198 | 2376.47 | 2.56 |
| 98 | 0.673 | 3.139 | 40 | 5.809 | 2.017 | 5.0447 | 0.198 | 2329.19 | 2.51 |
| 99 | 0.673 | 3.188 | 39 | 5.956 | 2.006 | 5.0447 | 0.198 | 2283.10 | 2.46 |
| 100 | 0.673 | 3.239 | 38 | 6.104 | 1.994 | 5.0447 | 0.198 | 2238.12 | 2.41 |
| 101 | 0.673 | 3.290 | 37 | 6.255 | 1.980 | 5.0447 | 0.198 | 2194.18 | 2.36 |
| 102 | 0.673 | 3.342 | 36 | 6.408 | 1.965 | 5.0447 | 0.198 | 2151.20 | 2.32 |
| 103 | 0.673 | 3.395 | 35 | 6.562 | 1.947 | 5.0447 | 0.198 | 2109.11 | 2.27 |
| 104 | 0.673 | 3.449 | 34 | 6.719 | 1.929 | 5.0447 | 0.198 | 2067.83 | 2.23 |

Table-22, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter \& heights.
For Coulomb wall ( $\phi=\mathbf{3 4}{ }^{\circ}$ and $\delta=\mathbf{2 2}^{\circ}$ )

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| log-spiral | $\mathrm{r}_{0}$ | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | $\mathbf{Q}_{\mathrm{u}}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathbf{Q}_{\mathbf{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 56 | 0.603 | 1.166 | 90 | 1.000 | 1.166 | 8.641 | 0.254 | 8191.07 | 39.87 |
| 57 | 0.603 | 1.180 | 89 | 1.041 | 1.180 | 8.641 | 0.254 | 7880.05 | 38.36 |
| 58 | 0.603 | 1.194 | 88 | 1.083 | 1.193 | 8.641 | 0.254 | 7586.09 | 36.93 |
| 59 | 0.603 | 1.208 | 87 | 1.126 | 1.206 | 8.641 | 0.254 | 7307.96 | 35.58 |
| 60 | 0.603 | 1.222 | 86 | 1.171 | 1.219 | 8.641 | 0.254 | 7044.58 | 34.29 |
| 61 | 0.603 | 1.237 | 85 | 1.216 | 1.232 | 8.641 | 0.254 | 6794.91 | 33.08 |
| 62 | 0.603 | 1.251 | 84 | 1.262 | 1.245 | 8.641 | 0.254 | 6558.03 | 31.92 |
| 63 | 0.603 | 1.266 | 83 | 1.309 | 1.257 | 8.641 | 0.254 | 6333.08 | 30.83 |
| 64 | 0.603 | 1.281 | 82 | 1.357 | 1.269 | 8.641 | 0.254 | 6119.26 | 29.79 |
| 65 | 0.603 | 1.296 | 81 | 1.406 | 1.280 | 8.641 | 0.254 | 5915.86 | 28.80 |
| 66 | 0.603 | 1.312 | 80 | 1.456 | 1.292 | 8.641 | 0.254 | 5722.20 | 27.86 |
| 67 | 0.603 | 1.327 | 79 | 1.506 | 1.303 | 8.641 | 0.254 | 5537.65 | 26.96 |
| 68 | 0.603 | 1.343 | 78 | 1.558 | 1.314 | 8.641 | 0.254 | 5361.64 | 26.10 |
| 69 | 0.603 | 1.359 | 77 | 1.611 | 1.324 | 8.641 | 0.254 | 5193.64 | 25.28 |
| 70 | 0.603 | 1.375 | 76 | 1.665 | 1.334 | 8.641 | 0.254 | 5033.15 | 24.50 |
| 71 | 0.603 | 1.391 | 75 | 1.720 | 1.344 | 8.641 | 0.254 | 4879.72 | 23.75 |
| 72 | 0.603 | 1.408 | 74 | 1.776 | 1.353 | 8.641 | 0.254 | 4732.91 | 23.04 |
| 73 | 0.603 | 1.424 | 73 | 1.833 | 1.362 | 8.641 | 0.254 | 4592.34 | 22.36 |
| 74 | 0.603 | 1.441 | 72 | 1.891 | 1.371 | 8.641 | 0.254 | 4457.62 | 21.70 |
| 75 | 0.603 | 1.458 | 71 | 1.950 | 1.379 | 8.641 | 0.254 | 4328.42 | 21.07 |
| 76 | 0.603 | 1.476 | 70 | 2.009 | 1.387 | 8.641 | 0.254 | 4204.41 | 20.47 |
| 77 | 0.603 | 1.493 | 69 | 2.070 | 1.394 | 8.641 | 0.254 | 4085.29 | 19.89 |
| 78 | 0.603 | 1.511 | 68 | 2.132 | 1.401 | 8.641 | 0.254 | 3970.78 | 19.33 |
| 79 | 0.603 | 1.529 | 67 | 2.195 | 1.407 | 8.641 | 0.254 | 3860.62 | 18.79 |
| 80 | 0.603 | 1.547 | 66 | 2.258 | 1.413 | 8.641 | 0.254 | 3754.55 | 18.28 |
| 81 | 0.603 | 1.565 | 65 | 2.323 | 1.418 | 8.641 | 0.254 | 3652.36 | 17.78 |
| 82 | 0.603 | 1.584 | 64 | 2.388 | 1.423 | 8.641 | 0.254 | 3553.82 | 17.30 |
| 83 | 0.603 | 1.602 | 63 | 2.455 | 1.428 | 8.641 | 0.254 | 3458.72 | 16.84 |
| 84 | 0.603 | 1.621 | 62 | 2.522 | 1.432 | 8.641 | 0.254 | 3366.88 | 16.39 |
| 85 | 0.603 | 1.640 | 61 | 2.591 | 1.435 | 8.641 | 0.254 | 3278.12 | 15.96 |
| 86 | 0.603 | 1.660 | 60 | 2.660 | 1.438 | 8.641 | 0.254 | 3192.26 | 15.54 |
| 87 | 0.603 | 1.680 | 59 | 2.730 | 1.440 | 8.641 | 0.254 | 3109.15 | 15.14 |
| 88 | 0.603 | 1.699 | 58 | 2.801 | 1.441 | 8.641 | 0.254 | 3028.63 | 14.74 |
| 89 | 0.603 | 1.720 | 57 | 2.873 | 1.442 | 8.641 | 0.254 | 2950.57 | 14.36 |
| 90 | 0.603 | 1.740 | 56 | 2.946 | 1.442 | 8.641 | 0.254 | 2874.83 | 13.99 |
| 91 | 0.603 | 1.761 | 55 | 3.020 | 1.442 | 8.641 | 0.254 | 2801.28 | 13.64 |
| 92 | 0.603 | 1.781 | 54 | 3.094 | 1.441 | 8.641 | 0.254 | 2729.80 | 13.29 |
| 93 | 0.603 | 1.803 | 53 | 3.170 | 1.440 | 8.641 | 0.254 | 2660.29 | 12.95 |
| 94 | 0.603 | 1.824 | 52 | 3.246 | 1.437 | 8.641 | 0.254 | 2592.63 | 12.62 |
| 95 | 0.603 | 1.845 | 51 | 3.323 | 1.434 | 8.641 | 0.254 | 2526.72 | 12.30 |
| 96 | 0.603 | 1.867 | 50 | 3.401 | 1.430 | 8.641 | 0.254 | 2462.46 | 11.99 |
| 97 | 0.603 | 1.889 | 49 | 3.479 | 1.426 | 8.641 | 0.254 | 2399.78 | 11.68 |
| 98 | 0.603 | 1.912 | 48 | 3.558 | 1.421 | 8.641 | 0.254 | 2338.56 | 11.38 |
| 99 | 0.603 | 1.934 | 47 | 3.639 | 1.415 | 8.641 | 0.254 | 2278.75 | 11.09 |
| 100 | 0.603 | 1.957 | 46 | 3.719 | 1.408 | 8.641 | 0.254 | 2220.26 | 10.81 |
| 101 | 0.603 | 1.981 | 45 | 3.801 | 1.400 | 8.641 | 0.254 | 2163.00 | 10.53 |
| 102 | 0.603 | 2.004 | 44 | 3.883 | 1.392 | 8.641 | 0.254 | 2106.93 | 10.26 |
| 103 | 0.603 | 2.028 | 43 | 3.966 | 1.383 | 8.641 | 0.254 | 2051.96 | 9.99 |
| 104 | 0.603 | 2.052 | 42 | 4.049 | 1.373 | 8.641 | 0.254 | 1998.04 | 9.73 |
| 105 | 0.603 | 2.076 | 41 | 4.134 | 1.362 | 8.641 | 0.254 | 1945.10 | 9.47 |
| 106 | 0.603 | 2.101 | 40 | 4.218 | 1.350 | 8.641 | 0.254 | 1893.09 | 9.22 |
| 107 | 0.603 | 2.125 | 39 | 4.304 | 1.338 | 8.641 | 0.254 | 1841.96 | 8.97 |
| 108 | 0.603 | 2.151 | 38 | 4.389 | 1.324 | 8.641 | 0.254 | 1791.64 | 8.72 |
| 109 | 0.603 | 2.176 | 37 | 4.476 | 1.310 | 8.641 | 0.254 | 1742.10 | 8.48 |
| 110 | 0.603 | 2.202 | 36 | 4.563 | 1.294 | 8.641 | 0.254 | 1693.27 | 8.24 |
| 111 | 0.603 | 2.228 | 35 | 4.650 | 1.278 | 8.641 | 0.254 | 1645.13 | 8.01 |
| 112 | 0.603 | 2.254 | 34 | 4.738 | 1.261 | 8.641 | 0.254 | 1597.63 | 7.78 |

Table-23, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Coulomb wall $\left(\phi=\mathbf{3 8}^{\circ}\right.$ and $\delta=\mathbf{2 2}^{\circ}$ )

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| log-spiral | $r_{0}$ | $r_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | $\mathbf{Q}_{\mathbf{u}}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathrm{Q}_{\mathrm{u}} / \mathbf{q}_{\mathrm{u}}$ |
| 52 | 0.635 | 1.289 | 90 | 1.000 | 1.289 | 11.466 | 0.217 | 9785.50 | 23.05 |
| 53 | 0.635 | 1.307 | 89 | 1.046 | 1.307 | 11.466 | 0.217 | 9385.47 | 22.11 |
| 54 | 0.635 | 1.325 | 88 | 1.092 | 1.324 | 11.466 | 0.217 | 9009.18 | 21.22 |
| 55 | 0.635 | 1.343 | 87 | 1.141 | 1.341 | 11.466 | 0.217 | 8654.81 | 20.39 |
| 56 | 0.635 | 1.362 | 86 | 1.190 | 1.358 | 11.466 | 0.217 | 8320.71 | 19.60 |
| 57 | 0.635 | 1.380 | 85 | 1.241 | 1.375 | 11.466 | 0.217 | 8005.35 | 18.86 |
| 58 | 0.635 | 1.399 | 84 | 1.293 | 1.392 | 11.466 | 0.217 | 7707.38 | 18.16 |
| 59 | 0.635 | 1.419 | 83 | 1.346 | 1.408 | 11.466 | 0.217 | 7425.53 | 17.49 |
| 60 | 0.635 | 1.438 | 82 | 1.400 | 1.424 | 11.466 | 0.217 | 7158.64 | 16.86 |
| 61 | 0.635 | 1.458 | 81 | 1.456 | 1.440 | 11.466 | 0.217 | 6905.66 | 16.27 |
| 62 | 0.635 | 1.478 | 80 | 1.513 | 1.455 | 11.466 | 0.217 | 6665.62 | 15.70 |
| 63 | 0.635 | 1.498 | 79 | 1.572 | 1.471 | 11.466 | 0.217 | 6437.62 | 15.17 |
| 64 | 0.635 | 1.519 | 78 | 1.631 | 1.485 | 11.466 | 0.217 | 6220.84 | 14.65 |
| 65 | 0.635 | 1.539 | 77 | 1.693 | 1.500 | 11.466 | 0.217 | 6014.53 | 14.17 |
| 66 | 0.635 | 1.561 | 76 | 1.755 | 1.514 | 11.466 | 0.217 | 5817.98 | 13.71 |
| 67 | 0.635 | 1.582 | 75 | 1.819 | 1.528 | 11.466 | 0.217 | 5630.55 | 13.26 |
| 68 | 0.635 | 1.604 | 74 | 1.884 | 1.542 | 11.466 | 0.217 | 5451.63 | 12.84 |
| 69 | 0.635 | 1.626 | 73 | 1.951 | 1.555 | 11.466 | 0.217 | 5280.66 | 12.44 |
| 70 | 0.635 | 1.648 | 72 | 2.019 | 1.567 | 11.466 | 0.217 | 5117.14 | 12.05 |
| 71 | 0.635 | 1.671 | 71 | 2.088 | 1.580 | 11.466 | 0.217 | 4960.58 | 11.69 |
| 72 | 0.635 | 1.694 | 70 | 2.159 | 1.592 | 11.466 | 0.217 | 4810.53 | 11.33 |
| 73 | 0.635 | 1.717 | 69 | 2.231 | 1.603 | 11.466 | 0.217 | 4666.59 | 10.99 |
| 74 | 0.635 | 1.740 | 68 | 2.304 | 1.614 | 11.466 | 0.217 | 4528.36 | 10.67 |
| 75 | 0.635 | 1.764 | 67 | 2.379 | 1.624 | 11.466 | 0.217 | 4395.47 | 10.35 |
| 76 | 0.635 | 1.789 | 66 | 2.455 | 1.634 | 11.466 | 0.217 | 4267.60 | 10.05 |
| 77 | 0.635 | 1.813 | 65 | 2.533 | 1.643 | 11.466 | 0.217 | 4144.43 | 9.76 |
| 78 | 0.635 | 1.838 | 64 | 2.611 | 1.652 | 11.466 | 0.217 | 4025.66 | 9.48 |
| 79 | 0.635 | 1.863 | 63 | 2.692 | 1.660 | 11.466 | 0.217 | 3911.01 | 9.21 |
| 80 | 0.635 | 1.889 | 62 | 2.774 | 1.668 | 11.466 | 0.217 | 3800.22 | 8.95 |
| 81 | 0.635 | 1.915 | 61 | 2.857 | 1.675 | 11.466 | 0.217 | 3693.06 | 8.70 |
| 82 | 0.635 | 1.941 | 60 | 2.941 | 1.681 | 11.466 | 0.217 | 3589.28 | 8.46 |
| 83 | 0.635 | 1.968 | 59 | 3.027 | 1.687 | 11.466 | 0.217 | 3488.68 | 8.22 |
| 84 | 0.635 | 1.995 | 58 | 3.114 | 1.692 | 11.466 | 0.217 | 3391.04 | 7.99 |
| 85 | 0.635 | 2.022 | 57 | 3.203 | 1.696 | 11.466 | 0.217 | 3296.19 | 7.76 |
| 86 | 0.635 | 2.050 | 56 | 3.293 | 1.699 | 11.466 | 0.217 | 3203.93 | 7.55 |
| 87 | 0.635 | 2.078 | 55 | 3.384 | 1.702 | 11.466 | 0.217 | 3114.10 | 7.34 |
| 88 | 0.635 | 2.107 | 54 | 3.476 | 1.704 | 11.466 | 0.217 | 3026.54 | 7.13 |
| 89 | 0.635 | 2.136 | 53 | 3.570 | 1.705 | 11.466 | 0.217 | 2941.10 | 6.93 |
| 90 | 0.635 | 2.165 | 52 | 3.666 | 1.706 | 11.466 | 0.217 | 2857.62 | 6.73 |
| 91 | 0.635 | 2.195 | 51 | 3.762 | 1.705 | 11.466 | 0.217 | 2775.98 | 6.54 |
| 92 | 0.635 | 2.225 | 50 | 3.860 | 1.704 | 11.466 | 0.217 | 2696.04 | 6.35 |
| 93 | 0.635 | 2.255 | 49 | 3.959 | 1.702 | 11.466 | 0.217 | 2617.68 | 6.17 |
| 94 | 0.635 | 2.286 | 48 | 4.060 | 1.699 | 11.466 | 0.217 | 2540.79 | 5.99 |
| 95 | 0.635 | 2.318 | 47 | 4.161 | 1.695 | 11.466 | 0.217 | 2465.26 | 5.81 |
| 96 | 0.635 | 2.349 | 46 | 4.264 | 1.690 | 11.466 | 0.217 | 2390.98 | 5.63 |
| 97 | 0.635 | 2.382 | 45 | 4.368 | 1.684 | 11.466 | 0.217 | 2317.84 | 5.46 |
| 98 | 0.635 | 2.414 | 44 | 4.473 | 1.677 | 11.466 | 0.217 | 2245.77 | 5.29 |
| 99 | 0.635 | 2.447 | 43 | 4.580 | 1.669 | 11.466 | 0.217 | 2174.66 | 5.12 |
| 100 | 0.635 | 2.481 | 42 | 4.688 | 1.660 | 11.466 | 0.217 | 2104.43 | 4.96 |
| 101 | 0.635 | 2.515 | 41 | 4.796 | 1.650 | 11.466 | 0.217 | 2035.00 | 4.79 |
| 102 | 0.635 | 2.550 | 40 | 4.906 | 1.639 | 11.466 | 0.217 | 1966.28 | 4.63 |
| 103 | 0.635 | 2.585 | 39 | 5.017 | 1.627 | 11.466 | 0.217 | 1898.21 | 4.47 |
| 104 | 0.635 | 2.620 | 38 | 5.129 | 1.613 | 11.466 | 0.217 | 1830.72 | 4.31 |
| 105 | 0.635 | 2.656 | 37 | 5.243 | 1.599 | 11.466 | 0.217 | 1763.73 | 4.15 |
| 106 | 0.635 | 2.693 | 36 | 5.357 | 1.583 | 11.466 | 0.217 | 1697.19 | 4.00 |
| 107 | 0.635 | 2.730 | 35 | 5.472 | 1.566 | 11.466 | 0.217 | 1631.02 | 3.84 |
| 108 | 0.635 | 2.767 | 34 | 5.588 | 1.547 | 11.466 | 0.217 | 1565.18 | 3.69 |

Table-24, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Coulomb wall $\left(\phi=42^{\circ}\right.$ and $\left.\delta=\mathbf{2 2}^{\circ}\right)$

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| log-spiral | $\mathrm{r}_{0}$ | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $K_{p}$ | $\mathrm{K}_{\mathrm{a}}$ | $\mathbf{Q}_{\mathrm{u}}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathbf{Q}_{\mathrm{u}} / \mathbf{q}_{\mathrm{u}}$ |
| 48 | 0.673 | 1.431 | 90 | 1.000 | 1.431 | 15.726 | 0.183 | 11994.45 | 12.91 |
| 49 | 0.673 | 1.453 | 89 | 1.051 | 1.453 | 15.726 | 0.183 | 11473.54 | 12.35 |
| 50 | 0.673 | 1.476 | 88 | 1.103 | 1.475 | 15.726 | 0.183 | 10986.36 | 11.83 |
| 51 | 0.673 | 1.500 | 87 | 1.157 | 1.498 | 15.726 | 0.183 | 10530.10 | 11.34 |
| 52 | 0.673 | 1.523 | 86 | 1.213 | 1.520 | 15.726 | 0.183 | 10102.22 | 10.88 |
| 53 | 0.673 | 1.547 | 85 | 1.270 | 1.542 | 15.726 | 0.183 | 9700.43 | 10.44 |
| 54 | 0.673 | 1.572 | 84 | 1.329 | 1.563 | 15.726 | 0.183 | 9322.64 | 10.04 |
| 55 | 0.673 | 1.597 | 83 | 1.389 | 1.585 | 15.726 | 0.183 | 8966.97 | 9.65 |
| 56 | 0.673 | 1.622 | 82 | 1.452 | 1.606 | 15.726 | 0.183 | 8631.68 | 9.29 |
| 57 | 0.673 | 1.648 | 81 | 1.516 | 1.628 | 15.726 | 0.183 | 8315.22 | 8.95 |
| 58 | 0.673 | 1.674 | 80 | 1.581 | 1.649 | 15.726 | 0.183 | 8016.15 | 8.63 |
| 59 | 0.673 | 1.700 | 79 | 1.649 | 1.669 | 15.726 | 0.183 | 7733.15 | 8.33 |
| 60 | 0.673 | 1.727 | 78 | 1.718 | 1.690 | 15.726 | 0.183 | 7465.04 | 8.04 |
| 61 | 0.673 | 1.755 | 77 | 1.789 | 1.710 | 15.726 | 0.183 | 7210.70 | 7.76 |
| 62 | 0.673 | 1.783 | 76 | 1.862 | 1.730 | 15.726 | 0.183 | 6969.11 | 7.50 |
| 63 | 0.673 | 1.811 | 75 | 1.937 | 1.749 | 15.726 | 0.183 | 6739.36 | 7.26 |
| 64 | 0.673 | 1.839 | 74 | 2.014 | 1.768 | 15.726 | 0.183 | 6520.58 | 7.02 |
| 65 | 0.673 | 1.869 | 73 | 2.093 | 1.787 | 15.726 | 0.183 | 6311.97 | 6.80 |
| 66 | 0.673 | 1.898 | 72 | 2.173 | 1.805 | 15.726 | 0.183 | 6112.79 | 6.58 |
| 67 | 0.673 | 1.928 | 71 | 2.256 | 1.823 | 15.726 | 0.183 | 5922.38 | 6.38 |
| 68 | 0.673 | 1.959 | 70 | 2.340 | 1.841 | 15.726 | 0.183 | 5740.08 | 6.18 |
| 69 | 0.673 | 1.990 | 69 | 2.426 | 1.858 | 15.726 | 0.183 | 5565.33 | 5.99 |
| 70 | 0.673 | 2.021 | 68 | 2.514 | 1.874 | 15.726 | 0.183 | 5397.58 | 5.81 |
| 71 | 0.673 | 2.053 | 67 | 2.605 | 1.890 | 15.726 | 0.183 | 5236.30 | 5.64 |
| 72 | 0.673 | 2.086 | 66 | 2.697 | 1.906 | 15.726 | 0.183 | 5081.04 | 5.47 |
| 73 | 0.673 | 2.119 | 65 | 2.791 | 1.920 | 15.726 | 0.183 | 4931.36 | 5.31 |
| 74 | 0.673 | 2.152 | 64 | 2.887 | 1.935 | 15.726 | 0.183 | 4786.82 | 5.15 |
| 75 | 0.673 | 2.187 | 63 | 2.985 | 1.948 | 15.726 | 0.183 | 4647.06 | 5.00 |
| 76 | 0.673 | 2.221 | 62 | 3.086 | 1.961 | 15.726 | 0.183 | 4511.70 | 4.86 |
| 77 | 0.673 | 2.256 | 61 | 3.188 | 1.973 | 15.726 | 0.183 | 4380.41 | 4.72 |
| 78 | 0.673 | 2.292 | 60 | 3.292 | 1.985 | 15.726 | 0.183 | 4252.87 | 4.58 |
| 79 | 0.673 | 2.328 | 59 | 3.398 | 1.996 | 15.726 | 0.183 | 4128.77 | 4.45 |
| 80 | 0.673 | 2.365 | 58 | 3.507 | 2.006 | 15.726 | 0.183 | 4007.83 | 4.32 |
| 81 | 0.673 | 2.403 | 57 | 3.617 | 2.015 | 15.726 | 0.183 | 3889.78 | 4.19 |
| 82 | 0.673 | 2.441 | 56 | 3.730 | 2.024 | 15.726 | 0.183 | 3774.38 | 4.06 |
| 83 | 0.673 | 2.480 | 55 | 3.844 | 2.031 | 15.726 | 0.183 | 3661.37 | 3.94 |
| 84 | 0.673 | 2.519 | 54 | 3.961 | 2.038 | 15.726 | 0.183 | 3550.55 | 3.82 |
| 85 | 0.673 | 2.559 | 53 | 4.080 | 2.043 | 15.726 | 0.183 | 3441.68 | 3.71 |
| 86 | 0.673 | 2.599 | 52 | 4.200 | 2.048 | 15.726 | 0.183 | 3334.58 | 3.59 |
| 87 | 0.673 | 2.640 | 51 | 4.323 | 2.052 | 15.726 | 0.183 | 3229.04 | 3.48 |
| 88 | 0.673 | 2.682 | 50 | 4.448 | 2.055 | 15.726 | 0.183 | 3124.88 | 3.36 |
| 89 | 0.673 | 2.725 | 49 | 4.575 | 2.056 | 15.726 | 0.183 | 3021.93 | 3.25 |
| 90 | 0.673 | 2.768 | 48 | 4.704 | 2.057 | 15.726 | 0.183 | 2920.03 | 3.14 |
| 91 | 0.673 | 2.812 | 47 | 4.835 | 2.056 | 15.726 | 0.183 | 2819.01 | 3.04 |
| 92 | 0.673 | 2.856 | 46 | 4.968 | 2.055 | 15.726 | 0.183 | 2718.71 | 2.93 |
| 93 | 0.673 | 2.901 | 45 | 5.103 | 2.052 | 15.726 | 0.183 | 2619.01 | 2.82 |
| 94 | 0.673 | 2.947 | 44 | 5.240 | 2.047 | 15.726 | 0.183 | 2519.76 | 2.71 |
| 95 | 0.673 | 2.994 | 43 | 5.379 | 2.042 | 15.726 | 0.183 | 2420.82 | 2.61 |
| 96 | 0.673 | 3.042 | 42 | 5.521 | 2.035 | 15.726 | 0.183 | 2322.08 | 2.50 |
| 97 | 0.673 | 3.090 | 41 | 5.664 | 2.027 | 15.726 | 0.183 | 2223.40 | 2.39 |
| 98 | 0.673 | 3.139 | 40 | 5.809 | 2.017 | 15.726 | 0.183 | 2124.67 | 2.29 |
| 99 | 0.673 | 3.188 | 39 | 5.956 | 2.006 | 15.726 | 0.183 | 2025.79 | 2.18 |
| 100 | 0.673 | 3.239 | 38 | 6.104 | 1.994 | 15.726 | 0.183 | 1926.63 | 2.07 |
| 101 | 0.673 | 3.290 | 37 | 6.255 | 1.980 | 15.726 | 0.183 | 1827.10 | 1.97 |
| 102 | 0.673 | 3.342 | 36 | 6.408 | 1.965 | 15.726 | 0.183 | 1727.11 | 1.86 |
| 103 | 0.673 | 3.395 | 35 | 6.562 | 1.947 | 15.726 | 0.183 | 1626.54 | 1.75 |
| 104 | 0.673 | 3.449 | 34 | 6.719 | 1.929 | 15.726 | 0.183 | 1525.31 | 1.64 |

Table-25, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Coulomb wall $\left(\phi=34^{\circ}\right.$ and $\left.\delta=\mathbf{2 5}^{\circ}\right)$

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| log-spiral | ro | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $\mathrm{K}_{\mathrm{p}}$ | $\mathrm{K}_{\mathrm{a}}$ | $\mathbf{Q}_{\mathbf{u}}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathrm{Q}_{\mathrm{u}} / \mathbf{q}_{\mathrm{u}}$ |
| 56 | 0.603 | 1.166 | 90 | 1.000 | 1.166 | 10.193 | 0.254 | 8221.40 | 40.02 |
| 57 | 0.603 | 1.180 | 89 | 1.041 | 1.180 | 10.193 | 0.254 | 7909.63 | 38.50 |
| 58 | 0.603 | 1.194 | 88 | 1.083 | 1.193 | 10.193 | 0.254 | 7614.87 | 37.07 |
| 59 | 0.603 | 1.208 | 87 | 1.126 | 1.206 | 10.193 | 0.254 | 7335.90 | 35.71 |
| 60 | 0.603 | 1.222 | 86 | 1.171 | 1.219 | 10.193 | 0.254 | 7071.61 | 34.43 |
| 61 | 0.603 | 1.237 | 85 | 1.216 | 1.232 | 10.193 | 0.254 | 6821.00 | 33.21 |
| 62 | 0.603 | 1.251 | 84 | 1.262 | 1.245 | 10.193 | 0.254 | 6583.12 | 32.05 |
| 63 | 0.603 | 1.266 | 83 | 1.309 | 1.257 | 10.193 | 0.254 | 6357.11 | 30.95 |
| 64 | 0.603 | 1.281 | 82 | 1.357 | 1.269 | 10.193 | 0.254 | 6142.19 | 29.90 |
| 65 | 0.603 | 1.296 | 81 | 1.406 | 1.280 | 10.193 | 0.254 | 5937.63 | 28.90 |
| 66 | 0.603 | 1.312 | 80 | 1.456 | 1.292 | 10.193 | 0.254 | 5742.76 | 27.96 |
| 67 | 0.603 | 1.327 | 79 | 1.506 | 1.303 | 10.193 | 0.254 | 5556.94 | 27.05 |
| 68 | 0.603 | 1.343 | 78 | 1.558 | 1.314 | 10.193 | 0.254 | 5379.61 | 26.19 |
| 69 | 0.603 | 1.359 | 77 | 1.611 | 1.324 | 10.193 | 0.254 | 5210.23 | 25.36 |
| 70 | 0.603 | 1.375 | 76 | 1.665 | 1.334 | 10.193 | 0.254 | 5048.31 | 24.58 |
| 71 | 0.603 | 1.391 | 75 | 1.720 | 1.344 | 10.193 | 0.254 | 4893.38 | 23.82 |
| 72 | 0.603 | 1.408 | 74 | 1.776 | 1.353 | 10.193 | 0.254 | 4745.03 | 23.10 |
| 73 | 0.603 | 1.424 | 73 | 1.833 | 1.362 | 10.193 | 0.254 | 4602.84 | 22.41 |
| 74 | 0.603 | 1.441 | 72 | 1.891 | 1.371 | 10.193 | 0.254 | 4466.45 | 21.74 |
| 75 | 0.603 | 1.458 | 71 | 1.950 | 1.379 | 10.193 | 0.254 | 4335.52 | 21.11 |
| 76 | 0.603 | 1.476 | 70 | 2.009 | 1.387 | 10.193 | 0.254 | 4209.71 | 20.49 |
| 77 | 0.603 | 1.493 | 69 | 2.070 | 1.394 | 10.193 | 0.254 | 4088.74 | 19.90 |
| 78 | 0.603 | 1.511 | 68 | 2.132 | 1.401 | 10.193 | 0.254 | 3972.31 | 19.34 |
| 79 | 0.603 | 1.529 | 67 | 2.195 | 1.407 | 10.193 | 0.254 | 3860.17 | 18.79 |
| 80 | 0.603 | 1.547 | 66 | 2.258 | 1.413 | 10.193 | 0.254 | 3752.06 | 18.27 |
| 81 | 0.603 | 1.565 | 65 | 2.323 | 1.418 | 10.193 | 0.254 | 3647.76 | 17.76 |
| 82 | 0.603 | 1.584 | 64 | 2.388 | 1.423 | 10.193 | 0.254 | 3547.04 | 17.27 |
| 83 | 0.603 | 1.602 | 63 | 2.455 | 1.428 | 10.193 | 0.254 | 3449.70 | 16.79 |
| 84 | 0.603 | 1.621 | 62 | 2.522 | 1.432 | 10.193 | 0.254 | 3355.56 | 16.34 |
| 85 | 0.603 | 1.640 | 61 | 2.591 | 1.435 | 10.193 | 0.254 | 3264.42 | 15.89 |
| 86 | 0.603 | 1.660 | 60 | 2.660 | 1.438 | 10.193 | 0.254 | 3176.13 | 15.46 |
| 87 | 0.603 | 1.680 | 59 | 2.730 | 1.440 | 10.193 | 0.254 | 3090.51 | 15.04 |
| 88 | 0.603 | 1.699 | 58 | 2.801 | 1.441 | 10.193 | 0.254 | 3007.42 | 14.64 |
| 89 | 0.603 | 1.720 | 57 | 2.873 | 1.442 | 10.193 | 0.254 | 2926.71 | 14.25 |
| 90 | 0.603 | 1.740 | 56 | 2.946 | 1.442 | 10.193 | 0.254 | 2848.26 | 13.87 |
| 91 | 0.603 | 1.761 | 55 | 3.020 | 1.442 | 10.193 | 0.254 | 2771.92 | 13.49 |
| 92 | 0.603 | 1.781 | 54 | 3.094 | 1.441 | 10.193 | 0.254 | 2697.60 | 13.13 |
| 93 | 0.603 | 1.803 | 53 | 3.170 | 1.440 | 10.193 | 0.254 | 2625.16 | 12.78 |
| 94 | 0.603 | 1.824 | 52 | 3.246 | 1.437 | 10.193 | 0.254 | 2554.50 | 12.44 |
| 95 | 0.603 | 1.845 | 51 | 3.323 | 1.434 | 10.193 | 0.254 | 2485.53 | 12.10 |
| 96 | 0.603 | 1.867 | 50 | 3.401 | 1.430 | 10.193 | 0.254 | 2418.14 | 11.77 |
| 97 | 0.603 | 1.889 | 49 | 3.479 | 1.426 | 10.193 | 0.254 | 2352.24 | 11.45 |
| 98 | 0.603 | 1.912 | 48 | 3.558 | 1.421 | 10.193 | 0.254 | 2287.75 | 11.14 |
| 99 | 0.603 | 1.934 | 47 | 3.639 | 1.415 | 10.193 | 0.254 | 2224.58 | 10.83 |
| 100 | 0.603 | 1.957 | 46 | 3.719 | 1.408 | 10.193 | 0.254 | 2162.67 | 10.53 |
| 101 | 0.603 | 1.981 | 45 | 3.801 | 1.400 | 10.193 | 0.254 | 2101.92 | 10.23 |
| 102 | 0.603 | 2.004 | 44 | 3.883 | 1.392 | 10.193 | 0.254 | 2042.27 | 9.94 |
| 103 | 0.603 | 2.028 | 43 | 3.966 | 1.383 | 10.193 | 0.254 | 1983.66 | 9.66 |
| 104 | 0.603 | 2.052 | 42 | 4.049 | 1.373 | 10.193 | 0.254 | 1926.03 | 9.38 |
| 105 | 0.603 | 2.076 | 41 | 4.134 | 1.362 | 10.193 | 0.254 | 1869.30 | 9.10 |
| 106 | 0.603 | 2.101 | 40 | 4.218 | 1.350 | 10.193 | 0.254 | 1813.43 | 8.83 |
| 107 | 0.603 | 2.125 | 39 | 4.304 | 1.338 | 10.193 | 0.254 | 1758.36 | 8.56 |
| 108 | 0.603 | 2.151 | 38 | 4.389 | 1.324 | 10.193 | 0.254 | 1704.03 | 8.30 |
| 109 | 0.603 | 2.176 | 37 | 4.476 | 1.310 | 10.193 | 0.254 | 1650.41 | 8.03 |
| 110 | 0.603 | 2.202 | 36 | 4.563 | 1.294 | 10.193 | 0.254 | 1597.43 | 7.78 |
| 111 | 0.603 | 2.228 | 35 | 4.650 | 1.278 | 10.193 | 0.254 | 1545.07 | 7.52 |
| 112 | 0.603 | 2.254 | 34 | 4.738 | 1.261 | 10.193 | 0.254 | 1493.26 | 7.27 |
| 113 | 0.603 | 2.281 | 33 | 4.826 | 1.242 | 10.193 | 0.254 | 1441.98 | 7.02 |

Table-26, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Coulomb wall $\left(\phi=\mathbf{3 8}^{\circ}\right.$ and $\left.\delta=\mathbf{2 5}^{\circ}\right)$

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Log-spiral | ro | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $\mathrm{K}_{\mathrm{p}}$ | $\mathrm{K}_{\mathrm{a}}$ | $Q_{u}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $Q_{u} / q_{u}$ |
| 52 | 0.635 | 1.289 | 90 | 1.000 | 1.289 | 13.901 | 0.217 | 9861.03 | 23.23 |
| 53 | 0.635 | 1.307 | 89 | 1.046 | 1.307 | 13.901 | 0.217 | 9459.94 | 22.28 |
| 54 | 0.635 | 1.325 | 88 | 1.092 | 1.324 | 13.901 | 0.217 | 9082.49 | 21.40 |
| 55 | 0.635 | 1.343 | 87 | 1.141 | 1.341 | 13.901 | 0.217 | 8726.86 | 20.56 |
| 56 | 0.635 | 1.362 | 86 | 1.190 | 1.358 | 13.901 | 0.217 | 8391.39 | 19.77 |
| 57 | 0.635 | 1.380 | 85 | 1.241 | 1.375 | 13.901 | 0.217 | 8074.57 | 19.02 |
| 58 | 0.635 | 1.399 | 84 | 1.293 | 1.392 | 13.901 | 0.217 | 7775.02 | 18.32 |
| 59 | 0.635 | 1.419 | 83 | 1.346 | 1.408 | 13.901 | 0.217 | 7491.48 | 17.65 |
| 60 | 0.635 | 1.438 | 82 | 1.400 | 1.424 | 13.901 | 0.217 | 7222.80 | 17.01 |
| 61 | 0.635 | 1.458 | 81 | 1.456 | 1.440 | 13.901 | 0.217 | 6967.91 | 16.41 |
| 62 | 0.635 | 1.478 | 80 | 1.513 | 1.455 | 13.901 | 0.217 | 6725.84 | 15.84 |
| 63 | 0.635 | 1.498 | 79 | 1.572 | 1.471 | 13.901 | 0.217 | 6495.70 | 15.30 |
| 64 | 0.635 | 1.519 | 78 | 1.631 | 1.485 | 13.901 | 0.217 | 6276.66 | 14.79 |
| 65 | 0.635 | 1.539 | 77 | 1.693 | 1.500 | 13.901 | 0.217 | 6067.96 | 14.29 |
| 66 | 0.635 | 1.561 | 76 | 1.755 | 1.514 | 13.901 | 0.217 | 5868.89 | 13.83 |
| 67 | 0.635 | 1.582 | 75 | 1.819 | 1.528 | 13.901 | 0.217 | 5678.82 | 13.38 |
| 68 | 0.635 | 1.604 | 74 | 1.884 | 1.542 | 13.901 | 0.217 | 5497.12 | 12.95 |
| 69 | 0.635 | 1.626 | 73 | 1.951 | 1.555 | 13.901 | 0.217 | 5323.26 | 12.54 |
| 70 | 0.635 | 1.648 | 72 | 2.019 | 1.567 | 13.901 | 0.217 | 5156.70 | 12.15 |
| 71 | 0.635 | 1.671 | 71 | 2.088 | 1.580 | 13.901 | 0.217 | 4996.97 | 11.77 |
| 72 | 0.635 | 1.694 | 70 | 2.159 | 1.592 | 13.901 | 0.217 | 4843.61 | 11.41 |
| 73 | 0.635 | 1.717 | 69 | 2.231 | 1.603 | 13.901 | 0.217 | 4696.22 | 11.06 |
| 74 | 0.635 | 1.740 | 68 | 2.304 | 1.614 | 13.901 | 0.217 | 4554.39 | 10.73 |
| 75 | 0.635 | 1.764 | 67 | 2.379 | 1.624 | 13.901 | 0.217 | 4417.77 | 10.41 |
| 76 | 0.635 | 1.789 | 66 | 2.455 | 1.634 | 13.901 | 0.217 | 4286.02 | 10.10 |
| 77 | 0.635 | 1.813 | 65 | 2.533 | 1.643 | 13.901 | 0.217 | 4158.81 | 9.80 |
| 78 | 0.635 | 1.838 | 64 | 2.611 | 1.652 | 13.901 | 0.217 | 4035.86 | 9.51 |
| 79 | 0.635 | 1.863 | 63 | 2.692 | 1.660 | 13.901 | 0.217 | 3916.87 | 9.23 |
| 80 | 0.635 | 1.889 | 62 | 2.774 | 1.668 | 13.901 | 0.217 | 3801.59 | 8.96 |
| 81 | 0.635 | 1.915 | 61 | 2.857 | 1.675 | 13.901 | 0.217 | 3689.78 | 8.69 |
| 82 | 0.635 | 1.941 | 60 | 2.941 | 1.681 | 13.901 | 0.217 | 3581.19 | 8.44 |
| 83 | 0.635 | 1.968 | 59 | 3.027 | 1.687 | 13.901 | 0.217 | 3475.62 | 8.19 |
| 84 | 0.635 | 1.995 | 58 | 3.114 | 1.692 | 13.901 | 0.217 | 3372.85 | 7.95 |
| 85 | 0.635 | 2.022 | 57 | 3.203 | 1.696 | 13.901 | 0.217 | 3272.70 | 7.71 |
| 86 | 0.635 | 2.050 | 56 | 3.293 | 1.699 | 13.901 | 0.217 | 3174.98 | 7.48 |
| 87 | 0.635 | 2.078 | 55 | 3.384 | 1.702 | 13.901 | 0.217 | 3079.52 | 7.25 |
| 88 | 0.635 | 2.107 | 54 | 3.476 | 1.704 | 13.901 | 0.217 | 2986.15 | 7.03 |
| 89 | 0.635 | 2.136 | 53 | 3.570 | 1.705 | 13.901 | 0.217 | 2894.72 | 6.82 |
| 90 | 0.635 | 2.165 | 52 | 3.666 | 1.706 | 13.901 | 0.217 | 2805.09 | 6.61 |
| 91 | 0.635 | 2.195 | 51 | 3.762 | 1.705 | 13.901 | 0.217 | 2717.12 | 6.40 |
| 92 | 0.635 | 2.225 | 50 | 3.860 | 1.704 | 13.901 | 0.217 | 2630.68 | 6.20 |
| 93 | 0.635 | 2.255 | 49 | 3.959 | 1.702 | 13.901 | 0.217 | 2545.63 | 6.00 |
| 94 | 0.635 | 2.286 | 48 | 4.060 | 1.699 | 13.901 | 0.217 | 2461.87 | 5.80 |
| 95 | 0.635 | 2.318 | 47 | 4.161 | 1.695 | 13.901 | 0.217 | 2379.29 | 5.60 |
| 96 | 0.635 | 2.349 | 46 | 4.264 | 1.690 | 13.901 | 0.217 | 2297.77 | 5.41 |
| 97 | 0.635 | 2.382 | 45 | 4.368 | 1.684 | 13.901 | 0.217 | 2217.22 | 5.22 |
| 98 | 0.635 | 2.414 | 44 | 4.473 | 1.677 | 13.901 | 0.217 | 2137.53 | 5.04 |
| 99 | 0.635 | 2.447 | 43 | 4.580 | 1.669 | 13.901 | 0.217 | 2058.62 | 4.85 |
| 100 | 0.635 | 2.481 | 42 | 4.688 | 1.660 | 13.901 | 0.217 | 1980.40 | 4.67 |
| 101 | 0.635 | 2.515 | 41 | 4.796 | 1.650 | 13.901 | 0.217 | 1902.79 | 4.48 |
| 102 | 0.635 | 2.550 | 40 | 4.906 | 1.639 | 13.901 | 0.217 | 1825.70 | 4.30 |
| 103 | 0.635 | 2.585 | 39 | 5.017 | 1.627 | 13.901 | 0.217 | 1749.07 | 4.12 |
| 104 | 0.635 | 2.620 | 38 | 5.129 | 1.613 | 13.901 | 0.217 | 1672.81 | 3.94 |
| 105 | 0.635 | 2.656 | 37 | 5.243 | 1.599 | 13.901 | 0.217 | 1596.87 | 3.76 |
| 106 | 0.635 | 2.693 | 36 | 5.357 | 1.583 | 13.901 | 0.217 | 1521.16 | 3.58 |
| 107 | 0.635 | 2.730 | 35 | 5.472 | 1.566 | 13.901 | 0.217 | 1445.64 | 3.41 |
| 108 | 0.635 | 2.767 | 34 | 5.588 | 1.547 | 13.901 | 0.217 | 1370.25 | 3.23 |
| 109 | 0.635 | 2.805 | 33 | 5.705 | 1.528 | 13.901 | 0.217 | 1294.91 | 3.05 |

Table-27, The variation of BCR with normalized shell height (h/D) \& shell Diameter for different shell diameter $\&$ heights.
For Coulomb wall $\left(\phi=42^{\circ}\right.$ and $\left.\delta=25^{\circ}\right)$

| Angle of | Radius of log-spiral |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Log-spiral | ro | $\mathrm{r}_{1}$ | 45- $\phi / 2$ | d/D | h/D | $\mathrm{K}_{\mathrm{p}}$ | $\mathrm{K}_{\mathrm{a}}$ | $\mathbf{Q}_{\mathbf{u}}$ | BCR |
| $\theta$ |  |  |  |  |  |  |  |  | $\mathrm{Q}_{\mathrm{u}} / \mathbf{q}_{\mathbf{u}}$ |
| 48 | 0.673 | 1.431 | 90 | 1.000 | 1.431 | 19.758 | 0.183 | 12186.29 | 13.12 |
| 49 | 0.673 | 1.453 | 89 | 1.051 | 1.453 | 19.758 | 0.183 | 11664.17 | 12.56 |
| 50 | 0.673 | 1.476 | 88 | 1.103 | 1.475 | 19.758 | 0.183 | 11175.56 | 12.03 |
| 51 | 0.673 | 1.500 | 87 | 1.157 | 1.498 | 19.758 | 0.183 | 10717.66 | 11.54 |
| 52 | 0.673 | 1.523 | 86 | 1.213 | 1.520 | 19.758 | 0.183 | 10287.91 | 11.08 |
| 53 | 0.673 | 1.547 | 85 | 1.270 | 1.542 | 19.758 | 0.183 | 9884.02 | 10.64 |
| 54 | 0.673 | 1.572 | 84 | 1.329 | 1.563 | 19.758 | 0.183 | 9503.89 | 10.23 |
| 55 | 0.673 | 1.597 | 83 | 1.389 | 1.585 | 19.758 | 0.183 | 9145.63 | 9.85 |
| 56 | 0.673 | 1.622 | 82 | 1.452 | 1.606 | 19.758 | 0.183 | 8807.51 | 9.48 |
| 57 | 0.673 | 1.648 | 81 | 1.516 | 1.628 | 19.758 | 0.183 | 8487.96 | 9.14 |
| 58 | 0.673 | 1.674 | 80 | 1.581 | 1.649 | 19.758 | 0.183 | 8185.53 | 8.81 |
| 59 | 0.673 | 1.700 | 79 | 1.649 | 1.669 | 19.758 | 0.183 | 7898.92 | 8.50 |
| 60 | 0.673 | 1.727 | 78 | 1.718 | 1.690 | 19.758 | 0.183 | 7626.90 | 8.21 |
| 61 | 0.673 | 1.755 | 77 | 1.789 | 1.710 | 19.758 | 0.183 | 7368.38 | 7.93 |
| 62 | 0.673 | 1.783 | 76 | 1.862 | 1.730 | 19.758 | 0.183 | 7122.33 | 7.67 |
| 63 | 0.673 | 1.811 | 75 | 1.937 | 1.749 | 19.758 | 0.183 | 6887.81 | 7.42 |
| 64 | 0.673 | 1.839 | 74 | 2.014 | 1.768 | 19.758 | 0.183 | 6663.96 | 7.18 |
| 65 | 0.673 | 1.869 | 73 | 2.093 | 1.787 | 19.758 | 0.183 | 6449.97 | 6.94 |
| 66 | 0.673 | 1.898 | 72 | 2.173 | 1.805 | 19.758 | 0.183 | 6245.10 | 6.72 |
| 67 | 0.673 | 1.928 | 71 | 2.256 | 1.823 | 19.758 | 0.183 | 6048.66 | 6.51 |
| 68 | 0.673 | 1.959 | 70 | 2.340 | 1.841 | 19.758 | 0.183 | 5860.02 | 6.31 |
| 69 | 0.673 | 1.990 | 69 | 2.426 | 1.858 | 19.758 | 0.183 | 5678.58 | 6.11 |
| 70 | 0.673 | 2.021 | 68 | 2.514 | 1.874 | 19.758 | 0.183 | 5503.79 | 5.93 |
| 71 | 0.673 | 2.053 | 67 | 2.605 | 1.890 | 19.758 | 0.183 | 5335.14 | 5.74 |
| 72 | 0.673 | 2.086 | 66 | 2.697 | 1.906 | 19.758 | 0.183 | 5172.14 | 5.57 |
| 73 | 0.673 | 2.119 | 65 | 2.791 | 1.920 | 19.758 | 0.183 | 5014.34 | 5.40 |
| 74 | 0.673 | 2.152 | 64 | 2.887 | 1.935 | 19.758 | 0.183 | 4861.33 | 5.23 |
| 75 | 0.673 | 2.187 | 63 | 2.985 | 1.948 | 19.758 | 0.183 | 4712.70 | 5.07 |
| 76 | 0.673 | 2.221 | 62 | 3.086 | 1.961 | 19.758 | 0.183 | 4568.10 | 4.92 |
| 77 | 0.673 | 2.256 | 61 | 3.188 | 1.973 | 19.758 | 0.183 | 4427.17 | 4.77 |
| 78 | 0.673 | 2.292 | 60 | 3.292 | 1.985 | 19.758 | 0.183 | 4289.58 | 4.62 |
| 79 | 0.673 | 2.328 | 59 | 3.398 | 1.996 | 19.758 | 0.183 | 4155.02 | 4.47 |
| 80 | 0.673 | 2.365 | 58 | 3.507 | 2.006 | 19.758 | 0.183 | 4023.22 | 4.33 |
| 81 | 0.673 | 2.403 | 57 | 3.617 | 2.015 | 19.758 | 0.183 | 3893.88 | 4.19 |
| 82 | 0.673 | 2.441 | 56 | 3.730 | 2.024 | 19.758 | 0.183 | 3766.75 | 4.06 |
| 83 | 0.673 | 2.480 | 55 | 3.844 | 2.031 | 19.758 | 0.183 | 3641.59 | 3.92 |
| 84 | 0.673 | 2.519 | 54 | 3.961 | 2.038 | 19.758 | 0.183 | 3518.16 | 3.79 |
| 85 | 0.673 | 2.559 | 53 | 4.080 | 2.043 | 19.758 | 0.183 | 3396.24 | 3.66 |
| 86 | 0.673 | 2.599 | 52 | 4.200 | 2.048 | 19.758 | 0.183 | 3275.62 | 3.53 |
| 87 | 0.673 | 2.640 | 51 | 4.323 | 2.052 | 19.758 | 0.183 | 3156.10 | 3.40 |
| 88 | 0.673 | 2.682 | 50 | 4.448 | 2.055 | 19.758 | 0.183 | 3037.49 | 3.27 |
| 89 | 0.673 | 2.725 | 49 | 4.575 | 2.056 | 19.758 | 0.183 | 2919.61 | 3.14 |
| 90 | 0.673 | 2.768 | 48 | 4.704 | 2.057 | 19.758 | 0.183 | 2802.28 | 3.02 |
| 91 | 0.673 | 2.812 | 47 | 4.835 | 2.056 | 19.758 | 0.183 | 2685.34 | 2.89 |
| 92 | 0.673 | 2.856 | 46 | 4.968 | 2.055 | 19.758 | 0.183 | 2568.64 | 2.77 |
| 93 | 0.673 | 2.901 | 45 | 5.103 | 2.052 | 19.758 | 0.183 | 2452.01 | 2.64 |
| 94 | 0.673 | 2.947 | 44 | 5.240 | 2.047 | 19.758 | 0.183 | 2335.32 | 2.51 |
| 95 | 0.673 | 2.994 | 43 | 5.379 | 2.042 | 19.758 | 0.183 | 2218.43 | 2.39 |
| 96 | 0.673 | 3.042 | 42 | 5.521 | 2.035 | 19.758 | 0.183 | 2101.20 | 2.26 |
| 97 | 0.673 | 3.090 | 41 | 5.664 | 2.027 | 19.758 | 0.183 | 1983.50 | 2.14 |
| 98 | 0.673 | 3.139 | 40 | 5.809 | 2.017 | 19.758 | 0.183 | 1865.21 | 2.01 |
| 99 | 0.673 | 3.188 | 39 | 5.956 | 2.006 | 19.758 | 0.183 | 1746.22 | 1.88 |
| 100 | 0.673 | 3.239 | 38 | 6.104 | 1.994 | 19.758 | 0.183 | 1626.40 | 1.75 |
| 101 | 0.673 | 3.290 | 37 | 6.255 | 1.980 | 19.758 | 0.183 | 1505.66 | 1.62 |
| 102 | 0.673 | 3.342 | 36 | 6.408 | 1.965 | 19.758 | 0.183 | 1383.88 | 1.49 |
| 103 | 0.673 | 3.395 | 35 | 6.562 | 1.947 | 19.758 | 0.183 | 1260.95 | 1.36 |
| 104 | 0.673 | 3.449 | 34 | 6.719 | 1.929 | 19.758 | 0.183 | 1136.80 | 1.22 |
| 105 | 0.673 | 3.504 | 33 | 6.877 | 1.908 | 19.758 | 0.183 | 1011.31 | 1.09 |

